Q. find the language generated by this grammar.

$P \Rightarrow S \to aSa$

$S \to bSb$

$S \to \epsilon$

Sol.

① $S \to aSa$

$S \to a\epsilon a \quad (S \to \epsilon)$

$S \to \underset{w}{\underline{a}}\,\underset{w^r}{\underline{a}}$

② $S \to aSa$

$S \to abSba \quad (S \to bSb)$

$S \to ab\epsilon ba \quad (S \to \epsilon)$

$S \to \underset{w}{\underline{ab}}\,\underset{w^r}{\underline{ba}}$

Language, $L = \{ww^r / w \in (a,b)\}$

Q. Solve using left-most derivation. $S \to aAS/a$

$A \to ab$

Sol.

$S \to aAS$

$\underset{lm}{\Longrightarrow} aabS \quad (A \to ab)$

$\underset{lm}{\Longrightarrow} aaba \quad (S \to a)$

Q. Solve using right-most derivation. $S \to aAS/a$

$A \to ab$

Sol.

$S \to aAS$

$\overset{rm}{\Longleftarrow} aAa \quad (S \to a)$

$\underset{rm}{\Longleftarrow} aaba \quad (A \to ab)$

Q. construct ambiguous grammar for the following.

$E \to I$

$E \to E+E$

$E \to E*E$

$I \to 0, 1, \ldots 9$

$w = 3*2+5$

→ we can solve this using lmd (or) rmd,

$E \to E * E$

$E \to I * E \ (E \to I)$

$E \to 3 * E \ (I \to 3)$

$E \to 3 * E + E \ (E \to E + E)$

$E \to 3 * I + E \ (E \to I)$

$E \to 3 * 2 + E \ (I \to 2)$

$E \to 3 * 2 + I \ (E \to I)$

$E \to 3 * 2 + 5 \ (I \to 5)$ //

Q. eliminate ϵ production. $S \to Babc$

$C \to b | \epsilon$

$B \to a | \epsilon$

Sol.

$S \to Babc / abC / Bab / ab$

$C \to b$ $\quad V_{nullable} = \{B, C\}$

$B \to a$ //

Q. eliminate unit production. $S \to AaB | C$ $\quad C \to a$

$A \to a | bC | B$ $\quad B \to A | bb$

Sol.

$\left.\begin{array}{l} S \to C \\ A \to B \\ B \to A \end{array}\right\}$ unit productions

$S \to AaB | a$

$A \to a | bC | bb$

$C \to a$

$B \to a | bC | bb$ //

Q. eliminate useless production. $S \to aS | A | C$

$A \to a$

$B \to aa$

$C \to aC$

Sol.

$S \to aS | A$ — ①

$S \to aS | A | a$ — ② $(\because A \to a)$

$S \to aS | A | a | aa$ — ③ $(\because S \to A \Rightarrow A \to a)$ //

Q. Convert the following CFG to CNF.

(%)

$S \rightarrow AAC$

$A \rightarrow aAb \mid \epsilon$

$C \rightarrow aC \mid a$

Sol.

① remove ϵ productions $\Rightarrow$ $S \rightarrow AAC \mid AC \mid C$

$A \rightarrow aAb \mid ab$

$V_{nullable} = \{A\}$

$C \rightarrow aC \mid a$

② remove unit production. $\Rightarrow$ $S \rightarrow AAC \mid AC \mid aC \mid a$

$A \rightarrow aAb \mid ab$

$S \rightarrow C$ is unit prod.

$C \rightarrow aC \mid a$

③ remove useless production.

$\rightarrow$ no useless production found

④ reconstruct (or) rename the variables.

$S \rightarrow AAC \mid AC \mid aC \mid a \Rightarrow P_1$

$A \rightarrow aAb \mid ab \Rightarrow P_2$

$C \rightarrow aC \mid a \Rightarrow P_3$

P_1: $S \rightarrow AAC \mid AC \mid aC \mid a$

$S \rightarrow AAC$ is not a CNF

$\left.\begin{array}{l} S \rightarrow AR_1 \\ R_1 \rightarrow AC \end{array}\right\}$ both are CNF

$S \rightarrow aC$

$\left.\begin{array}{l} S \rightarrow R_2C \\ R_2 \rightarrow a \end{array}\right\}$ both are CNF

$\therefore S \rightarrow AR_1 \mid AC \mid R_2C \mid a$ is in CNF form.

P_2: $A \to aAb \mid ab$

$A \to aAb$ is not a CNF

$A \to R_2 A R_3$

$R_2 \to a$

$R_3 \to b$

$A \to R_2 R_4$

$R_4 \to A R_3$

$\left.\begin{array}{l} R_2 \to a \\ R_3 \to b \\ A \to R_2 R_4 \end{array}\right\}$ all are in CNF form

$A \to ab$

$\left.\begin{array}{l} A \to R_2 R_3 \\ R_2 \to a \\ R_3 \to b \end{array}\right\}$ all are in CNF form

$\therefore A \to R_2 R_4 \mid R_2 R_3$ is in CNF form.

P_3: $C \to aC \mid a$

$C \to aC$ is not a CNF

$\left.\begin{array}{l} C \to R_2 C \\ R_2 \to a \end{array}\right\}$ both are CNF

$\therefore C \to R_2 C \mid a$ is in CNF form.

Thereform, final CNF form is,

$S \to AR_1 \mid AC \mid R_2 C \mid a$

$A \to R_2 R_4 \mid R_2 R_3$

$C \to R_2 C \mid a$ //

Q. ~~first~~ give the GNF for the grammar given.

$S \to AA \mid a$

$A \to SS \mid b$

Sol.

Step1 → it is in CNF format only.

Step2 → rename the variables,

$S \to R_1$, $A \to R_2$.

$\Rightarrow$ $R_1 \to R_2 R_2 \mid a$ → P_1

$R_2 \to R_1 R_1 \mid b$ → P_2

Step 3 → check i and j.

P_1: $R_1 \to R_2 R_2 \mid a$ $\Rightarrow i=1, j=2 \Rightarrow i<j$

"leave as it is"

$\Rightarrow R_1 \to R_2 R_2 \mid a$ — ①

P_2: $R_2 \to R_1 R_1 \mid b$ $\Rightarrow i=2, j=1 \to i>j$

apply lemma 1 (substitution)

$R_2 \to R_1 R_1 \mid b$ — ②

$R_2 \to \underbrace{R_2}_{A} \underbrace{R_2 R_1}_{\alpha} \mid \underbrace{aR_1}_{\beta_1} \mid \underbrace{b}_{\beta_2}$ } not in GNF format

now, $i=2, j=2 \to i=j$

apply lemma 2 (elimination)

$R_2 \to aR_1 \mid b$

$R_2 \to aR_1 Z_1 \mid bZ_1$

where $Z_1 \to \alpha \Rightarrow R_2 R_1$

$Z_1 \to R_2 R_1$

$Z_1 \to R_2 R_1 Z_1$

$\Rightarrow R_2 \to aR_1 \mid b \mid aR_1 Z_1 \mid bZ_1$ $\Rightarrow$ GNF form

$Z_1 \rightarrow R_2 R_1$

$Z_1 \rightarrow aR_1R_1 \mid bR_1 \mid aR_1Z_1R_1 \mid bZ_1R_1 \Rightarrow$ GNF form

$Z_1 \rightarrow R_2R_1Z_1$

$Z_1 \rightarrow aR_1R_1Z_1 \mid bR_1Z_1 \mid aR_1Z_1R_1Z_1 \mid bZ_1R_1Z_1 \Rightarrow$ GNF form

$R_1 \rightarrow R_2R_2 \mid a$

$R_1 \rightarrow aR_1R_2 \mid bR_2 \mid aR_1Z_1R_2 \mid bZ_1R_2 \mid a \Rightarrow$ GNF form.