Unit – 4

★ F – Test

To test the equality of population variance for which the small samples have been drawn.

$$H_0 : \sigma_1^2 = \sigma_2^2$$

If $\sigma_1^2 > \sigma_2^2$ then $F = \dfrac{\sigma_1^2}{\sigma_2^2}$

$$\text{dof} : (n_1 - 1, n_2 - 1) \qquad [(v_1, v_2)]$$

If $\sigma_2^2 > \sigma_1^2$, then $F = \dfrac{\sigma_2^2}{\sigma_1^2}$

$$\text{dof} = (n_2 - 1, n_1 - 1) \qquad [(v_1, v_2)]$$

$$\sigma_1^2 = \frac{n_1 s_1^2}{n_1 - 1}$$

$$\sigma_2^2 = \frac{n_2 s_2^2}{n_2 - 1}$$

Que – A sample of size 13 gave an estimated population variance of 3·0 while another sample of size 15 gave an estimate of 2·5. Could both sample be from population with the same variance.

Soln. $n_1 = 13$ $\qquad$ $n_2 = 15$

$\sigma_1^2 = 3$ $\qquad$ $\sigma_2^2 = 25$

$H_0 : \sigma_1^2 = \sigma_2^2$, $\quad H_1 = \sigma_1^2 \neq \sigma_2^2$

$$F = \frac{\sigma_1^2}{\sigma_2^2} = \frac{3}{2\cdot5} = 1\cdot2$$

$$\text{dof } \gamma = (n_1 - 1, n_2 - 1) = (12, 14)$$

$$\text{Tab } F = 2\cdot53$$

$$|F| < \text{Tab } F$$

Accept H_0 (No difference among variance)

Que – Two random samples gave the following data

	size	mean	variance
sample 1	8	9·6	1·2

sample 2 11 16·5 2·5

can we conclude that two samples have been drawn from the sample normal population ?

Soln. – Given $n_1 = 8$, $\bar{x}_1 = 9.6$, $s_1^2 = 1.2$

$n_2 = 11$, $\bar{x}_2 = 16.5$, $s_2^2 = 2.5$

To test for variance (F Test)

$H_0 : \sigma_1^2 = \sigma_2^2$, $H_1 : \sigma_1^2 \neq \sigma_2^2$

$$\sigma_1^2 = \frac{n_1 s_1^2}{n_1 - 1} = 1.37$$

$$\sigma_2^2 = \frac{n_2 s_2^2}{n_2 - 1} = 2.75$$

$$F = \frac{\sigma_2^2}{\sigma_1^2} = 2.007$$

dof $\gamma = (n_2 - 1, n_1 - 1)$

$= (10, 7)$

Tab F $= 3.65$

$|F| <$ tab F

Accept H_0

No difference among variance

★ To test for mean

$H_0 : \bar{x}_1 = \bar{x}_2$, $H_1 : \bar{x}_1 \neq \bar{x}_2$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \quad \text{where , } S = \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}}$$

$= 1.47$

$= -10.61$

dof : $n_1 + n_2 - 2$

$= 17$

tab t $= 2.11$

$|t| >$ tab t

Reject H_0.

Two samples could not be drawn from same normal pop.

Que – The nicotine contents in two random samples of tobacco are given below :-

sample 1	21	24	25	26	27	
sample 2	22	27	28	30	31	36

can you say that two samples come from same population?

soln. —

x_1	x_1^2	x_2	x_2^2
21	441	22	484
24	576	27	729
25	625	28	784
26	676	30	900
27	729	31	961
		36	1296
$\Sigma =$ 123	3048	174	5151

$$\bar{x}_1 = \frac{\Sigma x_1}{n_1} = \frac{123}{5} = 24.6$$

$$\bar{x}_2 = \frac{\Sigma x_2}{n_2} = \frac{174}{6} = 29$$

$$s_1^2 = \frac{\Sigma x_1^2}{n_1} - \left(\frac{\Sigma x_1}{n_1}\right)^2 = \frac{3048}{5} - (24.6)^2$$

$$= 4.24$$

$$s_2^2 = \frac{\Sigma x_2^2}{n_2} - \left(\frac{\Sigma x_2}{n_2}\right)^2 = \frac{5151}{6} - (29)^2$$

$$= 17.5$$

(i) To check for variance (F-test)

$H_0 : \sigma_1^2 = \sigma_2^2$

$H_1 : \sigma_1^2 \neq \sigma_2^2$

$$\sigma_1^2 = \frac{n_1 s_1^2}{n_1 - 1} = 5.3$$

$$\sigma_2^2 = \frac{n_2 s_2^2}{n_2 - 1} = 21.0$$

$$F = \frac{\sigma_2^2}{\sigma_1^2} = 3.9$$

dof : $(n_2 - 1, n_1 - 1) = (5, 4)$

Tab F $= 6.26$

$|F| <$ tab F

Accept H_0 i.e., no difference among variance.

(ii) To check for mean (t-test)

$H_0 : \bar{x}_1 = \bar{x}_2$

$H_1 : \bar{x}_1 \neq \bar{x}_2$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = -1.96$$

$$s = \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}} =$$

dof : $n_1 + n_2 - 2$

$= 9$

tab t $= 2.26$

$|t| <$ tab t

Accept H_0

Que — Two samples of size 9 and 8 gave the sum of square of deviation from the respective means equal to 160 and 91 respectively. Could both samples be from the population under the same variance.

Soln. — $s_1^2 = \frac{\Sigma x_1^2}{n} - \left(\frac{\Sigma x_1}{n_1}\right)^2 = \frac{\Sigma (x_1 - \bar{x}_1)^2}{n_1}$

$n_1 = 9, \quad n_2 = 8$

$\Sigma (x_1 - \bar{x}_1)^2 = 160 \quad \Rightarrow n_1 s_1^2 = 160$

$\Sigma (x_2 - \bar{x}_2)^2 = 91 \quad \Rightarrow n_2 s_2^2 = 91$

$\sigma_1^2 = \frac{n_1 s_1^2}{n_1 - 1} = \frac{160}{8} = 20$

$\sigma_2^2 = \frac{n_2 s_2^2}{n_2 - 1} = \frac{91}{7} = 13$

$\sigma_1^2 > \sigma_2^2 \qquad F = \frac{\sigma_1^2}{\sigma_2^2}$

$H_0 : \sigma_1^2 = \sigma_2^2$

$H_1 : \sigma_1^2 \neq \sigma_2^2$

$F = \frac{20}{13} = 1.53$

dof : $(n_1 - 1, n_2 - 1) = (8, 7)$

tab F $= 3.73$

tab F $> |F|$

Accept H_0

☆ Chi - Square Test :

1) Goodness of fit : To test whether there's a diff. b/w (abs & expected) frequency.

$\psi^2 = \Sigma \frac{(O-E)^2}{E}$ $\qquad$ O → observed freq, E → expected freq

dof $= n - 1$

Que — From the following data check whether the accidents are uniformly distributed over a week.

Days :	Mon	Tues	Wed	Thus	Fri	Sat
No. of acc.	15	19	13	12	16	15

H_0 : Accidents occur uniformly

Total no. of acc $= 90$

Expected freq $= \frac{90}{6} = 15$ $(n = 6)$

O	E	O − E	$(O-E)^2/E$	$\chi^2 = \sum \frac{(O-E)^2}{E}$
15	15	0	0	
19	15	4	1.06	
13	15	−2	0.26	
12	15	−3	0.6	
16	15	1	0.06	
15	15	0	0	
				1.98

dof $= n-1 = 5$

$\chi^2 = 1.98$

Tab $\chi^2 = ?$

(0.05) 2T

Tab $\chi^2 = 11.07$

$\chi^2 <$ tab χ^2

Accept H_0.

Que – A sample analysis, out of 500 students in exam, 220 students, 170 got 3rd class, 90 got 2nd class and 20 got Ist class. Do these dates commensurate with the result examination in ratio 4: 3: 2 : 1.

H_0 : The observed results commensurate in the general exam result.

Total $= 500$ students

To find expected freq divide 500 in the ratio $4:3:2:1$

$500 \times \frac{4}{10} : 500 \times \frac{3}{10} : 500 \times \frac{2}{10} : 500 \times \frac{1}{10}$

$= 200 : 150 : 100 : 50$

observed	expected	$(O-E)$	$(O-E)^2/E$
220	200	20	2
170	150	20	2.66
90	100	−10	1
20	50	−30	18

$\chi^2 = \sum \frac{(O-E)^2}{E}$

$\chi^2 = 23.66$

dof $= 3$

Tab $\chi^2 = 7.81$

$\chi^2 >$ tab χ^2

Reject H_0

Type -3

Independence of attributes :

To check whether the attributes are dependent or independent.

H_0 : Attributes are independent.

Observed freq table (given)

	Att B_1	Att B_2
Att A_1	a	b
Att A_2	c	d

To find expected freq :

	B_1	B_2
A_1	$\frac{(a+b)(a+c)}{N}$	$\frac{(a+b)(b+d)}{N}$
A_2	$\frac{(a+c)(c+d)}{N}$	$\frac{(b+d)(c+d)}{N}$

$N \rightarrow$ whole total

Qn - From the given data can you say there is a relation between smoking & literate ?

	Smokers	Non- smokers
Lit.	83	57
Illit.	45	68

Literates and smoking habits are indep.

	S	NS	
L	83	57	= 140
IL	45	68	= 113
	128	125	253

Expected freq:

	S	NS
L	$\frac{140 \times 128}{\ldots} = 71$	$\frac{140 \times 125}{\ldots} = 69$

253 253

IL $\frac{113 \times 128}{256} = 57$ $\frac{113 \times 125}{253} = 56$

O	E	$(O-E)$	$(O-E)^2/E$
83	71		
57	69		
45	57		
68	56		

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

$$\text{dof} = (m-1)(n-1)$$
$$= (2-1)(2-1) = 1$$

Qu - Fit a binomial $dist^n$ for the data and also test the goodness of fit.

x	0	1	2	3	4	5	6
f	5	18	28	12	7	6	4

Soln. — To fit a binomial distⁿ

fx : 0 18 56 36 28 30 24

Mean $\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{192}{80} = 2.4$

$(n=6)$ $np = 2.4 \Rightarrow p = 0.4$, $q = 0.6$

Theoretical frequency $= NP(x) = 80\left[{}^nC_x\, p^x q^{n-x}\right]$

x	0	1	2	3	4	5	6
$NP(x)$	3.73	4.9	24.88	22.12	11.06	2.95	0.33

whole no.:- 4 15 25 22 11 3 0

To test the goodness of fit

O	5	18	28	12	7	6	4
E	4	15	25	22	11	3	0

H_0 : The binomial fit for the given data is satisfactory.

Combine 1st and 2nd data and last three data's.

O_i: 23 28 12 17

E_i: 19 25 22 14

O	E	O − E	$\frac{(O-E)^2}{E}$
23	19	4	
28	25	3	
12	22	−10	
17	14	3	
			6.38

$\chi^2 = \Sigma \frac{(O-E)^2}{E}$

$= 6.38$

dof $v = n - k$

$= 4 - 2$

Tab $\chi^2 = 5.99$

$\chi^2 >$ Tab χ^2

Reject H_0

Que — Fit a poisson distⁿ for the data and also test the goodness of fit.

x	0	1	2	3	4	5
f	142	156	69	27	5	1

soln.

$f x$	0	156	138	81	20	5

$$\text{Mean} = \frac{\Sigma f x}{\Sigma f} = \frac{400}{400} = 1 = \lambda$$

$$NP(x) = 400 \frac{e^{-\lambda} \lambda^x}{x!}$$

x	0	1	2	3	4	5
$NP(x)$	147	147	74	25	6	1

(whole no.'s)

Test for goodness of fit

O	142	156	69	27	5	1
E	147	147	74	25	6	1

combine last three

O	142	156	69	33
E	147	147	74	32

O	E	O − E	$\frac{(O-E)^2}{E}$
142	147	−5	
156	147	9	
69	74	5	
33	32	1	
			1.09

$$\chi^2 = \Sigma \frac{(O-E)^2}{E}$$

$= 1.09$

dof $v = n - k = 4 - 2 = 2$

Tab $\chi^2 = 5.99$

$\chi^2 <$ Tab χ^2

Accept H_0

☆ Queuing Theory

1) Input (or arrival pattern)

The no. of arrivals per unit of time has a Poisson distⁿ with mean λ and the time between two consecutive arrivals has a exponential distⁿ with mean $\frac{1}{\lambda}$.

2) Service

no. of customers served per unit of time has poisson distⁿ with mean μ and the inter service time has a exponential distⁿ with mean $1/\mu$.

3) Queue Discipline

(i) FCFS (First come first service) (FIFO)

(ii) LCFS (last come first service)

4) System Capacity

No. of customers in the system may be finite or infinite.

5) Service Channel

Single channel system Multiple channel system.

6) Service stage

(i) Single stage

(ii) multiple stage

Difference eqn.

$P_n(t) \rightarrow$ Probability of n customers in the system at time t.

$$\lambda_{n-1} P_{n-1} - (\lambda_n + \mu_n) P_n + \mu_{n+1} P_{n+1} = 0 \quad \text{for } n > 0$$

$$-\lambda_0 P_0 + \mu_1 P_1 = 0$$

for $n = 0$

Kendall Notation

$(a|b|c : d/e)$

$a \rightarrow$ inter arrival time

$b \rightarrow$ service time

$c \rightarrow$ no. of service channel

$d \rightarrow$ queue size

$e \rightarrow$ queue discipline

Model - 1

Single server Model with infinite capacity

$(M|M|1 : \infty|FCFS)$

1) $P_n = \left(\frac{\lambda}{\mu}\right)^n P_0$ and $P_0 = 1 - \frac{\lambda}{\mu}$

2) Pbty that the system is busy $= 1 - P_0 = \lambda/\mu$

3) Average (expected) no. of customers

(i) in the system

$E(N_s) = \frac{\lambda}{\quad} = L_s$

$\mu - \lambda$

(ii) in the queue

$$E(N_q) = \frac{\lambda^2}{\mu(\mu-\lambda)}$$

4) expected length of non empty que

$$L_n = \frac{\mu}{\mu-\lambda}$$

5) Average waiting time of customer

(i) in system $E(W_s) = \dfrac{1}{\mu-\lambda}$

(ii) in queue $E(W_q) = \dfrac{\lambda}{\mu(\mu-\lambda)}$

6) Pbty of the no. of customer in the system

(i) exceeds k

$$P(n > k) = \left(\frac{\lambda}{\mu}\right)^{k+1}$$

(ii) greater than or equal to k.

$$P(n \geq k) = \left(\frac{\lambda}{\mu}\right)^{k}$$

7) Pbty of waiting time of customer

(i) in the system exceeds t

$$P(W_s > t) = e^{-(\mu-\lambda)t}$$

(ii) in the queue exceeds t

$$P(W_q > t) = \frac{\lambda}{\mu} e^{-(\mu-\lambda)t}$$

8) Pbty density function of waiting time

(i) in system

$$f(w) = (\mu-\lambda)e^{-(\mu-\lambda)w}$$

(ii) in queue

$$g(w) = \begin{cases} \frac{\lambda}{\mu}(\mu-\lambda)e^{-(\mu-\lambda)w} & w > 0 \\ 1 - \frac{\lambda}{\mu} & w = 0 \end{cases}$$

<u>Little Formula</u> (holds good for infinite capacity mode)

1) $L_s = \lambda W_s$ $\qquad$ $\lambda \longrightarrow$ mean arrival rate

2) $L_q = \lambda W_q$

3) $W_s = W_q + \frac{1}{\mu}$

4) $L_s = L_q + \frac{\lambda}{\mu}$

$\mu \to$ mean service rate

$n \to$ no. of customers

Given, $\frac{1}{\lambda} = 12 \Rightarrow \lambda = \frac{1}{12}$

$\frac{1}{\mu} = 4 \Rightarrow \mu = \frac{1}{4}$

(i) Average no. of customers waiting in the system,

$$E(N_s) = \frac{\lambda}{\mu - \lambda} = \frac{1}{2}$$

(ii) $P(W > 0) = 1 - P(W = 0)$

$$= 1 - P_0$$

$$= \frac{\lambda}{\mu} = \frac{1}{3}$$

(iii) $P(W_s > t) = e^{-(\mu - \lambda)t}$

$$P(\underset{\downarrow \, 10}{W_s > t}) = e^{-\frac{1}{6}(10)} = 0.18$$

(iv) P(Phone will be in use)

$$= 1 - P_0$$

$$= 1 - \frac{2}{} = \frac{1}{} \quad \left[P_0 = 1 - \frac{\lambda}{\mu} = 2 \right]$$

(v) The second phone will be installed if

$$E(w_q) > 3$$

$$\frac{\lambda}{\mu(\mu-\lambda)} > 3$$

$$\frac{\lambda_R}{\mu(\mu-\lambda_R)} > 3$$

$$\frac{\lambda_R}{\frac{1}{4}\left(\frac{1}{4}-\lambda_R\right)} > 3$$

$$\lambda_R > \frac{3}{16} - \frac{3}{4}\lambda_R$$

$$\frac{7\lambda_R}{4} > \frac{3}{16}$$

$$\lambda_R > \frac{3}{28}$$

$\therefore$ the arrival state should increase by $\frac{3}{28} - \frac{1}{12} = \frac{1}{42}$

Average length of non empty queue $= \frac{\mu}{\mu-\lambda}$

$$= \frac{3}{2}$$

2) $\frac{1}{\lambda} = 12 \Rightarrow \lambda = \frac{1}{12}$

$\frac{1}{\mu} = 10 \Rightarrow \mu = \frac{1}{10}$

(a) $E(N_s) = \frac{\lambda}{\mu-\lambda} = \frac{1/12}{\frac{1}{10}-\frac{1}{12}} = 5$

$E(N_q) = \frac{\lambda^2}{\mu(\mu-\lambda)} = 4.17$

(b) P(customer straight goes to the barber chair)

$= P$(No customer in system)

$= P_0 = 1 - \frac{\lambda}{\mu} = \frac{1}{6}$

Percentage $= 16.66$

(c) $E(W_s) = \frac{1}{\mu-\lambda} = 60$ min

(e) $E(W_q) = \frac{\lambda}{\mu(\mu-\lambda)} = 50$

(f) $P(W_s > t) = e^{-(\mu-\lambda)t}$

$$P(W_s > 30) = e^{-(\frac{1}{10} - \frac{1}{12})30}$$
$$= e^{-0.5}$$

(d) $E(W_s) > 75$

$$\frac{1}{\mu - \lambda_R} > 75$$

$$\frac{1}{\frac{1}{10} - \lambda_R} > 75$$

$$\lambda_R > \frac{13}{150}$$

Arrival rate must increase by $\frac{13}{150} - \frac{1}{12} = \frac{1}{300}$

g) P(customer has to wait) $= 1 - P_0$
$$= \frac{\lambda}{\mu} = \frac{5}{6}$$

Percentage $= 83.33\%$.

(h) $P(n > K) = \left(\frac{\lambda}{\mu}\right)^{K+1} \Rightarrow P(n>3) = \left(\frac{\lambda}{\mu}\right)^4 = 0.48$

$\lambda = 6/\text{min}$

$\frac{1}{\mu} = 7.5$ sec

$\mu = \frac{1}{7.5}$ per sec

$= \frac{60}{7.5}$ per min

$= 8$ per min

(3) (a) To find the total time required to purchase the ticket and reach the seal

$E(W_s) = \frac{1}{\mu - \lambda} = \frac{1}{2}$

(to purchase the ticket)

Total time = purchase + be seated
$$= \frac{1}{2} + 1.5$$
$$= 2 \text{ min}$$

Yes, he can expect.

(b) P(he will be seated for the start of picture)
= P(total time < 2 min)

$= P(W_s \le 0.5)$

$= 1 - P(W_s > 0.5)$

$= 1 - e^{-(\mu-\lambda)t}$

$= 1 - e^{-2(0.5)} = 1 - e^{-1}$

$= 0.632$

(c) $P(W_s \le t) = 0.99$

$1 - P(W_s > t) = 0.99$

$-P(W_s > t) = -0.01$

$e^{-(\mu-\lambda)t} = 0.01$

$-(\mu-\lambda)t = \log 0.01$

$-2t = -2.3$

$t = 1.15$ min

P(ticket purchasing time < 1.15) = 0.99

P(total time < 1.15 + 1.5) = 0.99

$\therefore$ The person must arrive atleast 2.65 min early so as to 99% sure of seeing the start of picture.

(4) $\lambda = 5$ job/hr

$\frac{1}{\mu} = 6$ min

$\mu = \frac{1}{6}$ per min

$= 10$ per hour

(a) P(machine is idle) $= P(N=0)$

$= P_0 = 1 - \frac{\lambda}{\mu} = \frac{1}{2}$

Percentage = 50%

(b) $E(W_s) = \frac{1}{\mu-\lambda} = \frac{1}{10-5} = \frac{1}{5}$ hr

(c) E(earning per day) = E(no. of jobs per day) x Earning per job

$= \underset{\text{hrs}}{8} \times \underset{\text{jobs}}{5} \times$ time in hr/job x earning / hr

$= 40 \times \frac{1}{5} \times 5 = 40$

Model – 2 (M|M|1 : K | FCFS)

It has finite no. of 'K' arrivals.

$$\mu_K = \mu \quad (n = 1, 2, 3, \ldots)$$

$$\lambda_n = \begin{cases} \lambda, & n = 0, 1, 2, \ldots K-1 \\ 0, & n = K, K+1, \ldots \end{cases}$$

1) $$P_0 = \begin{cases} \dfrac{1 - \lambda/\mu}{1 - \left(\frac{\lambda}{\mu}\right)^{K+1}}, & \lambda \neq \mu \\ \dfrac{1}{K+1}, & \lambda = \mu \end{cases}$$

2) Steady state Probability

$$P_n = \begin{cases} \left(\frac{\lambda}{\mu}\right)^n P_0, & \lambda \neq \mu \\ \dfrac{1}{K+1}, & \lambda = \mu \end{cases}$$

In general, $P_K = \left(\frac{\lambda}{\mu}\right)^n P_0$

i.e., P(system is full) $= P_K = \left(\frac{\lambda}{\mu}\right)^K P_0$

3) Average no. of customers in the system

$$L_s = E(N_s) = \frac{\lambda}{\mu - \lambda} - \frac{(K+1)(\lambda/\mu)^{K+1}}{1 - \left(\frac{\lambda}{\mu}\right)^{K+1}}, \quad \lambda \neq \mu$$

$$= \sum_{n=0}^{K} \frac{n}{K+1} = \frac{K}{2}, \quad \lambda = \mu$$

4) Average no. of customers in queue

$$L_q = E(N_q) = E(N_s) - (1 - P_0)$$

$$= E(N_s) - \left(\frac{\lambda}{\mu}\right)$$

5) Average waiting time in the system & in queue

$$E(W_s) = \frac{E(N_s)}{\lambda}$$

$$E(W_q) = \frac{E(N_q)}{\lambda}$$

Que –

Ans) $k = 5$, $\lambda = 10/hr$

$\frac{1}{\mu} = 2$ min, $\mu = 30/hr$

(i) $E(N_s) = \frac{\lambda}{\mu - \lambda} - \frac{(k+1)(\lambda/\mu)^{k+1}}{1-\left(\frac{\lambda}{\mu}\right)^{k+1}}$

$= 0.492$

(ii) P(system is full)

$= P(k = 5)$

$= \left(\frac{\lambda}{\mu}\right)^k P_0 = 0.00274$

$P_0 = \frac{1 - \lambda/\mu}{1-\left(\frac{\lambda}{\mu}\right)^{k+1}} = 0.667$

Que 2 –

At a railway – – – – – – results ?

Ans. $\lambda = 6/hr$, $\mu = 12/hr$

$k = 2+1 = 3$

(i) To find steady state pbty

$P_n = \left(\frac{\lambda}{\mu}\right)^n P_0 \quad [n = 1, 2, 3, \ldots]$

$P_0 = \frac{1 - \lambda/\mu}{1-\left(\frac{\lambda}{\mu}\right)^{k+1}} = 0.533$

$P_1 = \frac{\lambda}{\mu} P_0 = 0.2667$, $P_2 = \left(\frac{\lambda}{\mu}\right)^2 P_0 = 0.133$

$P_3 = \left(\frac{\lambda}{\mu}\right)^3 P_0 = 0.0667$

(ii) Average waiting time of train coming in yard

$E(W_s) = \frac{E(N_s)}{\lambda}$

Now, $\mu = 6/hr$, $\lambda = 6/hr$

Here, $\lambda = \mu$

$E(N_s) = \frac{k}{2} = 1.5$

$\lambda' = \mu(1 - P_0) = 4.5$

$\left[P_0 = \frac{1}{k+1} = \frac{1}{4} \right]$

Que 3 – The local one person – – – – – service time.

Ans. $k = 5$, $\lambda = 5/hr$

$\mu = 4/hr$

(a) P(barber is idle) = $P_0 = \dfrac{1-\lambda/\mu}{1-\left(\frac{\lambda}{\mu}\right)^{k+1}} = 0.088$

Percentage $= 8.8\%$

(b) P(system is full) $= P_K = \left(\dfrac{\lambda}{\mu}\right)^{K} P_0 = 0.2711$

(c) Expected no. of customer waiting for a haircut

$$E(N_q) = E(N_s) - (1-P_0) = 2.2$$

$$E(N_s) = \frac{\lambda}{\lambda-\mu} - \frac{(k+1)(\lambda/\mu)^{k+1}}{1-\left(\frac{\lambda}{\mu}\right)^{k+1}}$$

(d) $E(W_s) = \dfrac{E(N_s)}{\lambda'} = \dfrac{E(N_s)}{\mu(1-P_0)} = 51.5$

<u>Que 4</u> - Patients — — — — — — 20 per hour.

<u>Ans</u> - $k = 15$, $\lambda = 30$, $\mu = 20$

(a) $\lambda' = (1-P_0)\,\mu$ [$* P_0 = 0.0007$]

$= 19.98$

(b) P(patient not wait) $= P_0 = 0.00076$

(c) $E(W_s) = \dfrac{E(N_s)}{\lambda'} = 0.65$ hr $= 39$ min