Test for proportions

In the videos, you learnt how to perform hypothesis testing for the mean of a Gaussian population. Another very useful example is testing for a population proportion p.

An example

Imagine that you have a coin, but you don't know whether it's fair or not. The proportion you are interested in is $p = \mathbf{P}(H)$. A possible set of hypothesis for this problem is

$$H_0 : p = 0.5 \text{ vs. } H_1 : p \neq 0.5$$

Imagine you toss the coin 20 times, of which 7 turned out heads. Your random sample consists in one random variable X ="number of heads in 20 coin flips", which has a $Binomial(20, p)$ distribution. A good estimation for the proportion is the relative frequency of heads:

$$\hat{p} = \frac{X}{20}$$

Remember that under certain conditions, the Central Limit Theorem states that $\hat{p} \sim \mathcal{N}\left(p, \sqrt{\frac{p(1-p)}{20}}\right)$, or equivalently

$$Z = \frac{\frac{X}{20} - p}{\sqrt{p(1-p)}}\sqrt{20} \sim \mathcal{N}(0,1)$$

Z **will be your test statistic.** If $H_0$is true $(p = 0.5)$, then your test statistic becomes

$$Z = \frac{\frac{X}{20} - 0.5}{\sqrt{0.5(1-0.5)}}\sqrt{20} = \frac{\frac{X}{20} - 0.5}{0.5}\sqrt{20} \sim \mathcal{N}(0,1)$$

Consider a significance level $\alpha = 0.05$. Then to make a decision you need to get the *p*-value for your observed statistic. With the observed sample $x = 7$, the observed statistic is

$$z = \frac{\frac{7}{20} - 0.5}{\ldots}\sqrt{20} = -1.3416$$