1 κ-Minkowski Weyl Ordered $\star$-Product

Our definition of the κ-Minkowski algebra is of the real Lie algebra

$$
\begin{aligned}
[\hat{x}_0, \hat{x}_i] &= \lambda \hat{x}_i \\
[\hat{x}_i, \hat{x}_j] &= 0
\end{aligned}
$$

whereas your paper, and a lot of others use

$$
\begin{aligned}
[\hat{x}_0, \hat{x}_i] &= i\lambda \hat{x}_i \\
[\hat{x}_i, \hat{x}_j] &= 0
\end{aligned}
$$

This significantly changes the Weyl ordered $\star$-product. In our case we obtain

$$
\begin{aligned}
f \star g &= \frac{1}{(2\pi)^8} \iint e^{ir_n(k,k')x^i} dk dk' \\
r_0 &= k_0 + k_0' \\
r_l &= \frac{k_l \phi(ik_0) + k_l' \phi(ik_0') e^{i\lambda k_0}}{\phi(ik_0 + ik_0')}
\end{aligned}
$$

where l runs over $1 \ldots n-1$. Your $\star$-product should then be

$$
\begin{aligned}
f \star g &= \frac{1}{(2\pi)^8} \iint e^{ir_n(k,k')x^i} dk dk' \\
r_0 &= k_0 + k_0' \\
r_l &= \frac{k_l \phi(k_0) + k_l' \phi(k_0') e^{-\lambda k_0}}{\phi(k_0 + k_0')}
\end{aligned}
$$

in each case

$$
\phi(a) = \frac{1 - e^{-a\lambda}}{a\lambda}
$$

Essentially, in the paper on Generalised Weyl Systems, equation (4.1) requires only a minus sign in the exponent, and equation (3.10) needs to agree (which involves moving the exponent from the term on the left, to the term on the right).