ECO00013C
(slides)

Version: updated on February 20, 2023.

Block 1: Topics in consumer theory

1. Lecture 1: Consumer theory
2. Lecture 2: Time
3. Lecture 3: Risk
4. Lecture 4: Labour supply

Block 2: Supply and demand

5. Lecture 5: Competitive markets
6. Lecture 6: Comparative statics

Block 3: Market failures

7. Lecture 7: Monopoly
8. Lecture 8: Oligopoly
9. Lecture 9: Externalities
10. Lecture 10: Public goods
11. Lecture 11: Asymmetry of information

Block 4: Game theory

12. Lecture 12: Strategic games
13. Lecture 13: Sequential games

Block 1: Topics in consumer theory

Lecture 1: Consumer theory

Bundles:

$$(x_1, x_2, ..., x_n)$$

Budget constraint: (two goods only)

$$p_1x_1 + p_2x_2 \leq m$$

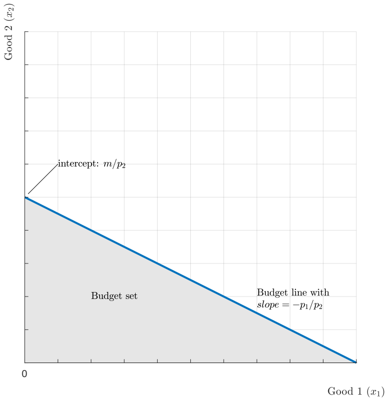
Budget line:

$$x_2 = \frac{m}{p_2} - \frac{p_1}{p_2}x_1$$

Lecture 1: Consumer theory

Budget line and budget set:

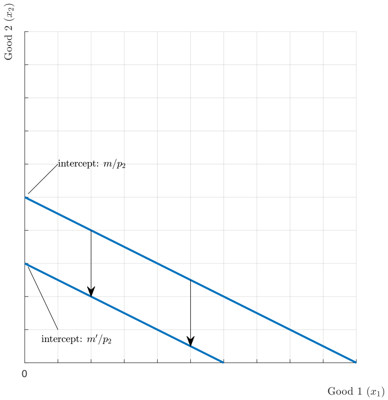
Figure 1: Consumer theory: the budget set and the budget line.



Note: In this figure, $m = 10$, $p_1 = 1$ and $p_2 = 2$.

Budget line shifts:

Figure 2: Consumer theory: shifting the budget line.



Note: In this figure, $m = 10$, $m' = 6$, $p_1 = 1$ and $p_2 = 2$.

Preferences: We often make the following assumptions on preferences:

1. Completeness
2. Reflexivity
3. Transitivity

Well behaved preferences: In addition we often assume

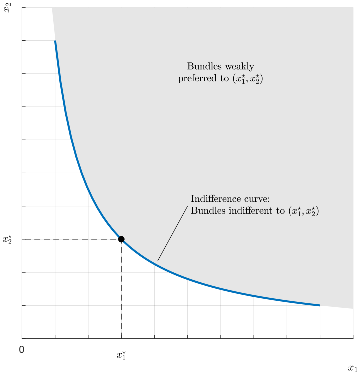
1. Monotonicity
2. Convexity

Indifference curves: An indifference curve graphs the set of bundles that are indifferent to some bundle

Lecture 1: Consumer theory

Indifference curves:

Figure 3: Consumer theory: indifference curves.



Utility function: Utility functions are simple a way to summarize preferences. A utility function $u(x_1, x_2)$ assigns numbers to bundles so that more preferred bundles gets higher numbers.

Utility function and indifference curve: we can find an indifference curve by by finding all the bundles that give the same utility level, that is, we need to find all the bundles (x_1, x_2) such that

$$u(x_1, x_2) = u^\star$$

to construct the indifference curve associated with the utility level $u^\star$.

Marginal utilities:

$$MU_1(x_1, x_2) = \frac{\partial u(x_1, x_2)}{\partial x_1} \tag{1.1}$$

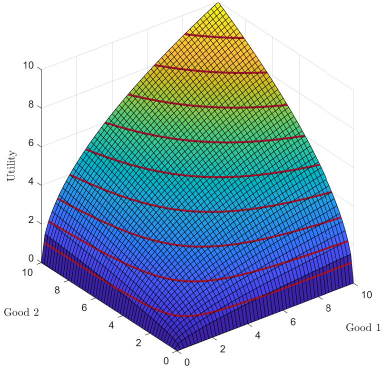
$$MU_2(x_1, x_2) = \frac{\partial u(x_1, x_2)}{\partial x_2} \tag{1.2}$$

Marginal rate of substitution (MRS): the MRS is the slope of the indifference curve at a given point, that is

$$\frac{dx_2}{dx_1} = -\frac{MU_1(x_1, x_2)}{MU_2(x_1, x_2)} \tag{1.3}$$

Utility functions and indifference curves:

Figure 4: Consumer theory: utility function and indifference curves.



Note: In this figure, $u(x_1, x_2) = \sqrt{x_1}\sqrt{x_2}$.

Optimal choice: The optimal choice is the bundle that is associated with the highest indifference curve that just touches the budget line.

Optimal choice (maths): We know that (unless we face a very particular case) the optimal choice is located at the tangency point between the highest indifference curve (the one that is the farther away from the origin) and the budget line. Thus

$$MRS = -\frac{p_1}{p_2}$$

This condition can be rewritten

$$\frac{\frac{\partial u(x_1,x_2)}{\partial x_1}}{\frac{\partial u(x_1,x_2)}{\partial x_2}} = \frac{p_1}{p_2}$$

The optimal solution must also satisfy the budget constraint

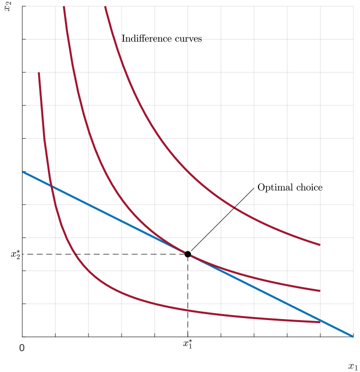
$$p_1 x_1 + p_2 x_2 = m$$

This gives us two equations to find two unknowns. We need to solve these two equations to find the optimal choice of x_1 and x_2.

Lecture 1: Consumer theory

Optimal choice:

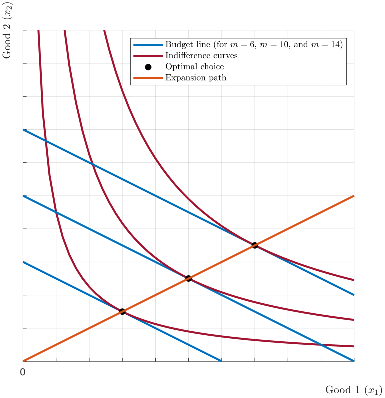
Figure 5: Consumer theory: the optimal choice.



Note: In this figure, $m = 10$, $p_1 = 1$ and $p_2 = 2$. The utility function is $u(x_1, x_2) = \sqrt{x_1 x_2}$.

The income expansion path:

Figure 6: Consumer theory: the income expansion path.

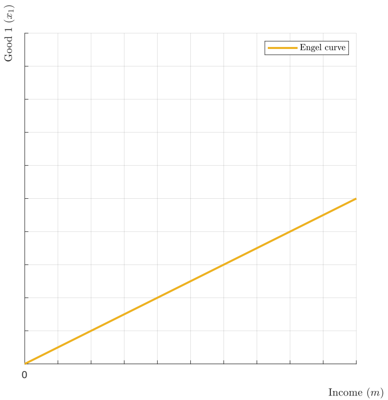


Note: In this figure, $p_1 = 1$ and $p_2 = 2$. We vary income such that $m = 6$, $m = 10$ or $m = 14$. The utility function is $u(x_1, x_2) = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$.

Lecture 1: Consumer theory

The Engel curve:

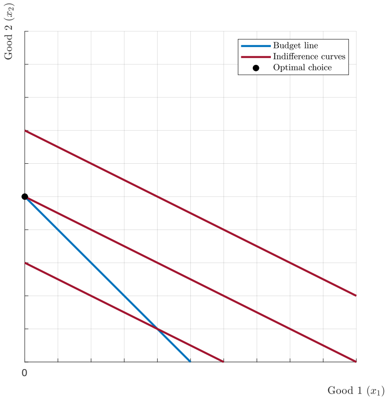
Figure 7: Consumer theory: the Engel curve.



Note: In this figure, $p_1 = 1$ and $p_2 = 2$. The utility function is $u(x_1, x_2) = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$.

Perfect substitutes:

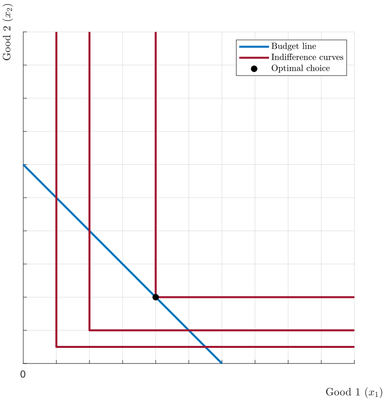
Figure 8: Consumer theory: perfect substitutes.



Note: In this figure, $p_1 = 1$, $p_2 = 1$ and $m = 5$. The utility function is $u(x_1, x_2) = x_1 + 2x_2$.

Perfect complements:

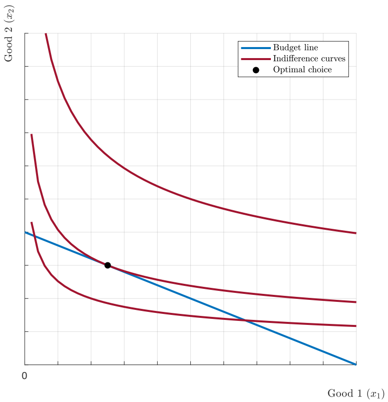
Figure 9: Consumer theory: perfect complements.



Note: In this figure, $p_1 = 1$, $p_2 = 1$ and $m = 5$. The utility function is $u(x_1, x_2) = \min\{\frac{1}{2}x_1, x_2\}$.

Cobb Douglas utility functions:

Figure 10: Consumer theory: the Cobb Douglas utility function.



Note: In this figure, $p_1 = 1$, $p_2 = 2.5$ and $m = 10$. The utility function is $u(x_1, x_2) = x_1^{\frac{1}{4}} x_2^{\frac{3}{4}}$.

Lecture 2: Time

Future value: The future value (i.e next period value) of some amount m received today is

$$(1+r)m$$

Present value: the present value (i.e today value) of some amount m received in the next period is

$$\frac{m}{1+r}$$

Present value (general formula):

$$m_1 + \frac{m_2}{1+r} + ... + \frac{m_{T-1}}{(1+r)^{T-2}} + \frac{m_T}{(1+r)^{T-1}}$$

Budget constraint (future value formulation):

$$(1+r)c_1 + c_2 = (1+r)m_1 + m_2$$

Budget constraint (present value formulation):

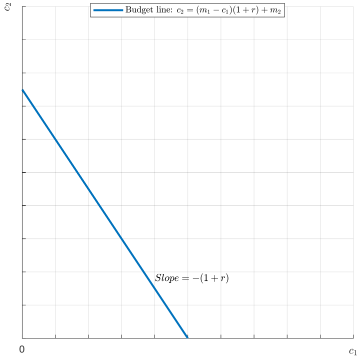
$$c_1 + \frac{c_2}{1+r} = m_1 + \frac{m_2}{1+r}$$

Budget line:

$$c_2 = (1+r)m_1 + m_2 - (1+r)c_1$$

Budget line:

Figure 11: Time: the intertemporal budget constraint.

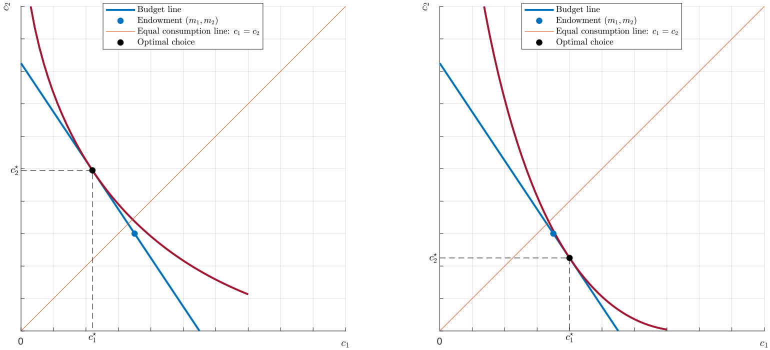


Note: In this figure, $m_1 = 3$, $m_2 = 3$ and $r = 0.5$.

Lecture 2: Time

Optimal intertemporal choice:

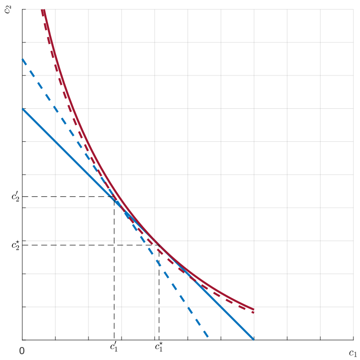
Figure 12: Time: optimal intertemporal choice.



Note: In this figure, $m_1 = 3.5$, $m_2 = 3$ and $r = 0.5$. On the left, $\rho = 0$; on the right $\rho = 1$. The utility function is $u(c_1, c_2) = \sqrt{c_1} + \frac{1}{1+\rho}\sqrt{c_2}$.

Comparative statics:

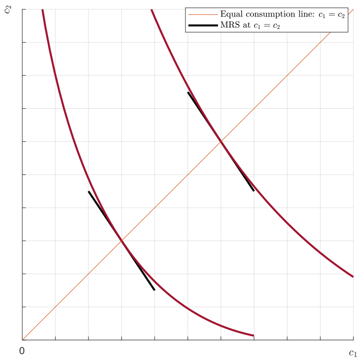
Figure 13: Time: comparative statics.



Note: In this figure, $m_1 = 3$, $m_2 = 4$, and $\rho = 0.2$. The utility function is $u(c_1, c_2) = \sqrt{c_1} + \frac{1}{1+\rho}\sqrt{c_2}$. The interest rate is set to $r = 0$ or $r = 0.5$.

Impatience:

Figure 14: Time: measuring impatience.



Note: In this figure, the utility function is $u(c_1, c_2) = \sqrt{c_1} + \frac{1}{1+\rho}\sqrt{c_2}$. We show the slope of the indifference curves (the MRS) along the equal consumption line.

The discounted utility model:

$$U(c_1, c_2) = u(c_1) + \frac{u(c_2)}{1+\rho}$$

where ρ is called the discount rate.

MRS: The marginal rate of substitution is

$$MRS = -(1+\rho)\frac{\frac{\partial u(c_1)}{\partial c_1}}{\frac{\partial u(c_2)}{\partial c_2}}$$

MRS (along the equal consumption line): With these preferences, we see that along the equal consumption line $c_1 = c_2$, the MRS is always the same and equal to $-(1+\rho)$.

Lecture 3: Risk

Lottery: A lottery specifies each possible outcome of the risky alternative it represents, as well as the probability that this outcome will occur.

Expected value:

$$EV = \pi_1 c_1 + \pi_2 c_2$$

Expected utility:

$$u(c_1, c_2, \pi_1, \pi_2) = \pi_1 v(c_1) + \pi_2 v(c_2)$$

Compound lottery: the compound lottery

$$L_3 = [L_1, L_2; \alpha, 1 - \alpha]$$

is a lottery where the outcomes are the simple lotteries L_1 and L_2, where the lottery L_1 occurs with probability α and the lottery L_2 occurs with probability $1 - \alpha$

Axioms of expected utility theory: we can represent individual preferences by a utility function that has the expected utility form if these assumptions are made

1. Continuity: a small changes in probabilities do not change the nature of the ordering of two lotteries.
2. Independence: if we mix each of two lotteries with a third lottery, then the preference ordering between the two resulting lotteries does not depend on (is independent of) this third lottery.

The Allais paradox: The result from this experiment are not compatible with expected utility theory, hence the name "paradox". Suppose that a player is playing a game in which he can earn either 0, 1 million or 5 million dollars. Suppose that a first lottery L_1 gives the player 1 million dollars with probability 1. A second lottery L_2 gives 0 with probability 0.01, 1 million dollar with probability 0.89, and 5 millions dollars with probability 0.1. Here, most people choose lottery 1 over lottery 2.
Lottery L_3 gives 0 with probability 0.89 and 1 million with probability 0.11.
Lottery L_4 gives 0 with probability 0.9 and 5 million with probability 0.1.
When given the choice between lotteries 3 and 4 only, most people choose lottery 4 over lottery 3.
But this is actually inconsistent with the expected utility theory. Suppose that $v(0) = 0$. The fact that people choose L_1 over L_2 means that $v(1) > 0.89v(1) + 0.1v(5)$. This implies

$$0.11v(1) > 0.1v(5)$$

The fact that most people choose L_4 over L_3 means that

$$0.1v(5) > 0.11v(1)$$

The two equations are not compatible!

Risk-averse: A consumer is said to be risk averse if he prefers a certain outcome to a lottery of equal expected value.

Risk-loving: the expected utility from the lottery is more than the utility from receiving (for certain) the expected value of the lottery.

Risk-neutral: The consumer is indifferent between taking the gamble or receiving for certain the expected value of the gamble.

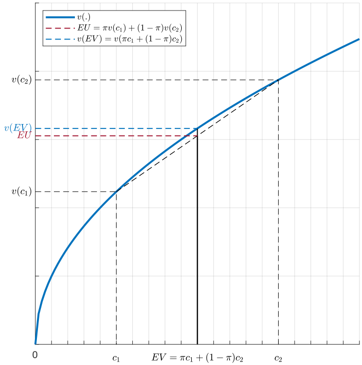
Risk premium: the amount a risk averse agent is willing to pay to avoid taking risks. It is is defined as the difference between the expected value of the lottery and the certainty equivalent.

Certainty equivalent: the amount of money (CE) that, if held with certainty, would provide the same utility as the lottery (EU).

$$v(CE) = EU$$

Risk-averse:

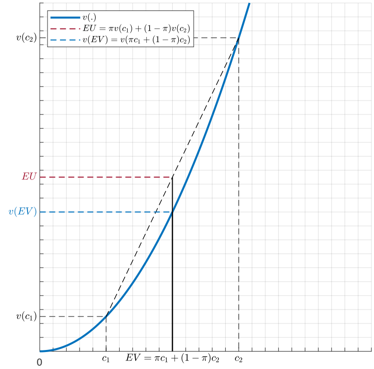
Figure 15: Risk: risk aversion.



Note: In this figure, $\pi = 0.5$, $c_1 = 5$ and $c_2 = 15$. We chose $v(x) = \sqrt{x}$.

Risk-loving:

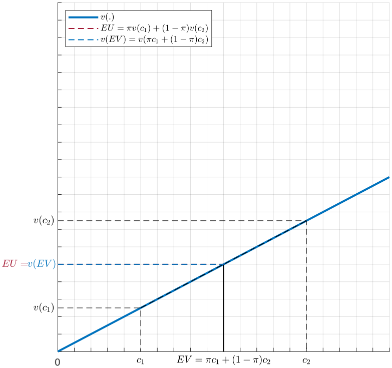
Figure 16: Risk: risk loving.



Note: In this figure, $\pi = 0.5$, $c_1 = 5$ and $c_2 = 15$. We chose $v(x) = 0.1x^2$.

Risk-neutral:

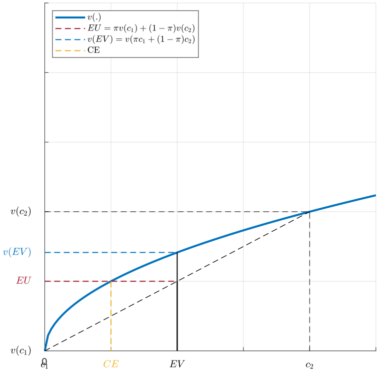
Figure 17: Risk: risk neutral.



Note: In this figure, $\pi = 0.5$, $c_1 = 5$ and $c_2 = 15$. We chose $v(x) = 0.5x$.

Risk premium and certainty equivalent:

Figure 18: Risk: risk premium and certainty equivalent.



Note: In this figure, $\pi = 0.5$, $c_1 = 0$ and $c_2 = 4$. We chose $v(x) = \sqrt{x}$.

Insurance: a consumer can buy K units of insurance at a unit price of γ. If the bad outcome occurs (with probability π, he gets

$$c_1 = \omega_1 + K - \gamma K$$

If the good outcome occurs (with probability $1 - \pi$), then

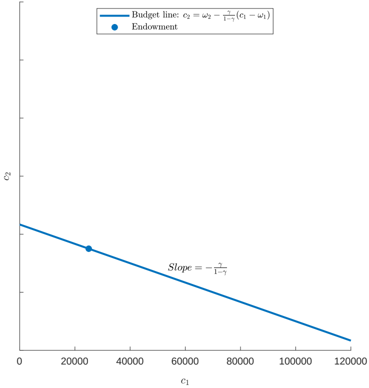
$$c_2 = \omega_2 - \gamma K$$

Budget line:

$$c_2 = \omega_2 - \frac{\gamma}{1-\gamma}(c_1 - \omega_1)$$

Budget line (insurance):

Figure 19: Risk: budget line with insurance.

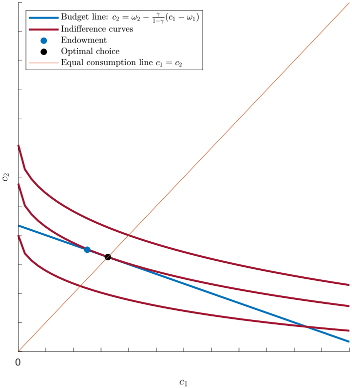


Note: In this figure, we represent the budget line $c_2 = \omega_2 - \frac{\gamma}{1-\gamma}(c1 - \omega_1)$, where $\omega_1 = 25,000\$$, $\omega_2 = 35,000\$$ and $\gamma = 0.25$.

Suppose that the consumer's preferences can be represented by an expected utility function and that the consumer is risk averse.

Optimal choice (insurance):

Figure 20: Risk: insurance choice.



Note: In this figure, $v(x) = \sqrt{x}$, so that the expected utility is $\pi v(c_1) + (1-\pi)v(c_2)$, and $\pi = 0.25$. In this example, the insurance policy is fair, that is $\pi = \gamma$.

Fair insurance: the insurance company charges the price $\gamma = \pi$.

Full insurance: if the consumer is risk averse, insurance is fair and his preferences are compatible with the expected utility model, then the consumer will choose to fully insure.

Lecture 4: Labour supply

Non-labor income: the amount M the consumer receives whether he works or not.

Consumption: the amount C the consumer decide to consume (at price p).

Labor: the number of hours of work the consumer decides to provide, for which he earns a wage w per hour worked.

Budget constraint:

$$pC = M + wL$$

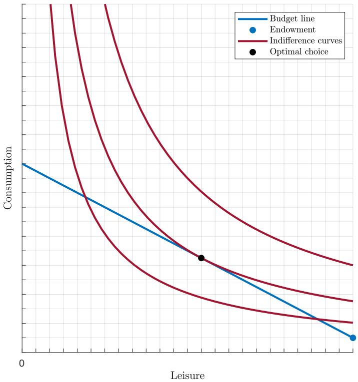
Budget constraint:

$$pC + wR = p\bar{C} + w\bar{L}$$

where $\bar{C} = M/p$.

Optimal choice (labor):

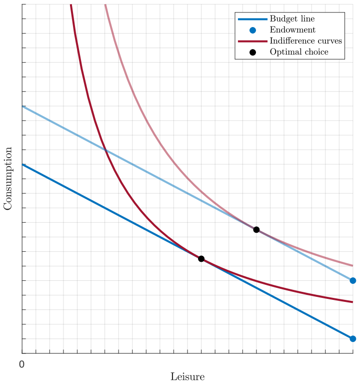
Figure 21: Labor: the optimal choice.



Note: In this figure, $M = 1$, $p = 1$, $w = 0.5$ and $\bar{L} = 24$. Preferences are represented by the utility function $u(R, C) = R^{\frac{1}{2}} C^{\frac{1}{2}}$.

Comparative statics (non labor income):

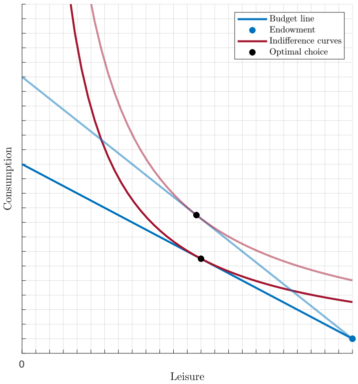
Figure 22: Labor: comparative statics (income).



Note: In this figure, $p = 1$, $w = 0.5$ and $\bar{L} = 24$. Preferences are represented by the utility function $u(R, C) = R^{\frac{1}{2}} C^{\frac{1}{2}}$. M increases from $M = 1$ (opaque blue line) to $M = 5$ (transparent blue line).

Comparative statics (wage):

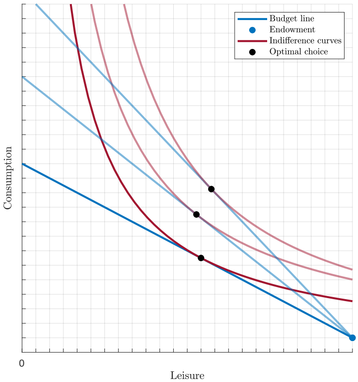
Figure 23: Labor: comparative statics (wage).



Note: In this figure, $M = 1$, $p = 1$ and $\bar{L} = 24$. Wage increases from $w = 0.5$ (opaque) to $w = 0.75$ (transparent). Preferences are represented by the utility function $u(R, C) = R^{\frac{1}{2}} C^{\frac{1}{2}}$.

Comparative statics (wage):

Figure 24: Labor: backward-bending labor supply.



Note: In this figure, $M = 1$, $p = 1$ and $\bar{L} = 24$. Wage increases from $w = 0.5$ (opaque) to $w = 0.75$ to $w = 1$ (transparent). Preferences such that we get a backward-bending labor supply.

Block 2: Competitive markets and comparative statics

Lecture 5: Competitive markets

Individual supply curve: The supply curve for an individual firm i is denoted $S_i(p)$, and shows the quantity supplied by firm i at each possible price p.

Market supply curve: the supply curve is simply

$$S(p) = \sum_{i=1}^{n} S_i(p)$$

i.e the sum of the individual supply curves (where n is the number of firms).

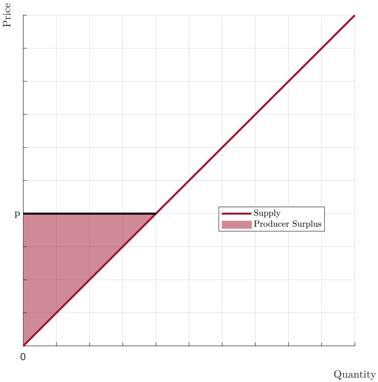
Inverse supply: the inverse supply function

$$P_S(q)$$

is the price at which producers would be willing to sell q units of goods.

Producer surplus: the producers' surplus is the area above the inverse supply curve and below the market price, up to the quantity producers sell.

Figure 25: Supply: The producers' surplus.



Note: The producers' surplus is showed as the filled area above the inverse supply curve and below the price.

Price elasticity of supply: the price elasticity of supply measures how the quantity supplied of a good changes with the price of that good. Formally, it is defined as

$$\epsilon_s = \frac{\partial S(p)}{\partial p} \frac{p}{S(p)}$$

Individual demand curve: The demand curve for an individual consumer j is denoted $D_j(p)$, and shows the quantity demanded by consumer j at each possible price p.

Market demand curve: the demand curve is the sum of all the individual demand curves

$$D(p) = \sum_{j=1}^{m} D_j(p)$$

where m is the number of consumers on the market.

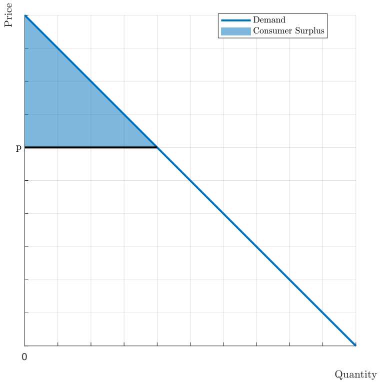
Inverse demand: the inverse demand function

$$P_D(q)$$

is the price at which consumers would be willing to buy q units of goods.

Consumer surplus: the consumers' surplus is the area below the inverse demand curve and above the market price, up to the quantity consumers buy.

Figure 26: Demand: The consumers' surplus.



Note: The consumers' surplus is showed as the filled area below the demand curve and above the price.

Price elasticity of demand: the price elasticity of demand measures how the quantity demanded of a good changes with the price of that good. Formally, it is defined as

$$\epsilon_d = \frac{\partial D(p)}{\partial p} \frac{p}{D(p)}$$

In other words, the price elasticity of demand is the percentage change in demand in response to a given percentage change in the price.

Perfect competition: a market environment is one of perfect competition if the following conditions are met:

1. Homogeneity of the good: The goods sold on the market by firms are strictly identical
2. Atomicity: the number of buyers and sellers on the market is sufficiently large so that none of their decisions can influence the equilibrium price.
3. Free entry: firms can enter and exit the market freely
4. Transparency: all buyers and sellers know the price at which they can buy or sell the good.

Partial equilibrium: examining equilibrium and changes in equilibrium on one market only, in isolation from other markets.

ebay ✓ (?)

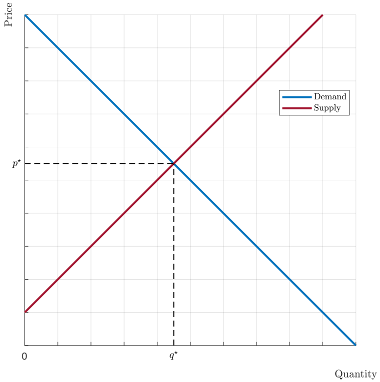
AIRBUS BOEING ✗

Equilibrium: a market is in equilibrium when the total quantity supplied by firms is equal to the total quantity demanded by consumers. The price for which supply equals demand is the equilibrium price. Formally, the equilibrium price $p^\star$ is such that

$$D(p^\star) = S(p^\star)$$

The equilibrium quantity of good produced (and consumed) is $q^\star = D(p^\star) = S(p^\star)$.

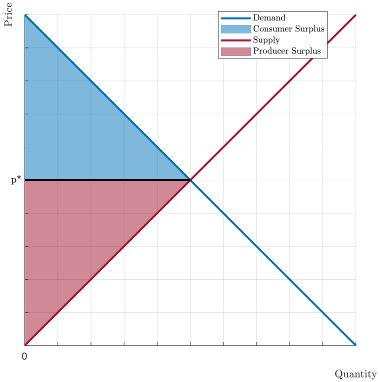
Figure 27: Partial Equilibrium: Market equilibrium.



Note: This figure illustrates the market equilibrium with supply curve $S(p) = c + dp$ ($c = -1$ and $d = 1$) and demand curve $D(p) = a - bp$ ($a = 10$ and $b = 1$) .

Total surplus: the total surplus is the sum of the consumers' and producers' surpluses. A perfectly competitive market maximizes the total surplus.

Figure 28: Partial Equilibrium: The total surplus.



<u>Note:</u> The total surplus is the sum of the blue and red areas (the sum of the consumers' and producers' surpluses).

Lecture 6: Comparative statics

Comparative statics: comparative statics consist of studying the effect of changes in the market environment on the equilibrium prices and quantities. For example:

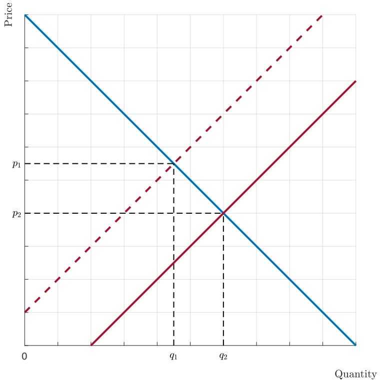
- Supply shift
- Demand shift
- Price ceilings and price floors
- Quotas
- Quantity taxes

In perfect competition, total surplus is maximized in equilibrium. Thus, market interventions will very often generate a deadweight loss: a net reduction in total surplus.

(a) Supply shift

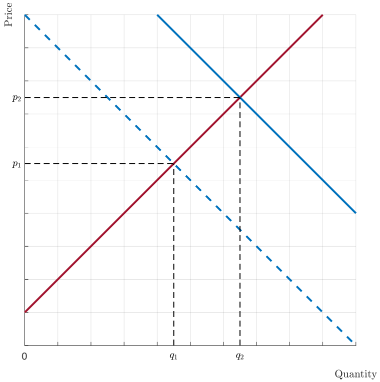
(b) Demand shift

Figure 30: Comparative statics: Supply shift.



Note: The figure illustrates a supply shift (from dashed to solid line: increased supply for all price levels). The equilibrium price decreases and the equilibrium quantity increases.

Figure 31: Comparative statics: Demand shift.



<u>Note:</u> The figure illustrates a demand shift (from dashed to solid: increased demand for all price levels). The equilibrium price and quantity increase.

Lecture 6: Comparative statics

(a) Price ceiling

(b) Price floor

(c) Quota

Let $p^\star$ and $q^\star$ be the equilibrium price and quantity in perfect competition.

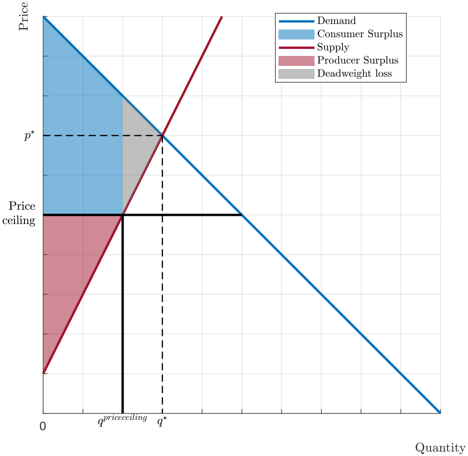
Price ceiling: a price ceiling, denoted $\bar{p}$, is the highest price at which consumers can legally buy the good. A price ceiling is such that $\bar{p} < p^\star$ (otherwise, it is useless!). Price ceilings generate excess demand and a deadweight loss.

Price floor: a price floor, or minimum price denoted $\underline{p}$, is the lowest price at which consumers can legally buy the good. A price floor is such that $\underline{p} > p^\star$ (otherwise, it is useless!). Price floors generate excess supply and a deadweight loss.

Quota: a quota, q^{quota}, sets the quantity of good provided. Whenever $q^{quota} > q^\star$, the goal is to force firms to produce more of the good. Whenever $q^{quota} < q^\star$, the goal is to limit the quantity of good produced. Very often, a quota is such that $q^{quota} < q^\star$. In this case, the quota generates a deadweight loss, and decreases the surplus of the consumers ; the producers' surplus usually increases.

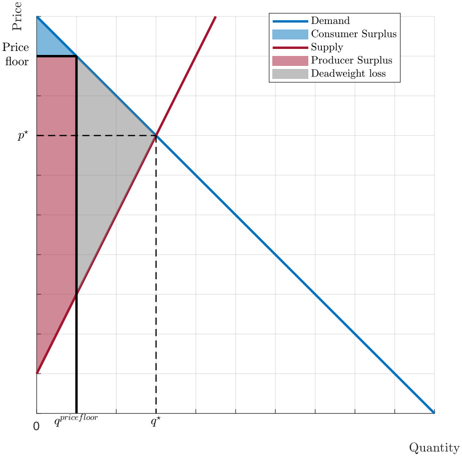
Lecture 6: Comparative statics

Figure 33: Comparative statics: Price ceiling.



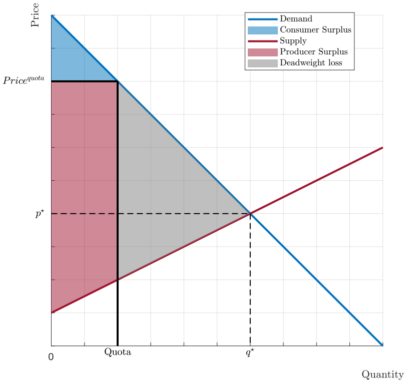
Note: The figure illustrates what happens when a price ceiling is introduced. Price ceilings generate excess demand and a deadweight loss.

Figure 34: Comparative statics: Price floor.



Note: The figure illustrates what happens when a price floor is introduced. Price floors generate excess supply and a deadweight loss.

Figure 35: Comparative statics: Quotas.



Note: The figure illustrates what happens when a quota is introduced. The quota generates a deadweight loss.

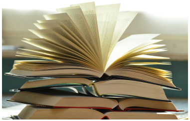
Lecture 6: Comparative statics

(a) Pollution

(b) Candy

(c) Gas

(d) Alcohol



(e) Books

Lecture 6: Comparative statics

Quantity tax: a quantity tax is a tax levied per unit of quantity bought or sold. Taxing the good introduces a wedge between the demand price (what the consumer pays, p_d) and the supply price (what the producer receives, p_s).

Demand and supply price: the demand and supply prices are related by the equation

$$p_s = p_d - t$$

Equivalently, we can obtain an expression for the demand price as $p_d = p_s + t$.

Equilibrium: equilibrium happens when

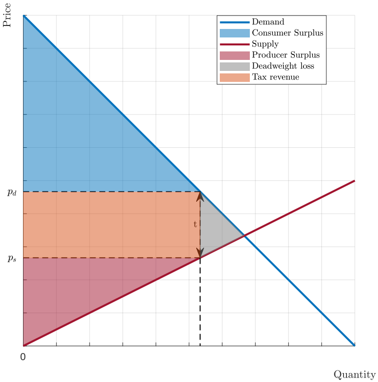
$$D(p_d) = S(p_s)$$

To solve for the equilibrium demand and supply prices and equilibrium quantity, combine the equations $p_s = p_d - t$ and $D(p_d) = S(p_s)$ and solve

$$D(p_d) = S(p_d - t)$$

Taxes generate a deadweight loss. The blue area represents the new consumer surplus, the red area is the new producer surplus, while the yellow rectangle represents the amount of money raised by the tax. This rectangle is the government surplus.

Figure 37: Comparative statics: Taxes.



Note: The imposition of a tax introduces a wedge t between the demand price p_d and the supply price p_s. Taxation generates a deadweight loss. Except in extreme cases, the tax is passed along to both the consumers and the producers. In this example, $D(p) = 10 - p$, $S(p) = 2p$ and $t = 2$.

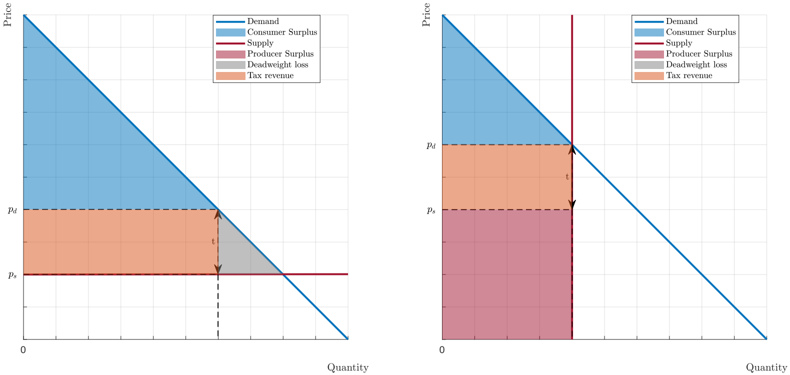
Passing along: that a tax is required to be paid by producers does not mean that they will end up paying the full tax. Conversely, that a tax is required to be paid by consumers does not mean they will end up paying the full tax. In fact, a tax will generally increase the price paid by consumers and decrease the price received by producers: the tax is passed along to both the consumers and the producers.

Elasticity: the steeper the demand (or supply) curve is, the less elastic it is. Demand (or supply) is said to be perfectly elastic if it is a horizontal line. Demand (or supply) is said to be perfectly inelastic if it is a vertical line.

Passing along: how much is passed along to consumers and producers depends on the elasticity of the demand and supply curves.

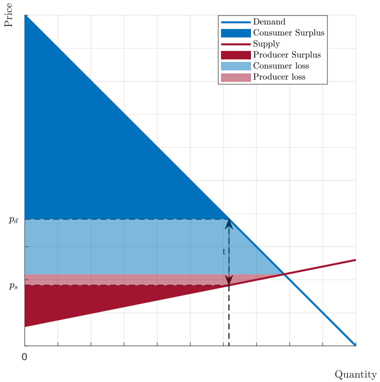
- in case demand (supply) is perfectly elastic, the tax is completely passed along to producers (consumers).
- in case demand (supply) is perfectly inelastic, the tax is completely passed along to consumers (producers).

Figure 38: Comparative statics: Taxes, perfectly (in)elastic supply curve.



<u>Note:</u> This illustrates the passing along of a tax when supply is perfectly elastic (left) or perfectly inelastic (right).

Figure 39: Comparative statics: Taxes with a relatively elastic supply curve.



Note: This illustrates the passing along of a tax when supply is quite elastic. The tax is mostly passed along to consumers.

Block 3: Market failures

Lecture 7: Monopoly

General equilibrium: the study of how equilibrium is determined in many (interconnected) markets simultaneously.

Competitive market economy: an economy in which every relevant good is traded in a market (complete markets) at publicly known prices and all agents act as price takers (perfect competition).

Fundamental welfare theorems: all competitive (or Walrasian) equilibria are Pareto efficient (first theorem) ; any Pareto efficient allocation can be reached by the use of competitive markets (second theorem).

Market failure: situation in which some of the assumptions of the welfare theorems do not hold and in which, as a consequence, market outcomes may be inefficient.

Types of market failure:

- Market power: some agents are no longer price takers.
- Externalities: the actions of one agent affect other agents (positively or negatively), without any market to account for it. Thus the complete markets assumption is violated
- Public goods: as above, in the presence of public goods, the complete markets assumption is violated
- Asymmetry of information: some agents have more information than others.

(a) Standard Oil Company

(b) The Pennsylvania Railroad Company

(c) Microsoft

Lecture 7: Monopoly

Monopoly: a market is a monopoly when there is a single firm offering the good.

Reasons for monopoly: natural monopolies, control of a rare resource/input or patent, institutional/state monopolies, predatory behavior.

Demand: the demand function

$$y = D(p)$$

gives us the amount y of good that can be sold on the market when the price is p.

Inverse demand: The inverse demand function

$$p(y)$$

defines the maximum price at which y units of good can be sold.

Lecture 7: Monopoly

Revenue: the revenue function of the monopoly firm is denoted

$$r(y) = p(y)y$$

Marginal revenue: the marginal revenue function

$$MR(y) = \frac{\partial r(y)}{\partial y}$$

defines the additional revenue the firm gets from selling one additional unit of output.

Profit maximization: the monopoly's profit maximization problem can be written

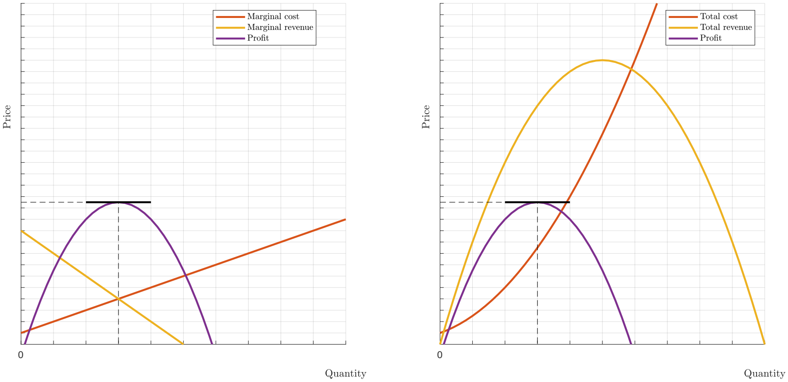
$$\max_y p(y)y - c(y)$$

where c(y) is the total cost function.

Optimal output: at the optimal choice of output $y^\star$, the marginal revenue should be equal to the marginal cost, that is

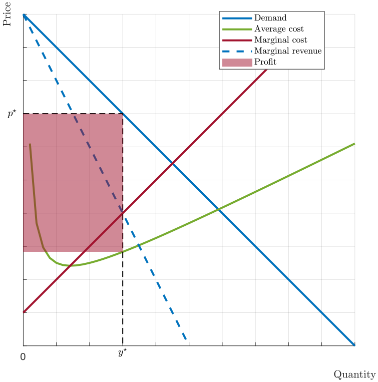
$$MR(y^\star) = MC(y^\star)$$

Figure 41: Monopoly: profit maximization.



Note: In this figure, we show the optimal output choice for a monopoly with the cost curve $c(y) = F + \alpha y + \beta \times y^2$ and facing the inverse demand $p_D(y) = a - b \times y$. We chose $F = 1$, $\alpha = 1$, $\beta = 0.5$, $a = 10$ and $b = 1$.

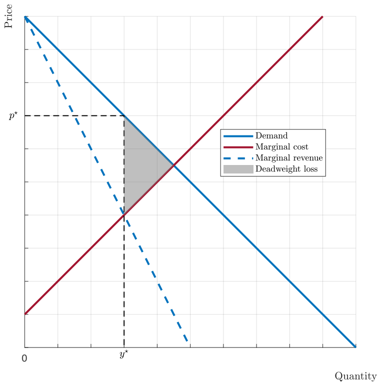
Figure 42: Monopoly: The monopolist behavior.



Note: In this figure, we show the optimal output choice for a monopoly with the cost curve $c(y) = F + \alpha y + \beta \times y^2$ and facing the inverse demand $p_D(y) = a - b \times y$. We chose $F = 1$, $\alpha = 1$, $\beta = 0.5$, $a = 10$ and $b = 1$.

Lecture 7: Monopoly

Figure 43: Monopoly: Deadweight loss of a monopoly.



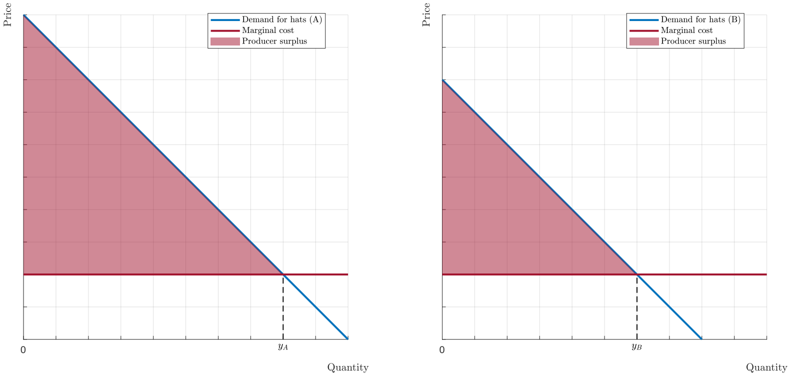
Note: In this figure, we show the deadweight loss generated by monopolies.

Pricing strategies: here are a few examples

- First degree price discrimination: each unit of the good is sold at the consumer who values it the most, and he is charged his willingness to pay.
- Second degree price discrimination: the firm sets a price per unit of output that is not constant but varies with the quantity bought by the consumer (quantity discounts and block pricing).
- Third degree price discrimination: the firm sells the good at different prices to different groups of consumers.
- Bundling: the firm is producing several goods and sells them as a bundle.
- Two-part tariffs: a two part tariff consists of a lump-sum fee p_1 (paid only once) plus a price p_2 per unit of good bought.

Pricing strategies examples: first degree price discrimination.

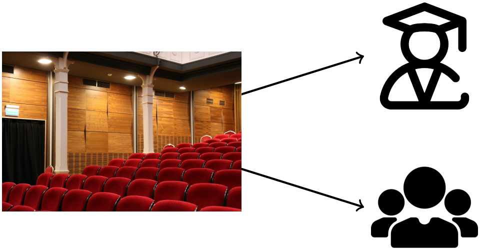
Figure 44: Monopoly: first degree price discrimination.



Note: This illustrates first degree price discrimination. The firm is selling each consumer (A on the left, B on the right) the good at the maximum price they are willing to pay for it. The firm is capturing all the consumer surplus.

Pricing strategies examples: third degree price discrimination.

$D_S(p) = 100 - 2p$



Movie theater

$(MC(y) = 20, F = 0)$

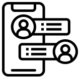
$D_N(p) = 100 - p$

- Without 3rd degree price discrimination: profit $= 1631$
- With 3rd degree price discrimination: profit $= 2050$

Pricing strategies examples: bundling.

A: \$12

B: \$9



A: \$5

B: \$7

A: \$17

B: \$16

In this example, $MC(y) = 0$.

- Without bundling: profit $= 28$
- With bundling: profit $= 32$

Lecture 8: Oligopoly

Lecture 8: Oligopoly

(a) Samsung

(b) Apple

(c) Huawei

(d) Xiaomi

(e) Oppo

Lecture 8: Oligopoly

(a) Airbus

(b) Boeing

Lecture 8: Oligopoly

Oligopoly: a market is an oligopoly when there are a few firms offering the good. When there are only two firms, the market is a duopoly.

Reasons: barriers to entry (institutional barriers, technological barriers (economies of scale), rare resources or patents, predatory behaviors), differentials in production costs, product differentiation.

A variety of settings:

- Firms compete sequentially by choosing quantity. The firm that sets its quantity first is the price leader, the other is the price follower.
- Firms compete sequentially by choosing price. The firm that sets its price first is the price leader, the other is the price follower.
- Firms compete simultaneously by choosing quantity.
- Firms compete simultaneously by choosing price.

Lecture 8: Oligopoly

(a) Organization of the Petroleum Exporting Countries

(b) Federation of Quebec Maple Syrup Producers

Cartels: when firms behave in a cooperative way, it is said that the firms are colluding, and that they form a cartel.

Profit maximization (cartels): in a cartel with two firms, they would choose output levels y_1 and y_2 jointly by solving

$$\max_{y_1, y_2} p(y_1 + y_2)\,[y_2 + y_2] - c(y_1) - c(y_2)$$

Instability of cartels: each firm has an incentive to cheat, i.e to increase its output by a little bit to increase its profit.

Maintaining cartels: cheating detection, enforcement mechanisms, institutional mechanisms, barriers to entry.

Figure 48: Stackelberg

<u>**Stackelberg model:**</u> a leader (firm 1, the Stackelberg leader) chooses quantity y_1 first, so as to maximize its profit, while taking into account the reaction behavior (output choice y_2) of the other firm (firm 2).

<u>**Firm 2's problem:**</u>

$$\max_{y_2} p(y_1 + y_2)y_2 - c(y_2)$$

<u>**Reaction function:**</u> solving firm 2's problem will typically provide us with a reaction function

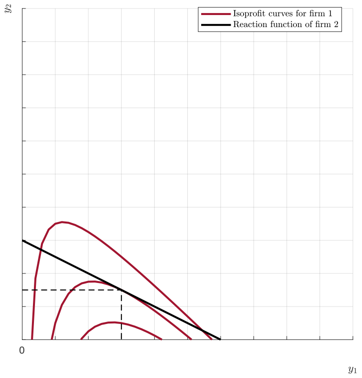
$$f_2(y_1)$$

which defines how much output firm 2 will produce given how much output firm 1 has produced.

<u>**Firm 1's problem:**</u> the leader's profit maximization problem, taking into account the follower's reaction function, is written

$$\max_{y_1} p(y_1 + f_2(y_1))y_1 - c(y_1)$$

Figure 49: Oligopoly: Stackelberg equilibrium.



Note: Firm 1 is the Stackelberg leader. He chooses the point on his competitor's reaction curve that gives him the highest profit.

Figure 50: Cournot

Lecture 8: Oligopoly

Cournot model: the two firms choose input simultaneously, while taking into account what they expect their competitor will do.

Beliefs: y_2^e is the amount firm 1 expect firm 2 to produce and y_1^e the amount firm 2 expect firm 1 to produce.

Profit maximization: firm 1 solves

$$\max_{y_1} p(y_1 + y_2^e)y_1 - c(y_1)$$

and firm 2 solves

$$\max_{y_2} p(y_1^e + y_2)y_2 - c(y_2)$$

Reaction function: solving these profit maximization problems will give us the reaction functions

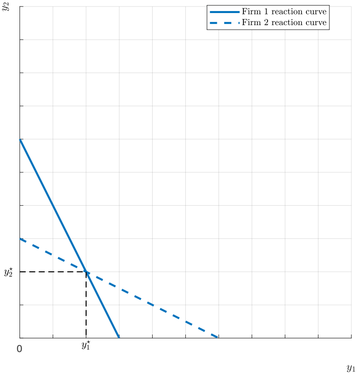
$$y_1 = f_1(y_2^e) \text{ and } y_2 = f_2(y_1^e)$$

Equilibrium: a combination of output $(y_1^\star, y_2^\star)$ forms a Cournot equilibrium whenever

$$y_1^\star = f_1(y_2^\star) \text{ and } y_2^\star = f_2(y_1^\star)$$

Lecture 8: Oligopoly

Figure 51: Oligopoly: Cournot equilibrium.



Note: Firm 1 and Firm 2 choose quantity simultaneously. The Cournot equilibrium lies at the intersection point between the two firms reaction curves.

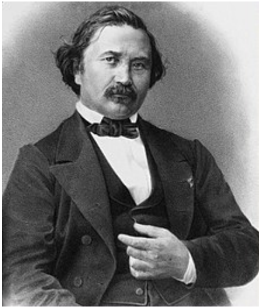


Figure 52: Bertrand

Bertrand model: the two firms choose price simultaneously.

Prices and costs: let p_1 be the price set by firm 1, and p_2 the price set by firm 2. Assume that the marginal cost of producing the good is constant and is the same for the two firms (we will denote it c).

Equilibrium: in this model, the only equilibrium is the competitive equilibrium, i.e $p_1 = p_2 = c$.

Lecture 9: Externalities

Externality: an externality is a situation in which (i) the welfare of a first agent is directly affected by the actions of a second agent (ii) there is no market to evaluate this effect and reward or penalize the second agent accordingly.

Examples: we distinguish

1. Negative externalities
2. Positive externalities

(a) Negative externality

(b) Positive externality

To study externalities, we will mostly make use of the supply and demand model from the first chapter.

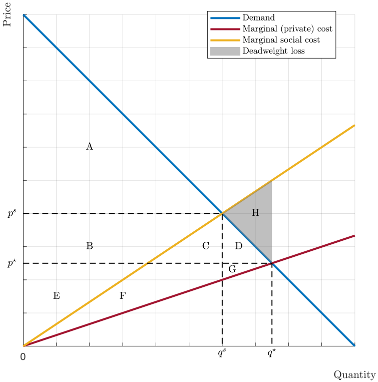
Marginal (private) cost: the private cost of producing one additional unit of good

Marginal social cost: the sum of the marginal (private) cost curve and of the marginal external cost of pollution.

Competitive equilibrium: the competitive equilibrium is, as usual, given by the intersection between the marginal social cost curve and the demand curve.

Social optimum: the social optimum is given by the intersection between the marginal cost curve and the demand curve.

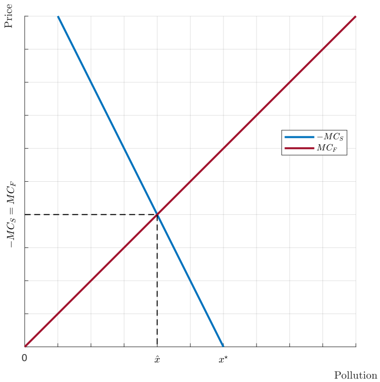
Figure 54: Externalities: Negative externalities (pollution).



Note: In this figure, the competitive equilibrium is determined by the intersection between the demand curve and the (private) marginal cost curve ; the social optimum is located at the intersection of the demand curve and the social marginal cost curve. In this example, the demand is $D(p) = 10 - p$, the marginal cost curve is $MC(y) = y/3$ and the social marginal cost curve is $SMC(y) = 2y/3$.
If externalities were ignored (competitive equilibrium), the consumer surplus would be $A + B + C + D$, the producer surplus would be $E + F + G$ and the total damage caused by pollution would be $F + G + C + D + H$. The total surplus is therefore $A + B + E - H$. If externalities are accounted for (socially optimal equilibrium), the consumer surplus is A, the producer surplus is $B + C + E + F$ and the total damage from pollution is $F + C$. Thus the total surplus is $A + B + E$. Thus, if ignored, the negative externality generates a deadweight loss equal to H.

Another interesting example is the production externality example from the lecture notes.

Figure 55: Externalities: Negative production externality.



Note: In this figure, we show that the scotch firm produces pollution up to the point where the marginal cost of pollution (cost reduction) is zero. The socially optimal level of pollution is at the point where the marginal cost of pollution for the scotch firm is equal to the marginal cost for the fishermen. In this example, $c_s(s,x) = 1 + s^2 + (6-x)^2$ and $c_f(f,x) = 1 + f^2 + \frac{1}{2}x^2$.

Another interesting example is the consumption externality example from the lecture notes.

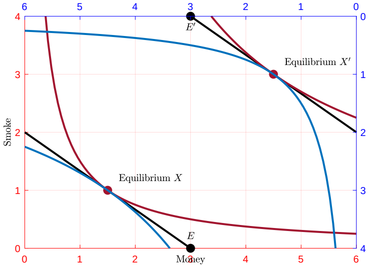
The economy: two roommates, A and B.

Preference: both like money, and A likes smoking while B likes a smoke free apartment.

Endowment: each roommate starts with 50\$. Depending on the legal system, either smoking is forbidden (endowment E) or permitted (endowment E').

Edgeworth box: this situation can be represented in a Edgeworth box.

Figure 56: Externalities: Consumption externalities, the Edgeworth box and equilibrium.



<u>Note:</u> In this figure, we show an Edgeworth box with consumption externalities: A enjoys money and smoking, while B enjoys money and "A not smoking". The equilibrium is either X or X', depending on the assignment of property rights.

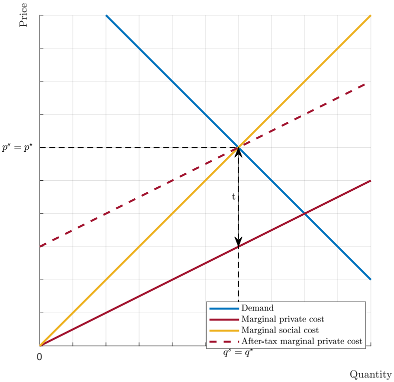
Solutions to the externality problem:

1. Emission standards (I): impose that the industry produces no more than the socially optimal level of output (we know the relationship between output and pollution).
2. Emission standards (II): impose that firms should not produce more that the socially optimal level of pollution (we can directly observe the amount of pollution produced by firms). The choice between I or II depends on the information the government has.
3. Taxes and subsidies: in case of a negative externality, we could tax the agent generating the negative externality. In case of a positive externality, we could subsidize the agent generating the positive externality.

4. Pollution permits/vouchers: we could allow firms to pollute, providing they buy a permit for each unit of pollution they emit. The total number of permits available sets the maximum amount of pollution that can be generated. Each firms is endowed with a certain number of permits. Clean firms can sell the permits to dirtier firms (those firms who find it more difficult to reduce pollution emissions).
5. Mergers: if one firm is polluting and, doing so, harms another firm, then the two firms could merge.
6. Bargaining: in the presence of negative externalities, bargaining between the parties could lead to an outcome in which everyone is better off.

The Coase theorem: if property rights (the exclusive right to use an asset) are well-defined and there are no transaction costs (parties can bargain easily), then economic agents will contract to achieve an efficient outcome, irrespective of the assignment of property rights.

Figure 57: Externalities: Internalizing the negative externality by taxation.



Note: In this figure, a tax t is imposed on the polluting firms ; as a result, the negative externality is internalized by firms, and the social optimum is reached.

Lecture 10: Public goods

Excludability: a good is said to be excludable if people can be excluded from consuming it.

Rivalry: a good is said to be rival if one person's consumption of that good reduces the amount available to others.

Types of goods:

	Rival	*Non Rival*
Excludable	Private good	Club good
Non Excludable	Common good	Public good

Lecture 10: Public goods

(a) Private good

(b) Club good

(c) Common good

(d) Public good

Lecture 10: Public goods

The economy: two consumers A and B, a public good that costs c. The reservation price r_A and r_B are the maximum amount of money A and B are willing to pay to have the public good available, respectively

Payment scheme: a payment scheme (g_A, g_B) specifies how much money A and B will contribute to buy the public good. The public good can only be acquired if

$$g_A + g_B \geq c$$

Necessary condition: suppose $g_A + g_B \geq c$, then purchasing the public good is a Pareto improvement if

$$r_A > g_A$$
$$r_B > g_B$$

Sufficient condition: we can always find a payment scheme such that the purchasing the public good is a Pareto improvement if

$$r_A + r_B > c$$

Figure 59: Public goods: the free rider problem.

		Player B	
		Buy	*Don't buy*
Player A	*Buy*	**(-50,-50)**	**(-50,100)**
	Don't buy	**(100,-50)**	**(0,0)**

Solutions to the free-rider problem.

1. Government provision
2. Voting
3. Auctions

The economy: b boats are sent to sea, and catch $f(b)$ fish. The price of sending a boat to the sea is a per boat.

Social optimum: the fishing port/society/etc wants to maximize the total wealth of the fishing town, and thus chooses b by solving

$$\max_b f(b) - ab$$

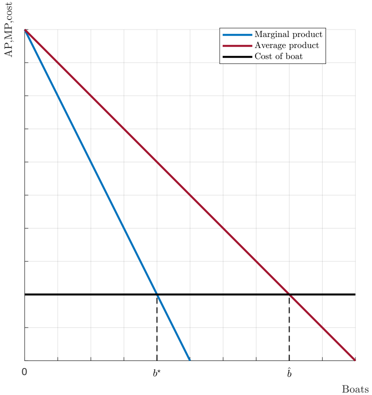
(the price of fish is just one for simplicity). Thus the optimal number of boats $b^\star$ is such that

$$MP(b^\star) = a$$

The fisherman problem: a fisherman contemplating the idea of sending a boat to the sea can expect to catch $f(b)/b = AP(b)$ fish, Thus, fishermen decide to go to the sea as long as $f(b)/b > a$. The number of boats sent to the sea will be $\hat{b}$ such that

$$f(\hat{b})/\hat{b} = a$$

Figure 60: Public goods: the tragedy of the commons



Note: The socially optimal number of boats sent to the sea is $b^\star$, when the marginal product of a boat is equal to its cost. But when fishermen individually decide whether to send a boat of not, the number of boats will be $\hat{b}$, when the average product of a boat is equal to its cost. This leads to overfishing.

Lecture 11: Asymmetry of information

Asymmetric information: a market with imperfect information, on which one side is better informed than the other, is a market with asymmetric information.

Adverse selection and moral hazard:

- Adverse selection: one party involved in a transaction has information about a hidden characteristic (regarding a person or a good) that is unknown to other parties, and takes advantage of this information. There is asymmetric information because of this hidden characteristic.
- Moral hazard: one informed party takes an action that another party cannot observe, and that harms the less-informed party. There is asymmetric information because of this hidden action.

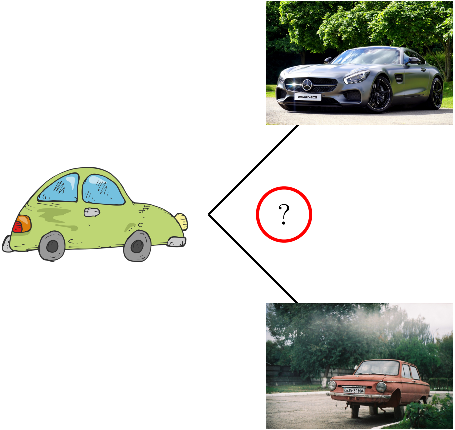


Figure 61: Adverse selection

Lecture 11: Asymmetry of information

We start with adverse selection and the market for lemons example.

The economy: 50 good cars and 50 bad (lemons) cars are for sale. Buyers don't observe quality of the car, but sellers know the quality.

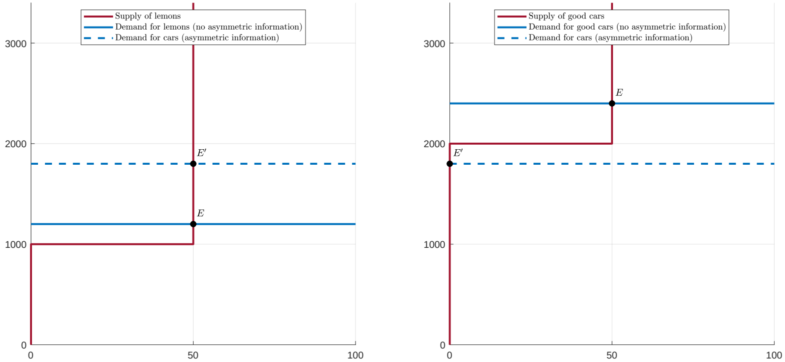
Sellers: willing to sell a good car for $2,000$\$ and a bad car for $1,000$\$.

Buyers: willing to buy a good car for $2,400$\$ and a bad car for $1,200$\$.

Pooling and separating equilibria: a market equilibrium in which both types of goods are traded and cannot be distinguished by the buyers is a pooling equilibrium. A market equilibrium in which only one of the two types of goods is traded, or both are traded and can be distinguished by the buyers, is a separating equilibrium.

The market for lemons: the pooling equilibrium cannot survive, as good quality cars are driven out of the market. The only equilibrium that remains is a separating equilibrium where only low quality cars are traded.

Figure 62: Asymmetric information: adverse selection.



Note: In this figure, we represent the problem of adverse selection on the market for used car. The quantity of lemons traded (without or with asymmetric information) and good cars traded (without or with asymmetric information) are shown on the left and right panels respectively.

Solutions to the adverse selection problem include:

- Screening: the less informed party gathers information on the more informed party. For example, medical insurance companies may ask to see medical records.
- Signaling: the more informed party sends information to the less informed party. For example, highly skilled workers may signal their quality by acquiring diplomas.
- Third party intervention: a third party, not involved in the transaction, such as a government, may collect information and sell it or give it to the less informed party. For example, a government or a consumer group may establish standards and certifications.
- Other government interventions: the government may impose disclosure requirements (for example, a seller of a house has to disclose all relevant information), product liability laws (laws that protect consumers from malfunctioning products), or universal coverage in the case of insurance markets (insurance is provided to or has to be bought by both low-risk and high-risk consumers).

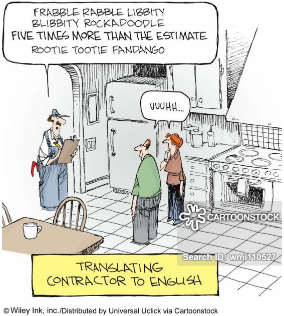


Figure 63: Moral hazard

We now have a look at moral hazard.

Moral hazard: moral hazard refers to the situation in which an informed party takes advantage of a less informed party, often through an unobserved action.

Principal-agent model: moral hazard is at the core of the so-called Principal-Agent model, in which a principal (a less-informed agent) contracts with an agent (the more informed agent), but cannot observe or evaluate the actions of the agent after the contract has been signed. Thus, the agent may take advantage of the principal.

Lecture 11: Asymmetry of information

Solutions to the moral hazard problem include:

- Incentives contracting: the principal designs an incentives contract to induce the agent to behave as expected.
- Monitoring: the principal may closely monitor the agent's actions to make sure he does not engage in opportunistic behavior.
- Bonding: the agent may be asked to deposit funds as a guarantee that they will behave properly. For example, tenants are asked to make a deposit to the landlord.
- Deferred payment: in the context of work contracts, the agent may receive a relatively low wage for some period of time, before receiving a higher wage later on, once his actions have been observed.
- Efficiency wage: in the context of work contracts, the agent may receive a relatively high wage, to discourage him to engage in opportunistic behavior (because if he is caught, he will be fired and this will be very costly given that the job was paying well).
- After the fact monitoring: it may be easier to monitor the agent after the facts. For example, an insurance company will refuse to reimburse a driver who got into an accident while drunk.

We take a look at signaling in markets with asymmetric information.

Workers: high skilled (in proportion q) generate output w_h and low skilled (in proportion $1 - q$) generate output w_l, where $w_h > w_l$. Skill is a hidden characteristic.

Without signals: the outcome will be a pooling equilibrium in which all workers are paid the average wage

$$w = qw_h + (1 - q)w_l$$

Signals: high-skill workers can signal their skill level by paying a cost c to acquire a degree.

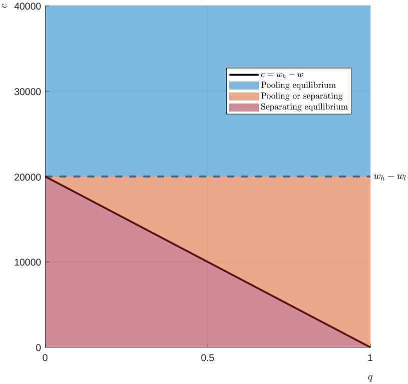
Separating equilibrium: high skilled workers invest in education, and receive a wage w_h, and low-skilled workers are paid w_l. This equilibrium can persist as long as

$$w_h - c > w_l$$

Pooling equilibrium: all workers receive the average wage w. This equilibrium can persist as long as

$$w_h - c < w$$

Figure 64: Asymmetric information: signaling.



Note: In this figure, we show the configurations of c and q that support a pooling or separating equilibrium (or both types of equilibria) in the signaling model. In this example, $w_h = 40,000$ and $w_l = 20,000$.

We take a look at a general principal-agent problem.

Principal-agent problem: principal is looking to hire an agent to produce output. Agent provides effort e to produce output $y = f(e)$. The agents incurs a cost of effort $c(e)$. Effort is not observed by the principal.

Principal's payoff: the principal receives the output $y = f(e)$ and must decide on the agent's reward, or incentive contract, $s(y)$. The principal's payoff is therefore

$$f(e) - s(f(e))$$

Agent's payoff: if the agent agrees to the contract, he receives

$$s(f(e)) - c(e)$$

Participation constraint the agent agrees to the contract if $s(f(e)) - c(e) \geq u$ where u is what he could get if he decided to walk away.

Optimal effort: the principal would like the agent to choose the effort that solves

$$\max_e f(e) - s(f(e))$$

subject to the participation constraint. The principal will design a contract such that the agent is exactly indifferent between walking away of signing the contract, so $s(f(e)) - c(e) = u$.

We conclude that the principal would like the agent to choose the effort that solves

$$\max_e f(e) - c(e) - u$$

The principal's payoff is maximized for the level of effort $e^\star$ such that

$$\frac{\partial f(e^\star)}{\partial e} = \frac{\partial c(e^\star)}{\partial e}$$

Incentive contracts: how can we make sure the agent will exert (close to) this level of effort?

Suppose effort is not observed.

Rental contracts: the payment to the worker is

$$s(f(e)) = f(e) - R$$

where R is a lump sum amount kept by the principal. R can be chosen to make the agent indifferent between walking away of signing the contract.

Sharecropping: the agent can keep a share $0 < \alpha < 1$ of the surplus, so that

$$s(e) = \alpha f(e) + F$$

This contract is not efficient, because the agent will choose $\hat{e}$ such that $\alpha \frac{\partial f(\hat{e})}{\partial e} = \frac{\partial c(\hat{e})}{\partial e}$ which is clearly different from $e^{\star}$.

Suppose effort is somewhat observed.

Wage labor: suppose effort is observed, then let

$$s(e) = we + K$$

The wage is chosen to be equal to $w = \frac{\partial f(e^\star)}{\partial e}$. The lump sum amount K is chosen so that the agent is indifferent between signing the contract or walking away.

Take-it-or-leave-it: the principal offers to give the agent an amount L if he chooses $e = e^\star$, and zero otherwise.

$$s(e) = \begin{cases} L \text{ if } e = e^\star \\ 0 \text{ otherwise} \end{cases}$$

The amount L is chosen so that the worker is indifferent between signing the contract or walking away.

Block 4: Game theory

Lecture 12: Strategic games

Strategic game: refers to any situation in which decision makers make simultaneous strategic decisions, i.e their decisions take into account the actions and responses of the other decision makers. A strategic game consists of

1. A set of players: the set of participants.
2. A set of actions: for each player, there is a set of possible actions/moves.
3. Payoffs: payoffs specify for each player the value of each possible outcome to the game.

Strategy: a rule or plan of action for playing the game.

Payoff matrix: it is often possible to represent a strategic game using a payoff matrix

Figure 65: Game theory: the payoff matrix.

		Player B	
		Left	*Right*
Player A	*Up*	**(3,9)**	**(2,8)**
	Down	**(0,2)**	**(1,1)**

Setting: two players A and B. A can choose any action in the set $\{r_1, ..., r_R\}$. Similarly, B can choose any action c in the set $\{c_1, ..., c_C\}$.

Best response function: the function $b_r(c)$ ($b_c(r)$) defines the best response for player A (B) to any choice the other player makes.

Dominant strategy: a strategy that is best no matter what the other players do.

Nash equilibrium: an outcome of the game is a Nash equilibrium if the strategy played by each player is a best reply to the strategies played by the other players.

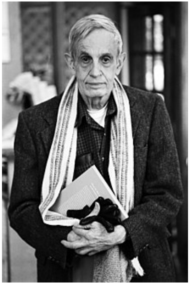


Figure 66: John Nash

Figure 67: Game theory: a strategic game example.

		Player B	
		Left	*Right*
Player A	*Up*	**(2,1)**	**(0,0)**
	Down	**(0,0)**	**(1,2)**

There are two Nash equilibria in pure strategy: (U, L) and (D, R).

Figure 68: The prisoner's dilemma

Figure 69: Game theory: the prisoner's dilemma.

		Player B	
		Confess	*Deny*
Player A	*Confess*	**(-3,-3)**	**(0,-6)**
	Deny	**(-6,0)**	**(-1,-1)**

The only Nash equilibrium in this game is (C, C), even though both players could actually be better off if they played (D, D). Thus it is clear from this example that a Nash equilibrium is not necessarily Pareto efficient!

Pure strategy: so far, we have only been looking at pure strategies, i.e. agents play purely one of the possible actions.

Mixed strategy: a strategy that consists of choosing a probability distribution over the possible actions.

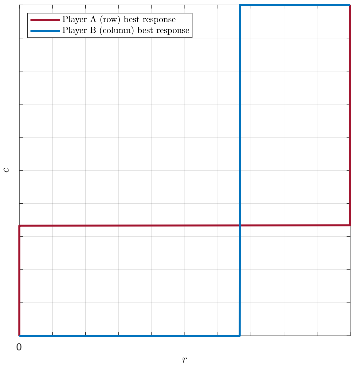
Existence of Nash equilibria: a strategic game with a finite number of players and actions has at least one Nash equilibrium (in pure or mixed strategy).

Figure 70: Game theory: a strategic game example.

		Player B	
		Left	*Right*
Player	*Up*	**(2,1)**	**(0,0)**
A	*Down*	**(0,0)**	**(1,2)**

There are two Nash equilibria in pure strategy: (U, L) and (D, R). There is one Nash equilibrium in mixed strategy: $(((2/3, 1/3), (1/3, 2/3)))$.

Figure 71: Game theory: best response functions with mixed strategies.



Note: Player A (or row player) plays Up with probability r and Down with probability $1-r$. Player B (or column player) plays Left with probability c and Right with probability $1-c$. The payoffs are given in figure 70. There is one Nash equilibrium in mixed strategy (in addition to two Nash equilibria in pure strategy): $((\frac{2}{3}, \frac{1}{3}), (\frac{1}{3}, \frac{2}{3}))$.

Lecture 12: Strategic games

We will take a look at different classes of games:

1. Coordination games: these are games in which the payoffs are the highest when the players coordinate their actions. Examples: battle of the sexes, assurance games, chicken.
2. Zero-sum games/competition games: these are games in which the payoff to one player is equal to the losses to the other player. Examples: matching pennies, penalty kicks.
3. Commitment games: these are sequential games in which commitment is an important strategic issue. Examples: the frog and the scorpion, entry deterrence.

Figure 72: Game theory: the battle of the sexes.

		Player B	
		Action	*Art*
Player A	*Action*	**(2,1)**	**(0,0)**
	Art	**(0,0)**	**(1,2)**

There are two Nash equilibria in pure strategy: $(Action, Action)$ and (Art, Art). There is one Nash equilibria in mixed strategy: $((\frac{2}{3}, \frac{1}{3}), (\frac{1}{3}, \frac{2}{3}))$.

Lecture 12: Strategic games

(a) The opera

(b) The Eiffel tower

Lecture 12: Strategic games

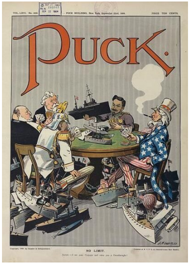


Figure 74: Arms race

Figure 75: Game theory: an assurance game.

		Player B	
		Refrain	*Build*
Player A	*Refrain*	**(4,4)**	**(1,3)**
	Build	**(3,1)**	**(2,2)**

There are two pure strategy Nash equilibria in this game: $(Refrain, Refain)$ and $(Build, Build)$.

Lecture 12: Strategic games

Figure 76: The chicken game

Figure 77: Game theory: the chicken game.

		Player B	
		Swerve	*Straight*
Player A	*Swerve*	**(0,0)**	**(-1,1)**
	Straight	**(1,-1)**	**(-2,-2)**

There are two pure strategy Nash equilibria in this game: $(Straight, Swerve)$ and $(Swerve, Straight)$.

Figure 78: Game theory: matching pennies.

		Player B	
		Heads	*Tails*
Player A	*Heads*	**(1,-1)**	**(-1,1)**
	Tails	**(-1,1)**	**(1,-1)**

There is only one Nash equilibrium in this game, and it is a mixed strategy Nash equilibrium: $((\frac{1}{2}, \frac{1}{2}), (\frac{1}{2}, \frac{1}{2}))$.

Lecture 12: Strategic games

(a) Sherlock

(b) Moriarty

Lecture 12: Strategic games

Figure 80: Penalty kicks

Figure 81: Game theory: penalty kicks.

		Player B	
		Left	*Right*
Player A	*Left*	**(50,-50)**	**(80,-80)**
	Right	**(90,-90)**	**(20,-20)**

There are no pure strategy Nash equilibria. There is one mixed strategy Nash equilibrium: $((0.7, 0.3), (0.6, 0.4))$.

Lecture 12: Strategic games

(a) A scorpion

(b) A frog

Lecture 13: Sequential games

Sequential game: a sequential game is a game in which players play sequentially. In a game with two players, the player who plays first is called the leader and the player who plays second is called the follower.

Extensive form: sequential games are often written in extensive form

Nash equilibrium: to solve for the Nash equilibrium in sequential games, a common technique is backward induction. We will solve the game backward, starting with the second player and then move on to the first player.

Figure 83: Game theory: a sequential game in extensive form

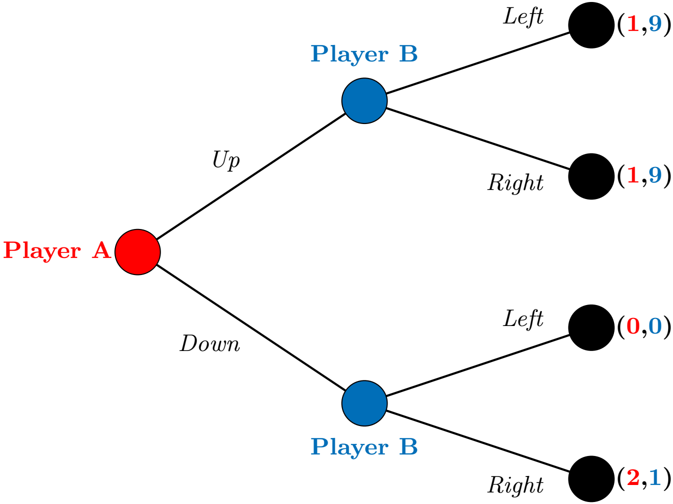
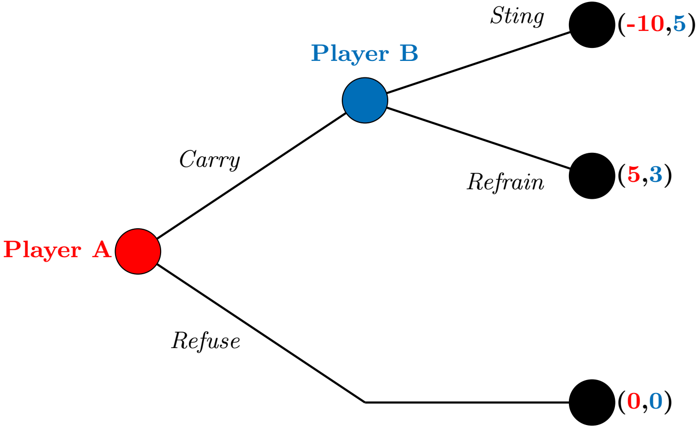
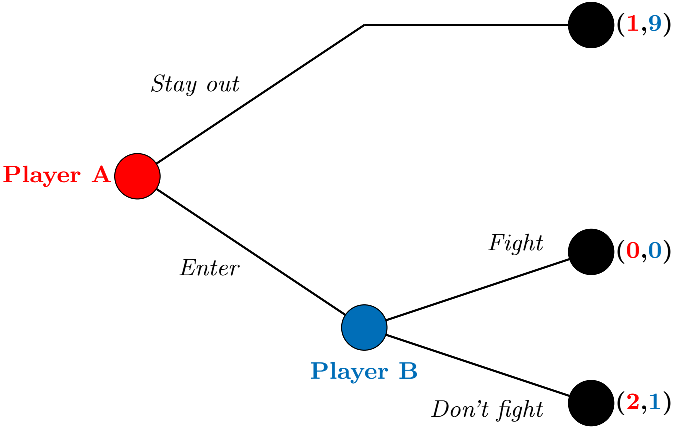


Figure 84: Game theory: the scorpion and the frog.



The only Nash equilibrium is $(Refuse)$.

Figure 85: Game theory: entry deterrence.



There is on Nash equilibrium $(Enter, Don'tfight)$.