PART – C (5 × 12 = 60 Marks)
Answer ALL Questions

28. a. Verify that the matrix $A=\begin{bmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ satisfies its characteristic equation and hence find A^4.

(OR)

b. Reduce the quadratic form $x_1^2+2x_2^2+x_3^2-2x_1x_2+2x_2x_3$ to the canonical form through an orthogonal transformation. Also find the rank, index, signature and nature of the quadratic form.

29. a. Expand $e^x \cos y$ in powers of x and y as far as the terms of the third degree.

(OR)

b. A rectangular box, open at the top, is to have a volume of 32cc. Find the dimensions of the box, that requires the least material for its construction.

30. a. Solve $\left(x^2D^2-xD+1\right)y=\left(\frac{\log x}{x}\right)^2$.

(OR)

b. Solve the equation $\frac{d^2y}{dx^2}+y=\tan x$, by the method of variation of parameters.

31. a. Find the equation of the circle of curvature of the parabola $y^2=12x$ at the point (3, 6).

(OR)

b. Find the evolute of the curve $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

32.a.i. Test the convergence of the following series $\frac{3}{4}+\frac{3.4}{4.6}+\frac{3.4.5}{4.5.8}+....\infty$.

ii. Examine the convergence of the following series $\frac{1}{2!}-\frac{2}{3!}+\frac{3}{4!}-....$

(OR)

b. Test the converges of the following series $\frac{3}{7}x+\frac{3.6}{7.10}x^2+\frac{3.6.9}{7.10.13}x^3+....(x>0)$.

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Reg. No.															

B.Tech. DEGREE EXAMINATION, NOVEMBER 2019
First Semester

18MAB101T - CALCULUS AND LINEAR ALGEBRA
(For the candidates admitted during the academic year 2018 – 2019 onwards)

Note:
(i) Part - A should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45[th] minute.
(ii) Part - B and Part - C should be answered in answer booklet.

Time: Three Hours | Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer ALL Questions

1. If $A=\begin{bmatrix} 2 & 5 & 1 \\ 0 & 4 & 6 \\ 0 & 0 & 1 \end{bmatrix}$, then the Eigen values of A^{-1} are
(A) $\frac{1}{2},\frac{1}{4},1$
(B) 1, 3, 4
(C) 2, 4, 1
(D) $\frac{1}{3},\frac{1}{4},1$

2. The index of the canonical form $-y_1^2+y_2^2+4y_3^2$ is
(A) 3
(B) 2
(C) 1
(D) 0

3. The sum of the Eigen values of the matrix $A=\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ is
(A) 6
(B) 4
(C) 5
(D) 2

4. The eigen vectors corresponding to the distinct eigen values of a real symmetric matrix are
(A) Imaginary
(B) Non-orthogonal
(C) Real
(D) Orthogonal

5. If $f(x, y)=0$ and y is an implicit function of x, then dy/dx is
(A) $\frac{-\partial f/\partial x}{\partial f/\partial y}$
(B) $\frac{\partial f/\partial y}{\partial f/\partial x}$
(C) $\frac{-\partial f/\partial y}{\partial f/\partial x}$
(D) $\frac{\partial f/\partial x}{\partial f/\partial y}$

6. If $J_1=J\left(\frac{x, y}{u, v}\right)$ and $J_2=J\left(\frac{u, v}{x, y}\right)$, then J_1J_2 is
(A) 0
(B) 1
(C) −1
(D) 2

7. u and v are functionally dependent if their Jacobian value is
(A) 0
(B) 1
(C) 2
(D) 3

8. The point at which there is no extreme value is
(A) Maximum point
(B) Minimum point
(C) Saddle point
(D) Stationary point

9. The complimentary function of $(D^2-2D+1)y=0$ is
(A) $C_1e^x+C_2e^{-x}$
(B) $(C_1+C_2x)e^x$
(C) $C_1e^{2x}+C_2e^{-2x}$
(D) $(C_1+C_2x)e^{-x}$

10. The particular integral of $(D^2+16)y=e^{-4x}$ is
(A) $\frac{x}{32}e^{-4x}$
(B) $\frac{1}{32}e^{-4x}$
(C) $\frac{1}{16}e^{-4x}$
(D) $\frac{x}{16}e^{-4x}$

11. The complementary function of $(x^2D^2-7xD+12)y=0$ is
(A) $\frac{c_1}{x^2}+\frac{c_2}{x^6}$
(B) $c_1x^2+\frac{c_2}{x^6}$
(C) $\frac{c_1}{x^2}+c_2x^6$
(D) $c_1x^2+c_2x^6$

12. In method of variation of parameters Complete Solution $=c_1f_1+c_2f_2$ + particular integral, where particular integral is
(A) $PI=Pf_1+Qf_2$
(B) $PI=Pf_2+Qf_1$
(C) $PI=PC_1+QC_2$
(D) $PI=PC_2+QC_1$

13. The curvature of a circle of radius 'r' is
(A) r
(B) $1/r$
(C) $1/r^2$
(D) r^2

14. The parametric equation of a parabola $y^2=4ax$ is
(A) $x=at^2, y=2at$
(B) $x=t, y=1/t$
(C) $x=at,\ y=2at$
(D) $x=2at, y=at^2$

15. The locus of centre of curvature is called
(A) Involute
(B) Evolute
(C) Radius of curvature
(D) Envelope

16. The radius of curvature at (3, 4) on the curve $x^2+y^2=25$ is
(A) 5
(B) 4
(C) 0
(D) 2

17. General term of $\frac{1}{3},\frac{-2}{3},\frac{3}{3^3},\frac{-4}{3^4},......$ is
(A) $u_n=\frac{(-1)^{n+1}n}{3^n}$
(B) $u_n=\frac{(-1)^{n-1}n}{3^n}$
(C) $u_n=\frac{(-1)^{n-1}n+1}{3^n}$
(D) $u_n=\frac{(-1)^{n}n+1}{3^n}$

18. The series $\Sigma\frac{1}{n^p}$ is converges if
(A) $P=1$
(B) $P<1$
(C) $P>1$
(D) $P=0$

19. The series Σu_n of positive terms is convergent if
(A) $\lim\limits_{n\to\infty}\frac{u_{n+1}}{u_n}<1$
(B) $\lim\limits_{n\to\infty}\frac{u_n}{u_{n+1}}=1$
(C) $\lim\limits_{n\to\infty}\frac{u_n}{u_{n+1}}<1$
(D) $\lim\limits_{n\to\infty}\frac{u_{n+1}}{u_n}>1$

20. n^{th} term of a series if in A.P.
(A) $t_n=a-(n-1)d$
(B) $t_n=a+(n+1)d$
(C) $t_n=a-(n+1)d$
(D) $t_n=a+(n-1)d$

PART – B (5 × 4 = 20 Marks)
Answer ANY FIVE Questions

21. Verify that the sum of the Eigen values of A equals the trace of A and that their product equal $|A|$, for the matrix $A=\begin{bmatrix}1&0&0\\0&3&-1\\0&-1&3\end{bmatrix}$.

22. If $x=r\cos\theta$, $y=r\sin\theta$, find $\frac{\partial(x,y)}{\partial(r,\theta)}$.

23. Solve the equation $(D^2-4D+3)y=\sin 3x$.

24. Find the radius of curvature of the curve $r=a(1+\cos\theta)$ at the point $\theta=\pi/2$.

25. Test the converges of the following series $\frac{1}{1.2}+\frac{2}{3.4}+\frac{3}{5.6}+......\infty$.

26. Determine the nature of the following quadratic form without reducing it to canonical form: $x_1^2+3x_2^2+6x_3^2+2x_1x_2+2x_2x_3+4x_3x_1$.

27. Find the envelope of the family of straight lines given by $y=mx\pm\sqrt{a^2m^2-b^2}$ where m is the parameter.

b. Verify Cayley-Hamilton theorem for the matrix $A=\begin{pmatrix}1&3&7\\4&2&3\\1&2&1\end{pmatrix}$ and hence find A^{-1}.

29. a. A rectangular box open at the top is to have volume of 32 cubic feet. Find the dimension in order that the total surface area is minimum.

(OR)

b.i. Find the Taylor's series expansion of $e^x \sin x$ near the point $\left(-1,\frac{\pi}{4}\right)$ upto 3rd degree terms.

ii. If $x=r\cos\theta, y=r\sin\theta$ verify that $\frac{\partial(x,y)}{\partial(r,\theta)}\times\frac{\partial(r,\theta)}{\partial(x,y)}=1$.

30. a. Solve the simultaneous equation $\frac{dx}{dt}+2y+\sin t=0$ and $\frac{dy}{dt}-2x-\cos t=0$.

(OR)

b. Solve $(D^2+4)y=4\tan 2x$, by using method of variation of parameters.

31. a. Find the equation of the circle of curvature of the parabola $y^2=12x$ at the point (3, 6)

(OR)

b. Find the evolute of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

32. a.i. Test the convergence of the series $\frac{1}{1.2.3}+\frac{3}{2.3.4}+\frac{5}{3.4.5}+\ldots\infty$.

ii. Test for convergence of the series $\Sigma(\log n)^{-2n}$.

(OR)

b. Test the convergence of the series $\Sigma\frac{1}{\sqrt{n+1}-1}$.

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Reg. No.

B.Tech. DEGREE EXAMINATION, MAY 2019
First / Second Semester

18MAB101T – CALCULUS AND LINEAR ALGEBRA
(For the candidates admitted during the academic year 2018 – 2019 onwards)

Note:
(i) Part - A should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.
(ii) Part - B and Part - C should be answered in answer booklet.

Time: Three Hours | Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer ALL Questions

1. The Eigen values of the matrix $\begin{pmatrix}1&2\\5&4\end{pmatrix}$ are
(A) 1, 6 (B) −1, 6
(C) 1, −6 (D) −1, −6

2. The inverse of the eigen values of the matrix $\begin{pmatrix}1&2\\0&3\end{pmatrix}$ is
(A) $1,\frac{1}{2}$ (B) 1, 2
(C) $1,\frac{1}{3}$ (D) 1, 3

3. The nature of the quadratic form, $x_1^2+2x_2^2+3x_3^2+2x_1x_2+2x_2x_3-2x_3x_1$ is
(A) Indefinite (B) Positive definite
(C) Negative definite (D) Negative semi definite

4. The signature of the quadratic form whose canonical form is $2y_1^2-y_2^2-y_3^2$
(A) 1 (B) −1
(C) 0 (D) 6

5. If u and v are functionally dependent then their Jacobian value is
(A) Zero (B) One
(C) Non-zero (D) > 0

6. If $rt-s^2<0$ at (a, b) then the point is
(A) Maximum point (B) Minimum point
(C) Saddle point (D) Zero

7. If $u=x^2+y^2+3xy$ then $\frac{\partial u}{\partial x}$ is
(A) $2y+3x$ (B) $3y$
(C) $2x+3y$ (D) $2x$

8. If $u=x^2, v=y^2$ then $\frac{\partial(u,v)}{\partial(x,y)}$ is

(A) $2xy$
(B) $4xy$
(C) $2y$
(D) xy

9. Complementary function of $(D^2-2D+1)y=0$ is

(A) $C_1e^{x}+C_2e^{-x}$
(B) $(C_1+C_2x)e^{x}$
(C) $C_1e^{2x}+C_2e^{-2x}$
(D) $(C_1+C_2x)e^{2x}$

10. Particular integral of $(D^2+9)y=e^{-2x}$ is

(A) $\frac{e^{-2x}}{15}$
(B) $\frac{e^{2x}}{15}$
(C) $\frac{e^{-2x}}{13}$
(D) $\frac{e^{2x}}{13}$

11. Solve of $(x^2D^2+xD+1)y=0$ is

(A) $Ae^{az}+Be^{bz}$
(B) $A\cos z+B\sin z$
(C) $(A+Bz)e^{z}$
(D) $(A+Bz)e^{-z}$

12. The complementary function of the second order differential equation having roots $\alpha+i\beta$ is

(A) $e^{-\alpha x}(c_1\cos\beta x+c_2\sin\beta x)$
(B) $c_1\cos\beta x+c_2\sin\beta x$
(C) $c_1\cos\beta x-c_2\sin\beta x$
(D) $e^{\alpha x}(c_1\cos\beta x+c_2\sin\beta x)$

13. The curvature of the straight line is

(A) 1
(B) −1
(C) 2
(D) 0

14. The radius of curvature of a curve $y=4\sin x$ at $x=\frac{\pi}{2}$ is

(A) 1/4
(B) −1/4
(C) 1/2
(D) −1/2

15. The evolute of a curve is the locus of ______.

(A) Centre of curvature
(B) Curvature
(C) Radius of curvature
(D) Line

16. The value of $\Gamma\left(\frac{1}{2}\right)$ is

(A) π
(B) $\pi/2$
(C) $\sqrt{\pi}$
(D) $\sqrt{\pi}/2$

17. The series $\sum_{n=1}^{\infty}\frac{1}{n^p}$ is convergent, if

(A) $p=1$
(B) $p=0$
(C) $p>1$
(D) $p<1$

18. The series $\sum_{n=1}^{\infty}\frac{1}{n!}$ is

(A) Convergent
(B) Divergent
(C) Oscillating
(D) Monotonic

19. A sines Σu_n is said to be absolutely convergent if the series

(A) $\Sigma|u_n|$ is convergent
(B) Σu_n is divergent
(C) $\Sigma|u_n|$ is divergent
(D) Σu_n is convergent

20. The series $\Sigma\frac{n^3}{3^n}$ is

(A) Conditionally convergent
(B) Absolutely convergent
(C) Convergent
(D) Divergent

PART – B ($5\times4=20$ Marks)
Answer ANY FIVE Questions

21. Find the Eigen values and Eigen vectors of $\begin{bmatrix}1 & 1\\ 3 & -1\end{bmatrix}$.

22. Find $\frac{du}{dt}$ if $u=x^2+y^2+z^2$ where $x=e^t, y=e^t\sin t, z=e^t\cos t$.

23. Find the envelope of a family of straight lines $x\cos\alpha+y\sin\alpha=a\sec\alpha$, α is the parameter.

24. Show that $\Gamma(n+1)=n\Gamma(n)$.

25. Solve $(D^2+3D+2)y=\sin x$.

26. If $u=f(y-z,z-x,x-y)$,, prove that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0$.

27. Test for convergence $u_n=\sqrt{\frac{3^n-1}{2^n+1}}$.

PART – C ($5\times12=60$ Marks)
Answer ALL Questions

28. a. Deduce the quadratic form $6x_1^2+3x_2^2+3x_3^2-4x_1x_2-2x_2x_3+4x_3x_1$ to a canonical form and hence find rank, index and signature.

(OR)

27. Test the convergence of the series $\sum_{n=1}^{\infty}\frac{1}{(n+1)(2n+1)}$.

PART – C (5 × 12 = 60 Marks)
Answer ALL Questions

28. a. Verify Cayley-Hamilton theorem for the matrix $A=\begin{pmatrix}1 & 3 & 7\\ 4 & 2 & 3\\ 1 & 2 & 1\end{pmatrix}$ and hence find A^{-1}.

(OR)

b. Reduce the quadratic form $6x_1^2+3x_2^2+3x_3^2-4x_1x_2-2x_2x_3+4x_3x_1$ to canonical form by an orthogonal transformation. Also state its nature.

29. a. Explain $e^x\cos y$ in powers of x and y as far as the terms of the third degree.

(OR)

b. Find the volume of the largest rectangular parallelepiped that can be inserted in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$.

30. a. Solve $(D^2-7D+12)y=e^{5x}+\cos 2x$.

(OR)

b. Solve $(x^2D^2+xD-9)y=\frac{5}{x^2}$.

31. a. Find the equation of the circle of curvature of the parabola $y^2=12x$ at the point (3, 6).

(OR)

b. Find the evolute of the parabola $y^2=4ax$.

32. a.i. State the axioms of convergence of series by comparison test.

ii. Test the convergence of the series $\frac{3}{4}+\frac{3.4}{4.6}+\frac{3.4.5}{4.6.8}+\ldots\infty$.

(OR)

b.i. State the Cauchy's criterion of convergence of an infinite series.

ii. Test the convergence of the series $\sum_{n=1}^{\infty}\left(\sqrt{n^4+1}-\sqrt{n^4-1}\right)$.

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Reg. No. | | | | | | | | | | | | | | |

B.Tech. DEGREE EXAMINATION, NOVEMBER 2018
First Semester

18MAB101T – CALCULUS AND LINEAR ALGEBRA
(For the candidates admitted during the academic year 2018 – 2019)

Note:
(i) **Part - A should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.**
(ii) **Part - B and Part - C should be answered in answer booklet.**

Time: Three Hours

Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer ALL Questions

1. The eigen values of A^2, given $A=\begin{pmatrix}3 & 1 & 4\\ 0 & 2 & 6\\ 0 & 0 & 5\end{pmatrix}$ are
   (A) 3, 2, 5
   (B) 9, 4, 25
   (C) 3, 1, 4
   (D) 9, 1, 16

2. The characteristics equation of the matrix $A=\begin{pmatrix}1 & 2\\ 4 & 3\end{pmatrix}$ is
   (A) $\lambda^2-4\lambda-5=0$
   (B) $\lambda^2+4\lambda-5=0$
   (C) $\lambda^2+4\lambda+5=0$
   (D) $\lambda^2+4\lambda-5=0$

3. The nature of the quadratic form, whose matrix is $\begin{pmatrix}1 & 1 & 2\\ 1 & 3 & 1\\ 2 & 1 & 6\end{pmatrix}$ is
   (A) Indefinite
   (B) Positive semi-definite
   (C) Positive definite
   (D) Negative definite

4. If the canonical form of a quadratic form is $-y_1^2+y_2^2+4y_3^2$, then the signature of QF is
   (A) 0
   (B) 3
   (C) 2
   (D) 1

5. If $u=xy^2+x^2y$, where $x=at^2$, $y=2at$, that $\frac{du}{dt}$
   (A) $2a^2t^2(8+5t)$
   (B) $2a^3t^3(8+5t)$
   (C) $4a^3t^3(8+5t)$
   (D) $4a^2t^2(8+5t)$

6. The stationary points of the function $3x^2-y^2+x^3$ are
   (A) (0, 0), (–2, 0)
   (B) (0, 0), (2, 0)
   (C) (1, 1), (–2, 0)
   (D) (1, 1) (2, 0)

7. The $x = r\cos\theta, y = r\sin\theta$, then the Jacobian of x, y with respect to r, θ $\left(i.e\ J\left(\frac{x,y}{r,\theta}\right)\right)$

(A) r (B) $1/r$
(C) $2r$ (D) $2/r$

8. Maclaurin series for e^{x+y} is

(A) $1+(x+y)-\frac{(x+y)^3}{3!}+....$ (B) $1-(x-y)-\frac{(x-y)^3}{2!}-...$
(C) $1+(x-y)-\frac{(x-y)^3}{2!}+....$ (D) $1+(x+y)+\frac{(x+y)^3}{2!}+...$

9. The solution of $(D^2+4)y = 0$ is

(A) $y = Ae^{2x} + Be^{2x}$ (B) $y = A\cos 2x + B\sin 2x$
(C) $y = Ae^{2x} + Be^{-2x}$ (D) $y = A\cos x + B\sin x$

10. The particular integral of the differential equation $(D^2 - 3D + 2)y$ cosx is

(A) $\frac{1}{10}(\cos x + 3\sin x)$ (B) $\frac{1}{9}(\cos x - 3\sin x)$
(C) $\frac{1}{10}(\cos x - 3\sin x)$ (D) $\cos x - 3\sin x$

11. The complementary function of $(D^2+4D+4)y = e^{-2x}$ is

(A) $CF = (Ax+B)e^{-2x}$ (B) $CF = (Ax+B)e^{2x}$
(C) $CF = A\cos 2x + B\sin 2x$ (D) $CF = Ae^{-2x} + Be^{-2x}$

12. The complementary function of $\left(x^2D^2 + 4xD + 2\right)y = x^2 + \frac{1}{x^2}$ is

(A) $CF = (Ax+B)e^{-2x}$ (B) $CF = (Ax-B)e^{-x}$
(C) $CF = Ae^{-x} + Be^{-2x}$ (D) $CF = \frac{A}{x} + \frac{B}{x^2}$

13. The radius of curvature of the curve $y = e^x$ at $x = 0$ is

(A) $2\sqrt{2}$ (B) $\sqrt{2}$
(C) 2 (D) 4

14. The radius of curvature of a circle at any point is same as its

(A) Chord (B) Radius
(C) Diameter (D) Tangent

15. The value of $\Gamma\left(\frac{1}{2}\right)$ is .

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$
(C) π (D) $\sqrt{\pi}$

16. When n is a +ve integer, $\Gamma(n+1) =$

(A) $(n+1)!$ (B) $n!$
(C) $(2n)!$ (D) $(n-1)!$

17. The series $\sum_{n=1}^{\infty}\frac{1}{n^p}$ is convergent, if

(A) $p > 1$ (B) $p < 1$
(C) $p = 1$ (D) $p = 0$

18. The series $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+....$ is

(A) Convergent (B) Oscillating
(C) Divergent (D) Monotonic

19. As per D'Alembert's ratio test, if Σu_n is a series of positive terms and $\lim_{n\to\infty}\frac{u_{n+1}}{u_n} = l$, the series is convergent, if

(A) $l > 1$ (B) $l = 0$
(C) $l = 1$ (D) $l < 1$

20. The series $\Sigma\frac{1}{n}\sin\left(\frac{1}{x}\right)$ is

(A) Convergent (B) Conditionally convergent
(C) Divergent (D) Absolutely convergent

PART – B (5 × 4 = 20 Marks)
Answer ANY FIVE Questions

21. Use Cayley-Hamilton theorem to find A^3, given that $A = \begin{pmatrix} -1 & 3 \\ 2 & 4 \end{pmatrix}$.

22. If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$ prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$

23. Find the Jacobian of y_1, y_2, y_3 with respect to x_1, x_2, x_3 if $y_1 = \frac{x_2x_3}{x_1}, y_2 = \frac{x_3x_1}{x_2}$ and $y_3 = \frac{x_1x_2}{x_3}$.

24. Find the eigenvalues of the matrix $A = \begin{pmatrix} 2 & -2 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{pmatrix}$.

25. Solve the equation $(D^2 - 4D + 3)y = \sin 3x$.

26. Find the envelope of the family of straight lines given by $x\cos\alpha + y\sin\alpha = a\sec\alpha$.

1. The number of positive terms in the canonical form is called

   A. Signature of the quadratic form

   B. Index of the quadratic form

   C. Quadratic form

   D. Positive form

   ANSWER: B

2. The matrix of the quadratic form $3x_1^2 + 3x_2^2 - 5x_3^2 - 2x_1x_2 - 6x_2x_3 - 6x_3x_1$ is

   A. $\begin{pmatrix} 3 & -1 & -3 \\ -3 & -3 & 5 \\ -1 & 3 & -3 \end{pmatrix}$

   $B.$ $\begin{pmatrix} -3 & -3 & -5 \\ 3 & -1 & -3 \\ -1 & 3 & -3 \end{pmatrix}$

   $C.$ $\begin{pmatrix} 3 & -1 & -3 \\ -1 & 3 & -3 \\ -3 & -3 & -5 \end{pmatrix}$

   $D.$ $\begin{pmatrix} 3 & 3 & -5 \\ -1 & -1 & -3 \\ -3 & -3 & -3 \end{pmatrix}$

   ANSWER: C

3. The eigen values of A^2, if $A = \begin{pmatrix} -1 & 0 & 0 \\ 2 & -3 & 0 \\ 1 & 4 & 2 \end{pmatrix}$ are

A. 1, 4, 9
B. 1, 9, 16
C. 2, 4, 6
D. 4, 9, 16

ANSWER: A

4. If two of the eigen values of a 3×3 matrix, whose determinant equals 4 are -1 and 2, then the third eigen value is

A. -8
B. -6
C. -4
D. -2

ANSWER: D

5. The nature of quadratic form $6x^2 + 3y^2 + 3z^2 - 4xy - 2yz + 4zx$ is

A. Positive Semidefinite
B. Indefinite
C. Positive definite
D. Negative definite

ANSWER: C

6. If 2 and 3 are the eigen values of $A = \begin{pmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{pmatrix}$, then the eigen values of A^{-1} are

A. $\frac{1}{2}, \frac{1}{3}, \frac{1}{3}$
B. $\frac{1}{2}, \frac{1}{2}, \frac{1}{4}$
C. $\frac{1}{2}, \frac{1}{2}, \frac{1}{3}$
D. $\frac{1}{2}, \frac{1}{3}, \frac{1}{6}$

ANSWER: C

7. The characteristic equation of the matrix $A = \begin{pmatrix} 1 & 2 \\ 5 & 4 \end{pmatrix}$ is

A. $\lambda^2 + 5\lambda + 6 = 0$
B. $\lambda^2 - 5\lambda - 5 = 0$
C. $\lambda^2 - 5\lambda - 6 = 0$
D. $\lambda^2 - 6\lambda + 5 = 0$

ANSWER: C

8. The sum and the product of the eigen values of $A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$ are

A. 3 and 2
B. 2 and 1
C. 5 and 2
D. 5 and 3

ANSWER: D

9. Every square matrix satisfies its own

   A. bilinear form
   B. inverse of the equation
   C. characteristic equation
   D. quadratic equation

   ANSWER: C

10. Let X_1 and X_2 be two column matrices, then X_1 and X_2 are orthogonal if

   A. $X_1 + X_2 = 0$
   B. $X_2 = 0$
   C. $X_1 = 0$
   D. $X_1^T X_2 = 0$

   ANSWER: D

11. If $u = (x-y)^4 + (y-z)^4 + (z-x)^4$, then $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} =$

   A. $4(y-z)^3 - 4(z-x)^3$
   B. $3(y-z)^4 - 3(z-x)^4$
   C. $4(x-y)^3 - 4(x-z)^3$
   D. $3(y-z)^4 - 3(z-x)^4$

   ANSWER: A

12. If $u = x^3y^4$ where $x = t^3$ and $y = t^2$ then $\frac{du}{dt} =$

A. 17t

B. $17t^{16}$

C. $16t^{17}$

D. 16t

ANSWER: B

13. If $f(x, y) = 0$ is an implicit function then $\frac{dy}{dx} =$

A. $\frac{-\partial f/\partial x}{\partial f/\partial y}$

B. $\frac{-\partial f/\partial y}{\partial f/\partial x}$

C. $\frac{\partial f/\partial x}{\partial f/\partial y}$

D. $\frac{\partial f/\partial y}{\partial f/\partial x}$

ANSWER: A

14. If $f(x, y) = \tan^{-1}(\frac{y}{x})$, then $f_y(1, 1)$ is

A. $\frac{-1}{2}$

B. $\frac{1}{2}$

C. -1

D. 1

ANSWER: B

15. If $rt - s^2 < 0$ at (a, b) where $r = \frac{\partial^2 f}{\partial x^2}$, $s = \frac{\partial^2 f}{\partial x \partial y}$ and $t = \frac{\partial^2 f}{\partial y^2}$ then the point is a

A. maximum point
B. minimum point
C. saddle point
D. doubtful point

ANSWER: C

16. If $x = r\cos\theta$ and $y = r\sin\theta$ then $\frac{\partial(x,y)}{\partial(r,\theta)} =$

A. r
B. r^2
C. 0
D. $2r$

ANSWER: A

17. The stationary points of $f(x, y) = x^3 + y^3 - 3axy$ are

A. (0, a) and (0, a)
B. (0, 0) and (a, a)
C. (a, 0) and (a, 0)
D. (0, a) and (a, 0)

ANSWER: B

18. If $r^2 = x^2 + y^2$ then $\frac{\partial r}{\partial x} =$

A. $\frac{y^2}{r}$
B. $\frac{y}{r}$
C. $\frac{x^2}{r}$

D. $\frac{x}{r}$

ANSWER: D

19. If $f(x, y) = x^2 y + \sin y + e^x$ then $f_{xy}(1, \pi) =$

A. $\pi + e$
B. $\pi - e$
C. 2
D. -2

ANSWER: C

20. If u, v and w are functionally dependent functions of three independent variables x, y and z then $\frac{\partial(u,v,w)}{\partial(x,y,z)} =$

A. 1
B. -1
C. neither 1 nor -1
D. 0

ANSWER: D

21. The particular integral of $(D^2 + 9)y = e^{-2x}$ is

A. $\frac{e^{-2x}}{13}$
B. $\frac{e^{-2x}}{14}$
C. $\frac{e^{2x}}{13}$
D. $\frac{e^{2x}}{14}$

ANSWER: A

22. The complete solution of $\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 12y = 0$ is

A. $y = c_1e^{-3x} + c_2e^{-4x}$
B. $y = c_1e^{3x} + c_2e^{4x}$
C. $y = ce^{3x}$
D. $y = ce^{4x}$

ANSWER: B

23. An equation of the form $x^n\frac{d^ny}{dx^n} + a_1x^{n-1}\frac{d^{n-1}y}{dx^{n-1}} + \ldots + a_ny = F(x)$ can be transformed to a linear differential equation with constant coefficients by the transformation

A. $x = e^{2z}$
B. $ax + b = e^z$
C. $x = e^z$
D. $ax^2 + b = e^z$

ANSWER: C

24. The equation of the form $(ax+b)^n\frac{d^ny}{dx^n} + p_1(ax+b)^{n-1}\frac{d^{n-1}y}{dx^{n-1}} + \ldots + p_ny = F(x)$ is known as

A. Maclaurin's saeries
B. Taylor's series
C. Cauchy homogeneous linear equation
D. Legendre linear differential equation

ANSWER: D

25. If $3i$ and $-3i$ are the roots of the differential equation $(D^2 + 9)y = 0$ then the complementary function is given by

A. $c_1 e^{3x} + c_2 e^{-3x}$
B. $c \sin 2x$
C. $c_1 \cos 3x + c_2 \sin 3x$
D. $c \cos 2x$

ANSWER: C

26. If $f_1 = \cos x$ and $f_2 = \sin x$, then the value of $f_1 f_2^{'} - f_2 f_1^{'}$ is

A. 1
B. -1
C. 2
D. -2

ANSWER: A

27. The solution of $(D^2 + 2D + 1)y = 8$ is

A. $y = (Ax + B)e^{-x} - 8$
B. $y = Ae^{x} + 8$
C. $y = Ae^{-x} + 8$
D. $y = (Ax + B)e^{-x} + 8$

ANSWER: D

28. The expansion of $(1-\frac{D}{2})^{-1}$ is

A. $(1+\frac{D}{2!}+\frac{D^2}{3!}+\frac{D^3}{4!}+\ldots)$
B. $(1+\frac{D}{2}+\frac{D^2}{4}+\frac{D^3}{8}+\ldots)$
C. $(1-\frac{D}{2}-\frac{D^2}{4}-\frac{D^3}{8}-\ldots)$
D. $(1+\frac{D}{2}+\frac{D^2}{3}+\frac{D^3}{4}+\ldots)$

ANSWER: B

29. The auxillary equation for $(x^2D^2+4xD+2)y=x\log x$ is

A. $m^2+2m+3=0$
B. $m^2+3m-2=0$
C. $m^2+3m+2=0$
D. $m^2-3m+2=0$

ANSWER: C

30. Using the transformation $z=\log x$, the differential equation $\frac{d^2y}{dx^2}+\frac{1}{x}\frac{dy}{dx}=\frac{12logx}{x^2}$ is transformed to a linear differential equation with constant coefficients as

A. $(D'^2-1)y=12z$
B. $(D'^2+D'+1)y=12z$
C. $(D'^2+1)y=12z$
D. $D'^2y=12z$

ANSWER: D

31. The curvature of a straight line is

A. 0

B. 1

C. -1

D. 2

ANSWER: A

32. The equation of the envelope of the family of curves $A\alpha^2+B\alpha+C=0$, where α being a parameter is

A. $\mathrm{B}^2+4AC=0$

$B.$ $B^2-AC=0$

$C.$ $B^2+AC=0$

$D.$ $B^2-4AC=0$

ANSWER: D

33. The radius of curvature in polar coordinates is

A. $\frac{(r^2-r_1^2)^{3/2}}{r^2+2r_1^2-rr_2}$

B. $\frac{(r^2+2r_1^2)^{3/2}}{r^2+2r_1^2-rr_2}$

C. $\frac{(r^2-2r_1^2)^{3/2}}{r^2+2r_1^2-rr_2}$

D. $\frac{(r^2+r_1^2)^{3/2}}{r^2+2r_1^2-rr_2}$

ANSWER: D

34. The equation of circle of curvature at any point (x, y) with center of curvature $(\bar{x}, \bar{y})$ and radius of curvature ρ is

A. $(x+\bar{x})^2+(y+\bar{y})^2=\rho^2$
B. $(x-\bar{x})^2+(y-\bar{y})^2=\rho^2$
C. $(x-\bar{x})^2+(y+\bar{y})^2=\rho^2$
D. $(x+\bar{x})^2+(y-\bar{y})^2=\rho^2$

ANSWER: B

35. Evolute of a curve is the envelope of the ______ to that curve

A. tangent
B. locus
C. parallel
D. normals

ANSWER: D

36. The curvature of a circle is the reciprocal of its

A. radius
B. diameter
C. locus
D. tangent

ANSWER: A

37. The locus of centre of curvature is called

A. involute
B. evolute
C. envelope
D. space

ANSWER: B

38. The radius of curvature for $y = e^x$ at $x = 0$ is

A. $2\sqrt{2}$
B. 2
C. $\sqrt{2}$
D. $\frac{1}{\sqrt{2}}$

ANSWER: A

39. The value of $\Gamma\frac{1}{2}$ is

A. $-\pi$
B. 2π
C. π
D. $\sqrt{\pi}$

ANSWER: D

40. The relation between Beta function and Gamma function is

A. $\beta(m,n) = \frac{\Gamma m}{\Gamma m+n}$

B. $\beta(m,n) = \frac{\Gamma m \Gamma n}{\Gamma(m+n)}$

C. $\beta(m,n) = \frac{\Gamma n}{\Gamma(m+n)}$

D. $\beta(m,n) = \Gamma m \Gamma n$

ANSWER: B

41. The sequence $a_n = 2^n$ is

A. convergent
B. divergent
C. oscillating
D. bounded

ANSWER: B

42. In the positive term series $\Sigma\, u_n$, if $\lim\limits_{n\to\infty} n(\frac{u_n}{u_{n+1}} - 1) = k$, then the series converges for

A. $k < 1$
B. $k > 1$
C. $k = 1$
D. $k \geq 1$

ANSWER: B

43. $\lim_{n\to\infty} \frac{2^n-2}{2^n+1} =$

A. ∞

B. 0

C. 1

D. 2

ANSWER: C

44. In a positive series $\Sigma\, u_n$, if $\lim\limits_{n\to\infty}(u_n)^{\frac{1}{n}} = \lambda$, then the series diverges for

A. $\lambda < 1$
B. $\lambda \geq 1$
C. $\lambda = 1$
D. $\lambda > 1$

ANSWER: D

45. An alternating series $u_1 - u_2 + u_3 - u_4 + \ldots$ converges if $\lim\limits_{n\to\infty} u_n = 0$ and

A. each term is numerically less than its preceding term
B. each term is numerically greater than its preceding term
C. conditionally Convergent
D. absolutely Convergent

ANSWER: A

46. The series $1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \frac{1}{5^2} - \ldots$ is

A. divergent
B. convergent
C. absolutely divergent
D. absolutely convergent

ANSWER: D

47. $\lim\limits_{n\to\infty} \frac{n^2}{3^n} \times \frac{3^{n+1}}{(n+1)^2} =$

A. 3

B. 4

C. 5

D. 6

ANSWER: A

48. The series $\frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \ldots$ converges for

A. $p < 1$

B. $p = 1$

C. $p > 1$

D. $p \geq 1$

ANSWER: C

49. The series $1 + \frac{2!}{2^2} + \frac{3!}{3^3} + \frac{4!}{4^4} + \ldots$ is

A. convergent

B. absolutely convergent

C. divergent

D. conditionally convergent

ANSWER: A

50. If $u_n = (\log n)^{-2n}$, then $\lim_{n\to\infty} (u_n)^{\frac{1}{n}} =$

A. ∞

B. 0

C. 1

D. 2

ANSWER: B

Unit – I: Matrices

PART A

MULTIPLE CHOICE QUESTIONS

1. The matrix of the quadratic form $x_1^2+5x_2^2+x_3^2+2x_1x_2+2x_2x_3+6x_3x_1$ is

$$\checkmark(a)\begin{pmatrix}1&1&3\\1&5&1\\3&1&1\end{pmatrix}\ (b)\begin{pmatrix}1&2&3\\1&5&2\\3&2&1\end{pmatrix}(c)\begin{pmatrix}1&4&4\\4&5&3\\4&3&1\end{pmatrix}(d)\begin{pmatrix}1&4&3\\4&5&4\\3&4&1\end{pmatrix}$$

2. The number of positive terms in the canonical form is called

   (a) Signature ✓(b) Index (c) Quadratic form (d) Positive definite

3. A homogeneous polynomial of second degree in any number of variables is

   (a) Canonical form ✓(b) Quadratic form (c) Orthogonal (d) Diagonal form

4. Find the eigen values of A^2 if $A=\begin{bmatrix}3&2&4\\0&2&6\\0&0&5\end{bmatrix}$

   (a) 6, 4, 10 ✓(b) 9, 4, 25 (c) 9, 2, 5 (d) 3, 2, 5

5. Find the sum and product of the eigen values of $A=\begin{bmatrix}2&1&1\\1&2&1\\0&0&1\end{bmatrix}$

   ✓(a) 5, 3 (b) 3, 5 (c) 2, 1 (d) 0, 1

6. The eigen values of an orthogonal matrix have the absolute value ______

   (a) 0 ✓(b) 1 (c) 2 (d) ±1

7. All the eigen values of a symmetric matrix with real elements are

   (a) Distinct ✓(b) Real (c) Equal (d) Conjugate complex numbers

8. Find the nature of the quadratic form $2x^2+3y^2+2z^2+2xy$

   ✓(a) Positive definite (b) Negative definite (c) Positive semi-definite (d) Indefinite

9. Write the Q.F. defined by the matrix $A = \begin{pmatrix} 6 & 1 & -7 \\ 1 & 2 & 0 \\ -7 & 0 & 1 \end{pmatrix}$

✓ (a) $6x_1^2 + 2x_2^2 + x_3^2 + 2x_1x_2 - 14x_1x_3$ (b) $6x_1^2 + x_2^2 + 6x_3^2 + x_1x_2 - 7x_1x_3$
(c) $6x_1^2 + 2x_2^2 + x_3^2 + 2x_1x_2 + 14x_1x_3$ (d) $6x_1^2 + x_2^2 + 6x_3^2 + x_1x_2 - 14x_1x_3$

10. Find the eigen values of the matrix $\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$

✓ (a) 1, 3 ✓(b) 3, 1 (c) 2, 1 (d) 1, 2

11. Find the eigen values of A^{10} if $A = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$

✓ (a) $1, 3^{10}$ (b) $3, 1$ (c) $3^2, 1^{10}$ (d) $0, 2$

12. If the sum of two eigen values and trace of a 3 x 3 matrix A are equal, find the value of $|A|$

✓ (a) 0 (b) 1 (c) -1 (d) 2

13. Find the characteristic equation of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}$

✓ $(a)\ \lambda^3 + \lambda^2 - 18\lambda - 40$ $(b)\ \lambda^3 - \lambda^2 + 18\lambda - 40$
$(c)\ \lambda^3 + \lambda^2 + 18\lambda + 40$ $(d)\ \lambda^3 + \lambda^2 - 18\lambda + 40$

14. Find the nature of the quadratic form $x_1^2 + 2x_2^2 + 3x_3^2 + 2x_1x_2 + 2x_2x_3 - 2x_3x_1$

✓ (a) Indefinite (b) Positive definite (c) Negative definite (d) Positive semidefinite

15. Find the eigen values of $A = \begin{bmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{bmatrix}$

✓ $(a)\ 1, 3, -4$ $(b)\ 1, -3, -4$ $(c)\ 1, -3, 4$ $(d)\ -1, 3, -4$

16. The matrix of the quadratic form $x^2 + xy$ is

✓ $(a)\begin{pmatrix} 1 & 1/2 \\ 1/2 & 0 \end{pmatrix}$ $(b)\begin{pmatrix} 1 & 2 \\ 2 & 0 \end{pmatrix}$ $(c)\begin{pmatrix} 0 & 1/2 \\ 1/2 & 1 \end{pmatrix}$ $(d)\begin{pmatrix} 1 & 2 \\ 1 & 0 \end{pmatrix}$

17. Two eigen values of the matrix $A=\begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{pmatrix}$ are 1 and 2. Find the third eigen value.

✓ (a) 3 (b) b (c) 2 (d) 1

18. Two of the eigen values of 3 x 3 matrix A are 2, 1 and |A| = 12. Find the third eigen value

✓(a) 6 (b) 3 (c) 2 (d) 1

19. If A is an orthogonal matrix then

(a) |A| = 0 (b) A is singular (c) $A^2 = I$ ✓(d) $A^T = A^{-1}$

20. Two eigen values of $A=\begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -5 & -2 \end{bmatrix}$ are equal and they are double the third. Find them.

✓(a) 1, 2, 2 (b) 2, 1, 1 (c) 2, 0, 1 (d) 1, 2, 3

21. Find the inverse of the eigen values of the matrix if $A=\begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$

✓$(a)\ -1, 1/6$ $(b)\ 1, 1/6$ $(c)\ 1, -1/6$ $(d)\ -1, -1/6$

22. Find rank and index of the QF whose canonical form is $3y_2^2 - 3y_3^2$

✓(a) 2, 1 (b) 1, 2 (c) 0, 1 (d) 0, 2

23. Find signature of the QF whose canonical form is $2y_1^2 - y_2^2 - y_3^2$,

(a) 1 ✓(b) -1 (c) 0 (d) 6

24. The eigen vectors corresponding to the distinct eigen values of a real symmetric matrix are

(a) imaginary (b) non-orthogonal (c) real ✓(d) orthogonal

25. Find the characteristic equation of the matrix $A=\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$

(a) $\lambda^2 - 3\lambda - 2 = 0$ ✓ (b) $\lambda^2 + 3\lambda + 2 = 0$ (c) $\lambda^2 - 3\lambda + 3 = 0$ (d) $\lambda^2 - 6\lambda + 3 = 0$

UNIT-II-FUNCTIONS OF SEVERAL VARIABLES

1. If Z= $x^2 + y^2$+3xy then what is$\frac{\partial z}{\partial x}$?

(i) 2y+3x (ii) 3y (iii) 2x+3y (iv) 2x

2. u=$\sin^{-1}\left(\frac{x^2+y^2}{x-y}\right)$ is homogeneous function of degree

(i) 2 (ii) 3 (iii) 1 (iv) 4

3.If u=ax^2+2hxy+by^2 then using Euler's theorem find x$\frac{\partial u}{\partial x}$ + y$\frac{\partial u}{\partial y}$ = ?

(i) u (ii) 2u (iii) 3u (iv) n(n-1)

4. If f(x , y) = e^{xy} then what is $f_{yyy}(1 , 1)$?

(i)-e (ii)$\frac{1}{e}$ (iii)e (iv)-$\frac{1}{e}$

5. if z= log (x^2+xy+y^2) then what is x$\frac{\partial z}{\partial x}$ +y$\frac{\partial z}{\partial y}$ =?

(i) 1 (ii) $\frac{2x+y}{x^2+xy+y^2}$ (iii) 2 (iv)$\frac{x+2y}{x^2+xy+y^2}$

6.If f(x,y) is an implicit function then $\frac{dy}{dx}$ =?

(i)$-\frac{(\partial f/\partial x)}{(\partial f/\partial y)}$ (ii) $\frac{(\partial f/\partial x)}{(\partial f/\partial y)}$ (iii)$\frac{(\partial f/\partial y)}{(\partial f/\partial x)}$ (iv)$-\frac{(\partial f/\partial y)}{(\partial f/\partial x)}$

7. If f(x , y) = $e^x\ cosy\ then\ what\ is\ f_{xy}\ (0,0)$?

(i) 1 (ii) -1 (iii) 0 (iv) 2

8. If f (x , y) = cosxcosy then f_{yy} (0, 0)=?

(i) 1 (ii) 0 (iii) -1 (iv)$^1/_2$

9. If f (x , y) = $\tan^{-1}\left({}^y/_x\right)$ then f_x (1 , 1) is

(i) $\frac{\pi}{4}$ (ii) ½ (iii) – ½ (iv) 0

10. If rt - $s^2 < 0$ at (a , b) then the point is

(i) Maximum point (ii) minimum point (iii) saddle point (iv) none of these

11. The stationary points of $x^2 + y^2 + 6x + 12$ are

(i) (-3 , 0) (ii) (0 , 3) (iii) (0 , -3) (iv) (3 , 0)

12. If x=$u^2 - v^2$ and y=2uv then J$\left(\frac{x\ ,y}{u\ ,v}\right)$ is

(i) $u^2 + v^2$ (ii) $2(u^2 + v^2)$ (iii) $4(u^2 + v^2)$ (iv) $4v^2$

13. If x=rcos θ and y=rsin θ Then what is $\frac{\partial(x\ ,y)}{\partial(r\ ,\theta)}$= ?

(i) r^2 (ii) r (iii) 2r (iv) 0

14. If v= $\tan^{-1}x$+$\tan^{-1}y$ then $\frac{\partial v}{\partial x}$ is

(i) 1+y^2 (ii) $\frac{1}{1+y^2}$ (iii) $\frac{1}{1+x^2}$ (iv) 1+x^2

15.u and v are functionally dependent if their jacobian value is

(i)zero (ii)one (iii)non-zero (iv)greater than zero

16.if $J_1 = J\left(\frac{x\ ,y}{u\ ,v}\right)$ and J_2=$J\left(\frac{u\ ,v}{x\ ,y}\right)$ then J_1J_2=?

(i)0 (ii)1 (iii)-1 (iv)2

17.The stationary points of f(x,y)=$sinx + siny + \sin(x+y)$ are

(i)$\left(\frac{\pi}{2},\frac{\pi}{3}\right)$ (ii)$\left(\frac{\pi}{3},\frac{\pi}{3}\right)$ (iii) $\left(\frac{\pi}{3},\frac{\pi}{2}\right)$ (iv) $\left(\frac{\pi}{2},\frac{\pi}{2}\right)$

18.The point (0,0) for f(x, y)=$x^3 + y^3 - 3axy$ is

(i)a maximum point (ii) a minimum point (iii)a saddle point (iv)none of these

19.If f(x, y)=$x^2 + y^2$ where x= rcosθ and y= rsinθ then$\frac{\partial f}{\partial \theta}$ is

(i) r (ii)r^2 (iii) 1 (iv) 0

20.If f(x, y)=x^2y+siny+e^x then $f_x(1,\pi)$ is

(i)2π-e (ii)2π (iii)2π+e (iv) 0

21. $u = \cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ is homogeneous function of degree

$(i)\ \frac{1}{2}$ (ii) 1 (iii) 2 (iv) 3

22. If $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$ then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} =$?

(i) sinu (ii) cosu (iii) sin2u (iv) tanu

23. If $u = \frac{y^2}{x}$, $v = \frac{x^2}{y}$ then $\frac{\partial(x,y)}{\partial(u,v)} =$?

(i) -3 (ii) 3 (iii) $-\frac{1}{3}$ (iv)$\frac{1}{3}$

24. $if\ x = r\cos\theta$, y= r $\sin\theta$, z=z then $\frac{\partial(x,y,z)}{\partial(r,\theta,z)} =$?

(i)2r (ii)r^2 (iii)$\frac{1}{r}$ (iv)r

25.If u=$x^2 - 2y$ and v=x+y then $\frac{\partial(u,v)}{\partial(x,y)} =$?

(i)2x (ii) 2x+2 (iii) 2y-2 (iv) 2x - y

ANSWERS

1.(iii) 2x+3y

2. (iii) 1

3. (ii) 2u

4.(iii)e

5. (iii) 2

6.i) $-\frac{(\partial f/\partial x)}{(\partial f/\partial y)}$

7. (iii) 0

8. (iii) – 1

9. (iii) - 1/2

10. (iii) saddle point

11. (i) (-3 , 0)

12. (iii) $4(u^2+v^2)$

13. (ii) r

14. (iii) $\frac{1}{1+x^2}$

15. (i)zero

16. (ii)1

17. (ii) $\left(\frac{\pi}{3},\frac{\pi}{3}\right)$

18. (iii) a saddle point

19. (iv) 0

20. (iii)2π+e

21. (i) $\frac{1}{2}$

22. (iii) sin2u

23. (iii) - $\frac{1}{3}$

24. (iv)r

25. (ii) 2x+2

SRM UNIVERSITY

Unit-III Ordinary Differential Equations

Multiple Choice Questions

1. Which of the following is the general solution to $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - 10y = 0$

   (a) $y = Ae^{2x} + Be^{-5x}$ (b) $y = Ae^{-2x} + Be^{5x}$ (c) $y = Ae^{-2x} + Be^{-5x}$ (d) $y = Ae^{2x} + Be^{5x}$

2. Solution of $(D^2 + 4)y = 0$ is

   (a) $y = A\cos 2x + B\sin 2x$ (b) $y = Ae^{2x} + Be^{-2x}$ (c) $y = A\cos\sqrt{2}x + B\sin\sqrt{2}x$
   (d) $y = (Ax + B)e^{2x}$

3. The P.I of $(D^2 + 4)y = \sin 2x$ is

   (a) $\frac{-x}{4}\cos 2x$ (b) $\frac{x}{4}\cos 2x$ (c) $\frac{x}{2}\cos 2x$ (d) $\frac{-x}{2}\cos 2x$

4. The equation $(a_0x^2D^2 + a_1xD + a_2)\,y = Q(x)$ is called, where $a_0, a_1, a_2 \in C$

   (a) Cauchy's equation (b) Legendre's equation (c) Taylor's equation (d) Clairaut's equation

5. Use the transformation $z = \log x$, convert the D.E $x^2y'' - xy' + y = x^2$ to an equation with constant coefficients

   (a) $(\theta^2 - 2\theta + 1)y = e^{2z}$ (b) $(\theta^2 - 2\theta + 1)y = e^{z}$ (c) $(\theta^2 + 2\theta + 1)y = e^{2z}$
   (d) $(\theta^2 + 2\theta + 1)y = e^{z}$

6. The solution of $(D^2 + 2D + 1)y = 7$ is

   (a) $y = (Ax + B)e^{-x} + 7$ (b) $y = (Ax + B)e^{-x} - 7$ (c) $y = (Ax + B)e^{x} + 7$
   (d) $y = (Ax + B)e^{x} - 7$

7. The P.I of $(D - 1)^2y = e^x\sin x$ is

   (a) $-e^x\cos x$ (b) $e^x\cos x$ (c) $e^x\sin x$ (d) $-e^x\sin x$

8. The P.I of $(D - 1)^2y = x$ is

   (a) $2 - x$ (b) $x + 2$ (c) x^2 (d) $-x^2$

9. If $1 \pm 2i$ are the roots of A.E of a differential equation $f(D)y = 0$ then the general solution is

   (a) $e^{-2x}(A\cos x - B\sin x)$ (b) $Ae^x + Be^{-2x}$ (c) $e^x(A\cos 2x + B\sin 2x)$ (d) $Ae^t + Be^{2x}$

10. Convert the equation $(5 + 2x)^2y'' - 6(5 + 2x)y' + 8y = 0$ to an equation with constant coefficient by using the transformation $z = \log(5 + 2x)$

    (a) $(\theta^2 + 4\theta + 2)y = 0$ (b) $(\theta^2 - 4\theta + 2)y = 0$ (c) $(\theta^2 + 4\theta + 4)y = 0$ (d) $(\theta^2 + 4\theta - 2)y = 0$

11. The P. I of $(D^2 + 4)y = \sinh 2x$ is

    (a) $y_p = \frac{\sinh 2x}{8}$ (b) $y_p = \frac{\sinh 2x}{4}$ (c) $y_p = \frac{-\sinh 2x}{8}$ (d) $y_p = \frac{-\sinh 2x}{4}$

12. The P.I of $(D^2+6D+5)y = e^{-x}$ is

(a) $y_p = \frac{xe^{-x}}{4}$ (b) $y_p = \frac{xe^{-x}}{2}$ (c) $y_p = \frac{e^{-x}}{2}$ (d) $y_p = \frac{e^{-x}}{4}$

13. The solution of $(D^2-2aD+a^2)y = 0$ is

(a) $Ae^{ax}+Be^{bx}$ (b) $Ae^{ax}+Be^{-ax}$ (c) $(Ax+B)e^{ax}$ (d) $(Ax+B)e^{-ax}$

14. The P.I of $(D^2+16)y = \cos 4x$ is

(a) $\frac{x}{2}\sin 2x$ (b) $\frac{x\sin 4x}{8}$ (c) $\frac{x}{2}\cos 2x$ (d) $\frac{x\cos 4x}{8}$

15. The C.F of $D^2y+y = cosecx$ is

(a) $Ae^{ax}+Be^{bx}$ (b) $A\cos x+B\sin x$ (c) $(Ax+B)e^{ax}$ (d) $(Ax+B)e^{-ax}$

16. If $y_1 = \cos ax, y_2 = \sin ax$ then the value of $y_1y_2^{'} - y_2y_1^{'}$ is

(a) -a (b) 0 (c) 1 (d) a

17. Solve $(D^2+1)\,y = 0$ given $y(0) = 0,\ y'(0) = 1$

(a) $y = \sin x$ (b) $y = \cos x$ (c) $y = A\cos x+B\sin x$ (d) $y = 0$

18. The P.I of $(D-2)^2y = e^{2x}$ is

(a) $\frac{x^2}{2}\,e^{2x}$ (b) $\frac{x}{4}\,e^{2x}$ (c) $\frac{x^2}{2}\,e^{-2x}$ (d) $\frac{x^2}{2}\,e^{-2x}$

19. The P.I of $(D^2+4)y = \sin(2x+5)$ is

(a) $-\frac{x}{2}\sin(2x+5)$ (b) $\frac{x}{4}\sin(2x+5)$ (c) $-\frac{x}{4}\cos(2x+5)$ (d) $\frac{x}{2}\cos(2x+5)$

20. Solve $(x^2D^2+xD+1)\,y = 0$ is

(a) $Ae^{az}+Be^{bz}$ (b) $A\cos z+B\sin z$ (c) $(Az+B)e^{az}$ (d) $(Az+B)e^{-az}$

21. The roots of the auxiliary equation $(m^2-4) = 0$ are

(a) ± 2 (b) $\pm 2i$ (c) $\pm\sqrt{2}$ (d) $1\pm 2i$

22. The solution of $(x^2D^2-7xD+12)\,y = 0$ is

(a) $Ae^{-2z}+Be^{6z}$ (b) $Ae^{2z}+Be^{-6z}$ (c) $Ae^{2z}+Be^{6z}$ (d) $Ae^{-2z}+Be^{-6z}$

23. If $y_1 = \cos x, y_2 = \sin x$ then the value of $y_1y_2^{'} - y_2y_1^{'}$ is

(a) -1 (b) 0 (c) 1 (d) $\frac{1}{2}$

24. If three roots of the auxiliary equation become equal to the real number a, then the corresponding C.F is

(a) $(Ax^2+Bx+C)\,e^{ax}$ (b) $Ae^{ax}+Be^{ax}+Ce^{ax}$ (c) $Ae^{ax}+(B\cos ax+C\sin ax)$ (d) a

25. The values of $\frac{e^{ax}}{D-a}$

(a) xe^{ax} (b) e^{ax} (c) x^2e^{ax} (d) $\frac{x^2}{2}e^{ax}$

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Answers:

1. *a* **2.** *a* **3.** *a* **4.** *a* **5.** *a* **6.** *a* **7.** *d* **8.** *b* **9.** *c* **10.** *b* **11.** *a*
12. *a* **13.** *c* **14.** *b* **15.** *b* **16.** *d* **17.** *a* **18.** *a* **19.** *c* **20.** *b* **21.** a
22. *c* **23.** *c* **24.** *a* **25.** *a*

SRM UNIVERSITY

Unit-IV Geometrical Applications of Differential Calculus
Multiple Choice Questions

1. If the radius of curvature and curvature of a curve at any point are ρ and κ respectively, then
   (a) $\rho = \frac{-1}{\kappa}$ (b) $\rho = \kappa$ (c) $\rho = -\kappa$ (d) $\rho = \frac{1}{\kappa}$
2. The locus of center of curvature is called
   (a) Involute (b) Evolute (c) Radius of curvature (d) Envelope
3. The envelope of the family of curves $A\alpha^2 + B\alpha + C = 0$ (α is parameter) is
   (a) $B^2 + 4AC = 0$ (b) $B^2 - AC = 0$ (c) $B^2 + AC = 0$ (d) $B^2 - 4AC = 0$
4. The curvature of the straight line is
   (a) 1 (b) 2 (c) -1 (d) 0
5. The radius of curvature of the curve $y = 4\sin x$ at $x = \frac{\pi}{2}$ is
   (a) $\frac{1}{2}$ (b) $\frac{-1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{-1}{4}$
6. The envelope of $ty - x = at^2$, t is the parameter is
   (a) $x^2 = 4ay$ (b) $y^2 = 4ax$ (c) $x^2 + y^2 = 1$ (d) $x^2 - y^2 = 1$
7. The curvature at any point of the circle is equal to $---$ of its raduis
   (a) Square (b) Same (c) Reciprocal (d) constant
8. What is the radius of curvature at (4, 3) on the curve $x^2 + y^2 = 25$
   (a) 5 (b) -5 (c) 25 (d) -25
9. What is the curvature of a circle of radius 3
   (a) 3 (b) -3 (c) $\frac{1}{3}$ (d) $\frac{-1}{3}$
10. Find the envelope of the curve $y = mx + \frac{a}{m}$ where m is a parameter
    (a) $y^2 - 4ax = 0$ (b) $y^2 + 4ax = 0$ (c) $x^2 + y^2 = 1$ (d) $xy = c^2$
11. The radius of curvature of $y = e^x$ at $x = 0$ is
    (a) $2\sqrt{2}$ (b) $\frac{2}{\sqrt{2}}$ (c) $\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$
12. The radius of curvature of the curve $y = log \sec x$ at any point of it is
    (a) $\sec x$ (b) $\tan x$ (c) $\cot x$ (d) $cosecx$
13. In an ellipse the radius of curvature at the end of which axis is equal to the semi latus rectum of the ellipse
    (a) Minor (b) Major (c) Vertical (d) Horizontal
14. The radius of curvature of the curve $x = t^2$, $y = t$ at $t = 1$ is
    (a) $5\frac{\sqrt{5}}{2}$ (b) $\frac{\sqrt{5}}{2}$ (c) $\frac{5}{2}$ (d) $\sqrt{5}$

15. Evolute of a curve is the envelope of $----$ of that curve

    (a) Tangent (b) Normal (c) Parallel (d) Locus

16. The evolute of the cycloid $x = a\,(\theta - \sin\theta)\,,\ \ y = a\,(1-\cos\theta)$ is

    (a) Astroid (b) Parabola (c) Cycloid (d) Circle

17. A curve which touches each member of a family of the curves is called $---$ of that family

    (a) Evolute (b) Envelope (c) Circle of curvature (d) Radius of curvature

18. The envelope of family of lines $y = mx + am^2$ $(where\,\mathrm{m}\,is the parameter)$ is

    (a) $x^2 + 2ay = 0$ (b) $x^2 + 4ay = 0$ (c) $y^2 + 2ax = 0$ (d) $y^2 + 4ax = 0$

19. The envelope of the family of lines $\frac{x}{t} + yt = 2c$, t being the parameter is

    (a) $x^2 + y^2 = c^2$ (b) $xy = c^2$ (c) $x^2y^2 = c^2$ (d) $x^2 - y^2 = c^2$

20. The radius of curvature at any point on the curve $r = e^\theta$ is

    (a) $\frac{\sqrt{2}}{r}$ (b) $\frac{r}{\sqrt{2}}$ (c) r (d) $\sqrt{2}r$

21. The radius of curvature in Cartesian coordinates is

    (a) $\rho = \dfrac{\left(1+y_1^2\right)^{3/2}}{y_2}$ (b) $\rho = \dfrac{\left(1-y_1^2\right)^{3/2}}{y_2}$ (c) $\rho = \dfrac{\left(1+y_1^2\right)^{2/3}}{y_2}$ (d) $\rho = \dfrac{\left(1+y_2^2\right)^{3/2}}{y_1}$

22. The radius of curvature in polar coordinates is

    (a) $\rho = \dfrac{\left(r^2 + (r')^2\right)^{3/2}}{r^2 - rr' + 2(r')^2}$ (b) $\rho = \dfrac{\left(r^2 - (r')^2\right)^{3/2}}{r^2 - rr' + 2(r')^2}$ (c) $\rho = \dfrac{\left(r^2 - (r'')^2\right)^{3/2}}{r^2 - rr' + 2(r')^2}$

    (d) $\rho = \dfrac{\left(r^2 + (r')^2\right)^{3/2}}{r^2 - rr'' + 2(r')^2}$

23. The radius of curvature in parametric coordinates is

    (a) $\rho = \dfrac{\left((x')^2 + (y')^2\right)^{3/2}}{x'y'' - y'x''}$ (b) $\rho = \dfrac{\left((x')^2 + (y')^2\right)^{3/2}}{x'y'' + y'x''}$ (c) $\rho = \dfrac{\left((x')^2 - (y')^2\right)^{3/2}}{x'y'' - y'x''}$

    (d) $\rho = \dfrac{\left((x')^2 - (y')^2\right)^{3/2}}{x'y'' + y'x''}$

24. The equation of circle of curvature at any point (x, y) with center of curvature $\overline{x}, \overline{y}$ and with radius of curvature ρ is

    (a) $(x + \overline{x})^2 + (y + \overline{y})^2 = \rho^2$ (b) $(x - \overline{x})^2 + (y - \overline{y})^2 = \rho^2$ (c) $(x - \overline{x})^2 - (y + \overline{y})^2 = \rho^2$
    (d) $(x + \overline{x})^2 + (y + \overline{y})^2 = \rho$

****

Answers:

1. *d* **2.** *b* **3.** *d* **4.** *d* **5.** *c* **6.** *b* **7.** *c* **8.** *a* **9.** *c* **10.** *b* **11.** *a* **12.** *a* **13.** *b* **14.** *a* **15.** *b* **16.** *c* **17.** *b* **18.** *b* **19.** *b* **20.** *d* **21.** *a* **22.** *d* **23.** *a* **24.** *b*

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SRM INSTITUTE OF SCIENCE & TECHNOLOGY
FACULTY OF ENGINEERING AND TECHNOLOGY
DEPARTMENT OF MATHEMATICS

Unit –IV Geometrical Applications of Differential Calculus
(Beta ,Gamma Functions)
Multiple Choice Questions

1. The value of $\beta(4,4)$ is -----------

✓(a). $\frac{36}{7!}$ (b). $\frac{6!}{7!}$ (c). $\frac{4!4!}{8!}$ (d). $\frac{3!}{7!}$

2. The value of $\beta\left(\frac{5}{2},\frac{3}{2}\right)$ is ----------------

(a). $\frac{\pi}{8}$ (b). $\frac{\sqrt{\pi}}{8}$ ✓(c). $\frac{\pi}{16}$ (d). $\frac{\pi^2}{16}$

3. $\beta(m,n)$ is equal to ------------

(a). $\frac{m!n!}{(m+n)!}$ (b). $\frac{m!n!}{(m-n)!}$ ✓(c). $\frac{\Gamma m\,\Gamma n}{\Gamma(m+n)}$ (d). $\frac{\Gamma m\,\Gamma n}{\Gamma(m-n)}$

4. The value of $\Gamma\left(\frac{1}{2}\right)$ is -----------

✓(a). $\sqrt{\pi}$ (b). π^2 (c). π (d). 2π

5. $\Gamma n\,\Gamma(1-n)$ is equal to --------

(a). $\int_0^\infty \frac{x^{1-n}}{1+x}dx$ ✓(b). $\Gamma(1)\,\beta(n,1-n)$ (c). $\Gamma(1)\,\beta(1-n,1-n)$ (d). $\Gamma(1)\,\beta(1-n,n)$

6. $\int_0^{\frac{\pi}{2}} \sin^p\theta\cos^q\theta\, d\theta$ is equal to -----------

✓(a) $.\frac{1}{2}\beta\left(\frac{p+1}{2},\frac{q+1}{2}\right)$ (b). $\frac{1}{2}\beta\left(\frac{p-1}{2},\frac{q-1}{2}\right)$ (c). $\frac{1}{2}\beta\left(\frac{p}{2},\frac{q}{2}\right)$ (d). $\frac{1}{2}\beta\left(\frac{1}{2},\frac{1}{2}\right)$

7. $\int_0^1 x^4[\log(\frac{1}{x})]^3$ is equal to -----------

(a). $\frac{6}{525}$ ✓(b). $\frac{6}{625}$ (c). $\frac{6!}{5!}$ (d). $\frac{5!}{6!}$

8. The value of $\int_0^{\frac{\pi}{2}} \sqrt{\cot\theta}\, d\theta$ is -----------

(a). $\frac{\pi}{2}$ ✓(b). $\frac{\pi}{\sqrt{2}}$ (c). $\frac{\sqrt{\pi}}{2}$ (d). $\sqrt{\frac{\pi}{2}}$

UNIT V
Sequence and Series

1. The series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if

   (a)p=1 (b)p=0 (c)p>1 (d)p<1

2. The series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is divergent if

   (a)p>1 (b)p=0 (c)p≤1 (d)p<1

3. If $\sum u_n$ is a series of positive term such that $\lim_{n\to\infty} (u_n)^{\frac{1}{n}} = l$ where $l>1$, then the series $\sum u_n$ is

   (a) Convergent (b) Divergent (c) Oscillating (d) Monotonic

4. If $\sum u_n$ is a series of positive term such that $\lim_{n\to\infty} (u_n)^{\frac{1}{n}} = l$, then the series $\sum u_n$ is convergent if

   (a)l<1 (b)l =0 (c)l>1 (d)l =1

5. The series $\sum_{n=1}^{\infty} \frac{1}{n!}$ is

   (a) Convergent (b) Divergent (c) Oscillating (d) Monotonic

6. The series $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+ ....to\infty$ is

   (a) Convergent (b) Divergent (c) Oscillating (d) Monotonic

7. By D'Alambert's Ratio test $\lim_{n\to\infty} (\frac{u_{n+1}}{u_n}) = l$, the series is convergent when

   (a)l<1 (b)l =0 (c)l>1 (d)l =1

8. By Raabe's test $\lim_{n\to\infty} \left[n(\frac{u_{n+1}}{u_n} - 1) \right] = l$, the series is divergent when

   (a)l<1 (b)l =0 (c)l>1 (d)l =1

9. The series $\sum_{n=1}^{\infty} \frac{2^n}{n!}$ is

   (a) Convergent (b) Divergent (c) Oscillating (d) Monotonic

10. A series $\sum u_n$ is said absolutely convergent if the series

    (a) $\sum |u_n|$ is convergent (c) $\sum u_n$ is divergent

    (c) $\sum u_n$ is convergent (b) $\sum |u_n|$ is divergent

11. A series $\sum u_n$ is saidconditionally convergent if the series

(a) $\sum |u_n|$ is convergent (b) $\sum u_n$ is divergent& $\sum |u_n|$ is convergent

(c) $\sum u_n$ is convergent& $\sum |u_n|$ is divergent (d) $\sum |u_n|$ is divergent

12. The series $1-\frac{1}{2}+\frac{1}{2^2}-\frac{1}{2^3}+$.......... is

(a) Convergent (b) Divergent (c) Conditionally convergent (d) absolutely convergent

13. The series $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+$.......... is

(a) Convergent (b) Divergent (c) Conditionally convergent (d) absolutely convergent

14. The series $\sum \frac{1}{n \log n}$ is

(a) Conditionally convergent (b) absolutely convergent (c) Convergent (d) Divergent

15. An absolutely convergent series is

(a) Conditionally convergent (b) absolutely convergent (c) Convergent (d) Divergent

16. The series $\sum \frac{n^3}{3^n}$ is

(a) Conditionally convergent (b) absolutely convergent (c) Convergent (d) Divergent

17. The series $\sum \frac{1}{(\log n)^n}$ is

(a) Convergent (b) Conditionally convergent (c) absolutely convergent (d) Divergent

ANSWERS

1. d	11. c
2. c	12. d
3. b	13. c
4. a	14. d
5. a	15. c
6. b	16.c
7. a	17.a
8. c	
9. a	
10. a	

SRM Institute of Science and Technology
Ramapuram Campus
Department of Mathematics
18MAB101T – Calculus And Linear Algebra

Year/Sem: I/I

Branch: Common to ALL B.Tech. except B.Tech. (Business Systems)

Unit – I

MATRICES

Part – A

1.	The sum of the eigen values of $A=\begin{pmatrix}1 & 2\\ 2 & 1\end{pmatrix}$ is	1 Mark	
	(a) 2 (b) 4 (c) -3 (d) 0	Ans (a)	(CLO – 1Apply)
2.	The eigen values of A^{-1}, if $A=\begin{pmatrix}1 & 5 & -1\\ 0 & 3 & 2\\ 0 & 0 & 4\end{pmatrix}$ are	1 Mark	
	(a) 2, 3, 4 (b) 2, 5, – 1 (c) 0, 0, 0 (d) $1,\frac{1}{3},\frac{1}{4}$	Ans (d)	(CLO -1Apply)
3.	If two eigen values of $A=\begin{pmatrix}8 & -6 & 2\\ -6 & 7 & -4\\ 2 & -4 & 3\end{pmatrix}$ are 3 and 15, then the third eigen value is __________.	1 Mark	
	(a) 1 (b) 0 (c) 2 (d) 3	Ans (b)	(CLO -1 Apply)
4.	If – 1, – 1, 2 are the eigen values of a matrix $A=\begin{pmatrix}0 & 1 & 1\\ 1 & 0 & 1\\ 1 & 1 & 0\end{pmatrix}$, then the eigen values of A^T are	1 Mark	
	(a) – 1, – 1, 2 (b) 1, 1, 1/2 (c) 1,1,4 (d) -1,-1,-2	Ans (a)	(CLO - 1 Apply)

5.	The sum of eigen values of the identity matrix of order 3 is	1 Mark	
	(a) 0 (b) 1 (c) 2 (d) 3	Ans (d)	(CLO - 1 Remember)
6.	The product of the two eigen values of the matrix $A = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$ is 4.Then the third eigen value is	1 Mark	
	(a) 2 (b) 1 (c) 3 (d) 8	Ans (d)	(CLO - 1 Apply)
7.	The index of the canonical form $Q = -{y_1}^2 + {y_2}^2 + 4{y_3}^2$ is	1 Mark	
	a) 3 (b) 2 (c) 1 (d) 0	Ans (b)	(CLO -1 Apply)
8.	If the eigen values of the matrix of the quadratic form $2{x_1}^2 + 6{x_2}^2 + 2{x_3}^2 + 8x_1x_3$ are – 2, 6, 6, then the nature of the quadratic form is ______.	1 Mark	
	(a) positive semi-definite (b) indefinite (c) negative definite (d) positive definite	Ans (b)	(CLO - 1 Apply)
9.	The matrix corresponding to the quadratic form ${x_1}^2 + 5{x_2}^2 + {x_3}^2 + 2x_2x_3 + 6x_3x_1 + 2x_1x_2$ is	1 Mark	
	(a) $\begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 2 & 3 \\ 1 & 5 & 2 \\ 3 & 2 & 1 \end{pmatrix}$ (c) $\begin{pmatrix} 1 & 4 & 4 \\ 4 & 5 & 3 \\ 4 & 3 & 1 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & 4 & 3 \\ 4 & 5 & 4 \\ 3 & 4 & 1 \end{pmatrix}$	Ans (a)	(CLO - 1 Apply)
10.	A homogeneous polynomial of the _________ degree in any number of variables is called a quadratic form.	1 Mark	

	(a) first (b) second (c) third (d) fourth	Ans (b)	(CLO - 1 Remember)
11.	A square matrix A is called orthogonal if	1 Mark	
	(a) $A = A^2$ (b) $A = A^{-1}$ (c) $A^T = A^{-1}$ (d) $AA^{-1} = I$	Ans (c)	(CLO - 1 Remember)
12.	The sum of the squares of the eigen values $\begin{pmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{pmatrix}$ is	1 Mark	
	(a) 10 (b) 38 (c) 45 (d) 20	Ans (b)	(CLO - 1 Apply)
13.	All the eigen values of a symmetric matrix with real elements are	1 Mark	
	(a) distinct (b) real (c) equal (d) conjugate complex numbers	Ans (b)	(CLO - 1 Remember)
14.	If the sum of two eigen values and trace of a 3 x 3 matrix A are equal, then the value of det (A) is	1 Mark	
	(a) 0 (b) 1 (c) -1 (d) 2	Ans (a)	(CLO - 1 Apply)
15.	If the canonical form of a quadratic form is $-y_1{}^2 + y_2{}^2 + 2y_3{}^2$, then the signature of the quadratic form is	1 Mark	
	(a) 2 (b) 1 (c) 0 (d) 3	Ans (b)	(CLO - 1Apply)
16.	Find the sum and product of the eigen values of A = $\begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$	1 Mark	

	(a) 5, 3 (b) 3, 5 (c) 2, 1 (d) 0, 1	Ans (a)	(CLO - 1 Apply)
17.	The eigen vectors corresponding to the distinct eigen values of a real symmetric matrix are	1 Mark	
	(a) imaginary (b) non-orthogonal (c) real (d) orthogonal	Ans (d)	(CLO - 1 Remember)
18.	The eigen values of a skew symmetric matrix are	1 Mark	
	(a) real (b) purely imaginary or zero (c) unitary (d) orthogonal	Ans (b)	(CLO - 1 Remember)
19.	Find the characteristic equation of the matrix A = $\begin{pmatrix} 5 & 4 \\ 1 & 2 \end{pmatrix}$	1 Mark	
	(a) $\lambda^2 - 7\lambda + 6 = 0$ (b) $\lambda^2 + 7\lambda + 6 = 0$ (c) $\lambda^2 - 7\lambda - 6 = 0$ (d) $\lambda^2 - 7\lambda + 5 = 0$	Ans (a)	(CLO - 1 Apply)
20.	The eigen values of an orthogonal matrix have the absolute value	1 Mark	
	(a) 0 (b) 1 (c) 2 (d) 3	Ans (b)	(CLO -1 Remember)
21.	The number of positive terms in the canonical form is called	1 Mark	
	(a) Signature (b) Index (c)quadratic (d)positive definite	Ans (b)	(CLO - 1Remember)
22.	The difference between the positive terms and negative terms in the canonical form is called	1 Mark	
	(a) Signature (b) Index (c)quadratic (d)positive definite	Ans (a)	(CLO - 1 Remember)

23.	Find the eigen values of A^2 if $A = \begin{pmatrix} 3 & 2 & 4 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{pmatrix}$.	1 Mark	
	(a) 6, 4, 10 (b) 9, 4, 25 (c) 9, 2, 5 (d) 3, 2, 5	Ans (b)	(CLO - 1 Apply)
24.	Find the nature of the quadratic form $2x^2 + 3y^2 + 2z^2 + 2xy$	1 Mark	
	(a)Positive definite (b) Negative definite (c) Positive semi-definite (d) Indefinite	Ans (a)	(CLO – 1 Apply)
25.	Find the eigen values of A^{10} if $A = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$	1 Mark	
	(a) $1,3^{10}$ (b) 1,3 (c) $3^2, 1^{10}$ (d)1,10	Ans (a)	(CLO - 1 Apply)
26.	Find the eigen values of the matrix $A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$.	1 Mark	
	(a) 1, -3 (b) 3, 1 (c) 2, 1 (d) 1, 2	Ans (b)	(CLO - 1 Apply)
27.	If the sum of two eigen values and trace of a 3 x 3 matrix A are equal, then the value of determinant of A is	1 Mark	
	(a) 0 (b) 1 (c) -1 (d) 2	Ans (a)	(CLO - 1 Apply)
28.	Find the eigen values of the matrix $A^3 = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$.	1 Mark	

	(a) 1, -3 (b) 3, 1 (c) 1, 27 (d) 1, -9	Ans (c)	(CLO - 1Apply)
29.	The eigen values of the matrix $A^{-1} = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$	1 Mark	
	(a) 1, 1/3 (b) 3, 1 (c) -2, 1 (d) 1, 2	Ans (a)	(CLO - 1 Apply)
30.	Find the sum and product of the eigen values of the matrix $A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$.	1 Mark	
	(a) 4, 3 (b) 3, 1 (c) -2, 1 (d) 1, 2	Ans (a)	(CLO - 1 Apply)
31.	Find the eigen values of $A = \begin{pmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{pmatrix}$.	1 Mark	
	(a) $1,3,-4$ (b) $1,-3,-4$ (c) $1,-3,4$ (d) $-1,3,-4$	Ans (a)	(CLO - 1 Apply)
32.	Two eigen values of $A = \begin{pmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -5 & -2 \end{pmatrix}$ are equal and they are double the third. Find them.	1 Mark	
	(a) 1, 2, 2 (b) 2, 1, 1 (c) 2, 0, 1 (d) 1, 2, 3	Ans (a)	(CLO - 1Apply)
33.	The eigen values of a diagonal matrix are the ________ elements of the matrix.	1 Mark	

	(a) diagonal (b) upper triangular (c) zero (d) unity	Ans (a)	(CLO - 1 Remember)
34.	Cayley-Hamilton theorem states that "Every __________ matrix satisfies its own characteristic equation".	1 Mark	
	(a) square (b) column (c) row (d) zero	Ans (a)	(CLO - 1Remember)
35.	Find rank and index of the QF whose canonical form is $3x^2 – 3y^2$.	1 Mark	
	(a) 2, 1 (b) 1, 2 (c) 0, 1 (d) 0, 2	Ans (a)	(CLO – 1 Apply)
36.	Write the Q.F. defined by the matrix $A = \begin{pmatrix} 6 & 1 & -7 \\ 1 & 2 & 0 \\ -7 & 0 & 1 \end{pmatrix}$	1 Mark	
	(a) $6x^2 + 2y^2 + z^2 + 2xy - 14xz$ (b) $6x + y^2 + 6z^2 + x\,y - 7xz$ (c) $6x^2 + 2y^2 + z^2 + 2xy + 14xz$ (b) $6x + y^2 + 6z^2 + x\,y - 14xz$	Ans (a)	(CLO -1Apply)

SRM Institute of Science and Technology
Ramapuram Campus
Department of Mathematics
18MAB101T – Calculus And Linear Algebra

Year/Sem: I/I

Branch: Common to ALL B.Tech. except B.Tech. (Business Systems)

Unit – I

MATRICES

Part – B

1. Find the characteristic equation of the matrix $A = \begin{pmatrix} 1 & 2 \\ 0 & 2 \end{pmatrix}$.

(A) $\lambda^2 - 3\lambda + 2 = 0$ **(B)** $\lambda^2 + 3\lambda + 2 = 0$

(C) $\lambda^2 - 3\lambda - 2 = 0$ **(D)** $\lambda^2 + 3\lambda - 2 = 0$

Solution: Let A = $\begin{pmatrix} 1 & 2 \\ 0 & 2 \end{pmatrix}$. Its characteristic equation is $\lambda^2 - S_1\lambda + S_2 = 0$ where $S_1 = sum\ of\ the\ main\ diagonal\ elements = 1 + 2 = 3$,

$S_2 = Determinant\ of\ A = |A|$ = 1(2) – 2(0) = 2

Therefore, the characteristic equation is $\lambda^2 - 3\lambda + 2 = 0$ **(Option A)**

2. Find the characteristic equation of $A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$.

(A) $\lambda^3 - 18\lambda^2 + 45\lambda = 0$ **(B)** $\lambda^3 - 28\lambda^2 + 45\lambda = 0$

(C) $\lambda^3 - 18\lambda^2 + 35\lambda = 0$ **(D)** $\lambda^3 - 18\lambda^2 - 45\lambda = 0$

Solution: Its characteristic equation is $\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$, where $S_1 = sum\ of\ the\ main\ diagonal\ elements$ = 8 + 7 + 3 = 18, $S_2 = Sum\ of\ the\ minors\ of\ the\ main\ diagonal\ elements = \begin{vmatrix} 7 & -4 \\ -4 & 3 \end{vmatrix} + \begin{vmatrix} 8 & 2 \\ 2 & 3 \end{vmatrix} + \begin{vmatrix} 8 & -6 \\ -6 & 7 \end{vmatrix} = 5 + 20 + 20 = 45$, $S_3 = Determinant\ of\ A = |A|$ = 8(5)+6(-10)+2(10) = 40 – 60 + 20 = 0

Therefore, the characteristic equation is $\lambda^3 - 18\lambda^2 + 45\lambda = 0$ **(Option A)**

3. Find the eigen values of the matrix $A = \begin{pmatrix} 1 & 1 \\ 3 & -1 \end{pmatrix}$.

(A) $2, -2$ **(B) $1, -1$**

(C) $3, -3$ **(D) $2, 2$**

Solution: Let A = $\begin{pmatrix} 1 & 1 \\ 3 & -1 \end{pmatrix}$ which is a non-symmetric matrix.

The characteristic equation of A is $\lambda^2 - S_1\lambda + S_2 = 0$ where $S_1 = sum\ of\ the\ main\ diagonal\ elements = 1 - 1 = 0$,

$S_2 = Determinant\ of\ A = |A|$ = 1(-1) – 1(3) = - 4

Therefore, the characteristic equation is $\lambda^2 - 4 = 0$ i.e., $\lambda^2 = 4$ or $\lambda = \pm 2$

Therefore, the eigen values are 2, -2. **(Option A)**

4. Find the sum and product of the eigen values of the matrix $A = \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$.

(A) $-3, 4$ **(B) $-3, -4$**

(C) $3, 4$ **(D) $-3, -4$**

Solution: Sum of the eigen values = Sum of the main diagonal elements = -3

Product of the eigen values = $|$ A $|$ = -1 (1 – 1) -1(-1 – 1) + 1(1- (-1)) = 2 + 2 = 4

(Option A)

5. Find the sum and product of eigen values of the matrix A^T where $A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$.

(A) $18, 0$ **(B) $18, 2$**

(C) $28, 0$ **(D) $18, -2$**

Solution: Since matrix A is symmetric , A and A^T have same eigen values.

Sum of Eigen value of A^T = trace(A) = 8+7+3=18

Product of Eigen value of $A^T = |A|$ = 8(5) + 6(−10)+2(10) = 40 −60 + 20 = 0 **(Option A)**

6. If 1, 1, 5 are the eigen values of A = $\begin{bmatrix} \mathbf{2} & \mathbf{2} & \mathbf{1} \\ \mathbf{1} & \mathbf{3} & \mathbf{1} \\ \mathbf{1} & \mathbf{2} & \mathbf{2} \end{bmatrix}$**, find the eigen values of 5A.**

(A) $\mathbf{5, 5, 2}$ **(B)** $\mathbf{5, 5, 25}$ **(C)** $\mathbf{2, 3, 2}$ **(D)** $\mathbf{7, 8, 7}$

Solution: By the property "If $\lambda_1, \lambda_2, \lambda_3$ are the eigen values of A, then $k\lambda_1, k\lambda_2, k\lambda_3$ are the eigen values of kA, the eigen values of 5A are 5(1), 5(1), 5(5) ie., 5,5,25**. (Option B)**

7. Find the eigen values of the matrix 2A^{-1} where A= $\begin{pmatrix} 3 & 8 & 9 \\ 0 & 1 & 3 \\ 0 & 0 & -2 \end{pmatrix}$.

(A) $\mathbf{\frac{2}{3}, 2, -1}$ **(B)** $\mathbf{\frac{1}{3}, 2, -4}$ **(C)** $\mathbf{\frac{2}{3}, 2, 1}$ **(D)** $\mathbf{\frac{2}{3}, 1, -2}$

Solution: Since given matrix is triangular matrix, the Eigen values are its diagonal elements.

$$\therefore \quad \lambda_1 = 3,\ \lambda_2 = 1,\ \lambda_3 = -2$$

Eigen values of 2A^{-1} are $\frac{2}{3}$, 2 , -1**(Option A)**

8. If the sum of two eigen values and trace of a 3 × 3 matrix A are equal, find the value of $|A|$**.**

(A) 5 **(B) 25** **(C) 2** **(D) 0**

Solution: Sum of the eigen values = $\lambda_1 + \lambda_2 + \lambda_3$ = sum of the diagonal elements

Given $\lambda_1 + \lambda_2$ = trace of A.

i.e., $\lambda_1 + \lambda_2 = \lambda_1 + \lambda_2 + \lambda_3$

Therefore $\lambda_3 = 0$. Then $|A|$ = Product of Eigen values = $\lambda_1 \lambda_2 \lambda_3 = 0$ **(Option D)**

9. Write the matrix corresponding to the quadratic form $\boldsymbol{x^2 + 2yz}$**.**

(A) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 1 & 0 \end{bmatrix}$ **(B)** $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 2 & 0 \end{bmatrix}$ **(C)** $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ **(D)** $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 2 & 0 \end{bmatrix}$

Solution: Given $\boldsymbol{X^T A X = x^2 + 2yz}$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \textbf{ (Option C)}$$

10. If 2, 3, –1 are the eigen values of 3 × 3 matrix, find rank, index and signature of the quadratic form.

(A) $\mathbf{5, 5, 2}$ **(B)** $\mathbf{5, 5, 25}$ **(C)** $\mathbf{3, 2, 1}$ **(D)** $\mathbf{1, 2, 3}$

Solution: Rank (r) = number of non zero terms in canonical form = 3

Index (p) = Number of positive terms in canonical form = 2

Signature (s) = Difference between number of positive terms and negative terms

= 2p – r

= 4 – 3

= 1 **(Option C)**

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SRM Institute of Science and Technology
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Department of Mathematics
18MAB101T-Calculus and linear algebra

Year/Sem: I/I **Part-A** **Branch: Common to All**

Unit -II

Functions of several variables

1.	If u and v are functionally dependent then their Jacobian value is	1 Mark	
	a)zero b) one c) non-zero d)greater than zero	Ans (a)	(CLO-2 / Remember)
2.	If $rt - s^2 < 0$ then the point is	1 Mark	
	a)maximum point b) minimum point c) saddle point d) fixed point	Ans (c)	(CLO-2 / Remember)
3.	If $z = x^2 + y^2 + 3xy$ then $\frac{\partial z}{\partial x} =$	1 Mark	
	a)2y+3x b) 3y c) 2x+3y d) 2x	Ans (c)	(CLO-2 / Apply)
4.	If $u = \sin^{-1}(\frac{x^2 + y^2}{x - y})$ is a homogeneous function of degree	1 Mark	
	a) 2 b) 3 c)1 d) 4	Ans (c)	(CLO-2 / Apply)
5.	If f(x,y) is an implicit function then $\frac{dy}{dx} =$	1 Mark	

	a) $-\dfrac{\left(\frac{\partial f}{\partial x}\right)}{\left(\frac{\partial f}{\partial y}\right)}$ b) $\dfrac{\left(\frac{\partial f}{\partial x}\right)}{\left(\frac{\partial f}{\partial y}\right)}$ c) $\dfrac{\left(\frac{\partial f}{\partial y}\right)}{\left(\frac{\partial f}{\partial x}\right)}$ d) $-\dfrac{\left(\frac{\partial f}{\partial y}\right)}{\left(\frac{\partial f}{\partial x}\right)}$	Ans (a)	(CLO-2 / Remember)
6.	If $x = r\cos\theta$ *and* $y = r\sin\theta$ then $\dfrac{\partial(x,y)}{\partial(r,\theta)} =$	1 Mark	
	a) r b) r^2 c) 2r d) 1/r	Ans (a)	(CLO-2 / Apply)
7.	If u is a homogeneous function of degree n then $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} =$	1 Mark	
	a) n b) nu c) u d) n^2u	Ans (b)	(CLO-2 / Remember)
8.	If $x = u^2 - v^2$ *and* $y = 2uv$ then $\dfrac{\partial(x,y)}{\partial(u,v)} =$	1 Mark	
	a) $u^2 + v^2$ b) $2(u^2 + v^2)$ c) $4(u^2 + v^2)$ d) $4v^2$	Ans (c)	(CLO-2 / Apply)
9.	If $J_1 = J\left(\dfrac{x,y}{u,v}\right)$ and $J_2 = J\left(\dfrac{u,v}{x,y}\right)$ then $J_1 J_2 =$	1 Mark	
	a) 0 b) -1 c) 2 d) 1	Ans (d)	(CLO-2 / Remember)
10.	The point (0.0) for $f(x,y) = x^3 + y^3 - 3axy$ is	1 Mark	
	a)maximum point b) minimum point	Ans (c)	(CLO-2 / Apply)

	c) saddle point d) fixed point		
11.	If $f(x,y)=x^2y+\sin y+e^x$ then $f_x(1,\pi)$ *is* is	1 Mark	
	a) $2\pi-e$ b) 2π c) $2\pi+e$ d) 0	Ans (c)	(CLO-2 / Apply)
12.	The stationary points of $x^2+y^2+6x+12$ are	1 Mark	
	a) (-3,0) b) (0,3) c) (0, -3) d) (3,0)	Ans (a)	(CLO-2 / Apply)
13.	The stationary points for $f(x,y)=\sin x+\sin y+\sin(x+y)$ are	1 Mark	
	a) $\left(\frac{\pi}{2},\frac{\pi}{3}\right)$ b) $\left(\frac{\pi}{3},\frac{\pi}{3}\right)$ c) $\left(\frac{\pi}{2},\frac{\pi}{2}\right)$ d) $\left(\frac{\pi}{3},\frac{\pi}{2}\right)$	Ans (b)	(CLO-2 / Apply)
14.	If $u=x^2-y^2$ *and* $v=2xy$ then $J\left(\frac{x,y}{u,v}\right) X\ J\left(\frac{u,v}{x,y}\right)=$	1 Mark	
	a) 0 b) -1 c) 2 d) 1	Ans (d)	(CLO-2 / Apply)
15.	If $f(x,y)=e^x\cos y$ then $f_{xy}(0,0)$ *is*	1 Mark	
	a) 0 b) -1 c) 2 d) 1	Ans (a)	(CLO-2 / Apply)
16.	If $u=ax^2+by^2+2hxy$ then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=$	1 Mark	

	a) u b) 2u c) 3u d) 4u	Ans (b)	(CLO-2 / Apply)
17.	If $x^y = y^x$, then $\frac{dy}{dx} =$	1 Mark	
	a) $(x\log y - y)y / x(y\log x - x)$ b) $(x\log x - x) / x(y\log y - y)$ c) $(x\log x - y)y / (y\log x - x)$ d) Does not exists	Ans (a)	(CLO-2 / Apply)
18.	If $f(x,y) = e^{xy}$ then $f_{yyy}(1,1)$ *is*	1 Mark	
	a) $-e$ b) $\frac{1}{e}$ c) e d) $-\frac{1}{e}$	Ans (c)	(CLO-2 / Apply)
19.	If $z = \log(x^2 + y^2 + xy)$ then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} =$	1 Mark	
	a) 1 b) 2 c) 0 d) 4	Ans (b)	(CLO-2 / Apply)
20.	If $f(x,y) = \tan^{-1}(y/x)$ then $f_x(1,1)$ *is*	1 Mark	
	a) 1/2 b) -1/2 c) 2 d) 1	Ans (b)	(CLO-2 / Apply)
21.	If V= x/y, then $x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y} =$	1 Mark	
	(a) 2V (b) 3V (c) 4V (d) 0V	Ans (d)	(CLO-2 / Apply)

22.	Saddle points are ------------	1 Mark	
	(a) a minimum (b) a maximum (c) neither a minimum nor a maximum (d) None	Ans (c)	(CLO-2 / Remember)
23.	If u = x+y/1-xy, v= $\tan^{-1}x+\tan^{-1}y$ then the functional relationship between u and v is	1 Mark	
	a) u = tan v (b) v = tan u (c) x = tany (d) y = tan x	Ans (a)	(CLO-2 / Apply)
24.	Lagrange's method of undetermined multipliers is to find the maximum or minimum value of a function of	1 Mark	
	a) Two variables (b) Three or more variables (c) One variable (d) None	Ans (b)	(CLO-2 / Remember)
25.	The condition for a function f(x,y) to have a maximum value is that	1 Mark	
	(a) $rt-s^2$ (b) $rt-s^2>0$, $r>0$ or $s>0$ (c) $rt-s^2>0$, $r<0$ or $s<0$ (d) $rt-s^2=0$, $r>0$	Ans (C)	(CLO-2 / Remember)

26.	The condition for a function f(x,y) to have a minimum value is that	1 Mark	
	(a) $rt-s^2$ (b) $rt-s^2>0$, $r>0$ or $s>0$ (c) $rt-s^2>0$, $r<0$ or $s<0$ (d) $rt-s^2=0, r>0$	Ans (b)	(CLO-2 / Remember)
27.	The condition for a function f(x,y) to have neither a maximum nor a minimum value is that	1 Mark	
	(a) $rt-s^2<0$ (b) $rt-s^2>0$, $r>0$ or $s>0$ (c) $rt-s^2>0$, $r<0$ or $s<0$ (d) $rt-s^2=0, r>0$	Ans (a)	(CLO-2 / Remember)
28.	The point (a,b) is called a stationary point if	1 Mark	
	(a) $f_x(a,b)=0$, $f_y(a,b)=0$ (b)$f_{xx}(a,b)=0$ (c) $f_{yy}(a,b)=0$ (d)$f_{xx}(a,b)=0$, $f_{yy}(a,b)=0$	Ans (a)	(CLO-2 / Remember)
29.	The minimum value of the function $x^2+y^2+6x+12$ is	1 Mark	
	(a) $\frac{1}{2}$ (b)2 (c)1 (d)3	Ans (d)	(CLO-2 / Apply)
30.	The maximum value of the function $x^3+y^3-12x-3y+20$ is	1 Mark	
	(a) 75 (b)27 (c)35 (d)38	Ans (d)	(CLO-2 / Apply)
31.	The points at which there is no extreme value are called __________.	1 Mark	

	(a) maximum points (b) minimum points (c) saddle points (d) none	Ans (C)	(CLO-2 / Remember)
32.	If $u = xe^y \sin x$ $v = xe^y \cos x$ $w = x^2 e^{2y}$ then the functional relationship is	1 Mark	
	(a) $u^2+w^2 = v$ (b) $v^2 + w^2 = u$ (c) $x^2+y^2 = u$ (d) $u^2+v^2 = w$	Ans (d)	(CLO-2 / Apply)
33.	In PDE, a real function depends	1 Mark	
	(a) One independent variable (b) More than one independent variable (c) No independent variable (d) None	Ans (b)	(CLO-2 / Remember)
34.	If z= x^2+y^2+2xy then $\partial z/\partial x$ is	1 Mark	
	(a)$2x^2+2y$ (b) $2x+2y$ (c) $2x-2y$ (d) $2y$	Ans (b)	(CLO-2 / Apply)
35.	If x= rcosθ y = rsinθ then $\frac{\partial(r,\theta)}{\partial(x,y)}$	1 Mark	
	(a) 0 (b) 1 (c) r (d) 1/r	Ans (d)	(CLO-2 / Apply)
36.	If $u = \frac{x}{y} + \frac{y}{z} + \frac{z}{x}$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} =$ ________.	1 Mark	

	(a) 1 (b) 3 u (c) – 1 (d) 0	Ans (d)	(CLO-2 / Apply)

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Unit – II

FUNCTIONS OF SEVERAL VARIABLES

Part – B

1. **If $u = (x - y)(y - z)(z - x)$, then find $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$.**

   (A) 0 **(B) 1** **(C) 2** **(D) 3**

   Solution:

   Given $u = (x - y)(y - z)(z - x)$

$$\frac{\partial u}{\partial x} = (y - z)[(x - y)(-1) + (z - x)(1)]$$
$$= -(x - y)(y - z) + (y - z)(z - x)$$
$$\frac{\partial u}{\partial y} = (z - x)[(x - y)(1) + (y - z)(-1)]$$
$$= (x - y)(z - x) - (y - z)(z - x)$$
$$\frac{\partial u}{\partial z} = (x - y)[(y - z)(1) + (z - x)(-1)]$$
$$= (x - y)(y - z) - (x - y)(z - x)$$
$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0 \quad (\textbf{Option A})$$

2. **If $x = r\cos\theta, y = r\sin\theta$ find $\frac{\partial x}{\partial r}, \frac{\partial y}{\partial \theta}$.**

   (A) cos θ, sin θ (B) cos θ, *r* cos θ (C) *r* cos θ, sin θ (D) *r*, θ

   Solution:

$$\frac{\partial x}{\partial r} = \cos\theta$$
$$\frac{\partial y}{\partial \theta} = r\cos\theta \quad (\textbf{Option B})$$

3. If $f(x,y) = \sin\left(\frac{x}{y}\right)$, then find $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y}$.

(A) 0 **(B) 1** **(C) 2** **(D) 3**

Solution:

$$\frac{\partial f}{\partial x} = \cos\left(\frac{x}{y}\right)\frac{1}{y}, \quad \frac{\partial f}{\partial y} = \cos\left(\frac{x}{y}\right)\left(-\frac{x}{y^2}\right)$$

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = 0 \text{ (Option A)}$$

4. Find $\frac{dy}{dx}$ when $x^3 + y^3 = 3axy$.

$(A) -\frac{x^2-ay}{y^2-ax}$ $(B)\frac{x^2-ay}{y^2-ax}$ $(C)\frac{y^2-ax}{x^2-ay}$ $(D) -\frac{y^2-ax}{x^2-ay}$

Solution:

Let $f(x,y) = x^3 + y^3 - 3axy$

$$\frac{dy}{dx} = -\frac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial y}} = -\frac{3x^2 - 3ay}{3y^2 - 3ax}$$

$$= -\frac{x^2 - ay}{y^2 - ax} \quad \text{(Option A)}$$

5. If $x = uv$, $y = \frac{u}{v}$, find $\frac{\partial(x,y)}{\partial(u,v)}$.

(A) $\frac{-2u}{v}$ **(B)** $\frac{2u}{v}$ **(C)** $\frac{-2v}{u}$ **(D)** $\frac{2v}{u}$

Solution:

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} v & u \\ \frac{1}{v} & \frac{-u}{v^2} \end{vmatrix} = -\frac{2u}{v} \quad \text{(Option A)}$$

6. If $f(x,y) = e^x \sin y$, then find $f_{yy}(0,0)$.

(A) 0 **(B) 1** **(C) 2** **(D) 3**

Solution:

$f_y = e^x \cos y$

$f_{yy}(x,y) = e^x(-\sin y)$

$f_{yy}(0,0) = 0$ **(Option A)**

7. If $x^y = y^x$, then find $\dfrac{dy}{dx}$.

(A) $\dfrac{y\,x^{y-1} - y^x \log y}{x\,y^{x-1} - x^y \log x}$ **(B)** $\dfrac{y\,x^{y-1} + y^x \log y}{x\,y^{x-1} - x^y \log x}$

(C) $\dfrac{y\,x^{y-1} + y^x \log y}{x\,y^{x-1} + x^y \log x}$ **(D)** $\dfrac{y\,x^{y-1} - y^x \log y}{x\,y^{x-1} + x^y \log x}$

Solution:

$f(x, y) = x^y - y^x = 0$

$$\frac{dy}{dx} = \frac{-f_x}{f_y} = \frac{y\,x^{y-1} - y^x \log y}{x\,y^{x-1} - x^y \log x}$$ **(Option A)**

8. If $f(x, y) = \tan^{-1}\left(\dfrac{y}{x}\right)$, then find $f_x(x, y)$ at the point (1, 1).

(A) -1/2 **(B) 1** **(C) 1/2** **(D) 3**

Solution:

$f_x(x, y) = \dfrac{-y}{x^2 + y^2}$ $f_x(1,1) = -\dfrac{1}{2}$ **(Option A)**

9. If $x = r\cos\theta$, $y = r\sin\theta$, then find $\dfrac{\partial(x, y)}{\partial(r, \theta)}$.

(A) *r* **(B) 1/ *r*** **(C) 1/2** **(D) 1**

Solution:

$$\text{Now}\quad \frac{\partial(x, y)}{\partial(r, \theta)} = \begin{vmatrix} \dfrac{\partial x}{\partial r} & \dfrac{\partial x}{\partial \theta} \\[2ex] \dfrac{\partial y}{\partial r} & \dfrac{\partial v}{\partial \theta} \end{vmatrix} = \begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix} = r\left(\sin^2\theta + \cos^2\theta\right) = r(1) = r$$

(Option A)

10. If $u = 2xy$, $v = x^2 - y^2$, then find $\dfrac{\partial(u,v)}{\partial(x,y)}$.

(A) $-4y^2 - 4x^2$ **(B) $-4y^2 + 4x^2$**
(C) $4y^2 - 4x^2$ **(D) $4y^2 + 4x^2$**

$$\frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} 2y & 2x \\ 2x & -2y \end{vmatrix} = -4y^2 - 4x^2$$ **(Option A)**

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SRM Institute of Science and Technology
Ramapuram campus
Department of Mathematics
18MAB101T - Calculus and Linear Algebra

Year/Sem: I/I **Part – A Branch: Common to ALL Branches**

Unit – III Ordinary Differential Equations

1.	The order and degree of $\frac{d^2y}{dx^2}+3\frac{dy}{dx}+2y=e^{-2x}$ are	1 Mark	
	a)2, 1 b) 1, 2 c) 2, 2 d)1, 1	Ans (a)	(CLO-3 Remember)
2.	The order and degree of $\frac{d^2y}{dx^2}+3\left(\frac{dy}{dx}\right)^2+2y=\sin 3x$ are	1 Mark	
	a) 1, 2 b) 2, 1 c) 2, 2 d) 1, 1	Ans (b)	(CLO-3 Remember)
3.	The order and degree of $\left(\frac{d^2y}{dx^2}\right)^2+2\frac{dy}{dx}+y=5x$ are	1 Mark	
	a) 1, 2 b) 2, 1 c) 2, 2 d) 1, 1	Ans (c)	(CLO-3 Remember)
4.	The order and degree of $\frac{dy}{dx}+3y=5x$ are	1 Mark	
	a) 1, 2 b) 2, 1 c) 2, 2 d) 1, 1	Ans (d)	(CLO-3 Remember)
5.	The number of arbitrary constants in the solution of a differential equation is equal to the _________ of that differential equation.	1 Mark	
	a) degree b) number of variables c) order d) number of terms	Ans (b)	(CLO-3 Remember)
6.	The number of arbitrary constants in the most general solution of n^{th} order differential equation is _________	1 Mark	
	a) 1 b) n – 1 c) n d) n + 1	Ans (c)	(CLO-3 Remember)
7.	The solution of $(D^3 - D^2 + D - 1)y = 0$ is	1 Mark	
	a) $y = Ae^x + B\cos x + C\sin x$ b) $y = Ae^x + B\cos x - C\sin x$ c) $y = Ae^{-x} + B\cos x + C\sin x$ d) $y = Ae^x + B\cosh x + C\sinh x$	Ans (a)	(CLO-3 Remember)
8.	The complementary function of $(D^2 + D + 1)y = 0$ is	1 Mark	

	a) $e^{\frac{1}{2}x}\left(C_1 \cos\frac{\sqrt{3}}{2}x + C_2 \sin\frac{\sqrt{3}}{2}x\right)$ b) -1, 2 c) $e^{\frac{-1}{2}x}\left(C_1 \cos\frac{\sqrt{3}}{2}x + C_2 \sin\frac{\sqrt{3}}{2}x\right)$ d) $\cos x + i\sin x$	Ans (c)	(CLO-3 Remember)
9.	The complementary function of $\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 15y = 0$	1 Mark	
	a) $C_1 e^{-5x} + C_2 e^{-3x}$ b) $C_1 e^{4x} + C_2 e^{4x}$ c) $C_1 e^{5x} + C_2 e^{3x}$ d) $C_1 e^{2x} + C_2 e^{6x}$	Ans (c)	(CLO-3 Remember)
10.	The complementary function of $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = 3e^{4x}$	1 Mark	
	a) $C_1 e^{-3x} + C_2 e^{-3x}$ b) $C_1 e^{3x} + C_2 e^{3x}$ c) $(C_1 + C_2 x)e^{-3x}$ d) $(C_1 + C_2 x)e^{3x}$	Ans (c)	(CLO-3 Remember)
11.	The complementary function of $(D^2 + 4)y = x \sin x$ is	1 Mark	
	a) $C_1 e^{-3x} + C_2 e^{-3x}$ b) $C_1 e^{3x} + C_2 e^{3x}$ c) $C_1 \cos 2x + C_2 \sin 2x$ d) $(C_1 + C_2 x)e^{3x}$	Ans (c)	(CLO-3 Remember)
12.	The particular integral of $(D^3 - D^2 + D - 1)y = 0$ is	1 Mark	
	a) 0 b) $Ae^x + B\cos x - C\sin x$ c) $B\cos x + C\sin x$ d) $Ae^x + B\cosh x + C\sinh x$	Ans (a)	(CLO-3 Remember)
13.	The particular integral of $(D^2 + 2D + 1)y = 5$ is	1 Mark	
	a) 0 b) 5 c) 2 d) 1	Ans (b)	(CLO-3 Remember)
14.	The particular integral of $(D^2 + 9)y = e^{-2x}$ is	1 Mark	
	a) $\frac{e^{-2x}}{15}$ b) $\frac{e^{2x}}{15}$ c) $\frac{e^{-2x}}{13}$ d) $\frac{e^{-2x}}{14}$	Ans (c)	(CLO-3 Remember)
15.	The particular integral of $(D^2 + 16)y = e^{-4x}$ is	1 Mark	
	a) $\frac{x}{32}e^{-4x}$ b) $\frac{1}{32}e^{-4x}$ c) $\frac{x}{16}e^{-4x}$ d) $\frac{1}{16}e^{-4x}$	Ans (b)	(CLO-3 Remember)
16.	The particular integral of $(D - 1)^2 y = e^x$ is	1 Mark	
	a) $\frac{x}{32}e^{-4x}$ b) $\frac{x^2}{2}e^{x}$ c) $\frac{x}{16}e^{-4x}$ d) $\frac{1}{16}e^{-4x}$	Ans (b)	(CLO-3 Remember)

17.	The particular integral of $(D^2 + a^2)y = \cos ax$ is	1 Mark	
	a) $\frac{-x}{2a}\sin ax$ b) $\frac{-x}{2a}\cos ax$ c) $\frac{x}{2a}\cos ax$ d) $\frac{x}{2a}\sin ax$	Ans (d)	(CLO-3 Remember)
18.	The particular integral of $(D^2 + 4)y = \sin 2x$ is	1 Mark	
	a) $\frac{x}{2}\sin x$ b) $\frac{-x}{2}\sin x$ c) $\frac{-x}{4}\cos 2x$ d) $\frac{x}{4}\cos 2x$	Ans (c)	(CLO-3 Remember)
19.	The particular integral of $(D^2 + 2)y = x^2$ is	1 Mark	
	a) $\frac{1}{2}x^2$ b) $\frac{1}{2}(x^2-1)$ c) $\frac{1}{2}(x^2+1)$ d) $\frac{-1}{2}x^2$	Ans (b)	(CLO-3 Remember)
20.	The method of variation of parameters is used to find the particular integral of a second order differential equation whose _________ is known.	1 Mark	
	a) Complementary function b) constant c) variable d) degree	Ans (a)	(CLO-3 Remember)
21.	The order and degree of $\left(\frac{d^2y}{dx^2}\right)^2+3\frac{dy}{dx}+2y=e^x$ are	1 Mark	
	a) 2, 1 b) 1, 2 c) 2, 2 d) 1, 1	Ans (c)	(CLO-3 Remember)
22.	The order and degree of $\left(\frac{d^2y}{dx^2}\right)^2+3\left(\frac{dy}{dx}\right)^4+2y=Sin2x$ are	1 Mark	
	a) 2, 1 b) 1, 2 c) 2, 2 d) 1, 1	Ans (c)	(CLO-3 Remember)
23.	The particular integral of $(D^3 - 1)y = 0$ is	1 Mark	
	a) 0 b) Ae^x+ Bcosh x c) Acos x + Bsin x d) Ae^x + Bcosh x + Csinh x	Ans (a)	(CLO-3 Remember)
24.	The particular integral of $(D^2 + 2D + 1)y = 1$ is	1 Mark	
	a) 0 b) 5 c) 2 d) 1	Ans (d)	(CLO-3 Remember)
25.	The particular integral of $(D^2 + 2)y = x$ is	1 Mark	
	a) $\frac{1}{2}x$ b) $\frac{1}{2}(x^2-1)$ c) $\frac{1}{2}(x^2+1)$ d) $\frac{-1}{2}x^2$	Ans (a)	(CLO-3 Remember)

26.	The particular integral of $(D^2 + 4)y = \cos 2x$ is	1 Mark	
	a) $\frac{x}{2}\sin x$ b) $\frac{-x}{2}\sin x$ c) $\frac{-x}{4}\cos 2x$ d) $\frac{x}{4}\sin 2x$	Ans (d)	(CLO-3 Remember)
27.	The particular integral of $(D^2 + 1)y = \cos 2x$ is	1 Mark	
	a) $\frac{x}{2}\sin x$ b) $\frac{1}{5}\cos 2x$ c) $\frac{-x}{4}\cos 2x$ d) $\frac{x}{4}\sin 2x$	Ans (b)	(CLO-3 Remember)
28.	The complementary function of $\frac{d^2y}{dx^2}+8\frac{dy}{dx}+15y=0$	1 Mark	
	a) $C_1 e^{-5x}+C_2e^{-3x}$ b) $C_1 e^{4x}+C_2e^{4x}$ c) $C_1 e^{5x}+C_2e^{3x}$ d) $C_1 e^{2x}+C_2e^{6x}$	Ans (a)	(CLO-3 Remember)
29.	The complementary function of $(D^2 + 4)y = \sin x$ is	1 Mark	
	a) $C_1 e^{-3x}+C_2e^{-3x}$ b) $C_1 e^{3x}+C_2e^{3x}$ c) $C_1 \cos 2x+C_2 \sin 2x$ d) $(C_1+C_2 x)e^{3x}$	Ans (c)	(CLO-3 Remember)
30.	The complementary function of $\frac{d^2y}{dx^2}-6\frac{dy}{dx}+9y=e^{4x}$	1 Mark	
	a) $C_1 e^{-3x}+C_2e^{-3x}$ b) $C_1 e^{3x}+C_2e^{3x}$ c) $(C_1+C_2 x)e^{-3x}$ d) $(C_1+C_2 x)e^{3x}$	Ans (d)	(CLO-3 Remember)
31.	The particular integral of $(D - 1)^2 y = e^{-x}$ is	1 Mark	
	a) $\frac{x}{32}e^{-4x}$ b) $\frac{x^2}{2}e^{x}$ c) $\frac{x}{16}e^{-4x}$ d) $\frac{1}{4}e^{-x}$	Ans (d)	(CLO-3 Remember)
32.	The particular integral of $(D - 3)^2y = 3^x$ is	1 Mark	
	a) $\frac{3^x}{(\log 3-3)^2}$ b) $\frac{2^x}{(\log 3-3)^2}$ c) $\frac{3^x}{(\log 2-2)^2}$ d) $\frac{2^x}{(\log 2-2)^2}$	Ans (a)	(CLO-3 Remember)
33.	The complementary function of $(D - 1)^2y = e^{-5x}$ is	1 Mark	
	a) $C_1 e^{-x}+C_2e^{-x}$ b) $C_1 e^{x}+C_2e^{x}$ c) $(C_1+C_2 x)e^{x}$ d) $(C_1+C_2 x)e^{-x}$	Ans (c)	(CLO-3 Remember)
34.	The particular integral of $(D + 1)^2 y = e^{-5x}$ is	1 Mark	

	a) $\frac{1}{16}e^{-5x}$ b) $\frac{x^2}{2}e^{x}$ c) $\frac{x}{36}e^{-5x}$ d) $\frac{1}{4}e^{-x}$	Ans (a)	(CLO-3 Remember)
35.	The particular integral of $(D^2 + 1)y = \cos x$ is	1 Mark	
	a) $\frac{x}{2}\sin x$ b) $\frac{-x}{3}\cos 2x$ c) $\frac{-x}{4}\cos 2x$ d) $\frac{x}{4}\sin 2x$	Ans (a)	(CLO-3 Remember)
36.	The particular integral of $(D^2 + 9)y = \sin 3x$ is	1 Mark	
	a) $\frac{x}{2}\sin x$ b) $\frac{-x}{6}\cos 3x$ c) $\frac{-x}{4}\cos 2x$ d) $\frac{x}{4}\sin 2x$	Ans (b)	(CLO-3 Remember)

SRM Institute of Science and Technology
Ramapuram Campus
Department of Mathematics
18MAB101T – Calculus And Linear Algebra

Year/Sem: I/I

Branch: Common to ALL B.Tech. except B.Tech. (Business Systems)

Unit – III

ORDINARY DIFFERENTIAL EQUATIONS

Part – B

1. Solve $(D^2 - 7D + 12)y = 0$.

$(a)\, y = Ae^{3x} + Be^{4x}$ $(b)\, y = Ae^{-3x} + Be^{4x}$

$(c)\, y = Ae^{3x} + Be^{-4x}$ $(d)\, y = Ae^{-3x} + Be^{-4x}$

$m^2 - 7m + 12 = 0$
$(m-3)(m-4) = 0$
$m = 3, 4$
$y = Ae^{3x} + Be^{4x}$ **(Option (a))**

2. Find the particular integral of $(D^2 - 9)y = e^{-2x}$.

$(a)\, PI = \frac{1}{13}e^{-2x}$ $(b)\, PI = -\frac{1}{5}e^{-2x}$

$(c)\, PI = \frac{x}{5}e^{-2x}$ $(d)\, PI = \frac{1}{5}e^{-2x}$

$$PI = \frac{1}{D^2 - 9} e^{-2x}$$
$$= \frac{1}{4-9} e^{-2x}$$
$$= -\frac{1}{5} e^{-2x}$$ **(Option (b))**

3. Find the particular integral of $(D^2 + 3D + 2)y = e^{-2x}$.

$(a) PI = -xe^{-2x}$ $(b) PI = xe^{-2x}$

$(c) PI = \dfrac{e^{-2x}}{12}$ $(d) PI = \dfrac{xe^{-2x}}{12}$

$$PI = \frac{1}{D^2 + 3D + 2} e^{-2x}$$
$$= \frac{1}{4-6+2} e^{-2x}$$
$$= x.\frac{1}{2D+3} e^{-2x}$$
$$PI = -xe^{-2x}$$ **(Option (a))**

4. Find the particular integral of $(D^2 + 4)y = \sin 2x$.

$(a) PI = -\dfrac{x \cos 2x}{4}$ $(b) PI = -\dfrac{\sin 2x}{8}$

$(c) PI = \dfrac{x \sin 2x}{4}$ $(d) PI = \dfrac{\sin 2x}{8}$

$$PI = \frac{1}{D^2+4}\sin 2x$$
$$= x.\frac{1}{2D}\sin 2x$$
$$= -x.\frac{\cos 2x}{4}$$ **(Option (a))**

5. Find the particular integral of $(D^2+D+1)y = 3x-1$.

$(a) PI = 3x-4$ $(b) PI = 3x$

$(c) PI = 3x-1$ $(d) PI = 3x^2-4$

$$PI = \frac{1}{D^2+D+1}(3x-1)$$
$$= [1+(D+D^2)]^{-1}(3x-1)$$
$$= (3x-1) - D(3x-1)$$
$$PI = 3x-4$$ **(Option (a))**

6. Find the particular integral of $(D^2+D+1)y = x$

$(a) PI = 3x-4$ $(b) PI = 3x$

$(c) PI = x-1$ $(d) PI = 3x^2-4$

$$PI = \frac{1}{D^2+D+1}(x)$$
$$= [1+(D+D^2)]^{-1}(x)$$
$$= [1-(D+D^2)](x)$$
$$= (x - D(x)) \quad = x-1$$
$$PI = x-1$$

(Option C)

7**. Solve** $(D^3 - 6D^2 + 11D - 6)y = 0$

$(a) y = Ae^{x} + Be^{2x} + Ce^{3x}$ $(b) y = Ae^{x} + Be^{-2x} + Ce^{3x}$

$(c) y = Ae^{x} + Be^{2x} + Ce^{-3x}$ $(d) y = Ae^{x} + Be^{-2x} + Ce^{-3x}$

$$m^3 - 6m^2 + 11m - 6 = 0$$
$$(m-1)(m-2)(m-3) = 0$$
$$m = 1,2,3$$
$$C.F = Ae^{x} + Be^{2x} + Ce^{3x}$$
Hence
$$y = Ae^{x} + Be^{2x} + Ce^{3x}$$

(Option A)

8**.** Find the particular integral of $(D^2 + D - 2)y = Sinx$

$(a) PI = \frac{-1}{10}(\cos x + 3\sin x)$ $(b) PI = \frac{1}{10}(\cos x + 3\sin x)$

$(c) PI = \frac{-1}{10}(\sin x + 3\cos x)$ $(d) PI = \frac{-1}{10}(\sin x - 3\cos x)$

P.I $= \frac{1}{D-3}\sin x = \frac{D+3}{D^2-9}\sin x$, Rationalizing the denominator

$= \frac{(D+3)\sin x}{-10}$, Putting $D^2 = -1$

$\therefore$ P.I. $= \frac{-1}{10}(D\, sin\, x + 3\, \sin x)$

$= \frac{-1}{10}(\cos x + 3\, \sin x)$

(Option A)

9. Find the complementary function of $(D^2+1)y = \cos ec\, x$.

$(a) CF = (A+Bx)e^x$ $(b) CF = (A+Bx)e^x$

$(c) CF = A\cos x + B\sin x$ $(d) CF = (A\cos x + B\sin x)e^x$

$m^2 + 1 = 0 \Rightarrow m = \pm i$

$CF = A\cos x + B\sin x$ **(Option (c))**

10. Solve $(D^2+4D+4)y = 0$.

$(a) y = Ae^{-2z} + Be^{-2z}$ $(b) y = (A+Bx)e^{-2x}$

$(c) y = \frac{A}{x} + \frac{B}{x^2}$ $(d) y = Ax + Bx^2$

$m^2 + 4m + 4 = 0$

$m = -2, -2$

$y = (A+Bx)e^{-2x}$ **(Option (B))**

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SRM Institute of Science and Technology

Ramapuram campus

Department of Mathematics

18MAB101T – Calculus and linear algebra

Year/Sem: I/I **Part-A** **Branch: Common to All**

Unit – IV

Differential Calculus

1.	The locus of centre of curvature is called	**1 Mark**	
	(a) Involute (b) Evolute (c) Radius of curvature (d) Envelope	Ans (b)	**(CLO-4 Remember)**
2.	The envelope of the family of curves $A\alpha^2+B\alpha+C=0$ (α is parameter) is	**1 Mark**	
	(a) $B^2+4AC=0$ (b) $B^2-4AC=0$ (c) $B^2+AC=0$ (d) $B^2-AC=0$	Ans (b)	**(CLO-4 Remember)**
3.	The curvature of the straight line is	**1 Mark**	
	(a) 1 (b) 2 (c) -1 (d) 0	Ans (d)	**(CLO-4 Remember)**
4.	Evolute of a curve is ______ of the normals of that curve	**1 Mark**	
	(a) Involute (b) Length (c) Envelope (d) End points	Ans (c)	**(CLO-4 Remember)**
5	The radius of curvature at (3,4) on the curve $x^2+y^2=25$ is	**1 Mark**	
	(a) 5 (b) 4 (c) 0 (d) 2	Ans (a)	**(CLO-4 Remember)**

6.	What is the curvature of a circle of radius 3?	**1 Mark**	
	(a) 3 (b) – 3 (c) $\frac{1}{3}$ (d) $\frac{-1}{3}$	Ans (c)	**(CLO-4 Remember)**
7.	In an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ the radius of curvature at the end of which axis is equal to the semi-latus rectum?	**1 Mark**	
	(a) minor (b) major (c) vertical (d) horizontal	Ans (b)	**(CLO-4 Remember)**
8.	Evolute of a curve is the envelope of ______ of that curve.	**1 Mark**	
	(a) tangent (b) normal (c) parallel (d) locus	Ans (b)	**(CLO-4 Remember)**
9.	The evolute of the cycloid $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ is	**1 Mark**	
	(a) astroid (b) parabola (c) cycloid (d) circle	Ans (c)	**(CLO-4 Remember)**
10.	A curve which touches each member of a family of the curves is called – – – of that family	**1 Mark**	
	(a) Evolute (b) Envelope (c) Circle of curvature (d) Radius of curvature	Ans (b)	**(CLO-4 Remember)**
11.	Envelope of the curve $y = mx + \frac{a}{m}$ (*where m is the parameter*) is	**1 Mark**	
	(a) $x^2 + ay = 0$ (b) $x + 4ay = 0$ (c) $y^2 - 4ax = 0$ (d) $y^2 + 4ax = 0$	Ans (c)	**(CLO-4 Remember)**

12.	If the radius of curvature and curvature of a curve at any point are ρ and k respectively, then	**1 Mark**	
	(a) $\rho = \frac{-1}{k}$ (b) $\rho = k$ (c) $\rho = -k$ (d) $\rho = \frac{1}{k}$	Ans (d)	**(CLO-4 Remember)**
13	The radius of curvature at the point (0, c) of the curve $y = c \cosh\left(\frac{x}{c}\right)$ is	**1 Mark**	
	(a) $\rho = c$ (b) $\rho = c^2$ (c) $\rho = kc$ (d) $\rho = kc^2$	Ans (a)	**(CLO-4 Remember)**
14	The radius of curvature of the curve $y = e^x$ at $x=0$ *is*	**1 Mark**	
	(a) $2\sqrt{2}$ (b) $\sqrt{2}$ (c) 2 (d) *4*	Ans (a)	**(CLO-4 Remember)**
15	The radius of curvature at the point (x, y) of the curve $y = c \log \sec\left(\frac{x}{c}\right)$ is	**1 Mark**	
	(a) $\rho = c\sec\left(\frac{x}{c}\right)$ (b) $\rho = c\cos\left(\frac{x}{c}\right)$ (c) $\rho = c\sin\left(\frac{x}{c}\right)$ (d) $\rho = c\tan\left(\frac{x}{c}\right)$	Ans (a)	**(CLO-4 Remember)**
16	The parametric form of the curve $y^2 = 4ax$ is	**1 Mark**	
	(a) $x = at^2; y = 2at$ (b) $x = at; y = 2at$ (c) $x = at^2; y = 2at^2$ (d) $x = 2at^2; y = 2at$	Ans (a)	**(CLO-4 Remember)**

17	The envelope of the curve $y = mx + \frac{a}{m}$ where m is the parameter is	**1 Mark**	
	(a) $y^2 - 4ax = 0$ (b) $y^2 + 4ax = 0$ (c) $x^2 + y^2 = 1$ (d) $xy = c^2$	Ans (a)	**(CLO-4 Remember)**
18	The radius of curvature of the curve $y = \log \sec x$ at any point on it is	**1 Mark**	
	(a) sec x (b) tan x (c) cot x (d) cosec x	Ans (a)	**(CLO-4 Remember)**
19	The radius of curvature of the curve $x = t^2$, $y = t$ at $t = 1$ is	**1 Mark**	
	(a) $5\frac{\sqrt{5}}{2}$ (b) $\frac{\sqrt{5}}{2}$ (c) $\frac{5}{2}$ (d) $\sqrt{5}$	Ans (a)	**(CLO-4 Remember)**
20	The radius of curvature of the parabola $y^2 = 12x$ at (3, 6) is	**1 Mark**	
	(a) $12\sqrt{2}$ (b) $2\sqrt{2}$ (c) $10\sqrt{2}$ (d) $\sqrt{2}$	Ans (a)	**(CLO-4 Remember)**
21	The radius of curvature of the curve $y = 4 \sin x$ at $x = \frac{\pi}{2}$ is	**1 Mark**	
	(a) $\frac{1}{2}$ (b) $\frac{-1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{-1}{3}$	Ans (c)	**(CLO-4 Remember)**
22	The envelope of family of lines $y = m x + a m^2$ (where m is the parameter) is	**1 Mark**	
	(a) $x^2 + 2ay = 0$ (b) $x^2 + 4ay = 0$ (c) $y^2 + 2ax = 0$ (d) $x^2 + 4ax = 0$	Ans (b)	**(CLO-4 Remember)**

23	The envelope of the family of lines $\frac{x}{t} + yt = 2c$, t being the parameter is	**1 Mark**	
	(a) $x^2 + y^2 = c^2$ (b) $xy = c^2$ (c) $x^2 y^2 = c^2$ (d) $x^2 - y^2 = c^2$	Ans (b)	**(CLO-4 Remember)**
24	The radius of curvature at any point on the curve $r = e^{\theta}$ is	**1 Mark**	
	(a) $\frac{\sqrt{2}}{r}$ (b) $\frac{r}{\sqrt{2}}$ (c) r (d) $\sqrt{2}\, r$	Ans (d)	**(CLO-4 Remember)**
25	The radius of curvature in Cartesian coordinates is	**1 Mark**	
	(a) $\rho = \frac{(1+y_1^2)^{\frac{3}{2}}}{y_2}$ (b) $\rho = \frac{(1-y_1^2)^{\frac{3}{2}}}{y_2}$ (c) $\rho = \frac{(1+y_1^2)^{\frac{2}{3}}}{y_2}$ (d) $\rho = \frac{(1+y_1^2)^{\frac{3}{2}}}{y_1}$	Ans (a)	**(CLO-4 Remember)**
26	The envelope of $ty - x = at^2$, t is the parameter is	**1 Mark**	
	(a) $x^2 = 4ay$ (b) $y^2 = 4ax$ (c) $x^2 + y^2 = 1$ (d) $x^2 - y^2 = 1$	Ans (b)	**(CLO-4 Remember)**
27	The radius of curvature in polar coordinates is	**1 Mark**	
	(a) $\rho = \frac{\left(r^2 + r'^2\right)^{\frac{3}{2}}}{r^2 - rr' + 2r'^2}$ (b) $\rho = \frac{\left(r^2 - r'^2\right)^{\frac{3}{2}}}{r^2 - rr' + 2r'^2}$ (c) $\rho = \frac{\left(r^2 - r''^2\right)^{\frac{3}{2}}}{r^2 - rr' + 2r'^2}$ (d) $\rho = \frac{\left(r^2 + r'^2\right)^{\frac{3}{2}}}{r^2 - rr'' + 2r'^2}$	Ans (d)	**(CLO-4 Remember)**

28	The radius of curvature in parametric coordinates is	**1 Mark**	
	(a) $\rho = \frac{\left(x'^2 + y'^2\right)^{\frac{3}{2}}}{x'y'' - y'x''}$ (b) $\rho = \frac{\left(x'^2 + y'^2\right)^{\frac{3}{2}}}{x'y'' + y'x''}$ (c) $\rho = \frac{\left(x'^2 - y'^2\right)^{\frac{3}{2}}}{x'y'' - y'x''}$ (d) $\rho = \frac{\left(x'^2 - y'^2\right)^{\frac{3}{2}}}{x'y'' + y'x''}$	Ans (a)	**(CLO-4 Remember)**
29	The equation of circle of curvature at any point (x, y) with center of curvature $(\bar{x}, \bar{y})$ and radius of curvature ρ is	**1 Mark**	
	(a) $\left(x+\bar{x}\right)^2 + \left(y+\bar{y}\right)^2 = \rho^2$ (b) $\left(x-\bar{x}\right)^2 + \left(y-\bar{y}\right)^2 = \rho^2$ (c) $\left(x-\bar{x}\right)^2 - \left(y+\bar{y}\right)^2 = \rho^2$ (d) $\left(x+\bar{x}\right)^2 + \left(y+\bar{y}\right)^2 = \rho^2$	Ans (b)	**(CLO-4 Remember)**
30	The curvature at any point of the circle is equal to___ of its radius	**1 Mark**	
	(a) same (b) ellipse (c) reciprocal (d) constant	Ans (c)	**(CLO-4 Remember)**
31	The parametric equations of rectangular hyperbola $xy = c^2$ is	**1 Mark**	
	(a) $x = ct, y = \frac{c}{t}$ (b) $x = ct, y = t$ (c) $x = \frac{c}{t}, y = t$ (d) $x = ct, y = \frac{1}{t}$	Ans (a)	**(CLO-4 Remember)**
32	The value of $\Gamma\left(\frac{1}{2}\right)$ is	**1 Mark**	
	(a) π (b) $\frac{\pi}{2}$ (c) $\sqrt{\pi}$ (d) $\frac{\sqrt{\pi}}{2}$	Ans (c)	**(CLO-4 Remember)**

33	The series $\sum_{n=1}^{\infty}\frac{1}{n^p}$ is convergent if	**1 Mark**	
	(a) p = 0 (b) p =1 (c) p > 1 (d) p < 1	Ans (c)	**(CLO-4 Remember)**
34	As per D' Alembert's ratio test, if $\sum u_n$ is a series of positive terms and $\sum_{n\to\infty}\frac{u_{n+1}}{u_n}=l$, then the series is convergent, if	**1 Mark**	
	(a) l = 0 (b) l =1 (c) l > 1 (d) l < 1	Ans (d)	**(CLO-4 Remember)**
35	If n is a positive integer, then $\Gamma(n+1)=$	**1 Mark**	
	(a) $(n + 1)!$ (b) $n!$ (c) $2n!$ (d) $(n - 1)!$	Ans (b)	**(CLO-4 Remember)**
36	The series $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+.........$ is	**1 Mark**	
	(a) convergent (b) divergent (c) oscillating (d) monotonic	Ans (b)	**(CLO-4 Remember**

SRM Institute of Science and Technology
Ramapuram Campus

Department of Mathematics

18MAB101T - Calculus And Linear Algebra

Year/Sem: I/I

Branch: Common to ALL B.Tech. except B.Tech. (Business Systems)

Unit – IV

DIFFERENTIAL CALCULUS

Part – B

1. Envelope of the curve $y = mx + \frac{a}{m}$ (where m is the parameter) is

(A) $x^2 + a\,y = 0$ (B) $x + 4\,a\,y = 0$

(C) $y^2 - 4\,a\,x = 0$ (D) $y^2 + 4ax = 0$

Solution: Given: $y = mx + \frac{a}{m}$

$$y = \frac{m^2 x + a}{m}$$

$$m^2 x - y\,m + a = 0$$

The above equation is a quadratic equation in 'm'.

The discriminant is $b^2 - 4ac = 0$.

The envelope of the curve is $y^2 - 4\,a\,x = 0$. **(Option C)**

2. The radius of curvature of the curve $y = e^x$ at $x = 0$ is

(A) $2\sqrt{2}$ (B) $\sqrt{2}$

(C) 2 (D) 4

Solution:

$y_1 = e^x$ at $x = 0$ is 1

$y_2 = e^x$ at $x = 0$ is 1

$$\rho = \frac{\left(1 + {y_1}^2\right)^{\frac{3}{2}}}{y_2}$$

$\rho = 2\sqrt{2}$ **(Option A)**

3. The radius of curvature of the curve y = 4 sin x at x = $\frac{\pi}{2}$ is

(A) $\frac{1}{2}$ (B) $\frac{-1}{2}$

(C) $\frac{1}{4}$ (D) $\frac{3}{4}$

Solution:

$y_1 = 4\cos x \ at\ x = \frac{\pi}{2} is\ 0$

$y_2 = -4\sin x\, at\ x = \frac{\pi}{2} is - 4$

$$\rho = \frac{\left(1 + {y_1}^2\right)^{\frac{3}{2}}}{y_2}$$

$|\rho| = \frac{1}{4}$ **(Option C)**

4. The envelope of family of lines y = $mx + am^2$ (where m is the parameter) is

(A) $x^2 + 2ay = 0$ (B) $x^2 + 4\,a\,y = 0$

(C) $y^2 + 2ax = 0$ (D) $x^2 + 4\,a\,x = 0$

Solution:

The given equation is quadratic in 'm'.

The discriminant is $b^2 - 4ac = 0$.

Envelope of the family of lines is $x^2 + 4ay = 0$. **(Option B)**

5. The envelope of the family of lines $\frac{x}{t} + y\,t = 2c$, t being the parameter is

(A) $x^2 + y^2 = c^2$ (B) $x\,y = c^2$

(C) $x^2\,y^2 = c^2$ (D) $x^2 - y^2 = c^2$

Solution:

Simplifying the equation $\frac{x}{t} + y\,t = 2c$, we get $yt^2 - 2ct + x = 0$

The discriminant is $b^2 - 4ac = 0$.

Envelope of the family of lines is $x\,y = c^2$. **(Option B)**

6. The radius of curvature of the curve $r = e^{\theta}$ at any point on it is

(a) $2\sqrt{2}$ (b) $\sqrt{2}\,r$

(c) 2 (d) 4

Solution:

$r' = e^{\theta}$

$r'' = e^{\theta}$

$$\rho = \frac{\left(r^2 + r'^2\right)^{\frac{3}{2}}}{r^2 - rr'' + 2r'^2}$$

$\rho = \sqrt{2}\,r$

7. The radius of curvature at the point (3, 4) on the curve $x^2 + y^2 = 25$ is

(A) 5 (B) 4 (C) 0 (D) 2

Solution:

We know that the radius of curvature of the circle is equal to its radius.

$\rho = 5$ **(Option A)**

8. B (5/2, 1/2) = _______.

(A) 1 (B) 4

(C) 3π/8 (D) π

Solution:

$$B(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$$

$$B(5/2,1/2) = \frac{\Gamma(5/2)\Gamma(1/2)}{\Gamma(3)} = \frac{3\pi}{8}$$ **(Option C)**

9. $\Gamma(-5/2)$ **= ____________.**

(A) 1 (B) 4

(C) 1/2 (D) $\frac{-8\sqrt{\pi}}{15}$

Solution:

$$\Gamma\left(-n+\frac{1}{2}\right) = \frac{(-2)^n}{1.3.5\cdots(2n-1)}\sqrt{\pi}$$

$$\Gamma\left(-\frac{5}{2}\right) = \Gamma\left(\frac{-6+1}{2}\right)$$

$$= \Gamma\left(-3+\frac{1}{2}\right) = \frac{-8}{15}\sqrt{\pi}$$

(Option D)

10. Evaluate $\int_0^\infty e^{-x} x^4 \, dx$.

(A) 1 (B) 24

(C) 1/2 (D) $\frac{-8\sqrt{\pi}}{3}$

Solution

$$\Gamma(n) = \int_0^\infty e^{-x} x^{n-1} \, dx$$

$$\int_0^\infty e^{-x} x^4 \, dx = \int_0^\infty e^{-x} x^{5-1} \, dx = \Gamma(5) = 4! = 24$$

(Option B)

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SRM Institute of Science and Technology

Ramapuram campus

Department of Mathematics

18MAB101T – Calculus and Linear Algebra

Year/Sem: I/I **Part – A** **Branch: Common to All**

Unit – V

Sequence and Series

1.	A sequence $\{a_n\}$ is said to be convergent if		**1 Mark**
	(a) $\lim_{n\to\infty} a_n = finite$ (b) $\lim_{n\to\infty} a_n = \infty$ (c) $\lim_{n\to\infty} a_n = -\infty$ (d) $\lim_{n\to\infty} a_n = infinite$	Ans (a)	**(CLO 5, Remember)**
2.	The sequence $\{(-1)^n\}$ is		**1 Mark**
	(a) oscillatory (b) monotonic (c) divergent to $-\infty$ (d) divergent to $+\infty$	Ans (a)	**(CLO 5, Remember)**
3.	A sequence which is monotonic and bounded is		**1 Mark**
	(a) conditionally convergent (b) absolutely convergent (c) convergent (d) divergent	Ans (c)	**(CLO 5, Remember)**
4.	The necessary condition for the convergence of $\sum u_n$ is		**1 Mark**
	(a) $\lim_{n\to\infty} u_n = 0$ (b) $\lim_{n\to\infty} u_n = \infty$ (c) $\lim_{n\to\infty} u_n = -\infty$ (d) $\lim_{n\to\infty} u_n \neq 0$	Ans (a)	**(CLO 5, Remember)**
5.	If $\{a_n\}$ and $\{b_n\}$ are two convergent sequences, then $\{a_n + b_n\}$ is		**1 Mark**
	(a) convergent (b) divergent (c) oscillatory (d) neither convergent nor divergent	Ans (a)	**(CLO 5, Remember)**
6.	The geometric series $1 + x + x^2 + x^3 + \cdots$ converges if		**1 Mark**
	(a) $-1 < x < 1$ (b) $x < 1$ (c) $x > 1$ (d) $x \geq 1$	Ans (a)	**(CLO 5, Remember)**
7.	The series $\frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \cdots$ converges if		**1 Mark**
	(a) $p > 0$ (b) $p < 1$ (c) $p > 1$ (d) $p \leq 1$	Ans (c)	**(CLO 5, Remember)**

8.	The series $\frac{1}{1^p}+\frac{1}{2^p}+\frac{1}{3^p}+\cdots$ diverges if	**1 Mark**	
	(a) $p>0$ (b) $p<1$ (c) $p>1$ (d) $p\le 1$	Ans (d)	**(CLO 5, Remember)**
9.	If $\sum u_n$ is a convergent series, then $\lim_{n\to\infty} u_n =$	**1 Mark**	
	(a) 1 (b) ± 1 (c) 0 (d) ∞	Ans (c)	**(CLO 5, Remember)**
10.	According to D' Alembert's ratio test, if $\sum u_n$ is a series of positive terms and $\lim_{n\to\infty}\frac{u_{n+1}}{u_n}=l$, then $\sum u_n$ converges if	**1 Mark**	
	(a) $l<1$ (b) $l>1$ (c) $l=1$ (d) $-1<l<1$	Ans (a)	**(CLO 5, Remember)**
11.	By Cauchy's root test, if $\sum u_n$ is a series of positive terms and $\lim_{n\to\infty}(u_n)^{\frac{1}{n}}=l$, then $\sum u_n$ converges if	**1 Mark**	
	(a) $l<1$ (b) $l>1$ (c) $l=1$ (d) $-1<l<1$	Ans (a)	**(CLO 5, Remember)**
12.	By Raabe's test, if $\lim\limits_{n\to\infty} n\left(\frac{u_n}{u_{n+1}}-1\right)=k$, then $\sum u_n$ diverges if	**1 Mark**	
	(a) $k<1$ (b) $k>1$ (c) $k=1$ (d) $-1<k<1$	Ans (a)	**(CLO 5, Remember)**
13.	By Logarithmic test, if $\lim\limits_{n\to\infty} n\log\frac{u_n}{u_{n+1}}=l$, then $\sum u_n$ converges if	**1 Mark**	
	(a) $l<1$ (b) $l>1$ (c) $l=1$ (d) $-1<l<1$	Ans (b)	**(CLO 5, Remember)**
14.	The series $\sum u_n$ containing positive and negative terms is __________, if $\sum\lvert u_n\rvert$ is divergent but $\sum u_n$ is convergent.	**1 Mark**	
	(a) divergent (b) oscillating finitely (c) oscillating infinitely (d) conditionally convergent	Ans (d)	**(CLO 5, Remember)**
15.	The series $\sum u_n$ containing positive and negative terms is absolutely convergent, if $\sum\lvert u_n\rvert$ is	**1 Mark**	
	(a) convergent (b) divergent to $-\infty$ (c) divergent to $+\infty$ (d) oscillatory	Ans (a)	**(CLO 5, Remember)**

16.	Every absolutely convergent series is necessarily	**1 Mark**	
	(a) divergent (b) convergent (c) oscillatory (d) conditionally convergent	Ans (b)	**(CLO 5, Remember)**
17.	The series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if	**1 Mark**	
	(a) $p = 0$ (b) $p = 1$ (c) $p > 1$ (d) $p < 1$	Ans (c)	**(CLO 5, Remember)**
18.	As per D' Alembert's ratio test, if $\sum u_n$ is a series of positive terms and $\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = l$, then the series is divergent if	**1 Mark**	
	(a) $l = 0$ (b) $l = 1$ (c) $l > 1$ (d) $l < 1$	Ans (c)	**(CLO 5, Remember)**
19.	A series of positive terms cannot ____________.	**1 Mark**	
	(a) oscillate (b) absolutely converge (c) converge (d) diverge	Ans (a)	**(CLO 5, Remember)**
20.	The series $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\ldots$ is	**1 Mark**	
	(a) divergent (b) conditionally convergent (c) oscillatory (d) neither convergent nor divergent	Ans (b)	**(CLO 5, Apply)**
21.	The series $1-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\ldots$ is	**1 Mark**	
	(a) divergent (b) absolutely convergent (c) oscillatory (d) neither convergent nor divergent	Ans (b)	**(CLO 5, Apply)**
22.	By Raabe's test, if $\lim_{n\to\infty} n\left(\frac{u_n}{u_{n+1}}-1\right) = k$, then $\sum u_n$ converges if	**1 Mark**	
	(a) $k < 1$ (b) $k > 1$ (c) $k = 1$ (d) $-1 < k < 1$	Ans (b)	**(CLO 5, Remember)**

23.	By Cauchy's root test, if $\sum u_n$ is a series of positive terms and $\lim_{n\to\infty}(u_n)^{\frac{1}{n}} = l$, then $\sum u_n$ diverges if	**1 Mark**	
	(a) $l < 1$ (b) $l > 1$ (c) $l = 1$ (d) $-1 < l < 1$	Ans (b)	**(CLO 5, Remember)**
24.	By Logarithmic test, if $\lim_{n\to\infty} n\log\frac{u_n}{u_{n+1}} = l$, then $\sum u_n$ diverges if	**1 Mark**	
	(a) $l < 1$ (b) $l > 1$ (c) $l = 1$ (d) $-1 < l < 1$	Ans (a)	**(CLO 5, Remember)**
25.	If $-1 < x < 1$, then the geometric series $1 + x + x^2 + x^3 + \cdots$ converges to	**1 Mark**	
	(a) $\frac{1}{1-x}$ (b) $\frac{1}{1+x}$ (c) e^x (d) $\frac{1}{x!}$	Ans (a)	**(CLO 5, Remember)**
26.	The geometric series $1 + x + x^2 + x^3 + \cdots$ oscillates finitely if	**1 Mark**	
	(a) $x = -1$ (b) $x < 1$ (c) $x > 1$ (d) $x \geq 1$	Ans (a)	**(CLO 5, Remember)**
27.	The geometric series $1 + x + x^2 + x^3 + \cdots$ oscillates infinitely if	**1 Mark**	
	(a) $x < -1$ (b) $x < 1$ (c) $x > 1$ (d) $x \geq 1$	Ans (a)	**(CLO 5, Remember)**
28.	If $\sum u_n$ is convergent, then $\sum k u_n$ (where k is constant) is	**1 Mark**	
	(a) convergent (b) divergent (c) oscillatory (d) neither convergent nor divergent	Ans (a)	**(CLO 5, Remember)**
29.	The series $1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + \cdots$ is	**1 Mark**	
	(a) divergent (b) neither convergent nor divergent (c) oscillatory (d) conditionally convergent	Ans (d)	**(CLO 5, Apply)**
30.	The convergence of the series $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$ is tested by	**1 Mark**	
	(a) Ratio test (b) Raabe's test (c) Leibnitz test (d) Cauchy Root test	Ans (c)	**(CLO 5, Remember)**

31.	A monotonic increasing sequence which is not bounded above is ______.	**1 Mark**	
	(a) oscillatory (b) convergent (c) divergent to $-\infty$ (d) divergent to $+\infty$	Ans (d)	**(CLO 5, Remember)**
32.	A monotonic decreasing sequence which is not bounded below is ______.	**1 Mark**	
	(a) oscillatory (b) convergent (c) divergent to $-\infty$ (d) divergent to $+\infty$	Ans (c)	**(CLO 5, Remember)**
33.	The series $\sum u_n$ of positive terms is convergent if	**1 Mark**	
	(a) $\lim\limits_{n\to\infty} \frac{u_{n+1}}{u_n} < 1$ (b) $\lim\limits_{n\to\infty} \frac{u_n}{u_{n+1}} = 1$ (c) $\lim\limits_{n\to\infty} \frac{u_n}{u_{n+1}} \le 1$ (d) $\lim\limits_{n\to\infty} \frac{u_{n+1}}{u_n} > 1$	Ans (a)	**(CLO 5, Remember)**
34.	The *n*th term of a series in Arithmetic Progression is	**1 Mark**	
	(a) $t_n = a-(n-1)d$ (b) $t_n = a+(n+1)d$ (c) $t_n = a-(n+1)d$ (d) $t_n = a+(n-1)d$	Ans (d)	**(CLO 5, Remember)**
35.	$\sum_{1}^{\infty} \frac{n^3}{3^n}$ is	**1 Mark**	
	(a) oscillatory (b) convergent (c) divergent to $-\infty$ (d) divergent to $+\infty$	Ans (b)	**(CLO 5, Apply)**
36.	The series $\sum \frac{1}{n} \sin\left(\frac{1}{n}\right)$ is	**1 Mark**	
	(a) oscillatory (b) convergent (c) divergent (d) conditionally convergent	Ans (b)	**(CLO 5, Apply)**
37.	If D'Alembert's ratio test fails, then use	**1 Mark**	
	(a) Comparison test (b) Leibnitz's test (c) Cauchy's integral test (d) Raabe's test	Ans (d)	**(CLO 5, Remember)**
38.	The series $\sum \frac{1}{n!}$ is	**1 Mark**	
	(a) oscillatory (b) convergent (c) divergent (d) conditionally convergent	Ans (b)	**(CLO 5, Apply)**

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Unit – V

SEQUENCE AND SERIES

Part – B

1. The sequence $\left\{\frac{1}{n}\right\}$ converges to _________.

(A) 0 **(B) 1** **(C) ½** **(D) ∞**

Solution:

$$a_n = \frac{1}{n}$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{1}{n} = 0$$

$\{a_n\}$ converges to 0. **(Option A)**

2. The sequence $\left\{\frac{n+1}{2n+3}\right\}$ converges to _________.

(A) 0 **(B) 1** **(C) ½** **(D) ∞**

Solution:

$$a_n = \frac{n+1}{2n+3}$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n\left(1+\frac{1}{n}\right)}{n\left(2+\frac{3}{n}\right)} = \frac{1}{2}$$

$\{a_n\}$ converges to ½ . **(Option C)**

3. Test the convergence of the series $\sum \frac{1}{\sqrt{n+1}}$ **.**

(A) converges **(B) diverges**
(C) oscillates finitely **(D) oscillates infinitely**

Solution:

$$u_n = \frac{1}{\sqrt{n+1}} = \frac{1}{\sqrt{n\left(1+\frac{1}{n}\right)}}$$

Let $v_n = \frac{1}{\sqrt{n}}$

Now $\frac{u_n}{v_n} = \frac{1}{\sqrt{1+\frac{1}{n}}}$

$$\lim_{n\to\infty} \frac{u_n}{v_n} = 1$$

$\sum v_n = \sum \frac{1}{\sqrt{n}} = \sum \frac{1}{n^{1/2}}$ is divergent.

Hence by comparison test, $\Sigma\, u_n$ is divergent. **(Option B)**

4. Test the convergence of the series $1+\frac{1}{3}+\frac{1}{5}+\cdots$ **.**

(A) converges **(B) diverges**
(C) oscillates finitely **(D) oscillates infinitely**

Solution:

$$u_n = \frac{1}{2n-1} = \frac{1}{n\left(2-\frac{1}{n}\right)}$$

Let $v_n = \frac{1}{n}$

Now $\frac{u_n}{v_n} = \frac{1}{2-\frac{1}{n}}$

$$\lim_{n\to\infty} \frac{u_n}{v_n} = \frac{1}{2}$$

$\sum v_n = \sum \frac{1}{n}$ is divergent.

Hence by comparison test, $\Sigma\, u_n$ is divergent. **(Option B)**

5. Test the convergence of the series $\sum \frac{x^n}{n\,!}$ where $x > 0$.

(A) converges **(B) diverges**
(C) oscillates finitely **(D) oscillates infinitely**

Solution:

$$u_n = \frac{x^n}{n\,!}, \quad u_{n+1} = \frac{x^{n+1}}{(n+1)\,!}$$

Now $\frac{u_{n+1}}{u_n} = \frac{x}{n+1} = \frac{x}{n\left(1+\frac{1}{n}\right)}$

$$\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = \lim_{n\to\infty} \frac{x}{n\left(1+\frac{1}{n}\right)} = 0 < 1$$

Hence by Ratio test, $\Sigma\, u_n$ is convergent. **(Option A)**

6. Test the convergence of the series $\sum \frac{n\,!}{n^n}$.

(A) converges **(B) diverges**
(C) oscillates finitely **(D) oscillates infinitely**

Solution:

$$u_n = \frac{n\,!}{n^n}, \quad u_{n+1} = \frac{(n+1)\,!}{(n+1)^{n+1}}$$

Now $\frac{u_{n+1}}{u_n} = \frac{1}{\left(1+\frac{1}{n}\right)^n}$

$$\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = \frac{1}{e} < 1$$

Hence by Ratio test, $\Sigma\, u_n$ is convergent. **(Option A)**

7. The series $1 + \frac{x}{1!} + \frac{x^2}{2!} + \cdots$ is ________________.

(A) absolutely convergent **(B) diverges to + ∞**
(C) oscillates finitely **(D) oscillates infinitely**

Solution:

$$u_n = \frac{x^{n-1}}{(n-1)\,!}, \quad u_{n+1} = \frac{x^n}{n\,!}$$

Now $\dfrac{u_{n+1}}{u_n} = \dfrac{x}{n}$

$\lim\limits_{n\to\infty}\left|\dfrac{u_{n+1}}{u_n}\right| = 0 < 1$

Hence the series is absolutely convergent. **(Option A)**

8. Test the convergence of the series $\sum \dfrac{n^3}{3^n}$.

(A) converges **(B) diverges**
(C) oscillates finitely **(D) oscillates infinitely**

Solution:

$$u_n = \frac{n^3}{3^n}, \quad (u_n)^{1/n} = \frac{(n^{1/n})^3}{3}$$

$$\lim_{n\to\infty}(u_n)^{1/n} = \frac{1}{3} < 1$$

Hence by Root test, $\Sigma\, u_n$ is convergent. **(Option A)**

9. Test the convergence of the series $\sum \dfrac{3^n\, n\,!}{n^n}$.

(A) converges **(B) diverges**
(C) oscillates finitely **(D) oscillates infinitely**

Solution:

$$u_n = \frac{3^n\, n\,!}{n^n}, \quad u_{n+1} = \frac{3^{n+1}\,(n+1)\,!}{(n+1)^{n+1}}$$

Now $\dfrac{u_{n+1}}{u_n} = \dfrac{3}{\left(1+\dfrac{1}{n}\right)^n}$

$$\lim_{n\to\infty}\frac{u_{n+1}}{u_n} = \frac{3}{e} > 1$$

Hence by Ratio test, $\Sigma\, u_n$ is divergent. **(Option B)**

10. Test the convergence of the series $\sum \dfrac{1}{n^2}$.

(A) converges **(B) diverges**
(C) oscillates finitely **(D) oscillates infinitely**

Solution:

By Harmonic Series test or p-test, $\sum \dfrac{1}{n^2}$ converges. **Option (A)**

* * * * *

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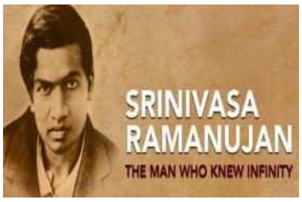
18MAB101T Calculus and Linear Algebra

MODULE – III



Tutorial Sheet -1

S.No.	Questions	Answers
	Part - A	
1.	Solve $(D^2+1)y=0$ given $y(0)=0$ and $y'(0)=1$	$y=\sin x$
2.	Solve $(D^2+5D+6)y=0$ given $y(0)=0$ and $y'(0)=15$	$y=15(e^{-2x}-e^{-3x})$
3.	Solve $(D^2-6D+9)y=0$ given $y(0)=2$ and $y'(0)=8$	$y=2(1+x)e^{3x}$
4.	Find PI of $(D^2+D+1)y=(1-e^x)^2$	$1-\frac{2}{3}e^x+\frac{e^{2x}}{7}$
5.	Solve $(D^2+4D+5)y=-2\cosh x$	$y=e^{-2x}(A\cos x+B\sin x)-\frac{e^x}{10}-\frac{e^{-x}}{2}$
	Part - B	
6.	Solve $(D^2+D)y=x^2+2x+4$	$y=A+Be^{-x}+\frac{x^3}{3}+4x$
7.	Solve $(D^2-3D+2)y=xe^{3x}$	$y=Ae^x+Be^{2x}+e^{3x}\left(\frac{x}{2}-\frac{3}{4}\right)$
8.	Solve $(D^2+4)y=x\sin x$	$y=A\cos 2x+B\sin 2x)+\frac{x}{3}\sin x-\frac{2}{9}\cos x$
9.	Solve $(D^2-4D+3)y=e^x\cos 2x$	$y=Ae^x+Be^{3x}-\frac{e^x}{8}(\sin 2x+\cos 2x)$
10.	Solve $(D^2+2D+1)y=\frac{e^{-x}}{x^2}$	$y=e^{-x}(A+Bx-\ln x)$

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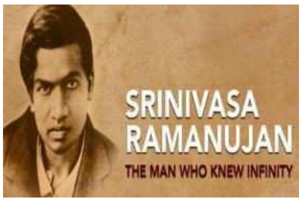
18MAB101T Calculus and Linear Algebra

MODULE – III



Tutorial Sheet -1

S.No.	Questions	Answers
	Part - A	
1.	Solve $(x^2D^2 + xD + 1)y = 0$	$y = A\cos(\log x) + B\sin(\log x)$
2.	Solve $(x^2D^2 - xD + 1)y = 0$	$y = (A\log x + B)x$
3.	Solve $(x^2D^2 - 20)y = 0$	$y = Ax^{-4} + Bx^5$
4.	Solve $((x+2)^2D^2 + 3(x+2)D - 3)y = 0$	$y = A(x+2) + B(x+2)^{-3}$
5.	Solve $((2x-1)^2D^2 + (2x-1)D - 2)y = 0$	$y = A(2x-1) + B(2x-1)^{-1/2}$
	Part - B	
6.	Solve $x^2y'' - xy' + y = \log x$	$y = (A\log x + B)x + \log x + 2$
7.	Solve $x^2y'' + 4xy' + 2y = e^x$	$y = Ax^{-1} + x^{-2}(e^x + B)$
8.	Solve $(x^2D^2 - 3xD + 4)y = x^2\cos(\log x)$	$y = (A\log x + B)x^2 - x^2\cos(\log x)$
9.	Solve $((x-2)^2D^2 - 3(x-2)D + 4)y = x$	$y = (x-2)^2(A\log(x-2) + B) + x - \frac{3}{2}$
10.	Solve $((1+x)^2D^2 + (1+x)D + 1)y = 2\sin(\log(1+x))$	$y = A\cos(\log(1+x)) + B\sin(\log(1+x)) - \log(1+x)\cos(\log(1+x))$

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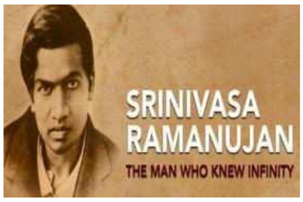
18MAB101T Calculus and Linear Algebra



MODULE - III

Tutorial Sheet -3

S.NO	Tutorial Sheet -3	Answers
1	**Solve** $\left(D^2+a^2\right)y=\tan ax$ **by the method of variation of parameter**	$y=(c_1\cos ax+c_2\sin ax)$ $-\frac{1}{a^2}\cos ax\log\left[\sec ax+\tan ax\right]$
2	**Solve** $\left(D^2+1\right)y=\sec ax$ **by the method of variation of parameter**	$y=(c_1\cos x+c_2\sin x)$ $-\cos x\log\left(\cos x\right)+x\sin x$
3	**Solve** $\left(D^2+1\right)y=\cos ecx$ **by the method of variation of parameter**	$y=(c_1\cos x+c_2\sin x)$ $+\sin x\log\left(\sin x\right)-x\cos x$
4	**Solve** $\left(D^2+2D+5\right)y=e^{-x}\tan x$ **by the method of variation of parameter**	$y=e^{-x}(c_1\cos 2x+c_2\sin 2x)$ $+\left[-\frac{1}{2}x+\frac{\sin 2x}{4}\right]e^{-x}\cos 2x$ $+\left[-\frac{-\cos 2x}{2}+\frac{1}{2}\log\left(\cos x\right)\right]e^{-x}\sin 2x$
5	**Solve** $\frac{dx}{dt}-y=0;\frac{dy}{dt}+x=0$	$x=A\cos t+B\sin t$ $y=-A\sin t+B\cos t$
6	**Solve** $\frac{dx}{dt}+y=e^t;x-\frac{dy}{dt}=t$	$x=-A\sin t+B\cos t+\frac{1}{2}e^t+t$ $y=A\cos t+B\sin t+\frac{1}{2}e^t-1$
7	$\frac{dx}{dt}+2x-3y=t;\frac{dy}{dt}-3x+2y=e^{2t}$	$x=Ae^t-Be^{-5t}-\frac{2}{5}t+\frac{3}{7}e^{2t}-\frac{13}{25}$ $x=Ae^t+Be^{-5t}-\frac{3}{5}t+\frac{4}{7}e^{2t}-\frac{12}{25}$

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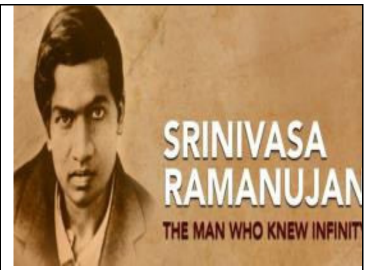
DEPARTMENT OF MATHEMATICS

18MAB201T- TRANSFORMS AND BOUNDARY VALUE PROBLEMS

UNIT - I Partial Differential Equations Tutorial Sheet - 1



Sl. No.	Questions	Answer
	Part - A	
1	**Form the PDE by eliminating arbitrary constants 'a' and 'b' from** $(x-a)^2+(x-b)^2+z^2=c^2$	$(p^2+q^2+1)z^2=c^2$
2	**Form the PDE by eliminating arbitrary constants 'a' and 'b' from** $\log(az-1)=x+ay+b$	$p=q(z-p)$
3	**Eliminate the arbitrary function 'f' from** $z=f(x^2+y^2)$	$py=qx$
4	**Solve** $\sqrt{p}+\sqrt{q}=1$	$z=ax+(1-\sqrt{a})^2y+c$
5	**Solve the equation** $pq+p+q=0$	$z=ax-\dfrac{a}{a+1}y+c$
	Part - B	
6	**Form the PDE by eliminating 'f' from** $f(x^2+y^2+z^2, xyz)=0$	$x(y^2-z^2)p+y(z^2-x^2)q$ $=z(x^2-y^2)$
7	**Form the PDE by eliminating 'f' from** $z=xy+f(x^2+y^2+z^2)$	$p(y+xz)-q(x+yz)$ $=y^2-x^2$
8	**Form the PDE by eliminating 'f' from** $xyz=f(x+y+z)$	$x(y-z)p+y(z-x)q$ $=z(x-y)$
9	**Form the PDE by eliminating 'f' and 'g' from** $z=f(x+ct)+g(x-ct)$	$q^2=c^2p^2$
10	**Obtain the PDE by eliminating 'a' , 'b' and 'c' from** $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}+\dfrac{z^2}{c^2}=1$	$zs+pq=0$

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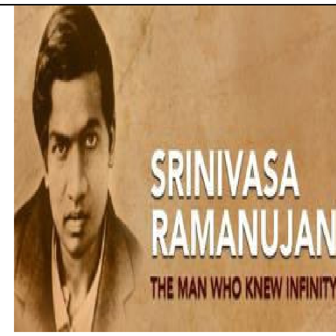
18MAB201T- TRANSFORMS AND BOUNDARY VALUE PROBLEMS

UNIT - I Partial Differential Equations
Tutorial Sheet - 2



Sl. No.	Questions	Answer
Part - A		
1	**Find the singular integral of the PDE** $z = px + qy + p^2 - q^2$	$4z = y^2 - x^2$
2	**Find the complete integral of the PDE** $\sqrt{p} + \sqrt{q} = \sqrt{x}$	$z = \frac{x^2}{2} + ax - \frac{4\sqrt{a}}{3} x^{\frac{3}{2}} + ay + c$
3	**Solve** $yp = 2xy + \log q$	$z = x^2 + ax + \frac{e^{ay}}{a} + c$
4	**Find the complete integral of** $p + q = \sin x + \sin y$	$z = ax - \cos x - \cos y - ay + c$
5	**Solve** $p \tan x + q \tan y = \tan z$	$\phi\left(\frac{\sin x}{\sin y}, \frac{\sin y}{\sin z}\right) = 0$
Part - B		
6	**Solve** $(3z - 4y)p + (4x - 2z)q = 2y - 3x$	$\phi\left(x^2 + y^2 + z^2, 2x + 3y + 4z\right) = 0$
7	**Solve** $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$	$\phi\left(\frac{x - y}{y - z}, xy + yz + zx\right) = 0$
8	**Solve** $(2z - y)p + (x + z)q + 2x + y = 0$	$\phi\left(x^2 + y^2 + z^2, z + 2y - x\right) = 0$
9	**Solve** $(y + z)p + (z + x)q = x + y$	$\phi\left(\frac{x - y}{y - z}, (x + y + z)(x - y)^2\right) = 0$
10	**Solve** $x^2(y - z)p + y^2(z - x)q = z^2(x - y)$	$\phi\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}, xyz\right) = 0$

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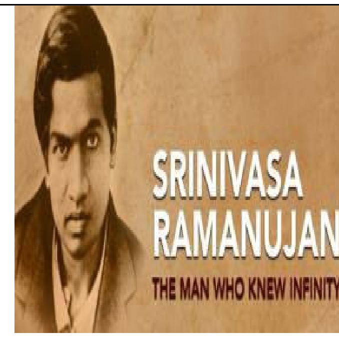
18MAB201T- TRANSFORMS AND BOUNDARY VALUE PROBLEMS

UNIT - I Partial Differential Equations
Tutorial Sheet - 3



Sl. No.	Questions	Answer
Part - A		
1	**Solve** $(D^2 - 3DD' + 2D'^2)z = 0.$	$z = \phi_1(y + x) + \phi_2(y + 2x)$
2	**Solve** $(D^2 - 4DD' + 4D'^2)z = e^{2x+y}.$	$z = \phi_1(y + 2x) + x\phi_2(y + 2x) + \frac{x^2}{2}e^{2x+y}$
3	**Solve** $(D^3 - 2D^2D')z = 4\sin(x + y).$	$z = \phi_1(y) + x\phi_2(y) + \phi_3(y + 2x) - 4\cos(x + y)$
4	**Solve** $(D^2 - 6DD' + 5D'^2)z = xy.$	$z = \phi_1(y + x) + \phi_2(y + 5x) + \frac{x^3 y}{6} + \frac{x^4}{4}$
5	**Solve** $(D^2 - DD')z = \sin x \sin 2y.$	$z = \phi_1(y) + \phi_2(y + x)$ $-\frac{1}{3}\left(2\cos x\cos 2y - \sin x\sin 2y\right)$
Part - B		
6	**Solve** $(D^2 + 2DD' + D'^2)z = 2\cos y - x\sin y.$	$z = \phi_1(y - x) + x\phi_2(y - x)$ $+ x\sin y + 2\cos y$
7	**Solve** $(D^3 + D^2D' - DD'^2 - D'^3)z = e^x\cos(2y).$	$z = \phi_1(y - x) + x\phi_2(y - x) + \phi_3(y + x)$ $+\frac{e^x}{25}\left(\cos 2y + 2\sin 2y\right)$
8	**Solve** $(D^3 - 2D^2D')z = \sin(x + 2y) + 3x^2 y.$	$z = \phi_1(y) + x\phi_2(y) + \phi_3(y + 2x)$ $-\frac{1}{3}\cos(x + 2y) + \frac{x^5 y}{20} + \frac{x^6}{60}$
9	**Solve** $(D^2 + DD' - 6D'^2)z = y\cos x.$	$z = \phi_1(y + 2x) + \phi_2(y - 3x)$ $+ \sin x - y\cos x$
10	**Solve** $(D^2 - 3DD' + 2D'^2)z = (2 + 4x)e^{x+2y}.$	$z = \phi_1(y + x) + \phi_2(y + 2x)$ $+\frac{2}{9}e^{x+2y}(11 + 6x)$

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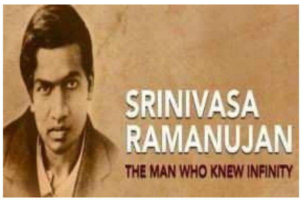
18MAB101T Calculus and Linear Algebra



MODULE –II

Functions of Several Variables

Sl.No.	Tutorial Sheet -3/ Part B	Answers
1	If x = r cosθ , y = r sin θ, Find $\frac{\partial(r,\theta)}{\partial(x,y)}$	$\frac{\partial(r,\theta)}{\partial(x,y)}$ = 1/r
2	If u = x+y, y = uv, Find $\frac{\partial(x,y)}{\partial(u,v)}$	$\frac{\partial(x,y)}{\partial(u,v)}$ = u
3	If x= u(1+v), y = v(1+u) , Find $\frac{\partial(x,y)}{\partial(u,v)}$	$\frac{\partial(x,y)}{\partial(u,v)}$ = 1+u+v
4	If u= x^2 , v = y^2 , Find $\frac{\partial(x,y)}{\partial(u,v)}$	$\frac{\partial(x,y)}{\partial(u,v)}$ = 1/4xy
5	If u = (2x-y)/2 , v = y/2, Find $\frac{\partial(u,v)}{\partial(x,y)}$	$\frac{\partial(u,v)}{\partial(x,y)}$ = ½
	Part C	
6	If u = x^2 -y^2 , v =2xy and x = r cosθ, y = r sin θ, Find $\frac{\partial(u,v)}{\partial(r,\theta)}$	$\frac{\partial(u,v)}{\partial(r,\theta)} = 4\,r^3$
7	If u = $x^2 - 2y$, v = x+y +z , w = x – 2y +3z, find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$	$\frac{\partial(u,v,w)}{\partial(x,y,z)} = 10x + 4$

8	Find the stationary points of the function f(x,y) = $x^3 + y^3 - 12x - 3y + 20$. Also find the nature of stationary points at (2,1) and (-2,-1)	Stationary points =(2,1),(2,-1),(-2,1),(-2,-1) Nature at (2,1) is minimum Nature at (-2,-1) is maximum
9	If u = yz/x , v= zx/y, w = xy/z, Show that $\frac{\partial(u,v,w)}{\partial(x,y,z)} = 4$	Ans :4
10	If u= x+y+z ,uv = y+z, uvw = z, find $\frac{\partial(x,y,z)}{\partial(u,v,w)}$	$\frac{\partial(x,y,z)}{\partial(u,v,w)} = u^2v$

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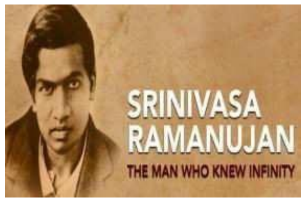
DEPARTMENT OF MATHEMATICS

18MAB101T Calculus and Linear Algebra

MODULE –II

Functions of Several Variables



Sl.No.	Tutorial Sheet -2/ Part B	Answers
1	If f(x,y) = x^y , find the value of f_x , f_y, at (1,1)	f_x=1 , f_y = 0,
2	If f(x,y) = $x^{1/3}+ y^{1/4}$-1, find the value of f_x , f_y, at (1,1)	$f_x = 1/3$, $f_y=1/4$
3	Find the stationary point of the function $f(x,y) = x^3+ y^3- 3axy$	Stationary points: (0,0),(a,a)
4	Find the stationary point of the function $f(x,y) = x^3+ y^3- 12x- 3y + 20$	Stationary points: (2,1),(2,-1),(-2,1),(-2,-1)
5	Find the stationary point of the function $f(x,y) = x^2+ 2y^2- x$	Stationary points (1/2,0)
Part C		
6	If f(x,y) = $tan^{-1}(y/x)$, find the value of f_x , f_y, f_{xx}, f_{xy}, f_{yy} at (1,1)	f_x =-1/2, $f_y = 1/2, f_{xx} = 1/2, f_{xy} = 0, f_{yy} = -1/2$
7	If $f(x,y) = sin(xy)$, find the value of f_x , f_y, f_{xx} at $(1,\pi/2)$	$f_x = 0$, $f_y = 0, f_{xx} = \frac{-\pi^2}{4}$

8	If f(x,y) = $e^x \, cosy$, find the value of f_x , f_y, f_{xx} at $(1,\pi/4)$	$f_x = \frac{e}{\sqrt{2}}$, $f_y = \frac{-1}{\sqrt{2}}, f_{xx} = \frac{e}{\sqrt{2}}$
9	If $f(x,y)$ = $x^2y + 3y - 2$, find the value of f_x , $f_y, f_{xx}, f_{xy}, f_{yy}$ $at\ (1,\text{-}2)$	$f_x = -4$, $f_y = 4, f_{xx} = -4, f_{xy} = 2, f_{yy} = 0$
10	If $f(x,y)$ = $tan^{-1}(xy)$, find the value of f_x , f_y, f_{xx} at $(1,-1)$	f_x = -1/2, $f_y = 1/2, f_{xx} = 1/2$

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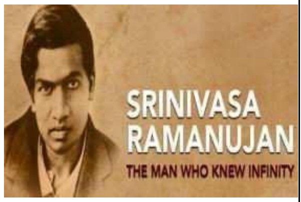
18MAB101T Calculus and Linear Algebra

MODULE –II

Functions of Several Variables



Sl.No.	Tutorial Sheet -1/ Part B	Answers
1	Find $\frac{dy}{dx}$ at (1,1) for 2xy -log xy =2	$\frac{dy}{dx}$ = -1
2	If x = r cos θ , y = r sin θ , Prove that $\frac{\partial r}{\partial x} = \frac{\partial x}{\partial r}$	Ans : cos θ
3	If u = $x^2y^3,\ x = \ log\ t,\ y\ = e^t$ find $\frac{\partial u}{\partial x}, \frac{dx}{dt}, \frac{dy}{dt}, \frac{\partial u}{\partial y}$	$\frac{\partial u}{\partial x} = 2\ log\ t\ .\ e^{3t}.\ ,\frac{dx}{dt} = 1/t, \frac{dy}{dt} = e^t\ , \frac{\partial u}{\partial y} = 3(log\ t)^2 e^{2t}$
4	If u = $x^2 + y^2,\ x = \ e^{2t},\ y\ = e^{2t}$ find $\frac{\partial u}{\partial x}, \frac{dx}{dt}, \frac{dy}{dt}, \frac{\partial u}{\partial y}$	$\frac{\partial u}{\partial x} = 2e^{2t}, \frac{dx}{dt} = 2e^{2t}, \frac{dy}{dt} = 2e^{2t}, \frac{\partial u}{\partial y} = 2e^{2t}$
5	Find the stationary points of $x^2 - xy + y^2 - 2x + y$.	Stationary points are (1,0)
	Part C	
6	A rectangular box, open at the top is to have a volume of 32 cc. Find dimensions of box which requires least amount of material for its construction.	X =4, y= 4, z= 2
7	Find the volume of the largest rectangular parallelopiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$	V = $\frac{8abc}{\sqrt{3}}$

8	Find the minimum value of x^2yz^3 subject to $2x + y + 3z = a$	X = a/6, y = a/6 , z = a/6
9	Find the maximum distance of the point (3,4,12) from the sphere $x^2 + y^2 + z^2 = 1$	Maximum distance = 14
10	Find the dimensions of the rectangular box open at the top, of maximum capacity whose surface is 432 sq.cm	Dimensions are 12,12,6 cm

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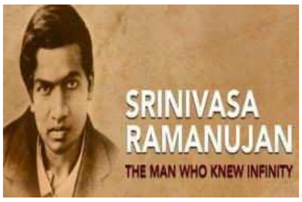
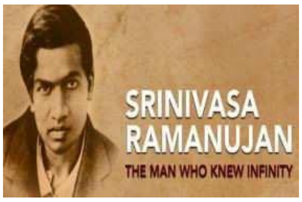
18MAB101T Calculus and Linear Algebra

UNIT –I Matrices



Sl.No.	Tutorial Sheet -1	Answers
	Part – A	
1	If A= $\begin{pmatrix} 3 & 5 & 3 \\ 0 & 4 & 6 \\ 0 & 0 & 1 \end{pmatrix}$, find the eigenvalues of (i) A (ii) A^{-1} (iii) adj A (iv) A^3	(i) 3,4,1 (ii) 1/3, ¼, 1 (iii) 12, 4, 3 (iv) 27, 64, 1
2	Two of the eigenvalues of A= $\begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{pmatrix}$ are 1 and 2. Find the eigenvalues of A^2.	1, 4, 9
3	Find the sum and product of the eigenvalues of the matrix A= $\begin{pmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{pmatrix}$	-1, 45
4	Find the eigenvalues and eigenvectors of the matrix A= $\begin{pmatrix} 4 & 1 \\ 3 & 2 \end{pmatrix}$	1, 5 $\begin{pmatrix} 1 \\ -3 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix}$
5	Find the characteristic equation of A= $\begin{pmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{pmatrix}$	$\lambda^3-11\lambda^2+36\lambda-36=0$
	Part – B	
6	Find the eigenvalues and eigenvectors of $\begin{pmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{pmatrix}$	2, 3, 5 $\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$
7	Find the eigenvalues and eigenvectors of $\begin{pmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{pmatrix}$	1, 1, 7 $\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$
8	Find the eigenvalues and eigenvectors of $\begin{pmatrix} 6 & -6 & 5 \\ 14 & -13 & 10 \\ 7 & -6 & 4 \end{pmatrix}$	-1, -1, -1 $\begin{pmatrix} 0 \\ 5 \\ 6 \end{pmatrix}, \begin{pmatrix} 5 \\ 0 \\ -7 \end{pmatrix}, \begin{pmatrix} 6 \\ 7 \\ 0 \end{pmatrix}$

9	**Find the eigenvalues and eigenvectors of** $\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$	**0, 3, 15** $\begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}, \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$
10	**Find the eigenvalues and eigenvectors of** $\begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$	**8, 2, 2** $\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$

	SRM Institute of Science and Technology Kattankulathur **DEPARTMENT OF MATHEMATICS** **18MAB101T Calculus and Linear Algebra** **UNIT –I Matrices**	
Sl.No.	**Tutorial Sheet -2**	**Answers**
	Part – A	
1	Verify Cayley Hamilton theorem and find A^4 when A=$\begin{bmatrix}1 & 2\\2 & -1\end{bmatrix}$.	$A^4 =\begin{bmatrix}25 & 0\\0 & 25\end{bmatrix}$
2	Two eigen values of the matrix A=$\begin{bmatrix}2 & 2 & 1\\1 & 3 & 1\\1 & 2 & 2\end{bmatrix}$ are equal to 1 each. Find the eigen values of A^{-1}	A= 1 , 1, 5 $A^{-1} = 1, 1, 1/5$
3	The matrix A is $\begin{bmatrix}-1 & 0 & 0\\2 & -3 & 0\\1 & 4 & 2\end{bmatrix}$. Find the eigen values of A^2	A= -1 , 3, 2 $A^2 = 1 , 9 , 4$
4	Verify Cayley Hamilton theorem and find A^{-1} when A=$\begin{bmatrix}3 & 1 & 1\\-1 & 5 & -1\\1 & -1 & 3\end{bmatrix}$.	$A^{-1} = 1/20\begin{bmatrix}7 & -2 & -3\\1 & 4 & 1\\-2 & 2 & 8\end{bmatrix}$
5	Verify Cayley Hamilton theorem and find A^{-1} when A=$\begin{bmatrix}1 & -1 & 1\\0 & 1 & 0\\2 & 0 & 3\end{bmatrix}$	$A^{-1} = \begin{bmatrix}3 & 3 & -1\\0 & 1 & 0\\-2 & -2 & 1\end{bmatrix}$
6	Obtain the matrix $A^6 - 25A^2 + 122A$ where A=$\begin{bmatrix}0 & 0 & 2\\2 & 1 & 0\\-1 & -1 & 3\end{bmatrix}$.	**Ans** : $\begin{bmatrix}-34 & 0 & -20\\-20 & -54 & 0\\10 & 10 & -74\end{bmatrix}$
7	If A =$\begin{bmatrix}1 & 2 & 2\\2 & 1 & 2\\2 & 2 & 1\end{bmatrix}$, Prove that $A^3 - 3A^2 - 9A - 5I = 0$. Hence find A^4 and A^{-1}.	. $A^4 = \begin{bmatrix}209 & 208 & 208\\208 & 209 & 208\\208 & 208 & 209\end{bmatrix}$
8	Diagonalise the matrix A=$\begin{bmatrix}2 & 1 & -1\\1 & 1 & -2\\-1 & -2 & 1\end{bmatrix}$ when	$A^{-1} = \begin{bmatrix}1 & 0 & 0\\0 & -1 & 0\\0 & 0 & 4\end{bmatrix}$

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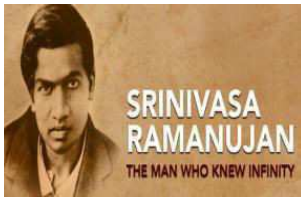
18MAB101T Calculus and Linear Algebra

UNIT –I Matrices



Sl.No.	Tutorial Sheet -3	Answers
Part – A		
1	**Write the Quadratic form $Q=x^2-2y^2+3z^2-4xz+5yz+6xz$ as product of matrices.**	**$Q=X^TAX$** **where $X^T=[x\ y\ z]$** $A=\begin{pmatrix} 1 & -2 & 3 \\ -2 & -2 & 5/2 \\ 3 & 5/2 & 3 \end{pmatrix}$
2	**Write the Q.F where** $A=\begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 9 \\ 3 & 9 & 3 \end{pmatrix}$	**$x^2+4y^2+3z^2+4xy+18yz+6xz$**
3	**Determine the nature of the quadratic form** **(i) $6x^2+3y^2+14z^2+4yz+18xz+4xy$** **(ii) $2xy+2yz-2xz$** **without reducing into canonical form.**	**(i) $D_1=6$, $D_2=14$, $D_3=$ -ve** **Q.F is indefinite.** **(ii) $D_1=0$, $D_2=-1$, $D_3=-2$** **Q.F is indefinite.**
Part – B		
4	**Reduce the quadratic form $Q=3x^2+5y^2+3z^2-2xy-2yz+2xz$ to canonical form and hence find its nature, rank, index and signature.**	$A=\begin{pmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{pmatrix}$ **$\lambda^3-11\lambda^2+36\lambda-36=0$** **$\lambda=2,3,6$** **$Q=2y_1^2+3y_2^2+6y_3^2$** nature=positive definite index=3 signature=3 rank=3
5	**Reduce the quadratic form $Q=x_1^2+2x_2x_3$ to canonical form and hence find its nature, rank, index and signature.**	**$\lambda^3-\lambda^2-\lambda+1=0$** **$\lambda=1,1,-1$** **$Q=y_1^2+y_2^2-y_3^2$** nature=indefinite index=2 signature=1 rank=3
6	**Reduce the quadratic form $Q=x_1^2+2x_2^2+x_3^2-2x_1x_2+2x_2x_3$ to canonical form and hence find its nature, rank, index and signature.**	**$\lambda^3-4\lambda^2+3\lambda=0$** **$\lambda=0,1,3$** **$Q=0y_1^2+y_2^2+3y_3^2$** nature=positive semi definite index=2 signature=2 rank=2

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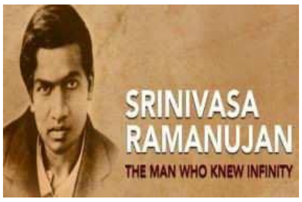
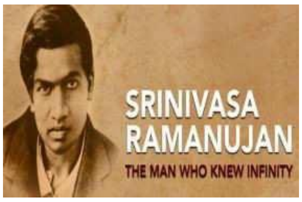
18MAB101T Calculus and Linear Algebra

UNIT –3 Ordinary Differential Equations



Sl.No.	Tutorial Sheet -1	Answers
	Part – A	
1	Solve $\left(D^2-7D+12\right)y=0$	$y=Ae^{3x}+Be^{4x}$
2	Solve $\left(D^2-2D+4\right)y=0$	$y=\left(Ax+B\right)e^{2x}$
3	Solve $\left(3D^2+D-14\right)y=0$	$y=Ae^{-(7/3)x}+Be^{2x}$
4	Solve $\left(D^2+2D+5\right)y=0$	$y=e^{-x}(A\cos 2x+B\sin 2x)$
5	Solve $\left(D^2+16\right)y=0$	$y=(A\cos 4x+B\sin 4x)$
6	Solve $\left(D^2+2D+2\right)y=e^{-2x}+\cos 2x$	$y=e^{-x}(A\cos x+B\sin x)+$ $\frac{1}{2}e^{-2x}+\frac{1}{5}\sin 2x-\frac{1}{10}\cos 2x$
7	Solve $\left(D^2-5D+6\right)y=x^2+3x-1$	$y=Ae^{2x}+Be^{3x}$ $+\frac{1}{6}\left[x^2+\frac{14}{3}x+\frac{26}{9}\right]$
8	Solve $\left(D^2+D+1\right)y=x^2e^{-x}$	$y=e^{-\frac{1}{2}x}(A\cos\frac{\sqrt{3}}{2}x+B\sin\frac{\sqrt{3}}{2}x)$ $+e^{-x}\left(x^2+2x\right)$
9	Solve $\left(D^2+4\right)y=x\sin x$	$y=(A\cos 2x+B\sin 2x)$ $+\frac{x}{3}\sin x-\frac{2}{9}\cos x$
10	$\left(D^2-2D+1\right)y=e^x\sin x$	$y=\left(Ax+B\right)e^x-e^x\sin x$

	SRM Institute of Science and Technology Kattankulathur **DEPARTMENT OF MATHEMATICS** **18MAB101T Calculus and Linear Algebra** **UNIT –III – Ordinary Differential Equations**	
Sl.No.	**Tutorial Sheet -2**	**Answers**
1	**Solve** $\left(x^2D^2-xD+1\right)y=0$	$y=x(A\log x+B)$
2	**Solve** $\left(x^2D^2+4xD+2\right)y=0$	$y=\frac{A}{x}+\frac{B}{x^2}$
3	**Solve** $\left(x^2D^2+1\right)y=0$	$y=\sqrt{x}\left[A\cos\left(\frac{\sqrt{3}}{2}\log x\right)+B\sin\left(\frac{\sqrt{3}}{2}\log x\right)\right]$
4	**Solve** $\left((x+2)^2D^2+4(x+2)D+1\right)y=0$	$y=(A\log(x+2)+B)(x+2)$
5	**Solve** $\left((2x+1)^2D^2-2(2x+1)D-12\right)y=6x+5$	$y=A(2x+1)^3+\frac{B}{2x+1}-\frac{3(2x+1)}{16}-\frac{1}{6}$
6	**Solve** $\left(x^2D^2+xD-9\right)y=\frac{5}{x^2}$	$y=Ax^3+\frac{B}{x^3}-\frac{1}{x^2}$
7	**Solve** $\left(x^2D^2+xD+1\right)y=4\sin\left(\log x\right)$	$y=(A\cos\left(\log x\right)x+B\sin\left(\log x\right))$ $-2\log x\left(\cos\left(\log x\right)\right)$
8	**Solve** $\left(x^2D^2-4xD+6\right)y=x^2+\log x$	$y=(Ax^2+Bx^3)-x^2\log x$ $+\frac{\log x}{6}+\frac{5}{36}$
9	**Solve** $\left(x^2D^2-xD+1\right)y=\frac{\log x}{x}$	$y=x(A\log x+B)+$ $\frac{1}{27x^2}\left[3\left(\log x\right)^2+4\left(\log x\right)+2\right]$

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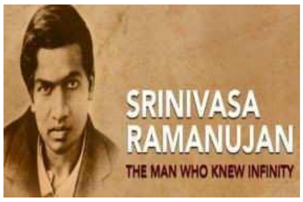
DEPARTMENT OF MATHEMATICS

18MAB101T Calculus and Linear Algebra

UNIT –III – Ordinary Differential Equations



	Tutorial Sheet -3	Answers
1	**Solve** $\left(D^2+a^2\right)y=\tan ax$ **by the method of variation of parameter**	$y=(c_1\cos ax+c_2\sin ax)$ $-\frac{1}{a^2}\cos ax\log\left[\sec ax+\tan ax\right]$
2	**Solve** $\left(D^2+1\right)y=\sec ax$ **by the method of variation of parameter**	$y=(c_1\cos x+c_2\sin x)$ $-\cos x\log\left(\cos x\right)+x\sin x$
3	**Solve** $\left(D^2+1\right)y=\cos ecx$ **by the method of variation of parameter**	$y=(c_1\cos x+c_2\sin x)$ $+\sin x\log\left(\sin x\right)-x\cos x$
4	**Solve** $\left(D^2+2D+5\right)y=e^{-x}\tan x$ **by the method of variation of parameter**	$y=e^{-x}(c_1\cos 2x+c_2\sin 2x)$ $+\left[-\frac{1}{2}x+\frac{\sin 2x}{4}\right]e^{-x}\cos 2x$ $+\left[-\frac{-\cos 2x}{2}+\frac{1}{2}\log\left(\cos x\right)\right]e^{-x}\sin 2x$
5	**Solve** $\frac{dx}{dt}-y=0;\frac{dy}{dt}+x=0$	$x=A\cos t+B\sin t$ $y=-A\sin t+B\cos t$
6	**Solve** $\frac{dx}{dt}+y=e^t;x-\frac{dy}{dt}=t$	$x=-A\sin t+B\cos t+\frac{1}{2}e^t+t$ $y=A\cos t+B\sin t+\frac{1}{2}e^t-1$
7	$\frac{dx}{dt}+2x-3y=t;\frac{dy}{dt}-3x+2y=e^{2t}$	$x=Ae^t-Be^{-5t}-\frac{2}{5}t+\frac{3}{7}e^{2t}-\frac{13}{25}$ $x=Ae^t+Be^{-5t}-\frac{3}{5}t+\frac{4}{7}e^{2t}-\frac{12}{25}$

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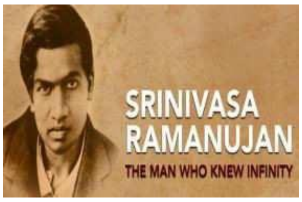
18MAB101T Calculus and Linear Algebra

UNIT - IV



	Tutorial Sheet -1	Answers
1.	Find the radius of the curve $y = e^x$ at (0, 1)	$\rho = 2\sqrt{2}$
2.	Find the radius of curvature at the point $\left(\frac{1}{4}, \frac{1}{4}\right)$ on the curve $\sqrt{x} + \sqrt{y} = 1$.	$\rho = 1/\sqrt{2}$
3.	Show that the radius of curvature at any point of the catenary $y = c\cosh(x/c)$ is y^2/c. Also find ρ at (0, c).	$\rho = C$
4.	Find the radius of curvature at the point (c, c) on the curve $xy = c^2$	$\rho = c\sqrt{2}$
5.	Find ρ at any point $P(at^2, 2at)$ on the parabola $y^2 = 4ax$.	$\rho = 2a(1+t^2)^{3/2}$
6.	Find the radius of curvature at any point $x = a\cos^3\theta, y = a\sin^3\theta$ of the curve $x^{2/3} + y^{2/3} = a^{2/3}$. Also show that $\rho^3 = 27axy$.	$\rho = 3a\sin 2\theta/2$
7.	Show that the radius of curvature at any point of the curve $x = ae^{\theta}(\sin\theta - \cos\theta)$, $y = ae^{\theta}(\sin\theta + \cos\theta)$ is twice the perpendicular distance of the tangent at the point from the origin.	
8.	Prove that the radius of curvature at any point of the cycloid $x = a(\theta + \sin\theta), y = a(1-\cos\theta)$ is $4a\cos\frac{\theta}{2}$.	
9.	Show that the line joining any point θ on $x = a(\theta + \sin\theta), y = a(1-\cos\theta)$ to its centre of curvature is bisected by the line y = 2a.	
10.	Find the circle of curvature of the curvature $\sqrt{x} + \sqrt{y} = \sqrt{a}$ at the point $\left(\frac{a}{4}, \frac{a}{4}\right)$.	$(x - \frac{3a}{4})^2 + (y - \frac{3a}{4})^2 = \frac{a^2}{2}$

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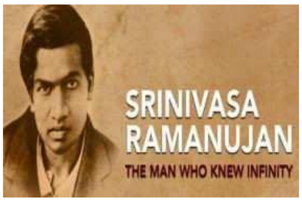
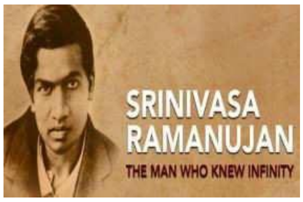
18MAB101T Calculus and Linear Algebra

UNIT - IV



	Tutorial Sheet -2	Answers
1.	State two properties of the evolute of the curve.	
2.	Find the envelope of the family of straight lines $y = mx + am^2$, m being the parameter	Ans: $x^2 + 4ay = 0$
3.	Define envelope of a family of curves.	
4.	Find the envelope of the family of straight lines $x\cos\alpha + y\sin\alpha = a\sec\alpha, \alpha$ being the parameters.	$\boldsymbol{Ans}: \boldsymbol{y^2 - 4a(a - x) = 0}$
5.	Define involutes and evolutes.	
6.	Find the equation of the circle of curvature at (c, c) on $xy = c^2$.	$\boldsymbol{Ans}: (\boldsymbol{x} - \boldsymbol{2c})^2 + (\boldsymbol{y} - \boldsymbol{2c})^2 = (\sqrt{2}\boldsymbol{c})^2$
7.	Find the equation of the evolute of the a) parabola $y^2 = 4ax$; b) ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$; c) hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$; d) rectangular hyperbola $xy = c^2$ and e) curve $x^{2/3} + y^{2/3} = a^{2/3}$.	Ans: a) $27ay^2 = 4(x - 2a)^3$ b) $(ax)^{\frac{2}{3}} + (by)^{\frac{2}{3}} = (a^2 - b^2)^{\frac{2}{3}}$ c) $(ax)^{\frac{2}{3}} - (by)^{\frac{2}{3}} = (a^2 + b^2)^{\frac{2}{3}}$ d) $(x + y)^{\frac{2}{3}} - (x - y)^{\frac{2}{3}} = (4c)^{\frac{2}{3}}$ e) $(x + y)^{\frac{2}{3}} + (x - y)^{\frac{2}{3}} = 2a^{\frac{2}{3}}$
8.	Show that the evolute of the cycloid $x = a(\theta - \sin\theta), y = a(1 - \cos\theta)$ is another equal cycloid.	
9.	Find the evolute of the tractrix $x = a\left(\cos t + \log\tan\left(\frac{t}{2}\right)\right), y = a\sin t$.	Ans: $y = a\cosh\frac{x}{a}$
10.	Show that the evolute of the curve $x = a(\cos\theta + \theta\sin\theta), y = a(\sin\theta - \theta\cos\theta)$ is a circle.	

	SRM Institute of Science and Technology Kattankulathur DEPARTMENT OF MATHEMATICS 18MAB101T Calculus and Linear Algebra UNIT - IV	
	Tutorial Sheet -3	Answers

1.	Show that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$	
2.	Evaluate $\int_0^1 x^6 (1-x)^9 dx$	$\frac{6!\ \ 9!}{16!}$
3.	Evaluate $\int_0^{\pi/2} \sin^6\theta \cos^{10}\theta d\theta$	$\frac{1}{512}\frac{225*63}{8!}\pi$
4	Evaluate $\int_0^{\pi/2} \sqrt{\cot\theta} d\theta$	$\frac{\pi}{\sqrt{2}}$
5.	Evaluate $\int_0^{\infty} e^{-x}\sqrt{x} dx$	$\frac{\sqrt{\pi}}{2}$
6.	Evaluate $\int_0^{\infty} e^{-4x} x^{16} dx$	$\frac{16!}{4^{17}}$
7.	Evaluate $\int_0^1 \frac{dx}{\sqrt{1-x^4}}$	$\frac{\sqrt{\pi}}{2} \frac{\Gamma\left(\frac{1}{4}\right)}{\Gamma\left(\frac{3}{4}\right)}$
8.	Evaluate $\int_0^{\infty} e^{-x^4} x^4 dx$	$\frac{1}{4}\Gamma\left(\frac{5}{4}\right)$

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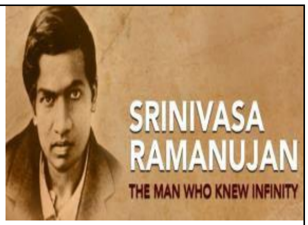
18MAB101T -CALCULUS AND LINEAR ALGEBRA

UNIT V: SEQUENCE & SERIES

Tutorial Sheet -1



Sl.No.	Questions	Answer
	Part – A	
1	Show that the sequence $\left\{\frac{n+1}{2n+7}\right\}$ is convergent.	
2	Examine the nature of the sequence: $\left\{2^n\right\}$	Divergent.
3	Examine the nature of the sequence: $\left\{3+(-1)^n\right\}$	Oscillatory.
4	Test for convergence of the series: $\frac{1}{4.7.10}+\frac{4}{7.10.13}+\frac{9}{10.13.16}+.......\infty$	Divergent.
5	Test for convergence of the series: $\frac{1}{1.2.3}+\frac{3}{2.3.4}+\frac{5}{3.4.5}+.......\infty$	Convergent.
	Part – B	
6	Test for convergence of the series: $\sum_{n=1}^{\infty}\frac{n^3+1}{2^n+1}$	Convergent.
7	Test for convergence of the series: $\frac{x}{1}+\frac{1}{2}.\frac{x^2}{3}+\frac{1.3}{2.4}.\frac{x^3}{5}+\frac{1.3.5}{2.4.6}.\frac{x^5}{7}+.....\infty, x>0$	Convergent for $0<x<1$. Divergent for $x>0$.
8	Test for convergence of the series: $\sum_{n=1}^{\infty}\sqrt{\frac{n}{n+1}}x^n, x>0$.	Convergent for $0<x<1$. Divergent for $x\geq 0$.
9	Test for convergence of the series: $\sum\frac{x^n}{n!}$	Convergent for all x.
10	Test for convergence of the series: $\sum\frac{x^n}{1+x^n}$	Convergent for $0<x<1$. Divergent for $x\geq 0$.

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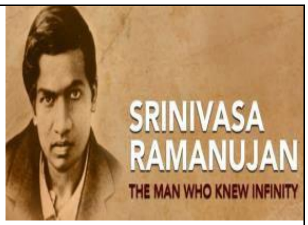
Kattankulathur

DEPARTMENT OF MATHEMATICS

18MAB101T -CALCULUS AND LINEAR ALGEBRA

UNIT V: SEQUENCE & SERIES

Tutorial Sheet -2



Sl.No.	Questions	Answer
	Part – A	
1	Test for convergence of the series: $\sum \frac{n^3}{3^n}$.	Convergent.
2	Test for convergence of the series: $\sum (\log n)^{-2n}$.	Convergent.
3	Test for convergence of the series: $\left(\frac{2^2}{1^2}-\frac{2}{1}\right)^{-1}+\left(\frac{3^3}{2^3}-\frac{3}{2}\right)^{-2}+\left(\frac{4^4}{3^4}-\frac{4}{3}\right)^{-3}+.....\infty$	Convergent.
4	Test for convergence of the series: $\sum \left(\frac{n+1}{2n+7}\right)^n$	Convergent.
5	Test for convergence of the series: $1+\frac{x}{2}+\frac{x^2}{3^2}+\frac{x^3}{4^3}+.....\infty, x>0$	Convergent.
	Part – B	
6	Test for convergence of the series: $\frac{2}{3.4}+\frac{2.4}{3.5.6}+\frac{2.4.6}{3.5.7.8}+\frac{2.4.6.8}{3.5.7.9.10}+.....\infty$	Convergent.
7	Test for convergence of the series: $\frac{3}{4}.\frac{x}{5}+\frac{3.6}{4.7}.\frac{x^2}{3}+\frac{3.6.9}{4.7.10}.\frac{x^3}{11}+....\infty, x>0$	Convergent for $0<x\le 1$. Divergent for $x>0$.
8	Test for convergence of the series: $\sum \frac{1.3.5...(2n-1)}{2.4.6...2n}x^n$.	Convergent for $0<x<1$. Divergent for $x\ge 0$.
9	Test for convergence of the series: $\sum \frac{(n!)^2}{(2n)!}x^n$.	Convergent for $x^2<4$. Divergent for $x^2\ge 4$.
10	Test for convergence of the series: $\frac{x}{1}+\frac{2^2x^2}{2!}+\frac{3^3x^3}{3!}+\frac{4^4x^4}{4!}+.....\infty$	Convergent for $x<\frac{1}{e}$.Divergent for $x\ge\frac{1}{e}$

SRM Institute of Science and Technology

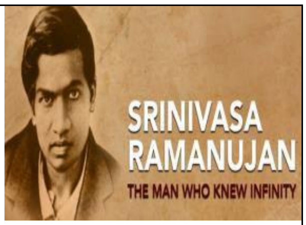
Kattankulathur

DEPARTMENT OF MATHEMATICS

18MAB101T -CALCULUS AND LINEAR ALGEBRA

UNIT V: SEQUENCE & SERIES

Tutorial Sheet -3



Sl.No.	Questions	Answer
	Part – A	
1	Define absolutely convergent with an example.	
2	Define conditionally convergent with an example.	
3	Test for convergence of the series: $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}}$	Convergent.
4	Test for convergence of the series: $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}(2n+3)}{2n}$	Oscillatory
5	Test whether the series is absolutely convergent or not: $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1}$.	Conditionally convergent
	Part – B	
6	Test for convergence of the series: $\sum_{n=2}^{\infty} \frac{(-1)^{n-1} x^n}{n(n-1)}, 0<x<1.$	Convergent.
7	State the values of x for which the series is convergent. $\frac{x}{1}-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+.....\infty.$	$-1<x\le 1$
8	Prove that the exponential series $1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+.....\infty$ is absolutely convergent and convergent for all values of x.	
9	Discuss the convergence of the series $\frac{x}{1+x}-\frac{x^2}{1+x^2}+\frac{x^3}{1+x^3}-.....\infty,$ if $0<x<1.$	Convergent.
10	Prove that the series $\frac{\sin x}{1^3}-\frac{\sin 2x}{2^3}+\frac{\sin 3x}{3^3}-.....\infty$ converges absolutely.	