Electron-positron pair production in curved spacetime

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2023-07-04

De sitter and Anti de sitter Spacetime

参考文章：（科普向）我们的宇宙到底是弯的还是直的：de Sitter 空间和 Anti-de sitter 空间速成 (2)

WKB 近似和 Whittaker 方程

参考资料：WKB 近似和 Whittaker 方程

Contents

One-loop effective action and Schwinger effect in (anti-) de Sitter space

1. Author: Rong-Gen Cai & Sang Pyo Kim
2. Date : 2014-09-11
3. Journal: Journal of High Energy Physics
4. Work : Study the Schwinger mechanism by a uniform electric field in dS_2 and AdS_2 and the curvature effect on the Schwinger effect, and further propose a thermal interpretation of the Schwinger formula in terms of the Gibbons-Hawking temperature and the Unruh temperature for an accelerating charge in dS_2 and an analogous expression in AdS_2.
5. Paper : One-loop effective action and Schwinger effect in (anti-) de Sitter space

The main formulas and mathematical methods 1

In a curved spacetime with the metric tensor g , a charged scalar field with the mass m and the charge q obeys the equation

$$\frac{1}{\sqrt{-g}} D_\mu \left(\sqrt{-g} g^{\mu\nu} D_\nu \right) \phi - m^2 \phi = 0 \tag{1}$$

where $D_\mu = \partial_\nu - iqA_\mu$ with $A_\mu = (-A_0, A_i)$ and the field tensor is $F_{\mu\nu} = \partial_{\nu,\mu} - A_{\mu,\nu}$ in the curved spacetime.

The main formulas and mathematical methods 2

In dS_2, the momentum mode $\phi_k = e^{-Ht/2}\varphi_k$ has the equation

$$\ddot{\varphi}_k(t) + \overline{\Omega}_k^2(t)\varphi_k(t) = 0 \tag{2}$$

where the dot denotes the derivative with respect to time and

$$\overline{\Omega}_k^2(t) = \gamma_{dS}^2 + \overline{k}^2 e^{-2Ht} + 2\frac{qE}{H}\overline{k}e^{-Ht}, \gamma_{dS}^2 = \left(\frac{qE}{H}\right)^2 + m^2 - \frac{H^2}{4} \tag{3}$$

NOTE

This type of equation can be solved using the WKB approximation method.

The main formulas and mathematical methods 3

The Schwinger formula for pairs with the given momentum or energy is approximately given by

$$N_S = e^{-2S} \tag{4}$$

where the instanton action for dS_2 is

$$S_k = -2ImS_k = \frac{2\pi}{H}\left(\gamma_{dS} - \frac{qE}{H}\right) \tag{5}$$

The main formulas and mathematical methods 4

The positive frequency solution $\phi_k^{(+)}$ and the negative frequency solution $\phi_k^{(-)}$ of the mode equation in dS_2

$$\ddot{\phi}_k + H\dot{\phi}_k + \omega_k^2(t)\phi_k = 0 \tag{6}$$

with

$$\omega_k^2(t) = m^2 + e^{-2Ht}\left(\bar{k} + \frac{qE}{H}e^{Ht}\right)^2 \tag{7}$$

satisfy the Wronskian condition

$$e^{Ht}Wr_{(t)}\left[\phi_k^{(+)}(t), \phi_k^{(-)}(t)\right] = i \tag{8}$$

The main formulas and mathematical methods 5

The out-vacuum is defined by the positive frequency solutions in the past infinity ($t = \infty$)

$$\phi^{(+)}_{out,k} = \frac{e^{i\pi/4} e^{\pi|\mu|/2}}{\sqrt{2\gamma}} \left(\frac{H}{2k}\right)^{3/2} e^{-Ht/2} z M_{\lambda,\mu}(z) \tag{9}$$

The in-vacuum is constructed by the positive frequency solutions in the future infinity ($t = -\infty$)

$$\phi^{(+)}_{in,k} = \frac{e^{-\pi|\lambda|/2}}{\sqrt{H}} \left(\frac{H}{2\bar{k}}\right)^{3/2} e^{-Ht/2} (-z) W_{-\lambda,\mu}(-z) \tag{10}$$

NOTE

Where $M_{\lambda,\mu}$ and $W_{\lambda,\mu}$ are the Whittaker functions.

The main formulas and mathematical methods 6

The logarithm of the gamma function

$$\ln\left(\Gamma(z)\right) = \int_0^\infty \frac{ds}{s}\left[\frac{e^{-zs}}{1-e^{-s}} - \cdots\right] \tag{11}$$

Gravitational correction to vacuum polarization

1. Author: Jentschura,U.D.
2. Date: 2015-2-13
3. Journal: Physical Review A
4. Work: Consider the gravitational correction to (electronic) vacuum polarization in the presence of a gravitational background field. The Dirac propagators for the virtual fermions are modified to include the leading gravitational correction (potential term) which corresponds to a coordinate dependent fermion mass.
5. Paper: Gravitational correction to vacuum polarization

Fermion wave equation

The preferred way to describe highly energetic fermions (neutrinos) which travel faster than is via the tachyonic Dirac equation, which in the local reference frame reads as

$$\left(i\hbar\gamma^{\mu}\frac{\partial}{\partial x^{\mu}} - \gamma^{5} m c_{loc} \right) \phi(t, \vec{r}) = 0 \tag{12}$$

The speed of light in a "local" reference frame

$$c_{loc} = c + \delta c_{\gamma} = c - |\delta c_{\gamma}| \tag{13}$$

DIRAC EQUATION AND GRAVITATIONAL COUPLING

The Hamiltonian which governs the gravitational interaction is given by

$$
\begin{aligned}
H &= \vec{\alpha} \cdot \vec{p} + \beta m \omega(r) \\
\omega &\approx 1 - \frac{r_s}{2r} = 1 - \frac{GM}{r} = 1 + \Phi_G
\end{aligned}
\tag{14}
$$

where $r_s = 2GM$ is the Schwarzschild radius.

Fermionic current and Schwinger effect in de Sitter spacetime

1. Author: Stahl, Clément & Strobel, Eckhard & Xue, She-Sheng
2. Date: 2016-1-6
3. Journal: Physical Review D
4. Work: Study the fermionic Schwinger effect in two dimensional de Sitter spacetime.
5. Paper: Fermionic current and Schwinger effect in de Sitter spacetime

THE DIRAC EQUATION IN dS SPACETIME

$$
\begin{aligned}
S = \int d^2x\sqrt{-g(x)} \Bigg[& -\frac{1}{k}R(x) \\
& + \frac{i}{2}\left[\overline{\psi}(x)\underline{\gamma}^{\mu}\nabla_{\mu}\psi(x) - \left(\nabla_{\mu}\overline{\psi}(x)\right)\underline{\gamma}^{\mu}\psi(x)\right] \\
& - m\overline{\psi}(x)\psi(x) - \frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x)\Bigg]
\end{aligned} \tag{15}
$$

The Dirac equation can be derived from the action (15) by varying with respect to the field $\psi(x)$, which gives

$$
\left(i\underline{\gamma}^{\mu}\nabla_{\mu} - m\right)\psi(x) = 0 \tag{16}
$$

The Schwinger mechanism in (Anti) de Sitter spacetimes

1. Author: Samantray,Prasant
2. Date: 2016-04-11
3. Journal: Journal of High Energy Physics
4. Work: Present a short and novel derivation of the Schwinger mechanism for particle pair production in $1+1$ dimensional de Sitter and Anti de Sitter spacetimes.
5. Paper:The Schwinger mechanism in (Anti) de Sitter spacetimes

Scalar current of created pairs by Schwinger mechanism in de Sitter spacetime

1. Author: Bavarsad,Ehsan & Stahl,Clément & Xue,She-Sheng
2. Date: 2016-11-4
3. Journal: Physical Review D
4. Work: Consider a charged scalar field in a D-dimensional de Sitter spacetime and investigate pair creation by a Schwinger mechanism in a constant electric field background.
5. Paper:Scalar current of created pairs by Schwinger mechanism in de Sitter spacetime

Effect of a magnetic field on Schwinger mechanism in de Sitter spacetime

1. Author: Bavarsad,Ehsan & Kim,Sang Pyo & Stahl,Clément & Xue,She-Sheng
2. Date: 2018-1-25
3. Journal: Physical Review D
4. Work: Investigate the effect of a uniform magnetic field background on scalar QED pair production in a four-dimensional de Sitter spacetime (dS4).
5. Paper:Effect of a magnetic field on Schwinger mechanism in de Sitter spacetime

Schwinger effect in inflaton-driven electric field

1. Author: Kitamoto,Hiroyuki
2. Date: 2018-11-12
3. Journal: Physical Review D
4. Work: Investigate the pair production of scalar particles in the inflaton-driven electric field. In particular, we evaluate the induced current due to the pair production.
5. Paper:Schwinger effect in inflaton-driven electric field