Summative Essay

Explain how the introduction of human capital into the standard Solow growth model affects steady state long-run economic growth. Using empirical evidence assess whether this "augmented" model represents an improvement in explaining observed rates of economic growth.

Macroeconomics II [ECO00002I-S1-A]

Department of Economics and Related Studies

University of York

United Kingdom

17th December 2023

Introduction

The Solow growth model is a neoclassical growth model that attempts put a framework around long run economic growth dynamics. It attempts to explain observed rates of economic growth through the lens of capital, labour, and technological progress by incorporating exogenous variables such as the savings rate and population growth to describe how an economy reaches a 'steady state' of income in the long run.

The Solow Model

Output Assumptions

Solow (1957) assumed a simple (one good is produced), closed (no trade with others) economy, and assumed that the output of the economy has constant returns to scale: if the factors of production are scaled by a given magnitude, then output would scale by the same magnitude i.e. $F(xK, xL) = xF(K, L)$.

Capital Assumptions

The economy accumulates a single commodity as capital (K) which exhibits diminishing marginal returns to capital, unlike the prior Harrod-Domar model which exhibited constant returns to capital. This change in approach stems from the observation that as capital increases in a fixed labour environment capital becomes under utilised.

Labour and Technology Assumptions

There is full employment with a complete inelastic labour supply: in essence, the labour force is the whole population. By extension, the labour force exhibits the same dynamics as population growth and is therefore assumed to increase over time exponentially at rate n with an initial state of L_0:

$$L(t) = L_0 \cdot e^{nt}$$

Similarly, technology is assumed to advance exponentially at rate g with an initial state of A_0.

$$A(t) = A_0 \cdot e^{gt}$$

It is assumed technological progress is strictly labour-augmenting, implying that technological progress only impacts the productivity of the worker.

With these assumptions, the output of the economy at any given point in time can be described by a production function of capital K and effective labour AL.[1]

$$Y(t) = F(K(t), A(t)L(t))$$

Output can be expressed in terms of effective labour, time notation is dropped as all functions in the model are themselves functions of time:

$$\frac{Y}{AL} = \frac{F}{AL}(K, AL)$$

Since output is assumed to have constant returns to scale,

$$\frac{Y}{AL} = F\left(\frac{K}{AL}, \frac{AL}{AL}\right)$$

To simplify notation, functions in terms of effective labour are expressed in lowercase:

$$y = F(k, 1)$$

For simplicity, a Cobb-Douglas production function is assumed as it adheres to the previous assumptions: its first derivative is positive whilst second derivative and the

1. Since technology is assumed to be strictly labour-augmenting, the combined productivity of labour and technology can be expressed as the composite of their two respective functions: AL. Simplifying the derivation of the steady state.

Inada conditions:

$$Y = K^{\alpha} \cdot AL^{1-\alpha}$$

In terms of effective labour,

$$y = k^{\alpha} \cdot 1$$

Time Path of Capital and the Steady State

Since the economy is assumed to be closed, output is equivalent to income (Y). It is assumed a fixed portion of this income is saved (not consumed) according to the savings proportion (s). Investment at a given time is equivalent to the amount saved at a given time (sY) through lending and depositing at a supposed perfect financial institution. Dividing all terms by AL to intensive form, positive capital accumulation equals sk^{α}.

However capital does not continuously accumulate. There are depreciating effects that reduce the per unit of labour value of capital over time. Firstly, *population growth*, as a population increases the same amount of capital is spread over more people, diluting the capital pool. Secondly, technological progress has an obsolescence effect on capital as technology improves the relative productivity of the current capital K decreases. Thirdly, there is a raw depreciation caused by wear and tear. Thus, the change in capital per effective unit of labour at any given time $\frac{dk}{dt} = \dot{k}$ is given by,

$$\dot{k} = \underbrace{sk^{\alpha}}_{\text{New Investment}} - \overbrace{nk}^{\text{Labour Spread}} - \underbrace{gk}_{\text{Obsolescence}} - \overbrace{\delta k}^{\text{Raw Depreciation}}$$

This implies that capital accumulates to a steady state (where $\dot{k} = 0$):

$$
\begin{aligned}
0 &= sk^{\alpha} - [n+g+\delta]k \\
[n+g+\delta]k &= sk^{\alpha} \\
[n+g+\delta]k^{1-\alpha} &= s \\
k^{1-\alpha} &= \frac{s}{n+g+\delta}
\end{aligned}
$$

Since the capital stock is non-negative,

$$k^* = \left[\frac{s}{n+g+\delta}\right]^{\dfrac{1}{1-\alpha}}$$

Since $y = k^{\alpha}$, the steady state output can be derived:

$$y^* = \left[\frac{s}{n+g+\delta}\right]^{\dfrac{\alpha}{1-\alpha}}$$

The relationship between the output per capita and the independent variables can be simplified using logarithms as y^* is expressed in terms of effective labour,

$$\ln\left(\frac{Y}{AL}\right) = \ln\left(\frac{s}{n+g+\delta}\right)^{\dfrac{\alpha}{1-\alpha}}$$

Using log power rule,

$$\ln\left(\frac{Y}{L}\right) - \ln A = \frac{\alpha}{1-\alpha}\ln\left(\frac{s}{n+g+\delta}\right)$$

Substituting in the progress function $A(t)$,

$$\ln\left(\frac{Y}{L}\right) = \frac{\alpha}{1-\alpha}\ln(s) - \frac{\alpha}{1-\alpha}\ln(n+g+\delta) + \ln A_0 + gt$$

The main prediction of this model on long-run economic growth is that countries with greater savings rates and lower population growth converge faster to their steady state income (assuming technology growth and accumulation are the same). This implication can be tested when observing economic growth rates of different economies; say an output shock decreases the per capita income below steady-state levels, then according to the simple Solow model, an economy an economy with greater savings rate once accounted for population growth and depreciation rate should converge faster than others. The last terms in the equation also imply that technological progress explains the difference in steady states between economies that have similar growth rates and population growth, since A_0 are not identical in every country.

Inaccuracies in the Solow Model

Mankiw et al. 1992 studied the efficacy of the Solow model to explain cross-country variation in income convergence. They observed differences in income levels (and by definition economic growth) over a 25 year time period (1960-1985) using empirical data for savings, population growth, and the depreciation rate to see how effective the Solow model is at explaining observed economic growth across different countries. Their results suggested the model implied a capital share of 0.6. Far greater than observed capital share of $\frac{1}{3}$ at time of their publication (OECD, 2015). Consequently, the observed relationship between the output per capita and savings rate is too high to be explained by the model.

One explanation for this anomaly is a possible unobserved variable. Human capital, the value of labour's skill-set to the economy, is a notable omission from the original model and could explain why the implied capital share is too high. One estimate puts the proportion of human capital of the United States' capital stock at 50% (Kendrick, 1976) emphasising the importance of its omission. Higher incomes should itself increase human

2. Diagram taken from slide 3, topic "Week 3: Exogenous Growth, Lecture Material and Readings, Mankiw Romer and Weil 1992", Macroeconomics II 2023/24, University of York.

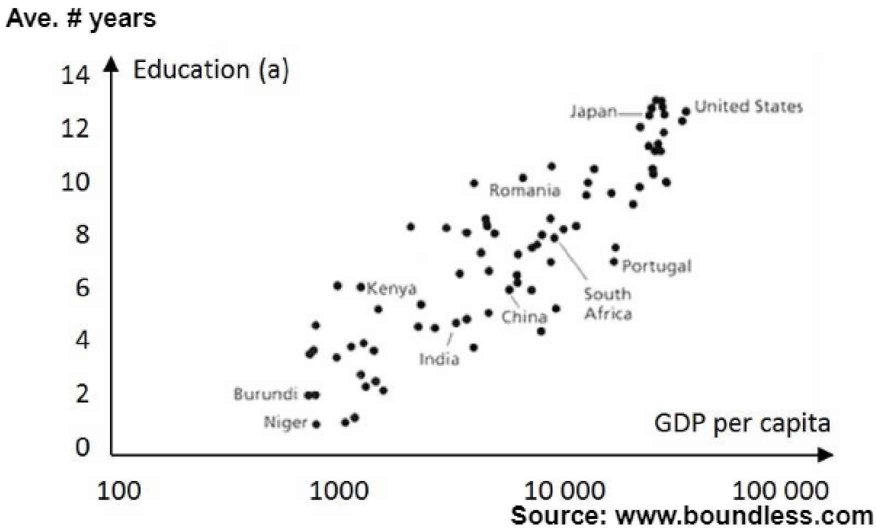

Figure 1: Correlation between Education level and Income per Capita.[2]

capital as workers have higher income to spend on improving their skill-set through education and training (as shown by the positive correlation in 1). Therefore, including human capital in the model could account for the additional impact of savings on human capital and explain the erroneous difference in observed elasticity of output to savings that are implied by the model.

The Augmented Solow Model

Under the same assumptions as the simple model except that capital is now two commodities. The augmented model adjusts the motion of capital and the production function to include human capital H as well as physical capital K. The augmented production function in Cobb-Douglas form becomes,

$$Y = K^{\alpha} \cdot H^{\beta} \cdot (AL)^{1-\alpha-\beta}$$

In terms of effective labour $\left(\frac{Y}{AL}\right)$ the function becomes,

$$y = k^{\alpha} \cdot h^{\beta} \cdot 1$$

The motion of capital is split into two, with savings now either going to human capital in proportion s_h and physical capital s_k. It is assumed depreciation of both types of capital is identical δ. Thus both capital types movement over time is defined by,

$$\dot{k} = s_k y - (n + g + \delta)k \qquad \dot{h} = s_h y - (n + g + \delta)h$$

Whereby $\dot{k}$ is the evolution of the physical capital stock in terms of effective labour with respect to time and $\dot{h}$ is the same but for human capital. Like the simple model, the augmented model implies that capital motion converges to a point where the net investment equals depreciation. Hence, the steady states of capital can be derived:

Substituting the augmented production function into the first order conditions,

$$\dot{k}: \qquad 0 = s_k k^{\alpha} \cdot h^{\beta} - (n + g + \delta)k \tag{1}$$

$$\dot{h}: \qquad 0 = s_h k^{\alpha} \cdot h^{\beta} - (n + g + \delta)h \tag{2}$$

Rearrange (1) for steady state physical capital in terms of human capital,

$$k = \left[\frac{s_k}{n + g + \delta} \cdot h^{\beta}\right]^{\dfrac{1}{1-\alpha}} \tag{3}$$

Substitute (3) into evolution of human capital (2) to isolate human capital steady state,

$$0 = s_h \left[\frac{s_h}{n + g + \delta} \cdot h^{\beta}\right]^{\dfrac{\alpha}{1-\alpha}} \cdot h^{\beta} - (n + g + \delta)$$

Solving for h,

$$h^* = \left[\frac{s_h^{1-\alpha} \cdot s_k^{\alpha}}{n + g + \delta}\right]^{\dfrac{1}{1-\alpha-\beta}} \tag{4}$$

Due to the symmetry of each capital motion function the physical capital steady state is,

$$k^* = \left[\frac{s_k^{1-\beta} \cdot s_h^{\beta}}{n + g + \delta}\right]^{\dfrac{1}{1-\alpha-\beta}} \tag{5}$$

Substituting each capital steady-state into the production function the steady state output can be derived,

$$y^* = \left[\frac{s_k^\alpha \cdot s_h^\beta}{(n+g+\delta)^{\alpha+\beta}} \right]^{\dfrac{1}{1-\alpha-\beta}}$$

Logarithms can be taken to express income in per capital terms, the concluding equation is reminiscent of the simple Solow model,

$$\ln\left(\frac{Y}{N}\right) = \frac{\alpha}{1-\alpha-\beta}\ln(s_k) + \frac{\beta}{1-\alpha-\beta}\ln(s_h) - \frac{\alpha+\beta}{1-\alpha-\beta}\ln(n+g+\delta) + \ln A_0 + gt$$

The augmented model remains consistent of Solow's original model in that income converges to a point where savings is in equilibrium with depreciation and population growth. It also comes to the conclusion when economies are not in the steady state, those with greater savings rates after accounting for population growth and depreciation would converge faster than other economies. However compared to the simple model, the rate of convergence now depends also on the share of human capital.

Since human capital is incorporated in the augmented model, the elasticity of savings into the physical capital stock is decreased as $\frac{\alpha}{1-\alpha} > \frac{\alpha}{1-\alpha-\beta}$. Mankiw et al. (1992) used the same cross-panel analysis used for the simple model using the duration of education as a proxy for human capital and found the implied share of physical capital and human capital to be roughly $\frac{1}{3}$ each, inline with the observed share of capital. Additionally, they found that the elasticity of output to savings under the augmented model to be 1, instead 0.5 under the simple model, again, more inline with the levels observed.

Conclusion

To conclude, augmenting the model to incorporate human capital provides a better explanation for observed rates of economic growth, empirical data by Mankiw supports the notion specifically stating that the inclusion of human capital accounts for 80 percent of the cross-variation in income. However, the limitations of the Solow model and its augmentations should not be understated. For example, the augmented model assumed that human capital depreciates at the same level as physical capital, this liberty although convenient may not be realistic since factors such as health impact the depreciation through effecting cognition and motor skills (McFadden, 2008). By using education as simple proxy for human capital, Mankiw et al may have missed the dynamics of healthcare on the depreciation and accumulation of human capital, which as identified in the augmented model, impacts income levels and conversely economic growth. Furthermore, more modern approaches incorporating more endogenous variables and heterogeneity could allow for more accurate explanations of economic growth, as it is unrealistic for the exogenous variables described in the Solow model to remain constant over time such as savings rate and depreciation rate.

Bibliography

Kendrick, John W. 1976. *The Formation and Stocks of Total Capital.* NBER. Accessed January 10, 2024. https://www.nber.org/books-and-chapters/formation-and-stocks-total-capital.

Mankiw, N. G., D. Romer, and D. N. Weil. 1992. "A Contribution to the Empirics of Economic Growth" [in en]. *The Quarterly Journal of Economics* 107, no. 2 (May): 407–437. ISSN: 0033-5533, 1531-4650, accessed December 17, 2023. https://doi.org/10.2307/2118477. https://academic.oup.com/qje/article-lookup/doi/10.2307/2118477.

McFadden, Daniel. 2008. "Human Capital Accumulation and Depreciation." *Review of Agricultural Economics* 30 (3): 379–385. ISSN: 1058-7195, accessed January 10, 2024. https://www.jstor.org/stable/30225880.

OECD. 2015. "The Labour Share In G20 Countries" (February). https://www.oecd.org/g20/topics/employment-and-social-policy/The-Labour-Share-in-G20-Economies.pdf.

Solow, Robert M. 1957. "Technical Change and the Aggregate Production Function." *The Review of Economics and Statistics* 39 (3): 312–320. ISSN: 0034-6535, accessed December 20, 2023. https://doi.org/10.2307/1926047. https://www.jstor.org/stable/1926047.