Consumption

- The approach taken:
  - Neoclassical theory: intertemporal choice and the Euler equation
  - Models with perfect foresight or quadratic preferences: the permanent income model
  - Buffer-stock models and precautionary savings

Consumption: Intertemporal Choice

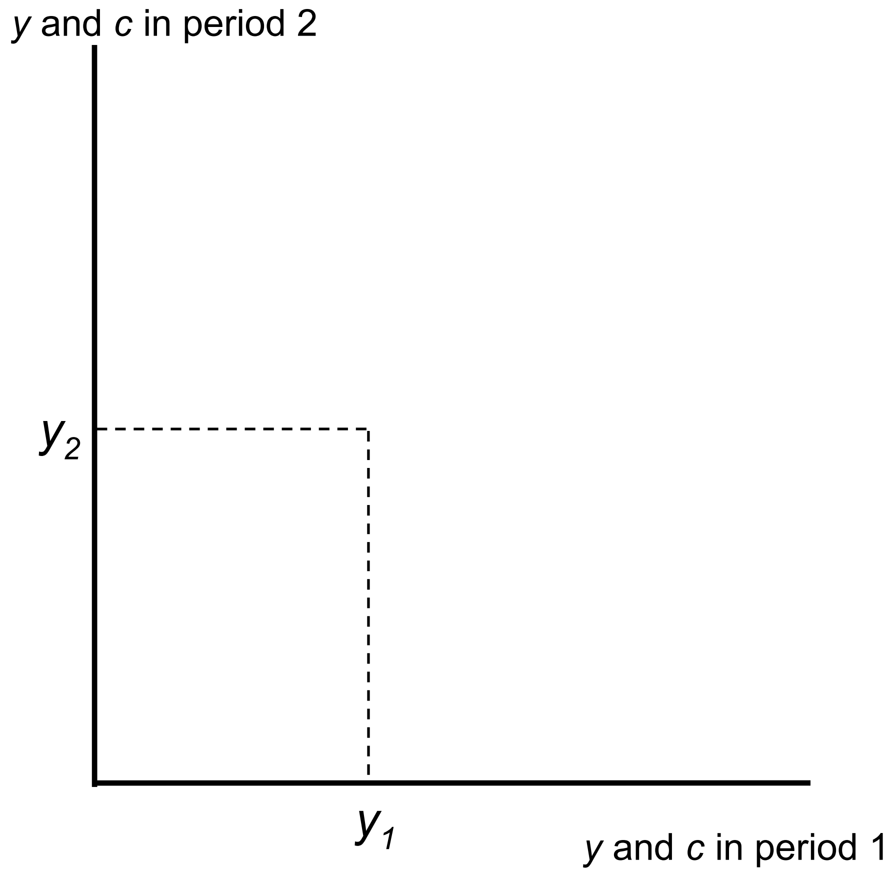
- "When the real income of the community increases or decreases, its consumption will increase or decrease" (Keynes, 1936).

- Basis of IS curve

- The less sensitive consumption is to income, the lower is the multiplier and the weaker is fiscal policy.

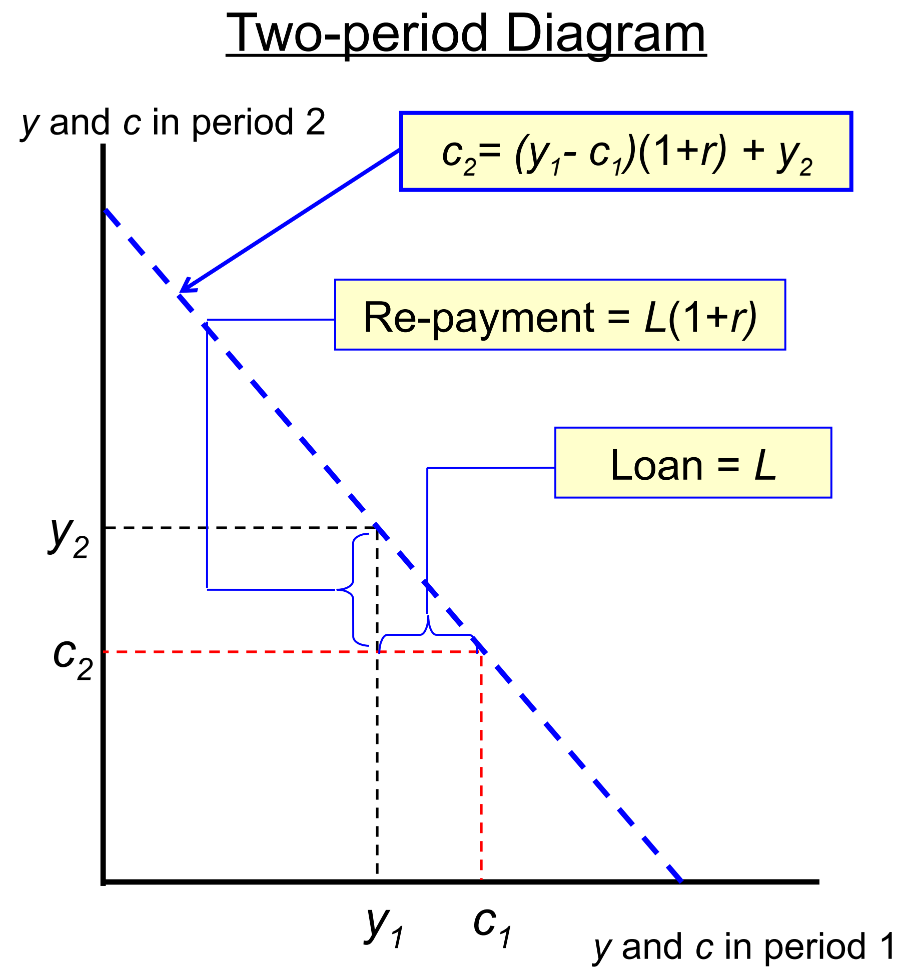
- To appreciate why the role of income may be weaker, consider an individual who:
  - lives for two periods (1 & 2)
  - earns y_1 in 1 and y_2 in 2

Two-period Diagram



Consumption: Intertemporal Choice

- The individual could consume her income in both periods.
- Or she could consume more than her income in period 1 by borrowing against future income.
- Or she could save in period 1 in order to consume more than y_2 in period 2.
- Her consumption need not match her income in each period – she faces an *intertemporal budget constraint* - with slope -(1+*r*).
- Where she chooses on this budget line depends on her preferences for consumption in the two periods.



Consumption: Intertemporal Choice

- Allocation of consumption over time
- Maximise *expected utility* subject to a *budget constraint*
- Economy with large numbers of identical individuals – **the representative agent**.

- **Expected Utility**:
  - *c* is consumption; δ is the subjective discount rate; *T* is planning horizon; *u*(.) is the 'sub-utility' function.
  - Income is received and consumption takes place at the end of the period.

Maths

$$\max_{c_s} U_t = E_t \left[\sum_{s=t}^{T} \left(\frac{1}{1+\delta} \right)^{s-t+1} u(c_s) \right]$$

Consumption: Intertemporal Choice

- Maximise U subject to a (lifetime) budget constraint
- W is present value of the individual's 'life-time' resources (expected at t)
- Resources are fully used on c
- A is financial wealth
- y is labour income
- 'Non-human' wealth
- 'Human' wealth

Maths

$$\max_{c_s} U_t = E_t\left[\sum_{s=t}^{T}\left(\frac{1}{1+\delta}\right)^{s-t+1} u(c_s)\right]$$

$$E_t\left[\sum_{s=t}^{T}\left(\frac{1}{1+r}\right)^{s-t+1} c_s\right] = W_t$$

$$W_t \equiv A_t + E_t\left[\sum_{s=t}^{T}\left(\frac{1}{1+r}\right)^{s-t+1} y_s\right]$$

Consumption: Intertemporal Choice

- E_t is the mathematical expectation given information (Ω) available at time *t.*
- Because of timing convention, current values are discounted.
- *Real interest rate* (*r*) is assumed to be constant.
- Utility is additive and time-separable - utility does not depend directly on consumption in other periods.

Maths

$$E_t(x) \equiv E(x \mid \Omega_t)$$

$$\text{Present value of } c_t = \frac{c_t}{1+r}$$

Consumption: Intertemporal Choice

Three Period Example

Period:	1	2	3
Labour Income	£100.00	£250.00	£0.00
Consumption	£110.00	£120.00	£125.48
Interest (paid or received)		£10.50	£125.48
Assets/Liabilities	£10.00	£119.50	£0.00

- A £10 loan is required to fund excess consumption in period 1
- Total income in period 2 is labour income (£250) minus repayment of loan (£10 x 1.05 = £10.50) = £239.50
- After subtracting consumption, the individual has assets of £119.50
- The principal and interest from the assets (£119.50 x 1.05 = £125.48) will cover consumption in period 3

Consumption: Intertemporal Choice

Three Period Example

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- Present Value of Consumption

$$\frac{c_1}{(1+r)} + \frac{c_2}{(1+r)^2} + \frac{c_3}{(1+r)^3} =$$

$$\frac{£110}{(1.05)} + \frac{£120}{(1.05)^2} + \frac{£125.475}{(1.05)^3} = £322$$

- Present Value of Income

$$\frac{y_1}{(1+r)} + \frac{y_2}{(1+r)^2} + \frac{y_3}{(1+r)^3} =$$

$$\frac{£100}{(1.05)} + \frac{£250}{(1.05)^2} + \frac{£0}{(1.05)^3} = £322$$

Consumption: Intertemporal Choice

Maths

$$L = E_t\left[\sum_{s=t}^{T}\left(\frac{1}{1+\delta}\right)^{s-t+1} u(c_s) - \lambda\left(\sum_{s=t}^{T}\left(\frac{1}{1+r}\right)^{s-t+1} c_s - W_t\right)\right]$$

$$= E_t\left[\frac{u(c_t)}{(1+\delta)} + \frac{u(c_{t+1})}{(1+\delta)^2} + \ldots - \lambda\left(\frac{c_t}{(1+r)} + \frac{c_{t+1}}{(1+r)^2} + \ldots - W_t\right)\right]$$

- Formally: maximise the *Lagrangian*.
- Given financial assets and expectations of future income (i.e. given *W*), the agent will formulate a consumption plan that maximises *L*.

Consumption: Intertemporal Choice

Maths

$$L = E_t\left[\frac{u(c_t)}{(1+\delta)} + \frac{u(c_{t+1})}{(1+\delta)^2} + \ldots - \lambda\left(\frac{c_t}{(1+r)} + \frac{c_{t+1}}{(1+r)^2} + \ldots - W_t\right)\right]$$

$$\frac{\partial L}{\partial c_t} = \frac{u'(c_t)}{(1+\delta)} - \frac{\lambda}{(1+r)} = 0 \qquad \frac{\partial L}{\partial c_{t+1}} = E_t\left[\frac{u'(c_{t+1})}{(1+\delta)^2}\right] - \frac{\lambda}{(1+r)^2} = 0$$

$$\lambda = \frac{(1+r)}{(1+\delta)}u'(c_t) \qquad \boxed{E_t\left[u'(c_{t+1})\right] = \frac{(1+\delta)}{(1+r)}u'(c_t)}$$

- First-order conditions: first with respect to c_t and then with respect to c_{t+1}.
- Solving the first for λ - and substituting into the second gives

Consumption: Intertemporal Choice

Maths

$$L = E_t\left[\frac{u(c_t)}{(1+\delta)} + \frac{u(c_{t+1})}{(1+\delta)^2} + \ldots - \lambda\left(\frac{c_t}{(1+r)} + \frac{c_{t+1}}{(1+r)^2} + \ldots - W_t\right)\right]$$

$$\frac{\partial L}{\partial c_{t+1}} = E_t\left[\frac{u'(c_{t+1})}{(1+\delta)^2}\right] - \frac{\lambda}{(1+r)^2} = 0 \qquad \lambda = \frac{(1+r)^2}{(1+\delta)^2}E_t\left[u'(c_{t+1})\right]$$

$$\frac{\partial L}{\partial c_{t+2}} = E_t\left[\frac{u'(c_{t+2})}{(1+\delta)^3}\right] - \frac{\lambda}{(1+r)^3} = 0 \qquad \boxed{E_t\left[u'(c_{t+2})\right] = E_t\left[\frac{(1+\delta)}{(1+r)}u'(c_{t+1})\right]}$$

- First-order condition with respect to c_{t+1} and with respect to $c_{t+2.}$
- Solving the first for λ - and substituting into the second gives.
- There are $T - t$ Euler equations, from which the optimal consumption sequence is derived.

Consumption: Intertemporal Choice

Euler Equation: $$u'(c_{t+1}) = \frac{(1+\delta)}{(1+r)} u'(c_t)$$

- Intuition:
  - Using the Euler equation, an individual selects consumption in periods t and $t+1$.
  - She cannot increase her utility (U) by consuming less (more) in one period and more (less) in another, remaining on her budget constraint.
  - No uncertainty assumed here - so no expectation term in the Euler equation.
  - Suppose the individual reduces her consumption by one unit (£1) in period t, enabling her (by the budget constraint) to spend £(1+r) more in period $t+1$.
  - If her consumption were optimal, her utility would not be improved.
  - The unit reduction in consumption in period t will reduce utility (U) by:

$$\frac{u'(c_t)}{(1+\delta)}$$ i.e. by her discounted marginal utility in period t.

Consumption: Intertemporal Choice

Euler Equation: $$u'(c_{t+1}) = \frac{(1+\delta)}{(1+r)} u'(c_t)$$

- Intuition:
  - Consumption in period *t+1* is increased by (1+*r*) and this raises *U* by:

$$\frac{(1+r)u'(c_{t+1})}{(1+\delta)^2} = \frac{\text{Change in consumption } (1+r) \text{ x marginal utility}}{(1+\delta)^2}$$

- By spending £1 less in period *t* utility is lowered by

$$\frac{u'(c_t)}{(1+\delta)} = \frac{(1+r)u'(c_{t+1})}{(1+\delta)^2} \quad \text{or} \quad u'(c_{t+1}) = \frac{(1+\delta)}{(1+r)} u'(c_t)$$

- By spending £1+*r* more in period *t+1* utility is raised by
- If her plan were optimal, these should be offsetting.

Consumption: Intertemporal Choice

- Summary:
  - The individual's problem is to allocate consumption over time for any expected path of labour income.
  - Her problem can be written formally:

$$\max_{c_s} L = E_t\left[\sum_{s=t}^{T}\left(\frac{1}{1+\delta}\right)^{s-t+1} u(c_s) - \lambda\left(\sum_{s=t}^{T}\left(\frac{1}{1+r}\right)^{s-t+1} c_s - W_t\right)\right]$$

  - The solution leads to pairs of *Euler* equations for each period of her planning horizon:

Euler Equation: $$u'(c_{t+1}) = \frac{(1+\delta)}{(1+r)} u'(c_t)$$

  - In order to use the model to explain behaviour, we need to be more precise about the nature of the utility function.

Permanent Income Hypothesis

- Euler Equation:

$$E_t\left[u'(c_{t+1})\right] = \frac{(1+\delta)}{(1+r)}u'(c_t)$$

- Use the Euler equation to derive the *Permanent Income Hypothesis* (PIH) of consumption.

- This model originally suggested by Friedman in 1957

- PIH requires either:
  - perfect certainty or
  - quadratic preferences

- Examine each in turn.

PIH: Perfect Certainty

- Euler Equation: $E_t\left[u'(c_{t+1})\right] = \frac{(1+\delta)}{(1+r)}u'(c_t)$

- With perfect certainty, individual knows exactly consumption in *t+1* [future income is known]
- Hence we can remove the expectation operator

$$E_t y_{t+s} = y_{t+s} \Rightarrow u'(c_{t+1}) = \frac{(1+\delta)}{(1+r)}u'(c_t)$$

- Solve this equation for consumption given any utility function *u*(.)

PIH: Perfect Certainty

$$\boxed{u'(c_{t+1}) = \frac{(1+\delta)}{(1+r)} u'(c_t)}$$

- For simplicity:

$$\delta = r$$ Subjective discount rate equals real interest rate.

$$u'(c_{t+1}) = u'(c_t)$$

$$f(c_{t+1}) = f(c_t)$$

- $f(.)$ is marginal utility

- Solve for c_{t+1}

$$c_{t+1} = f^{-1}\left[f(c_t)\right]$$

$$= c_t$$

PIH: Perfect Certainty

$$\boxed{u'(c_{t+1}) = u'(c_t) \quad [r = \delta]}$$

- Example: Constant Relative Risk Aversion:

$$u(c_s) = \frac{c_s^{(1-\rho)}}{(1-\rho)} \quad \text{so} \quad u'(c_s) = c_s^{-\rho}$$

- The Euler equation is then:

$$u'(c_{t+1}) = u'(c_t)$$

$$c_{t+1}^{-\rho} = c_t^{-\rho}$$

$$c_{t+1} \equiv c_t$$

PIH: Perfect Certainty

$$\boxed{u'(c_{t+1}) = u'(c_t) \quad [r = \delta]}$$

- From the Euler equation between periods *t*+2 and *t+1*:

$$c_{t+2} = c_{t+1} = c_t$$

- In general: $c_s = c_t$ for all $s > t$

- So the budget constraint can be re-written:

$$\sum_{s=t}^{T} \left(\frac{1}{1+r}\right)^{s-t+1} E_t(c_s) = \sum_{s=t}^{T} \left(\frac{1}{1+r}\right)^{s-t+1} c_s = W_t$$

$$\sum_{s=t}^{T} \left(\frac{1}{1+r}\right)^{s-t+1} c_t = c_t \sum_{s=t}^{T} \left(\frac{1}{1+r}\right)^{s-t+1} = W_t$$

PIH: Perfect Certainty

- Digression: sum of geometric series

$$k = \sum_{i=0}^{\infty} x^i \quad \text{where } 0 < x < 1$$

$$k = 1 + x + x^2 + x^3 + \ldots$$

$$kx = x + x^2 + x^3 + \ldots$$

- Subtract *kx from k*

$$k - kx = 1 + x^{\infty} = 1$$

$$k = \frac{1}{1 - x}$$

PIH: Perfect Certainty

- Simplify $c_t \sum_{s=t}^{T} \left(\frac{1}{1+r} \right)^{s-t+1} = W_t = c_t \left(\frac{1}{1+r} \right) \sum_{s=t}^{T} \left(\frac{1}{1+r} \right)^{s-t}$

- Sum of a geometric series for large T

$$c_t \left(\frac{1}{1+r} \right) \left(\frac{1}{1 - \frac{1}{1+r}} \right) = c_t \left(\frac{1}{1+r} \right) \left(\frac{1}{\frac{1+r-1}{1+r}} \right) = c_t \left(\frac{1}{1+r} \right) \left(\frac{1}{\frac{r}{1+r}} \right) = c_t \left(\frac{1}{r} \right) = W_t$$

So: $c_t = rW_t$ - consumption is proportional to wealth