The objective of this problem is to find the schedule that maximizes the total net profit. From the table supplied, it is clear that there are 9 variables in the equation. Fortunately the amount of variables can be reduced since they are dependent on one another.

	Hams	Bellies	Picnics
Regular Time	x_1	x_3	x_5
Overtime	x_2	x_4	x_6
Fresh	x_7	x_8	x_9

Our original equation for $z = \mathbf{c}^T\mathbf{x}$ will be

$$z = 14x_1 + 11x_2 + 8x_7 + 12x_3 + 7x_4 + 4x_8 + 13x_5 + 9x_6 + 4x_9 \tag{1}$$

Now, replace x_7, x_8, and x_9 with the variables they are dependent on.

$$\begin{aligned} x_1 + x_2 + x_7 &= 480 \\ x_7 &= 480 - x_1 - x_2 \end{aligned}$$

Similarly,

$$\begin{aligned} x_3 + x_4 + x_8 &= 400 \\ x_8 &= 400 - x_3 - x_4, \end{aligned}$$

and,

$$\begin{aligned} x_5 + x_6 + x_9 &= 230 \\ x_9 &= 230 - x_5 - x_6. \end{aligned}$$

Plug these values into (1), the new $\mathbf{c}^T\mathbf{x}$ will be

$$\begin{aligned} z &= 14x_1 + 11x_2 + 8(480 - x_1 - x_2) \\ &\quad + 12x_3 + 7x_4 + 4(400 - x_3 - x_4) \\ &\quad + 13x_5 + 9x_6 + 4(230 - x_5 - x_6) \\ z &= 14x_1 + 11x_2 + 3840 - 8x_1 - 8x_2 \\ &\quad + 12x_3 + 7x_4 + 1600 - 4x_3 - 4x_4 \\ &\quad + 13x_5 + 9x_6 + 920 - 4x_5 - 4x_6 \\ z &= 6x_1 + 3x_2 + 8x_3 + 3x_4 + 9x_5 + 5x_6 + 6360 \end{aligned} \tag{2}$$

The 6360 can just forgotten about for now. We will add it back on once we solve for z. Now, work on getting the constraints. Since there are only 480 Hams total, then $x_1 + x_2 \leq 480$ since $x_7 \geq 0$. Similarly, $x_3 + x_4 \leq 400$ since $x_8 \geq 0$, and $x_5 + x_6 \leq 230$ since $x_9 \geq 0$. The total number of hams, bellies, and picnics that can be smoked during regular hours is 420. That means $x_1 + x_3 + x_5 \leq 420$. The total that can be smoked during overtime is 250 so $x_2 + x_4 + x_6 \leq 250$. Since the factory cannot make a negative number of items it makes sense to constrain all the variables to positive values.

$$x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9 \geq 0$$

This finalizes our constraints, and they can now be written as

$$\begin{array}{cccccccccccc} x_1 & + & x_2 & & & & & & & \leq & 480 \\ & & & & x_3 & + & x_4 & & & & \leq & 400 \\ & & & & & & & x_5 & + & x_6 & \leq & 230 \\ x_1 & & + & & x_3 & & + & x_5 & & & \leq & 420 \\ & & x_2 & & + & & x_4 & + & & x_6 & \leq & 250 \\ & & x_1, & x_2, & x_3, & x_4, & x_5, & x_6, & \geq & 0 & & \end{array} \tag{3}$$

From the constraints, extract the **A** matrix and **b** vector from the coefficients and left hand side of the equations respectively. All together this gives everything needed to put this problem in standard form.

$$\mathbf{c} = \begin{bmatrix} 6 \\ 3 \\ 8 \\ 3 \\ 9 \\ 5 \end{bmatrix} \qquad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} \qquad \mathbf{A} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix} \qquad \mathbf{b} = \begin{bmatrix} 480 \\ 400 \\ 230 \\ 420 \\ 250 \end{bmatrix} \tag{4}$$

In order to solve this linear programming problem, it is necessary to introduce slack variables s_1, s_2, s_3, s_4, and s_5 to change the inequalities to equalities. For convenience these will be written in matrix form.

$$\begin{array}{c} \begin{array}{ccccccccccccc} z & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & s_1 & s_2 & s_3 & s_4 & s_5 & \end{array} \\ \begin{pmatrix} 1 & -6 & -3 & -8 & -3 & -9 & -5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 480 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 400 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 230 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 420 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 250 \end{pmatrix} \end{array} \tag{5}$$

The set $B_0 = \{s_1, s_2, s_3, s_4, s_5\}$ corresponds to the basic variables which will be switched with variables from $R_0 = \{x_1, x_2, x_3, x_4, x_5, x_6\}$, corresponding to the solution variables. At this point the basic feasible solution is

$$x_1 = x_2 = x_3 = x_4 = x_5 = x_6 = 0.$$

This results in $z = 0$. In order to maximize z, pick the highest value from R_0 to enter B_0. To do this take the first row of the matrix and solve it for z.

$$z = 0 + 6x_1 + 3x_2 + 8x_3 + 3x_4 + 9x_5 + 5x_6$$

x_5 is the highest coefficient so it will be the entering variable. Now, looking in the x_5 column, find the minimum solution to the rows.

$$\begin{aligned} s_1 &= 480 && \geq 0 \\ s_2 &= 400 && \geq 0 \\ s_3 &= 230 - x_5 && \geq 0 \implies x_5 \quad \leq 230 \\ s_4 &= 420 - x_5 && \geq 0 \implies x_5 \quad \leq 420 \\ s_5 &= 250 && \geq 0 \end{aligned} \tag{6}$$

s_3 will be the departing variable since 230 is the minimum solution. $\mathbf{A}_{4,6}$ will be the pivot element. Since it is already equal to 1 it is not necessary to scale it. Now add

$$\mathbf{A} + \begin{bmatrix} 9 \cdot \mathbf{A}_{4,} \\ 0 \cdot \mathbf{A}_{4,} \\ 0 \cdot \mathbf{A}_{4,} \\ 0 \cdot \mathbf{A}_{4,} \\ -1 \cdot \mathbf{A}_{4,} \\ 0 \cdot \mathbf{A}_{4,} \end{bmatrix} = \begin{array}{c} \begin{array}{ccccccccccccc} z & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & s_1 & s_2 & s_3 & s_4 & s_5 & \end{array} \\ \begin{pmatrix} 1 & -6 & -3 & -8 & -3 & 0 & 4 & 0 & 0 & 9 & 0 & 0 & 2070 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 480 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 400 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 230 \\ 0 & 1 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & -1 & 1 & 0 & 190 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 250 \end{pmatrix} \end{array}, \tag{7}$$

to get the next basic feasible solution. Solving for z in the first row gives

$$z = 2070 + 6x_1 + 3x_2 + 8x_3 + 3x_4 - 4x_6 - 9s_3 \tag{8}$$

Setting all the variables to 0 gives the basic feasible solution of 2070. Now, repeating the procedures, until all the coefficients in the the solution are negative, will give the maximum value for z. $B_1 = \{s_1, s_2, x_5, s_4, s_5\}$

and $R_1 = \{x_1, x_2, x_3, x_4, s_3, x_6\}$. For the next step choose x_3 to be the entering variable and s_4 to be the departing variable.

$$\mathbf{A} + \begin{bmatrix} 8 \cdot \mathbf{A}_{5,} \\ 0 \cdot \mathbf{A}_{5,} \\ -1 \cdot \mathbf{A}_{5,} \\ 0 \cdot \mathbf{A}_{5,} \\ 0 \cdot \mathbf{A}_{5,} \\ 0 \cdot \mathbf{A}_{5,} \end{bmatrix} = \begin{array}{c} \begin{array}{ccccccccccccc} z & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & s_1 & s_2 & s_3 & s_4 & s_5 & \end{array} \\ \begin{pmatrix} 1 & 2 & -3 & 0 & -3 & 0 & -4 & 0 & 0 & 1 & 8 & 0 & 3590 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 480 \\ 0 & -1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & -1 & 0 & 210 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 230 \\ 0 & 1 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & -1 & 1 & 0 & 190 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 250 \end{pmatrix} \end{array} \tag{9}$$

Solving for z gives

$$z = 3590 - 2x_3 + 3x_2 + 3x_4 + 4x_6 - s_3 - 8s_4 \tag{10}$$

$B_2 = \{s_1, s_2, x_5, x_3, s_5\}$ and $R_2 = \{x_1, x_2, s_4, x_4, s_3, x_6\}$. Choose x_6 to be the entering variable and s_2 to be the departing variable.

$$\mathbf{A} + \begin{bmatrix} 4 \cdot \mathbf{A}_{3,} \\ 0 \cdot \mathbf{A}_{3,} \\ 0 \cdot \mathbf{A}_{3,} \\ -1 \cdot \mathbf{A}_{3,} \\ 1 \cdot \mathbf{A}_{3,} \\ -1 \cdot \mathbf{A}_{3,} \end{bmatrix} = \begin{array}{c} \begin{array}{ccccccccccccc} z & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & s_1 & s_2 & s_3 & s_4 & s_5 & \end{array} \\ \begin{pmatrix} 1 & -2 & -3 & 0 & 1 & 0 & 0 & 0 & 4 & 5 & 4 & 0 & 4430 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 480 \\ 0 & -1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & -1 & 0 & 210 \\ 0 & 1 & 0 & 0 & -1 & 1 & 0 & 0 & -1 & 0 & 1 & 0 & 20 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 400 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 1 & 1 & 40 \end{pmatrix} \end{array} \tag{11}$$

Solving for z gives

$$z = 4430 + 2x_3 + 3x_2 - x_4 - 4s_2 - 5s_3 - 4s_4 \tag{12}$$

$B_3 = \{s_1, x_6, x_5, x_3, s_5\}$ and $R_3 = \{x_1, x_2, s_4, x_4, s_3, s_2\}$. Choose x_2 to be the entering variable and s_5 to be the departing variable.

$$\mathbf{A} + \begin{bmatrix} 3 \cdot \mathbf{A}_{6,} \\ -1 \cdot \mathbf{A}_{6,} \\ 0 \cdot \mathbf{A}_{6,} \\ 0 \cdot \mathbf{A}_{6,} \\ 0 \cdot \mathbf{A}_{6,} \\ 0 \cdot \mathbf{A}_{6,} \end{bmatrix} = \begin{array}{c} \begin{array}{ccccccccccccc} z & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & s_1 & s_2 & s_3 & s_4 & s_5 & \end{array} \\ \begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 2 & 7 & 3 & 4550 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & -1 & -1 & 440 \\ 0 & -1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & -1 & 0 & 210 \\ 0 & 1 & 0 & 0 & -1 & 1 & 0 & 0 & -1 & 0 & 1 & 0 & 20 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 400 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 1 & 1 & 40 \end{pmatrix} \end{array} \tag{13}$$

Solving for z gives

$$z = 4430 - x_1 - x_4 - s_2 - 2s_3 - 7s_4 - 3s_5 \tag{14}$$

$B_4 = \{s_1, x_6, x_5, x_3, x_2\}$ and $R_4 = \{x_1, s_5, s_4, x_4, s_3, s_2\}$. This is almost the maximum solution, since all the coefficients are negative. Add back the 6360 found in (2) and set all the variables to 0. This produces the maximum value for z.

$$\begin{aligned} z &= 4430 + 6360 - x_1 - x_4 - s_2 - 2s_3 - 7s_4 - 3s_5 \\ z &= 4430 + 6360 - 0 - 0 - 0 - 0 - 0 - 0 \\ z &= 10790 \end{aligned} \tag{15}$$

To solve for the variables in B_4 set the variables in R_4 to 0. This gives

$$\begin{aligned} s_1 &= 480 \\ x_6 &= 210 \\ x_5 &= 20 \\ x_3 &= 400 \\ x_2 &= 40 \end{aligned} \tag{16}$$

Now it is possible to find the x_7, x_8, and x_9 from earlier.

$$\begin{aligned} x_7 &= 480 - 0 - 40 &&= 440 \\ x_8 &= 400 - 400 - 0 &&= 0 \\ x_9 &= 230 - 20 - 210 &&= 0 \end{aligned} \tag{17}$$

This gives

	Hams	Bellies	Picnics
Regular Time	0	400	20
Overtime	40	0	210
Fresh	440	0	0