Week 1

Overview of Corporate Finance

Lecture outline

- What is corporate finance?
- The goal of financial management
- Some concepts

Learning outcomes

- Understand what is corporate finance
- Understand the goal of financial managers and the role of financial managers

Corporate finance in the news

0:14

NatWest boss facing government pressure to resign

By Faarea Masud
Business reporter

Dame Alison Rose admitted a "serious error" in talking about Nigel Farage's relationship with its private banking arm Coutts.

Read more >

Share

18:05 25 Jul

Magnum-maker's profits soar after it raises prices

The consumer goods giant sees profits rise by a fifth based almost entirely on increasing its prices.

Read more >

Share

13:40 25 Jul

Virgin Media O2 to cut more than a tenth of UK jobs

By Noor Nanji
Business reporter, BBC News

About 2,000 roles will go by the end of this year during what could be a "difficult" time, the firm says.

Read more >

What is corporate finance?

Investment	Financing	Liquidity
• Choose the best projects • Capital budgeting	• Choose sources of financing for investment • Capital structure	• Ensure you have enough cash and inventory • Short-term financial planning

A firm's balance sheet model

Net working capital

Current assets (e.g., cash, stock inventory)

Current liabilities (e.g., short-term debts)

Non-current assets:
- Tangible non-current assets (e.g., land, buildings, plants)
- Intangible non-current assets (e.g., trademarks, patents, copyrights)

Non-current liabilities (e.g., long-term debts)

Equity

Total assets (what the firm <u>owns</u>)

Total liabilities (what the firm <u>owes</u> others)

Example 1

- On 1 March 2023, Admiral Group plc, a British motor insurer, announced its financial results for the year 2022
- At the end of 2022, the company had:
  - £7.7 million in tangible non-current assets and £69.0 million in intangible non-current assets
  - Current assets amounted to £793.6 million and current liabilities were £632.7 million
- Admiral Group had no non-current liabilities
- What is Admiral Group's balance sheet model?

Admiral Group's balance sheet

Net working capital = 793.6-632.7 = £160.9 mil

Current assets = £793.6 mil

Non-current assets:

- Tangible non-current assets = £7.7 mil
- Intangible non-current assets = £69 mil

Total assets = 793.6+7.7+69 = £870.3 mil

Current liabilities = £632.7 mil

Non-current liabilities = 0

Equity = 879.3-632.7 = £237.6 mil

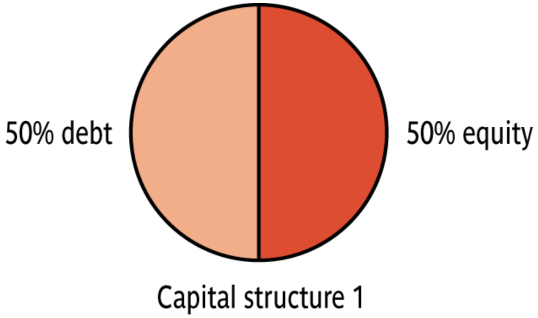
Total liabilities = £870.3 mil

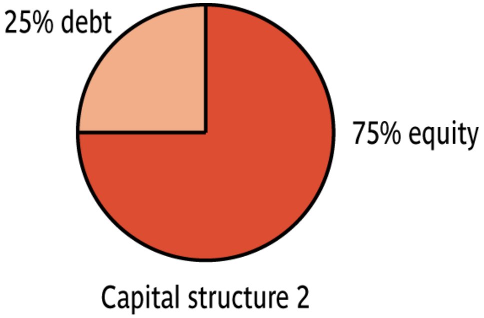
Capital structure

Debt	Equity
• Debt financing = borrowing money and paying back with interest • Examples: corporate bonds, loans • Providers of debts: bond holders, creditors, debt holders	• Equity financing = raising money by selling equity (shares) • Providers of equity: equity holders, stockholders, shareholders

If we think of a firm as a pie:

Firm value = Value of bonds + Value of shares

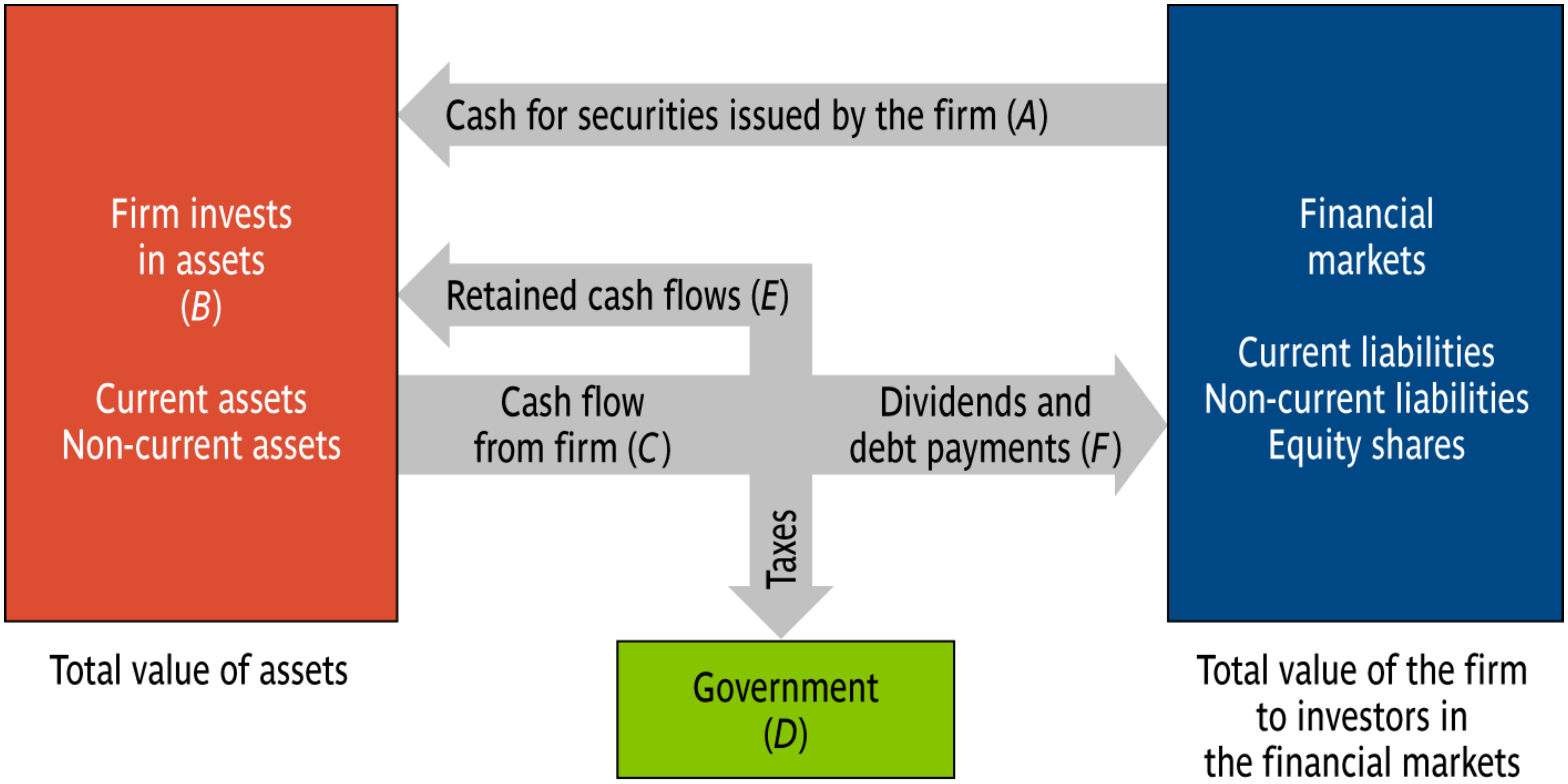




What do you do as a financial manager?

- Responsible for investment decisions
- Responsible for financing decisions
- Responsible for short-term financial planning
- Oversee accounting and audit function in firm
- Ensure the financial welfare of the firm

How cash flows…



Accounting flow vs. Cash flow (1)

- Midland plc is an Irish firm that refines and trades gold.
- At the end of the year, it sold 2,500 ounces of gold for €1.67 million.
- The company had acquired the gold for €1 million at the beginning of the year. The company paid cash for the gold when it was purchased.
- Unfortunately, it has yet to collect from the customer to whom the gold was sold.

Accounting flow vs. Cash flow (2)

- Accounting flow
- Cash flow

(Accounting View) Income Statement Year End December 31		(Financial View) Income Statement Year Ended December 31	
Sales	€1,670,000	Cash inflow	€ 0
Costs	-€1,000,000	Cash outflow	-€1,000,000
Profit	€ 670,000	Net cash flow	–€1,000,000

Timing of cash flows

The Italian firm, Montana SpA, is attempting to choose between two proposals for new products. Both proposals will provide additional cash flows over a four-year period and will initially cost €10,000. The cash flows from the proposals are as follows:

Which one is better?

Year	New Product *A*	New Product *B*
1	€ 0	€ 4,000
2	0	4,000
3	0	4,000
4	20,000	4,000
Total	€20,000	€16,000

The risk of cash flows

- The Norwegian firm, Fjell ASA, is considering expanding operations overseas, and it is evaluating the Netherlands and South Africa as possible sites. The Netherlands is relatively safe, whereas operating in South Africa is seen as considerably riskier. In both cases, the company would close down operations after one year.
- After undertaking a complete financial analysis, Fjell has come up with the following cash flows of the alternative plans for expansion under three scenarios—pessimistic, most likely, and optimistic:

	Pessimistic	Most Likely	Optimistic
Netherlands	750,000	1,000,000	1,250,000
South Africa	0	1,500,000	2,000,000

Which one is the best?

Book value vs market value

- Book value: the accounting value of a firm's assets
- Market value: the price at which willing buyers and sellers would trade the assets

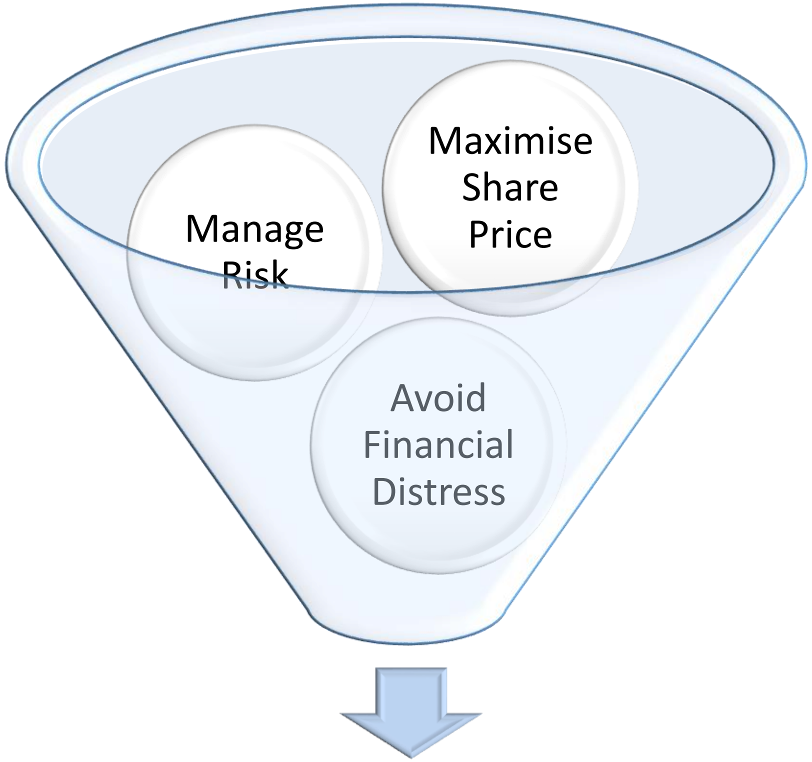
A firm’s income statement

$$Income = Revenue - Expenses$$

U.S. COMPOSITE CORPORATION
Income Statement
2012
($ in millions)

Total operating revenues	$2,262
Cost of goods sold	1,655
Selling, general, and administrative expenses	327
Depreciation	90
Operating income	$ 190
Other income	29
Earnings before interest and taxes (EBIT)	$ 219
Interest expense	49
Pretax income	$ 170
Taxes	84
Current: $71	
Deferred: 13	
Net income	$ 86
Addition to retained earnings:	$ 43
Dividends:	43

The goal of financial management



Maximise Value of Owner's
Equity

Corporate tax rates

- Average tax rate: tax bill divided by taxable income, i.e., the percentage of income that goes to pay taxes
- Marginal tax rate: the tax you would pay (in %) if you earned one more dollar

Real world data

Please use one-time sign in to get access to WRDS database.

Week 2

Overview of corporate governance

Lecture outline

- Corporate firms
- Agency theory
- The governance structure of corporations
- The OECD principles of good governance

Learning outcomes

- Be able to differentiate different types of corporations
- Be able to understand the agency theory
- Gain a basic understanding of the governance structure of corporations
- Understand the OECD principles of good governance

Uber broke laws, duped police and secretly lobbied governments, leak reveals

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- Files expose attempts to lobby Joe Biden, Olaf Scholz and George Osborne
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Factbox: Credit Suisse's scandals - spies, lies and money laundering

Reuters

October 3, 2022 6:57 PM GMT+1 · Updated 10 months ago

Aa

WIRELESS

Ericsson investigates in-house bribery scandal tied to ISIS

By **Monica Alleven** • Feb 16, 2022 12:52pm

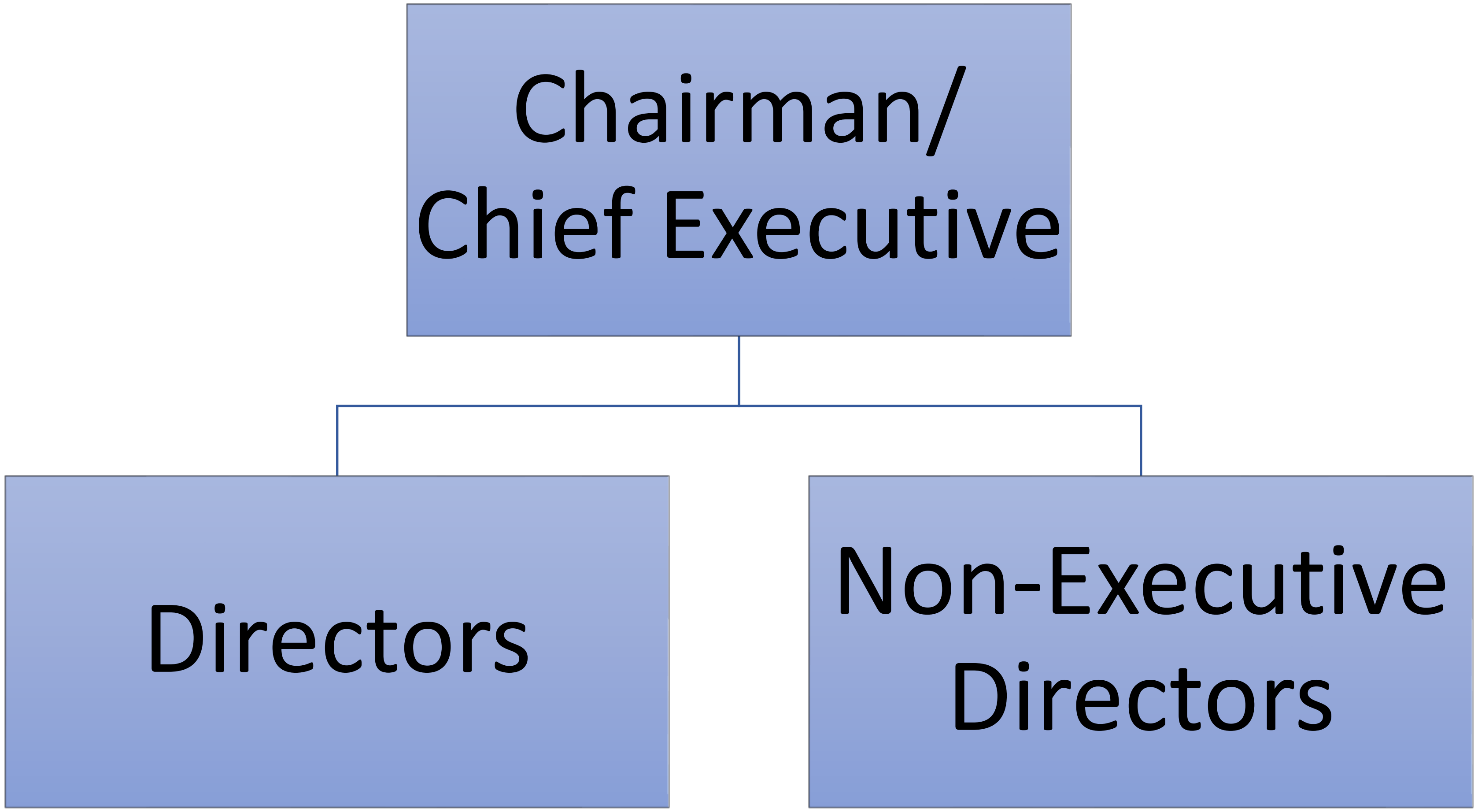
Types of corporations

Sole Proprietorship	Partnership	Limited Corporation
• Owned and managed by one person • Very easy to form • Unlimited liability • Amount of funding is limited by owner’s personal wealth • Profits taxed as personal income • Life of company linked to life of owner	• Controlled by general partners • Easy to form • Requires a partnership agreement • Difficult to raise cash • Profits taxed as personal income • Partnership is terminated when a partner dies or leaves the firm	• Board of directors • Articles and memorandum of incorporation required • Limited liability • Profits taxed at corporate tax rate • Life of company hypothetically unlimited

The Arts and Mems

Articles of Incorporation	Memorandum of Association
• Name of the corporation • Intended life of the corporation (it may be forever) • Business purpose • Number of shares that the corporation is authorized to issue, with a statement of limitations and rights of different classes of shares • Nature of the rights granted to shareholders • Number of members of the initial board of directors	• The rules by which the corporation is organised

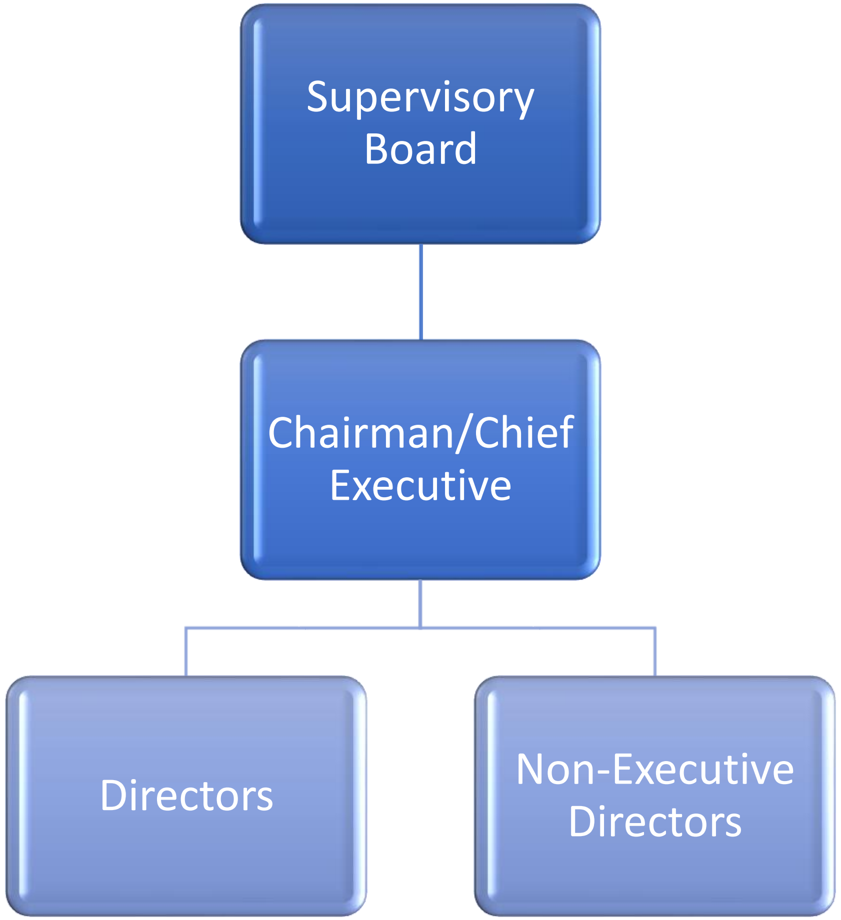
The Board of Directors (Single tier)



The Board of Directors (Two tiers)

Implementing policy

Policymaking



Unitary vs Dual board structures

Unitary

- Board reports to shareholders
- Shareholders elect directors at AGM

Two-Tier

- Board reports to supervisory board
- Supervisory board elects directors
- Supervisory board consists of representatives from banks, government, trade unions, other stakeholders

Partnerships vs Corporations

	Partnership	Corporation
Liquidity and marketability	Restricted trading	Traded easily sometimes on exchange
Voting rights	Partners have control	Each share gives a voting right
Taxation	Personal tax rate	Corporate tax rate
Reinvestment and dividend payout	All profits allocated to partners	Total freedom in dividend decisions
Liability	General partners have unlimited liability	Shareholders have limited liability
Continuity of existence	Limited life	Unlimited life

Corporate governance across countries

- Investor protection
- The financial system
- Control mechanisms
- Firm corporate governance systems

Investor protection: Legal systems

Common law	Civil law
• Law is developed through court rulings • Flexible and can adjust quickly to events ⇒ “Stronger” protection of outside investors	• Law is developed through regulation and code of laws • Based on code of principles • Does not change ⇒ “Weaker” protection of outside investors

Investor protection: Shareholder legal protection

- What are the country-level legal rights of shareholders?
- Main characteristics:
  - Proxy vote by mail is allowed
  - Votes are not blocked before the annual general meeting
  - Cumulative voting or proportional representation exists
  - Oppressed minorities mechanisms exist
  - Pre-emptive rights exist
  - There is a minimum percentage to call an extraordinary shareholders' meeting

Investor protection: Law enforcement

- Many countries have strong regulations but very weak enforcement
- To what extent does a government enforce its laws?
- Two issues to consider:
  - The efficiency of the judicial system
  - Is the rule of law and order followed?
- Useful data source: World Bank’s Governance Indicators

Bank- vs Market-based systems

<table>
<tr><th>Bank-based systems</th><th>Market-based systems</th></tr>
<tr><td>

Banks are central to the process of moving funds between demanders and suppliers of capital
More active monitoring
Examples: Germany, Japan
Indicators of banking development:

Bank liquid liabilities/GDP
Bank assets/GDP
Domestic bank deposits/GDP

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Securities markets are as important and can even be significantly more important
External market discipline
Examples: US, UK
Indicators of market development:

Market capitalisation/GDP
Total trading volume/GDP

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Control mechanisms: Ownership structure

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<tr><th>Widely held firms</th><th>Closely held firms</th></tr>
<tr><td>Separation between ownership and control
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Exit investment strategies</td><td>Manager and shareholder incentives aligned
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Agency theory (1)

- Principal-agent relationship: an agent is hired by a principal to do a job for them
- Managers are hired by shareholders
- Do managers act in the interests of shareholders?
  - Not always the case
  - Some real-life examples

Agency theory (2)

- Agency problems (agency costs) = conflict of interest that occurs when agents don't fully represent the best interests of principals
  - Hidden action: agents make decisions to maximize their own interests at the expense of the principals' interests without being detected by the principal
  - Hidden information (asymmetric information): agents have access to all available information about the firm while principals typically only receive some summary reports (e.g., annual report) which could be manipulated

Mechanisms to mitigate the agency costs

- Direct managerial/financial incentives
  - Option to buy stock or opportunity for promotion
  - Tie management compensation to market performance, e.g., earning-per-share growth
  - Although incentives can be used to align management and stockholder interests, they need to be structured carefully to make sure that they achieve their intended goal
- Control: if managers perform poorly, they can lose their job
- Two tier systems: trade unions and other groups can make managers act in the interests of stakeholders as well as shareholders

The OECD Principles of Corporate Governance (2015)

- Ensuring the basis for an effective corporate governance framework
- The rights and equitable treatment of shareholders and key ownership functions
- Institutional investors, stock markets, and other intermediaries
- The role of stakeholders in corporate governance
- Disclosure and transparency
- The responsibilities of the board

Ensuring the basis for an effective corporate governance framework

- The corporate governance framework:
  - Should promote transparent and fair markets and the efficient allocation of resources
  - Should be consistent with the rule of law and support effective supervision and enforcement

The rights and equitable treatment of shareholders and key ownership functions

- The corporate governance framework should protect and facilitate the exercise of shareholders' rights and ensure the equitable treatment of all shareholders, including minority and foreign shareholders
- All shareholders should have the opportunity to obtain effective redress for violation of their rights

Institutional investors, stock markets, and other intermediaries

- The corporate governance framework should provide sound incentives throughout the investment chain and provide for stock markets to function in a way that contributes to good corporate governance

The role of stakeholders in corporate governance

- The corporate governance framework should recognise the rights of stakeholders established by law or through mutual agreements and encourage active co-operation between corporations and stakeholders in creating wealth, jobs, and the sustainability of financially sound enterprises

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The responsibilities of the board

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Board of directors: Real world data

What do these statistics tell us?

- Education background
- Board independence
- Board diversity

- Social capital

- (In)effective monitoring
- Bring in expertise from outside

- Difference in experience
- Difference in risk-taking attitudes

Why do we care about these statistics?

What else matters?

- Personal attributes
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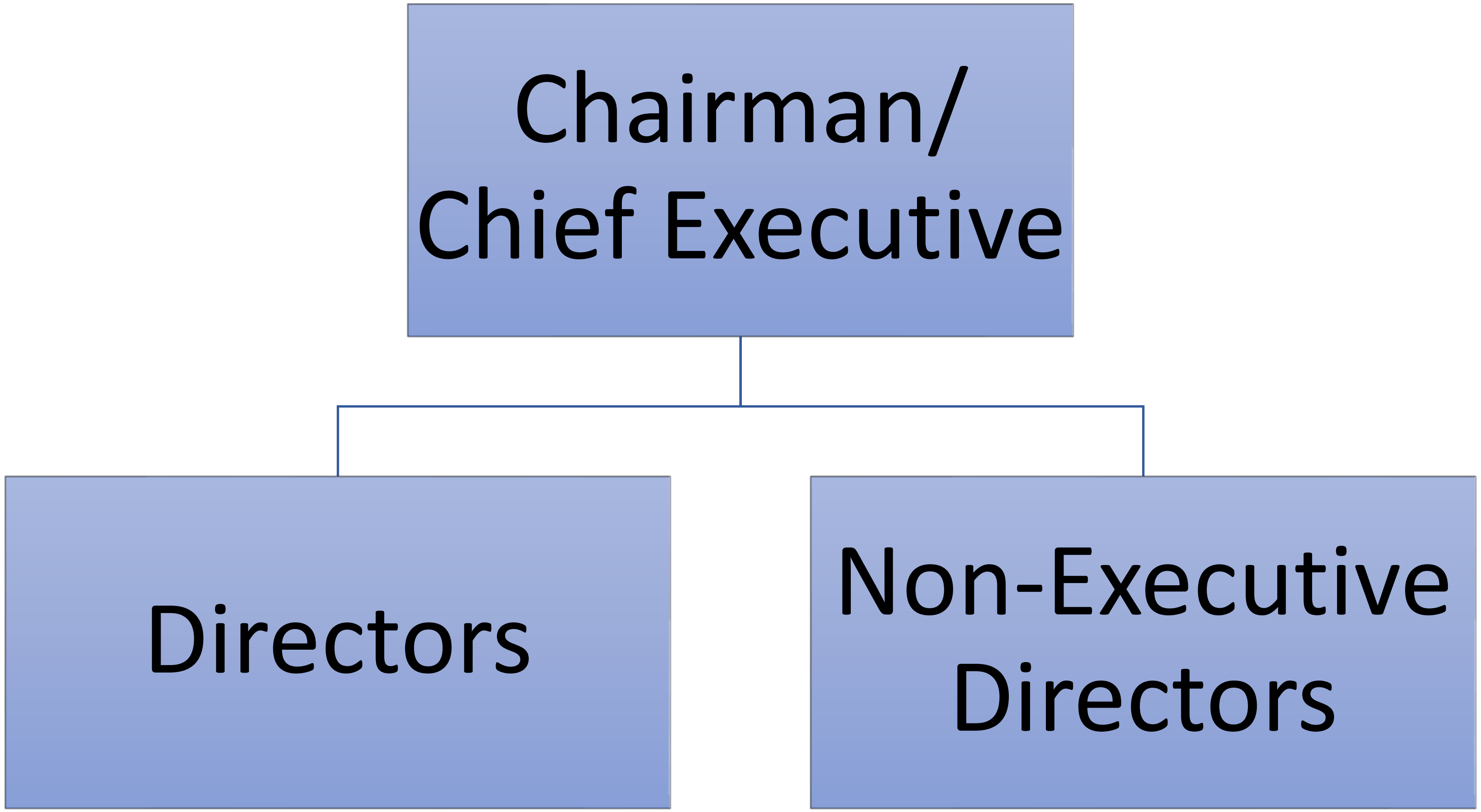

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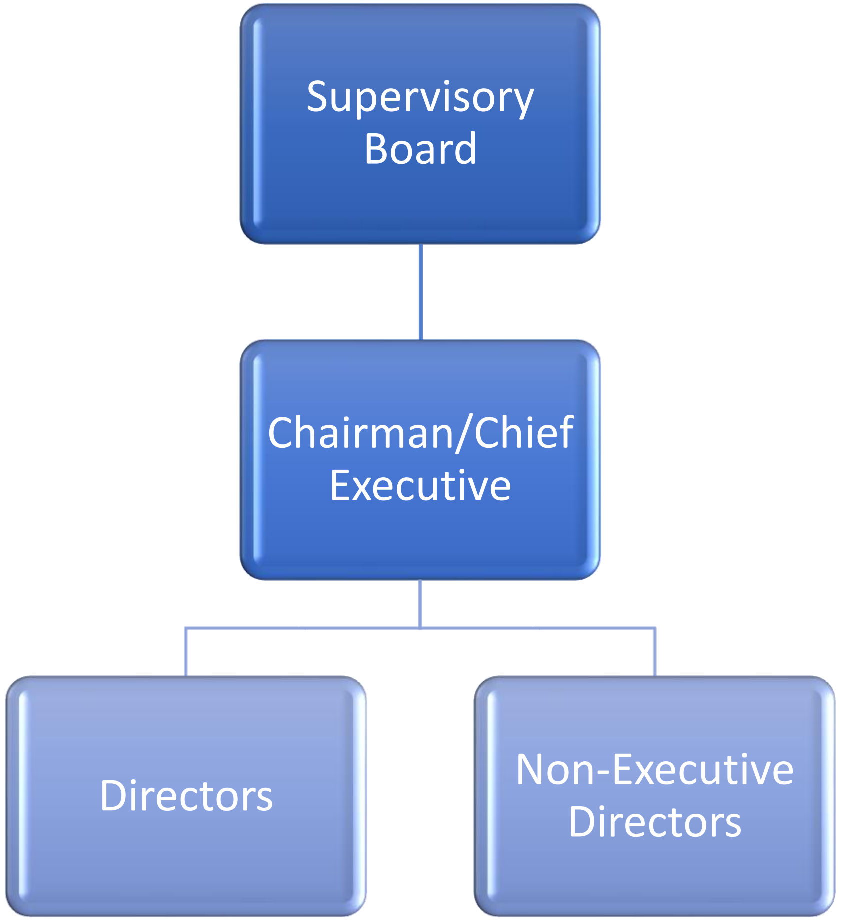

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Week 3

Agency problems

Lecture outline

- Agency costs of equity
- Agency costs of debt
- The principal-agent contract design
- Contract design in practice

Learning outcomes

- Understand the agency costs of debt and agency costs of equity
- Understand the basic principal-agent contract design
- Understand some issues/concerns about managers' compensation in reality

Principal – Agent relationship

- Principal is too busy to do a given job → hires agent

- Managers: maximize stockholder wealth → agents
- Stockholders have significant control over management: hire & fire managers via disciplinary mechanisms → principals
- But managers do not always act in the stockholders' best interest → conflict of interests → principal-agent problem (agency problem)

Agency costs (of equity) caused by

- **Separation of ownership from control** (hidden action)
  - Managers are in a position to maximise their own wealth without necessarily being “detected” by the owners of the company
- **Asymmetric information** (hidden information)
  - Managers have access to accounting data and financial reports whereas shareholders only receive annual reports which may be manipulated

Mechanisms to mitigate the agency costs of equity (1)

- Corporate governance is the system of rules, practices and processes by which a company is directed and controlled
- The role of corporate governance is to mitigate the conflict of interest without unduly burdening managers with the risk of the firm
- Corporate governance approaches:
  - Rules based (e.g., US)
  - Principals based (e.g., UK)

Mechanisms to mitigate the agency costs of equity (2)

- Direct managerial financial incentives
  - Incentives can be used to align management and stockholder interests
  - The incentives need to be structured carefully to make sure that they achieve their intended goal
  - Option to buy stock or opportunity for promotion
  - Tie management compensation to measures such as EPS growth

Agency costs of debt

- Conflict of interests between shareholders and debt holders arises if investment decisions have different consequences for the value of equity and the value of debt
- Such a conflict of interest is most likely to occur when there is a greater likelihood of financial distress
- We will illustrate the type of agency costs that may arise when managers take actions that benefit shareholders but harm debt holders and lower the total value of the firm

Example 1

- Consider Baxter, Inc., which is facing financial distress
  - Baxter has a loan of $1 million due at the end of the year
  - Without a change in its strategy, the market value of its assets will be only $900,000 at that time, and Baxter will default on its debt
  - Baxter is considering a new strategy which requires no upfront investment but it has only a 50% chance of success
    - If the new strategy succeeds, it will increase the value of the firm’s asset to $1.3 mil
    - If the new strategy fails, the value of the firm’s assets will fall to $300,000
    - Expected value of assets = $0.5 \times 1.3\,mil + 0.5 \times 300{,}000 = 800{,}000$
  - Can shareholders benefit if Baxter changes its strategy? What’s about debt holders?

Baxter's balance sheet

Equity holders get nothing

Bond holders get 900,000 → (100,000)

No action

Assets = 900,000	Liabilities = 1,000,000
	Equity = 0

New strategy – success (50%) Equity holders get 300,000 with 50% chance → Expected payoff = 150,000

Assets = 1,300,000	Liabilities = 1,000,000
	Equity = 300,000

New strategy – failure (50%)

Assets = 300,000	Liabilities = 1,000,000
	Equity = 0

Bond holders get 1 mil with 50% chance and 300,000 with 50% chance → Expected payoff = $0.5 \times 1\ mil + 0.5 \times 300{,}000 = 650{,}000$ → (250,000)

Excessive risk taking & Over-investment

- The debt holders' $250,000 loss corresponds to the $100,000 expected decline in firm value due to the risky strategy and the equity holders' $150,000 gain

$\Rightarrow$Equity holders are gambling with the debt holders' money

- When a firm faces financial distress, shareholders can gain at the expense of debt holders by taking a negative-NPV project (excessive risk taking)
- What can debt holders do?
  - pay less for the debt initially (and implement protective covenants restricting future actions)

Example 2

- Now assume Baxter does not pursue the risky strategy but instead the firm is considering an investment opportunity that requires an initial investment of $100,000 and will generate a risk-free return of 50%
- If the current risk-free rate is 50%, this investment clearly has a positive NPV
  - If Baxter raises $100,000 in new equity to make the investment, who benefits?
  - If you were Baxter's shareholders, would you invest in this new project?

Baxter's balance sheet (raising 100,000 in equity)

Without new project

Equity holders get nothing
Debt holders get 900,000

Assets = 900,000	Liabilities = 1,000,000
	Equity = 0

With new project

Assets = 900,000 + 150,000	Liabilities = 1,000,000
	Equity = 50,000

Equity holders get 50,000
Debt holders get 1,000,000

Debt overhang & Under-investment

- Shareholders have no incentives in making this new investment
- Debt overhang: A situation in which equity holders choose not to invest in a positive NPV project because the value of undertaking the investment opportunity will accrue to bondholders rather than themselves

Cashing out

- When a firm faces financial distress, shareholders have an incentive to withdraw money from the firm, if possible
  - E.g., if it is likely the company will default, the firm may sell assets below market value and use the funds to pay an immediate cash dividend to the shareholders
- This is another form of under-investment that occurs when a firm faces financial distress

Contract design

- Contract design can be used to align the managerial incentives with the interests of business owners in order to mitigate agency costs
- The first major insights were derived in the context of employment contracts involving risk-averse agents (employees) whose actions could not be directly observed by the principal (employer)
- Instead, the principal could only imperfectly observe a measure of the agent's performance

Basic principal-agent model – some definitions

- Rewards = outcomes that people care about (both financial and non-financial rewards)
- Effort = actions that people won't take without rewards (not just hours worked)
- Incentives = links between rewards and effort (not just compensation contract)
  - E.g., compensation sensitive to performance, concern about future labour market outcomes

Basic principal-agent model – setup

- 2 individuals: principal and agent
- The role of the principal is to supply capital, bear risk, and design an incentive scheme (a contract) for the agent
  - The principal can be thought of as a "representative shareholder" or the board of directors
- The role of the agent is to exert effort in order to make sound managerial decisions on behalf of the principal

The technology of production

- The agent's contribution to firm value (x)
  - Change in shareholders' wealth
- The action (effort) the agent takes to produce output (a)
- Other exogenous factors that are beyond the agent's control (ε). For simplicity, assume $E(\varepsilon) = 0$
- The production function: $x = a + \varepsilon$

Contract

- The agent's total compensation for the period of the contract, denoted by w, is a linear function of output:

$$w = s + b \times x$$

- In such a contract, s can be viewed as salary and b as the agent's bonus rate

Payoffs

- The principal's payoff (or "profit"):

$$\pi = x - w$$

For simplicity, we assume the principal is risk neutral

- The agent's payoff (or "utility"):

$$U = w - c(a)$$

where *c(a)* is the dollar amount necessary to compensate the agent for taking a particular action, *a*

For simplicity, the agent is risk-averse

Sequence of events

1. The Principal and the Agent sign a compensation contract
2. The Agent chooses an action (a) which could be either low effort (a_L) or high effort (a_H) (in most cases, the Principal cannot observe this choice)
3. Events beyond the Agent's control (ε) occur
4. Together, the action and the noise determine the Agent's output (x)
5. Output is observed by the Principal and the Agent: x_S(success) or x_F (failure)
6. The Agent receives the compensation specified by the contract

Unobservable effort

- The principal can base the contract on performance only (not on effort)

Effort/Performance	x_S (success)	x_F (failure)
a_H	P_H	$1 - P_H$
a_L	P_L	$1 - P_L$

What should be the optimal contract?

Optimal contract

- Maximize the Principal's expected payoff:

$$P_H(x_S - w_S) + (1 - P_H)(x_F - w_F)$$

s.t.:

- Incentive compatibility constraint (IC) (the agent should be compensated so they prefer to exert high effort)

$$P_H u(w_S) + (1 - P_H)u(w_F) - c(a_H) \geq P_L u(w_S) + (1 - P_L)u(w_F) - c(a_L)$$

- Individual rationality constraints (IR) (the contract should be attractive to the agent)

$$P_H u(w_S) + (1 - P_H)u(w_F) - c(a_H) \geq \bar{U}$$

where $\bar{U}$ is the Agent's reservation utility (opportunity cost, e.g., outside offer)

Optimization with inequality constraints

Max $f(x)$

s.t.: $g_j(x) \leq c_j$ for j = 1, 2, ..., m

The Lagrange function is:

$$L(x) = f(x) - \sum_{j=1}^{m} \lambda_j [g_j(x) - c_j]$$

Kuhn-Tucker (KT) conditions:

Complementary slackness conditions

$L_i'(x) = 0$ for i = 1, 2, ..., m

$\lambda_j \geq 0$; $g_j(x) \leq c_j$ and $\lambda_j [g_j(x) - c_j] = 0$ for j = 1, 2, ..., m

An (inequality) constraint is binding if $g_j(x) = c_j$

The KT conditions are both necessary and sufficient if

- The objective function is concave

AND

Either (1) each constraint is linear (i.e., each constraint is binding)

Or (2) each constraint function is convex, and some vectors of the variables satisfy all constraints strictly

In other words, we have to check for solutions when $\boldsymbol{\lambda_j > 0}$ and when $\boldsymbol{\lambda_j = 0}$ for all j!

Example

$$\text{Max } -(x-4)^2 - (y-4)^2$$

s.t.:

$$x + y \leq 4$$
$$x + 3y \leq 9$$

Solution (1)

The Lagrange function is:

$$-(x-4)^2 - (y-4)^2 - \lambda_1(x+y-4) - \lambda_2(x+3y-9)$$

KT conditions:

$$-2x + 8 - \lambda_1 - \lambda_2 = 0$$
$$-2y + 8 - \lambda_1 - 3\lambda_2 = 0$$
$$\lambda_1(x+y-4) = 0$$
$$\lambda_2(x+3y-9) = 0$$
$$x + y \leq 4$$
$$x + 3y \leq 9$$
$$\lambda_1; \lambda_2 \geq 0$$

Solution (2)

If $\lambda_1 = 0; \lambda_2 = 0$ (both constraints are not binding):

$$-2x + 8 = 0 \ (1)$$
$$-2y + 8 = 0 \ (2)$$
$$x + y < 4 \ (3)$$
$$x + 3y < 9 \ (4)$$

From (1) and (2): $x = y = 2$

Do not satisfy (3)

Solution (3)

If $\lambda_1 = 0; \lambda_2 > 0$ (the 1st constraint is not binding):

$$-2x + 8 - \lambda_2 = 0 \ (1)$$
$$-2y + 8 - 3\lambda_2 = 0 \ (2)$$
$$x + y < 4 \ (3)$$
$$x + 3y = 9 \ (4)$$

From (1) and (2):

$$y = 3x - 8$$

Combined with (3): $x = 3.3; y = 1.9$ do not satisfy (3)

Solution (4)

If $\lambda_1 > 0; \lambda_2 = 0$ (the 2nd constraint is not binding):

$$-2x + 8 - \lambda_1 = 0 \ (1)$$
$$-2y + 8 - \lambda_1 = 0 \ (2)$$
$$x + y = 4 \ (3)$$
$$x + 3y < 9 \ (4)$$

From (1) and (2):

$$x = y$$

Combined with (3): $x = y = 2$

This is a solution

Solution (5)

If $\lambda_1 > 0; \lambda_2 > 0$ (both constraints are binding):

$$-2x + 8 - \lambda_1 - \lambda_2 = 0 \ (1)$$
$$-2y + 8 - \lambda_1 - 3\lambda_2 = 0 \ (2)$$
$$x + y = 4 \ (3)$$
$$x + 3y = 9 \ (4)$$

From (3) and (4):

$$x = 1.5; y = 2.5$$

Combined with (1) and (2): $\lambda_2 = -1$ does not satisfy $\lambda_2 > 0$

Society & Equity | Social Impact | Compensation | Worker Rights | Exploration & Production

Britain's CEOs get 16% pay rise to 3.9 mln pounds even as workers struggle

By **Tommy Wilkes**

August 22, 2023 12:07 AM GMT+1 · Updated 23 days ago

Aa

Is higher compensation necessary good for firms?

Things to consider:

- Moral hazard
- Unequal pay
- Stakeholders

Corporate Social Responsibility

- ESG scoring
- Shareholders vs stakeholders podcasts

Executives vs. typical workers (Financial Times 2017; 2022)

CEOs vs. other executives

Female vs. male executives (CNBC 2021)

Quiz 1

The Board of Directors of Baxter, Inc. are designing an employment contract for the company's new CEO. The CEO can exert either a high effort a_H (the equivalent monetary value is 1) or a low effort a_L (the equivalent monetary value is 0) in managing the company. The Board of Directors are expecting that effort will have an impact on the company's earnings in the way given in the table below. They also know that the CEO has also offers from other companies providing her with a satisfaction level $\overline{U} = 2$.

The manager's utility function is given by:

$$U = \sqrt{w} - c(a)$$

Assume the manager's effort is not observable, derive the optimal contract for the manager.

Effort/Performance	$x_S = 100$ (success)	$x_F = 10$ (failure)
a_H	$P_H = 2/3$	$1 - P_H = 1/3$
a_L	$P_L = 1/3$	$1 - P_L = 2/3$

- Maximize the Principal's expected payoff:

$$\frac{2}{3}(100 - w_S) + \frac{1}{3}\ (10 - w_F)$$

s.t.:

- Incentive compatibility constraint (IC) (the agent should be compensated so as to prefer to exert high effort)

$$\frac{2}{3}\sqrt{w_S} + \frac{1}{3}\sqrt{w_F} - 1 \geq \frac{1}{3}\sqrt{w_S} + \frac{2}{3}\sqrt{w_F}$$

- Individual rationality constraints (IR) (the contract should be attractive to the agent)

$$\frac{2}{3}\sqrt{w_S} + \frac{1}{3}\sqrt{w_F} - 1 \geq 2$$

- IC

$$2\sqrt{w_S} + \sqrt{w_F} - 3 \geq \sqrt{w_S} + 2\sqrt{w_F}$$
$$\Leftrightarrow \sqrt{w_F} - \sqrt{w_S} \leq -3$$

- IR

$$2\sqrt{w_S} + \sqrt{w_F} - 3 \geq 6$$
$$\Leftrightarrow -2\sqrt{w_S} - \sqrt{w_F} \leq -9$$

The Lagrange function is:

$$70-\frac{2w_S}{3}-\frac{w_F}{3}-\lambda_1(\sqrt{w_F}-\sqrt{w_S}+3)-\lambda_2(-2\sqrt{w_S}-\sqrt{w_F}+9)$$

KT conditions:

$$-\frac{2}{3}+\frac{1}{2}\times\lambda_1\times {w_S}^{-\frac{1}{2}}+\lambda_2\times {w_S}^{-\frac{1}{2}}=0$$

$$-\frac{1}{3}-\frac{1}{2}\times\lambda_1\times {w_F}^{-\frac{1}{2}}+\lambda_2\times\frac{1}{2}\times {w_F}^{-\frac{1}{2}}=0$$

$$\lambda_1(\sqrt{w_F}-\sqrt{w_S}+3)=0$$

$$\lambda_2(-2\sqrt{w_S}-\sqrt{w_F}-9)=0$$

$$\lambda_1,\lambda_2\geq 0;\sqrt{w_F}-\sqrt{w_S}\leq -3;-2\sqrt{w_S}-\sqrt{w_F}\leq -9$$

We can easily rule out the case when both constraints are not binding

IC is binding; IR is not binding

Does not make sense as $\frac{\lambda_1}{\sqrt{w_F}}$ cannot be negative!

$$-\frac{2}{3}+\frac{1}{2}\times\lambda_1\times {w_S}^{-\frac{1}{2}}=0 \Leftrightarrow \frac{\lambda_1}{\sqrt{w_S}}=\frac{4}{3}$$

$$-\frac{1}{3}-\frac{1}{2}\times\lambda_1\times {w_F}^{-\frac{1}{2}}=0 \Leftrightarrow \frac{\lambda_1}{\sqrt{w_F}}=\frac{-2}{3}$$

$$\sqrt{w_F}-\sqrt{w_S}=-3$$

$$-2\sqrt{w_S}-\sqrt{w_F}<-9$$

$$\lambda_1>0$$

IC is not binding; IR is binding

$$-\frac{2}{3}+\lambda_2 \times w_S{}^{-\frac{1}{2}}=0 \Leftrightarrow \frac{\lambda_2}{\sqrt{w_S}}=\frac{2}{3}$$

$$-\frac{1}{3}+\lambda_2 \times \frac{1}{2} \times w_F{}^{-\frac{1}{2}}=0 \Leftrightarrow \frac{\lambda_2}{\sqrt{w_F}}=\frac{2}{3}$$

$$\sqrt{w_F}-\sqrt{w_S}<-3$$

$$-2\sqrt{w_S}-\sqrt{w_F}=-9$$

$$\lambda_2>0$$

Also does not make sense as $\sqrt{w_F}=\sqrt{w_S}$ does not satisfy $\sqrt{w_F}-\sqrt{w_S}<-3$

Both constraints are binding

(IC)

$$\sqrt{w_S} - \sqrt{w_F} = 3\ (1)$$

(IR)

$$2\sqrt{w_S} + \sqrt{w_F} = 9\ (2)$$

From (1) and (2) we get:

$$3\sqrt{w_S} = 12 \Leftrightarrow \sqrt{w_S} = 4 \Leftrightarrow w_S = 16$$
$$\sqrt{w_F} = \sqrt{w_S} - 1 = 1 \Leftrightarrow w_F = 1$$

On average, the manager's compensation for hard work is:

$$\frac{2}{3}w_S + \frac{1}{3}w_F = 11$$

The shareholders earn an expected value of:

$$70 - 11 = 59$$

Mathematics 2

Constrained optimisation

Constrained optimisation

In economics, we often face various **constraints**.

- E.g., the budget constraint.

Finding the best way to do something under constraints is called a **constrained optimisation problem**

A typical constrained optimisation problem could be either a maximisation problem or a minimisation problem. A constrained maximisation problem can be represented as follows:

$$\begin{aligned} \max \quad & f(\boldsymbol{x}) \\ \text{s.t.} \quad & g_i(\boldsymbol{x}) \leq 0, \;\; i \in \{1, 2, \ldots, m\} \\ & h_j(\boldsymbol{x}) = 0, \;\; j \in \{1, 2, \ldots, t\} \end{aligned} \tag{1}$$

Here, f is called the **objective function**.

$g_i(\boldsymbol{x})$ is called an **inequality constrained function**

- When an inequality constraint holds with equality ($g_i(\boldsymbol{x}) = 0$), it is said to be **binding**.

$h_j(\boldsymbol{x})$ is called an **equality constrained function**.

A minimisation problem can be written as:

$$\begin{aligned} \min \quad & f(\boldsymbol{x}) \\ \text{s.t.} \quad & g_i(\boldsymbol{x}) \leq 0, \;\; i \in \{1, 2, \ldots, m\} \\ & h_j(\boldsymbol{x}) = 0, \;\; j \in \{1, 2, \ldots, t\} \end{aligned} \tag{2}$$

The feasible region

We call a point $\boldsymbol{x}$ a **feasible solution** if it satisfies the constraints:

$$g_i(\boldsymbol{x}) \leq 0, \;\; i \in \{1, 2, \ldots, m\}$$
$$h_j(\boldsymbol{x}) = 0, \;\; j \in \{1, 2, \ldots, t\}$$

Let F denote the set of all feasible solutions.

- It is called the **feasible set** or the **feasible region**.

Maximisation

The maximisation problem (1) is to find some feasible solution $\boldsymbol{x}^* \in F$ such that $f(\boldsymbol{x}^*) \geq f(\boldsymbol{x})$ for all $\boldsymbol{x} \in F$.

- We've already encountered such problems when covering linear programming.
- The difference now is that f, g_i and h_j need not be linear functions.

The Lagrange function

A powerful method for solving constrained optimisation problems is to transform the maximisation or minimisation problem into a **Lagrange function** and then solve a system of equations.

- The Lagrangian for the **maximisation** problem is given by:

$$\mathcal{L}(\boldsymbol{x}, \lambda_1, \cdots, \lambda_m, \mu_1, \cdots, \mu_t) = f(\boldsymbol{x}) - \sum_{i=1}^{m} \lambda_i g_i(\boldsymbol{x}) - \sum_{j=1}^{t} \mu_j h_j(\boldsymbol{x})$$

- The Lagrangian for the **minimisation** problem is given by:

$$\mathcal{L}(\boldsymbol{x}, \boldsymbol{\lambda}, \boldsymbol{\mu}) = f(\boldsymbol{x}) + \sum_{i=1}^{m} \lambda_i g_i(\boldsymbol{x}) + \sum_{j=1}^{t} \mu_j h_j(\boldsymbol{x})$$

The Kuhn-Tucker condition

Step 1. Write down the following system of equations and inequalities, called the **Kuhn-Tucker condition**:

$$\begin{aligned} \nabla_{\boldsymbol{x}} \mathcal{L}(\boldsymbol{x}, \boldsymbol{\lambda}, \boldsymbol{\mu}) &= \mathbf{0} & & \\ g_i(\boldsymbol{x}) &\leq 0 & & i \in \{1, 2, \ldots, m\} \\ h_j(\boldsymbol{x}) &= 0 & & j \in \{1, 2, \ldots, t\} \\ \lambda_i g_i(\boldsymbol{x}) &= 0 & & i \in \{1, 2, \ldots, m\} \\ \lambda_i &\geq 0 & & i \in \{1, 2, \ldots, m\} \end{aligned} \tag{3}$$

For example, for the maximisation problem we have:

$$\nabla_{\boldsymbol{x}} \mathcal{L}(\boldsymbol{x}, \boldsymbol{\lambda}, \boldsymbol{\mu}) = \nabla f(\boldsymbol{x}) - \sum_{i=1}^{m} \lambda_i \nabla g_i(\boldsymbol{x}) - \sum_{j=1}^{t} \mu_j \nabla h_j(\boldsymbol{x}) = \mathbf{0}$$

The complementary slackness condition

Step 1. Solve the following system of equations, called the **Kuhn-Tucker condition**:

$$\begin{aligned} \nabla_{\boldsymbol{x}} \mathcal{L}(\boldsymbol{x}, \boldsymbol{\lambda}, \boldsymbol{\mu}) &= \mathbf{0} & & \\ g_i(\boldsymbol{x}) &\leq 0 & \quad & i \in \{1, 2, \ldots, m\} \\ h_j(\boldsymbol{x}) &= 0 & \quad & j \in \{1, 2, \ldots, t\} \\ \lambda_i g_i(\boldsymbol{x}) &= 0 & \quad & i \in \{1, 2, \ldots, m\} \\ \lambda_i &\geq 0 & \quad & i \in \{1, 2, \ldots, m\} \end{aligned} \tag{3}$$

We call $\lambda_i g_i(\boldsymbol{x}) = 0$ the **complementary slackness condition**.

- It implies that whenever inequality constraint i is not binding ($g_i(\boldsymbol{x}) < 0$), we must have $\lambda_i = 0$.

Kuhn-Tucker method, continued

Step 2. Find all feasible solutions $\boldsymbol{x}$ to the equations and inequalities in (3).

We may not know which inequality constraints will be binding.

- Many of them may be binding, or none of them.
- We therefore need to check for solutions where $\lambda_i > 0$ and where $\lambda_i = 0$ for all i.

Kuhn-Tucker method, continued

We may not know which inequality constraints will be binding.

- For example, suppose we have two inequality constraints and the following Lagrange function:

$$\mathcal{L}(x_1, x_2, \lambda_1, \lambda_2) =$$
$$-(x_1 - 10)^2 - (x_2 - 10)^2 + 200$$
$$-\lambda_1(x_1 + 2x_2 - a) - \lambda_2(x_1x_2 - b)$$

In Step 2, we then need to check the following cases for feasible solutions:

(a) $\lambda_1 = \lambda_2 = 0$
(b) $\lambda_1 > 0, \lambda_2 = 0$
(c) $\lambda_1 = 0, \lambda_2 > 0$
(d) $\lambda_1 > 0, \lambda_2 > 0$

And confirm that they do not violate the constraints.

Step 3. Evaluate f at all feasible solutions found in Step 2. The feasible solution with the largest function value is usually the solution to the constrained maximisation problem.

- I have uploaded *some* necessary and sufficient conditions on VLE.
- On the exam, I will not give you constrained optimisation problems where this method does not work.
  - Just be aware that there are various necessary and sufficient conditions for the method to produce a valid solution.

Example 1

Suppose a consumer has the utility function $u(x_1, x_2) = x_1^{1/3} x_2^{2/3}$, where x_1 and x_2 denote the quantities consumed of two goods. Suppose the consumer has a budget of $B > 0$. The prices of the two goods are $p_1 > 0$ and $p_2 > 0$. Find the optimal consumption levels of the goods for the consumer under his/her budget constraint.

Example 1

The problem can be written as:

$$\begin{aligned} \max \quad & u(x_1, x_2) = x_1^{1/3} x_2^{2/3} \\ \text{s.t.} \quad & p_1 x_1 + p_2 x_2 \leq B \end{aligned}$$

Example 1

Rewrite it as:

$$\begin{aligned} \max \quad & u(x_1, x_2) = x_1^{1/3} x_2^{2/3} \\ \text{s.t.} \quad & p_1 x_1 + p_2 x_2 - B \leq 0 \end{aligned}$$

Its corresponding Lagrange function is:

$$\mathcal{L}(x_1, x_2, \lambda) = x_1^{1/3} x_2^{2/3} - \lambda(p_1 x_1 + p_2 x_2 - B)$$

Its Kuhn-Tucker condition is:

$$\begin{aligned} \nabla_{x_1, x_2} \mathcal{L}(x_1, x_2, \lambda) = \nabla u(x_1, x_2) - \lambda \nabla g(x_1, x_2) & = \mathbf{0} \\ p_1 x_1 + p_2 x_2 - B & \leq 0 \\ \lambda(p_1 x_1 + p_2 x_2 - B) & = 0 \\ \lambda & \geq 0 \end{aligned}$$

Example 1

Rewrite it as:

$$\begin{aligned} \max \quad & u(x_1, x_2) = x_1^{1/3} x_2^{2/3} \\ \text{s.t.} \quad & p_1 x_1 + p_2 x_2 - B \leq 0 \end{aligned}$$

Its corresponding Lagrange function is:

$$\mathcal{L}(x_1, x_2, \lambda) = x_1^{1/3} x_2^{2/3} - \lambda(p_1 x_1 + p_2 x_2 - B)$$

Its Kuhn-Tucker condition is:

$$\nabla_{x_1, x_2} \mathcal{L}(x_1, x_2, \lambda) = \begin{pmatrix} \frac{1}{3} x_1^{-2/3} x_2^{2/3} \\ \frac{2}{3} x_1^{1/3} x_2^{-1/3} \end{pmatrix} - \lambda \begin{pmatrix} p_1 \\ p_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{aligned} p_1 x_1 + p_2 x_2 - B \quad & \leq 0 \\ \lambda(p_1 x_1 + p_2 x_2 - B) \quad & = 0 \\ \lambda \quad & \geq 0 \end{aligned}$$

Example 1

Or, equivalently, its Kuhn-Tucker condition is:

$$\begin{aligned}
\tfrac{1}{3}x_1^{-2/3}x_2^{2/3} - \lambda p_1 &= 0 \\
\tfrac{2}{3}x_1^{1/3}x_2^{-1/3} - \lambda p_2 &= 0 \\
p_1x_1 + p_2x_2 - B &\leq 0 \\
\lambda(p_1x_1 + p_2x_2 - B) &= 0 \\
\lambda &\geq 0
\end{aligned}$$

- Observe that since $p_1 > 0$, $p_2 > 0$ and $B > 0$, there exist $x_1 > 0$ and $x_2 > 0$ such that $p_1x_1 + p_2x_2 - B \leq 0$.
- Also, for any $x_1 > 0$ and $x_2 > 0$ satisfying $p_1x_1 + p_2x_2 - B \leq 0$, we have $u(x_1, x_2) > u(0, x_2) = u(x_1, 0) = 0$.

$\implies$ The solution (x_1, x_2) to the problem must be $x_1 > 0$ and $x_2 > 0$.

Example 1

It then follows from

$$\frac{1}{3}x_1^{-2/3}x_2^{2/3} - \lambda p_1 = 0$$

that $\lambda > 0$. This means we can ignore the $\lambda = 0$ case.

From

$$\lambda(p_1 x_1 + p_2 x_2 - B) = 0$$

we then know that $p_1 x_1 + p_2 x_2 - B = 0$.

- I.e., when $\lambda > 0$, the budget constraint is binding.

Example 1

Solving the equations

$$\frac{1}{3}x_1^{-2/3}x_2^{2/3} = \lambda p_1$$

$$\frac{2}{3}x_1^{1/3}x_2^{-1/3} = \lambda p_2$$

for x_2 yields $x_2 = \dfrac{2x_1p_1}{p_2}$.

Substituting $x_2 = \dfrac{2x_1p_1}{p_2}$ into $p_1x_1 + p_2x_2 = B$ gives the optimal consumption:

$$x_1^* = \frac{B}{3p_1}$$

$$x_2^* = \frac{2B}{3p_2}$$

λ is called a **Lagrange multiplier**.

- If we have multiple constraints, all λ_is and μ_js are called Lagrange multipliers.

$$\mathcal{L}(\boldsymbol{x}, \lambda) = x_1^{1/3} x_2^{2/3} - \lambda(p_1 x_1 + p_2 x_2 - B)$$

$$\nabla_{\boldsymbol{x}} \mathcal{L}(\boldsymbol{x}, \lambda) = \nabla u(\boldsymbol{x}) - \lambda \nabla g(\boldsymbol{x}) = \mathbf{0}$$

Lagrange multipliers

λ is called a **Lagrange multiplier**.

- If we have multiple constraints, all λ_is and μ_js are called Lagrange multipliers.

$$\begin{aligned}
\mathcal{L}(\boldsymbol{x}, \lambda) &= x_1^{1/3} x_2^{2/3} - \lambda(p_1 x_1 + p_2 x_2 - B) \\
\nabla_{\boldsymbol{x}} \mathcal{L}(\boldsymbol{x}, \lambda) &= \nabla u(\boldsymbol{x}) - \lambda \nabla g(\boldsymbol{x}) = \mathbf{0} \\
\nabla u(\boldsymbol{x}) &= \lambda \nabla g(\boldsymbol{x})
\end{aligned}$$

I.e., the gradient of the objective function u is proportional to the gradient of g (our constraint).

- λ is the proportionality constant.

Lagrange multipliers

$$\nabla u(\boldsymbol{x}) = \lambda \nabla g(\boldsymbol{x})$$

$$\nabla \mathcal{L}(\boldsymbol{x}, \lambda) = \begin{pmatrix} \frac{1}{3} x_1^{-2/3} x_2^{2/3} \\ \frac{2}{3} x_1^{1/3} x_2^{-1/3} \end{pmatrix} - \lambda \begin{pmatrix} p_1 \\ p_2 \end{pmatrix}$$

Here, $\nabla g(\boldsymbol{x})$ is just the price vector.

- The price vector determines how much more we can consume of each good if we relax the constraint (by increasing B).

λ reflects the difference between how much the utility increases when we increase consumption slightly ($\nabla u(\boldsymbol{x})$) and how much we can increase consumption when the constraint is relaxed ($\nabla g(\boldsymbol{x})$).

- It is the marginal utility of relaxing the constraint, called the **shadow price**.

Lagrange multipliers

In general, Lagrange multipliers measure how much the value of the objective function changes as a result of a marginal relaxation of the constraint.

- If the constraint is not binding, relaxing the constraint further does not improve the objective value.

$\Longrightarrow$ The Lagrange multiplier is 0.

Example 2

Solve the following problem:

$$\begin{aligned} \min \quad & x_1^2 + x_2^2 \\ \text{s.t.} \quad & (x_1 - 2)^2 + (x_2 - 1)^2 \leq 1 \end{aligned}$$

Solution: Let $f(\boldsymbol{x}) = x_1^2 + x_2^2$ and $g(\boldsymbol{x}) = (x_1 - 2)^2 + (x_2 - 1)^2 - 1$. We can write the problem into the standard form:

$$\begin{aligned} \min \quad & f(\boldsymbol{x}) \\ \text{s.t.} \quad & g(\boldsymbol{x}) \leq 0 \end{aligned}$$

Example 2

The Lagrange function is:

$$\mathcal{L}(\boldsymbol{x}, \lambda) = x_1^2 + x_2^2 + \lambda((x_1 - 2)^2 + (x_2 - 1)^2 - 1)$$

The Kuhn-Tucker condition is:

$$\begin{aligned} \nabla_{\boldsymbol{x}} \mathcal{L}(\boldsymbol{x}, \lambda) = \nabla f(\boldsymbol{x}) + \lambda \nabla g(\boldsymbol{x}) = \mathbf{0} \\ \lambda \geq 0,\ \lambda((x_1 - 2)^2 + (x_2 - 1)^2 - 1) = 0 \\ (x_1 - 2)^2 + (x_2 - 1)^2 - 1 \leq 0, \end{aligned}$$

where:

$$\nabla f(\boldsymbol{x}) = \begin{pmatrix} 2x_1 \\ 2x_2 \end{pmatrix}, \quad \nabla g(\boldsymbol{x}) = \begin{pmatrix} 2x_1 - 4 \\ 2x_2 - 2 \end{pmatrix}$$

Example 2

We can rewrite the Kuhn-Tucker condition as:

$$2x_1 + \lambda(2x_1 - 4) = 0$$
$$2x_2 + \lambda(2x_2 - 2) = 0$$
$$\lambda \geq 0,\ \lambda((x_1 - 2)^2 + (x_2 - 1)^2 - 1) = 0$$
$$(x_1 - 2)^2 + (x_2 - 1)^2 - 1 \leq 0$$

The first two equations give us:

$$\begin{aligned} x_1 &= \frac{2\lambda}{1+\lambda} \\ x_2 &= \frac{\lambda}{1+\lambda} \end{aligned}$$

We need to consider two cases: $\lambda = 0$ and $\lambda > 0$.

Example 2

$$\mathcal{L}(\boldsymbol{x}, \lambda) = x_1^2 + x_2^2 + \lambda((x_1 - 2)^2 + (x_2 - 1)^2 - 1)$$

Case 1 ($\lambda = 0$). By assuming that $\lambda = 0$, we assume that the constraint is not binding.

- The Lagrange function becomes:

$$\mathcal{L}(\boldsymbol{x}, \lambda) = f(\boldsymbol{x}) = x_1^2 + x_2^2$$

The first expression in the Kuhn-Tucker condition becomes:

$$\nabla_{\boldsymbol{x}} \mathcal{L}(\boldsymbol{x}, \lambda) = \nabla f(\boldsymbol{x}) = \mathbf{0}$$

I.e., in Case 1, we are just looking for stationary points!

- Same as in the unconstrained optimisation case.

Example 2

$$\begin{aligned} x_1 &= \frac{2\lambda}{1+\lambda} \\ x_2 &= \frac{\lambda}{1+\lambda} \end{aligned}$$

Case 1 ($\lambda = 0$). Inserting $\lambda = 0$ gives us $x_1 = x_2 = 0$.

- This violates the constraint $(x_1 - 2)^2 + (x_2 - 1)^2 - 1 \leq 0$, since $4 \nleq 0$.

$\implies$ Case 1 can be disregarded.

Example 2

Case 2 ($\lambda > 0$). As before, we have:

$$\begin{aligned} x_1 &= \frac{2\lambda}{1+\lambda} \\ x_2 &= \frac{\lambda}{1+\lambda} \end{aligned}$$

It follows from $\lambda((x_1 - 2)^2 + (x_2 - 1)^2 - 1) = 0$ that:

$$(x_1 - 2)^2 + (x_2 - 1)^2 = 1$$

I.e., the constraint is binding.

Example 2

Plug $x_1 = \dfrac{2\lambda}{1+\lambda}$ and $x_2 = \dfrac{\lambda}{1+\lambda}$ into $(x_1 - 2)^2 + (x_2 - 1)^2 = 1$.

- This yields $\lambda^2 + 2\lambda - 4 = 0$, which has two roots:

$$\begin{aligned} \lambda_A &= -1 - \sqrt{5} < 0 \\ \lambda_B &= -1 + \sqrt{5} > 0 \end{aligned}$$

Example 2

$$\begin{aligned} x_1 &= \frac{2\lambda}{1+\lambda} \\ x_2 &= \frac{\lambda}{1+\lambda} \\ \lambda_A &= -1-\sqrt{5} < 0 \\ \lambda_B &= -1+\sqrt{5} > 0 \end{aligned}$$

$\lambda_A < 0$ contradicts the assumtion $\lambda \geq 0$.

- So we have $\lambda^* = \lambda_B = \sqrt{5} - 1$ and the minimum point:

$$\begin{aligned} x_1^* &= \frac{2\sqrt{5}-2}{\sqrt{5}} \\ x_2^* &= \frac{\sqrt{5}-1}{\sqrt{5}} \end{aligned}$$

The minimum value is: $f(x^*) = 6 - 2\sqrt{5}$

Example 2

- The first expression of the Kuhn-Tucker condition is:

$$\nabla_{\boldsymbol{x}} \mathcal{L}(\boldsymbol{x}, \lambda) = \nabla f(\boldsymbol{x}) + \lambda \nabla g(\boldsymbol{x}) = \mathbf{0}, \text{ or}$$
$$-\nabla f(\boldsymbol{x}) = \lambda \nabla g(\boldsymbol{x})$$

Since $\lambda^* > 0$, we have found a point where:

(a) The constraint is binding.
(b) The gradient of f is proportional to the gradient of g, and pointing in the opposite direction.

- The level sets of f and g will be tangent when the gradients are proportional, since they are perpendicular to the gradients.

This is where the sign in front of λ matters. There is a point where the constraint is binding while $f(\boldsymbol{x})$ and $\nabla g(\boldsymbol{x})$ point in the same direction.

Minimisation and maximisation

Consider the constrained minimisation problem:

$$
\begin{aligned}
\min \quad & f(x) \\
\text{s.t.} \quad & g_i(x) \leq 0, \;\; i \in \{1, 2, \ldots, m\} \\
& h_j(x) = 0, \;\; j \in \{1, 2, \ldots, t\}
\end{aligned}
\tag{4}
$$

Any minimisation problem can be reformulated as a maximisation problem and vice versa.

Minimisation and maximisation

Lemma 1 The following two problems have the same optimal solutions:

$$\max_{x \in F} f(x), \qquad\qquad \min_{x \in F} -f(x)$$

Proof. Let $x^* \in F$ be an optimal solution to $\max_{x \in F} f(x)$. Then we have:

$$f(x^*) \geq f(x) \quad \text{for all } x \in F$$

Equivalently:

$$-f(x^*) \leq -f(x) \quad \text{for all } x \in F$$

This means that x^* is also an optimal solution to $\min_{x \in F} -f(x)$. $\square$

Readings

Study Chapters 13 and 14 in **Essential Mathematics for Economic Analysis**.

Week 4

Financial Statement Analysis and Discounted Cash Flow Valuation

Lecture outline

- Annual report
- Financial statement analysis
- Time value of money
- Discounted cash flow valuation

Learning outcomes

- Gain a basic understanding of a firm's annual report
- Be able to perform financial statement analysis and use financial statement information
- Understand time value of money
- Be able to evaluate a project/investment based on discounted cash flow evaluation

Documents and Events

Quarterly Disclosure | Other Documents & Events

Year	Shareholder Meeting	Investor Events	Product Reveals	Other Events
2023	Annual Report & Webcast	2023 Investor Day 2023 Investor Day Presentation		
2022	Annual Report & Webcast		Tesla Semi Delivery Event	3-1 stock split Filing for 3-for-1 Stock Split AI Day 2
2021	Annual Report & Webcast		Tesla Model S Plaid Delivery Event	Tesla AI Day
2020	Annual Report & Webcast	Tesla Battery Day		5-1 stock split Filing for 5-for-1 Stock Split
2019	Annual Report & Webcast	Tesla Autonomy Investor Day	35K Model 3 Solar Roof	Maxwell Technologies, Inc. Transaction

Source: tesla.com

An annual report contains

- The statement of financial positions (balance sheet)
- The income statement
- The statement of cash flows

Balance sheet (1)

Current assets (e.g., cash, stock inventory)

Non-current assets:

- Tangible non-current assets (e.g., land, buildings, plants)
- Intangible non-current assets (e.g., trademarks, patents, copyrights)

Current liabilities (e.g., short-term debts)

Non-current liabilities (e.g., long-term debts)

Equity

Balance sheet (2)

- 3 important considerations:
  - Liquidity
  - Debt vs. equity
  - Value vs. cost

Income statement (1)

Summarizes performance over a specific period

Revenue - Expenses ≡ Income

Income statement (2)

- 3 important considerations:
  - Non-cash items: expenses charged against revenues that do not directly affect cash flow (e.g., depreciation)
  - Time
  - Costs

Statement of cash flows (1)

CF from assets	CF from creditors
	CF from shareholders

- Total CF
  - CF from operating activities: cash generated from normal business activities
  - CF from investing activities: cash generated or expended from long-term investments
  - CF from financing activities: cash raised from investors

Statement of cash flows (2)

- 3 important considerations:
  - Operating cash flows
  - Cash flow is NOT the same as profit
  - Free cash flow: cash flow available after all taxes have been paid and after all positive <u>net present value</u> projects have been financed

Financial statement analysis

- Ratio analysis
- The Du Pont Identity
- Choose a Benchmark

Some of inventories might be damaged/lost; large inventories = short-term trouble

Short-term solvency ratios (liquidity ratios)

>1: the firm can meet its immediate financial obligations

- Determine the firm's ability to pay its bills over the short run without undue stress

$$Current\ ratio = \frac{Current\ assets}{Current\ liabilities}$$

$$Quick\ ratio\ (Acid\ test\ ratio) = \frac{Current\ assets - Inventory}{Current\ liabilities}$$

$$Cash\ ratio = \frac{Cash\ and\ cash\ equivalents}{Current\ liabilities}$$

>1: the firm has more cash than it needs to pay off the current debts. Is it good?

Long-term solvency ratios (financial leverage ratios)

- Determine the firm's long-term ability to meet its obligations, or, more generally, its financial leverage

$$Total\ debt\ ratio = \frac{Total\ assets\ \ - Total\ equity}{Total\ assets}$$

$$Debt - equity\ ratio = Total\ debt/Total\ equity$$

$$Equity\ multiplier = Total\ assets/Total\ equity$$

$$Times\ interest\ earned\ ratio(Interest\ coverage\ ratio) = EBIT\ /Interest\ payable$$

$$Cash\ coverage\ ratio = \frac{EBIT + Depreciation}{Interest\ payable}$$

Higher ratio = the firm is efficient in managing inventory BUT excessively high ratio might mean inadequate stocking

Turnover ratios (asset management ratios)

$$Inventory\ turnover = Cost\ of\ goods\ sold/Inventory$$

$$Days'sales\ in\ inventory = 365\ days/Inventory\ turnover$$

$$Receivables\ turnover = Sales/Trade\ receivables$$

$$Days'sales\ in\ receivables = 365\ days/Receivables\ turnover$$

$$Total\ asset\ turnover = Sales/Total\ assets$$

How fast the firms collect sales

Profitability ratios

$$Profit\ margin = Net\ income/Sales$$

$$Return\ on\ assets\ (ROA) = Net\ income/Total\ assets$$

$$Return\ on\ equity\ (ROE) = Net\ income/Total\ equity$$

Market value ratios

$$Earnings\ per\ Share\ (EPS) = \ Net\ income/Outstanding\ shares$$

$$Price\ earnings\ ratio\ \ = \ Price\ per\ share/Earnings\ per\ share$$

$$Market - to - book\ ratio = \frac{Market\ value\ per\ share}{Book\ value\ per\ share}$$

Du Pont identity

$$ROE = \frac{Net\ income}{Sales} \times \frac{Sales}{Assets} \times \frac{Assets}{Total\ equity} = ROA \times \frac{Assets}{Total\ equity}$$

Profit margin = operating efficiency

Total asset turnover = asset use efficiency

Equity multiplier = financial leverage

After calculations: Choose a benchmark

Time Trend Analysis

- Look at the same ratio over years

Peer Group Analysis

- Compare ratio with similar firms
- Companies in same industry

Everything we have discussed so far is directly related to cash flows (in/out). In the next part, we will discuss how to determine the value today of a cash flow expected in the future.

Time value of money: A "simple" concept

£100 received on your birthday this year ≠ £100 you will receive on your birthday next year

Notations

FV	Future value
PV	Present value
r	Interest rate or discount rate
t	Number of periods

Time value of money: The single-period case

$$PV = \frac{FV}{1 + r}$$

$$NPV = PV - Cost$$

$$FV = PV \times (1 + r)$$

Example 1

- You are offered the following opportunity:
  - Today, you will pay £400,000 for an artwork.
  - In 1 year from now, I will buy back the said artwork at the price £480,000
- The bank interest rate is 25%. Will you accept my offer?

$$NPV(Artwork) = -400{,}000 + \frac{480{,}000}{1.25} = -£16{,}000$$

Example 2

- Suppose you deposit £100 for 1 year at an interest rate of 10%.
- How much will it amount to in 1 year?

$$100 \times (1 + 0.1) = 110$$

- How much will it amount to if you leave it in the account for another year, assuming the interest rate stays the same?

$$110 \times (1 + 0.1) = 121$$

$$121 = 100 + 10 + 10 + 1$$

Interest earned in yr 2 **on the interest paid in yr 1**: £10 × 0.1 = £1

Original amount

Interest earned in yr 1

Interest earned in yr 2

- This process of leaving your money and any accumulated interest in an investment for more than one period, and thereby reinvesting the interest, is called **compounding**
- **Compounding** the interest means earning interest on interest
- **Compound interest** = interest earned on both the initial principal and the interest reinvested from prior periods
- Simple interest = interest earned only on the original principal

Time value of money: The multi-period case

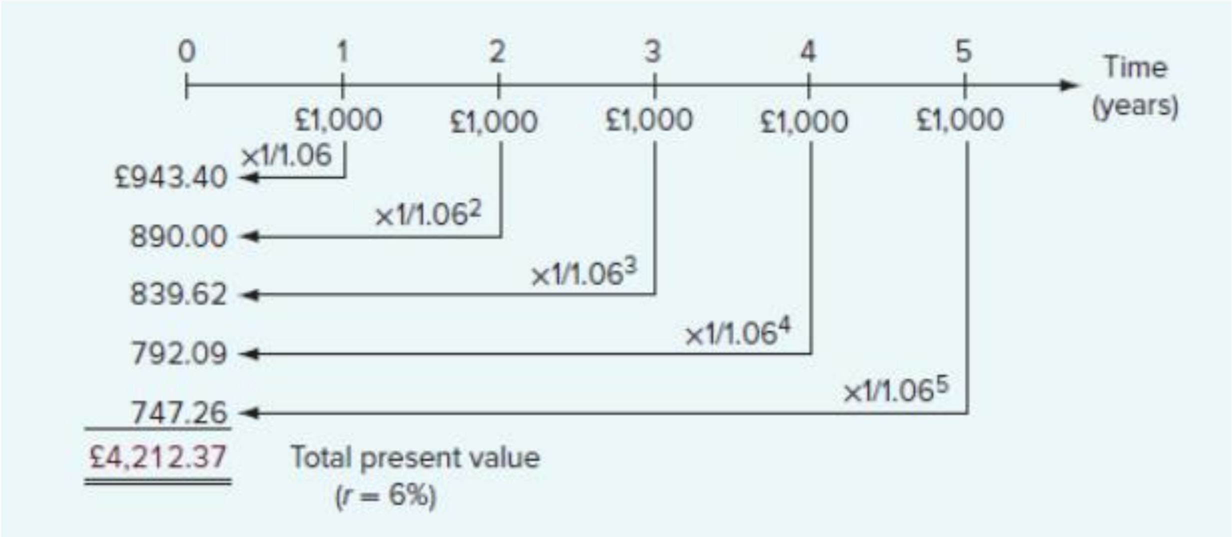
$$PV = \frac{FV_t}{(1+r)^t}$$

$$FV = PV \times (1+r)^t$$

Example 3 (1)

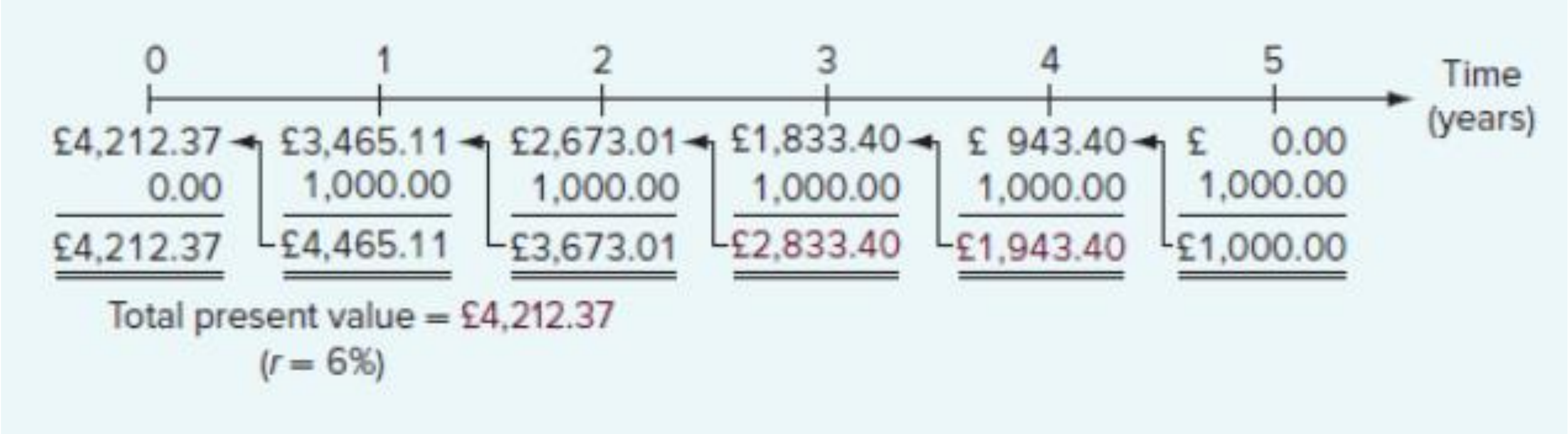
Suppose we had an investment that was going to pay £1,000 at the end of every year for the next 5 years. Interest rate is 6%. What is the PV of these cash flows?

- PV calculated by discounting each cash flow separately:



Example 3 (2)

PV calculated by discounting back one period at a time:



NPV of multiple cash flows: The Algebraic formula

$$NPV = -C_0 + \frac{C_1}{1+r} + \frac{C_2}{(1+r)^2} + \cdots + \frac{C_t}{(1+r)^t}$$

$$= -C_0 + \sum_{i=1}^{t} \frac{C_i}{(1+r)^i}$$

Compounding periods

Sometimes interest is charged more frequently than once per year

Semi-Annually (2 times a year)	Quarterly (4 times a year)	Monthly (12 times a year)	Weekly (52 times a year	Daily (365 times a year)	Continuous

Compounding more than once a year over many years

$$FV = C_0 \times (1 + \frac{r}{m})^{mt}$$

where

- C_0 is the initial investment
- *r* is the **stated annual interest rate** (the annual interest rate without consideration of compounding) – also called quoted interest rate, nominal interest rate
- *m* is the number of times the investment is compounded in a year
- *t* is the number of years

Example 4

Mr. Bean is investing £5,000 at a stated annual interest rate of 12% per year, compounded quarterly, for five years. What is his wealth at the end of five years?

$$5{,}000 \times (1 + \frac{0.12}{4})^{4\times 5} = 5{,}000 \times 1.03^{20} = 9{,}030.5$$

Continuous compounding

$$FV = C_0 \times e^{rt}$$

Simplifications

Perpetuity

- A constant stream of cash flows that never ends

Growing Perpetuity

- A stream of cash flows that grow at a constant rate infinitely

Annuity

- A constant stream of cash flows that last for a fixed number of periods

Growing Annuity

- A stream of cash flows that grow at a constant rate for a fixed number of periods

Perpetuity and Growing Perpetuity

- Perpetuity

$$PV = \frac{C}{r}$$

- Growing perpetuity

$$PV = \frac{C}{r - g}$$

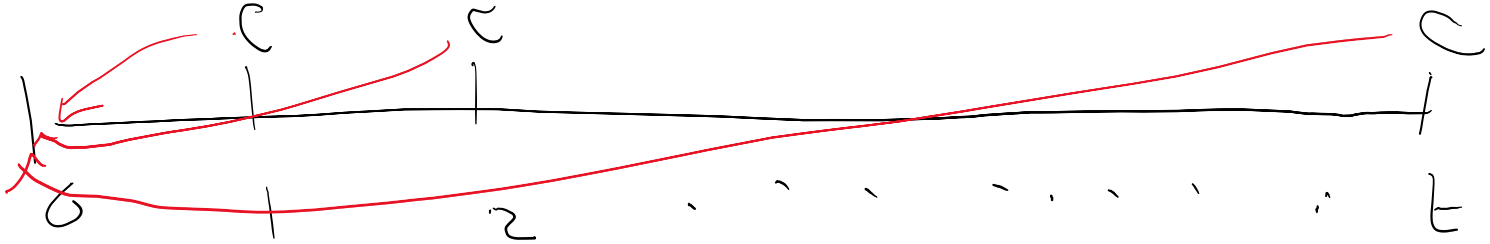
where g is the growth rate (assume g<r)

Annuity

$$PV = C \times \left[\frac{1}{r} - \frac{1}{r \times (1+r)^t}\right] = \frac{C}{r} \times \left[1 - \frac{1}{(1+r)^t}\right]$$

$$FV = C \times \left[\frac{(1+r)^t - 1}{r}\right] = \frac{C}{r} \times [(1+r)^t - 1]$$

Proof



$$PV = \frac{C}{1+r} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \cdots + \frac{C}{(1+r)^t}$$

$$PV \times (1+r) = C + \frac{C}{1+r} + \frac{C}{(1+r)^2} + \cdots + \frac{C}{(1+r)^{t-1}} = C + PV - \frac{C}{(1+r)^t}$$

$$PV \times r = C \times \left[1 - \frac{1}{(1+r)^t}\right]$$

$$PV = \frac{C}{r} \times \left[1 - \frac{1}{(1+r)^t}\right]$$

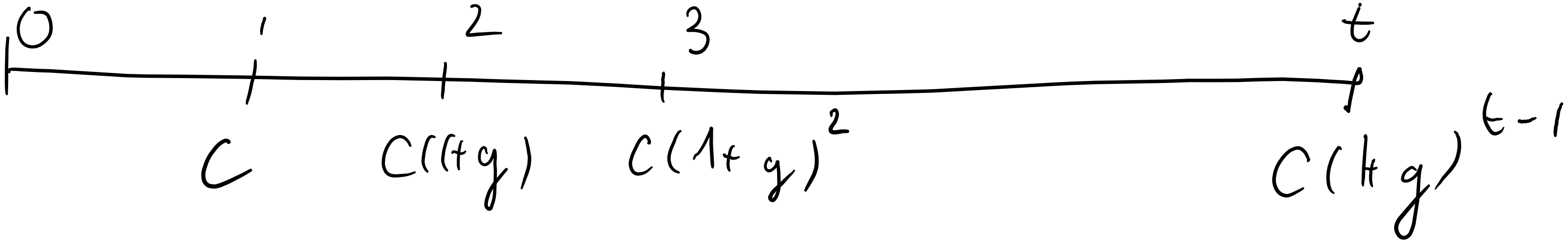
Growing Annuity

$$PV = \frac{C}{r-g} \times \left[1 - (\frac{1+g}{1+r})^t\right]$$

$$FV = \frac{C}{r-g} \times [(1+r)^t - (1+g)^t]$$

Assume g<r

Proof



$$PV = \frac{C}{1+r} + \frac{C(1+g)}{(1+r)^2} + \frac{C(1+g)^2}{(1+r)^3} + \cdots + \frac{C(1+g)^{t-1}}{(1+r)^t}$$

$$PV \times \frac{1+r}{1+g} = \frac{C}{1+g} + \frac{C}{1+r} + \frac{C(1+g)}{(1+r)^2} + \cdots + \frac{C(1+g)^{t-2}}{(1+r)^{t-1}} = \frac{C}{1+g} + PV - \frac{C(1+g)^{t-1}}{(1+r)^t}$$

$$PV \times (\frac{1+r}{1+g} - 1) = C \times \left[\frac{1}{1+g} - \frac{(1+g)^{t-1}}{(1+r)^t}\right]$$

$$PV \times (r-g) = C \times \left[1 - \frac{(1+g)^t}{(1+r)^t}\right]$$

$$PV = \frac{C}{r-g} \times \left[1 - (\frac{1+g}{1+r})^t\right]$$

Week 5

Net Present Value and Other Investment Rules

Lecture outline

- The NPV rule
- The payback rule
- The discounted payback
- The average accounting return
- The internal rate of return
- The profitability index
- The incremental NPV

Learning outcomes

- Be able to perform various analyses to make investment decisions
- Be able to perform discounted cash flow analysis to evaluate a project
- Gain a basic understanding of other considerations when doing capital budgeting

Recall Week 1

Investment	Financing	Liquidity
• **Choose the best projects** • **Capital budgeting**	• Choose sources of financing for investment • Capital structure	• Ensure you have enough cash and inventory • Short-term financial planning

In the news

CFO JOURNAL

Companies Review Capital Budgets for Savings in Uncertain Economy

Chief financial officers are looking for potential cuts after a recent increase in capital spending

CFO JOURNAL

Companies' Capital Spending Forecast to Slow in 2023 Amid Recession Fears

The prospect of a downturn is making companies more cautious about overextending themselves as they face potential revenue declines

- What is a project that is worth investing in?
    - If it creates value for the firm’s owners
- How do we know if it will create value?

The NPV rule

- Use the discounted cash flow valuation to calculate NPV
- An investment should be accepted if the NPV is positive and rejected if it is negative

Strengths of the NPV rule

Uses Cash Flows	Uses all Cash Flows	Discounts Cash Flows
• Cash flows are better than earnings	• Other approaches ignore cash flows beyond a certain date	• Fully incorporates the time value of money

The payback rule

- Payback period is the amount of time required for an investment to generate cash flows sufficient to recover its initial cost (the time it takes to break even in <u>an accounting sense</u>)
- An investment is acceptable if its calculated payback period is less than some <u>pre-specified</u> number of years

Example 1

Below is the expected cash flows for 5 projects. Using the payback rule to determine which one we should invest in.

Year	A	B	C	D	E
0	-100	-200	-200	-200	-50
1	30	40	40	100	100
2	40	20	20	100	-50,000,000
3	50	10	10	-200	
4	60		130	200	

After 2 years, we recover 70 from project A, so we will collect the remaining 30 sometime in year 3

$$Payback\ period\ A = 2 + \frac{30}{50} = 2.6\ years$$

Project D has 2 payback periods: 2 years and 4 years

Project E has a payback of 0.5 years but is unrealistic

Strengths of the payback rule

- Simple
- Applicable to
  - Very small-scale investments
  - Firms with severe capital rationing

Shortcomings of the payback rule

- *Ignore the time value of money*
- Ignore risks: very risky and very safe projects are calculated in the same way
- The pre-defined cut-off period is arbitrarily chosen

The discounted payback rule

- Discounted payback period is the length of time required for an investment's discounted cash flows to equal its initial cost (the time it takes to break even in an <u>economic or financial sense</u>)
- An investment is acceptable if its discounted payback is less than some pre-specified number of years

Example 2

Suppose we require a 12.5% return on new investments. We have an investment that costs £300 and has cash flows of £100 per year for five years. Calculate the payback period and discounted payback period.

	CF		Accumulated CF	
Year	Ordinary	Discounted	Ordinary	Discounted
1	100	89	100	89
2	100	79	200	168
3	100	70	300	238
4	100	62	400	300
5	100	55	500	355

The average accounting return (AAR)

$$AAR = \frac{Average\ net\ income}{Average\ book\ value}$$

- A project is acceptable if its average accounting return exceeds a target average accounting return
- Shortcomings:
  - Not a true rate of return; ignore time value of money
  - Arbitrary benchmark
  - Based on accounting (book) values, not cash flows and market values

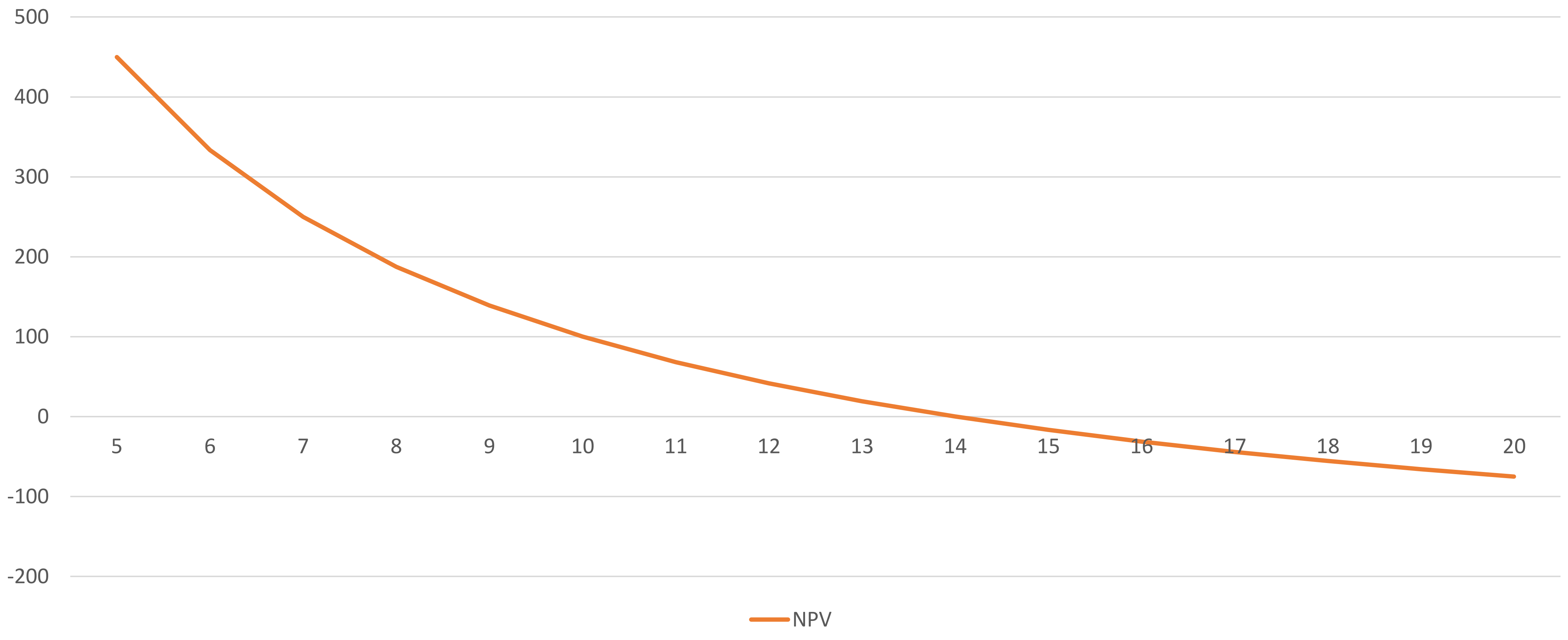
The internal rate of return (IRR)

- IRR is the discount rate that makes the NPV of an investment zero
- An investment is acceptable if the IRR exceeds the required return and rejected otherwise
- NPV profile: a graphical representation of the relationship between an investment NPVs and various discount rates. IRR is where the curve intersects with the y-axis

Example 3

Consider a project that requires -$250 mil initial investment and generates $35 mil each year forever. What is the project's IRR? What is its NPV profile? If the required return is 10%, will you invest in this project?

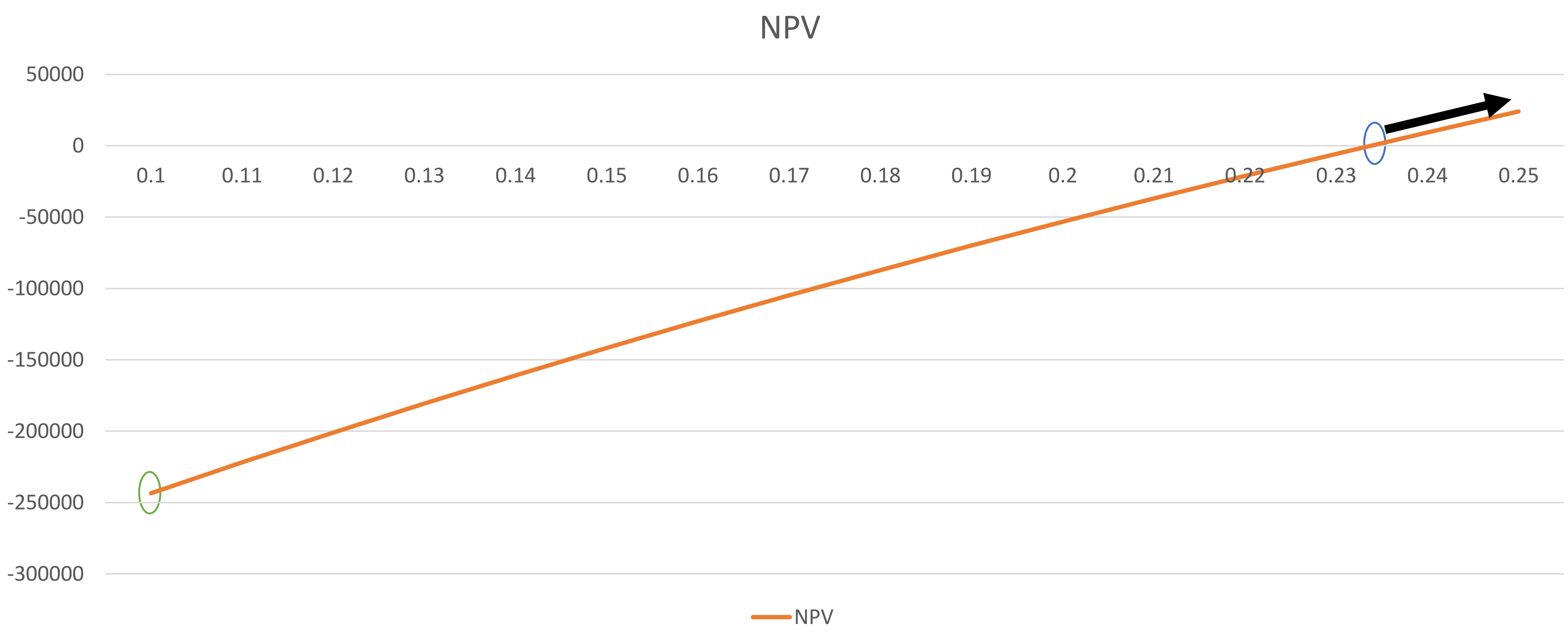
$$-250 + \frac{35}{IRR} = 0, so\ IRR = 14\%$$



500
400
300
200
100
0
-100
-200
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
NPV

Case study IRR (1)

Mrs. Brown has been offered a publishing deal of £1 mil upfront if she agrees to write a book about her kids. She estimates that it will take her 3 years to write the book and during that time, she will have to forgo alternative sources of income amounting to £500,000 per year. The estimated opportunity cost of capital is 10%. According to the IRR rule, should she accept this offer? Is it really a good deal?

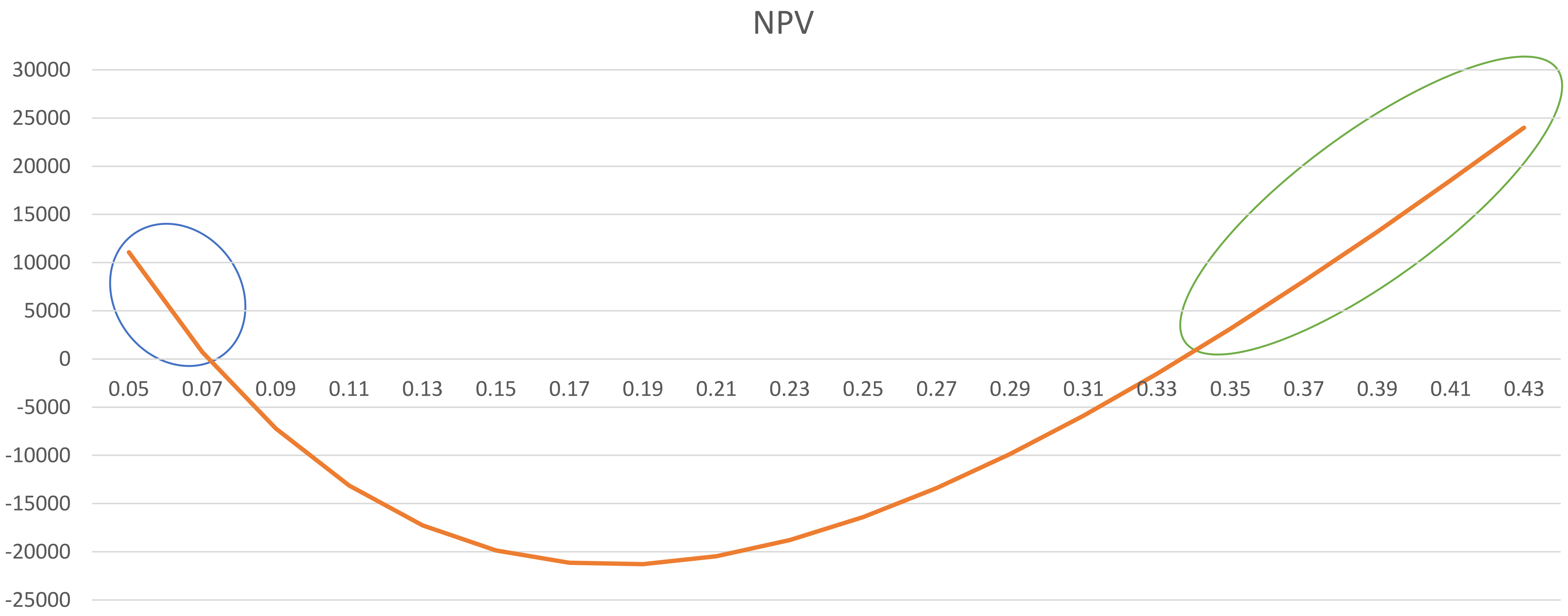


NPV
50000
0
-50000
-100000
-150000
-200000
-250000
-300000
0.1
0.11
0.12
0.13
0.14
0.15
0.16
0.17
0.18
0.19
0.2
0.21
0.22
0.23
0.24
0.25
NPV

Case study IRR (2)

Mrs. Brown bargains to get a better offer. In response, the publisher offers her a smaller up-front payment of £550,000 but will give her £1 mil when the book is published and sold in 4 years from now. Should Mrs. Brown accept this offer, according to the IRR rule?

IRR rule could lead to misleading results if the CFs are unconventional!



Shortcomings of IRR

- Difficult to compute and might mislead in case of non-conventional cash flows
- Might give a different answer compared to NPV in case of mutually exclusive investments

The profitability index (PI)

$$PI\ (benefit - cost\ ratio) = \frac{PV\ of\ future\ CFs}{Initial\ cost}$$

- Shortcomings:
  - May lead to incorrect decisions in comparisons of mutually exclusive investments
- Advantages:
  - Useful in case of capital rationing (next lecture) or more generally, limited resources

- Making capital investment decisions is not just about investing in one specific projects
  - Capital is scarce: investing in one project might lead to cutting budget for others
- In the next part, we will perform a detailed discounted (incremental) cash flow analysis to assess whether a project should be undertaken

Relevant cash flows

- The incremental cash flows for project evaluation consist of any changes in the firm's future cash flows that are a <u>direct consequence</u> of taking the project
- Thus, any cash flow that exists regardless of whether or not a project is undertaken is not relevant

Some basic rules (1)

- Sunk costs:
  - Cash flows that have already occurred
  - Ignore all sunk costs
- Opportunity costs
  - Lost revenues that you forgo because of making the proposed investment
  - Incorporate opportunity costs into your analysis
- Depreciation:
  - Straight-line depreciation: the asset’s cost (less any expected salvage value) is divided equally over its estimated useful life

Some basic rules (2)

- Side effects: either **erosion** or **synergy**
  - Erosion is when a new product reduces the cash flows of existing products
  - Synergy occurs when a new project increases the cash flows of existing projects
  - Include side effects in the analysis
- Allocated cost
  - An accounting measure to reflect expenditure or an asset’s use across the whole company
  - Should be viewed as a cash outflow only if it is an incremental cost of the project
- Interest paid (or financial costs in general) is NOT a relevant cash flow for project evaluation

Operating cash flow (OCF)

$$OCF = Net\ income + Depreciation - Increase\ (or + Decrease)\ in\ net\ working\ capital$$

If we ignore net working capital:

- Top-down approach:

$$OCF = Sales - Cash\ costs\ - Taxes$$

- Bottom-up approach:

$$OCF = (Sales - Cash\ costs\ - Taxes)(1 - t_C) + Depreciation$$

- Tax shield approach:

$$OCF = (Sales - Cash\ costs)(1 - t_C) + Depreciation \times t_C$$

Other considerations

- Inflation:
  - Nominal interest rate vs. real interest rate

$$Real\ IR = \frac{1 + Nominal\ IR}{1 + Inflation} - 1$$

$$Real\ IR \cong Nominal\ IR - Inflation$$

  - Nominal CF vs. real CF

Let's get started with a simple case

- Suppose we think we can sell 50,000 cans of shark attractant per year at a price of £4 per can.
- It costs us about £2.50 per can to make the attractant, and a new product such as this one typically has only a 3-year life.
- We require a 20% return on new products.
- Fixed costs for the project, including such things as rent on the production facility, will run to £12,000 per year.
- We shall need to invest a total of £90,000 in manufacturing equipment which will be 100% depreciated straight-line over the 3-year life of the project. The cost of removing the equipment will roughly equal its actual value in 3 years, so it will be essentially worthless on a market value basis as well.
- The project will require £20,000 investment in net working capital in years 0-2 and no working capital in year 3, and the tax rate is 34%.

Projected income statement in each year during years 1-3

Sales	50,000 × 4 = 200,000
Variable costs	50,000 × 2.5 = 125,000
Gross profit	75,000
Fixed costs	12,000
Depreciation	90,000 ÷ 3 = 30,000
Profit before tax	33,000
Tax	33,000 × 0.34 = 11,220
Net income	21,780

Consider incremental earnings: changes in earnings as a result of the project/investment

Changes in NWC

	Year			
	0	1	2	3
Net working capital	20,000	20,000	20,000	0
ΔNWC	+20,000	0	0	-20,000

Capital spending (or capital expenditure) = money paid to acquire/upgrade fixed, physical or non-consumable assets

Free cash flow (Total cash flow)

	Year			
	0	1	2	3
Net income		21,780	21,780	21,780
+ Depreciation		30,000	30,000	30,000
- Increase (+ Decrease) in NWC	-20,000			20,000
Operating cash flow	-20,000	51,780	51,780	71,780
- Capital spending	-90,000			
Total CF	-110,000	51,780	51,780	71,780

$$NPV = -110{,}000 + \frac{51{,}780}{1.2} + \frac{51{,}780}{1.2^2} + \frac{71{,}780}{1.2^3} = 10{,}648$$

Using IRR function in Excel, we can find the IRR of 25.8%

Case study 2 (Difficulty level: High)

See the attached pdf file on VLE

- In each year during years 1-4:

$$\text{Sales} = 260 \times 100{,}000 - 100 \times 100{,}000 \times 25\% = 23.5 \text{ mil}$$

$$\text{Variables costs} = 110 \times 100{,}000 - 60 \times 100{,}000 \times 25\% = 9.5 \text{ mil}$$

$$\text{Fixed costs} = 2{,}800{,}000 + 200{,}000 = 3 \text{ mil}$$

$$\text{Depreciation} = 7{,}500{,}000/5 = 1.5 \text{ mil}$$

- Year 0:

$$\text{Fixed costs} = 5{,}000{,}000 + 50 \times 200{,}000 = 15 \text{ mil}$$

- Year 5:

$$\text{Depreciation} = 1.5 \text{ mil}$$

Projected income statement (millions)

	Year					
	0	1	2	3	4	5
Sales		23.5	23.5	23.5	23.5	
Variable costs		9.5	9.5	9.5	9.5	
Gross profit		14	14	14	14	
Fixed costs	15	3	3	3	3	
Depreciation		1.5	1.5	1.5	1.5	1.5
EBIT	-15	9.5	9.5	9.5	9.5	-1.5
Taxes@20%	-3	1.9	1.9	1.9	1.9	-0.3
Net income	-12	7.6	7.6	7.6	7.6	-1.2

Changes in NWC

	Year					
	0	1	2	3	4	5
NWC	0	2.1	2.1	2.1	2.1	0
ΔNWC	0	+2.1	0	0	0	-2.1

Free cash flow (millions)

	Year					
	0	1	2	3	4	5
Net income	-12	7.6	7.6	7.6	7.6	-1.2
+Depreciation		1.5	1.5	1.5	1.5	1.5
-ΔNWC		-2.1				2.1
Operating CF	-12	7	9.1	9.1	9.1	2.4
-Capital spending	-7.5					
Free CF	-19.5	7	9.1	9.1	9.1	2.4

$$NPV = -19.5 + \frac{7}{1.12} + \frac{9.1}{1.12^2} + \frac{9.1}{1.12^3} + \frac{9.1}{1.12^4} + \frac{2.4}{1.12^5} = 7.63\ mil$$
$$IRR = 27.9\%$$

Week 6

Risk Analysis for Capital Budgeting

Lecture outline

- Worst-best scenarios
- Sensitivity analysis
- Break-even analysis
- Other considerations

Learning outcomes

- Be able to perform scenario analysis
- Be able to perform break-even analysis
- Gain a basic understanding of other issues in capital budgeting

Scenario analysis

The determination of what happens to NPV estimates when we ask what-if questions

Case study 1

Assume we are considering a new investment which costs €200,000, has a 5-year life and has no salvage value. For simplicity, depreciation is straight-line to 0. The required return is 12%, and the tax rate is 34%. In addition, we have compiled the following information:

	Base case	Lower bound	Upper bound
Unit sales	6,000	5,500	6,500
Price per unit	80	75	85
Variable costs per unit	60	58	62
Fixed costs per year	50,000	45,000	55,000

Base case NPV

Sales	6,000 × 80 = 480,000
Variable costs	6,000 × 60 = 360,000
Fixed costs	50,000
Depreciation	200,000 ÷ 5 = 40,000
Profit before tax	30,000
Taxes	30,000 × 0.34 = 10,200
Net income	19,800

Operating CF per year:

$$19{,}800 + 40{,}000 = 59{,}800$$

NPV (base case):

$$-200{,}000 + 59{,}800 \times \left[\frac{1}{0.12} - \frac{1}{0.12 \times 1.12^5}\right] = 15{,}567$$

Best case vs. worst case

	Base case	Worst case	Best case
Unit sales	6,000	5,500	6,500
Price per unit	80	75	85
Variable costs per unit	60	62	58
Fixed costs per year	50,000	55,000	45,000

Worst case NPV

Sales	5,500 × 75 = 412,500
Variable costs	5,500 × 62 = 341,000
Fixed costs	55,000
Depreciation	200,000 ÷ 5 = 40,000
Profit before tax	-24,000
Taxes	-24,000 × 0.34 = -7,990
Net income	-15,510

Operating CF per year:

$$-15{,}510 + 40{,}000 = 24{,}490$$

NPV (worst case):

$$-200{,}000 + 24{,}490 \times \left[\frac{1}{0.12} - \frac{1}{0.12 \times 1.12^5}\right] = -111{,}719$$

Best case NPV

Sales	6,500 × 85 = 552,500
Variable costs	6,500 × 58 = 377,000
Fixed costs	45,000
Depreciation	200,000 ÷ 5 = 40,000
Profit before tax	90,500
Taxes	90,500 × 0.34 = 30,770
Net income	59,730

Operating CF per year:

$$59{,}730 + 40{,}000 = 99{,}730$$

NPV (worst case):

$$-200{,}000 + 99{,}730 \times \left[\frac{1}{0.12} - \frac{1}{0.12 \times 1.12^5}\right] = 159{,}507$$

In comparison

Scenario	Net income	CF	NPV	IRR (%)
Base	19,800	59,800	15,567	15.1
Worst	-15,510	24,490	-111,719	-14.4
Best	59,730	99,730	159,507	40.9

- However, using the worst-best scenarios might be misleading/not enough
- Instead, we could go with several optimistic and pessimistic cases

Sensitivity analysis

- Investigation of what happens to NPV when only one variable is changed

Case study 1 (cont.)

- Back to our case study
- Repeat the analysis when we only change the unit sales

	Base case	Worst case	Best case
Unit sales	6,000	5,500	6,500
Price per unit	80	80	80
Variable costs per unit	60	60	60
Fixed costs per year	50,000	50,000	50,000

Case study 1 (cont.)

- Back to our case study
- Repeat the analysis when we only change the fixed costs

	Base case	Worst case	Best case
Unit sales	6,000	6,000	6,000
Price per unit	80	80	80
Variable costs per unit	60	60	60
Fixed costs per year	50,000	55,000	45,000

Only change the unit sales

Scenario	Base	Worst	Best
Net income	19,800	13,200	26,400
CF	59,800	53,200	66,400
NPV	15,567	-8,225	39,359
IRR	15.1%	10.3%	19.7%

Only change the fixed costs

Scenario	Base	Worst	Best
Net income	19,800	16,500	23,100
CF	59,800	56,500	63,100
NPV	15,567	3,671	27,463
IRR	15.1%	12.7%	17.4%

Break-even analysis

- Variable costs
  - Costs that change when the quantity of output changes
  - $VC = Quantity\ of\ output\ \times Cost\ per\ unit\ of\ output$
- Fixed costs
  - Costs that do not change when the quantity of output changes during a particular time period
- Total costs = variable costs + fixed costs
- Marginal (incremental) cost: the change in costs that occurs when there is a small change in output
- Marginal (incremental) revenue: the change in revenue that occurs when there is a small change in output

Accounting break-even

- The sales level that results in zero project net income

Case study 2

Suppose we retail USB flash drives for £5 apiece. We can buy drives from a wholesale supplier for £3 apiece. We have accounting expenses of £600 in fixed costs and £300 in depreciation. How many drives do we have to sell to break even – that is, for net income to be zero?

Sales	Q × 5
Variables costs	Q × 3
Fixed costs	600
Depreciation	300
Profit before tax	2Q – 900
Taxes	(2Q – 900) × 0.28
Net income	(2Q – 900) × 0.72

Set (2Q – 900) × 0.72 equal to 0 and solve the equation, we can find that the quantity of sales needed to break even is 450.

Why accounting break even?

- Relatively easy to calculate
- Help reduce forecasting risk
- A project that does not break even in an accounting sense reduces total earnings
- A project that just breaks even on an accounting basis loses money in a financial or opportunity cost sense

More generally…

- If ignoring taxes, operating cash flow (OCF) can be written as EBIT plus depreciation:

$$OCF = [(P - v) \times Q - FC - D] + D = (P - v) \times Q - FC$$

$$Q = \frac{OCF + FC}{P - v}$$

- *Q* is the sales volume needed to achieve any given *OCF*

Other break-even measures

- Cash break-even: the sales level that results in a zero operating cash flow
- Financial break-even: the sales level that results in a zero NPV

Operating leverage

- Operating leverage is the degree to which a firm or project relies on fixed costs
- Degree of operating leverage (DOL): the percentage change in operating cash flow relative to the percentage change in quantity sold

$$DOL = 1 + \frac{FC}{OCF}$$

Implications of operating leverage

- Fixed costs act as a lever in the sense that a small percentage change in operating revenue can be magnified into a large percentage change in operating cash flow and NPV
- The higher the degree of operating leverage, the greater is the potential danger from forecasting risk
- From a managerial perspective, one way of coping with highly uncertain projects is to keep the degree of operating leverage as low as possible. This will generally have the effect of maintaining the break-even point (however measured) at its minimum level

Capital rationing

- Capital rationing refers to the situation that exists if a firm has positive-NPV projects but cannot find the necessary financing
- Soft rationing refers to the situation that occurs when units in a business are allocated a certain amount of financing for capital budgeting
- Hard rationing refers to the situation that occurs when a business cannot raise financing for a project under any circumstances

Example

Consider the following investment opportunities:

Project	Cash flows (£mil)		
	Year 1	Year 2	Year 3
A	-20	70	10
B	-10	15	40
C	-10	-5	60

The required rate of return is 12%. What would be the investment decision if there is no capital rationing? How would the decision change if the firm applies capital rationing by restricting the total fund for investments to £20 mil?

Project	Cash flows (£mil)			NPV	Rank (NPV)
	Year 1	Year 2	Year 3		
A	-20	70	10	50.47	1
B	-10	15	40	35.28	2
C	-10	-5	60	33.37	3

No capital rationing: invest in all 3 projects

Project	Cash flows (£mil)			NPV	Rank (NPV)
	Year 1	Year 2	Year 3		
A	-20	70	10	50.47	1
B	-10	15	40	35.28	2
C	-10	-5	60	33.37	3

With capital rationing: If we rely on NPV to decide, we have to forgo B and C (i.e., invest in A only)

Project	Cash flows (£mil)			NPV	Rank (NPV)	PI	Rank (PI)
	Year 1	Year 2	Year 3				
A	-20	70	10	50.47	1	3.52	3
B	-10	15	40	35.28	2	4.53	1
C	-10	-5	60	33.37	3	4.34	2

With capital rationing: If we rely on PI to decide, we should invest in B and C (i.e., forgo A)

Advantages of capital rationing

- Ensure a budget is followed
- Optimal utilization of resources
- More effective and efficient project management

Disadvantages of capital rationing

- Applying capital rationing = focusing on short-term rather than long-term growth

Weeks 7&8

Capital structure policy

Lecture outline

- Cost of capital: the basics
- Raising capital
- Capital structure
  - The Modigliani-Miller (M&M) propositions
  - Static theory
  - Pecking order theory

Learning outcomes

- Be able to understand the key differences between debt financing and equity financing
- Be able to understand the M&M propositions with/without taxes, with/without bankruptcy costs
- Gain a basic understanding of the static theory and pecking order theory and applying them in choosing capital structure

Basic concepts

- A firm can raise funds by issuing both debt and equity securities - this mix is known as the capital structure
  - How should the financial manager choose the right combination between debt and equity to maximize the firm value?
  - Is equity better than debt, or vice versa?

Cost of capital

- Cost of equity: the return that equity investors require on their investment in the firm
  - Ordinary equity: equity without priority for dividends or in bankruptcy
  - Preference shares: equity with dividend priority over ordinary shares, normally with a fixed dividend rate, sometimes without voting rights
- Cost of debt: the return that lenders require on the firm’s debt
  - Borrowing from banks
  - Issuing corporate bonds

Cost of equity: Constant dividend growth approach

$$R_E = \frac{D_1}{P_0} + g$$

where:

R_E: Cost of equity

D_1: next period's projected dividend

P_0: share price

g: dividend's constant growth rate

Cost of equity: The security market line (SML) approach

$$R_E = R_f + \beta_E \times (R_M - R_f)$$

where:

R_E: Cost of equity

R_f: risk-free rate (frequently used proxy: returns on short-term government bonds)

R_M: expected return on the overall market (frequently used proxy: returns on FTSE 100 index, returns on S&P 500 index)

β_E: systematic risk of the equity

Cost of preference shares

$$R_P = \frac{D}{P_0}$$

where D is the fixed dividend and P_0 is the current preference share price

Weighted average cost of capital (WACC)

- WACC: the weighted average of the cost of equity and the after-tax cost of debt, also can be interpreted as overall return on assets
- If firm does not use preference shares

$$WACC = \frac{E}{V} \times R_E + \frac{D}{V} \times R_D \times (1 - t_C)$$

where E is the market value of equity; D is the market value of debt; V is total market value of debt and equity (V=E+D); R_E is cost of equity; R_D is cost of debt; t_C is tax rate.

- If firm uses preference shares

$$WACC = \frac{E}{V} \times R_E + \frac{P}{V} \times R_P + \frac{D}{V} \times R_D \times (1 - t_C)$$

You have a brilliant idea for a new financial product targeting the elderlies. Where should you start to create your business?

Early-stage financing and venture capital

- Go to private equity and venture capital (VC) market
  - VC: financing for new, often high-risk, ventures
- A large amount of private equity investment is undertaken by professional private equity managers representing large institutional investors such as mutual funds and pension funds

Stages of financing

- Seed money:
  - A small amount of financing needed to prove a concept or develop a product. Marketing is not included in this stage.
- Start-up:
  - Financing for firms that started within the past year. Funds are likely to pay for marketing and product development expenditures.
- Later stage capital:
  - Additional money to begin sales and manufacturing after a firm has spent its start-up funds.
- Growth capital:
  - Funds earmarked for a firm to enable it to reach its potential and achieve successful growth.
- Replacement capital:
  - Financing for a company to buy out other investors in the firm.
- Buyout financing:
  - Money provided for managers and outside investors to acquire a fully functioning firm.

Going public

SEPTEMBER 14 2023

Arm Ltd

Arm shares jump by 25% on first day of trading

SoftBank-backed chip designer's stock climbed as high as $63.59 in early trading as valuation exceeds $65bn

Unhedged podcast 17 min listen

Three IPOs test the market

Arm, Instacart and Birkenstock are going public in the coming weeks. How will the market respond?

Source: Financial Times (2023)

The basic procedure

[IPO process explained]

Alternative issue methods

Method	Type	Definition
Public traditional negotiated cash offer	Firm commitment cash offer	The company negotiates an agreement with an investment firm or bank to underwrite and distribute the new shares. A specified number of shares are bought by underwriters and sold at a higher price.
	Best efforts cash offer	The company has an investment firm or bank sell as many of the new shares as possible at the agreed-upon price. There is no guarantee concerning how much cash will be raised.
	Dutch auction cash offer	The company has an investment firm or bank auction shares to determine the highest offer price obtainable for a given number of shares to be sold.
Privileged subscription	Direct rights issue	The company offers the new equity directly to its existing shareholders.
	Standby rights issue	Like the direct rights issue, this contains a privileged subscription arrangement with existing shareholders. The net proceeds are guaranteed by the underwriters.
Non-traditional cash offer	Shelf cash offer	Qualifying companies can authorize all shares they expect to sell over a two-year period and sell them when needed.
	Competitive firm cash offer	The company can elect to award the underwriting contract through a public auction instead of negotiation.
Private	Direct placement	Securities are sold directly to the purchaser, who, at least until recently, generally could not resell securities for at least two years.

The capital structure question

Now that your firm is up and running. How should you choose the "right" debt-equity ratio (i.e., the "optimal" capital structure)?

- How does financial leverage affect ROE/EPS?
- How does financial leverage affect the cost of capital/cost of equity?

Debt vs. Equity

- Interest is tax deductible → lowers the effective cost of debt
- Debt holders are limited to a fixed return → stockholders do not have to share profits if the business does exceptionally well
- Debt holders do not have voting rights
- Higher debt ratios lead to greater risk and higher required interest rates (to compensate for the additional risk)

Maximizing firm value vs. maximizing shareholders' interests

The market value of J. J. Sprint plc is £1,000. The company currently has no debt, and each of J. J. Sprint's 100 shares sells for £10. Suppose that J. J. Sprint plans to borrow £500 and pay the £500 proceeds to shareholders as an extra cash dividend of £5 per share, which leads to 3 possible outcomes below. In which scenario should the firm restructure?

	No debt (original capital structure) £	Value of debt plus equity after payment of dividend (three possibilities) (a) £	(b) £	(c) £
Debt	0	500	500	500
Equity	1,000	750	500	250
Firm value	1,000	1,250	1,000	750

Maximizing firm value vs. maximizing shareholders’ interests

	Pay-offs to shareholders after restructuring		
	(a)	(b)	(c)
Equity value change (Capital gain)	-250	-500	-750
Dividends	500	500	500
Net effect	+250	0	-250

- Changes in capital structure benefit the shareholders **if and only if** the value of the firm increases
- Optimal capital structure
  - maximize firm value
  - lowest possible WACC

Financial leverage and EPS: Example (1)

	Current	Proposed
Assets (£)	8,000,000	8,000,000
Debt (£)	0	4,000,000
Equity (£)	8,000,000	4,000,000
Debt-equity ratio	0	1
Share price (£)	20	20
Shares outstanding	400,000	200,000
Interest rate (10%)	10	10

Financial leverage magnifies losses and gains

Financial leverage and EPS: Example (2)

	Unlevered firm (no debt)		
	Recession	Expected	Expansion
EBT (£)	500,000	1,000,000	1,500,000
Interest (£)	0	0	0
Net income (£)	500,000	1,000,000	1,500,000
ROE (%)	6.25	12.5	18.75
EPS (£)	1.25	2.5	3.75

	Levered firm (use debt)		
	Recession	Expected	Expansion
EBT (£)	500,000	1,000,000	1,500,000
Interest (£)	-400,000	-400,000	-400,000
Net income (£)	100,000	600,000	1,100,000
ROE (%)	2.50	15.00	27.50
EPS (£)	0.50	3.00	5.50

Financial leverage and firm value: Example (3)

With-debt line is steeper: EPS is more sensitive to changes in EBIT because of financial leverage

Advantage to debt

Disadvantage to debt

What do we learn from the example?

- The effect of financial leverage depends on the company's EBIT
  - When EBIT is relatively high, leverage is beneficial
- Leverage:
  - magnifies returns (losses) to shareholders relative to no leverage case
  - Shareholders are exposed to more risk because the EPS and ROE are much more sensitive to changes in EBIT
- Because of the impact that financial leverage has on both the expected return to shareholders and the riskiness of the equity, capital structure is an important consideration

Does conclusion #4 necessarily follow?

- NO

Corporate borrowing and homemade leverage (1)

Assume you have a chance to invest in the firm discussed in the earlier example and you can choose 1 of the following strategies:

- Strategy A: buy 100 shares of the levered equity
- Strategy B:
  - Borrow £2,000 from a bank
  - Use the borrowed proceeds plus your own investment of £2,000 (a total of £4,000) to buy 200 shares of the current unlevered equity @ £20 per share
- Which strategy to you prefer?

Corporate borrowing and homemade leverage (2)

	Recession	Expected	Expansion
Strategy A:			
EPS of levered equity	0.50	3.00	5.50
Earnings for 100 shares	50	300	550
Net cost = 100 shares × £20/share = £2,000			

Strategy B:			
Earnings for 200 shares	1.25×200=250	2.5×200=500	3.75×200=750
Interest @10% on £2,000	-200	-200	-200
Net earnings	50	300	550
Initial cost = Cost of stock – Amount borrowed = 200 shares × £20/share – £2,000 = £2,000			

So what?

- Capital structure seems to be irrelevant because shareholders can adjust the amount of financial leverage by borrowing and lending on their own
  - Homemade leverage: the use of personal borrowing to change the overall amount of financial leverage to which the individual is exposed

Notation

D = market value of debt in levered firm

E = market value of equity in levered firm

V_U = value of unlevered firm

$V_L = D + E$ = value of levered firm

EBIT = net operating income before interest and tax

r_E= return required on equity in levered firm

r_D = return required on debt in levered firm

WACC = weighted average cost of capital

t_C = rate of corporation tax

The Modigliani-Miller (M&M) propositions

Assumption: perfect capital market

- Investors and firms can trade the same set of securities at competitive market prices equal to the PV of their future cash flows
- There are no taxes, transaction costs, or issuance costs associated with security trading
- A firm’s financing decisions do not change the cash flows generated by its investments, nor do they reveal new information about them

MM1 - Proof (1)

- Consider two firms in the same risk class:
  - one all-equity financed (unlevered) with value V_U
  - the other levered with debt (D) and equity (E) and value V_L
- Now consider two alternative portfolio strategies:
  - Investment 1: Buy 10% of shares in unlevered firm U
  - Investment 2: Buy 10% of equity + 10% of debt of levered firm L

MM1 - Proof (2)

- Investment 1

Initial cost of investment	Expected return
$0.1V_U$	**0.1EBIT**

- Investment 2

	Initial cost of investment	Expected return
Debt	0.1D	$0.1 \times r_D \times D$
Equity	0.1E	$0.1 \times (EBIT - r_D \times D)$
Total	$0.1(D+E)=0.1V_L$	**0.1EBIT**

- Investments 1 and 2 offer the same payoff
- In a perfect capital market, 2 investments with same payoff (and risk) must have the same initial cost: $0.1V_U = 0.1V_L \rightarrow \mathbf{V_U = V_L}$

M&M Proposition I

- In a perfect capital market, the firm's value is *independent* of its capital structure
- In other words, the firm's overall cost of capital, WACC, is *NOT* affected by the capital structure

MM2 - Proof (1)

$$r_E \times \frac{E}{D+E} + r_D \times \frac{D}{D+E} = WACC$$

$$\frac{\cancel{D+E}}{\cancel{E}} \times r_E \times \frac{\cancel{E}}{\cancel{D+E}} + \frac{\cancel{D+E}}{E} \times r_D \times \frac{D}{\cancel{D+E}} = \frac{D+E}{E} \times WACC$$

$$r_E + \frac{D}{E} \times r_D = \frac{D+E}{E} \times WACC$$

$$r_E = WACC + \frac{D}{E}(WACC - r_D)$$

MM2 - Proof (2)

- When the firm has only equity:

$$WACC = r_U \ (1)$$

According to MM1, WACC is not affected by capital structure, so that WACC in (1) = WACC in (2)

- When the firm has both equity and debt:

$$r_E = WACC + \frac{D}{E}(WACC - r_D)(2)$$

Additional risk due to leverage

$$r_E = r_U + \frac{D}{E}(r_U - r_D)$$

Cost of capital without leverage = Return on assets

M&M Proposition II

- The cost of equity increases with the firm's debt-equity ratio
- In other words
  - The expected rate of return on the common stocks of a levered firm increases with the debt/equity ratio

To sum up

- In a perfect capital market:
  - The overall return on firm, or overall return on assets, (WACC) remains constant (i.e., capital structure is irrelevant)
  - The required return on equity (r_E) increases with debt to match the increased risk
    - This is because increasing debt increases the risk exposure of shareholders (recall they are residual claimants in case of default)
    - They will require a higher rate of return for this extra risk

$\Rightarrow$Implication: *In a perfect capital market, any attempt to raise the value of the firm by issuing debt will be exactly offset by the increase in the cost of equity*

Recap

- In a perfect capital market (everyone borrows & lends at the same interest rate, no transaction/bankruptcy costs, no taxes, symmetric information):
  - Capital structure is irrelevant to shareholders because of homemade leverage. That is, if shareholders prefer a capital structure different from the one chosen by the firm, they can add leverage to their own portfolio to achieve the preferred leverage
  - Overall cost of capital, WACC, remains constant and is equal to the overall return on assets
  - Cost of equity increases with the debt-equity ratio

What if the market is not perfect?

Tax and MM1 (1)

	Unlevered	Levered
EBIT	10,000	10,000
D	0	8,000
r_D	0	10%
Interest paid ($r_D \times D$)	0	800
Pre-tax income	10,000	9,200
Tax@35%	3,500	3,220
Net income	6,500	5,980

	Unlevered	Levered
EBIT	10,000	10,000
Tax@35%	3,500	3,220
CF	6,500	6,780

$$CF_L = CF_U + 250$$

$$250 = 800 \times 0.35 = r_D \times D \times tax\ rate \dashrightarrow \textbf{Debt interest tax shield (DITS)}$$

Tax and MM1 (2)

Assume that debt is permanent and constant (i.e., the firm pays a constant interest in perpetuity)

$$PV(DITS) = \frac{t_C \times r_D \times D}{r_D} = t_C \times D$$

$$V_U = \frac{EBIT \times (1 - t_C)}{r_U}$$

$$V_L = V_U + t_C \times D$$

$$WACC = \frac{E}{D+E} \times r_E + (1 - t_C) \times \frac{D}{D+E} \times r_D$$

D increases → the firm's value increases & WACC decreases

Tax and MM2

$$WACC = \frac{E}{D+E} \times r_E + (1 - t_C) \times \frac{D}{D+E} \times r_D$$

$t_C \times \frac{D}{D+E} \times r_D$ is the reduction in WACC due to the interest tax shield

t_C increases → tax advantage of debt increases

$$r_E = r_U + (1 - t_C) \times (r_U - r_D) \times \frac{D}{E}$$

D increases → cost of equity increases

Default and bankruptcy in a perfect market

- Financial distress: when a firm has difficulty meeting its debt obligations
- Default: when a firm fails to make the required interest or principal payments on its debt, or violates a debt covenant
- After the firm defaults, debt holders are given certain rights to the assets of the firm and may even take legal ownership of the firm’s assets through bankruptcy

$\Rightarrow$An important consequence of leverage is the risk of bankruptcy

$\Rightarrow$Equity financing does not carry this risk

Bankruptcy and capital structure

- With perfect capital markets:
  - The risk of bankruptcy is not a disadvantage of debt
  - Bankruptcy simply shifts the ownership of the firm from equity holders to debt holders without changing the total value available to all investors
- In reality:
  - Bankruptcy is often a long and complicated process that imposes both **<u>direct</u>** and **<u>indirect</u>** costs on the firm and its investors

Direct costs of bankruptcy

- The bankruptcy process is complex, time-consuming, and costly
  - Costly outside experts are often hired by the firm to assist with the bankruptcy process
  - Creditors also incur costs during the bankruptcy process
    - They may wait several years to receive payment
    - They may hire their own experts for legal and professional advice

$\Rightarrow$The direct costs of bankruptcy reduce the value of assets that the firm's investors will ultimately receive

Indirect costs of bankruptcy

- While the indirect costs are difficult to measure accurately, they are often much larger than the direct costs of bankruptcy
  - Loss of customers
  - Loss of suppliers
  - Loss of employees
  - Fire sale of assets
  - etc

British retailer Arcadia Group, owner of Topshop, files for bankruptcy.

Debenhams set to shut shop after 242 years as pandemic hammers UK retail

By James Davey

4 MIN READ

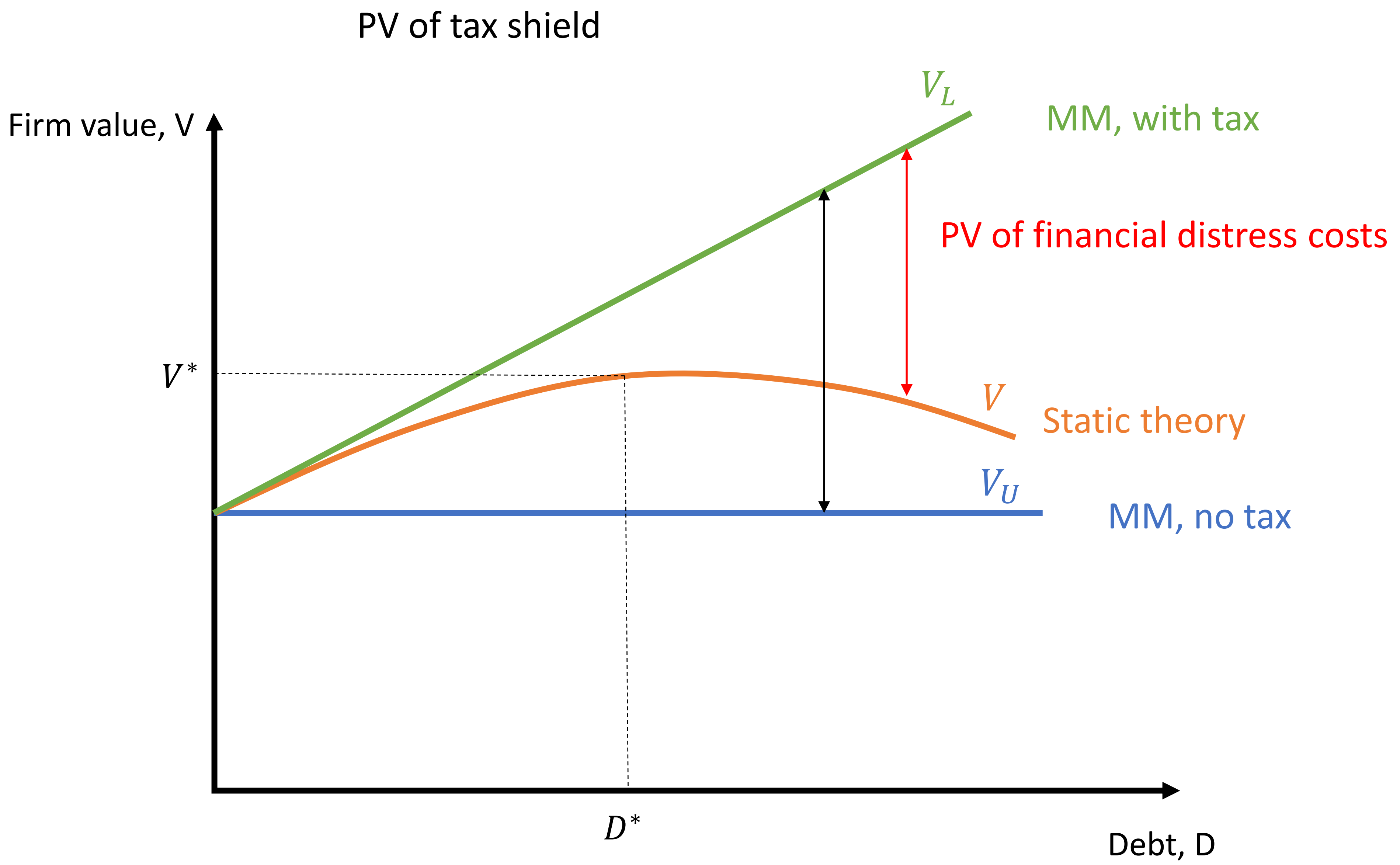
LONDON (Reuters) - British department store group Debenhams is set to close all its UK shops after 242 years in business, putting 12,000 jobs at risk in the country's second major corporate failure in as many days.

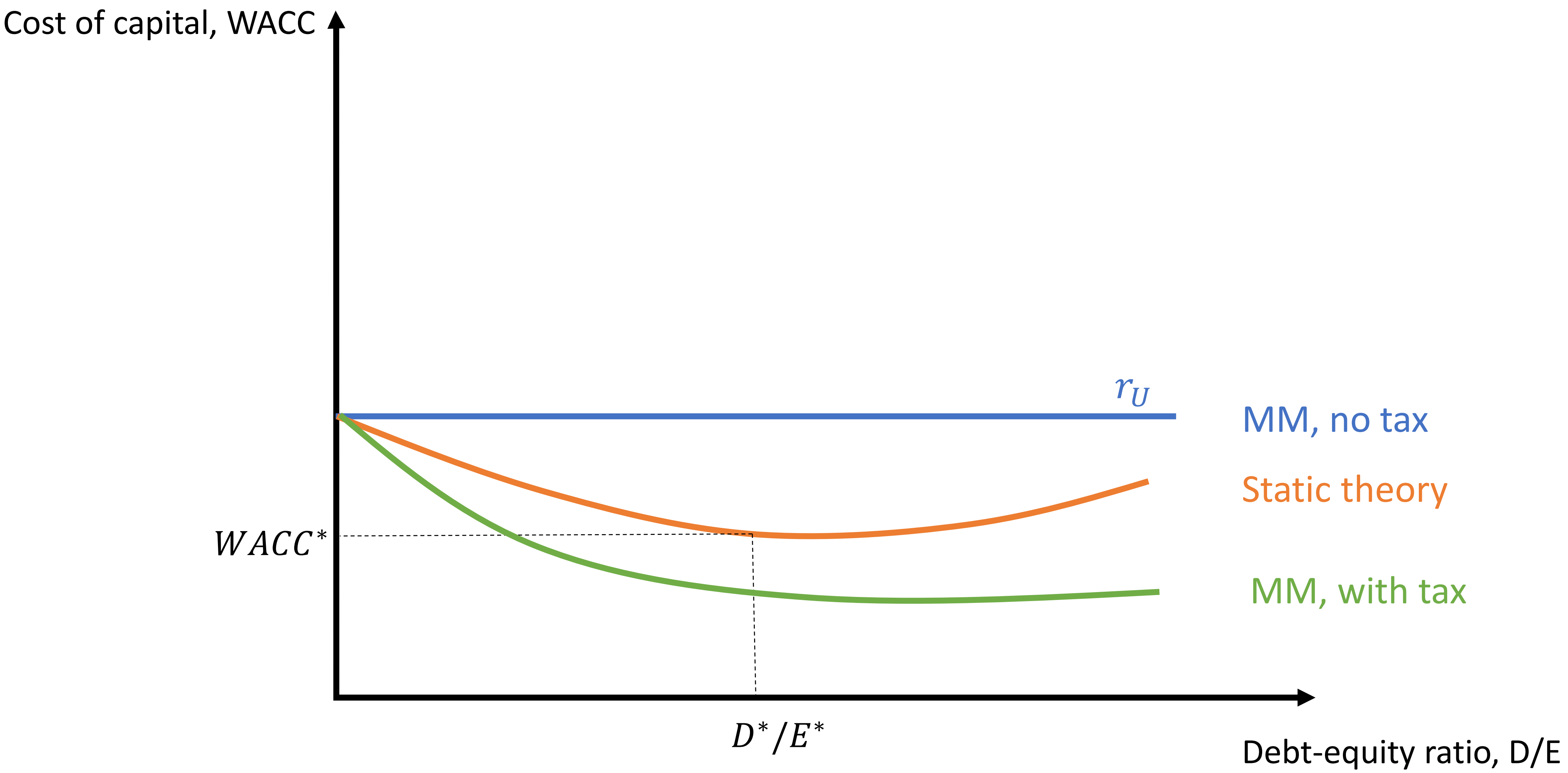
Sources: Reuters (2020); The New York Times (2020)

Static theory of capital structure

- At relatively low debt levels, the probability of bankruptcy and financial distress is low, and the (tax shield) benefit from debt outweighs the cost
- At very high debt levels, the possibility of financial distress is a chronic, ongoing problem for the firm, so the benefit from debt financing may be more than offset by the financial distress costs

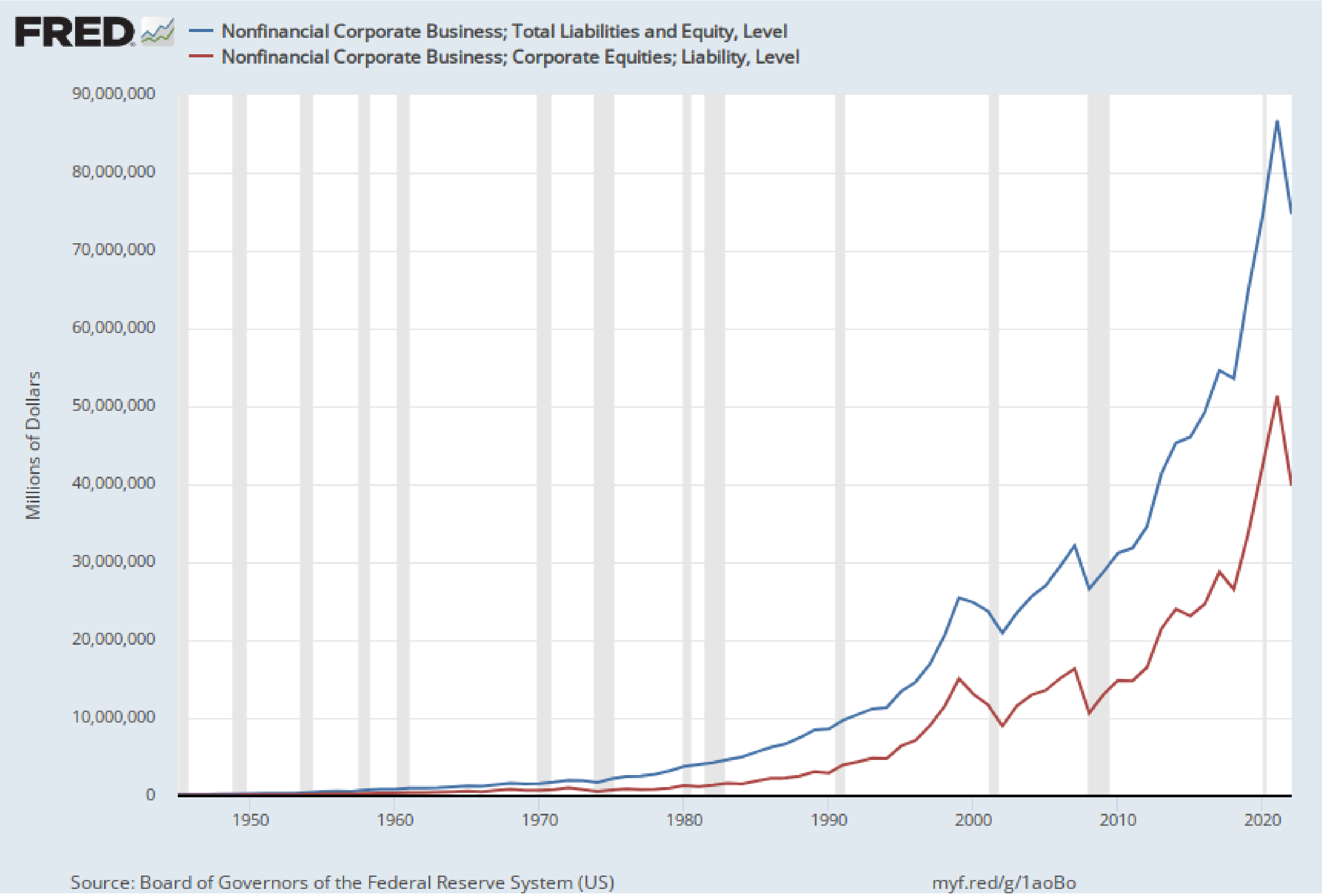
=> Static theory of capital structure: a firm borrows up to the point where the tax benefit from an extra pound or euro in debt is exactly equal to the cost that comes from the increased probability of financial distress


PV of tax shield
Firm value, V
V_L
MM, with tax
PV of financial distress costs
V^*
V
Static theory
V_U
MM, no tax
D^*
Debt, D


Cost of capital, WACC
r_U
MM, no tax
Static theory
MM, with tax
$WACC^*$
D^*/E^*
Debt-equity ratio, D/E

In reality, many large, financially sophisticated and highly profitable firms use little debt. This is different from what we would expect from the static theory. Why?

Pecking order theory



Managers have private information have incentives to issue overpriced risky securities but also understand that issuing such securities will result in a negative price reaction because rational investors, who are at an information disadvantage, will discount the prices of any risky securities the firm issues → riskier securities are more subjective to adverse selection

Retained earnings

- No adverse selection

Debt

- Minor adverse selection

Equity

- Serious adverse selection

Pecking order theory

- Pecking order theory:
    - Firms prefer internal to external finance
    - When external fund is necessary, firms prefer debt to equity
- Implications:
    - No target capital structure
    - Profitable firms use less debt
    - Companies will want financial slack

Quiz 3

ACB Group is a global power and automation technologies firm that is listed on the London Stock Exchange. You have been hired to assist the new management in evaluating two funding options for a major expansion project that will cost £1.5 billion. The company is considering whether to issue 100% equity, 100% debt with a 5% cost of debt, or 50/50 debt/equity. Assume no taxes.

You have been given the following data:

Expected Earnings next year: £3 billion

Dividends: No dividends are paid by the company

Number of Shares Outstanding: 10 billion

Current Cost of Equity: 10%

Net Cash Flow from Major Expansion Project: A permanent £0.3 billion per annum

a. What is the net present value of the project to ACB Group's shareholders?
b. Show the market value balance sheet of ACB Group upon completion of each of the 3 funding scenarios.
c. What is the expected return on equity for each scenario?
d. What is the share price of the firm under each scenario?
e. Are your results consistent with Modigliani and Miller's Proposition I and II? Explain.

Week 9

Dividends and payout policy

Lecture outline

- Cash dividends
  - Does dividend policy matter?
  - Real-world factors
- Share repurchases
- Stock dividends and stock splits

Learning outcomes

- Be able to understand the basics of different dividends and payout policies
- Be able to understand the factors that affect dividend policy
- Be able to understand and analyse the effects of dividend policy/dividend announcement on stock price

Dividend: a payment made out of a firm’s earnings to its owners, in the form of either cash or stock

Forbes

5 Best Dividend Stocks For Passive Income In September 2023

Discover the best dividend stocks for passive income. Find out which stocks made the list, as well as the analysis and reasoning behind it.

1 day ago

Investors Chronicle

Aviva's £800mn sale shores up its dividend

further burnished its dividend credentials after sealing the £800mn cash sale of its 25.9 per cent stake in Singapore Life Holdings to...

23 hours ago

The Telegraph

A drop in profits and a dividend cut sound disastrous – but you should keep buying this housebuilder

Questor share tip: MJ Gleeson has a strong position in an attractive niche market and the share price already factors in a lot of bad news.

1 day ago

Cash dividends

- Regular cash dividends
  - A cash payment made by a firm to its owners in the normal course of business
- Extra dividends
  - May or may not be repeated in the future
- Special dividends
  - Truly unusual/one-time event, won't be repeated
- Liquidating dividends
  - Some or all of the business has been sold off

Standard Method of Cash Dividend Payment

Declaration date

- the date on which the board of directors passes a resolution to pay a dividend

Ex-dividend date

- the date two business days before the date of record, establishing those individuals entitled to a dividend

Date of record

- the date by which a holder must be on record to be designated to receive a dividend

Date of payment

- the date on which the dividend is paid

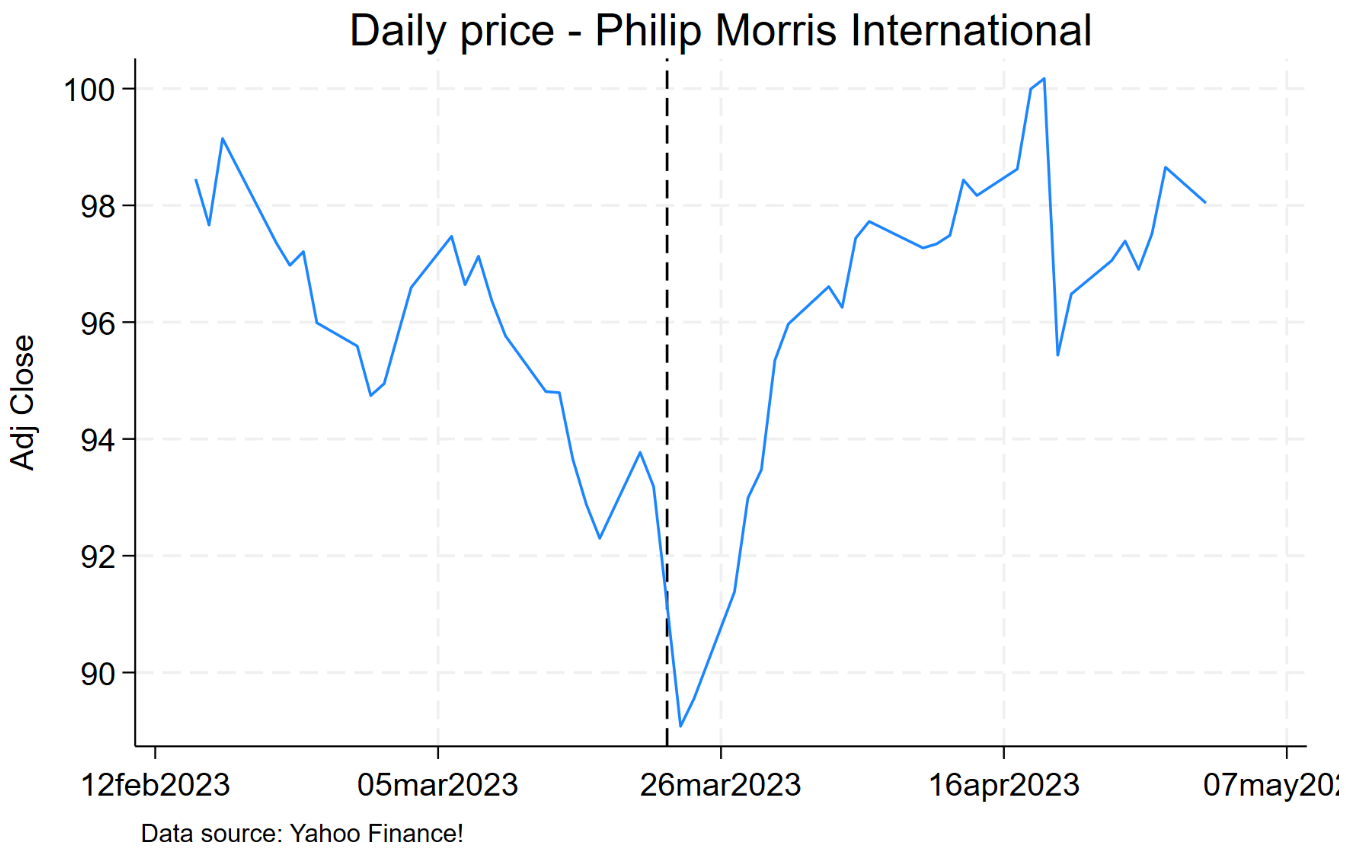
Price behaviour around the ex-dividend date (1)

- Suppose we have an equity that sells for £10 per share. The board of directors declares a dividend of £1 per share, and the record date is set to be Tuesday 12 June. The ex-date will be 2 business (not calendar) days earlier, on Friday 8 June.
- If you buy the equity on Thursday 7 June, just as the market closes, you'll get the £1 dividend, because the equity is trading cum dividend.
- If you wait and buy it just as the market opens on Friday, you won't get the £1 dividend. What happens to the value of the equity overnight?

With dividend

Price behaviour around the ex-dividend date (2)

- We would expect that the share price will go down by ***about*** the dividend amount when the equity goes ex-dividend
- Determining the value of the actual price drop is complicated because of the different tax rates and tax rules that apply for different buyers


Daily price - Philip Morris International
Adj Close
100
98
96
94
92
90
12feb2023
05mar2023
26mar2023
16apr2023
07may202
Data source: Yahoo Finance!

Dividend vs dividend policy

- Dividend policy
  - Time pattern of dividend payout
  - Should the firm pay out a large dividend now or a small (or even zero) dividend?
- Dividend = cash matters
  - Investors prefer higher dividends to lower dividends at any single date if the dividend level is held constant at every other date
  - If the dividend per share at a given date is raised while the dividend per share at every other date is held constant, the share price will rise
- How’s about dividend policy?

Case study (1)

Bristol is an all-equity firm that has existed for 10 years. The current financial managers plan to dissolve the firm in 2 years. The total cash flows the firm will generate, including the proceeds from liquidation, will be £10,000 in each of the next 2 years.

Case study (2)

Current Policy: Dividends Set Equal to Cash Flow

At present, dividends at each date are set equal to the cash flow of £10,000. There are 100 shares outstanding and the required rate of return is 10%.

$$P_0 = \frac{D_1}{1+r} + \frac{D_2}{(1+r)^2} = \frac{100}{1.1} + \frac{100}{1.1^2} = £173.55$$

The firm is worth 100 × £173.55 = £17,355

Several members of the board of Bristol have expressed dissatisfaction with the current dividend policy and have asked you to analyse an alternative policy.

Case study (3)

Another possible policy is for the firm to pay a dividend of £110 per share on the first date (Date 1), which makes a total dividend of £11,000. Because the cash flow is only £10,000, an extra £1,000 must somehow be raised.

Assume that equity is issued. The new shareholders will desire enough cash flow at Date 2 that they earn the required 10% return on their Date 1 investment. This means the new shareholders will get the Date 2 CF of £1,000×1.1 = £1,100, leaving £8,900 to be distributed to the old shareholders.

$$P_0 = \frac{D_1}{1+r} + \frac{D_2}{(1+r)^2} = \frac{110}{1.1} + \frac{89}{1.1^2} = £173.55$$

The irrelevance of dividend policy

- The previous results suggest that dividend policy **might** not be matter to the firm value
- Explanations:
  - Any increase in a dividend at some point in time is exactly offset by a decrease somewhere else, so the net effect, once we account for time value, is zero.
  - Shareholders can alter the firm's dividend policy to suit themselves
    - Homemade dividend policy: the tailored dividend policy created by individual investors who undo corporate dividend policy by reinvesting dividends or selling shares of equity

In reality, there are several factors that could affect the irrelevance of dividend policy

Real world factors favouring a low dividend policy

- Taxes
  - Effective tax rates on dividend income are different from tax rates on capital gains
- Flotation costs
  - Selling new equity can be expensive
- Dividend restrictions
  - A corporation may face restrictions on its ability to pay dividends. E.g., a corporation may be prohibited by law from paying dividends if the dividend amount exceeds the firm’s retained earnings.

Real world factors favouring a high dividend policy

- Desire for current income
- Information content of dividends and dividend signalling
- Agency costs
- Behavioural finance

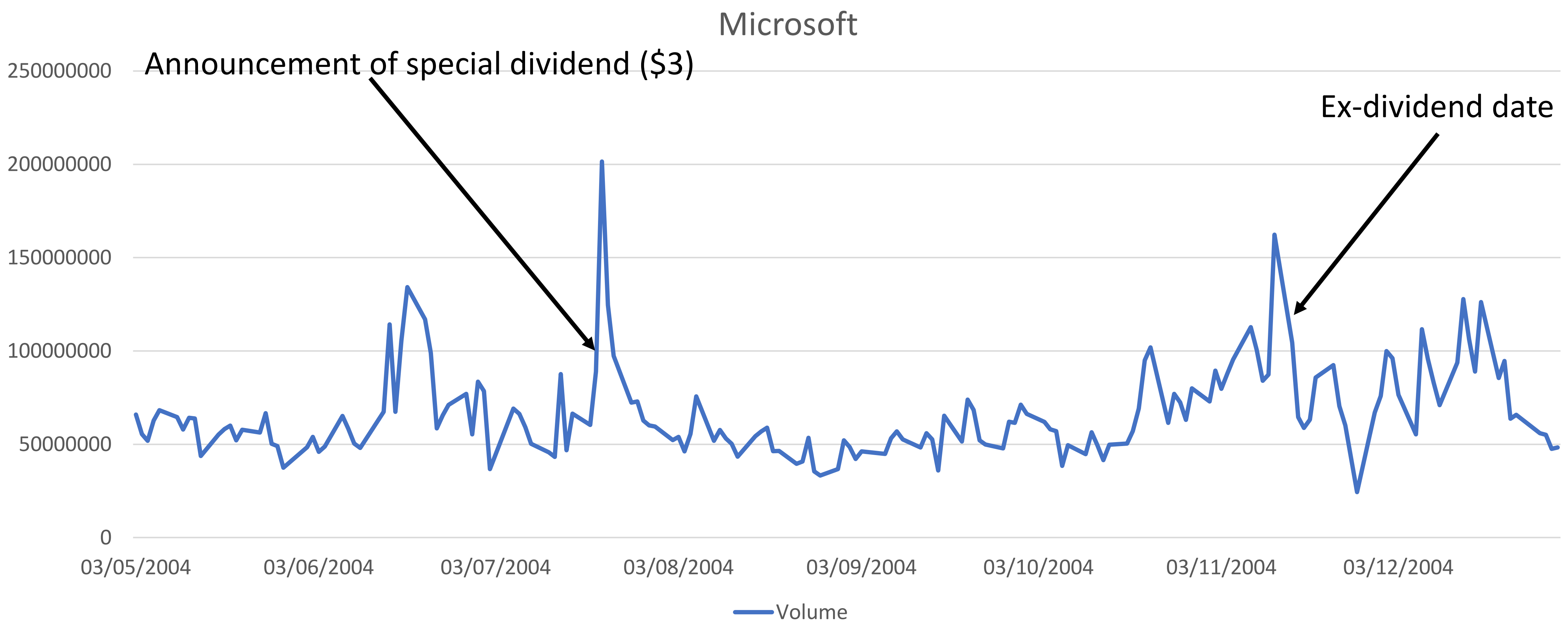
E.g.: Firms pay dividends because investors exercise better self-control with their spendings if they get a "check in the mail" in the form of a dividend (Shefrin and Statman, 1984)

- Managers have superior information about firms' performance
- Announcement of an increase in dividends gives a good signal about future earnings and cash flows
- It also signals that the firm won't be hoarding cash, thus, reducing agency costs and enhancing shareholder wealth

The clientele effect

- Different investors prefer different dividend policies
  - Tax-free or low-tax paying investors prefer high dividends
  - High-tax paying investors prefer low dividends or no dividends
- Firms' dividend policy is optimized for the tax preference of its investor clientele
- Dynamic clientele effect (dividend-capture theory):
  - Ignoring transaction costs, investors can trade shares at the time of the dividend so that non-taxed investors receive the dividend. That is, non-taxed investors need not hold the high-dividend-paying stocks all the time, it is necessary only that they hold them when the dividend is actually paid

Low-tax investors buy the stock in anticipation of the dividend and then reverse those trades afterwards



The catering theory of dividends

- Baker and Wurgler (Journal of Finance, 2004):
  - The decision to pay dividends is driven by prevailing investor demand for dividend payers
  - Managers (rationally) cater to investors by paying dividends when investors put a stock price premium on payers, and by not paying when investors prefer nonpayers

Share repurchases

- Share repurchase (buyback): the purchase, by a corporation, of its own shares of equity
  - Firms buy their own equity from open market
  - Tender offer
  - Firms buy shares from specific individual shareholders (targeted purchase)

Most common in US

The firm announces the intention to all shareholders to repurchase a fixed number of shares for a fixed price, conditional on shareholders agreeing to tender their shares. If not enough shares are tendered, the deal can be cancelled

Similar to tender offer but only targeted shareholders can tender their shares

Cash dividends vs Repurchases: Conceptual example (1)

Telephonic Industries has excess cash of £300,000 (or £3 per share) and is considering an immediate payment of this amount as an extra dividend. The firm forecasts that, after the dividend, earnings will be £450,000 per year, or £4.50 for each of the 100,000 shares outstanding. The price–earnings ratio is 6 for comparable companies.

Alternatively, the firm could use the excess cash to repurchase some of its own equity. Imagine that a tender offer of £30 a share is made.

Cash dividends vs Repurchases: Conceptual example (2)

- Cash dividends:
  - Share price after dividend is paid = £27 (= £4.50 $\times$ 6)
  - With dividends, each (existing) shareholder owns a share worth £27 and receives £3 in dividends, making the total value of £30
- Repurchase:
  - Repurchased shares = 10,000 (= £300,000/£30)
  - Remaining shares = 90,000 (= 100,000-10,000)
  - Earnings per (remaining) shares = £5 (= £450,000/90,000)
  - Share price after the repurchase is £30 (= £5 $\times$ 6)

"Improvement" in EPS

Dividends vs Repurchases: Real world considerations

- Flexibility:
  - Firms often view dividends as a commitment to their stockholders and are quite hesitant to reduce an existing dividend. Repurchases do not represent a similar commitment.
- Executive compensation:
  - Executives are frequently given stock options as part of their overall compensation. They would prefer a repurchase to a dividend because the stock price will be greater after a repurchase than after a dividend.
- Undervaluation
  - Firms might buy back stock when they believe that the stock price is temporarily depressed.
- Taxes
  - Repurchases provide a tax advantage over dividends.

Pros and cons of paying dividends

Pros	Cons
• Cash dividends can underscore good results and provide support for the share price. • Dividends may attract institutional investors who prefer some return in the form of dividends. A mix of institutional and individual investors may allow a firm to raise capital at a lower cost because of the ability of the firm to reach a wider market. • Share prices usually increase with the announcement of a new or increased dividend. • Dividends absorb excess cash flow and may reduce the agency costs that arise from conflicts between management and shareholders.	• Dividends are taxed. • Dividends can reduce internal sources of financing. Dividends may force the firm to forgo positive-NPV projects (which, of course, firms SHOULD NOT do) or rely on costly external equity financing. • Once dividends are established, dividend cuts are hard to make without adversely affecting a firm’s share price.

Dividend policies in reality

- In Europe:
  - Aggregate dividends paid are quite large
  - But the total real dividends (share repurchases + cash dividends) were decreasing
  - Percentage of firms paying dividends has also declined
- Dividend smoothing
  - Dividend-paying firms tend to raise dividends only after earnings have risen (i.e., dividend growth lags earnings growth)
  - Firms don’t increase or cut dividends in response to temporary earning fluctuations (i.e., dividend growth tends to be smoother than earnings growth)

Stock dividends and stock splits

- Stock dividend: a payment made by a firm to its owners in the form of equity, diluting the value of each share outstanding
  - A 20% stock dividend means that a shareholder receives one new share for every five currently owned
  - Changes the composition of equity accounts (a portion of retained earnings is transferred to paid-in capital but the total stockholders’ equity remains the same)
- Stock split: an increase in a firm’s shares outstanding without any change in owners’ equity
  - A three-for-one stock split means each old share is split into three new shares
  - Does not have any effect on total paid-in capital, retained earnings, and total stockholders’ equity
- Reverse split: a stock split in which a firm’s number of shares outstanding is reduced