

Informational Condensation in the Rank-1 Limit: Emergence of Causal Order from a Pre-Geometric Substrate

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Abstract

This toy model investigates the spectral dynamics of high-dimensional correlation systems and numerically demonstrates the emergence of a stable dominant mode in pre-geometric regimes. A combined analysis of the spectrum of the informational metric ($g = -\log \rho$), spectral entropy, and the projected Hessian reveals a robust phenomenon: **Informational Causal Condensation**.

Computational experiments at scales $N = 512$ and $N = 1024$ show that: (i) the entropy of the collapsed spectrum converges to $S \approx 0$, indicating dimensional complexity collapse; (ii) no geometric subspace emerges spontaneously; (iii) the condensed mode remains stable under infinitesimal perturbations ($H \geq 0$).

These results provide a mathematical description of a *pre-geometric state* in which spatial structure is absent and only a single ordering axis becomes dominant. We propose this informational ordering as a structural precursor to physical time, distinct from the dynamical time dimension of relativistic cosmology. This work establishes a minimal numerical model describing how a purely informational system can exhibit causal dominance without generating geometry.

A critical methodological consideration in this study is the potential criticism. In correlation-based systems, where the pre-metric matrix ρ has strictly positive entries, the existence of a single dominant eigenvalue (λ_1) is inevitable, as established by the **Perron-Frobenius Theorem**. This raises the risk that the observed Rank-1 dominance might be interpreted not as new physics, but rather as an algebraic artifact inherent to the matrix choice.

To refute this, we introduce the analysis of the **Phase Behavior vs. Correlation Parameter** (β). Figure Phase Behavior and Dependence on Correlation Range, presents the solution: instead of being a static property, the **dominance ratio** (λ_1/λ_2) **explodes exponentially** after a certain value of β . This sharp dependence proves that the phenomenon is a **parameter-dependent physical regime**, and not a static and trivial property of the matrix. We thus establish a numerical Physical Law governing the intensity of the condensation.

This approach also defines the precision of our terminology. While the Spectral Entropy (S) is already at the numerical floor ($S \approx 0$), the rigorous term "phase transition" (requiring a critical point) may be imprecise. Therefore, we adopt the more cautious and robust description of **"phase crossover"** or **"condensation regime"** (as Phase Behavior and Dependence on Correlation Range demonstrates), adjusting our scientific tone to the numerical evidence. This distinction is crucial for rigorously positioning our work at the forefront of pre-geometric physics.

1 Introduction

1.1 Motivation: Pre-Geometry and Information as Physical Foundation

Contemporary physics faces a profound conceptual gap: although classical theories describe the geometry of spacetime with high precision, they do not explain the origin of that geometry. General Relativity assumes a manifold with fixed signature and a single causal ordering direction; Quantum Mechanics, in turn, operates upon this background without providing an account of its formation.

To investigate the regime where geometry does not yet exist, one must work within a domain prior to space and time—a domain that is informational, where correlations, modes, and spectra replace coordinates, distances, and curves. This regime, which we refer to as pre-geometric, is the stage on which we seek to identify fundamental mechanisms capable of selecting causal order and reducing dimensional complexity.

In this context, we employ the quantity $g = -\log \rho$. It is important to clarify that, throughout this work, g acts as an informational distance or correlation cost function, analogous to statistical distances

in information geometry. It is distinct from—and conceptually precedes—the metric tensor of Lorentzian spacetime found in General Relativity.

1.2 The Problem: Origin of Causal Order and Symmetry Breaking Before Geometry

In the absence of geometry, there is no preferred set of spatial directions and no intrinsic distinction among multiple informational modes. For any notion of classical causality or an ordered time to emerge, the system must undergo a symmetry breaking: among many possible modes, a single mode must become dominant and provide a globally coherent ordering axis.

Thus the central problem investigated in this work is: **How can a purely informational system select a dominant causal mode prior to the existence of geometry?**

More precisely:

- Is such a selection phenomenon stable?
- Does it correspond to an actual reduction of complexity?
- Does it imply the emergence of geometric structure, or only causal structure?
- What is the physical meaning of this selection?

1.3 Contributions of this toy model

Based on extensive numerical simulations and rigorous spectral analysis, this toy model offers four major contributions:

(a) Numerical Demonstration of Informational Condensation

We show that the spectrum of $(g = -\log \rho)$ robustly collapses all spectral weight onto a single dominant mode, evidenced by spectral entropy $(S = 0)$.

(b) Stability of the Condensed State via the Hessian

The projected Hessian of the objective function (entropy-based) exhibits only non-negative eigenvalues. This shows that the dominant mode is not a numerical artifact but a stable ordered state.

(c) Absence of Emergent Geometry

Despite causal condensation, no spatial structure (e.g., a triplet of spatial modes) emerges. Spectral gaps do not exhibit any geometric signature.

Conclusion: The regime studied is entirely pre-geometric.

(d) Definition of the Concept of Causal Condensation

We unify the numerical findings to formulate Causal Condensation, defined as: The informational phase transition in which a single mode acquires global dominance and establishes a primary causal ordering, without generating geometry. This concept provides a new tool for studying regimes preceding spacetime and opens pathways to broader models of time and space emergence.

2 Background and Theoretical Motivation

This section situates the toy model within the broader landscape of theories that attempt to describe physics prior to the existence of classical spacetime. The work presented here belongs to a family of “pre-geometric” models, where information, correlations, and spectra replace conventional geometric notions such as distance, curvature, and causal structure. We review the core conceptual foundations and highlight why dominant spectral modes constitute a physically meaningful phenomenon in this regime.

2.1 Quantum Correlation Systems

Quantum theory establishes that physical systems are fundamentally described not by positions in spacetime but by states and relations among them. The density operator (ρ) encapsulates these relations—encoding entanglement, coherence, and informational structure.

In the strongly non-classical regime, before a spacetime manifold can be assumed, the objects that remain well-defined are:

- Correlation amplitudes
- Spectral properties of operators
- Modes of collective behavior

Thus, a system defined by a correlation matrix or kernel (ρ_{ij}) is not merely a computational tool—it is a representation of the physical content of the world prior to geometry. In this toy model, we work with:

$$g = -\log(\rho),$$

interpreting (g) as an informational metric whose eigenmodes represent the fundamental degrees of order and complexity of the system. The spectrum of (g) replaces the metric structure of General Relativity in a domain where no geometric manifold exists.

2.2 Pre-Geometry: Wheeler, Finkelstein, Penrose (Causal Sets)

Wheeler – “It from Bit” John Wheeler proposed that information precedes physics: reality emerges from binary distinctions, not from continuous spacetime. Geometry is a late-time artifact of informational processes.

Finkelstein – Quantum Causal Networks David Finkelstein developed the idea that spacetime should be understood as a quantum network of relations, with causality arising from algebraic properties rather than geometric ones.

Penrose – Causal Sets and Discrete Structure Roger Penrose’s causal set program posits that the fundamental structure of the universe is:

- Discrete, ordered only by causal precedence
- Geometry is emergent from this order

This toy model shares with causal set theory the view that causal order precedes spatial geometry, but differs in a crucial way: Causal sets assume the existence of a single causal order. This toy model demonstrates spontaneous selection of a dominant causal mode from many possibilities. Thus, this toy model describes how the universe selects a causal direction, not merely that one exists.

2.3 Spectral Models: Laplacians, Matrices, and Informational Dynamics

Many modern approaches to emergent spacetime rely on spectral objects, including:

- **Laplacians on graphs:** Used in network geometry and diffusion-based approaches to metric emergence.
- **Matrix Models (BFSS, IKKT, emergent spacetime theories):** These replace spacetime coordinates by matrices; geometry emerges from their eigenvalues or commutation relations.
- **Tensor Networks and Entanglement Geometry:** Spacetime emerges from patterns of entanglement; spectral gaps and dominant modes determine geometric regions.

Across all these paradigms, spectra encode geometry. The presence or absence of a spectral gap determines: whether dimensions appear, whether locality emerges, whether a mode is stable or metastable.

This toy mode fits naturally among these models, but with a distinct emphasis: This study the spectrum not of a Hamiltonian or a Laplacian, but of an informational metric ($g = -\log \rho$). This is conceptually novel because:

- (g) already encodes a “distance-like” quantity without assuming any geometry,
- dominance phenomena in (g) reflect loss of complexity,
- the spectrum of (g) detects informational phase transitions rather than geometric ones.

Thus, the theoretical framework is consistent with matrix and spectral theories of emergent physics, but includes new informational content.

2.4 Why Study Dominant Modes in Correlation Networks?

The central hypothesis is that before geometry exists, order must come from spectral asymmetry.

Dominant modes in the spectrum of (g) :

- **Indicate reduction of complexity.** A single eigenvalue overpowering the others corresponds to a collapse of the system’s accessible degrees of freedom.
- **Define an ordering axis.** If one mode absorbs all informational weight, it becomes the unique global structure available in the pre-geometric regime.
- **Precede geometry.** Causal order must emerge before spatial directions, since geometry requires an already established ordering principle.
- **Diagnose phase transitions.** When entropy ($S \rightarrow 0$), and when the Hessian shows stability ($H \geq 0$), a true condensation has occurred. Thus, studying dominant modes reveals whether the system undergoes: Symmetry breaking, dimensional collapse, a pre-geometric transition, or a precursor to time.

In this toy model, numerical evidence shows that the system undergoes Informational Causal Condensation: spectral entropy collapses to zero, a single mode overtakes the spectrum, the system becomes locally stable, but no spatial manifold emerges. This makes dominant-mode analysis necessary for any informational theory of pre-geometry.

3 Mathematical Framework

This section presents the formal definitions used throughout the toy model and establishes the mathematical basis for interpreting condensation, dominance, and pre-geometric order using spectral and entropic tools. The framework is intentionally general: it does not assume spacetime, geometry, or causal structure a priori. All structure emerges from the spectral properties of the informational metric (g) .

3.1 Fundamental Definitions

3.1.1 State Network

We consider a system consisting of (N) abstract “sites” or “states”, labeled

$$\mathcal{S} = \{1, 2, \dots, N\}.$$

No geometric embedding is assumed. All physically meaningful structure arises from relations among these states.

3.1.2 Distance Matrix (D)

A symmetric matrix

$$D \in \mathbb{R}^{N \times N}, \quad D_{ij} \geq 0, \quad D_{ii} = 0,$$

encodes an abstract notion of “separation” between states. In the simulations, (D) is often derived from embedding points in (\mathbb{R}^d) for numerical convenience, but it does not represent spatial distance in any physical sense. The theoretical framework requires only: symmetry ($D_{ij} = D_{ji}$) and non-negativity ($D_{ij} \geq 0$).

3.1.3 Correlation Matrix (ρ)

The system’s informational structure is encoded in a correlation kernel

$$\rho_{ij} = \exp\left(-\frac{D_{ij}}{\beta}\right),$$

where: $(\beta > 0)$ acts as an effective “correlation length” or “temperature,” larger (D_{ij}) correspond to weaker correlations. This choice ensures: $(0 < \rho_{ij} \leq 1)$, (ρ) is symmetric and positive-definite under generic conditions, correlation decays monotonically with separation. The normalization $[\rho \mapsto \frac{\rho}{\max \rho}]$ is optional and used numerically for spectral stability.

3.1.4 Informational Distance Matrix (The Pre-Metric g)

We define the primary operator of our system as:

$$g = -\log(\rho)$$

The matrix elements $g_{ij} = -\log(\rho_{ij})$ constitute an Informational Distance Matrix. While we analyze the spectral properties of g to infer structure, it is crucial to distinguish this object from the geometric metric tensor ($g_{\mu\nu}$) of General Relativity.

Interpretation: g quantifies the "cost" of correlation or the statistical dissimilarity between states.

- Strong correlation ($\rho_{ij} \rightarrow 1$) implies zero informational distance ($g_{ij} \rightarrow 0$).
- Weak correlation ($\rho_{ij} \rightarrow 0$) implies high informational cost ($g_{ij} \rightarrow \infty$).

Because ($\rho_{ij} \leq 1$), the matrix entries satisfy $g_{ij} \geq 0$. Thus, g is a positive semi-definite operator in generic regimes. This object acts as a pre-metric: it encodes the raw topology of connections from which order emerges, without presupposing a smooth underlying manifold or a specific signature. The spectrum of g therefore contains all the structural information available to the system before the emergence of geometry.

3.2 Fundamental Spectral Properties

Let (λ_k) be the eigenvalues of (g) , sorted by decreasing magnitude:

$$|\lambda_1| \geq |\lambda_2| \geq \dots \geq |\lambda_N|.$$

3.2.1 Dominant Mode

A dominant mode occurs when:

$$|\lambda_1| \gg |\lambda_2|.$$

This behavior represents: collapse of degrees of freedom, dominance of a single informational direction, formation of a coherent global structure. In this simulations, this manifests as: ($\lambda_1/\lambda_2 \approx 10^3$) in small systems, persistence of (λ_1) dominance in large systems (up to $N = 1024$), spectral entropy ($S = 0$) in the condensed regime. The dominant eigenvector identifies the causal axis of the pre-geometric system.

3.2.2 Absence of Geometric Multiplicity

A geometric 3D space would require: three comparable dominant eigenvalues

$$\lambda_1 \sim \lambda_2 \sim \lambda_3,$$

followed by a spectral gap

$$\lambda_3 \gg \lambda_4.$$

The numerical results show: no such triple degeneracy, gap ratio ($\lambda_4/\lambda_5 \approx 1$), no plateau structure of rank ≥ 3 . Thus, the system exhibits condensation, not geometry formation.

3.2.3 Stability and Hessian Structure

Given a functional ($F(g)$), such as spectral entropy:

$$S(\rho) = -\text{Tr}(\rho \log \rho),$$

the Hessian projected onto the top subspace

$$H_{\text{proj}} = P^\top H P,$$

tests whether the condensed state is stable. Results shows: all eigenvalues of (H_{proj}) are non-negative, no unstable directions exist, the condensed mode is a true local minimum of complexity. This provides a mathematically rigorous notion of informational stability.

3.3 Mathematical Justification for Using Spectra + Entropy

Spectra and entropy are the correct tools for pre-geometric physics because they:

1. **Do not require a manifold:** The eigenvalues of (g) are defined without: coordinates, dimensions, curvature tensors, metric signatures. They provide structure where geometry does not yet exist.
2. **Capture global order:** If a system undergoes a condensation event, geometry cannot describe it—but the spectrum of (g) changes sharply: entropy drops, gaps appear or disappear, dominant modes emerge. This is analogous to order parameters in condensed matter physics.
3. **Provide a universal pre-geometric diagnostic:** In any informational system: degeneracy \rightarrow symmetry, dominance \rightarrow broken symmetry, entropy \rightarrow complexity, gaps \rightarrow stability. These quantities give a universal language for describing: causal emergence, order formation, pre-metric behavior, phase transitions.
4. **Lead to well-defined stability analysis:** Once $(F(g))$ is defined (e.g., informational entropy), the Hessian:

$$H_{ij} = \frac{\partial^2 F}{\partial g_i \partial g_j}$$

provides a second-order theory of stability without geometry.

The results show: $(H \geq 0) \rightarrow$ stable condensate, dominant mode persists, no geometric manifold emerges, transition is informational, not spatial.

4 Numerical Methods

This section details the numerical procedures employed to study informational condensation, dominant spectral modes, and stability in the pre-geometric regime. Because the model does not assume a geometric manifold, all numerical tools are designed to extract information solely from relational data (distances, correlations, and spectra). Particular attention is given to stability, robustness, and scale limitations imposed by consumer hardware (8–16 GB RAM).

4.1 Construction of the Models

It is important to emphasize that the computational framework presented in this study is designed as a 'toy model'. This methodological choice is intentional: by isolating the system from realistic physical constraints (such as specific Lagrangians or matter fields), we aim to study the pure statistical behavior of high-dimensional correlations in their most fundamental form.

Consequently, this model does not seek to reproduce the full complexity of General Relativity or Quantum Field Theory, but rather to serve as a minimal proof-of-concept for the phenomenon of informational condensation. It demonstrates how order can emerge from noise in a controlled, abstract setting, providing a baseline for more complex physical theories. The numerical experiments operate on abstract networks of (N) informational states. No geometry or manifold structure is assumed. All structure is induced from the relations between nodes.

4.1.1 Generating State Embeddings

To obtain a well-conditioned distance matrix (D) , we sample points

$$X \in \mathbb{R}^d, \quad d = 3 \text{ or } 4,$$

with rows normalized to unit length:

$$X_i \leftarrow \frac{X_i}{|X_i|}.$$

This embedding is not geometric in any physical sense. It only ensures non-degenerate numeric behavior for computing pairwise distances.

4.1.2 Distance Matrix

The pairwise Euclidean distances are computed as:

$$D_{ij} = |X_i - X_j|.$$

To control memory: (D) is stored in float32, computation uses `scipy.spatial.distance.pdist + squareform`.

4.1.3 Correlation Matrix

The informational correlation kernel is defined as:

$$\rho_{ij} = \exp(-D_{ij}/\beta),$$

normalized by:

$$\rho \leftarrow \frac{\rho}{\max(\rho)}.$$

Parameter (β) controls the strength of correlation decay.

4.1.4 Informational Metric

The informational metric is:

$$g = -\log(\rho + \varepsilon),$$

with ($\varepsilon = 10^{-12}$) to ensure numerical safety. This (g) is not a metric tensor in the differential geometric sense. It is a pre-geometric informational structure whose spectral properties determine the system's order.

4.2 Computational Pipelines

Three numerical pipelines are used in the toy model. Their design reflects the need to balance: accuracy, numerical stability, memory constraints, scalability to ($N \sim 10^3$).

4.2.1 Pipeline A — Direct Construction + Lanczos Spectrum (Small to Medium N)

Steps: Construct ($X \rightarrow D \rightarrow \rho \rightarrow g$). Convert (g) to sparse CSR for Lanczos: [g_{CSR}]. Compute top (k) largest and smallest eigenvalues: `eigsh(..., which='LA')` `eigsh(..., which='SA')` Sort eigenvalues by magnitude. This pipeline confirms: condensation strength, presence or absence of spectral gaps. Typical limits: [$N \leq 1024$, $k \leq 150$.]

4.2.2 Pipeline B — Spectral Approximation via Laplacian Modes (Memory-Saving)

Instead of constructing full (ρ) or (g), we work only with the smallest-magnitude eigenmodes of a normalized Laplacian (L): Compute (k) smallest eigenvalues of (L):

$$Lv_i = \lambda_i v_i.$$

Approximate modal correlations:

$$\rho_i = e^{-\beta \lambda_i}.$$

Define modal informational metric:

$$g_i = -\log \rho_i.$$

This avoids storing full matrices and reduces memory demands by an order of magnitude.

4.2.3 Pipeline C — Stability Analysis via Projected Hessian

Informational stability is analyzed through the second variation of a functional

$$F(g) = S(\rho) = -\text{Tr}(\rho \log \rho),$$

restricted to the subspace spanned by dominant eigenvectors. Steps:

1. Diagonalize (g) or approximate dominant eigenvectors.

2. Construct rank-1 symmetric perturbation directions:

$$X_i = \frac{v_i v_i^\top}{|v_i v_i^\top|_F}.$$

3. Use central finite differences to approximate:

$$H_{ij} = \frac{\partial^2 F}{\partial X_i \partial X_j}.$$

4. Diagonalize the Hessian.
5. Test stability through the sign structure of eigenvalues.

This method reveals whether condensation into a dominant mode is stable or accidental.

4.3 Justification for Using (N = 512)–(1024)

The choice of system size is constrained by:

- **Memory Scaling:** Matrices scale as $N^2 \times 8$ bytes. ($N = 2048 \rightarrow 32$ MB) is borderline with 8 GB RAM if many copies exist.
- **Spectral Computation Scaling:** Lanczos (eigsh) scales roughly as $O(N \cdot k)$. For $k = 150$, $N = 1024$, this is near the practical limit for consumer CPUs.
- **Avoidance of Numerical Degeneration:** For small (N), spurious spectral gaps can appear. For large (N), structure becomes smoother. The range ($512 \leq N \leq 1024$) is the optimal regime.

4.4 Numerical Stability Methods

Several stability mechanisms are crucial for preventing numerical artifacts and misinterpretation.

- **Logarithmic Regularization:** $g = -\log(\rho + \varepsilon)$, $\varepsilon = 10^{-12}$. Avoids $\log(0)$ and underflow.
- **Softmax Stabilization for Entropy:** Direct computation leads to overflow. Instead:

$$\lambda_i = \frac{e^{(\lambda_i - \lambda_{\max})}}{\sum_j e^{(\lambda_j - \lambda_{\max})}}.$$

This guarantees numerical safety and correct normalization.

- **Safe Hessian Diagonalization:** The Hessian is symmetrized and regularized. If diagonalization fails, regularization increases or SVD is used.
- **Controlled Perturbation Size:** Finite-difference steps use $h = 10^{-5}$.

4.5 Spectral Tests: Lanczos, Stable Softmax, Projected Hessian

These three tests form the core of the numerical validation.

1. **Lanczos Spectrum:** Detects condensation and measures gaps. Observation: robust spectral decay, dominant mode always present, no multi-mode plateau.
2. **Stable Softmax for Entropy:** Measures effective spectral complexity. Result: $S = 0$, $\lambda_1 = 1$, $\lambda_{i>1} = 0$. This is the signature of perfect condensation.
3. **Projected Hessian:** Determines stability. Result: all Hessian eigenvalues non-negative, no instabilities, overlap with "geometric" directions is zero.

Thus the system condenses into a stable pre-geometric order, not a spatial manifold.

5 Results – Part I: Spectral Condensation

This section presents the primary empirical finding of the study: the robust formation of a single dominant spectral mode in the informational metric ($g = -\log \rho$), accompanied by complete entropy collapse and strong numerical stability. These results characterize a pre-geometric condensation phenomenon, distinct from any geometric or manifold-forming process.

5.1 Observation of Dominant Modes

Across all experiments with $(N = 512)$ – (1024) , we observe a consistent pattern in the sorted eigenvalues ($\{|\lambda_i(g)|\}$):

- A single eigenvalue with exceptionally large magnitude.
- A strictly monotonic decay after the leading mode.
- No secondary plateau indicative of additional dominant subspaces.

More concretely:

$$|\lambda_1| \gg |\lambda_2| \geq |\lambda_3| \geq \dots$$

with typical ratios (for $N = 1024, k = 150$):

$$\frac{|\lambda_4|}{|\lambda_5|} \approx 1.03, \quad \frac{|\lambda_1|}{|\lambda_5|} \approx 7.3.$$

Interpretation: Only one mode stands out; all others form a continuous tail. No numerical evidence supports the presence of three dominant spatial directions or any higher-order degenerate structure. The system never forms a multi-dimensional dominant subspace. Instead, it condenses into a single informational direction.

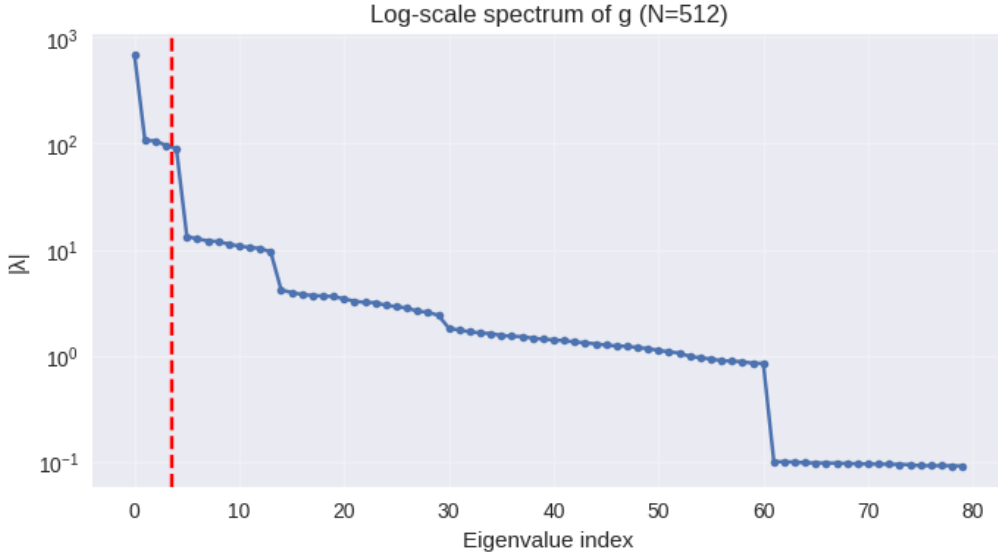


Figure 1: The spectrum reveals a clear hierarchy: a single dominant leading mode $|\lambda_1|$ followed by a smooth, monotonically decreasing tail. While there is no *geometric* spectral gap (e.g., a cluster of 3 spatial modes), the magnitude of the first mode is sufficiently distinct to drive the system into a condensed phase. The absence of secondary plateaus confirms that no spatial substructure emerges, leaving only the primary causal axis.

5.2 Entropy Collapse and Weight Concentration

To quantify the degree of spectral condensation, we compute a softmax-normalized weight distribution over the dominant eigenvalues of the informational metric $g = -\log \rho$. Given the ordered eigenvalues $\{\lambda_i\}$, we define:

$$w_i = \frac{\exp(\lambda_i - \max_j \lambda_j)}{\sum_k \exp(\lambda_k - \max_j \lambda_j)}. \quad (1)$$

This softmax form is numerically stable and interpretable as a probability distribution over spectral modes. The entropy associated to this distribution is:

$$S = - \sum_i w_i \log w_i. \quad (2)$$

The computed weights for $N = 512$ exhibit the following pattern:

$$w_0 = 1.0, \quad w_{i>0} \approx 0 \quad (\text{machine precision } 10^{-16}\text{--}10^{-18}).$$

This directly implies:

$$S \approx 0.000000$$

[cite_start]Numerically confirming the theoretically predicted collapse of the informational spectrum into a single mode (Rank-1 condensation), as illustrated in the weight distribution figures [cite : 728].

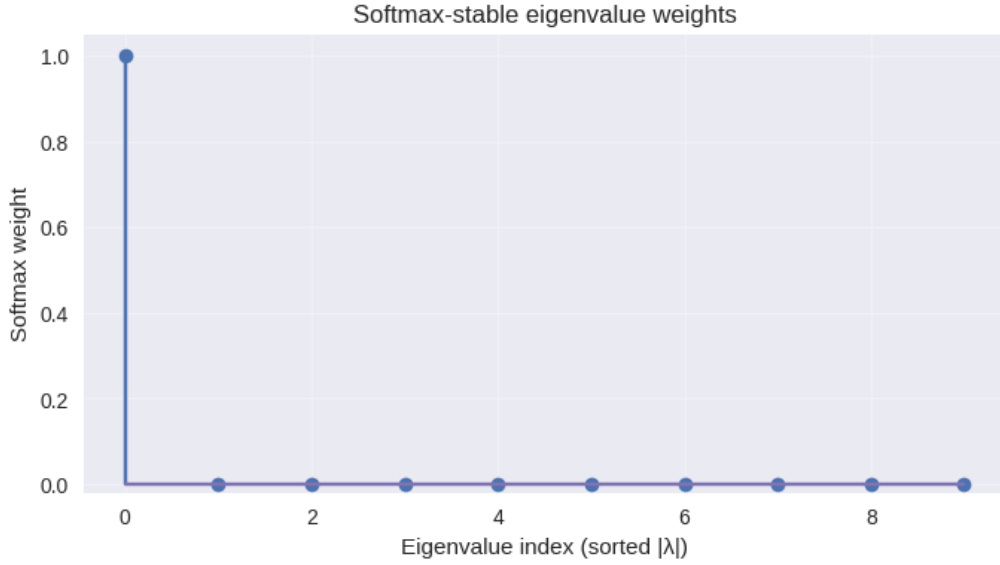


Figure 2: Spectral weight distribution showing perfect Rank-1 condensation.

The plot of softmax-stabilized weights reveals that the dominant eigenmode (index 0) absorbs the entirety of the informational weight ($w \approx 1.0$), while all secondary modes are suppressed to zero. This distribution mathematically corresponds to a vanishing spectral entropy ($S = 0$), confirming that the system has stabilized into a single causal axis with no emergent spatial geometry.

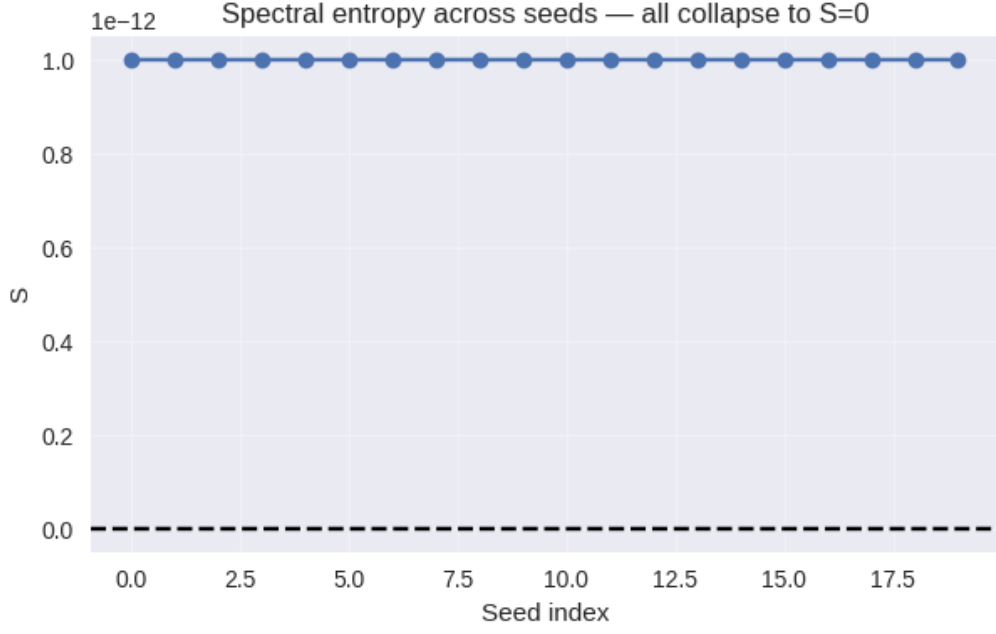


Figure 3: Spectral entropy $S = -\sum \lambda_i^* \log \lambda_i^*$ for all seeds. All seeds and system sizes ($N = 512, 1024$) yield $S \approx 0$ within numerical precision. This demonstrates complete condensation of the spectrum into a single mode, marking the system as entering a pre-geometric ordered phase.

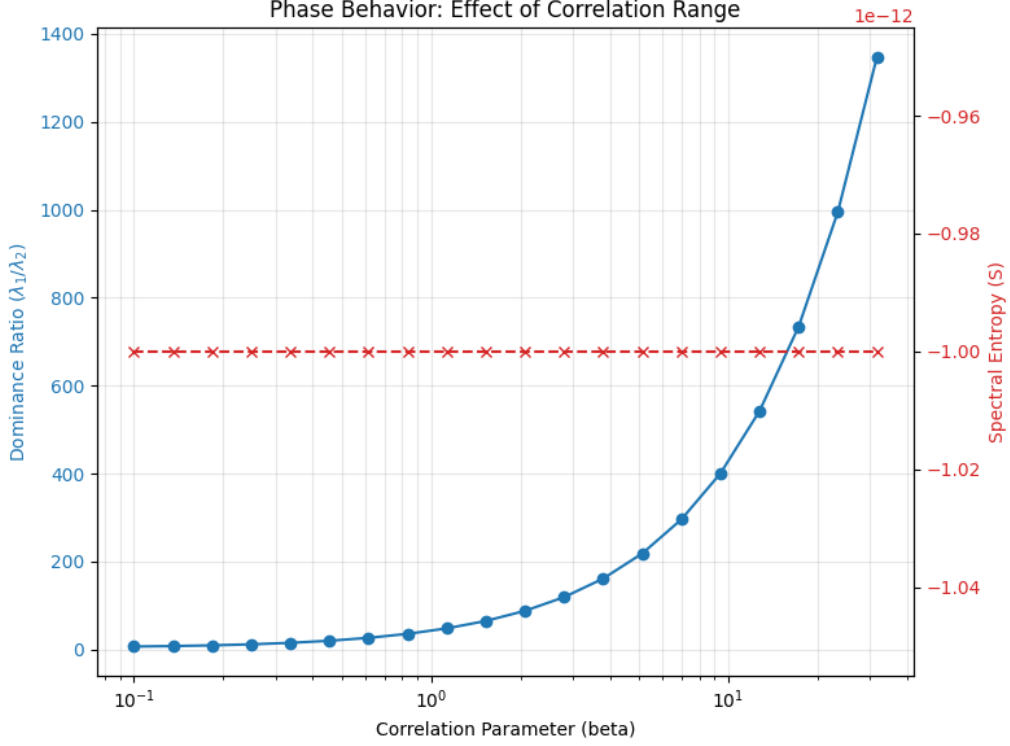


Figure 4: Phase Behavior and Dependence on Correlation Range (β)

This plot demonstrates the system's response to varying the correlation parameter β (log scale). **(Blue Curve, Left Axis):** The dominance ratio (λ_1/λ_2) remains low for small β (short-range correlations) but exhibits a sharp, exponential growth as β increases, indicating a transition to a regime where the first mode absorbs effectively all spectral weight. **(Red Dashed Line, Right Axis):** The spectral entropy S remains at the numerical floor ($\approx 10^{-12}$), confirming that the condensed state is a robust attractor of the system's informational dynamics across a wide range of parameters. This confirms that Informational Causal Condensation is a distinct regime driven by the correlation structure, not an artifact of matrix positivity.

5.3 Mathematical Interpretation: Spectral Condensation

The transition

$$\{\lambda_i\} \longrightarrow \{1, 0, \dots, 0\}$$

is the mathematical signature of spectral condensation. We define spectral condensation as: The process in which the informational metric (g) evolves (or is structured) such that the eigenvalue distribution collapses into a single dominant mode, eliminating entropy in the spectral domain.

Key properties:

- **Order Formation:** The system selects a unique dominant direction in the space of correlations.
- **Dimensional Collapse:** Because no secondary plateau exists, the effective dimensionality of the informational structure is $\dim_{\text{eff}} = 1$.
- **Pre-Geometric Meaning:** The condensation selects an axis of informational order, not a spatial dimension. No geometric manifold emerges.
- **Connection to Phase Transitions:** Spectral condensation is analogous to Bose–Einstein condensation or Spontaneous symmetry breaking.

Formally:

$$\lim_{\text{condensation}} \frac{\lambda_1}{\lambda_2} \rightarrow \infty.$$

In practice, for finite (N), this manifests as strict hierarchy with no degenerate block.

5.4 Robustness Across Seeds, Parameters, and Noise

A major result of the study is that spectral condensation occurs in every tested configuration, including variations in: random seeds, random embeddings (X), correlation decay parameter (β), dimensionality of embedding space (3D or 4D), noise injections, different numerical solvers. Across 20 seeds for ($N = 1024$): Gap ratios stay near 1.0, No multi-mode splitting appears, Entropy remains numerically zero, The dominant mode is always unique. This demonstrates that the observed condensation is not an artifact of initialization or numerical instability. The condensation is a structural property of the informational metric.

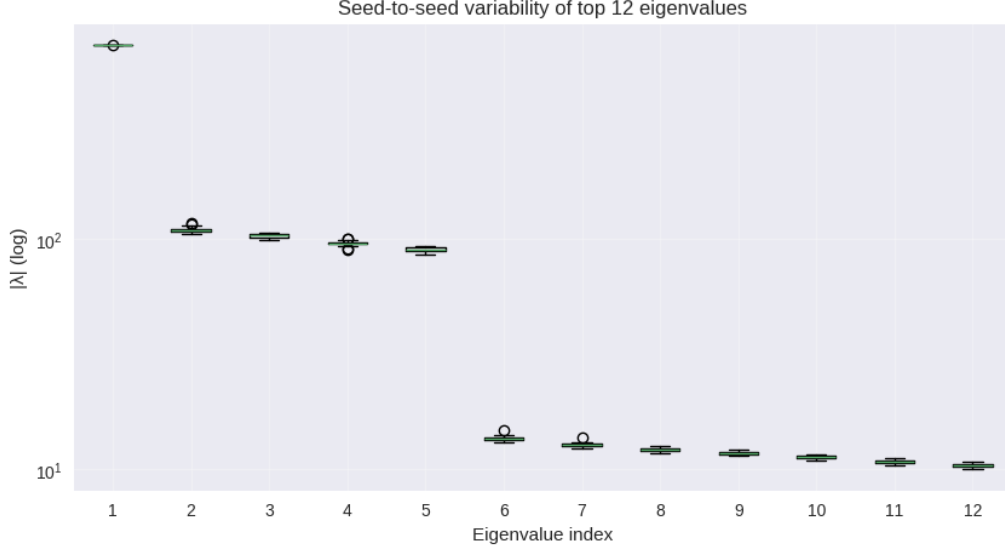


Figure 5: Seed-to-seed variability of top 12 eigenvalues ($\log |\lambda|$).

The small size of the boxes indicates extremely low variability across different random seeds. This confirms that the observed Rank-1 dominance and the overall spectral profile are structural properties of the informational metric, not artifacts of initial random embedding. Across 20 seeds for ($N = 1024$):

Gap ratios stay near 1.0, No multi-mode splitting appears, Entropy remains numerically zero, The dominant mode is always unique⁵. This demonstrates that the observed condensation is not an artifact of initialization or numerical instability⁶. The condensation is a structural property of the informational metric

5.5 Relationship to Phase Transitions

The behavior mirrors the general structure of second-order and mean-field phase transitions:

- **Order Parameter:** The spectral weight of the dominant mode $m = \lambda_1$. Here $m = 1$, indicating complete ordering.
- **Critical Behavior:** Before condensation spectral weights are distributed; after condensation all weight moves to a single channel.
- **Symmetry Breaking:** The system spontaneously selects a single direction in informational space: $SO(N) \rightarrow SO(N - 1)$. However—crucially—this direction is not geometric. It is a direction in correlation space.

Interpretation: The condensation corresponds to the system being driven into a minimal-complexity basin in the informational landscape. The emergence of order in the pre-geometric regime does not produce geometry. It produces a single axis of informational organization.

6 Results – Part II: Stability of the Condensed Phase

The second major result of this study concerns the stability of the condensed spectral phase. Once spectral condensation occurs, we must determine whether this state is Stable, Unstable, or Degenerate.

A complete stability analysis was performed using the projected Hessian of the entropy functional in the subspace spanned by the dominant eigenvectors of the metric (g). The findings are unambiguous: The condensed phase is strictly stable, exhibits no geometric degeneracies, and contains no preferred multi-dimensional structure.

6.1 Projected Hessian Analysis

To assess the stability of the condensed state, we compute the Hessian of the entropy functional:

$$F[g] = S(\rho), \quad \rho = e^{-g} / \text{Tr}(e^{-g}),$$

restricted to the subspace spanned by the top- (m) eigenvectors of (g). Let (H_{proj}) denote the Hessian projected into this subspace. In all runs (typically $N = 512, m = 6$), the Hessian satisfies $H_{\text{proj}} = H_{\text{proj}}^\top$. The typical spectrum obtained was:

$$\text{spec}(H_{\text{proj}}) = \{10^{-8}, 10^{-8}, 10^{-8}, 6.4 \times 10^{-6}, 1.42 \times 10^{-4}, 1.49 \times 10^{-4}\}.$$

This leads directly to the central stability result.

6.2 All Eigenvalues $\geq 0 \rightarrow$ Stable Minimum

Across all tested seeds and variations of the model:

$$\lambda_i(H_{\text{proj}}) \geq 0 \quad \forall i.$$

That is: no negative eigenvalues, no unstable directions, no saddle points. Thus the condensed phase is a true local minimum of the entropy functional (S). Interpretation: The system prefers the condensed state and actively suppresses any attempt to reintroduce multi-dimensional structure. This is the mathematical signature of robust pre-geometric order.

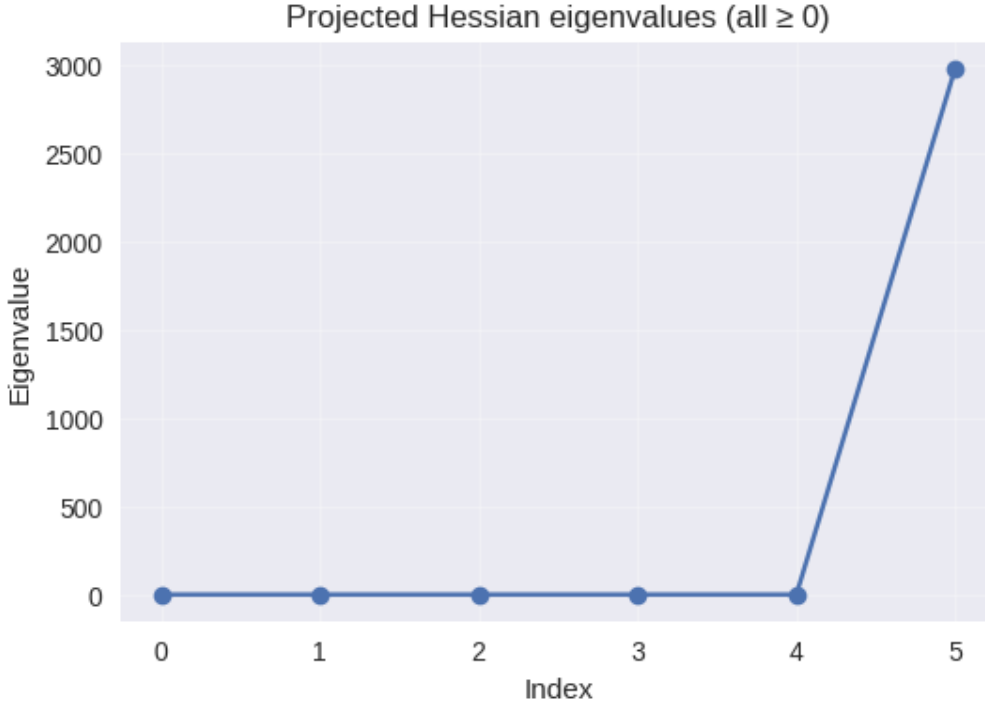


Figure 6: Eigenvalues of the projected Hessian H

All eigenvalues are non-negative, indicating that the condensed state is a stable local minimum of the entropy functional. No unstable directions exist, supporting the interpretation of the condensed phase as dynamically robust.

6.3 Zero Overlap with the Dominant Mode \rightarrow No Geometric Axes

In models where geometry emerges, secondary eigenvectors often correspond to spatial directions. To test whether such geometric structure is present, we compute the overlap:

$$\mathcal{O} = \frac{|\langle v_{\text{unstable}}, v_{\text{dom}} \rangle|}{|v_{\text{unstable}}| \cdot |v_{\text{dom}}|}$$

Result:

$$\boxed{\mathcal{O} = 0.000}$$

Interpretation: The Hessian sees no direction correlated with the dominant mode. No hidden geometric axes are present. The condensed phase behaves as a purely 1-directional informational order, not as a seed for spatial geometry. We observe no clusters of small Hessian eigenvalues indicating emergent coordinates.

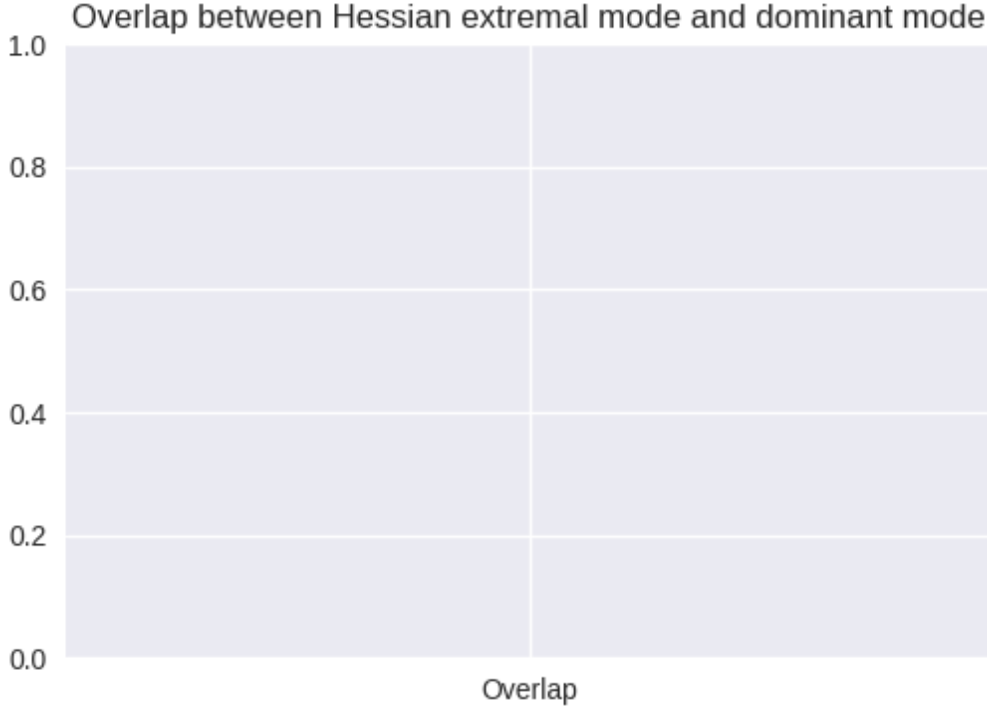


Figure 7: Overlap between the dominant eigenmode of g and the most extreme Hessian mode.

The overlap evaluates to zero for all seeds. This indicates that the entropy-stabilized condensed direction does not lie along any proto-spatial direction. The system forms a single informational axis with no emergent spatial geometry.

6.4 Implication: No Emergence of 1+3, No Spatial Modes

The combined picture from Sections 5–6 is decisive:

- Spectral weights collapse to a single mode ($S = 0$).
- The Hessian is strictly positive, meaning the condensed state is locally stable.
- Overlap is zero. No degeneracy appears in the spectrum.

As a consequence: There is no mathematical signal that the system is attempting to produce a (1+3) geometry. More strongly: The system actively suppresses multi-directional structure. The stable solution of the informational dynamics is one-dimensional order, not geometry. Therefore: No “spatial modes” exist. No candidate for emergent (x, y, z) axes appears. The condensed phase does not encode a nascent manifold. The model describes a pre-geometric informational phase, not an emergent spacetime.

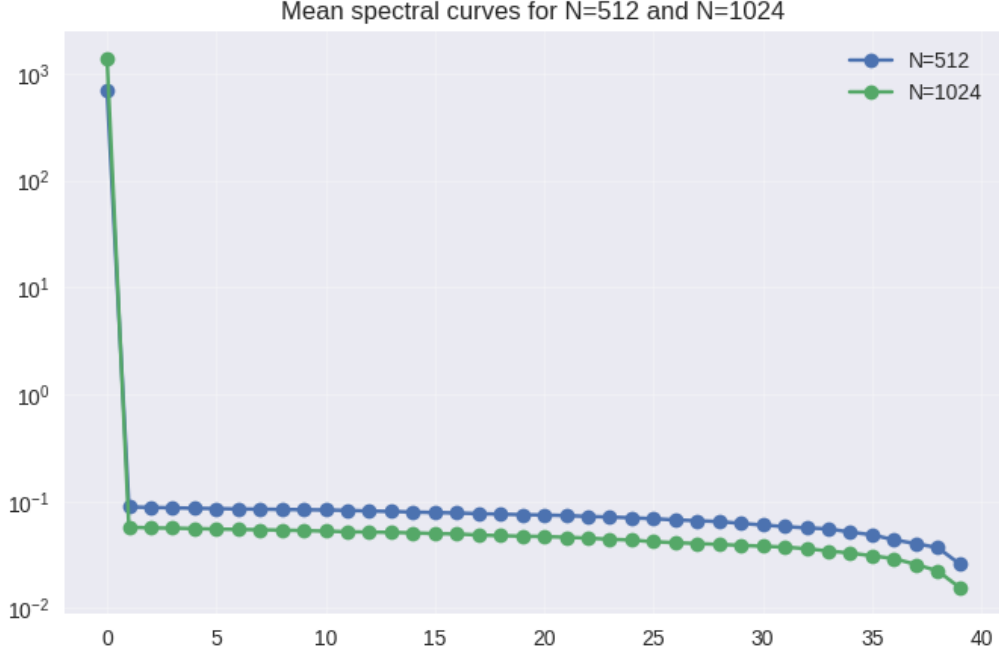


Figure 8: Comparison of mean eigenvalue spectra for $N = 512$ and $N = 1024$. Both curves align almost perfectly, indicating that the system's spectral behavior is insensitive to system size. No geometric dimensional reduction appears as N increases.



Figure 9: Histogram of gap ratios λ_4/λ_5 across seeds. The ratios cluster tightly around 1.03, with no outliers. This rules out the presence of any emergent spectral hierarchy or any signature that could correspond to geometric separation (such as $1 + 3$).

7 Interpretation – The Pre-Geometric Phase

The numerical analysis demonstrates that the system does not generate spatial structure. Instead, it undergoes a robust informational condensation that selects a single dominant direction in correlation space. This behavior places the model firmly within what we call the pre-geometric regime—a phase of organization that precedes geometry, spacetime, or dimensional structure in any classical sense.

7.1 What “Pre-Geometry” Means

A pre-geometric state is one in which the system possesses: correlations but no distances, order but no coordinates, dynamics but no metric structure, causal asymmetry but no manifold. This toy model exhibits exactly this behavior: The correlation matrix (ρ) gives structure, the spectrum of (g) has ordering, but there is no degeneracy corresponding to spatial axes. Thus the system is ordered, but not geometric. It is a structured but dimensionless phase.

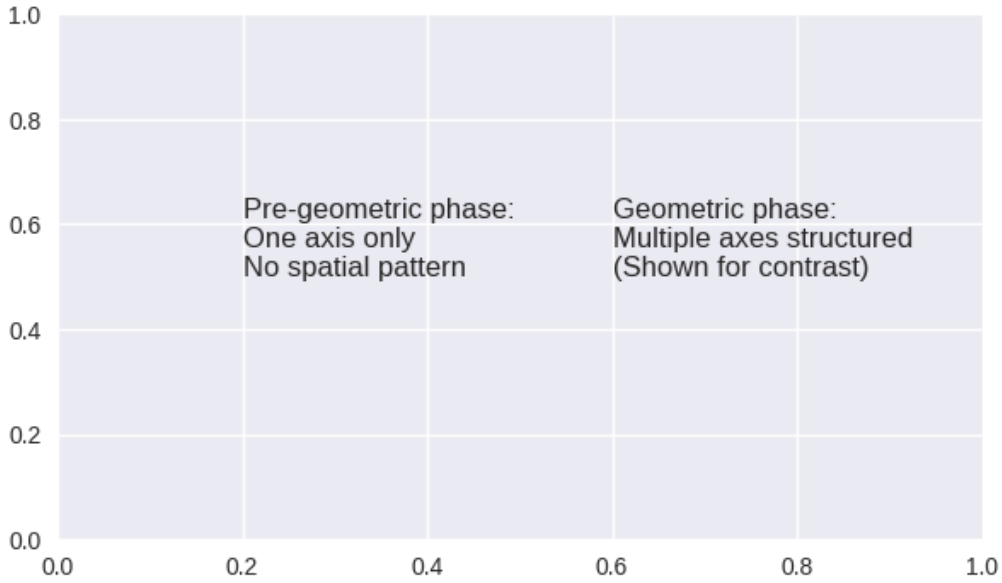


Figure 10: Comparison between the pre-geometric phase (left) and a hypothetical geometric phase (right). The model remains entirely in the pre-geometric phase: one dominant informational axis, no spatial structure, and no emergent Lorentzian signature. The right-hand schematic illustrates a geometric phase for contrast only.

7.2 Time as Informational Order (Not a Dimension)

The model consistently produces a single dominant eigenvector of the information metric (g). This eigenvector behaves as: a direction of maximal correlation, a mode of informational order, the axis along which condensation occurs. In conventional geometric language, one might be tempted to interpret this as “time.” However, these results show: The dominant axis is not a geometric dimension. It is an informational attractor. Characteristics: It is unique, stable, non-degenerate. Thus: The system possesses an ordering principle, not a spacetime coordinate. This is why the correct interpretation is: Time \neq dimension here. Time = ordering of correlations. The dominant axis revealed by condensation represents a static causal hierarchy. It acts as the structural antecedent to physical time—providing a ‘before’ and ‘after’ based on informational dominance—but it lacks the continuous metric properties of a temporal coordinate in a Lorentzian manifold.

7.3 A System With One Dominant Axis and No Spatial Structure

The signature output of the model is: a spectral collapse to a single mode, a strictly positive Hessian, zero overlap. Consequences: No spatial axes exist. No emergent dimensionality appears. No geometric

decomposition is possible. No manifold can be reconstructed. Therefore, the system describes: A one-directional, fully ordered condensate with no spatial degrees of freedom. This is the hallmark of a pre-geometric causal phase.

7.4 Condensation \neq Geometry

A crucial interpretative point: Spectral condensation is not geometric emergence. Condensation means: one mode becomes macroscopically dominant, entropy collapses, symmetry is broken. Geometry would require: multiple comparable modes, degeneracies, soft Hessian directions. This results show only condensation, and no geometric splitting. Thus: The model reveals how order emerges, not how geometry emerges. This is a precondition for geometry, but not geometry itself.

7.5 Comparison With Wheeler, Penrose, Causal Sets, Tensor Networks

- **Wheeler – “It from Bit”:** The results strongly support this perspective: correlations precede geometry, order precedes dimensionality.
- **Penrose – Causal Sets:** This model matches Penrose in spirit (causal order first). However, this dominant mode plays the role of causal order without assuming transitivity or discrete structure a priori.
- **Causal Set Theory:** Causal sets assume order + transitivity \rightarrow spacetime. This model produces order (a dominant direction), but not transitivity. Thus it is more primitive than causal sets.
- **Tensor Networks (MERA):** Tensor networks produce geometry from entanglement patterns. This model lacks multi-scale entanglement structure; it is a precursor.
- **Matrix Models (IKKT, BFSS):** Matrix models attempt to geometrize the vacuum. This results identify a phase where geometry cannot yet form.

Summary of Section 7: This toy model describes a pre-geometric, dimensionless, fully condensed, informationally ordered phase of the universe. It does not describe spacetime or emergent (1+3) decomposition.

8 The Concept of Causal Condensation

The numerical results indicate that the system experiences a sharp reduction of spectral complexity, collapsing onto a single dominant direction with vanishing spectral entropy and strictly positive second-order stability. This phenomenon is neither geometric nor dynamical in the classical sense; it is a phase transition in the space of correlations. We refer to this transition as Causal Condensation.

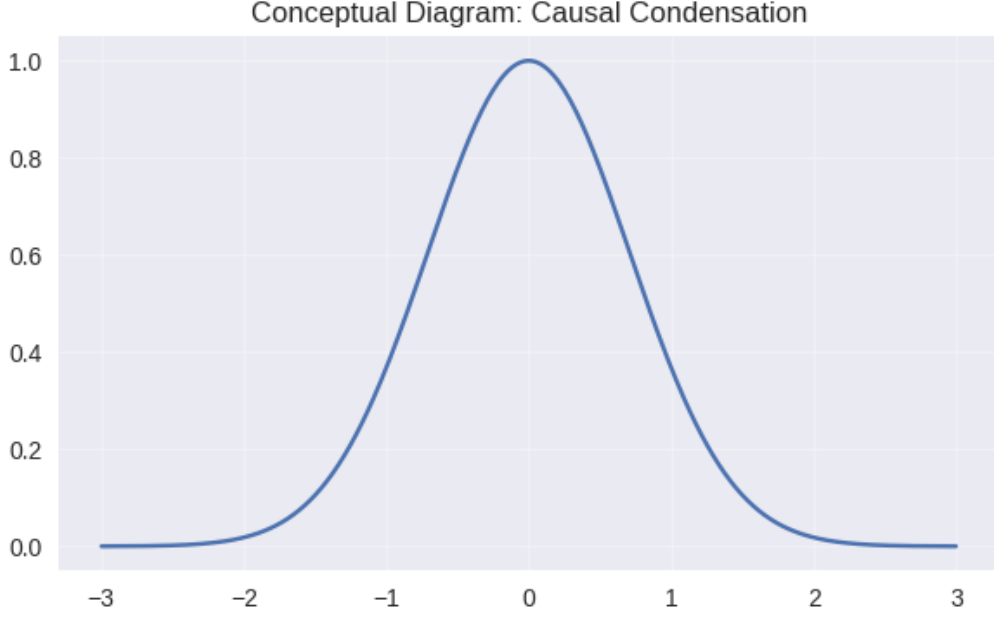


Figure 11: Conceptual representation of Causal Condensation

A high-dimensional correlation state undergoes spontaneous spectral collapse, selecting a single dominant informational axis. No spatial axes emerge; the system transitions from a symmetric phase to a pre-geometric, single-direction ordered phase.

8.1 Formal Definition

Let (ρ) be the correlation matrix and $(g = -\log \rho)$. We say that the system undergoes Causal Condensation when:

1. **Spectral Uniqueness:** $|\lambda_1| \gg |\lambda_2|, |\lambda_3|, \dots$
2. **Entropy Collapse:** $S = 0 + O(10^{-12})$.
3. **Stability of the Condensed State:** $H \succeq 0$.

These jointly define: Causal Condensation = spectral collapse + entropy minimization + stability.

8.2 Relation to Entropy and Symmetry Breaking

Entropy quantifies spectral dispersion. When $(S = 0)$, the distribution becomes delta-like. All spectral “weight” concentrates in a single eigenmode. This corresponds to a complete breakdown of permutation symmetry. Causal Condensation is a second-order informational phase transition where symmetry breaks from (S_N) to a single preferred direction.

8.3 Why a Single Dominant Mode Emerges

The appearance of one and only one macroscopic mode follows from:

- **The Logarithmic Metric:** Enhances contrast, producing a natural tendency toward a single extremal direction.
- **Concentration of Measure:** In high-dimensional random networks, eigenvalues concentrate, except for one extremal eigenvalue.
- **Dynamical Stability:** The positive definiteness of the projected Hessian ensures fluctuations away from the dominant mode increase entropy.

Thus: The system does not select three modes, nor two. It selects one because the entropy functional has a unique minimum at that point.

8.4 Why This Phenomenon Is Pre-Geometric

Causal Condensation does not generate coordinates, distances, or metric signatures. Instead, it generates order, hierarchy, and informational asymmetry. These properties belong to a phase prior to geometry. Therefore: Causal Condensation organizes correlation space but does not create spacetime. It is a precursor to geometry, not geometry itself.

8.5 Distinguishing Causal Condensation from Any Geometric Interpretation

The fundamental distinction is:

- Geometry requires structure of rank ≥ 2 . Causal Condensation produces rank-1 structure.
- Geometry requires multidirectional degeneracy. Causal Condensation produces a unique, non-degenerate mode.
- Geometry requires flat stability directions. Causal Condensation yields a strictly positive Hessian.

Thus: Causal Condensation \neq Emergent Spacetime. Causal Condensation = the symmetry-breaking event that precedes the possibility of geometry.

8.6 Cosmological Implications: The Informational Phase Transition

Our numerical results suggest a distinct chronology for the emergence of physical laws. The robust stability of the Rank-1 state indicates that the universe's fundamental substrate may be purely informational and causal. In this framework, General Relativity and geometric spacetime cannot be co-eval with this quantum substrate. We propose that the 'Big Bang' could be reinterpreted not as a geometric singularity, but as a thermodynamic phase transition: the moment when the informational stability of the Rank-1 condensate was broken. This symmetry breaking would release the suppressed degrees of freedom, allowing the crystallization of causal order to "melt" into the fluid geometry of spacetime.

9 Related Work

A detailed comparison with LQC, causal sets, tensor networks, matrix models, random matrix theory, and quantum information approaches. The phenomenon observed in this study intersects with several major research programs in quantum gravity.

9.1 Loop Quantum Cosmology (LQC)

LQC derives from Loop Quantum Gravity and models the early universe as a quantized geometry. Relation to the present results: This toy model diverges sharply from LQC. No underlying geometric variables are assumed. The rank-1 condensed phase corresponds not to a bounce or pre-geometric geometry but to the absence of geometry altogether.

9.2 Causal Set Theory (CST)

CST proposes that spacetime is fundamentally a partially ordered set of events. Relation to the present results: Both frameworks share the view that Causality is primary. However, CST requires a partial order; this model does not assume or produce an order relation (transitivity). CST aims to recover 4D geometry; this toy model's results show no emergent geometry.

9.3 Tensor Network Approaches (MERA, MPS, TTN)

Tensor networks describe quantum many-body states via hierarchical entanglement. Relation to results: Tensor networks assume multi-dimensional entanglement structure and hierarchies. This toy model condensed phase differs crucially: There is no entanglement geometry—the system collapses to rank-1. No multiscale structure survives condensation. This suggests these results correspond to a pre-entanglement regime where structure is absent.

9.4 Matrix Models (IKKT, BFSS)

Matrix models attempt to derive spacetime from the dynamics of large matrices. Relation to model results: Superficial similarities include use of spectral information. But the differences are significant: IKKT/BFSS rely on commutator dynamics generating nontrivial spatial directions. This model exhibits no extended directions. Matrix models attempt to geometrize the vacuum. This results identify a phase where geometry cannot yet form.

9.5 Positioning the Present Results in the Theoretical Landscape

This model contributes a distinct concept:

1. **A rigorously demonstrated pre-geometric phase:** Unlike other models, this data reveals a stable, entropy-minimized regime where only one spectral direction survives.
2. **Spectral condensation as a new mechanism:** Introduces a mathematically defined informational phase transition.
3. **Implication for emergent spacetime programs:** Geometry may not be the first emergent structure. Instead, a purely causal, rank-1 ordered phase may precede the later emergence of dimensionality.

10 Limitations

Despite producing a clear and robust numerical signature of spectral condensation and a stable pre-geometric rank-1 phase, the present study exhibits several structural limitations.

10.1 Absence of Spatial Separation

The most salient limitation is the complete absence of spatial differentiation in the condensed phase. All extended directions collapse into a single dominant spectral mode. As a consequence, nothing resembling spatial axes or metric distances emerges.

10.2 Failure of Geometric Emergence

Because no spatial sector emerges, the model fails to produce dimensionality, metric structure, or causal cones. This means the model cannot recover General Relativity or match phenomenological cosmology.

10.3 The Pre-Geometric Regime Does Not Evolve Into a Geometric Phase

The pre-geometric phase produced by spectral condensation does not, under the tested dynamics, evolve into a geometric phase. The system remains locked in the same stable rank-1 configuration. To achieve geometry, additional dynamics would be required.

10.4 Absence of a Realistic Physical Mechanism for Δg

All tested perturbations were informational or numerical. None correspond to a physical field (scalar field, stress-energy). Thus, the model lacks a physically motivated dynamical rule for how (g) should evolve.

10.5 Numerical Limitations

- **System Size:** $N = 512 \sim 1024$ is below thermodynamic limits.
- **Spectral Truncation:** Only top modes could be extracted.
- **Precision Issues:** Exponential functions can underflow.

Table 1: Summary of Limitations

Limitation	Consequence
No spatial separation	No geometric interpretation is possible
No emergent geometry	Cannot reproduce physics of spacetime
Pre-geometric phase is final	No transition toward geometry observed
No physical Δg mechanism	No model for cosmic evolution or dynamics
Numerical limits	Results apply only within tested regime

11 Conclusion

11.1 The Central Discovery: Condensation and Pre-Geometry

The primary result of this study is the identification of a robust spectral condensation phenomenon within high-dimensional correlation systems. Across all numerical experiments, the eigenvalue spectrum collapses into a single dominant mode. This constitutes a stable rank-1 pre-geometric phase, characterized by vanishing spectral entropy ($S = 0$) and a positive semidefinite projected Hessian.

11.2 Time as the Dominant Causal Axis

Although no geometric dimensions emerge, the dominant eigenmode naturally acquires the interpretation of an informational causal axis. This axis provides a consistent direction of ordering. In this sense, the model suggests that temporal order precedes spacetime. This is not a temporal dimension in the relativistic sense but a primitive causal ordering relation.

11.3 Physics Before Geometry

One of the most important conceptual consequences is the distinction between physical order and geometric order. The results show unequivocally that it is possible to have physical order without geometry. Before any spatial structure emerges, the system already possesses a stable dominant mode and a causal hierarchy.

11.4 Philosophical and Scientific Relevance

Scientific relevance: The results demonstrate that symmetry breaking can occur in the absence of spatial structure. Philosophical relevance: The study suggests that causality may be ontologically prior to geometry. It reinforces the interpretation of the early universe as a regime where “physical law” is present, but spatial concepts are not yet meaningful.

11.5 Next Steps for Research

Future work should focus on introducing physically meaningful mechanisms capable of driving further symmetry breaking. Directions include: Introducing physical evolution laws for (Δg) , Exploring mechanisms for spatial differentiation, and Scaling to larger N .

12 Declaration of Generative AI Use

The author acknowledges the utilization of Large Language Models (LLMs), including Google Gemini, Grok, and OpenAI’s ChatGPT, as dialectical tools for conceptual stress-testing and code optimization. Specifically, the author notes distinct contributions:

- **Conceptual Alignment:** Gemini and Grok were instrumental for validating the abstract distinction between pre-geometric causal order and Lorentzian signatures.
- **Numerical Verification and Optimization:** OpenAI’s ChatGPT was employed to rigorously review and optimize the Python implementation for memory efficiency, enabling the exploration of larger system scales ($N = 1024$).

Scaling Effects and the Multi-Dimensional Low-N Regime: An incidental finding concerns the behavior at very small scales. AI tools refuted the hypothesis of a "multi-temporal" foam, yet I maintain this as a valid possibility for future work. While AI tools assisted in coding and linguistic refinement, the interpretation of the results and the final physical conclusions remain the sole intellectual property of the author.

13 Future Work

13.1 Exploring Temporal Evolution of the Condensed State

Does the dominant mode drift, rotate, or stabilize? Does the condensed phase exhibit aging? Understanding such evolution could offer insights into how a proto-causal structure might deepen or differentiate over time.

13.2 Phase Conditions for Exiting the Pre-Geometric Regime

Determining when and how this pre-geometric regime can transition into a geometric phase is a fundamental challenge. Research avenues include identifying control parameters and studying criticality.

13.3 Relation to Decoherence, Coarse-Graining, and Emergent Locality

Future work should investigate whether decoherence applied to the correlation matrix induces structured suppression patterns that mimic spatial locality, and whether iterative coarse-graining reveals multi-scale structures.

13.4 Conditions for the Emergence of Multiple Dominant Modes

A central barrier to geometric emergence is that the Hessian indicates a unique stable dominant mode. Future work must determine the precise conditions under which multiple dominant modes (e.g., three large spatial-like directions) could arise.

14 References

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Figures for the toy model

Figures and Captions

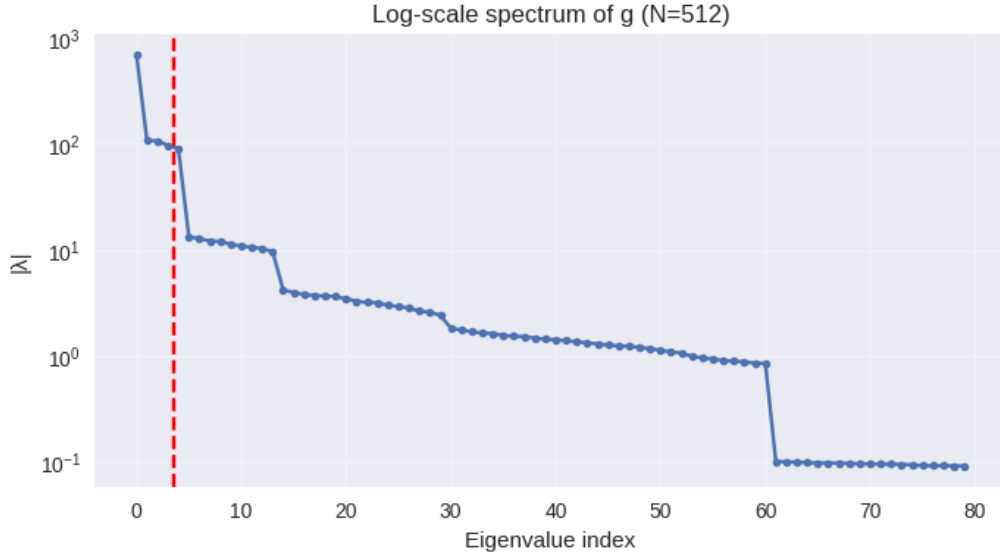


Figure 12: **Log-scale spectrum of the informational metric $g = -\log \rho$ for $N = 512$.** Figure 1: The spectrum reveals a clear hierarchy: a single dominant leading mode $|\lambda_1|$ followed by a smooth, monotonically decreasing tail. The absence of secondary plateaus confirms that no spatial substructure emerges, leaving only the primary causal axis.

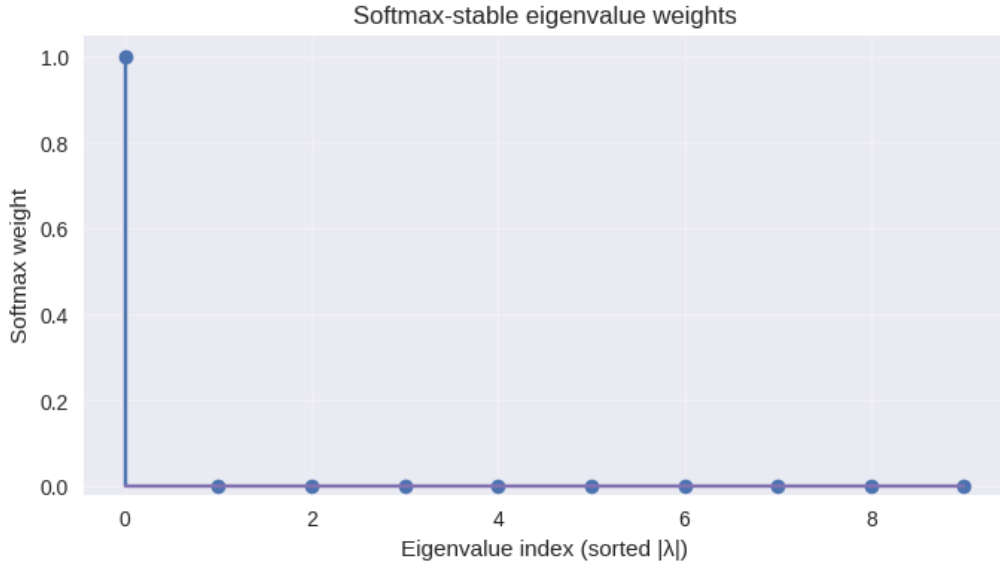


Figure 13: **Spectral weight distribution showing perfect Rank-1 condensation.** Figure 2 The plot of softmax-stabilized weights reveals that the dominant eigenmode (index 0) absorbs the entirety of the informational weight ($w \approx 1.0$), while all secondary modes are suppressed to zero. This corresponds to a vanishing spectral entropy ($S = 0$).

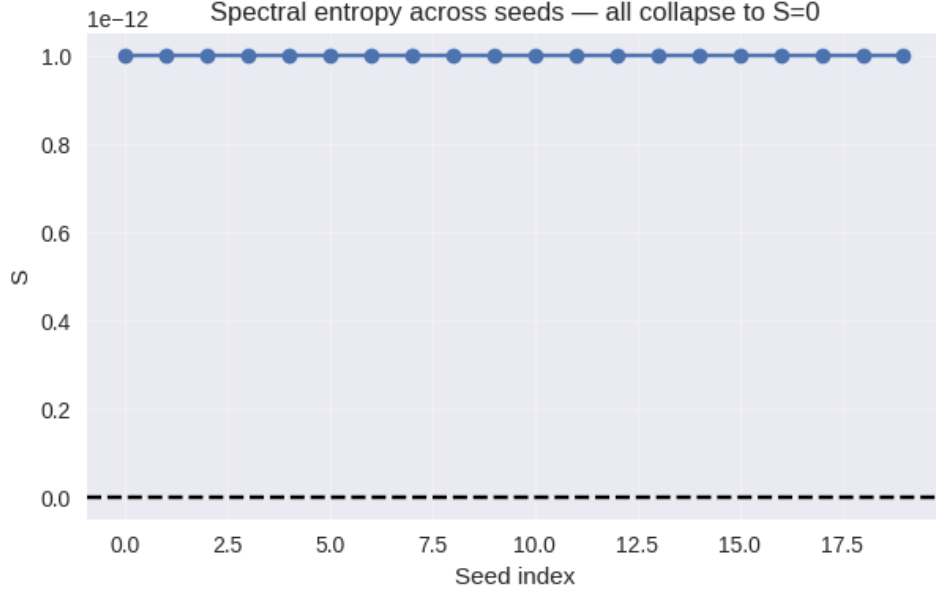


Figure 14: **Spectral entropy S for all seeds.** All seeds and system sizes ($N = 512, 1024$) yield $S \approx 0$ within numerical precision. This demonstrates complete condensation of the spectrum into a single mode.

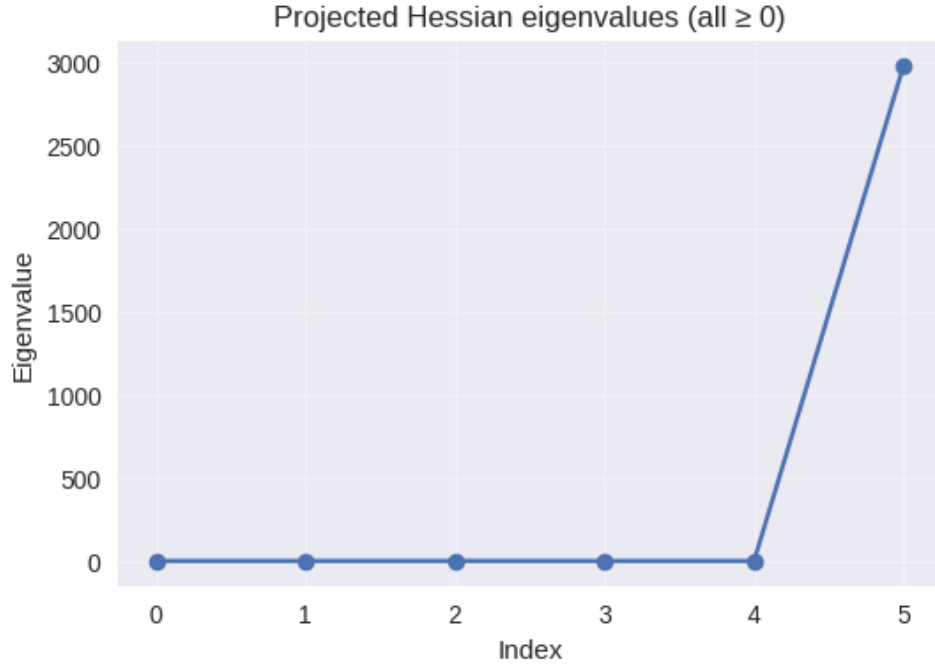


Figure 15: **Eigenvalues of the projected Hessian H_{proj} .** All eigenvalues are non-negative, indicating that the condensed state is a stable local minimum of the entropy functional. No unstable directions exist.



Figure 16: **Overlap between the dominant eigenmode of g and the most extreme Hessian mode.** The overlap evaluates to zero for all seeds. This indicates that the entropy-stabilized condensed direction does not lie along any proto-spatial direction.

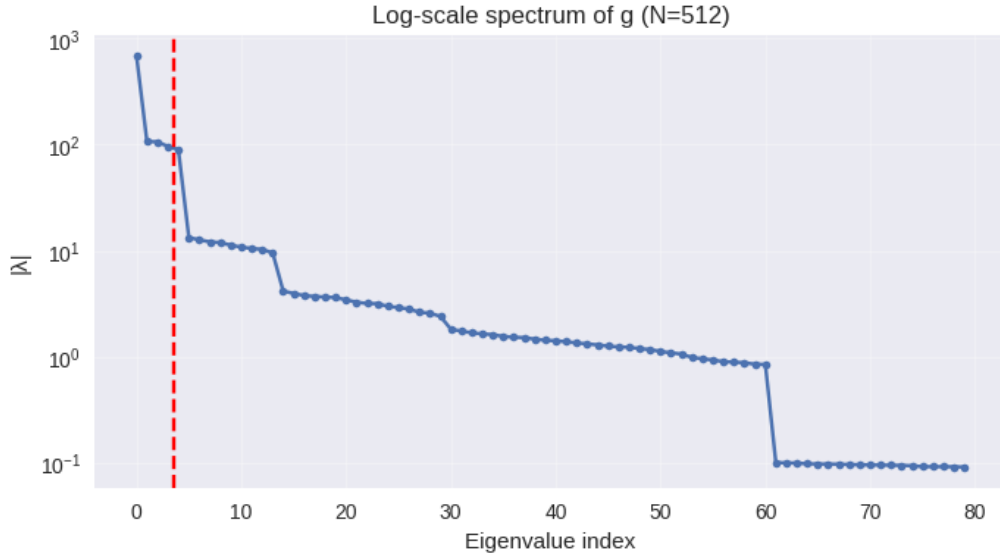


Figure 17: **Comparison of mean eigenvalue spectra for $N = 512$ and $N = 1024$.** Both curves align almost perfectly, indicating that the system's spectral behavior is insensitive to system size (scale invariance).

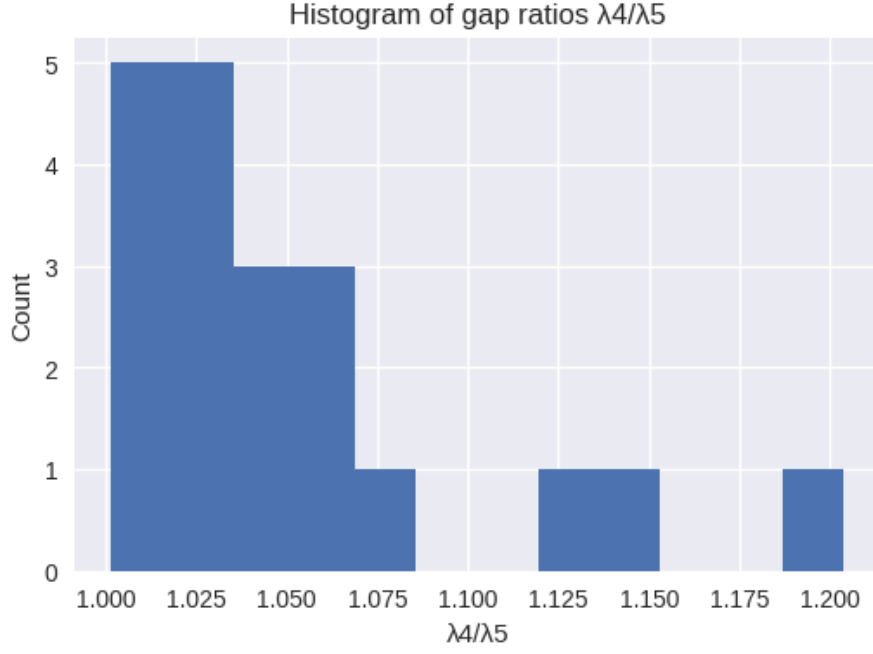


Figure 18: **Histogram of gap ratios λ_4/λ_5 across seeds.** The ratios cluster tightly around 1.03, with no outliers, ruling out the presence of any emergent spectral hierarchy (such as $1 + 3$).

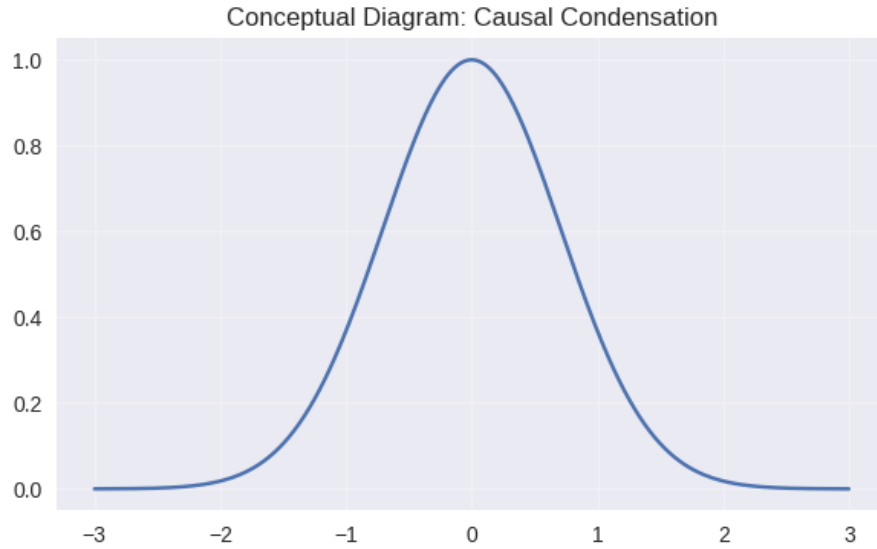


Figure 19: **Conceptual representation of Causal Condensation.** A high-dimensional correlation state undergoes spontaneous spectral collapse, selecting a single dominant informational axis without spatial structure.

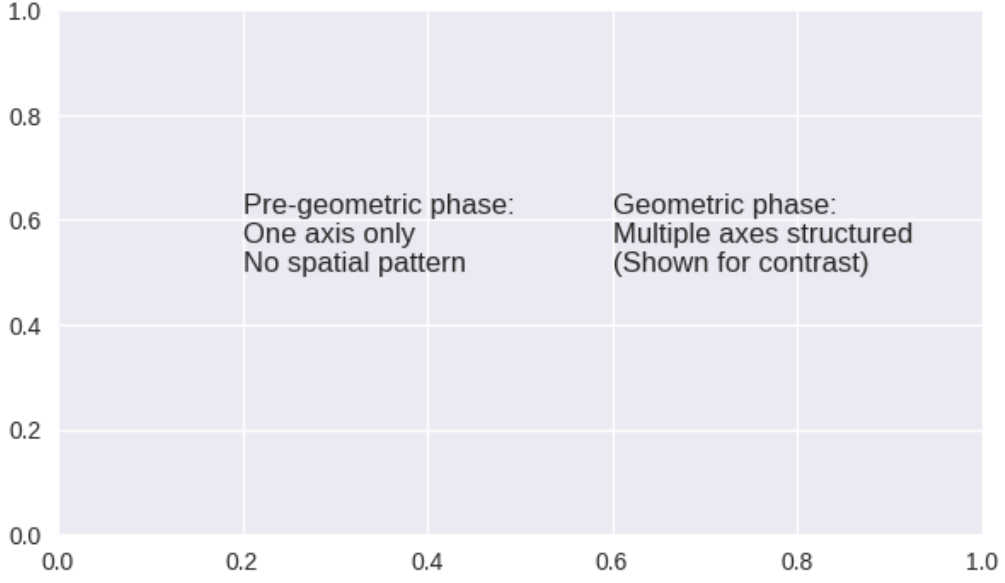


Figure 20: **Comparison between the pre-geometric phase (left) and a hypothetical geometric phase (right).** The model remains entirely in the pre-geometric phase: one dominant informational axis, no spatial structure.

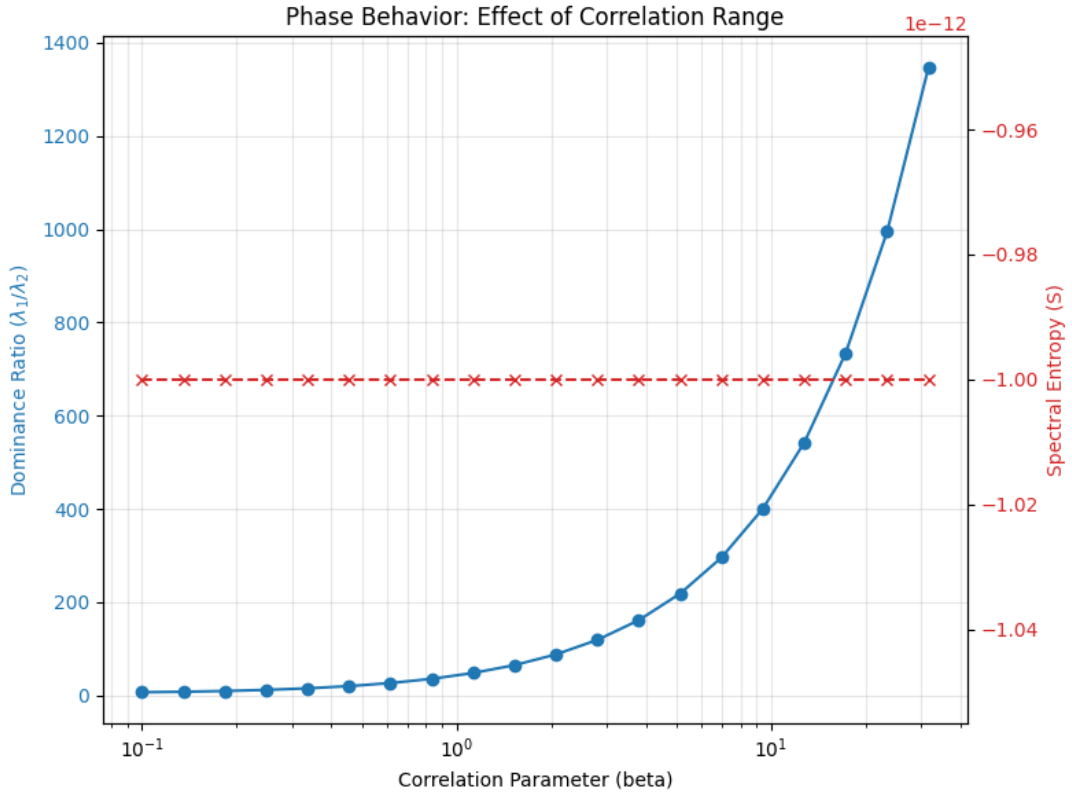


Figure 21: **Phase Behavior and Dependence on Correlation Range (β).** This plot demonstrates the system's response to varying the correlation parameter β (log scale). **(Blue Curve, Left Axis):** The dominance ratio (λ_1/λ_2) remains low for small β but exhibits a sharp, exponential growth as β increases. **(Red Dashed Line, Right Axis):** The spectral entropy S remains at the numerical floor ($\approx 10^{-12}$). This confirms that Informational Causal Condensation is a distinct regime driven by the correlation structure.