Randomnized Algorithm for Finding Min-Cut

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Overview

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2. Analysis of Contract Algorithm
   - Error Probability
   - Time Complexity
   - Time Complexity
   - Time Complexity
   - Time Complexity
3. Second Section

Waiting for first Success

Bob has only p=10% possibility to pass a math exam, but you can take the exam as many times as you like.

- What's possibility of taking the exam only once?
- How about 10 times, 100 times

- Using the following formula

$$\Pr(\text{Within N exams, Bob passes}) = 1 - (1-p)^N$$

With N=5, P = 0.41. With N=10, P = 0.65. With N=100, P = 0.99
So after a large number of tries, Bob will finally pass the exam for almost sure even if he know nothing.

Randomnized Algorithms

what's randomnized algorithms

Randomnized algorithm is an algorithm that makes random decision when it processes the input. So this kind of algorithm cannot always get the right answer. However, we have showed even if it can only succeed with small probability, we can get the right answer by running it many times.

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Why we need that

1. Simpler
2. New way to analysis complex system

Analysis of Contract Algorithm

Success Rate of Contract Algorithm

The Contraction Algorithm returns a global min-cut of G with probability at least $1/\binom{n}{2}$

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Proof

Let's assume the global min-cut in graph G(V,E) is (A,B) and it has size k. The set of cut edges is F

Case.1 An edge in F is contracted. Then game over.

Case.2 The contracted edge is not in F. We can continue contracting.

So, how large or small is the possibility p_1 that we end our game at first step?

Analysis of Contract Algorithm

Proof Cont'd

When contracting the first edge randomly,

1. $\forall v \in V, deg(v) \geq k$
2. $|E| \geq \frac{1}{2}nk$
3. $p_1 \leq \frac{k}{\frac{1}{2}nk} = \frac{2}{n}$

After j iteration, there are n-j super-nodes in the current graph G'.

1. $\left|E'\right| \geq \frac{1}{2}(n-j)k$
2. $p_j \leq \frac{k}{\frac{1}{2}(n-j)k} = \frac{2}{n-j}$

$$\Pr(\text{Correct}) \geq \prod_{i=0}^{n-3}(1-p_n) = \prod_{i=0}^{n-3}\left(1-\frac{2}{n-i}\right) = \binom{n}{2}^{-1}$$

Analysis of Contract Algorithm

Success Rate of Contract Algorithm

The Contraction Algorithm returns a global min-cut of G with probability at least $1/\binom{n}{2}$

Time Complexity

After contracting $n^2 \log n$ times, the probability that we do not find the optimal solution

$$p_e = (1 - \binom{n}{2}^{-1})^{n^2 \log n} \approx 1/n$$

so the running time is $\mathcal{O}(n^2 \log n \times |E|)$

Bullet Points

- Lorem ipsum dolor sit amet, consectetur adipiscing elit
- Aliquam blandit faucibus nisi, sit amet dapibus enim tempus eu
- Nulla commodo, erat quis gravida posuere, elit lacus lobortis est, quis porttitor odio mauris at libero
- Nam cursus est eget velit posuere pellentesque
- Vestibulum faucibus velit a augue condimentum quis convallis nulla gravida

Blocks of Highlighted Text

Block 1

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Block 2

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Block 3

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Multiple Columns

Heading

1. Statement
2. Explanation
3. Example

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Integer lectus nisl, ultricies in feugiat rutrum, porttitor sit amet augue. Aliquam ut tortor mauris. Sed volutpat ante purus, quis accumsan dolor.

Table

Treatments	Response 1	Response 2
Treatment 1	0.0003262	0.562
Treatment 2	0.0015681	0.910
Treatment 3	0.0009271	0.296

Table: Table caption

Theorem

Theorem (Mass–energy equivalence)

$E = mc^2$

Verbatim

Example (Theorem Slide Code)

```
\begin{frame}
\frametitle{Theorem}
\begin{theorem}[Mass--energy equivalence]
$E = mc^2$
\end{theorem}
\end{frame}
```

Figure

Uncomment the code on this slide to include your own image from the same directory as the template .TeX file.

Citation

An example of the `\cite` command to cite within the presentation:

This statement requires citation [Smith, 2012].

References

John Smith (2012)
Title of the publication
Journal Name 12(3), 45 – 678.

The End