

THEORY BB

IMPORT THEORY Interval

TYPE PARAMETERS S,T,U

DATA TYPES

BB2D(S, T)

constructors

Cons_BB2D($\text{proj}x2d : \text{Interval}(S), \text{proj}y2d : \text{Interval}(T)$)

BB3D(S, T, U)

constructors

Cons_BB3D($\text{proj}x3d : \text{Interval}(S), \text{proj}y3d : \text{Interval}(T), \text{proj}z3d : \text{Interval}(U)$)

OPERATORS

projBB23dtoXY *expression* ($bb3d : BB3D(S, T, U)$)

direct definition

$\text{Cons_BB2D}(\text{proj}x3d(bb3d), \text{proj}y3d(bb3d))$

projBB23dtoXZ *expression* ($bb3d : BB3D(S, T, U)$)

direct definition

$\text{Cons_BB2D}(\text{proj}x3d(bb3d), \text{proj}z3d(bb3d))$

projBB23dtoYZ *expression* ($bb3d : BB3D(S, T, U)$)

direct definition

$\text{Cons_BB2D}(\text{proj}y3d(bb3d), \text{proj}z3d(bb3d))$

NotEmpty2D *predicate* ($bb2d : BB2D(S, T), \text{comp}S : \text{Comparable}(S), \text{comp}T : \text{Comparable}(T)$)

well-definedness condition $\text{CompForIntervalWellCons}(\text{proj}x2d(bb2d), \text{comp}S),$

$\text{CompForIntervalWellCons}(\text{proj}y2d(bb2d), \text{comp}T)$

direct definition

$\text{ItoSet}(\text{proj}x2d(bb2d), \text{comp}S) \neq \emptyset \wedge$

$\text{ItoSet}(\text{proj}y2d(bb2d), \text{comp}T) \neq \emptyset$

NotEmpty3D *predicate* ($bb3d : BB3D(S, T, U), \text{comp}S : \text{Comparable}(S), \text{comp}T : \text{Comparable}(T),$
 $\text{comp}U : \text{Comparable}(U)$)

well-definedness condition $\text{CompForIntervalWellCons}(\text{proj}x3d(bb3d), \text{comp}S),$

$\text{CompForIntervalWellCons}(\text{proj}y3d(bb3d), \text{comp}T), \text{CompForIntervalWellCons}(\text{proj}z3d(bb3d), \text{comp}U)$

direct definition

$\text{ItoSet}(\text{proj}x3d(bb3d), \text{comp}S) \neq \emptyset \wedge \text{ItoSet}(\text{proj}y3d(bb3d), \text{comp}T) \neq \emptyset \wedge$

$\text{ItoSet}(\text{proj}z3d(bb3d), \text{comp}U) \neq \emptyset$

NotEmpty2DInt *predicate* ($bb2d : BB2D(\mathbb{Z}, \mathbb{Z})$)

direct definition

$\text{ItoSetInt}(\text{proj}x2d(bb2d)) \neq \emptyset \wedge \text{ItoSetInt}(\text{proj}y2d(bb2d)) \neq \emptyset$

NotEmpty3DInt *predicate* ($bb3d : BB3D(\mathbb{Z}, \mathbb{Z}, \mathbb{Z})$)

direct definition

$\text{ItoSetInt}(\text{proj}x3d(bb3d)) \neq \emptyset \wedge \text{ItoSetInt}(\text{proj}y3d(bb3d)) \neq \emptyset \wedge \text{ItoSetInt}(\text{proj}z3d(bb3d)) \neq \emptyset$

THEOREMS

thm1 :

$\forall bb2d \cdot bb2d \in BB2D(\mathbb{Z}, \mathbb{Z})$

$\Rightarrow (\text{NotEmpty2DInt}(bb2d) \Leftrightarrow \text{NotEmpty2D}(bb2d, \text{Cons_Comp}(\mathbb{Z}, \{x \mapsto y \mid x \leq y\}), \text{Cons_Comp}(\mathbb{Z}, \{x \mapsto y \mid x \leq y\})))$

thm2 :

$\forall bb3d \cdot bb3d \in BB3D(\mathbb{Z}, \mathbb{Z}, \mathbb{Z})$

$\Rightarrow (\text{NotEmpty3DInt}(bb3d) \Leftrightarrow$

$\text{NotEmpty3D}(bb3d, \text{Cons_Comp}(\mathbb{Z}, \{x \mapsto y \mid x \leq y\}), \text{Cons_Comp}(\mathbb{Z}, \{x \mapsto y \mid x \leq y\}), \text{Cons_Comp}(\mathbb{Z}, \{x \mapsto y \mid x \leq y\})))$

END