## BPL\_TEST2\_Batch\_calibration - demo

This notebook shows the possibilities for calibration of the model BPL\_TEST2\_Batch using scipy.optimize.minimize() routine. There are several different methods to choose between. In this notebook we work with simulated data.

The text-book model of batch cultivation we simulate is the following where S is substrate, X is cell concentration, and V is volume of the broth

$$\frac{d(VS)}{dt} = -q_S(S) \cdot VX$$

$$\frac{d(VX)}{dt} = \mu(S) \cdot VX$$

and where specific cell growth rate  $\mu$  and substrate uptake rate  $q_S$  are

$$\mu(S) = Y \cdot q_S(S)$$

$$q_S(S) = q_S^{max} rac{S}{K_s + S}$$

where Y is the yield,  $q_S^{max}$  is the maximal specific substrate uptake rate and  $K_s$  is the corresponding saturation constant.

The parameter estimation is done with optimization methods that only require evaluation of the missmatch between simulation with given parameters and data. At start the allowed range for each parameter is given. The method used for optimization is SLSQP but can easily be changed [1].

In the near future the FMU may provide first derivative gradient information, that will make it possible to choose corresponding method of minimize() for improved performance. This possibility is related to the upgrade to the FMI-standard ver 3.0 for the Modelica compiler.

The Python package PyFMI [2] that is the base for FMU-explore has a simplified built-in functionality for parameter estimation that also use scipy.optimize.minimize(). However, there is estimated and the purpose seems to only address smaller examples. Therefore we here define a no possibility to include parameter changes to the compiled model that should not be Python function evaluation() that facilitate the formulation of the parameter estimation and bring flexibility to choice of optimization method.

```
In [1]: run -i BPL_TEST2_Batch_explore.py
```

Linux - run FMU pre-comiled JModelica 2.4

units

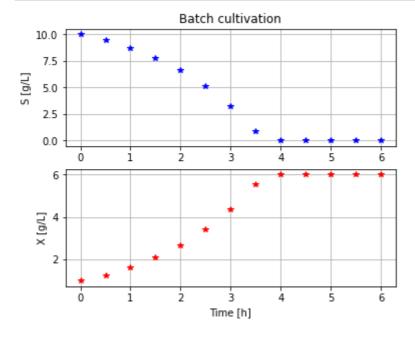
Note that both disp() and describe() takes values from the last simulation

Brief information about a command by help(), eg help(simu)

Key system information is listed with the command system info()

```
In [2]: # Adjust the size of diagrams
plt.rcParams['figure.figsize'] = [15/2.54, 12/2.54]
```

### Generate data later used for parameter estimation



```
In [5]: # Store data in a DataFrame for later use
    data = pd.DataFrame(data={'time':sim_res['time'], 'X':sim_res['bioreactor.c[]
    data
```

Out[5]:		time	Х	S
	0	0.0	1.000000	1.000000e+01
	1	0.5	1.280773	9.438453e+00
	2	1.0	1.640079	8.719842e+00
	3	1.5	2.099615	7.800770e+00
	4	2.0	2.686770	6.626459e+00
	5	2.5	3.435479	5.129043e+00
	6	3.0	4.385325	3.229350e+00
	7	3.5	5.559252	8.814967e-01
	8	4.0	6.000000	1.048673e-08

	time	Х	S
9	4.5	6.000000	-6.547559e-11
10	5.0	6.000000	6.182144e-11
11	5.5	6.000000	-4.234324e-12
12	6.0	6.000000	-1.961610e-13

# Simulation with initial guess of parameters compared with data

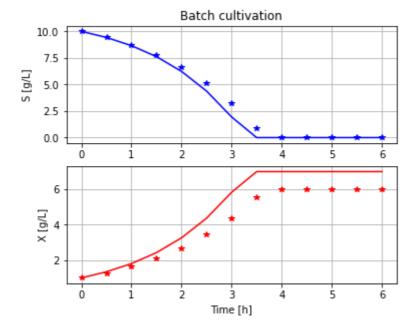
Here we define the parameters that should be estimated and specify allowed ranges. Nominal parameters are chosen as the mid-point of the allowed parameter range.

Simulation with these nominal parameter set and compare with data give an idea of who well the model fit data.

```
In [6]: # Parameters to be estimated using parDict names and their bounds
    parEstim = ['Y', 'qSmax', 'Ks']
    parBounds = [(0.4, 0.8), (0.7, 1.3), (0.05, 0.20)]
    parEstim_0 = [np.mean(parBounds[k]) for k in range(len(parBounds))]

In [7]: # Simulation with nominal parameters
    newplot(plotType='Demo_1')
    par(Y=parEstim_0[0], qSmax=parEstim_0[1], Ks=parEstim_0[2])
    simu(simulationTime)

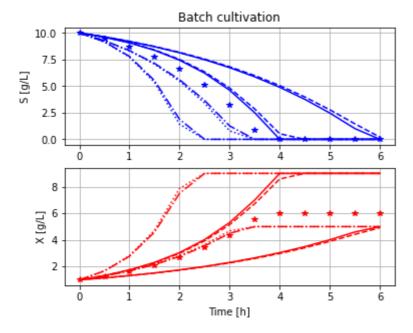
# Show data
    ax1.plot(data['time'], data['S'],'b*')
    ax2.plot(data['time'], data['X'],'r*')
    plt.show()
```



```
In [8]: # Simulation over the parameter ranges given
newplot(plotType='Demo_1')
for Y_value in parBounds [0]:
    for qSmax_value in parBounds[1]:
        for Ks_value in parBounds[2]:
            par(Y=Y_value, qSmax_value, Ks=Ks_value)
```

```
simu(simulationTime)

# Show data
ax1.plot(data['time'], data['S'],'b*')
ax2.plot(data['time'], data['X'],'r*')
plt.show()
```



Simulation over the different parameter combinations of the parameter bounds shows that data is "covered" and we have good hope to find a parameter combination that fits data well.

#### Parameter estimation

Here we use the scipy.optimize.minimize() procedure which contain a family of different methods [1]. Since we has chosen to work with bounds on the parameters to be estimated there are only three methods to choose between. Here the method Sequential Least SQuares Programming SLSQP is chosen.

Note that we in the definition of evaluation() make use of PyFMI-functions to administrate the simulation parameters as well as running it, instead of using the simplified simu() function we are used to.

```
# For clarity of code here we import again
 In [9]:
          import scipy.optimize
          # Parameters to be estimated using parDict names and their bounds
In [10]:
          extra args = (parEstim, data, fmu model, simulationTime, parDict, parLocation
          # Modified evaluation function tailored for Python optimization algorithms
In [11]:
          def evaluation(x, parEstim, data=data, fmu_model=fmu_model, simulationTime=si
                          parDict=parDict, parLocation=parLocation):
              """The parameter list is tailored for scipy optimization algorithms inter
                 where the first parameter x is an array with parameters that are tuned
                 and evalauted and parEstim is a list of the names of these parameters.
              # Load model
              global model
              if model is None:
                  model = load_fmu(fmu_model)
              model.reset()
```

```
# Change parameters and initial values from default
              for i, p in enumerate(parEstim): model.set(parLocation[p], x[i])
              for p in set(parDict)-set(parEstim): model.set(parLocation[p], parDict[p]
              # Simulation options
              opts = model.simulate options()
              opts['ncp'] = 12
              opts['result handling'] = 'memory'
              opts['silent mode'] = True
              # Simulate
              sim res = model.simulate(start time=0.0, final time=simulationTime, optic
              # Calculate loss function V
              V['X'] = np.linalg.norm(data['X'] - np.interp(data['time'], sim_res['time'])
              V['S'] = np.linalg.norm(data['S'] - np.interp(data['time'], sim_res['time'])
              return V['X'] + V['S']
          import time
In [12]:
         # Run minimize()
In [13]:
          start time = time.time()
          result = scipy.optimize.minimize(evaluation, x0=parEstim 0, args=extra args,
                                            method='SLSQP', bounds=parBounds, options={'
          print('CPU-time =', time.time()-start time)
         Optimization terminated successfully
                                                  (Exit mode 0)
                     Current function value: 3.236649117036713e-05
                     Iterations: 28
                     Function evaluations: 146
                     Gradient evaluations: 28
         CPU-time = 0.39418745040893555
          result
In [14]:
              fun: 3.236649117036713e-05
Out[14]:
              jac: array([-17.95259042, -5.35182525, -0.01872408])
          message: 'Optimization terminated successfully'
             nfev: 146
              nit: 28
             niev: 28
           status: 0
```

The estimated parameters result.x are very close to the original values and no surprise.

x: array([0.49999975, 0.99999427, 0.09996165])

Test of the three methods available that handle parameter bounds: TNC, L-BFGS-B and SLSQP. It turns out that SLSQP is by far the fastest. It is 3 times faster than L-BFGS-B which is faster than TNC. Can be that SLSQP is less robust though. The nit (number of iterations does not differ that much though: 24 vs 30. The nfev (number of function evaluations) is perhaps more important 127 vs 256. A more precise timer function is likely timeit for this short times.

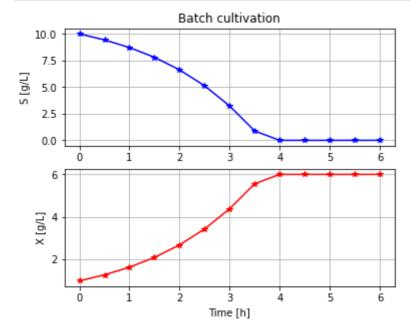
The Nelder-Mead algorithm has a good reputation to be very robust, but more slow, and with this method we cannot have bounds on the parameters.

# Simulation with estimated parameters compared with data

success: True

```
In [15]: newplot(plotType='Demo_1')
    par(Y=result.x[0], qSmax=result.x[1], Ks=result.x[2])
    simu(simulationTime)

# Show data
    ax1.plot(data['time'], data['S'], 'b*')
    ax2.plot(data['time'], data['X'], 'r*')
    plt.show()
```



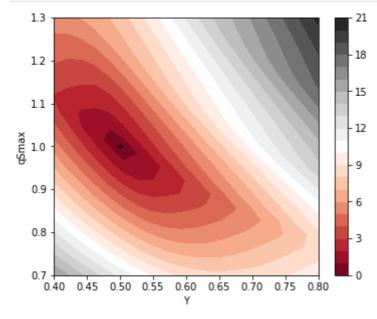
```
In [16]: # The estimated parameters are
    for i in range(len(parEstim)): print(parEstim[i],':', result.x[i])
```

Y: 0.4999997464088378 qSmax: 0.999994265760737 Ks: 0.099961646658623

### Analysis of the loss function

The problem is small and analysis of the loss function brings some insight. From the diagram above showing parameter sweep over combinations min- and max-parameters we see that the parameter  $K_s$  has little influence. Let use set that a fixed value and then plot the loss function in the parameters Y and qSmax. We do this by go through all the parametera combinations and evaluate each of them.

```
plt.contourf(Y, qSmax, V, 20, cmap='RdGy')
plt.plot(result.x[0], result.x[1], 'k+')
plt.colorbar()
plt.ylabel('qSmax')
plt.xlabel('Y')
plt.show()
```



We see the following in the contour diagram of the loss function simplified:

- The minima is unique in the range of parmaters we study. This is good news.
- The contour plot is ellipsoid and rather narrow. The more narrow the ellipsoid the more difficult and more time it takes to converge to the minima.
- The direction of the ellipsoid axis indicate the correlation you may get between the two parameters during the minimization process.

Note that the form of the contour plot change with the parameters (and initial values) of the actual proces. You can see the impact by changing the parameters in "cell # 4" where data is generated and then just choose to run that cell and the cells below. No need to restart the notebook.

### Summary

A choice was made to work with allowed ranges of parameters to be estimated and a start value was defined as the center point in this parameter space. There are only three methods available in optimize.minimize() that can handle bounds on parameters.

An evaluation() function was created that define how the difference between simulation and data is measured. The function is rather transparent and easy to modify and you may want to change weight on the loss in S and X, for instance. Here they have so far equal weight.

The FMU-explore workspace dictionaries partDict[] and parLocation[] are useful also here and simplify the code for the evaluation() function. But we also use the detailed PyFMI-functions to administrate and set parameters of the actual simulation.

The call optimize.minimize() has several parameters and can easily be modified, for instance change of method.

The estimated parameters were close to perfect!

The contour plot of the simplified loss function shows that the minima is unique and should not be difficult too difficut to obtain.

#### References

- [1] Scipy Reference guide on optimize.minimize() here
- [2] Andersson, C., Åkesson, J., Fuhrer C.: "PyFMI: A Python package for simulation of coupled dynamic models with the functional mock-up interface", Centre for Mathematical Sciences, Lund University, Report LUTFNA-5008-2016, 2016.

### **Appendix**

```
In [18]: describe('parts')
         ['bioreactor', 'bioreactor.culture', 'liquidphase', 'MSL']
         describe('MSL')
In [19]:
         MSL: 3.2.2 build 3 - used components:
         scipy.__version__
In [20]:
Out[20]: '1.5.2'
         system info()
In [21]:
         System information
          -OS: Linux
          -Python: 3.8.2
          -PyFMI: 2.7.4
          -FMU by: JModelica.org
          -FMI: 2.0
          -Type: FMUModelCS2
          -Name: BPL TEST2.Batch
          -Generated: 2022-09-19T14:20:20
          -MSL: 3.2.2 build 3
          -Description: Bioprocess Library version 2.1.0 beta
          -Interaction: FMU-explore ver 0.9.3
In [ ]:
```