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DEPARTMENT OF MATHEMATICS

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MATHEMATICAL LOGIC

* **Proposition :** It is a declarative statement which is either true or false but not both

Any proposition can have two possible truth values namely "True" and "False".

Eg:
1. Chennai is the capital of TN – preposition – T.
2. 2+3=6 – Proposition – F
3. My name is Suresh – Proposition – T or F.
4. Take a cup of Coffee – Not Preposition.

Atomic Preposition :

It is a preposition which cannot be further split into simpler sentences.

Compound Preposition :

It is a proposition formed by combining two or more than two atomic prepositions.

* **Logic Operators or Connectives :**

- AND – $\wedge$ – Conjunction
- OR – $\vee$ – Disjunction
- NOT – $\neg, \sim, p', \bar{p}$ – Negation
- If .... then – $\rightarrow$ – Conditional
- If and only if (iff) – $\leftrightarrow$ – Biconditional

Negation : It is a proposition which gives opposite meaning of given proposition.

Truth Table :

Any proposition has two possible values - T & F

Negation :

p	$\neg p$
T	F
F	T

Conjunction, Disjunction, Conditional, Biconditional :

If p and q are any two propositions then their conjunction, Disjunction, Conditional and Biconditional will be given by $p \wedge q$, $p \vee q$, $p \rightarrow q$, $p \leftrightarrow q$ respectively.

Truth Table :

p	q	$p \wedge q$	$p \vee q$	$p \rightarrow q$	$p \leftrightarrow q$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	F	T	T	F
F	F	F	F	T	T

1. Symbolize the statements :

R : Manasa is rich

H : Manasa is Happy

i) Manasa is poor but happy : $\neg R \wedge H$

ii) Manasa is rich or poor : $R \vee \neg R$

iii) Manasa is neither rich nor happy : $\neg R \wedge \neg H$.

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2. Symbolize the statements :

p : It is raining

q : The sun is shining

(i) If it is raining, then there are clouds in the sky : $p \to r$ 3.

(ii) If it is not raining, then the sun is not shining and there are clouds in the sky : $\neg p \to (\neg q \wedge r)$

(iii) The sun is shining iff it is not raining : $q \leftrightarrow \neg p$

. If either Ram takes C++ or Kumar takes Pascal then Latha will take Lotus.

R : Ram takes C++

K : Kumar takes Pascal

L : Latha takes Lotus.

$(R \vee K) \to L$.

1. Write the Negation of the statement.

"2 is even and -3 is negative".

2 is not even or -3 is not negative.

Problems using Truth Table :

1. Construct truth table for the following Compound proposition :

a) $p \wedge (p \to q) \to q$

b) $(p \to q) \leftrightarrow (\neg p \vee q)$

c) $\neg(q \to r) \wedge r \wedge (p \to r)$

d) $p \to (q \to r)$

e) $p \to (p \to (q \to p) \to p)$

$p \to q$	$\neg p$	$\neg p \vee q$
T	F	T
F	F	F
T	T	T
T	T	T

T	T	T
T	T	T
T	F	T
T	F	T

Contingency:

A statement formula which is neither a tautology nor a contradiction is called Contingency.

Eg: $p \vee q$, $p \wedge q$, $p \to q$, $p \leftrightarrow q$.

2. Indicate which ones are Tautology and Contradiction.

a) $(p \to \neg p) \to \neg p$ — Tautology

b) $p \to (p \vee q)$ — Tautology

c) $(\neg Q \wedge P) \wedge Q$ — Contradiction.

d) $\neg(q \to r) \wedge r \wedge (p \to q)$ — Contradiction

e) $p \wedge (p \to q) \to q$ — Tautology

a) $(p \to \neg p) \to \neg p$

p	$\neg p$	$(p \to \neg p)$	$(p \to \neg p) \to \neg p$
T	F	F	T
F	T	T	T

e) $p \wedge (p \to q) \to q$

p	q	$p \to q$	$p \wedge (p \to q)$	q
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

b) $p \to (p \vee q)$

p	q	$p \vee q$	$p \to (p \vee q)$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

d) $\underbrace{\neg(q \to r)}_{\star} \wedge r \wedge (p \to q)$

p	q	r	$q \to r$	$\neg\star$	$p \to q$	A	B
T	T	T	T	F	T	F	F
T	T	F	F	T	T	F	F
T	F	T	T	F	F	F	F
T	F	F	T	F	F	F	F
F	T	T	T	F	T	F	F
F	T	F	F	T	T	F	F
F	F	T	T	F	T	F	F
F	F	F	T	F	T	F	F

c) $(\neg Q \wedge P) \wedge Q$

P	Q	$\neg Q$	$(\neg Q \wedge P)$	$(\neg Q \wedge P) \wedge Q$
T	T	F	F	F
T	F	T	T	F
F	T	F	F	F
F	F	T	F	F

* Tautological Implication :

$A \Rightarrow B$

A compound proposition A is said to imply tautologically B if B is true whenever A is true.

(or)

In other words, $A \Rightarrow B$ if $A \to B$ is a Tautology

1. Prove the following Implications :

a) $P \to Q \Rightarrow (\neg Q \to \neg P)$

b) $P \to (Q \to R) \Rightarrow (P \to Q) \to (P \to R)$

c) $(P \vee \neg(q \wedge r)) \wedge \neg P) \Rightarrow \neg q \vee \neg r$

d) $(P \vee \neg P) \to q \to ((P \vee \neg P) \to r) \Rightarrow q \to r$

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a) $P \to Q \Rightarrow (\neg Q \to \neg P)$

Let $P \to Q = A$

$\neg Q \to \neg P = B$

To prove $A \Rightarrow B$

we need to prove $A \to B$ is a Tautology.

Truth Table :

P	Q	$P \to Q$ (A)	$\neg Q$	$\neg P$	$\neg Q \to \neg P$ (B)	$A \to B$
T	T	T	F	F	T	T
T	F	F	T	F	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T

P	Q	R	$P\to Q$	$Q\to R$	$P\to R$	$\overset{A}{P\to(Q\to R)}$	$\overset{B}{(P\to Q)\to(P\to R)}$	$A\to B$
T	T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F	T
T	F	T	F	T	T	T	T	T
T	F	F	F	T	F	T	T	T
F	T	T	T	T	T	T	T	T
F	T	F	T	F	T	T	T	T
F	F	T	T	T	T	T	T	T
F	F	F	T	T	T	T	T	T

∴ The given argument is a Tautology.

∴ $P\to(Q\to R) \Rightarrow (P\to Q)\to(P\to R)$

c) $((P \vee \neg(q\wedge r)) \wedge \neg P)) \Rightarrow \neg q \vee \neg r$

Let $A = P \vee \neg(q\wedge r)$ $\quad C = (P \vee \neg(q\wedge r)) \wedge \neg P$

$B = \neg q \vee \neg r$

T	T	T	F	T	T	T	T	T	T
T	T	F	F	T	F	F	T	F	T
T	F	T	F	T	T	T	F	T	T
T	F	F	F	T	F	T	F	T	T
F	T	T	T	T	T	T	T	T	T
F	T	F	T	T	F	F	T	F	T
F	F	T	T	T	T	T	F	T	T
F	F	F	T	T	F	T	F	T	T

Since the truth values in the last column are all true, the given argument is a Tautology.

$\therefore (p \vee \neg p) \rightarrow q \rightarrow ((p \vee \neg p) \rightarrow r) \Rightarrow q \rightarrow r.$

* Equivalence of Formulae : $\Leftrightarrow$

Two propositions A and B are said to be logically equivalent if and only if $A \leftrightarrow B$ is a tautology (or) A & B have identical truth values.

i.e., $A \Leftrightarrow B$ iff $A \leftrightarrow B$ is a Tautology.

1. Prove the following Equivalences :

a) $p \vee q \Leftrightarrow \neg(\neg p \wedge \neg q)$

b) $(p \rightarrow q) \wedge (p \rightarrow r) \Leftrightarrow p \rightarrow (q \wedge r)$

c) $p \rightarrow q \Leftrightarrow (\neg p \vee q)$

d) $\neg(p \leftrightarrow q) \Leftrightarrow (p \wedge \neg q) \vee (\neg p \vee q)$

e) $\neg\neg p \Leftrightarrow p.$

T	T	F	F	T	F	T	T
T	F	F	T	T	F	T	T
F	T	T	F	T	F	T	T
F	F	T	T	F	T	F	T

Since, the last column has all the truth values are True, the given argument is a Tautology.

$\therefore$ $p \vee q \Leftrightarrow \neg(\neg p \wedge \neg q)$.

b) $(p \to q) \wedge (p \to r) \Leftrightarrow p \to (q \wedge r)$

Let $A = (p \to q) \wedge (p \to r)$, $B = p \to (q \wedge r)$

p	q	r	$p \to q$	$p \to r$	$q \wedge r$	$(p \to q) \wedge (p \to r)$	$p \to (q \wedge r)$	$A \Leftrightarrow B$
T	T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F	T
T	F	T	F	T	F	F	F	T
T	F	F	F	F	F	F	F	T
F	T	T	T	T	T	T	T	T
F	T	F	T	T	F	T	T	T
F	F	T	T	T	F	T	T	T
F	F	F	T	T	F	T	T	T

p	q	$\neg p$	$p \to q$	$\neg p \vee q$	$p \to q \Leftrightarrow (\neg p \vee q)$
T	T	F	T	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

$p \to q \leftrightarrow (\neg p \vee q)$ is a tautology

$\therefore\ p \to q \Leftrightarrow (\neg p \vee q)$

d) $\neg(p \leftrightarrow q) \Leftrightarrow (p \wedge \neg q) \vee (\neg p \wedge q)$

Let $A = \neg(p \leftrightarrow q)$

$B = (p \wedge \neg q) \vee (\neg p \wedge q)$

p	q	$\neg p$	$\neg q$	$p \leftrightarrow q$	$\neg(p \leftrightarrow q)$	$(p \wedge \neg q)$	$(\neg p \wedge q)$	$(p \wedge \neg q) \vee (\neg p \wedge q)$	$A \leftrightarrow B$
T	T	F	F	T	F	F	F	F	T
T	F	F	T	F	T	T	F	T	T
F	T	T	F	F	T	F	T	T	T
F	F	T	T	T	F	F	F	F	T

$\neg(p \leftrightarrow q) \leftrightarrow (p \wedge \neg q) \vee (\neg p \wedge q)$ is a Tautology

$\therefore\ \neg(p \leftrightarrow q) \Leftrightarrow (p \wedge \neg q) \vee (\neg p \wedge q)$.

e) $\neg\neg p \Leftrightarrow p$

p	$\neg p$	$\neg\neg p$	$\neg\neg p \leftrightarrow p$
T	F	T	T
F	T	F	T

$\neg\neg p \Leftrightarrow p$ is a Tautology

1.	Idempotent law	$p \vee p \Leftrightarrow p$
2.	Identity law	$p \vee F \Leftrightarrow p$
3.	Dominant law	$p \vee T \Leftrightarrow T$
4.	Complement law	$p \vee \neg p \Leftrightarrow T$
5.	Associative law	$p \vee (q \vee r) \Leftrightarrow (p \vee q) \vee r$
6.	Distributive law	$p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$
7.	Absorption Law	$p \vee (p \wedge q) \Leftrightarrow p$
8.	Commutative law	$p \vee q \Leftrightarrow q \vee p$
9.	De Morgan's law	$\neg (p \vee q) \Leftrightarrow \neg p \wedge \neg q$

1. $p \to q \Leftrightarrow \neg p \vee q$ (Conditional Equivalence)

2. $p \to q \Leftrightarrow \neg q \to \neg p$ (Contrapositive)

3. $(p \to q) \wedge (p \to r) \Leftrightarrow p \to (q \wedge r)$

4. $(p \to r) \wedge (q \to r) \Leftrightarrow (p \vee q) \to r$

5. $(p \to r) \vee (q \to r) \Leftrightarrow (p \wedge q) \to r$

6. $(p \leftrightarrow q) \Leftrightarrow (p \to q) \wedge (q \to p)$

7. $p \to (q \to r) \Leftrightarrow (p \wedge q) \to r$

Note: If $p \to q$, then the inverse is $q \to p$

then the converse is $\neg p \to \neg q$

then the contrapositive is $\neg q \to \neg p$.

Implication Formulae :

1.	$p, q \Rightarrow p \wedge q$	
2.	$\neg p, p \vee q \Rightarrow q$	Disjunctive Syllogism
3.	$p, p \to q \Rightarrow q$	Modus Pollens
4.	$\neg q, p \to q \Rightarrow \neg p$	Modus Tollens
5.	$p \to q, q \to r \Rightarrow p \to r$	Chain rule
6.	$p \vee q, p \to r, q \to r \Rightarrow r$	Dilemma
7.	$p \vee q, \neg p \vee r \Rightarrow q \vee r$	Resolution
8.	$p \wedge q \Rightarrow p, p \wedge q \Rightarrow q$	Simplification

{always take , as 'and' } i.e, conjunction.

Problems without using Truth Table (Using Laws of Logic):

1. Show that the equivalence

$P \to (Q \to R) \Leftrightarrow (P \wedge Q) \to R$ using laws of logic.

LHS: $P \to (Q \to R) \Leftrightarrow P \to (\neg Q \vee R)$ {Conditional Equivalence}

$\Leftrightarrow \neg P \vee (\neg Q \vee R)$ {Conditional Equivalence}

$\Leftrightarrow (\neg P \vee \neg Q) \vee R$ {Associative law}

$\Leftrightarrow \neg (P \wedge Q) \vee R$ {De Morgan's law}

$\Leftrightarrow (P \wedge Q) \to R$ {Conditional Equivalence}

: RHS

$\therefore P \to (Q \to R) \Leftrightarrow (P \wedge Q) \to R$.

2. Show that the equivalence

$P \to (Q \vee R) \Leftrightarrow (P \to Q) \vee (P \to R)$

LHS: $P \to (Q \vee R) \Leftrightarrow \neg P \vee (Q \vee R)$ {Conditional ~~Equivalence~~}

$\Leftrightarrow (\neg P \vee Q) \vee (\neg P \vee R)$ {Associative, Distributive}

$\Leftrightarrow (P \to Q) \vee (P \to R)$ {Conditional Equivalence}

: RHS

$\therefore P \to (Q \vee R) \Leftrightarrow (P \to Q) \vee (P \to R)$.

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1. Prove that $\neg (P \wedge Q) \to (\neg P \vee (\neg P \vee Q)) \Leftrightarrow \neg P \vee Q$ without constructing truth table.

: $\neg (P \wedge Q) \to (\neg P \vee (\neg P \vee Q)) \Leftrightarrow \neg (P \wedge Q) \to ((\neg P \vee \neg P) \vee Q)$ {Associative Law}

$\Leftrightarrow \neg (P \wedge Q) \to (\neg P \vee Q)$ {Idempotent law}

$\Leftrightarrow \neg\neg (P \wedge Q) \vee (\neg P \vee Q)$ {Conditional Equivalence}

$\Leftrightarrow (P \wedge Q) \vee (\neg P \vee Q)$ ($\because \neg\neg P = P$)

$\Leftrightarrow (Q \wedge P) \vee (Q \vee \neg P)$ {Commutative law}

$\Leftrightarrow (Q \vee \neg P) \vee (Q \wedge P)$ {Commutative law}

A ∨ (B ∧ C)

$\Leftrightarrow ((Q \vee \neg P) \vee Q) \wedge ((Q \vee \neg P) \vee P)$ {Distributive law}

$\Leftrightarrow (Q \vee (\neg P \vee Q)) \wedge (Q \vee (\neg P \vee P))$ {Associative law}

$\Leftrightarrow (Q \vee (Q \vee \neg P)) \wedge (Q \vee T)$

$\Leftrightarrow ((Q \vee Q) \vee \neg P) \wedge T$ {Dominant law}

$\Leftrightarrow (Q \vee \neg P) \wedge T$ {Identity law}

$\Leftrightarrow (Q \vee \neg P)$

$\Leftrightarrow (\neg P \vee Q)$ {Commutative law}

$\therefore$ LHS = RHS

{In AND, OR operators, always use AND first}

{$\Leftrightarrow$, $\equiv$ operators represent equivalent}

2. Prove that $(\neg p \vee q) \wedge (p \wedge (p \wedge q)) \equiv p \wedge q$.

: $(\neg p \vee q) \wedge (p \wedge (p \wedge q)) \Leftrightarrow (\neg p \vee q) \wedge ((p \wedge p) \wedge q)$ {Associative law}

$\Leftrightarrow (\neg p \vee q) \wedge (p \wedge q)$ {Idempotent law}

$\Leftrightarrow (q \vee \neg p) \wedge (q \wedge p)$ {Commutative law}

$\Leftrightarrow \underset{A}{(q \wedge p)} \wedge \underset{(B \vee C)}{(q \vee \neg p)}$

$\Leftrightarrow ((q \wedge p) \wedge q) \vee ((q \wedge p) \wedge \neg p)$ {Distributive law}

$\Leftrightarrow (q \wedge p) \vee (q \wedge (p \wedge \neg p))$ {Associative law}

$\Leftrightarrow (q \wedge p) \vee (q \wedge F)$

$\Leftrightarrow (q \wedge p) \vee F$

$\Leftrightarrow (q \wedge p)$

$\Leftrightarrow (p \wedge q)$ {Commutative law}

$\therefore$ LHS = RHS

\# 3. Show ...

$(\sim p \wedge (\sim q \wedge r)) \vee (q \wedge r) \vee (p \wedge r)$

1 :

$\Leftrightarrow ((\sim p \wedge \sim q) \wedge r) \vee (q \wedge r) \vee (p \wedge r)$ {Associative law}

$\Leftrightarrow ((\sim p \wedge \sim q) \wedge r) \vee ((q \vee p) \wedge r)$ {Distributive law}

$(A \cap B) \cup (C \cap B)$

$(A \cup C) \cap B$

$\Leftrightarrow (((\sim p \wedge \sim q) \vee (q \vee p)) \wedge r)$

$\Leftrightarrow ((\sim (p \vee q) \vee (p \vee q)) \wedge r)$ {Associative law, De Morgan's Law}

$\Leftrightarrow (T \wedge r)$ {complement law}

$\Leftrightarrow r$

$\therefore$ LHS = RHS. 6

4. Show that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology, without using a Truth Table.

: $(p \wedge q) \rightarrow (p \vee q)$

$\Leftrightarrow \sim (p \wedge q) \vee (p \vee q)$ {Conditional equivalence}

$\Leftrightarrow (\sim p \vee \sim q) \vee (p \vee q)$ {De Morgan's law}

$\Leftrightarrow (\sim q \vee \sim p) \vee (p \vee q)$ {~~Associative~~ law} Commutative

$\Leftrightarrow (\overset{A}{\sim q} \vee (\sim p \vee \overset{B}{p}) \vee \overset{C}{q})$ 7

Part ★ $\Leftrightarrow (\sim q \vee \sim p) \vee p \vee (\sim q \vee \sim p) \vee q$ {~~Associative~~ law}

$\Leftrightarrow (\sim q \vee (\sim p \vee p)) \vee ((\sim q \vee q) \vee \sim p)$ {Distributive law}

$\Leftrightarrow (\sim q \vee T) \vee (T \vee \sim p)$ {Associative law}

$\Leftrightarrow T \vee T$ {Dominant law}

$\Leftrightarrow T.$

$\therefore (p \wedge q) \rightarrow (p \vee q)$ is a Tautology.

$a \vee (b \vee c) \Leftrightarrow (a \vee b) \vee c$

5. Write the dual of $\neg P \to (\neg P \to (\neg P \wedge Q))$

: $\neg P \to (\neg P \to (\neg P \wedge Q)) \Leftrightarrow \neg P \to (\neg\neg P \vee (\neg P \wedge Q))$ {Conditional Equivalence}

$\Leftrightarrow \neg P \to (P \vee (\neg P \wedge Q))$

$\Leftrightarrow \neg P \to [(P \vee \neg P) \wedge (P \vee Q)]$ {Distributive law}

$\Leftrightarrow \neg P \to [T \wedge (P \vee Q)]$ {Identity law}

$\Leftrightarrow \neg P \to (P \vee Q)$

$\Leftrightarrow \neg\neg P \vee (P \vee Q) \Leftrightarrow P \vee (P \vee Q)$

$\Leftrightarrow (P \vee P) \vee Q$

$\Leftrightarrow (P \vee Q)$

∴ The dual of $P \vee Q$ is $P \wedge Q$.

6. Prove that $\neg(P \leftrightarrow Q) \Leftrightarrow (P \wedge \neg Q) \vee (\neg P \wedge Q)$.

: By Equivalence formula,

$P \leftrightarrow Q \Leftrightarrow (P \to Q) \wedge (Q \to P)$

$\neg(P \leftrightarrow Q) \Leftrightarrow \neg[(P \to Q) \wedge (Q \to P)]$

$\Leftrightarrow \neg(P \to Q) \vee \neg(Q \to P)$

$\Leftrightarrow \neg(\neg P \vee Q) \vee \neg(\neg Q \vee P)$ {Conditional Equivalence}

$\Leftrightarrow (P \wedge \neg Q) \vee (Q \wedge \neg P)$ {De Morgan's law}

$\Leftrightarrow (P \wedge \neg Q) \vee (\neg P \wedge Q)$ {Commutative law}

∴ LHS = RHS.

7. $p \to (q \to p) \Leftrightarrow \neg p \to (p \to q)$

: LHS: $p \to (q \to p) \Leftrightarrow \neg p \vee (\neg q \vee p)$

$\Leftrightarrow \neg p \vee (p \vee \neg q)$

$\Leftrightarrow (\neg p \vee p) \vee \neg q$

$\Leftrightarrow T \vee \neg q$

$\Leftrightarrow T$.

RHS: $\neg p \to (p \to q) \Leftrightarrow \neg\neg p \vee (\neg p \vee q)$

$\Leftrightarrow p \vee (\neg p \vee q)$

$\Leftrightarrow (p \vee \neg p) \vee q$

$\Leftrightarrow T \vee q \Leftrightarrow T$.

∴ LHS = RHS

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Problems on Implication:

1. $[(p\vee\neg p)\to q]\to[(p\vee\neg p)\to r]\Leftrightarrow q\to r.$

: LHS: $[(p\vee\neg p)\to q]\to[(p\vee\neg p)\to r]$

$$\Leftrightarrow [T\to q]\to[T\to r]$$

$$\Leftrightarrow (\neg T\vee q)\to(\neg T\vee r)$$

$$\Leftrightarrow (F\vee q)\to(F\vee r)$$

$$\Leftrightarrow q\to r.$$

$(q\to r)\to r$
$\neg q\vee r\to r$
$q\wedge\neg r\vee r$
$q\wedge F = q$

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2. $(p\vee q)\wedge(p\to r)\wedge(q\to r)\Rightarrow r.$

: LHS: $(p\vee q)\wedge((p\vee q)\to r)$ {Equivalence formula No. 4}

$$\Rightarrow \underset{A}{(p\vee q)}\underset{\cap}{\wedge}\underset{(B\,\cup\,C)}{[\neg(p\vee q)\vee r]}$$

$$\Rightarrow [(p\vee q)\wedge\neg(p\vee q)]\vee[(p\vee q)\wedge r]$$

$$\Rightarrow F\vee[(p\vee q)\wedge r]$$

$$\Rightarrow (p\vee q)\wedge r$$

To prove $[(p\vee q)\wedge r]\to r$ is a Tautology

$$[(p\vee q)\wedge r]\to r$$

$$\Leftrightarrow \neg[(p\vee q)\wedge r]\vee r$$

$$\Leftrightarrow [\neg(p\vee q)\vee\neg r]\vee r$$

$$\Leftrightarrow \neg(p\vee q)\vee(\neg r\vee r)$$

$$\Leftrightarrow \neg(p\vee q)\vee T$$

$$\Leftrightarrow T.$$

3. $p\to(q\to r) \Rightarrow (p\to q)\to(p\to r)$

LHS :

$p\to(q\to r)$

$\Leftrightarrow p\to(\neg q\vee r)$

$\Leftrightarrow \neg p\vee(\neg q\vee r)$

$\Leftrightarrow (\neg p\vee\neg q)\vee r$

$\Leftrightarrow \sim(p\wedge q)\vee r$

$\Leftrightarrow (p\wedge q)\to r \to$ ①

RHS: $(p\to q)\to r$

$(p\to q)\to(p\to r)$

$\Leftrightarrow (\neg p\vee q)\to(\neg p\vee r)$

$\Leftrightarrow \neg(\neg p\vee q)\vee(\neg p\vee r)$

$\cancel{(\neg\neg p\wedge\neg q)\vee(\neg p\vee r)}$

$\cancel{(p\wedge\neg q)\vee(\neg p\vee r)}$

$\Leftrightarrow (p\wedge\neg q)\vee(\neg p\vee r)$

$\Leftrightarrow \underbrace{(\neg p\vee r)}_{A}\ \underset{\cup}{\vee}\ (\underset{B}{p}\ \underset{\cap}{\wedge}\ \underset{C}{\neg q})$

$\Leftrightarrow ((\neg p\vee r)\vee p)\wedge((\neg p\vee r)\vee\neg q))$

$\Leftrightarrow T\wedge(\neg p\vee(r\vee\neg q))$

$\Leftrightarrow \neg p\vee(\neg q\vee r)$

$\Leftrightarrow \neg(p\wedge q)\vee r$

$\Leftrightarrow (p\wedge q)\to r \to$ ②

Since ① and ② are equivalent

LHS $\Rightarrow$ RHS.

Part-C

* Inference Theory :

Inference theory deals with inferring a conclusion from the given set of premises or hypothesis.

Premise: It is a statement which is assumed to be true.

Types of Inference :

- Direct proof (Direct method)
- Indirect proof (Indirect method)

Rules of inference :

Rule P : A premise may be introduced at any step in the derivation.

Rule T : A tautological formula T may be introduced using the preceeding premises already used (combining one or more premises together).

Problems using Direct Proof :

1. Show that R is a valid inference from the premises $P \to Q$, $Q \to R$ and P.

Step No.	Statement	Rule

2. Show that $P \to S$ logically follows from the premises $\neg P \vee Q$, $\neg Q \vee R$ and $R \to S$.

Step No.	Statement	Rule
1.	$\neg P \vee Q$	Rule P
2.	$P \to Q$	Rule T (1) Conditional Equivalence
3.	$\neg Q \vee R$	Rule P
4.	$Q \to R$	Rule T(3) Conditional Equivalence
5.	$P \to R$	Rule T(2)(4) Hypothetical Syllogism
6.	$R \to S$	Rule P
7.	$P \to S$	Rule T (5)(6) Hypothetical Syllogism.

3. Demonstrate S is a valid inference from $P \to \neg q$, $q \vee r$, $\neg s \to P$ and $\neg r$.

Step No.	Statement	Rule

1.	$P \vee Q$	Rule P
2.	$\neg P \rightarrow Q$	Rule T (1)
3.	$Q \rightarrow S$	Rule P
4.	$\neg P \rightarrow S$	Rule T (2) (3)
5.	$\neg S \rightarrow P$	Rule T (4) Contrapositive
6.	$P \rightarrow R$	Rule P
7.	$\neg S \rightarrow R$	Rule T (5)(6)
8.	$S \vee R$	Rule T (7)

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Rule CP (Conditional Premise):

If the conclusion is of the form $\boxed{r \to s}$, then take r as an additional new premise and derive s as conclusion.

1. Derive $P \to \neg S$ from $P \to (Q \vee R)$, $Q \to \neg P$, $S \to \neg R$ using CP Rule.
The premises are $P \to (Q \vee R)$, $Q \to \neg P$, $S \to \neg R$ & P (additional premise)

Step No.	Statement	Rule
1.	$P \to (Q \vee R)$	Rule P
2.	P	Rule CP
3.	$Q \vee R$	Rule T (1)(2) Modus Pollens
4.	$\neg Q \to R$	Rule T(3) Conditional Equivalence
5.	$Q \to \neg P$	Rule P
6.	$P \to \neg Q$	Rule T (5) Contrapositive
7.	$S \to \neg R$	Rule P
8.	$P \to R$	Rule T (6)(4) Hypothetical Syllogism.
9.	$\neg R \to \neg P$	Rule T(8) Contrapositive
10.	$S \to \neg P$	Rule T(7)(9) Hypothetical Syllogism
11.	$P \to \neg S$	Rule T(10) Contrapositive
12.	$\neg S$	Rule T(2)(11) Modus Pollens.

8.	$\neg P \rightarrow S$	Rule T (7) Contrapositive
9.	S	Rule T (5)(8) Modus Pollens

Show that $S \vee R$ logically follows from $P \vee Q$, $P \rightarrow R$ & $Q \rightarrow S$

(or)

Prove that $(P \vee Q), P \rightarrow R, Q \rightarrow S \Rightarrow S \vee R$ by Direct Method.

Step No.	Statement	Rule
1.	$P \vee Q$	Rule P
2.	$\neg P \rightarrow Q$	Rule T (1)
3.	$Q \rightarrow S$	Rule P
4.	$\neg P \rightarrow S$	Rule T (2) (3)
5.	$\neg S \rightarrow P$	Rule T (4) Contrapositive
6.	$P \rightarrow R$	Rule P
7.	$\neg S \rightarrow R$	Rule T (5)(6)
8.	$S \vee R$	Rule T (7)

$P \to (Q \to R)$	Rule P
P	Rule CP
$Q \to R$	Rule T (1)(2) Modus Pollens
$Q \to (R \to S)$	Rule P
$Q \to R \wedge (R \to S)$	Rule T(3)(4) Equivalence formula
$\neg Q \vee (R \wedge (\neg R \vee S))$	Rule T Conditional Equivalence
$\neg Q \vee ((R \wedge \neg R) \vee (R \wedge S))$	Rule T (6) Distributive Law (7)

3. Prove that $P \to (q \to r)$; $\neg S \vee P$, $q \Rightarrow S \to r$ using CP rule.

Take S as a new premise

Step No	Statement	Rule
1.	$\neg S \vee P$	Rule P
2.	$S \to P$	Rule T(1) Conditional Equivalence
3.	S	Rule CP
4.	P	Rule T(2)(3) Modus Pollens
5.	$P \to (q \to r)$	Rule P
6.	$q \to r$	Rule T(4)(5) Modus Pollens
7.	q	Rule P
8.	r	Rule T(6)(7) Modus Pollens.
9.	$S \to r$	Rule CP.

Indirect Method:

1. Using indirect method, derive r from $\neg q$, $p \to q$, $p \vee r$.

The premises are $\neg q$, $p \to q$, $p \vee r$, $\neg r$ (negated conclusion)

Step No	Statement	Rule
1.	$p \to q$	Rule P
2.	$\neg q$	Rule P
3.	$\neg P$	Rule T(1)(2) Modus Tollens
4.	$P \vee r$	Rule P
5.	$\neg P \to r$	Rule T(4) Conditional Equivalence
6.	r	Rule T(3)(5) Modus Pollens
7.	$\neg r$	Rule P Negated conclusion.
8.	$r \wedge \neg r = F$	Rule T(6)(7) Complement law.

∴ It is a Contradiction

⇒ The premises give the conclusion r.

2. Show That $(a \to b, c \to b, d \to (a \vee c); d) \Rightarrow b$ by Indirect method.

Take $\neg b$ as an additional premise

Step No	Statement	Rule
1.	$a \to b$	Rule P
2.	$c \to b$	Rule P
3.	$(a \vee c) \to b$	Rule T(1)(2) Equivalence Formula
4.	$d \to (a \vee c)$	Rule P
5.	$d \to b$	Rule T (3)(4) Hypothetical Syllogism
6.	d	Rule P
7.	b	Rule T(6)(7) Modus Pollens
8.	$\neg b$	Negated conclusion
9.	$b \wedge \neg b = F$	Rule T(7)(8) Complement law.

3. Show that $p \to q, q \to r, p \vee r \Rightarrow r$ by Indirect Method.

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Inconsistent :

A set of premises is said to be inconsistent if their conjunction is a contradiction. i.e., the premise $H_1, H_2, H_3, \cdots H_n$ are said to be inconsistent if $H_1 \wedge H_2 \wedge \cdots \wedge H_n$ $\Rightarrow F$ (contradiction).

Consistent :

A set of premises is said to be consistent if it is not inconsistent.

Problems on inconsistency :

Show that the following premises are inconsistent.

i. If Rama gets his degree, he will go for a job.

ii. If he goes for a job, he will get married soon.

iii. If he goes for higher study, he will not get married.

iv. Rama gets his degree & goes for higher studies.

From the given information, the premises are

p : Rama gets his degree

q : He will go for a job

r : He will get married

s : He goes for higher studies.

The symbolic form (i.e) the premises are

i.) $P \rightarrow q$ ii) $q \rightarrow r$, iii) $s \rightarrow \neg r$, iv) $P \wedge S$.

$P \to q$	Rule P
$q \to r$	Rule P
$P \to r$	Rule T (1)(2) Hypothetical Syllogism
$S \to \neg r$	Rule P
$r \to \neg S$	Rule T (4) Contrapositive
$P \to \neg S$	Rule T(3)(5) Hypothetical Syllogism
$\neg P \vee \neg S$	Rule T(6) Conditional Equivalence
$\neg(P \wedge S)$	Rule T(7) De Morgan's law
$P \wedge S$	Rule P
F Contradiction	Rule T (8)(9) Complement law.

$P \to Q$, $P \to R$, $Q \to \neg R$, P are inconsistent.

Statement	Rule
$P \to Q$	Rule P
$Q \to \neg R$	Rule P
$P \to \neg R$	Rule T (1)(2)
P	Rule P
$\neg R$	Rule T (3)(4)
$P \to R$	

Show that the following are inconsistent.

i. If Jack misses many classes due to illness then he fails in High School.

ii. If Jack fails in High School then he is uneducated.

iii. If Jack reads lot of books then he is not uneducated.

iv. Jack misses many classes & reads lot of books.

The premises are

i. $p \to q$
ii. $q \to r$
iii. $s \to \neg r$
iv. $p \wedge s$.

S.T. the premises "one student in this class knows how to write Programs in Java" and "everyone who knows how to write programs in Java can get a high paying job" imply the conclusion "Someone in this class can get a high paying job"

Premises : $C(x)$: x is in the class
$J(x)$: x knows Java. $H(x)$: x can get high paying job

$\exists x \, [C(x) \wedge J(x)]$. $\forall x \, (J(x) \to H(x)) \Rightarrow \exists x \, (C(x) \wedge H(x))$.

Predicate Calculus:

The logic based on predicates is called Predicate Calculus.

Quantifiers:

The word which indicates quantity is called a Quantifier (Eg: All, some, few)

Types of Quantifiers:

- Universal Quantifier $\forall$ (for all, for every)
- Existential Quantifier $\exists$ (There exists)

Rules:

1. Universal Specification (US)

$$\forall x, p(x) \Leftrightarrow P(y)$$

2. Universal Generalisation (UG)

$$P(y) \Rightarrow (\forall x) P(x)$$

3. Existential Specification (ES)

$$(\exists x) P(x) \Rightarrow P(y)$$

4. Existential Generalisation (EG)

$$P(y) \Rightarrow (\exists x) P(x)$$

1. Write in Symbolic form

i. Something is green

ii. Everything is green

iii. Nothing is green

iv. Something is not green.

: Let $G(x)$ = x is green.

i) $(\exists x)\, G(x)$

ii) $(\forall x)\, G(x)$

iii) $(\forall x)\, \neg G(x)$

iv) $(\exists x)\, \neg G(x)$.

2. Write in symbolic form

a) All men are giants

b) No men are giants

c) Some men are giants

d) Some men are not giant.

: Let $M(x)$: x is man

$G(x)$: x is a giant

a) $(\forall x)\, (M(x) \rightarrow G(x))$

b) $(\forall x)\, (M(x) \rightarrow \neg G(x))$

c) $(\exists x)\, (M(x) \wedge G(x))$

d) $\exists x\, (M(x) \wedge \neg G(x))$

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Therefore there are dangerous animals.

: Symbolised form : $L(x)$: x is Lion

$D(x)$: x is dangerous

Premises are : $\forall x\,(L(x) \rightarrow D(x)),\ (\exists x)\,L(x) \Rightarrow (\exists x)\,D(x)$

Step No	Statement	Rule
1.	$\forall x\,(L(x) \rightarrow D(x))$	Rule P
2.	$L(y) \rightarrow D(y)$	Universal Specification(1)
3.	$\exists x\, L(x)$	Rule P
4.	$L(y)$	ES(3)
5.	$D(y)$	Rule T (2)(4) Modus Pollens
6.	$(\exists x)\,D(x)$	EG(5)

Hence the argument is valid.

3. IQ

** UQ 2. Prove that $\forall x\,(P(x) \vee Q(x)) \Rightarrow \forall x\,(P(x)) \vee (\exists x)\,Q(x)$ by Indirect method. : Take $\neg\,[(\forall x)\,P(x) \vee (\exists x)\,Q(x)]$ as additional Premise.

:

Step No.	Statement	Rule

7.	$\neg Q(y)$	US (5)
8.	$\neg P(y) \land \neg Q(y)$	Rule T (6)(7) (Conjunction)
9.	$\neg(P(y) \lor Q(y))$	Rule T (8) (De Morgan's law)
10.	$(\forall x)(P(x) \lor Q(x))$	Rule P
11.	$P(y) \lor Q(y)$	US
12.	$(\neg(P(y) \lor Q(y))) \land (P(y) \lor Q(y)) = F$	Rule T (Conjugation & Complement law).

3. Use indirect method to prove that $\exists z\, Q(z)$ follows from $\forall x(P(x) \rightarrow Q(x))$ and $\exists y\, P(y)$.

Let us take $\neg(\exists(z)) Q(z)$ as an additional premise.

Step No	Statement	Rule
1.	$\forall x(P(x) \rightarrow Q(x))$	Rule P
2.	$\forall x(\neg P(x) \lor Q(x))$	Rule T (1)
3.	$\neg P(a) \lor Q(a)$	U.S. (2)
4.	$\exists y P(y)$	Rule P
5.	$P(a)$	E.S (4)
6.	$\neg(\exists z)\, Q(z)$	Rule P (Negated conclusion) (Additional Premise)
7.	$(\forall z)\, \neg(Q(z))$	Rule T (Negation law) (6)
8.	$\neg Q(a)$	U.S (7)
9.	$P(a) \land \neg Q(a)$	Rule T (5)(8) (Conjunction)
10.	$\neg(\neg P(a)) \lor Q(a)$	Rule T (9) (De Morgan's law)
11.	F	Rule T (3) (10) (Compliment law).

Q: Eg: 1. Negate & Simplify.

$\exists(x)\ (P(x) \vee Q(x))$

$\neg(\exists x\ (P(x) \vee Q(x)))$

$\forall x\ (\neg P(x) \wedge \neg Q(x))$

$\exists(x)\ [P(x) \wedge Q(x))$

2. Let $A = \{1, 2, 3, 4, 5, 6\}$ write truth values of

i) $(\exists x \in A)\ (x^2 > 25)$ True

ii) $(\exists x \in A)\ [x + 6 > 12)$ False

iii) $(\forall x \in A)\ (x^2 - x < 30)$ False.

12M. *** Mathematical Induction:

Let $S(n)$ be any mathematical statement having more than one occurances and $n \geq 1$.

Step 1: First prove $S(n)$ is true for $n = 1$, $n \in Z^+$ (set of +ve integers)

Step 2: Assume that $S(k)$ is true for $K \in Z^+$

Step 3: Prove that $S(k+1)$ is true for all $K \in Z^+$.

* imp: $\boxed{\text{i.e., If } n \geq 1,\ S(n) \text{ tautologically implies } S(n+1)}$

1. Use M.I to prove that $1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$

Sol: Let $S(n): 1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$

Step 1: To prove $S(1)$ is true

$$S(1):\ 1^2 = \frac{1(1+1)(2(1)+1)}{6} = 1$$

$$1 = 1$$

$S(1)$ is true.

Step 2: Assume $S(k)$ is true for $k \in Z^+$

$$S(k):\ 1^2 + 2^2 + \cdots + k^2 = \frac{k(k+1)(2k+1)}{6} \longrightarrow (1)$$

Step 3: To prove S(k+1) is true.

Consider $S(k+1): 1^2+2^2+3^2+\dots+k^2+(k+1)^2$

$$= \frac{k(k+1)(2k+1)}{6}+(k+1)^2 \quad \{\text{from ① as } S(k) \text{ is true}\}$$

$$= \frac{k(k+1)(2k+1)+6(k+1)^2}{6}$$

$$= (k+1)\left[\frac{k(2k+1)+6(k+1)}{6}\right]$$

$$= \frac{(k+1)[2k^2+7k+6]}{6}$$

$$= \frac{(k+1)(k+2)(2k+3)}{6}$$

$$\Rightarrow 1^2+2^2+3^2+\dots+k^2+(k+1)^2 = \frac{(k+1)(k+2)(2k+3)}{6}$$

$\therefore$ S(k+1) is true for all $k \in Z^+$.

2. Use MI to prove that $\frac{1}{1\cdot2}+\frac{1}{2\cdot3}+\frac{1}{3\cdot4}+\dots+\frac{1}{n(n+1)} = \frac{n}{n+1}$.

: Let $S(n) = \frac{1}{1\cdot2}+\frac{1}{2\cdot3}+\frac{1}{3\cdot4}+\dots+\frac{1}{n(n+1)} = \frac{n}{n+1}$

Step 1: To prove S(1) is true

$S(1): \frac{1}{1\cdot2} = \frac{1}{2}$

$\frac{1}{2} = \frac{1}{2}$

S(1) is true

Step 2: Assume S(k) is true for $k \in Z^+$

$$S(k): \frac{1}{1\cdot2}+\frac{1}{2\cdot3}+\dots+\frac{1}{k(k+1)} = \frac{k}{(k+1)} \rightarrow ①$$

Step 3: To prove S(k+1) is true

Consider $S(k+1): \frac{1}{1\cdot2}+\frac{1}{2\cdot3}+\dots+\frac{1}{k(k+1)}+\frac{1}{(k+1)(k+2)}$

$$= \frac{k}{k+1}+\frac{1}{(k+1)(k+2)}$$

$$= \frac{k+1}{k+2}$$ $\therefore$ S(k+1) is true for all $k \in Z^+$

3. Use MI to prove that 3^n+7^n-2 is divisible by 8.

: Let $S(n) = 3^n+7^n-2$

Step 1: To prove $S(1)$ is true.

$S(1): 3^1+7^1-2 = 8$ which is divisible by 8.

$S(1)$ is true

Step 2: Assume $S(k)$ is true for $k \in Z^+$

$S(k)$: ~~8~~ 3^k+7^k-2 is divisible by 8 $\longrightarrow$ (1)

Step 3: To prove $S(k+1)$ is true

Consider $S(k+1)$: $3^{k+1}+7^{k+1}-2$

$= 3^k \cdot 3+7^k \cdot 7-2$

$= 3^k \cdot 3+(3+4)7^k-2$

$= 3 \cdot 3^k+3 \cdot 7^k+4 \cdot 7^k-2$

$= 3(3^k+7^k-2)+4 \cdot 7^k+4$

$= 3(3^k+7^k-2)+4(7^k+1)$.

7^k+1 is an even number

$4(7^k+1)$ will be divisible by 8

3^k+7^k-2 is also divisible by 8, by step 2.

$3^{k+1}+7^{k+1}-2$ is also divisible by 8

$\therefore$ $S(k+1)$ is true.

4. Use M.I to prove that the following are true.

i) $\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+\cdots+\frac{1}{\sqrt{n}} > \sqrt{n}$ for $n \geq 2$.

ii) $1 \cdot 2+2 \cdot 3+3 \cdot 4+\cdots+n(n+1) = \frac{n(n+1)(n+2)}{3}$

iii) 8^n-3^n is divisible by 5 for all $n \in N$.

iv) a^n-b^n is divisible by $a-b$ $\forall n \in N$.

v) 5^n-4n-1 is divisible by 16.

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4.

(i) $\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\cdots+\frac{1}{\sqrt{n}}>\sqrt{n}$ for $n\geq 2$.

Let $S(n)=\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+\cdots+\frac{1}{\sqrt{n}}>\sqrt{n}$, $n\geq 2$

Step 1: To prove $S(2)$ is true.

$S(2): \frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}=1+\frac{1}{\sqrt{2}}=1.7>1.414=\sqrt{2}$.

$$\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}>\sqrt{2}$$

$S(2)$ is true.

Step 2: Assume $S(K)$ is true

$S(K): \frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+\cdots+\frac{1}{\sqrt{K}}>\sqrt{K}$ is true for all $K\in Z^+$.

Step 3: To prove $S(K+1)$ is true

$$S(K+1): \frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+\cdots+\boxed{\frac{1}{\sqrt{K}}}+\frac{1}{\sqrt{K+1}}>\sqrt{K}+\frac{1}{\sqrt{K+1}}$$

$>\sqrt{K}$

$$>\frac{\sqrt{K(K+1)}+1}{\sqrt{K+1}}$$

$$>\frac{K+1}{\sqrt{K+1}}=\sqrt{K+1}$$

$K(K+1)$

$K^2+K>K^2$

$\sqrt{K(K+1)}>\sqrt{K^2}$

$\sqrt{K(K+1)}>K$

(X)

$\therefore \frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+\cdots+\frac{1}{\sqrt{K+1}}>\sqrt{K+1}$.

$S(K+1)$ is true

$\therefore S(n)$ is true for all $n\in Z^+$.

5. T. $n\geq 2^{n-1}$ for $n=1,2,3\ldots$

$a^n - b^n$ is divisible by $a-b \ \forall \ n \in N$.

Let $s(n) = a^n - b^n$ divisible by $a-b$.

<u>Step 1:</u> To prove $s(1)$ is true.

$s(1)$: $a^1 - b^1 = a-b$

Divisible by $a-b$

$s(1)$ is true.

<u>Step 2:</u> Assume $s(k)$ is true

$s(k)$: $a^k - b^k$ is divisible by $a-b \ \forall \ k \in N$

<u>Step 3:</u> To prove $s(k+1)$ is true.

$s(k+1)$: $a^{k+1} - b^{k+1}$ ~~is di~~

$a^k \cdot a - b^k \cdot b$

$= a^k \cdot a - b \cdot b^k + b^k a - ab^k$ {Add & subtract ~~ab^k~~}

$= a(a^k - b^k) + b^k(a-b)$.

From step 1, 2 $a^k - b^k$ & $a-b$ are divisibly by $a-b$.

$s(k+1)$ is divisible by $a-b$.

$\therefore$ $a^n - b^n$ is divisible by $a-b \ \forall \ n \in N$.

PROGRAMMING FUNDAMENTALS

Logical Operators

Kenneth Leroy Busbee and Dave Braunschweig

Overview

A **logical operator** is a symbol or word used to connect two or more expressions such that the value of the compound expression produced depends only on that of the original expressions and on the meaning of the operator.[1] Common logical operators include AND, OR, and NOT.

Discussion

Within most languages, expressions that yield Boolean data type values are divided into two groups. One group uses the relational operators within their expressions and the other group uses logical operators within their expressions.

The logical operators are often used to help create a test expression that controls program flow. This type of expression is also known as a Boolean expression because they create a Boolean answer or value when evaluated. There are three common logical operators that give a Boolean value by manipulating other Boolean operand(s). Operator symbols and/or names vary with different programming languages:

Language	AND	OR	NOT
C++	`&&`	`||`	`!`
C#	`&&`	`||`	`!`
Java	`&&`	`||`	`!`
JavaScript	`&&`	`||`	`!`
Python	`and`	`or`	`not`
Swift	`&&`	`||`	`!`

The vertical dashes or piping symbol is found on the same key as the backslash \. You use the SHIFT key to get it. It is just above the Enter key on most keyboards. It may be a solid vertical line on some keyboards and show as a solid vertical line on some print fonts.

In most languages there are strict rules for forming proper logical expressions. An example is:

```
6 > 4 && 2 <= 14
6 > 4 and 2 <= 14
```

This expression has two relational operators and one logical operator. Using the precedence of operator rules the two “relational comparison” operators will be done before the “logical and” operator. Thus:

```
true && true
True and True
```

The final evaluation of the expression is: true.

We can say this in English as: It is true that six is greater than four and that two is less than or equal to fourteen.

When forming logical expressions programmers often use parentheses (even when not technically needed) to make the logic of the expression very clear. Consider the above complex Boolean expression rewritten:

```
(6 > 4) && (2 <= 14)
(6 > 4) and (2 <= 14)
```

Most programming languages recognize any non-zero value as true. This makes the following a valid expression:

```
6 > 4 && 8
6 > 4 and 8
```

But remember the order of operations. In English, this is six is greater than four and eight is not zero. Thus,

```
true && true
True and True
```

To compare 6 to both 4 and 8 would instead be written as:

```
6 > 4 && 6 > 8
6 > 4 and 6 > 8
```

This would evaluate to false as:

```
true && false
True and False
```

Truth Tables

A common way to show logical relationships is in truth tables.

Logical and (&&)

x	Y	x and y
false	false	false
false	true	false
true	false	false
true	true	true

Logical or (||)

x	Y	x or y
false	false	false
false	true	true
true	false	true
true	true	true

Logical not (!)

x	not x
false	true
true	false

Examples

I call this example of why I hate “and” and love “or”.

Every day as I came home from school on Monday through Thursday; I would ask my mother, “May I go outside and play?” She would answer, “If your room is clean and your homework is done then you may go outside and play.” I learned to hate the word “and”. I could manage to get one of the tasks done and have some time to play before dinner, but both of them… well, I hated “and”.

On Friday my mother took a more relaxed viewpoint and when asked if I could go outside and play she responded, “If your room is clean or your homework is done then you may go outside and play.” I learned to clean my room quickly on Friday afternoon. Well, needless to say, I loved “or”.

For the next example, just imagine a teenager talking to their mother. During the conversation, mom says, “After all, your Dad is reasonable!” The teenager says, “Reasonable. (short pause) Not.”

Maybe college professors will think that all their students studied for the exam. Ha ha! Not. Well, I hope you get the point.

Examples:

- 25 < 7 || 15 > 36
- 15 > 36 || 3 < 7
- 14 > 7 && 5 <= 5
- 4 > 3 && 17 <= 7
- ! false
- ! (13 != 7)
- 9 != 7 && ! 0
- 5 > 1 && 7

More examples:

- 25 < 7 or 15 > 36

- 15 > 36 or 3 < 7
- 14 > 7 and 5 <= 5
- 4 > 3 and 17 <= 7
- not False
- not (13 != 7)
- 9 != 7 and not 0
- 5 > 1 and 7

Key Terms

logical operator
: An operator used to create complex Boolean expressions.

truth tables
: A common way to show logical relationships.