Assignment 5

February 26, 2024

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0.0.1 Problem constant growth

Note that though the daisy growth rate per unit time depends on the amount of available fertile land, it is not otherwise coupled to the environment (i.e. β_i is not a function of temperature. Making the growth a function of bare ground, however, keeps the daisy population bounded and the daisy population will eventually reach some steady state. The next python cell has a script that runs a fixed timestep Runge Kutte routine that calculates area coverage of white and black daisies for fixed growth rates β_w and β_b . Try changing these growth rates (specified in the derivs5 routine) and the initial white and black concentrations (specified in the fixed_growth.yaml file discussed next).

- 1. For a given set of growth rates try various (non-zero) initial daisy populations.
- 2. For a given set of initial conditions try various growth rates. In particular, try rates that are both greater than and less than the death rate.
- 3. Can you determine when non-zero steady states are achieved? Explain.

0.0.2 Running the constant growth rate demo

In the appendix we discuss the design of the integrator class and the adaptive Runge-Kutta routine. For this demo, we need to be able to change variables in the configuration file. For this demonstration you are asked to:

1. Change the inital white and black daisy concentrations by changing these lines in the fixed_growth.yaml input file (you can find this file in this lab directory):

initvars:

whiteconc: 0.2 blackconc: 0.7

2. Change the white and black daisy growth rates by editing the variables beta_w and beta_b in the derivs5 routine in the next cell

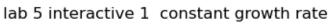
To hand in: plot graphs to illustrate how these changes have affected the fractional coverage of black and white daisies over time compared to the original. Comment on the changes that you see.

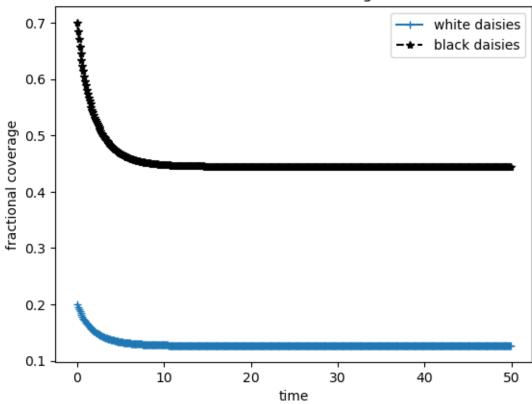
The Integrator class contains two different timeloops, both of which use embedded Runge Kutta Cash Carp code given in Lab 4 and coded here as rkckODE5. The simplest way to loop through the timesteps is just to call the integrator with a specified set of times. This is done in timeloop5fixed.

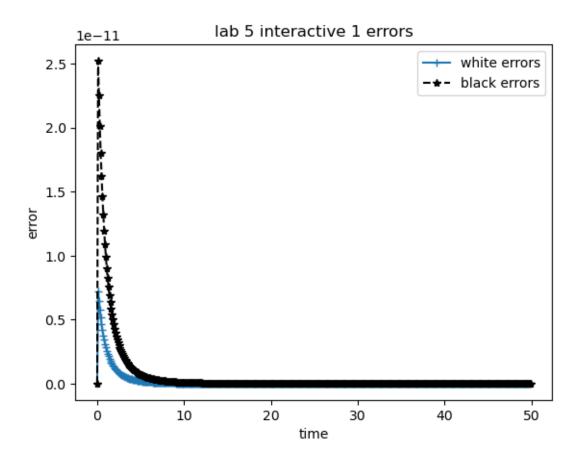
Below we will describe how to use the error extimates returned by rkckODE5 to tune the size of the timesteps, which is done in timeloop5Err.

```
[8]: #
     # 4.1 integrate constant growth rates with fixed timesteps
     import context
     from numlabs.lab5.lab5_funs import Integrator
     from collections import namedtuple
     import numpy as np
     import matplotlib.pyplot as plt
     # 0.7 beta
     class Integ51(Integrator):
         def set_yinit(self):
             # read in 'albedo white chi SO L albedo black R albedo ground'
             uservars = namedtuple('uservars', self.config['uservars'].keys())
             self.uservars = uservars(**self.config['uservars'])
             # read in 'whiteconc blackconc'
             initvars = namedtuple('initvars', self.config['initvars'].keys())
             self.initvars = initvars(**self.config['initvars'])
             self.yinit = np.array(
                 [self.initvars.whiteconc, self.initvars.blackconc])
             self.nvars = len(self.yinit)
             return None
         # Construct an Integ51 class by inheriting first intializing
         # the parent Integrator class (called super). Then do the extra
         # initialization in the set_yint function
         def __init__(self, coeffFileName):
             super().__init__(coeffFileName)
             self.set_yinit()
         def derivs5(self, y, t):
             """y[0]=fraction white daisies
                y[1]=fraction black daisies
                Constant growty rates for white
                and black daisies beta_w and beta_b
```

```
returns dy/dt
        user = self.uservars
        # bare ground
       x = 1.0 - y[0] - y[1]
        # growth rates don't depend on temperature
       beta_b = 0.7 # growth rate for black daisies
       beta_w = 0.7 # growth rate for white daisies
        # create a 1 x 2 element vector to hold the derivitive
       f = np.empty([self.nvars], 'float')
       f[0] = y[0] * (beta_w * x - user.chi)
       f[1] = y[1] * (beta_b * x - user.chi)
       return f
theSolver = Integ51('fixed_growth.yaml')
timeVals, yVals, errorList = theSolver.timeloop5fixed()
plt.close('all')
thefig, theAx = plt.subplots(1, 1)
theLines = theAx.plot(timeVals, yVals)
theLines[0].set marker('+')
theLines[1].set_linestyle('--')
theLines[1].set_color('k')
theLines[1].set_marker('*')
theAx.set_title('lab 5 interactive 1 constant growth rate')
theAx.set_xlabel('time')
theAx.set_ylabel('fractional coverage')
theAx.legend(theLines, ('white daisies', 'black daisies'), loc='best')
thefig, theAx = plt.subplots(1, 1)
theLines = theAx.plot(timeVals, errorList)
theLines[0].set marker('+')
theLines[1].set_linestyle('--')
theLines[1].set color('k')
theLines[1].set_marker('*')
theAx.set title('lab 5 interactive 1 errors')
theAx.set_xlabel('time')
theAx.set ylabel('error')
out = theAx.legend(theLines, ('white errors', 'black errors'), loc='best')
```





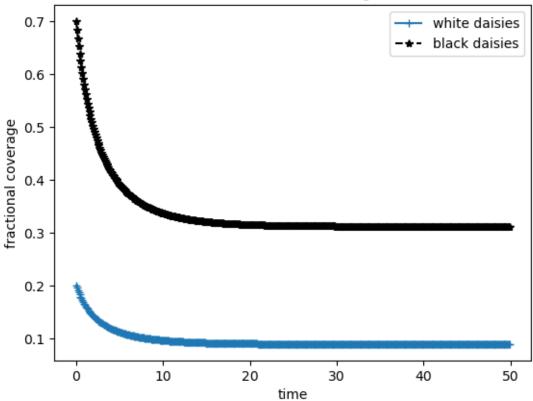


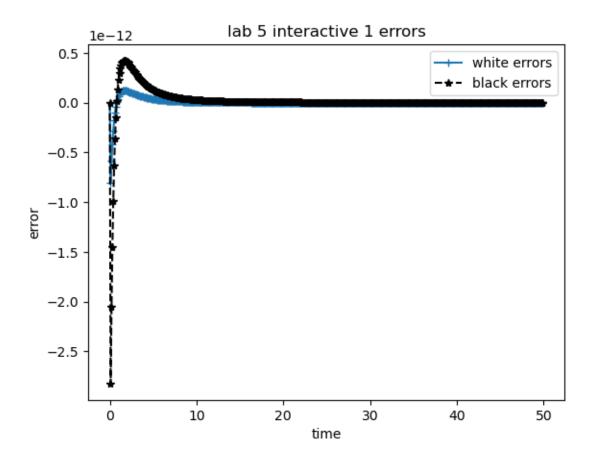
```
# Construct an Integ51 class by inheriting first intializing
    # the parent Integrator class (called super). Then do the extra
    # initialization in the set_yint function
   def __init__(self, coeffFileName):
        super().__init__(coeffFileName)
        self.set_yinit()
   def derivs5(self, y, t):
        """y[0]=fraction white daisies
           y[1]=fraction black daisies
           Constant growty rates for white
           and black daisies beta_w and beta_b
           returns dy/dt
       user = self.uservars
        # bare ground
       x = 1.0 - y[0] - y[1]
        # growth rates don't depend on temperature
       beta_b = 0.5 # growth rate for black daisies
       beta w = 0.5 # growth rate for white daisies
        # create a 1 x 2 element vector to hold the derivitive
       f = np.empty([self.nvars], 'float')
        f[0] = y[0] * (beta_w * x - user.chi)
        f[1] = y[1] * (beta_b * x - user.chi)
       return f
theSolver = Integ51('fixed_growth.yaml')
timeVals, yVals, errorList = theSolver.timeloop5fixed()
plt.close('all')
thefig, theAx = plt.subplots(1, 1)
theLines = theAx.plot(timeVals, yVals)
theLines[0].set marker('+')
theLines[1].set_linestyle('--')
theLines[1].set_color('k')
theLines[1].set_marker('*')
theAx.set_title('lab 5 interactive 1 constant growth rate')
theAx.set_xlabel('time')
theAx.set_ylabel('fractional coverage')
```

```
theAx.legend(theLines, ('white daisies', 'black daisies'), loc='best')

thefig, theAx = plt.subplots(1, 1)
theLines = theAx.plot(timeVals, errorList)
theLines[0].set_marker('+')
theLines[1].set_linestyle('--')
theLines[1].set_color('k')
theLines[1].set_marker('*')
theAx.set_title('lab 5 interactive 1 errors')
theAx.set_xlabel('time')
theAx.set_ylabel('error')
out = theAx.legend(theLines, ('white errors', 'black errors'), loc='best')
```

lab 5 interactive 1 constant growth rate



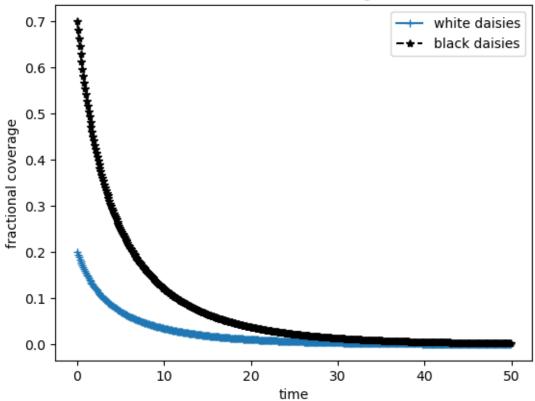


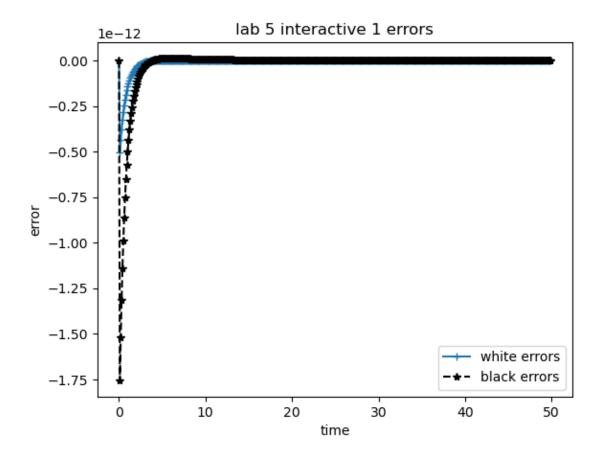
```
# Construct an Integ51 class by inheriting first intializing
    # the parent Integrator class (called super). Then do the extra
    # initialization in the set_yint function
   def __init__(self, coeffFileName):
        super().__init__(coeffFileName)
        self.set_yinit()
   def derivs5(self, y, t):
        """y[0]=fraction white daisies
           y[1]=fraction black daisies
           Constant growty rates for white
           and black daisies beta_w and beta_b
           returns dy/dt
       user = self.uservars
        # bare ground
       x = 1.0 - y[0] - y[1]
        # growth rates don't depend on temperature
       beta_b = 0.2 # growth rate for black daisies
       beta w = 0.2 # growth rate for white daisies
        # create a 1 x 2 element vector to hold the derivitive
       f = np.empty([self.nvars], 'float')
        f[0] = y[0] * (beta_w * x - user.chi)
        f[1] = y[1] * (beta_b * x - user.chi)
       return f
theSolver = Integ51('fixed_growth.yaml')
timeVals, yVals, errorList = theSolver.timeloop5fixed()
plt.close('all')
thefig, theAx = plt.subplots(1, 1)
theLines = theAx.plot(timeVals, yVals)
theLines[0].set marker('+')
theLines[1].set_linestyle('--')
theLines[1].set_color('k')
theLines[1].set_marker('*')
theAx.set_title('lab 5 interactive 1 constant growth rate')
theAx.set_xlabel('time')
theAx.set_ylabel('fractional coverage')
```

```
theAx.legend(theLines, ('white daisies', 'black daisies'), loc='best')

thefig, theAx = plt.subplots(1, 1)
theLines = theAx.plot(timeVals, errorList)
theLines[0].set_marker('+')
theLines[1].set_linestyle('--')
theLines[1].set_color('k')
theLines[1].set_marker('*')
theAx.set_title('lab 5 interactive 1 errors')
theAx.set_xlabel('time')
theAx.set_ylabel('error')
out = theAx.legend(theLines, ('white errors', 'black errors'), loc='best')
```

lab 5 interactive 1 constant growth rate





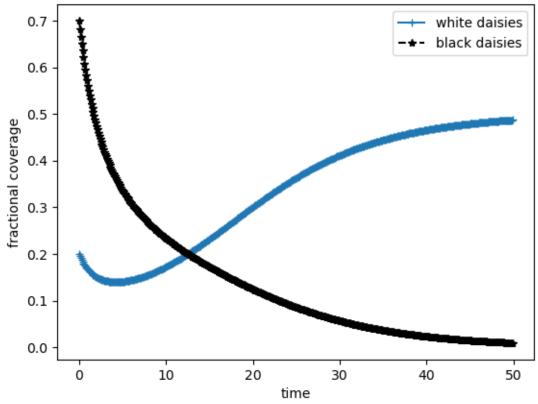
As growth rate for both black and white daisies decreases, the steady state population of black and white daisies decreases. Past a certain point (beta=0.2), the growth rate of daisies fall below their death rate and their population drops to zero over time instead of stabilizing at a finite value.

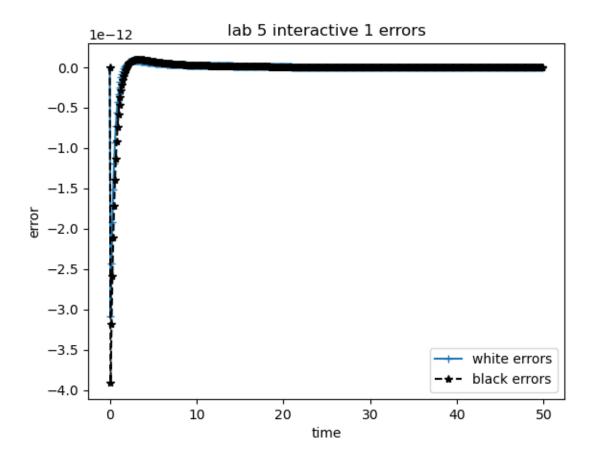
```
self.nvars = len(self.yinit)
        return None
    # Construct an Integ51 class by inheriting first intializing
    # the parent Integrator class (called super). Then do the extra
    # initialization in the set_yint function
    def __init__(self, coeffFileName):
        super().__init__(coeffFileName)
        self.set_yinit()
    def derivs5(self, y, t):
        """y[0]=fraction white daisies
           y[1]=fraction black daisies
           Constant growty rates for white
           and black daisies beta_w and beta_b
           returns dy/dt
        11 11 11
        user = self.uservars
        # bare ground
        x = 1.0 - y[0] - y[1]
        # growth rates don't depend on temperature
        beta_b = 0.4 # growth rate for black daisies
        beta_w = 0.6 # growth rate for white daisies
        # create a 1 x 2 element vector to hold the derivitive
        f = np.empty([self.nvars], 'float')
        f[0] = y[0] * (beta_w * x - user.chi)
        f[1] = y[1] * (beta_b * x - user.chi)
        return f
theSolver = Integ51('fixed_growth.yaml')
timeVals, yVals, errorList = theSolver.timeloop5fixed()
plt.close('all')
thefig, theAx = plt.subplots(1, 1)
theLines = theAx.plot(timeVals, yVals)
theLines[0].set_marker('+')
theLines[1].set_linestyle('--')
theLines[1].set_color('k')
```

```
theLines[1].set_marker('*')
theAx.set_title('lab 5 interactive 1 constant growth rate')
theAx.set_xlabel('time')
theAx.set_ylabel('fractional coverage')
theAx.legend(theLines, ('white daisies', 'black daisies'), loc='best')

thefig, theAx = plt.subplots(1, 1)
theLines = theAx.plot(timeVals, errorList)
theLines[0].set_marker('+')
theLines[1].set_linestyle('--')
theLines[1].set_color('k')
theLines[1].set_marker('*')
theAx.set_title('lab 5 interactive 1 errors')
theAx.set_xlabel('time')
theAx.set_ylabel('error')
out = theAx.legend(theLines, ('white errors', 'black errors'), loc='best')
```





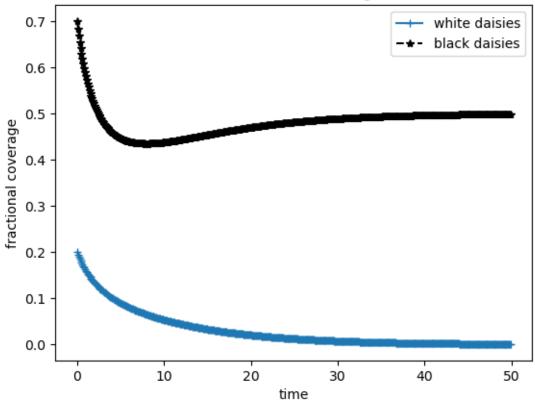


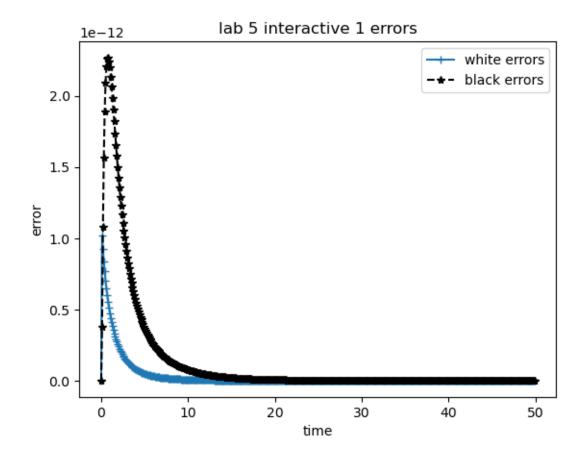
```
# Construct an Integ51 class by inheriting first intializing
    # the parent Integrator class (called super). Then do the extra
    # initialization in the set_yint function
   def __init__(self, coeffFileName):
        super().__init__(coeffFileName)
        self.set_yinit()
   def derivs5(self, y, t):
        """y[0]=fraction white daisies
           y[1]=fraction black daisies
           Constant growty rates for white
           and black daisies beta_w and beta_b
           returns dy/dt
       user = self.uservars
        # bare ground
       x = 1.0 - y[0] - y[1]
        # growth rates don't depend on temperature
       beta_b = 0.6 # growth rate for black daisies
       beta w = 0.4 # growth rate for white daisies
        # create a 1 x 2 element vector to hold the derivitive
       f = np.empty([self.nvars], 'float')
        f[0] = y[0] * (beta_w * x - user.chi)
        f[1] = y[1] * (beta_b * x - user.chi)
       return f
theSolver = Integ51('fixed_growth.yaml')
timeVals, yVals, errorList = theSolver.timeloop5fixed()
plt.close('all')
thefig, theAx = plt.subplots(1, 1)
theLines = theAx.plot(timeVals, yVals)
theLines[0].set marker('+')
theLines[1].set_linestyle('--')
theLines[1].set_color('k')
theLines[1].set_marker('*')
theAx.set_title('lab 5 interactive 1 constant growth rate')
theAx.set_xlabel('time')
theAx.set_ylabel('fractional coverage')
```

```
theAx.legend(theLines, ('white daisies', 'black daisies'), loc='best')

thefig, theAx = plt.subplots(1, 1)
theLines = theAx.plot(timeVals, errorList)
theLines[0].set_marker('+')
theLines[1].set_linestyle('--')
theLines[1].set_color('k')
theLines[1].set_marker('*')
theAx.set_title('lab 5 interactive 1 errors')
theAx.set_xlabel('time')
theAx.set_ylabel('error')
out = theAx.legend(theLines, ('white errors', 'black errors'), loc='best')
```

lab 5 interactive 1 constant growth rate





When the growth rates for black or white daisies are adjusted such that one is higher than the other, the daisy with the higher growth rate tends to outcompete the other by taking up most of the available space, thus limiting the growth rate of the other daisy type. This limitation might be severe enough that the other daisy type's reproduct rate falls below the replacement level, causing its population to drop to zero over time.

0.0.3 Problem Initial

The feedback means a stable daisy population (a steady state) and the environmental conditions are in a delicate balance. The code below produces a steady state which arises from a given initial daisy population,

- 1. Add a small initial fraction of black daisies (say, 0.01) to the value in initial.yaml and see what effect this has on the temperature and final daisy populations. Do you still have a final non-zero daisy population?
- 2. Attempt to adjust the initial white daisy population to obtain a non-zero steady state. Do you have to increase or decrease the initial fraction? What is your explanation for this behavior?
- 3. Experiment with other initial fractions of daisies and look for non-zero steady states.

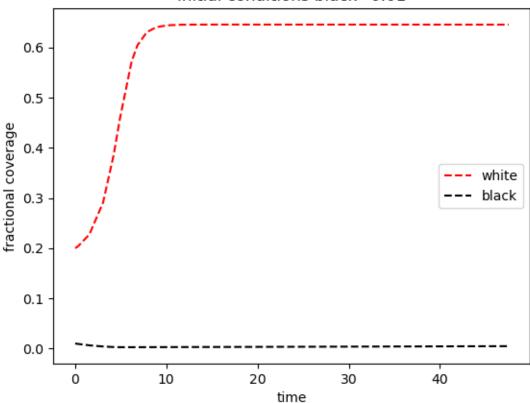
```
[31]: # functions for problem initial
      from numlabs.lab5.lab5_funs import Integrator
      class Integ54(Integrator):
          def set_yinit(self):
              # read in 'albedo_white chi SO L albedo_black R albedo_ground'
              uservars = namedtuple('uservars', self.config['uservars'].keys())
              self.uservars = uservars(**self.config['uservars'])
              # read in 'whiteconc blackconc'
              initvars = namedtuple('initvars', self.config['initvars'].keys())
              self.initvars = initvars(**self.config['initvars'])
              self.yinit = np.array(
                  [self.initvars.whiteconc, self.initvars.blackconc])
              self.nvars = len(self.yinit)
              return None
          def __init__(self, coeff_file_name):
              super().__init__(coeff_file_name)
              self.set_yinit()
          def find_temp(self, yvals):
                  Calculate the temperatures over the white and black daisies
                  and the planetary equilibrium temperature given the daisy fractions
                  input: yvals -- array of dimension [2] with the white [0] and
       ⇔black [1]
                          daisy fractiion
                  output: white temperature (K), black temperature (K), equilibrium
       \hookrightarrow temperature (K)
              .....
              sigma = 5.67e-8 # Stefan Boltzman constant W/m^2/K^4
              user = self.uservars
              bare = 1.0 - yvals[0] - yvals[1]
              albedo_p = bare * user.albedo_ground + \
                  yvals[0] * user.albedo_white + yvals[1] * user.albedo_black
              Te_4 = user.S0 / 4.0 * user.L * (1.0 - albedo_p) / sigma
              temp e = Te 4**0.25
              eta = user.R * user.L * user.S0 / (4.0 * sigma)
              temp_b = (eta * (albedo_p - user.albedo_black) + Te_4)**0.25
              temp_w = (eta * (albedo_p - user.albedo_white) + Te_4)**0.25
              return (temp_w, temp_b, temp_e)
```

```
def derivs5(self, y, t):
    """y[0]=fraction white daisies
       y[1]=fraction black daisies
       no feedback between daisies and
       albedo_p (set to ground albedo)
    temp_w, temp_b, temp_e = self.find_temp(y)
    if (temp b \ge 277.5 \text{ and } temp b \le 312.5):
        beta b = 1.0 - 0.003265 * (295.0 - temp b)**2.0
    else:
        beta_b = 0.0
    if (temp_w >= 277.5 \text{ and } temp_w <= 312.5):
        beta_w = 1.0 - 0.003265 * (295.0 - temp_w)**2.0
    else:
        beta_w = 0.0
    user = self.uservars
    bare = 1.0 - y[0] - y[1]
    \# create a 1 x 2 element vector to hold the derivitive
    f = np.empty_like(y)
    f[0] = y[0] * (beta_w * bare - user.chi)
    f[1] = y[1] * (beta b * bare - user.chi)
    return f
```

```
[52]: # Solve and plot for problem initial
      import matplotlib.pyplot as plt
      import pandas as pd
      theSolver = Integ54('initial.yaml')
      timevals, yvals, errorlist = theSolver.timeloop5Err()
      daisies = pd.DataFrame(yvals, columns=['white', 'black'])
      temp_w, temp_b, temp_e = theSolver.find_temp(yvals)
      print("temp_w=",temp_w)
      print("temp_b=",temp_b)
      print("temp_e=",temp_e)
      thefig, theAx = plt.subplots(1, 1)
      line1, = theAx.plot(timevals, daisies['white'])
      line2, = theAx.plot(timevals, daisies['black'])
      line1.set(linestyle='--', color='r', label='white')
      line2.set(linestyle='--', color='k', label='black')
      theAx.set_title('Initial conditions black=0.01')
      theAx.set_xlabel('time')
      theAx.set_ylabel('fractional coverage')
      out = theAx.legend(loc='center right')
```

```
temp_w= [315.30710523 315.28209314]
temp_b= [324.91908158 324.89622455]
temp_e= [320.22381474 320.19668091]
```





[42]: daisies['black'].mean()

[42]: 0.004250433681174635

The final population of black daisies is nonzero. The equilibrium temperature is only negligibly impacted by the introduction of a small amount of black daisies, though technically we expect there to be a small corresponding increase in temp_b given a higher steady-state proportion of black daisies, all else being equal.

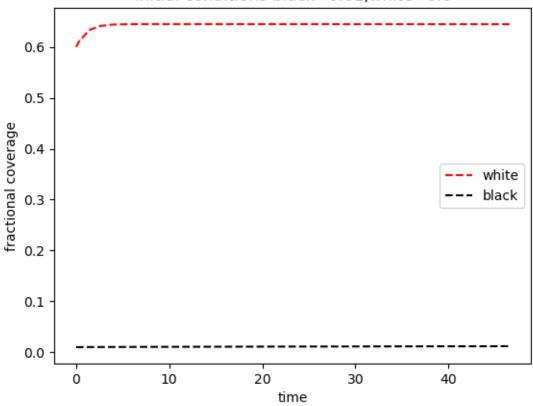
```
[53]: theSolver = Integ54('initial2.yaml')
   timevals, yvals, errorlist = theSolver.timeloop5Err()
   daisies = pd.DataFrame(yvals, columns=['white', 'black'])
   temp_w, temp_b, temp_e = theSolver.find_temp(yvals)
   print("temp_w=",temp_w)
   print("temp_b=",temp_b)
   print("temp_e=",temp_e)
```

```
thefig, theAx = plt.subplots(1, 1)
line1, = theAx.plot(timevals, daisies['white'])
line2, = theAx.plot(timevals, daisies['black'])
line1.set(linestyle='--', color='r', label='white')
line2.set(linestyle='--', color='k', label='black')
theAx.set_title('Initial conditions black=0.01, white=0.6')
theAx.set_xlabel('time')
theAx.set_ylabel('fractional coverage')
out = theAx.legend(loc='center right')

temp_w= [315.36100312 315.28779338]
```

```
temp_w= [315.36100312 315.28779338]
temp_b= [324.96833768 324.9014336 ]
temp_e= [320.28228324 320.20286473]
```

Initial conditions black=0.01, white=0.6



```
[46]: daisies['black'].mean()
```

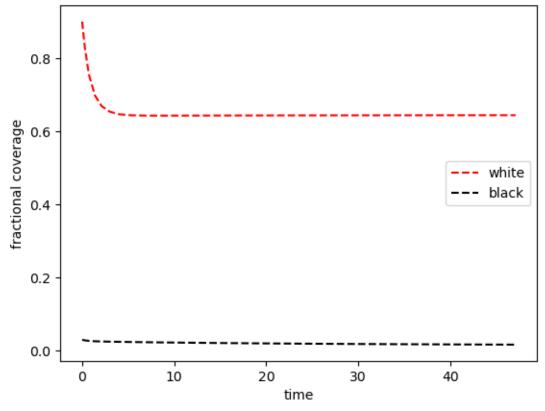
[46]: 0.010792035616259665

The proportion of white daisies need to be increased to increase the proportion of steady-state black daisies. This is because a greater proportion of white daisies increases the planetary albedo and decreases the overall temperature, making it more favourable for black daisies to grow.

```
[61]: theSolver = Integ54('initial3.yaml')
      timevals, yvals, errorlist = theSolver.timeloop5Err()
      daisies = pd.DataFrame(yvals, columns=['white', 'black'])
      temp_w, temp_b, temp_e = theSolver.find_temp(yvals)
      print("temp_w=",temp_w)
      print("temp_b=",temp_b)
      print("temp_e=",temp_e)
      thefig, theAx = plt.subplots(1, 1)
      line1, = theAx.plot(timevals, daisies['white'])
      line2, = theAx.plot(timevals, daisies['black'])
      line1.set(linestyle='--', color='r', label='white')
      line2.set(linestyle='--', color='k', label='black')
      theAx.set_title('Initial conditions black=0.03, white=0.9')
      theAx.set_xlabel('time')
      theAx.set_ylabel('fractional coverage')
      out = theAx.legend(loc='center right')
     temp w = [314.792049]
                           315.27506246]
```

temp_w= [314.792049 315.27506246] temp_b= [324.44852751 324.88979975] temp_e= [319.6649778 320.18905375]

Initial conditions black=0.03,white=0.9



```
[62]: daisies['black'].mean()
```

[62]: 0.022562030429514206

In general, a small population of black daisies requires a much larger proportion of white daisies to offset the heating effect it introduces.

0.0.4 Problem adaptive

The demos in the previous section solved the Daisyworld equations using the embedded Runge-Kutta methods with adaptive timestep control.

- 1. Run the code and find solutions of Daisyworld with the default settings found in adapt.yaml using the timeloop5Err adaptive code
- 2. Find the solutions again but this time with fixed stepsizes and compare the size of the timesteps and number of the timesteps.
- 3. How much faster does the fixed timeloop need to be to give the same performance as the adaptive timeloop for this case?

```
[86]: theSolver = Integ54('adapt.yaml')
    timeVals, yVals, errorList = theSolver.timeloop5Err()

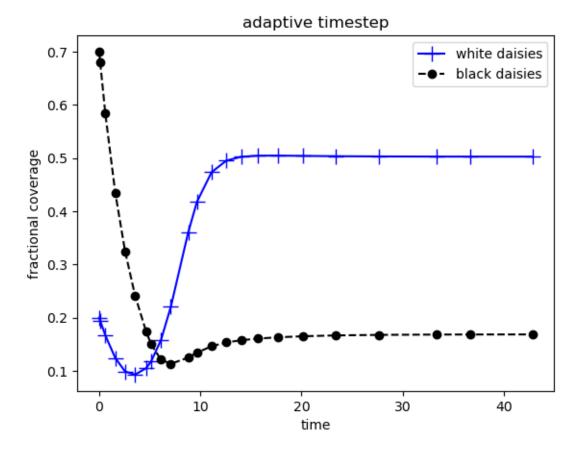
yvals = pd.DataFrame.from_records(yVals, columns=['white', 'black'])

thefig, theAx = plt.subplots(1, 1)

points, = theAx.plot(timeVals, yvals['white'], '-b+', label='white daisies')
    points.set_markersize(12)
    theLine1, = theAx.plot(timeVals, yvals['black'], '--ko', label='black daisies')
    theAx.set_title('adaptive timestep')
    theAx.set_xlabel('time')
    theAx.set_ylabel('fractional coverage')
    out = theAx.legend(loc='best')

# timeVals,yVals,errorList=theSolver.timeloop5fixed()
# whiteDaisies=[frac[0] for frac in yVals]

timevals_adaptive = timeVals
```



```
[87]: theSolver = Integ54('adapt.yaml')
    timeVals, yVals, errorList = theSolver.timeloop5fixed()

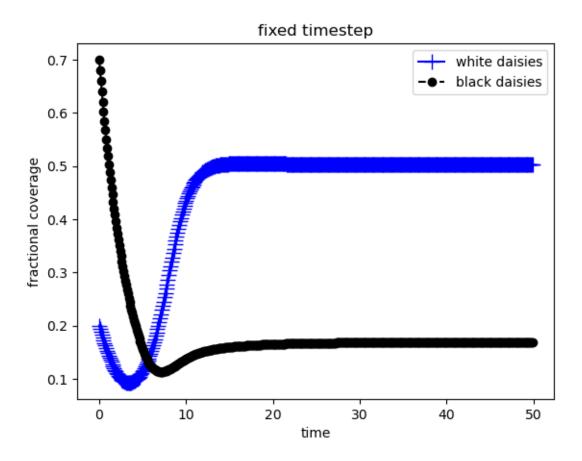
yvals = pd.DataFrame.from_records(yVals, columns=['white', 'black'])

thefig, theAx = plt.subplots(1, 1)

points, = theAx.plot(timeVals, yvals['white'], '-b+', label='white daisies')
    points.set_markersize(12)
    theLine1, = theAx.plot(timeVals, yvals['black'], '--ko', label='black daisies')
    theAx.set_title('fixed timestep')
    theAx.set_xlabel('time')
    theAx.set_ylabel('fractional coverage')
    out = theAx.legend(loc='best')

# timeVals,yVals,errorList=theSolver.timeloop5fixed()
# whiteDaisies=[frac[0] for frac in yVals]

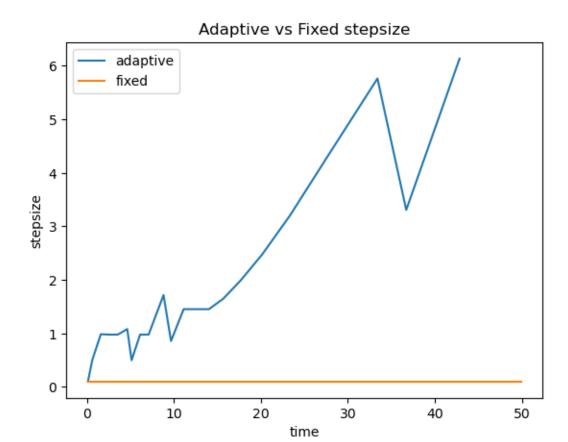
timevals_fixed = timeVals
```



```
[88]: adaptive_len = len(timevals_adaptive)
    fixed_len = len(timevals_fixed)
    print("adaptive:", adaptive_len, ", fixed:", fixed_len)

adaptive: 23 , fixed: 500

[89]: adaptive_stepsize = np.diff(timevals_adaptive)
    fixed_stepsize = np.diff(timevals_fixed)
    plt.plot(timevals_adaptive[1:], adaptive_stepsize, label='adaptive')
    plt.plot(timevals_fixed[1:], fixed_stepsize, label='fixed')
    plt.legend()
    plt.title("Adaptive vs Fixed stepsize")
    plt.xlabel("time")
    plt.ylabel("stepsize")
[89]: Text(0, 0.5, 'stepsize')
```



The fixed timeloop needs to run about 20 times faster than the adaptive timeloop to be comparable in performance.

[]: