B-Splines Cúbica C^2

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Abstract

Algorithm to compute the control points of a C^2 Cubic B-Spline and its correspondent Bézier's Curve.

1 Algorithm

Consider the vector $[u_0, u_1, ..., u_L]$ Let $p_0, p_1, ..., p_{L+3}$ be the vertexes of the control polygon and $d_0, d_1, ..., d_{L+3}$ the control points of the C^2 Cubic B-Spline $S(c)$ such that:

1. $d_0 = p_0$
2. $d_1 = p_1$
3. $d_2 = (1 - \alpha)p_1 + \alpha p_2$, where $\alpha = \frac{\Delta u_1}{\Delta u_1 + \Delta u_2}$
4. $d_{3L-3} = (1 - \alpha)p_{3L-4} + \alpha p_{3L-2}$, where $\alpha = \frac{\Delta u_L}{\Delta u_L + \Delta u_{L+1}}$
5. $d_{3L-2} = (1 - \alpha)p_L + \alpha p_{L+1}$, where $\alpha = \frac{\Delta u_L}{\Delta u_{L-1} + \Delta u_L}$
6. $d_{3L-1} = p_{L+1}$
7. $d_{3L} = p_{L+2}$

8. For $i = 1, ..., L - 2$
   (a) $d_{3i} = (1 - \alpha)p_{3i-1} + \alpha p_{3i+1}$, where $\alpha = \frac{\Delta u_i}{\Delta u_i + \Delta u_{i+1}}$
   (b) $d_{3i+1} = (1 - \alpha)p_{i+1} + \alpha p_{i+2}$, where $\alpha = \frac{\Delta u_i}{\Delta u_i + \Delta u_{i+1} + \Delta u_{i+2}}$
   (c) $d_{3i+2} = (1 - \alpha)p_{i+1} + \alpha p_{i+2}$, where $\alpha = \frac{\Delta u_i + \Delta u_{i+1}}{\Delta u_i + \Delta u_{i+1} + \Delta u_{i+2}}$

Also consider, for $j = 0, ..., L$:

1. $u_0 = dist(d_0, d_2)$
2. $u_j = u_{j-1} + dist(d_j, d_{j+2})$

e $\Delta u_i = u_i - u_{i-1}$, where $dist(a, b)$ is the distance between the points a and b.