Foundations of Linear Models, Assignment 1

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Problem 1

1	0	1
0	2	2
1	2	3

(i) Is the matrix of full rank?

The determinant of the matrix is 0 which implies that one of the rows of the matrix is linearly dependent on another row (row3 = row2 + 1), hence the matrix is not of full rank. The rank of the matrix is 2.

(ii) Determine the eigenvalues and eigenvectors

Since the determinant of the matrix is 0, the product of the eigen values is 0, i.e. $\prod_{i=1}^{3} \lambda_i = 0$ and the sum of the eigen values gives 6 (trace of the matrix), i.e. $\sum_{i=1}^{3} \lambda_i = 6$. The eigen values for the matrix are; 4.732, 1.268, 0 and the eigen vectors are;

-0.2113249	0.7886751	-0.5773503
-0.5773503	-0.5773503	-0.5773503
-0.7886751	0.2113249	0.5773503

(iii) Check for the validity of Lemma 2(iii)

Lemma 2(iii) is valid if $UDU^{-1} = A$ where U is the eigen vector of the matrix A, U^{-1} is the inverse of the eign vectors of the matrix A and D is a diagonal matrix of the eigenvalues of A.

$$U = \begin{pmatrix} -0.2113249 & 0.7886751 & -0.5773503 \\ -0.5773503 & -0.5773503 & -0.5773503 \\ -0.7886751 & 0.2113249 & 0.5773503 \end{pmatrix}$$

$$D = \begin{pmatrix} 4.732051 & 0.000000 & 0.000000e+00 \\ 0.000000 & 1.267949 & 0.000000e+00 \\ 0.000000 & 0.000000 & 5.329071e-15 \end{pmatrix}$$

and

$$U^{-1} = \begin{pmatrix} -0.2113249 & -0.5773503 & -0.7886751 \\ 0.7886751 & -0.5773503 & 0.2113249 \\ -0.5773503 & -0.5773503 & 0.5773503 \end{pmatrix}$$

The result of the multiplication gives the matrix below which is exactly A, hence the Lemma holds.

1	0	1
0	2	2
1	2	3

(iv) Check for the Validity of Lemma 2(iv)

rank(A) = number of nonzero eigenvalues. This lemma is true for the matrix in question as the number of non-zero eugen values is 2 which is the same as the rank of the matrix.

Problem 2