Problem Set 3

Microeconomics ECO00037I

Isaac Beight-Welland

University of York

United Kingdom

November 7, 2023

Problem 3.1

(a) The marginal product of labour and capital for the given production function $f(K,L) = K^{\frac{1}{2}}L^{\frac{1}{2}}$ is given respectively by:

$$\frac{\partial L}{\partial L} = \frac{K^{\frac{1}{2}}}{2L^{\frac{1}{2}}} = \sqrt{\frac{K}{L}} \qquad \frac{\partial f}{\partial K} = \sqrt{\frac{L}{K}}$$

The marginal rate of substitution is the rate of change of capital with respect to labour:

$$MRS = \frac{\partial K}{\partial L} = \frac{\partial K}{\partial f} \div \frac{\partial L}{\partial f}$$

For PaperInc, the technical rate of substitution is given by:

$$-\sqrt{\frac{L}{K}} \div \sqrt{\frac{K}{L}} = -\sqrt{\frac{L}{K}} \times \sqrt{\frac{L}{K}} = -\frac{L}{K}$$

(b) To find the optimal bundle for a 10 units of output we must minimise cost subject to the production function:

$$\min_{c} \; c = 4K + L \quad \text{s.t} \quad 10 = \sqrt{KL}$$

Rearranging the constraint function for capital and labour:

$$100 = KL$$

$$L = 100K^{-1}$$

Substitute into the cost function:

$$c = 4K + 100K^{-1}$$

Cost minimised when the marginal output of capital is zero:

$$\frac{dc}{dK} = 4 - 100K^{-2} = 0$$

$$4 = \frac{100}{K^2}$$

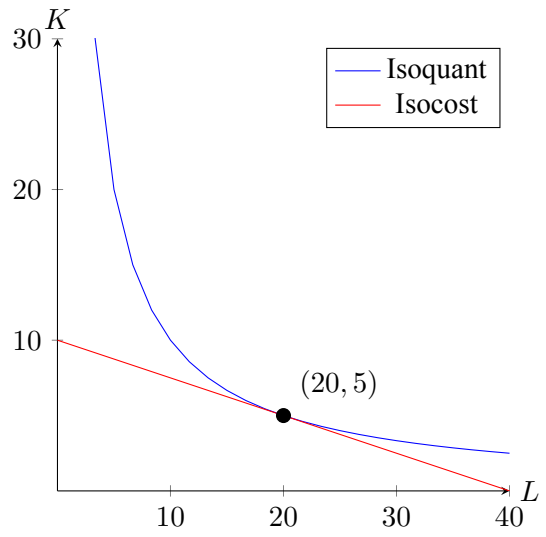
$$K = 5$$

Substitute back into constraint function for labour:

$$10 = \sqrt{5L}$$

$$L = 20$$

(c) The graph below shows the optimal bundle for PaperInc's input costs at the production of 10 output.



(d) Supposing capital was fixed at 16 in the short run. The cost function, the total cost of for a given output $c(y)$, is given by substituting 16 into the production function, rearranging for L and substituting into the input function:

$$f(16, L) = 4\sqrt{L}$$

$$y = 4\sqrt{L}$$

$$L = \frac{y^2}{16}$$

Substitute into cost constraint

$$c(y) = 64 + \frac{y^2}{16}$$

(e) In the long run, all inputs are variable, thus like part (b), it is assumed that PaperInc minimises its costs.