

# Power Grid Failures - Renewable Energy

How fluctuations in solar and wind affect grid security in  
Germany

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**Fons van der Plas**

*July 2019*

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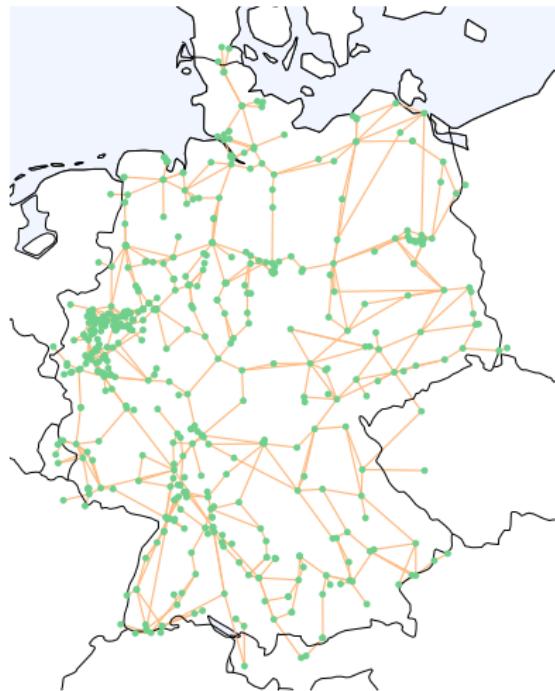
*supervisor:* Dr. Henk Don

Radboud University





# Transmission network of Germany



**Figure 1:** The  $n = 585$  nodes and  $k = 695$  lines of the SciGRID dataset.

# Power injection

At each node  $i \in \{1, \dots, n\}$ :

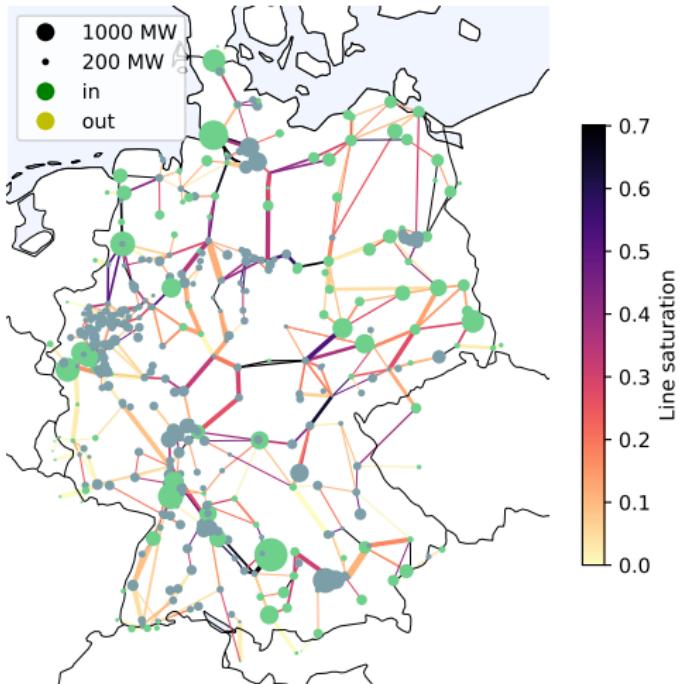
$$p_i := S_i^{\text{green}} + S_i^{\text{grey}} - S_i^{\text{load}}$$

	green	grey
<i>Operation costs</i>	<b>0-10 €/MWh</b>	30-35 €/MWh
<i>CO<sub>2</sub>eq-emissions</i>	<b>0 kg/MWh</b>	490 - 820 kg/MWh <sup>1</sup>
<i>Controllable?</i>	wind & hydro	<b>yes</b>
<i>Output</i>	stochastic	<b>deterministic</b>

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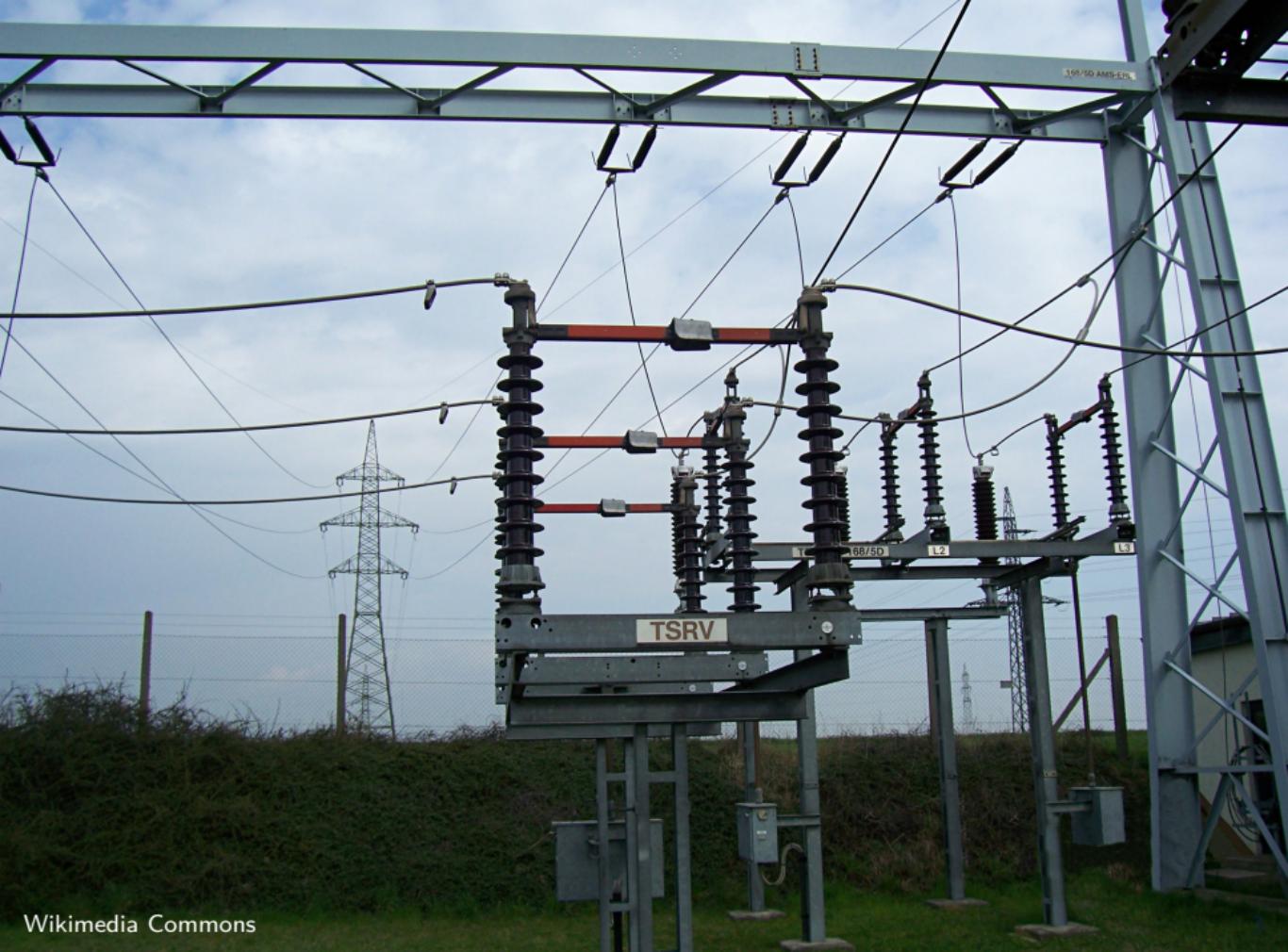
<sup>1</sup>IPCC 2014

# Line limits



**Figure 2:** Line saturation at 11am, 1 Jan 2011.







# Summary

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Given:

- Grid structure
- Hourly load
- Hourly generation

Predict:

- ‘**Top 10**’ of lines most likely to fail
- For each risky line: the most likely  
**cascading line failures**

[Voorpagina](#)[Nieuws & Achtergrond](#)[Columns & Opinie](#)[Video](#)[Wetenschap](#)**NIEUWS STROOMNETTEN**

## Elektriciteitsnetwerk kan stroom uit lokale groene projecten niet aan

Lokale groene energieprojecten kunnen niet doorgaan doordat zij hun stroom niet kwijt kunnen. Netbeheerder Enexis moet in Groningen en Drenthe 'nee' verkopen aan boeren, bedrijven en verenigingen die zonnepanelen op hun daken willen, door een gebrek aan capaciteit op het elektriciteitsnet. Ook aansluiting van nieuwe grootschalige zonneweiden is in bepaalde gebieden niet meer mogelijk.

**Jurre van den Berg** 11 januari 2019, 2:00

# Emergent failures and cascades in power grids: a statistical physics perspective

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(Dated: April 23, 2018)

We model power grids transporting electricity generated by intermittent renewable sources as complex networks, where line failures can emerge indirectly by noisy power input at the nodes. By combining concepts from statistical physics and the physics of power flows, and taking weather correlations into account, we rank line failures according to their likelihood and establish the most likely way such failures occur and propagate. Our insights are mathematically rigorous in a small-noise limit and are validated with data from the German transmission grid.

PACS numbers: 89.75.Hc, 89.20.-a, 88.80.-q

Understanding cascading failures in complex networks is of great importance and has received a lot of attention in recent years [1–17]. Despite proposing different mechanisms for their evolution, a common feature is that cascades are triggered by some *external* event. This initial attack is chosen either (i) deliberately, to target the most vulnerable or crucial network component or (ii) uniformly at random, to understand the average network reliability. This distinction led to the insight that complex networks are resilient to random attacks, but vulnerable to targeted attacks [7, 18, 19]. However, both lead to the *direct* failure of the attacked network component.

In this Letter, we focus on networks in which edge failures occur in a fundamentally different manner. Specifically, we consider networks where fluctuations of the node

Previous works applying large-deviations techniques to problems in complex networks dynamics, such as epidemic extinction and biophysical networks, include [23, 24].

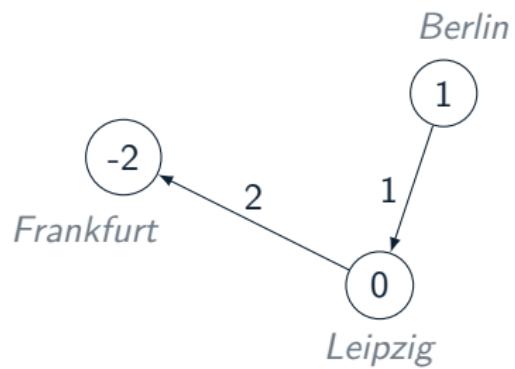
We model a transmission network by a connected graph  $G$  with  $n$  nodes representing the *buses* and  $m$  directed edges modeling *transmission lines*. The nominal values of net power injections at the nodes are given by  $\mu = \{\mu_i\}_{i=1,\dots,n}$ . We model the stochastic fluctuation of the power injections around  $\mu$ , due to variability in renewable generation, by means of the random vector  $\mathbf{p} = \{p_i\}_{i=1,\dots,n}$ , which is assumed to follow a multivariate Gaussian distribution with density

$$\varphi(\mathbf{x}) = \frac{\exp(-\frac{1}{2}(\mathbf{x} - \boldsymbol{\mu})^T (\varepsilon \boldsymbol{\Sigma}_p)^{-1} (\mathbf{x} - \boldsymbol{\mu}))}{(2\pi)^{\frac{n}{2}} \det(\varepsilon \boldsymbol{\Sigma}_p)^{\frac{1}{2}}}, \quad (1)$$

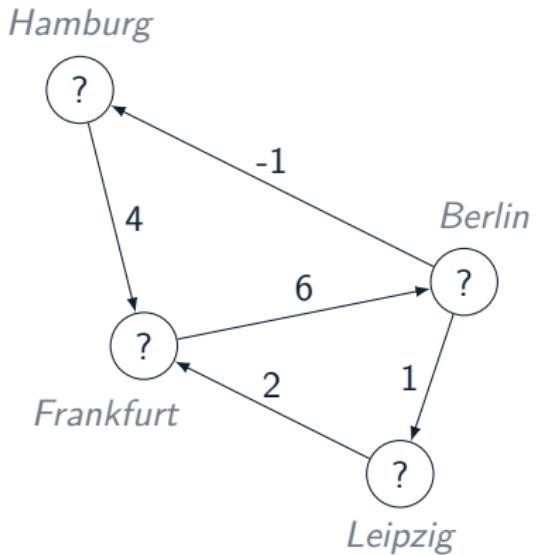
# Power Flow

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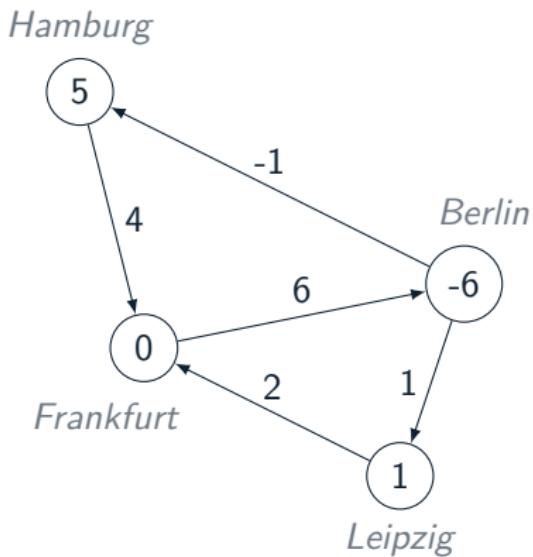
# Digraph flow



# Puzzle!



# Solution



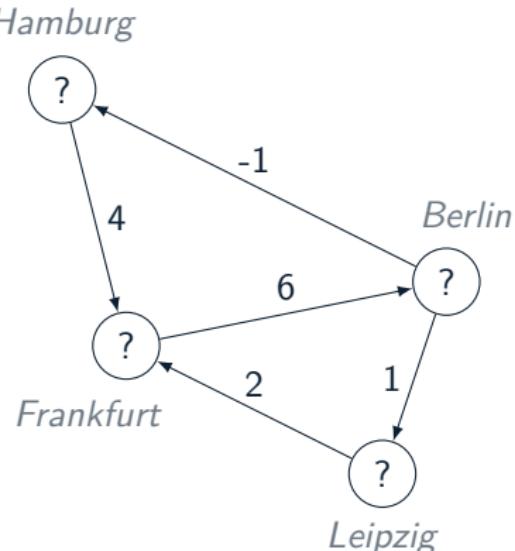
# Transmission

$$\mathbf{f} = \begin{pmatrix} 1 \\ 2 \\ 6 \\ -1 \\ 4 \end{pmatrix} \in \mathbb{R}^k, \quad \mathbf{p} = \begin{pmatrix} ? \\ ? \\ ? \\ ? \\ ? \end{pmatrix} \in \mathbb{R}^n$$

Power transmission is linear

$$\mathbf{p} = \mathbf{C} \cdot \mathbf{f}$$

$$\mathbf{C} \in \mathbb{R}^{n \times k}$$



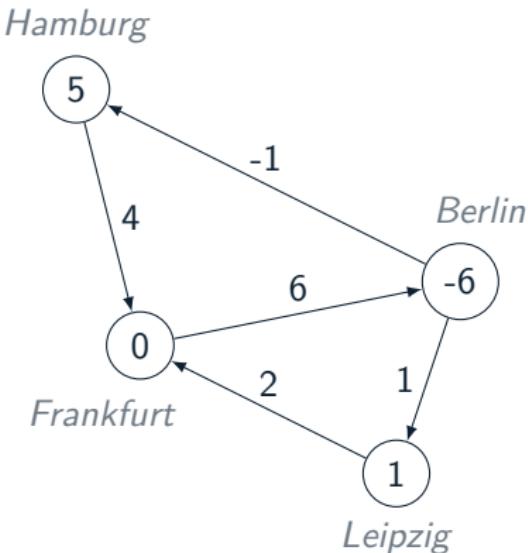
# Transmission

Power transmission is linear

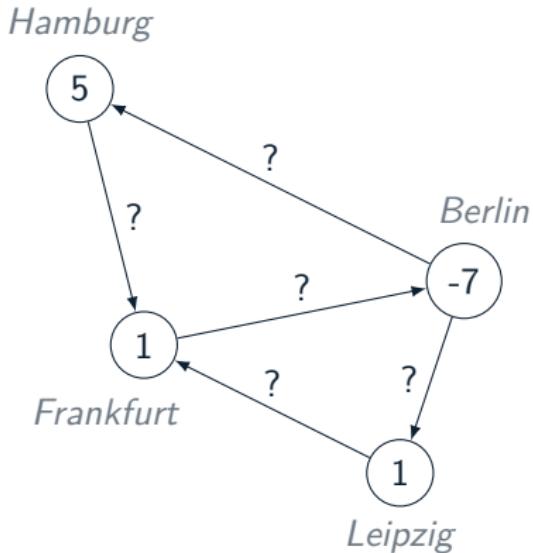
$$p = C \cdot f$$

$$C = \begin{pmatrix} +1 & 0 & -1 & +1 & 0 \\ 0 & 0 & 0 & -1 & +1 \\ 0 & -1 & +1 & 0 & -1 \\ -1 & +1 & 0 & 0 & 0 \end{pmatrix}$$

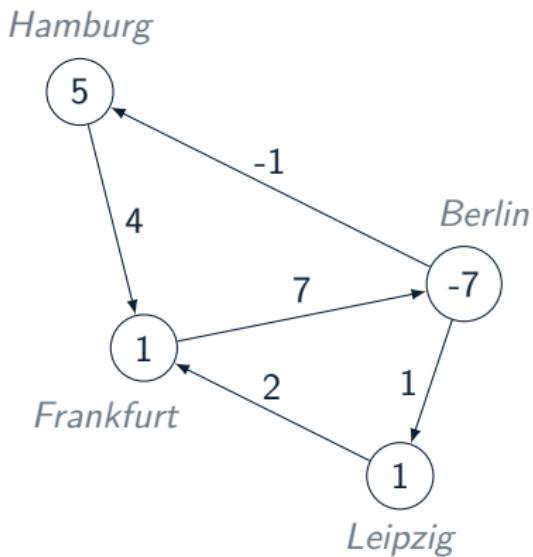
$$\text{we get: } p = C \cdot f = \begin{pmatrix} -6 \\ 5 \\ 0 \\ 1 \end{pmatrix}$$



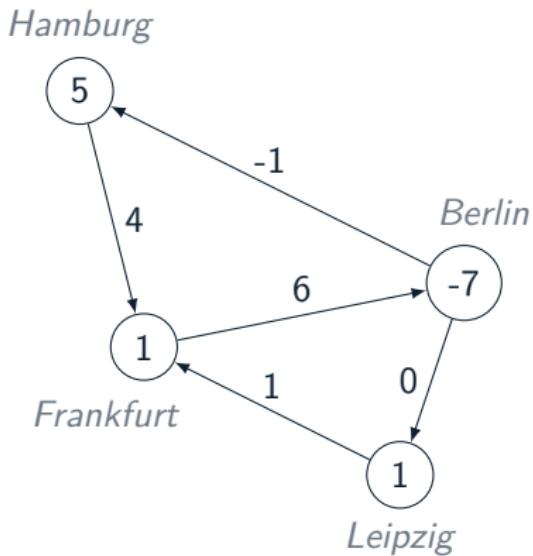
# Inverse problem: Linear Power Flow (LPF)



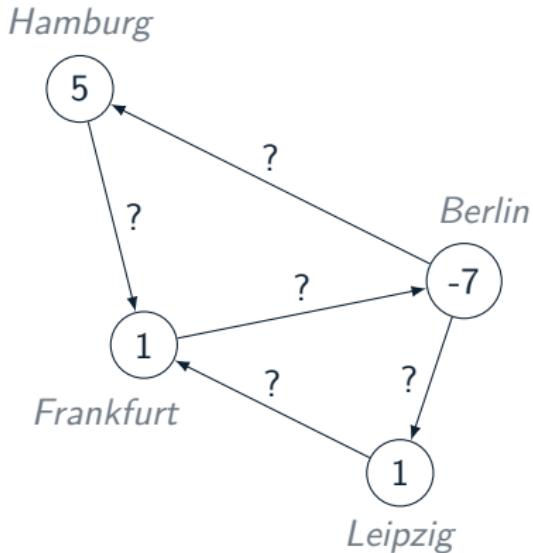
# Solution



## Another solution



# Inverse problem: Linear Power Flow (LPF)



## Inverse problem: Linear Power Flow (LPF)

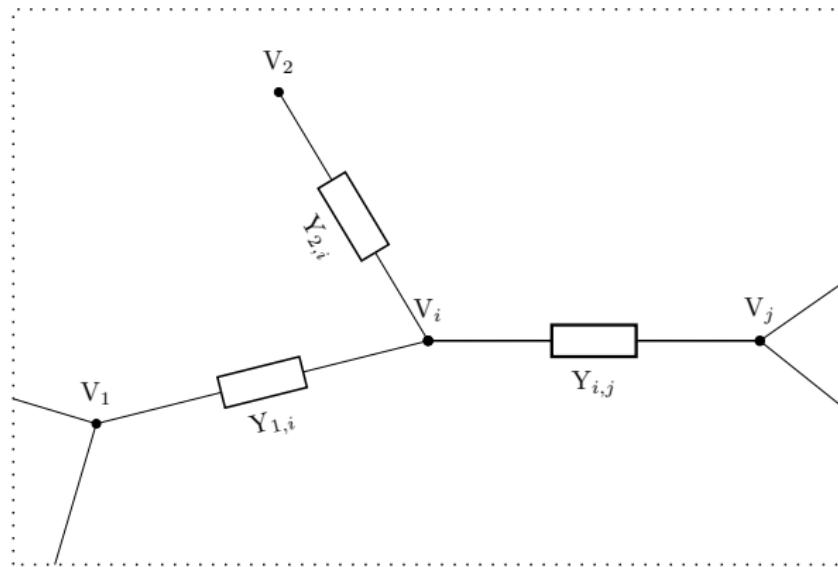
The **LPF** is a (linear) *right-inverse* of  $\mathbf{C}$ :

$$\mathbf{C} \cdot \mathbf{LPF} = \mathbf{I}.$$

Many linear maps satisfy this property, but there is only one *right LPF* that satisfies **Kirchoff's Circuit Laws** and **Ohm's Law**.

The **LPF** is difficult to compute: requires *Singular Value Decomposition* of a  $k \times k$  matrix.

## Electric model - HV Layer



**Figure 3:** Top layer of the electric circuit used in the model. Transmission lines are modelled as impedances.

High-Voltage

Ground

$V_2$

$V_1$

$V_i$

$V_3$

$\sim$

$\sim$

$\sim$

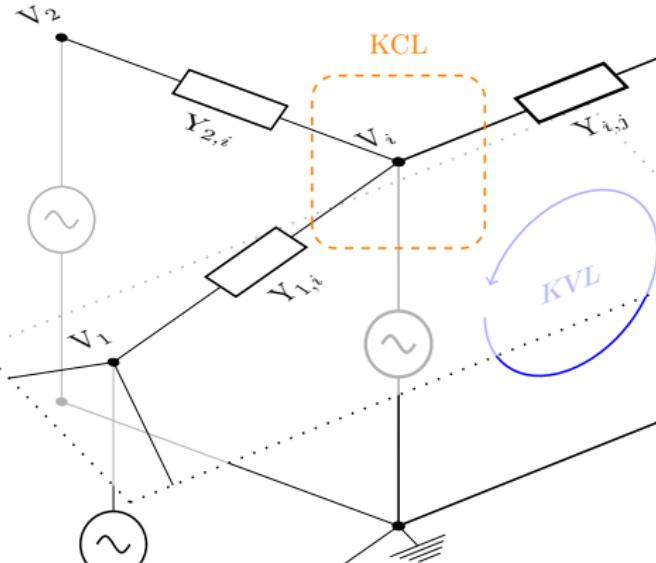
$Y_{2,i}$

$Y_{1,i}$

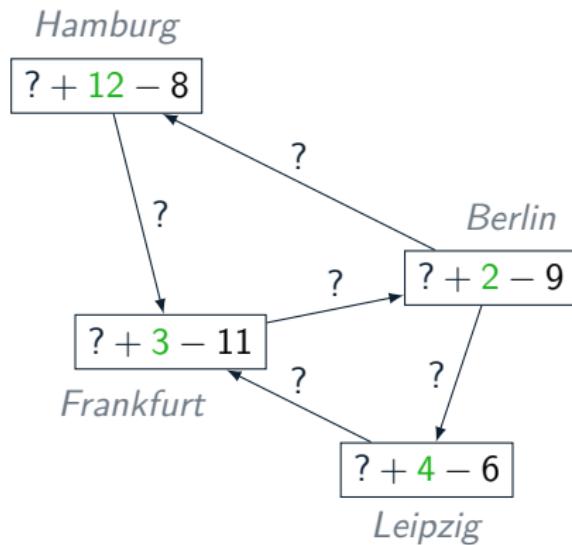
$Y_{3,i}$

KCL

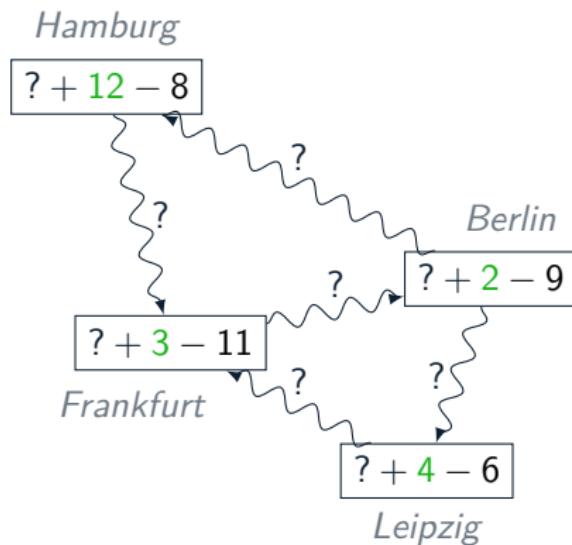
KVL

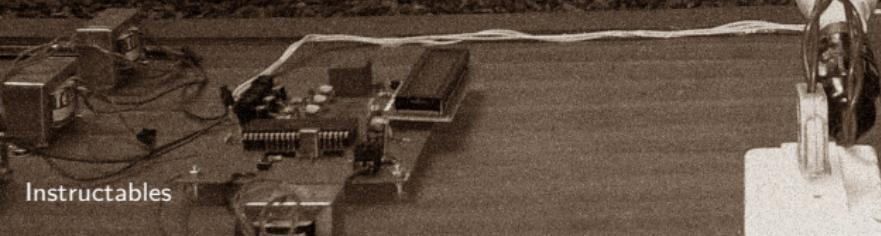
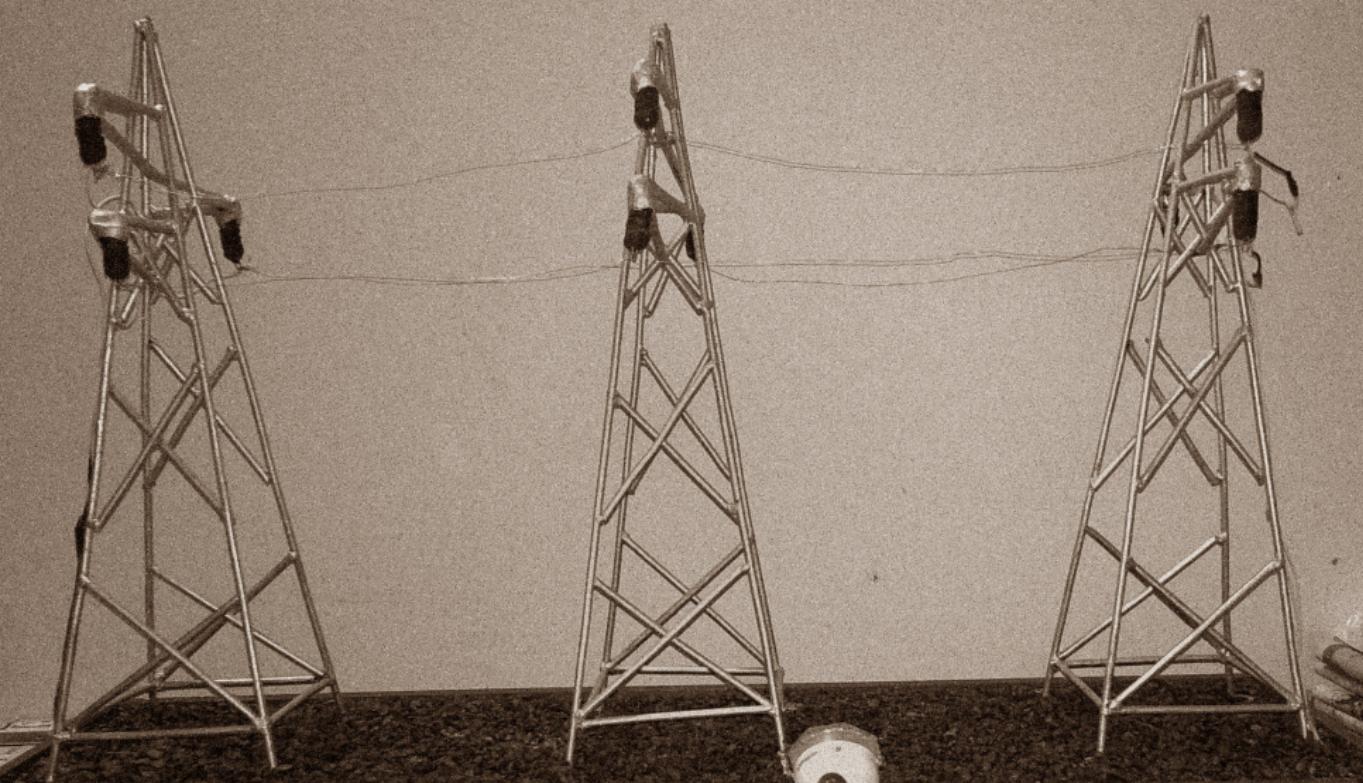


# Unknown generation: Linear Optimum Power Flow (LOPF)



# Unknown generation: Optimum Power Flow (OPF)





# OPF to supplement data

Available:

- Grid structure
- Hourly load
- Hourly **solar and wind** generation

Not available:

- Hourly **grey** generation



Available:

- Grid structure
- Hourly load
- Hourly generation

## OPF accuracy

OPF is used to generate an **hourly power injection dataset**, by assuming that the cheapest generation profile is used at all times.

**OPF is a fair approximation, because**

*grid operators actually use OPF algorithms to minimise costs*

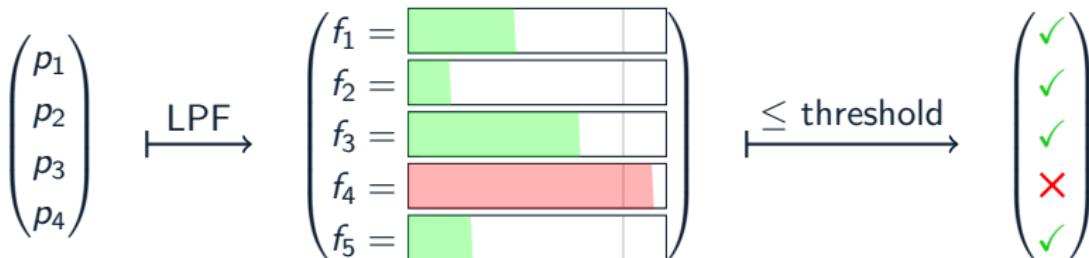
## **Stochastic power injections**

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# Deterministic power injection $p$

Given a power injection  $\mathbf{p} \in \mathbb{R}^n$ :

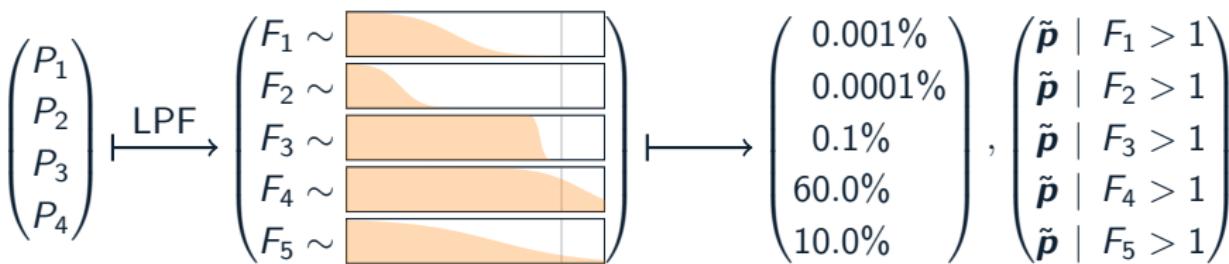
- **Linearised Power Flow** computes *line current*
- $> 100\%$  saturated  $\Rightarrow$  **line failure**



# Stochastic power injection $P$

When the power injection  $P$  is **stochastic**:

- LPF now gives the line current *distribution*
- For each line, we have:
  - the failure *probability*
  - the *most likely power injection*  $\tilde{p}$  that would cause that failure



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## Stochastic power injection $P$

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When the power injection  $P$  is **Gaussian**:

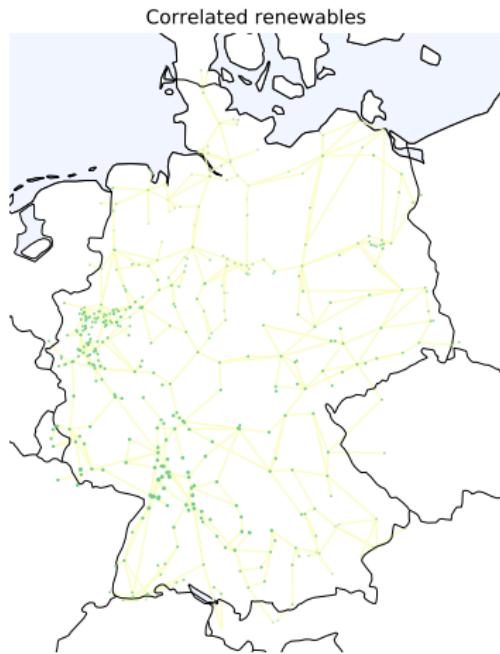
$$P \sim \mathcal{N}(\mu, \Sigma),$$

then the line flow vector  $F$  is also Gaussian:

$$F = LPF \cdot P$$

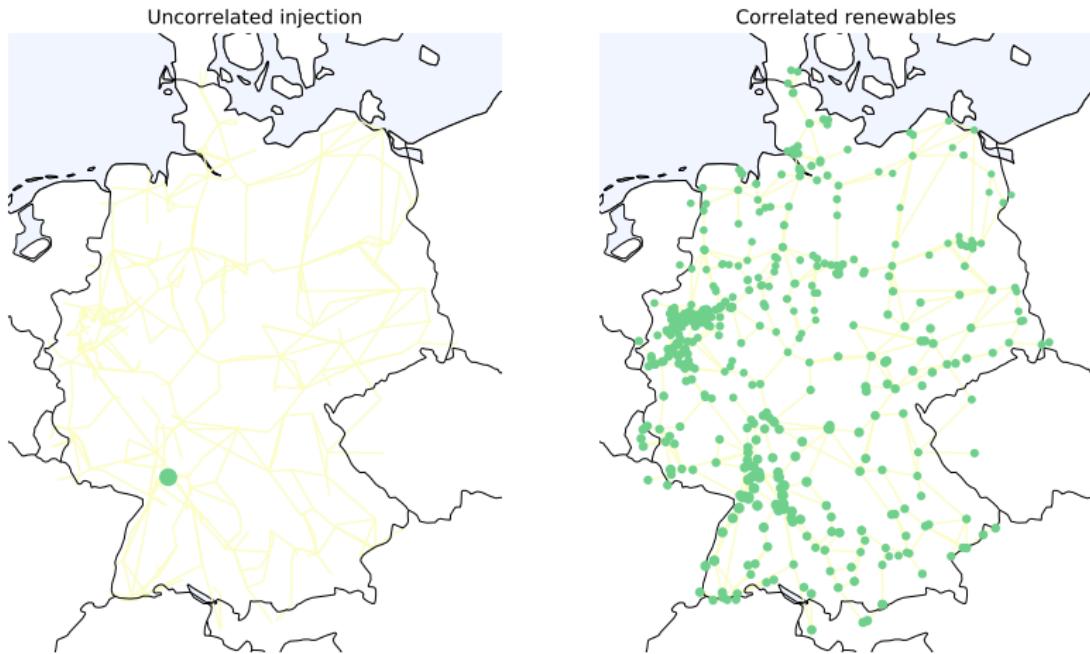
$$F \sim \mathcal{N}(LPF \cdot \mu, LPF \cdot \Sigma \cdot LPF^T).$$

# Correlated injections, night-time



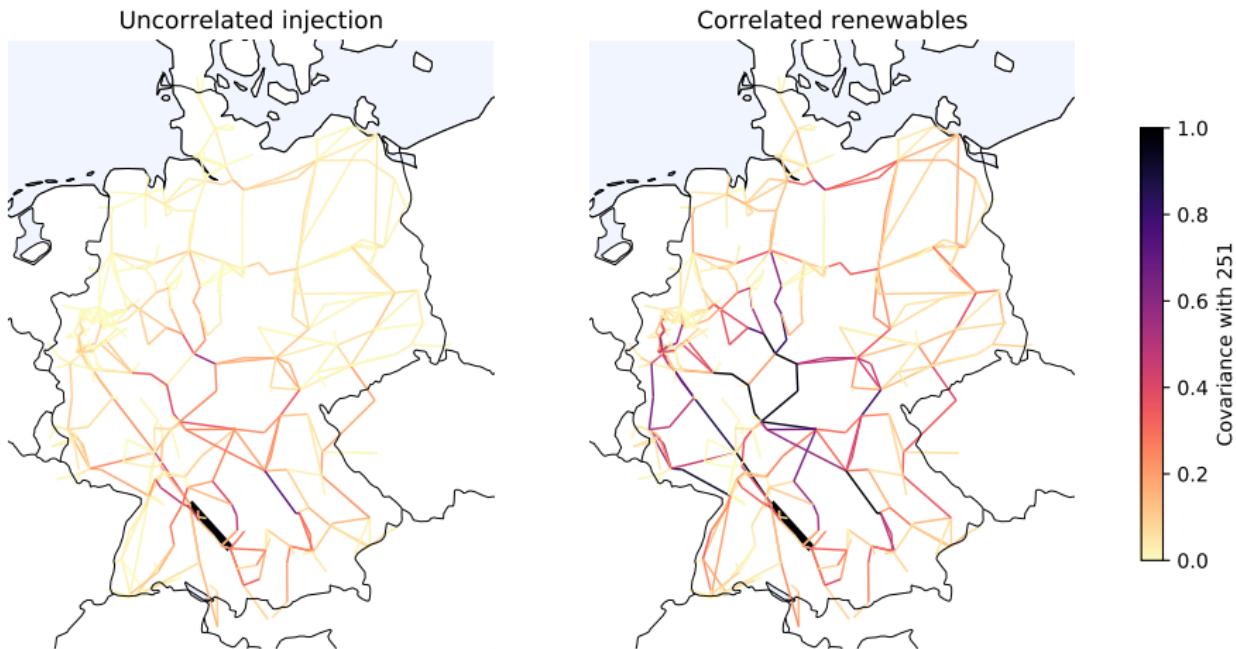
**Figure 4:** Covariance relative to bus 345, wind only.

# Correlated injections, daytime



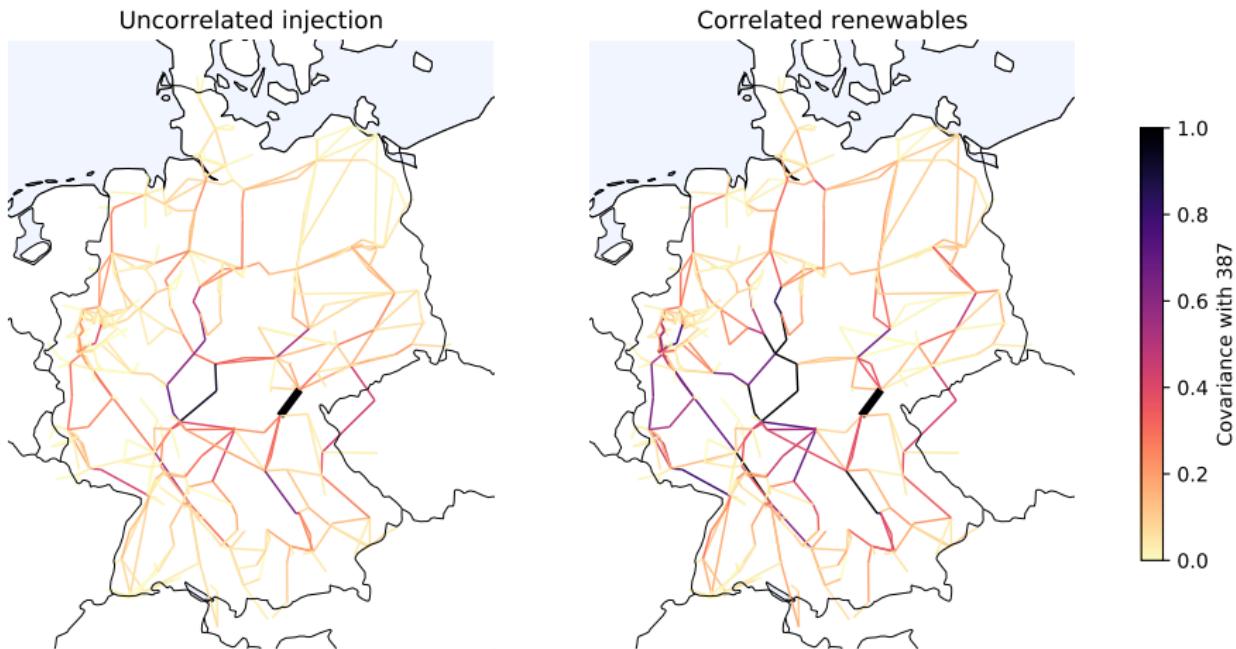
**Figure 5:** Covariance relative to bus 345, wind & solar.

# Correlated flow, daytime



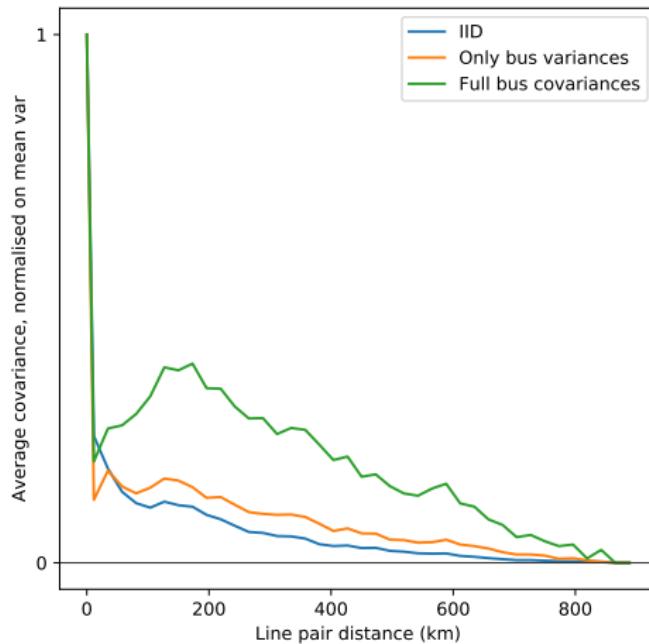
**Figure 6:** Covariance relative to line 251.

# Correlated flow, daytime



**Figure 7:** Covariance relative to line 387.

# Long-range correlations



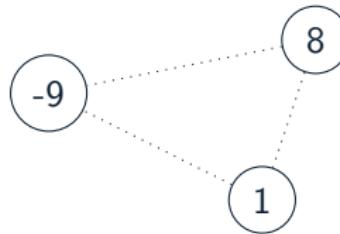
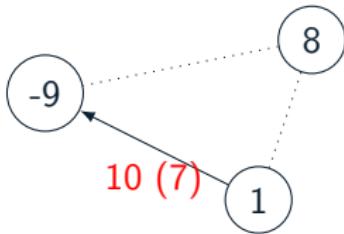
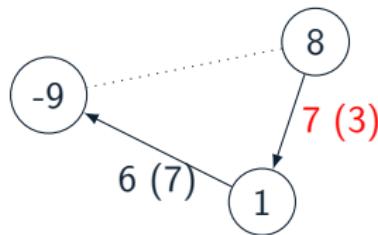
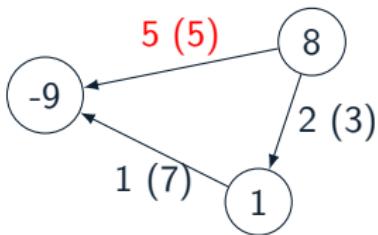
**Figure 8:** Covariances of  $10^5$  random line pairs, versus their distance.

## Cascading failures

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# Cascades

Flow redistributions cause new failures!



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## Top 10 risky lines!

line	$\mathbb{P}[ F_l  \geq 1]$	cascaded
651	9.56 %	1
652	8.96 %	1
411	7.06 %	1
54	6.21 %	1
298	3.45 %	13
473	3.13 %	1
213	2.37 %	1
25	2.11 %	2
645	2.02 %	32
74	1.96 %	1

## Less risky lines: lower top 50

line	$\mathbb{P}[ F_l  \geq 1]$	cascaded
459	$9.51 \cdot 10^{-7}$	137
280	$7.90 \cdot 10^{-7}$	185
481	$7.61 \cdot 10^{-7}$	216
337	$5.82 \cdot 10^{-7}$	3
22	$3.47 \cdot 10^{-7}$	125
416	$3.12 \cdot 10^{-7}$	178
144	$2.69 \cdot 10^{-7}$	198
654	$1.26 \cdot 10^{-7}$	175
188	$1.02 \cdot 10^{-7}$	152
18	$6.43 \cdot 10^{-8}$	162

## Conclusions

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# Conclusions

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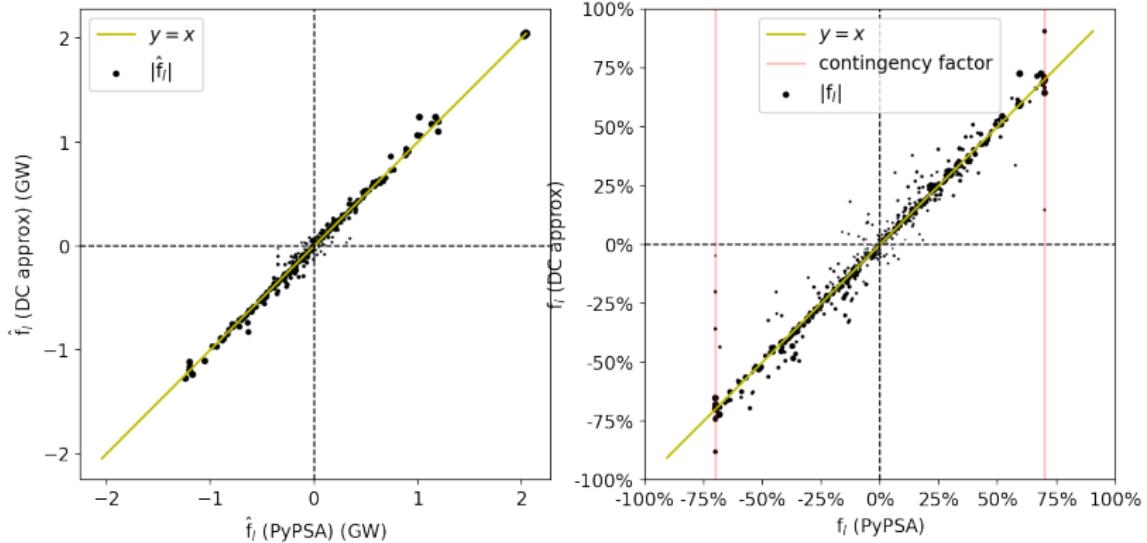
- Long-distance correlations in line currents are increased because of correlated weather.
  - If an emergent failure occurs, it can coincide with or cause other failures.
- 
- Complex behaviour of transmission networks arises from simple, *elegant* mathematical structure



# Appendix

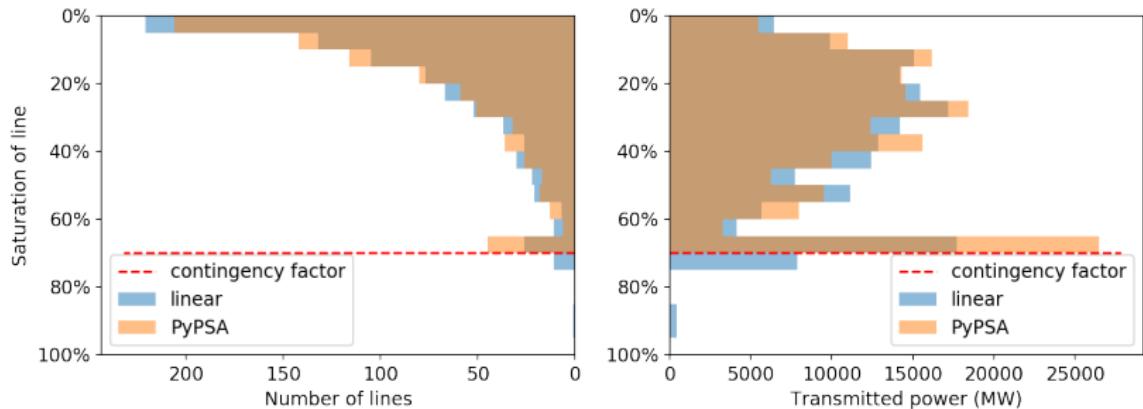
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# LPF accuracy



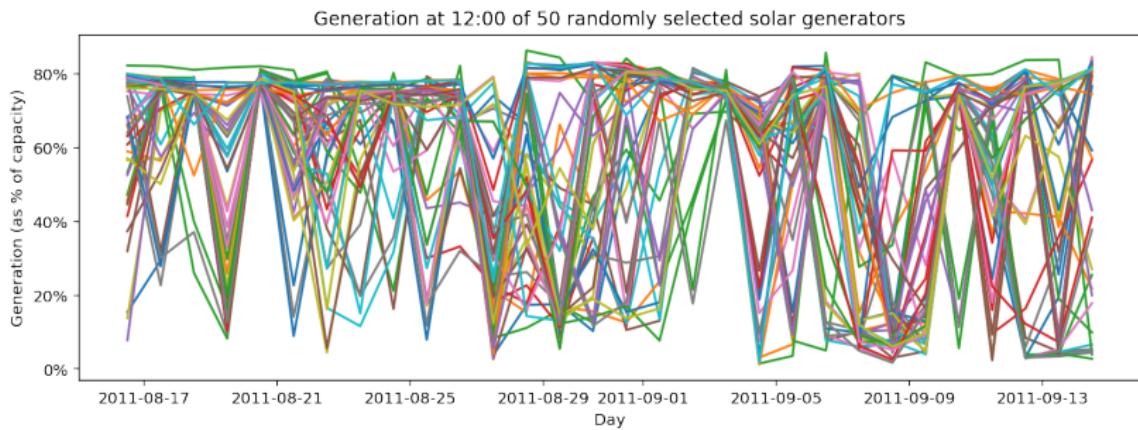
**Figure 9:** Line flow (left) and saturation (right) computed using LPF and (non-linear) PF

# Solar model



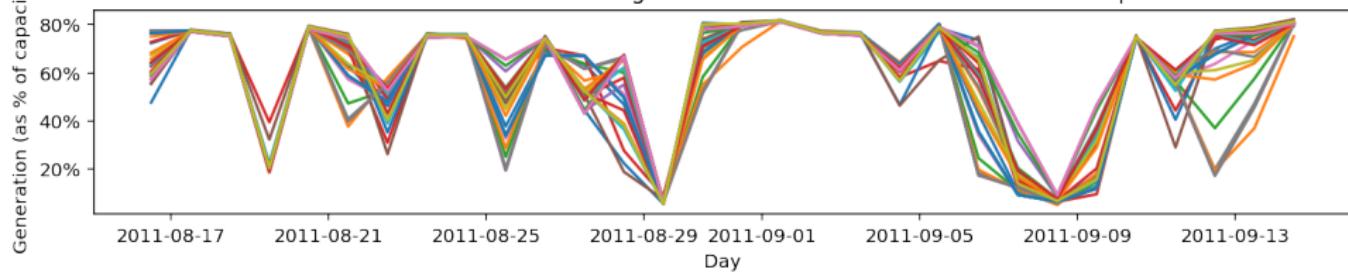
**Figure 10:** Histogram of line flow (left) and saturation (right) computed using LPF and (non-linear) PF

# Solar model

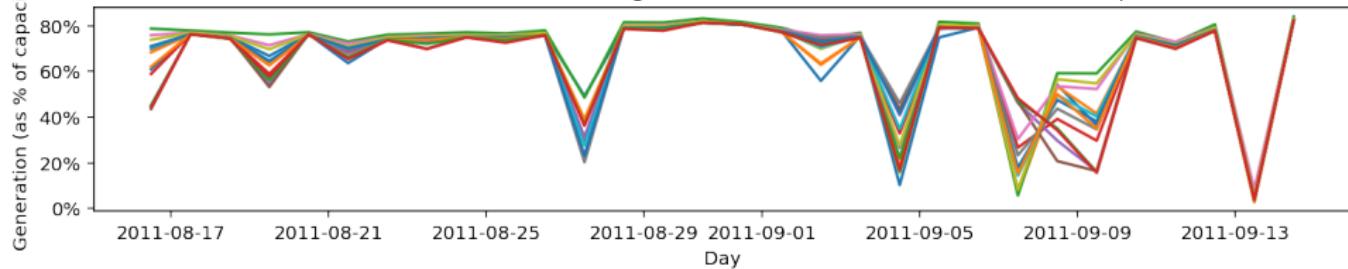


**Figure 11:** Daily solar generation at random nodes

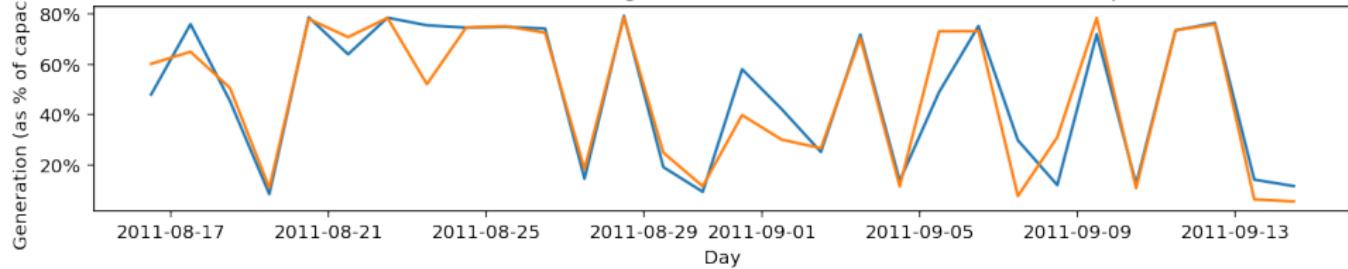
Generation at 12:00 of 39 solar generators that are all \*less than 100 km apart\*



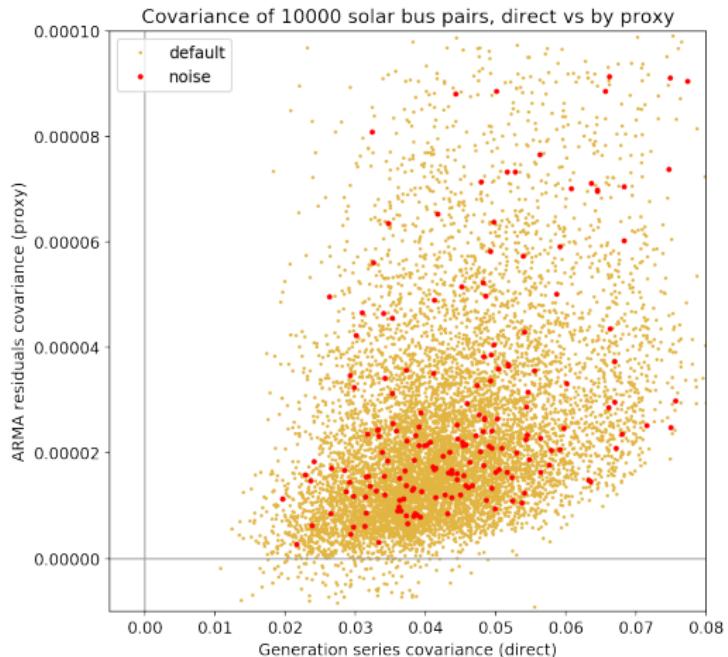
Generation at 12:00 of 14 solar generators that are all \*less than 100 km apart\*



Generation at 12:00 of 2 solar generators that are all \*less than 100 km apart\*

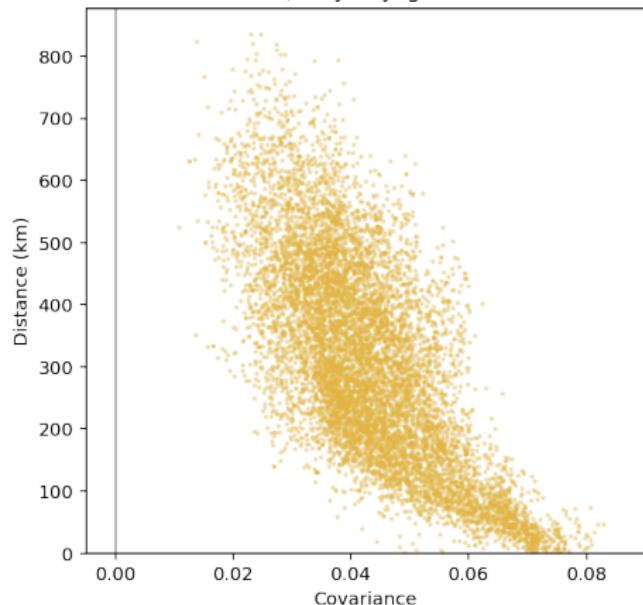


# ARMA?

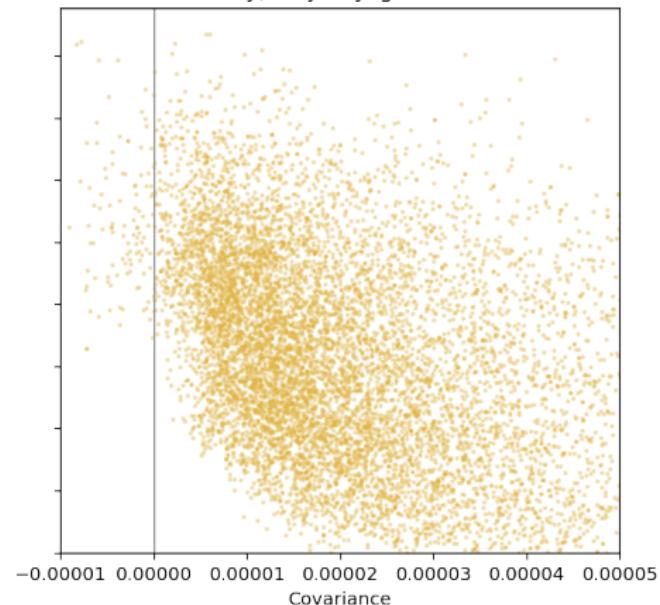


**Figure 12:** Two methods for estimating bus covariances

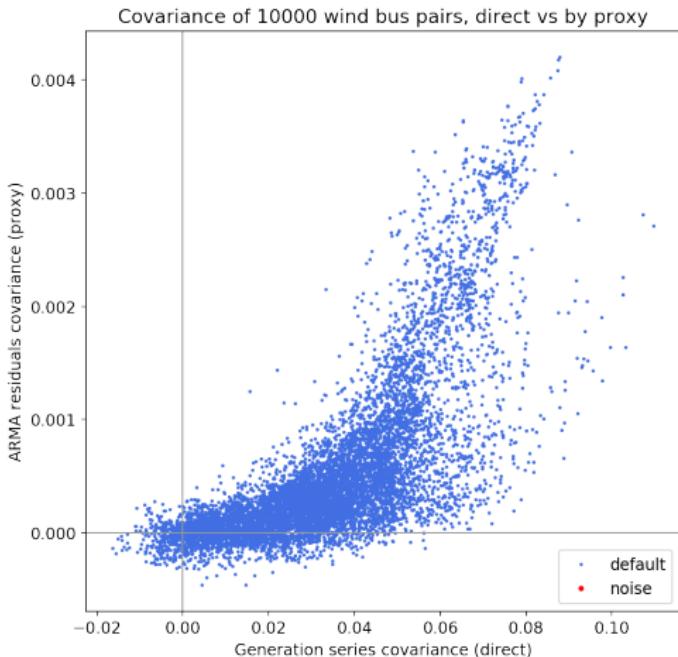
Direct, only daylight hours



Proxy, only daylight hours

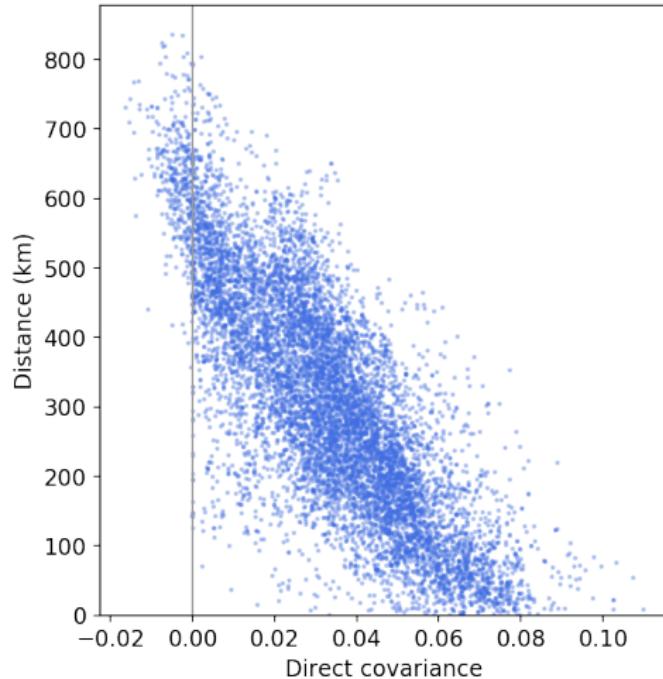


# ARMA?

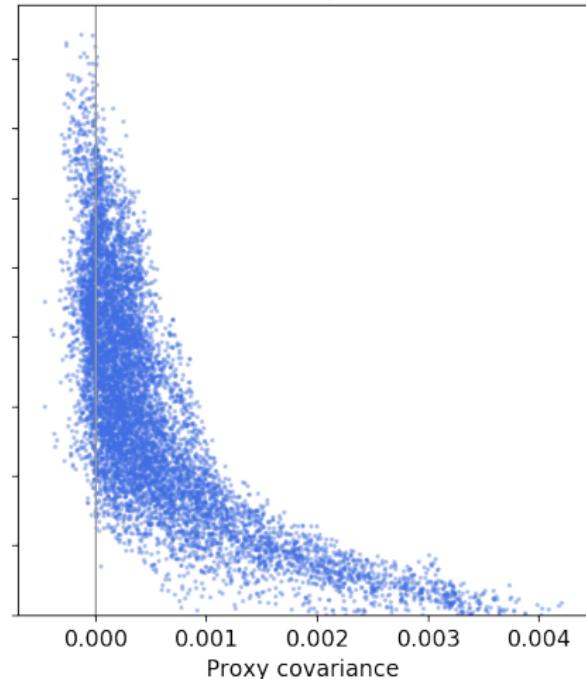


**Figure 13:** Two methods for estimating bus covariances

Direct



Proxy



**Github**

[github.com/fonsp/grid-analysis](https://github.com/fonsp/grid-analysis)