PIH: Quadratic Preferences

- Perfect certainty unrealistic
- Re-introduce uncertainty but assume *quadratic preferences*:

$$u(c_s) = -\frac{1}{2}\left(c_s - c^*\right)^2 \qquad u'(c_s) = c^* - c_s$$

'Bliss' c – marginal utility zero when c is at bliss point

- The Euler equation is now (again assuming $r = \delta$):

$$E_t\left[u'(c_{t+1})\right] = u'(c_t)$$

$$E_t\left[c^* - c_{t+1}\right] = c^* - c_t$$

$$c^* - E_t(c_{t+1}) = c^* - c_t$$

$$E_t(c_{t+1}) = c_t$$

Using Euler equations for each pair of adjacent periods:

$E_t(c_s) = c_t$ for all $s > t$

PIH: Quadratic Preferences

- As with perfect certainty the budget constraint is:

$$\sum_{s=t}^{T}\left(\frac{1}{1+r}\right)^{s-t+1} E_t(c_s) = c_t \sum_{s=t}^{T}\left(\frac{1}{1+r}\right)^{s-t+1} = W_t$$

  - and this can be simplified to:

$$c_t = rW_t$$

- *Definition:* Permanent income is the *constant* income stream with the *same present value* as expected future income:

$$\sum_{s=t}^{T}\left(\frac{1}{1+r}\right)^{s-t+1} y_t^p = \sum_{s=t}^{T}\left(\frac{1}{1+r}\right)^{s-t+1} E_t(y_s) = W_t$$

PIH: Quadratic Preferences

$$\sum_{s=t}^{T}\left(\frac{1}{1+r}\right)^{s-t+1} y_t^p = \sum_{s=t}^{T}\left(\frac{1}{1+r}\right)^{s-t+1} E_t(y_s) = W_t$$

- Solve for permanent income:

$$y_t^p \sum_{s=t}^{T}\left(\frac{1}{1+r}\right)^{s-t+1} = y_t^p \left(\frac{1}{1+r}\right) \sum_{s=t}^{T}\left(\frac{1}{1+r}\right)^{s-t} = W_t$$

$$y_t^p \left(\frac{1}{1+r}\right)\left(\frac{1+r}{r}\right) = W_t \qquad y_t^p = rW_t$$

- Thus with quadratic preferences and $r = \delta$:

$$\boxed{c_t = y_t^p = rW_t}$$

PIH: Random Walk Model

- The Euler equation with $r = \delta$: $E_t(c_{t+1}) = c_t$
- Actual consumption in *t+1* will differ from expected by unpredictable shocks:

$$\begin{aligned} c_{t+1} &= E_t(c_{t+1}) + \omega_{t+1} \\ &= c_t + \omega_{t+1} \end{aligned}$$

- The term ω reflects:
  - new information available in period *t+1*
  - that could not be predicted given information available in *t*
- The PIH predicts that consumption should follow a *random walk*

PIH: Random Walk Model

- To test the model we estimate the following *statistical* model:

$$c_t = \alpha c_{t-1} + \beta \Delta \hat{y}_t^d + \varepsilon_t \qquad \text{PIH: } c_t = c_{t-1} + \omega_t \text{ or } \Delta c_t = \omega_t$$

predicted change in disposable income

- and compare it with the PIH model.

- If the PIH model were true, we would expect to find:

$\hat{\alpha} = 1$ consumption will have a *unit root*.

$\hat{\beta} = 0$ the change in consumption will be *unpredictable*.

PIH: Random Walk Model

- Estimation Results:

$$c_t = \hat{\alpha} c_{t-1} + \hat{\beta} \Delta \hat{y}_t^d + \hat{\varepsilon}_t \quad \Rightarrow \quad \hat{\alpha} = 1 \quad \Rightarrow \quad \Delta c_t = \hat{\beta} \Delta \hat{y}_t^d + \hat{\varepsilon}_t$$

- Importantly the change in consumption is significantly influenced by the predicted change in disposable income: $\hat{\beta} > 0$

  - Marjorie Flavin (1981), using US data, found consumption to be ‘excessively sensitive’ to income.

  - John Campbell and Greg Mankiw (1991) confirmed this for the US and other countries.

PIH: Random Walk Model

- Campbell and Mankiw (1991) explanation

- Two types of consumers:
  - Rational (R) for whom PIH applies
  - Income constrained (IR)

- Rational consumers: $\Delta c_t^R = \varepsilon_t$

- Income constrained consumers: $\Delta c_t^{IC} = \lambda \Delta \hat{y}_t^d$

Proportion income constrained

- In aggregate: $\Delta c_t = \Delta c_t^R + \Delta c_t^{IC} = \lambda \Delta \hat{y}_t^d + \varepsilon_t$

- They find λ **in the range 0.351 to 0.698**

PIH: Summary

- Under quadratic preferences the Euler equation implies that consumption will follow a random walk

- This also implies that individuals will consume their *permanent income* – the *constant* income stream with the same present value as their *expected actual* income.

- The change in consumption is not random but predictable from past data, hence the PIH is rejected by the data.

- Campbell and Mankiw argue that some consumers are income constrained and this explains the empirical failure of PIH.

Buffer Stock Model

- Permanent Income Hypothesis: $u(c_s) = -\frac{1}{2}(c_s - c^*)^2 \quad u'(c_s) = c^* - c_s$
  (quadratic preferences)

- Buffer Stock Model: $u(c_s) = \frac{c_s^{(1-\rho)}}{(1-\rho)}$ so $u'(c_s) = c_s^{-\rho}$
  (constant relative risk aversion preferences)

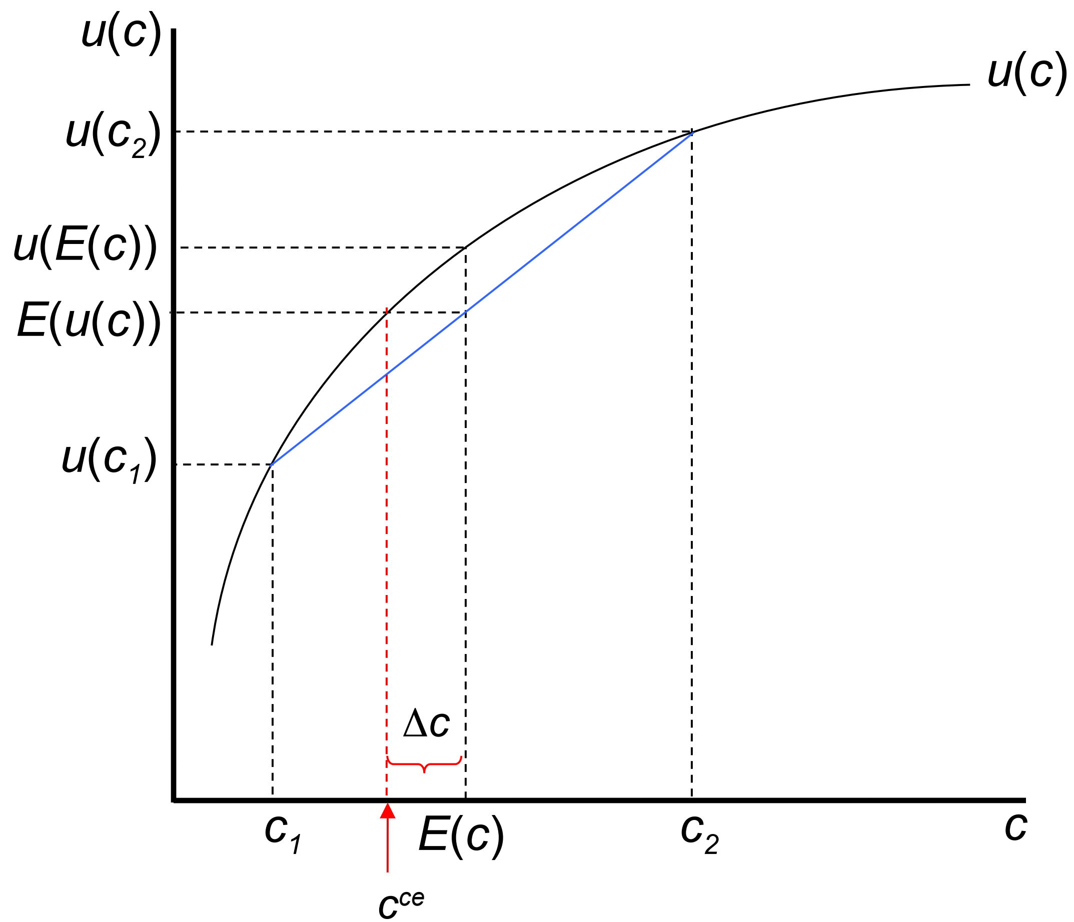
Marginal utility linear in consumption

Marginal utility non-linear in consumption

- CRRA: leads to 'buffer stock model' (Carroll, Deaton)
- Agents prefer to hold positive buffer stocks in case the worst happens
- Consumption more correlated with income

Buffer Stock Model

- Individual can consume c_1 with probability 0.5
- and c_2, also with probability 0.5
- Expected consumption is $0.5(c_1 + c_2) = E(c)$
- *If* individual consumed $E(c)$ for sure, utility is $u(E(c))$
- However expected utility is $0.5(u(c_1)+ u(c_2)) = E(u(c))$
- The individual would be prepared to pay up to Δc to receive $E(c)$ for sure …
- and this depends on curvature of u
- c^{ce} is the 'certainty equivalent' to uncertain c



Buffer Stock Model

- The certainty equivalent c is defined by: $$u\left(c^{ce}\right)=E(u)$$
- or $$c^{ce}=u^{-1}\left(E(u)\right)$$
- In the CRRA case

$$\frac{c^{ce\,(1-\rho)}}{(1-\rho)}=0.5\left[\frac{c_1^{(1-\rho)}}{(1-\rho)}+\frac{c_2^{(1-\rho)}}{(1-\rho)}\right] \Rightarrow c^{ce}=\left\{0.5(1-\rho)\left[\frac{c_1^{(1-\rho)}}{(1-\rho)}+\frac{c_2^{(1-\rho)}}{(1-\rho)}\right]\right\}^{\frac{1}{1-\rho}}$$

- Assume c_1 = £50,000, c_2 = £100,000, $E(c)$ = £75,000

	ρ	c^{ce}	Δc
Risk Neutral	0	£75,000	£0
Risk Averse	2	£66,667	£8,333
	5	£58,566	£16,434
	50	£50,712	£24,288

Buffer Stock Model

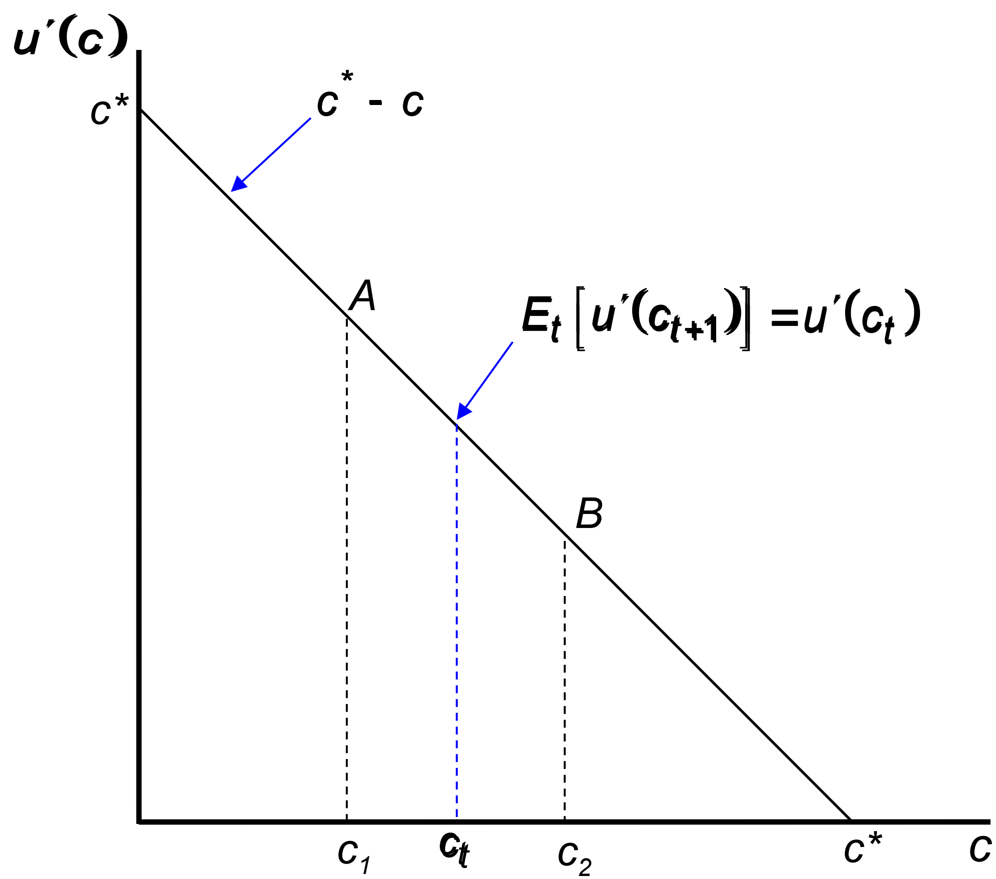
- Quadratic preferences

$$u(c) = -\frac{1}{2}\left(c - c^*\right)^2$$

$$u'(c) = c^* - c$$

- In period *t+1*
  - c_{t+1} = c_1 with probability 0.5
  - = c_2 with probability 0.5
- Expected marginal utility in *t+1* is therefore mid-way between *A* and *B*
- The Euler equation requires that marginal utility at *t* equals expected marginal utility.

$$E_t\left[u'(c_{t+1})\right] = u'(c_t)$$



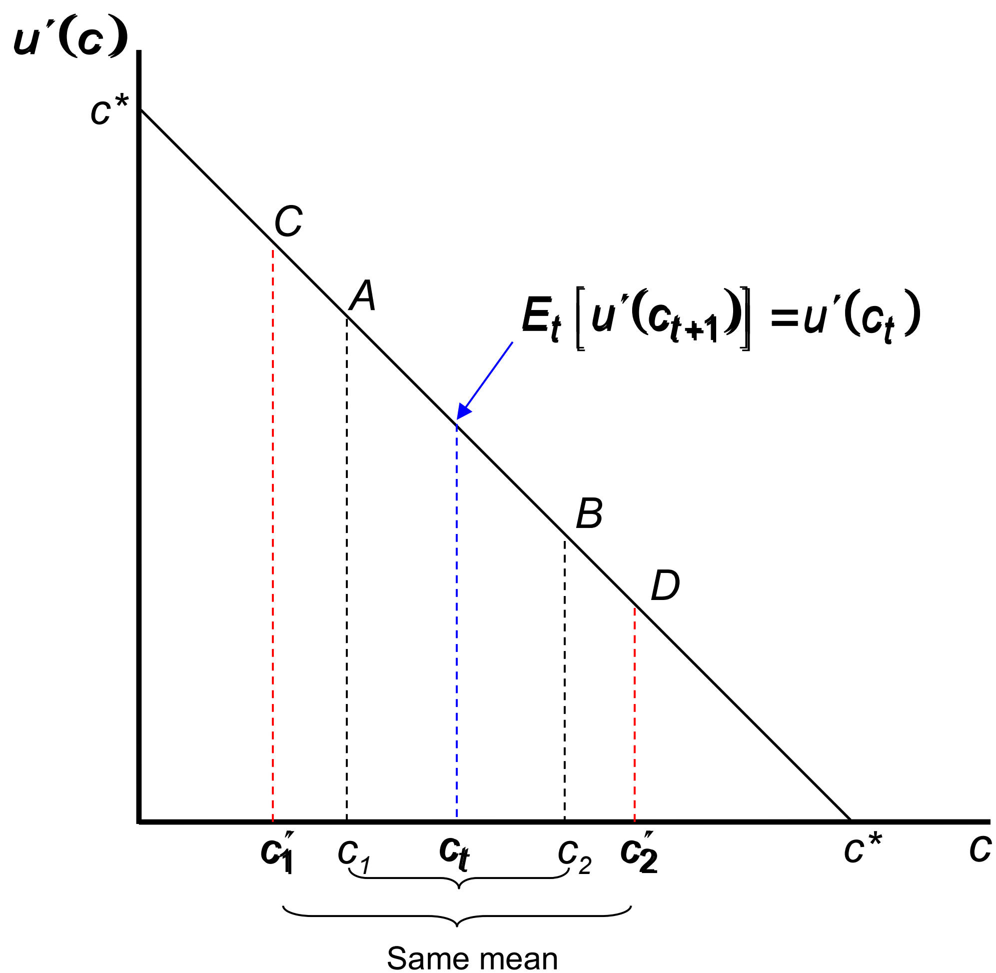
Buffer Stock Model

- Now consider the effects of a *mean-preserving* increase in uncertainty in period *t+1* so that

$$c_{t+1} = c_1'' \text{ with probability } 0.5$$
$$= c_2'' \text{ with probability } 0.5$$

- Expected marginal utility in *t+1* is now mid-way between *C* and *D*
- Because the marginal utility function is linear, this point is the same as that between *A* & *B*.
- Hence a mean-preserving increase in uncertainty has no affect on current consumption.

$$E_t\left[u'(c_{t+1})\right] = u'(c_t)$$



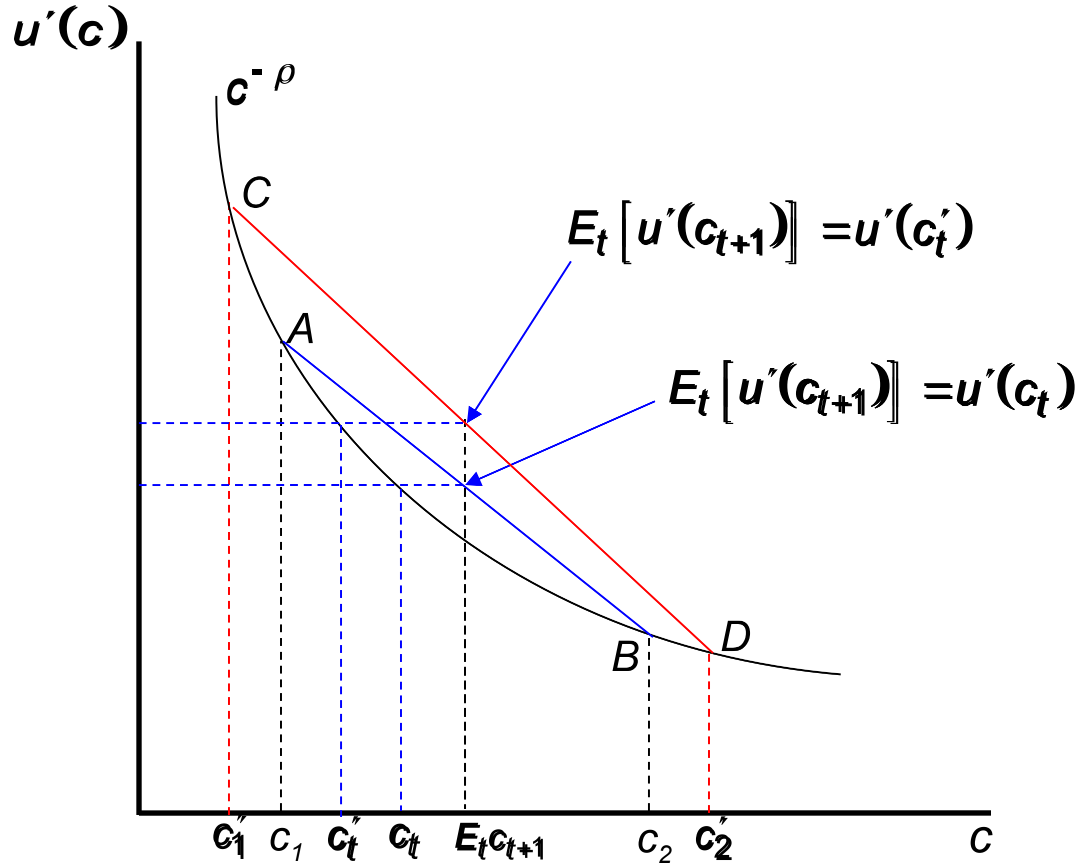
Buffer Stock Model

- Assume the same uncertain consumption in *t+1* but with CRRA marginal utility:

$$u'(c)=c^{-\rho}$$

- Expected marginal utility in *t+1* is now mid-way on a straight line between *A* and *B*
- Current consumption must satisfy the Euler equation – at c_t
- Now imagine the same mean-preserving increase in uncertainty
- Expected utility in *t+1* is midway between C and D
- The Euler equation is now satisfied at a lower level of consumption.

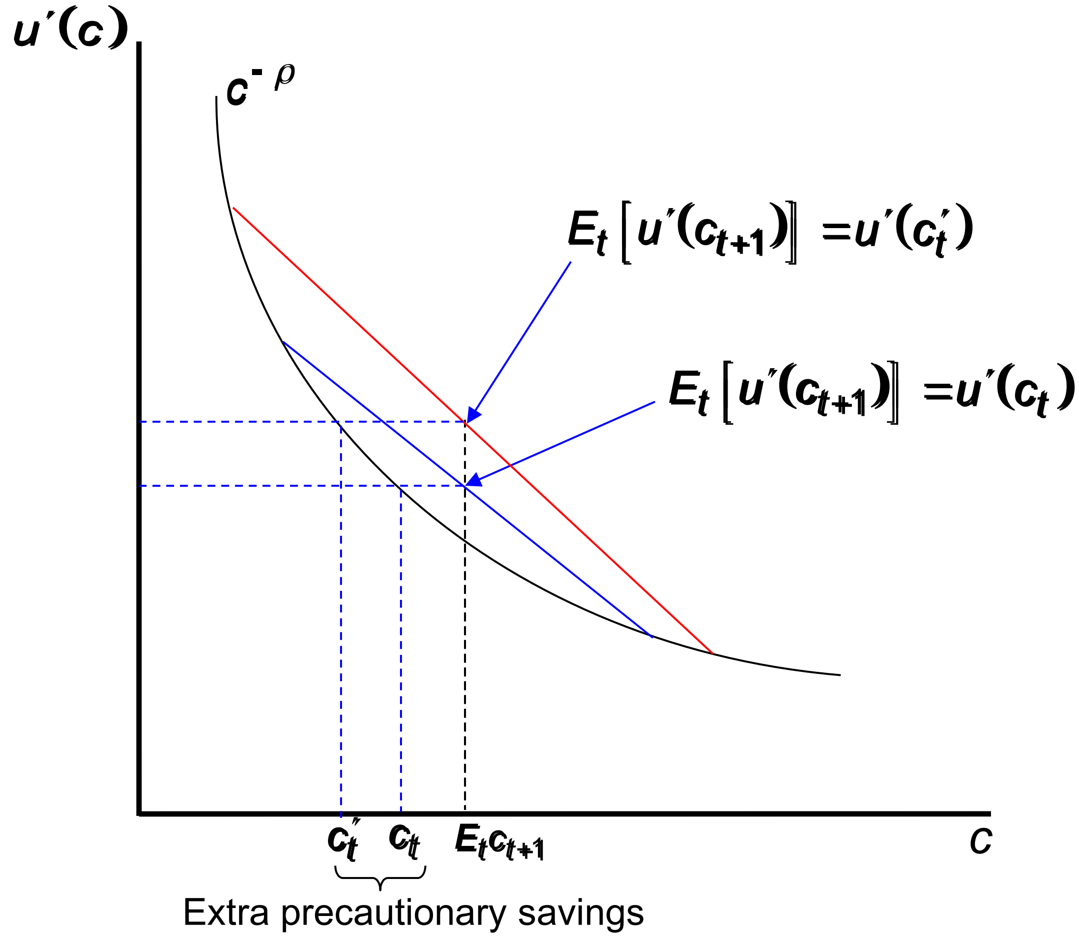
$$E_t\left[u'(c_{t+1})\right]=u'(c_t)$$



Buffer Stock Model

$$E_t\left[u'(c_{t+1})\right] = u'(c_t)$$

- An increase in uncertainty about future income causes the individual to consume less
- If current income is unaffected by the increased uncertainty, the individual will save more.
- The extra savings in period *t* are *precautionary* savings



Buffer Stock Model

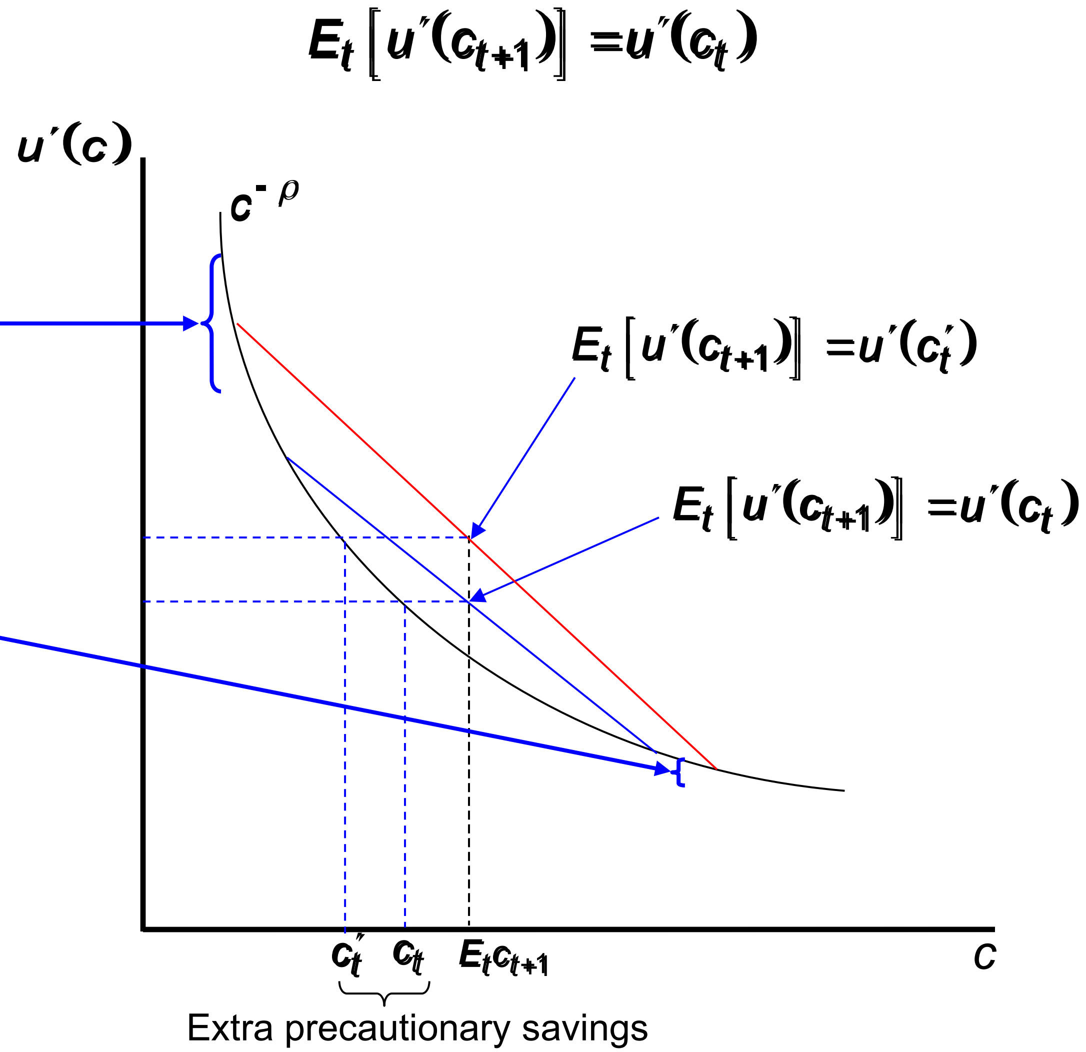
- Buffer stocks:

- Because marginal utility is non-linear in *c*, *utility losses* are greater when there is a unit fall in *c* at a low level of *c* …

- than *utility gains* from a unit rise in *c* at higher levels.

- Because agents will lose a great deal from an unexpected adverse shock, they will hold precautionary *buffer stocks*

- They will be unlikely to run up debt



Buffer Stock Model

- Carroll Model:
  - Individuals are impatient, with a strong preference for current consumption. They may be tempted to borrow.
  - However they are concerned about the possibility that they may be hit by an adverse shock in the future – their income may drop to zero.
  - They need to balance these competing pressures: they would like to consume their current income but they also need to build up some precautionary buffer stocks in case the worst happens.
  - At low levels of income, impatience may dominate, so consumption is close to income (hence ‘excess sensitivity’ of *c* to *y*)
  - At higher levels of income, the precautionary motive becomes more important and individuals save more.