THEORY Interval

IMPORT THEORY Comparable

TYPE PARAMETERS S

DATA TYPES

Interval(S)

constructors

Cons_Interval($lower : S, upper : S$)

OPERATORS

CompForIntervalWellCons ***predicate*** $(in : Interval(S), comp : Comparable(S))$

well−definedness condition $ComparableWellCons(comp)$

direct definition

$upper(in) \in SetDef(comp) \wedge lower(in) \in SetDef(comp)$

Ile ***predicate*** $(in1 : Interval(S), in2 : Interval(S), comp : Comparable(S))$

well−definedness condition $CompForIntervalWellCons(in1, comp), CompForIntervalWellCons(in2, comp)$

direct definition

$upper(in1) \mapsto lower(in2) \in CompRel(comp)$

Ioverlap ***predicate*** $(in1 : Interval(S), in2 : Interval(S), comp : Comparable(S))$

well−definedness condition $CompForIntervalWellCons(in1, comp), CompForIntervalWellCons(in2, comp)$

direct definition

$lower(in1) \mapsto upper(in2) \in CompRel(comp) \wedge lower(in2) \mapsto upper(in1) \in CompRel(comp)$

ItoSet ***expression*** $(in : Interval(S), comp : Comparable(S))$

well−definedness condition $CompForIntervalWellCons(in, comp)$

direct definition

$\{x \mid x \in SetDef(comp) \wedge lower(in) \mapsto x \in CompRel(comp) \wedge x \mapsto upper(in) \in CompRel(comp)\}$

ItoSetInt ***expression*** $(in : Interval(\mathbb{Z}))$

direct definition

$ItoSet(in, Cons\_Comp(\mathbb{Z}, \{x \mapsto y \mid x \leq y\}))$

IleInt ***predicate*** $(in1 : Interval(\mathbb{Z}), in2 : Interval(\mathbb{Z}))$

direct definition

$Ile(in1, in2, Cons\_Comp(\mathbb{Z}, \{x \mapsto y \mid x < y\}))$

IoverlapInt ***predicate*** $(in1 : Interval(\mathbb{Z}), in2 : Interval(\mathbb{Z}))$

direct definition

$Ioverlap(in1, in2, Cons\_Comp(\mathbb{Z}, \{x \mapsto y \mid x \leq y\}))$

IWInt ***expression*** $(in : Interval(\mathbb{Z}))$

direct definition

$upper(in) - lower(in)$

THEOREMS

thm5 :

$\forall comp \cdot comp \in Comparable(S) \wedge ComparableWellCons(comp) \wedge order(comp) \Rightarrow transitive($
$Cons\_Comp(\{i \mid i \in Interval(SetDef(comp)) \wedge ItoSet(i, comp) \neq \emptyset\},$
$\{in1 \mapsto in2 \mid in1 \in Interval(SetDef(comp)) \wedge ItoSet(in1, comp) \neq \emptyset$
$\wedge in2 \in Interval(SetDef(comp)) \wedge ItoSet(in2, comp) \neq \emptyset \wedge Ile(in1, in2, comp)\}))$

thm4 :

$\forall s, u \cdot s \subseteq S \wedge s \subseteq u \Rightarrow (\forall i \cdot i \in Interval(s) \Rightarrow i \in Interval(u))$

thm6 :

$\forall x, y \cdot ItoSet(Cons\_Interval(x, y), Cons\_Comp(\mathbb{Z}, \{x \mapsto y \mid x \leq y\})) = x..y$

END