Economic Growth

Exogenous Growth Models: The Solow-Swan Model and Evidence

The Solow-Swan Model

At k^*:

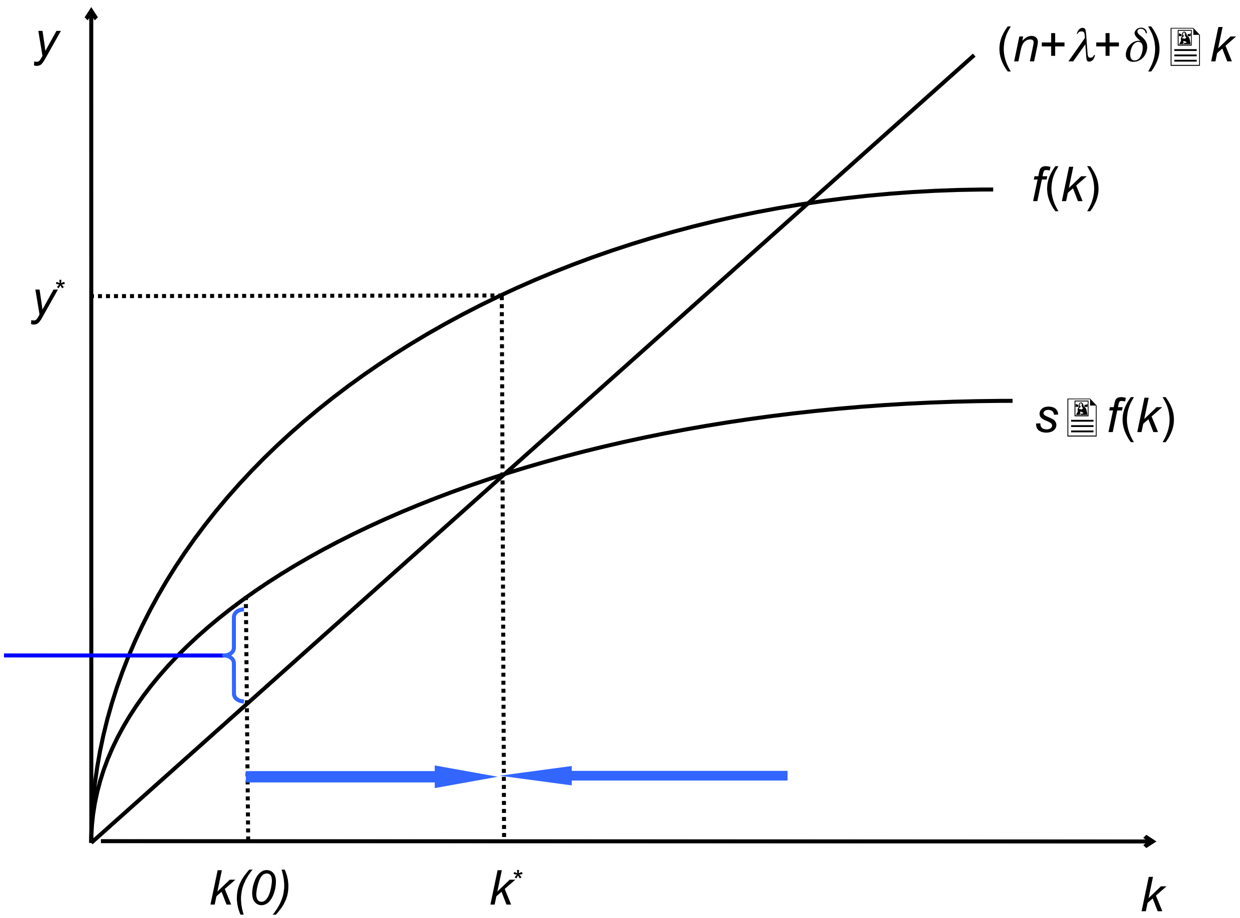
$$s \cdot f(k^*) = (n + \lambda + \delta)k^*$$

and $\dot{k} = 0$

At $k(0)$:

$$s \cdot f(k(0)) > (n + \lambda + \delta)k(0)$$

and $\dot{k} > 0$



Empirical evidence

1. Convergence?
   - Is there evidence that countries ‘catch up’ with other countries?
   - How *fast* is this process?
2. Quantitative relationship between GDP per capita and potential ‘drivers’
   - Savings rate
   - ‘Institutional quality’

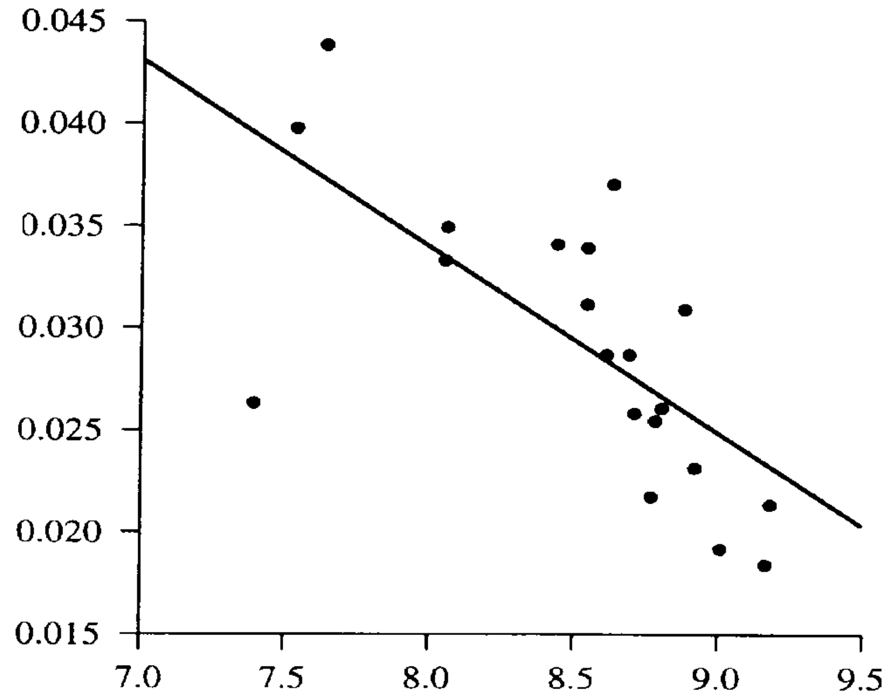
1. Convergence

- Similar graph of 20 OECD countries does indicate a measure of convergence

- These countries are more likely to be sharing the same steady-state levels of y – same values for s and n for example.

- Evidence of convergence *conditional on the steady-state level.*

GDP pc Growth
1960-1985



Real GDP pc 1960

Source: Barro and Sala-i-Martin (1995)

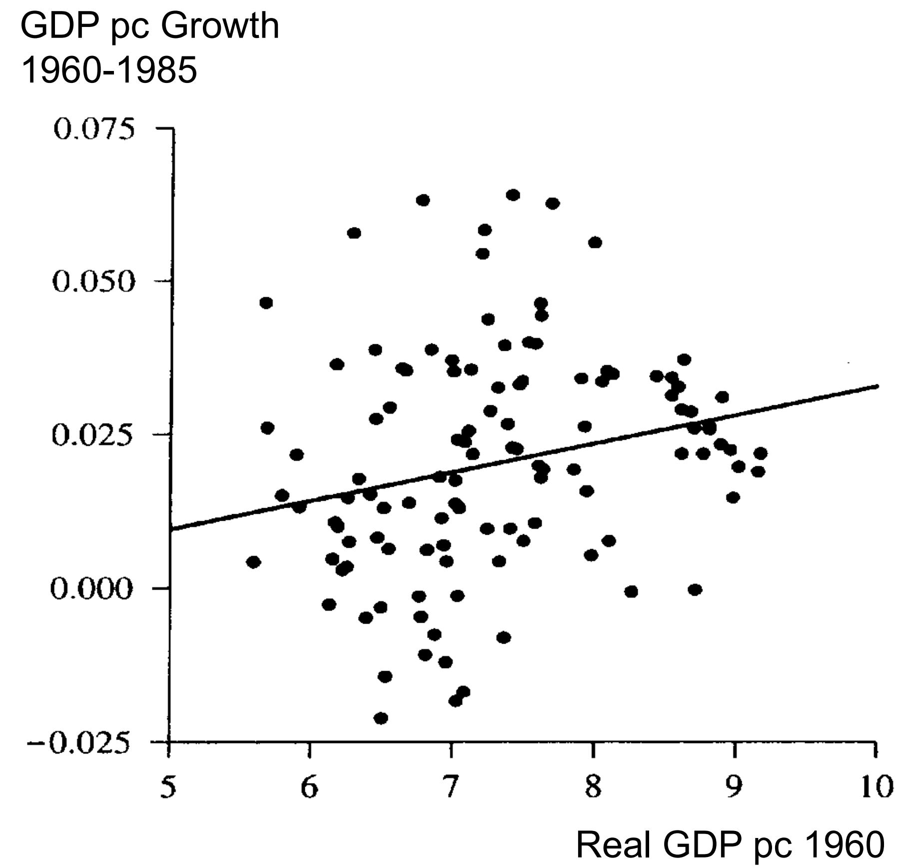
(Lack of) Convergence?

- Solow-Swan model predicts that, if all countries have same steady-state *y,* then rich countries will grow at slower rate than poor – *absolute convergence*.

- If countries have different steady-state *y* (due, say, to different *s* or *n*), then there is *conditional convergence*

- Real GDP growth rates of 118 countries and their initial levels of GDP - suggest absolute convergence is rejected

See also Pritchett (1997)



Source: Barro and Sala-i-Martin (1995)

Convergence

- Empirical cross-country estimates of convergence provide mixed results.
- A more positive take is here:

https://www.cgdev.org/blog/everything-you-know-about-cross-country-convergence-now-wrong

- Evidence fairly convincing in for ‘homogenous’ samples of countries/states – such as the OECD, or State-level data in the US.
- But even here, there are at least open questions about the *speed* of convergence.
- The Solow model predicts much faster convergence than that which occurs in reality.

2. The Solow-Swan Model Drivers of income per capita

- Consider the Cobb-Douglas case:

$f(k) = k^{\alpha}$ **where** α **is the share of capital (0.3)**

The steady-state solution is

$$sk^{\alpha} = (n+\lambda+\delta)k \quad \Rightarrow \quad k = \left(\frac{s}{n+\lambda+\delta}\right)^{\frac{1}{1-\alpha}}$$

- and the solution for y is

$$y = k^{\alpha} = \left(\frac{s}{n+\lambda+\delta}\right)^{\frac{\alpha}{1-\alpha}}$$

- Re-write in terms of output per worker

$$\frac{Y}{N} = yA = \left(\frac{s}{n+\lambda+\delta}\right)^{\frac{\alpha}{1-\alpha}} A$$

The Solow-Swan Model

$$\frac{Y}{N} = \left(\frac{s}{n+\lambda+\delta}\right)^{\frac{\alpha}{1-\alpha}} A$$

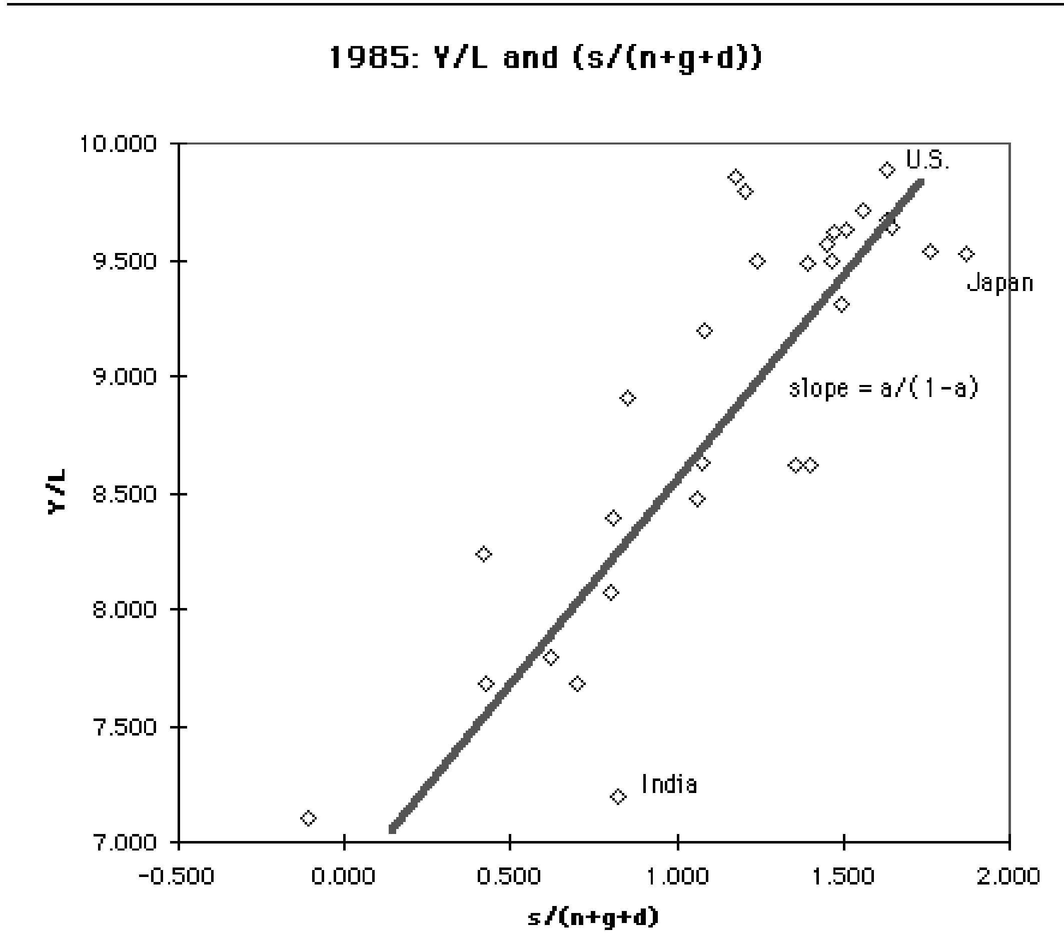
- Take logs $$\log\left(\frac{Y}{N}\right) = \frac{\alpha}{1-\alpha}\log\left(\frac{s}{n+\lambda+\delta}\right) + \log(A)$$

- In a cross-section of different countries:
  - with different values for *s* and *n* (assuming same values for λ, and δ)
  - with same technology *A*
  - in a plot of log *(Y/N)* on log(s/(*n*+ λ+ δ)))
    - find a positive slope
    - if α = 0.3 the slope should be around 0.4

The Solow-Swan Model

$$\log\left(\frac{Y}{N}\right) = \frac{\alpha}{1-\alpha}\log\left(\frac{s}{n+\lambda+\delta}\right) + \log(A)$$

- (Source: Brad De Long)
- Slope is greater than 0
- Though slope is approximately 5 not 0.4, implying α of 0.8…
- Relationship between Y/N and *s much* stronger in reality than implied by basic Solow



The Empirical Relationship between income per capita and savings/investment

- Much stronger than implied by Solow alone
- Implies *other* channels through which investment may drive (or be associated with) income per capita
- Education (Mankiw, Romer and Weil)
- Endogenous growth (also coming up)