Microeconomics
ECO00037I

University of York

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Outline:

1. Consumer theory
2. Producer theory
3. Perfect competition
4. Topics in microeconomic theory

Consumer theory

Bundles. A consumption bundle is denoted

$$x = (x_1, ..., x_n) \in X$$

where n is the number of commodities and X is the consumer's consumption set.

Budget constraint: (two goods only)

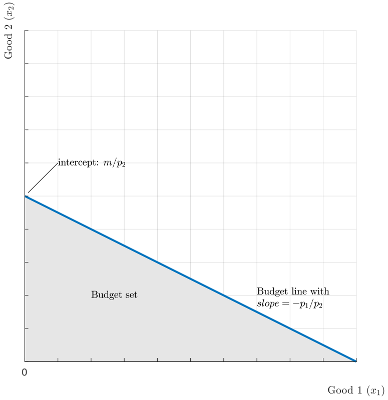
$$p_1 x_1 + p_2 x_2 \leq m$$

Budget line:

$$x_2 = \frac{m}{p_2} - \frac{p_1}{p_2} x_1$$

Budget line and budget set:

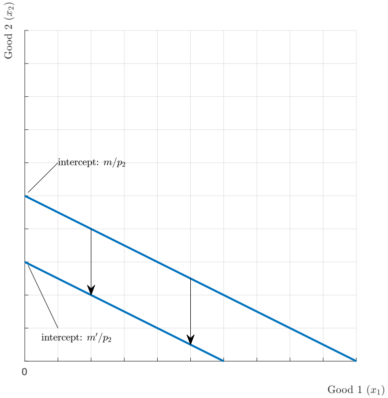
Figure: Consumer theory: the budget set and the budget line.



Note: In this figure, $m = 10$, $p_1 = 1$ and $p_2 = 2$.

Budget line shifts:

Figure: Consumer theory: shifting the budget line.



Note: In this figure, $m = 10$, $m' = 6$, $p_1 = 1$ and $p_2 = 2$.

Preference relations

- $x \succsim y$: "x is at least as good as y"
- $x \succ y$: "x is preferred to y"
- $x \sim y$: "x is indifferent to y"

Preferences: We often make the following assumptions on preferences:

1. Completeness
2. Reflexivity
3. Transitivity

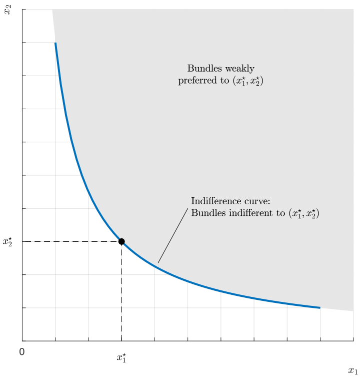
Well behaved preferences: In addition we often assume

1. Monotonicity
2. Convexity

Indifference curves: An indifference curve graphs the set of bundles that are indifferent to some bundle

Indifference curves:

Figure: Consumer theory: indifference curves.



Properties of indifference curves

1. Indifference curves cannot cross (by transitivity).
2. Bundles on indifference curves father from the origin are preferred to those on indifference curves closer to the origin (by monotonicity).
3. Indifference curves slope downward (by monotonicity).
4. Indifference curves are convex (by convexity).

Utility function: Utility functions are simple a way to summarize preferences. A utility function $u(x_1, x_2)$ assigns numbers to bundles so that more preferred bundles gets higher numbers.

Utility function and indifference curve: we can find an indifference curve by by finding all the bundles that give the same utility level, that is, we need to find all the bundles (x_1, x_2) such that

$$u(x_1, x_2) = u^\star$$

to construct the indifference curve associated with the utility level $u^\star$.

Marginal utility:

$$MU_1(x_1, x_2) = \frac{\partial u(x_1, x_2)}{\partial x_1}$$

$$MU_2(x_1, x_2) = \frac{\partial u(x_1, x_2)}{\partial x_2}$$

Marginal rate of substitution (MRS): the MRS is the slope of the indifference curve at a given point, that is

$$MRS(x_1, x_2) = \frac{dx_2}{dx_1} = -\frac{MU_1(x_1, x_2)}{MU_2(x_1, x_2)}$$

The utility maximization problem. The consumer's decision problem can be stated as the following utility maximization problem:

$$\max_{x \geq 0} u(x)$$

$$\text{subject to } p \cdot x \leq m$$

Indirect utility function. The *indirect utility function* is the function

$$v(p, m)$$

that gives the maximum level utility achievable given prices p and income m.

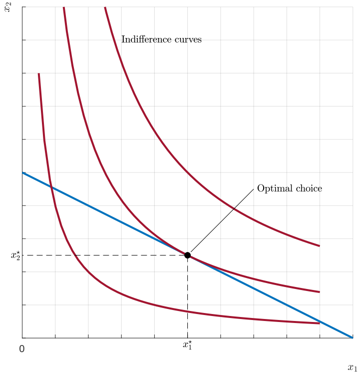
Marshallian demand function. The rule that assigns the set of optimal consumption bundles in the utility maximization problem to each price and income (p, m) is called the Marshallian demand function

$$x(p, m)$$

Interior solutions. An optimal bundle at which the consumer buys (strictly) positive quantities of both goods (in the two goods case) is called an interior solution.

Corner solutions. An optimal bundle at which the consumer buys only one of the two goods (in the two goods case) is called a corner solution.

Figure: Consumer theory: the optimal choice.



Note: In this figure, $m = 10$, $p_1 = 1$ and $p_2 = 2$. The utility function is $u(x_1, x_2) = \sqrt{x_1 x_2}$.

Finding interior solutions using the Lagrangian. To solve the utility maximization problem, we introduce the Lagrangian:

$$\mathcal{L}(\lambda, x) = u(x) - \lambda(p \cdot x - m)$$

For an interior solution $x^\star$, the first order conditions are

$$\frac{\partial u(x^\star)}{\partial x_i} - \lambda p_i = 0 \qquad \text{for } i = 1, 2$$

Combining the equations for good 1 and 2, we obtain

$$-\frac{\frac{\partial u(x^\star)}{\partial x_1}}{\frac{\partial u(x^\star)}{\partial x_2}} = -\frac{p_1}{p_2}$$

which, combined with the budget constraint, will allow us to find the optimal point.

Finding an interior solution - geometric interpretation. At the optimal bundle, the budget line is tangent to the indifference curve. This is not true whenever there is a corner solution or the indifference curve exhibits a kink at the optimal choice. However, if we rule out these two particular cases, then tangency is a necessary condition for optimality. And whenever indifference curves are in addition convex, then tangency is a sufficient condition: any point that satisfies the tangency condition must be an optimal point.

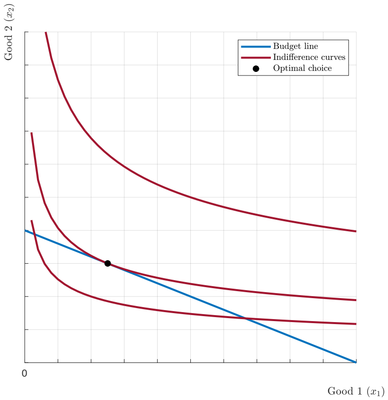
The tangency condition is written

$$-\frac{\frac{\partial u(x_1,x_2)}{\partial x_1}}{\frac{\partial u(x_1,x_2)}{\partial x_2}} = -\frac{p_1}{p_2}$$

which is the condition we obtained using the Lagrangian.

Example: a case with an interior solution (Cobb Douglas).

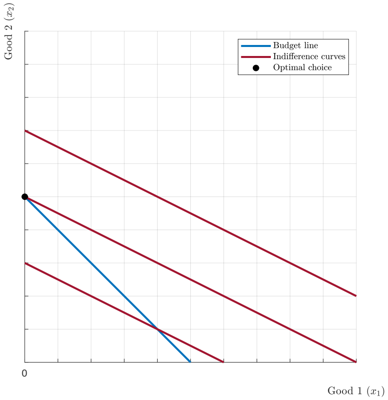
Figure: Consumer theory: the Cobb Douglas utility function.



Note: In this figure, $p_1 = 1$, $p_2 = 2.5$ and $m = 10$. The utility function is $u(x_1, x_2) = x_1^{\frac{1}{4}} x_2^{\frac{3}{4}}$.

Example: a case with a corner solution (perfect substitutes):

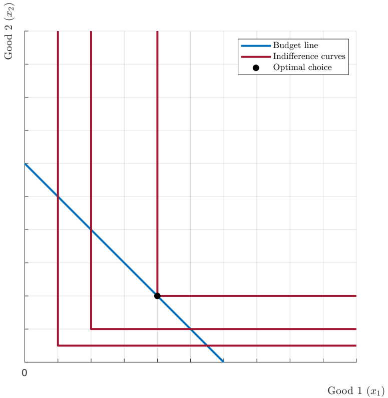
Figure: Consumer theory: perfect substitutes.



Note: In this figure, $p_1 = 1$, $p_2 = 1$ and $m = 5$. The utility function is $u(x_1, x_2) = x_1 + 2x_2$.

Example: a case with kinky preferences (perfect complements):

Figure: Consumer theory: perfect complements.



Note: In this figure, $p_1 = 1$, $p_2 = 1$ and $m = 5$. The utility function is $u(x_1, x_2) = \min\{\frac{1}{2}x_1, x_2\}$.

Expenditure minimization problem. The expenditure minimization problem is stated as follows:

$$\min_{x \geq 0} p \cdot x$$

$$\text{subject to } u(x) \geq u$$

Expenditure function. The *expenditure function* is the function

$$e(p, u)$$

that gives the minimum cost of achieving a given level of utility.

Hicksian demand function. The rule that assigns the set of optimal consumption bundles in the expenditure minimization problem to each price and utility level (p, u) is called the Hicksian demand function

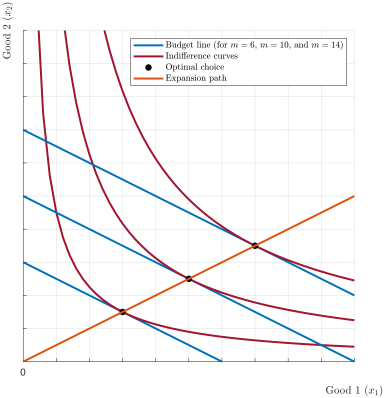
$$h(p, u)$$

Income change. Consider a Walrasian demand function for some good i, $x_i(p, m)$, and let us fix prices. What will be the effect of a change in income m on the demand for good i? We can distinguish among different types of goods, in particular:

- *Normal goods*: a good i is normal if the demand for that good is nondecreasing in income (that is, if $\partial x_i(p, m)/\partial m \geq 0$).
- *Inferior goods*: a good i is inferior if the demand for that good is decreasing in income.

The income expansion path:

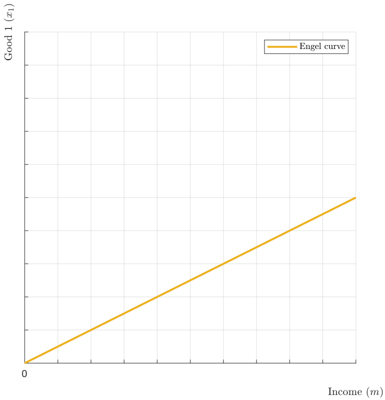
Figure: Consumer theory: the income expansion path.



Note: In this figure, $p_1 = 1$ and $p_2 = 2$. We vary income such that $m = 6$, $m = 10$ or $m = 14$. The utility function is $u(x_1, x_2) = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$.

The Engel curve:

Figure: Consumer theory: the Engel curve.



Note: In this figure, $p_1 = 1$ and $p_2 = 2$. The utility function is $u(x_1, x_2) = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$.

Homothetic preferences. Preferences are *homothetic* if when a consumer is indifferent between bundle (x_1, x_2) and (y_1, y_2), then he is indifferent between (tx_1, tx_2) and (ty_1, ty_2), for any positive value of t.

When preferences are homothetic, the income expansion path is a straight line: when we scale up income by some amount, then the demanded bundle scales up by the same amount.

Price change

Consider a Walrasian demand function for some good i, $x_i(p, m)$, and let us fix income. Let us now consider the effect of a change in the price of good i on the demand of good i, keeping all other prices fixed. There are two particular types of goods to keep in mind:

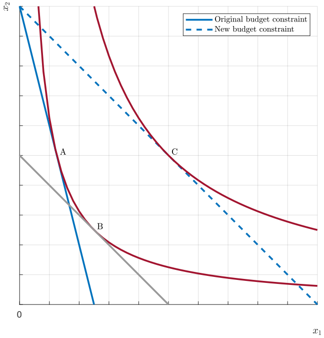
- *ordinary goods*, which are goods whose demand increases as the price of the good decreases
- *Giffen goods*, which are goods whose demand decreases as the price of the good decreases.

The change in the demand of good i following the change in price implicitly combines two effects:

- a *substitution effect*: this is the change in demand due to the change in the rate at which the consumer can exchange good i for other goods
- an *income effect*: this is the change in demand due to having more (if the price decreases) or less (if the price increases) purchasing power

Graphical analysis

Figure: Consumer theory: income and substitution effects



Note: This illustrates the decomposition into the substitution and income effects in two different examples.

Slutsky equation. The Slutsky equation is

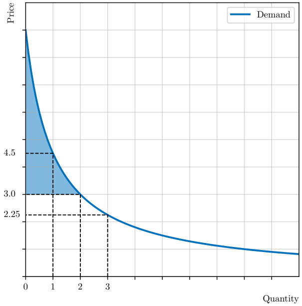
$$\frac{\partial x_j(p,m)}{\partial p_i} = \frac{\partial h_j(p,u)}{\partial p_i} - \frac{\partial x_j(p,m)}{\partial m} x_i(p,m)$$

Let m be the consumer's income. Consider a change in prices from p to p'. Let u be the utility of the consumer before the price change and u' the utility of the consumer after the price change.

How can we evaluate the impact of such a policy change on the consumer's welfare?

An individual's consumer surplus is defined as the area below that consumer's demand function and above the price, between quantity 0 and the quantity consumed.

Figure: Consumer theory: demand, inverse demand, and individual consumer surplus

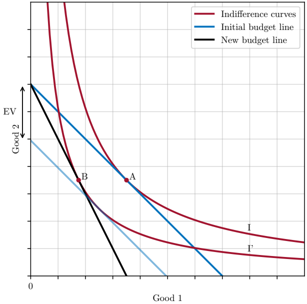


Equivalent variation: the equivalent variation (EV) is how much money should be given to the consumer before the change in prices to give the consumer the same level of utility they would get after the change. Formally, the equivalent variation EV is such that

$$v(p, m + EV) = u'$$

or alternately, $EV = e(p, u') - m$.

Figure: Consumer theory: equivalent variation



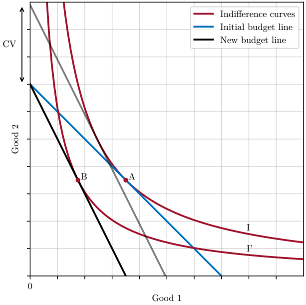
Note: In this figure, $u(x_1, x_2) = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$, $m = 7$, $p_1 = p_2 = 1$ and $p_1' = 2$.

Compensating variation: the compensating variation (CV) is how much money should be taken away from the consumer after the change in prices to maintain the same level of utility they had before the change. Formally, the compensating variation CV is such that

$$v(p', m - CV) = u$$

or alternately, $CV = m - e(p', u)$.

Figure: Consumer theory: compensating variation



Note: In this figure, $u(x_1, x_2) = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$, $m = 7$, $p_1 = p_2 = 1$ and $p_1' = 2$.

Producer theory

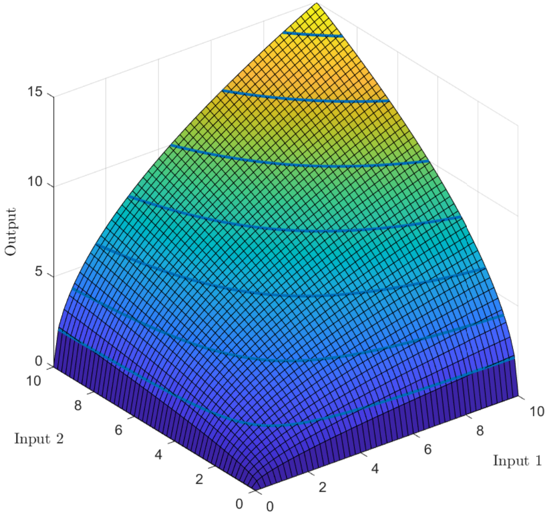
Factors of production

$$(x_1, ..., x_n).$$

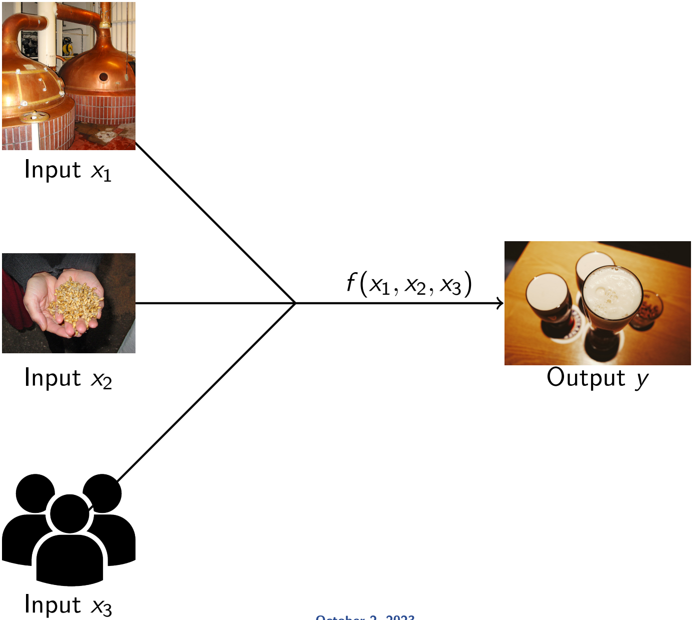
Production function

$$y = f(x)$$

where y is the maximum amount of output that can be produced using the bundle of inputs x.



Note: The production function illustrated here is $f(x_1, x_2) = \frac{3}{2}\sqrt{x_1 x_2}$.


Input x_1
Input x_2
Input x_3
$f(x_1, x_2, x_3)$
Output y

Properties of technology

- *Monotonicity*: if the amount of at least one input is increased, output will (weakly) increase.
- *Convexity*: if two production techniques x and z both produce y units of output, then any weighted average $tx + (1 - t)z$ of these two production techniques will produce at least y units of output.

Isoquants: an *isoquant* is the set of all possible combinations of inputs that yield the same amount of output.

Properties of isoquants:

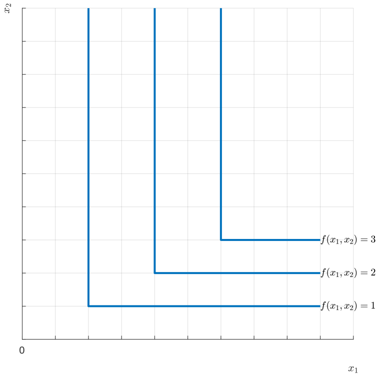
- The farther an isoquant is from the origin, the greater the level of output. This comes from the monotonicity assumption on technology, which means that production is an increasing function of input: if we increase the amount of some input (all else equal), we should be able to produce at least as much good as before.
- Isoquants do not cross.
- Isoquants slope downward.
- Isoquant curves are convex (by the convexity assumption)

(a) Perfect complements

(b) Perfect substitutes

Perfect complements

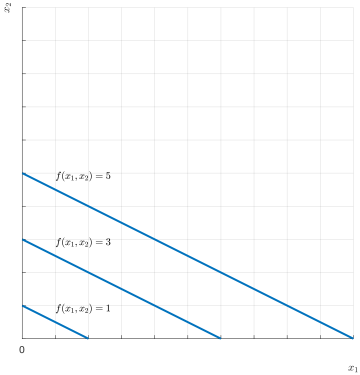
Figure: Technology: Perfect complements.



Note: In this figure, the production function is $f(x_1, x_2) = \min\{\frac{1}{2}x_1, x_2\}$.

Perfect substitutes

Figure: Technology: Perfect substitutes.



Note: In this figure, the production function is $f(x_1, x_2) = \frac{1}{2}x_1 + x_2$.

Returns to scale

1. If $f(kx) = kf(x)$, then the production is said to exhibit constant returns to scale. Doubling the amount of inputs doubles the output.
2. If $f(kx) > kf(x)$, then the production is said to exhibit increasing returns to scale. Doubling the amount of inputs more than doubles the output.
3. If $f(kx) < kf(x)$, then the production is said to exhibit decreasing returns to scale. Doubling the amount of inputs less than doubles the output.

Profit maximization: we will assume that firms act so as to maximize their profits. Note that cost minimization is a necessary condition for profit maximization!

The cost minimization problem. Firms minimize the costs of producing a given level of output. The problem to solve is

$$\min_x w \cdot x$$
$$\text{subject to } f(x) = y$$

<u>**Calculus analysis**</u> To solve the cost-minimization problem, let us introduce the Lagrangian:

$$\mathcal{L}(\lambda, x) = w \cdot x - \lambda(f(x) - y)$$

<u>**First order conditions**</u>

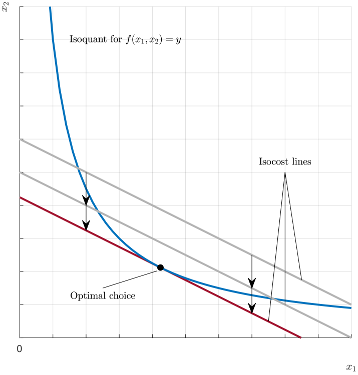
$$w_i - \lambda \frac{\partial f(x^\star)}{\partial x_i} = 0 \qquad \text{for } i = 1, ..., n$$
$$f(x^\star) = y$$

For some inputs i and j, we get the condition

$$-\frac{w_i}{w_j} = -\frac{\frac{\partial f(x^\star)}{\partial x_i}}{\frac{\partial f(x^\star)}{\partial x_j}}$$

Geometric solution

Figure: Cost minimization: Optimal inputs choice.



Note: In this figure, the production function is $f(x_1, x_2) = \sqrt{x_1 x_2}$ and the desired level of output is $y = 3$. Factor prices are $w_1 = 1$ and $w_2 = 2$.

Conditional factor demand functions give the optimal choices of inputs that minimize costs as a function of the factor prices and of the desired level of output.

$$x_i(w, y)$$

for each input $i = 1, ..., n$.

The total cost function. The cost function is the minimal cost of producing output y at the input prices w and is equal to

$$c(w, y) = w \cdot x(w, y)$$

Short-run total cost function. The short-run cost function is

$$c(w, y, x_f) = w_v \cdot x_v(w, y, x_f) + w_f \cdot x_f$$

Long-run total cost function in the long-run. In the long-run, all factors are variable so the cost function is simply

$$c(w, y) = w \cdot x(w, y).$$

Notable costs functions in the short-run.

- The (short-run) *average cost*

$$\frac{c(w, y, x_f)}{y}$$

is the cost per unit of output.

- The (short-run) *average variable cost* function

$$\frac{w_v \cdot x_v(w, y, x_f)}{y}$$

is the variable cost per unit of output.

- The (short-run) *average fixed cost* function

$$\frac{w_f \cdot x_f}{y}$$

is the fixed cost per unit of output.

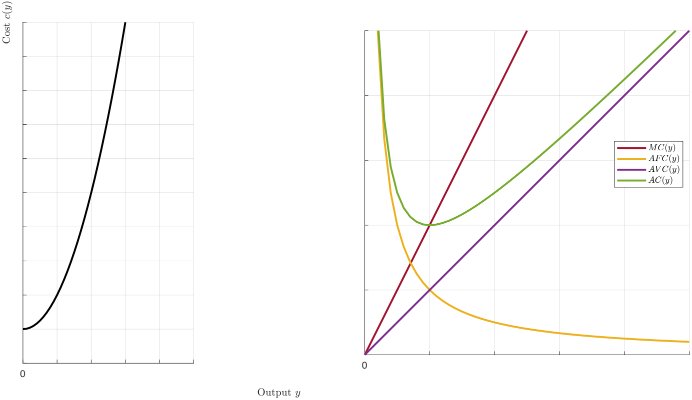
Notable costs functions in the short-run.

- The (short-run) *marginal cost*

$$\frac{\partial c_v(w, y, x_f)}{\partial y}$$

measures the change in costs arising from a small change in output. It is the derivative of the cost with respect to output produced.

Figure: Cost function: Cost functions.



Note: On the left is represented the cost function $c(y) = 1 + y^2$. On the right are represented the corresponding average cost, average variable cost, average fixed cost and marginal cost curves.

Notable costs functions in the long-run.

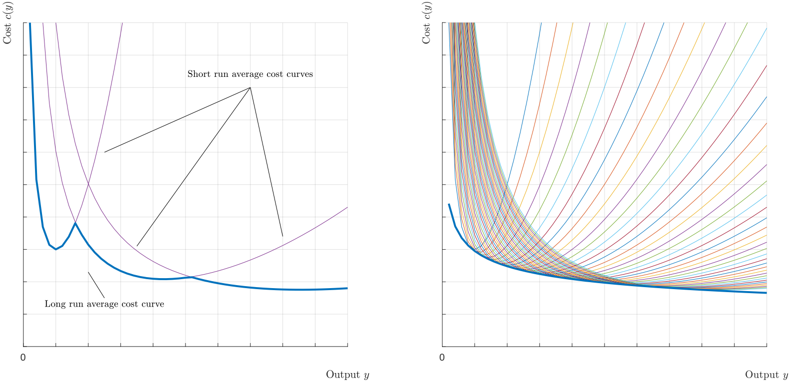
- the (long-term) average cost $\frac{c(w,y)}{y}$.
- the (long-term) marginal cost $\frac{\partial c(w,y)}{\partial y}$.

Costs functions: some properties.

1. By definition, we have that the average cost is the sum of the average variable costs and the average fixed costs.
2. The average fixed cost function is always decreasing with output (it's a constant divided by y)
3. The marginal cost curve lies below the average cost curve when the average costs are declining, and above when the average costs are increasing. The marginal cost curve lies below the average variable cost curve when the average variable costs are declining, and above when the average variable costs are increasing. Thus, the marginal cost curve passes through the minimum point of both the average variable cost and the average cost curves.
4. The marginal cost is in fact the marginal variable cost. Remember, fixed costs do not vary with output, so the derivative of the fixed cost with respect to y is zero.

Long-run average cost

Figure: Cost function: The long-run average cost curve.



Economies of scale. In the long-run, the cost function is said to exhibit

- *economies of scale* whenever the average cost function is decreasing
- *diseconomies of scale* whenever the average cost function is increasing

Note that increasing returns to scale imply economies of scale and decreasing returns to scale imply diseconomies of scale.

Perfect competition

1. *Homogeneity* of the good: The goods sold on the market by firms are strictly identical
2. *Atomicity*: the number of buyers and sellers on the market is sufficiently large so that none of their decisions can influence the equilibrium price.
3. *Free entry*: firms can enter and exit the market freely
4. *Transparency*: all buyers and sellers know the price at which they can buy or sell the good.

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Profit maximization

$$\max_{y} \pi(y) = py - c(y)$$

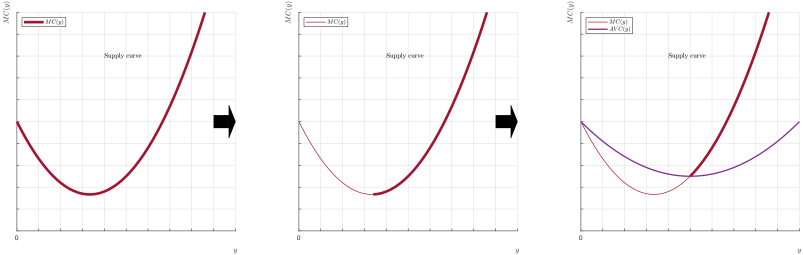
The first order and second order conditions for an interior solution $y^{\star}$ are

$$p = \frac{\partial c(y^{\star})}{\partial y}$$
$$\frac{\partial^2 c(y^{\star})}{\partial y^2} \geq 0$$

The supply function, $y(p)$, gives the level of output that maximizes the firm's profits for each price p. It must satisfy the first and second order conditions above.

Supply curve

Figure: Firm's supply: The firm's supply curve.



Note: This illustrates the construction of the firm's supply curve. The firm chooses its optimal level of output on the increasing part of the marginal cost curve, and above the average variable cost curve.

Perfect competition

Industry supply function and industry inverse supply function. Let $y_i(p)$ the supply curve of the ith firm in an industry with m firms. Then, the *industry supply function* is

$$S(p) = \sum_{i=1}^{m} y_i(p)$$

The *industry inverse supply curve* is the inverse of the industry supply curve: it gives the minimum price at which the industry is willing to supply a certain amount of good.

Elasticity of supply.

$$\epsilon_s = \frac{\partial S(p)}{\partial p}\frac{p}{S(p)}$$

Producers's surplus. The *producers' surplus* is the area above the inverse supply curve and below the market price, up to the quantity producers sell.

Industry demand function. Let $x_j(p)$ be the demand function of the jth consumer for the good produced by the industry. Then, the *industry demand function* is

$$D(p) = \sum_{j=1}^{n} x_j(p)$$

where n is the number of consumers.

The *industry inverse demand function* is the inverse of the industry demand function: it indicates what the price of the good should be for the demanders to demand some given amount.

Elasticity of demand.

$$\epsilon_d = \frac{\partial D(p)}{\partial p} \frac{p}{D(p)}$$

Perfect competition

1. Homogeneity of the good: The goods sold on the market by firms are strictly identical
2. Atomicity: the number of buyers and sellers on the market is sufficiently large so that none of their decisions can influence the equilibrium price.
3. Free entry: firms can enter and exit the market freely
4. Transparency: all buyers and sellers know the price at which they can buy or sell the good.

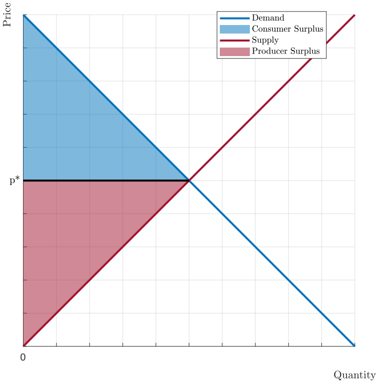
Partial equilibrium: we will examining equilibrium and changes in equilibrium on one market only, in isolation from other markets.

Equilibrium. A market is in equilibrium when the total quantity supplied by firms is equal to the total quantity demanded by consumers. The price for which supply equals demand is the equilibrium price. Formally, the equilibrium price $p^\star$ is such that

$$D(p^\star) = S(p^\star)$$

Total surplus: the total surplus is the sum of the consumers' and producers' surpluses. A perfectly competitive market maximizes the total surplus.

Figure: Partial Equilibrium: The total surplus.



Note: The total surplus is the sum of the blue and red areas (the sum of the consumers' and producers' surpluses).

Comparative statics

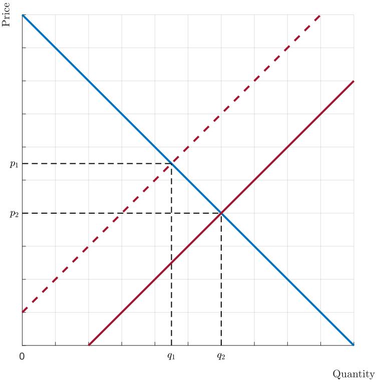
- Supply shift.
- Demand shift.
- Price ceilings and price floors.
- Quotas.
- Quantity taxes.
- Subsidies.

In perfect competition, total surplus is maximized in equilibrium. Thus, market interventions will very often generate a deadweight loss: a net reduction in total surplus.

(a) Supply shift

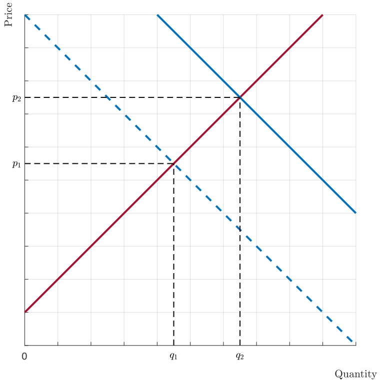
(b) Demand shift

Figure: Comparative statics: Supply shift.



Note: The figure illustrates a supply shift (from dashed to solid line: increased supply for all price levels). The equilibrium price decreases and the equilibrium quantity increases.

Figure: Comparative statics: Demand shift.



Note: The figure illustrates a demand shift (from dashed to solid: increased demand for all price levels). The equilibrium price and quantity increase.

(a) Price ceiling

(b) Price floor

(c) Quota

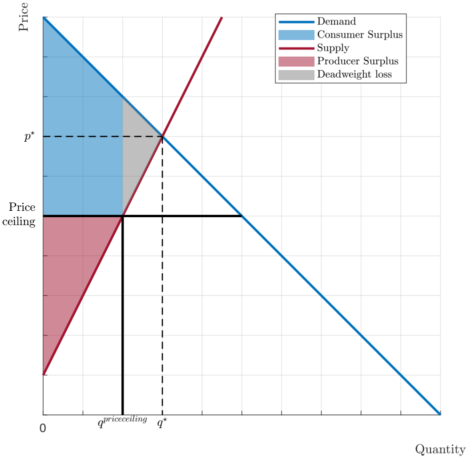
Let $p^\star$ and $q^\star$ be the equilibrium price and quantity in perfect competition.

Price ceiling: a price ceiling, denoted $\bar{p}$, is the highest price at which consumers can legally buy the good. A price ceiling is such that $\bar{p} < p^\star$ (otherwise, it is useless!). Price ceilings generate excess demand and a deadweight loss.

Price floor: a price floor, or minimum price denoted $\underline{p}$, is the lowest price at which consumers can legally buy the good. A price floor is such that $\underline{p} > p^\star$ (otherwise, it is useless!). Price floors generate excess supply and a deadweight loss.

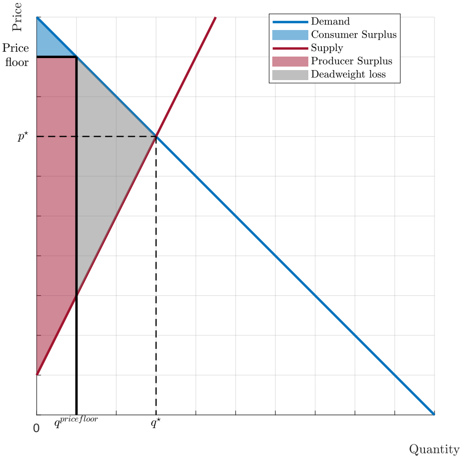
Quota: a quota, q^{quota}, sets the quantity of good provided. Whenever $q^{quota} > q^\star$, the goal is to force firms to produce more of the good. Whenever $q^{quota} < q^\star$, the goal is to limit the quantity of good produced. Very often, a quota is such that $q^{quota} < q^\star$. In this case, the quota generates a deadweight loss, and decreases the surplus of the consumers ; the producers' surplus usually increases.

Figure: Comparative statics: Price ceiling.



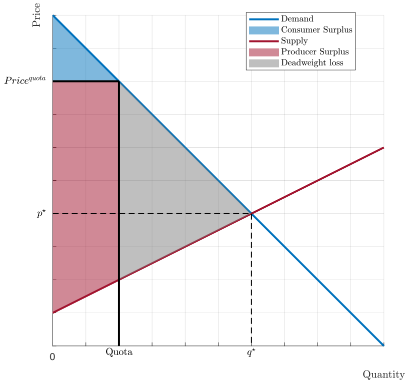
Note: The figure illustrates what happens when a price ceiling is introduced. Price ceilings generate excess demand and a deadweight loss.

Figure: Comparative statics: Price floor.



Note: The figure illustrates what happens when a price floor is introduced. Price floors generate excess supply and a deadweight loss.

Figure: Comparative statics: Quotas.



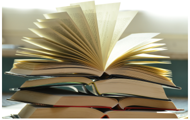
Note: The figure illustrates what happens when a quota is introduced. The quota generates a deadweight loss.

(a) Pollution

(b) Candy

(c) Gas

(d) Alcohol



(e) Books

Quantity taxes: introduces a wedge between the demand price (what the consumer pays, p_d) and the supply price (what the producer receives, p_s).

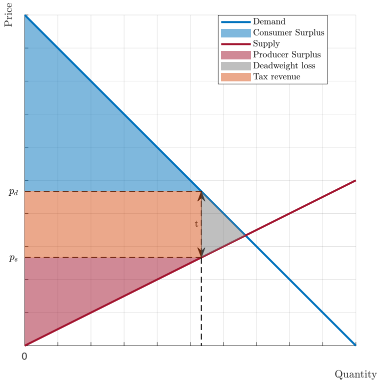
$$p_s = p_d - t$$

Equilibrium happens when

$$D(p_d) = S(p_s)$$

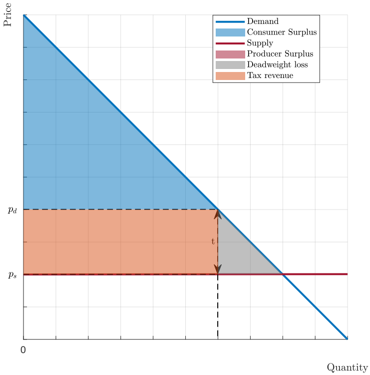
Graphical analysis

Figure: Comparative statics: Taxes.



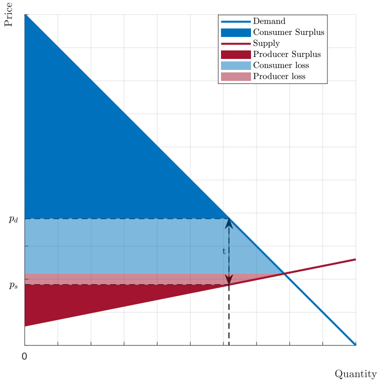
Passing along. A tax will generally increase the price paid by consumers and decrease the price received by producers: the tax is *passed along* to both the consumers and the producers. How much is passed along depends on the relative elasticity of supply and demand.

Figure: Comparative statics: Taxes, perfectly (in)elastic supply curve.



Note: This illustrates the passing along of a tax when supply is perfectly elastic (left) or perfectly inelastic (right).

Figure: Comparative statics: Taxes with a relatively elastic supply curve.



Note: This illustrates the passing along of a tax when supply is quite elastic. The tax is mostly passed along to consumers. In this example, $t = 2$, $S(p) = -3 + 5p$ and $D(p) = 10 - p$.

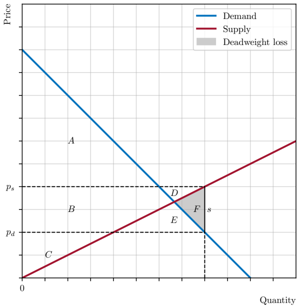
<u>**Subsidies**</u>. Let s be the subsidy. The demand and supply prices are related by the equation

$$p_s = p_d + s$$

Equilibrium is characterized by the equation

$$D(p_d) = S(p_s)$$

Figure: Comparative statics: Subsidies.



Note: This illustrates the effect of introducing a subsidy on the market. In this example, $s = 2$, $S(p) = 2p$ and $D(p) = 10 - p$. After the introduction of the subsidy, the consumers' surplus is $A + B + E$, the producers' surplus is $C + B + D$, the goverment expenditures are $B + D + E + F$ and the deadweight loss is F.

Topics in microeconomic theory

General equilibrium analysis: general equilibrium analysis (as opposed to partial equilibrium) is the study of how equilibrium is determined in many (interconnected) markets simultaneously.

Competitive market economies: an economy in which every relevant good is traded in a market (complete markets) at publicly known prices and all agents act as price takers (perfect competition).

Pareto efficiency. An allocation of goods is said to be Pareto efficient if there is no reallocation of goods that would make all agents at least as well off, and at least one agent strictly better off.

Competitive (or Walrasian) equilibrium Consider a perfectly competitive economy in which several goods are produced using a variety of inputs. If there exists a set of prices (for the goods and the inputs) such that

1. each consumer is choosing his most preferred affordable bundle
2. firms are maximizing their profits
3. the demand for each good is equal to the amount produced
4. the demand for each input is equal to the amount available

then this constitutes a competitive (or Walrasian) equilibrium.

The first welfare theorem The first welfare theorem states all Walrasian equilibria are Pareto efficient.

The second welfare theorem The second welfare theorem states that any Pareto efficient allocation can be reached by the use of competitive markets.

Market failures

Market failures are situations in which some of the assumptions of the welfare theorems do not hold and in which, as a consequence, market outcomes may be inefficient. Examples of market failures include:

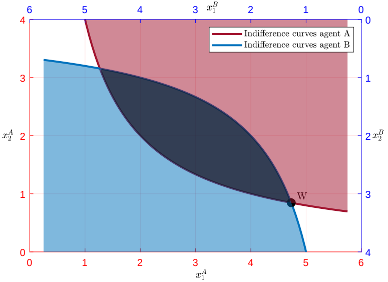
- Market power: some agents are no longer price takers. For example, we will study monopolies and oligopolies.
- Externalities: the actions of one agent affect other agents (positively or negatively), without any market to account for it. Thus the complete markets assumption is violated
- Public goods: as above, in the presence of public goods, the complete markets assumption is violated
- Asymmetry of information: some agents have more information than others.

Pure exchange economy. We consider a simple economy with only two consumers, A and B, and two goods, 1 and 2. Each consumer is endowed with a certain amount of each good, and we completely ignore production for now. This is known as a pure exchange economy.

Endowments. Let the total amount of good 1 available in the economy be X_1, and the total amount of good 2 be X_2. We assume that the two consumers are endowed with some amount of these goods. Consumer A is endowed with a bundle (ω_1^A, ω_2^A), while consumer B gets (ω_1^B, ω_2^B). Of course, it must be the case that $\omega_1^A + \omega_1^B = X_1$ and $\omega_2^A + \omega_2^B = X_2$.

Allocations. An allocation for consumer A is a bundle of good 1 and 2, denoted (x_1^A, x_2^A). An allocation for consumer B is a bundle of good 1 and 2, denoted (x_1^B, x_2^B).

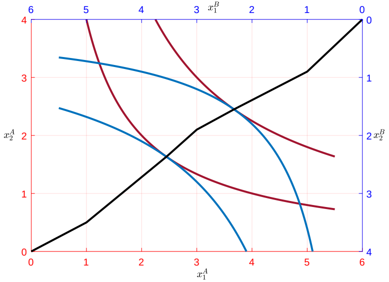
Figure: General equilibrium: the Edgeworth box



Note: This illustrates an Edgeworth box. There is 6 units of good 1 available and 4 units of good 2 available. The initial endowment is represented by the point W.

The Pareto set. In this exchange economy, an allocation of goods is Pareto efficient if there are no mutually beneficial trades at such allocation.

Figure: General equilibrium: the contract curve

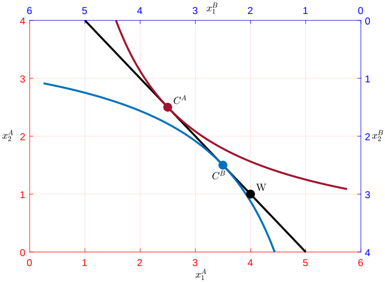


Note: In this Edgeworth box, we have represented the set of Pareto efficient allocations, or contract curve, as a solid black line connecting all the Pareto efficient allocations.

Prices. Let p_1 and p_2 be the prices of the good 1 and 2, respectively.

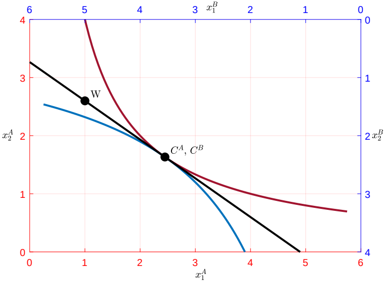
Budget line. In the Edgeworth box, we can draw the budget line faced by the consumers as the straight line slope $-p_1/p_2$ that cuts through the initial allocation W.

Figure: General equilibrium: Disequilibrium.



Note: In this Edgeworth box, the prices are such that there is disequilibrium. There is excess demand of good 2.

Figure: General equilibrium: Competitive equilibrium (without production).



Note: In this Edgeworth box, the prices are such that there is equilibrium. There is no excess demand of good 1 or good 2, and the consumers are maximizing their utility.

Exchange economy with production. The economy is very similar to the one examined previous. There are two consumers, A and B, and two goods, 1 and 2. The quantities X_1 and X_2 of goods available in the economy are no longer fixed. Instead, these will be produced.

For simplicity, we assume that the goods are produced using as inputs the labour supplied by A and B. A and B supply the same amount of labour and receive the same wage. In addition, we assume that A and B own the firms producing the good and that all profits get equally redistributed to them.

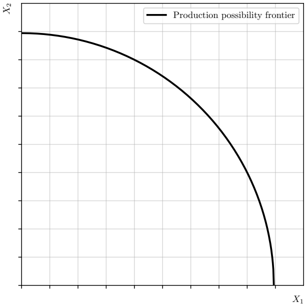
Production possibility frontier. The production possibility frontier (PPF) describes all the combinations of goods (X_1, X_2) that can be produced in this economy, by using the available resources (e.g. here the time spent by A and B on production activities).

Marginal rate of transformation. The marginal rate of transformation (MRT) is the slope of the production possibility frontier. It indicates by how much we should cut the production of good 2 in order to produce an additional unit of good 1. The MRT is equal to

$$MRT = -\frac{MC_1}{MC_2}$$

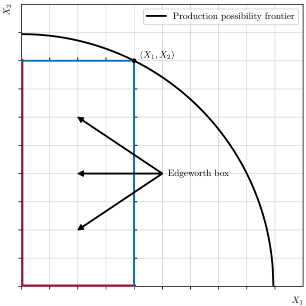
where MC_1 and MC_2 are the marginal costs of good 1 and 2, respectively.

Figure: General equilibrium: Production possibility frontier.



Note: This figures shows the production possibility frontier, i.e. all the combinations of goods (X_1, X_2) that can be produced in this economy.

Figure: General equilibrium: Production possibility frontier and Edgeworth box.



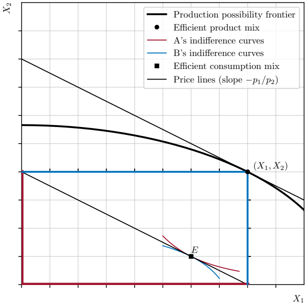
Note: This figures shows the production possibility frontier and, for a chosen product mix (X_1, X_2), the corresponding Edgeworth box.

In this economy, if there exists a set of prices (for the goods and the inputs) such that

1. each consumer is choosing his most preferred affordable bundle
2. firms are maximizing their profits
3. the demand for each good is equal to the amount produced
4. the demand for each input is equal to the amount available

then this constitutes a competitive (or Walrasian) equilibrium.

Figure: General equilibrium: Competitive equilibrium (with production).



Note: In this figure, the prices are such that there is a general equilibrium. The product mix is located at the tangency point between the PPF and a price line (with slope $-p_1/p_2$). The consumers' choices lead to the same point E, located on the contract curve, such that supply equals demand on all markets.

<u>Characterization of the general equilibrium</u>. The previous figure suggests that the competitive equilibrium is characterized by the equality between the MRS of each consumer, the MRT and the price ratio, that is

$$MRS = -\frac{p_1}{p_2} = MRT$$

Non-labour income: the amount M the consumer receives whether he works or not.

Consumption: the amount C the consumer decide to consume (at price p).

Labour: the number of hours of work the consumer decides to provide, denoted ℓ, for which he earns a wage w per hour worked. The consumer can provide at most $\bar{T}$ hours of work

Leisure: leisure is therefore $L = \bar{T} - \ell$

Budget constraint:

$$pC = M + w\ell$$

Budget constraint:

$$pC + w(\bar{T} - \ell) = p\bar{C} + w\bar{L}$$

where $\bar{C} = M/p$.

Budget constraint:

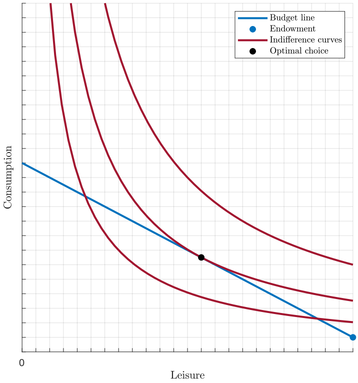
$$pC + wL = p\bar{C} + w\bar{L}$$

Budget line:

$$C = \bar{C} + (w/p)\bar{L} - \frac{w}{p}L$$

Optimal choice (labour):

Figure: Labour: the optimal choice.



Note: In this figure, $M = 1$, $p = 1$, $w = 0.5$ and $\bar{L} = 24$. Preferences are represented by the utility function $u(L, C) = L^{\frac{1}{2}} C^{\frac{1}{2}}$.

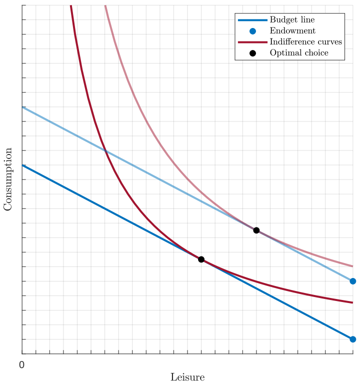
Slutsky equation. The Slutsky equation in the labour supply model is

$$\frac{\partial L(p,w,m)}{\partial w} = \frac{\partial L(p,w,u)}{\partial w} + \frac{\partial L(p,w,m)}{\partial m}[\bar{L} - L]$$

Effect of a change in wage. In the Slutsky equation, the first term is negative, but the second term is nonnegative as $\bar{L} - L \geq 0$ (and if leisure is a normal good). Thus, the effect of an increase in wage on leisure is inherently ambiguous.

Comparative statics (non labour income):

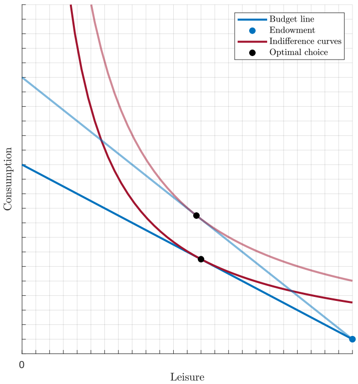
Figure: Labor: comparative statics (income).



Note: In this figure, $p = 1$, $w = 0.5$ and $\bar{L} = 24$. Preferences are represented by the utility function $u(L, C) = L^{\frac{1}{2}} C^{\frac{1}{2}}$. M increases from $M = 1$ (opaque blue line) to $M = 5$ (transparent blue line).

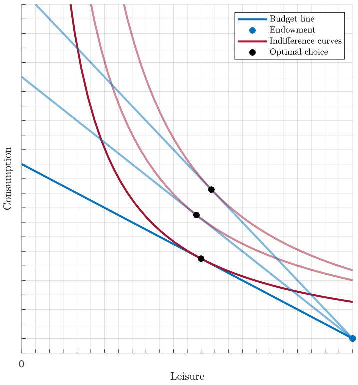
Comparative statics (wage):

Figure: Labor: comparative statics (wage).



Comparative statics (wage):

Figure: Labor: backward-bending labor supply.



Lottery: A lottery specifies each possible outcome of the risky alternative it represents, as well as the probability that this outcome will occur.

Expected value:

$$EV = \pi_1 c_1 + \pi_2 c_2$$

Expected utility:

$$u(c_1, c_2, \pi_1, \pi_2) = \pi_1 v(c_1) + \pi_2 v(c_2)$$

Compound lottery: the compound lottery

$$L_3 = [L_1, L_2; \alpha, 1 - \alpha]$$

is a lottery where the outcomes are the simple lotteries L_1 and L_2, where the lottery L_1 occurs with probability α and the lottery L_2 occurs with probability $1 - \alpha$

Axioms of expected utility theory: we can represent individual preferences by a utility function that has the expected utility form if these assumptions are made

1. Continuity: a small changes in probabilities do not change the nature of the ordering of two lotteries.
2. Independence: if we mix each of two lotteries with a third lottery, then the preference ordering between the two resulting lotteries does not depend on (is independent of) this third lottery.

The Allais paradox: The result from this experiment are not compatible with expected utility theory, hence the name "paradox". Suppose that a player is playing a game in which he can earn either 0, 1 million or 5 million dollars. Suppose that a first lottery L_1 gives the player 1 million dollars with probability 1. A second lottery L_2 gives 0 with probability 0.01, 1 million dollar with probability 0.89, and 5 millions dollars with probability 0.1. Here, most people choose lottery 1 over lottery 2.
Lottery L_3 gives 0 with probability 0.89 and 1 million with probability 0.11. Lottery L_4 gives 0 with probability 0.9 and 5 million with probability 0.1. When given the choice between lotteries 3 and 4 only, most people choose lottery 4 over lottery 3. But this is actually inconsistent with the expected utility theory. Suppose that $v(0) = 0$. The fact that people choose L_1 over L_2 means that $v(1) > 0.89v(1) + 0.1v(5)$. This implies

$$0.11v(1) > 0.1v(5)$$

The fact that most people choose L_4 over L_3 means that

$$0.1v(5) > 0.11v(1)$$

The two equations are not compatible!

Risk-averse: A consumer is said to be risk averse if he prefers a certain outcome to a lottery of equal expected value.

Risk-loving: the expected utility from the lottery is more than the utility from receiving (for certain) the expected value of the lottery.

Risk-neutral: The consumer is indifferent between taking the gamble or receiving for certain the expected value of the gamble.

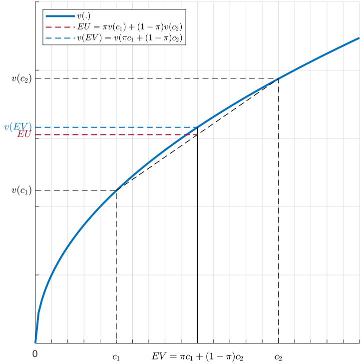
Risk premium: the amount a risk averse agent is willing to pay to avoid taking risks. It is is defined as the difference between the expected value of the lottery and the certainty equivalent.

Certainty equivalent: the amount of money (CE) that, if held with certainty, would provide the same utility as the lottery (EU).

$$v(CE) = EU$$

Risk-averse:

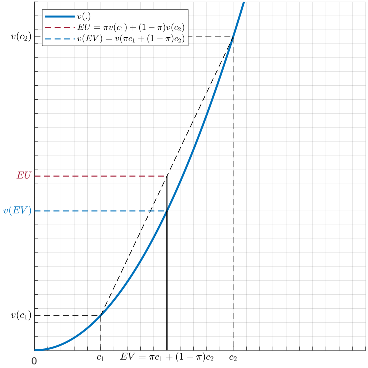
Figure: Risk: risk aversion.



Note: In this figure, $\pi = 0.5$, $c_1 = 5$ and $c_2 = 15$. We chose $v(x) = \sqrt{x}$.

Risk-loving:

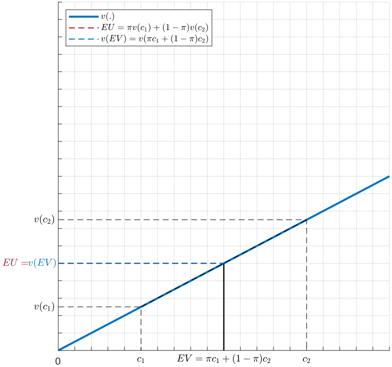
Figure: Risk: risk loving.



Note: In this figure, $\pi = 0.5$, $c_1 = 5$ and $c_2 = 15$. We chose $v(x) = 0.1x^2$.

Risk-neutral:

Figure: Risk: risk neutral.



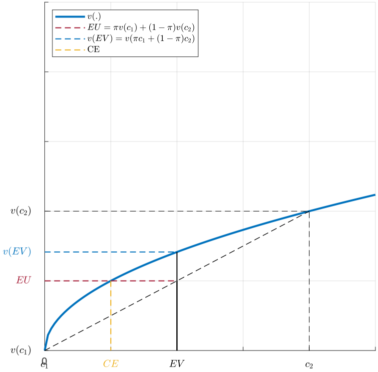
Note: In this figure, $\pi = 0.5$, $c_1 = 5$ and $c_2 = 15$. We chose $v(x) = 0.5x$.

Measuring the degree of risk aversion. The Arrow-Pratt measure is a commonly used measure of risk aversion and is defined as follows, for some consumption level c:

$$\rho(c) = -\frac{\frac{\partial^2 v(c)}{\partial c^2}}{\frac{\partial v(c)}{\partial c}}$$

Risk premium and certainty equivalent:

Figure: Risk: risk premium and certainty equivalent.



Note: In this figure, $\pi = 0.5$, $c_1 = 0$ and $c_2 = 4$. We chose $v(x) = \sqrt{x}$.

Insurance: a consumer can buy K units of insurance at a unit price of γ. If the bad outcome occurs (with probability π), he gets

$$c_1 = \omega_1 + K - \gamma K$$

If the good outcome occurs (with probability $1 - \pi$), then

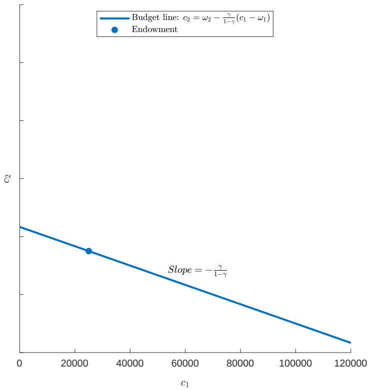
$$c_2 = \omega_2 - \gamma K$$

Budget line:

$$c_2 = \omega_2 - \frac{\gamma}{1-\gamma}(c_1 - \omega_1)$$

Budget line (insurance):

Figure: Risk: budget line with insurance.

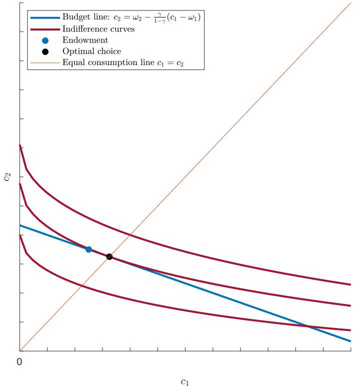


Note: In this figure, we represent the budget line $c_2 = \omega_2 - \frac{\gamma}{1-\gamma}(c1 - \omega_1)$, where $\omega_1 = 25,000\$$, $\omega_2 = 35,000\$$ and $\gamma = 0.25$.

Suppose that the consumer's preferences can be represented by an expected utility function and that the consumer is risk averse.

Optimal choice (insurance):

Figure: Risk: insurance choice.



Fair insurance: the insurance company charges the price $\gamma = \pi$.

Full insurance: if the consumer is risk averse, insurance is fair and his preferences are compatible with the expected utility model, then the consumer will choose to fully insure.