Assignment 7a

March 25, 2024

1 Assignment 7a

[ ]:
```
#%matplotlib qt
# Needed to ge the animation visualization working

%matplotlib inline
import numpy as np
import matplotlib.pyplot as plt
from matplotlib import animation
import rain_mod
from IPython.display import Image
```

Figure Staggered Grid: The staggered grid for the drop in the pond problem.

The original equations, boundary and initial conditions are changed to reflect the staggered case. The equations are changed to the following:

(Staggered, Eqn 1)

$$\frac{u_i(t+dt) - u_i(t-dt)}{2dt} + g\frac{h_{i+1}(t) - h_i(t)}{dx} = 0$$

(Staggered, Eqn 2)

$$\frac{h_i(t+dt) - h_i(t-dt)}{2dt} + H\frac{u_i(t) - u_{i-1}(t)}{dx} = 0$$

The initial conditions are: At $t = 0$ and $t = dt$, all points have zero elevation except at h_3, where

$$h_3(0) = h_0$$

$$h_3(dt) = h_3(0) - h_0 Hg\frac{dt^2}{dx^2}$$

At $t = 0$ and $t = dt$, all points have zero velocity except at u_2 and u_3, where

$$u_2(dt) = -h_0 g\frac{dt}{dx}$$

$$u_3(dt) = -u_2(dt)$$

This time we assume there is a wall at u_1 and u_4, so we will ignore the value of h_1. The boundary conditions are:

$$u_1(t) = 0$$

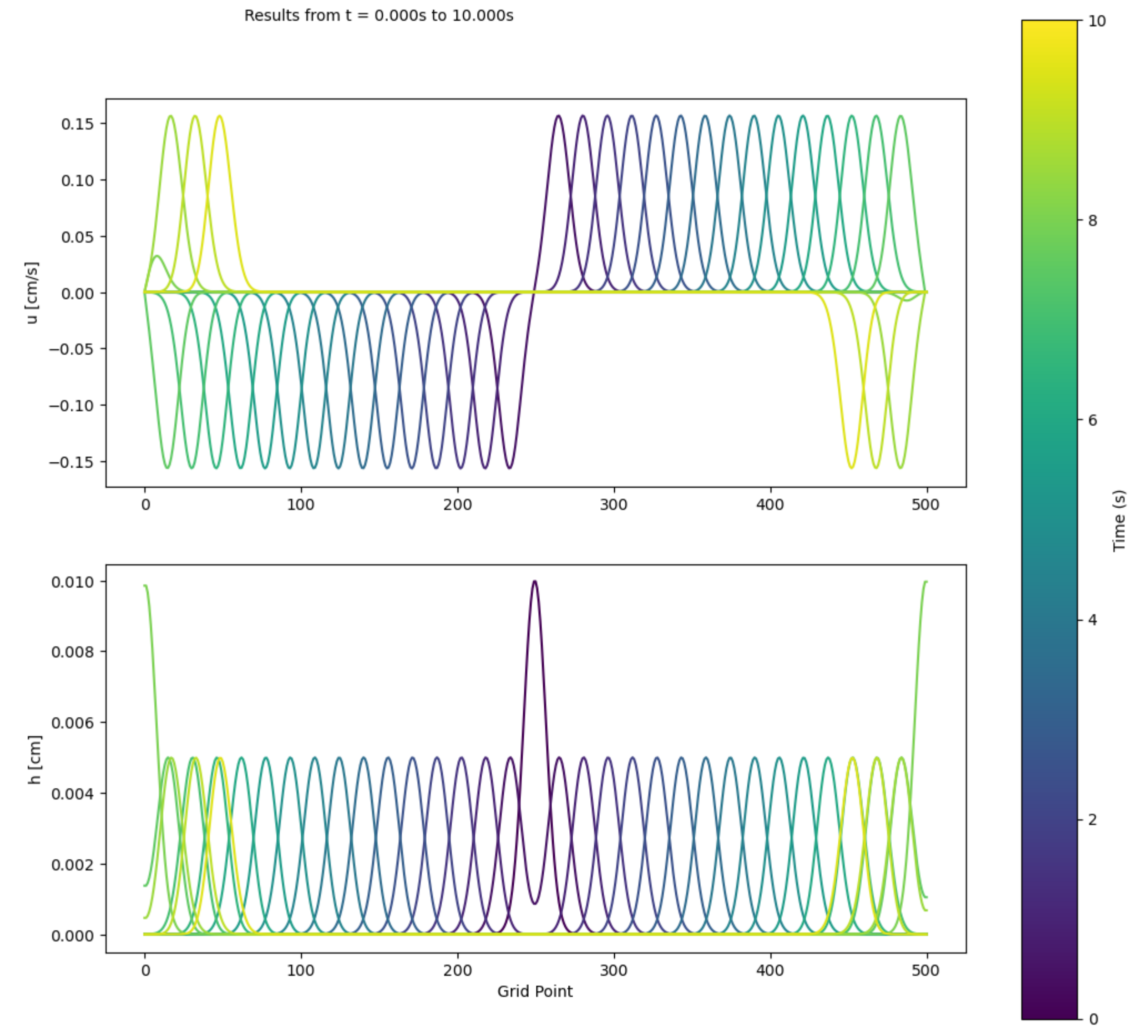
$$u_4(t) = 0$$

1.0.1 Problem One

Modify *rain.py* to solve this problem (Simple equations on a staggered grid). Submit your code and a final plot for one case.

In order to test the correctness of the code, I introduced a gaussian initial condition setting. Travelling gaussians remain gaussian as it propagates in time and space. For an initial stationary gaussian perturbation, it can be decomposed into two left- and right- propagating gaussian wavepackets. At a stiff boundary (no flux through the surface) we expect the gaussian waveforms to be reflected in a way that retains its gaussian profile. In addition, since shallow water waves are non-dispersive, we expect the width of the gaussian to remain constant over time.

[ ]:
```
u, h, dt, n_time = rain_mod.rain([1000, 501], setting='gaussian')
```



[ ]:
```
'''
fig, (ax_u, ax_h) = plt.subplots(2,1, figsize=(10,10))
```

```
the_title = fig.text(0.25, 0.95, 'Results from t = %.3fs to %.3fs' % (0,␣
↪dt*n_time))
# Set the figure title, and the axes labels.

def animate(time):
    ax_u.clear()
    ax_h.clear()
    ax_u.set_title('t = %.3f' % (2*dt*time))
    ax_h.set_title('t = %.3f' % (2*dt*time))
    ax_u.plot(u.store[:, 2*time])
    ax_h.plot(h.store[:, 2*time])
    ax_u.set_ylabel('u [cm/s]')
    ax_h.set_ylabel('h [cm]')
    ax_h.set_xlabel('Grid Point')
    ax_u.set_ylim(-0.16, 0.16)
    ax_h.set_ylim(-0.004, 0.012)

anim = animation.FuncAnimation(fig, animate, frames=n_time//2, interval=1,␣
↪blit=False, repeat=True)
plt.show()
writergif = animation.PillowWriter(fps=20)
anim.save(r'gaussian.gif',writer=writergif)
'''
```

[ ]:

```
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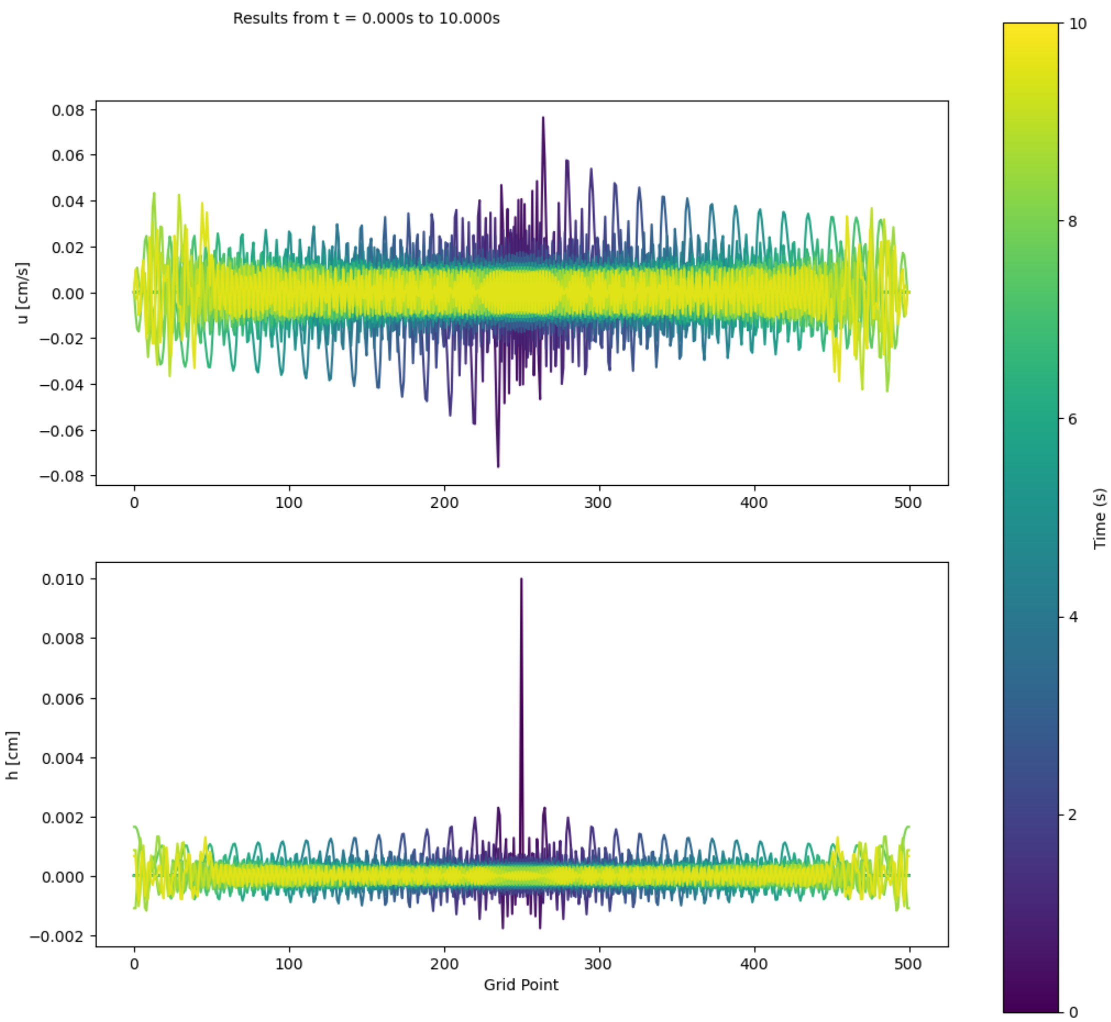
```

As seen in the plot and animation, the output of the simulation is as expected. Now running the original initial condition (a dirac-delta distribution):

[ ]:

```
u, h, dt, n_time = rain_mod.rain([1000, 501], setting='point')
```



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    ax_u.plot(u.store[:, 2*time])
    ax_h.plot(h.store[:, 2*time])
    ax_u.set_ylabel('u [cm/s]')
    ax_h.set_ylabel('h [cm]')
```

```
    ax_h.set_xlabel('Grid Point')
    ax_u.set_ylim(-0.16, 0.16)
    ax_h.set_ylim(-0.004, 0.012)

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```

The fourier transform of a dirac-delta function is constant in k-space, therefore a dirac-delta perturbation has significant high-wavenumber modes that cannot be resolved with a finite grid, resulting in narrow triangle waves.