Homework 1

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Problem 5

Consider the color TV problem without constraints (Example 2.1). Because the company's assembly plant is located overseas, the U.S. government has imposed a tariff of $25 per unit.
(a) Find the optimal production levels, taking the tariff into consideration. What does the tariff cost the company? How much of this cost is paid directly to the government, and how much represents lost sales?
(b) Would it be worthwhile for the company to relocate production facilities to the U.S. in order to avoid the tariff? Assume that the overseas facility can be leased to another manufacturer for $200,000 annually. The construction costs have been amortized ovet the expected life of the new facility.
(c) The perpose of the tariff is to motivate manufacturing companies to operate plants in the U.S. What is the minimum tariff that would make it worthwhile for the company to relocate its facility?
(d) Given that the tariff is large enough to motivate the company to move its facility, how important is the actual tariff amount? Consider the sensitivity of both production levels and profit to the amount of the tariff.

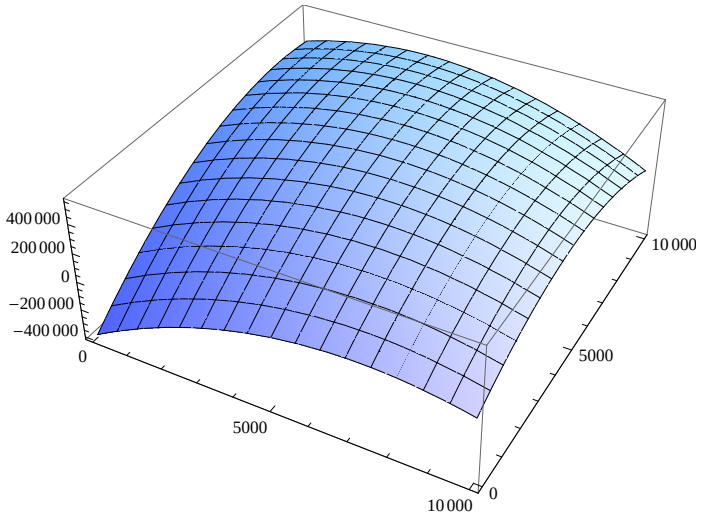
- **(A)**

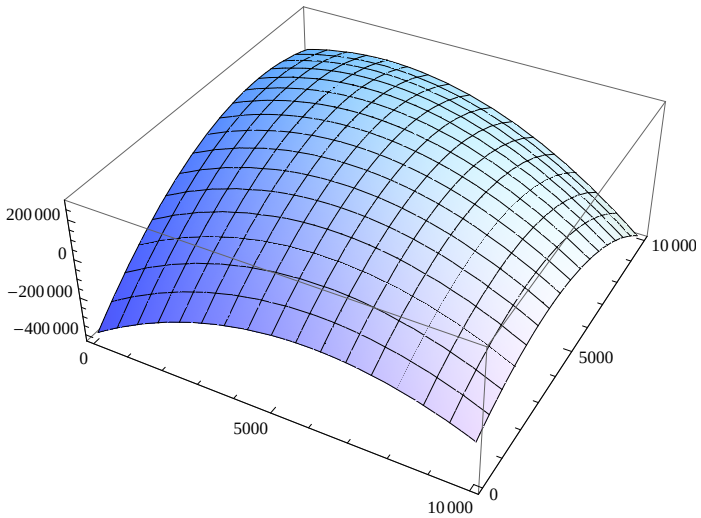
y = profit without considering tariff
yt = profit when considering tariff
x1 = number of 19 inch tvs
x2 = number of 21 inch tvs

```
Clear[y, yt, s, st, a, b, c]
```

```
y[x1_, x2_] = (339 - .01 * x1 - .003 * x2) * x1 +
    (399 - .004 * x1 - .01 * x2) * x2 - (400 000 + 195 * x1 + 225 * x2);
yt[x1_, x2_] = (339 - .01 * x1 - .003 * x2) * x1 + (399 - .004 * x1 - .01 * x2) * x2 -
    (400 000 + 195 * x1 + 225 * x2) - 25 (x1 + x2);
```

```
Plot3D[y[x1, x2], {x1, 0, 10 000}, {x2, 0, 10 000}]
Plot3D[yt[x1, x2], {x1, 0, 10 000}, {x2, 0, 10 000}]
```






```
(*
s = the optimal number of tvs to make to maximize profit without the tariff
   st = the optimal number of tvs to make to maximize profit with the tariff
*)
s = Solve[{D[y[x1, x2], x1] == 0, D[y[x1, x2], x2] == 0}, {x1, x2}]
st = Solve[{D[yt[x1, x2], x1] == 0, D[yt[x1, x2], x2] == 0}, {x1, x2}]
```

```
{{x1 → 4735.04, x2 → 7042.74}}
```

```
{{x1 → 3809.12, x2 → 6116.81}}
```

```
a = y[x1, x2] /. s (* the profit without considering the tariff *)
b = yt[x1, x2] /. st (* the profit considering the tariff *)
```

```
{553 641.}
```

```
{282 345.}
```

```
c = 25 (x1 + x2) /. st (* the amount of money the company pays the government *)
```

```
{248 148.}
```

```
a - b (* how much profit is lost due to the tariff *)
```

```
{271 296.}
```

```
a - b - c (* the amount of the lost profit minus the amount paid
  to the government directly. The cost that represents lost sales. *)
```

```
{23 148.1}
```

■ (B)

y = profit of the company if it moves to America
yt = the profit of the company if it stays

```
Clear[y, yt, s, st, a, b]
```

```
y[x1_, x2_] = 200 000 + (339 - .01 * x1 - .003 * x2) * x1 +
    (399 - .004 * x1 - .01 * x2) * x2 - (950 000 + 195 * x1 + 225 * x2);
yt[x1_, x2_] = (339 - .01 * x1 - .003 * x2) * x1 + (399 - .004 * x1 - .01 * x2) * x2 -
    (400 000 + 195 * x1 + 225 * x2) - (25) (x1 + x2);
```

```
(* the optimal solutions to the two equations *)
s = Solve[{D[y[x1, x2], x1] == 0, D[y[x1, x2], x2] == 0}, {x1, x2}]
st = Solve[{D[yt[x1, x2], x1] == 0, D[yt[x1, x2], x2] == 0}, {x1, x2}]
```

```
{{x1 → 4735.04, x2 → 7042.74}}
```

```
{{x1 → 3809.12, x2 → 6116.81}}
```

```
a = y[x1, x2] /. s (* the profit the company will make if they move to america *)
b = yt[x1, x2] /. st(* the profit the company will make if they stay overseas *)
```

```
{203 641.}
```

```
{282 345.}
```

It is not worth it for the company to move to America. With the tariff at twenty-five dollars they make more money overseas than they would by moving to America.

■ (C)

r = tariff in dollars

```
Clear[yt, st, f]
```

```
yt[x1_, x2_, r] = (339 - .01 * x1 - .003 * x2) * x1 +
    (399 - .004 * x1 - .01 * x2) * x2 - (400 000 + 195 * x1 + 225 * x2) - r (x1 + x2);
```

```
st = Solve[{D[yt[x1, x2, r], x1] == 0, D[yt[x1, x2, r], x2] == 0}, {x1, x2}]
 (* find an optimal solution in terms of tariff *)
```

```
{{x1 → 4735.04 - 37.037 r, x2 → 7042.74 - 37.037 r}}
```

```
f = yt[x1, x2, r] /. st;
(* plug the solution in terms of tariff into the original equation *)
```

```
Solve[f == 203 641, {r}]
(* solve for when the cost with tariff equals the cost without the tariff *)
```

```
{{r → 33.1787}, {r → 284.821}}
```

We see that in order for the company to justify a move to America the tariff must be at least thirty-three

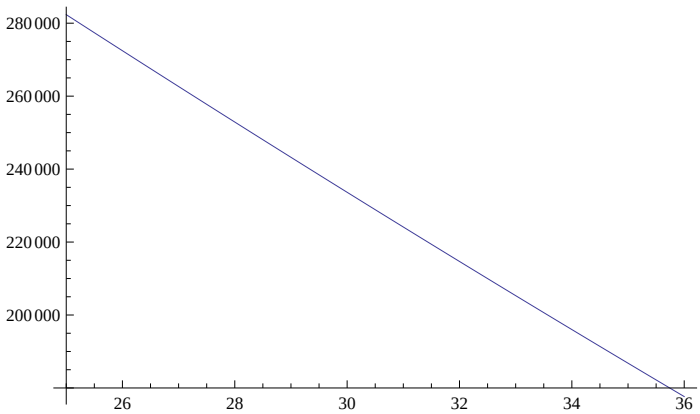
dollars. This would cause the profit from operating overseas to be equivalent to the cost for operating in America

- **(D)**

```
Clear[yt, s, x1opt, x2opt, yopt, dx1optda, dyopt, r]

yt[x1_, x2_, r_] = (339 - .01 * x1 - .003 * x2) * x1 +
    (399 - .004 * x1 - .01 * x2) * x2 - (400 000 + 195 * x1 + 225 * x2) - r (x1 + x2);

s = Solve[{D[yt[x1, x2, r], x1] == 0, D[yt[x1, x2, r], x2] == 0}, {x1, x2}]
```

```
{{x1 → 4735.04 - 37.037 r, x2 → 7042.74 - 37.037 r}}
```

```
x1opt = x1 /. s; (* optimal solution for 19 inch tvs in terms of r *)
x2opt = x2 /. s; (* optimal solution for 21 inch tvs in terms of r *)
yopt = yt[x1, x2, r] /. s;(* optimal solution of profit in terms of r *)

Plot[yopt, {r, 25, 36}]
```




```
Clear[r]
dx1optda = D[x1opt, r]
 (* derivative of the optimal solution for x1 with respect to r *)
```

```
{-37.037}
```

```
r = 25;
S = dx1optda * (r / x1opt) (* sensitivity of the tariff with respect to 19 inch tvs *)
```

```
{-0.243082}
```

Increasing the tariff by ten percent will decrease the optimal number of 19 inch tvs by 2 percent

```
Clear[r]
dx2optda = D[x2opt, r]
 (* derivative of the optimal solution for x2 with respect to r *)
```

```
{-37.037}
```

```
r = 25;
S = dx2optda * (r / x2opt)(* sensitivity of the tariff with respect to 21 inch tvs *)
```

```
{-0.151374}
```

Increasing the tariff by ten percent will decrease the optimal number of 21 inch tvs by 1 percent

```
Clear[r]
dyopt = D[yopt, r]
 (* derivative of the optimal solution for profit with respect to r *)
```

```
{3777.78 - 37.037 (399 - 0.004 (4735.04 - 37.037 r) - 0.01 (7042.74 - 37.037 r)) -
  37.037 (339 - 0.01 (4735.04 - 37.037 r) - 0.003 (7042.74 - 37.037 r)) +
  0.481481 (4735.04 - 37.037 r) + 0.518519 (7042.74 - 37.037 r) + 148.148 r}
```

```
r = 25;
dyopt * (r / yopt)(* sensitivity of the tariff with respect to the profit *)
```

```
{-0.878884}
```

Increasing the tariff by ten percent will decrease the maximum profit by 8 percent

The actual tariff amount is very important we see that raising it by just 8 dollars makes it worthwhile to move the company to America.