

Investment

- This topic is related to consumption
 - Emphasis at this level on microfoundations
- Material quite closely follows chapter section on investment in Carlin and Soskice (2006) – scanned copy is available through the VLE reading list.

Investment

Types of Investment

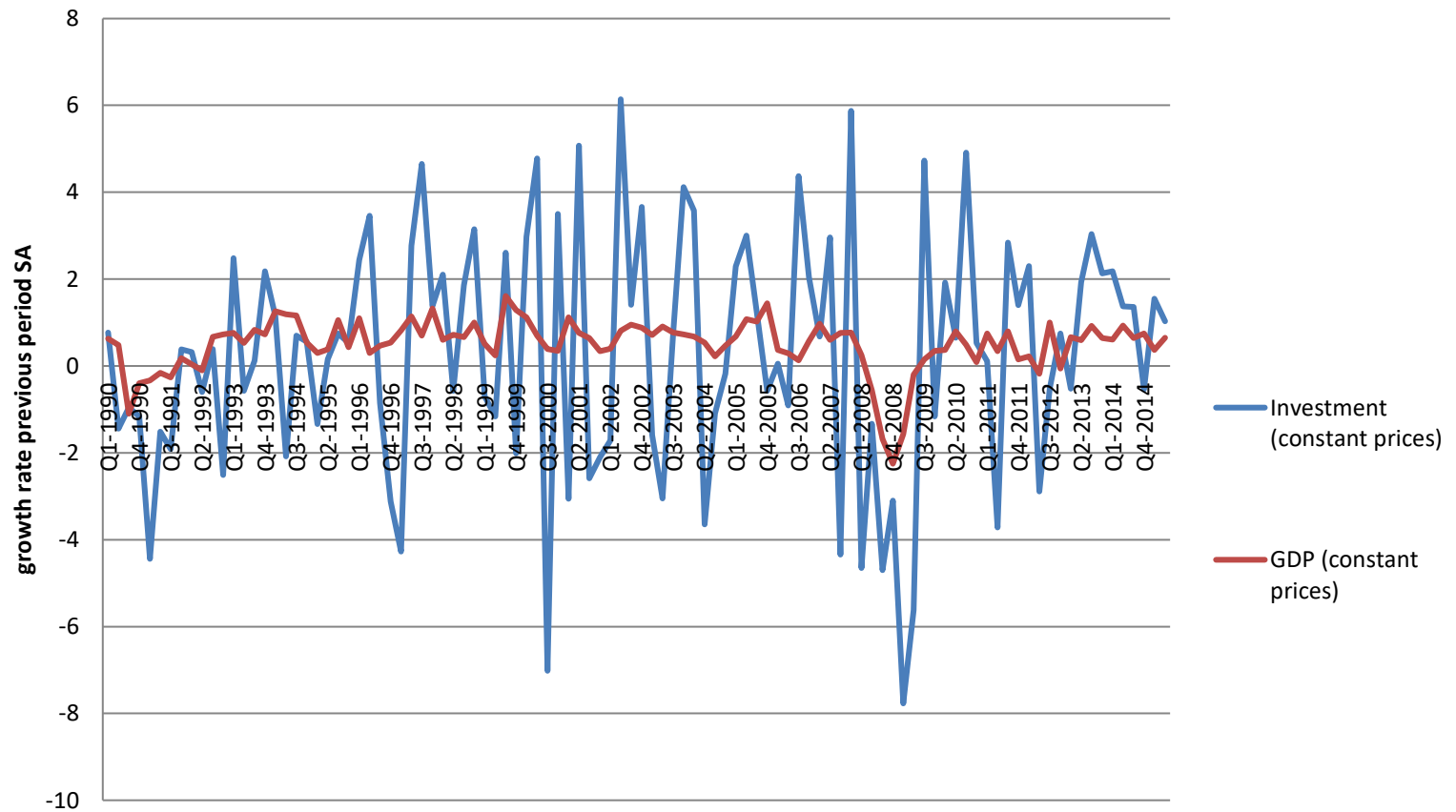
- Business fixed investment (plant and machinery) – “Fixed Capital Formation”
- Residential Investment (dwellings)
- Inventory Investment (stocks of goods)

We shall concentrate on the first of these

- Of all the components of GDP, investment is the most volatile.
- See next slide for UK data

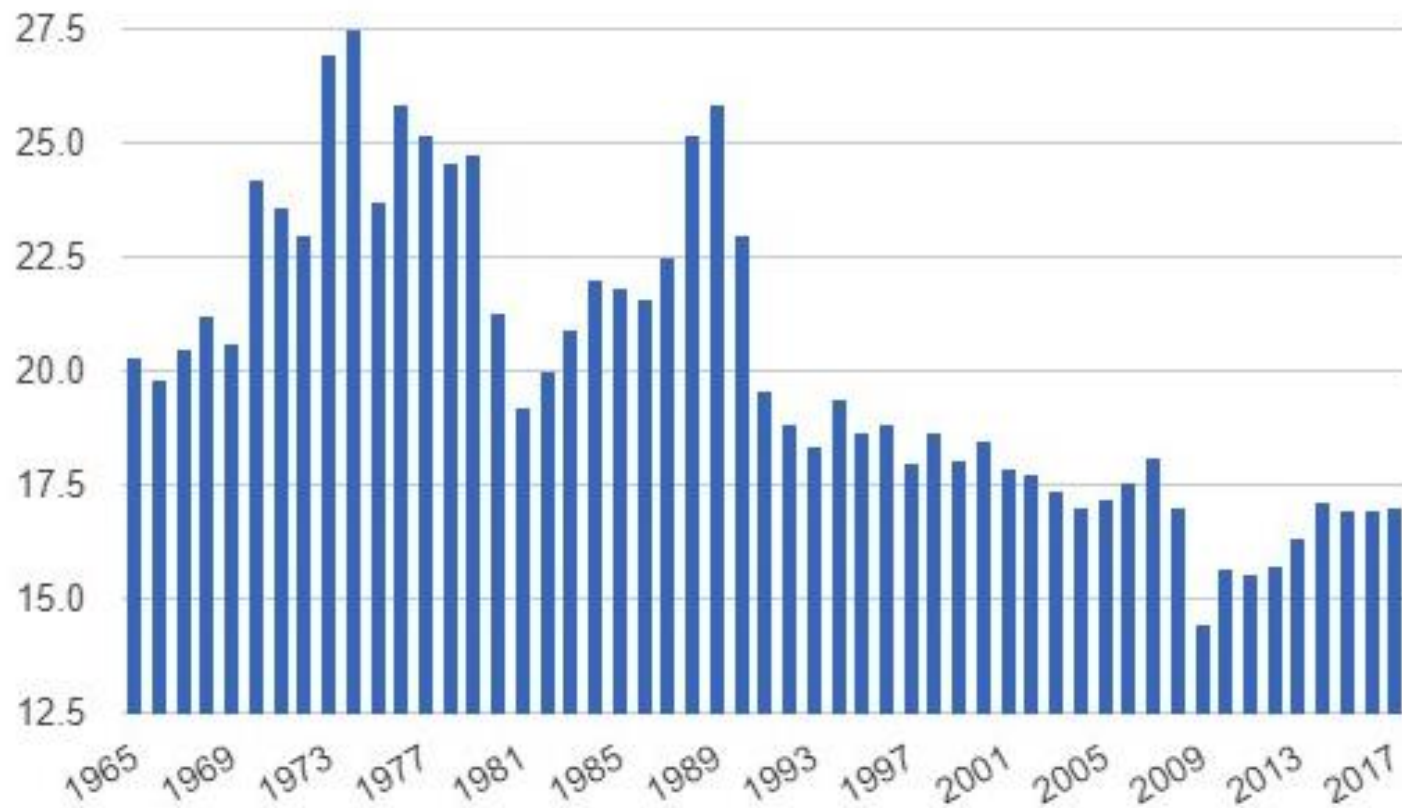
Investment

UK Investment and GDP



Investment

United Kingdom - Capital investment, percent of GDP



Microfounded Investment

Firms are assumed to be profit-maximisers
(One justification: this is a legal requirement for PLCs)

Suppose revenue in all periods into the future from a capital stock of K is:

$$Pf(K)$$

where P is the (fixed) price level and f is the production function (labour does not enter here by Occam's Razor)

Microfounded Investment

If the real interest rate is constant, then the Net Present Value of this revenue stream is:

$$V_0 = \frac{1}{1+r} Pf(K) + \frac{1}{(1+r)^2} Pf(K) + \frac{1}{(1+r)^3} Pf(K) + \dots$$

Hence

$$V_0 = \frac{Pf(K)}{r}$$

Microfounded Investment

Firms choose *investment* to maximize profits:

$$\Pi = \frac{Pf(I_t + K_{t-1})}{r} - C(I)$$

where $C(I)$ are investment costs.

We will consider three cases:

1. Perfect competition and no adjustment costs
2. Perfect competition and adjustment costs – Tobin's Q model of investment
3. Imperfect competition and no adjustment costs

Microfounded Investment

1. Perfect competition and no adjustment costs

Investment costs:

$$C(I) = P_I I$$

Where P_I is the exogenous price of investment goods and I is the volume of investment undertaken.

Microfounded Investment

1. Perfect competition and no adjustment costs

Profit-maximization at $t=0$ means maximization of:

$$\frac{Pf(I + K_{-1})}{r} - P_I I$$

Which requires

$$\frac{Pf'(K)}{r} - P_I = 0$$

hence

$$\frac{Pf'(K)}{rP_I} = 1$$

and $\frac{Pf'(K)}{rP_I} \equiv q$ (or 'marginal q ')
 rP_I

Microfounded Investment

1. Perfect competition and no adjustment costs

$$\frac{Pf'(K)}{rP_I} \equiv q \text{ (or 'marginal } q\text{')}$$

- The numerator – the marginal *value* product of capital
- The denominator – the ‘user cost of capital’

Microfounded Investment

1. Perfect competition and no adjustment costs

At the optimum,

$$\frac{Pf'(K^*)}{rP_I} = 1$$

$$f'(K^*) = \frac{rP_I}{P}$$

e.g. $f(K) = K^{0.5}$, then

$$0.5(K^*)^{-0.5} = \frac{rP_I}{P}$$

Microfounded Investment

1. Perfect competition and no adjustment costs
and

$$(K^*)^{-0.5} = \frac{2rP_I}{P}$$
$$K^* = \left(\frac{2rP_I}{P}\right)^{-\frac{1}{0.5}} = \left(\frac{2rP_I}{P}\right)^{-2} = \left(\frac{P}{2rP_I}\right)^2$$

Hence

$$I^* = K^* - K_{-1} = \left(\frac{P}{2rP_I}\right)^2 - K_{-1}$$

Microfounded Investment

1. Perfect competition and no adjustment costs

In general

$$I = I(r, P_I, P)$$

(underpinning the IS curve)

Investment:

- Falls with the interest rate
- Falls in the price of investment goods
- Increases in the price of final output goods

Microfounded Investment

2. Perfect competition and adjustment costs: Tobin's Q model of Investment

- e.g. costs associated with training workers to use new capital
- Adjustment costs normally assumed to be quadratic in the amount of investment:

$$C(I) = P_I I + \left(\frac{a}{2}\right) P_I I^2$$

Microfounded Investment

2. Perfect competition and adjustment costs: Tobin's Q model of Investment

Analysis now incorporates depreciation, hence

$$I_t = K_t - (1 - \delta)K_{t-1}$$

To simplify, assume linear production: $y = f_K K$, where f_K is the constant marginal product of capital.

Microfounded Investment

2. Perfect competition and adjustment costs: Tobin's Q model of Investment

Firm now chooses I to maximize:

$$\frac{Pf_K K}{1+r} + \frac{Pf_K K(1-\delta)}{(1+r)^2} + \dots - P_I I - \left(\frac{a}{2}\right) P_I I^2$$

Which simplifies to

$$\frac{Pf_K (I_t + (1-\delta)K_{t-1})}{r + \delta} - P_I I - \left(\frac{a}{2}\right) P_I I^2$$

Microfounded Investment

2. Perfect competition and adjustment costs: Tobin's Q model of Investment

The first-order condition is

$$\frac{Pf_K}{r + \delta} - P_I - aP_I I = 0$$

Which implies

$$I = \frac{1}{a} \left(\frac{Pf_K}{P_I(r + \delta)} - 1 \right)$$

Microfounded Investment

2. Perfect competition and adjustment costs: Tobin's Q model of Investment

i.e.

$$I = \frac{1}{a} (q - 1)$$

Note that q is a *sufficient* determinant of I .

Microfounded Investment

2. Perfect competition and adjustment costs: Tobin's Q model of Investment

- Greater adjustment costs (a)
 - *dampen* response of I to all other drivers
 - Could cause higher or lower I depending on whether $q \gtrless 1$.
- Greater value of marginal product of capital (Pf_K)
 - Increases investment (e.g. induced by technological progress or demand)

Microfounded Investment

2. Perfect competition and adjustment costs: Tobin's Q model of Investment

- Increases in *Relative* prices of capital goods (P_I/P)
 - reduces I
 - e.g. investment subsidies
- Increases in the real interest rate (r)
 - Reduces investment
- Increases in the depreciation rate (δ)
 - Reduces investment

Microfounded Investment

2. Perfect competition and adjustment costs: Tobin's Q model of Investment

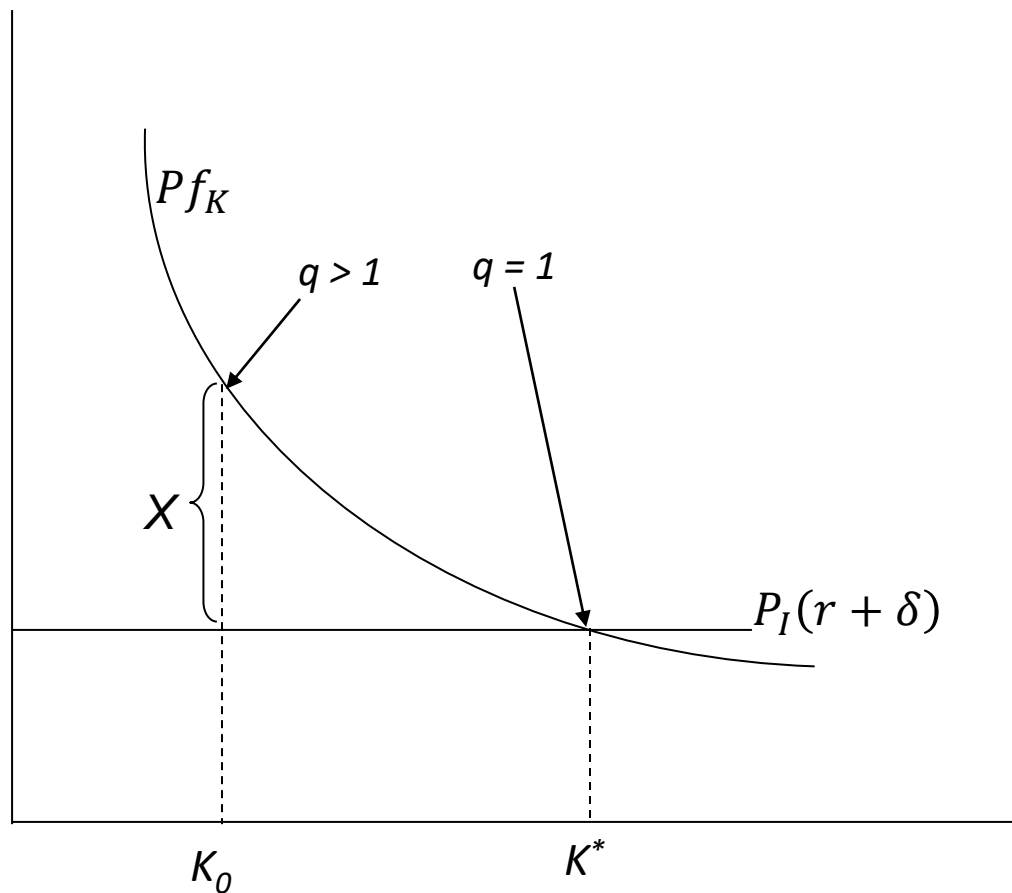
Measuring q

Recall $I = \frac{1}{a}(q - 1)$ and $q = \frac{Pf_K}{P_I(r+\delta)}$.

Suppose a small change in the capital stock of a firm. Then

$$q = \frac{\Delta \text{Market Value}}{\text{purchase price of additional capital goods}}$$

Tobin's q Model of investment



- Firms with a capital stock of K_0 receive revenue of X above cost of capital on each additional unit of K
- The market value of firm depends on expected future profits.
- Because of X the market will place a high value on firm - so ($q > 1$)
- Investment will take K to K^* where $q = 1$

Microfounded Investment

2. Perfect competition and adjustment costs: Tobin's Q model of Investment

Defining (average) Q :

$$Q = \frac{\text{Market Value}}{\text{replacement cost of capital stock}}$$

Q and q are related, but also Q is observable (i.e. at the level of the firm.)

Microfounded Investment

2. Perfect competition and adjustment costs: Tobin's Q model of Investment

$$Q = \frac{\text{Market Value}}{\text{replacement cost of capital stock}}$$

The role of Q in a simple example:

- Average market price of house: £200,000
- Cost to build a new house: £120,000
- Tobin's $Q = \frac{£200,000}{£120,000} = 1.67$
- Since $Q > 1$ investment (building) will take place

Microfounded Investment

3. Imperfect competition and no adjustment costs

- Price now endogenous, with demand curve:

$$y_i = P_i^{-\eta} \left(\frac{Y}{n} \right) \Leftrightarrow P_i = y_i^{-\frac{1}{\eta}} \left(\frac{Y}{n} \right)^{\frac{1}{\eta}}$$

Where Y is aggregate demand, η is the elasticity of demand and n is the number of sectors (each firm is a sector: $i=1, \dots, n$).

Microfounded Investment

3. Imperfect competition and no adjustment costs

Issue: if the firm increases K_i , it now results in a fall in price.

Choice of investment level maximizes:

$$\frac{P_i f(K_i)}{r + \delta} - P_I I_i$$

With First Order Condition (FOC):

$$\frac{1}{r + \delta} \frac{\partial (P_i f(K_i))}{\partial I_i} - P_I = 0$$

Microfounded Investment

3. Imperfect competition and no adjustment costs

In the (simplifying) situation of $y_i = f(K_i) = K_i$

$$\begin{aligned} \frac{\partial(P_i f(K_i))}{\partial I_i} &= \frac{\partial \left(y_i^{-\frac{1}{\eta}} \left(\frac{Y}{n} \right)^{\frac{1}{\eta}} K_i \right)}{\partial I_i} \\ &= \frac{\partial \left(K_i^{-\frac{1}{\eta}} \left(\frac{Y}{n} \right)^{\frac{1}{\eta}} K_i \right)}{\partial I_i} = \frac{\partial \left(K_i^{1-\frac{1}{\eta}} \left(\frac{Y}{n} \right)^{\frac{1}{\eta}} \right)}{\partial I_i} \end{aligned}$$

Microfounded Investment

3. Imperfect competition and no adjustment costs

Note that $K_i = (1 - \delta)K_{i,t-1} + I_i$, hence

$$\frac{\partial(P_i f(K_i))}{\partial I_i} = \frac{(\eta - 1)}{\eta} \left(\frac{Y}{n}\right)^{\frac{1}{\eta}} K_i^{-\frac{1}{\eta}}$$

Substituting this into the FOC above, then

$$\frac{1}{r + \delta} \frac{(\eta - 1)}{\eta} \left(\frac{Y}{n}\right)^{\frac{1}{\eta}} K_i^{-\frac{1}{\eta}} - P_I = 0$$

Microfounded Investment

3. Imperfect competition and no adjustment costs

Hence

$$K_i = \left(\frac{\eta - 1}{\eta(r + \delta)P_I} \right)^\eta \frac{Y}{n}$$

And so using $I_i = K_i - (1 - \delta)K_{i,t-1}$ then

$$I_i = \frac{1}{n} \left(\frac{\eta - 1}{\eta(r + \delta)P_I} \right)^\eta (Y - (1 - \delta)Y_{t-1})$$

This is the accelerator model of investment.

Evidence

- Microfounded theories argue for the importance of:
 - q (or Q) – which depends on the ratio of the market value of the firm and the replacement cost of capital (in the case of competition in goods markets)
 - Economic growth – or ‘scale’ effects (in the case of imperfect competition)
- Empirically the former seem to be unimportant relative to the latter

Evidence

Why?

- Real (goods) markets are imperfect?
- Observed market value (share price*number of shares – or ‘market capitalization’) a poor measure of true market value (i.e. capital markets arguably not fully efficient, and subject to noise trade)
- Investment is ‘lumpy’ – hence *observed* $Q > 1$, but investment may still not take place
 - Observed data on investment is also *very* discontinuous

Evidence

Why?

- Cash flow & Credit constraints
 - Reminiscent of empirical consumption findings: ‘excess sensitivity’ to cashflow
- The role of uncertainty & risk aversion
 - Entails ‘option value’ to *not* investing
 - Hence expected rate of return must be meaningfully higher than investment costs

Conclusions

- Investment remains difficult to understand, specify or certainly forecast.
- The emphasis here is on microfounded decisions – whether embedded in:
 - Competitive markets – Tobin's Q model of investment – arguing that $Q = \frac{\text{Market Value}}{\text{replacement cost of capital stock}}$ is a sufficient determinant of investment
 - Imperfect competition – emphasising scale-economies – and giving an underpinning for the Accelerator model