Tutorial-1 [PHN-624]

Part A (Basic)

Q1: Write code for calculating the factorial and double factorial of a number.

Q2: Write down a code that will calculate sum of square of N numbers i.e.

$$Y = 1^2 + 2^2 + ... + N^2$$

Q3: The single-particle quadrupole moment (Q_{sp}) and magnetic moment (μ) are given by the the following expressions:

$$Q_{sp} = -\frac{(2j-1)}{2(j+1)} < r^2 >,$$

$$\mu = (j - \frac{1}{2})g_l + \frac{1}{2}g_s \quad \text{for} \quad j = l + \frac{1}{2},$$

$$\mu = \frac{j}{j+1}[(j + \frac{3}{2})g_l - \frac{1}{2}g_s] \quad \text{for} \quad j = l - \frac{1}{2},$$

where $< r^2 >= \frac{3}{5}(1.2A^{1/3})^2$, and j is the spin. Write down a code to calculate single-particle quadrupole moment (Q_{sp}) and magnetic moment (μ) for ground state of ^{7}Li. [Given $g_l^p = 1$, $g_l^n = 0$ and $g_s^p = 5.585$]

Q4: The oscillator length parameter (b) can be written as, $b = \frac{197.33}{\sqrt{940 \times \hbar\omega[\text{MeV}]}}$fm. Write a code to determine 'b' with $\hbar\omega = 41A^{-1/3}$ MeV and $\hbar\omega = 45A^{-1/3} - 25A^{-2/3}$ MeV separately for $A = 36$. How much they are differ from each other.

Q5: The Woods- Saxon potential can be given by

$$V(r) = \frac{V_0}{1 + exp(r - R)/a},$$

where, depth potential $V_0 = -25$ MeV, surface diffuseness $a = 0.5$ fm, and radius $R = 5$ fm. Write a code for potential $V(r)$ and plot the data for the range (in fm) $-10 < r < 10$.

Q6: Integrate the function $f(x) = x^2e^{-x^2}$ for $0 \leq x \leq 2$ using numerical integration methods.

Part B (Based on Unit-1)

Q1: Write a code that will calculate n^{th} order Hermite Polynomials ($H_n(x)$) for a given value of x using recurrence relations.

Q2: Use the above code to plot harmonic oscillator wave functions of n^{th} order in the range $-8 \leq x \leq 8$

$$\Psi_n(x) = \frac{1}{\sqrt{2^n n!}} \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} \exp\left(-\frac{m\omega x^2}{2\hbar}\right) H_n\left(\sqrt{\frac{m\omega}{\hbar}}x\right).$$

Where in natural units $\sqrt{\frac{m\omega}{\hbar}} = 1$. Find the number of nodes in it from the figure.

Q3: Write a code to print $n \times n$ matrices for H.O. Potential and Kinetic energy. [Hint: Use the relation, $\psi_n''(x) = 2\sqrt{n(n-1)}\psi_{n-2}(x) - \sqrt{8n}x\psi_{n-1}(x) + (x^2 - 1)\psi_n(x)$.]

Q4: Write down codes to calculate Legendre and associated Legendre polynomials and plot them.

Q5: Then calculate the spherical harmonics ($Y_l^m(\theta,\phi)$) for a given value of l and m using the following expression.

$$Y_l^m(\theta,\phi) = (-1)^m \sqrt{\frac{(2l+1)}{4\pi}\frac{(l-m)!}{(l+m)!}}\; P_l^m(cos\theta)\exp{(im\phi)}.$$

Where $0 \leq \theta \leq \pi$ and $-\pi \leq \phi \leq \pi$.

Q6: Write down codes to calculate Laguerre and associated Laguerre polynomials and plot them.

Q7: The total wavefunction of hydrogen atom is given by the product of the radial part and the angular part as follows

$$\psi_{nlm}(r,\theta,\phi) = \sqrt{\left(\frac{2}{na}\right)^3 \frac{(n-l-1)!}{2n[(n+l)!]^3}} \exp\left(\frac{-r}{na}\right)\left(\frac{2r}{na}\right)^l [L_{n-l-1}^{2l+1}(2r/na)]\, Y_l^m(\theta,\phi),$$

where $a = 0.529 \times 10^{-10} m$. Write down a code to calculate wave-function for Hydrogen-atom and plot $|\psi_{200}|^2$, $|\psi_{210}|^2$, $|\psi_{211}|^2$, and $|\psi_{310}|^2$.

Q8: Write down a code to calculate Clebsch–Gordan (C. G.) coefficients for coupling of two angular momenta. Find the C. G. coefficients associated with the coupling of the spins of the electron and the proton of a hydrogen atom in its ground state ($j_1 = 1/2$ and $j_2 = 1/2$). [Code based on C.G. Coefficients and Wigner 3j symbol]

Q9: For the generalization of previous problem, write down a code to calculate Wigner $6j$ symbol for the coupling of three angular momenta and $9j$ symbol for the coupling of four angular momenta.

Q10: For an electric radiation of multipole order 2, the transition strength $B(E2)$ in terms of the transition quadrupole moments (Q_t) according to the rotational formula is given by

$$B(E2; I \to I-2) = \frac{5}{16\pi} Q_t^2 \begin{pmatrix} I & 2 & I-2 \\ K & 0 & K \end{pmatrix}^2.$$

Calculate the $B(E2)$ values corresponding to $17/2^+ \to 13/2^+$ (given $Q_t = 224\ efm^2$) and $21/2^+ \to 17/2^+$ (given $Q_t = 202\ efm^2$) transitions.

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