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## Elsevier TCS: Author response

Dear Paul Spirakis, Dear anonymous reviewers:

Thank you very much for your time and good feedback on our work.

We now prepared a major revision of our submission, trying to address all your comments and good suggestions. Attached, we include a detailed discussion of all the changes made.

We very much hope that our revision addresses your concerns, and will be happy to take any further inputs.

Kind regards, on behalf of all authors,

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### Response to Reviewer #2:

QUESTIONS / FUTURE WORK that the authors or others might consider interesting:

1) All edge lengths in the physical network are unit. What would happen if they were not? Presumably the flow-based algorithms would still work, but perhaps something changes in the dynamic programming approach?
I.e., perhaps some of them become pseudopolynomial?

%%%%%%%%% RESPONSE %%%%%%%%%%%%

2) Perhaps some relaxation of the requirement that replicas are distributed uniformly between nodes makes sense?

%%%%%%%%% RESPONSE %%%%%%%%%%%%

The reviewer has a good point, and indeed, such a generalization would be interesting to study. However, for this paper and due to space and time constraints, we decided

3) As some of the problems are NP-hard, and moreover of practical interest, considering approximation algorithms seems a worthwhile direction.

#### %%%%%%%%% RESPONSE %%%%%%%%%%%%

3) I suppose that one might argue that in the applications the underlying physical network is (more or less) fixed. In such case, when the instance specifies only the data distribution, number of nodes to be embedded, and the b\_1 and b\_2 coefficients, would some of the previously NP-hard problems become tractable?

#### %%%%%%%%% RESPONSE %%%%%%%%%%%

Assume that the physical network is fixed and name it T\_fixed. We presented hardness results for two variants:

- 1) RS+MA+NI using the construction that we name (for sake of this response) the C1
- 2) RS+MA+FP using the construction that we name (for sake of this response) the C2
- 3) RS(2)+MA+FP using the construction that we name (for sake of this response) the C3
- 4) RS(2)+NI+FP+BW using the construction that we name (for sake of this response) the C4

The remaining hardness results (RS+MA+FP+NI, RS+MA+FP+BW, RS+FP+NI+BW, RS+MA+FP+NI+BW) are the consequences of hardness results (1) and (2).

Consider the hardness result of RS+MA+NI. If the construction C1 can be embedded in T\_fixed (which requires certain height of the tree as well as the number of nodes and the existance of desired links), then the hardness result holds. The carefully-crafted threshold value can be satisfied only by solutions that co-locates nodes with replicas. Hence, the reduction is correct with possible additional leaves in Triple Gadgets.

Consider the hardness result of RS+MA+FP. If the construction C2 can be embedded in T\_fixed, then the hardness result holds. The threshold value can be satisfied only by solutions that place exactly one node in every Triple Gadget. Hence, the reduction is correct with possible additional leaves in any Triple Gadget, as well as additional leaves outside Triple Gadgets.

Consider the hardness result of RS(2)+MA+FP. If the construction C2 can be embedded in T\_fixed, then the hardness result holds. Due to the value of threshold, leaves without chunk replicas cannot be considered candidates for hosting the node.

Consider the hardness result of RS(2)+NI+FP+BW. If the construction C2 can be embedded in T\_fixed and number of nodes to be placed is at least two, then the hardness result holds. In presence of bandwidth capacities (the BW variant), each edge has the specified bandwidth capacity. For each edge of T\_fixed that is outside of C2, we set its capacity to zero, disallowing the placement of any node in such leaf.

We added a discussion.

#### SPECIFIC COMMENTS:

SECTION 5 in general: You keep changing the names for instances of the two problems. Be consistent!

%%%%%%%%% RESPONSE %%%%%%%%%%%%

Thanks for pointing this out! We changed every occurrence of FP+RS(2)+MA(4) to RS(2)+MS(4)+FP. The naming convention is to list all the variants in the following order: RS, MA, FP, NI, BW.

#### SECTION 5.1:

Let me reiterate that I firmly believe that this section requires a major revision. Even though the argument should be easy to follow, this is impossible in the current version. When you do, please provide the calculations. And check if the threshold value is what it should be, because it appears to me that it is slightly off.

- Observation 2, points 1 and 2: some chunk types are unique to either of the two subtrees, so clearly, they do not have exactly one replica in the other; please refomulate.
- "Reduction" (page 26):
  - \* In point 2: (1) the n nodes should be placed each in a distinct leaf, not all in the same one, as your writing suggests; (2) the 3D-Matching solution selects a triple, not a Triple Gadget --- the latter corresponds to the former
  - \* In point 3, you refer to "Element Subtree" but, strictly speaking, you have never defined it.
- "Proof of correctness" (right after the "reduction"):
  - \* Correctness of what?
  - \* (1) The two costs you introduce presumably correspond to something, but you never explain that. (2) Moreover, t is undefined, and should probably not be used at all, since on page 23 you state that t will be used for a triple from T. (3) Similarly, it is only in Lemma 6 that you specify the domain for u.
- Lemma 7: Just like the "Element Subtree", "Unique Subtree" was not defined.

And neither was sigma.

- Lemma 8: \ell was not defined. I can only assume it is \*not\* the \ell from the previous Lemma, since that one was to prove that \ell=0.

SECTION 5.2:

- I believe that you should provide a figure with the tree, similar to the one in Section 5.1

- Element Gadget: you write that it has 4\*(deg(e)-1)+1 leaves, but it seems that this was copied verbatim from Section 5.1 and that the right number is deg(e)

- The threshold value formula (bottom of page 29):
  - \* Could you please increase the font size?

- \* It appears that every single occurrence of |V| in the formula should be replaced by n
- \* Shouldn't the summand for e in the sum on the right end of the first line be \binom{deg(e)-1}{2} rather than deg(e)-1?
- \* Are you sure of the coefficient (i.e., their lack) next to the terms in the last two lines?

%%%%%%%%% RESPONSE %%%%%%%%%%%%

We provided the correct value in easier to follow way. E.g. we didn't expand the binomial coefficiants.

- Lemma 10:
  - \* last line of statement: n should be replaced by |V|

%%%%%%%%%% RESPONSE %%%%%%%%%%%%%

\* could you explain in detail how you arrive at the final inequalities?

- Lemma 12 and its proof: It appears that you mean the "Element Gadget", even though you are referring to the "Triple Gadget"

#### MINOR COMMENTS:

throughout article: I think you never define/explain the term "uplink"

p3l54-55: "constituting parts" --> "constituent parts"

p4l14-17: The sentence "The standard datacenter topologies today are (multi-rooted) fat-tree resp. Clos topologies [2, 21], hierarchical networks recursively made of sub-trees at each level; servers are located at the tree leaves." sounds gibberish --- please reformulate.

p8l48: remove the first 'a' from "can now be computed a from a solution to the"

p8l40: insert space in "withthe"

p11l54: change "we can computed" to either "can be computed" or "we can compute"

%%%%%%%%%%% RESPONSE %%%%%%%%%%% We changed "we can computed" to "can be computed"

p1626-27: was "communicated" supposed to be "connected"?

%%%%%%%%% RESPONSE %%%%%%%%%%%%

We changed "communicated" to "connected" for clarity purposes. However, please note that in our model the connection implies the communication and the reservation of bandwith for such communication.

p16l44: remove the repetition of "formula"

p19l23 (end of the paragraph): all elements of the \*union\* of X,Y,Z have to be covered (once), not their Cartesian product.

NP-hardness proofs: I think you should use a single symbol, perhaps \tau, for the threshold; \$Th\$ rather confusingly looks like two disconnected symbols.

p22-23: Lemma 4 and Theorem 5 --- since you do the easy calculation of the cost a specified solution in proof of Theorem 5, I believe you should also properly explain the argument in the proof of Lemma 4: it is straightforward but definitely no easier than the mere calculations in Theorem 5, as the claim is that that specific solution minimizes the cost.

# Response to Reviewer #3:

Given the number of problems (32), I would have liked the authors to perhaps create a table; it is hard for a reviewer to keep track of which problems can be solved efficiently and which are hard. Also, the optimization function - the footprint, is just stated and not discussed at all. Perhaps this is a well-agreed upon function in this field (which is new to me). Why is this the idea function to optimize?

%%%%%%%%%%% RESPONSE %%%%%%%%%%%% Summary of results is presented in Table 1 at page 34.

The footprint is the standard and most common optimization function in the literature on virtual network embeddings. However, while the function is standard, different researchers use different terms to denote it. Often, it is also called "min-sum" objective function in the literature. There is some literature also on the min-max objective function, however, it is rare in the context of virtual network embeddings. (In contrast to say flow allocation problems.)

Lastly, for the problems that are hard, I would also like the authors to mention how hard they are to approximate, or whether one can get a nice approximate solution.

%%%%%%%%% RESPONSE %%%%%%%%%%%%

Quality approximation algorithms for the stated problems are broad topic on its own. Such solutions are under ongoing research, and we are afraid that we cannot accommodate them in the current journal version within reasonable time and space (note also the space constraints). Also note that due to link capacity constraints, sometimes, without augmentation, no approximations exist.