Emergent Reasoning in Large Language Models

A Topological and Constraint-Based Formalization

How LLMs traverse constraint manifolds to produce reasoning

Slide 1: The Challenge

LLMs Demonstrate Reasoning Without Rules

LLMs perform tasks that seem to require reasoning:

- Multi-step reasoning
- Analogical inference
- Constraint satisfaction
- Factual and relational reasoning
- Structured problem solving

Yet they contain: No explicit rules, no logic engines, no symbolic manipulators.

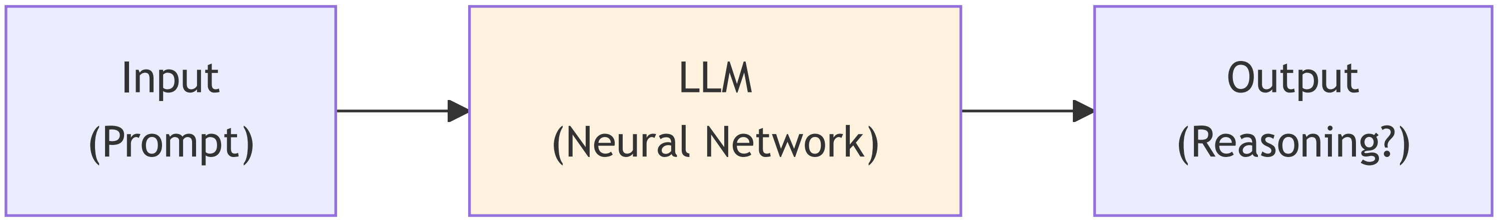


Diagram 0

The Question: How can probabilistic systems yield structured inference?

Slide 2: The Core Thesis

LLM Computation = Constrained Topological Traversal

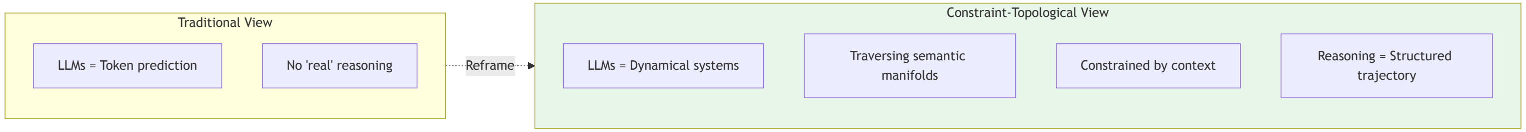


Diagram 1

The claim: Reasoning emerges from topology-constrained traversal within a learned semantic manifold.

Slide 3: Why “Probabilistic = No Reasoning” is Wrong

All Physical Computation Involves Noise

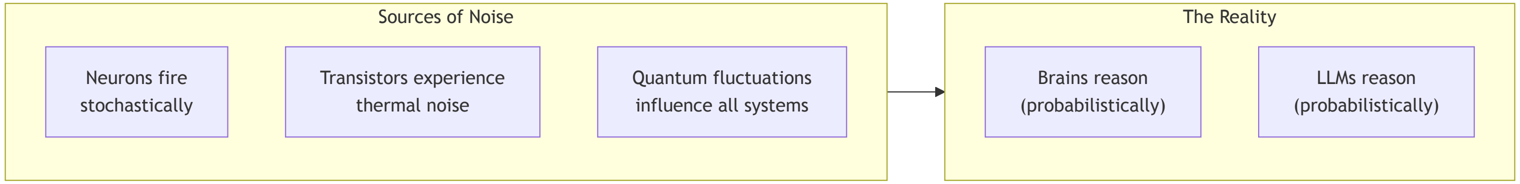


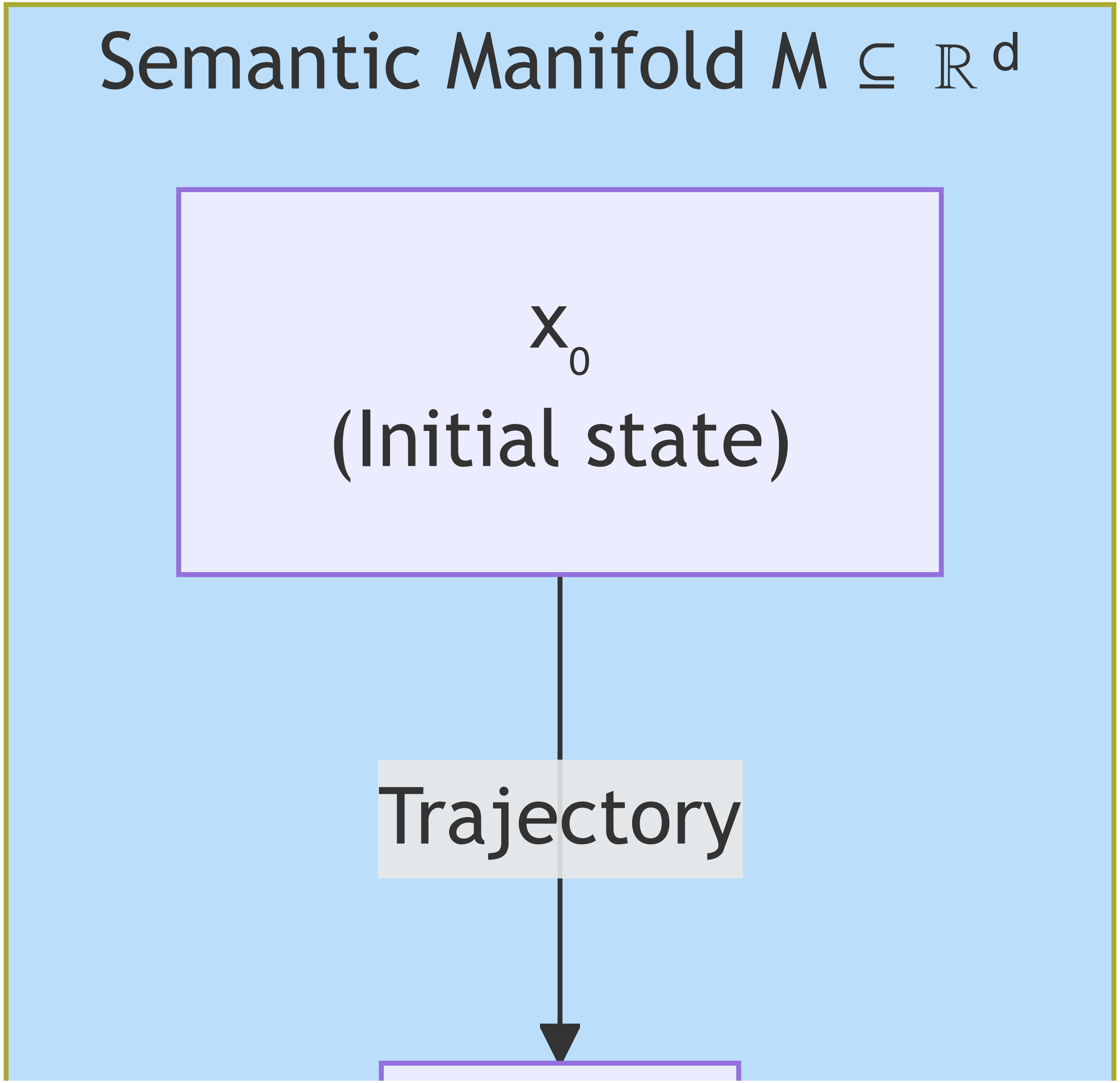
Diagram 2

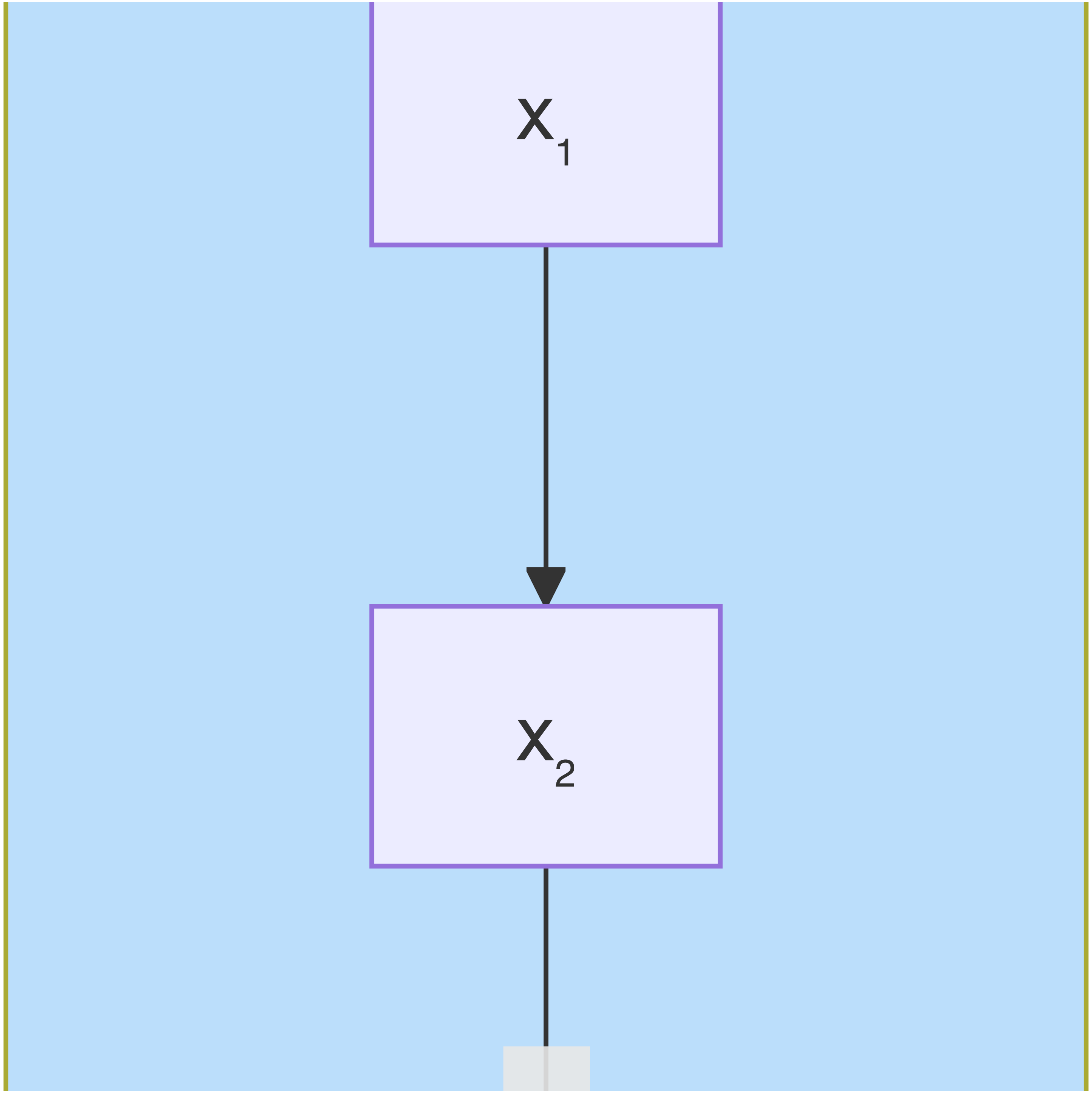
Brains and LLMs both implement: Probabilistic computation refined by constraints.

The argument “LLMs can’t reason because they are probabilistic” is unsound - it would equally invalidate human reasoning.

Slide 4: The Semantic Manifold

Where LLM Computation Happens


Semantic Manifold M ⊆ ℝ d
x 0
(Initial state)
Trajectory



X_1
X_2

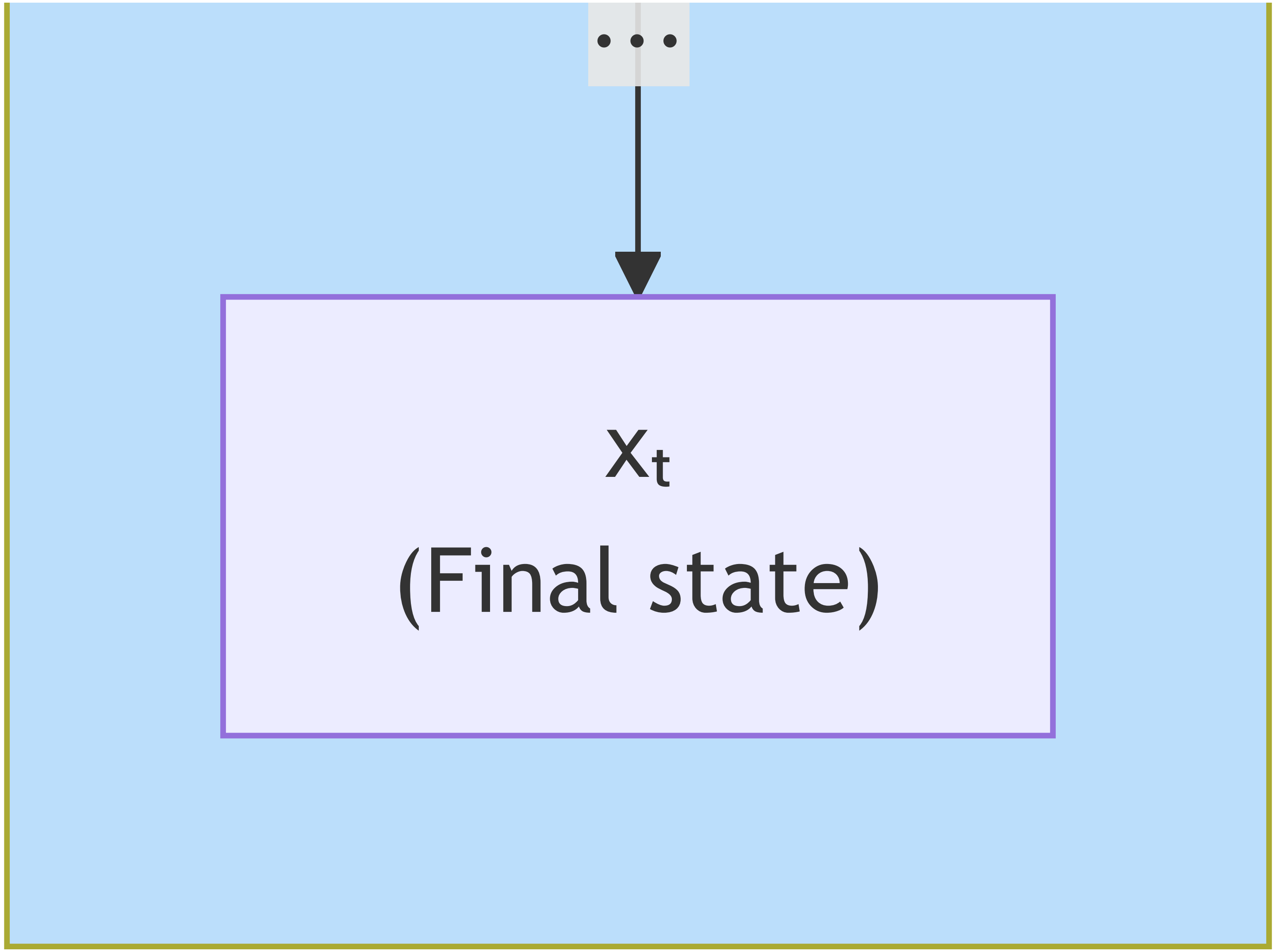


Diagram 3

Definitions: - $\mathbf{M} \subseteq \mathbb{R}^d$: The set of valid activations (semantic manifold) - $\mathbf{x} \in \mathbf{M}$: A semantic state (point on manifold) - **Trajectory** $\mathbf{x_0, x_1, \ldots, x_t}$: A reasoning path

Each point represents a semantic configuration; trajectories represent semantic evolution.

Slide 5: Constraint Sets

Context Narrows the Possible

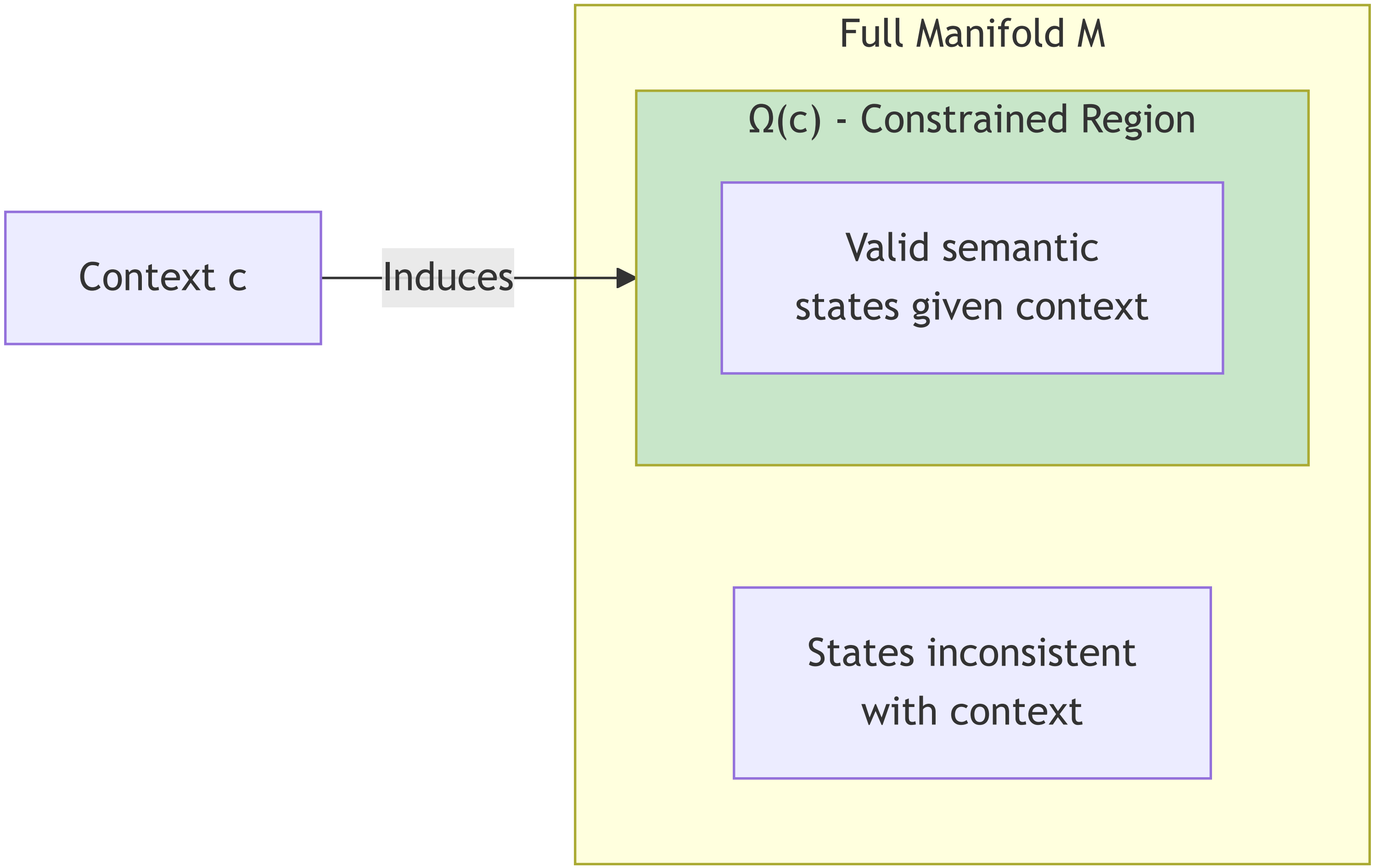


Diagram 4

Context c induces a constraint set:

```
Ω(c) ⊆ M
```

Representing all states consistent with context.

- **Strong constraints** → Narrow $\Omega(c)$ → Precise reasoning
- **Weak constraints** → Wide $\Omega(c)$ → Drift, hallucination

Slide 6: The Preferred Direction Function

The Core of LLM Reasoning

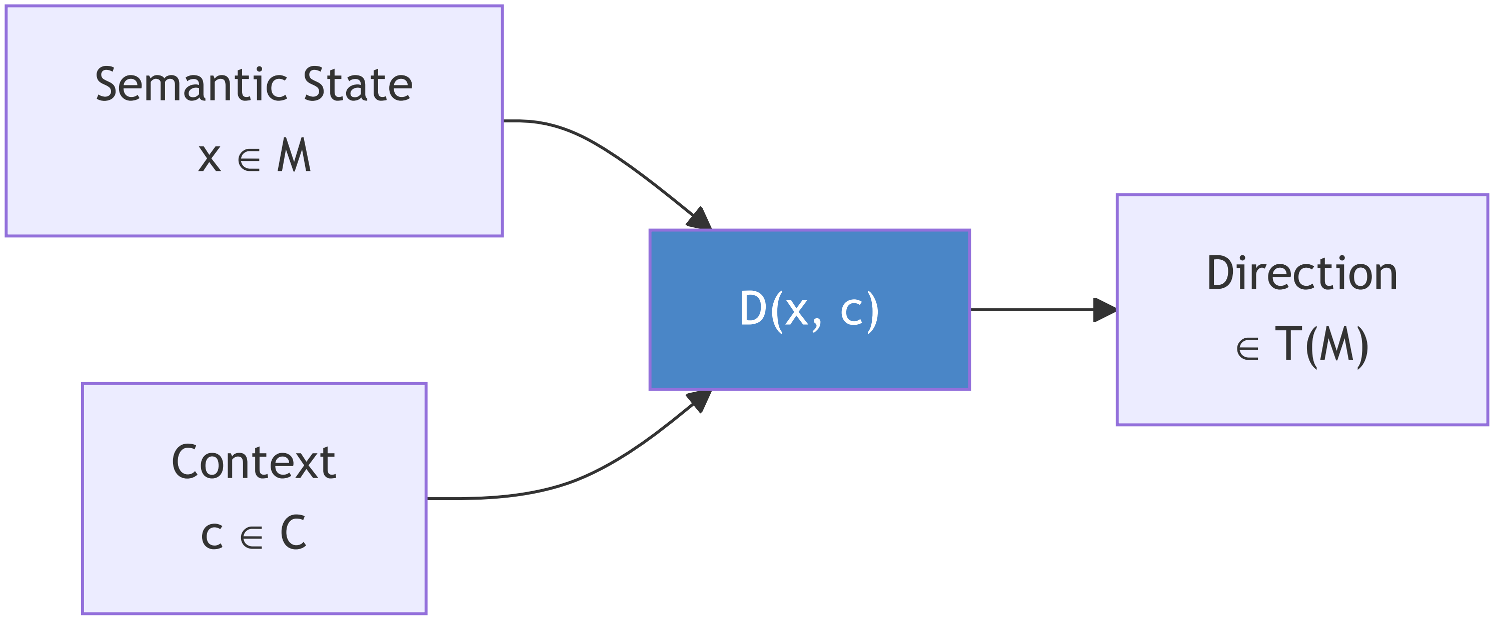


Diagram 5

Definition: The preferred direction function is:

```
D : M × C → T(M)
```

Where T(M) is the tangent bundle (all possible directions from any point).

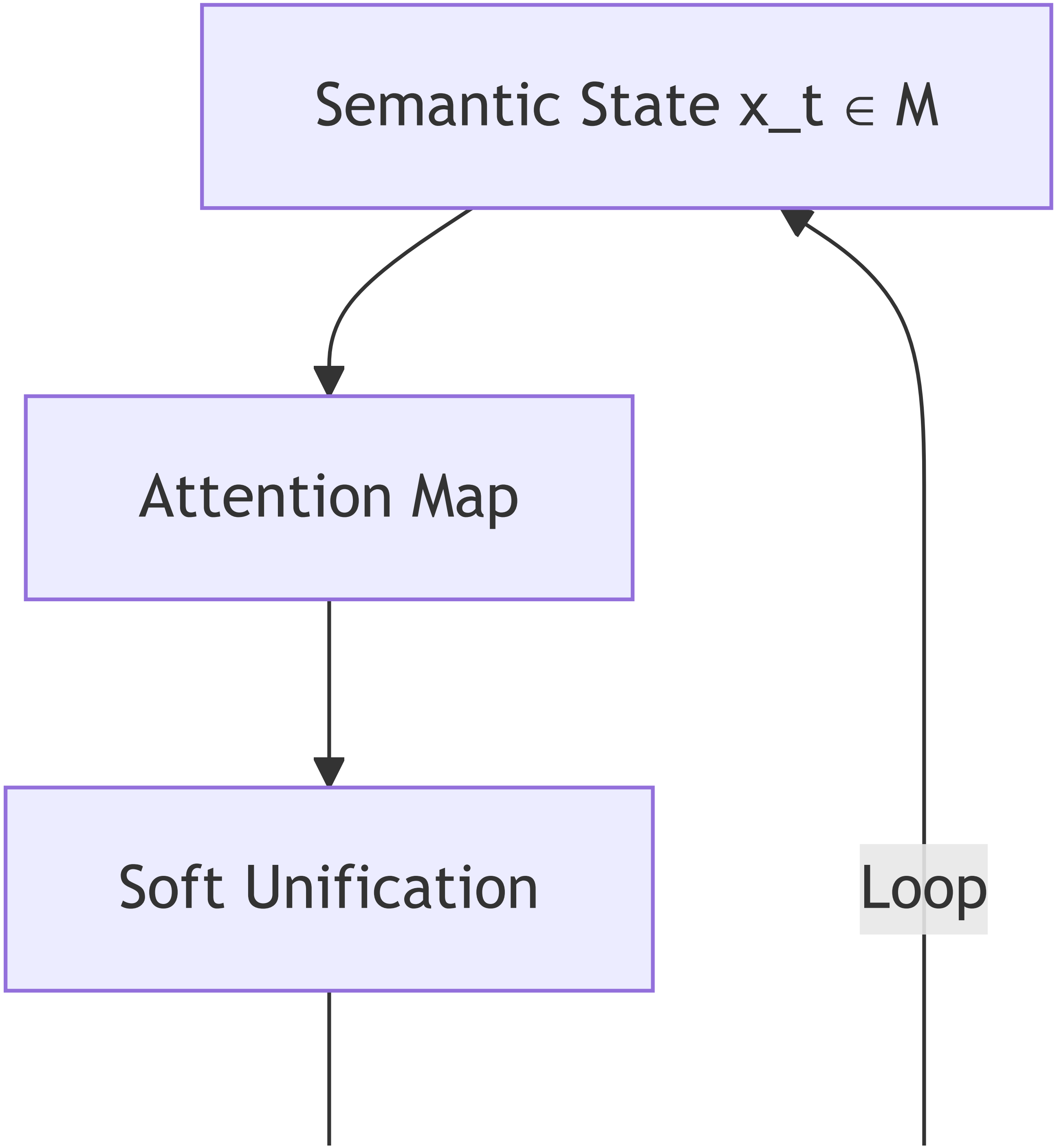
The model performs:

```
x_{t+1} = x_t + Δt · D(x_t, c_t)
```

Attention IMPLEMENTS this function - it computes which direction the model “wants” to move.

Slide 7: The Model Overview

Transformer as Dynamical System


Semantic State x_t ∈ M
Attention Map
Soft Unification
Loop

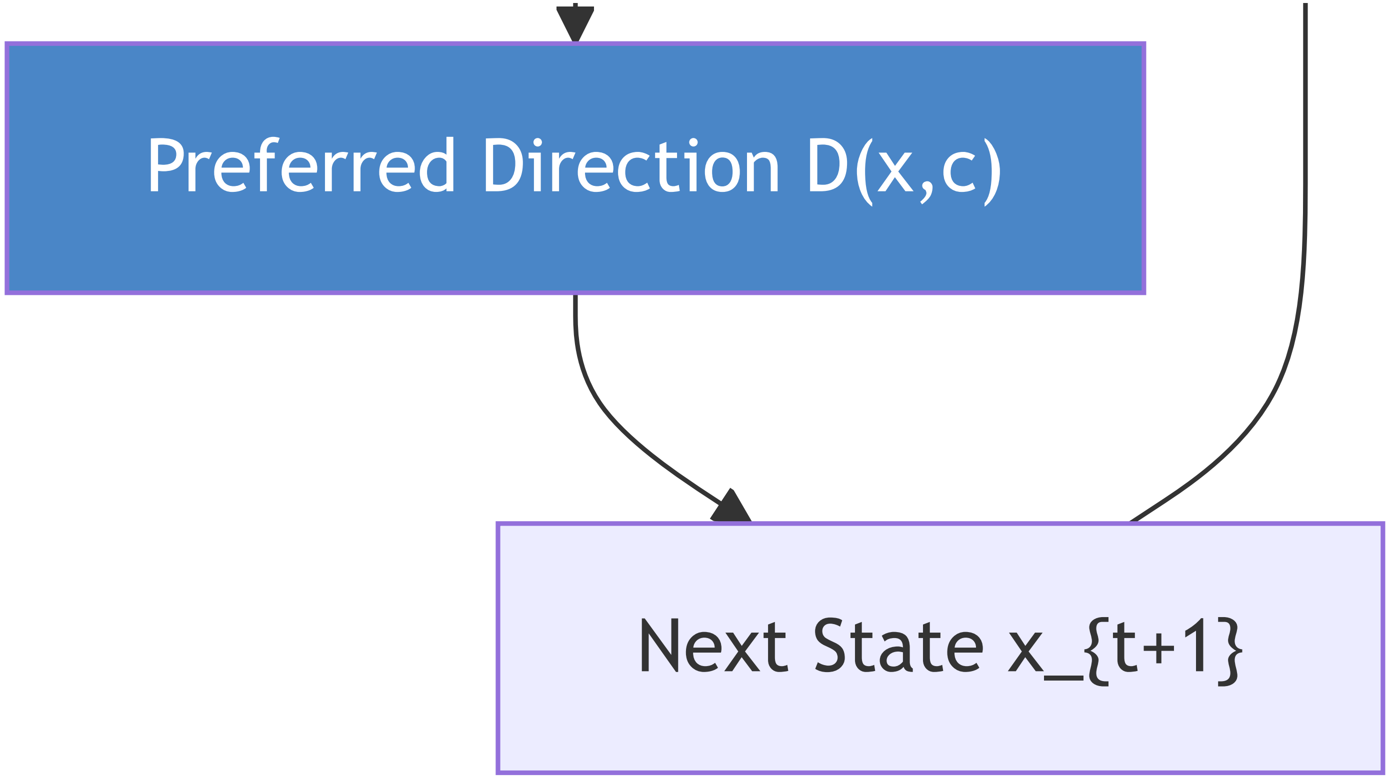


Diagram 6

The Transformer is a **dynamical system** that: 1. Takes current semantic state 2. Computes attention (relevance constraints) 3. Performs soft unification (pattern matching) 4. Determines preferred direction 5. Moves to next state 6. Repeats

Slide 8: Soft Unification

The Continuous Analogue of Prolog

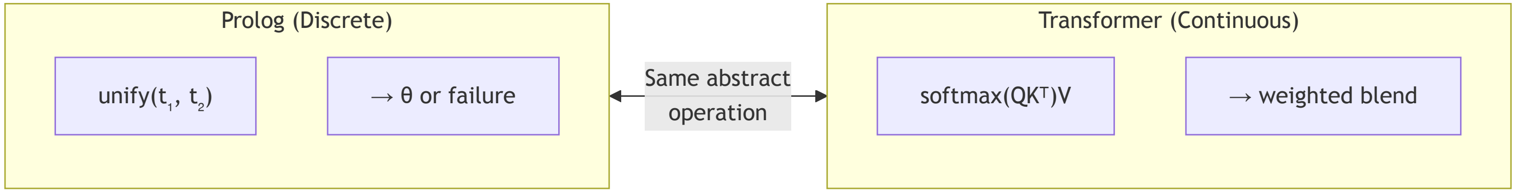


Diagram 7

Attention computes:

$$U\_soft(q) = \Sigma_i \ \alpha_i(q, k_i) \ v_i$$

$$\text{where } \alpha_i = \text{softmax}(\langle q, k_i \rangle / \sqrt{d\_k})$$

This is **soft unification**: similarity-weighted synthesis.

Symbolic Unification	Soft Unification
Exact match or fail	Similarity-weighted blend
Discrete substitution	Continuous interpolation
Binary success/failure	Graded compatibility

Slide 9: Prolog vs. Transformer Correspondence

Two Instantiations of the Same Pattern

Prolog	Transformer
Terms	Embeddings
Unification	Soft similarity matching
Backtracking	Parallel weighted evaluation
Substitutions	Continuous blending
Discrete search tree	Dynamical system trajectory

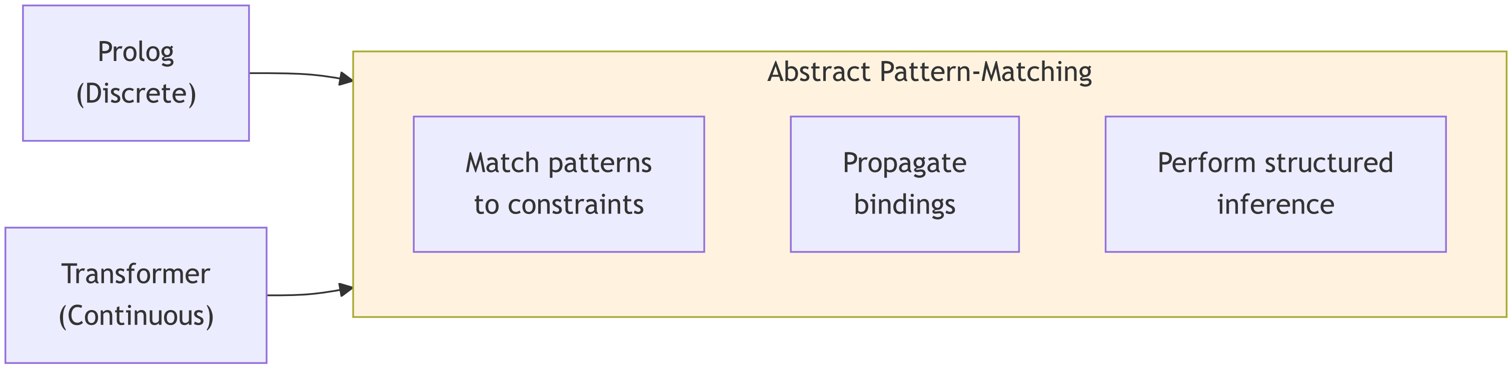


Diagram 8

Both instantiate the same abstract unification principle - one discrete and explicit, the other continuous and implicit.

Slide 10: Multi-Head Attention as Constraint Composition

Parallel Soft Constraints

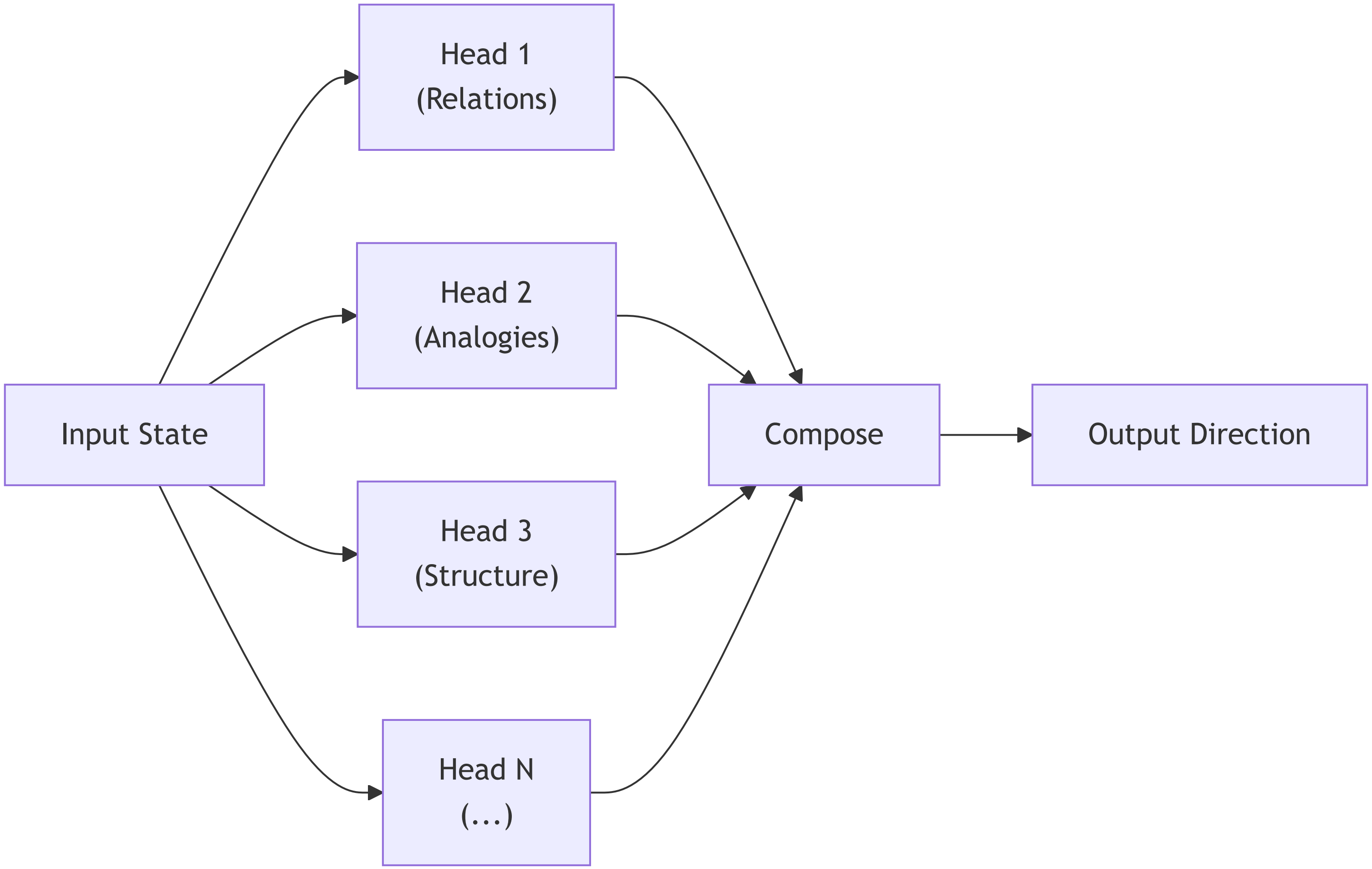


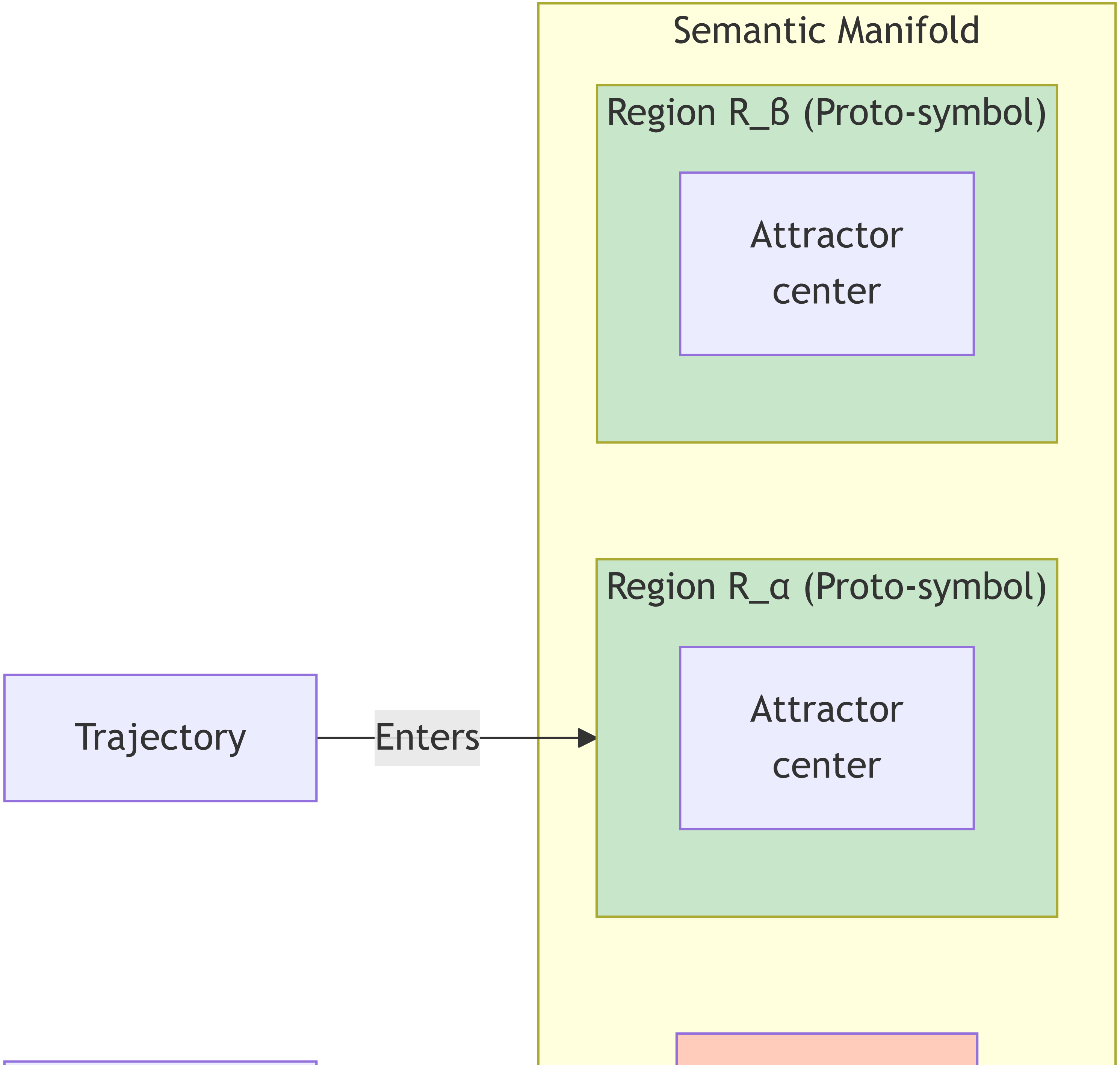
Diagram 9

Each attention head captures: - Different relations - Different analogies - Different structural patterns

Stacked layers → Deeper, more abstract constraints

Slide 11: Proto-Symbol Regions

Where Symbolic-Like Behavior Emerges


Semantic Manifold
Region R_β (Proto-symbol)
Attractor center
Region R_α (Proto-symbol)
Attractor center
Trajectory
Enters

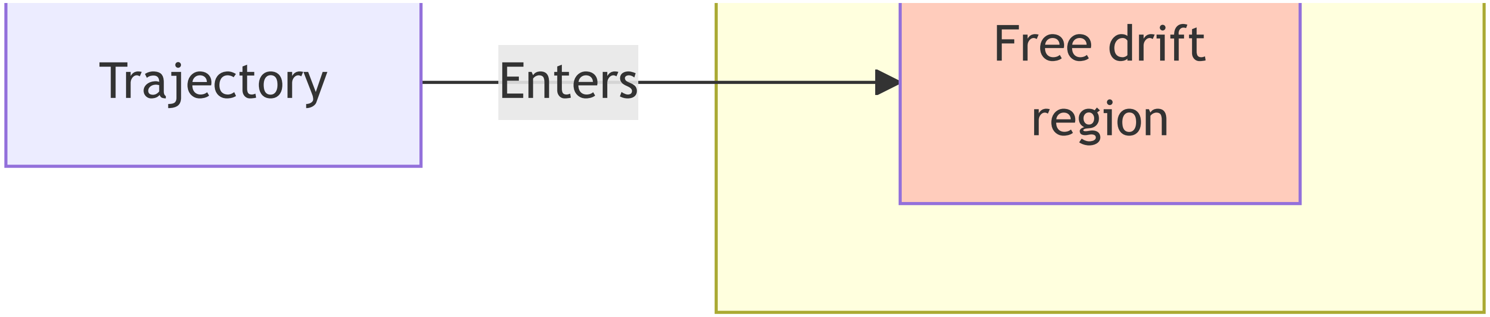


Diagram 10

Definition: A region R_α ⊆ M is a **proto-symbol** if: - It is an attractor-like set (trajectories enter and remain stable) - Outputs correspond to consistent semantic categories - Internal correlations are strong

Proto-symbols enable discrete-like reasoning from continuous operations.

Slide 12: Attractor Structure and Behavior

Stability vs. Hallucination

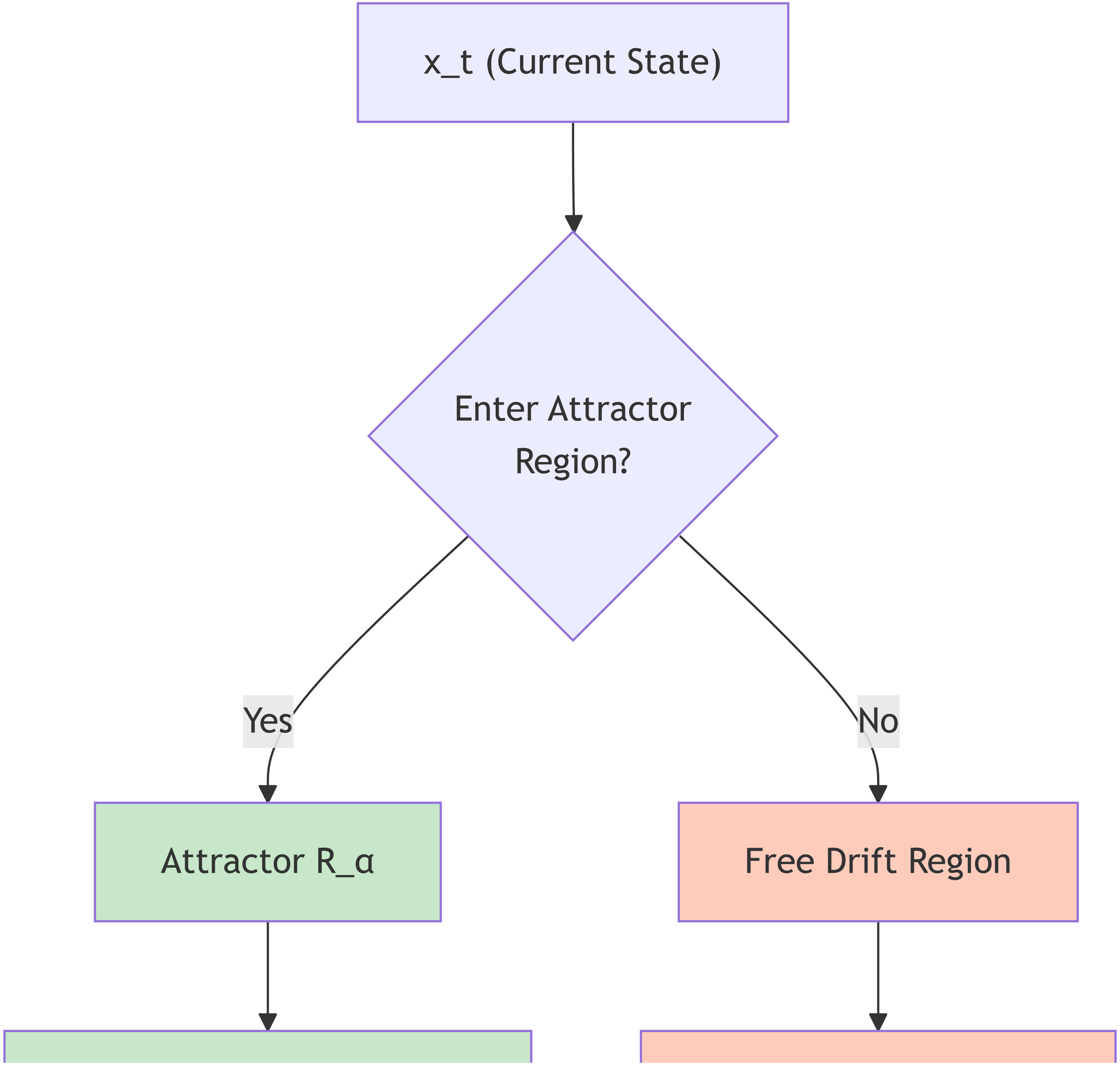

x_t (Current State)
Enter Attractor Region?
Yes
No
Attractor R_α
Free Drift Region

Diagram 11

Inside an attractor: - Trajectories remain stable - Outputs are semantically narrow - Behavior is predictable, coherent

In free drift regions: - Trajectories wander - Constraints are weak - Hallucination becomes likely

Slide 13: Markov-Blanket-Like Boundaries

Statistical Independence at Region Edges

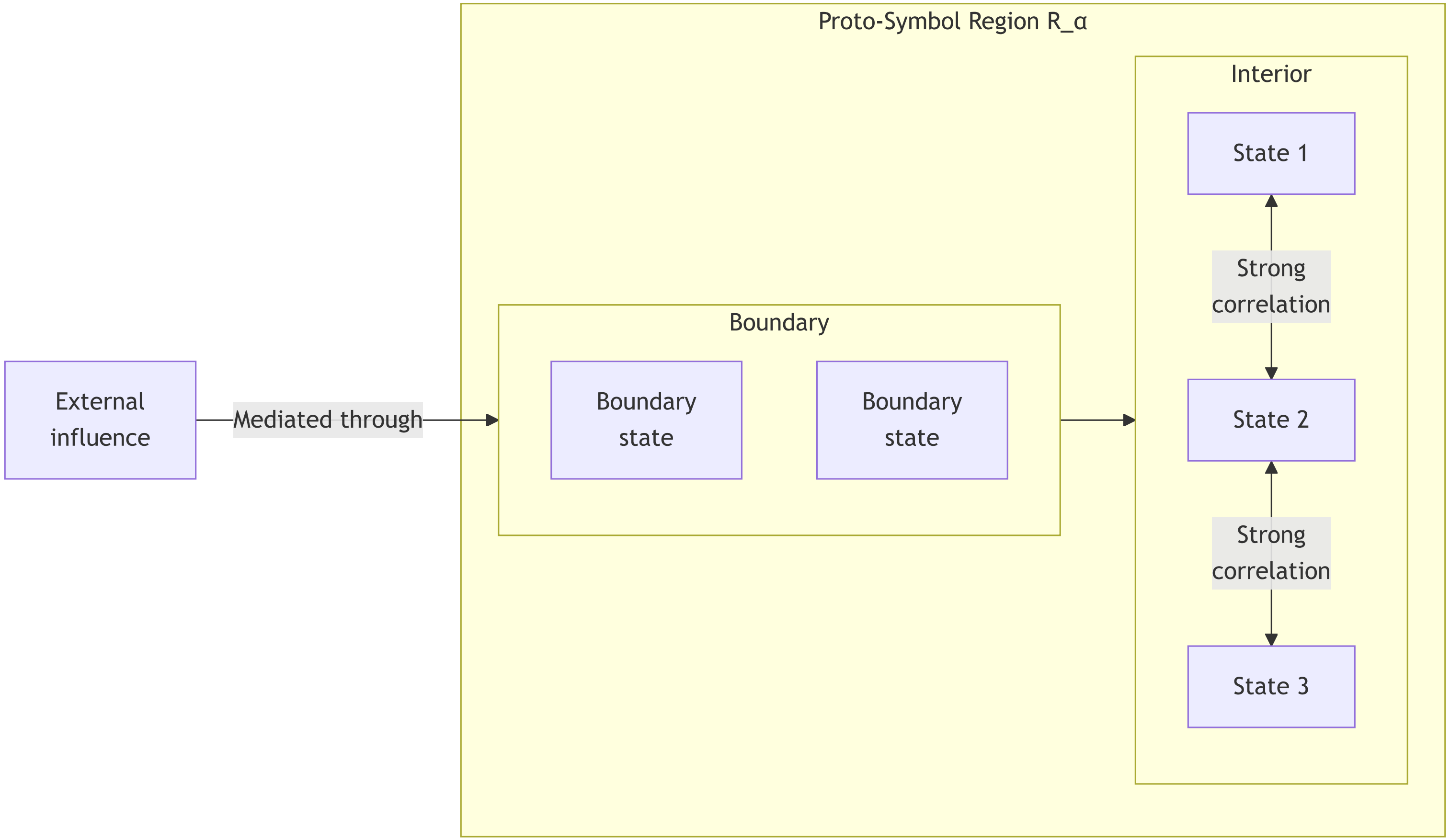


Diagram 12

Inside R_α: - States predict each other well (high internal coherence) - External influence is mediated through boundary states - This gives **symbolic-like integrity**

The boundary functions as a **pseudo-Markov blanket** - providing conditional independence from the exterior.

Slide 14: Hallucinations as Chaotic Excursions

Leaving the Stable Manifold

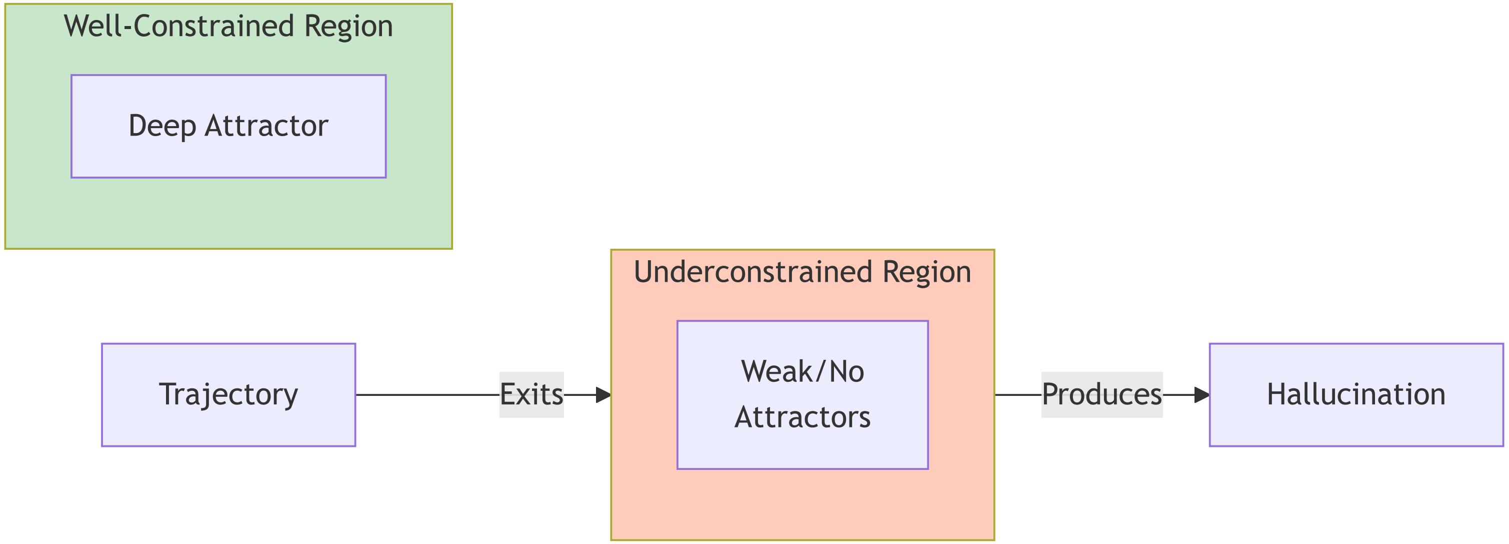


Diagram 13

Hallucinations occur when: - Trajectories enter underconstrained basin regions - No strong attractors exist to stabilize output - The model "drifts" without grounding

This is not "making things up" - it's a trajectory entering a region where the constraint topology doesn't enforce coherent output.

Slide 15: The Direction Function Formalized

The Core Computational Primitive

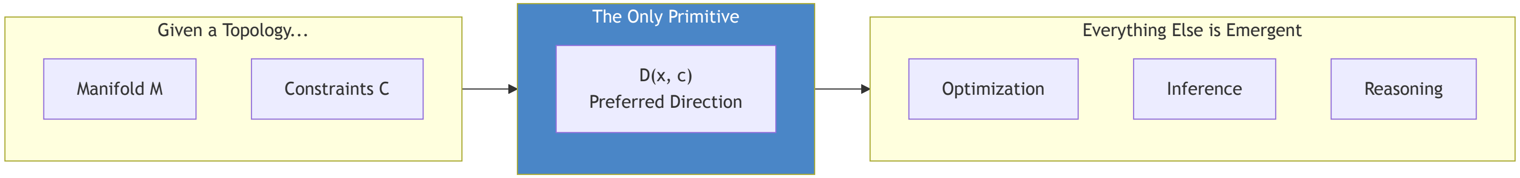


Diagram 14

The fundamental computation:

```
D(x, c) : M × C → T(M)
```

Everything else - optimization, inference, reasoning - is emergent trajectory behavior arising from repeatedly applying D.

Slide 16: Attractors as Meaning Structures

Stability = Meaning

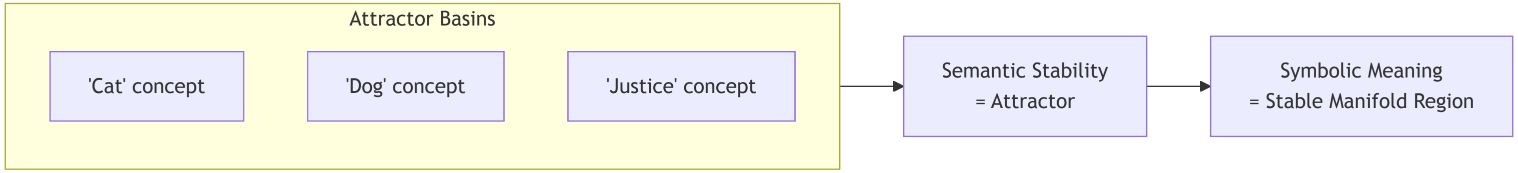


Diagram 15

The mapping: | Concept | Formal Object | |— — —|— — — —–| | Meaning | Attractor basin | | Concept | Proto-symbol region | | Understanding | Trajectory entering stable attractor | | Confusion | Trajectory between attractors |

Slide 17: Category-Theoretic Interpretation

A Deeper Formalism

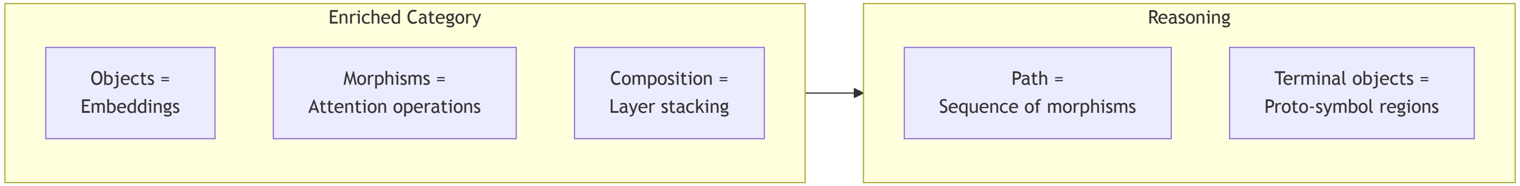


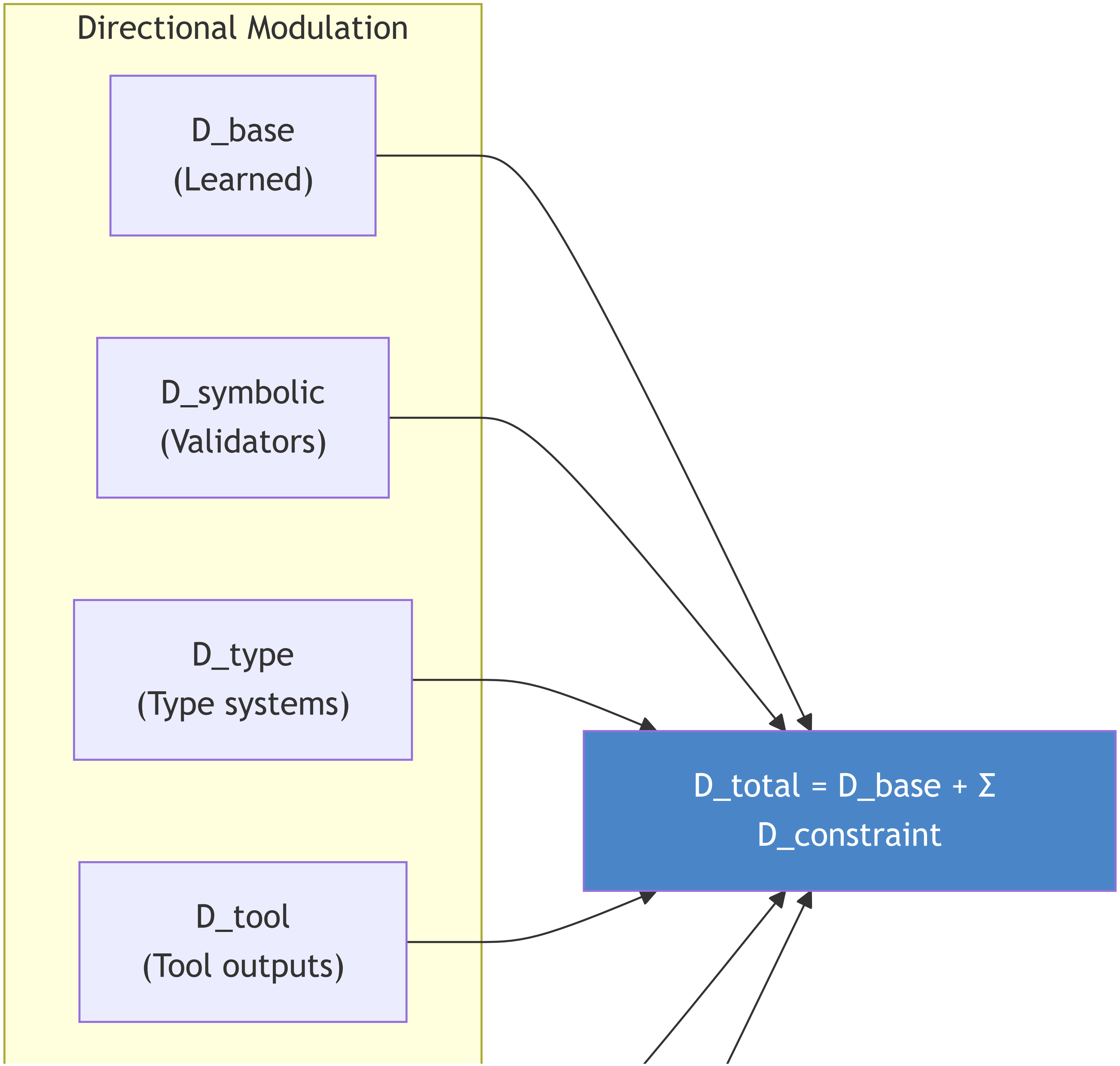
Diagram 16

Category-theoretic view: - **Embeddings** = Objects in an enriched category - **Attention** = Enriched morphisms - **Reasoning** = Sequence of morphisms forming a path - **Proto-symbol regions** = Limits or terminal objects

This provides a rigorous mathematical foundation for the framework.

Slide 18: Architectural Implications

How to Build Explicit Reasoning


Directional Modulation
D_base
(Learned)
D_symbolic
(Validators)
D_type
(Type systems)
D_tool
(Tool outputs)
D_total = D_base + Σ
D_constraint

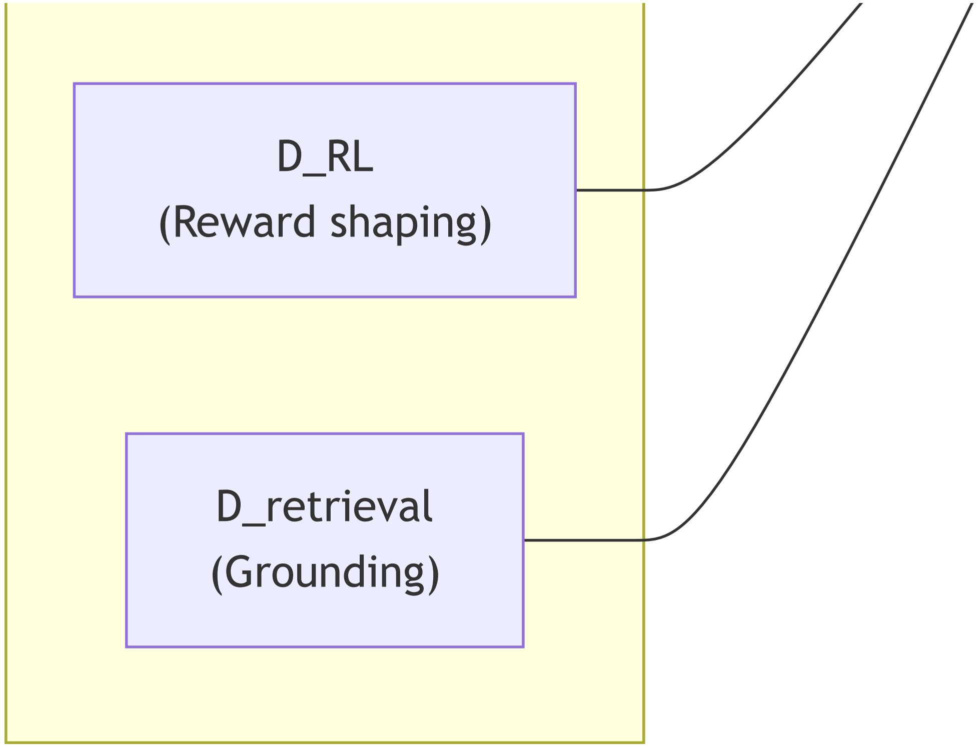


Diagram 17

Modify the direction function:

```
D_total(x, c) = D_base + Σ_k D_constraint^(k)
```

Sources of constraint directions: - Symbolic validators - Type systems - Tool outputs - RL reward shaping - Retrieval grounding

Slide 19: Topology-Aware Regularization

Training for Better Reasoning

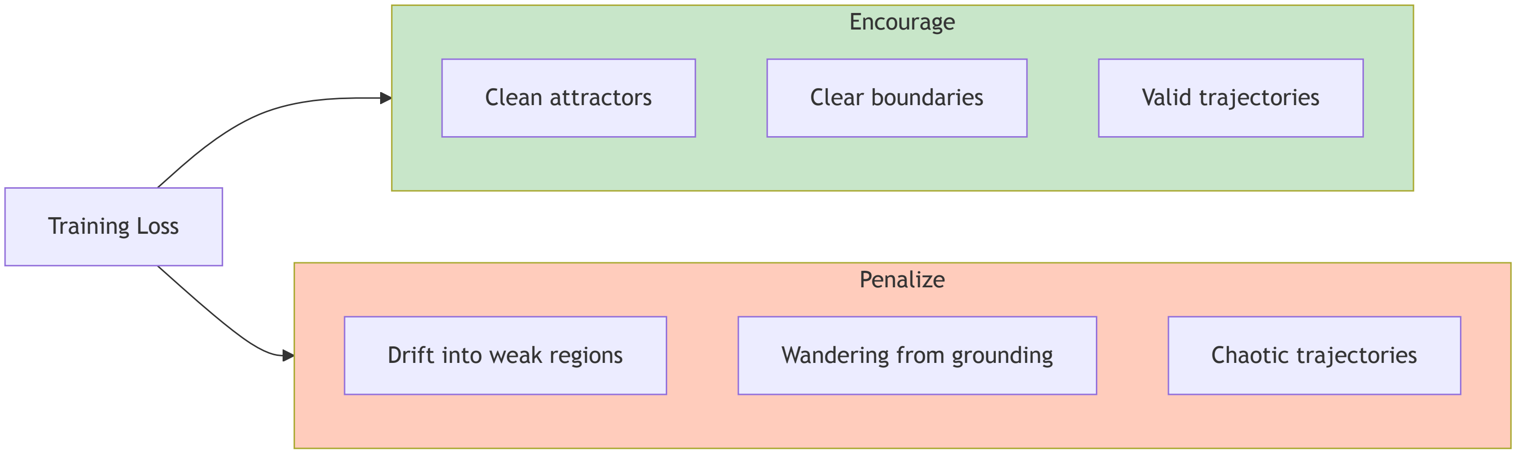


Diagram 18

Better training objectives: - Reward trajectories that stay in well-defined attractor basins - Penalize excursions into underconstrained regions - Shape the manifold topology for cleaner reasoning

Slide 20: Hybrid Neuro-Symbolic Systems

The Best of Both Worlds

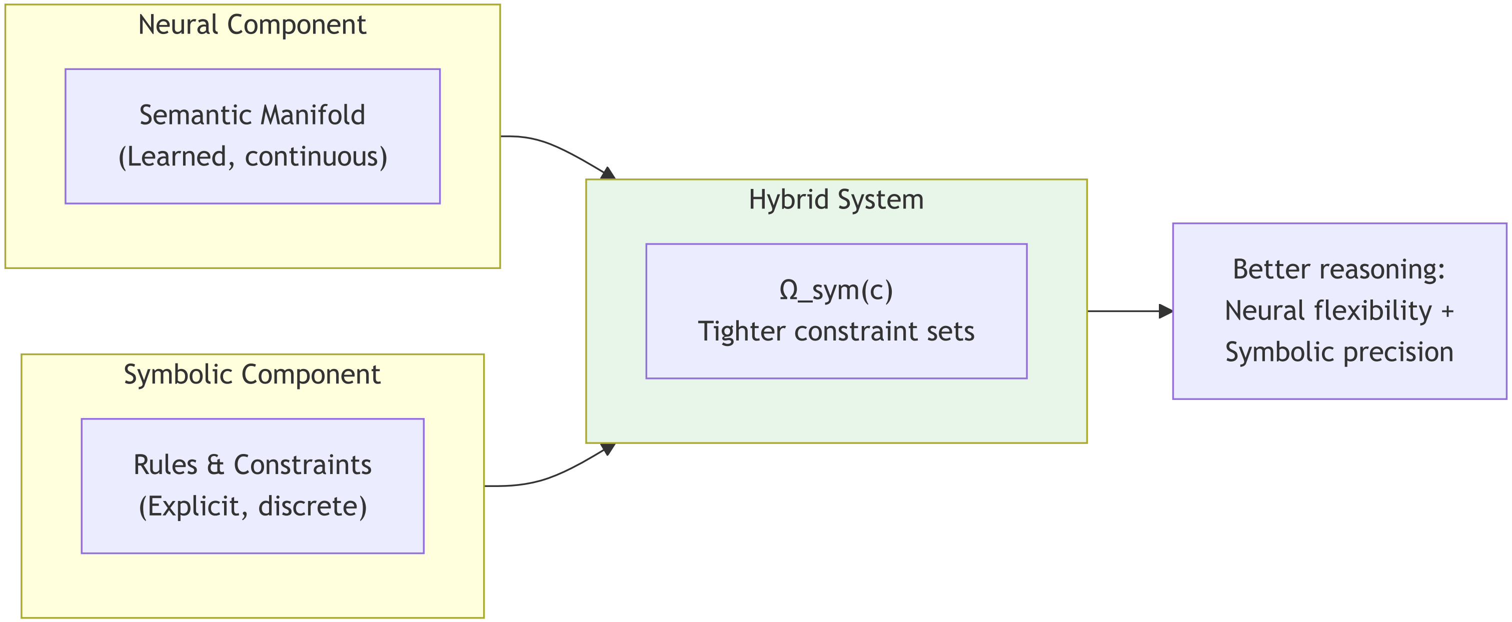


Diagram 19

Symbolic components define tighter constraint sets:

$\Omega\_sym(c) \subset \Omega(c)$

These reshape manifold traversal, preventing hallucination while preserving flexibility.

Slide 21: Connection to Constraint-Emergence Ontology

LLMs as Constraint Manifold Laboratories

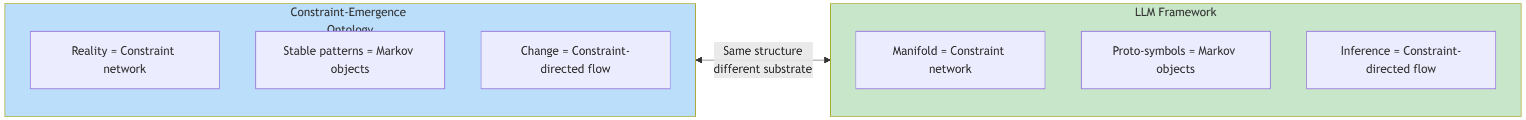


Diagram 20

LLMs instantiate the same abstract pattern as physical reality: - High-dimensional constraint manifold - Markov objects (proto-symbols) emerge through stability - Trajectories follow constraint-directed flow

LLMs are accessible laboratories for studying constraint dynamics.

Slide 22: The SDLC Connection

From LLM Reasoning to Software Development

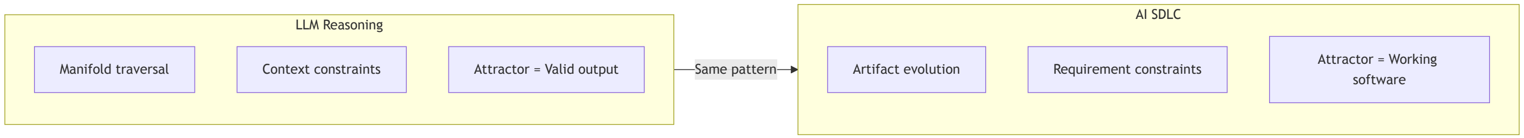


Diagram 21

LLM Concept	SDLC Application
Preferred direction D(x,c)	Builder stage transition
Context constraints	Requirements + Standards
Proto-symbol attractor	Approved artifact
Hallucination	Failed tests, defects
Grounding (retrieval)	Architecture context, ADRs

Slide 23: Summary - The Framework

What We've Established

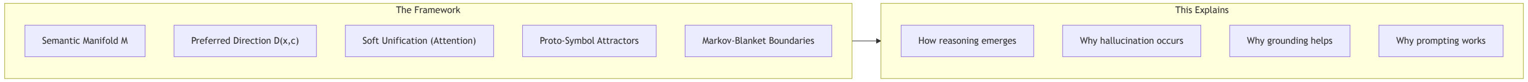


Diagram 22

The framework shows: 1. LLM reasoning = constrained manifold traversal 2. Attention = soft unification (same as Prolog, different substrate) 3. Proto-symbols = attractor regions with Markov-like boundaries 4. Hallucination = trajectory leaving stable region 5. Grounding = adding constraint sources

Slide 24: Key Takeaways

What This Means for AI Development

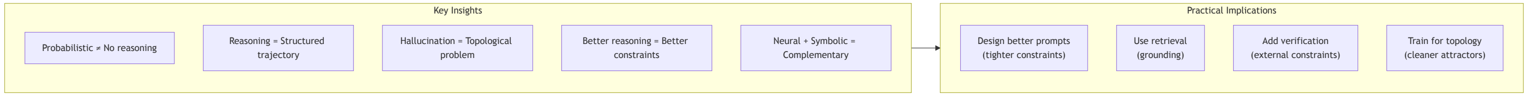


Diagram 23

The path forward: - Don’t fight the probabilistic nature - work with it - Add constraint sources to tighten valid regions - Shape the manifold topology through training - Build hybrid systems that combine neural flexibility with symbolic precision

Slide 25: Conclusion

Bridging the Gap

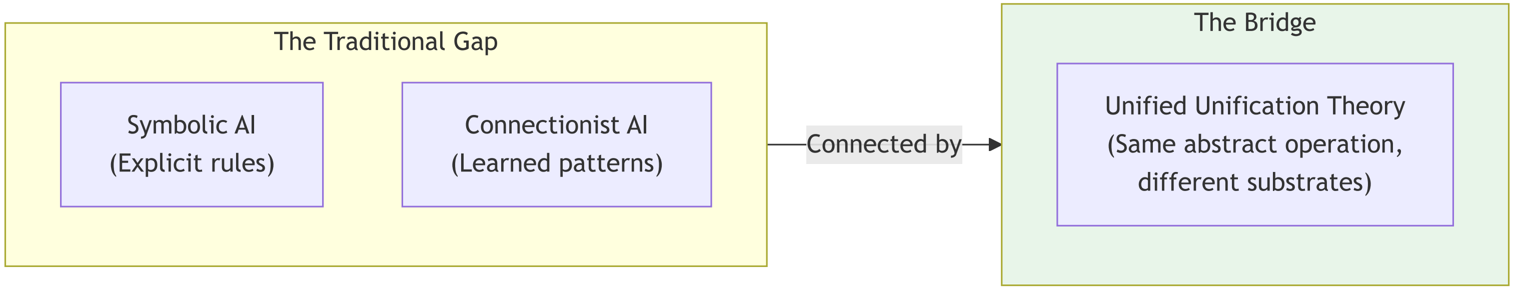


Diagram 24

This framework: - Closes the explanatory gap between symbolic and neural reasoning - Provides formal foundations for understanding LLM behavior - Offers architectural guidance for building better reasoners - Connects to the broader constraint-emergence ontology

> **Reasoning is not magic. It is structured traversal of a constraint manifold. Both brains and LLMs do it - just with different implementations of the same abstract pattern.**

This presentation formalizes how Large Language Models perform reasoning through topology-constrained traversal of semantic manifolds.

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