SRM Institute of Science and Technology
Ramapuram Campus
Department of Mathematics
18MAB101T – Calculus And Linear Algebra

Year/Sem: I/I

Branch: Common to ALL B.Tech. except B.Tech. (Business Systems)

Unit – I

MATRICES

Part – A

1.	The sum of the eigen values of $A=\begin{pmatrix}1 & 2\\ 2 & 1\end{pmatrix}$ is	1 Mark	
	(a) 2 (b) 4 (c) – 3 (d) 0	Ans (a)	(CLO – 1Apply)
2.	The eigen values of A^{-1}, if $A=\begin{pmatrix}1 & 5 & -1\\ 0 & 3 & 2\\ 0 & 0 & 4\end{pmatrix}$ are	1 Mark	
	(a) 2, 3, 4 (b) 2, 5, – 1 (c) 0, 0, 0 (d) $1,\frac{1}{3},\frac{1}{4}$	Ans (d)	(CLO -1Apply)
3.	If two eigen values of $A=\begin{pmatrix}8 & -6 & 2\\ -6 & 7 & -4\\ 2 & -4 & 3\end{pmatrix}$ are 3 and 15, then the third eigen value is __________.	1 Mark	
	(a) 1 (b) 0 (c) 2 (d) 3	Ans (b)	(CLO -1 Apply)
4.	If – 1, – 1, 2 are the eigen values of a matrix $A=\begin{pmatrix}0 & 1 & 1\\ 1 & 0 & 1\\ 1 & 1 & 0\end{pmatrix}$, then the eigen values of A^T are	1 Mark	
	(a) – 1, – 1, 2 (b) 1, 1, 1/2 (c) 1,1,4 (d) -1,-1,-2	Ans (a)	(CLO - 1 Apply)

5.	The sum of eigen values of the identity matrix of order 3 is	1 Mark	
	(a) 0 (b) 1 (c) 2 (d) 3	Ans (d)	(CLO - 1 Remember)
6.	The product of the two eigen values of the matrix $A=\begin{pmatrix}6&-2&2\\-2&3&-1\\2&-1&3\end{pmatrix}$ is 4.Then the third eigen value is	1 Mark	
	(a) 2 (b) 1 (c) 3 (d) 8	Ans (a)	(CLO - 1 Apply)
7.	The index of the canonical form $Q=-{y_1}^2+{y_2}^2+4{y_3}^2$ is	1 Mark	
	a) 3 (b) 2 (c) 1 (d) 0	Ans (b)	(CLO -1 Apply)
8.	If the eigen values of the matrix of the quadratic form $2{x_1}^2+6{x_2}^2+2{x_3}^2+8x_1x_3$ are – 2, 6, 6, then the nature of the quadratic form is ______.	1 Mark	
	(a) positive semi-definite (b) indefinite (c) negative definite (d) positive definite	Ans (a)	(CLO - 1 Apply)
9.	The matrix corresponding to the quadratic form ${x_1}^2+5{x_2}^2+{x_3}^2+2x_2x_3+6x_3x_1+2x_1x_2$ is	1 Mark	
	(a) $\begin{pmatrix}1&1&3\\1&5&1\\3&1&1\end{pmatrix}$ (b) $\begin{pmatrix}1&2&3\\1&5&2\\3&2&1\end{pmatrix}$ (c) $\begin{pmatrix}1&4&4\\4&5&3\\4&3&1\end{pmatrix}$ (d) $\begin{pmatrix}1&4&3\\4&5&4\\3&4&1\end{pmatrix}$	Ans (a)	(CLO - 1 Apply)
10.	A homogeneous polynomial of the _________ degree in any number of variables is called a quadratic form.	1 Mark	

	(a) first (b) second (c) third (d) fourth	Ans (b)	(CLO - 1 Remember)
11.	A square matrix A is called orthogonal if	1 Mark	
	(a) $A = A^2$ (b) $A = A^{-1}$ (c) $A^T = A^{-1}$ (d) $AA^{-1} = I$	Ans (c)	(CLO - 1 Remember)
12.	The sum of the squares of the eigen values $\begin{pmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{pmatrix}$ is	1 Mark	
	(a) 10 (b) 38 (c) 45 (d) 20	Ans (b)	(CLO - 1 Apply)
13.	All the eigen values of a symmetric matrix with real elements are	1 Mark	
	(a) distinct (b) real (c) equal (d) conjugate complex numbers	Ans (a)	(CLO - 1 Remember)
14.	If the sum of two eigen values and trace of a 3 x 3 matrix A are equal, then the value of det (A) is	1 Mark	
	(a) 0 (b) 1 (c) -1 (d) 2	Ans (a)	(CLO - 1 Apply)
15.	If the canonical form of a quadratic form is $-{y_1}^2 + {y_2}^2 + 2{y_3}^2$, then the signature of the quadratic form is	1 Mark	
	(a) 2 (b) 1 (c) 0 (d) 3	Ans (b)	(CLO - 1Apply)
16.	Find the sum and product of the eigen values of A = $\begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$	1 Mark	

	(a) 5, 3 (b) 3, 5 (c) 2, 1 (d) 0, 1	Ans (b)	(CLO - 1 Apply)
17.	The eigen vectors corresponding to the distinct eigen values of a real symmetric matrix are	1 Mark	
	(a) imaginary (b) non-orthogonal (c) real (d) orthogonal	Ans (d)	(CLO - 1 Remember)
18.	The eigen values of a skew symmetric matrix are	1 Mark	
	(a) real (b) imaginary (c) unitary (d) orthogonal	Ans (b)	(CLO - 1 Remember)
19.	Find the characteristic equation of the matrix A = $\begin{pmatrix} 5 & 4 \\ 1 & 2 \end{pmatrix}$	1 Mark	
	(a) $\lambda^2 - 7\lambda + 6 = 0$ (b) $\lambda^2 + 7\lambda + 6 = 0$ (c) $\lambda^2 - 7\lambda - 6 = 0$ (d) $\lambda^2 - 7\lambda + 5 = 0$	Ans (a)	(CLO - 1 Apply)
20.	The eigen values of an orthogonal matrix have the absolute value	1 Mark	
	(a) 0 (b) 1 (c) 2 (d) 3	Ans (b)	(CLO -1 Remember)
21.	The number of positive terms in the canonical form is called	1 Mark	
	(a) Signature (b) Index (c)quadratic (d)positive definite	Ans (b)	(CLO - 1Remember)
22.	The difference between the positive terms and negative terms in the canonical form is called	1 Mark	
	(a) Signature (b) Index (c)quadratic (d)positive definite	Ans (a)	(CLO - 1 Remember)

23.	Find the eigen values of A^2 if $A = \begin{pmatrix} 3 & 2 & 4 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{pmatrix}$.	1 Mark	
	(a) 6, 4, 10 (b) 9, 4, 25 (c) 9, 2, 5 (d) 3, 2, 5	Ans (b)	(CLO - 1 Apply)
24.	Find the nature of the quadratic form $2x^2 + 3y^2 + 2z^2 + 2xy$	1 Mark	
	(a)Positive definite (b) Negative definite (c) Positive semi-definite (d) Indefinite	Ans (b)	(CLO – 1 Apply)
25.	Find the eigen values of A^{10} if $A = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$	1 Mark	
	(a) $1, 3^{10}$ (b) 1,3 (c) $3^2, 1^{10}$ (d)1,10	Ans (a)	(CLO - 1 Apply)
26.	Find the eigen values of the matrix $A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$.	1 Mark	
	(a) 1, -3 (b) 3, 1 (c) 2, 1 (d) 1, 2	Ans (b)	(CLO - 1 Apply)
27.	If the sum of two eigen values and trace of a 3 x 3 matrix A are equal,then the value of determinant of A is	1 Mark	
	(a) 0 (b) 1 (c) -1 (d) 2	Ans (a)	(CLO - 1 Apply)
28.	Find the eigen values of the matrix $A^3 = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$.	1 Mark	

	(a) 1, -3 (b) 3, 1 (c) 1,9 (d) 1, -9	Ans (c)	(CLO - 1Apply)
29.	The eigen values of the matrix $A^{-1} = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$	1 Mark	
	(a) 4,3 (b) 3, 1 (c) -2, 1 (d) 1, 2	Ans (a)	(CLO - 1 Apply)
30.	Find the sum and product of the eigen values of the matrix $A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$.	1 Mark	
	(a) 4,3 (b) 3, 1 (c) -2, 1 (d) 1, 2	Ans (a)	(CLO - 1 Apply)
31.	Find the eigen values of $A = \begin{pmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{pmatrix}$.	1 Mark	
	(a) $1,3,-4$ (b) $1,-3,-4$ (c) $1,-3,4$ (d) $-1,3,-4$	Ans (a)	(CLO - 1 Apply)
32.	Two eigen values of $A = \begin{pmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -5 & -2 \end{pmatrix}$ are equal and they are double the third. Find them.	1 Mark	
	(a) 1, 2, 2 (b) 2, 1, 1 (c) 2, 0, 1 (d) 1, 2, 3	Ans (a)	(CLO - 1Apply)
33.	The eigen values of a diagonal matrix are the ________ elements of the matrix	1 Mark	

	(a) diagonal (b) upper triangular (c) zero (d) unity	Ans (a)	(CLO - 1 Remember)
34.	Cayley-Hamilton theorem states that "Every __________ matrix satisfies its own characteristic equation".	1 Mark	
	(a) square (b) column (c) row (d) zero	Ans (a)	(CLO - 1Remember)
35.	Find rank and index of the QF whose canonical form is $3x^2 - 3y^2$.	1 Mark	
	(a) 2, 1 (b) 1, 2 (c) 0, 1 (d) 0, 2	Ans (a)	(CLO – 1 Apply)
36.	Write the Q.F. defined by the matrix $A = \begin{pmatrix} 6 & 1 & -7 \\ 1 & 2 & 0 \\ -7 & 0 & 1 \end{pmatrix}$	1 Mark	
	(a) $6x^2 + 2y^2 + z^2 + 2xy - 14xz$ (b) $6x + y^2 + 6z^2 + x\,y - 7xz$ (c) $6x^2 + 2y^2 + z^2 + 2xy + 14xz$ (b) $6x + y^2 + 6z^2 + x\,y - 14xz$	Ans (a)	(CLO -1Apply)

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Unit -II

Functions of several variables

1.	If u and v are functionally dependent then their Jacobian value is	1 Mark	
	a)zero b) one c) non-zero d)greater than zero	Ans (a)	(CLO-2 / Remember)
2.	If $rt - s^2 < 0$ then the point is	1 Mark	
	a)maximum point b) minimum point c) saddle point d) fixed point	Ans (c)	(CLO-2 / Remember)
3.	If $z = x^2 + y^2 + 3xy$ then $\frac{\partial z}{\partial x} =$	1 Mark	
	a)2y+3x b) 3y c) 2x+3y d) 2x	Ans (c)	(CLO-2 / Apply)
4.	If $u = \sin^{-1}(\frac{x^2 + y^2}{x - y})$ is a homogeneous function of degree	1 Mark	
	a) 2 b) 3 c)1 d) 4	Ans (c)	(CLO-2 / Apply)
5.	If f(x,y) is an implicit function then $\frac{dy}{dx} =$	1 Mark	

	a) $-\dfrac{\left(\frac{\partial f}{\partial x}\right)}{\left(\frac{\partial f}{\partial y}\right)}$ b) $\dfrac{\left(\frac{\partial f}{\partial x}\right)}{\left(\frac{\partial f}{\partial y}\right)}$ c) $\dfrac{\left(\frac{\partial f}{\partial y}\right)}{\left(\frac{\partial f}{\partial x}\right)}$ d) $-\dfrac{\left(\frac{\partial f}{\partial y}\right)}{\left(\frac{\partial f}{\partial x}\right)}$	Ans (a)	(CLO-2 / Remember)
6.	If $x = r\cos\theta$ and $y = r\sin\theta$ then $\dfrac{\partial(x,y)}{\partial(r,\theta)} =$	1 Mark	
	a) r b) r^2 c) 2r d) 1/r	Ans (a)	(CLO-2 / Apply)
7.	If u is a homogeneous function of degree n then $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} =$	1 Mark	
	a) n b) nu c) u d) n^2u	Ans (b)	(CLO-2 / Remember)
8.	If $x = u^2 - v^2$ and $y = 2uv$ then $\dfrac{\partial(x,y)}{\partial(u,v)} =$	1 Mark	
	a) $u^2 + v^2$ b) $2(u^2 + v^2)$ c) $4(u^2 + v^2)$ d) $4v^2$	Ans (c)	(CLO-2 / Apply)
9.	If $J_1 = J\left(\dfrac{x,y}{u,v}\right)$ and $J_2 = J\left(\dfrac{u,v}{x,y}\right)$ then $J_1J_2 =$	1 Mark	
	a) 0 b) -1 c) 2 d) 1	Ans (d)	(CLO-2 / Remember)
10.	The point (0.0) for $f(x,y) = x^3 + y^3 - 3axy$ is	1 Mark	
	a)maximum point b) minimum point	Ans (c)	(CLO-2 / Apply)

	c) saddle point d) fixed point		
11.	If $f(x,y)=x^2y+\sin y+e^x$ then $f_x(1,\pi)$ *is* is	1 Mark	
	a) $2\pi-e$ b) 2π c) $2\pi+e$ d) 0	Ans (c)	(CLO-2 / Apply)
12.	The stationary points of $x^2+y^2+6x+12$ are	1 Mark	
	a) (-3,0) b) (0,3) c) (0, -3) d) (3,0)	Ans (a)	(CLO-2 / Apply)
13.	The stationary points for $f(x,y)=\sin x+\sin y+\sin(x+y)$ are	1 Mark	
	a) $\left(\frac{\pi}{2},\frac{\pi}{3}\right)$ b) $\left(\frac{\pi}{3},\frac{\pi}{3}\right)$ c) $\left(\frac{\pi}{2},\frac{\pi}{2}\right)$ d) $\left(\frac{\pi}{3},\frac{\pi}{2}\right)$	Ans (b)	(CLO-2 / Apply)
14.	If $u=x^2-y^2$ *and* $v=2xy$ then $J\left(\frac{x,y}{u,v}\right) X\, J\left(\frac{u,v}{x,y}\right)=$	1 Mark	
	a) 0 b) -1 c) 2 d) 1	Ans (d)	(CLO-2 / Apply)
15.	If $f(x,y)=e^x\cos y$ then $f_{xy}(0,0)$ *is*	1 Mark	
	a) 0 b) -1 c) 2 d) 1	Ans (a)	(CLO-2 / Apply)
16.	If $u=ax^2+by^2+2hxy$ then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=$	1 Mark	

	a) u b) 2u c) 3u d) 4u	Ans (b)	(CLO-2 / Apply)
17.	If $x^y = y^x$, then $\frac{dy}{dx} =$	1 Mark	
	a) $(x\log y - y)y / x(y\log x - x)$ b) $(x\log x - x)/x(y\log y - y)$ c) $(x\log x - y)y/(y\log x - x)$ d) Does not exists	Ans (a)	(CLO-2 / Apply)
18.	If $f(x,y) = e^{xy}$ then $f_{yyy}(1,1)$ *is*	1 Mark	
	a) $-e$ b) $\frac{1}{e}$ c) e d) -$\frac{1}{e}$	Ans (c)	(CLO-2 / Apply)
19.	If $z = \log(x^2 + y^2 + xy)$ then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} =$	1 Mark	
	a) 1 b) 2 c) 0 d) 4	Ans (b)	(CLO-2 / Apply)
20.	If $f(x,y) = \tan^{-1}(y/x)$ then $f_x(1,1)$ *is*	1 Mark	
	a) 1/2 b) -1/2 c) 2 d) 1	Ans (b)	(CLO-2 / Apply)
21.	If V= x/y, then $x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y} =$	1 Mark	
	(a) 2V (b) 3V (c) 4V (d) 0V	Ans (d)	(CLO-2 / Apply)

22.	Saddle points are ------------	1 Mark	
	(a) a minimum (b) a maximum (c) neither a minimum nor a maximum (d) None	Ans (c)	(CLO-2 / Remember)
23.	If $u = x+y/1-xy$, $v= \tan^{-1}x+\tan^{-1}y$ then the functional relationship between u and v is	1 Mark	
	a) $u = \tan v$ (b) $v = \tan u$ (c) $x = \tan y$ (d) $y = \tan x$	Ans (a)	(CLO-2 / Apply)
24.	Lagrange's method of undetermined multipliers is to find the maximum or minimum value of a function of	1 Mark	
	a) Two variables (b) Three or more variables (c) One variable (d) None	Ans (b)	(CLO-2 / Remember)
25.	The condition for a function f(x,y) to have a maximum value is that	1 Mark	
	(a) $rt-s^2$ (b) $rt-s^2>0$, $r>0$ or $s>0$ (c) $rt-s^2>0$, $r<0$ or $s<0$ (d) $rt-s^2 = 0$, $r>0$	Ans (C)	(CLO-2 / Remember)

26.	The condition for a function f(x,y) to have a minimum value is that	1 Mark	
	(a) $rt-s^2$ (b) $rt-s^2>0, r>0$ or $s>0$ (c) $rt-s^2>0, r<0$ or $s<0$ (d) $rt-s^2 = 0, r>0$	Ans (b)	(CLO-2 / Remember)
27.	The condition for a function f(x,y) to have neither a maximum nor a minimum value is that	1 Mark	
	(a) $rt-s^2<0$ (b) $rt-s^2>0, r>0$ or $s>0$ (c) $rt-s^2>0, r<0$ or $s<0$ (d) $rt-s^2 = 0, r>0$	Ans (a)	(CLO-2 / Remember)
28.	The point (a,b) is called a stationary point if	1 Mark	
	(a) $f_x(a,b)=0, f_y(a,b)=0$ (b) $f_{xx}(a,b)=0$ (c) $f_{yy}(a,b)=0$ (d) $f_{xx}(a,b)=0, f_{yy}(a,b)=0$	Ans (a)	(CLO-2 / Remember)
29.	The minimum value of the function $x^2+y^2+6x+12$ is	1 Mark	
	(a) $\frac{1}{2}$ (b) 2 (c) 1 (d) 3	Ans (d)	(CLO-2 / Apply)
30.	The maximum value of the function $x^3+y^3-12x-3y+20$ is	1 Mark	
	(a) 75 (b) 27 (c) 35 (d) 38	Ans (d)	(CLO-2 / Apply)
31.	The points at which there is no extreme value are called __________.	1 Mark	

	(a) maximum points (b) minimum points (c) saddle points (d) none	Ans (C)	(CLO-2 / Remember)
32.	If $u = xe^y \sin x$ $v = xe^y \cos x$ $w = x^2e^{2y}$ then the functional relationship is	1 Mark	
	(a) $u^2+w^2 = v$ (b) $v^2 + w^2 = u$ (c) $x^2+y^2 = u$ (d) $u^2+v^2 = w$	Ans (d)	(CLO-2 / Apply)
33.	In PDE, a real function depends	1 Mark	
	(a) One independent variable (b) More than one independent variable (c) No independent variable (d) None	Ans (b)	(CLO-2 / Remember)
34.	If $z= x^2+y^2+2xy$ then $\partial z/\partial x$ is	1 Mark	
	(a) $2x^2+2y$ (b) $2x+2y$ (c) $2x-2y$ (d) $2y$	Ans (b)	(CLO-2 / Apply)
35.	If $x= r\cos\theta$ $y = r\sin\theta$ then $\frac{\partial(r,\theta)}{\partial(x,y)}$	1 Mark	
	(a) 0 (b) 1 (c) r (d) 1/r	Ans (d)	(CLO-2 / Apply)
36.	If $u = \frac{x}{y} + \frac{y}{z} + \frac{z}{x}$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} =$ ________.	1 Mark	

	(a) 1 (b) 3 u (c) – 1 (d) 0	Ans (d)	(CLO-2 / Apply)

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18MAB101T - Calculus and Linear Algebra

Year/Sem: I/I **Part – A Branch: Common to ALL Branches**

Unit – III Ordinary Differential Equations

1.	The order and degree of $\frac{d^2y}{dx^2}+3\frac{dy}{dx}+2y=e^{-2x}$ are	1 Mark	
	a)2, 1 b) 1, 2 c) 2, 2 d)1, 1	Ans (a)	(CLO-3 Remember)
2.	The order and degree of $\frac{d^2y}{dx^2}+3\left(\frac{dy}{dx}\right)^2+2y=\sin 3x$ are	1 Mark	
	a) 1, 2 b) 2, 1 c) 2, 2 d) 1, 1	Ans (b)	(CLO-3 Remember)
3.	The order and degree of $\left(\frac{d^2y}{dx^2}\right)^2+2\frac{dy}{dx}+y=5x$ are	1 Mark	
	a) 1, 2 b) 2, 1 c) 2, 2 d) 1, 1	Ans (c)	(CLO-3 Remember)
4.	The order and degree of $\frac{dy}{dx}+3y=5x$ are	1 Mark	
	a) 1, 2 b) 2, 1 c) 2, 2 d) 1, 1	Ans (d)	(CLO-3 Remember)
5.	The number of arbitrary constants in the solution of a differential equation is equal to the _________ of that differential equation.	1 Mark	
	a) degree b) number of variables c) order d) number of terms	Ans (b)	(CLO-3 Remember)
6.	The number of arbitrary constants in the most general solution of n^{th} order differential equation is _________	1 Mark	
	a) 1 b) n – 1 c) n d) n + 1	Ans (c)	(CLO-3 Remember)
7.	The solution of $(D^3 - D^2 + D - 1)y = 0$ is	1 Mark	
	a) $y = Ae^x + B\cos x + C\sin x$ b) $y = Ae^x + B\cos x - C\sin x$ c) $y = Ae^{-x} + B\cos x + C\sin x$ d) $y = Ae^x + B\cosh x + C\sinh x$	Ans (a)	(CLO-3 Remember)
8.	The complementary function of $(D^2 + D + 1)y = 0$ is	1 Mark	

	a) $e^{\frac{1}{2}x}\left(C_1 \cos\frac{\sqrt{3}}{2}x + C_2 \sin\frac{\sqrt{3}}{2}x\right)$ b) -1, 2 c) $e^{\frac{-1}{2}x}\left(C_1 \cos\frac{\sqrt{3}}{2}x + C_2 \sin\frac{\sqrt{3}}{2}x\right)$ d) $\cos x + i \sin x$	Ans (c)	(CLO-3 Remember)
9.	The complementary function of $\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 15y = 0$	1 Mark	
	a) $C_1 e^{-5x} + C_2 e^{-3x}$ b) $C_1 e^{4x} + C_2 e^{4x}$ c) $C_1 e^{5x} + C_2 e^{3x}$ d) $C_1 e^{2x} + C_2 e^{6x}$	Ans (c)	(CLO-3 Remember)
10.	The complementary function of $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = 3e^{4x}$	1 Mark	
	a) $C_1 e^{-3x} + C_2 e^{-3x}$ b) $C_1 e^{3x} + C_2 e^{3x}$ c) $(C_1 + C_2 x)e^{-3x}$ d) $(C_1 + C_2 x)e^{3x}$	Ans (c)	(CLO-3 Remember)
11.	The complementary function of $(D^2 + 4)y = x \sin x$ is	1 Mark	
	a) $C_1 e^{-3x} + C_2 e^{-3x}$ b) $C_1 e^{3x} + C_2 e^{3x}$ c) $C_1 \cos 2x + C_2 \sin 2x$ d) $(C_1 + C_2 x)e^{3x}$	Ans (c)	(CLO-3 Remember)
12.	The particular integral of $(D^3 - D^2 + D - 1)y = 0$ is	1 Mark	
	a) 0 b) $Ae^x + B\cos x - C\sin x$ c) $B\cos x + C\sin x$ d) $Ae^x + B\cosh x + C\sinh x$	Ans (a)	(CLO-3 Remember)
13.	The particular integral of $(D^2 + 2D + 1)y = 5$ is	1 Mark	
	a) 0 b) 5 c) 2 d) 1	Ans (b)	(CLO-3 Remember)
14.	The particular integral of $(D^2 + 9)y = e^{-2x}$ is	1 Mark	
	a) $\frac{e^{-2x}}{15}$ b) $\frac{e^{2x}}{15}$ c) $\frac{e^{-2x}}{13}$ d) $\frac{e^{-2x}}{14}$	Ans (c)	(CLO-3 Remember)
15.	The particular integral of $(D^2 + 16)y = e^{-4x}$ is	1 Mark	
	a) $\frac{x}{32}e^{-4x}$ b) $\frac{1}{32}e^{-4x}$ c) $\frac{x}{16}e^{-4x}$ d) $\frac{1}{16}e^{-4x}$	Ans (b)	(CLO-3 Remember)
16.	The particular integral of $(D - 1)^2 y = e^x$ is	1 Mark	
	a) $\frac{x}{32}e^{-4x}$ b) $\frac{x^2}{2}e^{x}$ c) $\frac{x}{16}e^{-4x}$ d) $\frac{1}{16}e^{-4x}$	Ans (b)	(CLO-3 Remember)

17.	The particular integral of $(D^2 + a^2)y = \cos ax$ is	1 Mark	
	a) $\frac{-x}{2a}\sin ax$ b) $\frac{-x}{2a}\cos ax$ c) $\frac{x}{2a}\cos ax$ d) $\frac{x}{2a}\sin ax$	Ans (d)	(CLO-3 Remember)
18.	The particular integral of $(D^2 + 4)y = \sin 2x$ is	1 Mark	
	a) $\frac{x}{2}\sin x$ b) $\frac{-x}{2}\sin x$ c) $\frac{-x}{4}\cos 2x$ d) $\frac{x}{4}\cos 2x$	Ans (c)	(CLO-3 Remember)
19.	The particular integral of $(D^2 + 2)y = x^2$ is	1 Mark	
	a) $\frac{1}{2}x^2$ b) $\frac{1}{2}(x^2-1)$ c) $\frac{1}{2}(x^2+1)$ d) $\frac{-1}{2}x^2$	Ans (b)	(CLO-3 Remember)
20.	The method of variation of parameters is used to find the particular integral of a second order differential equation whose _________ is known.	1 Mark	
	a) Complementary function b) constant c) variable d) degree	Ans (a)	(CLO-3 Remember)
21.	The order and degree of $\left(\frac{d^2y}{dx^2}\right)^2 + 3\frac{dy}{dx} + 2y = e^x$ are	1 Mark	
	a) 2, 1 b) 1, 2 c) 2, 2 d) 1, 1	Ans (c)	(CLO-3 Remember)
22.	The order and degree of $\left(\frac{d^2y}{dx^2}\right)^2 + 3\left(\frac{dy}{dx}\right)^4 + 2y = Sin2x$ are	1 Mark	
	a) 2, 1 b) 1, 2 c) 2, 2 d) 1, 1	Ans (c)	(CLO-3 Remember)
23.	The particular integral of $(D^3 - 1)y = 0$ is	1 Mark	
	a) 0 b) $Ae^x + B\cosh x$ c) $A\cos x + B\sin x$ d) $Ae^x + B\cosh x + C\sinh x$	Ans (a)	(CLO-3 Remember)
24.	The particular integral of $(D^2 + 2D + 1)y = 1$ is	1 Mark	
	a) 0 b) 5 c) 2 d) 1	Ans (d)	(CLO-3 Remember)
25.	The particular integral of $(D^2 + 2)y = x$ is	1 Mark	
	a) $\frac{1}{2}x$ b) $\frac{1}{2}(x^2-1)$ c) $\frac{1}{2}(x^2+1)$ d) $\frac{-1}{2}x^2$	Ans (a)	(CLO-3 Remember)

26.	The particular integral of $(D^2 + 4)y = \cos 2x$ is	1 Mark	
	a) $\frac{x}{2}\sin x$ b) $\frac{-x}{2}\sin x$ c) $\frac{-x}{4}\cos 2x$ d) $\frac{x}{4}\sin 2x$	Ans (d)	(CLO-3 Remember)
27.	The particular integral of $(D^2 + 1)y = \cos 2x$ is	1 Mark	
	a) $\frac{x}{2}\sin x$ b) $\frac{1}{5}\cos 2x$ c) $\frac{-x}{4}\cos 2x$ d) $\frac{x}{4}\sin 2x$	Ans (b)	(CLO-3 Remember)
28.	The complementary function of $\frac{d^2y}{dx^2}+8\frac{dy}{dx}+15y=0$	1 Mark	
	a) $C_1 e^{-5x}+C_2e^{-3x}$ b) $C_1 e^{4x}+C_2e^{4x}$ c) $C_1 e^{5x}+C_2e^{3x}$ d) $C_1 e^{2x}+C_2e^{6x}$	Ans (a)	(CLO-3 Remember)
29.	The complementary function of $(D^2 + 4)y = \sin x$ is	1 Mark	
	a) $C_1 e^{-3x}+C_2e^{-3x}$ b) $C_1 e^{3x}+C_2e^{3x}$ c) $C_1 \cos 2x+C_2 \sin 2x$ d) $(C_1+C_2 x)e^{3x}$	Ans (c)	(CLO-3 Remember)
30.	The complementary function of $\frac{d^2y}{dx^2}-6\frac{dy}{dx}+9y=e^{4x}$	1 Mark	
	a) $C_1 e^{-3x}+C_2e^{-3x}$ b) $C_1 e^{3x}+C_2e^{3x}$ c) $(C_1+C_2 x)e^{-3x}$ d) $(C_1+C_2 x)e^{3x}$	Ans (d)	(CLO-3 Remember)
31.	The particular integral of $(D-1)^2 y = e^{-x}$ is	1 Mark	
	a) $\frac{x}{32}e^{-4x}$ b) $\frac{x^2}{2}e^{x}$ c) $\frac{x}{16}e^{-4x}$ d) $\frac{1}{4}e^{-x}$	Ans (d)	(CLO-3 Remember)
32.	The particular integral of $(D-3)^2y = 3^x$ is	1 Mark	
	a) $\frac{3^x}{(\log 3-3)^2}$ b) $\frac{2^x}{(\log 3-3)^2}$ c) $\frac{3^x}{(\log 2-2)^2}$ d) $\frac{2^x}{(\log 2-2)^2}$	Ans (a)	(CLO-3 Remember)
33.	The complementary function of $(D-1)^2y = e^{-5x}$ is	1 Mark	
	a) $C_1 e^{-x}+C_2e^{-x}$ b) $C_1 e^{x}+C_2e^{x}$ c) $(C_1+C_2 x)e^{x}$ d) $(C_1+C_2 x)e^{-x}$	Ans (c)	(CLO-3 Remember)
34.	The particular integral of $(D+1)^2 y = e^{-5x}$ is	1 Mark	

	a) $\frac{1}{16}e^{-5x}$ b) $\frac{x^2}{2}e^{x}$ c) $\frac{x}{36}e^{-5x}$ d) $\frac{1}{4}e^{-x}$	Ans (a)	(CLO-3 Remember)
35.	The particular integral of $(D^2 + 1)y = \cos x$ is	1 Mark	
	a) $\frac{x}{2}\sin x$ b) $\frac{-x}{3}\cos 2x$ c) $\frac{-x}{4}\cos 2x$ d) $\frac{x}{4}\sin 2x$	Ans (a)	(CLO-3 Remember)
36.	The particular integral of $(D^2 + 9)y = \sin 3x$ is	1 Mark	
	a) $\frac{x}{2}\sin x$ b) $\frac{-x}{6}\cos 3x$ c) $\frac{-x}{4}\cos 2x$ d) $\frac{x}{4}\sin 2x$	Ans (b)	(CLO-3 Remember)

SRM Institute of Science and Technology

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Department of Mathematics

18MAB101T – Calculus and linear algebra

Year/Sem: I/I **Part-A** **Branch: Common to All**

Unit – IV

Differential Calculus

1.	The locus of centre of curvature is called	**1 Mark**	
	(a) Involute (b) Evolute (c) Radius of curvature (d) Envelope	Ans (b)	**(CLO-4 Remember)**
2.	The envelope of the family of curves $A\alpha^2+B\alpha+C=0$ (α is parameter) is	**1 Mark**	
	(a) $B^2 + 4AC = 0$ (b) $B^2 - 4AC = 0$ (c) $B^2 + AC = 0$ (d) $B^2 - AC = 0$	Ans (b)	**(CLO-4 Remember)**
3.	The curvature of the straight line is	**1 Mark**	
	(a) 1 (b) 2 (c) -1 (d) 0	Ans (d)	**(CLO-4 Remember)**
4.	Evolute of a curve is ______ of the normals of that curve	**1 Mark**	
	(a) Involute (b) Length (c) Envelope (d) End points	Ans (c)	**(CLO-4 Remember)**
5	The radius of curvature at (3,4) on the curve $x^2 + y^2 = 25$ is	**1 Mark**	
	(a) 5 (b) 4 (c) 0 (d) 2	Ans (a)	**(CLO-4 Remember)**

6.	What is the curvature of a circle of radius 3?	**1 Mark**	
	(a) 3 (b) – 3 (c) $\frac{1}{3}$ (d) $\frac{-1}{3}$	Ans (c)	**(CLO-4 Remember)**
7.	In an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ the radius of curvature at the end of which axis is equal to the semi-latus rectum?	**1 Mark**	
	(a) minor (b) major (c) vertical (d) horizontal	Ans (b)	**(CLO-4 Remember)**
8.	Evolute of a curve is the envelope of ______ of that curve.	**1 Mark**	
	(a) tangent (b) normal (c) parallel (d) locus	Ans (b)	**(CLO-4 Remember)**
9.	The evolute of the cycloid $x = a\,(\theta - \sin\theta)$, $y = a\,(1 - \cos\theta)$ is	**1 Mark**	
	(a) astroid (b) parabola (c) cycloid (d) circle	Ans (c)	**(CLO-4 Remember)**
10.	A curve which touches each member of a family of the curves is called $---$ of that family	**1 Mark**	
	(a) Evolute (b) Envelope (c) Circle of curvature (d) Radius of curvature	Ans (b)	**(CLO-4 Remember)**
11.	Envelope of the curve $y = mx + \frac{a}{m}$ (*where m is the parameter*) is	**1 Mark**	
	(a) $x^2 + ay = 0$ (b) $x + 4ay = 0$ (c) $y^2 - 4ax = 0$ (d) $y^2 + 4ax = 0$	Ans (c)	**(CLO-4 Remember)**

12.	If the radius of curvature and curvature of a curve at any point are ρ and k respectively, then	**1 Mark**	
	(a) $\rho = \frac{-1}{k}$ (b) $\rho = k$ (c) $\rho = -k$ (d) $\rho = \frac{1}{k}$	Ans (d)	**(CLO-4 Remember)**
13	The radius of curvature at the point $(0, c)$ of the curve $y = c \cosh\left(\frac{x}{c}\right)$ is	**1 Mark**	
	(a) $\rho = c$ (b) $\rho = c^2$ (c) $\rho = kc$ (d) $\rho = kc^2$	Ans (a)	**(CLO-4 Remember)**
14	The radius of curvature of the curve $y = e^x$ at $x=0$ *is*	**1 Mark**	
	(a) $2\sqrt{2}$ (b) $\sqrt{2}$ (c) 2 (d) *4*	Ans (a)	**(CLO-4 Remember)**
15	The radius of curvature at the point (x, y) of the curve $y = c \log \sec\left(\frac{x}{c}\right)$ is	**1 Mark**	
	(a) $\rho = c\sec\left(\frac{x}{c}\right)$ (b) $\rho = c\cos\left(\frac{x}{c}\right)$ (c) $\rho = c\sin\left(\frac{x}{c}\right)$ (d) $\rho = c\tan\left(\frac{x}{c}\right)$	Ans (a)	**(CLO-4 Remember)**
16	The parametric form of the curve $y^2 = 4ax$ is	**1 Mark**	
	(a) $x = at^2; y = 2at$ (b) $x = at; y = 2at$ (c) $x = at^2; y = 2at^2$ (d) $x = 2at^2; y = 2at$	Ans (a)	**(CLO-4 Remember)**

17	The envelope of the curve $y = mx + \frac{a}{m}$ where m is the parameter is	**1 Mark**	
	(a) $y^2 - 4ax = 0$ (b) $y^2 + 4ax = 0$ (c) $x^2 + y^2 = 1$ (d) $xy = c^2$	Ans (a)	**(CLO-4 Remember)**
18	The radius of curvature of the curve $y = \log \sec x$ at any point on it is	**1 Mark**	
	(a) $\sec x$ (b) $\tan x$ (c) $\cot x$ (d) $\operatorname{cosec} x$	Ans (a)	**(CLO-4 Remember)**
19	The radius of curvature of the curve $x = t^2$, $y = t$ at $t = 1$ is	**1 Mark**	
	(a) $5\frac{\sqrt{5}}{2}$ (b) $\frac{\sqrt{5}}{2}$ (c) $\frac{5}{2}$ (d) $\sqrt{5}$	Ans (a)	**(CLO-4 Remember)**
20	The radius of curvature of the parabola $y^2 = 12x$ at (3, 6) is	**1 Mark**	
	(a) $12\sqrt{2}$ (b) $2\sqrt{2}$ (c) $10\sqrt{2}$ (d) $\sqrt{2}$	Ans (a)	**(CLO-4 Remember)**
21	The radius of curvature of the curve $y = 4 \sin x$ at $x = \frac{\pi}{2}$ is	**1 Mark**	
	(a) $\frac{1}{2}$ (b) $\frac{-1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{-1}{3}$	Ans (c)	**(CLO-4 Remember)**
22	The envelope of family of lines $y = m x + a m^2$ (where m is the parameter) is	**1 Mark**	
	(a) $x^2 + 2ay = 0$ (b) $x^2 + 4ay = 0$ (c) $y^2 + 2ax = 0$ (d) $x^2 + 4ax = 0$	Ans (b)	**(CLO-4 Remember)**

23	The envelope of the family of lines $\frac{x}{t} + yt = 2c$, t being the parameter is	**1 Mark**	
	(a) $x^2 + y^2 = c^2$ (b) $xy = c^2$ (c) $x^2 y^2 = c^2$ (d) $x^2 - y^2 = c^2$	Ans (b)	**(CLO-4 Remember)**
24	The radius of curvature at any point on the curve $r = e^{\theta}$ is	**1 Mark**	
	(a) $\frac{\sqrt{2}}{r}$ (b) $\frac{r}{\sqrt{2}}$ (c) r (d) $\sqrt{2}\, r$	Ans (d)	**(CLO-4 Remember)**
25	The radius of curvature in Cartesian coordinates is	**1 Mark**	
	(a) $\rho = \frac{(1+y_1^2)^{\frac{3}{2}}}{y_2}$ (b) $\rho = \frac{(1-y_1^2)^{\frac{3}{2}}}{y_2}$ (c) $\rho = \frac{(1+y_1^2)^{\frac{2}{3}}}{y_2}$ (d) $\rho = \frac{(1+y_1^2)^{\frac{3}{2}}}{y_1}$	Ans (a)	**(CLO-4 Remember)**
26	The envelope of $ty - x = at^2$, t is the parameter is	**1 Mark**	
	(a) $x^2 = 4ay$ (b) $y^2 = 4ax$ (c) $x^2 + y^2 = 1$ (d) $x^2 - y^2 = 1$	Ans (b)	**(CLO-4 Remember)**
27	The radius of curvature in polar coordinates is	**1 Mark**	
	(a) $\rho = \frac{\left(r^2 + r'^2\right)^{\frac{3}{2}}}{r^2 - rr' + 2r'^2}$ (b) $\rho = \frac{\left(r^2 - r'^2\right)^{\frac{3}{2}}}{r^2 - rr' + 2r'^2}$ (c) $\rho = \frac{\left(r^2 - r''^2\right)^{\frac{3}{2}}}{r^2 - rr' + 2r'^2}$ (d) $\rho = \frac{\left(r^2 + r'^2\right)^{\frac{3}{2}}}{r^2 - rr'' + 2r'^2}$	Ans (d)	**(CLO-4 Remember)**

28	The radius of curvature in parametric coordinates is	**1 Mark**	
	(a) $\rho=\frac{\left(x'^2+y'^2\right)^{\frac{3}{2}}}{x'y''-y'x''}$ (b) $\rho=\frac{\left(x'^2+y'^2\right)^{\frac{3}{2}}}{x'y''+y'x''}$ (c) $\rho=\frac{\left(x'^2-y'^2\right)^{\frac{3}{2}}}{x'y''-y'x''}$ (d) $\rho=\frac{\left(x'^2-y'^2\right)^{\frac{3}{2}}}{x'y''+y'x''}$	Ans (a)	**(CLO-4 Remember)**
29	The equation of circle of curvature at any point (x, y) with center of curvature $(\bar{x},\bar{y})$ and radius of curvature ρ is	**1 Mark**	
	(a) $(x+\bar{x})^2+(y+\bar{y})^2=\rho^2$ (b) $(x-\bar{x})^2+(y-\bar{y})^2=\rho^2$ (c) $(x-\bar{x})^2-(y+\bar{y})^2=\rho^2$ (d) $(x+\bar{x})^2+(y+\bar{y})^2=\rho^2$	Ans (b)	**(CLO-4 Remember)**
30	The curvature at any point of the circle is equal to___ of its radius	**1 Mark**	
	(a) same (b) ellipse (c) reciprocal (d) constant	Ans (c)	**(CLO-4 Remember)**
31	The parametric equations of rectangular hyperbola $xy = c^2$ is	**1 Mark**	
	(a) $x=ct, y=\frac{c}{t}$ (b) $x=ct, y=t$ (c) $x=\frac{c}{t}, y=t$ (d) $x=ct, y=\frac{1}{t}$	Ans (a)	**(CLO-4 Remember)**
32	The value of $\Gamma\left(\frac{1}{2}\right)$ is	**1 Mark**	
	(a) π (b) $\frac{\pi}{2}$ (c) $\sqrt{\pi}$ (d) $\frac{\sqrt{\pi}}{2}$	Ans (c)	**(CLO-4 Remember)**

33	The series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if	**1 Mark**	
	(a) p = 0 (b) p =1 (c) p > 1 (d) p < 1	Ans (c)	**(CLO-4 Remember)**
34	As per D' Alembert's ratio test, if $\sum u_n$ is a series of positive terms and $\sum_{n\to\infty} \frac{u_{n+1}}{u_n} = l$, then the series is convergent, if	**1 Mark**	
	(a) $l = 0$ (b) $l = 1$ (c) $l > 1$ (d) $l < 1$	Ans (d)	**(CLO-4 Remember)**
35	If n is a positive integer, then $\Gamma(n+1) =$	**1 Mark**	
	(a) $(n + 1)!$ (b) $n!$ (c) $2n!$ (d) $(n - 1)!$	Ans (b)	**(CLO-4 Remember)**
36	The series $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + ...$ is	**1 Mark**	
	(a) convergent (b) divergent (c) oscillating (d) monotonic	Ans (b)	**(CLO-4 Remember**

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18MAB101T – Calculus and Linear Algebra

Year/Sem: I/I | **Part – A** | **Branch: Common to All**

Unit – V

Sequence and Series

1.	A sequence $\{a_n\}$ is said to be convergent if	**1 Mark**	
	(a) $\lim_{n\to\infty} a_n = finite$ (b) $\lim_{n\to\infty} a_n = \infty$ (c) $\lim_{n\to\infty} a_n = -\infty$ (d) $\lim_{n\to\infty} a_n = infinite$	Ans (a)	**(CLO 5, Remember)**
2.	The sequence $\left\{(-1)^n\right\}$ is	**1 Mark**	
	(a) oscillatory (b) monotonic (c) divergent to $-\infty$ (d) divergent to $+\infty$	Ans (a)	**(CLO 5, Remember)**
3.	A sequence which is monotonic and bounded is	**1 Mark**	
	(a) conditionally convergent (b) absolutely convergent (c) convergent (d) divergent	Ans (c)	**(CLO 5, Remember)**
4.	The necessary condition for the convergence of $\sum u_n$ is	**1 Mark**	
	(a) $\lim_{n\to\infty} u_n = 0$ (b) $\lim_{n\to\infty} u_n = \infty$ (c) $\lim_{n\to\infty} u_n = -\infty$ (d) $\lim_{n\to\infty} u_n \neq 0$	Ans (a)	**(CLO 5, Remember)**
5.	If $\{a_n\}$ and $\{b_n\}$ are two convergent sequences, then $\{a_n + b_n\}$ is	**1 Mark**	
	(a) convergent (b) divergent (c) oscillatory (d) neither convergent nor divergent	Ans (a)	**(CLO 5, Remember)**
6.	The geometric series $1 + x + x^2 + x^3 + \cdots$ converges if	**1 Mark**	
	(a) $-1 < x < 1$ (b) $x < 1$ (c) $x > 1$ (d) $x \geq 1$	Ans (a)	**(CLO 5, Remember)**
7.	The series $\frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \cdots$ converges if	**1 Mark**	
	(a) $p > 0$ (b) $p < 1$ (c) $p > 1$ (d) $p \leq 1$	Ans (c)	**(CLO 5, Remember)**

8.	The series $\frac{1}{1^p}+\frac{1}{2^p}+\frac{1}{3^p}+\cdots$ diverges if	**1 Mark**	
	(a) $p>0$ (b) $p<1$ (c) $p>1$ (d) $p\le 1$	Ans (d)	**(CLO 5, Remember)**
9.	If $\sum u_n$ is a convergent series, then $\lim_{n\to\infty} u_n =$	**1 Mark**	
	(a) 1 (b) ± 1 (c) 0 (d) ∞	Ans (c)	**(CLO 5, Remember)**
10.	According to D' Alembert's ratio test, if $\sum u_n$ is a series of positive terms and $\lim_{n\to\infty}\frac{u_{n+1}}{u_n}=l$, then $\sum u_n$ converges if	**1 Mark**	
	(a) $l<1$ (b) $l>1$ (c) $l=1$ (d) $-1<l<1$	Ans (a)	**(CLO 5, Remember)**
11.	By Cauchy's root test, if $\sum u_n$ is a series of positive terms and $\lim_{n\to\infty}(u_n)^{\frac{1}{n}}=l$, then $\sum u_n$ converges if	**1 Mark**	
	(a) $l<1$ (b) $l>1$ (c) $l=1$ (d) $-1<l<1$	Ans (a)	**(CLO 5, Remember)**
12.	By Raabe's test, if $\lim\limits_{n\to\infty} n\left(\frac{u_n}{u_{n+1}}-1\right)=k$, then $\sum u_n$ diverges if	**1 Mark**	
	(a) $k<1$ (b) $k>1$ (c) $k=1$ (d) $-1<k<1$	Ans (a)	**(CLO 5, Remember)**
13.	By Logarithmic test, if $\lim\limits_{n\to\infty} n\log\frac{u_n}{u_{n+1}}=l$, then $\sum u_n$ converges if	**1 Mark**	
	(a) $l<1$ (b) $l>1$ (c) $l=1$ (d) $-1<l<1$	Ans (b)	**(CLO 5, Remember)**
14.	The series $\sum u_n$ containing positive and negative terms is __________, if $\sum\lvert u_n\rvert$ is divergent but $\sum u_n$ is convergent.	**1 Mark**	
	(a) divergent (b) oscillating finitely (c) oscillating infinitely (d) conditionally convergent	Ans (d)	**(CLO 5, Remember)**
15.	The series $\sum u_n$ containing positive and negative terms is absolutely convergent, if $\sum\lvert u_n\rvert$ is	**1 Mark**	
	(a) convergent (b) divergent to $-\infty$ (c) divergent to $+\infty$ (d) oscillatory	Ans (a)	**(CLO 5, Remember)**

16.	Every absolutely convergent series is necessarily	1 Mark	
	(a) divergent (b) convergent (c) oscillatory (d) conditionally convergent	Ans (b)	**(CLO 5, Remember)**
17.	The series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if	1 Mark	
	(a) $p = 0$ (b) $p = 1$ (c) $p > 1$ (d) $p < 1$	Ans (c)	**(CLO 5, Remember)**
18.	As per D' Alembert's ratio test, if $\sum u_n$ is a series of positive terms and $\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = l$, then the series is divergent if	1 Mark	
	(a) $l = 0$ (b) $l = 1$ (c) $l > 1$ (d) $l < 1$	Ans (c)	**(CLO 5, Remember)**
19.	A series of positive terms cannot ____________.	1 Mark	
	(a) oscillate (b) absolutely converge (c) converge (d) diverge	Ans (a)	**(CLO 5, Remember)**
20.	The series $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots$ is	1 Mark	
	(a) divergent (b) conditionally convergent (c) oscillatory (d) neither convergent nor divergent	Ans (b)	**(CLO 5, Apply)**
21.	The series $1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots$ is	1 Mark	
	(a) divergent (b) absolutely convergent (c) oscillatory (d) neither convergent nor divergent	Ans (b)	**(CLO 5, Apply)**
22.	By Raabe's test, if $\lim_{n\to\infty} n\left(\frac{u_n}{u_{n+1}} - 1\right) = k$, then $\sum u_n$ converges if	1 Mark	
	(a) $k < 1$ (b) $k > 1$ (c) $k = 1$ (d) $-1 < k < 1$	Ans (b)	**(CLO 5, Remember)**

23.	By Cauchy's root test, if $\sum u_n$ is a series of positive terms and $\lim_{n\to\infty}(u_n)^{\frac{1}{n}} = l$, then $\sum u_n$ diverges if	**1 Mark**	
	(a) $l < 1$ (b) $l > 1$ (c) $l = 1$ (d) $-1 < l < 1$	Ans (b)	**(CLO 5, Remember)**
24.	By Logarithmic test, if $\lim_{n\to\infty} n\log\frac{u_n}{u_{n+1}} = l$, then $\sum u_n$ diverges if	**1 Mark**	
	(a) $l < 1$ (b) $l > 1$ (c) $l = 1$ (d) $-1 < l < 1$	Ans (a)	**(CLO 5, Remember)**
25.	If $-1 < x < 1$, then the geometric series $1 + x + x^2 + x^3 + \cdots$ converges to	**1 Mark**	
	(a) $\frac{1}{1-x}$ (b) $\frac{1}{1+x}$ (c) e^x (d) $\frac{1}{x!}$	Ans (a)	**(CLO 5, Remember)**
26.	The geometric series $1 + x + x^2 + x^3 + \cdots$ oscillates finitely if	**1 Mark**	
	(a) $x = -1$ (b) $x < 1$ (c) $x > 1$ (d) $x \geq 1$	Ans (a)	**(CLO 5, Remember)**
27.	The geometric series $1 + x + x^2 + x^3 + \cdots$ oscillates infinitely if	**1 Mark**	
	(a) $x < -1$ (b) $x < 1$ (c) $x > 1$ (d) $x \geq 1$	Ans (a)	**(CLO 5, Remember)**
28.	If $\sum u_n$ is convergent, then $\sum k u_n$ (where k is constant) is	**1 Mark**	
	(a) convergent (b) divergent (c) oscillatory (d) neither convergent nor divergent	Ans (a)	**(CLO 5, Remember)**
29.	The series $1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + \cdots$ is	**1 Mark**	
	(a) divergent (b) neither convergent nor divergent (c) oscillatory (d) conditionally convergent	Ans (d)	**(CLO 5, Apply)**
30.	The convergence of the series $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$ is tested by	**1 Mark**	
	(a) Ratio test (b) Raabe's test (c) Leibnitz test (d) Cauchy Root test	Ans (c)	**(CLO 5, Remember)**

31.	A monotonic increasing sequence which is not bounded above is ______.	**1 Mark**	
	(a) oscillatory (b) convergent (c) divergent to $-\infty$ (d) divergent to $+\infty$	Ans (d)	**(CLO 5, Remember)**
32.	A monotonic decreasing sequence which is not bounded below is ______.	**1 Mark**	
	(a) oscillatory (b) convergent (c) divergent to $-\infty$ (d) divergent to $+\infty$	Ans (c)	**(CLO 5, Remember)**
33.	The series $\sum u_n$ of positive terms is convergent if	**1 Mark**	
	(a) $\lim\limits_{n\to\infty} \frac{u_{n+1}}{u_n} < 1$ (b) $\lim\limits_{n\to\infty} \frac{u_n}{u_{n+1}} = 1$ (c) $\lim\limits_{n\to\infty} \frac{u_n}{u_{n+1}} \le 1$ (d) $\lim\limits_{n\to\infty} \frac{u_{n+1}}{u_n} > 1$	Ans (a)	**(CLO 5, Remember)**
34.	The nth term of a series in Arithmetic Progression is	**1 Mark**	
	(a) $t_n = a-(n-1)d$ (b) $t_n = a+(n+1)d$ (c) $t_n = a-(n+1)d$ (d) $t_n = a+(n-1)d$	Ans (d)	**(CLO 5, Remember)**
35.	$\sum_{1}^{\infty} \frac{n^3}{3^n}$ is	**1 Mark**	
	(a) oscillatory (b) convergent (c) divergent to $-\infty$ (d) divergent to $+\infty$	Ans (b)	**(CLO 5, Apply)**
36.	The series $\sum \frac{1}{n} \sin\left(\frac{1}{n}\right)$ is	**1 Mark**	
	(a) oscillatory (b) convergent (c) divergent (d) conditionally convergent	Ans (b)	**(CLO 5, Apply)**
37.	If D'Alembert's ratio test fails, then use	**1 Mark**	
	(a) Comparison test (b) Leibnitz's test (c) Cauchy's integral test (d) Raabe's test	Ans (d)	**(CLO 5, Remember)**
38.	The series $\sum \frac{1}{n!}$ is	**1 Mark**	
	(a) oscillatory (b) convergent (c) divergent (d) conditionally convergent	Ans (b)	**(CLO 5, Apply)**

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Unit – I

MATRICES

Part – B

1. Find the characteristic equation of the matrix $A = \begin{pmatrix} 1 & 2 \\ 0 & 2 \end{pmatrix}$.

(A) $\lambda^2 - 3\lambda + 2 = 0$ **(B)** $\lambda^2 + 3\lambda + 2 = 0$

(C) $\lambda^2 - 3\lambda - 2 = 0$ **(D)** $\lambda^2 + 3\lambda - 2 = 0$

Solution: Let A = $\begin{pmatrix} 1 & 2 \\ 0 & 2 \end{pmatrix}$. Its characteristic equation is $\lambda^2 - S_1\lambda + S_2 = 0$ where $S_1 = sum\ of\ the\ main\ diagonal\ elements = 1 + 2 = 3$,

$S_2 = Determinant\ of\ A = |A|$ = 1(2) – 2(0) = 2

Therefore, the characteristic equation is $\lambda^2 - 3\lambda + 2 = 0$ **(Option A)**

2. Find the characteristic equation of $A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$.

(A) $\lambda^3 - 18\lambda^2 + 45\lambda = 0$ **(B)** $\lambda^3 - 28\lambda^2 + 45\lambda = 0$

(C) $\lambda^3 - 18\lambda^2 + 35\lambda = 0$ **(D)** $\lambda^3 - 18\lambda^2 - 45\lambda = 0$

Solution: Its characteristic equation is $\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$, where $S_1 = sum\ of\ the\ main\ diagonal\ elements$ = 8 + 7 + 3 = 18, $S_2 = Sum\ of\ the\ minors\ of\ the\ main\ diagonal\ elements = \begin{vmatrix} 7 & -4 \\ -4 & 3 \end{vmatrix} + \begin{vmatrix} 8 & 2 \\ 2 & 3 \end{vmatrix} + \begin{vmatrix} 8 & -6 \\ -6 & 7 \end{vmatrix} = 5 + 20 + 20 = 45$, $S_3 = Determinant\ of\ A = |A|$ = 8(5)+6(-10)+2(10) = 40 – 60 + 20 = 0

Therefore, the characteristic equation is $\lambda^3 - 18\lambda^2 + 45\lambda = 0$ **(Option A)**

3. Find the eigen values of the matrix $\boldsymbol{A = \begin{pmatrix} 1 & 1 \\ 3 & -1 \end{pmatrix}}$.

(A) $\boldsymbol{2, -2}$ **(B) $\boldsymbol{1, -1}$**

(C) $\boldsymbol{3, -3}$ **(D) $\boldsymbol{2, 2}$**

Solution: Let A = $\begin{pmatrix} 1 & 1 \\ 3 & -1 \end{pmatrix}$ which is a non-symmetric matrix.

The characteristic equation of A is $\lambda^2 - S_1\lambda + S_2 = 0$ where $S_1 = sum\ of\ the\ main\ diagonal\ elements = 1 - 1 = 0$,

$S_2 = Determinant\ of\ A = |A|$ = 1(-1) – 1(3) = - 4

Therefore, the characteristic equation is $\lambda^2 - 4 = 0$ i.e., $\lambda^2 = 4$ or $\lambda = \pm 2$

Therefore, the eigen values are 2, -2. **(Option A)**

4. Find the sum and product of the eigen values of the matrix $\boldsymbol{A = \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}}$.

(A) $\boldsymbol{-3, 4}$ **(B) $\boldsymbol{-3, -4}$**

(C) $\boldsymbol{3, 4}$ **(D) $\boldsymbol{-3, -4}$**

Solution: Sum of the eigen values = Sum of the main diagonal elements = -3

Product of the eigen values = $|A|$ = -1 (1 – 1) -1(-1 – 1) + 1(1- (-1)) = 2 + 2 = 4

(Option A)

5. Find the sum and product of eigen values of the matrix A^T where $\boldsymbol{A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}}$.

(A) $\boldsymbol{18, 0}$ **(B) $\boldsymbol{18, 2}$**

(C) $\boldsymbol{28, 0}$ **(D) $\boldsymbol{18, -2}$**

Solution: Since matrix A is symmetric , A and A^T have same eigen values.

Sum of Eigen value of A^T = trace(A) = 8+7+3=18

Product of Eigen value of $A^T = |A|$ = 8(5) + 6(−10)+2(10) = 40 −60 + 20 = 0 **(Option A)**

6. If 1, 1, 5 are the eigen values of A = $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$, find the eigen values of 5A.

(A) $5, 5, 2$ **(B) $5, 5, 25$** **(C) $2, 3, 2$** **(D) $7, 8, 7$**

<u>Solution</u>: By the property "If $\lambda_1, \lambda_2, \lambda_3$ are the eigen values of A, then $k\lambda_1, k\lambda_2, k\lambda_3$ are the eigen values of kA, the eigen values of 5A are 5(1), 5(1), 5(5) ie., 5,5,25**. (Option B)**

7. Find the eigen values of the matrix 2A^{-1} where A= $\begin{pmatrix} 3 & 8 & 9 \\ 0 & 1 & 3 \\ 0 & 0 & -2 \end{pmatrix}$.

(A) $\frac{2}{3}, 2, -1$ **(B) $\frac{1}{3}, 2, -4$** **(C) $\frac{2}{3}, 2, 1$** **(D) $\frac{2}{3}, 1, -2$**

<u>Solution</u>: Since given matrix is triangular matrix, the Eigen values are its diagonal elements.

$\therefore \quad \lambda_1 = 3, \ \lambda_2 = 1, \ \lambda_3 = -2$

Eigen values of 2A^{-1} are $\frac{2}{3}$, 2 , -1 **(Option A)**

8. If the sum of two eigen values and trace of a 3 × 3 matrix A are equal, find the value of $|A|$.

(A) 5 **(B) 25** **(C) 2** **(D) 0**

<u>Solution:</u> Sum of the eigen values = $\lambda_1 + \lambda_2 + \lambda_3$ = sum of the diagonal elements

Given $\lambda_1 + \lambda_2$ = trace of A.

i.e., $\lambda_1 + \lambda_2 = \lambda_1 + \lambda_2 + \lambda_3$

Therefore $\lambda_3 = 0$. Then $|A|$ = Product of Eigen values = $\lambda_1 \lambda_2 \lambda_3 = 0$ **(Option D)**

9. Write the matrix corresponding to the quadratic form $x^2 + 2yz$.

(A) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 1 & 0 \end{bmatrix}$ **(B)** $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 2 & 0 \end{bmatrix}$ **(C)** $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ **(D)** $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 2 & 0 \end{bmatrix}$

Solution: Given $\boldsymbol{X^T A X = x^2 + 2yz}$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \textbf{ (Option C)}$$

10. If 2, 3, –1 are the eigen values of 3 × 3 matrix, find rank, index and signature of the quadratic form.

(A) $\boldsymbol{5, 5, 2}$ **(B)** $\boldsymbol{5, 5, 25}$ **(C)** $\boldsymbol{3, 2, 1}$ **(D)** $\boldsymbol{1, 2, 3}$

Solution: Rank (r) = number of non zero terms in canonical form = 3

Index (p) = Number of positive terms in canonical form = 2

Signature (s) = Difference between number of positive terms and negative terms

= 2p – r

= 4 – 3

= 1 **(Option C)**

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Unit – II

FUNCTIONS OF SEVERAL VARIABLES

Part – B

1. **If $u = (x-y)(y-z)(z-x)$, then find $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}$.**

   (A) 0 (B) 1 (C) 2 (D) 3

   Solution:

   Given $u = (x-y)(y-z)(z-x)$

$$\frac{\partial u}{\partial x} = (y-z)[(x-y)(-1)+(z-x)(1)]$$
$$= -(x-y)(y-z)+(y-z)(z-x)$$
$$\frac{\partial u}{\partial y} = (z-x)[(x-y)(1)+(y-z)(-1)]$$
$$= (x-y)(z-x)-(y-z)(z-x)$$
$$\frac{\partial u}{\partial z} = (x-y)[(y-z)(1)+(z-x)(-1)]$$
$$= (x-y)(y-z)-(x-y)(z-x)$$
$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z} = 0 \quad (\textbf{Option A})$$

2. **If $x = r\cos\theta, y = r\sin\theta$ find $\frac{\partial x}{\partial r}, \frac{\partial y}{\partial \theta}$.**

   (A) cos θ, sin θ (B) cos θ, *r* cos θ (C) *r* cos θ, sin θ (D) *r*, θ

   Solution:

$$\frac{\partial x}{\partial r} = \cos\theta$$
$$\frac{\partial y}{\partial \theta} = r\cos\theta \quad (\textbf{Option B})$$

3. **If $f(x,y) = \sin\left(\frac{x}{y}\right)$, then find $x\frac{\partial f}{\partial x} + y\ \frac{\partial f}{\partial y}$.**

(A) 0 **(B) 1** **(C) 2** **(D) 3**

Solution:

$$\frac{\partial f}{\partial x} = \cos\left(\frac{x}{y}\right)\frac{1}{y}, \quad \frac{\partial f}{\partial y} = \cos\left(\frac{x}{y}\right)\left(-\frac{x}{y^2}\right)$$

$$x\frac{\partial f}{\partial x} + y\ \frac{\partial f}{\partial y} = 0 \text{ (Option A)}$$

4. **Find $\frac{dy}{dx}$ when $x^3 + y^3 = 3axy$.**

$(A) -\frac{x^2-ay}{y^2-ax}$ $(B)\frac{x^2-ay}{y^2-ax}$ $(C)\frac{y^2-ax}{x^2-ay}$ $(D) -\frac{y^2-ax}{x^2-ay}$

Solution:

Let $f(x,y) = x^3 + y^3 - 3axy$

$$\frac{dy}{dx} = -\frac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial y}} = -\frac{3x^2 - 3ay}{3y^2 - 3ax}$$

$$= -\frac{x^2 - ay}{y^2 - ax} \quad \text{(Option A)}$$

5. **If $x = uv,\ y = \frac{u}{v}$, find $\frac{\partial(x,y)}{\partial(u,v)}$.**

(A) $\frac{-2u}{v}$ **(B)** $\frac{2u}{v}$ **(C)** $\frac{-2v}{u}$ **(D)** $\frac{2v}{u}$

Solution:

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} v & u \\ \frac{1}{v} & \frac{-u}{v^2} \end{vmatrix} = -\frac{2u}{v} \quad \text{(Option A)}$$

6. If $f(x,y) = e^x \sin y$, then find $f_{yy}(0,0)$.

(A) 0 **(B) 1** **(C) 2** **(D) 3**

Solution:

$f_y = e^x \cos y$

$f_{yy}(x,y) = e^x(-\sin y)$

$f_{yy}(0,0) = 0$ **(Option A)**

7. If $x^y = y^x$, then find $\frac{dy}{dx}$.

(A) $\frac{y\,x^{y-1} - y^x \log y}{x\,y^{x-1} - x^y \log x}$ **(B)** $\frac{y\,x^{y-1} + y^x \log y}{x\,y^{x-1} - x^y \log x}$

(C) $\frac{y\,x^{y-1} + y^x \log y}{x\,y^{x-1} + x^y \log x}$ **(D)** $\frac{y\,x^{y-1} - y^x \log y}{x\,y^{x-1} + x^y \log x}$

Solution:

$f(x, y) = x^y - y^x = 0$

$$\frac{dy}{dx} = \frac{-f_x}{f_y} = \frac{y\,x^{y-1} - y^x \log y}{x\,y^{x-1} - x^y \log x}$$ **(Option A)**

8. If $f(x, y) = \tan^{-1}\left(\frac{y}{x}\right)$, then find $f_x(x, y)$ at the point (1, 1).

(A) -1/2 **(B) 1** **(C) 1/2** **(D) 3**

Solution:

$f_x(x, y) = \frac{-y}{x^2 + y^2}$ $f_x(1,1) = -\frac{1}{2}$ **(Option A)**

9. If $x = r\cos\theta$, $y = r\sin\theta$, then find $\frac{\partial(x, y)}{\partial(r, \theta)}$.

(A) r **(B) 1/ r** **(C) 1/2** **(D) 1**

Solution:

Now $$\frac{\partial(x, y)}{\partial(r, \theta)} = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial v}{\partial \theta} \end{vmatrix} = \begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix} = r\left(\sin^2\theta + \cos^2\theta\right) = r(1) = r$$

(Option A)

10. If $u = 2xy$, $v = x^2 - y^2$, then find $\frac{\partial(u,v)}{\partial(x,y)}$.

(A) $-4y^2 - 4x^2$ **(B) $-4y^2 + 4x^2$**
(C) $4y^2 - 4x^2$ **(D) $4y^2 + 4x^2$**

$$\frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} 2y & 2x \\ 2x & -2y \end{vmatrix} = -4y^2 - 4x^2$$ **(Option A)**

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Unit – III

ORDINARY DIFFERENTIAL EQUATIONS

Part – B

1. Solve $(D^2 - 7D + 12)y = 0$.

$(a) y = Ae^{3x} + Be^{4x}$ $(b) y = Ae^{-3x} + Be^{4x}$

$(c) y = Ae^{3x} + Be^{-4x}$ $(d) y = Ae^{-3x} + Be^{-4x}$

$m^2 - 7m + 12 = 0$
$(m-3)(m-4) = 0$
$m = 3, 4$
$y = Ae^{3x} + Be^{4x}$ **(Option (a))**

2. Find the particular integral of $(D^2 - 9)y = e^{-2x}$.

$(a) PI = \frac{1}{13}e^{-2x}$ $(b) PI = -\frac{1}{5}e^{-2x}$

$(c) PI = \frac{x}{5}e^{-2x}$ $(d) PI = \frac{1}{5}e^{-2x}$

$$PI = \frac{1}{D^2 - 9} e^{-2x}$$
$$= \frac{1}{4-9} e^{-2x}$$
$$= -\frac{1}{5} e^{-2x}$$ **(Option (b))**

3. Find the particular integral of $(D^2 + 3D + 2)y = e^{-2x}$.

$(a) PI = -xe^{-2x}$ $(b) PI = xe^{-2x}$

$(c) PI = \frac{e^{-2x}}{12}$ $(d) PI = \frac{xe^{-2x}}{12}$

$$PI = \frac{1}{D^2 + 3D + 2} e^{-2x}$$
$$= \frac{1}{4-6+2} e^{-2x}$$
$$= x.\frac{1}{2D+3} e^{-2x}$$
$$PI = -xe^{-2x}$$ **(Option (a))**

4. Find the particular integral of $(D^2 + 4)y = \sin 2x$.

$(a) PI = -\frac{x\cos 2x}{4}$ $(b) PI = -\frac{\sin 2x}{8}$

$(c) PI = \frac{x\sin 2x}{4}$ $(d) PI = \frac{\sin 2x}{8}$

$$PI = \frac{1}{D^2+4}\sin 2x$$
$$= x.\frac{1}{2D}\sin 2x$$
$$= -x.\frac{\cos 2x}{4}$$ **(Option (a))**

5. Find the particular integral of $(D^2+D+1)y = 3x-1$.

$(a) PI = 3x-4$ $(b) PI = 3x$

$(c) PI = 3x-1$ $(d) PI = 3x^2-4$

$$PI = \frac{1}{D^2+D+1}(3x-1)$$
$$= [1+(D+D^2)]^{-1}(3x-1)$$
$$= (3x-1) - D(3x-1)$$
$$PI = 3x-4$$ **(Option (a))**

6. Find the particular integral of $(D^2+D+1)y = x$

$(a) PI = 3x-4$ $(b) PI = 3x$

$(c) PI = x-1$ $(d) PI = 3x^2-4$

$$PI = \frac{1}{D^2+D+1}(x)$$
$$= [1+(D+D^2)]^{-1}(x)$$
$$= [1-(D+D^2)](x)$$
$$= (x - D(x)) \quad = x-1$$
$$PI = x-1$$

(Option C)

7**. Solve** $(D^3-6D^2+11D-6)y=0$

$(a) y = Ae^x + Be^{2x} + Ce^{3x}$ $(b) y = Ae^x + Be^{-2x} + Ce^{3x}$

$(c) y = Ae^x + Be^{2x} + Ce^{-3x}$ $(d) y = Ae^x + Be^{-2x} + Ce^{-3x}$

$$m^3-6m^2+11m-6=0$$
$$(m-1)(m-2)(m-3)=0$$
$$m=1,2,3$$
$$C.F = Ae^x + Be^{2x} + Ce^{3x}$$
Hence
$$y = Ae^x + Be^{2x} + Ce^{3x}$$

(Option A)

8**.** Find the particular integral of $(D^2+D-2)y=Sinx$

$(a) PI = \frac{-1}{10}(\cos x + 3\sin x)$ $(b) PI = \frac{1}{10}(\cos x + 3\sin x)$

$(c) PI = \frac{-1}{10}(\sin x + 3\cos x)$ $(d) PI = \frac{-1}{10}(\sin x - 3\cos x)$

$$\text{P.I} = \frac{1}{D-3}\sin x = \frac{D+3}{D^2-9}\sin x$$, Rationalizing the denominator

$$= \frac{(D+3)\sin x}{-10},\ \text{Putting } D^2 = -1$$

$$\therefore \text{P.I.} = \frac{-1}{10}(D\sin x + 3\sin x)$$

$$= \frac{-1}{10}(\cos x + 3\sin x)$$

(Option A)

9. Find the complementary function of $(D^2+1)y = \operatorname{cosec} x$.

$(a) CF = (A+Bx)e^x$ $\quad$ $(b) CF = (A+Bx)e^x$

$(c) CF = A\cos x + B\sin x$ $\quad$ $(d) CF = (A\cos x + B\sin x)e^x$

$m^2+1=0 \Rightarrow m = \pm i$

$CF = A\cos x + B\sin x$ **(Option (c))**

10. Solve $(D^2+4D+4)y = 0$.

$(a) y = Ae^{-2z} + Be^{-2z}$ $\quad$ $(b) y = (A+Bx)e^{-2x}$

$(c) y = \frac{A}{x} + \frac{B}{x^2}$ $\quad$ $(d) y = Ax + Bx^2$

$m^2+4m+4=0$

$m = -2, -2$

$y = (A+Bx)e^{-2x}$ **(Option (B))**

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Unit – IV

DIFFERENTIAL CALCULUS

Part – B

1. Envelope of the curve $y = mx + \frac{a}{m}$ (where m is the parameter) is

(A) $x^2 + a\,y = 0$ (B) $x + 4\,a\,y = 0$

(C) $y^2 - 4\,a\,x = 0$ (D) $y^2 + 4ax = 0$

Solution: Given: $y = mx + \frac{a}{m}$

$$y = \frac{m^2 x + a}{m}$$

$$m^2 x - y\,m + a = 0$$

The above equation is a quadratic equation in 'm'.

The discriminant is $b^2 - 4ac = 0$.

The envelope of the curve is $y^2 - 4\,a\,x = 0$. **(Option C)**

2. The radius of curvature of the curve $y = e^x$ at $x = 0$ is

(A) $2\sqrt{2}$ (B) $\sqrt{2}$

(C) 2 (D) 4

Solution:

$y_1 = e^x$ at $x = 0$ is 1

$y_2 = e^x$ at $x = 0$ is 1

$$\rho = \frac{\left(1 + y_1{}^2\right)^{\frac{3}{2}}}{y_2}$$

$\rho = 2\sqrt{2}$ **(Option A)**

3. The radius of curvature of the curve y = 4 sin x at x = $\frac{\pi}{2}$ is

(A) $\frac{1}{2}$ (B) $\frac{-1}{2}$

(C) $\frac{1}{4}$ (D) $\frac{3}{4}$

Solution:

$y_1 = 4\cos x \ at\ x = \frac{\pi}{2} is\ 0$

$y_2 = -4\sin x\, at\ x = \frac{\pi}{2} is - 4$

$$\rho = \frac{\left(1 + y_1{}^2\right)^{\frac{3}{2}}}{y_2}$$

$|\rho| = \frac{1}{4}$ **(Option C)**

4. The envelope of family of lines y = $mx + am^2$ (where m is the parameter) is

(A) $x^2 + 2ay = 0$ (B) $x^2 + 4\,a\,y = 0$

(C) $y^2 + 2ax = 0$ (D) $x^2 + 4\,a\,x = 0$

Solution:

The given equation is quadratic in 'm'.

The discriminant is $b^2 - 4ac = 0$.

Envelope of the family of lines is $x^2 + 4ay = 0$. **(Option B)**

5. The envelope of the family of lines $\frac{x}{t} + y\,t = 2c$, t being the parameter is

(A) $x^2 + y^2 = c^2$ (B) $x\,y = c^2$

(C) $x^2\,y^2 = c^2$ (D) $x^2 - y^2 = c^2$

Solution:

Simplifying the equation $\frac{x}{t} + y\,t = 2c$, we get $yt^2 - 2ct + x = 0$

The discriminant is $b^2 - 4ac = 0$.

Envelope of the family of lines is $x\,y = c^2$. **(Option B)**

6. The radius of curvature of the curve $r = e^{\theta}$ at any point on it is

(a) $2\sqrt{2}$ (b) $\sqrt{2}\,r$

(c) 2 (d) 4

Solution:

$r' = e^{\theta}$

$r'' = e^{\theta}$

$$\rho = \frac{\left(r^2 + r'^2\right)^{\frac{3}{2}}}{r^2 - rr'' + 2r'^2}$$

$\rho = \sqrt{2}\,r$

7. The radius of curvature at the point (3, 4) on the curve $x^2 + y^2 = 25$ is

(A) 5 (B) 4 (C) 0 (D) 2

Solution:

We know that the radius of curvature of the circle is equal to its radius.

$\rho = 5$ **(Option A)**

8. B (5/2, 1/2) = _______.

(A) 1 (B) 4

(C) 3π/8 (D) π

Solution:

$$B(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$$

$$B(5/2,1/2) = \frac{\Gamma(5/2)\Gamma(1/2)}{\Gamma(3)} = \frac{3\pi}{8}$$ **(Option C)**

9. $\Gamma(-5/2)$ **=____________.**

(A) 1 (B) 4

(C) 1/2 (D) $\frac{-8\sqrt{\pi}}{15}$

Solution:

$$\Gamma\left(-n+\frac{1}{2}\right) = \frac{(-2)^n}{1.3.5\cdots(2n-1)}\sqrt{\pi}$$

$$\Gamma\left(-\frac{5}{2}\right) = \Gamma\left(\frac{-6+1}{2}\right)$$

$$= \Gamma\left(-3+\frac{1}{2}\right) = \frac{-8}{15}\sqrt{\pi}$$

(Option D)

10. Evaluate $\int_0^\infty e^{-x}\, x^4\, dx$.

(A) 1 (B) 24

(C) 1/2 (D) $\frac{-8\sqrt{\pi}}{3}$

Solution

$\Gamma(n) = \int_0^\infty e^{-x}\, x^{n-1}\, dx$

$\int_0^\infty e^{-x}\, x^4\, dx = \int_0^\infty e^{-x}\, x^{5-1}\, dx = \Gamma(5) = 4! = 24$

(Option B)

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18MAB101T - Calculus And Linear Algebra

Year/Sem: I/I

Branch: Common to ALL B.Tech. except B.Tech. (Business Systems)

Unit – V

SEQUENCE AND SERIES

Part – B

1. The sequence $\left\{\frac{1}{n}\right\}$ converges to _________.

(A) 0 **(B) 1** **(C) ½** **(D) ∞**

Solution:

$$a_n = \frac{1}{n}$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{1}{n} = 0$$

$\{a_n\}$ converges to 0. **(Option A)**

2. The sequence $\left\{\frac{n+1}{2n+3}\right\}$ converges to _________.

(A) 0 **(B) 1** **(C) ½** **(D) ∞**

Solution:

$$a_n = \frac{n+1}{2n+3}$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n\left(1+\frac{1}{n}\right)}{n\left(2+\frac{3}{n}\right)} = \frac{1}{2}$$

$\{a_n\}$ converges to ½ . **(Option C)**

3. Test the convergence of the series $\sum \frac{1}{\sqrt{n+1}}$ **.**

(A) converges **(B) diverges**
(C) oscillates finitely **(D) oscillates infinitely**

Solution:

$$u_n = \frac{1}{\sqrt{n+1}} = \frac{1}{\sqrt{n\left(1+\frac{1}{n}\right)}}$$

Let $v_n = \frac{1}{\sqrt{n}}$

Now $\frac{u_n}{v_n} = \frac{1}{\sqrt{1+\frac{1}{n}}}$

$$\lim_{n\to\infty} \frac{u_n}{v_n} = 1$$

$\sum v_n = \sum \frac{1}{\sqrt{n}} = \sum \frac{1}{n^{1/2}}$ is divergent.

Hence by comparison test, $\Sigma\, u_n$ is divergent. **(Option B)**

4. Test the convergence of the series $1+\frac{1}{3}+\frac{1}{5}+\cdots$ **.**

(A) converges **(B) diverges**
(C) oscillates finitely **(D) oscillates infinitely**

Solution:

$$u_n = \frac{1}{2n-1} = \frac{1}{n\left(2-\frac{1}{n}\right)}$$

Let $v_n = \frac{1}{n}$

Now $\frac{u_n}{v_n} = \frac{1}{2-\frac{1}{n}}$

$$\lim_{n\to\infty} \frac{u_n}{v_n} = \frac{1}{2}$$

$\sum v_n = \sum \frac{1}{n}$ is divergent.

Hence by comparison test, $\Sigma\, u_n$ is divergent. **(Option B)**

5. Test the convergence of the series $\sum \frac{x^n}{n\,!}$ where $x > 0$.

(A) converges **(B) diverges**
(C) oscillates finitely **(D) oscillates infinitely**

Solution:

$$u_n = \frac{x^n}{n\,!}, \quad u_{n+1} = \frac{x^{n+1}}{(n+1)\,!}$$

Now $\frac{u_{n+1}}{u_n} = \frac{x}{n+1} = \frac{x}{n\left(1+\frac{1}{n}\right)}$

$$\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = \lim_{n\to\infty} \frac{x}{n\left(1+\frac{1}{n}\right)} = 0 < 1$$

Hence by Ratio test, $\Sigma\, u_n$ is convergent. **(Option A)**

6. Test the convergence of the series $\sum \frac{n\,!}{n^n}$.

(A) converges **(B) diverges**
(C) oscillates finitely **(D) oscillates infinitely**

Solution:

$$u_n = \frac{n\,!}{n^n}, \quad u_{n+1} = \frac{(n+1)\,!}{(n+1)^{n+1}}$$

Now $\frac{u_{n+1}}{u_n} = \frac{1}{\left(1+\frac{1}{n}\right)^n}$

$$\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = \frac{1}{e} < 1$$

Hence by Ratio test, $\Sigma\, u_n$ is convergent. **(Option A)**

7. The series $1 + \frac{x}{1!} + \frac{x^2}{2!} + \cdots$ is ________________.

(A) absolutely convergent **(B) diverges to + ∞**
(C) oscillates finitely **(D) oscillates infinitely**

Solution:

$$u_n = \frac{x^{n-1}}{(n-1)\,!}, \quad u_{n+1} = \frac{x^n}{n\,!}$$

Now $\frac{u_{n+1}}{u_n} = \frac{x}{n}$

$\lim_{n\to\infty} \left| \frac{u_{n+1}}{u_n} \right| = 0 < 1$

Hence the series is absolutely convergent. **(Option A)**

8. Test the convergence of the series $\sum \frac{n^3}{3^n}$**.**

(A) converges **(B) diverges**

(C) oscillates finitely **(D) oscillates infinitely**

Solution:

$$u_n = \frac{n^3}{3^n}, \quad (u_n)^{1/n} = \frac{(n^{1/n})^3}{3}$$

$$\lim_{n\to\infty} (u_n)^{1/n} = \frac{1}{3} < 1$$

Hence by Root test, $\Sigma\, u_n$ is convergent. **(Option A)**

9. Test the convergence of the series $\sum \frac{3^n\, n\,!}{n^n}$**.**

(A) converges **(B) diverges**

(C) oscillates finitely **(D) oscillates infinitely**

Solution:

$$u_n = \frac{3^n\, n\,!}{n^n}, \quad u_{n+1} = \frac{3^{n+1}\,(n+1)\,!}{(n+1)^{n+1}}$$

Now $\frac{u_{n+1}}{u_n} = \frac{3}{\left(1+\frac{1}{n}\right)^n}$

$$\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = \frac{3}{e} > 1$$

Hence by Ratio test, $\Sigma\, u_n$ is divergent. **(Option B)**

10. Test the convergence of the series $\sum \frac{1}{n^2}$**.**

(A) converges **(B) diverges**

(C) oscillates finitely **(D) oscillates infinitely**

Solution:

By Harmonic Series test or p-test, $\sum \frac{1}{n^2}$ converges. **Option (A)**

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Unit – I: Matrices

PART A

MULTIPLE CHOICE QUESTIONS

1. The matrix of the quadratic form $x_1^2+5x_2^2+x_3^2+2x_1x_2+2x_2x_3+6x_3x_1$ is

$$\checkmark(a)\begin{pmatrix}1&1&3\\1&5&1\\3&1&1\end{pmatrix}\ (b)\begin{pmatrix}1&2&3\\1&5&2\\3&2&1\end{pmatrix}(c)\begin{pmatrix}1&4&4\\4&5&3\\4&3&1\end{pmatrix}(d)\begin{pmatrix}1&4&3\\4&5&4\\3&4&1\end{pmatrix}$$

2. The number of positive terms in the canonical form is called

   (a) Signature ✓(b) Index (c) Quadratic form (d) Positive definite

3. A homogeneous polynomial of second degree in any number of variables is

   (a) Canonical form ✓(b) Quadratic form (c) Orthogonal (d) Diagonal form

4. Find the eigen values of A^2 if $A=\begin{bmatrix}3&2&4\\0&2&6\\0&0&5\end{bmatrix}$

   (a) 6, 4, 10 ✓(b) 9, 4, 25 (c) 9, 2, 5 (d) 3, 2, 5

5. Find the sum and product of the eigen values of $A=\begin{bmatrix}2&1&1\\1&2&1\\0&0&1\end{bmatrix}$

   ✓(a) 5, 3 (b) 3, 5 (c) 2, 1 (d) 0, 1

6. The eigen values of an orthogonal matrix have the absolute value ______

   (a) 0 ✓(b) 1 (c) 2 (d) ±1

7. All the eigen values of a symmetric matrix with real elements are

   (a) Distinct ✓(b) Real (c) Equal (d) Conjugate complex numbers

8. Find the nature of the quadratic form $2x^2+3y^2+2z^2+2xy$

   ✓(a) Positive definite (b) Negative definite (c) Positive semi-definite (d) Indefinite

9. Write the Q.F. defined by the matrix $A = \begin{pmatrix} 6 & 1 & -7 \\ 1 & 2 & 0 \\ -7 & 0 & 1 \end{pmatrix}$

✓ (a) $6x_1^2 + 2x_2^2 + x_3^2 + 2x_1x_2 - 14x_1x_3$ (b) $6x_1^2 + x_2^2 + 6x_3^2 + x_1x_2 - 7x_1x_3$
(c) $6x_1^2 + 2x_2^2 + x_3^2 + 2x_1x_2 + 14x_1x_3$ (d) $6x_1^2 + x_2^2 + 6x_3^2 + x_1x_2 - 14x_1x_3$

10. Find the eigen values of the matrix $\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$

✓ (a) 1, 3 ✓(b) 3, 1 (c) 2, 1 (d) 1, 2

11. Find the eigen values of A^{10} if $A = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$

✓ (a) $1, 3^{10}$ (b) $3,1$ (c) $3^2, 1^{10}$ (d) $0, 2$

12. If the sum of two eigen values and trace of a 3 x 3 matrix A are equal, find the value of $|A|$

✓ (a) 0 (b) 1 (c) -1 (d) 2

13. Find the characteristic equation of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}$

✓ $(a)\ \lambda^3 + \lambda^2 - 18\lambda - 40$ $(b)\ \lambda^3 - \lambda^2 + 18\lambda - 40$
$(c)\ \lambda^3 + \lambda^2 + 18\lambda + 40$ $(d)\ \lambda^3 + \lambda^2 - 18\lambda + 40$

14. Find the nature of the quadratic form $x_1^2 + 2x_2^2 + 3x_3^2 + 2x_1x_2 + 2x_2x_3 - 2x_3x_1$

✓ (a) Indefinite (b) Positive definite (c) Negative definite (d) Positive semidefinite

15. Find the eigen values of $A = \begin{bmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{bmatrix}$

✓ $(a)\ 1, 3, -4$ $(b)\ 1, -3, -4$ $(c)\ 1, -3, 4$ $(d)\ -1, 3, -4$

16. The matrix of the quadratic form $x^2 + xy$ is

✓ $(a)\begin{pmatrix} 1 & 1/2 \\ 1/2 & 0 \end{pmatrix}$ $(b)\begin{pmatrix} 1 & 2 \\ 2 & 0 \end{pmatrix}$ $(c)\begin{pmatrix} 0 & 1/2 \\ 1/2 & 1 \end{pmatrix}$ $(d)\begin{pmatrix} 1 & 2 \\ 1 & 0 \end{pmatrix}$

17. Two eigen values of the matrix $A = \begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{pmatrix}$ are 1 and 2. Find the third eigen value.

✓ (a) 3 (b) b (c) 2 (d) 1

18. Two of the eigen values of 3 x 3 matrix A are 2, 1 and |A| = 12. Find the third eigen value

✓(a) 6 (b) 3 (c) 2 (d) 1

19. If A is an orthogonal matrix then

(a) |A| = 0 (b) A is singular (c) $A^2 = I$ ✓(d) $A^T = A^{-1}$

20. Two eigen values of $A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -5 & -2 \end{bmatrix}$ are equal and they are double the third. Find them.

✓(a) 1, 2, 2 (b) 2, 1, 1 (c) 2, 0, 1 (d) 1, 2, 3

21. Find the inverse of the eigen values of the matrix if $A = \begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$

✓$(a)\ -1, 1/6$ $(b)\ 1, 1/6$ $(c)\ 1, -1/6$ $(d)\ -1, -1/6$

22. Find rank and index of the QF whose canonical form is $3y_2^2 - 3y_3^2$

✓(a) 2, 1 (b) 1, 2 (c) 0, 1 (d) 0, 2

23. Find signature of the QF whose canonical form is $2y_1^2 - y_2^2 - y_3^2$,

(a) 1 ✓(b) -1 (c) 0 (d) 6

24. The eigen vectors corresponding to the distinct eigen values of a real symmetric matrix are

(a) imaginary (b) non-orthogonal (c) real ✓(d) orthogonal

25. Find the characteristic equation of the matrix $A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$

(a) $\lambda^2 - 3\lambda - 2 = 0$ ✓ (b) $\lambda^2 + 3\lambda + 2 = 0$ (c) $\lambda^2 - 3\lambda + 3 = 0$ (d) $\lambda^2 - 6\lambda + 3 = 0$

UNIT-II-FUNCTIONS OF SEVERAL VARIABLES

1. If Z= x^2+y^2+3xy then what is $\frac{\partial z}{\partial x}$?

(i) 2y+3x (ii) 3y (iii) 2x+3y (iv) 2x

2. u=$\sin^{-1}\left(\frac{x^2+y^2}{x-y}\right)$ is homogeneous function of degree

(i) 2 (ii) 3 (iii) 1 (iv) 4

3.If u=ax^2+2hxy+by^2 then using Euler's theorem find x$\frac{\partial u}{\partial x}$ + y$\frac{\partial u}{\partial y}$ = ?

(i) u (ii) 2u (iii) 3u (iv) n(n-1)

4. If f(x , y) = e^{xy} then what is $f_{yyy}(1,1)$?

(i)-e (ii)$\frac{1}{e}$ (iii)e (iv)-$\frac{1}{e}$

5. if z= log (x^2+xy+y^2) then what is x$\frac{\partial z}{\partial x}$ +y$\frac{\partial z}{\partial y}$ =?

(i) 1 (ii) $\frac{2x+y}{x^2+xy+y^2}$ (iii) 2 (iv)$\frac{x+2y}{x^2+xy+y^2}$

6.If f(x,y) is an implicit function then $\frac{dy}{dx}$ =?

(i)$-\frac{(\partial f/\partial x)}{(\partial f/\partial y)}$ (ii) $\frac{(\partial f/\partial x)}{(\partial f/\partial y)}$ (iii)$\frac{(\partial f/\partial y)}{(\partial f/\partial x)}$ (iv)$-\frac{(\partial f/\partial y)}{(\partial f/\partial x)}$

7. If f(x , y) = $e^x\ cosy\ then\ what\ is\ f_{xy}\ (0,0)$?

(i) 1 (ii) -1 (iii) 0 (iv) 2

8. If f (x , y) = cosxcosy then f_{yy} (0, 0)=?

(i) 1 (ii) 0 (iii) -1 (iv)$^1/_2$

9. If f (x , y) = $\tan^{-1}\left({}^{y}/_{x}\right)$ then f_x (1 , 1) is

(i) $\frac{\pi}{4}$ (ii) ½ (iii) – ½ (iv) 0

10. If rt - $s^2 < 0$ at (a , b) then the point is

(i) Maximum point (ii) minimum point (iii) saddle point (iv) none of these

11. The stationary points of $x^2 + y^2 + 6x + 12$ are

(i) (-3 , 0) (ii) (0 , 3) (iii) (0 , -3) (iv) (3 , 0)

12. If x=$u^2 - v^2$ and y=2uv then J$\left(\frac{x\ ,y}{u\ ,v}\right)$ is

(i) $u^2 + v^2$ (ii) $2(u^2 + v^2)$ (iii) $4(u^2 + v^2)$ (iv) $4v^2$

13. If x=rcos θ and y=rsin θ Then what is $\frac{\partial(x\ ,y)}{\partial(r\ ,\theta)}$= ?

(i) r^2 (ii) r (iii) 2r (iv) 0

14. If v= $\tan^{-1} x$+$\tan^{-1} y$ then $\frac{\partial v}{\partial x}$ is

(i) 1+y^2 (ii) $\frac{1}{1+y^2}$ (iii) $\frac{1}{1+x^2}$ (iv) 1+x^2

15.u and v are functionally dependent if their jacobian value is

(i)zero (ii)one (iii)non-zero (iv)greater than zero

16.if $J_1 = J\left(\frac{x\ ,y}{u\ ,v}\right)$ and J_2=$J\left(\frac{u\ ,v}{x\ ,y}\right)$ then J_1J_2=?

(i)0 (ii)1 (iii)-1 (iv)2

17.The stationary points of f(x,y)=$sinx + siny + \sin(x + y)$ are

(i)$\left(\frac{\pi}{2},\frac{\pi}{3}\right)$ (ii)$\left(\frac{\pi}{3},\frac{\pi}{3}\right)$ (iii) $\left(\frac{\pi}{3},\frac{\pi}{2}\right)$ (iv) $\left(\frac{\pi}{2},\frac{\pi}{2}\right)$

18.The point (0,0) for f(x, y)=$x^3 + y^3 - 3axy$ is

(i)a maximum point (ii) a minimum point (iii)a saddle point (iv)none of these

19.If f(x, y)=$x^2 + y^2$ where x= rcosθ and y= rsinθ then$\frac{\partial f}{\partial \theta}$ is

(i) r (ii) r^2 (iii) 1 (iv) 0

20.If f(x, y)=x^2y+siny+e^x then $f_x(1,\pi)$ is

(i)2π-e (ii)2π (iii)2π+e (iv) 0

21. $u = \cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ is homogeneous function of degree

$(i)\,\frac{1}{2}$ (ii) 1 (iii) 2 (iv) 3

22. If $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$ then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = ?$

(i) sinu (ii) cosu (iii) sin2u (iv) tanu

23. If $u = \frac{y^2}{x}$, $v = \frac{x^2}{y}$ then $\frac{\partial(x,y)}{\partial(u,v)} = ?$

(i) -3 (ii) 3 (iii) $-\frac{1}{3}$ (iv) $\frac{1}{3}$

24. $if\ x = r\cos\theta$, y= r sin θ, z=z then $\frac{\partial(x,y,z)}{\partial(r,\theta,z)} = ?$

(i)2r (ii)r^2 (iii)$\frac{1}{r}$ (iv)r

25.If u=$x^2 - 2y$ and v=x+y then $\frac{\partial(u,v)}{\partial(x,y)} = ?$

(i)2x (ii) 2x+2 (iii) 2y-2 (iv) 2x - y

ANSWERS

1.(iii) 2x+3y

2. (iii) 1

3. (ii) 2u

4.(iii)e

5. (iii) 2

6.i) $-\frac{(\partial f/\partial x)}{(\partial f/\partial y)}$

7. (iii) 0

8. (iii) – 1

9. (iii) - 1/2

10. (iii) saddle point

11. (i) (-3 , 0)

12. (iii) 4$(u^2 + v^2)$

13. (ii) r

14. (iii) $\frac{1}{1+x^2}$

15. (i)zero

16. (ii)1

17. (ii)$\left(\frac{\pi}{3}, \frac{\pi}{3}\right)$

18. (iii) a saddle point

19. (iv) 0

20. (iii)2π+e

21. (i) $\frac{1}{2}$

22. (iii) sin2u

23. (iii) - $\frac{1}{3}$

24. (iv)r

25. (ii) 2x+2

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Unit-III Ordinary Differential Equations

Multiple Choice Questions

1. Which of the following is the general solution to $\frac{d^2y}{dx^2}+3\frac{dy}{dx}-10y=0$

   (a) $y=Ae^{2x}+Be^{-5x}$ (b) $y=Ae^{-2x}+Be^{5x}$ (c) $y=Ae^{-2x}+Be^{-5x}$ (d) $y=Ae^{2x}+Be^{5x}$

2. Solution of $(D^2+4)y=0$ is

   (a) $y=A\cos 2x+B\sin 2x$ (b) $y=Ae^{2x}+Be^{-2x}$ (c) $y=A\cos\sqrt{2}x+B\sin\sqrt{2}x$
   (d) $y=(Ax+B)e^{2x}$

3. The P.I of $(D^2+4)y=\sin 2x$ is

   (a) $\frac{-x}{4}\cos 2x$ (b) $\frac{x}{4}\cos 2x$ (c) $\frac{x}{2}\cos 2x$ (d) $\frac{-x}{2}\cos 2x$

4. The equation $(a_0x^2D^2+a_1xD+a_2)\,y=Q(x)$ is called, where $a_0,a_1,a_2\in C$

   (a) Cauchy's equation (b) Legendre's equation (c) Taylor's equation (d) Clairaut's equation

5. Use the transformation $z=\log x$, convert the D.E $x^2y''-xy'+y=x^2$ to an equation with constant coefficients

   (a) $(\theta^2-2\theta+1)y=e^{2z}$ (b) $(\theta^2-2\theta+1)y=e^{z}$ (c) $(\theta^2+2\theta+1)y=e^{2z}$
   (d) $(\theta^2+2\theta+1)y=e^{z}$

6. The solution of $(D^2+2D+1)y=7$ is

   (a) $y=(Ax+B)e^{-x}+7$ (b) $y=(Ax+B)e^{-x}-7$ (c) $y=(Ax+B)e^{x}+7$
   (d) $y=(Ax+B)e^{x}-7$

7. The P.I of $(D-1)^2y=e^x\sin x$ is

   (a) $-e^x\cos x$ (b) $e^x\cos x$ (c) $e^x\sin x$ (d) $-e^x\sin x$

8. The P.I of $(D-1)^2y=x$ is

   (a) $2-x$ (b) $x+2$ (c) x^2 (d) $-x^2$

9. If $1\pm 2i$ are the roots of A.E of a differential equation $f(D)y=0$ then the general solution is

   (a) $e^{-2x}\,(A\cos x-B\sin x)$ (b) $Ae^{x}+Be^{-2x}$ (c) $e^{x}\,(A\cos 2x+B\sin 2x)$ (d) $Ae^{t}+Be^{2x}$

10. Convert the equation $(5+2x)^2y''-6(5+2x)y'+8y=0$ to an equation with constant coefficient by using the transformation $z=\log(5+2x)$

    (a) $(\theta^2+4\theta+2)y=0$ (b) $(\theta^2-4\theta+2)y=0$ (c) $(\theta^2+4\theta+4)y=0$ (d) $(\theta^2+4\theta-2)y=0$

11. The P. I of $(D^2+4)y=\sinh 2x$ is

    (a) $y_p=\frac{\sinh 2x}{8}$ (b) $y_p=\frac{\sinh 2x}{4}$ (c) $y_p=\frac{-\sinh 2x}{8}$ (d) $y_p=\frac{-\sinh 2x}{4}$

12. The P.I of $(D^2+6D+5)y=e^{-x}$ is

(a) $y_p=\frac{xe^{-x}}{4}$ (b) $y_p=\frac{xe^{-x}}{2}$ (c) $y_p=\frac{e^{-x}}{2}$ (d) $y_p=\frac{e^{-x}}{4}$

13. The solution of $(D^2-2aD+a^2)y=0$ is

(a) $Ae^{ax}+Be^{bx}$ (b) $Ae^{ax}+Be^{-ax}$ (c) $(Ax+B)e^{ax}$ (d) $(Ax+B)e^{-ax}$

14. The P.I of $(D^2+16)y=\cos 4x$ is

(a) $\frac{x}{2}\sin 2x$ (b) $\frac{x\sin 4x}{8}$ (c) $\frac{x}{2}\cos 2x$ (d) $\frac{x\cos 4x}{8}$

15. The C.F of $D^2y+y=cosecx$ is

(a) $Ae^{ax}+Be^{bx}$ (b) $A\cos x+B\sin x$ (c) $(Ax+B)e^{ax}$ (d) $(Ax+B)e^{-ax}$

16. If $y_1=\cos ax, y_2=\sin ax$ then the value of $y_1y_2^{'}-y_2y_1^{'}$ is

(a) -a (b) 0 (c) 1 (d) a

17. Solve $(D^2+1)\,y=0$ given $y(0)=0,\ \ y'(0)=1$

(a) $y=\sin x$ (b) $y=\cos x$ (c) $y=A\cos x+B\sin x$ (d) $y=0$

18. The P.I of $(D-2)^2y=e^{2x}$ is

(a) $\frac{x^2}{2}\,e^{2x}$ (b) $\frac{x}{4}\,e^{2x}$ (c) $\frac{x^2}{2}\,e^{-2x}$ (d) $\frac{x^2}{2}\,e^{-2x}$

19. The P.I of $(D^2+4)y=\sin(2x+5)$ is

(a) $-\frac{x}{2}\sin(2x+5)$ (b) $\frac{x}{4}\sin(2x+5)$ (c) $-\frac{x}{4}\cos(2x+5)$ (d) $\frac{x}{2}\cos(2x+5)$

20. Solve $(x^2D^2+xD+1)\,y=0$ is

(a) $Ae^{az}+Be^{bz}$ (b) $A\cos z+B\sin z$ (c) $(Az+B)e^{az}$ (d) $(Az+B)e^{-az}$

21. The roots of the auxiliary equation $(m^2-4)=0$ are

(a) ± 2 (b) $\pm 2i$ (c) $\pm\sqrt{2}$ (d) $1\pm 2i$

22. The solution of $(x^2D^2-7xD+12)\,y=0$ is

(a) $Ae^{-2z}+Be^{6z}$ (b) $Ae^{2z}+Be^{-6z}$ (c) $Ae^{2z}+Be^{6z}$ (d) $Ae^{-2z}+Be^{-6z}$

23. If $y_1=\cos x, y_2=\sin x$ then the value of $y_1y_2^{'}-y_2y_1^{'}$ is

(a) -1 (b) 0 (c) 1 (d) $\frac{1}{2}$

24. If three roots of the auxiliary equation become equal to the real number a, then the corresponding C.F is

(a) $(Ax^2+Bx+C)\,e^{ax}$ (b) $Ae^{ax}+Be^{ax}+Ce^{ax}$ (c) $Ae^{ax}+(B\cos ax+C\sin ax)$ (d) a

25. The values of $\frac{e^{ax}}{D-a}$

(a) xe^{ax} (b) e^{ax} (c) x^2e^{ax} (d) $\frac{x^2}{2}e^{ax}$

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Answers:

1. *a* **2.** *a* **3.** *a* **4.** *a* **5.** *a* **6.** *a* **7.** *d* **8.** *b* **9.** *c* **10.** *b* **11.** *a*
12. *a* **13.** *c* **14.** *b* **15.** *b* **16.** *d* **17.** *a* **18.** *a* **19.** *c* **20.** *b* **21.** a
22. *c* **23.** *c* **24.** *a* **25.** *a*

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Unit-IV Geometrical Applications of Differential Calculus

Multiple Choice Questions

1. If the radius of curvature and curvature of a curve at any point are ρ and κ respectively, then

   (a) $\rho = \frac{-1}{\kappa}$ (b) $\rho = \kappa$ (c) $\rho = -\kappa$ (d) $\rho = \frac{1}{\kappa}$

2. The locus of center of curvature is called

   (a) Involute (b) Evolute (c) Radius of curvature (d) Envelope

3. The envelope of the family of curves $A\alpha^2 + B\alpha + C = 0$ (α is parameter) is

   (a) $B^2 + 4AC = 0$ (b) $B^2 - AC = 0$ (c) $B^2 + AC = 0$ (d) $B^2 - 4AC = 0$

4. The curvature of the straight line is

   (a) 1 (b) 2 (c) -1 (d) 0

5. The radius of curvature of the curve $y = 4\sin x$ at $x = \frac{\pi}{2}$ is

   (a) $\frac{1}{2}$ (b) $\frac{-1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{-1}{4}$

6. The envelope of $ty - x = at^2$, t is the parameter is

   (a) $x^2 = 4ay$ (b) $y^2 = 4ax$ (c) $x^2 + y^2 = 1$ (d) $x^2 - y^2 = 1$

7. The curvature at any point of the circle is equal to $---$ of its raduis

   (a) Square (b) Same (c) Reciprocal (d) constant

8. What is the radius of curvature at (4, 3) on the curve $x^2 + y^2 = 25$

   (a) 5 (b) -5 (c) 25 (d) -25

9. What is the curvature of a circle of radius 3

   (a) 3 (b) -3 (c) $\frac{1}{3}$ (d) $\frac{-1}{3}$

10. Find the envelope of the curve $y = mx + \frac{a}{m}$ where m is a parameter

    (a) $y^2 - 4ax = 0$ (b) $y^2 + 4ax = 0$ (c) $x^2 + y^2 = 1$ (d) $xy = c^2$

11. The radius of curvature of $y = e^x$ at $x = 0$ is

    (a) $2\sqrt{2}$ (b) $\frac{2}{\sqrt{2}}$ (c) $\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$

12. The radius of curvature of the curve $y = log \sec x$ at any point of it is

    (a) $\sec x$ (b) $\tan x$ (c) $\cot x$ (d) $cosecx$

13. In an ellipse the radius of curvature at the end of which axis is equal to the semi latus rectum of the ellipse

    (a) Minor (b) Major (c) Vertical (d) Horizontal

14. The radius of curvature of the curve $x = t^2$, $y = t$ at $t = 1$ is

    (a) $5\frac{\sqrt{5}}{2}$ (b) $\frac{\sqrt{5}}{2}$ (c) $\frac{5}{2}$ (d) $\sqrt{5}$

15. Evolute of a curve is the envelope of $----$ of that curve
    (a) Tangent (b) Normal (c) Parallel (d) Locus
16. The evolute of the cycloid $x = a\,(\theta - \sin\theta)\,,\ \ y = a\,(1 - \cos\theta)$ is
    (a) Astroid (b) Parabola (c) Cycloid (d) Circle
17. A curve which touches each member of a family of the curves is called $---$ of that family
    (a) Evolute (b) Envelope (c) Circle of curvature (d) Radius of curvature
18. The envelope of family of lines $y = mx + am^2$ $(where\,\mathrm{m}\,istheparameter)$ is
    (a) $x^2 + 2ay = 0$ (b) $x^2 + 4ay = 0$ (c) $y^2 + 2ax = 0$ (d) $y^2 + 4ax = 0$
19. The envelope of the family of lines $\frac{x}{t} + yt = 2c,$ t being the parameter is
    (a) $x^2 + y^2 = c^2$ (b) $xy = c^2$ (c) $x^2y^2 = c^2$ (d) $x^2 - y^2 = c^2$
20. The radius of curvature at any point on the curve $r = e^{\theta}$ is
    (a) $\frac{\sqrt{2}}{r}$ (b) $\frac{r}{\sqrt{2}}$ (c) r (d) $\sqrt{2}r$
21. The radius of curvature in Cartesian coordinates is
    (a) $\rho = \dfrac{\left(1 + y_1^2\right)^{3/2}}{y_2}$ (b) $\rho = \dfrac{\left(1 - y_1^2\right)^{3/2}}{y_2}$ (c) $\rho = \dfrac{\left(1 + y_1^2\right)^{2/3}}{y_2}$ (d) $\rho = \dfrac{\left(1 + y_2^2\right)^{3/2}}{y_1}$
22. The radius of curvature in polar coordinates is
    (a) $\rho = \dfrac{\left(r^2 + (r')^2\right)^{3/2}}{r^2 - rr' + 2(r')^2}$ (b) $\rho = \dfrac{\left(r^2 - (r')^2\right)^{3/2}}{r^2 - rr' + 2(r')^2}$ (c) $\rho = \dfrac{\left(r^2 - (r'')^2\right)^{3/2}}{r^2 - rr' + 2(r')^2}$
    (d) $\rho = \dfrac{\left(r^2 + (r')^2\right)^{3/2}}{r^2 - rr'' + 2(r')^2}$
23. The radius of curvature in parametric coordinates is
    (a) $\rho = \dfrac{\left((x')^2 + (y')^2\right)^{3/2}}{x'y'' - y'x''}$ (b) $\rho = \dfrac{\left((x')^2 + (y')^2\right)^{3/2}}{x'y'' + y'x''}$ (c) $\rho = \dfrac{\left((x')^2 - (y')^2\right)^{3/2}}{x'y'' - y'x''}$
    (d) $\rho = \dfrac{\left((x')^2 - (y')^2\right)^{3/2}}{x'y'' + y'x''}$
24. The equation of circle of curvature at any point (x, y) with center of curvature $\overline{x}, \overline{y}$ and with radius of curvature ρ is
    (a) $(x + \overline{x})^2 + (y + \overline{y})^2 = \rho^2$ (b) $(x - \overline{x})^2 + (y - \overline{y})^2 = \rho^2$ (c) $(x - \overline{x})^2 - (y + \overline{y})^2 = \rho^2$
    (d) $(x + \overline{x})^2 + (y + \overline{y})^2 = \rho$

****

Answers:

1. *d* **2.** *b* **3.** *d* **4.** *d* **5.** *c* **6.** *b* **7.** *c* **8.** *a* **9.** *c* **10.** *b* **11.** *a* **12.** *a* **13.** *b* **14.** *a* **15.** *b* **16.** *c* **17.** *b* **18.** *b* **19.** *b* **20.** *d* **21.** *a* **22.** *d* **23.** *a* **24.** *b*

SRM INSTITUTE OF SCIENCE & TECHNOLOGY
FACULTY OF ENGINEERING AND TECHNOLOGY
DEPARTMENT OF MATHEMATICS
Unit –IV Geometrical Applications of Differential Calculus
(Beta ,Gamma Functions)
Multiple Choice Questions

1. The value of $\beta(4,4)$ is -----------

✓(a). $\frac{36}{7!}$ (b). $\frac{6!}{7!}$ (c). $\frac{4!4!}{8!}$ (d). $\frac{3!}{7!}$

2. The value of $\beta\left(\frac{5}{2},\frac{3}{2}\right)$ is ----------------

(a). $\frac{\pi}{8}$ (b). $\frac{\sqrt{\pi}}{8}$ ✓(c). $\frac{\pi}{16}$ (d). $\frac{\pi^2}{16}$

3. $\beta(m,n)$ is equal to ------------

(a). $\frac{m!n!}{(m+n)!}$ (b). $\frac{m!n!}{(m-n)!}$ ✓(c). $\frac{\Gamma m \Gamma n}{\Gamma(m+n)}$ (d). $\frac{\Gamma m \Gamma n}{\Gamma(m-n)}$

4. The value of $\Gamma\left(\frac{1}{2}\right)$ is -----------

✓(a). $\sqrt{\pi}$ (b). π^2 (c). π (d). 2π

5. $\Gamma n \Gamma(1-n)$ is equal to --------

(a). $\int_0^\infty \frac{x^{1-n}}{1+x}dx$ ✓(b). $\Gamma(1)\beta(n,1-n)$ (c). $\Gamma(1)\beta(1-n,1-n)$ (d). $\Gamma(1)\beta(1-n,n)$

6. $\int_0^{\frac{\pi}{2}} \sin^p\theta \cos^q\theta\, d\theta$ is equal to -----------

✓(a) $.\frac{1}{2}\beta\left(\frac{p+1}{2},\frac{q+1}{2}\right)$ (b). $\frac{1}{2}\beta\left(\frac{p-1}{2},\frac{q-1}{2}\right)$ (c). $\frac{1}{2}\beta\left(\frac{p}{2},\frac{q}{2}\right)$ (d). $\frac{1}{2}\beta\left(\frac{1}{2},\frac{1}{2}\right)$

7. $\int_0^1 x^4[\log(\frac{1}{x})]^3$ is equal to -----------

(a). $\frac{6}{525}$ ✓(b). $\frac{6}{625}$ (c). $\frac{6!}{5!}$ (d). $\frac{5!}{6!}$

8. The value of $\int_0^{\frac{\pi}{2}} \sqrt{\cot\theta}\, d\theta$ is -----------

(a). $\frac{\pi}{2}$ ✓(b). $\frac{\pi}{\sqrt{2}}$ (c). $\frac{\sqrt{\pi}}{2}$ (d). $\sqrt{\frac{\pi}{2}}$

UNIT V
Sequence and Series

1. The series $\sum_{n=1}^{\infty}\frac{1}{n^p}$ is convergent if

   (a)p=1 (b)p=0 (c)p>1 (d)p<1

2. The series $\sum_{n=1}^{\infty}\frac{1}{n^p}$ is divergent if

   (a)p>1 (b)p=0 (c)p≤1 (d)p<1

3. If $\sum u_n$ is a series of positive term such that $\lim_{n\to\infty}(u_n)^{\frac{1}{n}} = l$ where $l>1$, then the series $\sum u_n$ is

   (a) Convergent (b) Divergent (c) Oscillating (d) Monotonic

4. If $\sum u_n$ is a series of positive term such that $\lim_{n\to\infty}(u_n)^{\frac{1}{n}} = l$, then the series $\sum u_n$ is convergent if

   (a)l<1 (b)l =0 (c)l>1 (d)l =1

5. The series $\sum_{n=1}^{\infty}\frac{1}{n!}$ is

   (a) Convergent (b) Divergent (c) Oscillating (d) Monotonic

6. The series $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+..........to\infty$ is

   (a) Convergent (b) Divergent (c) Oscillating (d) Monotonic

7. By D'Alambert's Ratio test $\lim_{n\to\infty}(\frac{u_{n+1}}{u_n}) = l$, the series is convergent when

   (a)l<1 (b)l =0 (c)l>1 (d)l =1

8. By Raabe's test $\lim_{n\to\infty}\left[n(\frac{u_{n+1}}{u_n}-1)\right] = l$, the series is divergent when

   (a)l<1 (b)l =0 (c)l>1 (d)l =1

9. The series $\sum_{n=1}^{\infty}\frac{2^n}{n!}$ is

   (a) Convergent (b) Divergent (c) Oscillating (d) Monotonic

10. A series $\sum u_n$ is said absolutely convergent if the series

    (a) $\sum |u_n|$ is convergent (c) $\sum u_n$ is divergent

    (c) $\sum u_n$ is convergent (b) $\sum |u_n|$ is divergent

11. A series $\sum u_n$ is saidconditionally convergent if the series

(a) $\sum|u_n|$ is convergent (b) $\sum u_n$ is divergent& $\sum|u_n|$ is convergent

(c) $\sum u_n$ is convergent& $\sum|u_n|$ is divergent (d) $\sum|u_n|$ is divergent

12. The series $1-\frac{1}{2}+\frac{1}{2^2}-\frac{1}{2^3}+..........$ is

(a) Convergent (b) Divergent (c) Conditionally convergent (d) absolutely convergent

13. The series $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+..........$ is

(a) Convergent (b) Divergent (c) Conditionally convergent (d) absolutely convergent

14. The series $\sum\frac{1}{n\log n}$ is

(a) Conditionally convergent (b) absolutely convergent (c) Convergent (d) Divergent

15. An absolutely convergent series is

(a) Conditionally convergent (b) absolutely convergent (c) Convergent (d) Divergent

16. The series $\sum\frac{n^3}{3^n}$ is

(a) Conditionally convergent (b) absolutely convergent (c) Convergent (d) Divergent

17. The series $\sum\frac{1}{(\log n)^n}$ is

(a) Convergent (b) Conditionally convergent (c) absolutely convergent (d) Divergent

ANSWERS

1. d	11. c
2. c	12. d
3. b	13. c
4. a	14. d
5. a	15. c
6. b	16.c
7. a	17.a
8. c	
9. a	
10. a	

SRM Institute of Science and Technology
Ramapuram Campus
Department of Mathematics
18MAB101T – Calculus And Linear Algebra

Year/Sem: I/I

Branch: Common to ALL B.Tech. except B.Tech. (Business Systems)

Unit – I

MATRICES

Part – A

1.	The sum of the eigen values of $A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$ is	1 Mark	
	(a) 2 (b) 4 (c) – 3 (d) 0	Ans (a)	(CLO – 1Apply)
2.	The eigen values of A^{-1}, if $A = \begin{pmatrix} 1 & 5 & -1 \\ 0 & 3 & 2 \\ 0 & 0 & 4 \end{pmatrix}$ are	1 Mark	
	(a) 2, 3, 4 (b) 2, 5, – 1 (c) 0, 0, 0 (d) $1, \frac{1}{3}, \frac{1}{4}$	Ans (d)	(CLO -1Apply)
3.	If two eigen values of $A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$ are 3 and 15, then the third eigen value is __________.	1 Mark	
	(a) 1 (b) 0 (c) 2 (d) 3	Ans (b)	(CLO -1 Apply)
4.	If – 1, – 1, 2 are the eigen values of a matrix $A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$, then the eigen values of A^T are	1 Mark	
	(a) – 1, – 1, 2 (b) 1, 1, 1/2 (c) 1,1,4 (d) -1,-1,-2	Ans (a)	(CLO - 1 Apply)

5.	The sum of eigen values of the identity matrix of order 3 is	1 Mark	
	(a) 0 (b) 1 (c) 2 (d) 3	Ans (d)	(CLO - 1 Remember)
6.	The product of the two eigen values of the matrix $A = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$ is 4.Then the third eigen value is	1 Mark	
	(a) 2 (b) 1 (c) 3 (d) 8	Ans (a)	(CLO - 1 Apply)
7.	The index of the canonical form $Q = -{y_1}^2 + {y_2}^2 + 4{y_3}^2$ is	1 Mark	
	a) 3 (b) 2 (c) 1 (d) 0	Ans (b)	(CLO -1 Apply)
8.	If the eigen values of the matrix of the quadratic form $2{x_1}^2 + 6{x_2}^2 + 2{x_3}^2 + 8x_1x_3$ are – 2, 6, 6, then the nature of the quadratic form is _______.	1 Mark	
	(a) positive semi-definite (b) indefinite (c) negative definite (d) positive definite	Ans (a)	(CLO - 1 Apply)
9.	The matrix corresponding to the quadratic form ${x_1}^2 + 5{x_2}^2 + {x_3}^2 + 2x_2x_3 + 6x_3x_1 + 2x_1x_2$ is	1 Mark	
	(a) $\begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 2 & 3 \\ 1 & 5 & 2 \\ 3 & 2 & 1 \end{pmatrix}$ (c) $\begin{pmatrix} 1 & 4 & 4 \\ 4 & 5 & 3 \\ 4 & 3 & 1 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & 4 & 3 \\ 4 & 5 & 4 \\ 3 & 4 & 1 \end{pmatrix}$	Ans (a)	(CLO - 1 Apply)
10.	A homogeneous polynomial of the __________ degree in any number of variables is called a quadratic form.	1 Mark	

	(a) first (b) second (c) third (d) fourth	Ans (b)	(CLO - 1 Remember)
11.	A square matrix A is called orthogonal if	1 Mark	
	(a) $A = A^2$ (b) $A = A^{-1}$ (c) $A^T = A^{-1}$ (d) $AA^{-1} = I$	Ans (c)	(CLO - 1 Remember)
12.	The sum of the squares of the eigen values $\begin{pmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{pmatrix}$ is	1 Mark	
	(a) 10 (b) 38 (c) 45 (d) 20	Ans (b)	(CLO - 1 Apply)
13.	All the eigen values of a symmetric matrix with real elements are	1 Mark	
	(a) distinct (b) real (c) equal (d) conjugate complex numbers	Ans (a)	(CLO - 1 Remember)
14.	If the sum of two eigen values and trace of a 3 x 3 matrix A are equal, then the value of det (A) is	1 Mark	
	(a) 0 (b) 1 (c) -1 (d) 2	Ans (a)	(CLO - 1 Apply)
15.	If the canonical form of a quadratic form is $-{y_1}^2 + {y_2}^2 + 2{y_3}^2$, then the signature of the quadratic form is	1 Mark	
	(a) 2 (b) 1 (c) 0 (d) 3	Ans (b)	(CLO - 1Apply)
16.	Find the sum and product of the eigen values of A = $\begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$	1 Mark	

	(a) 5, 3 (b) 3, 5 (c) 2, 1 (d) 0, 1	Ans (b)	(CLO - 1 Apply)
17.	The eigen vectors corresponding to the distinct eigen values of a real symmetric matrix are	1 Mark	
	(a) imaginary (b) non-orthogonal (c) real (d) orthogonal	Ans (d)	(CLO - 1 Remember)
18.	The eigen values of a skew symmetric matrix are	1 Mark	
	(a) real (b) imaginary (c) unitary (d) orthogonal	Ans (b)	(CLO - 1 Remember)
19.	Find the characteristic equation of the matrix A = $\begin{pmatrix} 5 & 4 \\ 1 & 2 \end{pmatrix}$	1 Mark	
	(a) $\lambda^2 - 7\lambda + 6 = 0$ (b) $\lambda^2 + 7\lambda + 6 = 0$ (c) $\lambda^2 - 7\lambda - 6 = 0$ (d) $\lambda^2 - 7\lambda + 5 = 0$	Ans (a)	(CLO - 1 Apply)
20.	The eigen values of an orthogonal matrix have the absolute value	1 Mark	
	(a) 0 (b) 1 (c) 2 (d) 3	Ans (b)	(CLO -1 Remember)
21.	The number of positive terms in the canonical form is called	1 Mark	
	(a) Signature (b) Index (c)quadratic (d)positive definite	Ans (b)	(CLO - 1Remember)
22.	The difference between the positive terms and negative terms in the canonical form is called	1 Mark	
	(a) Signature (b) Index (c)quadratic (d)positive definite	Ans (a)	(CLO - 1 Remember)

23.	Find the eigen values of A^2 if $A = \begin{pmatrix} 3 & 2 & 4 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{pmatrix}$.	1 Mark	
	(a) 6, 4, 10 (b) 9, 4, 25 (c) 9, 2, 5 (d) 3, 2, 5	Ans (b)	(CLO - 1 Apply)
24.	Find the nature of the quadratic form $2x^2 + 3y^2 + 2z^2 + 2xy$	1 Mark	
	(a)Positive definite (b) Negative definite (c) Positive semi-definite (d) Indefinite	Ans (b)	(CLO – 1 Apply)
25.	Find the eigen values of A^{10} if $A = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$	1 Mark	
	(a) $1, 3^{10}$ (b) 1,3 (c) $3^2, 1^{10}$ (d)1,10	Ans (a)	(CLO - 1 Apply)
26.	Find the eigen values of the matrix $A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$.	1 Mark	
	(a) 1, -3 (b) 3, 1 (c) 2, 1 (d) 1, 2	Ans (b)	(CLO - 1 Apply)
27.	If the sum of two eigen values and trace of a 3 x 3 matrix A are equal,then the value of determinant of A is	1 Mark	
	(a) 0 (b) 1 (c) -1 (d) 2	Ans (a)	(CLO - 1 Apply)
28.	Find the eigen values of the matrix $A^3 = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$.	1 Mark	

	(a) 1, -3 (b) 3, 1 (c) 1,9 (d) 1, -9	Ans (c)	(CLO - 1Apply)
29.	The eigen values of the matrix $A^{-1} = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$	1 Mark	
	(a) 4,3 (b) 3, 1 (c) -2, 1 (d) 1, 2	Ans (a)	(CLO - 1 Apply)
30.	Find the sum and product of the eigen values of the matrix $A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$.	1 Mark	
	(a) 4,3 (b) 3, 1 (c) -2, 1 (d) 1, 2	Ans (a)	(CLO - 1 Apply)
31.	Find the eigen values of $A = \begin{pmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{pmatrix}$.	1 Mark	
	(a) $1,3,-4$ (b) $1,-3,-4$ (c) $1,-3,4$ (d) $-1,3,-4$	Ans (a)	(CLO - 1 Apply)
32.	Two eigen values of $A = \begin{pmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -5 & -2 \end{pmatrix}$ are equal and they are double the third. Find them.	1 Mark	
	(a) 1, 2, 2 (b) 2, 1, 1 (c) 2, 0, 1 (d) 1, 2, 3	Ans (a)	(CLO - 1Apply)
33.	The eigen values of a diagonal matrix are the ________ elements of the matrix	1 Mark	

	(a) diagonal (b) upper triangular (c) zero (d) unity	Ans (a)	(CLO - 1 Remember)
34.	Cayley-Hamilton theorem states that "Every ________ matrix satisfies its own characteristic equation".	1 Mark	
	(a) square (b) column (c) row (d) zero	Ans (a)	(CLO - 1Remember)
35.	Find rank and index of the QF whose canonical form is $3x^2 - 3y^2$.	1 Mark	
	(a) 2, 1 (b) 1, 2 (c) 0, 1 (d) 0, 2	Ans (a)	(CLO – 1 Apply)
36.	Write the Q.F. defined by the matrix $A = \begin{pmatrix} 6 & 1 & -7 \\ 1 & 2 & 0 \\ -7 & 0 & 1 \end{pmatrix}$	1 Mark	
	(a) $6x^2 + 2y^2 + z^2 + 2xy - 14xz$ (b) $6x + y^2 + 6z^2 + x y - 7xz$ (c) $6x^2 + 2y^2 + z^2 + 2xy + 14xz$ (b) $6x + y^2 + 6z^2 + x y - 14xz$	Ans (a)	(CLO -1Apply)

SRM Institute of Science and Technology
Ramapuram Campus
Department of Mathematics
18MAB101T – Calculus And Linear Algebra

Year/Sem: I/I

Branch: Common to ALL B.Tech. except B.Tech. (Business Systems)

Unit – I

MATRICES

Part – B

1. Find the characteristic equation of the matrix $A = \begin{pmatrix} 1 & 2 \\ 0 & 2 \end{pmatrix}$.

(A) $\lambda^2 - 3\lambda + 2 = 0$ **(B)** $\lambda^2 + 3\lambda + 2 = 0$

(C) $\lambda^2 - 3\lambda - 2 = 0$ **(D)** $\lambda^2 + 3\lambda - 2 = 0$

<u>**Solution**</u>: Let A = $\begin{pmatrix} 1 & 2 \\ 0 & 2 \end{pmatrix}$. Its characteristic equation is $\lambda^2 - S_1\lambda + S_2 = 0$ where $S_1 = sum\ of\ the\ main\ diagonal\ elements = 1 + 2 = 3$,

$S_2 = Determinant\ of\ A = |A|$ = 1(2) – 2(0) = 2

Therefore, the characteristic equation is $\lambda^2 - 3\lambda + 2 = 0$ **(Option A)**

2. Find the characteristic equation of $A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$.

(A) $\lambda^3 - 18\lambda^2 + 45\lambda = 0$ **(B)** $\lambda^3 - 28\lambda^2 + 45\lambda = 0$

(C) $\lambda^3 - 18\lambda^2 + 35\lambda = 0$ **(D)** $\lambda^3 - 18\lambda^2 - 45\lambda = 0$

<u>**Solution**</u>: Its characteristic equation is $\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$, where $S_1 = sum\ of\ the\ main\ diagonal\ elements$ = 8 + 7 + 3 = 18, $S_2 = Sum\ of\ the\ minors\ of\ the\ main\ diagonal\ elements = \begin{vmatrix} 7 & -4 \\ -4 & 3 \end{vmatrix} + \begin{vmatrix} 8 & 2 \\ 2 & 3 \end{vmatrix} + \begin{vmatrix} 8 & -6 \\ -6 & 7 \end{vmatrix} = 5 + 20 + 20 = 45$, $S_3 = Determinant\ of\ A = |A|$ = 8(5)+6(-10)+2(10) = 40 – 60 + 20 = 0

Therefore, the characteristic equation is $\lambda^3 - 18\lambda^2 + 45\lambda = 0$ **(Option A)**

3. Find the eigen values of the matrix $A = \begin{pmatrix} 1 & 1 \\ 3 & -1 \end{pmatrix}$.

(A) $\mathbf{2, -2}$ **(B)** $\mathbf{1, -1}$

(C) $\mathbf{3, -3}$ **(D)** $\mathbf{2, 2}$

Solution: Let A = $\begin{pmatrix} 1 & 1 \\ 3 & -1 \end{pmatrix}$ which is a non-symmetric matrix.

The characteristic equation of A is $\lambda^2 - S_1\lambda + S_2 = 0$ where $S_1 =$ $sum\ of\ the\ main\ diagonal\ elements = 1 - 1 = 0,$

$S_2 = Determinant\ of\ A = \ |A|$ = 1(-1) – 1(3) = - 4

Therefore, the characteristic equation is $\lambda^2 - 4 = 0$ i.e., $\lambda^2 = 4$ or $\lambda = \pm 2$

Therefore, the eigen values are 2, -2. **(Option A)**

4. Find the sum and product of the eigen values of the matrix $A = \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$.

(A) $\mathbf{-3, 4}$ **(B)** $\mathbf{-3, -4}$

(C) $\mathbf{3, 4}$ **(D)** $\mathbf{-3, -4}$

Solution: Sum of the eigen values = Sum of the main diagonal elements = -3

Product of the eigen values = $|A|$ = -1 (1 – 1) -1(-1 – 1) + 1(1- (-1)) = 2 + 2 = 4

(Option A)

5. Find the sum and product of eigen values of the matrix A^T where $A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$.

(A) $\mathbf{18, 0}$ **(B)** $\mathbf{18, 2}$

(C) $\mathbf{28, 0}$ **(D)** $\mathbf{18, -2}$

Solution: Since matrix A is symmetric , A and A^T have same eigen values.

Sum of Eigen value of A^T = trace(A) = 8+7+3=18

Product of Eigen value of $A^T = |A|$ = 8(5) + 6(−10)+2(10) = 40 −60 + 20 = 0 **(Option A)**

6. If 1, 1, 5 are the eigen values of A = $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$**, find the eigen values of 5A.**

(A) $\mathbf{5, 5, 2}$ **(B)** $\mathbf{5, 5, 25}$ **(C)** $\mathbf{2, 3, 2}$ **(D)** $\mathbf{7, 8, 7}$

Solution: By the property "If $\lambda_1, \lambda_2, \lambda_3$ are the eigen values of A, then $k\lambda_1, k\lambda_2, k\lambda_3$ are the eigen values of kA, the eigen values of 5A are 5(1), 5(1), 5(5) ie., 5,5,25**. (Option B)**

7. Find the eigen values of the matrix 2A^{-1} where A= $\begin{pmatrix} 3 & 8 & 9 \\ 0 & 1 & 3 \\ 0 & 0 & -2 \end{pmatrix}$.

(A) $\mathbf{\frac{2}{3}, 2, -1}$ **(B)** $\mathbf{\frac{1}{3}, 2, -4}$ **(C)** $\mathbf{\frac{2}{3}, 2, 1}$ **(D)** $\mathbf{\frac{2}{3}, 1, -2}$

Solution: Since given matrix is triangular matrix, the Eigen values are its diagonal elements.

$$\therefore \quad \lambda_1 = 3,\ \lambda_2 = 1,\ \lambda_3 = -2$$

Eigen values of 2A^{-1} are $\frac{2}{3}$, 2 , -1 **(Option A)**

8. If the sum of two eigen values and trace of a 3 × 3 matrix A are equal, find the value of $|A|$**.**

(A) 5 **(B) 25** **(C) 2** **(D) 0**

Solution: Sum of the eigen values = $\lambda_1 + \lambda_2 + \lambda_3$ = sum of the diagonal elements

Given $\lambda_1 + \lambda_2$ = trace of A.

i.e., $\lambda_1 + \lambda_2 = \lambda_1 + \lambda_2 + \lambda_3$

Therefore $\lambda_3 = 0$. Then $|A|$ = Product of Eigen values = $\lambda_1 \lambda_2 \lambda_3 = 0$ **(Option D)**

9. Write the matrix corresponding to the quadratic form $\boldsymbol{x^2 + 2yz}$**.**

(A) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 1 & 0 \end{bmatrix}$ **(B)** $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 2 & 0 \end{bmatrix}$ **(C)** $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ **(D)** $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 2 & 0 \end{bmatrix}$

Solution: Given $\boldsymbol{X^T A X = x^2 + 2yz}$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$ **(Option C)**

10. If 2, 3, –1 are the eigen values of 3 × 3 matrix, find rank, index and signature of the quadratic form.

(A) $\mathbf{5, 5, 2}$ **(B)** $\mathbf{5, 5, 25}$ **(C)** $\mathbf{3, 2, 1}$ **(D)** $\mathbf{1, 2, 3}$

Solution: Rank (r) = number of non zero terms in canonical form = 3

Index (p) = Number of positive terms in canonical form = 2

Signature (s) = Difference between number of positive terms and negative terms

= 2p – r

= 4 – 3

= 1 **(Option C)**

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18MAB101T - Calculus And Linear Algebra

Year/Sem: I/I

Branch: Common to ALL B.Tech. except B.Tech. (Business Systems)

Unit – I

MATRICES

Part – C

1. Find the eigen values of $A = \begin{bmatrix} 2 & 2 & -7 \\ 2 & 1 & 2 \\ 0 & 1 & -3 \end{bmatrix}$.

__Solution__:

Its characteristic equation can be written as $\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$ where

$S_1 = sum\ of\ the\ main\ diagonal\ elements = 2 + 1 - 3 = 0$

$S_2 = Sum\ of\ the\ minors\ of\ the\ main\ diagonal\ elements$

$$= \begin{vmatrix} 1 & 2 \\ 1 & -3 \end{vmatrix} + \begin{vmatrix} 2 & -7 \\ 0 & -3 \end{vmatrix} + \begin{vmatrix} 2 & 2 \\ 2 & 1 \end{vmatrix} = -5 + (-6) + (-2) = -5 - 6 - 2 = -13$$

$S_3 = Determinant\ of\ A = |A|$ = 2 (-5)-2 (-6) $-7(2)$ = -10 + 12 – 14 = -12

Therefore, the characteristic equation of A is $\lambda^3 - 13\lambda + 12 = 0$

$$\begin{array}{c|cccc} 3 & 1 & 0 & -13 & 12 \\ & 0 & 3 & 9 & -12 \\ \hline & 1 & 3 & -4 & \multicolumn{1}{|c}{0} \end{array}$$

$(\lambda - 3)(\lambda^2 + 3\lambda - 4) = 0$

$$\Rightarrow \lambda = 3\ , \lambda = \frac{-3 \pm \sqrt{3^2 - 4(1)(-4)}}{2(1)} = \frac{-3\ \pm \sqrt{25}}{2} = \frac{-3 \pm 5}{2} = \frac{-3+5}{2}, \frac{-3-5}{2} = 1, -4$$

Therefore, the eigen values are $\lambda =$ 3, 1 and -4.

2. The product of two eigen values of the matrix A = $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ is 16. Find the third eigen value.

Solution: Let the eigen values of the matrix be $\lambda_1, \lambda_2, \lambda_3$.

Given $\lambda_1\lambda_2 = 16$

We know that $\lambda_1\lambda_2\lambda_3 = |A|$ (Since product of the eigen values is equal to the determinant of the matrix)

$\lambda_1\lambda_2\lambda_3 = \begin{vmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{vmatrix}$ = 6(9-1)+2(-6+2) +2(2-6) = 48-8-8 = 32

$Therefore, \lambda_1\lambda_2\lambda_3 = 32 \Rightarrow 16\lambda_3 = 32 \Rightarrow \lambda_3 = 2$

3. Show that the matrix $A = \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$ satisfies its own characteristic equation.

Solution: Let A $= \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$. The characteristic equation of A is $\lambda^2 - S_1\lambda + S_2 = 0$ where

$S_1 = Sum\ of\ the\ main\ diagonal\ elements = 1 + 1 = 2,$

$S_2 = |A| = 1 - (-4) = 5$

The characteristic equation is $\lambda^2 - 2\lambda + 5 = 0$

To prove $A^2 - 2A + 5I = 0$

$A^2 = A(A) = \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} -3 & -4 \\ 4 & -3 \end{bmatrix}$

$A^2 - 2A + 5I = \begin{bmatrix} -3 & -4 \\ 4 & -3 \end{bmatrix} - \begin{bmatrix} 2 & -4 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Therefore, the given matrix satisfies its own characteristic equation.

4. If A = $\begin{bmatrix} 1 & 0 \\ 0 & 5 \end{bmatrix}$ write A^2 interms of A and I, using Cayley – Hamilton theorem.

Solution: Cayley-Hamilton theorem states that every square matrix satisfies its own characteristic equation.

The characteristic equation of A is $\lambda^2 - S_1\lambda + S_2 = 0$ where

$S_1 = Sum\ of\ the\ main\ diagonal\ elements = 6$, $S_2 = |A| = 5$

Therefore, the characteristic equation is $\lambda^2 - 6\lambda + 5 = 0$

By Cayley-Hamilton theorem, $A^2 - 6A + 5I = 0$

(i.e.) $A^2 = 6A - 5I$

5. Determine A^4 If A = $\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$, using Cayley – Hamilton theorem.

<u>Solution</u>: Cayley-Hamilton theorem states that every square matrix satisfies its own characteristic equation.

The characteristic equation of A is $\lambda^2 - S_1\lambda + S_2 = 0$ where

$S_1 = Sum\ of\ the\ main\ diagonal\ elements = 0$

$S_2 = |A| = -5$

Therefore, the characteristic equation is $\lambda^2 - 5 = 0$

By Cayley-Hamilton theorem, $A^2 - 5I = 0$ (i.e.) $A^2 = 5I$

$$A^2 = \begin{bmatrix} 5 & 0 \\ 0 & -5 \end{bmatrix}$$

$$\text{Therefore } A^4 = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 25 & 0 \\ 0 & 25 \end{bmatrix}$$

6. Given A = $\begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$, Find A^{-1} using Cayley – Hamilton theorem.

<u>Solution:</u> The characteristic equation of A is $\lambda^2 - S_1 \lambda + S_2 = 0$,

Here, $S_1 = 4$ and $S_2 = -5 \Rightarrow \lambda^2 - 4\lambda - 5 = 0$.

By Cayley – Hamilton theorem $A^2 - 4A - 5I = 0$.

Multiply by A^{-1}, we get $A - 4I - 5A^{-1} = 0$ $\quad \therefore A^{-1} = \frac{1}{5}[A - 4I] = \begin{bmatrix} \frac{-3}{5} & \frac{2}{5} \\ \frac{4}{5} & \frac{-1}{5} \end{bmatrix}$

7. Determine the nature of the following quadratic form $f(x_1, x_2, x_3) = x_1^2 + 2x_2^2$.

<u>Solution</u>: The matrix of the quadratic form is Q = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

The eigen values of the matrix are 1, 2, 0

Therefore, the quadratic form is Positive Semi-definite.

8. Discuss the nature of the quadratic form $2x^2+3y^2+2z^2+2xy$ without reducing it to canonical form.

Solution: The matrix of the quadratic form is Q = $\begin{bmatrix} 2 & 1 & 0 \\ 1 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$.

$D_1 = 2(+ve)$

$D_2 = \begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} = 5(+ve)$

$D_3 = \begin{vmatrix} 2 & 1 & 0 \\ 1 & 3 & 0 \\ 0 & 0 & 2 \end{vmatrix} = 2(6-0)-1(2-0)+0 = 12-2 = 10(+ve)$

Therefore, the quadratic form is positive definite.

9. Find the quadratic form of the matrix A= $\begin{bmatrix} 2 & 0 & -1 \\ 0 & 3 & 2 \\ -1 & 2 & 5 \end{bmatrix}$.

Solution: Let A = $\begin{bmatrix} 2 & 0 & -1 \\ 0 & 3 & 2 \\ -1 & 2 & 5 \end{bmatrix}$

Quadratic form is $X^T A X$, where $X^T = (x, y, z)$

Therefore, Q.F=$\begin{pmatrix} x & y & z \end{pmatrix}\begin{bmatrix} 2 & 0 & -1 \\ 0 & 3 & 2 \\ -1 & 2 & 5 \end{bmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = 2x^2 + 3y^2 + 5z^2 - 2zx + 4yz$

10. If the eigen vectors of a 2 × 2 matrix A are $X_1=\begin{pmatrix} 1 \\ -2 \end{pmatrix}$, $X_2=\begin{pmatrix} 2 \\ 1 \end{pmatrix}$, then verify that they are mutually orthogonal. Also find normalized matrix N.

Solution: X_1 and X_2 are said to be mutually orthogonal if $X_1^T X_2 = 0$.

$$\begin{pmatrix} 1 & -2 \end{pmatrix}\begin{pmatrix} 2 \\ 1 \end{pmatrix} = 0$$

Modal matrix M= $\begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$. Normalized matrix N= $\begin{bmatrix} \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ \frac{-2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{bmatrix}$

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Unit – II

FUNCTIONS OF SEVERAL VARIABLES

Part – B

1. **If $u = (x - y)(y - z)(z - x)$, then find $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$.**

   (A) 0 **(B) 1** **(C) 2** **(D) 3**

   Solution:

   Given $u = (x - y)(y - z)(z - x)$

$$\frac{\partial u}{\partial x} = (y - z)[(x - y)(-1) + (z - x)(1)]$$
$$= -(x - y)(y - z) + (y - z)(z - x)$$
$$\frac{\partial u}{\partial y} = (z - x)[(x - y)(1) + (y - z)(-1)]$$
$$= (x - y)(z - x) - (y - z)(z - x)$$
$$\frac{\partial u}{\partial z} = (x - y)[(y - z)(1) + (z - x)(-1)]$$
$$= (x - y)(y - z) - (x - y)(z - x)$$
$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0 \quad (\textbf{Option A})$$

2. **If $x = r\cos\theta, y = r\sin\theta$ find $\frac{\partial x}{\partial r}, \frac{\partial y}{\partial \theta}$.**

   (A) cos θ, sin θ (B) cos θ, r cos θ (C) r cos θ, sin θ (D) r, θ

   Solution:

$$\frac{\partial x}{\partial r} = \cos\theta$$
$$\frac{\partial y}{\partial \theta} = r\cos\theta \quad (\textbf{Option B})$$

3. **If $f(x,y) = \sin\left(\frac{x}{y}\right)$, then find $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y}$.**

(A) 0 **(B) 1** **(C) 2** **(D) 3**

Solution:

$$\frac{\partial f}{\partial x} = \cos\left(\frac{x}{y}\right)\frac{1}{y}, \quad \frac{\partial f}{\partial y} = \cos\left(\frac{x}{y}\right)\left(-\frac{x}{y^2}\right)$$

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = 0 \text{ (Option A)}$$

4. **Find $\frac{dy}{dx}$ when $x^3 + y^3 = 3axy$.**

$(A)\ -\frac{x^2-ay}{y^2-ax}$ $(B)\ \frac{x^2-ay}{y^2-ax}$ $(C)\ \frac{y^2-ax}{x^2-ay}$ $(D)\ -\frac{y^2-ax}{x^2-ay}$

Solution:

Let $f(x,y) = x^3 + y^3 - 3axy$

$$\frac{dy}{dx} = -\frac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial y}} = -\frac{3x^2 - 3ay}{3y^2 - 3ax}$$

$$= -\frac{x^2 - ay}{y^2 - ax} \quad \text{(Option A)}$$

5. **If $x = uv$, $y = \frac{u}{v}$, find $\frac{\partial(x,y)}{\partial(u,v)}$.**

(A) $\frac{-2u}{v}$ **(B)** $\frac{2u}{v}$ **(C)** $\frac{-2v}{u}$ **(D)** $\frac{2v}{u}$

Solution:

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} v & u \\ \frac{1}{v} & \frac{-u}{v^2} \end{vmatrix} = -\frac{2u}{v} \quad \text{(Option A)}$$

6. If $f(x,y) = e^x \sin y$, then find $f_{yy}(0,0)$.

(A) 0 **(B) 1** **(C) 2** **(D) 3**

Solution:

$f_y = e^x \cos y$

$f_{yy}(x,y) = e^x(-\sin y)$

$f_{yy}(0,0) = 0$ **(Option A)**

7. If $x^y = y^x$, then find $\frac{dy}{dx}$.

(A) $\dfrac{y\,x^{y-1} - y^x \log y}{x\,y^{x-1} - x^y \log x}$ **(B)** $\dfrac{y\,x^{y-1} + y^x \log y}{x\,y^{x-1} - x^y \log x}$

(C) $\dfrac{y\,x^{y-1} + y^x \log y}{x\,y^{x-1} + x^y \log x}$ **(D)** $\dfrac{y\,x^{y-1} - y^x \log y}{x\,y^{x-1} + x^y \log x}$

Solution:

$f(x, y) = x^y - y^x = 0$

$$\frac{dy}{dx} = \frac{-f_x}{f_y} = \frac{y\,x^{y-1} - y^x \log y}{x\,y^{x-1} - x^y \log x}$$ **(Option A)**

8. If $f(x, y) = \tan^{-1}\left(\frac{y}{x}\right)$**, then find** $f_x(x, y)$ **at the point (1, 1).**

(A) -1/2 **(B) 1** **(C) 1/2** **(D) 3**

Solution:

$f_x(x, y) = \dfrac{-y}{x^2 + y^2}$ $f_x(1,1) = -\dfrac{1}{2}$ **(Option A)**

9. If $x = r\cos\theta$, $y = r\sin\theta$ **, then find** $\dfrac{\partial(x, y)}{\partial(r, \theta)}$**.**

(A) *r* **(B) 1/ *r*** **(C) 1/2** **(D) 1**

Solution:

$$\text{Now} \quad \frac{\partial(x, y)}{\partial(r, \theta)} = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial v}{\partial \theta} \end{vmatrix} = \begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix} = r\left(\sin^2\theta + \cos^2\theta\right) = r(1) = r$$

(Option A)

10. If $u = 2xy$, $v = x^2 - y^2$, then find $\dfrac{\partial(u,v)}{\partial(x,y)}$.

(A) $-4y^2 - 4x^2$ **(B) $-4y^2 + 4x^2$**
(C) $4y^2 - 4x^2$ **(D) $4y^2 + 4x^2$**

$$\frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} 2y & 2x \\ 2x & -2y \end{vmatrix} = -4y^2 - 4x^2$$ **(Option A)**

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Unit – II

FUNCTIONS OF SEVERAL VARIABLES

Part – C

1. Find the Taylor's series expansion of $f(x,y)=xe^y$ upto first degree terms near the point (0, 0).

Solution:

$$f(x,y)=xe^y \qquad f(0,0)=0$$
$$f_x=e^y \qquad f_x(0,0)=1$$
$$f_y=xe^y \qquad f_y(0,0)=0$$

$$f(x,y)=0+1.(x-0)+0.(y-0)=x$$

2. *If* $x=r\cos\theta$ *and* $y=r\sin\theta$, *then* find $\frac{\partial r}{\partial x}$.

Solution: Given $x=r\cos\theta$, $y=r\sin\theta$, then $r^2=x^2+y^2 \Rightarrow r=\sqrt{x^2+y^2}$

Now $\frac{\partial r}{\partial x}=\frac{1}{2\sqrt{x^2+y^2}}(2x)=\frac{x}{\sqrt{x^2+y^2}}=\frac{x}{r}$

3. Find $\frac{du}{dt}$, if $u = x\,y^2 + x^2\,y$, where $x = a\,t^2$, $y = 2\,a\,t$.

Solution:

$$\frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt}$$

$$\frac{du}{dt} = (y^2 + 2\,x\,y)(2\,a\,t) + (2\,x\,y + x^2)(2\,a) = 16\,a^3\,t^3 + 10\,a^3\,t^4.$$

4. Find $\frac{du}{dt}$, if $u = x^3\,y^4$, where $x = t^3$, $y = t^2$.

Solution:

$$\frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt}$$

$$\frac{du}{dt} = (3x^2\,y^4)(3t^2) + (4\,x^3\,y^3)(2t) = 17\,t^{16}.$$

5. Expand $e^x \sin y$ as Maclaurin's series upto first degree terms.

Solution:

$f(x, y) = e^x \sin y$ $\quad\quad$ $f(0,0) = 0$

$f_x(x, y) = e^x \sin y$ $\quad\quad$ $f_x(0,0) = 0$

$f_y(x, y) = e^x \cos y$ $\quad\quad$ $f_y(0,0) = 1$

Maclaurin's series

$$f(x, y) = 0 + x.0 + y.1 = y$$

6. If $f(x, y) = x^y$, then find $f_{yy}(1,1)$.

Solution:

$$f_y = x^y (\log x)$$

$$f_{yy}(x, y) = x^y (\log x)^2$$

$f_{yy}(1,1) = 0$

7. A rectangular box open at the top is to have a volume of 32 cubic feet. How do you define the auxiliary function using Lagrange's method of multipliers to find the dimensions of the box that requires the least material for its construction?

Solution:

Volume is 32 i.e., xyz = 32

Condition g(x,y,z)=xyz – 32

f(x,y,z) = Total surface area = xy + 2yz + 2zx

$F(x, y, z) = (xy + 2yz + 2zx) + \lambda (xyz – 32)$

8. How do you define the auxiliary function using Lagrange's method of multipliers to find the volume of the greatest rectangular parallelopiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$**?**

Solution:

Volume of the rectangular is 2x .2y .2z = 8xyz =f(x,y,z)

Condition $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}-1 = g(x,y,z)$$

$$F(x,y,z)=8xyz+\lambda\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}-1\right)$$

9. Find the stationary points of the function $f(x,y)=3(x^2-y^2)-x^3+y^3$**.**

Solution:

$$f(x,y)=3(x^2-y^2)-x^3+y^3$$

$$p=\frac{\partial f}{\partial x}=6x-3x^2;\ \mathrm{q}=\frac{\partial f}{\partial y}=-6y+3y^2;$$

$$r=\frac{\partial^2 f}{\partial x^2}=6-6x;\quad s=\frac{\partial^2 f}{\partial x\partial y}=0 \text{ and } t=\frac{\partial^2 f}{\partial y^2}=-6+6y.$$

p = 0 implies x = 0 and x = 2.

and q = 0 implies y = 0 and y = 2

Therefore the stationary points are (0, 0), (0, 2), (2, 0) and (2, 2).

10. Expand e^{xy} in powers of x and y up to first degree term at the point (0, 0) using Taylor's series expansion.

Solution:

$f(x,y) = e^{xy}$ $\qquad f(0,0) = 1$

$f_x(x,y) = y e^{xy}$ $\qquad f_x(0,0) = 0$

$f_y(x,y) = x e^{xy}$ $\qquad f_y(0,0) = 0$

Taylor's series

$f(x,y) = 1$

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Unit – III

ORDINARY DIFFERENTIAL EQUATIONS

Part – B

1. Solve $\left(D^2 - 7D + 12\right)y = 0$.

$(a) y = Ae^{3x} + Be^{4x}$ $(b) y = Ae^{-3x} + Be^{4x}$

$(c) y = Ae^{3x} + Be^{-4x}$ $(d) y = Ae^{-3x} + Be^{-4x}$

$m^2 - 7m + 12 = 0$
$(m-3)(m-4) = 0$
$m = 3, 4$
$y = Ae^{3x} + Be^{4x}$ **(Option (a))**

2. Find the particular integral of $\left(D^2 - 9\right)y = e^{-2x}$.

$(a) PI = \frac{1}{13}e^{-2x}$ $(b) PI = -\frac{1}{5}e^{-2x}$

$(c) PI = \frac{x}{5}e^{-2x}$ $(d) PI = \frac{1}{5}e^{-2x}$

$$PI = \frac{1}{D^2-9}e^{-2x}$$
$$= \frac{1}{4-9}e^{-2x}$$
$$= -\frac{1}{5}e^{-2x}$$ **(Option (b))**

3. Find the particular integral of $(D^2+3D+2)y = e^{-2x}$.

$(a) PI = -xe^{-2x}$ $(b) PI = xe^{-2x}$

$(c) PI = \frac{e^{-2x}}{12}$ $(d) PI = \frac{xe^{-2x}}{12}$

$$PI = \frac{1}{D^2+3D+2}e^{-2x}$$
$$= \frac{1}{4-6+2}e^{-2x}$$
$$= x.\frac{1}{2D+3}e^{-2x}$$
$$PI = -xe^{-2x}$$ **(Option (a))**

4. Find the particular integral of $(D^2+4)y = \sin 2x$.

$(a) PI = -\frac{x\cos 2x}{4}$ $(b) PI = -\frac{\sin 2x}{8}$

$(c) PI = \frac{x\sin 2x}{4}$ $(d) PI = \frac{\sin 2x}{8}$

$$\begin{aligned} PI &= \frac{1}{D^2+4}\sin 2x \\ &= x.\frac{1}{2D}\sin 2x \\ &= -x.\frac{\cos 2x}{4} \textbf{ (Option (a))} \end{aligned}$$

5. Find the particular integral of $(D^2+D+1)y = 3x-1$.

$(a) PI = 3x-4$ $\quad$ $(b) PI = 3x$

$(c) PI = 3x-1$ $\quad$ $(d) PI = 3x^2-4$

$$\begin{aligned} PI &= \frac{1}{D^2+D+1}(3x-1) \\ &= [1+(D+D^2)]^{-1}(3x-1) \\ &= (3x-1)-D(3x-1) \\ PI &= 3x-4 \textbf{ (Option (a))} \end{aligned}$$

6. Find the particular integral of $(D^2+D+1)y = x$

$(a) PI = 3x-4$ $\quad$ $(b) PI = 3x$

$(c) PI = x-1$ $\quad$ $(d) PI = 3x^2-4$

$$\begin{aligned} PI &= \frac{1}{D^2+D+1}(x) \\ &= [1+(D+D^2)]^{-1}(x) \\ &= [1-(D+D^2)](x) \\ &= (x-D(x)) \quad = x-1 \\ PI &= x-1 \end{aligned}$$

(Option C)

7. **Solve** $(D^3 - 6D^2 + 11D - 6)y = 0$

$(a)\, y = Ae^{x} + Be^{2x} + Ce^{3x}$ $(b)\, y = Ae^{x} + Be^{-2x} + Ce^{3x}$

$(c)\, y = Ae^{x} + Be^{2x} + Ce^{-3x}$ $(d)\, y = Ae^{x} + Be^{-2x} + Ce^{-3x}$

$$m^3 - 6m^2 + 11m - 6 = 0$$
$$(m-1)(m-2)(m-3) = 0$$
$$m = 1,2,3$$
$$C.F = Ae^{x} + Be^{2x} + Ce^{3x}$$
Hence
$$y = Ae^{x} + Be^{2x} + Ce^{3x}$$

(Option A)

8. Find the particular integral of $(D^2 + D - 2)y = Sinx$

$(a)\, PI = \frac{-1}{10}(\cos x + 3\sin x)$ $(b)\, PI = \frac{1}{10}(\cos x + 3\sin x)$

$(c)\, PI = \frac{-1}{10}(\sin x + 3\cos x)$ $(d)\, PI = \frac{-1}{10}(\sin x - 3\cos x)$

$$\text{P.I} = \frac{1}{D-3}\sin x = \frac{D+3}{D^2-9}\sin x$$, Rationalizing the denominator

$$= \frac{(D+3)\sin x}{-10},$$ Putting $D^2 = -1$

$$\therefore \text{P.I.} = \frac{-1}{10}(D\, sin\, x + 3\sin x)$$

$$= \frac{-1}{10}(\cos x + 3\sin x)$$

(Option A)

9. Find the complementary function of $(D^2+1)y = \cos ec\, x$.

$(a) CF = (A+Bx)e^x$ $(b) CF = (A+Bx)e^x$

$(c) CF = A\cos x + B\sin x$ $(d) CF = (A\cos x + B\sin x)e^x$

$m^2+1=0 \Rightarrow m = \pm i$

$CF = A\cos x + B\sin x$ **(Option (c))**

10**.** Solve $(D^2+4D+4)y = 0$.

$(a) y = Ae^{-2z} + Be^{-2z}$ $(b) y = (A+Bx)e^{-2x}$

$(c) y = \dfrac{A}{x} + \dfrac{B}{x^2}$ $(d) y = Ax + Bx^2$

$m^2+4m+4=0$

$m = -2,-2$

$y = (A+Bx)e^{-2x}$ **(Option (B))**

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SRM Institute of Science and Technology
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Department of Mathematics
18MAB101T - Calculus And Linear Algebra

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Branch: Common to ALL B.Tech. except B.Tech. (Business Systems)

Unit – III

ORDINARY DIFFERENTIAL EQUATIONS

Part – C

1. Find the particular integral of $(D^2+3D+2)y=e^{-x}$

Ans:

$$PI=\frac{1}{D^2+3D+2}e^{-x}$$
$$=\frac{1}{1-3+2}e^{-x}$$
$$=x.\frac{1}{2D+3}e^{-x}$$
$$=x.\frac{1}{1}e^{-x}$$
$$PI=xe^{-x}$$

2. Solve $x^2\frac{d^2y}{dx^2}-7x\frac{dy}{dx}+12y=0$

Ans:

$$Let\ x=e^z \Rightarrow z=\log x$$
$$subsitute\ \mathrm{xD}=\mathrm{D}^{'};x^2D^2=\mathrm{D}'(\mathrm{D}'-1)$$
$$(\mathrm{D}'^2-8\mathrm{D}'+12)\mathrm{y}=0$$
$$\mathrm{m}^2-8m+12=0\Rightarrow m=2,6$$
$$C.F=Ae^{2z}+Be^{6z}$$
$$y=Ax^2+Bx^6$$

3. Find the particular integral of $\frac{d^2y}{dx^2}+\frac{dy}{dx}-2y=e^x$.

Ans:

$$\text{P.I.} = \frac{1}{f(D)}\,F(x) = \frac{1}{f(D)}e^x \text{, putting } D = 1,\ f(1) = 0$$

$$\therefore \text{P.I} = x\frac{1}{f'(D)}e^x \qquad \because \text{P.I.} = x\frac{1}{f'(a)}e^{ax} \text{ if } f(a) = 0$$

$$\Rightarrow \text{P.I.} = x\frac{1}{2D+1}e^x = \frac{1}{f'(1)}e^x,\ f'(1) \neq 0$$

$$\Rightarrow \text{P.I.} = \frac{xe^x}{3}$$

4. Find the particular integral of $(D^2+4)y = Cos2x$

Ans:

$$PI = \frac{1}{D^2+4}\cos 2x$$

$$= \frac{1}{-4+4}\cos 2x$$

$$= x.\frac{1}{2D}\cos 2x$$

$$= \frac{x}{4}\sin 2x$$

$$PI = \frac{x}{4}\sin 2x$$

5. Find the particular integral of $(D^2+9)y = x\cos x$

Ans:

$$\text{P.I.} = \frac{1}{f(D)}\,F(x) = \frac{1}{D^2+9}\,x\cos x$$

$$= x\frac{1}{D^2+9}\cos x + \frac{-2D}{(D^2+9)^2}\cos x$$

$$= x\frac{1}{-1+9}\cos x + \frac{-2D}{(-1+9)^2}\cos x, \qquad \text{Putting } D^2 = -1$$

$$= \frac{x\cos x}{8} - \frac{2D\cos x}{64}$$

$$= \frac{x\cos x}{8} - \frac{2D\cos x}{64}$$

$$\therefore \text{P.I.} = \frac{x\cos x}{8} + \frac{\sin x}{32}$$

6. Find the particular integral of $\frac{d^2y}{dx^2} - y = 5x - 2.$

Ans:

$$\text{P.I.} = \frac{1}{f(D)}\,F(x) = \frac{1}{D^2-1}(5x-2)$$

$$= \frac{1}{-(1-D^2)}(5x-2)$$

$$= -(1-D^2)^{-1}(5x-2)$$

$$= -[1 + D^2 + \cdots](5x-2)$$

$$= -(5x-2)$$

7. Solve $x^2\dfrac{d^2y}{dx^2}-4x\dfrac{dy}{dx}+6y=0$

Ans:

$$Let\ x=e^z \Rightarrow z=\log x$$
$$subsitute\ \mathrm{xD}=\mathrm{D}';\ x^2D^2=\mathrm{D}'(\mathrm{D}'-1)$$
$$(\mathrm{D}'^2-5\mathrm{D}'+6)\mathrm{y}=0$$
$$\mathrm{m}^2-5m+6=0\Rightarrow m=-2,-3$$
$$C.F=Ae^{-2z}+Be^{-3z}$$
$$y=\frac{A}{x^2}+\frac{A}{x^3}$$

8. Solve $x^2\dfrac{d^2y}{dx^2}-x\dfrac{dy}{dx}+y=0$.

Ans:

$$(\mathrm{D}'^2-2\mathrm{D}'+1)\mathrm{y}=0$$
$$\mathrm{m}^2-2m+1=0\Rightarrow m=1,1$$
$$C.F=(A+Bz)e^z$$
$$y=(A+B\log x)x$$

9. Solve $x\dfrac{dy}{dx}+2y=0$

Ans:

$$(\mathrm{D}'+2)\mathrm{y}=0$$
$$m+2=0\Rightarrow m=-2$$
$$y=\frac{A}{x^2}$$

10. Find the particular integral of $(x^2D^2 + xD - 1)y = \sin(\log x)$.

Ans

$$\boxed{\begin{aligned} PI &= \frac{1}{D'^2 - 1}\sin z \\ &= \frac{\sin z}{-2} \\ &= -\frac{\sin(\log x)}{2} \end{aligned}}$$

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SRM Institute of Science and Technology
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18MAB101T - Calculus And Linear Algebra

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Unit – IV

DIFFERENTIAL CALCULUS

Part – B

1. Envelope of the curve $y = mx + \frac{a}{m}$ (where m is the parameter) is

(A) $x^2 + a\,y = 0$ (B) $x + 4\,a\,y = 0$

(C) $y^2 - 4\,a\,x = 0$ (D) $y^2 + 4ax = 0$

Solution: Given: $y = mx + \frac{a}{m}$

$$y = \frac{m^2 x + a}{m}$$

$$m^2 x - y\,m + a = 0$$

The above equation is a quadratic equation in 'm'.

The discriminant is $b^2 - 4ac = 0$.

The envelope of the curve is $y^2 - 4\,a\,x = 0$. **(Option C)**

2. The radius of curvature of the curve $y = e^x$ at $x = 0$ is

(A) $2\sqrt{2}$ (B) $\sqrt{2}$

(C) 2 (D) 4

Solution:

$y_1 = e^x$ at $x = 0$ is 1

$y_2 = e^x$ at $x = 0$ is 1

$$\rho = \frac{\left(1 + y_1^{\,2}\right)^{\frac{3}{2}}}{y_2}$$

$\rho = 2\sqrt{2}$ **(Option A)**

3. The radius of curvature of the curve $y = 4 \sin x$ at $x = \frac{\pi}{2}$ is

(A) $\frac{1}{2}$ (B) $\frac{-1}{2}$

(C) $\frac{1}{4}$ (D) $\frac{3}{4}$

Solution:

$y_1 = 4\cos x \ at \ x = \frac{\pi}{2} is\ 0$

$y_2 = -4\sin x\, at\ x = \frac{\pi}{2} is - 4$

$$\rho = \frac{\left(1 + y_1^{\,2}\right)^{\frac{3}{2}}}{y_2}$$

$|\rho| = \frac{1}{4}$ **(Option C)**

4. The envelope of family of lines y = $mx + am^2$ (where *m* is the parameter) is

(A) $x^2 + 2ay = 0$ (B) $x^2 + 4\,a\,y = 0$

(C) $y^2 + 2ax = 0$ (D) $x^2 + 4\,a\,x = 0$

Solution:

The given equation is quadratic in 'm'.

The discriminant is $b^2 - 4ac = 0$.

Envelope of the family of lines is $x^2 + 4ay = 0$. **(Option B)**

5. The envelope of the family of lines $\frac{x}{t} + y\,t = 2c$, t being the parameter is

(A) $x^2 + y^2 = c^2$ (B) $x\,y = c^2$

(C) $x^2\,y^2 = c^2$ (D) $x^2 - y^2 = c^2$

Solution:

Simplifying the equation $\frac{x}{t} + y\,t = 2c$, we get $yt^2 - 2ct + x = 0$

The discriminant is $b^2 - 4ac = 0$.

Envelope of the family of lines is $x\,y = c^2$. **(Option B)**

6. The radius of curvature of the curve $r = e^{\theta}$ at any point on it is

(a) $2\sqrt{2}$ (b) $\sqrt{2}\,r$

(c) 2 (d) 4

Solution:

$r' = e^{\theta}$

$r'' = e^{\theta}$

$$\rho = \frac{\left(r^2 + r'^2\right)^{\frac{3}{2}}}{r^2 - rr'' + 2r'^2}$$

$\rho = \sqrt{2}\,r$

7. The radius of curvature at the point (3, 4) on the curve $x^2 + y^2 = 25$ is

(A) 5 (B) 4 (C) 0 (D) 2

Solution:

We know that the radius of curvature of the circle is equal to its radius.

$\rho = 5$ **(Option A)**

8. B (5/2, 1/2) = _______.

(A) 1 (B) 4

(C) $3\pi/8$ (D) π

Solution:

$$B(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$$

$$B(5/2,1/2) = \frac{\Gamma(5/2)\Gamma(1/2)}{\Gamma(3)} = \frac{3\pi}{8}$$ **(Option C)**

9. $\Gamma(-5/2)$ **=___________.**

(A) 1 (B) 4

(C) 1/2 (D) $\frac{-8\sqrt{\pi}}{15}$

Solution:

$$\Gamma\left(-n+\frac{1}{2}\right) = \frac{(-2)^n}{1.3.5\cdots(2n-1)}\sqrt{\pi}$$

$$\Gamma\left(-\frac{5}{2}\right) = \Gamma\left(\frac{-6+1}{2}\right)$$

$$= \Gamma\left(-3+\frac{1}{2}\right) = \frac{-8}{15}\sqrt{\pi}$$

(Option D)

10. Evaluate $\int_0^\infty e^{-x} x^4 \, dx$.

(A) 1 (B) 24

(C) 1/2 (D) $\frac{-8\sqrt{\pi}}{3}$

Solution

$$\Gamma(n) = \int_0^\infty e^{-x} x^{n-1} \, dx$$

$$\int_0^\infty e^{-x} x^4 \, dx = \int_0^\infty e^{-x} x^{5-1} \, dx = \Gamma(5) = 4! = 24$$

(Option B)

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Unit – IV

DIFFERENTIAL CALCULUS

Part – C

1. Find the radius of curvature of the curve $y^2 = 12x$ at the point (3, 6).

Solution:

$$\frac{dy}{dx} = \frac{6}{y}$$

At (3, 6), $\frac{dy}{dx} = 1$

$$\frac{d^2y}{dx^2} = 6\left(\frac{-1}{y^2}\right)\frac{dy}{dx}$$

At (3, 6), $\frac{d^2y}{dx^2} = 6\left(\frac{-1}{36}\right)1 = \frac{-1}{6}$

$$\rho = \frac{\left(1 + {y_1}^2\right)^{\frac{3}{2}}}{y_2}$$

$$\rho = \frac{(1+1)^{\frac{3}{2}}}{-1/6} = -12\sqrt{2}$$

2. Find the radius of curvature of the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ at the point (a/4, a/4).

Solution:

$$\frac{dy}{dx} = -\sqrt{\frac{y}{x}}$$

At (a/4, a/4), $\frac{dy}{dx} = -1$

$$\frac{d^2y}{dx^2} = -\left[\frac{\sqrt{x}.\frac{1}{2}y^{-1/2}\frac{dy}{dx} - \sqrt{y}.\frac{1}{2}x^{-1/2}}{x}\right]$$

At (a/4, a/4), $\frac{d^2y}{dx^2} = \frac{4}{a}$

$$\rho = \frac{\left(1 + {y_1}^2\right)^{\frac{3}{2}}}{y_2}$$

$$\rho = \frac{a}{\sqrt{2}}$$

3. Find the radius of curvature of the curve $xy = c^2$ at the point (c, c).

Solution:

$$x.\frac{dy}{dx} + y.1 = 0$$

$$\frac{dy}{dx} = \frac{-y}{x}$$

At (c, c), $\frac{dy}{dx} = -1$

$$\frac{d^2y}{dx^2} = -\left[\frac{x\frac{dy}{dx} - y.1}{x^2}\right]$$

At (c, c), $\frac{d^2y}{dx^2} = -\left(\frac{-2c}{c^2}\right) = \frac{2}{c}$

$$\rho = \frac{\left(1+y_1^{\ 2}\right)^{\frac{3}{2}}}{y_2}$$

$$\rho = \frac{(1+1)^{\frac{3}{2}}}{2/c} = \sqrt{2}\,c$$

4. If $\boldsymbol{x = a\cos}\theta$, $y = \boldsymbol{b\sin}\,\theta$, then find $\frac{dy}{dx}$.

Solution:

$$\frac{dx}{d\theta} = -a\sin\theta\,,\ \frac{dy}{d\theta} = -a\cos\theta\,,\ \frac{dy}{dx} = -\frac{b}{a}\cot\theta$$

5. Find the envelope of x cos θ + y sin θ = 1, θ being the parameter.

Solution:

$x\cos\theta + y\sin\theta = 1$ ____________________(1)

Differentiate partially w.r.t. θ.

$x(-\sin\theta) + y(\cos\theta) = 0$ ____________________(2)

Squaring and adding (1) and (2)

$x^2 + y^2 = 1$

6. Find the envelope of x cos α + y sin α = a sec α, α being the parameter.

Solution:

$x\cos\alpha + y\sin\alpha = a\sec\alpha$

Divide by cos α.

$x + y\tan\alpha = a\sec^2\alpha$

$x + y\tan\alpha = a(1 + \tan^2\alpha)$

$a\tan^2\alpha - y\tan\alpha + (a - x) = 0$

Here $A = a$, $B = -y$, $C = a - x$

Envelope is given by $B^2 - 4AC = 0$.

$$y^2 = 4a\,(a - x)$$

7. Find $\int_0^1 x^6 (1-x)^9\, dx$**.**

Solution:

$m = 7$, $n = 10$

$$B(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}\, dx$$

$$= \int_0^1 x^{7-1}(1-x)^{10-1}\, dx$$

$$= \frac{\Gamma(7)\Gamma(10)}{\Gamma(17)} = \frac{6!9!}{16!}$$

8. Prove that $\dfrac{B(m+1,n)}{B(m,n+1)} = \dfrac{m}{n}$**.**

Solution:

$$\frac{B(m+1,n)}{B(m,n+1)} = \frac{\dfrac{\Gamma(m+1)\Gamma(n)}{\Gamma(m+n+1)}}{\dfrac{\Gamma(m)\Gamma(n+1)}{\Gamma(m+n+1)}} = \frac{m\Gamma(m)\Gamma(n)}{n\Gamma(m)\Gamma(n)} = \frac{m}{n}$$

9. Find $\int_0^{\pi/2} \sqrt{\tan\theta}\, d\theta$**.**

Solution:

$$\int_0^{\pi/2} \sqrt{\tan\theta}\, d\theta = \int_0^{\pi/2} \sqrt{\frac{\sin\theta}{\cos\theta}}\, d\theta = \int_0^{\pi/2} \sin^{1/2}\theta \cos^{-1/2}\theta\, d\theta$$

$$= \frac{1}{2} B\left(\frac{3/2}{2}, \frac{1/2}{2}\right)$$

$$= \frac{1}{2} B\left(\frac{3}{4}, \frac{1}{4}\right)$$

$$= \frac{1}{2}\frac{\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{1}{4}\right)}{\Gamma(1)}$$

Formula $\Gamma(n)\Gamma(1-n) = \frac{\pi}{\sin n\pi}$

$$= \frac{\pi}{\sqrt{2}}$$

10. Find $\int_0^{\pi/2} \sin^6\theta\cos^6\theta\,d\theta$ **.**

Solution: m = 6, n = 6

$$\int_0^{\pi/2} \sin^6\theta\cos^6\theta\,d\theta = \frac{1}{2}B\left(\frac{m+1}{2}, \frac{n+1}{2}\right)$$

$$= \frac{1}{2}B\left(\frac{7}{2}, \frac{7}{2}\right)$$

$$= \frac{1}{2}\frac{\Gamma\left(\frac{7}{2}\right)\Gamma\left(\frac{7}{2}\right)}{\Gamma(7)}$$

$$= \frac{1}{2}\frac{\left(\frac{15}{8}\sqrt{\pi}\right)\left(\frac{15}{8}\sqrt{\pi}\right)}{6!}$$

11. Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.

Solution:

$$B(m,n) = 2\int_0^{\pi/2} \sin^{2m-1}\theta\,\cos^{2n-1}\theta\,d\theta$$

Put m = ½, n = ½ .

$$B\left(\frac{1}{2}, \frac{1}{2}\right) = 2\int_0^{\pi/2} d\theta$$

$$\frac{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{1}{2}\right)}{\Gamma(1)} = 2.\frac{\pi}{2}$$

$$\left(\Gamma\left(\frac{1}{2}\right)\right)^2 = \pi$$

Hence $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.

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18MAB101T - Calculus And Linear Algebra

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Unit – V

SEQUENCE AND SERIES

Part – B

1. The sequence $\left\{\frac{1}{n}\right\}$ **converges to _________.**

(A) 0 **(B) 1** **(C) ½** **(D) ∞**

Solution:

$$a_n = \frac{1}{n}$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{1}{n} = 0$$

$\{a_n\}$ converges to 0. **(Option A)**

2. The sequence $\left\{\frac{n+1}{2n+3}\right\}$ **converges to _________.**

(A) 0 **(B) 1** **(C) ½** **(D) ∞**

Solution:

$$a_n = \frac{n+1}{2n+3}$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n\left(1+\frac{1}{n}\right)}{n\left(2+\frac{3}{n}\right)} = \frac{1}{2}$$

$\{a_n\}$ converges to ½ . **(Option C)**

3. Test the convergence of the series $\sum \frac{1}{\sqrt{n+1}}$ **.**

(A) converges **(B) diverges**

(C) oscillates finitely **(D) oscillates infinitely**

Solution:

$$u_n = \frac{1}{\sqrt{n+1}} = \frac{1}{\sqrt{n\left(1+\frac{1}{n}\right)}}$$

Let $v_n = \frac{1}{\sqrt{n}}$

Now $\frac{u_n}{v_n} = \frac{1}{\sqrt{1+\frac{1}{n}}}$

$$\lim_{n\to\infty} \frac{u_n}{v_n} = 1$$

$\sum v_n = \sum \frac{1}{\sqrt{n}} = \sum \frac{1}{n^{1/2}}$ is divergent.

Hence by comparison test, $\Sigma\, u_n$ is divergent. **(Option B)**

4. Test the convergence of the series $1+\frac{1}{3}+\frac{1}{5}+\cdots$ **.**

(A) converges **(B) diverges**

(C) oscillates finitely **(D) oscillates infinitely**

Solution:

$$u_n = \frac{1}{2n-1} = \frac{1}{n\left(2-\frac{1}{n}\right)}$$

Let $v_n = \frac{1}{n}$

Now $\frac{u_n}{v_n} = \frac{1}{2-\frac{1}{n}}$

$$\lim_{n\to\infty} \frac{u_n}{v_n} = \frac{1}{2}$$

$\sum v_n = \sum \frac{1}{n}$ is divergent.

Hence by comparison test, $\Sigma\, u_n$ is divergent. **(Option B)**

5. Test the convergence of the series $\sum \frac{x^n}{n\,!}$ where $x > 0$.

(A) converges **(B) diverges**
(C) oscillates finitely **(D) oscillates infinitely**

Solution:

$$u_n = \frac{x^n}{n\,!}, \quad u_{n+1} = \frac{x^{n+1}}{(n+1)\,!}$$

Now $\frac{u_{n+1}}{u_n} = \frac{x}{n+1} = \frac{x}{n\left(1+\frac{1}{n}\right)}$

$$\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = \lim_{n\to\infty} \frac{x}{n\left(1+\frac{1}{n}\right)} = 0 < 1$$

Hence by Ratio test, $\Sigma\, u_n$ is convergent. **(Option A)**

6. Test the convergence of the series $\sum \frac{n\,!}{n^n}$.

(A) converges **(B) diverges**
(C) oscillates finitely **(D) oscillates infinitely**

Solution:

$$u_n = \frac{n\,!}{n^n}, \quad u_{n+1} = \frac{(n+1)\,!}{(n+1)^{n+1}}$$

Now $\frac{u_{n+1}}{u_n} = \frac{1}{\left(1+\frac{1}{n}\right)^n}$

$$\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = \frac{1}{e} < 1$$

Hence by Ratio test, $\Sigma\, u_n$ is convergent. **(Option A)**

7. The series $1+\frac{x}{1!}+\frac{x^2}{2!}+\cdots$ is ________________.

(A) absolutely convergent **(B) diverges to + ∞**
(C) oscillates finitely **(D) oscillates infinitely**

Solution:

$$u_n = \frac{x^{n-1}}{(n-1)\,!}, \quad u_{n+1} = \frac{x^n}{n\,!}$$

Now $\dfrac{u_{n+1}}{u_n} = \dfrac{x}{n}$

$$\lim_{n\to\infty}\left|\frac{u_{n+1}}{u_n}\right| = 0 < 1$$

Hence the series is absolutely convergent. **(Option A)**

8. Test the convergence of the series $\sum \dfrac{n^3}{3^n}$.

(A) converges **(B) diverges**
(C) oscillates finitely **(D) oscillates infinitely**

Solution:

$$u_n = \frac{n^3}{3^n}, \quad (u_n)^{1/n} = \frac{(n^{1/n})^3}{3}$$

$$\lim_{n\to\infty}(u_n)^{1/n} = \frac{1}{3} < 1$$

Hence by Root test, $\Sigma\, u_n$ is convergent. **(Option A)**

9. Test the convergence of the series $\sum \dfrac{3^n\, n\,!}{n^n}$.

(A) converges **(B) diverges**
(C) oscillates finitely **(D) oscillates infinitely**

Solution:

$$u_n = \frac{3^n\, n\,!}{n^n}, \quad u_{n+1} = \frac{3^{n+1}\,(n+1)\,!}{(n+1)^{n+1}}$$

Now $\dfrac{u_{n+1}}{u_n} = \dfrac{3}{\left(1+\dfrac{1}{n}\right)^n}$

$$\lim_{n\to\infty}\frac{u_{n+1}}{u_n} = \frac{3}{e} > 1$$

Hence by Ratio test, $\Sigma\, u_n$ is divergent. **(Option B)**

10. Test the convergence of the series $\sum \dfrac{1}{n^2}$.

(A) converges **(B) diverges**
(C) oscillates finitely **(D) oscillates infinitely**

Solution:

By Harmonic Series test or p-test, $\sum \dfrac{1}{n^2}$ converges. **Option (A)**

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Unit – V

SEQUENCE AND SERIES

Part – C

Question 1

Show that the series $\frac{1}{1.2.3}+\frac{1}{2.3.4}+\frac{1}{3.4.5}+\cdots$ **is convergent.**

Solution

$$u_n=\frac{1}{n(n+1)(n+2)}=\frac{1}{n\,n\left(1+\frac{1}{n}\right)n\left(1+\frac{2}{n}\right)},\ \text{and let } v_n=\frac{1}{n^3}.$$

Now $\frac{u_n}{v_n}=\frac{1}{\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)}$

$\lim\limits_{n\to\infty}\frac{u_n}{v_n}=1$, which is finite and non – zero.

$\therefore$ Both $\Sigma\, u_n$ and $\Sigma\, v_n$ converge or diverge together.

But $\sum v_n=\sum\frac{1}{n^3}$ is convergent by Harmonic series test or p - series test.

Hence by comparison test, $\Sigma\, u_n$ is convergent.

Question 2:

Show that the series $1+\frac{1}{2^2}+\frac{2^2}{3^3}+\frac{3^3}{4^3}+\cdots$ **is divergent.**

Solution

Neglect the first term. Then the series is $\frac{1}{2^2}+\frac{2^2}{3^3}+\frac{3^3}{4^3}+\cdots$

$$u_n=\frac{n^n}{(n+1)^{n+1}}=\frac{n^n}{n^{n+1}\left(1+\frac{1}{n}\right)^n\left(1+\frac{1}{n}\right)^1} \text{ and let } v_n=\frac{1}{n}.$$

Now $$\frac{u_n}{v_n}=\frac{1}{\left(1+\frac{1}{n}\right)^n\left(1+\frac{1}{n}\right)}$$

$$\lim_{n\to\infty}\frac{u_n}{v_n}=\frac{1}{e}, \text{which is finite and non-zero.}$$

$\therefore$ Both $\Sigma\, u_n$ and $\Sigma\, v_n$ converge or diverge together.

But $\sum v_n=\sum\frac{1}{n}$ is divergent by Harmonic series test or p-series test.

Hence by comparison test, $\Sigma\, u_n$ is divergent.

Question 3:

Show that the series $\sum_{n=1}^{\infty}\sin\left(\frac{1}{n}\right)$ is divergent.

Solution

$$u_n=\sin\left(\frac{1}{n}\right)$$

Let $v_n=\frac{1}{n}$

Now $$\frac{u_n}{v_n}=\frac{\sin\left(\frac{1}{n}\right)}{\frac{1}{n}}$$

$$\lim_{n\to\infty}\frac{u_n}{v_n}=\lim_{\frac{1}{n}\to 0}\frac{\sin\left(\frac{1}{n}\right)}{\frac{1}{n}}=1, \text{which is finite and non-zero.}$$

Formula : $\lim_{x\to 0}\frac{\sin x}{x}=1$

$\therefore$ Both $\Sigma\, u_n$ and $\Sigma\, v_n$ converge or diverge together.

$\sum v_n = \sum \frac{1}{n}$ is divergent by Harmonic series test or p - series test.

Hence by comparison test, Σu_n is divergent.

Question 4:

Show that the series $1+\frac{2^p}{2!}+\frac{3^p}{3!}+\frac{4^p}{4!}+\cdots$ **is convergent for all values of *p*.**

Solution

$$u_n = \frac{n^p}{n!}, \quad u_{n+1} = \frac{(n+1)^p}{(n+1)!}$$

$$\frac{u_n}{u_{n+1}} = \frac{n^p}{n!} \times \frac{(n+1)!}{(n+1)^p} = \frac{n+1}{\left(1+\frac{1}{n}\right)^p} = \frac{n\left(1+\frac{1}{n}\right)}{\left(1+\frac{1}{n}\right)^p}$$

$$\lim_{n\to\infty} \frac{u_n}{u_{n+1}} = \infty > 1$$

Hence by Ratio test, Σu_n is convergent.

Question 5:

Establish the convergence of the series $\frac{1}{3}+\left(\frac{2}{5}\right)^2+\left(\frac{3}{7}\right)^3+\cdots$

Solution

$$u_n = \left(\frac{n}{2n+1}\right)^n$$

$$\lim_{n\to\infty} (u_n)^{1/n} = \lim_{n\to\infty} \frac{n}{2n+1} = \frac{1}{2} < 1$$

Hence by Cauchy's root test, Σu_n is convergent.

Question 6:

Test the convergence of the series $\sum_{n=2}^{\infty} \frac{1}{n \log n}$.

Solution

$f(n) = \frac{1}{n \log n}$ Clearly $f(n)$ is a monotonic decreasing sequence.

$$\int_2^{\infty} f(x)dx = \int_2^{\infty} \frac{1}{x \log x} dx = \int_2^{\infty} \frac{1/x}{\log x} dx = \left(\log(\log(x)\right)_2^{\infty} = \infty$$

By Cauchy's integral test, the given series is divergent.

Question 7:

Test the convergence of the series $1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + \cdots$.

Solution

$u_n = \frac{1}{\sqrt{n}}$ $\qquad$ $u_{n+1} = \frac{1}{\sqrt{n+1}}$

(i) Clearly $u_n > u_{n+1}$.

(ii) $\lim_{n \to \infty} u_n = \lim_{n \to \infty} \frac{1}{\sqrt{n}} = 0$

$\therefore$ By Leibnitz's test, the given series is convergent.

Question 8:

Test the convergence of the series $\frac{1}{1.2} - \frac{1}{3.4} + \frac{1}{5.6} - \cdots$.

Solution

$u_n = \frac{1}{(2n-1)(2n)}$ $\qquad$ $u_{n+1} = \frac{1}{(2n+1)(2n+2)}$

(i) Since $\frac{1}{(2n-1)(2n)} > \frac{1}{(2n+1)(2n+2)}$ always, clearly $u_n > u_{n+1}$.

(ii) $\lim_{n\to\infty} u_n = \lim_{n\to\infty} \frac{1}{n\left(2-\frac{1}{n}\right)2n} = 0$

$\therefore$ By Leibnitz's test, the given series is convergent.

Question 9:

Prove that the series $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\cdots$ **is conditionally convergent.**

Solution

$u_n = \frac{1}{n}$ $\qquad$ $u_{n+1} = \frac{1}{n+1}$

(i) Clearly $u_n > u_{n+1}$.

(ii) $\lim_{n\to\infty} u_n = \lim_{n\to\infty} \frac{1}{n} = 0$

$\therefore$ By Leibnitz's test, the given series is *convergent*.

Also $\Sigma\,|\,u_n| = 1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\cdots = \sum \frac{1}{n}$ is *divergent* by Harmonic series test (or) p-series test.

Hence the given series is conditionally convergent.

Question 10:

Test the convergence of the series $\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\cdots$

Solution

The series of absolute terms $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\cdots = \sum \frac{1}{n^2}$ is clearly *convergent* by Harmonic series test (or) p-series test.

$\therefore$ The series is absolutely convergent.

Since every absolutely convergent series is convergent, the given series is convergent.

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