Foundations of Linear Models, Assignment 1

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Problem 1

1	0	1
0	2	2
1	2	3

(i) Is the matrix of full rank?

The matrix is not of full rank since the matrix has $rank = 2$ which does not span the maximum space in dimension, which is 3. Also, the determinant of the matrix is 0 which implies that one of the rows of the matrix is linearly dependent on another row (row3 = row2 + 1), leading to the same conclusion that the matrix is not of full rank.

(ii) Determine the eigenvalues and eigenvectors

The eigen values for the matrix are; 4.732, 1.268, 0 and the eigen vectors are;

-0.2113249	0.7886751	-0.5773503
-0.5773503	-0.5773503	-0.5773503
-0.7886751	0.2113249	0.5773503

Since the determinant of the matrix is 0 and it is as well symmetric, the product of the eigen values should be 0, i.e. $\prod_{i=1}^{3} \lambda_i = 0$ and the sum of the eigen values should give 6 (trace of the matrix), i.e. $\sum_{i=1}^{3} \lambda_i = 6$.

(iii) Check for the validity of Lemma 2(iii)

Lemma 2(iii) is valid if $UDU^{-1} = A$ where U is the eigen vector of the matrix A, U^{-1} is the inverse of the eign vectors of the matrix A and D is a diagonal matrix of the eigenvalues of A.

$$U = \begin{pmatrix} -0.2113 & 0.7887 & -0.5774 \\ -0.5774 & -0.5774 & -0.5774 \\ -0.7887 & 0.2113 & 0.5774 \end{pmatrix}$$

$$D = \begin{pmatrix} 4.732 & 0.000 & 0.000 \\ 0.000 & 1.268 & 0.000 \\ 0.000 & 0.000 & 0.000 \end{pmatrix}$$

and

$$U^{-1} = \begin{pmatrix} -0.2113 & -0.5774 & -0.7887 \\ 0.7887 & -0.5774 & 0.2113 \\ -0.5774 & -0.5774 & 0.5774 \end{pmatrix}$$

Multiplying the matrix, gives;

$$\begin{pmatrix} -0.2113 & 0.7886751 & -0.5774 \\ -0.5774 & -0.5773503 & -0.5774 \\ -0.7887 & 0.2113 & 0.5774 \end{pmatrix} \times \begin{pmatrix} 4.732 & 0.000 & 0.000 \\ 0.000 & 1.268 & 0.000 \\ 0.000 & 0.000 & 0.000 \end{pmatrix} \times \begin{pmatrix} -0.2113 & -0.5774 & -0.7887 \\ 0.7887 & -0.5774 & 0.2113 \\ -0.5774 & -0.5774 & 0.5774 \end{pmatrix}$$

The result of the multiplication gives the matrix below which is exactly A, hence the Lemma holds.

1	0	1
0	2	2
1	2	3

(iv) Check for the Validity of Lemma 2(iv)

rank(A) = number of nonzero eigenvalues. This lemma is true for the matrix in question as the number of non-zero eugen values is 2 which is the same as the rank of the matrix.

Problem 2

Problem 4