Lecture notes in microeconomics
ECO00037I

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Preamble - Why these notes?

These notes provide an **overview** of the main concepts, ideas and problems that you will encounter throughout the course. They are **not**, by any means, a replacement for the textbooks, problem sets or any other resources that will be used in the course. Please make sure to check these other materials carefully!

The notes were designed with the following features in mind:

- They are **concise**. I hope this will help you remember the main ideas and concepts. But please remember that the notes are not exhaustive!
- They feature explanations drawn from various sources and from my own understanding. I hope you will find these helpful.
- They can be improved continuously (typos can be fixed, paragraphs can be rewritten, graphs can be changed), based on your feedback!

Acknowledgement

The explanations, notations, examples, graphs, etc featured in these notes are the product of a combination of my own work and understanding and of the (excellent) work of others, including:

- "Microeconomics: Theory and Applications with Calculus", Global Edition, 5th Edition, by Jeffrey M. Perloff.
- "Intermediate Microeconomics with Calculus: A Modern Approach", International Student Edition, by Hal R. Varian.
- "Microeconomics", International Student Version, 5th Edition, by David Besanko and Ronald Braeutigam.
- "Intermediate Microeconomics, A Tool-Building Approach" by Samiran Banerjee.
- "Microeconomic Theory And Public Policy" (lecture notes), by David Autor.
- "Microeconomic Analysis", by Hal R. Varian.
- "Microeconomic Theory", by Andreu Mas-Colell, Michael Dennis Whinston, and Jerry R. Green.
- "Eléments de microéconomie", by Pierre Picard.

Therefore, these notes are for your **personal use only. Please do not circulate.** I am grateful to current and past students for their feedback that helped improve these notes over the years.

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1 Consumer theory

In this chapter, we will study the consumer's problem of choosing the best bundles of goods (according to their preferences) they can afford (depending on their budget constraint).

Outline:

1. Budget constraint. We begin by describing the bundles of goods that the consumer can afford. A bundle of goods is affordable if its cost does not exceed the consumer's income.

2. Preferences. We then describe the preferences of consumers over bundles of goods and introduce the key assumption that these preferences must satisfy.

3. Utility. We introduce the concept of utility and utility functions, which are a useful way of summarizing a consumer's preferences.

4. Utility maximisation problem. We introduce and solve the consumer's utility maximisation problem, i.e. the process of choosing the most preferred bundle from the set of affordable bundles.

Finally, we explore additional topics of interest, including the consumer's demand function, comparative statics, the consumer's welfare.

Key ideas & concepts

preferences, completeness, transitivity, monotonicity, indifference curve, utility, utility function, marginal utility, marginal rate of substitution, utility maximization, indirect utility function, expenditure function, Marshallian demand, Hicksian demand, Engel curve, compensating variation, equivalent variation, consumer's surplus.

1.1 Budget constraint

Bundles

A *consumption bundle* x is a vector $x = (x_1, x_2, ..., x_n)$, where n is the number of different goods that can be consumed and x_i is the quantity of good i in the bundle. For now, we focus on the case where $n = 2$, that is, there are only two goods to consume: bananas and oranges, books and DVDs, etc. We use only two goods because we can then represent the consumer's choice graphically.

Budget constraint

Let $p = (p_1, p_2, ..., p_n)$ be the vector of prices the consumer is facing. In the case of two goods, we have p_1 and p_2 as the prices of good 1 and 2, respectively. Assume that the consumer has m dollars to spend on consumption. The *budget constraint* is

$$p_1x_1 + p_2x_2 \leq m \tag{1}$$

Any bundle (x_1, x_2) that satisfies this equation is affordable and all these affordable bundles make up the budget set of the consumer.

To represent graphically the *budget set*, we first need to plot the budget line. The *budget line* is formed by all the bundles that use all the money available, that is

$$p_1x_1 + p_2x_2 = m$$

The trick is to put x_2 on the left side of the equal sign, and put everything else on the other side, like so:

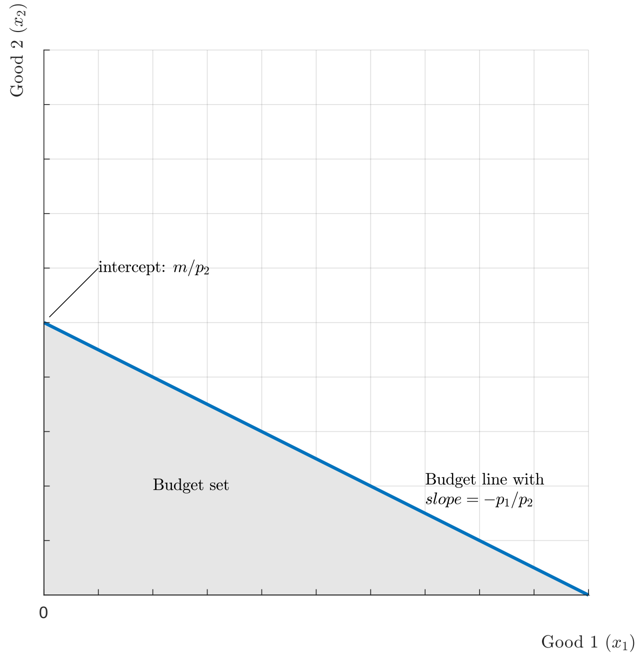
$$x_2 = \frac{m}{p_2} - \frac{p_1}{p_2}x_1$$

We recognize the equation of an affine function with intercept $\frac{m}{p_2}$ and slope $-\frac{p_1}{p_2}$. The intercept is the maximal amount of good 2 we can buy. The slope of the budget line is the *opportunity cost* of good 1: it is how much good 2 the consumer must give up in order to consume one more unit of good 1. In figure 1, we show a budget set and the budget line.

EXAMPLE. *From the example in figure 1, the budget constraint is $x_1 + 2x_2 = 10$. Hence, the budget line is $x_2 = 5 - \frac{1}{2}x_1$. The slope is $-\frac{p_1}{p_2} = -\frac{1}{2}$.*

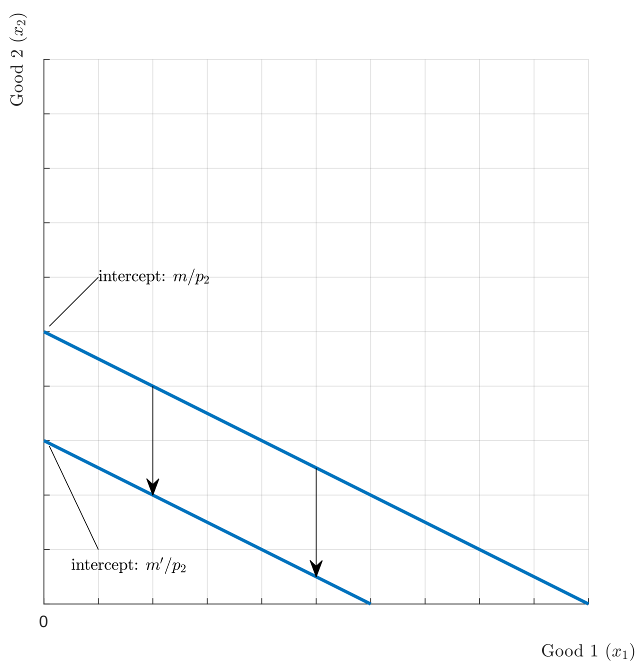
Changing the prices of the goods will change the slope of the budget line. Increasing the amount of money available will shift the budget line away from the origin (conversely, decreasing the amount of money will shift the budget line closer to the origin). Figure 2 shows what happens when we decrease the amount of money available from m to m'.

Figure 1: Consumer theory: the budget set and the budget line.



Note: In this figure, $m = 10$, $p_1 = 1$ and $p_2 = 2$.

Figure 2: Consumer theory: shifting the budget line.



Note: In this figure, $m = 10$, $m' = 6$, $p_1 = 1$ and $p_2 = 2$.

1.2 Preferences

Preferences are relationships between bundles. Consider two bundles, say (x_1, x_2) and (y_1, y_2). Whenever (x_1, x_2) is strictly preferred to (y_1, y_2), we write $(x_1, x_2) \succ (y_1, y_2)$. Whenever (x_1, x_2) and (y_1, y_2) are regarded as indifferent, we write $(x_1, x_2) \sim (y_1, y_2)$. Finally, whenever (x_1, x_2) is as least as good as (y_1, y_2), we write $(x_1, x_2) \succeq (y_1, y_2)$.

We will make the following assumptions on preferences:

1. *Completeness*. Any two bundles can be compared.

2. *Reflexivity*. Any bundle is at least as good as itself.

3. *Transitivity*. Take any three bundles x, y, z. Then if $x \succeq y$ and $y \succeq z$, then $x \succeq z$.

These are the three essential assumptions on preferences.

Throughout the lecture, we will put special emphasis on well behaved preferences, which satisfy these two additional properties:

1. *Monotonicity*, or "more-is-better". This property states that, all else equal, more of a commodity is better than less of it.

2. *Convexity*: averages are preferred to extremes. Convexity is defined as follows: given three bundles x, y and z such that $x \succeq z$ and $y \succeq z$, then $tx + (1-t)y \succeq z$ for all $0 \leq t \leq 1$.

Indifference curves

In our two goods case, there is a nice way of representing preferences in a graphical way through indifference curves. An *indifference curve* graphs the set of bundles that are indifferent to some bundle, as shown in figure 3.

With well-behaved preferences, indifference curves will have the following properties:

1. Bundles on indifference curves father from the origin are preferred to those on indifference curves closer to the origin (by monotonicity).

2. Indifference curves slope downward (by monotonicity).

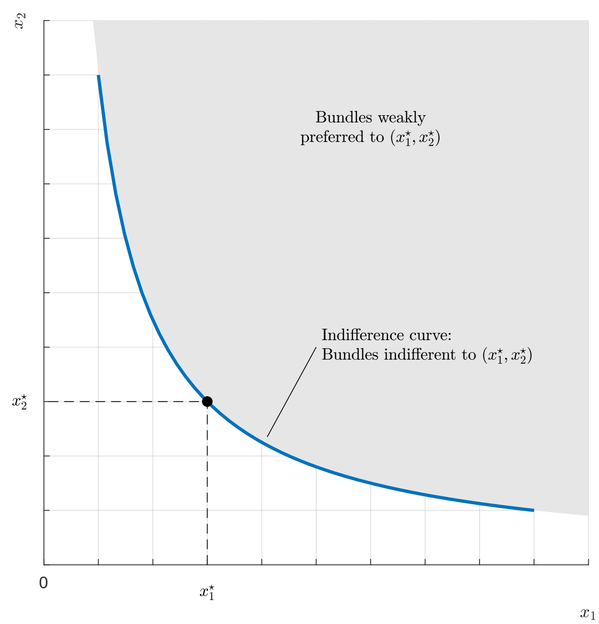
3. Indifference curves cannot cross (by transitivity).

4. Indifference curves are convex (by convexity).

Marginal rate of substitution

The *marginal rate of substitution* (MRS) is the slope of the indifference curve. At a given point on a given indifference curve, the MRS tells us how much of good 2 the consumer is willing to give up to increase his consumption of good 1, while staying on the same indifference curve.

Figure 3: Consumer theory: indifference curves.



<u>Note:</u> In this figure, the blue curve is an indifference curve. The consumer is indifferent between any two bundles located on this curve. The utility function is $u(x_1, x_2) = \sqrt{x_1}\sqrt{x_2}$, and the utility level associated with the indifference curve is equal to 3.

1.3 Utility

Utility functions are simple a way to summarize preferences. A *utility function* $u(x_1, x_2)$ assigns numbers to bundles so that more preferred bundles gets higher numbers. We could easily construct a utility function from indifference curves by assigning to each indifference curve a number: the farther away from the origin the indifference curve is, the higher the number.

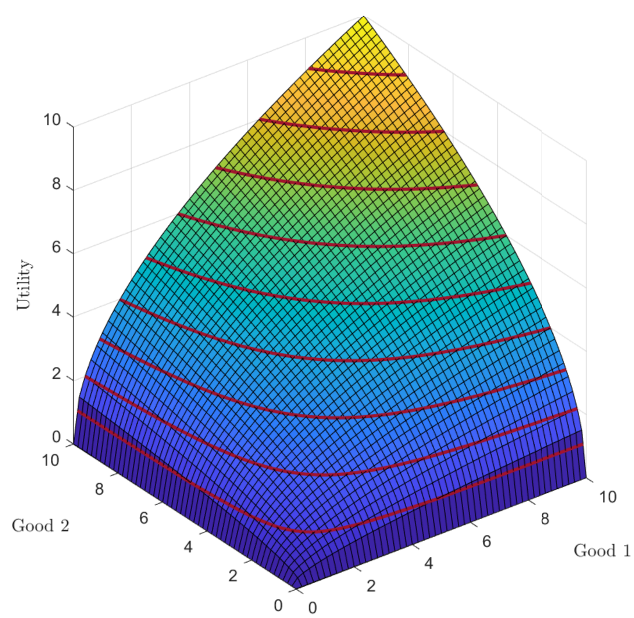
Conversely, we can construct an indifference curve from the utility function by finding all the bundles that give the same utility level, that is, we need to find all the bundles (x_1, x_2) such that

$$u(x_1, x_2) = u^\star$$

to construct the indifference curve associated with the utility level $u^\star$.

In figure 4, we represent the utility from consuming two goods, good 1 and good 2, and the indifference curves.

Figure 4: Consumer theory: utility function and indifference curves.

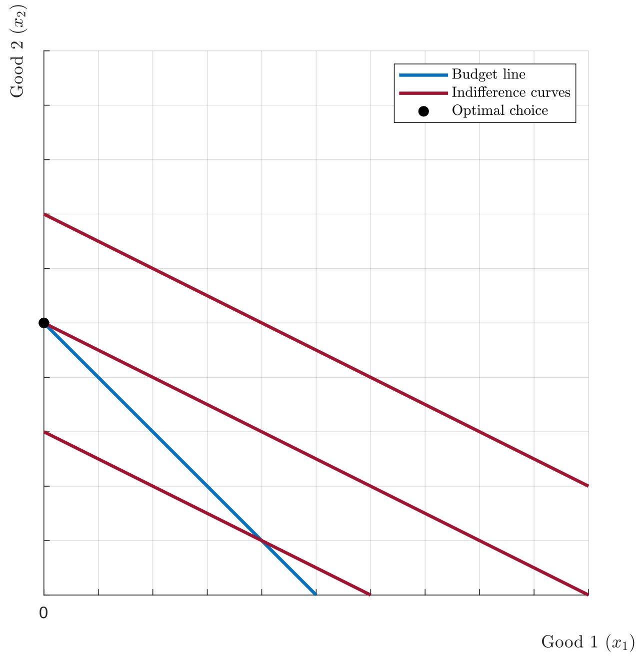


Note: In this figure, $u(x_1, x_2) = \sqrt{x_1}\sqrt{x_2}$.

Special preferences

Perfect substitutes. Two goods are *perfect substitutes* when the marginal rate of substitution of one for the other is a constant. Graphically, whenever goods are perfect substitutes the indifference curves are straight lines, as in figure 5.

Figure 5: Consumer theory: perfect substitutes.



Note: In this figure, $p_1 = 1$, $p_2 = 1$ and $m = 5$. The utility function is $u(x_1, x_2) = x_1 + 2x_2$.

Perfect complements. Two goods are *perfect complements* when consumers always want to consume them together in fixed proportion to each other. Graphically, the indifference curves are L-shaped (they comprise straight line segments at right angle), as shown in figure 6.

Cobb-Douglas. The *Cobb-Douglas* utility function is commonly used, and is of the form

$$u(x_1, x_2) = x_1^c x_2^d$$

Cobb Douglas indifference curves are well-behaved, as shown in figure 7.

Marginal utility and marginal rate of substitution

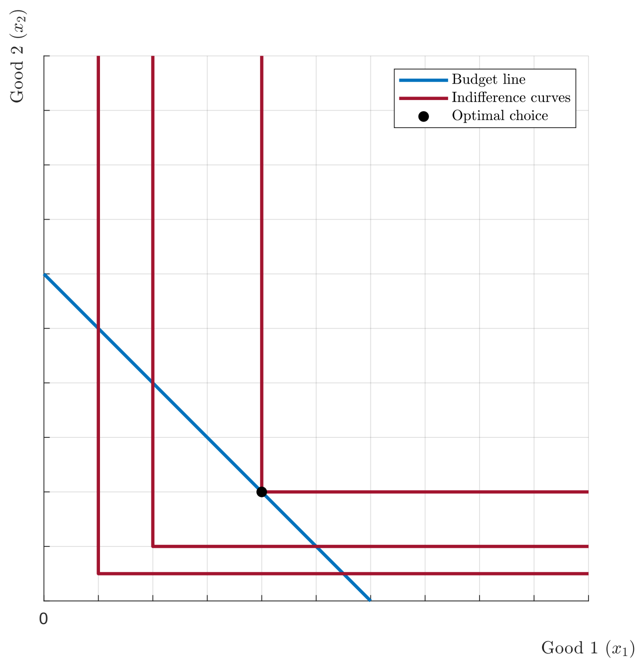
The *marginal utility* from some good is the extra utility from consuming a little bit more of that good, holding the consumption of the other goods fixed. In mathematical terms, it is a partial derivative. The marginal utility from good 1 is

$$MU_1(x_1, x_2) = \frac{\partial u(x_1, x_2)}{\partial x_1} \tag{2}$$

and the marginal utility from good 2 is

$$MU_2(x_1, x_2) = \frac{\partial u(x_1, x_2)}{\partial x_2} \tag{3}$$

Figure 6: Consumer theory: perfect complements.



Note: In this figure, $p_1 = 1$, $p_2 = 1$ and $m = 6$. The utility function is $u(x_1, x_2) = \min\{\frac{1}{2}x_1, x_2\}$.

When the utility function is known, we can actually compute the MRS with the help of marginal utilities. To find the MRS, remember that we want to find variations in quantities of good 1 and 2 (dx_1, dx_2) such that utility remains constant, that is

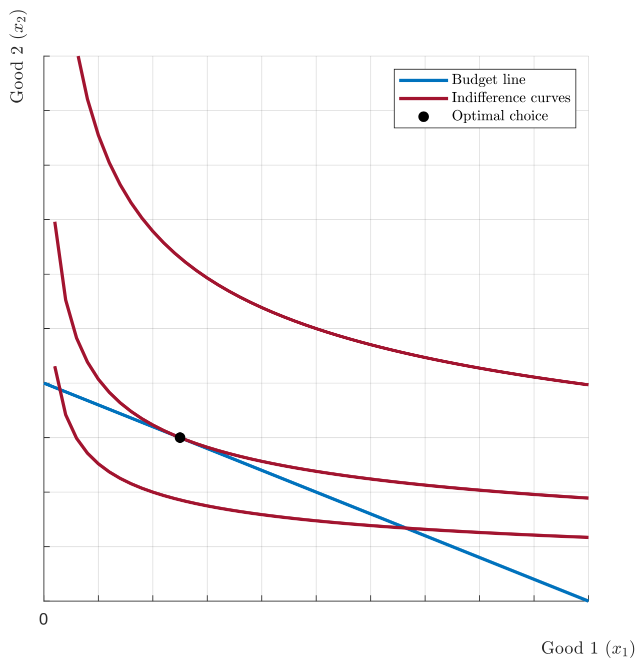
$$MU_1(x_1, x_2)dx_1 + MU_2(x_1, x_2)dx_2 = 0$$

hence

$$\frac{dx_2}{dx_1} = -\frac{MU_1(x_1, x_2)}{MU_2(x_1, x_2)} \tag{4}$$

The left hand side is what we know as the MRS, i.e the slope of the indifference curve. The right hand side tells us how to compute the MRS using marginal utilities.

Figure 7: Consumer theory: the Cobb Douglas utility function.



Note: In this figure, $p_1 = 1$, $p_2 = 2.5$ and $m = 10$. The utility function is $u(x_1, x_2) = x_1^{\frac{1}{4}} x_2^{\frac{3}{4}}$.

1.4 Choice

Utility maximisation problem

We have the ingredients to derive the *optimal choice* of the consumer: we know their preferences over bundles, and we know which bundles are affordable. The optimal choice the most preferred bundle from the budget set. The consumer's decision problem can be stated as the following *utility maximization problem*:

$$\max_{x \geq 0} u(x)$$
$$\text{subject to } p \cdot x \leq m$$

Indirect utility function

The *indirect utility function* is the function

$$v(p, m)$$

that gives the maximum level utility achievable given prices p and income m. Note that because the consumer will use all his income, so we can define the indirect utility function as

$$v(p, m) = \max_{x \geq 0} u(x)$$
$$\text{subject to } p \cdot x = m$$

Marshallian demand function

The rule that assigns the set of optimal consumption bundles in the utility maximization problem to each price and income (p, m) is called the *Marshallian* (or uncompensated or *Walrasian* or market or ordinary) demand function, and is denoted

$$x(p, m)$$

This is the usual demand function: it gives us demand given the price, holding income constant. Note that it depends only on prices and income, which are usually observable.

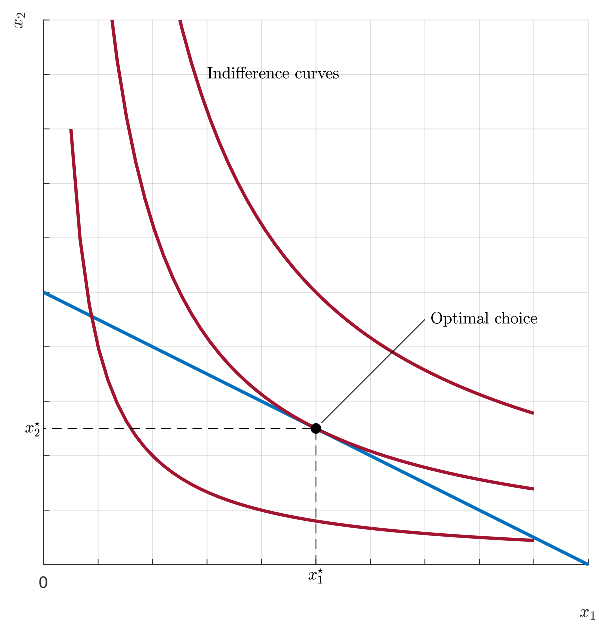
Interior and corner solutions

An optimal bundle at which the consumer buys (strictly) positive quantities of both goods (in the two goods case) is called an *interior solution*. An optimal bundle at which the consumer buys only one of the two goods (in the two goods case) is called a *corner solution*.

Solving the utility maximisation problem - introduction

Let us first find the optimal bundle using graphical reasoning. We can focus on the bundles that are on the budget line: we know the optimal bundle will be there (and not below it), because we have assumed that more is better (and so the consumer will want to spend all of their money). The optimal choice is the bundle that is associated with the highest indifference curve (the farthest away from the origin as possible) that just touches the budget line. This is illustrated in figure 8.

Figure 8: Consumer theory: the optimal choice.



Note: In this figure, $m = 10$, $p_1 = 1$ and $p_2 = 2$. The utility function is $u(x_1, x_2) = \sqrt{x_1 x_2}$.

Solving the utility maximisation problem - interior solutions

Finding an interior solution using the Lagrangian. The Lagrangian associated to the utlity maximisation problem is

$$\mathcal{L}(\lambda, x_1, x_2) = u(x_1, x_2) - \lambda(p_1 x_1 + p_2 x_2 - m)$$

which we now differentiate with respect to x_i (where $i = 1, 2$ here) and λ, the Lagrange multiplier. For an interior solution $x^\star$, the first order conditions are

$$\frac{\partial u(x^\star)}{\partial x_i} - \lambda p_i = 0 \qquad \text{for } i = 1, 2$$

Combining the equations for good 1 and 2, we get the optimality condition

$$-\frac{\frac{\partial u(x^\star)}{\partial x_1}}{\frac{\partial u(x^\star)}{\partial x_2}} = -\frac{p_1}{p_2}$$

The optimal solution must also satisfy the budget constraint

$$p_1 x_1 + p_2 x_2 = m$$

To find the optimal choice $x^\star$, we can combine the optimality condition and the budget constraint and solve for $x^\star$.

Geometric interpretation. At the optimal bundle, the budget line is tangent to the indifference curve. It is important to note that this is not true whenever there is a corner solution or the indifference curve exhibits a kink at the optimal choice.

However, if we rule out these two particular cases (corner solutions and kinky preferences), then tangency is a necessary condition for optimality. And whenever indifference curves are in addition convex (curving away from the origin), then tangency is a sufficient condition: any point that satisfies the tangency condition must be an optimal point.

In mathematical terms, the tangency condition means that the slopes of the indifference curve and the budget line at the optimal point are equal, that is, $MRS = -\frac{p_1}{p_2}$. Thus, we obtain

$$-\frac{\frac{\partial u(x_1,x_2)}{\partial x_1}}{\frac{\partial u(x_1,x_2)}{\partial x_2}} = -\frac{p_1}{p_2}$$

which is the condition we obtained using the Lagrangian.

EXAMPLE. *To understand the optimality condition intuitively, recall that the right hand side tells us that if we want to consume one more unit of good 1, we must decrease our consumption of good 2 by* $-\frac{p_1}{p_2}$ *in order to stay on the budget line. The left hand side tells us by how much we should decrease our consumption of good 2 in order to increase our consumption of good 1 by one unit and still maintain the same level of utility.*

Now we see why the equality should hold. Assume that the MRS at a given point is equal to -2 *and the prices are equal to 1 so that* $-p_1/p_2 = -1$. *Consider increasing our consumption of good 1 by 1 unit and decreasing our consumption of good 2 by 1 unit. This would be a good trade: we would still be on the budget line, but on a higher indifference curve. Indeed, we would have had to give up two units of good 2 to stay on the same indifference curve as before, and we only gave up one unit. Thus the current choice is not optimal: we can do better!*

EXAMPLE. *Consider the example from figure 8. The slope of the budget line is* $-p_1/p_2 = -0.5$. *The marginal utilities are* $MU_1 = \frac{\partial u(x_1,x_2)}{\partial x_1} = \frac{\sqrt{x_2}}{2\sqrt{x_1}}$ *and* $MU_2 =$

$\frac{\partial u(x_1,x_2)}{\partial x_2} = \frac{\sqrt{x_1}}{2\sqrt{x_2}}$. *The consumer will choose the bundle such that* $MRS = -p_1/p_2$, *that is* $\frac{x_2}{x_1} = 0.5$, *thus* $x_1 = 2x_2$. *Using the budget constraint, we have* $1 \times 2x_2 + 2x_2 = 10$, *thus* $x_2 = 2.5$, *and* $x_1 = 5$.

EXAMPLE*. A consumer likes to buy round-trips to a domestic location (D) and to a foreign location (F). The price of a round trip to a domestic location is* $p_D = 100$ *while the price of a round trip to a foreign location is* $p_F = 400$. *His preferences are represented by the utility function* $u(D, F) = DF$. *The total amount he can spend on round trips is* 4000. *Thus his budget constraint is* $100D + 400F = 4000$. *This gives us the budget line* $F = 10 - 0.25D$, *which is a straight line of slope* $-p_D/p_F = -0.25$. *The marginal utilities are* $MU_1 = \frac{\partial u(D,F)}{\partial D} = F$ *and* $MU_2 = \frac{\partial u(D,F)}{\partial F} = D$. *The consumer will choose the bundle such that* $MRS = -p_D/p_F$, *that is* $\frac{F}{D} = 0.25$, *thus* $D = 4F$. *Using the budget constraint, we have* $100 \times 4F + 400F = 4000$, *thus* $F = 5$ *and* $D = 20$.

Solving the utility maximisation problem - corner solutions

The methods used so far to solve the consumer utility maximisation problem assume the existence of an interior solution. But there are cases, e.g. with perfect substitutes or quasilinear utility functions, in which corner solutions can arise.

Perfect substitutes. Two goods are perfect substitutes when the marginal rate of substitution (MRS) of one for the other is a constant. If $MRS > -p_1/p_2$, the consumer only buys good 2. If $MRS < -p_1/p_2$, the consumer only buys good 1. If $MRS = p_1/p_2$, then any bundle located on the interior of this budget line is an optimal bundle, and so are the corners (the two extremities of the budget line).[1]

Quasi-linear utility. A *quasi linear utility function* is a utility function of the form $u(x_1, x_2) = x_1 + v(x_2)$. To find the optimal bundle in this case, we follow a two step procedure. First, we look for an interior solution using the usual methods. If we cannot find one, then we look for a corner solution.

EXAMPLE*. Perfect substitutes. Suppose the goods are perfect substitutes for the consumer. Suppose that the prices are such that the indifference curves and the budget line have different slopes, as in figure 5. In this particular example, the MRS is equal to* -0.5 *and the slope of the budget line is* -1. *Graphically, we see that as we try to move to an indifference curve that's as far away from the origin as possible while still maintaining a point of contact with the budget line, we will reach the optimal bundle in which the consumer buys 5 units of good 2 and 0 units of good 1 (a corner*

[1] If $MRS = -p_1/p_2$, the indifference curves and budget line have the same slope. As we try to move to an indifference curve that's as far away from the origin as possible while still maintaining a point of contact with the budget line, we will reach an indifference curve that coincides with the budget line. Any bundle located on the interior of this budget line / indifference curve is an optimal bundle, and so are the corners (the two extremities of the budget line).

solution). What is the intuition in this particular example? The MRS tells us that if the consumer gives up one unit of good 1, they need only 0.5 units of good 2 to maintain utility constant. At the same time, the slope of the budget line tells us that if the consumer gives up one unit of good 1, they would have enough money to buy one unit of good 2. This is more than what is needed to maintain utility constant, so clearly, this is a good deal! The consumer will want to keep exchanging units of good 1 for more units of good 2 until they consume 0 units of good 1.

***EXAMPLE**. Quasilinear utility. Let $u(x_1, x_2) = x_1 + 10\sqrt{x_2}$, $p_1 = p_2 = 1$ and $m = 2$. The marginal utility with respect to good 1 is 1 and the marginal utility with respect to good 2 is $10/(2\sqrt{x_2})$. Therefore, the MRS is $-\frac{1}{10/(2\sqrt{x_2})}$ which is $-\sqrt{x_2}/5$. The slope of the budget line is $-p_1/p_2 = -1$. Therefore, the optimality condition is $\sqrt{x_2}/5 = 1$, thus $x_2 = 25$. Using the budget constraint, the quantity of good 1 is $x_1 = m/p_1 - p_2 x_2/p_1$, thus $x_1 = 2 - 25 = -23$. The quantity of good 1 is negative, so clearly something is wrong. We should look for a corner solution instead. If the consumer spends all of their income on good 1, they can buy $m/p_1 = 2$ units of good 1 and get utility $u(2, 0) = 2$. If the consumer spends all of their income on good 2 instead, they can buy $m/p_2 = 2$ units of good 1 and get utility $u(0, 2) = 10\sqrt{2}$, which yields more utility. The solution is therefore the corner solution in which the consumer only buys good 2.*

The expenditure minimization problem

Consider this alternative consumer decision problem: given prices p, what is the minimum level of expenditures needed to attain a certain level of utility u? The *expenditure minimization problem* is defined as follows:

$$\min_{x \geq 0} p \cdot x$$
$$\text{subject to } u(x) \geq u$$

The expenditure function

The *expenditure function* is the function

$$e(p, u)$$

that indicates the minimum cost of achieving a given level of utility. It is defined as follows:

$$e(p, u) = \min_{x \geq 0} p \cdot x$$
$$\text{subject to } u(x) \geq u$$

The expenditure function is very useful because it provides a monetary value, and can be used to evaluate policies.

Hicksian demand function

The rule that assigns the set of optimal consumption bundles in the expenditure minimization problem to each price and utility level (p, u) is called the *Hicksian* (or compensated) demand function, and is denoted

$$h(p, u)$$

This is a demand function that gives us demand given prices, holding utility constant. This is why it is called compensated demand function, because the income changes in the background to compensate for any change in prices (to maintain the same level of utility). Note that the Hicksian demand function is not directly observable.

1.5 Demand

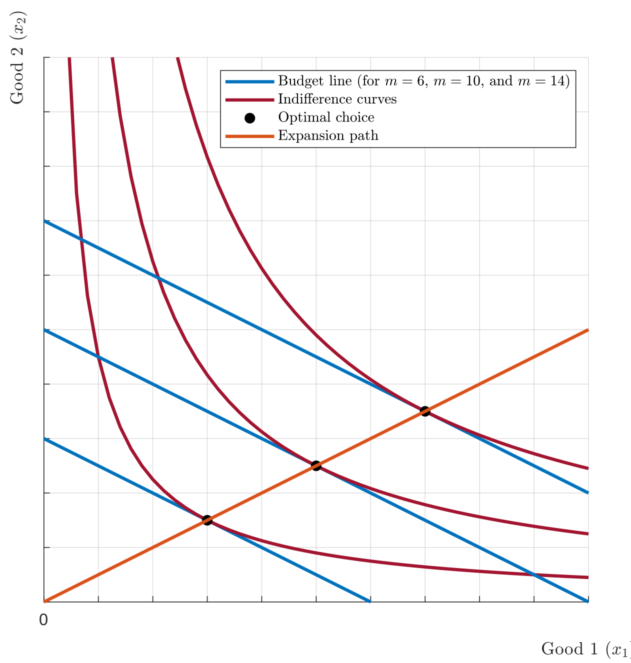
The effects of a change in income

Consider a Marshallian demand function for some good i, $x_i(p,m)$, and let us fix prices. What will be the effect of a change in income m on the demand for good i? We can distinguish among different types of goods, in particular:

- *Normal goods*: a good i is normal if the demand for that good is nondecreasing in income (that is, if $\partial x_i(p,m)/\partial m \geq 0$).
- *Inferior goods*: a good i is inferior if the demand for that good is decreasing in income.

The Engel curve

Figure 9: Consumer theory: the income expansion path.

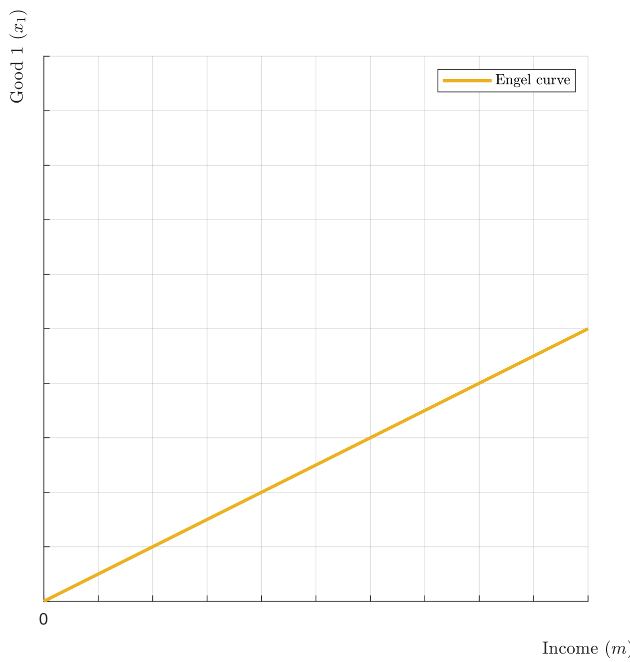


Note: In this figure, $p_1 = 1$ and $p_2 = 2$. We vary income such that $m = 6$, $m = 10$ or $m = 14$. The utility function is $u(x_1, x_2) = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$.

Building on the previous analysis, we can determine the demanded bundles for various levels of prices and income. Let us focus on income first. The *income expansion path* (or income offer curve) is the curve that connects the optimal choice at different level of income, holding prices fixed. This is illustrated in figure 9.

The *Engel curve* for a given good represents how demand for that good changes as income changes (holding prices fixed). Thus the Engel curve is easily constructed from the income expansion path. This is represented in figure 10, which is derived from figure 9.

Figure 10: Consumer theory: the Engel curve.



Note: In this figure, $p_1 = 1$ and $p_2 = 2$. The utility function is $u(x_1, x_2) = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$.

Homothetic preferences. Preferences are *homothetic* if when a consumer is indifferent between bundle (x_1, x_2) and (y_1, y_2), then he is indifferent between (tx_1, tx_2) and (ty_1, ty_2), for any positive value of t.

When preferences are homothetic, the income expansion path is a straight line: when we scale up income by some amount, then the demanded bundle scales up by the same amount. This is illustrated in figure 9 since the Cobb Douglas utility function is homothetic.

The effects of a change in prices

Consider a Marshallian demand function for some good i, $x_i(p, m)$, and let us fix income. Let us now consider the effect of a change in the price of good i on the demand of good i, keeping all other prices fixed. The change in the demand of good i implicitly combines two effects:

- a *substitution effect*: this is the change in demand due to the change in the rate at which the consumer can exchange good i for other goods.

- an *income effect*: this is the change in demand due to having more (if the price decreases) or less (if the price increases) purchasing power.

The substitution effect is always of the opposite sign of the price change, and the income effect can be positive or negative. The net effect can therefore be positive or negative. There are two particular types of goods to keep in mind: (i) *ordinary goods*, which are goods whose demand increases as the price of the good decreases ; and (ii) *Giffen goods*, which are goods whose demand decreases as the price of the good decreases.

Slutsky decomposition

The Slutsky equation allow us to decompose the net change in demand into a substitution and an income effect. It makes use of the Marshallian demand function (which gives us the net effect of the change in price, and the income effect) and the Hicksian demand (which gives us the substitution effect, because it holds utility fixed). For now, we will consider the graphical representation of the Slutsky equation.

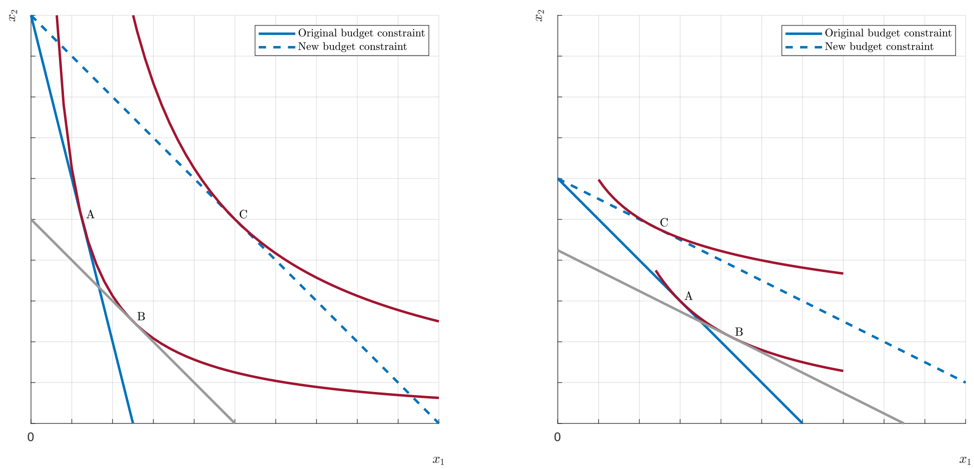
Suppose for simplicity that there are only two goods. We can represent the *Slutsky decomposition* using the usual indifference curves and budget lines. This is illustrated in figure 11. In the left panel, the price of good one decreases. The initial budget line (before the price change) is represented as a solid blue line, and the optimal consumption choice is bundle A. The new budget line (after the price change) is represented as a dashed blue line, and the new optimal consumption is bundle C. To isolate the substitution effect, we draw a new budget line (in grey) that features the new price ratio, but we change the level of income to maintain utility to its original level. The optimal consumption choice under this budget line is bundle B. Moving from bundle A to bundle B shows the substitution effect ; moving from bundle B to C shows the income effect. In this example, the good is normal, so the substitution effect and the income effect go in the same direction, so the demand for good 1 increases as its price decreases.

In the right panel, good 1 is a Giffen good. Once again, we look at a case in which the price of good one decreases. The initial budget line (before the price change) is represented as a solid blue line, and the optimal consumption choice is bundle A. The new budget line (after the price change) is represented as a dashed blue line, and the new optimal consumption is bundle C. Moving from bundle A to bundle B shows the substitution effect ; moving from bundle B to C shows the income effect. In this example, the good is inferior, so the substitution effect and the income effect go in different directions. The net effect is that the demand for good 1 decreases as its price decreases (hence this is a Giffen good).

The Slutsky equation

Consider the standard consumer decision problem, and suppose that the utility function is continuous, that preferences satisfy local nonsatiation (a weaker assumption

Figure 11: Consumer theory: income and substitution effects



Note: This illustrates the decomposition into the substitution and income effects in two different examples.

than monotonicity) and that both the utility maximization problem and expenditure minimization problems have a solution. In particular, let $x^\star$ solve

$$\max u(x)$$
$$\text{s.t } p \cdot x \leq m$$

and denote $u = u(x^\star)$. Then $x^\star$ solves

$$\min p \cdot x$$
$$\text{s.t } u(x) \geq u$$

Similarly, if $x^\star$ solves the expenditure minimization problem and we let $m = p \cdot x^\star$ then, assuming $m > 0$, $x^\star$ solves the utility maximization problem. Thus, under the conditions stated above, utility maximization implies expenditure minimization, and vice-versa. This result has an important implication, in that we can relate the Hicksian and Marshallian demand functions through

$$h(p, u) = x(p, e(p, u))$$

If $h(p, u)$ solves the expenditure minimization problem for prices p and utility level u, then

$$h_i(p, u) = \frac{\partial e(p, u)}{\partial p_i}$$

for all goods $i = 1, ..., n$, assuming the derivative exists and $p_i > 0$. This result is know as *Shephard's lemma*.

GOING FURTHER. *There are several ways to prove this result. One is to apply the envelop theorem to contrained minimization problems. Another way is to write that* $\frac{\partial e(p,u)}{\partial p_i} = h_i(p,u) + \sum_j p_j \frac{\partial h_j(p,u)}{\partial p_i}$, *since* $e(p,u) = p \cdot h(p,u)$. *The first order conditions from the expenditure minimization problem tell us that* $p_j = \lambda \frac{\partial u(h(p,u))}{\partial x_j}$. *Thus* $\frac{\partial e(p,u)}{\partial p_i} = h_i(p,u) + \lambda \sum_j \frac{\partial u(h(p,u))}{\partial x_j} \frac{\partial h_j(p,u)}{\partial p_i}$. *But because the constraint* $u(h(p,u)) = u$ *holds for all* p, *differentiating this with respect to* p_j *leads to* $\sum_j \frac{\partial u(h(p,u))}{\partial x_j} \frac{\partial h_j(p,u)}{\partial p_i} = 0$, *which gives the result.*

The *Slutsky equation* is

$$\frac{\partial x_j(p,m)}{\partial p_i} = \frac{\partial h_j(p,v(p,m))}{\partial p_i} - \frac{\partial x_j(p,m)}{\partial m} x_i(p,m)$$

GOING FURTHER. *To show this result, let* $x^\star$ *maximize utility for prices and income* $(p^\star, m^\star)$, *and denote* $u^\star = u(x^\star)$. *Recall that for all* (p,u), *we have* $h_j(p,u) = x_j(p, e(p,u))$. *Differentiating this with respect to* p_i *and evaluating the derivative at* $(p^\star, u^\star)$, *we get*

$$\frac{\partial h_j(p^\star, u^\star)}{\partial p_i} = \frac{\partial x_j(p^\star, e(p^\star, u^\star))}{\partial p_i} + \frac{\partial x_j(p^\star, e(p^\star, u^\star))}{\partial m} \frac{\partial e(p^\star, u^\star)}{\partial p_i}$$

but that last term is just $x_i^\star$, *so rearranging the equation yields the Slutsky equation.*

The Slutsky equation decomposes the demand change induced by a price change into a substitution effect and an income effect (see graphs above).

1.6 Consumer welfare

Suppose that the price of some good, say good 1, is changing from p_1 to p_1' following the implementation of some policy. How can we evaluate the impact of such a policy change on the consumer's welfare? We could compare the utility of the consumer under both regimes, but this approach has two main drawbacks: 1) we usually don't know the consumer's utility function and 2) utility numbers are difficult to interpret (they are not monetary values) and to compare across different consumers. Let us explore other options to conduct this welfare analysis.

An individual's consumer surplus

An *individual's consumer surplus* is defined as the area below that consumer's demand function and above the price, between quantity 0 and the quantity consumed.

This concept builds on the idea of comparing the amount the consumer is willing to pay for an extra unit of good (the marginal willingness to pay) to the price of that extra unit to measure the consumer's well being. This information is actually contained in the demand curve of the consumer, or rather, the inverse demand curve. The inverse demand function is obtained by inverting the demand function, so that price is now a function of quantity. The inverse demand function indicates the consumer's marginal willingness to pay for different amounts of good (see figure 12 for an illustration). Note that graphically, the demand curve and inverse demand curve are the same, they just read in different directions: the demand curve indicates how many units of good the consumer wants to buy at a given price, while the inverse demand will reflect the maximum amount the consumer is willing to pay for an extra unit (his marginal willingness to pay).

The consumer surplus is a widely used method to conduct welfare analysis, that is relatively easily measured. All that is required is to know the Marshallian (or uncompensated) demand of the consumer, which we can construct if we observe the quantity of good demanded by the consumer at different prices.

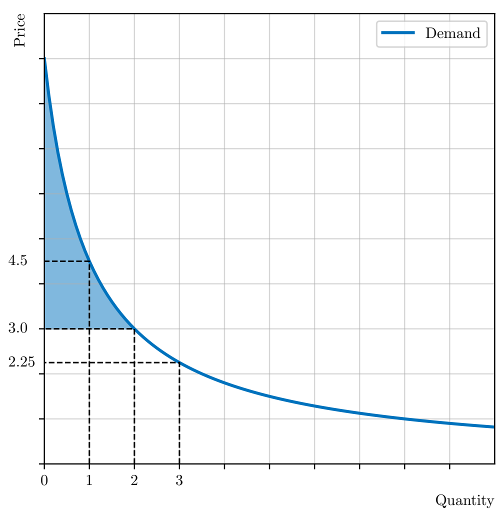
Indifference curve analysis

An alternative approach is to go back to the optimal choice of the consumer and make use of the indifference curves to derive monetary measures of welfare changes. We will examine two methods: the compensating variation and the equivalent variation.

Equivalent variation

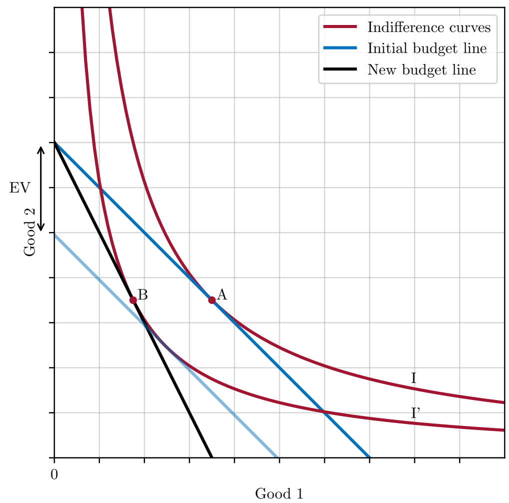
Let m be the consumer's income. Consider a change in prices from p to p'. Let u be the utility of the consumer before the price change and u' the utility of the consumer after the price change. The *equivalent variation* (EV) is how much money should be given to the consumer before the change in prices to give the consumer the same level of utility

Figure 12: Consumer theory: demand, inverse demand, and individual consumer surplus



Note: This figure shows information about a consumer's marginal willingness is contained in the inverse demand. Here, the consumer is willing to pay up to £4.5 for a first unit, £3 for a second unit, and £2.25 for a third unit. Indeed, if the price of the good was 4.5, then the consumer would be willing to buy only one unit. If the price was 3, the consumer would be willing to buy the first unit as well as the second, for a total demand of 2. If the price was 2.25, the consumer would be willing to buy the first unit as well as the second and the third, for a total demand of 3. The shaded area shows the individual consumer surplus when $p = 3$.

Figure 13: Consumer theory: equivalent variation



Note: In this figure, $u(x_1, x_2) = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$, $m = 7$, $p_1 = p_2 = 1$ and $p_1' = 2$.

they would get after the change. It is a negative number if the consumer is made worse off by the change, and positive otherwise. Formally, the equivalent variation EV is such that

$$v(p, m + EV) = u'$$

or alternately, $EV = e(p, u') - m$.

Figure 13 illustrates the case of an increase in the price of good 1. Before the change in prices, the optimal choice of the consumer is point A, located on the indifference curve named I. After the change in prices, the new optimal choice is point B, located on the indifference curve named I'. In this particular example (a price increase), to find the equivalent variation we need to shift the initial budget line downwards up to the point where it is tangent to the new indifference curve I' (this shifted budget line is the light blue line). The change in income needed to achieve this shift is the equivalent variation.

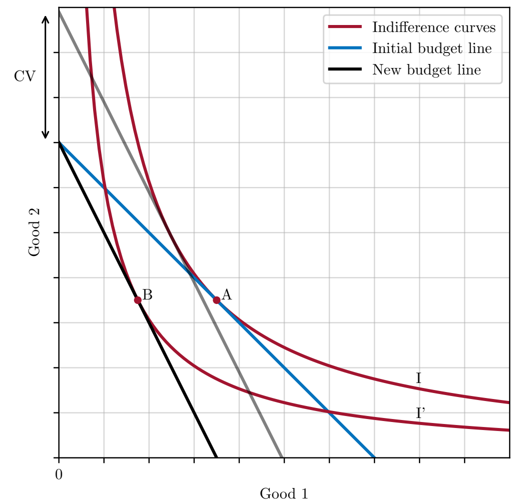
Compensating variation

Let m be the consumer's income. Consider a change in prices from p to p'. Let u be the utility of the consumer before the price change and u' the utility of the consumer after the price change. The *compensating variation* (CV) is how much money should be taken away from the consumer after the change in prices to maintain the same level of utility they had before the change. It is a negative number if the consumer is made worse off by the change, positive otherwise. Formally, the compensating variation CV is such that

$$v(p', m - CV) = u$$

or alternately, $CV = m - e(p', u)$.

Figure 14: Consumer theory: compensating variation



Note: In this figure, $u(x_1, x_2) = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$, $m = 7$, $p_1 = p_2 = 1$ and $p_1' = 2$.

Figure 14 illustrates the case of an increase in the price of good 1. Before the change in prices, the optimal choice of the consumer is point A, located on the indifference curve named I. After the change in prices, the new optimal choice is point B, located on the indifference curve named I'. In this particular example (a price increase), to find the compensating variation we need to shift the new budget line upwards up to the point where it is tangent to the initial indifference curve I (this shifted budget line is the light grey line). The (negative of the) change in income needed to achieve this shift is the compensating variation.

EXAMPLE. *Suppose the utility function of the consumer is given by* $u(x_1, x_2) = x_1^{0.5} x_2^{0.5}$. *Recall that in this example, the optimality condition we get from the consumer utility maximisation problem is* $x_2/x_1 = p_1/p_2$. *Let* $m = 10$ *be the amount of income available to this consumer.*

Suppose that $p_1 = 1$ *and* $p_2 = 1$. *Thus the optimality condition reads* $x_2/x_1 = 1$, *that is* $x_1 = x_2$. *Plugging this in the budget constraint, we get* $x_1 + x_1 = 10$, *and thus* $x_1 = x_2 = 5$. *At this optimal point, the utility of the consumer is* $5^{0.5} 5^{0.5} = 5$.

Suppose now that $p_1 = 4$ *and* $p_2 = 1$. *Thus the optimality condition reads* $x_2/x_1 = 4$, *that is* $x_2 = 4x_1$. *Plugging this in the budget constraint, we get* $4x_1 + 4x_1 = 10$, *and thus* $x_1 = 1.25$ *and* $x_2 = 5$. *At this optimal point, the utility of the consumer is* $1.25^{0.5} 5^{0.5} = 2.5$.

Let us determine what level of income m *would give the consumer a utility level of* 5, *after the price change (when* $p_1 = 4$*). At these prices, recall that the optimality condition gives* $x_2 = 4x_1$ *and the budget constraint is* $4x_1 + x_2 = m$, *so combining both we get* $4x_1 + 4x_1 = m$, *so the optimal quantity of good 1 is* $x_1 = m/8$ *and the optimal quantity of good 2 is* $x_2 = m/2$. *The utility of the consumer is therefore*

$(m/8)^{0.5}(m/2)^{0.5} = m/4$. *We want to find* m *such that utility is equal to* 5, *that is* $m/4 = 5$, *thus* $m = 20$. *Therefore, we should give this consumer an extra* £10 *(from* $m = 10$ *to* $m = 20$*) so that they attain the same level of utility as before the price change. Thus* $CV = -£10$.

Let us determine what level of income m *would give the consumer a utility level of* 2.5, *before the price change (so when* $p_1 = 1$*). At these prices, recall that the optimality condition gives* $x_1 = x_2$ *and that the budget constraint is* $x_1 + x_2 = m$, *so combining combining both we get* $x_1 + x_1 = m$, *so the optimal quantity of good 1 is* $x_1 = m/2$ *and the optimal quantity of good 2 is* $x_2 = m/2$. *The utility of the consumer is therefore* $(m/2)^{0.5}(m/2)^{0.5} = m/2$. *We want to find* m *such that utility is equal to* 2.5, *that is* $m/2 = 2.5$, *thus* $m = 5$. *Therefore, we should take* £5 *from this consumer (from* $m = 10$ *to* $m = 5$*) to give them the same utility as after the price change. Thus* $EV = -£5$.

2 Producer theory

In this chapter, we will study the firm's behavior, an entity that combines inputs to produce output. To help us model its behavior, we will make two key assumptions:

1. The firm's objective is to maximize its profit.

2. The prices at which the firm can buy inputs and sell output are given.

In this framework, the firm's behavior can be summarized as:

- The firm chooses how to produce.

- The firm chooses how much to produce.

Outline:

1. Technology. To produce a certain amount of output, the firm faces technological constraints (which combinations of inputs can produce the desired level of output?). We will introduce the production function, a convenient way to summarize these technological constraints.

2. Cost minimization. Technological constraints lead to economic constraints. Indeed, inputs are costly to use (they have a price!). Our profit maximization assumption implies that the firm will choose inputs so as to minimize the cost of producing the desired level of output. We will introduce the cost function, a convenient way to summarize these economic constraints. With these two chapters, we have dealt with "how to produce".

3. Cost function. There is a whole family of cost functions and cost curves. We will study their properties in this chapter.

4. Profit function and the firm's supply. The firm chooses the amount of output that maximizes its profit. The price at which it sells output is given by the market (market constraints). With this chapter, we have dealt with "how much to produce".

Key ideas & concepts

production function, isoquant, perfect substitutes, perfect complements, marginal product, returns to scale, technical rate of substitution, cost minimization, cost function, marginal cost, average cost, profit, prodit maximization, supply.

2.1 Technology

The firm

A *firm* produces *output(s)* from various combinations of *inputs* using technologies. A *technology* is a process by which inputs are converted to output. Overall, *technology* dictates which combinations of inputs and outputs are feasible.

Factors of production

Inputs are sometimes called *factors of production*. These inputs fall into three board categories:

1. Labour: workers, etc. Often denoted L.
2. Materials: natural resources, raw goods, processed products, etc.
3. Capital: land, buildings, equipment, etc. Often denoted K.

A bundle of n inputs is denoted as a vector $x = (x_1, ..., x_n)$, where x_1 is the quantity of factor 1, x_2 the quantity of factor 2, ..., x_n the quantity of factor n. Throughout this course, we will stick to the case where $n = 2$, that is, there are only two factors of production. Therefore, a bundle of inputs will be a vector (x_1, x_2).

EXAMPLE*. You combine hours doing exercises (E) and hours reading books (B) to get a grade at the final exam in economics. PaperInc. combine hours of work (L) and machines (K) to produce paper. Farmers use hours of work (L) and machines and land (K) to produce corn.*

The production function

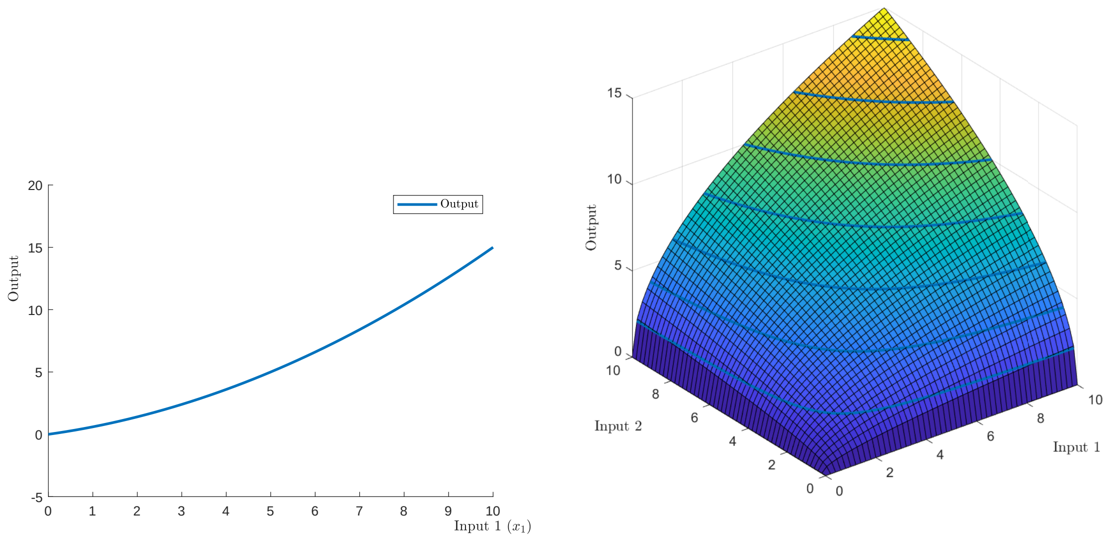
The simplest way to describe the technology of a firm is the *production function*. The production function measures the maximum amount of good the firm can produce from using a given combination of inputs. The production function for a firm using two inputs is

$$y = f(x_1, x_2) \tag{5}$$

where y is the maximum amount of output that can be produced using x_1 units of input 1 and x_2 units of input 2.

EXAMPLE*. The grade production function could be something like* $f(E, B) = 10B + 10E^{0.5}$. *This tells us that if you spend 3 hours per week reading books and 9 hours per week doing exercises, your final grade is a* 60. *Let's say the production function for beer is* $f(L, K) = \frac{3}{2}L^{0.5}K^{0.5}$ *where L the number of hours of work supplied by workers and K is the number of distillation machines used. With 4 machines and 4 hours of work we get* $\frac{3}{2}\sqrt{4}\sqrt{4} = 6$ *litres of beer. This production function is represented graphically in the right panel of figure 15.*

Figure 15: Producer theory: two production functions.



Note: On the left, the production function is $f(x_1) = \frac{1}{2}x_1 + \frac{1}{10}x_1^2$. On the right, the production function is $f(x_1, x_2) = \frac{3}{2}\sqrt{x_1 x_2}$.

Properties of technology

It is common to assume certain properties about technology. A well-behaved technology meets these two properties:

- *Monotonicity*: if the amount of at least one input is increased, output will (weakly) increase.
- *Convexity*: if two production techniques (x_1, x_2) and (z_1, z_2) both produce y units of output, then a weighted average of these two production techniques will produce at least y units of output.

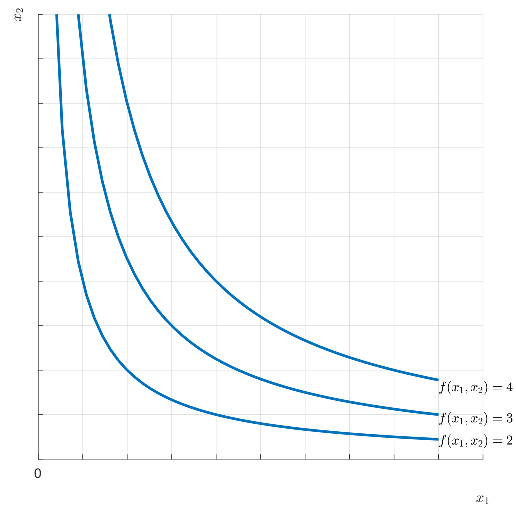
Isoquants

An *isoquant* is the set of all possible combinations of inputs that yield the same amount of output. The isoquant is an important conceptual tool and is very similar to the indifference curves for consumers. Isoquants are very handy when there are only two inputs because they convey the same information as the production function but can be represented in a 2-D space (see figure).

EXAMPLE. *Say you want to achieve* 75 *points at the final exam in economics. To "produce" this grade, you can use two inputs. Input 1 is the number of hours you spend doing exercises (E). Input 2 is the number of hours you spend reading books (B).*

The technology available to you is such that you can get 75 *points by spending 20 hours on exercises and 20 hours reading the textbook. But of course, you could get the*

Figure 16: Producer theory: isoquants.



Note: In this figure, the production function is $f(x_1, x_2) = \sqrt{x_1 x_2}$.

same result by spending 35 hours on exercises and 10 hours on reading. You could get yet again the same grade by spending only 10 hours on exercises and 30 hours on reading. By connecting these bundles together by a curve, we have just constructed the isoquant for getting a 75 *in the final exam!*

Given the assumptions that we made on technology, isoquants will have the following properties:

- The farther an isoquant is from the origin, the greater the level of output. This comes from the monotonicity assumption on technology, which means that production is an increasing function of input: if we increase the amount of some input (all else equal), we should be able to produce at least as much good as before.
- Isoquants do not cross.
- Isoquants slope downward.
- Isoquant curves are convex (by the convexity assumption)

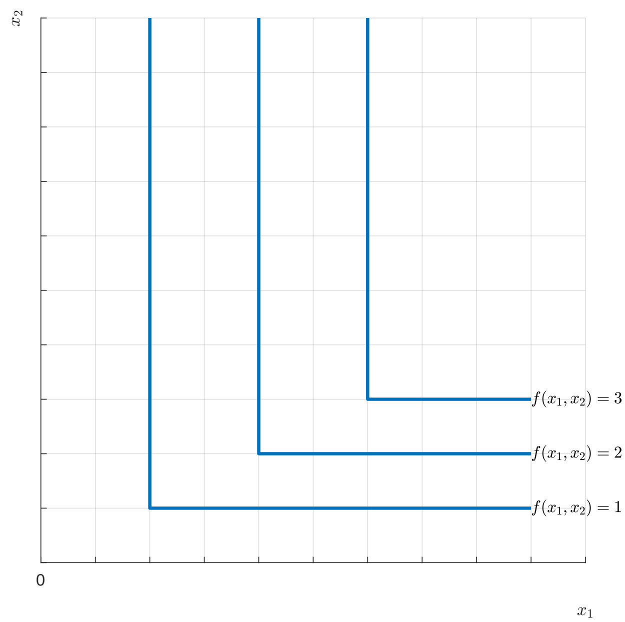
GOING FURTHER. *To see how the isoquants relates to the production function, recall that the isoquant corresponding to a certain level of ouput, say $y^\star$, is the set of all points (x_1, x_2) such that*

$$f(x_1, x_2) = y^\star$$

Substitutability

The curvature of the isoquant shows how easily firms can substitute one input for another while producing the same amount of output. The following figures 17 and 18 show two extreme cases when inputs are *perfect substitutes* (isoquants are straight lines) or *perfect complements* (isoquants are L-shaped).

Figure 17: Producer theory: perfect complements.



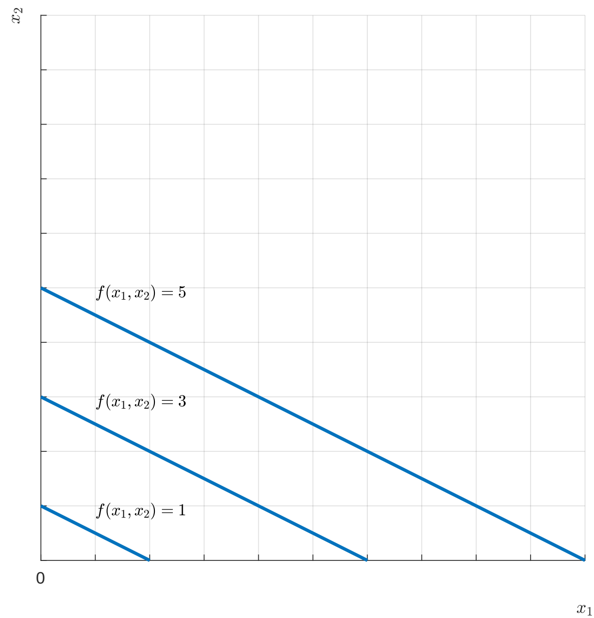
Note: In this figure, the production function is $f(x_1, x_2) = \min\{\frac{1}{2}x_1, x_2\}$.

EXAMPLE. *Go back to the grade production example. We see that hours spent on exercises and hours reading the book are imperfect substitutes/complements. Indeed, we can always substitute one input for the other to get the same grade, but not always at the same rate.*

However, the ingredients to produce a burger are perfect complements. Let's say we need two buns (BUN) and a steak (S) to produce a burger. With three buns and a steak, the maximum number of burgers we can produce is still one. With two buns and two steaks, the maximum number of burgers we can produce is still one, but with four buns and two steaks we can produce two burgers. What's important is that we need two buns for each steak we use to increase the number of burgers produced. The production function is $f(BUN, S) = min\{\frac{1}{2}BUN, S\}$.

One example of perfect substitutes could be robots (K) and humans (L). For example, two humans can substitute for one robot in producing some output. In that case, the production function would be $f(L, K) = L + 2K$.

Figure 18: Producer theory: perfect substitutes.



Note: In this figure, the production function is $f(x_1, x_2) = \frac{1}{2}x_1 + x_2$.

Marginal product

We are already familiar with the concept of marginal utility in consumer theory. There is a similar one in producer theory, which is called marginal product. The *marginal product* with respect to input 1 is how much extra output we can produce from increasing input 1 by a little bit, holding all other inputs fixed. In mathematical terms, the marginal product of factor 1 is

$$MP_1(x_1, x_2) = \frac{\partial f(x_1, x_2)}{\partial x_1} \tag{6}$$

and the marginal product of factor 2 is

$$MP_2(x_1, x_2) = \frac{\partial f(x_1, x_2)}{\partial x_2} \tag{7}$$

Usually, we will study production technologies for which marginal products are diminishing. This means that more and more of a single input produces more output, but at a decreasing rate. Note that when we talk about the marginal product with respect to one input, we hold all other inputs constant!

EXAMPLE. *Let's freeze input 2 (hours reading book), and fix it to 10 hours. Assume you decide to spend 1 hour doing exercises. By how much will your grade* *increase if you decide to spend one more hour doing exercises (from 1 to 2 hours)? Probably quite a lot, perhaps 5 points. Now, say you decide to spend 50 hours on exercises. By how much will your grade increase if you decide to spend one more hour on exercises (from 50 to 51 hours)? Probably not much, perhaps a point. That's because 50 hours on exercises is already plenty, and spending one more hour won't help you much. Conversely, if you spend only one hour doing exercises, one more hour could save you at the final exam. That's because this additional hour will allow you to cover a lot of new material.*

EXAMPLE. *The following example is borrowed from Perloff, p. 211. In the 18th century, Malthus predicted that population would grow faster than food production, leading to mass starvation. The reasoning was that because land is finite, the marginal product of labor would be decreasing, making it harder and harder to nourish the growing population. But remember that when talking about marginal product of an input, we keep all other inputs fixed. The marginal product of labour may rise indefinitely if the quantity of the other inputs is changing as well or if new inputs are used (like machines, fertilizers, biotechnologies).*

Returns to scale

The concept of *returns to scale* describes how output changes in response to a proportional change in all of the inputs.

To understand the concept of returns to scale, remember the following example: by how much is output going to increase if we double the quantity of all inputs? If output doubles as well, then we have constant returns to scale. If output more than doubles, then we have increasing returns to scale. If output less than doubles, then we have decreasing returns to scale.

GOING FURTHER. *Assume the quantity of all inputs is multiplied by k (so that $k = 2$ means we double all inputs). We have a new inputs bundle (kx_1, kx_2) and output is $f(kx_1, kx_2)$. Three cases are distinguished:*

1. *If $f(kx_1, kx_2) = kf(x_1, x_2)$, then the production is said to exhibit constant returns to scale. Doubling the amount of inputs doubles the output.*

2. *If $f(kx_1, kx_2) > kf(x_1, x_2)$, then the production is said to exhibit increasing returns to scale. Doubling the amount of inputs more than doubles the output.*

3. *If $f(kx_1, kx_2) < kf(x_1, x_2)$, then the production is said to exhibit decreasing returns to scale. Doubling the amount of inputs less than doubles the output.*

Note that in many examples, the production function will exhibit increasing returns to scale for low amounts of inputs/output and decreasing returns to scale for large amounts of inputs/output. Increasing returns to scale may come from: better specialization of workers and equipment as the firm grows ; technology itself ; the indivisibility of some inputs becomes less problematic. Decreasing returns to scale may come from: organisational inefficiencies as the firms grows too large ; technology itself.

EXAMPLE. *In small startups, employees will frequently take on multiple roles within the firm. An engineer will probably be doing finance and HR tasks for example. This is inefficient ; the engineer should be doing engineering tasks only. As the firm becomes larger, it can hire people specialized in HR or finance, and thus the firm becomes more efficient. But if the firm becomes too big, there may be some organizational inefficiencies.*

The technical rate of substitution

Let's say the firm uses x_1 units of input 1 and x_2 units of input 2 to produce y units of output. Say the firm decides to decrease the amount of input 1 used by one unit. By how much should the firm increase the amount of input 2 used to still be able to produce y units of output? This is exactly what the *technical rate of substitution* (TRS) tells us.

Graphically, in the case with two inputs, the technical rate of substitution is the slope of the isoquant. Formally, the technical rate of substitution is equal to the ratio of the marginal products of the inputs (see note). That is:

$$TRS(x_1, x_2) = \frac{dx_2}{dx_1} = -\frac{MP_1(x_1, x_2)}{MP_2(x_1, x_2)} \tag{8}$$

GOING FURTHER. *Let's derive the formula for the TRS. Remember that we want to find variations in quantities of input 1 and 2 (dx_1, dx_2) such that output remains constant, that is*

$$MP_1(x_1, x_2)dx_1 + MP_2(x_1, x_2)dx_2 = 0$$

hence

$$TRS(x_1, x_2) = \frac{dx_2}{dx_1} = -\frac{MP_1(x_1, x_2)}{MP_2(x_1, x_2)} \tag{9}$$

The TRS is usually decreasing (this comes from the convexity assumption). This means that the more of input 1 the firm is using, the more difficult it is to replace input 2 with input 1 to maintain output constant. Similarly, the more of input 2 the firm is using, the more difficult it is to replace input 1 with input 2 to maintain output constant

EXAMPLE. *Go back to the grade production example. Say you have chosen the input bundle $(20, 20)$ that allows you to get 75 points at the exam. But now you decide* *to decrease the amount of time spent on exercises by one hour (from 20 to 19). The TRS will tell you how much more time you should spend reading books to still be able to get a* 75*. Perhaps it will be one hour, perhaps half an hour, or perhaps 5 hours: it depends on your technology!*

2.2 Cost minimization

Rationale

Firms maximize their profits. This assumption implies that for any given level of output y, firms minimize the cost of producing that output. If it is not the case, then there exists a cheaper way of producing y, meaning the firm is not maximizing its profit - a contradiction! Therefore, *cost minimization* is a key objective of the firm.

Digression on costs

The *explicit cost* of a good or resource is the direct, out of pocket, payment for acquiring that good or resource.

The economic or *opportunity cost* of a good or resource is the value of the best alternative of that good or resource.

Sunk costs are past expenditures on a good or resource that cannot be recovered.

EXAMPLE. *When entering the classroom, I could charge you £5. This would be the explicit cost of the lecture. There is also an opportunity cost, which is the value of the best alternative use of your time: reading a book, go to the pub, watching a movie. To illustrate the concept of sunk costs, consider the following example. Assume there is a firm that is selling 10000 units of some good at price of £10. On Day 1, the firm acquires Technology 1 for £10,000, which allows the firm to produce the good at a unit cost of £5. The firm does not have the time to produce the good that on the next day, Day 2, Technology 2 is introduced. It can be acquired for £20,000 and allows the firm to produce at a unit cost of £1. What should the firm do? It does not matter that the firm has already bought Technology 1: the firm should buy Technology 2 because this will increase its profit (compute it!). The fact that the firm has bought Technology 1 the day before is irrelevant: this cost is sunk.*

Factor prices

The firm can buy factors of production at fixed prices. In this chapter, we assume that there are two factors of production, x_1 and x_2, whose prices are w_1 and w_2 respectively. These prices are called *factor prices*.

EXAMPLE. *A firm producing beer can buy machines at a unit price of* $w_K = £50$, *while workers are paid at a hourly wage of* $w_L = £10$.

Isocost

The cost of using x_1 units of input 1 and x_2 units of input 2 is

$$w_1x_1 + w_2x_2$$

To find all the combinations of inputs (x_1, x_2) that have some given level of cost, say C, we write

$$w_1 x_1 + w_2 x_2 = C \tag{10}$$

which the isocost line associated to the level of cost C. In other words, the *isocost* is the set of all input bundles that have the same cost. See figure 19.

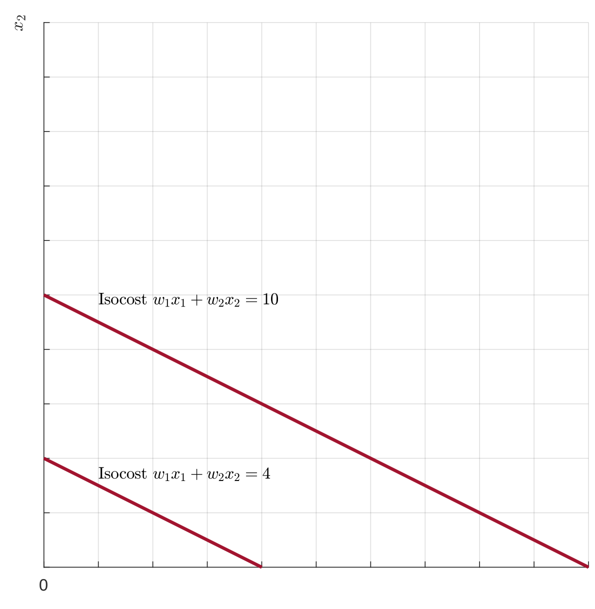
EXAMPLE. *For a cost of $C = £200$, a beer producer can buy 4 machines and 0 hours of work from workers ; or 3 machines and 5 hours of work ; ... ; or 0 machine and 20 hours of work. If you plot all these input bundles in the (L, K) plane, you will see the isocost line appear!*

Putting x_2 on one side of the equal sign and everything else on the other side, we get

$$x_2 = \frac{C}{w_2} - \frac{w_1}{w_2} x_1$$

The isocost line is a straight line with intercept $\frac{C}{w_2}$ and slope $-\frac{w_1}{w_2}$. Changing the prices of the inputs will change the slope of the isocost line. Changing the cost level will shift the entire isocost line: away from the origin for a higher cost level, towards the origin for a lower cost level.

Figure 19: Producer theory: isocost lines.



Note: In this figure, factor prices are $w_1 = 1$ and $w_2 = 2$. We consider two cost levels, $C = 4$ and $C = 10$.

The cost minimization problem

Firms minimize the costs of producing a given level of output. The firm solves the following *cost minimization problem*:

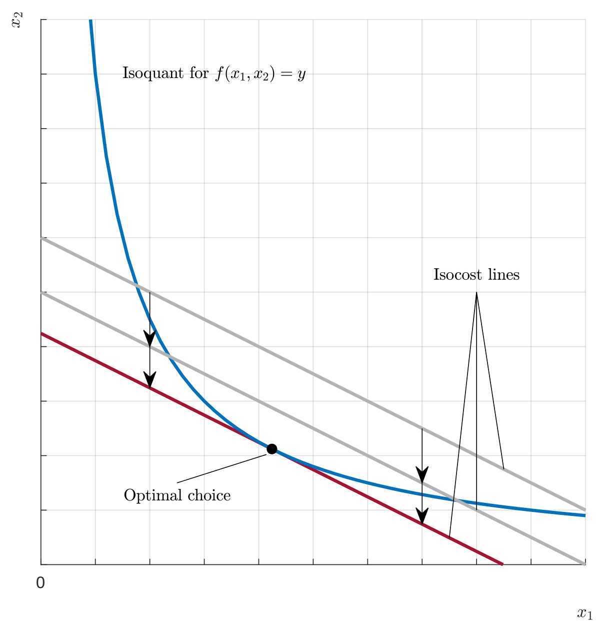
$$\min_{x_1,x_2} w_1x_1 + w_2x_2 \text{ subject to } f(x_1,x_2) = y \tag{11}$$

EXAMPLE*. Say we want to know how to optimally produce 10 bottles of beer. The production function is* $f(L,K)$*. To this end, they solve the cost minimization problem* $\min_{L,K} w_L L + w_K K$ *subject to the constraint* $f(L,K) = 10$.

Solving the cost minimization problem: geometric solution

The optimal input bundle is the input bundle that minimize production costs while allowing to produce y units of output. The optimal choice can be determined by finding the point on the isoquant that has the lowest associated isocost curve. This geometric solution to the cost minimization problem is illustrated figure 20.

Figure 20: Producer theory: optimal inputs choice.



Note: In this figure, the production function is $f(x_1,x_2) = \sqrt{x_1x_2}$ and the desired level of output is $y = 3$. Factor prices are $w_1 = 1$ and $w_2 = 2$.

At the optimal solution, the isocost line is tangent to the isoquant curve. It is important to note that this is not always true. For example, isoquants may have kinks ; or sometimes the optimal choice will be a boundary optimum (that is, the optimal choice of inputs involves using zero units of one of the goods), in which case tangency is not satisfied. However, if we rule out these two particular cases (kinky technologies and boundary optima), then tangency is a necessary condition for optimality. And if in addition isoquant curves are convex (curving away from the origin), then tangency is a sufficient condition: any point that satisfies the tangency condition must be an optimal point.

In mathematical terms, the tangency condition means that the slopes of the isoquant curve and the isocost line at the optimal point $(x_1^\star, x_2^\star)$ are equal, that is the technical rate of substitution must equal the factor price ratio:

$$-\frac{MP_1(x_1^\star, x_2^\star)}{MP_2(x_1^\star, x_2^\star)} = -\frac{w_1}{w_2} \tag{12}$$

EXAMPLE. *Assume that there are two factors of production x_1 and x_2 and that the production function is $f(x_1, x_2) = \sqrt{x_1}\sqrt{x_2}$. The factor prices are w_1 and w_2. Say we want to produce y units of output. We want to find a combination of inputs (x_1, x_2) that allows us to produce y units AND that minimizes the cost of producing y units.*

Thus we want x_1 and x_2 to be such that

$$\sqrt{x_1}\sqrt{x_2} = y$$

In addition, we know that at the optimal choice of inputs should satisfy the tangency condition, i.e.

$$\frac{MP_1(x_1, x_2)}{MP_2(x_1, x_2)} = \frac{w_1}{w_2} \tag{13}$$

Great! We now have two unknowns (x_1 and x_2) and two equations.

Let us compute the marginal products. We have $MP_1(x_1, x_2) = \frac{\sqrt{x_2}}{2\sqrt{x_1}}$ and $MP_2(x_1, x_2) = \frac{\sqrt{x_1}}{2\sqrt{x_2}}$. Using this new information, we see that the tangency condition tells us that

$$\begin{aligned} x_2 &= x_1 w_1/w_2 \textit{ or} \\ x_1 &= x_2 w_2/w_1 \end{aligned}$$

We can now use the requirement that $\sqrt{x_1}\sqrt{x_2} = y$ and substitute out x_1. We get $\sqrt{x_2 w_2/w_1}\sqrt{x_2} = y$, and thus

$$x_2 = \sqrt{w_1/w_2}y$$

Going back to $\sqrt{x_1}\sqrt{x_2} = y$, we could also substitute out x_2. We get $\sqrt{x_1}\sqrt{x_1 w_1/w_2} = y$, and thus

$$x_1 = \sqrt{w_2/w_1}y$$

These are the optimal choices of x_1 and x_2 when the factor prices are w_1 and w_2 and the quantity we want to produce is y. Anticipating the end of this chapter, these are *called the factor demand functions, denoted*

$$
\begin{aligned}
x_1(w_1, w_2, y) &= \sqrt{\tfrac{w2}{w1}} y \\
x_2(w_1, w_2, y) &= \sqrt{\tfrac{w1}{w2}} y
\end{aligned}
$$

GOING FURTHER. *Alternatively, here is the calculus analysis of the cost minimization problem. Recall that the problem of finding a cost-minimizing way of producing y units of output can be written:*

$$\min_{x_1,x_2} w_1x_1 + w_2x_2 \text{ subject to } f(x_1, x_2) = y$$

To solve this problem, we will use the method of Lagrange multipliers. First, we write down the Lagrangian

$$\mathcal{L} = w_1x_1 + w_2x_2 - \lambda(f(x_1, x_2) - y)$$

Second, we differentiate with respect to x_1, x_2 and λ to get the first order conditions

$$
\begin{aligned}
w_1 - \lambda \frac{\partial f(x_1, x_2)}{\partial x_1} &= 0 \\
w_2 - \lambda \frac{\partial f(x_1, x_2)}{\partial x_2} &= 0 \\
f(x_1, x_2) - y &= 0
\end{aligned}
$$

The last equation is simply the constraint $f(x_1, x_2) = y$. We can combine the first two equations to eliminate λ to get

$$\frac{w_1}{w_2} = \frac{\frac{\partial f(x_1,x_2)}{\partial x_1}}{\frac{\partial f(x_1,x_2)}{\partial x_2}}$$

You recognize the tangency condition that we derive earlier using a geometric argument. We now have two equations (the tangency condition and the constraint) and two unknowns, x_1 and x_2. A bit of algebra (see previous example) should allow us to recover the optimal choice of x_1 and x_2.

Conditional factor demand functions

The optimal choices of inputs that minimize costs are a function of the factor prices and of the desired level of output. We call these functions the *conditional factor demand functions*, and write these as

$$
\begin{aligned}
&x_1(w_1, w_2, y) \\
&x_2(w_1, w_2, y)
\end{aligned}
$$

EXAMPLE. *Using the conditional factor demand functions, a beer producer know they it should buy $x_L(w_L, w_K, 10)$ units of labour and $x_K(w_L, w_K, 10)$ units of capital if it wants to produce* 10 *bottles of beer at the minimum cost.*

The total cost function

The smallest total cost of producing output level y is therefore

$$c(w_1, w_2, y) = w_1 x_1(w_1, w_2, y) + w_2 x_2(w_1, w_2, y)$$

The function $c(w_1, w_2, y)$ is called the *total cost function*.

Graphically, the total cost function can be derived using the expansion path. The *expansion path* is the curve that goes through the tangency points between isocosts and isoquants. The points on the expansion path are the cost minimizing combinations of labor and capital for each output level.

2.3 Cost function

The total cost function

In the previous chapter, we showed how firms choose inputs optimally so as to minimize the cost of producing a given amount of output. We can now determine the cost of producing any level of output. Recall that the conditional factor demand functions are denoted $x_1(w_1, w_2, y)$ and $x_2(w_1, w_2, y)$. The total cost function is written as

$$c(w_1, w_2, y) = w_1 x_1(w_1, w_2, y) + w_2 x_2(w_1, w_2, y)$$

The total cost function measures the minimal cost of producing a given level of output (given factor prices).

For the remainder of this chapter, we will keep the factor prices fixed, so we can drop them from the notation. The total cost function is simply written

$$c(y)$$

Short-run vs. long-run

So far, we have been implicitly reasoning in the *long-run*: all the inputs (here, x_1 and x_2) can vary freely. In the *short run* however, some factors of production are *fixed* (as opposed to *variable* factors of production, which can be adjusted freely).

EXAMPLE. *In the short-run, a beer producer can freely adjust the number of machines and hours of work they use to produce beer. However, it is not the case for the number of distilleries in which they operate: this number is likely to be fixed in the short-run. In the long-run however, they can find and buy new places to produce even larger volumes of beer.*

GOING FURTHER. *Cost minimization in the short run, with fixed inputs. Let's assume that in the short run, the firm cannot adjust freely the quantity of input 2. This quantity is fixed, and we will denote it $\bar{x_2}$. As before, the production function is $f(x_1, x_2) = \sqrt{x_1}\sqrt{x_2}$ and the factor prices are w_1 and w_2. The cost minimization problem is*

$$\min_{x_1} w_1 x_1 + w_2 \bar{x_2} \text{ subject to } \sqrt{x_1}\sqrt{\bar{x_2}} = y$$

There is no need to use the Lagrangian to solve this problem. The optimal amount of input 1 is unambiguously defined by the constraint, from which we get

$$x_1 = \frac{y^2}{\bar{x_2}}$$

As before, we can compute the cost function as

$$c(y) = w_1 \frac{y^2}{\bar{x_2}} + w_2 \bar{x_2}$$

In the short run, the cost function is the sum of two components: variable costs, which depend on the quantity of output produced (here $w_1 \frac{y^2}{\bar{x_2}}$ depends on y) and fixed costs, which do not depend on the quantity of output produced (here, $w_2 \bar{x_2}$ is fixed).

Costs functions

Whether we are looking at the firm's behavior in the short run or in the long run, the cost function of the firm can always be written as the sum of the variable costs, denoted $c_v(y)$, and the fixed costs, denoted F:

$$c(y) = c_v(y) + F \tag{14}$$

The *variable costs* of the firm

$$c_v(y)$$

are production costs that change with the level of output. Variable costs are costs that involve payments to factors the firm can freely adjust.

The *fixed costs* of the firm

$$F$$

are production costs that do not change with the level of output. In the short run, fixed costs are costs that involve payments to factors the firm cannot adjust (these includes expenditures on land, office space, production facilities). In the long run, all factors are variable and there are no fixed costs. The firm can always go out of business to produce zero output at zero cost. However, there may still be quasi-fixed factors (factors that must be used in a fixed amount, independent of the output, as long as output is positive: for example electricity for lighting) in the long-run. Thus, there may be quasi-fixed costs (costs that are independent of output but have to be paid as long as output is positive) in the long-run.

EXAMPLE. *For a beer producer, variable costs comes from buying machines and remunerating workers. Fixed costs may come from paying the rent for their distillery, say £500 a month, which they have to pay whether they produce 1 bottle of beer or 1,000 bottles.*

Now that we have a general expression for the cost function, we can define four important cost curves.

- The *average cost*

$$AC(y) = \frac{c(y)}{y} = \frac{c_v(y)}{y} + \frac{F}{y} \tag{15}$$

  is the cost per unit of output.

- The *average variable cost* function

$$AVC(y) = \frac{c_v(y)}{y}$$

  is the variable cost per unit of output.

- The *average fixed cost* function

$$AFC(y) = \frac{F}{y}$$

is the fixed cost per unit of output.

- The *marginal cost*

$$MC(y) = \frac{\partial c_v(y)}{\partial y} \tag{16}$$

measures the change in costs arising from a small change in output. It is the derivative of the cost with respect to output produced.

EXAMPLE. *Say the cost function of producing y bottles of beer is $c(y) = 1 + y^2$. In this example, the fixed cost is $F = 1$. The variable cost function is $c_v(y) = y^2$. The average cost, average variable cost and average fixed cost functions are, respectively, $AC(y) = \frac{1}{y} + y$, $AVC(y) = y$ and $AFC(y) = \frac{1}{y}$. The marginal cost is $MC(y) = 2y$.*

Costs functions: some properties.

These cost curves raise a few comments:

1. By definition, we have

$$AC(y) = AVC(y) + AFC(y)$$

2. The average fixed cost function is always decreasing with output (it's a constant divided by y)

3. The marginal cost curve lies below the average cost curve when the average costs are declining, and above when the average costs are increasing. The marginal cost curve lies below the average variable cost curve when the average variable costs are declining, and above when the average variable costs are increasing. Thus, the marginal cost curve passes through the minimum point of both the average variable cost and the average cost curves.

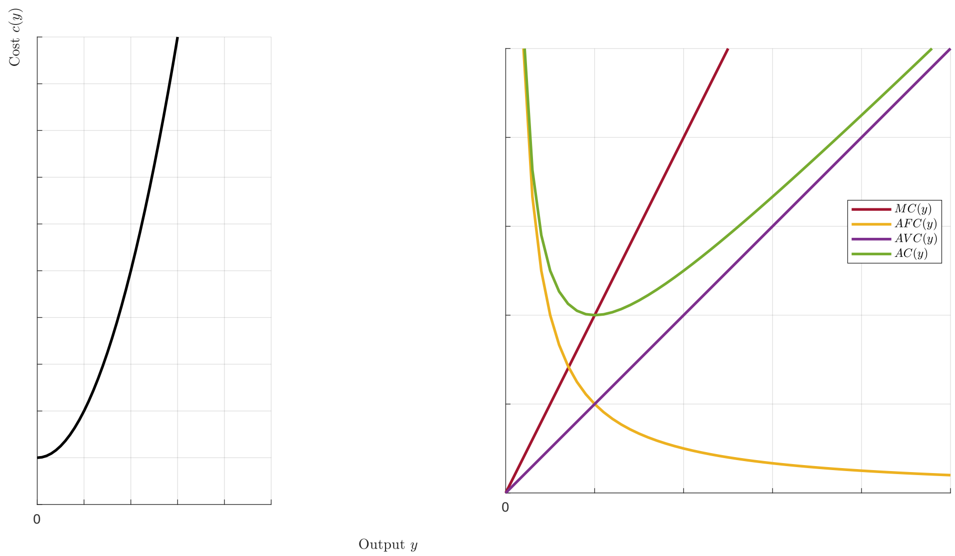
4. The marginal cost is in fact the marginal variable cost. Remember, fixed costs do not vary with output, so the derivative of the fixed cost with respect to y is zero.

EXAMPLE. *Consider once again the cost function of beer production*

$$c(y) = 1 + y^2$$

In figure 21, we plot the cost, the average cost, the average variable cost, the average fixed cost and the marginal cost.

Figure 21: Producer theory: cost functions.



Note: On the left is represented the cost function $c(y) = 1 + y^2$. On the right are represented the corresponding average cost, average variable cost, average fixed cost and marginal cost curves.

Costs in the short run

We said that in the short run, some factors of production are fixed, and therefore generate fixed costs. The cost function is called the short-run cost function, and measures the minimum cost to produce a given level of output, only adjusting the variable factors of production.

Costs in the long run

In the long run, all factors of production are variable. The cost function is called the long-run cost function, and measures the minimum cost to produce a given level of output, adjusting all the variable factors of production.

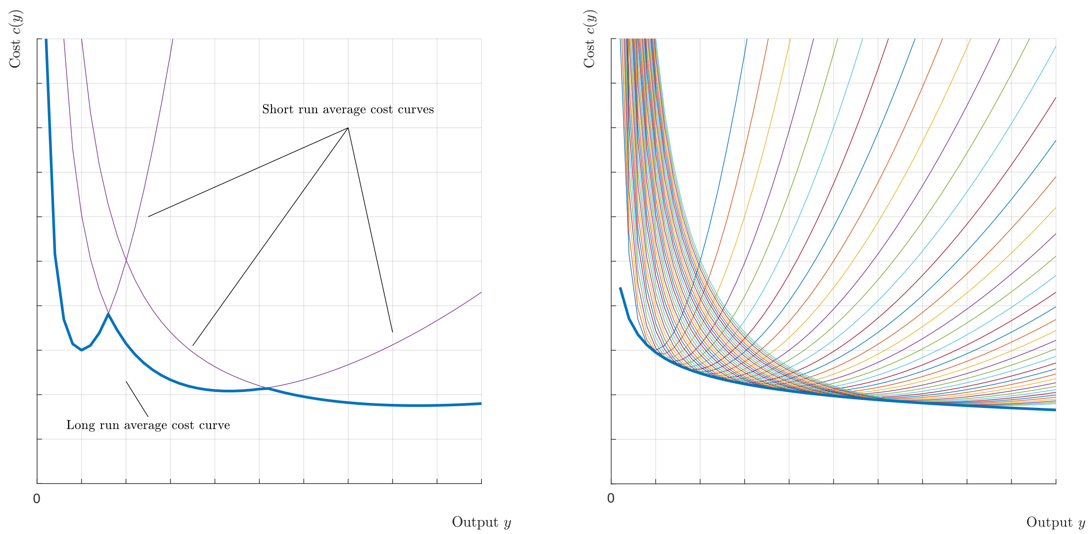
In the long run, it is particularly interesting to look at the shape of the average cost function. Whenever it is decreasing, the cost function is said to exhibit *economies of scale*. Whenever it is increasing, the cost function is said to exhibit *diseconomies of scale*.

A firm may benefit from economies of scale because when operating at a larger scale: workers may become more specialized ; the firm has extra flexibility ; the firm can negotiate lower input prices.

A firm may suffer from diseconomies of scale because when operating at a larger scale: factory space may become a problem ; the firm may become more complex and inefficient.

But more importantly, increasing returns to scale imply economies of scale and decreasing returns to scale imply diseconomies of scale. Remember that in the long

Figure 22: Producer theory: the long-run average cost curve.



Note: The production function is $f(x_1, x_2) = x_1^{\frac{1}{3}} x_2$. Input 2 is fixed to either $\bar{x_2} = 1$, or $\bar{x_2} = 3$ or $\bar{x_2} = 5$ which gives us the three short run average cost functions represented on the left. The long-run average cost function is the lower envelope of all the short-run average cost functions ("all" means three here, but on the right side I have plotted as many short-run average cost functions as I could, as well as the long-run average cost function).

run, all factors are variable. Therefore doubling the inputs will double the production cost. If there are increasing returns to scale, output will more than double. Thus, the average cost will fall. If there are decreasing returns to scale, output will less than double. Thus, the average cost will rise.

GOING FURTHER. *Recall that the long run total cost measures the minimum cost to produce a given level of output, adjusting all the variable factors of production. Thus the long run total cost must always be lower or equal to the short run total cost (precisely: it is equal to the short run total cost, when the fixed inputs have been set to their long run optimal values). In fact, graphically, the* long run total cost curve *is the* lower envelope *of all of the firm's short run total cost curve.*

The same reasoning applies for average costs: the average cost curve is the lower envelope of all of the firm's short run average cost curves. This is shown in figure 22.

2.4 Profit maximization and the firm's supply

So far, we have studied how firms produce a given amount of output. We now study how they decide on how much to produce. Recall the two fundamental assumptions we made at the beginning of this block: the market price is given (firms are price takers), and firms maximize their profits. Firms set their output where their profit is maximized. We will first consider the how firms maximize profit in general, before looking at the firm's supply in the short run and in the long run.

Perfect competition

We will assume that the market environment (the way firms interact with each other) is one of *perfect (or pure) competition*. A market is perfectly competitive is the following conditions are met:

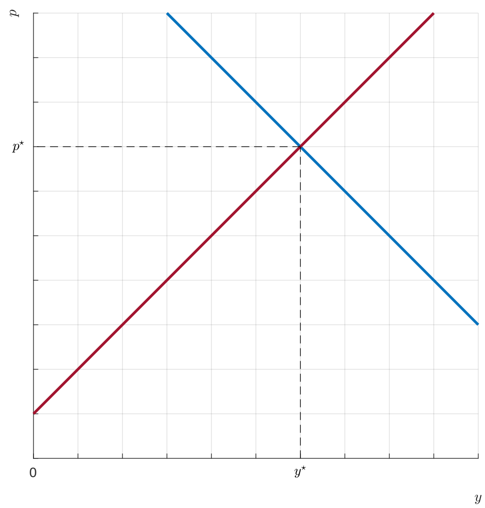
1. *Homogeneity* of the good: The goods sold on the market by firms are strictly identical
2. *Atomicity*: the number of buyers and sellers on the market is sufficiently large so that none of their decisions can influence the equilibrium price.
3. *Free entry*: firms can enter and exit the market freely
4. *Transparency*: all buyers and sellers know the price at which they can buy or sell the good.

One major implication of the perfect competition assumption is that both the buyers and sellers are *price takers*: the price is given ; buyers and sellers only worry about which quantity to buy or sell at this given price.

EXAMPLE. *Consider whisky production in the US in 1919. The market environment is roughly one of pure competition. Homogeneity of the good means that the whisky produced by Vito Corleone or by Al Capone is strictly identical (indistinguishable for consumers). Atomicity means that there are so many producers of whisky out there, that whatever Corleone chooses to produce cannot have an impact on the equilibrium price of whisky. Free entry is satisfied as we assume all is required to start producing whisky is a couple of machines, basic raw materials and hours of work. The equilibrium price of a litre of whisky is known to all consumers and producers so transparency is verified as well.*

As a reminder, the price is given by the equilibrium of market supply and market demand, as shown in figure 23. Market demand is given to us, and we will soon see how market supply is constructed from firms' individual supply (spoiler: it is the sum of all the individual firm's supply).

Figure 23: Producer theory: the firm's price-taking behaviour.



Note: The market demand function is $D(p) = 13 - p$ and the market supply function is $S(p) = -1 + p$. The equilibrium price is $p^\star = 7$. This price is given to firms in pure competition.

Profit

The *profit* from selling y units of output is

$$\pi(y) = py - c(y) \tag{17}$$

Profit is the difference between the firm's revenue py (which is price times quantity sold) and the firm's production cost (the cost of producing the quantity that is sold).

EXAMPLE. *Let's assume the price of a litre of whisky is £7. It is given by the market equilibrium. Corleone's cost function for producing whisky is* $c(y) = 1 + y^2$. *If Corleone chooses to produce and sell* 1 *bottle, he will make a profit of* $7 \times 1 - 1 - 1^2 =$ £5. *Producing and selling* 5 *bottles will net him a profit of* $7 \times 5 - 1 - 5^2 =$ £9. *In fact, using the tools introduced in the next paragraph, we can check that the amount of output that maximizes the profit is* 3.5 *bottles of whisky, in which case he makes a* £11.25 *profit.*

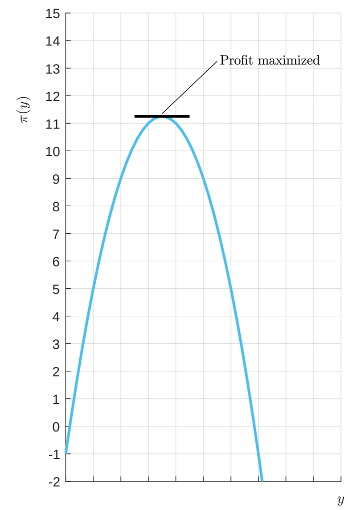
Profit maximization

Firms are price takers. They choose the level of output y that maximize their profits (see figure 24). They solve the *profit maximization* problem

$$\max_y \pi(y) = py - c(y) \tag{18}$$

subject to $y \geq 0$.

Figure 24: Producer theory: profit maximization.



Note: The firm chooses the level of output that maximizes its profit. Here, the cost function is $c(y) = 1 + y^2$. The price is given and equal to 7.

The first order condition tells us that the optimal level of output $y^\star$ should satisfy

$$p - \frac{\partial c}{\partial y}(y^\star)$$

In other words, the firm chooses output y such that the marginal cost at y is equal to the market price.

$$p = MC(y) \tag{19}$$

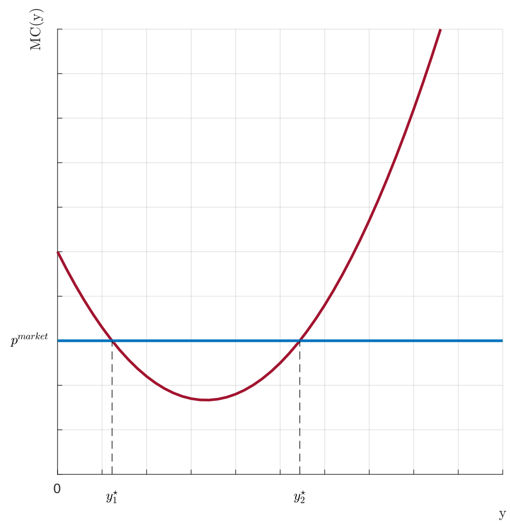
EXAMPLE. *Let's look at the intuition behind this result. Say Corleone is producing y units of output and the marginal cost at y is £6. The market price is $p =$ £7. Producing one more unit of good would cost £6 to Corleone, and he could sell this additional unit for £7. Thus he would increase its profit by £1 by producing and selling one more unit: so he should produce more of the good! Assume now that the market price falls to $p =$ £5. Remember that producing that last unit of output has cost Corleone £6. But now he can only sell it for £5, making a 1 dollar loss. Thus he would actually increase its profit by producing and selling one less unit: Corleone should produce less of the good!*

Firm's supply

The firm chooses the level of output y such that

$$p = MC(y) \tag{20}$$

Figure 25: Producer theory: necessary condition for profit maximization.



Note: There are cases (as illustrated in this figure) in which there are two levels of output, $y_1^\star$ and $y_2^\star$ for which $p = MC(y)$. However, it is clear that at $y_1^\star$, the firm is not maximizing profit!

Thus, the marginal cost curve of the firm is its *supply curve*. Indeed, for any given price, the above relationship $p = MC(y)$ defines which quantity of output y the firm will want to produce for any given price: this is a supply curve!

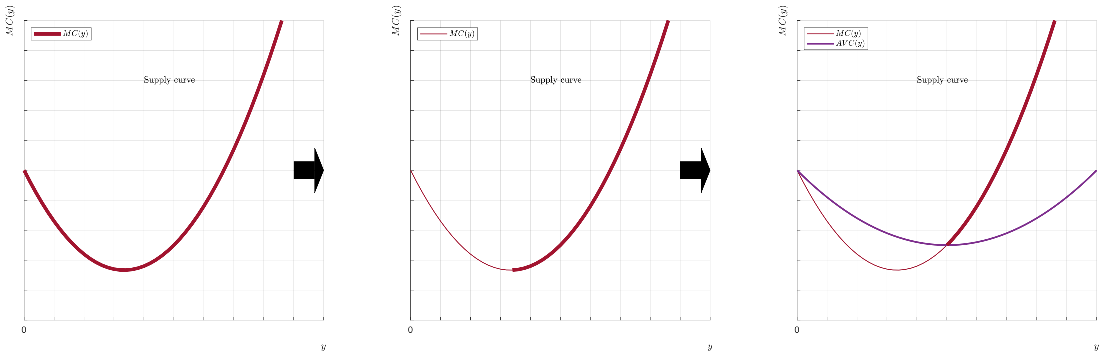
EXAMPLE. *The cost function of some firm is $c(y) = 1 + y^2$. The marginal cost function is $MC(y) = 2y$. Thus, for any given price p, the firm will choose to produce y units of output where y is such as $p = 2y$. In other words, for any price p, the firm will choose to supply $p/2$ units of good.*

However, we need to qualify the statement that the marginal cost curve is the supply curve of the firm. The condition $p = MC(y)$ is only a necessary condition for profit maximization. It is not a sufficient one. Thus, when constructing the supply curve, we must take care of the following issues:

1. Issue 1: multiple levels of output. It is possible that several levels of output satisfy the condition $p = MC(y)$, as illustrated in figure 25.

   However, it is clear the quantity of output y_1 does not maximize profit, because at this point, the marginal cost curve slopes downward: by increasing its output, the firm could produce a few additional units of output at a cost that is below the market price. Thus, the optimal level of output should be located where the marginal cost curve is increasing.

Figure 26: Producer theory: the firm's supply curve.



Note: This illustrates the construction of the firm's supply curve. The firm chooses its optimal level of output on the increasing part of the marginal cost curve, and above the average variable cost curve.

2. Issue 2: should the firm actually produce something? It is very well possible that the firm can maximize its profit (or rather minimize its losses) by not producing at all. If the firm produces nothing, its profit is

$$\pi(0) = p \times 0 - c_v(0) - F = -F$$

By producing y units of output, the firm gets a profit of $py - c_v(y) - F$. Thus, the firm should produce nothing at all if

$$-F > py - c_v(y) - F$$

that is, if

$$AVC(y) > p \tag{21}$$

Thus is called the *shutdown condition*, because if a point where price equals marginal cost is beneath the average variable cost curve, then the firm would be better off producing nothing at all. This is because selling the output at this price won't even cover for the variable costs. The implication of the shutdown condition is that the optimal output choice should be located on the portion of marginal cost curve that lies above the average variable cost curve.

Putting everything together, we now know that the firm chooses its optimal level of output on the increasing part of the marginal cost curve, and above the average variable cost curve, as shown in figure 26.

EXAMPLE. *With the cost function $c(y) = 1 + y^2$, we don't face either problems. The marginal cost curve ($MC(y) = 2y$) is always increasing, and always above the average variable cost ($AVC(y) = y$).*

Assume now that the cost function for whisky production is $c(y) = 1+5y-y^2+\frac{1}{10}y^3$. The marginal cost curve is $MC(y) = 5 - 2y + \frac{3}{10}y^2$. We can check that it is increasing *for output greater than or equal to* $20/6$*. The average variable cost curve is* $AVC(y) = 5 - y + \frac{1}{10}y^2$*. We know that the marginal cost curve cuts the average variable cost curve at its minimum. So above the quantity for which the average variable cost is minimal, we know that the marginal cost will be greater than the average variable cost. We can check that the average variable reaches its minimum for* $y = 5$*.*

Putting it all together, when the cost function is $c(y) = 1 + 5y - y^2 + \frac{1}{10}y^3$*, we know that Corleone's supply curve will be the marginal cost curve for output greater than or equal to* $20/6$ *and* 5*. So Corleone's supply curve will be the marginal cost curve for output greater than or equal to* 5*. See figure 26.*

The firm's supply in the short-run and in the long-run

The preceding analysis applies to the short-run: a competitive firm's short-run supply curve is the portion of its marginal cost curve that is upward sloping and that lies above its average variable cost curve.

Things are pretty similar in the long-run. The competitive firm's long run supply curve is the portion of its long-run marginal cost (the derivative of the long-run total cost function) that is upward sloping and that lies above its long-run average cost curve. Indeed, in the long run, a firm can always earn zero profit by going out of business, thus if a point where price equals long-run marginal cost curve is beneath the long-run average cost curve, then the firm would rather go out of business. This is consistent with the shutdown condition in the short run, since in the long run all costs are variable, and so the average variable cost curve and average cost curve are the same.

Market supply

In the short-run, the number of firms on the market is fixed. The short-run market supply curve shows the quantity supplied in the aggregate by all firms in the market for each possible market price. It is obtained by summing the individual firms' short-term supply curves (graphically, the market supply curve is obtained by summing horizontally the individual firms' short-term supply curves). Since each firm's supply curve is its marginal cost curve (with the two caveats that we explored before), the market supply curve tells us the marginal cost of producing the last unit supplied on the market. This is because each producer produces until the marginal cost of the last unit supplied is equal to the market price.

In the long-run, supply can vary as firms enter and exit the market. Thus, we cannot obtain the market supply curve by summing the individual firm long-run supply curves. If firms have identical costs, can freely enter or exit the market and if input prices are constant, then the long-run market supply is flat at minimum of the long-run average cost. This is because free entry drives the market price to the minimum level of long-run average cost. As long as the market price is above the minimum level of long-run average cost, firms in the market will earn a positive profit. This attracts new firms, and shifts the short-run supply to the right (market output expands), driving the market price down. Entry will continue until the price falls to the minimum level of long-run average cost (and profits are driven to zero). The equilibrium number of firm on the market in the long run will be such that the quantity supplied is equal to the quantity demanded at the equilibrium price.

In the long-run, competitive firms earn zero profit. This is a bit paradoxical, but it is important to note that the cost we use to define the profit is an opportunity cost: it includes the value of the next best investment the firm could make. Thus, if the firm earns zero profit, it only means that it earns the normal business profit that it could have gained by investing somewhere else.

3 Perfect competition

In this chapter, we will study supply and demand in a simplified framework. We will assume that the market, where buyers meet sellers, works in perfect competition and in isolation from other markets (partial equilibrium). We will then introduce and study a wide range of market interventions such as quotas, taxes, etc.

Outline:

1. Supply: our second ingredient is the supply curve. We will discuss some of its properties.
2. Demand: our first ingredient is the demand curve. We will discuss some of its properties.
3. Market: our last ingredient is the market, where supply meets demand. We will define perfect competition and partial equilibrium.
4. Comparative statics: now that we have our basic framework, we will play around with it and consider a wide range of market interventions such as taxes, quotas, and so on.

Key ideas & concepts

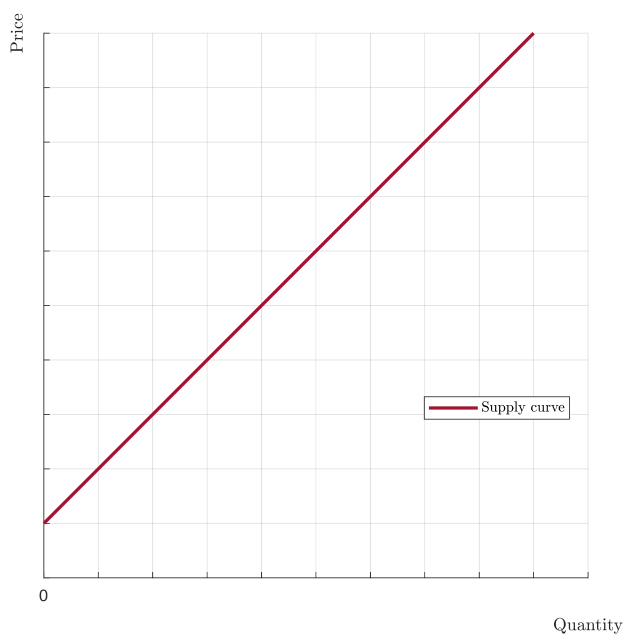
perfect competition, partial equilibrium, supply curve, demand curve, consumer surplus, producer surplus, total surplus, supply or demand shift, price ceiling and price floors, quota, taxes, deadweight loss, passing along of a tax, subsidy.

3.1 Supply

The supply curve

The *supply curve* for an individual firm i is denoted $S_i(p)$, and shows the quantity supplied by firm i at each possible price p. In producer theory, we studied how to construct these individual supply curves.

Figure 27: Perfect competition: the supply curve.



Note: This illustrates the supply curve $S(p) = c + dp$, where $c = -1$ and $d = 1$.

In perfect competition, there are many firms identical to firm i on the market. If there are n such firms, then the supply curve is simply

$$S(p) = \sum_{i=1}^{n} S_i(p)$$

i.e the sum of the individual supply curves. This is the "short run" supply curve because we assume that n is fixed (but large). A supply curve is shown in figure 27.

Elasticity of supply

The *price elasticity of supply* measures how the quantity supplied of a good changes with the price of that good. Formally, it is defined as

$$\epsilon_s = \frac{\partial S(p)}{\partial p} \frac{p}{S(p)}$$

In other words, the price elasticity of supply is the percentage change in supply in response to a given percentage change in the price.

Graphically (see figure 27), elasticity of supply is somehow related to how steep the supply curve is. The steepest the supply curve is, the less sensitive quantity supplied is to price, the smallest is the elasticity. When the supply curve is vertical, it is said that supply is perfectly inelastic (since quantity does not change with price). When the supply curve is horizontal, it is said that supply is perfectly elastic (since a small change in price would generate an infinite change in quantity).

Inverse supply

The demand and supply functions measure the amount of good consumers and firms are willing to buy or produce at a given price. Sometimes, it is useful to invert these relationships to get price as a function of quantity.

The *inverse supply* function

$$P_S(q)$$

is the price at which producers would be willing to sell q units of goods.

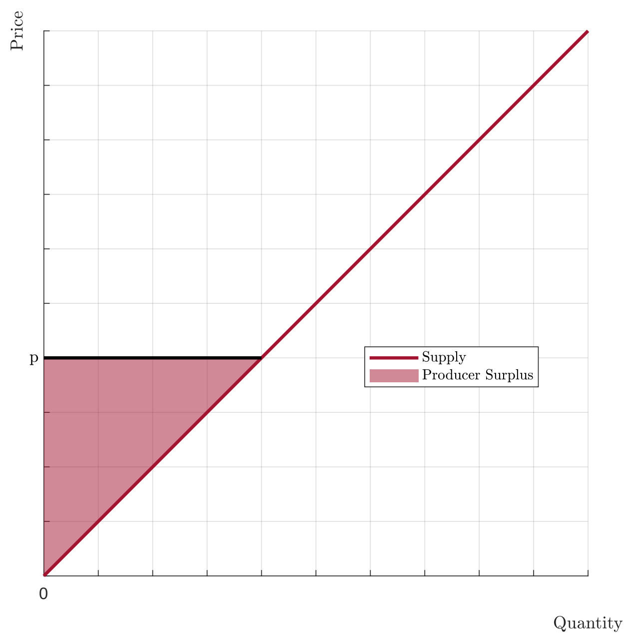
The inverse demand and supply curve are useful to define the consumer and producer surplus.

EXAMPLE. *Assume that the beer supply curve is* $S(p) = p - 1$. *Hence, the inverse supply function is* $P_S(q) = 1 + q$. *This tells us that the beer industry is willing to supply* 7 *(for example) bottles of beer at a minimum price of* $1 + 7 =$ *£*8. *Assume that the beer demand curve is* $D(p) = 10 - p$. *Hence, the inverse demand function is* $P_D(q) = 10 - q$. *This tells us that beer consumers are willing to pay* $10 - 6 =$ *£*4 *to buy* 6 *(for example) bottles of beer.*

Producers's surplus

The *producers' surplus* is the area above the inverse supply curve and below the market price, up to the quantity producers sell. This is represented in figure 28.

Figure 28: Perfect competition: the producers' surplus.



Note: The producers' surplus is showed as the filled area above the inverse supply curve and below the price. In this example, $S(p) = p$.

3.2 Demand

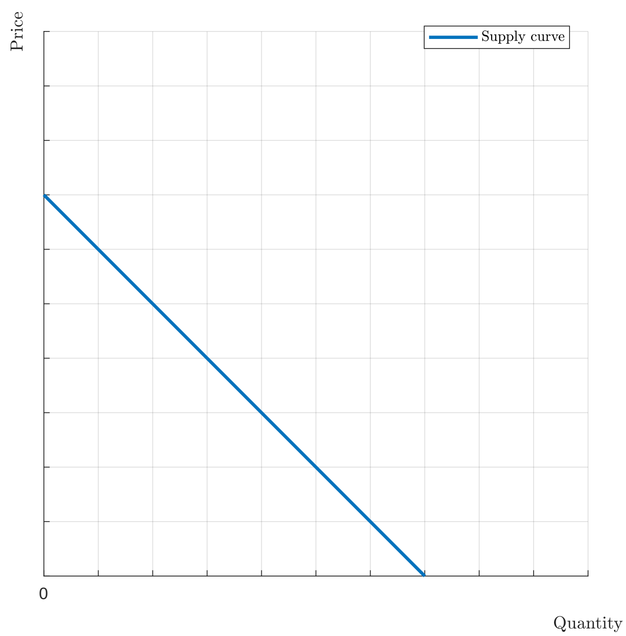
The demand curve

The *demand curve* for an individual consumer j is denoted $D_j(p)$, and shows the quantity demanded by consumer j at each possible price p. The market demand curve is the sum of all the individual demand curves, so that

$$D(p) = \sum_{j=1}^{m} D_j(p)$$

where m is the number of consumers on the market.

Figure 29: Perfect competition: the demand curve.



Note: This illustrates the demand curve $D(p) = a - bp$, where $a = 7$ and $b = 1$.

The demand curve measures the total quantity of good that consumers are willing to buy at price p. A demand curve is shown in figure 29.

Elasticity of demand

The *price elasticity of demand* measures how the quantity demanded of a good changes with the price of that good. Formally, it is defined as

$$\epsilon_d(y) = \frac{\partial D(p)}{\partial p} \frac{p}{D(p)}$$

In other words, the price elasticity of demand is the percentage change in demand in response to a given percentage change in the price.

Graphically (see figure 29), the elasticity of demand is somehow related to how steep the demand curve is. The steepest the demand curve is, the less sensitive quantity demanded is to price, the smallest is the elasticity. When the demand curve is vertical, it is said that demand is perfectly inelastic (since quantity does not change with price). When the demand curve is horizontal, it is said that demand is perfectly elastic (since a small change in price would generate an infinite change in quantity demanded).

The elasticity of demand depends on many things, in particular how many and how close substitutes to the good other goods are.

Inverse demand and inverse supply

The demand and supply functions measure the amount of good consumers and firms are willing to buy or produce at a given price. Sometimes, it is useful to invert these relationships to get price as a function of quantity.

The *inverse demand* function

$$P_D(q)$$

is the price at which consumers would be willing to buy q units of goods.

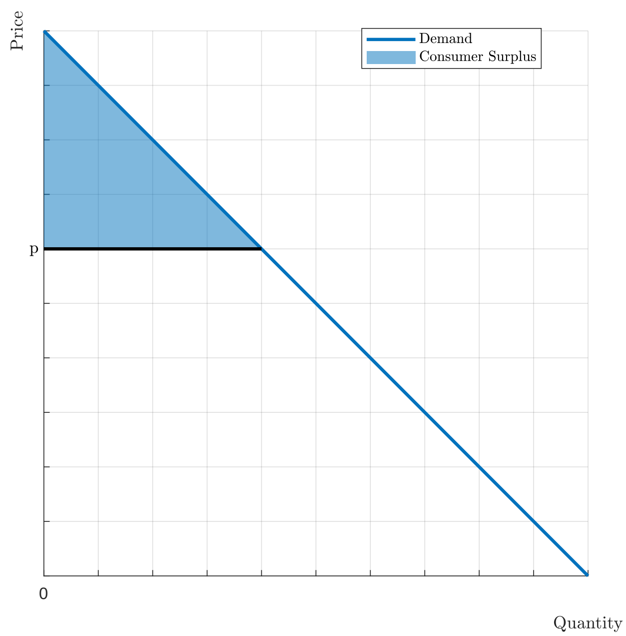
The inverse demand and supply curve are useful to define the consumer and producer surplus.

EXAMPLE. *Assume that the beer supply curve is* $S(p) = p - 1$. *Hence, the inverse supply function is* $P_S(q) = 1 + q$. *This tells us that the beer industry is willing to supply* 7 *(for example) bottles of beer at a minimum price of* $1 + 7 =$ £8. *Assume that the beer demand curve is* $D(p) = 10 - p$. *Hence, the inverse demand function is* $P_D(q) = 10 - q$. *This tells us that beer consumers are willing to pay* $10 - 6 =$ £4 *to buy* 6 *(for example) bottles of beer.*

Consumers' surplus

The *consumers' surplus* is the area below the inverse demand curve and above the market price, up to the quantity consumers buy. This is represented in figure 30.

Figure 30: Perfect competition: the consumers' surplus.



Note: The consumers' surplus is showed as the filled area below the demand curve and above the price. In this example, $D(p) = 10 - p$.

3.3 Partial equilibrium

The supply-and-demand framework we are building will be a very powerful tool to analyze a specific type of market environment: the so-called perfectly competitive markets. We will start by defining a perfectly competitive market and then move on to study equilibrium on such markets.

Perfect competition

A market environment is one of perfect competition if the following conditions are met:

1. Homogeneity of the good: The goods sold on the market by firms are strictly identical
2. Atomicity: the number of buyers and sellers on the market is sufficiently large so that none of their decisions can influence the equilibrium price.
3. Free entry: firms can enter and exit the market freely
4. Transparency: all buyers and sellers know the price at which they can buy or sell the good.

Partial equilibrium

By *partial equilibrium*, we mean that we will examining equilibrium and changes in equilibrium on one market only, in isolation from other markets. Later on, we will study general equilibrium, where markets are interconnected and what happens on one market can have repercussions on the other markets.

EXAMPLE. *Say the government decides to impose a quota on whisky production. We will study the impact of such a policy on the whisky market (partial equilibrium analysis). But one side effect of the policy is that many consumers may turn to substitutes to whisky, for example vodka. It is likely that the demand for vodka will increase. Thus the policy not only has an effect on equilibrium prices and quantities on the whisky market, but also on the vodka market.*

Equilibrium

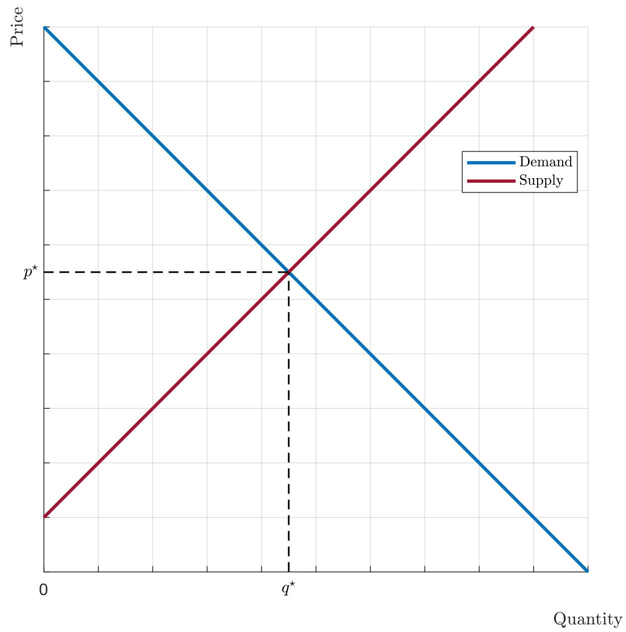
A market is in *equilibrium* when the total quantity supplied by firms is equal to the total quantity demanded by consumers.

The price for which supply equals demand is the equilibrium price. Formally, the equilibrium price $p^\star$ is such that

$$D(p^\star) = S(p^\star)$$

The equilibrium quantity of good produced (and consumed) is $q^\star = D(p^\star) = S(p^\star)$.

Figure 31: Perfect competition: market equilibrium.



Note: This figure illustrates the market equilibrium with supply curve $S(p) = c + dp$ ($c = -1$ and $d = 1$) and demand curve $D(p) = a - bp$ ($a = 10$ and $b = 1$) .

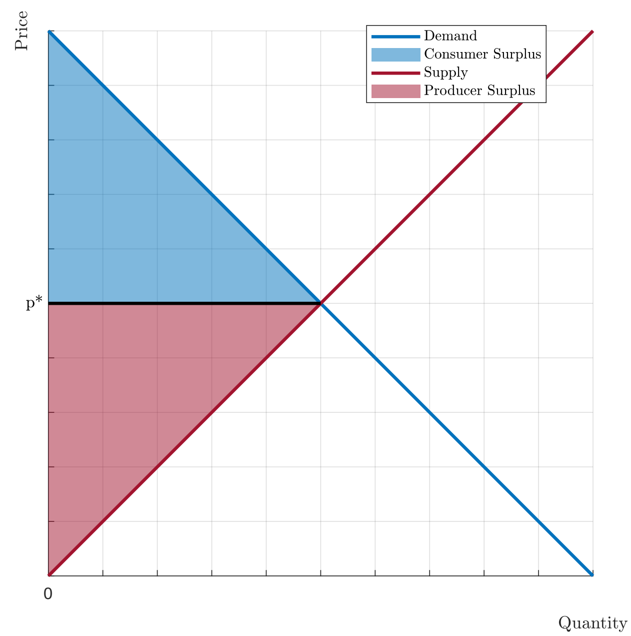
EXAMPLE. *Suppose that demand is* $D(p) = a - bp$ *and supply is* $S(p) = c + dp$. *The equilibrium price* $p^\star$ *is such that* $a - bp^\star = c + dp^\star$. *Thus, we can deduce that* $p^\star = \frac{a-c}{b+d}$ *and* $q^\star = \frac{ad+bc}{b+d}$. *See figure 31.*

Total surplus and efficiency

The *total surplus* is the sum of the consumers' and producers' surpluses. This is shown in figure 32.

A perfectly competitive market maximizes the total surplus. Perfect competition is therefore linked to a certain form of *efficiency*.

Figure 32: Perfect competition: the total surplus.



Note: The total surplus is the sum of the blue and red areas (the sum of the consumers' and producers' surpluses). In this example, $D(p) = 10 - p$ and $S(p) = p$.

3.4 Comparative statics

In the context of partial equilibrium analysis, *comparative statics* consist of studying the effect of changes in the market environment on the equilibrium prices and quantities. In the remainder of this section, we will be looking at:

- Supply shift
- Demand shift
- Price ceilings and price floors
- Quotas
- Quantity taxes
- Subsidies

Deadweight loss

In perfect competition, we saw that total surplus is maximized in equilibrium. Thus, market interventions will very often generate a *deadweight loss*: a net reduction in total surplus. The deadweight loss measures a loss due to inefficiency.

Supply shift

The supply curve may shift to the left or to the right. In the first case, less good is supplied at any price ; in the second case, more good is supplied at any price.

Policies that could shift the supply curve to the left include:

- Direct limitation of the number of firms on the market (licensing)
- Indirect limitation of the number of firms on the market (increase in the cost of entry or cost of exit)

As a consequence of a right (left) *supply shift*, the equilibrium price should decrease (increase) and the equilibrium quantity should rise (fall). See figure 33.

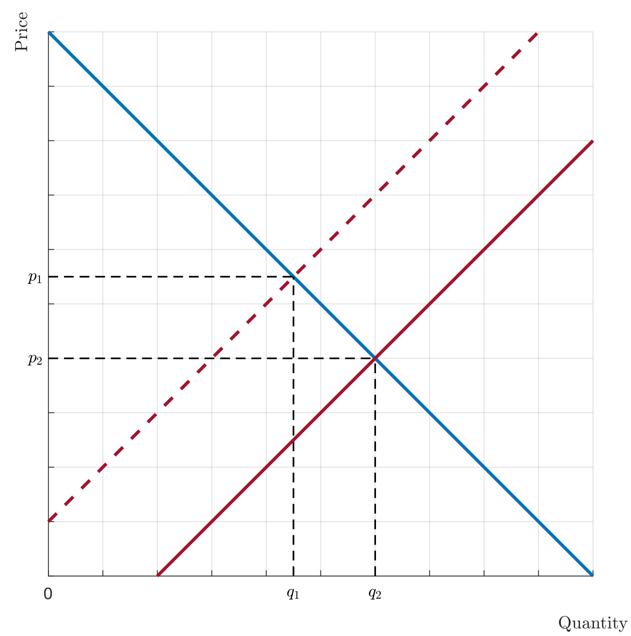
Demand shift

The demand curve may shift to the left or to the right. In the first case, less good is demanded at any price ; in the second case, more good is demanded at any price.

Policies that could shift the demand curve to the right include cash transfers to all consumers, a booming economy or policies aimed at strengthening the consumers's taste for the good.

As a consequence of a right (left) *demand shift*, the equilibrium price and quantity should rise (fall). See figure 34.

Figure 33: Perfect competition: supply shift.



Note: The figure illustrates a supply shift (from dashed to solid line: increased supply for all price levels). The equilibrium price decreases and the equilibrium quantity increases. In this example, $D(p) = 10 - p$ and $S(p) = 2 + p$ (solid) or $S(p) = -1 + p$ (dashed).

EXAMPLE. *Assume that a new machine allows all firms to produce beer at a lower cost. This would result in a supply curve to the right. Assume that the government is posting ads everywhere reminding consumers that alcohol consumption is potentially harmful. If the policy is effective in changing consumer's tastes, this will result in a demand curve shift to the left.*

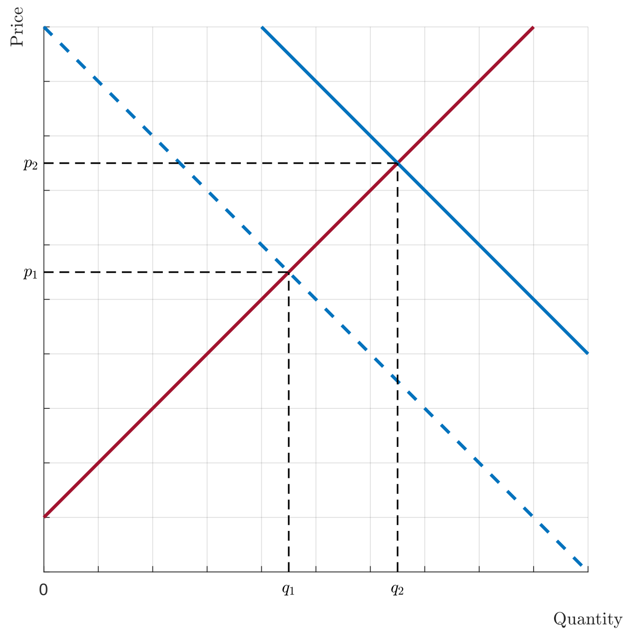
EXAMPLE. *Assume that the demand for beer in the UK is* $D(p) = 10 - p$ *while supply is* $S(p) = -1 + p$. *The equilibrium price is* $p^\star = 5.5$ *and equilibrium quantity is* $q^\star = 4.5$. *Now, the Queen posts on Instagram a picture of her drinking beer. Popular demand for beer immediately rises, so demand is now* $D(p) = 14 - p$. *The new equilibrium price is* $p^\star = 7.5$ *and equilibrium quantity* $q^\star = 6.5$. *See figure 34.*

Price ceiling and price floors

Let $p^\star$ and $q^\star$ be the equilibrium price and quantity in perfect competition.

A *price ceiling*, denoted $\bar{p}$, is the highest price at which consumers can legally buy the good. A price ceiling is such that $\bar{p} < p^\star$ (otherwise, it is useless!). Price ceilings generate excess demand and a deadweight loss. Figure 35 illustrate the impact of a price ceiling. In that figure, it is implicitly assumed that the consumers with

Figure 34: Perfect competition: demand shift.



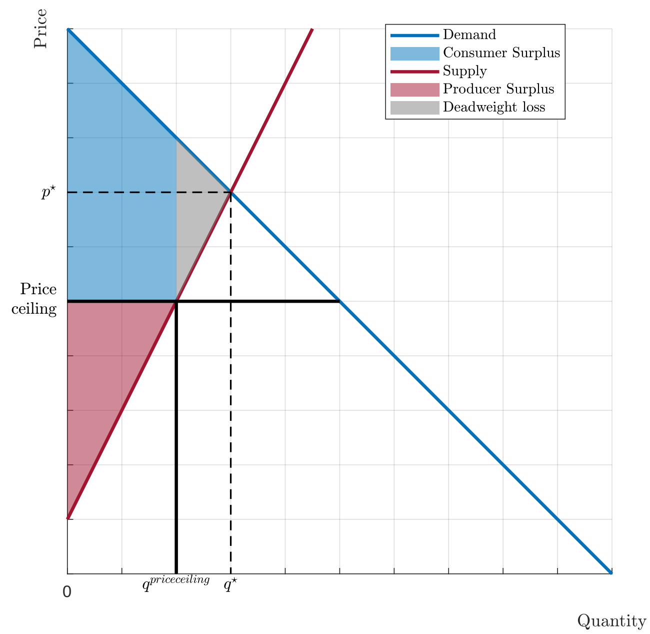
Note: The figure illustrates a demand shift (from dashed to solid: increased demand for all price levels). The equilibrium price and quantity increase. In this example, $S(p) = -1 + p$ and $D(p) = 14 - p$ (solid) or $D(p) = 10 - p$ (dashed).

the highest willingness to pay purchase all the available units of the good (this is a reasonable assumption if one assumes that consumers can easily resell the good to other consumers with a higher willingness to pay). Thus, the deadweight loss is likely underestimated in figure 35.

A *price floor*, or minimum price denoted $\underline{p}$, is the lowest price at which consumers can legally buy the good. A price floor is such that $\underline{p} > p^\star$ (otherwise, it is useless!). Price floors generate excess supply and a deadweight loss. Figure 36 illustrate the impact of a price floor. In that figure, it is implicitly assumed that the most efficient producers supply the good. Thus, the deadweight loss is likely underestimated in figure 36.

EXAMPLE. *Assume that the supply curve for beer is $S(p) = -0.5 + 0.5p$ and demand is $D(p) = 10 - p$, as in figure 36. The equilibrium price is $p^\star = 7$ and equilibrium quantity is $q^\star = 3$. With the linear supply and demand curves that we chose, it is quite easy to compute surplus. The consumer surplus is $\frac{3\times 3}{2} = 4.5$ and the producer surplus is $\frac{3\times 6}{2} = 9$. Introduce a price floor of $\underline{p} = 9$. At this price, the quantity bought by consumers is equal to $q^{floor} = 1$. The consumer surplus is now $\frac{1\times 1}{2}$, thus their surplus has decreased by 4. For producers, we see that their surplus is increasing by $(\underline{p} - p^\star) \times q^{floor} = 2$ (the red rectangle above $p^\star$). But at the same time, it is decreasing*

Figure 35: Perfect competition: price ceiling.



Note: The figure illustrates what happens when a price ceiling is introduced. Price ceilings generate excess demand and a deadweight loss. In this example, $D(p) = 10 - p$ and $S(p) = -\frac{1}{2} + \frac{1}{2}p$, and the price ceiling is 5. In this figure, it is implicitly assumed that the consumers with the highest willingness to pay purchase all the available units of the good.

by $\left(\frac{q^\star - q^{floor}}{2}\right) \times (p^\star - P_S(q^{floor})) = 4$ *(the grey rectangle to the right of* q^{floor} *and below* $p^\star$*). So overall the producer surplus is decreasing by* 2*.*

Quotas

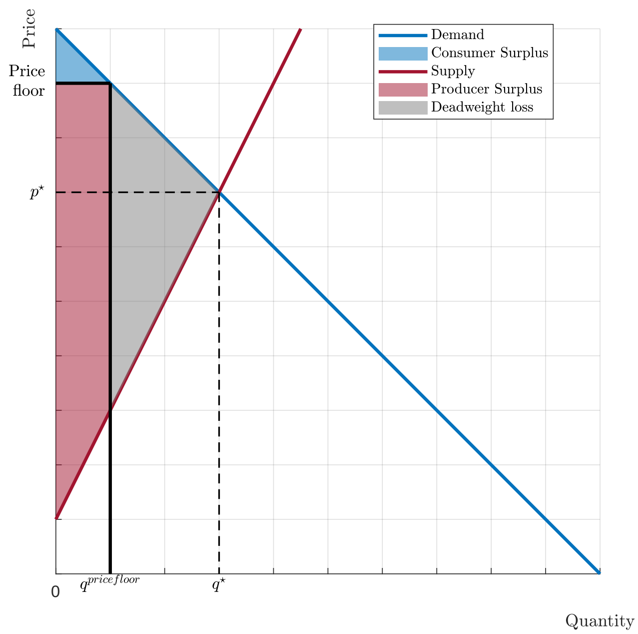
Instead of regulating prices, governments can regulate quantities by imposing quotas. A *quota*, q^{quota}, sets the quantity of good provided. Whenever $q^{quota} > q^\star$, the goal is to force firms to produce more of the good. Whenever $q^{quota} < q^\star$, the goal is to limit the quantity of good produced.

Very often, a quota is such that $q^{quota} < q^\star$. This is illustrated in figure 37. In this case, the quota generates a deadweight loss, and decreases the surplus of the consumers ; the producers' surplus usually increases. In that figure, it is implicitly assumed that the most efficient producers supply the good. Thus, the deadweight loss is likely underestimated in figure 37.

Quantity taxes

A *quantity tax* is a tax levied per unit of quantity bought or sold. Taxing the good introduces a wedge between the *demand price* (what the consumers pay, p_d) and the

Figure 36: Perfect competition: price floor.



Note: The figure illustrates what happens when a price floor is introduced. Price floors generate excess supply and a deadweight loss. In this example, $D(p) = 10 - p$ and $S(p) = -\frac{1}{2} + \frac{1}{2}p$, and the price floor is 9. In this figure, it is implicitly assumed that the most efficient producers supply the good.

supply price (what the producers receive, p_s).

Let t be the quantity tax. The demand and supply prices are related by the equation

$$p_s = p_d - t$$

Equivalently, we can obtain an expression for the demand price as $p_d = p_s + t$.

Equilibrium is characterized by the equation

$$D(p_d) = S(p_s)$$

To solve for the equilibrium demand and supply prices and equilibrium quantity, combine the equations $p_s = p_d - t$ and $D(p_d) = S(p_s)$ and solve

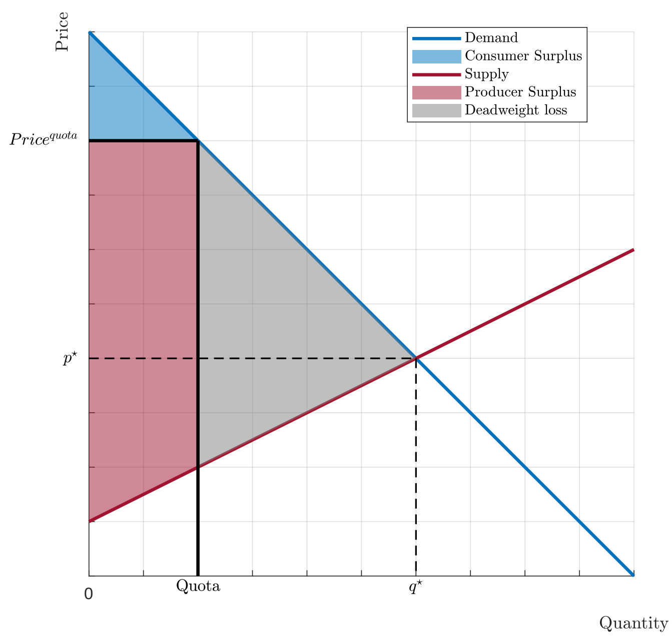
$$D(p_d) = S(p_d - t)$$

This is illustrated in figure 38, and in the following example.

EXAMPLE. *Suppose that demand for beer is $D(p) = a - bp$ and supply is $S(p) = c + dp$. A tax t is imposed on the good. Thus we have a supply price and a demand price such that $p_s = p_d - t$. In equilibrium, we must have $D(p_d) = S(p_s)$, that is $a - bp_d = c + dp_s$. Substituting out p_s, we solve*

$$a - b(p_s + t) = c + dp_s$$

Figure 37: Perfect competition: quotas.



Note: The figure illustrates what happens when a quota is introduced. The quota generates a deadweight loss. In this example, $D(p) = 10 - p$ and $S(p) = -2 + 2p$ and the quota is 2. In this figure, it is implicitly assumed that the most efficient producers supply the good.

We can deduce that in equilibrium, $p_s^\star = \frac{a-c-bt}{d+b}$ *and* $p_d^\star = \frac{a-c+dt}{d+b}$. *The equilibrium quantity is* $q^\star = D(p_d^\star) = S(p_s^\star)$.

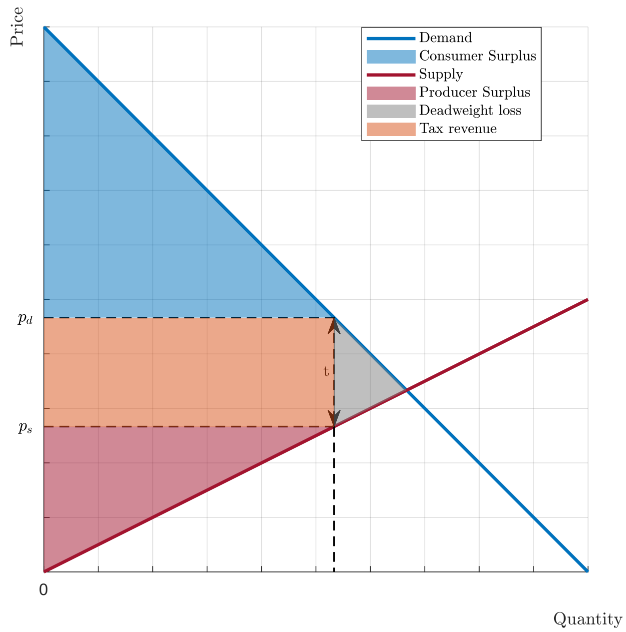
Taxes generate a deadweight loss, represented by the grey triangle in figure 38. The blue area represents the new consumer surplus, the red area is the new producer surplus, while the yellow rectangle represents the amount of money raised by the tax. This rectangle is the government surplus.

Passing along a tax

That a tax is required to be paid by producers does not mean that they will end up paying the full tax. Conversely, that a tax is required to be paid by consumers does not mean they will end up paying the full tax. In fact, a tax will generally increase the price paid by consumers and decrease the price received by producers: the tax is *passed along* to both the consumers and the producers.

How much is passed along to consumers and producers depends on the characteristics of the demand and supply curves. In particular, it depends on the *elasticity* of supply and demand. The steeper the demand (or supply) curve is, the less elastic it is. Demand (or supply) is said to be perfectly elastic if it is a horizontal line. Demand

Figure 38: Perfect competition: taxes.



Note: The imposition of a tax introduces a wedge t between the demand price p_d and the supply price p_s. Taxation generates a deadweight loss. Except in extreme cases, the tax is passed along to both the consumers and the producers. In this example, $D(p) = 10 - p$, $S(p) = 2p$ and $t = 2$.

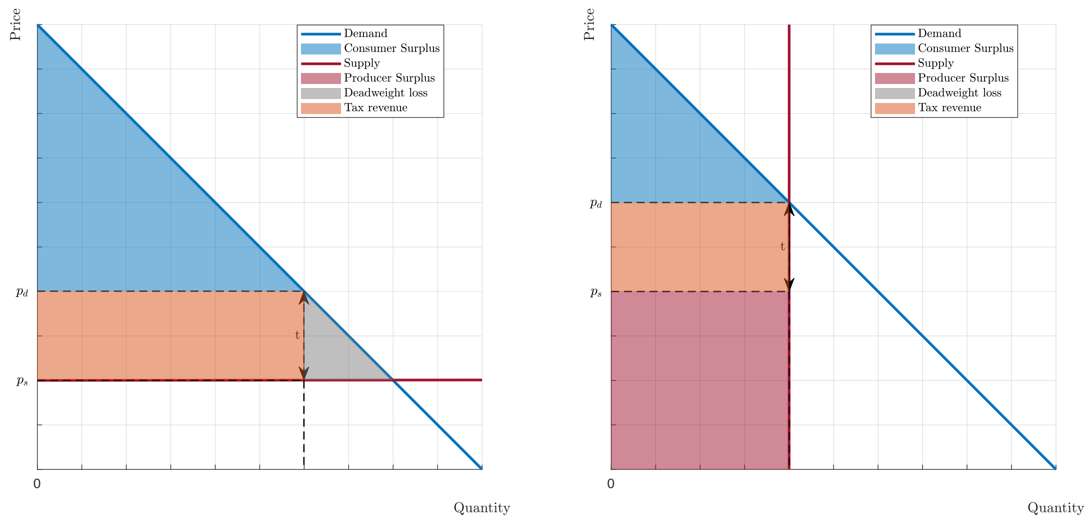
(or supply) is said to be perfectly inelastic if it is a vertical line.

In case demand (supply) is perfectly elastic, the tax is completely passed along to producers (consumers). In case demand (supply) is perfectly inelastic, the tax is completely passed along to consumers (producers). In figure 39, we illustrate the case of a perfectly elastic and inelastic supply curves.

The above examples are special cases. Usually, the tax is passed along to both the producers and the consumers. How much is passed along depends on the relative elasticity of supply and demand. In the example in figure 40, supply is quite elastic: much of the tax is passed to consumers.

GOING FURTHER. *Passing along and elasticities. Let p and q be the equilibrium price and quantity on the market without tax. Suppose a small amount of tax t is introduced on the market, and let dp_d be the change in price paid by demanders and dp_s be the change in price received by suppliers. We can approximate the change in quantity demanded dq by $dq = \frac{\partial D(p)}{\partial p} dp_d$ which can be rewritten as $dq = \epsilon_d \frac{q}{p} dp_d$ where ϵ_d is the elasticity of demand. Similarly, we can approximate the change in quantity supplied dq by $dq = \frac{\partial S(p)}{\partial p} dp_s$ which can be rewritten as $dq = \epsilon_s \frac{q}{p} dp_d$ where ϵ_s is the elasticity of supply. Because the market will reach a new equilibrium after the tax is introduced,*

Figure 39: Perfect competition: taxes, perfectly (in)elastic supply curve.



Note: This illustrates the passing along of a tax when supply is perfectly elastic (left) or perfectly inelastic (right).

these quantity changes should be equal to one another. Hence, $\epsilon_d dp_d = \epsilon_s dp_s$. *Therefore*

$$\frac{dp_d}{dp_s} = \frac{\epsilon_s}{\epsilon_d}$$

The impact of the tax will be relatively more important for consumers than for suppliers the more elastic supply is relatively to demand (in absolute values).

Subsidies

Subsidies can be treated in the same way as quantity taxes (they are negative taxes). Like quantity taxes, a *subsidy* introduces a wedge between the demand price (what the consumers pay, p_d) and the supply price (what the producers receive, p_s).

Let s be the subsidy. The demand and supply prices are related by the equation

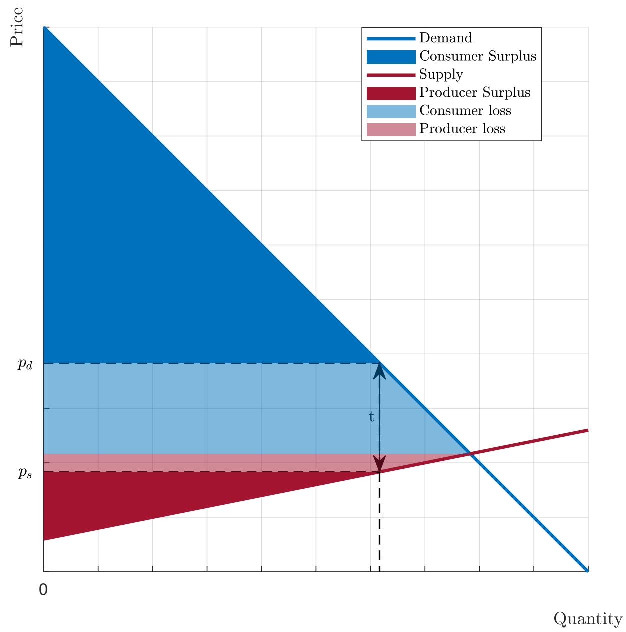
$$p_s = p_d + s$$

Equilibrium is characterized by the equation

$$D(p_d) = S(p_s)$$

Figure 41 shows the effect of introducing a subsidy on the market. The quantity exchanged on the market will be more than in the equilibrium without subsidy. Suppliers will receive a higher price than in the equilibrium without subsidies and consumers will pay a lower price than in the equilibrium without subsidies. Both the consumers'

Figure 40: Perfect competition: taxes with a relatively elastic supply curve.



Note: This illustrates the passing along of a tax when supply is quite elastic. The tax is mostly passed along to consumers. In this example, $t = 2$, $S(p) = -3 + 5p$ and $D(p) = 10 - p$.

surplus and producers' surplus will increase. The government will lose money paying for the subsidy. Overall, subsidies generate a deadweight loss, represented as a grey area in figure 41.

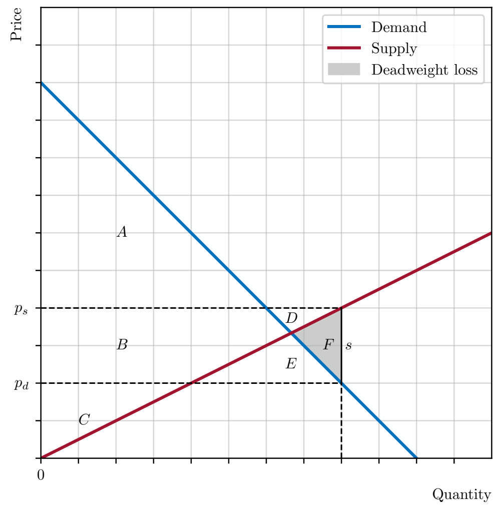
Inefficiency and redistribution

On perfectly competitive markets, we saw that the total surplus is maximized without government intervention. Therefore, any form of government intervention such as price ceilings, price floors, quotas, taxes, subsidies, etc will be a source of inefficiency as it will generate a deadweight loss. However, it is important to note that this is only true if we accept the assumption that markets are perfectly competitive. In addition, it is worth pointing out that government interventions will, in general, redistribute surplus: depending on the type of intervention, consumers (some of them or all of them) may win or lose, producers (some of them or all of them) may win or lose, and the government itself may win or lose money. Thus, at a cost of a deadweight loss, government interventions are a means to redistribute income.

Who wins and who loses from government intervention, and how big the deadweight loss will end up being, depends crucially on the properties of supply and demand (in particular, their elasticity).

In summary:

Figure 41: Perfect competition: subsidies.



Note: This illustrates the effect of introducing a subsidy on the market. In this example, $s = 2$, $S(p) = 2p$ and $D(p) = 10 - p$. After the introduction of the subsidy, the consumers' surplus is $A + B + E$, the producers' surplus is $C + B + D$, the goverment expenditures are $B + D + E + F$ and the deadweight loss is F.

- In the case of a price floor, the quantity exchanged on the market will be less than the efficient level. There is excess supply. The consumer surplus will decrease. The effect on the producer surplus is ambiguous.
- In the case of a price ceiling, the quantity exchanged on the market will be less than the efficient level. There is excess demand. The producer surplus will decrease. The effect on the consumer surplus is ambiguous.
- In the case of a quota, the quantity exchanged on the market will (usually) be less than the efficient level. The consumer surplus will decrease. The effect on the producer surplus is ambiguous.
- In the case of a unit tax, the quantity exchanged on the market will be less than the efficient level. Both the consumer surplus and producer surplus will decrease. The government will earn money.
- In the case of a unit subsidy, the quantity exchanged on the market will be more than the efficient level. Both the consumer surplus and producer surplus will increase. The government will lose money.

4 Topics in microeconomic theory

In this chapter, we explore various topics in microeconomic theory, including:

1. General equilibrium: as opposed to partial equilibrium analysis, we will study economies made of mutiple (interconnected) markets.

2. Labour supply: in this application, we examine the consumer's choice between consumption and leisure.

3. Risk: in this application, we examine the consumer's choice between risky alternatives.

Key ideas & concepts

general equilibrium, competitive market economies, Pareto efficiency, Edgeworth box, first welfare theorem, second welfare theorem, leisure, labour supply, backward-bending labour supply, risk, lottery, expected value, expected utility, risk aversion, risk premium, insurance.

4.1 General equilibrium: introduction

General equilibrium analysis

General equilibrium analysis (as opposed to partial equilibrium) is the study of how equilibrium is determined in many (interconnected) markets simultaneously.

Competitive market economies

The environment we will place ourselves in is one of a (perfectly) *competitive market economy*. A (perfectly) competitive market economy is an economy in which every relevant good is traded in a market (complete markets) at publicly known prices and all agents act as price takers (perfect competition).

Pareto efficiency

Pareto efficiency is a very important concept in economics. An allocation of goods is said to be *Pareto efficient* if there is no reallocation of goods that would make all agents at least as well off, and at least one agent strictly better off. Note that Pareto efficiency does not have anything to do with fairness. Determining whether an allocation is fair or not is beyond economics.

EXAMPLE*. Suppose that A and B find 100 gold coins in the ground. They have to decide how to allocate the coins between the two of them. The allocation that we perceive as fair* $(50, 50)$ *is Pareto efficient. But so is* $(100, 0)$ *and* $(0, 100)$*. In fact, in this particular example, any allocation of coins is Pareto efficient.*

Competitive (or Walrasian) equilibrium

Consider a perfectly competitive economy in which several goods are produced using a variety of inputs. If there exists a set of prices (for the goods and the inputs) such that

1. each consumer is choosing his most preferred affordable bundle
2. firms are maximizing their profits
3. the demand for each good is equal to the amount produced
4. the demand for each input is equal to the amount available

then this constitutes a *competitive (or Walrasian) equilibrium*.

The first welfare theorem

The *first welfare theorem* states all Walrasian equilibria are Pareto efficient.

The second welfare theorem

The *second welfare theorem* states that any Pareto efficient allocation can be reached by the use of competitive markets.

Market failures

Market failures are situations in which some of the assumptions of the welfare theorems do not hold and in which, as a consequence, market outcomes may be inefficient. Examples of market failures include:

- Market power: some agents are no longer price takers. For example, we will study monopolies and oligopolies.
- Externalities: the actions of one agent affect other agents (positively or negatively), without any market to account for it. Thus the complete markets assumption is violated
- Public goods: as above, in the presence of public goods, the complete markets assumption is violated
- Asymmetry of information: some agents have more information than others.

4.2 General equilibrium: exchange

We now explore in greater details some of the themes introduced in the previous section, by examining first a simplified pure exchange economy.

Pure exchange economy

We consider a simple economy with only two consumers, A and B, and two goods, 1 and 2. Each consumer is endowed with a certain amount of each good, and we completely ignore production for now. This is known as a *pure exchange economy*.

Endowments

Let the total amount of good 1 available in the economy be X_1, and the total amount of good 2 be X_2. We assume that the two consumers are endowed with some amount of these goods. Consumer A is endowed with a bundle (ω_1^A, ω_2^A), while consumer B gets (ω_1^B, ω_2^B). Of course, it must be the case that $\omega_1^A + \omega_1^B = X_1$ and $\omega_2^A + \omega_2^B = X_2$.

Allocations

An *allocation* for consumer A is a bundle of good 1 and 2, denoted (x_1^A, x_2^A). An allocation for consumer B is a bundle of good 1 and 2, denoted (x_1^B, x_2^B).

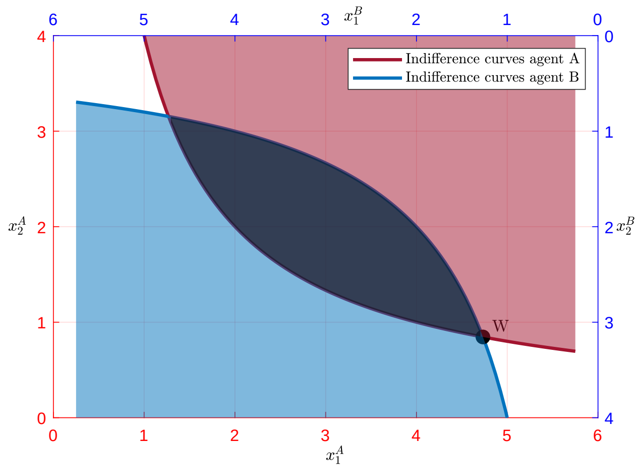
The Edgeworth box

This simplified economy can be represented by the *Edgeworth box*. The box is a rectangle: the width of the rectangle represents the total amount of good 1 available in the economy (X_1), while the height of the box represents the total amount of good 2 available in the economy (X_2). The box is formed of two systems of axes. The bottom and left edges of the box are the axes for consumer A, while the top and right edges of the box are the axes for consumer B. The initial *endowment* W is the point with the coordinates (ω_1^A, ω_2^A) on consumer A's axes, or with coordinates (ω_1^B, ω_2^B) on consumer B's axes. See figure 42.

In the Edgeworth box, we can represent indifference curves for both A and B. The indifference curve for consumer A that goes through W represents all the allocations (or bundles) (x_1^A, x_2^A) that are indifferent to the initial endowment for consumer A. Similarly, the indifference curve for consumer B that goes through W represents all the allocations (or bundles) (x_1^B, x_2^B) that are indifferent to the initial endowment for consumer B.

For consumer A, the indifference curves face away from the bottom left corner: the farther away indifference curves are from the origin (the bottom left corner), the higher the utility. Any allocation of goods that lies above the indifference curve that goes through the initial allocation point W is preferred to that initial endowment.

Figure 42: General equilibrium: the Edgeworth box.



Note: This illustrates an Edgeworth box. There is 6 units of good 1 available and 4 units of good 2 available. The initial endowment is represented by the point W.

For consumer B, the indifference curves face away from the top right corner: the farther away indifference curves are from the origin (the top right corner), the higher the utility. Any allocation of goods that lies below the indifference curve that goes through the initial allocation point W is preferred to that initial endowment. A few indifference curves are represented in figure 42.

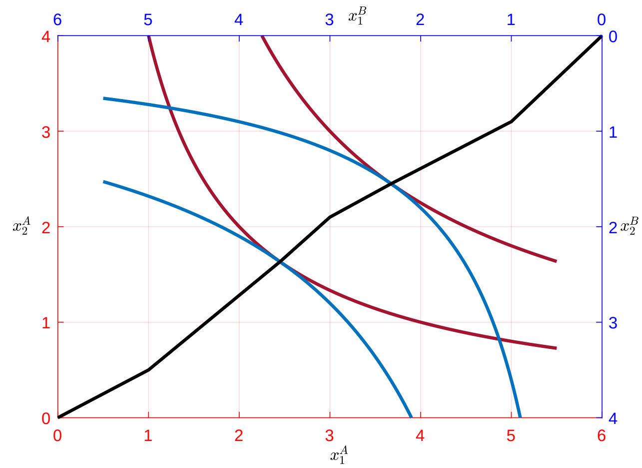
The Pareto set

In this exchange economy, an allocation of goods is Pareto efficient if there are no mutually beneficial trades at such allocation.

Is the initial endowment point Pareto efficient? It could be if we are lucky, but in general, it won't be the case. In figure 42, we can see that the initial allocation W is not Pareto efficient. Consumer A and B could trade goods to reach any allocation in the grey (dark) area in figure 42. This area is the set of mutually beneficial allocations: all the points in this area lie above the indifference curve of consumer A that cuts through W, and below the indifference curve of consumer B that cuts through W. The two consumers will trade until there is no longer any mutual improvements possible. The resulting allocation will be Pareto efficient, since there will be no way to make both people better off.

Geometrically, the indifference curves of the two agents must be tangent at any Pareto efficient allocation. The set of all Pareto efficient points in the Edgeworth box is the *Pareto set*, sometimes called the *contract curve*. See figure 43.

Figure 43: General equilibrium: the contract curve.



Note: In this Edgeworth box, we have represented the set of Pareto efficient allocations, or contract curve, as a solid black line connecting all the Pareto efficient allocations.

Prices

The trading process described above is a good start, but it won't really tell us where on the contract curve the two agents will end up.

To reach a more precise characterization of the equilibrium, we introduce prices for each of the two goods, p_1 and p_2 respectively. We assume that the prices are given by some central planner.

Assume that the prices have been set to p_1 and p_2. The two consumers can exchange the goods between each other at these given prices.

Budget line

Consumer A can afford any allocation (x_1^A, x_2^A) such that $p_1 x_1^A + p_2 x_2^A \leq p_1 \omega_1^A + p_2 \omega_2^A$. This defines the budget line for consumer A. Consumer B can afford any allocation (x_1^B, x_2^B) such that $p_1 x_1^B + p_2 x_2^B \leq p_1 \omega_1^B + p_2 \omega_2^B$. This defines the budget line for consumer B.

In the Edgeworth box, we can draw the budget line faced by the consumers as the straight line slope $-p_1/p_2$ that cuts through the initial allocation W. See figure 44.

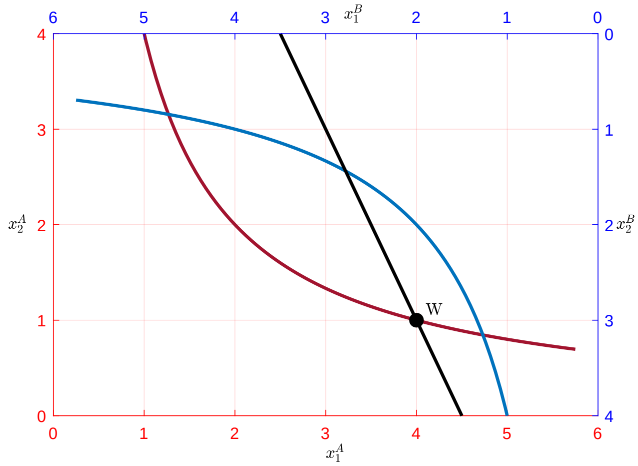
Competitive (or Walrasian) equilibrium

In this exchange economy, if there exists a set of prices (for the goods and the inputs) such that

1. each consumer is choosing his most preferred affordable bundle

2. the demand for each good is equal to the amount produced

Figure 44: General equilibrium: the Edgeworth box, with the budget line.



Note: In this Edgeworth box, we also represented a budget line, when prices are p_1 and p_2. The budget line cuts through the point W.

then this constitutes a competitive (or Walrasian) equilibrium.

From consumer theory, we know that consumer A will want to consume the bundle (x_1^A, x_2^A) at the tangency point (C^A) between the budget line and the indifference curve that is farthest from the origin. Similarly, we know that consumer B will want to consume the bundle (x_1^B, x_2^B) at the tangency point (C^B) between the budget line and the indifference curve that is farthest from the origin.

Unfortunately, there is no warrantee that for the arbitrary chosen prices (p_1, p_2) the total demand for good 1 will not exceed to total amount of good 1 available ; or that the total demand for good 2 will not exceed to total amount of good 2 available. In this case, for those prices, the economy is in *disequilibrium*. See figure 45.

However, if there exists a set of prices (p_1, p_2) such that each consumer is choosing his most preferred affordable bundle, and that the total demand of each good is equal to the total amount of each good available in the economy, then the market is in equilibrium. In that case, we say this is a competitive equilibrium, or Walrasian equilibrium. See figure 46.

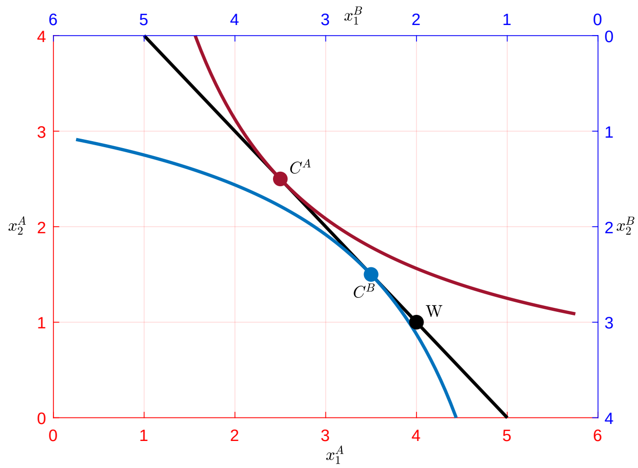
The question of the existence of such set of prices is beyond this course, but we can say that under some assumptions, this set of prices exists.

The first welfare theorem

The first welfare theorem states that all Walrasian equilibria are Pareto efficient.

To see this, go back to the Edgeworth box. In equilibrium, we know that consumers choose their most preferred affordable bundle, say (x_1^A, x_2^A) and (x_1^B, x_2^B). We also know that in equilibrium, there is no excess demand or supply of any of the goods, so the

Figure 45: General equilibrium: disequilibrium.



Note: In this Edgeworth box, the prices are such that there is disequilibrium. There is excess demand of good 2.

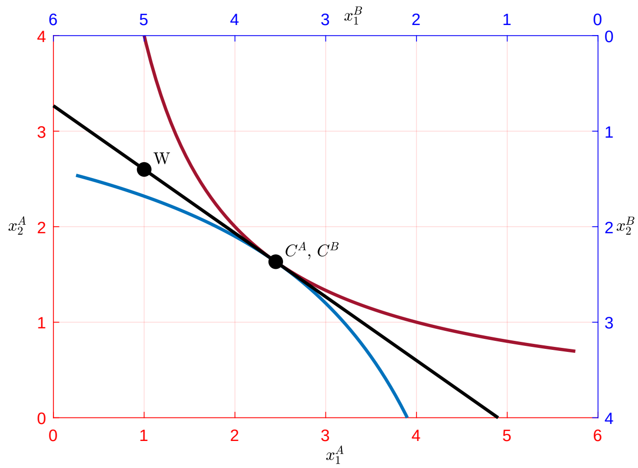
point (x_1^A, x_2^A) plotted from A's point of view and the point (x_1^B, x_2^B) plotted from B's point of view coincide in the box. Since these bundles are the most preferred, the set of bundles that consumer A would prefer to (x_1^A, x_2^A) must lie above the budget line, while the set of bundles that consumer B would prefer to (x_1^B, x_2^B) must lie below the budget line. These two sets cannot intersect, meaning that there are no allocations that both agents would prefer to the equilibrium allocation. Hence, the equilibrium allocation is Pareto-efficient.

The second welfare theorem

The second welfare theorem states that any Pareto efficient allocation can be reached by the use of competitive markets. Precisely, the theorem states that if preferences are convex, then there always exists a set of prices such that each Pareto efficient allocation is a market equilibrium for an appropriate assignment of endowments.

To see this, go back to the Edgeworth box. Assume that we want to reach some Pareto efficient allocation in equilibrium. This Pareto efficient allocation lies, by definition, on the contract curve. At this Pareto efficient allocation, the indifference curves of consumer A and B are tangent. We want to find equilibrium prices such that each consumer is choosing his most preferred affordable bundle, and that the total demand of each good is equal to the total amount of each good available in the economy. We can do that by drawing a straight line cutting through the Pareto efficient allocation and tangent to both indifference curves. The slope of this line gives us the relative equilibrium prices, and this line is the budget line. Any initial endowment that puts the two consumers on this line will lead us to the desired Pareto efficient allocation by trading.

Figure 46: General equilibrium: competitive equilibrium (without production).



Note: In this Edgeworth box, the prices are such that there is equilibrium. There is no excess demand of good 1 or good 2, and the consumers are maximizing their utility.

Production

We have completely ignored the production side of the economy (in our analysis, the quantities of good 1 and 2 are given to us, they are not produced). Production can be added back into the mix, as we shall see now.

4.3 General equilibrium: production

Exchange economy with production

The economy is very similar to the one examined in the previous section. There are two consumers, A and B, and two goods, 1 and 2. The quantities X_1 and X_2 of goods available in the economy are no longer fixed. Instead, these will be produced and production will react to changes in prices. Note that, interestingly, production introduces another way to "trade" goods for one another in this economy, by producing more of a good and less of the other.

For simplicity, we assume that the goods are produced using as inputs the labour supplied by A and B. A and B supply the same amount of labour and receive the same wage. In addition, we assume that A and B own the firms producing the good and that all profits get equally redistributed to them.

Production possibility frontier

The *production possibility frontier* (PPF) describes all the combinations of goods (X_1, X_2) that can be produced in this economy, by using the available resources (e.g. here the time spent by A and B on production activities). Graphically, the PPF connects all such combinations of goods. An illustration is provided in figure 47.

The PPF is decreasing: to produce more of a good, we must decrease the production of the other good. The shape of the PPF will depend on the technology used to produce the goods. Usually, the PPF will be concave, as shown in figure 47.

Marginal rate of transformation

The *marginal rate of transformation* (MRT) is the slope of the production possibility frontier. It indicates by how much we should cut the production of good 2 in order to produce an additional unit of good 1.

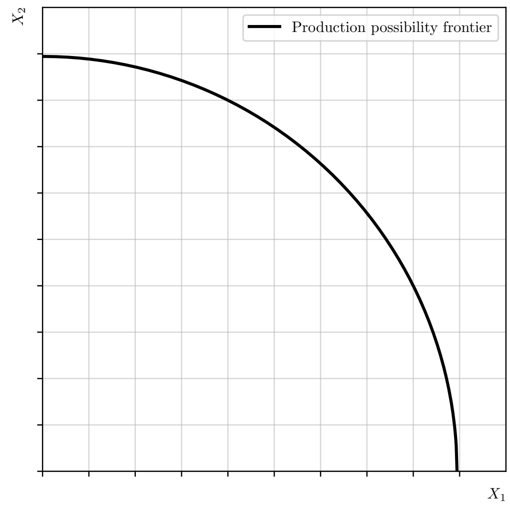
Because the PPF is decreasing, note that the MRT is negative. And if the PPF is concave, then the MRT is also decreasing: it becomes more and more difficult to produce good 1 as we produce more of it (i.e. we must give up on increasingly large amounts of good 2 in order to produce an extra unit of good 1 as we produce more and more good 1).

The MRT reflects the marginal cost of producing one good relative to the marginal cost of producing the other. In fact, the MRT is equal to minus the ratio of the marginal costs, i.e.

$$MRT = -\frac{MC_1}{MC_2}$$

where MC_1 and MC_2 are the marginal costs of good 1 and 2, respectively.

Figure 47: General equilibrium: production possibility frontier.



Note: This figures shows the production possibility frontier, i.e. all the combinations of goods (X_1, X_2) that can be produced in this economy.

EXAMPLE. *MRT and marginal costs. Suppose that at some point* (X_1, X_2) *on the PPF, the marginal cost (expressed in some unit, say pounds) of good 1 is* £1 *while the marginal cost of good 2 is* £2. *This means that producing one more unit of good 1 would cost an extra pound. This can be made possible by cutting down the production of good 2 by half a unit. Hence, the MRT at this point is* $-1/2$, *which is minus the ratio of the marginal costs.*

Edgeworth box

For each product mix (X_1, X_2) on the PPF, we can draw the corresponding Edgeworth box as shown in figure 48.

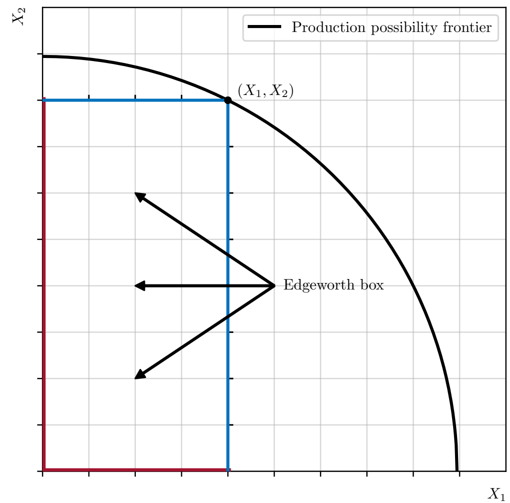
Competitive (or Walrasian) equilibrium

In this economy, if there exists a set of prices (for the goods and the inputs) such that

1. each consumer is choosing his most preferred affordable bundle
2. firms are maximizing their profits
3. the demand for each good is equal to the amount produced
4. the demand for each input is equal to the amount available

then this constitutes a competitive (or Walrasian) equilibrium.

Figure 48: General equilibrium: production possibility frontier and Edgeworth box.



Note: This figures shows the production possibility frontier and, for a chosen product mix (X_1, X_2), the corresponding Edgeworth box.

The competitive equilibrium in both consumption and production is illustrated in figure 49. Let p_1 and p_2 be the prices of good 1 and 2, respectively. Recall that competitive, profit-maximizing, firms will choose quantities to produce such that price equals marginal cost, and thus $p_1/p_2 = MC_1/MC_2$. Therefore,

$$MRT = -\frac{p_1}{p_2}$$

that is, the marginal rate of transformation is equal to the ratio of the prices. For these relative prices, the product mix (X_1, X_2) is shown on the PPF in figure 49 at the tangency point between the PPF and a price line reflecting the price ratio (the price line has slope $-p_1/p_2$).

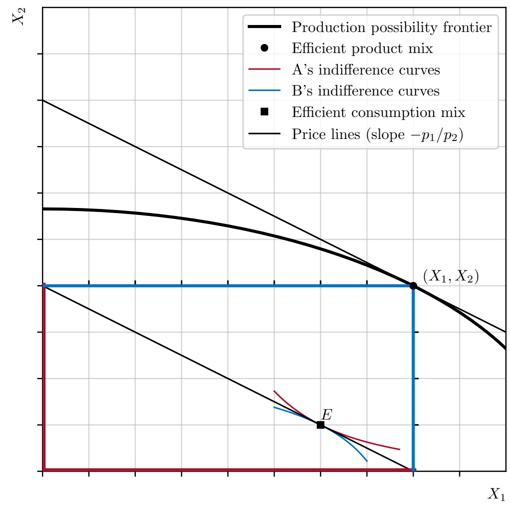
For these prices and product mix (X_1, X_2), we can draw the corresponding Edgeworth box and budget line on the same graph (which, under the simplifying assumption that the consumers receive the same wages and half the profits, is a line of slope $-p_1/p_2$ that cuts through the center of the box). If, for these relative prices, the choices of the consumers yield to the same point in the box (like point E in figure 49), then we have a general equilibrium. At this point, supply equal demand on all markets.

Characterization of the general equilibrium

Figure 49 suggests that the competitive equilibrium is characterized by the equality between the MRS of each consumer, the MRT and the price ratio, that is

$$MRS = -\frac{p_1}{p_2} = MRT$$

Figure 49: General equilibrium: competitive equilibrium (with production).



Note: In this figure, the prices are such that there is a general equilibrium. The product mix is located at the tangency point between the PPF and a price line (with slope $-p_1/p_2$). The consumers' choices lead to the same point E, located on the contract curve, such that supply equals demand on all markets.

The equality between the MRS of each consumer is not surprising. Given the product mix (X_1, X_2), we know (from the section looking at the pure exchange economy) that the consumers will trade until all mutually beneficial trades are exhausted, i.e. until they reach an allocation located on the contract curve. Thus, the competitive equilibrium is said to achieve *consumption efficiency*.

The equality between the MRS of each consumer and the MRT can be understood intuitively. Suppose that for one consumer, $MRS > MRT$ (the same reasoning can be applied if $MRS < MRT$), e.g. $MRS = -1$ and $MRT = -\frac{1}{2}$. If this consumer were to consume one more unit of good 1 and one less unit of good 2, their utility would remain the same. But what the MRT is telling us is that to produce this extra unit of good 1, we only need to cut production of good 2 by half a unit. So clearly, changing the product mix in this way would make the consumer better off, without making any other consumers worse off! Therefore, the competitive equilibrium is said to achieve an *efficient product mix*: the rate at which each consumer is willing to exchange one good for the other is equal to the rate at which firms can transform that good into the other.

4.4 Labour supply

Non labour income

In this model, we assume that the consumer starts with some level of income, that he received whether he works or not. This amount M is called the *non-labour income* of the consumer.

Consumption

The consumer can decide to use some of his income to consume goods. Let C be the amount of consumption chosen, and p the price of consumption.

Labour

But M is not the only source of income in this model. The consumer can decide to supply L hours of work, with a wage rate of w. Thus, the budget constraint of the consumer in this model is

$$pC = M + wL$$

Leisure

Suppose that the consumer can provide at most $\bar{L}$ hours of work (for example, 24 hours if we look at daily labor supply). Then, adding $w\bar{L}$ on both sides of the budget constraint, we get

$$pC + w\bar{L} = M + wL + w\bar{L}$$

Rearranging terms, we have

$$pC + w(\bar{L} - L) = M + w\bar{L}$$

The quantity $\bar{L} - L$ is in fact what we call *leisure*! It is the maximum number of hours available (in a day, a week, a month) minus the number of hours actually worked. We denote the number of hours of leisure by R.

The budget line

The final bit of notation we introduce is $\bar{C} = M/p$, which is the maximum amount of consumption the consumer can buy if he does not work. Using all these notations, we get the budget constraint

$$pC + wR = p\bar{C} + w\bar{L}$$

There are two interesting things to say about the budget constraint written like this. First, the right-hand side is what is called the full or implicit income of the consumer: it is the maximum amount of consumption he could get if he used all of his time to work. Second, it appears w is not only the wage rate, it is also in fact the price of leisure! This is not very surprising since the wage rate is in fact the opportunity cost of leisure (taking one more hour of leisure means working one hour less, means "losing" a wage w).

Rearranging the budget constraint, we get the equation for the budget line

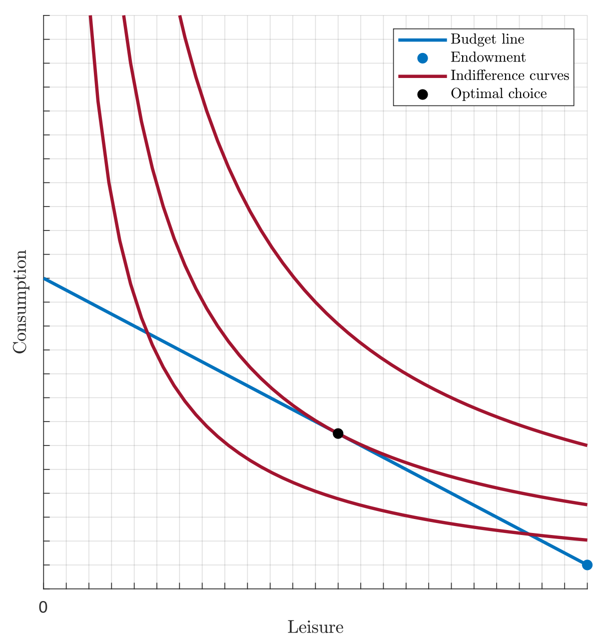
$$C = \bar{C} + (w/p)\bar{L} - \frac{w}{p}R$$

This is a straight line of slope $-\frac{w}{p}$, where $\frac{w}{p}$ is called the real wage (this is the amount of consumption an hour of work allow the consumer to buy). The budget line cuts through the point $(\bar{L}, \bar{C})$, also called the endowment point (if the worker decides not to work at all, then he gets $\bar{L}$ hours of leisure, and $\bar{C}$ units of consumption).

Optimal choice

As usual, with well behaved preferences over leisure and consumption, then the optimal choice occurs where the marginal rate of substitution equals the slope of the budget line, as shown in figure 50.

Figure 50: Labour supply: optimal choice.



Note: In this figure, $M = 1$, $p = 1$, $w = 0.5$ and $\bar{L} = 24$. Preferences are represented by the utility function $u(R, C) = R^{\frac{1}{2}}C^{\frac{1}{2}}$.

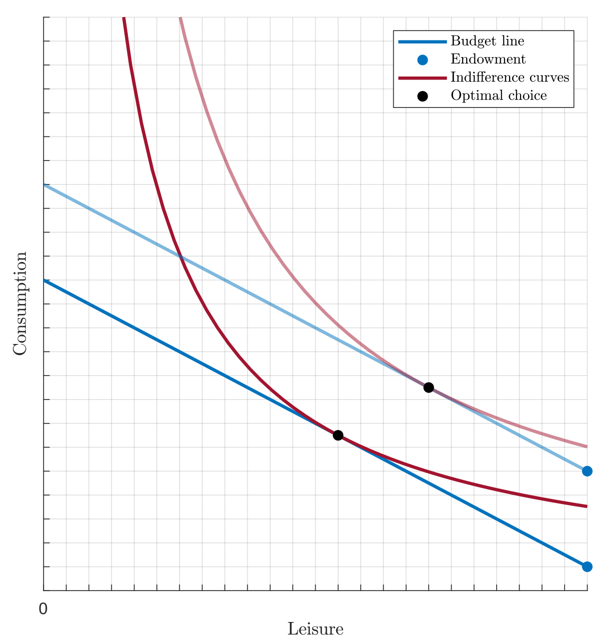
EXAMPLE. *Consider the problem illustrated in figure 50. A consumer receives nonlabor income $M = 1$ and can supply L hours of work in which case he receives a wage*

$w = 0.5$ per unit of labor. The maximum amount of labor supply possible is $\bar{L} = 24$. The amount of consumption the consumer has is denoted C, and the price of consumption is $p = 1$. His preferences over consumption and leisure R are represented by the utility function $u(R, C) = R^{\frac{1}{2}} C^{\frac{1}{2}}$. The budget line is $C = \frac{M + w\bar{L}}{p} - \frac{w}{p} R$, which is a straight line of slope $-\frac{w/p}{=} - 0.5$. The marginal utility with respect to leisure is $MU_1 = \frac{\partial u(R,C)}{\partial R} = \frac{C^{\frac{1}{2}}}{2R^{\frac{1}{2}}}$. The marginal utility with respect to consumption is $MU_2 = \frac{\partial u(R,C)}{\partial C} = \frac{R^{\frac{1}{2}}}{2C^{\frac{1}{2}}}$. Thus the MRS is $-MU_1/MU_2 = \frac{C}{R}$. At the optimal choice of consumption and leisure, we must have $MRS = -\frac{w}{p}$, that is $\frac{C}{R} = 0.5$, thus $R = 2C$. Using the budget constraint we have $C + w2C = M + w\bar{L}$, that is $2C = 13$, hence $C = 6.5$ and $R = 13$. This person chooses to work $24 - 13 = 11$ hours a day.

Comparative statics

We will consider two types of comparative statics. First, what happens if nonlabor income increases (while prices and wages remain fixed)? Since leisure is usually a normal good, we can expect consumption and leisure to increase as income increases. This is illustrated in figure 51.

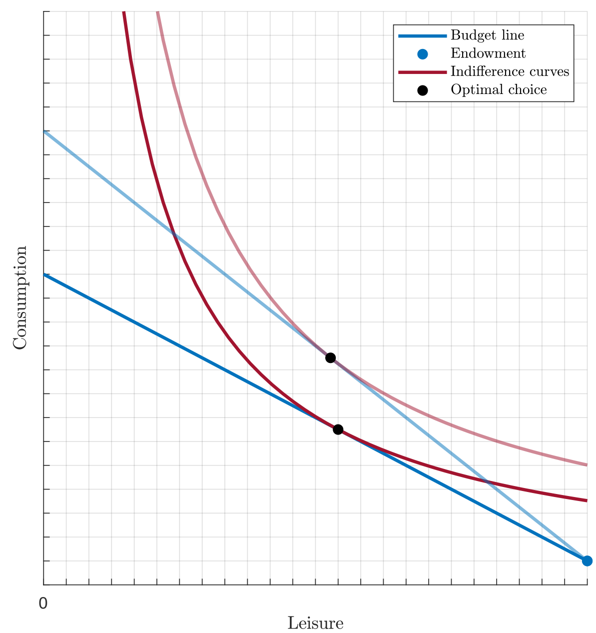
Figure 51: Labour supply: comparative statics (non labour income).



Note: In this figure, $p = 1$, $w = 0.5$ and $\bar{L} = 24$. Preferences are represented by the utility function $u(R, C) = R^{\frac{1}{2}} C^{\frac{1}{2}}$. M increases from $M = 1$ (opaque blue line) to $M = 5$ (transparent blue line).

Second, consider the effect of an increase in the wage rate (keeping everything else fixed). We could try to use naively (and incorrectly here) the ideas of substitution and income effects. As the wage rate increases, the price of leisure increases, so people will want less of it (substitution effect). In addition, their purchasing power will go down, which will decrease their consumption of leisure if leisure is a normal good (income effect). Thus, an increase the price of leisure will lead to a decrease in the demand for leisure and the supply of labor will go up. This is illustrated in figure 52.

Figure 52: Labour supply: comparative statics (wage).

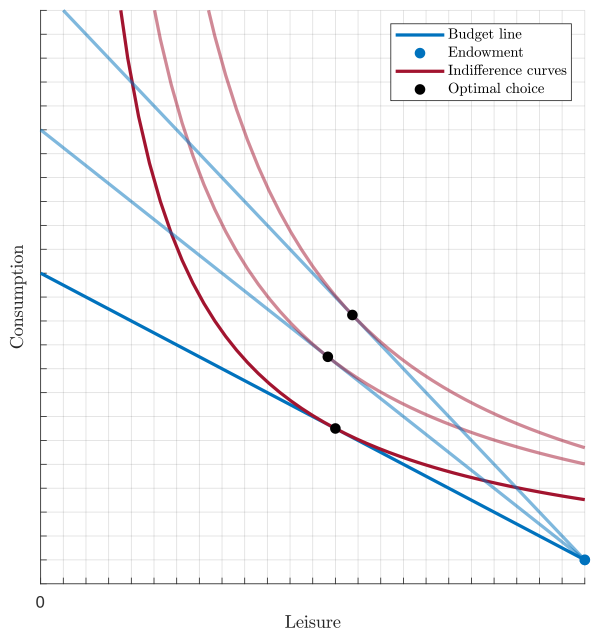


Note: In this figure, $M = 1$, $p = 1$ and $\bar{L} = 24$. Wage increases from $w = 0.5$ (opaque) to $w = 0.75$ (transparent). Preferences are represented by the utility function $u(R, C) = R^{\frac{1}{2}} C^{\frac{1}{2}}$.

EXAMPLE. *Suppose a consumer receives nonlabor income $M = 0$ and can supply L hours of work in which case he receives a wage w per unit of labor. The maximum amount of labor supply possible is $\bar{L} = 24$. The amount of consumption the consumer has is denoted C, and the price of consumption is $p = 1$. His preferences over consumption and leisure R are represented by the utility function $u(R, C) = C + 2L^{\frac{1}{2}}$. The budget line is $C = \frac{M + w\bar{L}}{p} - \frac{w}{p} R$, which is a straight line of slope $-\frac{w}{p} = -w$. The marginal utility with respect to leisure is $MU_1 = \frac{\partial u(R,C)}{\partial R} = \frac{1}{R^{\frac{1}{2}}}$. The marginal utility with respect to consumption is $MU_2 = \frac{\partial u(R,C)}{\partial C} = 1$. Thus the MRS is $-MU_1/MU_2 = \frac{1}{R^{\frac{1}{2}}}$. At the optimal choice of consumption and leisure, we must have $MRS = -\frac{w}{p}$, that is $\frac{1}{R^{\frac{1}{2}}} = w$, thus $R = \frac{1}{w^2}$. Using the budget constraint we have $C = M + w\bar{L} - wR$, that is $C = 24w - \frac{1}{w}$ since $M = 0$. This consumer's labor supply is simply $L = \bar{L} - R = 24 - \frac{1}{w^2}$, which is increasing in wage.*

However, the story does not stop here. Actually, we were a bit imprecise in the previous paragraph, because when increasing the wage rate, we not only increased the price of leisure, but we also changed the income of the consumer. There is an extra income-effect at play here! The consumer gets some extra income (because of the higher wage), and may very well spend it on more leisure. Thus, it can happen that an increase in the wage rate results in a decrease in the supply of labor. This could yield to a backward-bending labor supply, as represented in figure 53.

Figure 53: Labour supply: backward-bending labour supply.



Note: In this figure, $M = 1$, $p = 1$ and $\bar{L} = 24$. Wage increases from $w = 0.5$ (opaque) to $w = 0.75$ to $w = 1$ (transparent). Preferences are such that we get a backward-bending labor supply.

4.5 Risk

Decision under risk

We talk about decision under *risk* when consumers face a choice among a number of risky alternatives.

There are several possible outcomes to each risky alternative, and each of these outcomes will occur with some probability.

Lotteries

Lotteries are used in consumer theory to represent/describe risky alternatives. A *lottery* specifies each possible outcome of the risky alternative it represents, as well as the probability that this outcome will occur.

EXAMPLE*. Consider a consumer who can make a risky investment that will give him £0 with probability 0.5 and £4 with probability 0.5. This risky investment can be represented by lottery with two outcomes (winning £0 or winning £4), where each of these outcomes occur with probability 0.5. Note that a safe investment that brings £2 with certainty can also be represented by a lottery, with only one outcome (winning £2) which occurs with probability 1.*

States of nature

Whether an outcome realizes or not depends on the state of nature that will occur (e.g. the house burns down or not, weather is bad or good, etc).

Thus, lotteries can also be used to represent *contingent consumption plans*, that is, plans that specify what will be consumed in each state of nature.

EXAMPLE*. Suppose that the wealth of a consumer is equal to the value of his house W. There is a risk that the house will burn, in which case the consumer will have a wealth of zero. Without any insurance, the consumer is de facto choosing this lottery: with some probability π, the house does not burn and the consumer has wealth W ; with probability $1-\pi$ the house burns down and the consumer gets wealth 0. The consumer, depending on his preferences, may choose to change this, for example by buying insurance. If he fully insure himself, then he de facto chooses the lottery: with probability π he will get W' and with probability $1-\pi$ he will get W' as well.*

In the following, we will consider lotteries with only two outcomes, 1 and 2. Consumption in each outcome is denoted c_1 and c_2, respectively. The probabilities that each outcome occurs π_1 and π_2, respectively.

Expected value

How do consumers choose between different lotteries? One first guess is that consumers compare the *expected value* of the different lotteries

$$EV = \pi_1 c_1 + \pi_2 c_2$$

This is the weighted average of the values of each possible outcome.

If consumers did not care about risk, then it is true that they could make decisions based on expected values. However, this may not be the case, e.g. consumers may be risk-averse or risk-loving.

EXAMPLE. *Consider a first lottery* $L1$ *that gives* $£10,000$ *with probability* 0.6 *and* $£0$ *with probability* 0.4. *A second lottery* $L2$ *gives* $£5,000$ *with probability* 1. *The expected value of lottery 1 (*$EV1 = £6,000$*) is higher than that of lottery 2 (*$EV2 = £5,000$*). However, it is clear than many (most?) people would choose the second lottery.*

Expected utility

Under some assumptions, it can be shown that consumers' preferences over risky alternatives can be represented by a utility function of the form

$$u(c_1, c_2, \pi_1, \pi_2) = \pi_1 v(c_1) + \pi_2 v(c_2)$$

Such utility functions are called *expected utility* functions, or sometimes *von Neumann-Morgenstern utility functions*. This is because in this model the utility associated to a lottery is the weighted average (the expectation) of the utility from its outcomes.

Compound lottery

Suppose that L_1 and L_2 are two simple lotteries (e.g like the ones we studied before). Then the *compound lottery*

$$L_3 = [L_1, L_2; \alpha, 1 - \alpha]$$

is a lottery where the outcomes are the simple lotteries L_1 and L_2, where the lottery L_1 occurs with probability α and the lottery L_2 occurs with probability $1 - \alpha$

EXAMPLE. *Suppose that with lottery* $L1$*, we can earn* 0 *with probability* 0.4 *and* 100 *with probability* 0.6. *Suppose that with lottery* $L2$*, we can earn* 0 *with probability* 0.5 *and* 100 *with probability* 0.5. *Suppose we construct lottery* $L3$ *as above with* $\alpha = 0.5$. *In this lottery, what is the probability of winning* 0*? It is* $0.5 \times 0.4 + 0.5 \times 0.5 = 0.45$. *And what is the probability of winning* 100*? It is* $0.5 \times 0.6 + 0.5 \times 0.5 = 0.55$.

Axioms of expected utility theory

In which case is it possible to represent preferences over risky alternatives by a utility function that has the expected utility form? In fact, this is possible if two main axioms are satisfied:

1. *Continuity*: this axiom means that small changes in probabilities do not change the nature of the ordering of two lotteries.

2. *Independence*. Suppose that lottery L_1 is at least as good as lottery L_2. Then the compound lottery $[L_1, L_3; \alpha, 1-\alpha]$ is at least as good as the compound lottery $[L_2, L_3; \alpha, 1-\alpha]$ for any lottery L_3. Conversely, if the lottery $[L_1, L_3; \alpha, 1-\alpha]$ is at least as good as the lottery $[L_2, L_3; \alpha, 1-\alpha]$, then the lottery L_1 is at least as good as the lottery L_2.

The independence axiom means that if we mix each of two lotteries with a third lottery, then the preference ordering between the two resulting lotteries does not depend on (is independent of) this third lottery.

EXAMPLE. *The* Allais paradox. *The result from this experiment are not compatible with expected utility theory, hence the name "paradox". Suppose that a player is playing a game in which he can earn either 0, 1 million or 5 million dollars. Suppose that a first lottery L_1 gives the player 1 million dollars with probability 1. A second lottery L_2 gives 0 with probability* 0.01*, 1 million dollar with probability* 0.89*, and 5 millions dollars with probability* 0.1*. Here, most people choose lottery 1 over lottery 2.*

Lottery L_3 gives 0 *with probability* 0.89 *and 1 million with probability* 0.11*. Lottery L_4 gives 0 with probability* 0.9 *and* 5 *million with probability* 0.1*. When given the choice between lotteries 3 and 4 only, most people choose lottery 4 over lottery 3.*

But this is actually inconsistent with the expected utility theory. Suppose that $v(0) = 0$. The fact that people choose L_1 over L_2 means that $v(1) > 0.89v(1) + 0.1v(5)$. This implies

$$0.11v(1) > 0.1v(5)$$

The fact that most people choose L_4 over L_3 means that

$$0.1v(5) > 0.11v(1)$$

The two equations are not compatible!

Risk aversion

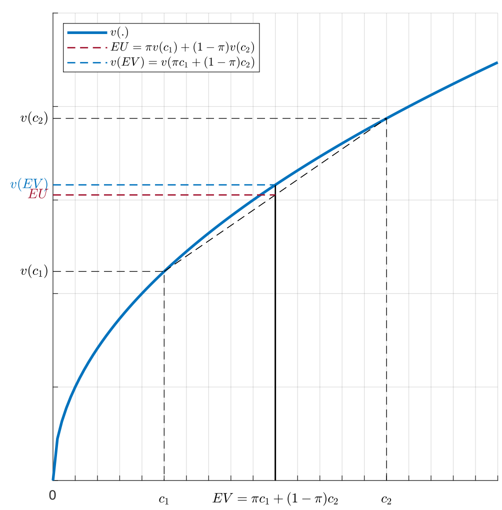
A consumer is said to be *risk averse* if he prefers a certain outcome to a lottery of equal expected value.

Whenever the utility function $v(.)$ is concave, the consumer is risk averse. Such utility function is represented in figure 54. Suppose that a consumer can choose a lottery that gives him £5 with probability 0.5 and £15 with probability 0.5. We can see from the plot (or from what we know about concave functions) that

$$v\left(\frac{1}{2}5 + \frac{1}{2}15\right) > \frac{1}{2}v(5) + \frac{1}{2}v(15)$$

In other words, the expected utility from the lottery is less than the utility from receiving the expected value of the lottery for certain. That is, the consumer prefers to receive $\frac{1}{2}5 + \frac{1}{2}15 = £10$ for certain than taking the gamble of receiving £5 with probability 0.5 and £15 with probability 0.5.

Figure 54: Risk: risk aversion.



Note: In this figure, $\pi = 0.5$, $c_1 = 5$ and $c_2 = 15$. We chose $v(x) = \sqrt{x}$.

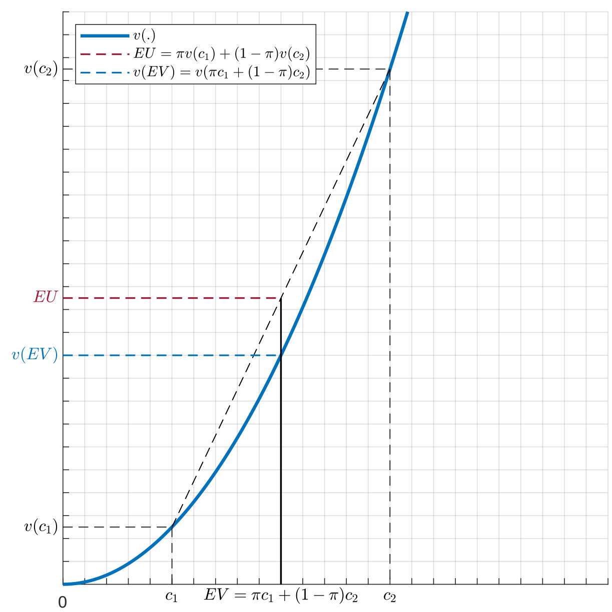
EXAMPLE. *Consider a first lottery L1 that gives* $£10,000$ *with probability* 0.6 *and* £0 *with probability* 0.4. *A second lottery L2 gives* $£5,000$ *with probability* 1. *The fact that many (most?) people would choose the second lottery suggest that they are risk averse. We could describe their choice using the expected utility model with a concave utility function, for example* $v(x) = \sqrt{x}$. *In this case,* $EU1 = 0.6\sqrt{10000} + 0.4\sqrt{0} = 60$ *and* $EU2 = 1\sqrt{5000} \approx 70$.

Whenever the utility function $v(.)$ is convex, the consumer is a *risk lover*. Such utility function is represented in figure 55. Suppose that the consumer can once again choose a lottery that gives him £5 with probability 0.5 and £15 with probability 0.5. We can see from the plot (or from what we know about convex functions) that

$$v\left(\frac{1}{2}5 + \frac{1}{2}15\right) < \frac{1}{2}v(5) + \frac{1}{2}v(15)$$

In other words, the expected utility from the lottery is more than the utility from receiving (for certain) the expected value of the lottery. That is, the consumer prefers taking the gamble of receiving £5 with probability 0.5 and £15 with probability 0.5 than to receive $\frac{1}{2}5 + \frac{1}{2}15 = £10$ for certain.

Figure 55: Risk: risk loving.



Note: In this figure, $\pi = 0.5$, $c_1 = 5$ and $c_2 = 15$. We chose $v(x) = 0.1x^2$.

Whenever the utility function $v(.)$ is linear, the consumer is *risk neutral*. This is shown in figure 56. The consumer is indifferent between taking the gamble or receiving for certain the expected value of the gamble.

The risk premium

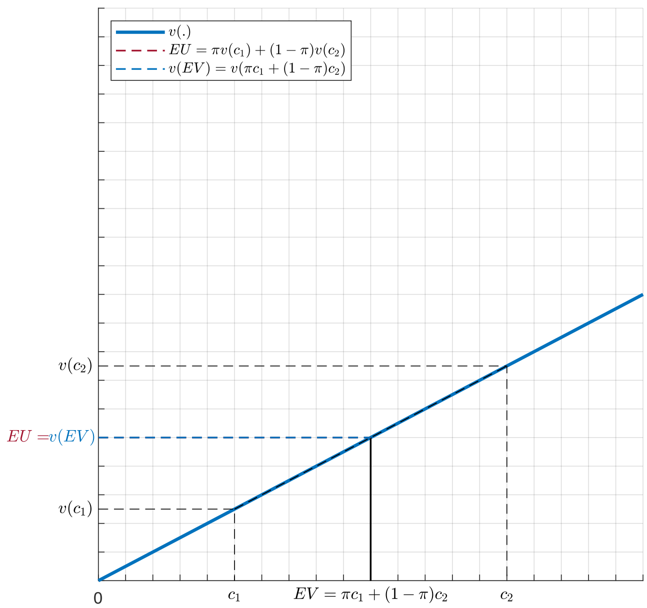
For a risk averse agent, the *risk premium* is the amount this agent is willing to pay to avoid taking risks. For a given lottery, it is is defined as the difference between the expected value of the lottery and the certainty equivalent.

The *certainty equivalent* CE is defined as the amount of money that, if held with certainty, would provide the same utility as the lottery. Suppose that the expected utility from a given lottery is EU, then the certainty equivalent is defined as the amount CE such as

$$v(CE) = EU$$

This is illustrated in figure 57.

Figure 56: Risk: risk neutral.



Note: In this figure, $\pi = 0.5$, $c_1 = 5$ and $c_2 = 15$. We chose $v(x) = 0.5x$.

EXAMPLE. *Suppose that a consumer is risk averse, with* $v(c) = \sqrt{c}$. *The consumer can choose a lottery which will give him a payoff of* £144 *with probability* 2/3 *and* £225 *with probability* 1/3. *The expected value from this lottery is* $EV = (2/3) \times 144 + (1/3) \times 225 =$ £171. *The expected utility is* $EU = (2/3) \times \sqrt{144} + (1/3) \times \sqrt{225} = 13$. *The certainty equivalent is* CE *such that* $v(CE) = 13$, *that is* $CE =$ £169. *The risk premium is therefore* $171 - 169 =$ £2. *This consumer is willing to give up* £2 *in order to receive* £169 *with certainty instead of choosing the lottery.*

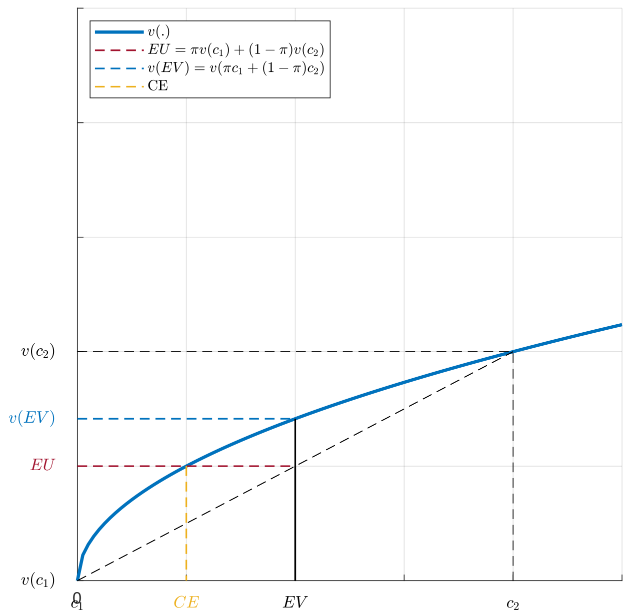
Measuring the degree of risk aversion

We have seen that a consumer's preferences towards risk are reflected in the shape of the utility function $v(.)$. The *Arrow-Pratt measure* is a commonly used measure of risk aversion and is defined as follows, for some consumption level c:

$$\rho(c) = -\frac{\frac{\partial^2 v(c)}{\partial c^2}}{\frac{\partial v(c)}{\partial c}}$$

Note that by assumption, $\partial v(c)/\partial c > 0$. The consumer is risk averse if $v(.)$ is concave, in which case $\partial^2 v(c)/\partial c^2 < 0$, and the Arrow-Pratt measure is positive. The consumer is risk neutral if $v(.)$ is linear, in which case $\partial^2 v(c)/\partial c^2 = 0$, and the Arrow-Pratt measure is zero. The consumer is risk loving if $v(.)$ is convex, in which case $\partial^2 v(c)/\partial c^2 > 0$, and the Arrow-Pratt measure is negative.

Figure 57: Risk: risk premium and certainty equivalent.



Note: In this figure, $\pi = 0.5$, $c_1 = 0$ and $c_2 = 4$. We chose $v(x) = \sqrt{x}$.

The demand for insurance

Suppose an agent is a lumberjack and gets income from cutting wood from the nearby forest. With some probability π, there is a fire in the forest (state 1), in which case the agent's income will be ω_1 (for example £25,000). With probability $1-\pi$, there is no fire (state 2), in which case the agent's income is ω_2 (for example £35,000). Initially, this is the only "lottery" the agent has access to, in which he would get ω_1 with probability π and ω_2 with probability $1-\pi$

Suppose now that the agent can buy K dollars of *insurance*, at a price of γ per dollar of insurance. If the fire does occur, then the agent's wealth becomes

$$c_1 = \omega_1 + K - \gamma K$$

If there is no fire, then the agent's wealth is

$$c_2 = \omega_2 - \gamma K$$

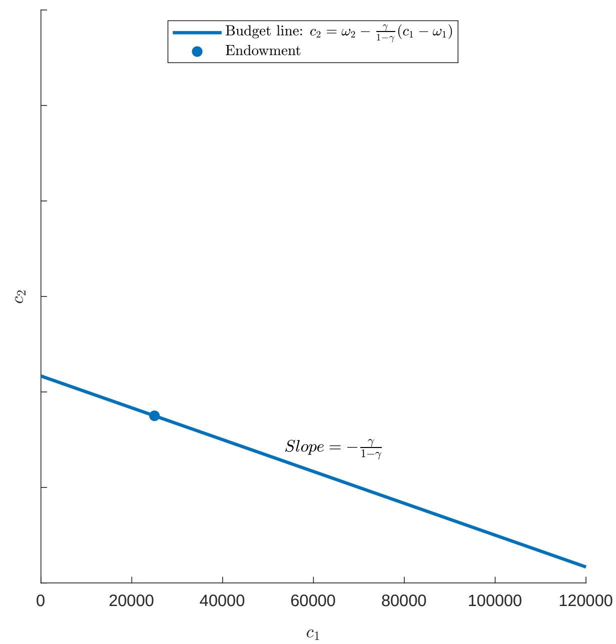
The insurance is a way for the agent to have access to new lotteries (not only the initial lottery where the agent would get ω_1 with probability π and ω_2 with probability $1-\pi$).

The budget line associated with the purchase of insurance is represented in figure 58. It cuts through the point (ω_1, ω_2), which is the endowment of the consumer (what he would get in case of a fire and in case of no fire if he does not buy any insurance). Combining the equations $c_1 = \omega_1 + K - \gamma K$ and $c_2 = \omega_2 - \gamma K$ to eliminate K, we get

$$c_2 = \omega_2 - \frac{\gamma}{1-\gamma}(c_1 - \omega_1)$$

The budget line is a straight line of slope $-\frac{\gamma}{1-\gamma}$ (with c_1 on the x-axis and c_2 on the y-axis).

Figure 58: Risk: budget line with insurance.



Note: In this figure, we represent the budget line $c_2 = \omega_2 - \frac{\gamma}{1-\gamma}(c1 - \omega_1)$, where $\omega_1 = £25,000$, $\omega_2 = £35,000$ and $\gamma = 0.25$.

Suppose that the agent's preferences can be represented by an expected utility function, so that the expected utility of the agent is

$$\pi v(c_1) + (1-\pi)v(c_2)$$

and suppose that the agent is risk-averse. As usual, we can study the agent's optimal choice of insurance using indifference curves. Since the agent is risk averse, the indifference curves are convex. We can see that the optimal choice is located on the indifference curve that is farthest from the origin and on the budget line. Ruling out corner solutions, the optimal choice lies at the tangency point between the indifference curve that is farthest from the origin and the budget line. At this tangency point, the slope of the indifference curve is equal to the slope of the budget line, that is:

$$-\frac{\pi \partial v(c_1)/\partial c_1}{(1-\pi)\partial v(c_2)/\partial c_2} = -\frac{\gamma}{1-\gamma}$$

Fair insurance

Given the price of insurance γ, the expected profits of the insurance company are

$$P = \gamma K - (\pi K + (1-\pi) \times 0) = \gamma K - \pi K$$

We say that an insurance policy is *fair* if the insurance premium is equal to the expected value of the promised insurance payment. In this case, the insurance company makes a zero expected profit, that is

$$\gamma K - \pi K = 0$$

which implies that the insurance company chooses a price $\gamma = \pi$.

If insurance is fair, then at the optimal insurance choice, we have

$$-\frac{\pi \partial v(c_1)/\partial c_1}{(1-\pi)\partial v(c_2)/\partial c_2} = -\frac{\pi}{1-\pi}$$

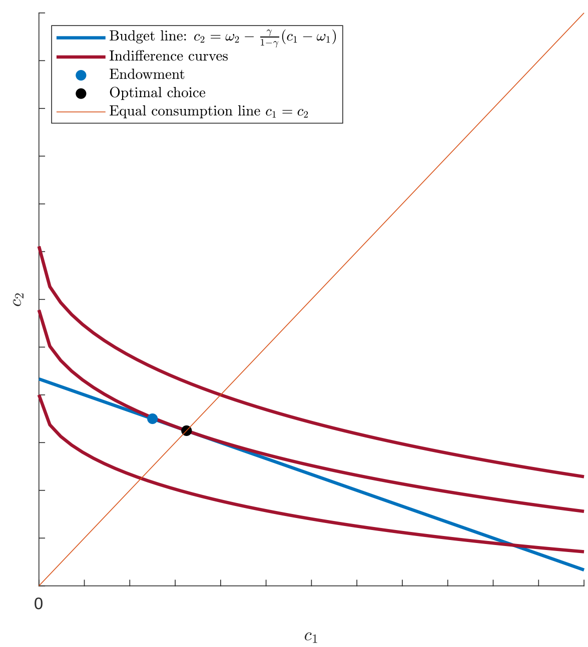
and thus

$$\frac{\partial v(c_2)}{\partial c_2} = \frac{\partial v(c_1)}{\partial c_1}$$

which, under basic assumptions on $v(.)$, implies that $c_1 = c_2$. In other words, whenever the agent is risk-averse and is facing a fair insurance policy, he will choose to *fully insure*: he wants his wealth to be the same whether a crash occurs or not. This is illustrated in figure 59.

EXAMPLE. *Consider the example from figures 58 and 59. A consumer's (who is a lumberjack) wealth is equal to the value of the wood he cuts from the nearby forest, which is* $\omega_1 = £25,000$ *if there is a fire in the forest, or* $\omega_2 = £35,000$ *if there is no fire. A fire occurs with probability* $\pi = 0.25$*. The consumer can purchase* K *units of insurance at a price of* $\gamma = 0.25$ *per unit of insurance (hence, insurance is fair). In case of a fire, his wealth is* $c_1 = \omega_1 + K - \gamma K$*. In the absence of a fire, his wealth is* $c_2 = \omega_2 - \gamma K$*. Note that if the consumer does not buy any insurance* $(K = 0)$*, then his wealth is indeed* $c_1 = \omega_1$ *if a fire occurs, and* $c_2 = \omega_2$ *if not. The budget line is* $c_2 = \omega_2 - \frac{\gamma}{1-\gamma}(c1 - \omega_1)$*, that is* $c_2 = 35,000 - \frac{1}{3}(c1 - 25,000)$*. The consumer's preferences over the wealth he gets if a fire occurs or not are represented by the utility function* $U(c_1, c_2) = \pi\sqrt{c_1} + (1-\pi)\sqrt{c_2}$*. The marginal utilities are* $MU_1 = \frac{\partial U(c_1,c_2)}{\partial c_1} = \frac{\pi}{2\sqrt{c_1}}$ *and* $MU_2 = \frac{\partial U(c_1,c_2)}{\partial c_2} = \frac{1-\pi}{2\sqrt{c_2}}$*. The consumer will choose the bundle such that* $MRS = -\frac{\gamma}{1-\gamma}$*, that is* $\frac{\pi}{1-\pi}\frac{\sqrt{c_2}}{\sqrt{c_1}} = \frac{\gamma}{1-\gamma}$*, thus* $\frac{1}{3}\frac{\sqrt{c_2}}{\sqrt{c_1}} = \frac{1}{3}$*. We get* $c_1 = c_2$*. Using the budget constraint, we have* $c_2 = 35,000 - \frac{1}{3}(c_2 - 25,000)$*, thus* $\frac{4}{3}c_2 = 35,000 + \frac{25,000}{3}$*. We obtain* $c_2 = 32,500$ *and* $c_1 = c_2 = 32,500$*. The consumer chooses to fully insure against the risk of a fire. Since* $c_2 = \omega_2 - \gamma K$*, we can conclude that the consumer chooses to buy* $K = (\omega_2 - c_2)/\gamma = 10,000$ *dollars of insurance.*

Figure 59: Risk: insurance choice.



Note: In this figure, $v(x) = \sqrt{x}$, so that the expected utility is $\pi v(c_1) + (1 - \pi)v(c_2)$ and $\pi = 0.25$. The insurance policy is fair, so $\pi = \gamma$.