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B.Tech Degree Examination - DECEMBER 2022

Open Book Examination

18 CSC301T - Formal Language and Automata

Answer Key.

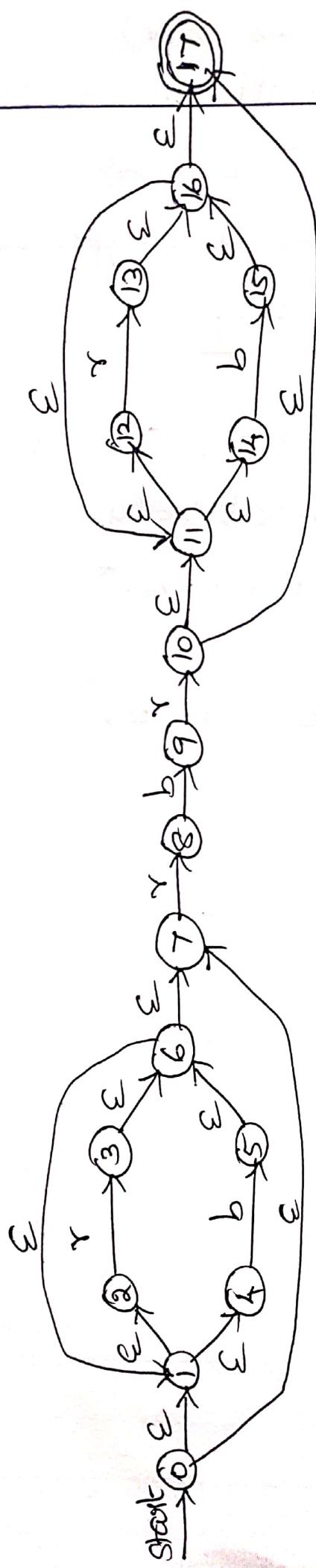
1. red ball and blue ball can be represented as 'r' and 'b' respectively.

(i)  $RE = (r+b)^* r b r (r+b)^*$  (2 Marks)

(Or)

$$RE = (r|b)^* r b r (r|b)^*$$

(ii)  $\epsilon$ -NFA (4 Marks)



ECLOSE

(3 Marks)

(iii)

$$\text{ECLOSE}(0) = \{0, 1, 2, 4, 7\}$$

$$\text{ECLOSE}(1) = \{1, 2, 4\}$$

$$\text{ECLOSE}(2) = \{2\}$$

$$\text{ECLOSE}(3) = \{3, 6, 1, 7, 2, 4\}$$

$$\text{ECLOSE}(4) = \{4\}$$

$$\text{ECLOSE}(5) = \{5, 6, 1, 7, 2, 4\}$$

$$\text{ECLOSE}(6) = \{6, 1, 7, 2, 4\}$$

$$" (7) = \{7\}$$

$$" (8) = \{8\}$$

$$" (9) = \{9\}$$

$$" (10) = \{10, 11, 12, 14, 17\}$$

$$" (11) = \{11, 12, 14\}$$

$$" (12) = \{12\}$$

$$" (13) = \{13, 16, 11, 17, 12, 14\}$$

$$" (14) = \{14\}$$

$$" (15) = \{15, 16, 11, 17, 12, 14\}$$

$$" (16) = \{16, 11, 17, 12, 14\}$$

$$" (17) = \{17\}$$

## DFA

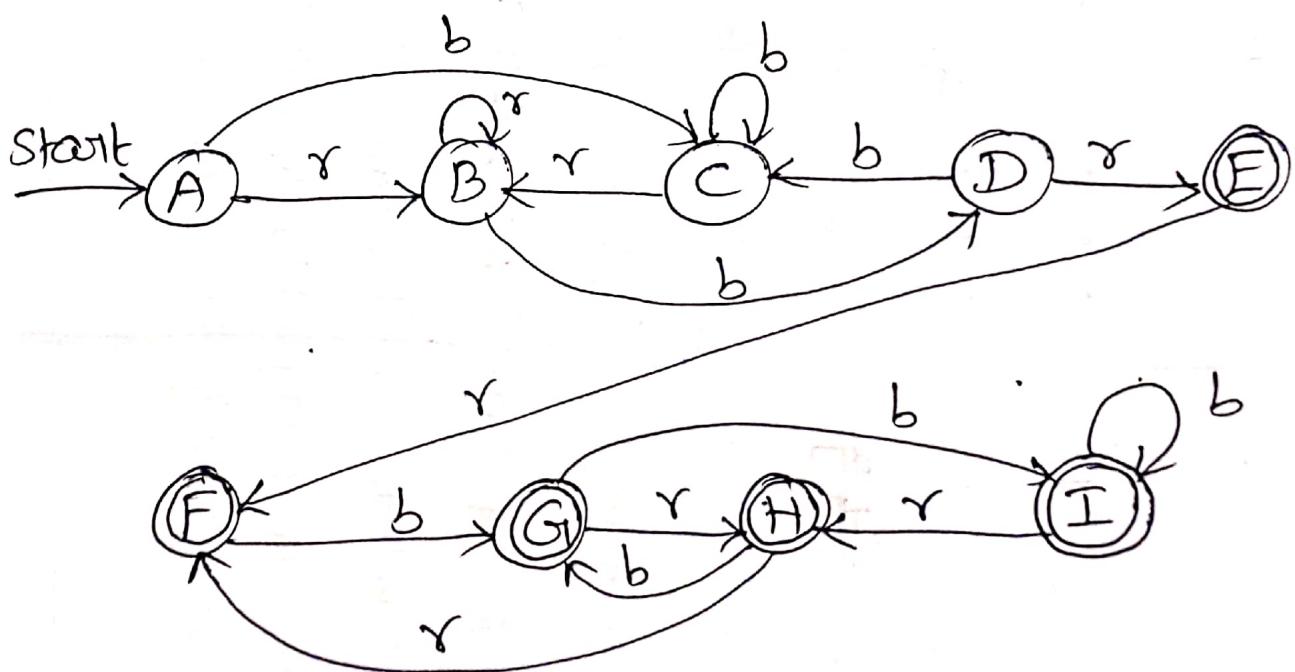
(5 Marks)

States	$\delta$	$b$
$\rightarrow \{0, 1, 2, 4, 7\}$	$\{1, 2, 3, 4, 6, 7, 8\}$	$\{1, 2, 4, 5, 6, 7\}$
$\{1, 2, 3, 4, 6, 7, 8\}$	$\{1, 2, 3, 4, 6, 7, 8\}$	$\{1, 2, 4, 5, 6, 7, 9\}$
$\{1, 2, 4, 5, 6, 7\}$	$\{1, 2, 3, 4, 6, 7, 8\}$	$\{1, 2, 4, 5, 6, 7\}$
$\{1, 2, 4, 5, 6, 7, 9\}$	$\{1, 2, 3, 4, 6, 7, 8\}$	$\{1, 2, 4, 5, 6, 7\}$
$* \{1, 2, 3, 4, 6, 7, 8, 10, 11, 12, 14, 17\}$	$\{1, 2, 3, 4, 6, 7, 8, 11, 12, 13, 14, 16, 17\}$	$\{1, 2, 4, 5, 6, 7, 9, 11, 12, 14, 15, 16, 17\}$
$* \{1, 2, 3, 4, 6, 7, 8, 11, 12, 13, 14, 16, 17\}$	$\{1, 2, 3, 4, 6, 7, 8, 11, 12, 13, 14, 16, 17\}$	$\{1, 2, 4, 5, 6, 7, 9, 11, 12, 14, 15, 16, 17\}$
$* \{1, 2, 3, 4, 6, 7, 9, 11, 12, 14, 15, 16, 17\}$	$\{1, 2, 3, 4, 6, 7, 8, 10, 11, 12, 14, 17, 13, 16\}$	$\{1, 2, 4, 5, 6, 7, 11, 12, 14, 15, 16, 17\}$
$* \{1, 2, 3, 4, 6, 7, 8, 10, 11, 12, 13, 14, 16, 17\}$	$\{1, 2, 3, 4, 6, 7, 8, 11, 12, 13, 14, 16, 17\}$	$\{1, 2, 4, 5, 6, 7, 9, 11, 12, 14, 15, 16, 17\}$
$* \{1, 2, 3, 4, 6, 7, 11, 12, 14, 15, 16, 17\}$	$\{1, 2, 3, 4, 6, 7, 8, 11, 12, 13, 14, 16, 17\}$	$\{1, 2, 4, 5, 6, 7, 11, 12, 14, 15, 16, 17\}$
$* \{1, 2, 3, 4, 6, 7, 11, 12, 14, 15, 16, 17\}$	$\{1, 2, 3, 4, 6, 7, 11, 12, 13, 14, 16, 17\}$	$\{1, 2, 4, 5, 6, 7, 11, 12, 14, 15, 16, 17\}$

iv) Minimize the DFA.

DFA

	$\gamma$	$b$
$\rightarrow A$	B	C
B	B	D
C	B	C
*	E	U
*	F	G
*	G	I
*	H	G
*	I	I



$$\pi_0 = \{ \{A, B, C, D\}_1, \{E, F, G, H, I\}_2 \}$$

	A	B	C	D	E	F	G	H	I
r	1	1	1	2	2	2	2	2	2
b	1	1	1	1	1	2	2	2	2

$$\pi_1 = \{ \{A, B, C\}_3, \{D\}_4, \{E, F, G, H, I\}_2 \}$$

	A	B	C	D	E	F	G	H	I
r	3	3	3	2	2	2	2	2	2
b	3	4	3	3	2	2	2	2	2

$$\pi_2 = \{ \{A, C\}_5, \{B\}_6, \{D\}_4, \{E, F, G, H, I\}_2 \}$$

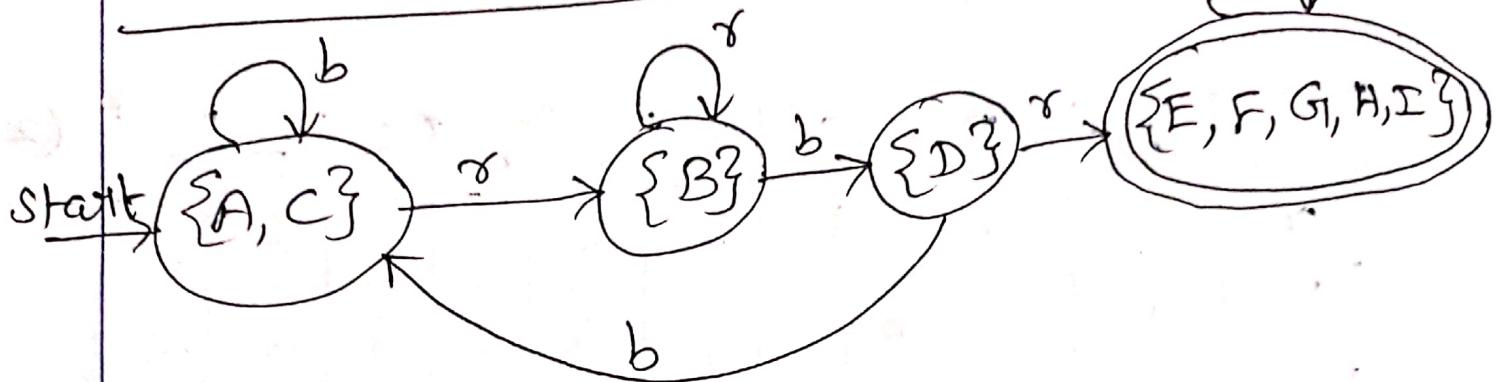
	A	B	C	D	E	F	G	H	I
r	6	6	6	2	2	2	2	2	2
b	5	4	5	5	2	2	2	2	2

$$\Pi_3 = \{ \{A, C\}, \{B\}, \{D\}, \{E, F, G, H, I\} \}$$

$\pi_3$  is same as  $\pi_2$

Minimized DFA

(4 Marks)



- v) (B)  
vi) (C)

2. Routes A, B, C and D are considered as a, b, c and d respectively.

(i) CFG

$$CFG_1, G_1 = (\{S, A\}, \{a, b, c, d\}, P, S) \quad (1 M)$$

$$P = \{ \begin{array}{l} S \rightarrow aSd \mid aAd \\ A \rightarrow bAc \mid bc \end{array} \} \quad (2 M)$$

(ii). Language for the CFG. (3 Marks)

$$L(G) = \{a^n b^m c^m d^n \mid n \geq 1, m \geq 1\}$$

(iii) Any string can be chosen. Let us take  
string = "aabc dd"

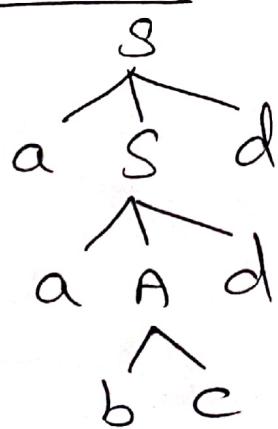
Left most derivation (2 marks)

$$\begin{aligned} S &\xrightarrow{\text{lm}} aSd \\ &\xrightarrow{\text{lm}} aaAdd \\ &\xrightarrow{\text{lm}} aabc dd \end{aligned}$$

Rightmost derivation (2 marks)

$$\begin{aligned} S &\xrightarrow{\text{rm}} aSd \\ &\xrightarrow{\text{rm}} aaAdd \\ &\xrightarrow{\text{rm}} aabc dd \end{aligned}$$

Parse Tree (2 marks)

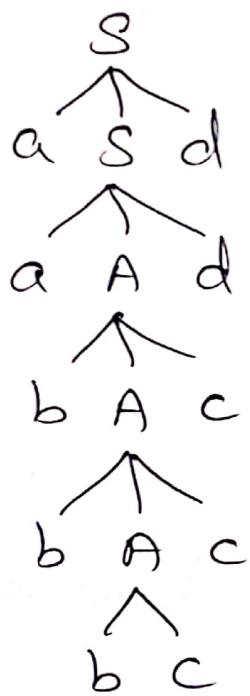


(iv)

Ambiguous or not.

Let string = "aabbbcccdd"

(4 marks)



(2 marks)

We can construct only 1 parse tree for this string. Hence the grammar is unambiguous.

v) (C)

vi) (B)

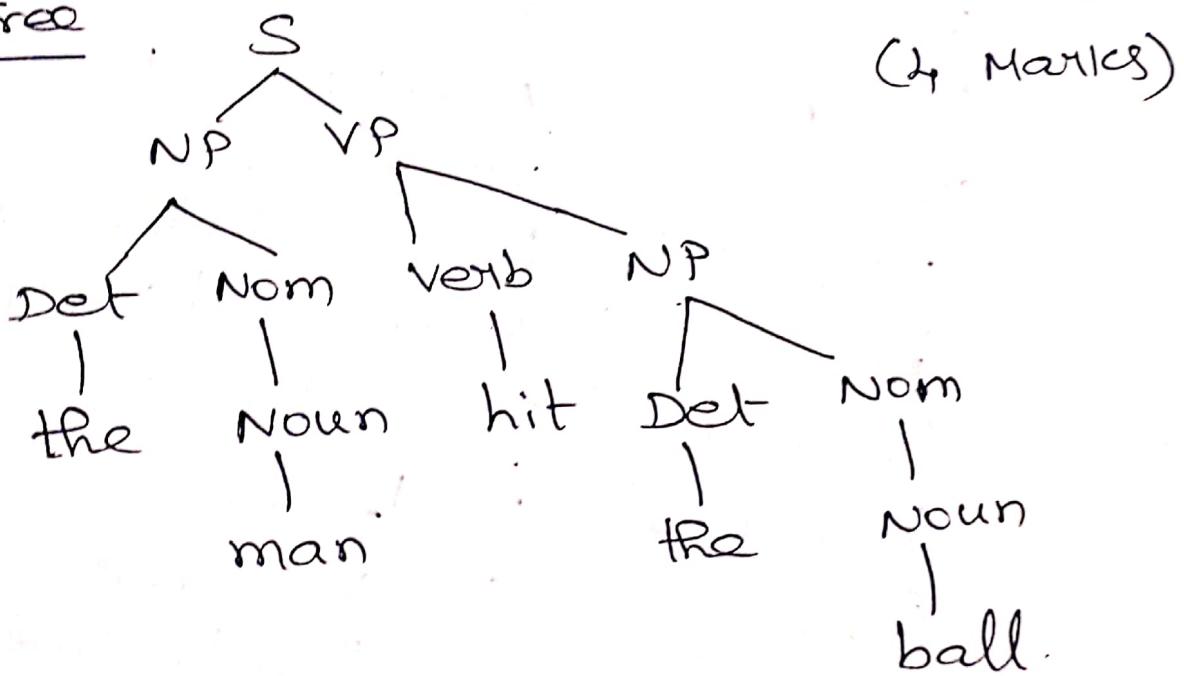
3.

(i) Terminals = {that, this, a, the, ball, flight, meal, man, book, include, hit, does} (1.5 Marks)

Non-terminals = {S, NP, VP, Nom, Det, Noun, Verb, Aux} (1.5 Marks)

(ii) Left-most derivation or rightmost derivation or parse tree can be used for proving.

Parse Tree



### (iii) Simplification of Grammar.

#### Elimination of ε-productions

(1 Mark)

No ε-productions in this grammar.

#### Elimination of Unit productions

(4 Marks)

<u>Unit pairs</u>	<u>Not unit productions</u>
(S, S)	$S \rightarrow NP\ VP   Aux\ NP\ VP$
(NP, NP)	$NP \rightarrow Det\ Nom$
(Nom, Nom)	$Nom \rightarrow Noun\ Nom$
(VP, VP)	$VP \rightarrow Verb\ NP$
(Det, Det)	$Det \rightarrow that\   this\   a\   the$
(Noun, noun)	$Noun \rightarrow ball\   flight\   meal\   man$
(verb, verb)	$Verb \rightarrow book\   include\   hit$
(Aux, Aux)	$Aux \rightarrow does$
(S, VP)	$S \rightarrow Verb\ NP$
(S, verb)	$S \rightarrow book\   include\   hit$
(Nom, Noun)	$Nom \rightarrow ball\   flight\   meal\   man$
(VP, Verb)	$VP \rightarrow book\   include\   hit$

$S \rightarrow NP\ VP$  |  $Aux\ NP\ VP$  |  $verb\ NP$  |  $book$  |  $include$  |  $hit$

$NP \rightarrow Det\ Nom$

$Nom \rightarrow noun\ noun$  |  $ball$  |  $flight$  |  $meal$  |  $man$

$VP \rightarrow verb\ NP$  |  $book$  |  $include$  |  $hit$

$Det \rightarrow that$  |  $this$  |  $a$  |  $the$

$Noun \rightarrow ball$  |  $flight$  |  $meal$  |  $man$

$Verb \rightarrow book$  |  $include$  |  $hit$

$Aux \rightarrow does$ .

### Elimination of useless symbols. (2 Marks)

Generating symbols = {that, this, a, the, ball, flight, meal, man, book, include, hit, does, VP, Nom, Aux, Verb, Noun, Det, NP, S}

Reachable symbols = {S, NP, VP, Aux, Verb, book, include, hit, Det, Nom, Noun, ball, flight, meal, man, that, this, a, the, does}

There are no non-generating and non-reachable symbols. So there are no useless symbols.

iv) Chomsky Normal Form (4 Marks)

$C_1 \rightarrow NP\ VP$

$S \rightarrow NP\ VP \mid Aux\ C_1 \mid \text{verb}\ NP \mid \text{book} \mid \text{include} \mid \text{hit}$

$NP \rightarrow \text{Det}\ Nom$

$Nom \rightarrow \text{Noun}\ Nom \mid \text{ball} \mid \text{flight} \mid \text{meal} \mid \text{man}$

$VP \rightarrow \text{verb}\ NP \mid \text{book} \mid \text{include} \mid \text{hit}$

$\text{Det} \rightarrow \text{that} \mid \text{this} \mid \text{a} \mid \text{the}$

$\text{Noun} \rightarrow \text{ball} \mid \text{flight} \mid \text{mean} \mid \text{man}$

$\text{Verb} \rightarrow \text{book} \mid \text{include} \mid \text{hit}$

$\text{Aux} \rightarrow \text{does}$

v) (A)

vi) (C)

4. case (i) Red, green, violet, yellow  $\Rightarrow r, g, v, y$

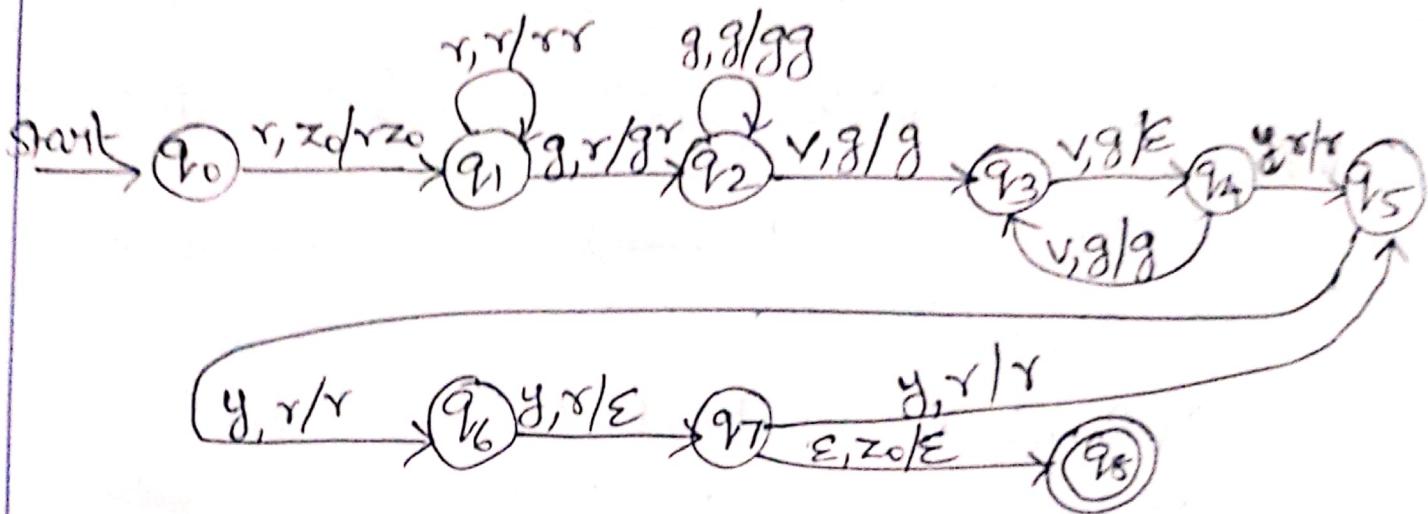
(i)  $L_1 = \{r^n g^m v^2m y^{3n} \mid n \geq 1, m \geq 1\}$  (3 Marks)

case (ii)

$L_2 = \{r^n g^{4n} \mid n \geq 1\}$

(2 Marks)

(ii) NFA,  $P = \{q_0, q_1, \dots, q_8\}, \{r, g, v, y\}, \{r, g, z_0\}, \delta, q_0, z_0, \{q_8\}$   
 (1 mark)



$$\delta(q_0, r, z_0) = (q_1, rz_0)$$

$$\delta(q_1, r, r) = (q_1, rr)$$

$$\delta(q_1, g, r) = (q_2, gr)$$

$$\delta(q_2, g, g) = (q_2, gg)$$

$$\delta(q_2, v, g) = (q_3, g)$$

$$\delta(q_3, v, g) = (q_4, \epsilon)$$

$$\delta(q_4, v, g) = (q_3, g)$$

$$\delta(q_4, y, r) = (q_5, r)$$

$$\delta(q_5, y, r) = (q_6, r)$$

$$\delta(q_6, y, r) = (q_7, \epsilon)$$

$$\delta(q_7, y, r) = (q_5, \epsilon)$$

$$\delta(q_7, \epsilon, z_0) = (q_8, \epsilon)$$

b) PDA,  $P = (\{q_0, q_1, \dots, q_6\}, \{\gamma, g\}, \{\gamma, z_0\}, \delta, q_0, z_0, \{q_6\})$

$$\delta(q_0, \gamma, z_0) = (q_1, \gamma z_0) \quad (4 \text{ Marks})$$

$$\delta(q_1, \gamma, \gamma) = (q_1, \gamma \gamma)$$

$$\delta(q_1, g, \gamma) = (q_2, \gamma)$$

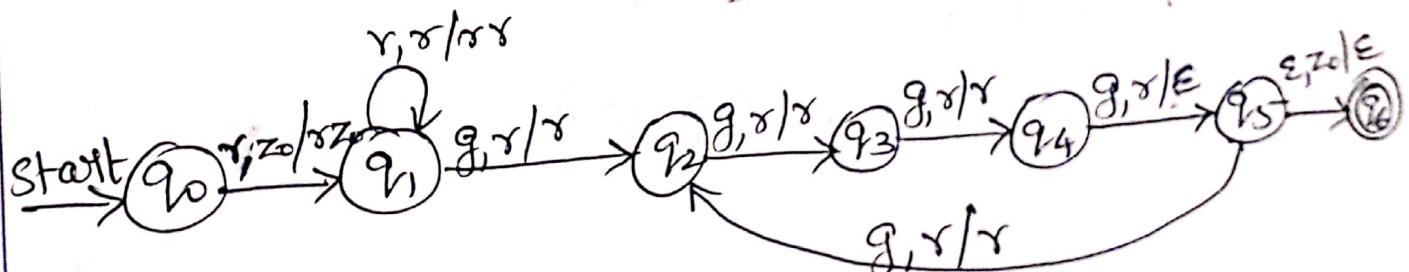
$$\delta(q_2, g, \gamma) = (q_3, \gamma)$$

$$\delta(q_3, g, \gamma) = (q_4, \gamma)$$

$$\delta(q_4, g, \gamma) = (q_5, \epsilon)$$

$$\delta(q_5, g, \gamma) = (q_2, \gamma)$$

$$\delta(q_5, \epsilon, z_0) = (q_6, \epsilon)$$



(iii)  $(q_0, \gamma g g g g, z_0) \vdash (q_1, g g g g, \gamma z_0)$

$\vdash (q_2, g g g, \gamma z_0)$

$\vdash (q_3, g g, \gamma z_0)$

$\vdash (q_4, g, \gamma z_0)$

$\vdash (q_5, \epsilon, z_0)$

$\vdash (q_6, \epsilon, \epsilon)$

(5 Marks)

So 1 red flower followed by 4 green flowers are picked.

iv) (B)

v) (D)

5. Thread mill and cycle  $\Rightarrow t \& c$

(i) CFG,  $G = (\{S\}, \{t, c\}, P, S)$  (2 Marks)  
 $P = \{ S \rightarrow tScc \mid tcc \}$

(ii) PDA  
PDA,  $P = (\{q\}, \{t, c\}, \{S, t, c\}, \delta, q_1, S)$  (2 Marks)  
 $\delta(q_1, \epsilon, S) = \{(q_1, tScc), (q_1, tcc)\}$  (4 Marks)  
 $\delta(q_1, t, t) = (q_1, \epsilon)$   
 $\delta(q_1, c, c) = (q_1, \epsilon)$

(iii)  $(q_1, ttcccc, S) \vdash (q_1, ttcccc, tScc)$  (4 Marks)  
 $\vdash (q_1, tcccc, Scc)$   
 $\vdash (q_1, tcccc, tcccc)$   
 $\vdash (q_1, cccc, cccc)$   
 $\vdash (q_1, ccc, ccc)$   
 $\vdash (q_1, cc, cc)$

$$\vdash (q, c, c)$$

$$\vdash (q, \epsilon, \epsilon)$$

iv)  $L = \{a^n b^m c^n d^m \mid n, m \geq 1\}$  (6 Marks)

Proof:

- \* Let  $L$  be a CFL
- \* Let  $n$  be a constant
- \* Let  $z = a^n b^m c^n d^m$ ,  $|z| \geq n$
- \* Split  $z = uvwxy$ , such that
  - (i)  $|vwx| \leq n$
  - (ii)  $vx \neq \epsilon$
  - (iii)  $\forall i \geq 0$ ,  $uv^i wx^i y \in L$

\*  $u = a^n b^m$

$$vwx = c^n \text{ where } |vwx| \leq n$$

$$vx = c^{n-p} \text{ where } |vx| \geq 1$$

$$y = d^m$$

$$\begin{aligned} uv^i wx^i y &= u v v^{i-1} w x x^{i-1} y \\ &= u v w x (vx)^{i-1} y \\ &= a^n b^m c^n (c^{n-p})^{i-1} d^m \end{aligned}$$

$$\begin{aligned}
 &= a^n b^m c^n c^{n-i-n-p_i+p} d^m \\
 &= a^n b^m c^{n-p_i+p} d^m \\
 &= a^n b^m c^{n-p_i+p} d^m
 \end{aligned}$$

Pick  $i=0$ ,  $a^n b^m c^p d^m \notin L$

Hence the language  $L = \{a^n b^m c^n d^m \mid n, m \geq 1\}$   
is not a CFL.

- (v) (D)
- (vi) (B)

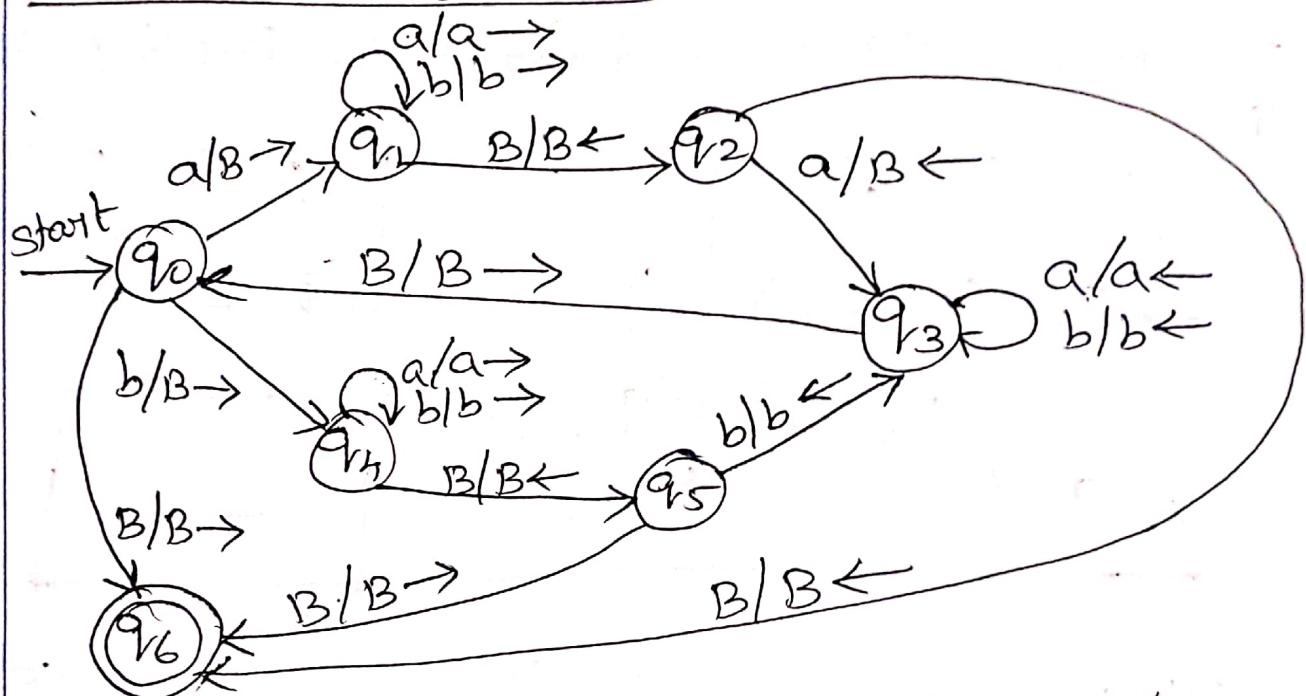
6.(i) Let us assume that the two symbols are a & b.

S	a	b	B	(8 Marks)
$\rightarrow q_0$	$(q_1, B, R)$	$(q_4, B, R)$	$(q_6, B, R)$	
$q_1$	$(q_1, a, R)$	$(q_1, b, R)$	$(q_2, B, L)$	
$q_2$	$(q_3, B, L)$	—	$(q_6, B, R)$	
$q_3$	$(q_3, a, L)$	$(q_3, b, L)$	$(q_0, B, R)$	
$q_4$	$(q_4, a, R)$	$(q_4, b, R)$	$(q_5, B, L)$	
$q_5$	—	$(q_3, b, L)$	$(q_6, B, R)$	
* $q_6$	—	—	—	

$$TM, M = (\{q_0, q_1, \dots, q_6\}, \{a, b\}, \{a, b, B\}, S, q_0, B, \{q_6\})$$

(or)

Transition Diagram



(ii)  $TM, M = (\{q_0, q_1, \dots, q_7\}, \{a, b\}, \{a, b, x, B\}, S, q_0, B, \{q_7\})$  (10 Marks)

S	a	b	x	B
$\rightarrow q_0$	$(q_1, B, R)$	$(q_6, B, R)$	-	-
$q_1$	$(q_1, a, R)$	$(q_2, b, R)$	-	-
$q_2$	$(q_3, x, R)$	$(q_5, b, L)$	-	-
$q_3$	$(q_3, a, R)$	$(q_3, b, R)$	-	$(q_4, a, L)$
$q_4$	$(q_4, a, L)$	$(q_4, b, L)$	$(q_2, x, R)$	-
$q_5$	$(q_5, a, L)$	$(q_5, b, L)$	$(q_5, a, L)$	$(q_0, B, R)$
$q_6$	$(q_6, B, R)$	$(q_7, b, R)$	-	-
*	$q_7$	-	-	-

- (iii) (C)  
 (iv) (B)

7. TM, M = ( $\{q_0, q_1, q_2\}$ ,  $\{0, 1\}$ ,  $\{0, 1, B\}$ ,  $S, q_0, B, \{q_2\}$ )

(i)

(4 Marks)

$S$	0	1	B
$\rightarrow q_0$	$(q_0, 0, R)$	$(q_0, 1, R)$	$(q_1, B, L)$
$q_1$	$(q_2, 0, R)$	$(q_1, 1, R)$	—
* $q_2$	—	—	—

(ii) MPCP Instance,  $w = 100$

(4 Marks)

Rule	List A	List B	Source
(1)	#	# $q_0 100 \#$	.
(2)	0 1 #	0 1 #	.
(3)	$q_0 0$ $q_0 1$ $0 q_0 \#$ $1 q_0 \#$	$0 q_0$ $1 q_0$ $q_1 0 \#$ $q_1 1 \#$	$S(q_0, 0) = (q_0, 0, R)$ $S(q_0, 1) = (q_0, 1, R)$ $S(q_0, B) = (q_1, B, L)$ $S(q_0, B) = (q_1, B, L)$

(4)	$q_1, 0$ $q_1, 1$	$0 q_2$ $1 q_1$	$s(q_1, 0) = (q_2, 0, R)$ $s(q_1, 1) = (q_1, 1, R)$
(5)	$0 q_2, 0$ $0 q_2, 1$ $1 q_2, 0$ $1 q_2, 1$ $0 q_2$ $1 q_2$ $q_2, 0$ $q_2, 1$	$q_2$ $q_2$ $q_2$ $q_2$ $q_2$ $q_2$ $q_2$ $q_2$	
(6)	$q_2 \# \#$	#	

ID for 100

$q_0 100 \rightarrow q_0 00B$   
 $\rightarrow 10q_0 0B$   
 $\rightarrow 100q_0 B$   
 $\rightarrow 10q_1 0 B$   
 $\rightarrow 100 q_2 B$

(2 marks)

TM halts. since  $q_2$  is a final state "100" is accepted.

A: #9<sub>0</sub>100#19<sub>0</sub>00#109<sub>0</sub>0#1009<sub>0</sub>#109<sub>1</sub>0#1009<sub>2</sub>#109<sub>2</sub>#19<sub>2</sub>#9<sub>2</sub>#  
 B: #9<sub>0</sub>100#19<sub>0</sub>00#109<sub>0</sub>0#1009<sub>0</sub>#109<sub>1</sub>0#1009<sub>2</sub>#109<sub>2</sub>#19<sub>2</sub>#9<sub>2</sub>#  
 ∴ MPCP has solution.

(iii) code for TM

$$\begin{array}{ll}
 q_0 \Rightarrow q_1 & 0 \Rightarrow x_1 \quad L \Rightarrow D_1 \\
 q_1 \Rightarrow q_2 & 1 \Rightarrow x_2 \quad R \Rightarrow D_2 \\
 q_2 \Rightarrow q_3 & B \Rightarrow x_3
 \end{array}$$

$$g(q_1, x_1) = (q_1, x_1, \mathcal{D}_2)$$

$$g(q_1, x_2) = (q_1, x_2, D^2)$$

$$g(q_1, x_3) = (q_2, x_3, D_1)$$

$$s(q_2, x_1) = (q_3, x_1, D^2)$$

$$s(q_2, x_2) = (q_2, x_2, D_2)$$

$$c_1 = 0101010100$$

$$c_2 = 010010100100$$

$$c_3 = 0100010010001^0$$

$$C_4 = 0010100010100$$

$$c_5 = 00100100100100$$

Code for TM is

$c_1 \parallel c_2 \parallel c_3 \parallel c_4 \parallel c_5$

(2 marks)

$c_1 \parallel c_2 \parallel c_3 \parallel \dots$   
01010101001101001010010011010001001  
000101100101000101001100100100100100100

(iv) (c)

(v) (d)