| $x \backslash y$ | 0 | 1 |
|---|---|---|
| 0 | 0: $x'y'$ | 1: $x'y$ |
| 1 | 2: $xy'$ | 3: $xy$ |

| $x \backslash yz$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 0: $x'y'z'$ | 1: $x'y'z$ | 3: $x'yz$ | 2: $x'yz'$ |
| 1 | 4: $xy'z'$ | 5: $xy'z$ | 7: $xyz$ | 6: $xyz'$ |

| $wx \backslash yz$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0: $w'x'y'z'$ | 1: $w'x'y'z$ | 3: $w'x'yz$ | 2: $w'x'yz'$ |
| 01 | 4: $w'xy'z'$ | 5: $w'xy'z$ | 7: $w'xyz$ | 6: $w'xyz'$ |
| 11 | 12: $wxy'z'$ | 13: $wxy'z$ | 15: $wxyz$ | 14: $wxyz'$ |
| 10 | 8: $wx'y'z'$ | 9: $wx'y'z$ | 11: $wx'yz$ | 10: $wx'yz'$ |

| Boolean functions | Operator symbol | Name | Comments |
|---|---|---|---|
| $F_0 = 0$ | | Null | Binary constant 0 |
| $F_1 = xy$ | $x \cdot y$ | AND | $x$ and $y$ |
| $F_2 = xy'$ | $x/y$ | Inhibition | $x$, but not $y$ |
| $F_3 = x$ | | Transfer | $x$ |
| $F_4 = x'y$ | $y/x$ | Inhibition | $y$, but not $x$ |
| $F_5 = y$ | | Transfer | $y$ |
| $F_6 = xy' + x'y$ | $x \oplus y$ | Exclusive-OR | $x$ or $y$, but not both |
| $F_7 = x + y$ | $x + y$ | OR | $x$ or $y$ |
| $F_8 = (x + y)'$ | $x \downarrow y$ | NOR | Not-OR |
| $F_9 = xy + x'y'$ | $(x \oplus y)'$ | Equivalence | $x$ equals $y$ |
| $F_{10} = y'$ | $y'$ | Complement | Not $y$ |
| $F_{11} = x + y'$ | $x \subset y$ | Implication | If $y$, then $x$ |
| $F_{12} = x'$ | $x'$ | Complement | Not $x$ |
| $F_{13} = x' + y$ | $x \supset y$ | Implication | If $x$, then $y$ |
| $F_{14} = (xy)'$ | $x \uparrow y$ | NAND | Not-AND |
| $F_{15} = 1$ | | Identity | Binary constant 1 |

$r_{rc} = r^n - N$. ADD 1 TO $r-1_c$ lsd $(r^{-m})$

$r_{(r-1)c} = r^n - r^{-m} - N$

**$r-1_c$ SUBTRACTION**

1. $M + N_{(r-1)c}$
2. → IF EAC ADD TO LSD
   → IF NO EAC, TAKE COMPLEMENT OF RESULT OF 1 AND ADD NEG SIGN IN FRONT.

**Axioms + Properties for Boolean Algebra** (B, +, ·, '; 0,1)

1) +, · are **closed in B**
2) **identity elements:**
   x + 0 = x = 1 · x, ∀ x
3) +, · are **commutative**
4) +, · are **distributive** one relative to the other one:
   (x + y) · z = x · z + y · z
   **(x · y) + z = (x + z)(y + z)**, ∀ x, y, z
5) ∀ x ∈ B, ∃ x' ∈ B, the **complement** of x, such that:
   x + x' = 1 and x · x' = 0
6) **0 ≠ 1**
7) +, · are **idempotent:**
   x + x = x and x · x = x, ∀ x
8) x + 1 = 1 and x · 0 = 0, ∀ x
9) x'' = x, ∀ x
10) **DeMorgan's Law:**
   (x + y)' = x'y', and (x · y)' = x' + y', ∀ x, y
11) +, · are **associative**
12) **Absorption** for +, · :
   x + xy = x
   x (x + y) = x, ∀ x, y

**CONTROLLED INPUT MULTIPLEXER e.g.**
4 INPUT EXTERNAL

1 OUTPUT Y = AT TIMES $I_0$, $I_1$, $I_2$, $I_3$

| $S_1$ | $S_2$ | Y |
|---|---|---|
| 0 | 0 | $I_0$ |
| 0 | 1 | $I_1$ |
| 1 | 0 | $I_2$ |
| 1 | 1 | $I_3$ |

4 OUTPUTS NEED 2 SELECTOR (CONTROLLERS)

$Y = S_1'S_2'I_0 + S_1'S_2I_1 + S_1S_2'I_2 + S_1S_2I_3$

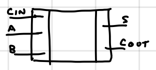
**DEMULTIPLEXER e.g.**
1 INPUT (EXT) I.
4 OUTPUT $Y_0\ Y_1\ Y_2\ Y_3$

| $S_1$ | $S_2$ | $Y_0$ | $Y_1$ | $Y_2$ | $Y_3$ |
|---|---|---|---|---|---|
| 0 | 0 | I | 0 | 0 | 0 |
| 0 | 1 | 0 | I | 0 | 0 |
| 1 | 0 | 0 | 0 | I | 0 |
| 1 | 1 | 0 | 0 | 0 | I |

$Y_0 = S_1'S_2'I$
$Y_1 = S_1'S_2I$
$Y_2 = S_1S_2'I$
$Y_3 = S_1S_2I$

**HALF + FULL ADDER**

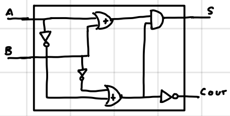
```
  1 0 1 1 +
  0 1 0 1
1 0 0 0 0
```
OVER FLOW, FA, HA



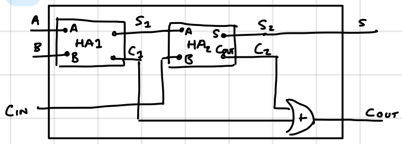

**HA**
CAPACITY IS ADDING 2 INTEGER

| A | B | S | $C_{OUT}$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 |

$S = A'B + AB' = (A'+B')(A+B)$
$C_{OUT} = AB = (A'+B')'$




**FA**




FA: $C_{OUT}$, A, B +, $C_{in}$, S

SOLUTION: $HA_1$: $C_1$, $S_1$; A +, B; $HA_2$: $C_2$, $S_1$ +, $C_{in}$, $S_2$

$\boxed{S = A'B'C + A'BC' + AB'C' + ABC}$
$\boxed{C_{OUT} = AB + AC + BC}$

**HALF + FULL SUBTRACTOR**

HS: $B_{OUT}$; A − B; D

| A | B | D | β |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 |

$D = A'B + AB'$
$\beta = A'B$

FS: $B_{OUT}$ ← $B_{in}$; A − B; D

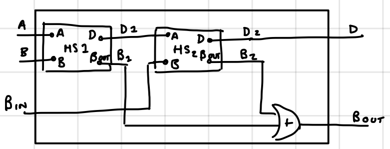
| A | B | $B_{IN}$ | D | $B_{OUT}$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 |

FS: $B_{OUT}$; A −; B −; $B_{IN}$; D

SOLUTION: $HS_1$: $B_1$; A −, B, $D_1$. $HS_2$: $B_2$; $D_1$, $B_{IN}$, $D_2$

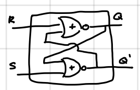
$B_1$ AND $B_2$ BOTH CAN'T EQUAL 1

$B_1 = 1 \rightarrow \{A = 0 \text{ AND } B = 1\} \rightarrow D_1 = 1 \rightarrow B_2 = 0$

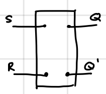



**SR LATCH**

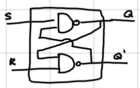
NOR




| S | R | Q | Q' |
|---|---|---|---|
| 0 | 0 | 0 / 1 | 1 / 0 } PS |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| ~~1~~ | ~~1~~ | ~~INVALID~~ | |



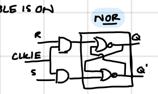

NAND



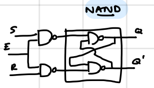

| S | R | Q | Q' |
|---|---|---|---|
| ~~0~~ | ~~0~~ | ~~INVALID~~ | |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 0 / 1 | 1 / 0 } PS |

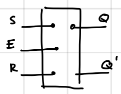
CLOCKED SR IS SIMILAR BUT Q/Q' ONLY CHANGES WHEN ENABLE IS ON

NOR



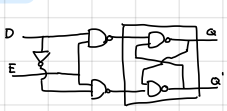

NAND

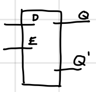



ISSUE IS S=R MAY CAUSE RACE CONDITION, UNPREDICTABLE

**D LATCH** INVERTER ON D PREVENTS S=R

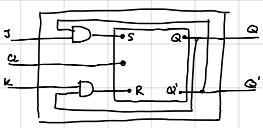



| D | Q(t+1) | |
|---|---|---|
| 0 | 0 | RESET |
| 1 | 1 | SET |

| CLK/ENABLE | Q |
|---|---|
| 0 | PS |
| 1 | D |

**JK FLIP FLOP**




J=K=1, CIRCUIT BECOMES A T FLIP FLOP, OUTPUT Q 2X SLOWER THAN CL

| J | K | Q(t+1) |
|---|---|---|
| 0 | 0 | Q(t) |
| 0 | 1 | 0 |
| 1 | 0 | 1 |
| 1 | 1 | Q'(t) |

CL = 1

| J | K | Q |
|---|---|---|
| 0 | 0 | PS |
| 0 | 1 | J |
| 1 | 0 | J |
| 1 | 1 | T FLIP FLOP 2X SLOWER THAN CL |

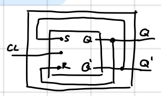
IF CL=0, Q = P(S) FOR ALL VALUES OF J AND Q

FOR OSCILLATING CL

OTHERWISE FOR CL=1

| | J | K | Q |
|---|---|---|---|
| | 0 | 0 | PS |
| J=0 → S=0 | 0 | 1 | 0 = J |
| K=0 → R=0 | 1 | 0 | 1 = J |

**T FLIP FLOP**




| T | Q(t+1) |
|---|---|
| 0 | Q(t) |
| 1 | Q'(t+1) |


Q OSCILLATE 2X SLOWER THAN CL INPUT

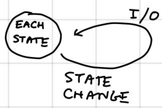
TRIGGERED BY RISING EDGE OF CL

**FINITE STATE MACHINES**

→ MEALY
OUTPUT AFF BY PRES + NEXT INPUT; UNSTABLE

→ MOORE
OUTPUT WAITS FOR NEXT VALUE; INSULATED

moore no dash in I/O




| PRESENT STATE | | NEXT STATE x=0 | | NEXT STATE x=1 | | OUTPUT x=0 | OUTPUT x=1 |
|---|---|---|---|---|---|---|---|
| A | B | A | B | A | B | y | y |
| 0 | 0 | | | | | | |
| 0 | 1 | | | | | | |
| 1 | 0 | | | | | | |
| 1 | 1 | | | | | | |

Given $f = \Sigma(0,2,5,6) + d\,\Sigma(3,4)$.

| | 4 | 2 | 1 |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 2 | 0 | 1 | 0 |
| 3 | 0 | 1 | 1 |
| 4 | 1 | 0 | 0 |
| 5 | 1 | 0 | 1 |
| 6 | 1 | 1 | 0 |

→ For every min term that is matched we will put tick mark.

Step 1:-

| Group | Minterm | Variable A B C |
|---|---|---|
| 0 | 0 ✓ | 0 0 0 |
| 1 | 2 ✓ | 0 1 0 |
| | 4 ✓ | 1 0 0 |
| 2 | 3 ✓ | 0 1 1 |
| | 5 ✓ | 1 0 1 |
| | 6 ✓ | 1 1 0 |

Step 2:-

| Group | matched Pair | Variable A B C |
|---|---|---|
| 0 | 0,2 ✓ | 0 _ 0 |
| | 0,4 ✓ | _ 0 0 |
| 1 | 2,3 | 0 1 _ ⇒ $\bar{A}B$ |
| | 2,6 ✓ | _ 1 0 |
| | 4,5 | 1 0 _ ⇒ $A\bar{B}$ |
| | 4,6 ✓ | 1 _ 0 |

Step 3:-

| Group | matched Pair | Variable A B C |
|---|---|---|
| 0 | 0,2,4,6 | _ _ 0 |
| | 0,4,2,6 | _ _ 0 |

} ⇒ $\bar{C}$

Prime Implicants table:-

| P.I | 0 | 2 | 5 | 6 | minterms involved |
|---|---|---|---|---|---|
| $\bar{C}$ | ⊗ | X | | ⊗ | 0,2,4,6 |
| $\bar{A}B$ | | X | | | 2,3 |
| $A\bar{B}$ | | | ⊗ | | 4,5 |

⇒ $\boxed{Y = \bar{C} + A\bar{B}}$

b) K-Map

| A \ BC | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 1 | | X | 1 |
| 1 | X | 1 | | 1 |

→ $\bar{C}$

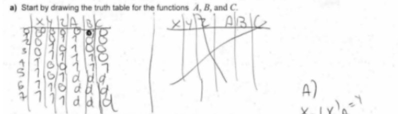
$A\bar{B}$

⇒ $\boxed{Y = \bar{C} + A\bar{B}}$

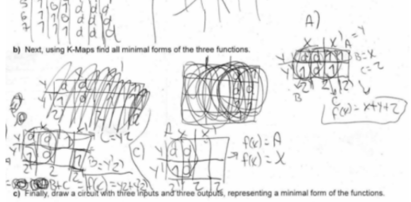
→ The output of K-Map is same as the output of Tabulation Method.

1. Design a combinational circuit with three inputs: $x, y, z$ and three outputs: $A, B, C$, such that,
- When the binary input $xyz$ represents the decimal digits $1, 3,$ or $4$, the binary output $ABC$ should represent the decimal digit that is 3 greater than the input: that is $4, 6,$ or $7$ respectively.
- Similarly, when the binary input represents the digits $0$ or $2$, the binary output should represent the digit that is 2 greater than the input.
- The remaining three, binary representations of the decimal digits $5, 6,$ and $7$ never occur.

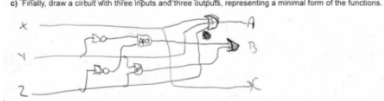
a) Start by drawing the truth table for the functions $A, B,$ and $C$.



b) Next, using K-Maps find all minimal forms of the three functions.

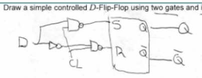



c) Finally, draw a circuit with three inputs and three outputs, representing a minimal form of the functions.




1

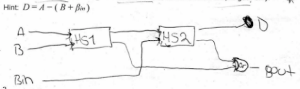
5. Draw a simple controlled $D$-Flip-Flop using two gates and inverters, and one SR flipflop.




3

3. Draw a full-subtractor using two half-subtractors, and one more simple gate only.

Hint: $D = A - (B + \beta_{in})$




2

| a | A | B | C | Y1 | Y2 | Y3 |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| 0 | 1 | 0 | 1 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 1 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 0 | 0 | 0 | 0 | 1 |
| 1 | 1 | 0 | 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 | 0 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |

Using K-Map for each output we will find function.

K-Map

$Y_1$ →

| aA \ BC | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0 | 0 | 0 | 0 |
| 01 | 1 | 1 | 1 | 1 |
| 11 | 0 | 1 | 1 | 0 |
| 10 | 0 | 1 | 1 | 0 |

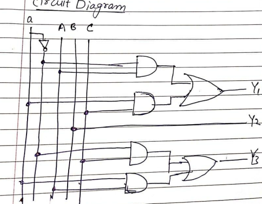
$Y_1 = \bar{a}A + aC$

$Y_2$ =

| | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0 | 0 | 1 | 1 |
| 01 | 0 | 0 | 1 | 1 |
| 11 | 0 | 0 | 1 | 1 |
| 10 | 0 | 0 | 1 | 1 |

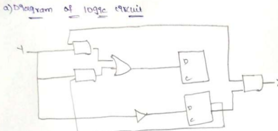
$Y_2 = B$

Circuit Diagram




Answer

a) Diagram of logic circuit




b) Truth table (or) State table

| Present Value A | Present Value B | Input Y | Next state A | Next state B | Output Z |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 | 1 |
| 0 | 0 | 1 | 1 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 | 0 |
| 0 | 1 | 1 | 1 | 0 | 0 |
| 1 | 0 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 1 | 0 | 0 |
| 1 | 1 | 0 | 0 | 1 | 0 |
| 1 | 1 | 1 | 1 | 0 | 0 |