Maths

Unit - 1

Set Theory

* Set → A set is a well defined collection of object.
* Representation of Set :-
  (a) Raster form :- eg. Set of binary digit = $\{0, 1\}$
  (b) Set Builder form :- eg. $A = \{x : x$ is a natural no. less than $9\}$

$A = \{x : x$ is +ve natural no. not exceeding $9\}$

Set Builder form.

Roster form → $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

Builder form is converted into this Roster form.

Types of Sets :-

(1) Universal Sets :- A set which contain all the objects is called Universal Set.

(2) NULL Set or Empty Set :- A set which does not contain any element is called NULL set.

e.g. $x = \{x : x^2 + 1 = 0, x$ is real$\}$
$= \phi$.

(3) Singleton Set :- A set containing one element is called singleton set.

eg. $A = \{0\}$,
$A = \{\phi\}$, etc

(4) Finite & Infinite Set :- A set which contain finite no. of elements is called finite set and a set which contain infinite no. of elements is called infinite set.

eg. $A = \{1, 3, 5\}$
$A = \{x : x^2 < 100, x \in Z^+\}$
$\rightarrow A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

e.g. $A = \{x : x$ is an even +ve integer$\}$
$\rightarrow \{2, 4, 6, 8, 10, 12 - - - - - \}$

(5) Equal Set :- Two set are said to be equal when element of one set is equal to the elements of another set.

e.g. $A = \{1, 2, 3\}$, $B = \{3, 2, 1\}$
$A = B$

* Not :- Two set is not equal by having equal size. e.g. $A = \{4, 5, 6\} \rightarrow$ Size $= 3$
$B = \{1, 2, 3\} \rightarrow$ Size $= 3$.

$A \neq B$

* **Subset** :- A set is said to be a subset of another set if and only if every element of that set is also available in that another set.

e.g. $A = \{1, 2, 3, 4, 5\}$
$B = \{1, 2, 3\}$

$\Rightarrow B \subset A$

(a) Proper Subset :- $B \subset A$
$\Rightarrow$ B is a subset of A but $A \neq B$.

(b) Improper Subset :- $B \subseteq A$.

Note :-

1. A is not subset of B is represented by $A \not\subseteq B$
2. Every Subset A is a subset of itself $A \subseteq A$
3. If $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$.
4. If $A \subseteq B$, then A is called subset of B and B is called superset of A.

* **Power Set** :- The set of all subset of set S is called Power Set of S.

$S = \{a, b, c\}$

$P(S) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{c, a\}, \{a, b, c\}\}$

MCQ

* Total no. of Power Set Element $= 2^n$ ← no. of element in Set S

* Cartesian Product :-

$A = \{1, 2\}$, $B = \{3, 4\}$

$A \times B = \{\{1,3\}, \{1,4\}, \{2,3\}, \{2,4\}\}$

$B \times A = \{\{3,1\}, \{3,2\}, \{4,1\}, \{4,2\}\}$

Not same because order matter.

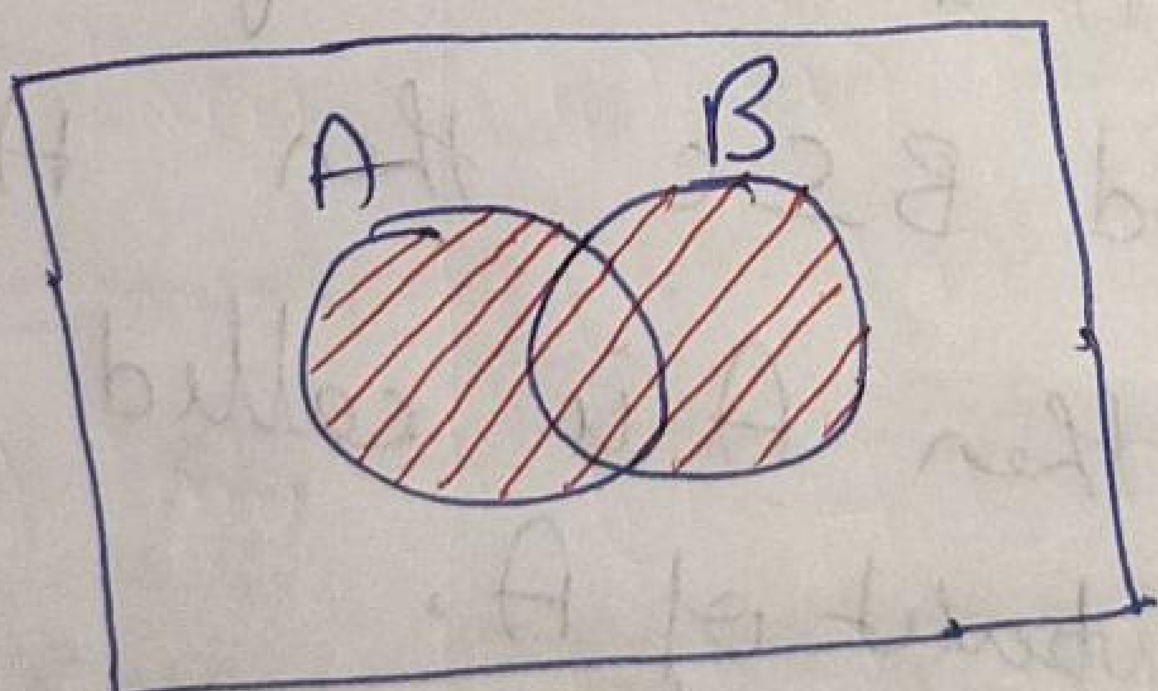
* $(a, b) \neq (b, a)$ unless $a = b$.

* Operations On Set :-

(I) Union :- Union of two set A and B denoted by $A \cup B$ is set of element belonging to A or to B or to both.

$A \cup B = \{x : x \in A \text{ or } x \in B\}$.

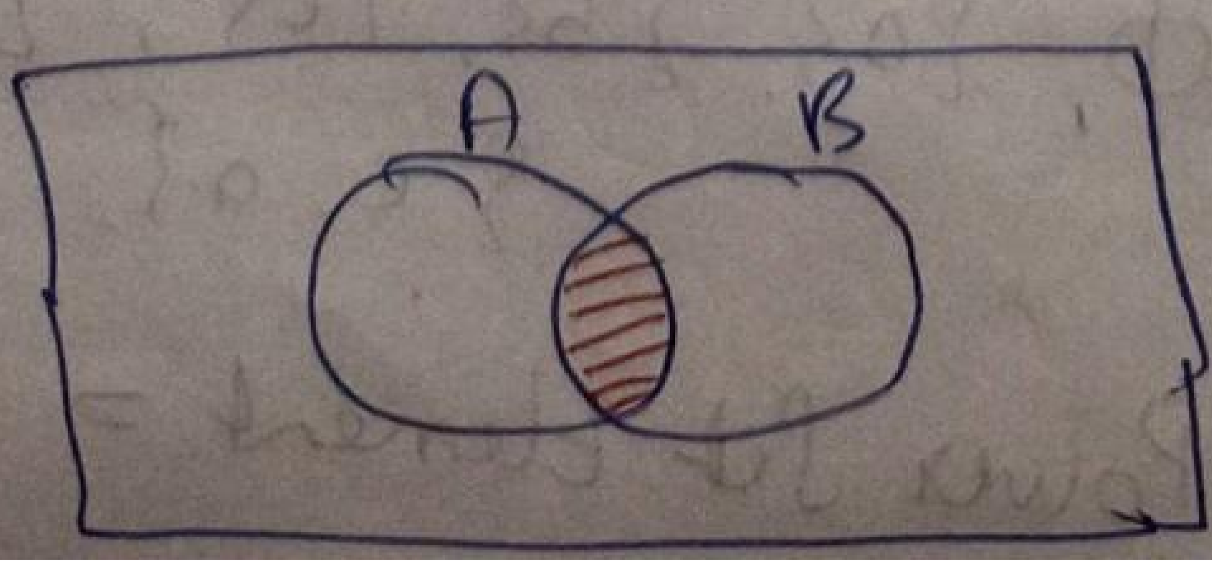
Eg. $A = \{1, 2, 3\}$
$B = \{4, 2\}$
$A \cup B = \{1, 2, 3, 4\}$



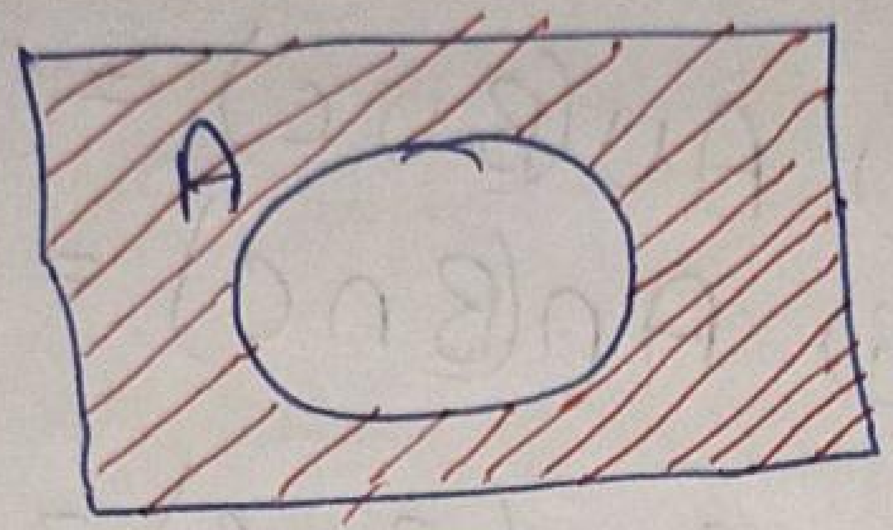
(II) Intersection :- Intersection of two set A and B, denoted by $A \cap B$, is the set of elements that belong to both A and B.

$A \cap B = \{x : x \in A \text{ and } x \in B\}$

Eg. $A = \{1, 2, 3, 4\}$
$B = \{2, 3, 5, 6\}$
$A \cap B = \{2, 3\}$



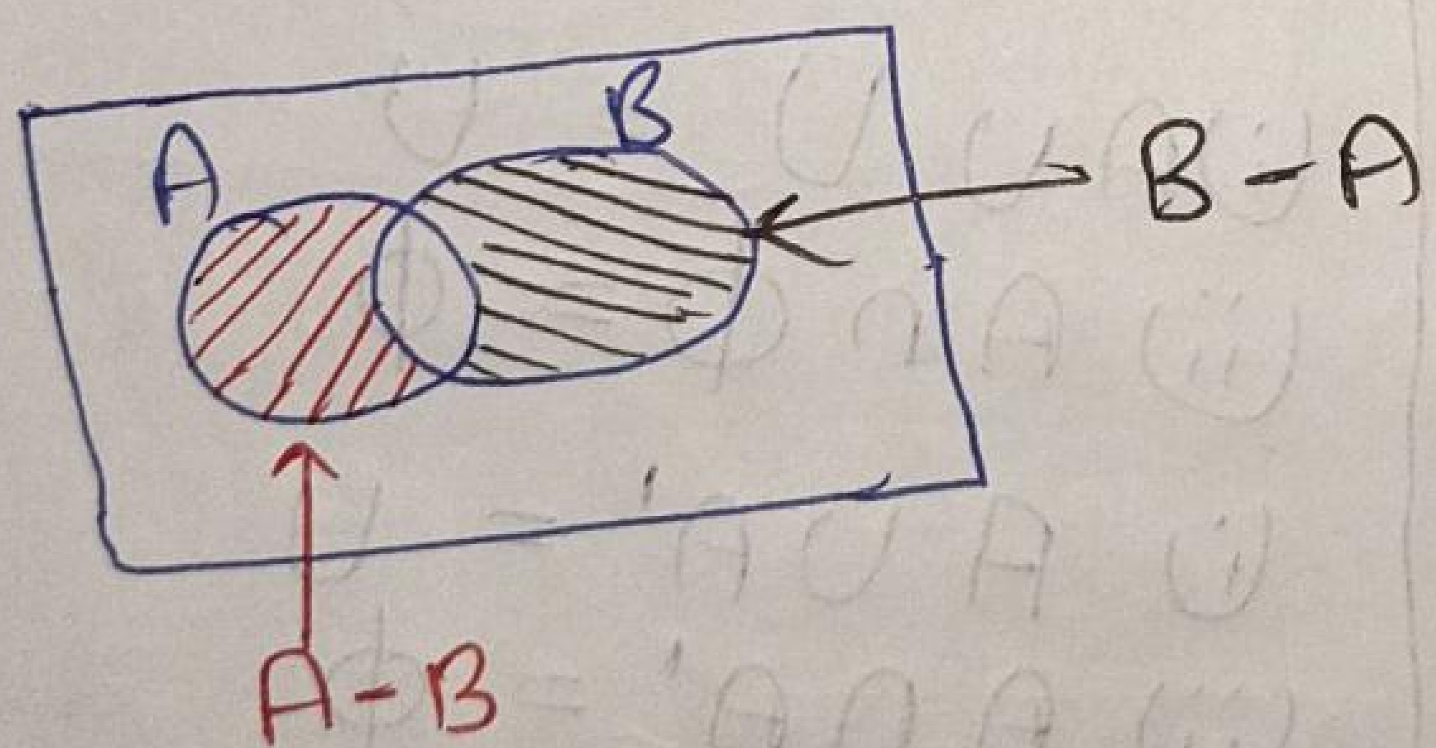
(III) Complement :- If U is Universal set and A is any set, then, Complement of A is the set of element that does not belong to A but belong to U. Denoted by (A', A^- or $\bar{A}$)



$$\bar{A} = \{x : x \in U \text{ and } x \notin A\}$$

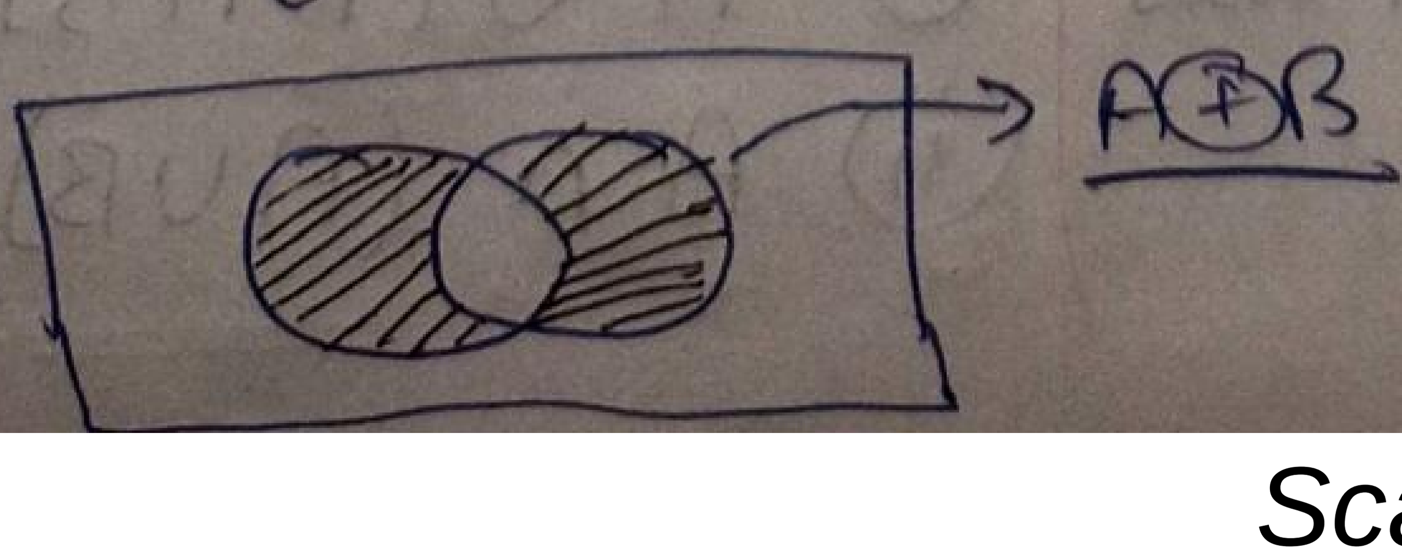
(IV) Difference :- If A and B are any two sets then the set of elements that belong to A but do not belong to B is called difference of A and B

$$A - B = \{x : x \in A \text{ and } x \notin B\}$$



(V) Symmetric Difference :- If A and B are any two set, the set of element that belong to A or B, but not to both is called the symmetric difference of A and B.

$$A \oplus B = (A-B) \cup (B-A)$$



Algebric Law of Set Theory :-

Name of Law	
1. Commutative	(i) $A \cup B = B \cup A$ (ii) $A \cap B = B \cap A$
2. Associative	(i) $A \cup (B \cup C) = (A \cup B) \cup C$ (ii) $A \cap (B \cap C) = (A \cap B) \cap C$
3. Distributive	(i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ (ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
4. Identity Law	(i) $A \cup \phi = A$ (ii) $A \cap U = A$
5. Idempotent Law	(i) $A \cup A = A$ (ii) $A \cap A = A$
6. Dominant Law	(i) $A \cup U = U$ (ii) $A \cap \phi = \phi$
7. Complement Law	(i) $A \cup A' = U$ (ii) $A \cap A' = \phi$
8. Double Complement Law	$\overline{\overline{A}} = A$
9. De Morgan's Law	(i) $\overline{A \cup B} = \bar{A} \cap \bar{B}$ (ii) $\overline{A \cap B} = \bar{A} \cup \bar{B}$
10. Absorption Law	(i) $A \cup (A \cap B) = A$ (ii) $A \cap (A \cup B) = A$

Q. Prove that $(A-C) \cap (C-B) = \phi$ analytically. where A, B & C are set. Verify graphically.

Sol $\Rightarrow$ $(A-C) \cap (C-B) = \{x : x \in (A-C) \cap (C-B)\}$

$= \{x : x \in (A-C) \text{ and } x \in (C-B)\}$

$= \{x : (x \in A \text{ and } x \notin C) \text{ and } (x \in C \text{ and } x \notin B)\}$

$= \{x : x \in A \text{ and } (x \notin C \text{ and } x \in C) \text{ and } x \notin B\}$

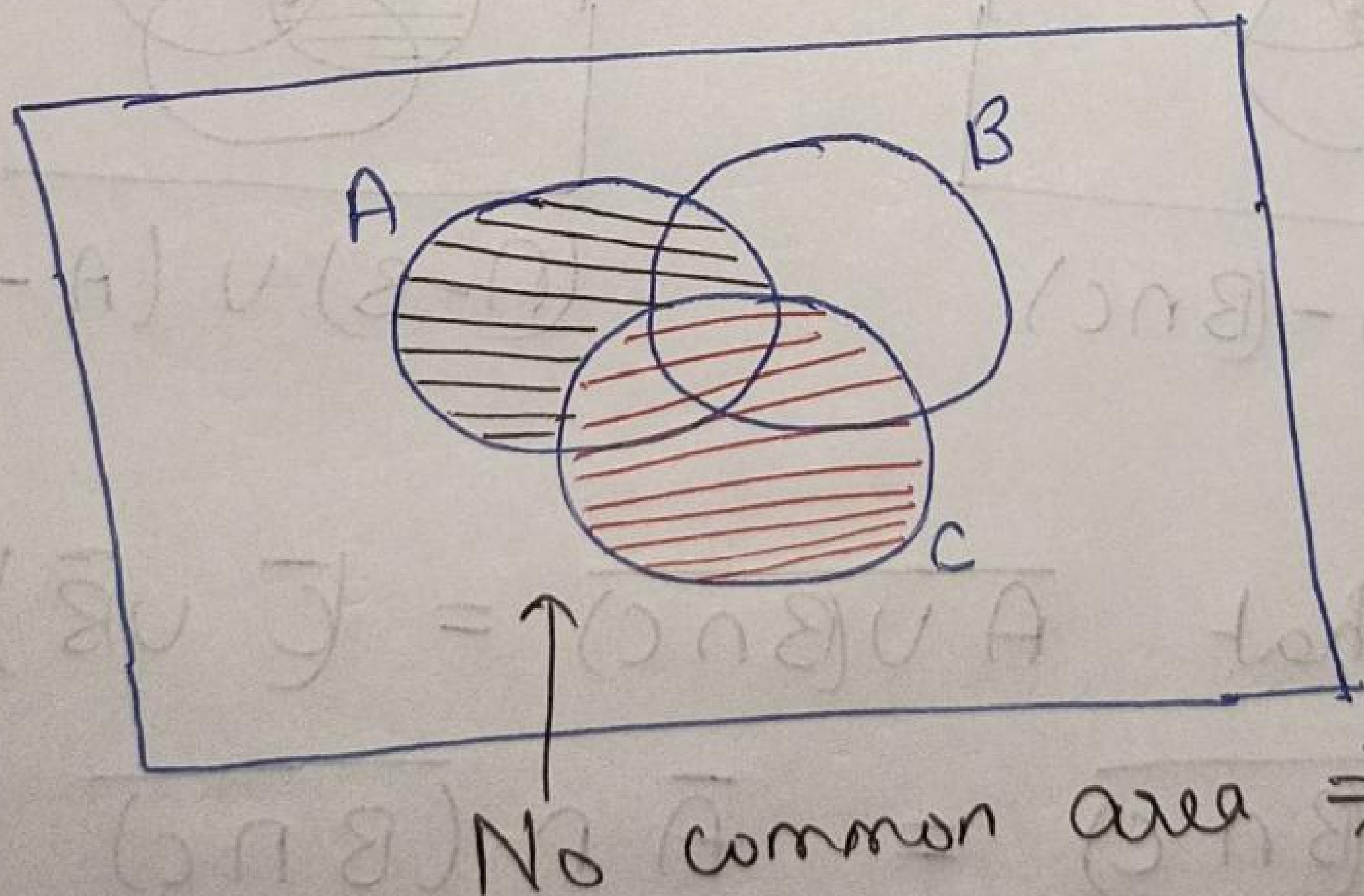
$= \{x : (x \in A \text{ and } x \in \phi) \text{ and } x \notin B\}$

$= \{x : (x \in \phi \text{ and } x \notin B)\}$

$= \{x : x \in \phi\}$

$= \phi$.

LHS = RHS



No common area $\Rightarrow \phi$.

2) Prove $A-(B \cap C) = (A-B) \cup (A-C)$ analytically where A, B, C are set and then verify graphically.

Sol $\Rightarrow$ $A-(B \cap C)$

$= \{x : x \in A-(B \cap C)\}$

$= \{x : x \in A \text{ and } x \notin (B \cap C)\}$

$= \{x : x \in A \text{ and } (x \notin B \text{ } \cancel{\text{and}} \text{ or } x \notin C)$

$= \{x : x \in A \text{ and } (x \notin B \text{ or } x \notin C)\}$

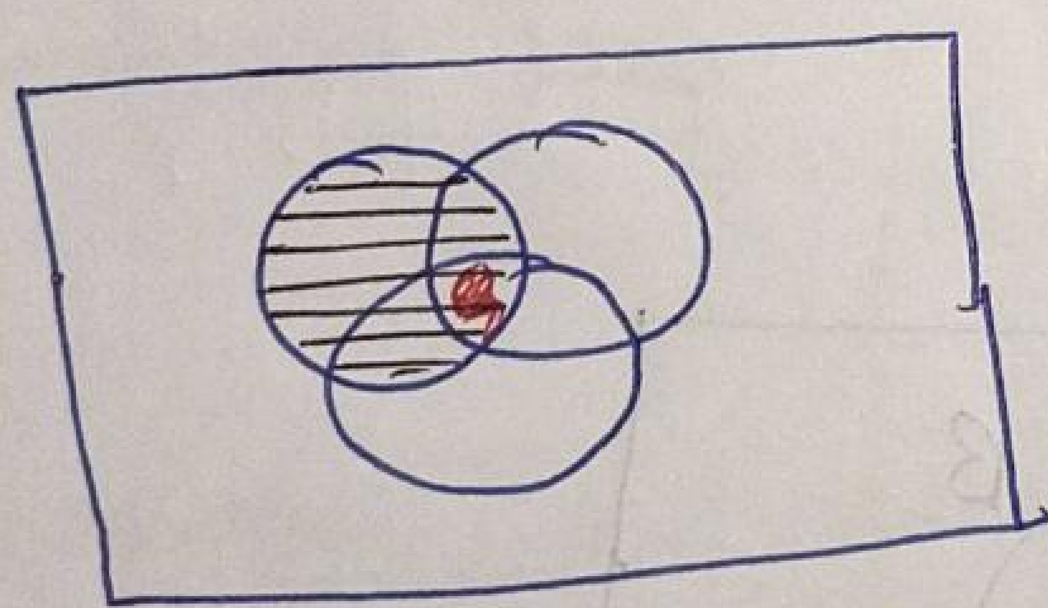
$= \{x : (x \in A \text{ and } x \notin B) \text{ or } (x \in A \text{ and } x \notin C)\}$

$= \{x : x \in (A-B) \text{ or } x \in (A-C)\}$

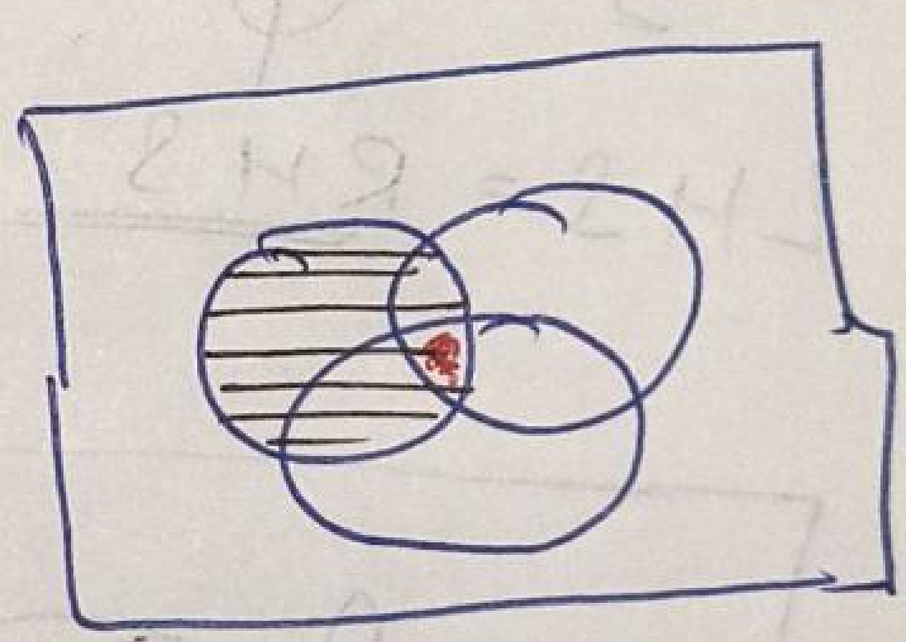
$= \{x : x \in (A-B) \cup (A-C)\}$

$= (A-B) \cup (A-C)$

LHS = RHS



$A-(B \cap C)$



$(A-B) \cup (A-C)$

3) Prove that $\overline{A \cup (B \cap C)} = (\bar{C} \cup \bar{B}) \cap \bar{A}$

Sol → $\overline{A \cup (B \cap C)} = \bar{A} \cap \overline{(B \cap C)}$

$= \bar{A} \cap (\bar{B} \cup \bar{C})$

$= \bar{A} \cap (\bar{C} \cup \bar{B})$

$= (\bar{C} \cup \bar{B}) \cap \bar{A}$

LHS = RHS

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4) Prove $A \cap (B - C) = (A \cap B) - (A \cap C)$ analytically & graphically.

Sol → $A \cap (B - C)$

$= \{x : x \in A \text{ and } x \in (B - C)\}$

$= \{x : x \in A \text{ and } x \in B \text{ and } x \notin C\}$

$= \{x : x \in A \cap B \cap \bar{C}\}$

$= A \cap B \cap \bar{C}$

$(A \cap B) - (A \cap C)$

$= \{x : x \in (A \cap B) \text{ and } x \notin (A \cap C)\}$

$= \{x : x \in A \text{ and } x \in B \text{ and } x \in (\bar{A} \cup \bar{C})\}$

$= \{x : x \in (A \cap B) \text{ and } (x \in \bar{A} \text{ or } x \in \bar{C})\}$

$= \{x : x \in (A \cap B) \text{ and } x \in \bar{A} \text{ or } x \in (A \cap B) \text{ and } x \in \bar{C}\}$

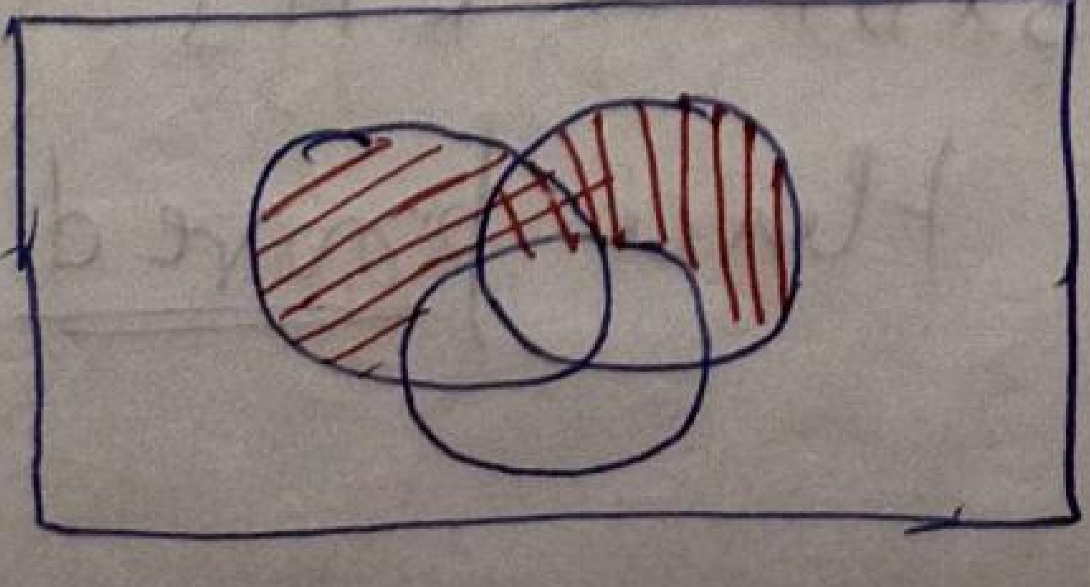
$= \{x : (x \in (A \text{ and } x \in \bar{A}) \text{ and } x \in B) \text{ or } (x \in A \cap B \cap \bar{C})\}$

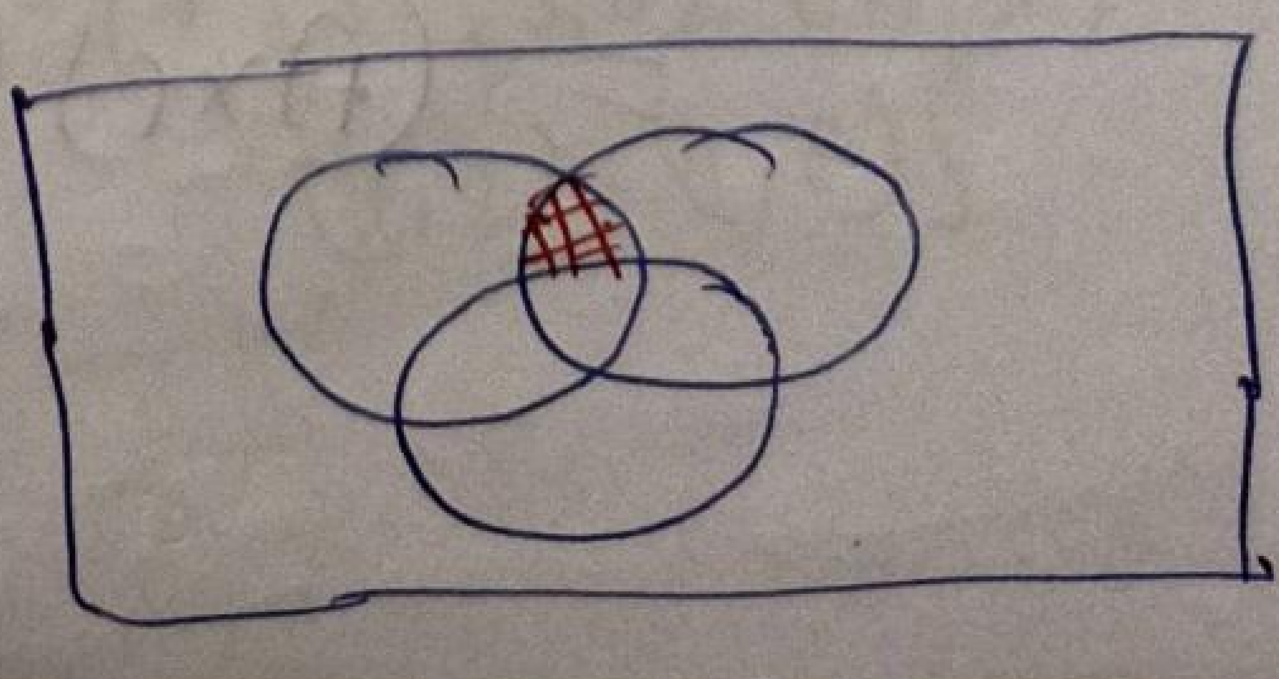
$= \{x : (\phi \text{ and } x \in B) \text{ or } x \in (A \cap B \cap \bar{C})\}$

$= \{x : \phi \text{ or } x \in (A \cap B \cap \bar{C})\}$

$= A \cap B \cap \bar{C} = LHS.$

Hence proved





5) Prove $A\times(B\cap C) = (A\times B)\cap(A\times C)$ analytically.

Sol → $A\times(B\cap C)$

$= \{(x,y) : x\in A \text{ and } y\in(B\cap C)\}$

$= \{(x,y) : x\in A \text{ and } (y\in B \text{ and } y\in C)\}$

$= \{(x,y) : (x\in A \text{ and } y\in B) \text{ and } (x\in A \text{ and } y\in C)\}$

$= \{(x,y) : A\times B \cap A\times C\}$

$= A\times B \cap A\times C = RHS.$

Hence proved.

6) If A, B, C, D are set. Prove $(A\cap B)\times(C\cap D) = (A\times C)\cap(B\times D)$.

Sol → $(A\cap B)\times(C\cap D)$

$= \{(x,y) : x\in A\cap B \text{ and } y\in(C\cap D)\}$

$= \{(x,y) : x\in A \text{ and } x\in B \text{ and } y\in C \text{ and } y\in D\}$

$= \{(x,y) : (x\in A \text{ and } y\in C) \text{ and } (x\in B \text{ and } y\in D)\}$

$= \{x,y : (x,y)\in(A\times C) \text{ and } (x,y)\in(B\cap D)\}$

$= \{x,y : (x,y)\in(A\times C)\cap(B\times D)\}$

$= (A\times C)\cap(B\times D) = RHS$

Hence proved

Relation

$A = \{1, 2\}$

$B = \{3, 4\}$

$A \times B = \{(1,3), (1,4), (2,3), (2,4)\}$

$R \subseteq A \times B$

Let A and B are two non-empty sets, then a binary relation from A to B is a subset of $A \times B$ i.e. $R \subseteq A \times B$.

$a \in A,\ b \in B$

$a^R b \rightarrow$ a is related to b.

$a^{\not R} b \rightarrow$ a is not related to b.

1) $A = \{1, 2, 3, 4\}$. $B = \{1, 7, 8\}$. R is defined such that "Less than".

Sol → Long Method :-

$A \times B = \{(1,1), (1,7), (1,8), (2,1), (2,7), (2,8), (3,1), (3,7), (3,8), (4,1), (4,7), (4,8)\}$

$a^R b = \{(1,7), (1,8), (2,7), (2,8), (3,7), (3,8), (4,7), (4,8)\}$

Short Method :-

$R = \{(1,7), (1,8), (2,7), (2,8), (3,7), (3,8), (4,7), (4,8)\}$

2 $A = \{0, 1, 2, 3, 4\}$, $B = \{0, 1, 2, 3\}$

$a^R b$ iff $a + b = 4$.

Sol → $a^R b = \{(1,3), (2,2), (3,1), (4,0)\}$

Domain → Range

Domain : $\{1, 2, 3, 4\}$

Range : $\{3, 2, 1, 0\}$.

Types of Relation :-

1) Universal Relation :- Any relation in which all the relation is included. $A \times B$ is having all the relation.

$\boxed{\text{or}}$

A relation R on set A is called universal relation if $R = A \times A$.

2) Void Relation :- A relation R on set A is called void Relation if R is a null set

Eg. $A = \{3, 4, 5\}$ & $a^R b$ iff $a + b > 10$

$R = \phi$.

3) Identity Relation :- A relation R on set A is called Identity Relation iff $a^R a$ $a \in R$.

or $R = \{(a, a) ; a \in A\}$.

Eg. $A = \{1, 2, 3\}$

$R = \{(1,1), (2,2), (3,3)\}$

4 Inverse Relation :- Let R be any relation from A to B then the inverse is denoted by R^{-1} is relation from B to A.

i.e. iff $a^R b$ then $b^{R^{-1}} a$.

or

$$R = \{(a,b) : a \in A,\ b \in B\}$$
$$R^{-1} = \{(b,a) : a \in A,\ b \in B\}$$

Eg. $A = \{2,3,5\}$, $B = \{6,8,10\}$ &

$a^R b$ iff a divides b.

$R = \{(2,6), (2,8), (2,10), (3,6), (5,10)\}$

$R^{-1} = \{(6,2), (8,2), (10,2), (6,3), (10,5)\}$

Q. $A = \{0,1,2,3,4\}$, $B = \{0,1,2,3\}$ find R iff $\mathrm{Lcm}(a,b) = 2$.

Sol → $a^R b = \{(1,2), (2,1), (2,2)\}$

Q The relation R on set $A = \{1,2,3,4,5\}$ is defined by $((a,b) \in R)$ iff 3 divides $(a-b)$.

(i) Find R & R^{-1}

(ii) Domain and Range of R & R^{-1}

(iii) List the element of the complement of R.

Sol → (i) $R = \{(1,1), (1,4), (2,2), (2,5), (3,3), (4,1), (5,2)\}$

$R^{-1} = \{(1,1), (4,1), (2,2), (5,2), (3,3), (1,4), (2,5)\}$

(ii) Domain of $R = \{1, 2, 3, 4, 5\}$ Ans
Range of $R = \{1, 2, 3, 4, 5\}$
Domain of $R^{-1} = \{1, 2, 3, 4, 5\}$ Ans
Range of $R^{-1} = \{1, 2, 3, 4, 5\}$

(iii) Complement of $R = \{(1,2), (1,3), (1,5), (2,1), (2,3), (2,4), (3,1), (3,2), (3,4), (3,5), (4,2), (4,3), (4,5), (5,1), (5,3), (5,4)\}$. Ans

Q If $R = \{(1,2), (2,4), (3,3)\}$ $S = \{(1,3), (2,4), (4,2)\}$
Find (i) $R \cup S$ (ii) $R \cap S$ (iii) $R - S$ (iv) $S - R$
(v) $R \oplus S$. Also verify $\text{dom}(R \cup S) = \text{dom}(R) \cup \text{dom}(S)$ and $\text{range}(R \cap S) \subseteq \text{range}(R) \cap \text{range}(S)$

Sol → (i) $R \cup S = \{(1,2), (2,4), (3,3), (1,3), (4,2)\}$
(ii) $R \cap S = \{(2,4)\}$
(iii) $R - S = \{(1,2), (3,3)\}$
(iv) $S - R = \{(1,3), (4,2)\}$
(v) $R \oplus S = \{(1,2), (3,3), (1,3), (4,2)\}$
→ $R \oplus S = (R - S) \cup (S - R)$.

$\text{dom}(R \cup S) = \{1, 2, 3, 4\}$
$\text{dom}(R) = \{1, 2, 3\}$ $\text{dom}(S) = \{1, 2, 4\}$
$\text{domain}(R) \cup \text{domain}(S) = \{1, 2, 3, 4\}$

range of $R \cap S = \{4\}$.

Range of $R = \{2, 4, 3\}$

Range of $S = \{3, 4, 2\}$

Range of (R) $\cap$ Range of (S) $= \{2, 4, 3\}$

$\Rightarrow$ range of $(R \cap S) \subseteq$ Range of R $\cap$ Range of S.

Properties of Relation :-

① Reflexive Property :- A relation R on set A is said to be reflexive if $a^R a \ \forall \ a \in A$.

e.g. If R is the relation on $A = \{1, 2, 3\}$.

$a^R b$ iff $a \le b$, $a, b \in A$.

$R = \{\underline{(1,1)}, (1,2), (1,3), \underline{(2,2)}, (2,3), \underline{(3,3)}\}$

R is reflexive

Note → Reflexive relation can be identity but identity can't be reflexive.

② Symmetric :- A relation R on set A is said to be symmetric if $a^R b$ then $b^R a$.

Eg. $A = \{1, 2, 3\}$ $B = \{1, 2, 3\}$ if $a + b = 3$.

$R = \{(1,2), (2,1)\}$

③ Antisymmetric :- A relation R on set A is said to be antisymmetric whenever (a, b) & (b, a) $\in R$ then $a = b$.

Identity is only example

i.e. $a^R b$ & $b^R a \Rightarrow a = b$.

(4) Transitive :- A relation R on set A is said to be transitive if
$a^R b$ & $b^R c$ then $a^R c$.

Equivalence Relation :- A relation R on set A is an equivalence relation if R is reflexive, symmetric and transitive.

Partial-order Relation :- A relation R on set A is an Partial order relation if R is reflexive, anti symmetric and transitive.

Q Determine whether the relation R on the set of all integer, is reflexive, symmetric, antisymmetric and transitive. where $a^R b$ iff
(i) $a \neq b$ (ii) $ab \geq 0$.

Sol → (i) $a^R b$ iff $a \neq b$.

Reflexive :- $a^R a$
$\Rightarrow a \neq a$
which is not possible
$\Rightarrow$ It is not reflexive

Symmetric : $a^R b$.
$a \neq b$.
$b^R a$
$b \neq a$
which is true
$\Rightarrow$ It is symmetric

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Anti-symmetric :- $a^R b$ & $b^R a$
$a \neq b$ & $b \neq a$
$\Rightarrow R$ is not antisymmetric

Transitive :- $a^R b$ & $b^R c$
$a \neq b$ & $b \neq c$.
which is not necessarily true.
$\Rightarrow R$ is not transitive.
$\Rightarrow$ Symmetric Only.

(ii) $ab \geq 0$.

Reflexive : $a^R a$
$\Rightarrow a \cdot a \geq 0$
$a^2 \geq 0$
$\Rightarrow$ True
$\Rightarrow$ Reflexive

Symmetric $\Rightarrow a^R b$
$ab \geq 0$
$b^R a$
$ba \geq 0$
$\Rightarrow$ true
$\Rightarrow$ Symmetric

Antisymmetric :- $a^R b$ & $b^R a$
$ab \geq 0$ & $ba \geq 0$
$a = b$ is not necessarily true
So, not a antisymmetric relation

Transitive :- $a^R b$ & $b^R c$

$ab \geq 0$ & $bc \geq 0$

$\Rightarrow$ Not transitive

$\Rightarrow$ Reflexive & Symmetric

Q If R is the relation on the set of ordered pairs of +ve integer such that $(a,b),(c,d) \in R$ whenever $ad = bc$. Prove that R is an equivalence Relation.

Sol $\rightarrow$ $(a,b)^R(c,d)$ iff $ad = bc$

Reflexive :- $a^R a$

$(a,b)^R(a,b)$.

$\Rightarrow ab = ab$

which is true

$\Rightarrow$ Reflexive

Symmetric :- $(a,b)^R(c,d)$

$\Rightarrow ad = bc$

$\Rightarrow bc = ad$

$\Rightarrow cb = da$

$\Rightarrow (c,d)^R(a,b)$

Hence Symmetric

Transitive :- $(a,b)^R (c,d)$ & $(c,d)^R (e,f)$

$ad = bc$ & $cf = de$

$\frac{d}{c} = \frac{b}{a}$ & $\frac{d}{c} = \frac{f}{e}$.

$\Rightarrow \frac{b}{a} = \frac{f}{e}$.

$\Rightarrow be = af$

$\Rightarrow af = be$

$= (a,b)^R (e,f)$.

$\Rightarrow$ transitive

Since R is Reflexive, Symmetric & transitive
$\therefore$ It is an equivalence relation.

Matrix Representation of Relations :-

$$m_{ij} = \begin{cases} 1 & \text{if } (a_i, b_j) \in R \\ 0 & \text{if } (a_i, b_j) \notin R. \end{cases}$$

Q Let $P = \{1,2,3,4\}$, $Q = \{a,b,c,d\}$.
$R = \{(1,a), (2,c), (2,d), (4,b)\}$. Find M_R.

Sol →

$$M_R = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

Ans

Q If R & S be relation on a set S, represented by matrices

$$M_R = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \;\&\; M_S = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Find matrices that represent
(a) $R \cup S$ (b) $R \cap S$ (c) $R \cdot S$ (d) $S \cdot R$ (e) $R \oplus S$.

Sol → (a) $R \cup S \Rightarrow M_{R \cup S} = M_R \vee M_S$.

$$M_{R \cup S} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \vee \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \text{ Ans}$$

(b) $R \cap S \Rightarrow M_{R \cap S} = M_R \wedge M_S$.

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \wedge \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \text{ Ans}$$

(c) $M_{RS} = M_R \cdot M_S$

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

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(d) $M_{SR} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ Ans

(e) $M_{R \oplus S} = M_{R \cup S} - M_{R \cap S}$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$ Ans

Q. If R is the relation of $A = \{1, 2, 3\}$ such that $(a, b) \in R$ iff $a+b$ = even. find relation matrix M_R. find also relation matrices of R^{-1}, $\bar{R}$ & R^2.

Sol→ $A = \{1, 2, 3\}$

$R = \{(1,1), (1,3), (2,2), (3,1), (3,3)\}$

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$M_{R^{-1}} = (M_R)^T = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$ Ans

$M_{\bar{R}} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ $M_{R^2} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ Ans

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Warshall's Algorithm :-

→ With this we convert non-transitive relation to transitive relation

Q Using Warshall's algorithm, find all transitive closure of relation $R = \{(4,4), (4,10), (6,6), (6,8), (8,10)\}$ on set $A = \{4, 6, 8, 10\}$.

Sol →

$$W_0 = M_R = \begin{array}{c|c} & \begin{array}{cccc} 4 & 6 & 8 & 10 \end{array} \\ \begin{array}{c} 4 \\ 6 \\ 8 \\ 10 \end{array} & \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

We will now compute W_1. To compute W_1. (i) transfer all 1's from W_0 to W_1
(ii) Location of non-zero entries in C_1
(iii) Location of non-zero entries in R_1

Non-zero entry in $C_1 = 4$
Non-zero entry in $R_1 = 4, 10$
Mark (4,4), (4,10) as 1

$$W_1 = \begin{array}{c|c} & \begin{array}{cccc} 4 & 6 & 8 & 10 \end{array} \\ \begin{array}{c} 4 \\ 6 \\ 8 \\ 10 \end{array} & \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Now following same step we will find W_2, W_3, W_4

Non-zero entry in C_2 : 6

Non-zero entry in R_2 : (6, 8)

Mark (6, 6), (6, 8) as 1

$$W_2 = \begin{array}{c} 4 \\ 6 \\ 8 \\ 10 \end{array} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Non-zero entry in C_3 : 6

Non-zero entry in R_3 : 10

Mark (6, 10) as 1

$$W_3 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Non-zero entry in C_4 : 4, 6, 8.

Non-zero entry in R_4 : —

$$W_4 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R^+ = \{(4,4), (4,10), (6,6), (6,8), (6,10), (8,10)\}$

Ans

Q Using Warshall Algorithm find transitive Closure relation of $R = \{(1,2), (2,3), (3,3)\}$ where $A = \{1,2,3\}$.

Sol →

$$W_0 = M_R = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

C_1 : —
R_1 : 2

$$W_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

C_2 : 1
R_2 : 3
mark $(1,3)$ as 1

$$W_2 = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$C_3 = 1, 2, 3$
$R_3 = 3$.
mark $(1,3), (2,3), (3,3)$ as 1

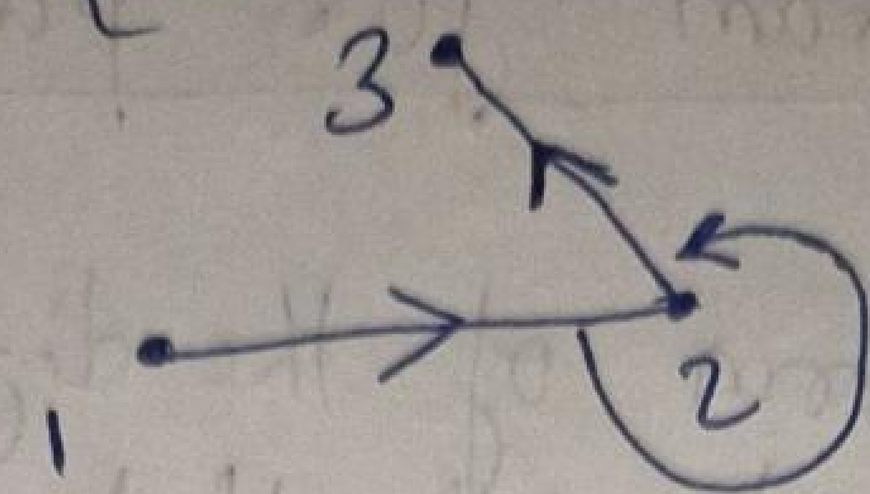
$$W_3 = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R^+ = \{(1,2), (1,3), (2,3), (3,3)\}$$ Ans

Representation of Relation by graph :-

To represent R on set A graphically, each element of A is represented by a point known as node or vertices. To show a is related to b, an ~~arch~~ arc is drawn from a to b. This arc is called ~~edt~~ edges or arcs. Direction is indicated by an arrow. Resulting diagram is called directed graph or di-graph.

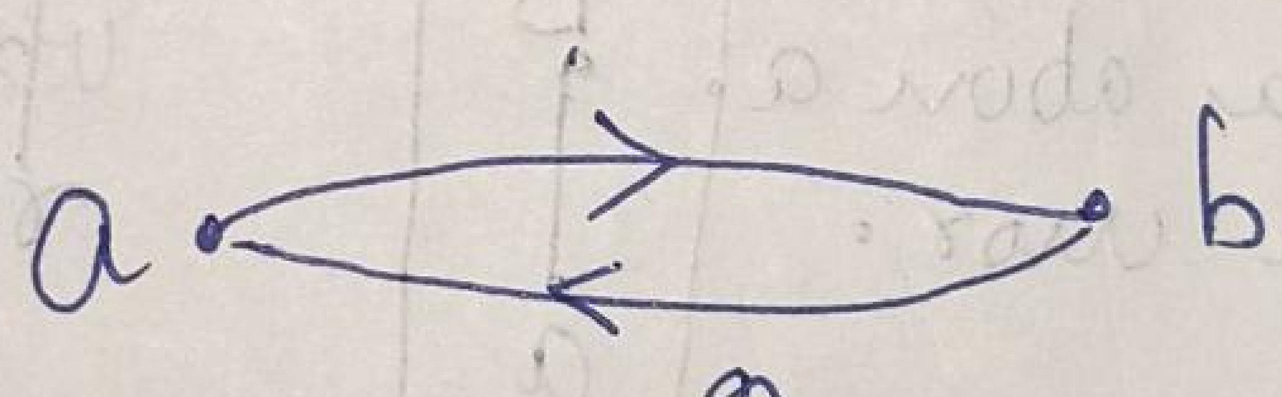
Eg. $R = \{(1,2), (2,3), (2,2)\}$



* How to Identify types of relation by looking at di-graph ?

(i) Reflexive :- A relation R is reflexive if and only if there is a self-loop at every vertex.

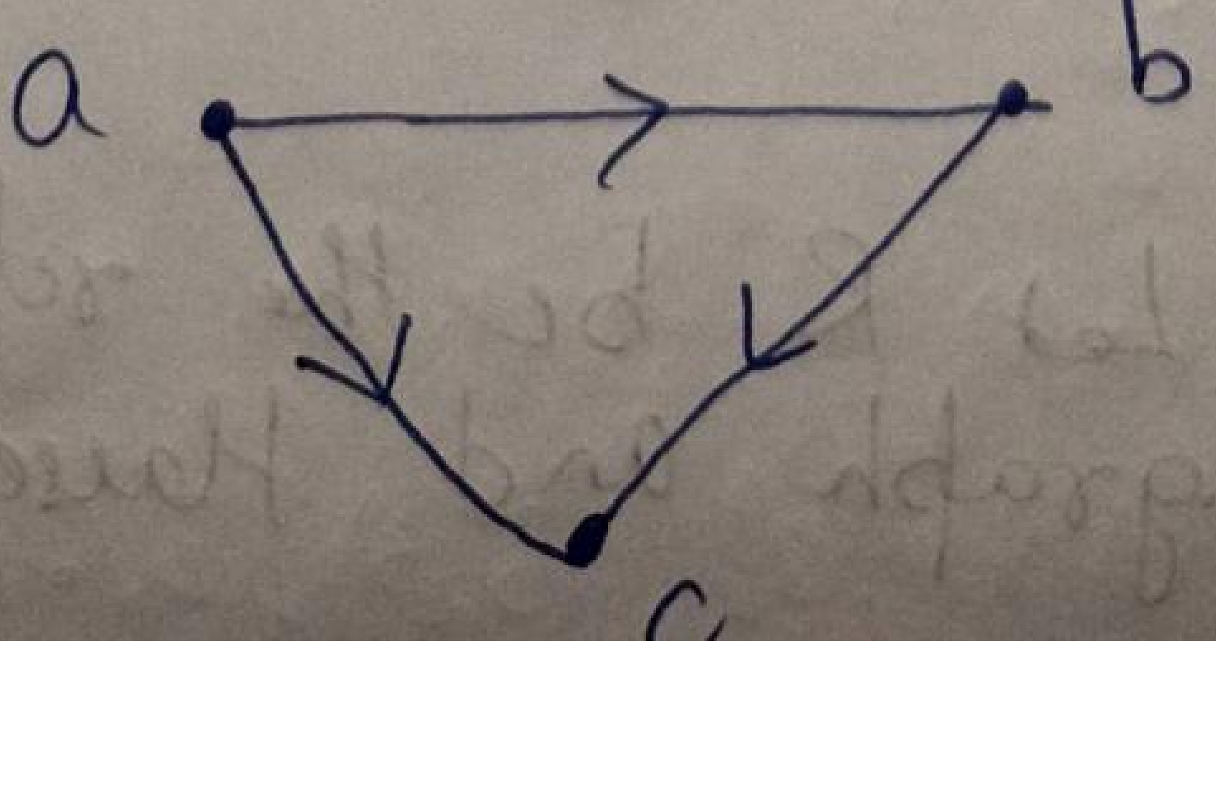
(ii) Symmetric :- A relation R is symmetric iff there is any parallel opposite edge between a and b.



symmetric

(iii) Antisymmetric :- iff there is no parallel edge

(iv) Transitive :- if and only if whenever there is an edge between a ~~and~~ to b and from b to c then there is edge between a to c.



Imp

Hasse Diagram for partial ordering

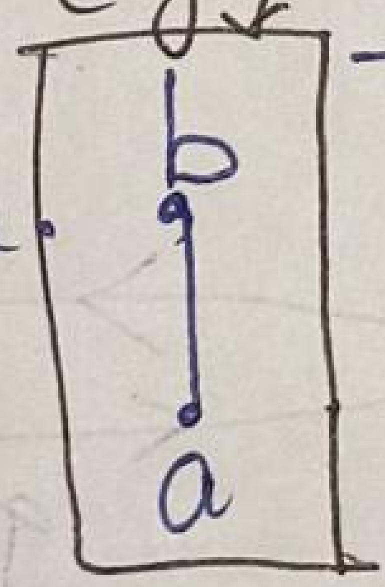
The simplified form of the digraph of a partial ordering on a finite set that contain sufficient information about the partial ordering is called Hasse diagram.

How to simplify?

Step 1 → Since partial ordering is reflexive, so remove all self loop from all vertex.

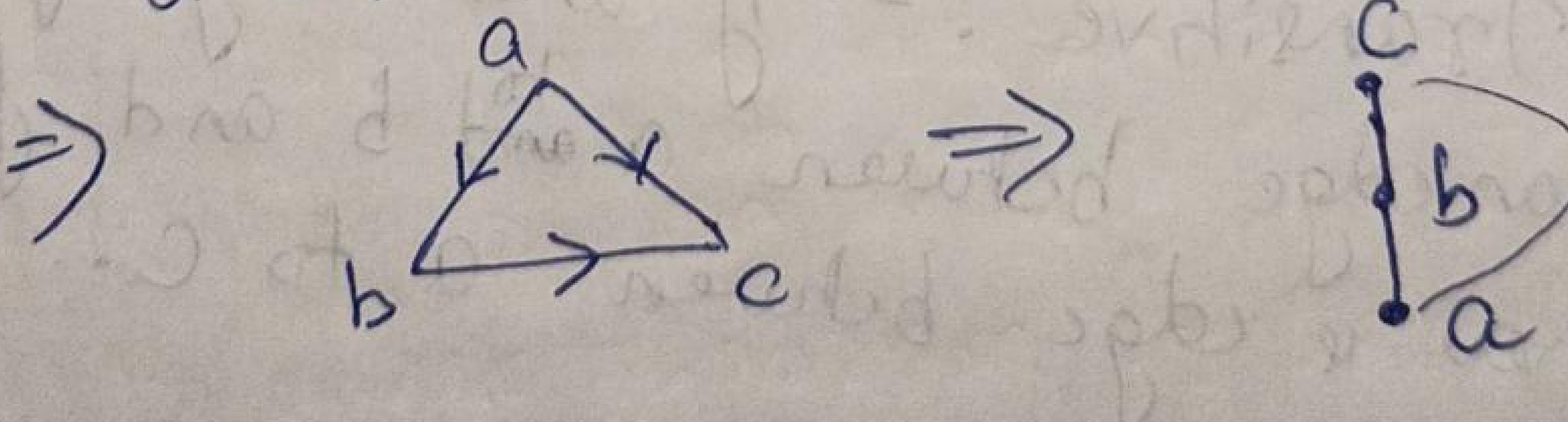
Step 2 → If $a^R b$, b appear above a & remove arrow.

e.g. → All edge are directed upward. No direction required.



Step 3 :- Remove all edge whose existence is implied by transitive property.

$a^R b$, $b^R c$, $a^R c$.



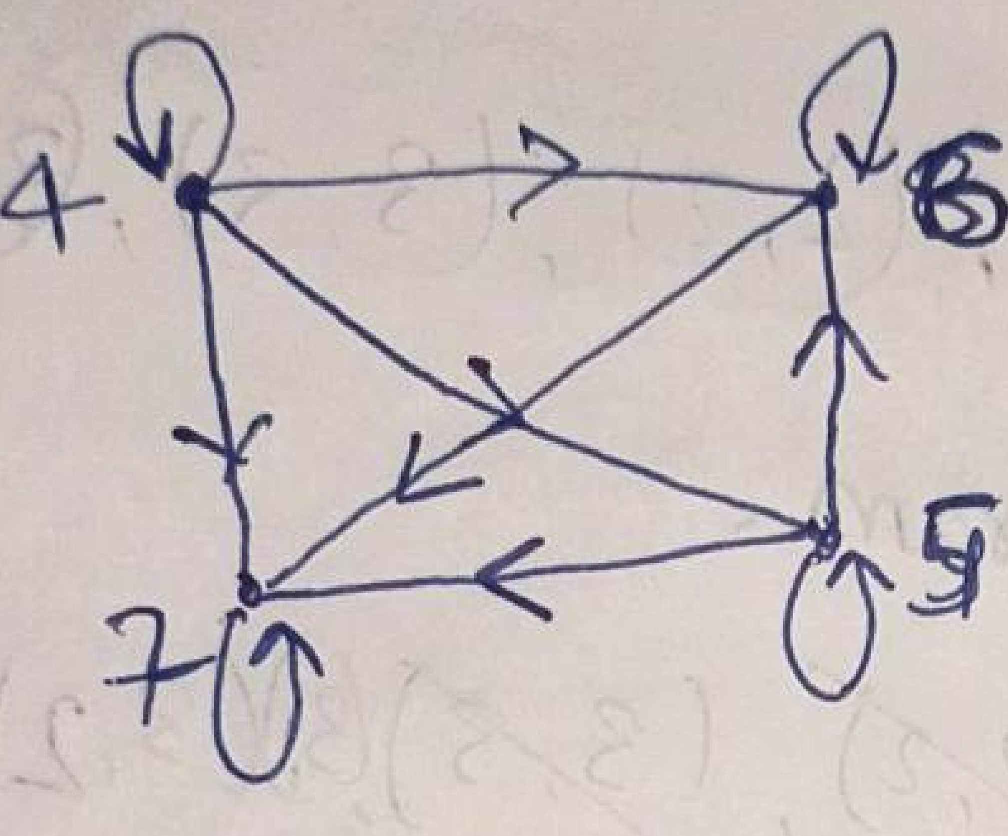
← No need to have edge from a to c.

Q. $A = \{4, 5, 6, 7\}$, Let R be the relation $\leq$ on A. Draw the digraph and Hasse diagram of R.

Sol → $A = \{4, 5, 6, 7\}$

$R = \{(4,4), (4,5), (4,6), (4,7), (5,5), (5,6), (5,7), (6,6), (6,7), (7,7)\}$

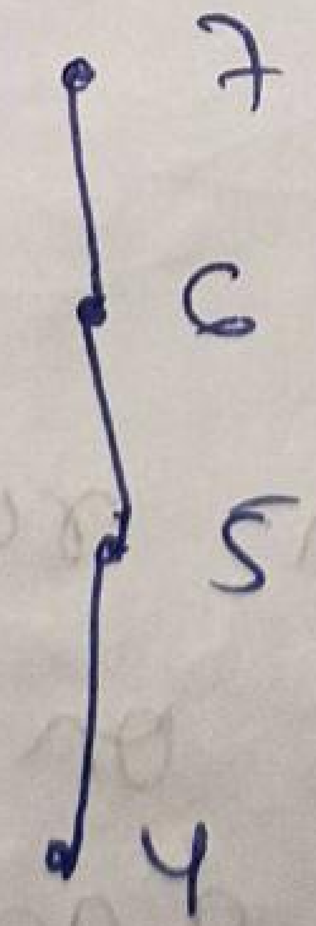
Di-graph :-



Hasse Diagram :-

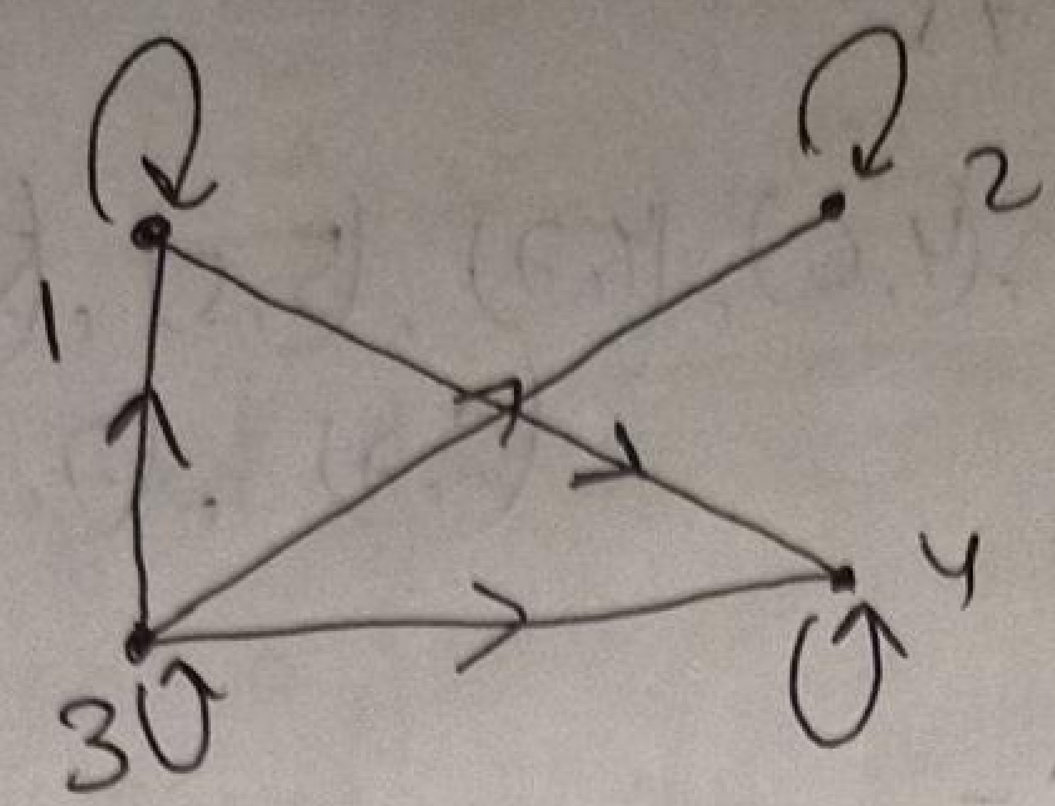
$R = \{\cancel{(4,4)}, (4,5), \cancel{(4,6)}, \cancel{(4,7)}, \cancel{(5,5)}, (5,6), \cancel{(5,7)}, \cancel{(6,6)}, (6,7), \cancel{(7,7)}\}$

$R = \{(4,5), (5,6), (6,7)\}$



Ans

Q ~~A = {1, 2, 3, 4}~~. Draw Hasse Diagram from the given digraph G for partial ordering relation on set $A = \{1, 2, 3, 4\}$



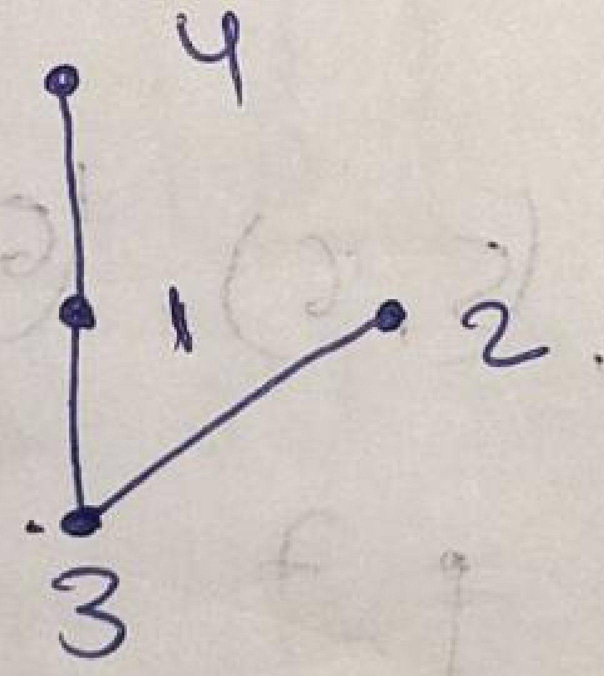
Sol → from graph.

$R = \{(1,1), (1,4), (2,2), (3,3), (3,1), (3,4), (4,4)\}$

For Hasse diagram.

$R = \{\cancel{(1,1)}, (1,4), \cancel{(2,2)}, \cancel{(3,3)}, (3,1), (3,2), \cancel{(3,4)}, \cancel{(4,4)}\}$

$R = \{(1,4), (3,2), (3,1), (3,4)\}$

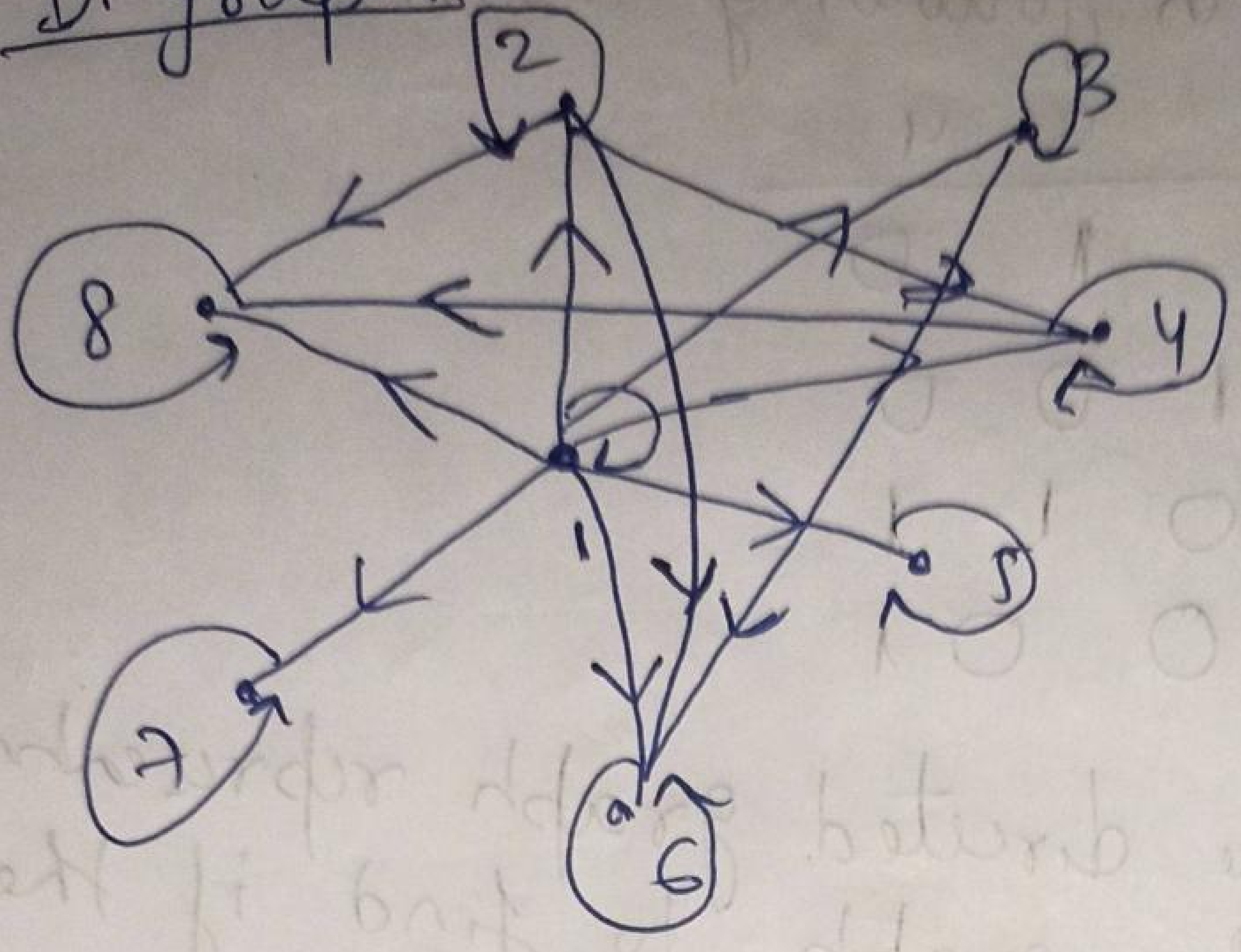


Ans

Q Draw the digraph representing the relation (a, b) a divides b on the set $A = \{1,2,3,4,5,6,7,8\}$. Reduce it to Hasse diagram representing the partial ordering.

Sol → $R = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (1,7), (1,8), (2,2), (2,4), (2,6), (2,8), (3,3), (3,6), (4,4), (5,5), (4,8), (6,6)\}$

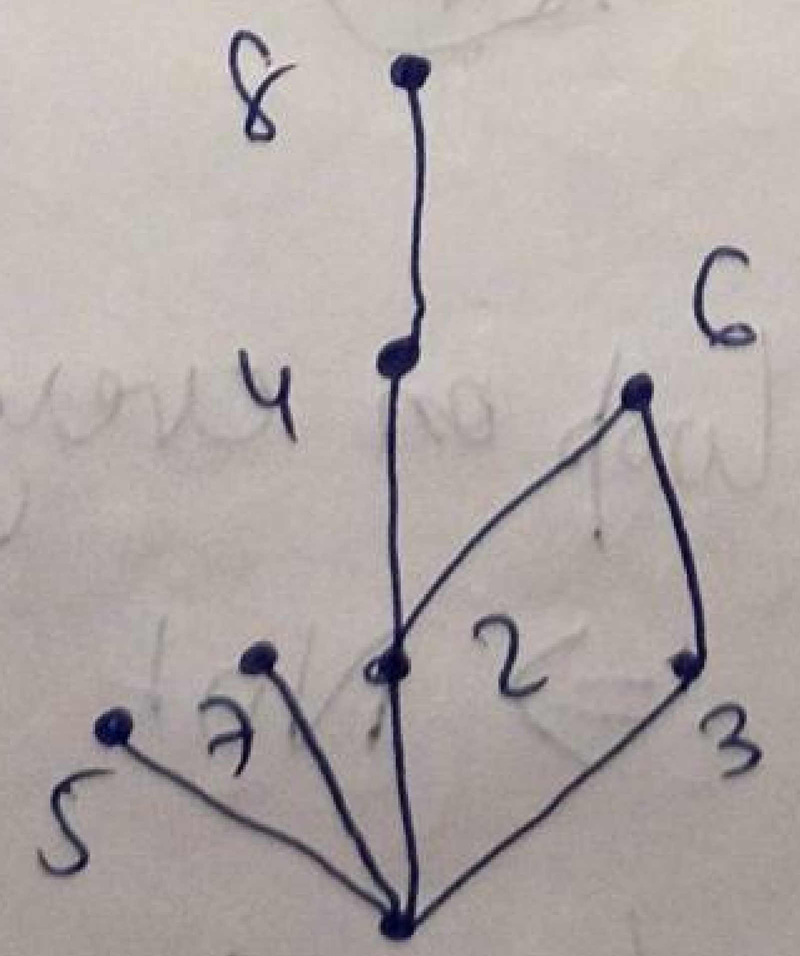
Di-graph :-



Hasse Diagram :-

$$R = \{ \cancel{(1,1)}, (1,2), (1,3), \cancel{(1,4)}, (1,5), \cancel{(1,6)}, (1,7), \cancel{(1,8)}, \cancel{(2,2)}, (2,4), (2,6), \cancel{(2,8)}, \cancel{(3,3)}, (3,6), \cancel{(4,4)}, (4,8), \cancel{(5,5)}, \cancel{(6,6)}, \cancel{(7,7)}, \cancel{(8,8)} \}$$

$$R = \{ (1,2), (1,3), (1,5), (1,7), (2,4), (2,6), (3,6), (4,8) \}$$

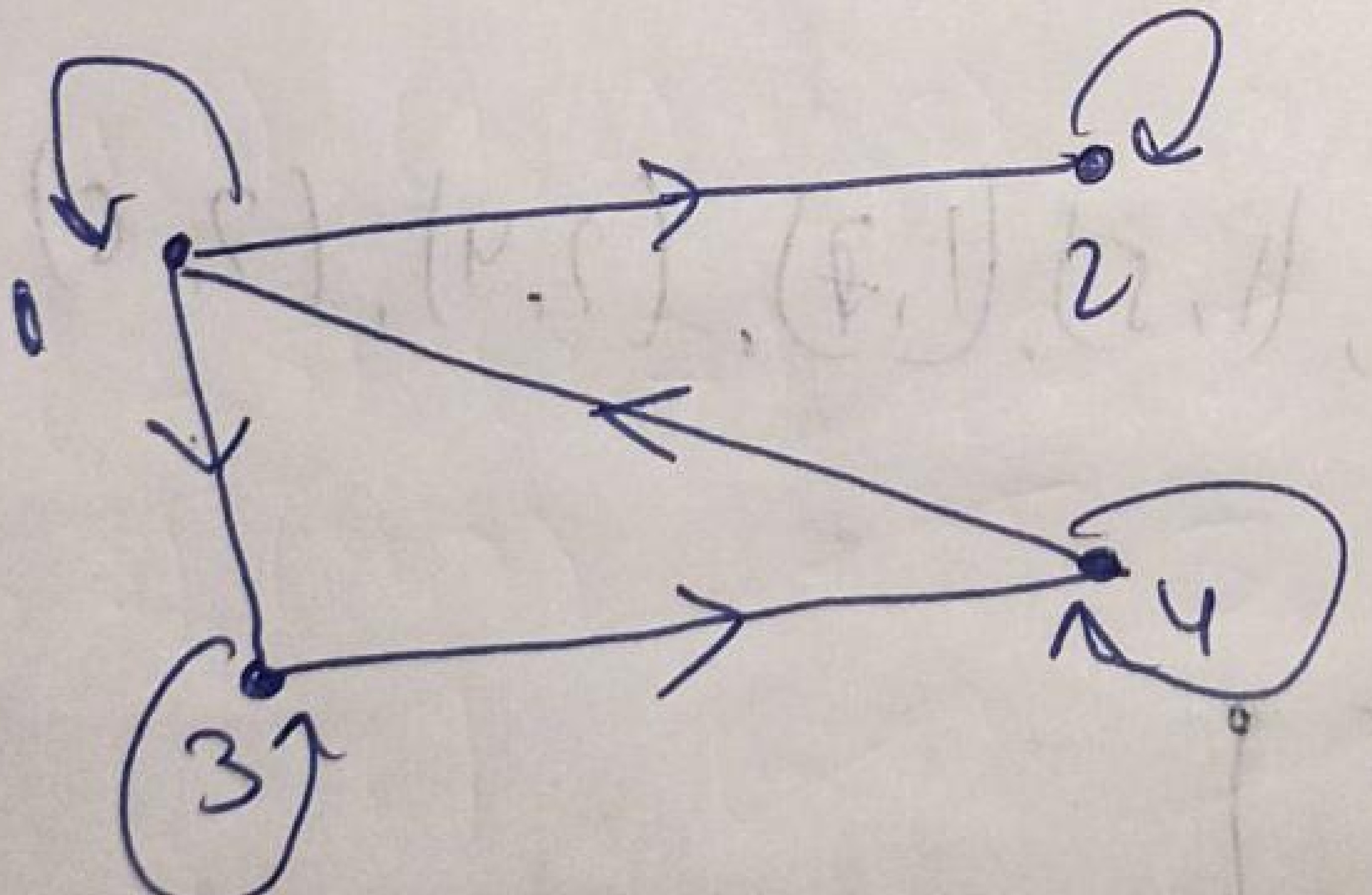


Q List the ordered pair in relation on {1, 2, 3, 4} Corresponding in following matrix

	1	2	3	4
1	1	1	1	0
2	0	1	0	0
3	0	0	1	1
4	1	0	0	1

Also draw the directed graph representing this relation. Use the graph to find if the relation is reflexive symmetric or transitive.

Sol → $R = \{(1,1), (1,2), (1,3), (2,2), (3,3), (3,4), (4,1), (4,4)\}$

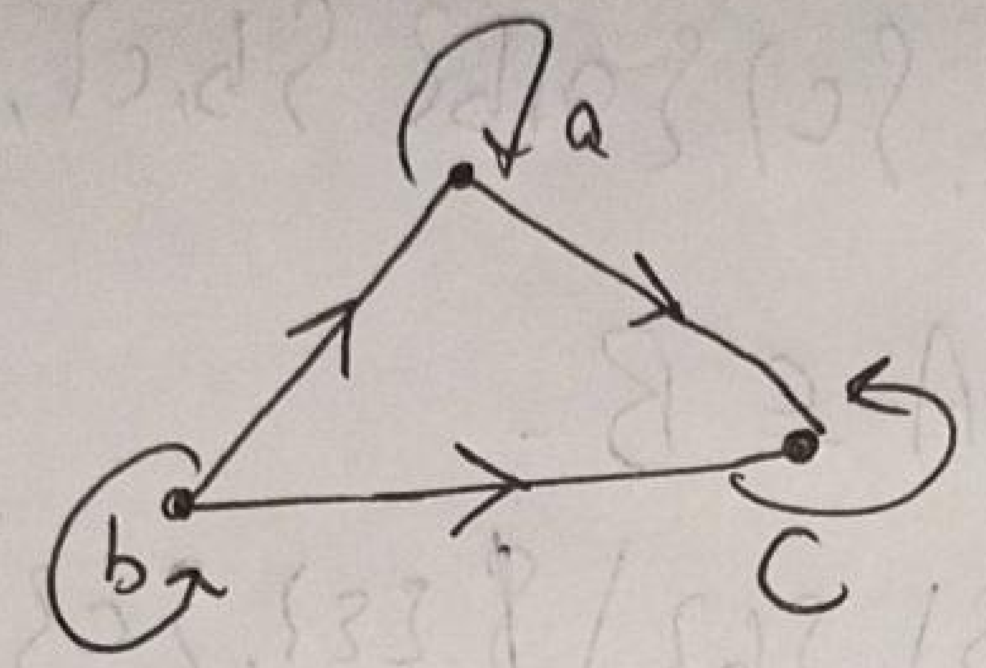


Since there is self loop on every vertex ⇒ Reflexive

No parallel edge ⇒ Not Symmetric

(1,3), (3,4) but (1,4) is not there. So, Not transitive.

Q List the ordered pair in the relation represented by di-graph given in the figure. Also use the graph to prove that relation is partial ordering. Also draw the directed graph representing R^{-1} and $\bar{R}$.



Sol → $R = \{(a,a), (a,c), (b,a), (b,b), (c,c), (b,c)\}$

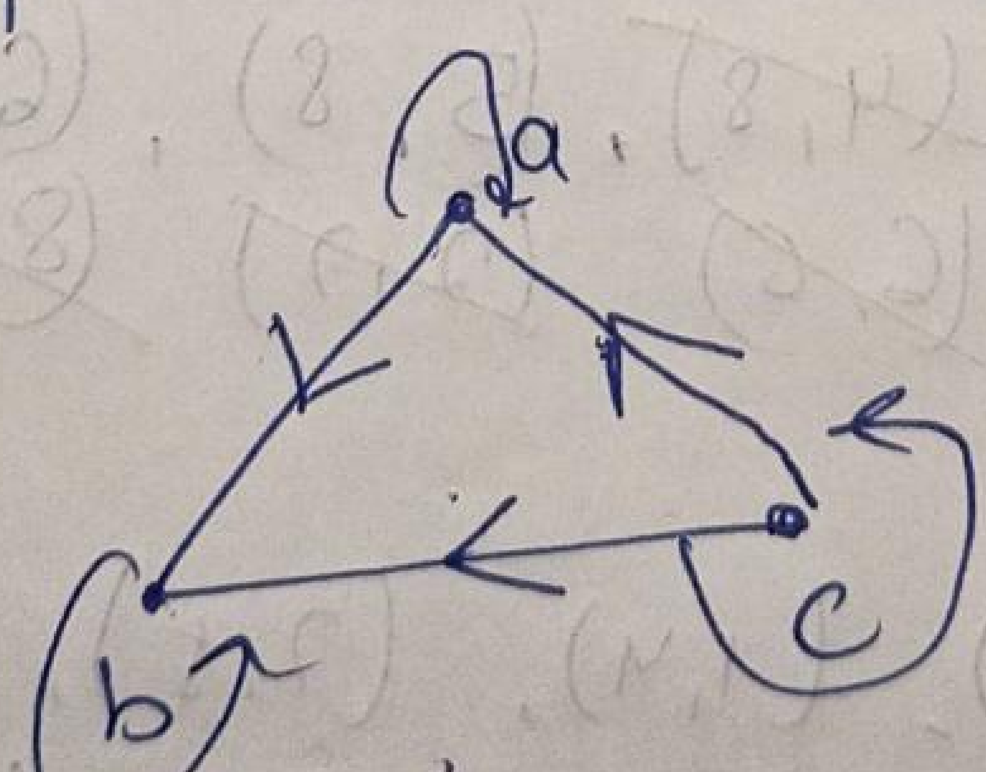
Since there is self loop on every node ⇒ Reflexive

Since there is no parallel edge ⇒ Anti-symmetric

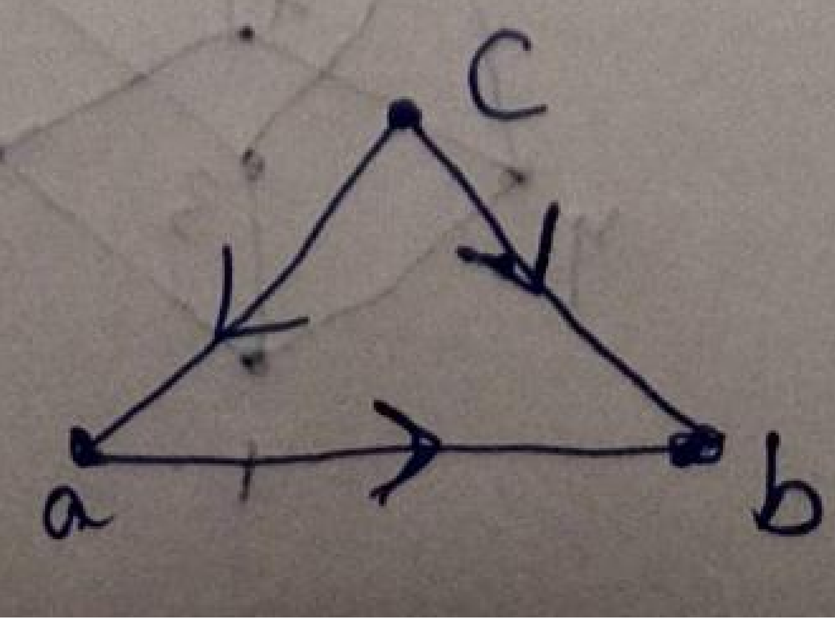
$(b,a), (a,c)$ & (b,c) so transitive.

So, graph relation is partial ordering.

R^{-1} :



$\bar{R}$: $\{(a,b), (c,a), (c,b)\}$



Q Draw the Hasse diagram representing the partial ordering $\{(A, B) : A \subseteq B\}$ on power set $P(S)$ where $S = \{a, b, c, d\}$.

Sol → Power set of S $P(S)$.

$= \{\{\}\}, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\}$

Relation $(a, b) : A \subseteq B$.

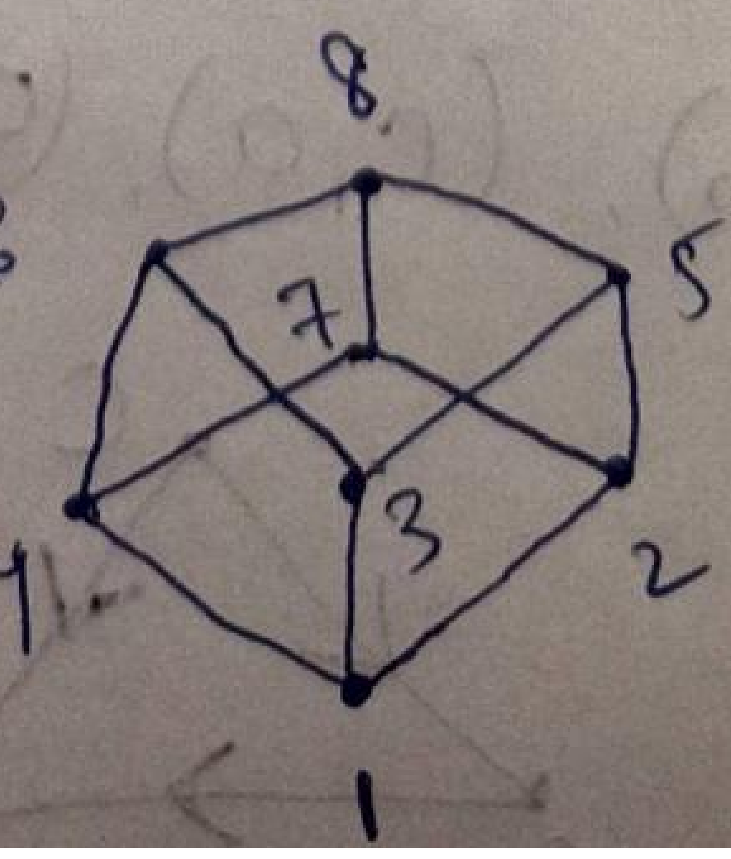
$(\{\{\}\}, \{\{\}\})$ $(\{\}, \{a\})$ $(\{\}, \{b\})$ $(\{\}, \{c\})$, $(\{\}, \{a, b\})$ ——— so on

or

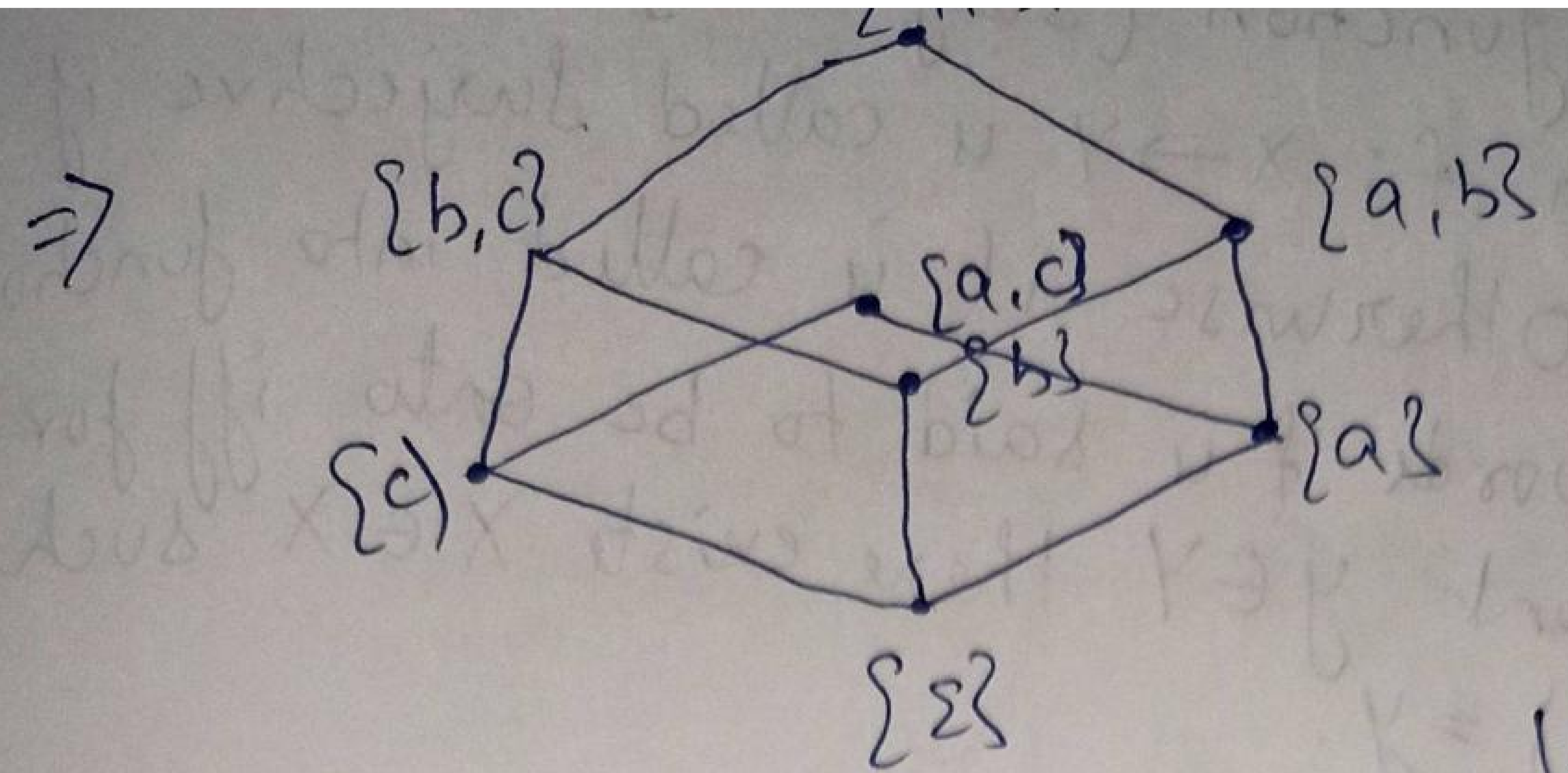
$A = (1, 2, 3, 4, 5, 6, 7, 8)$

$R = \{\cancel{(1,1)}, (1,2), (1,3), (1,4), \cancel{(1,5)}, \cancel{(1,6)}, \cancel{(1,7)}, \cancel{(1,8)}$
$(2,5), (2,7), \cancel{(2,8)}, (3,5), (3,6), (4,6),$
$\cancel{(3,8)}, (4,7), \cancel{(4,8)}, (5,8), (6,8), (7,8)\}$
$\cancel{(2,2)}\cancel{(3,3)}\ \cancel{(4,4)}, \cancel{(5,5)}, \cancel{(6,6)}\ \cancel{(7,7)}\ \cancel{(8,8)}\}$

$R = \{(1,2), (1,3), (1,4), (2,5), (2,7), (3,5), (3,6),$
$(4,6), (4,7), (5,8), (6,8), (7,8)\}$



⇒

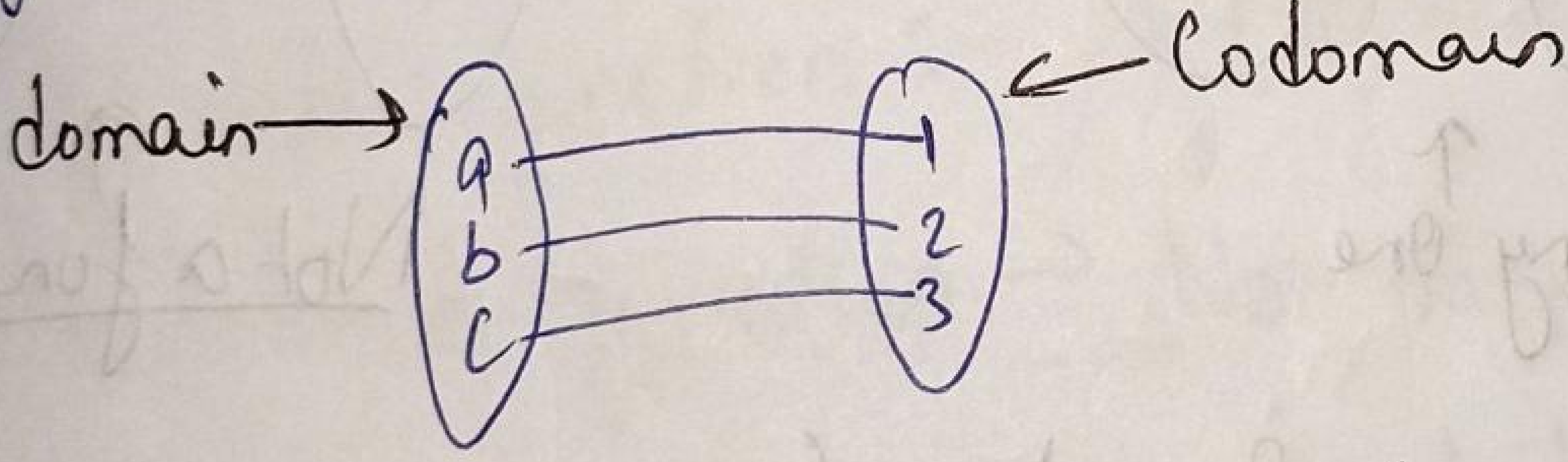


Ans

Functions (mapping).

$f : X \rightarrow Y$.

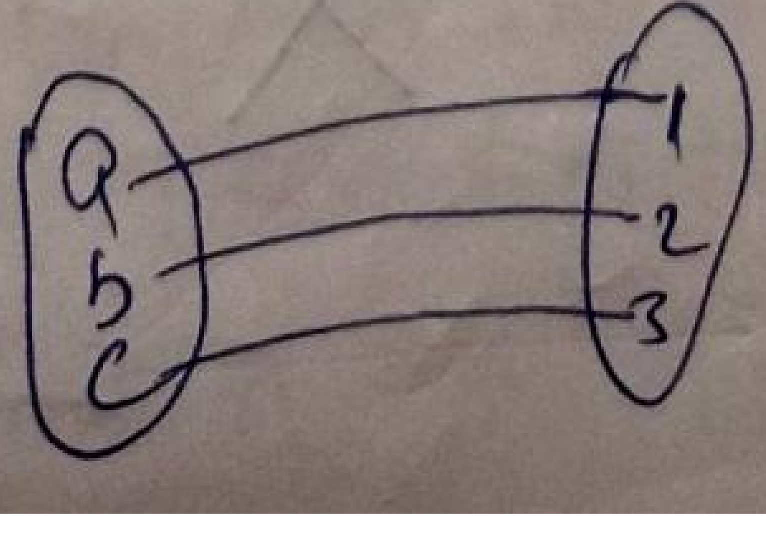
A relation f from a set X to another Y is called a function if for every $x \in X$ there is a unique $y \in Y$ such that $(x, y) \in f$.



Range $\subseteq$ Co-Domain

Types of function :-

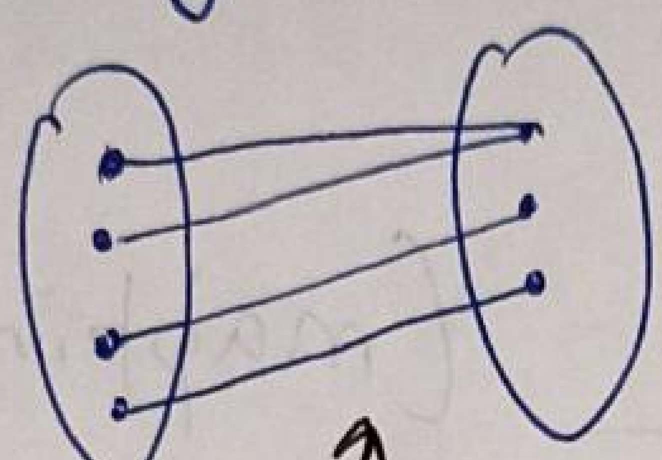
(i) One-One :- A function $f : X \rightarrow Y$ is called one-one (injective) if distinct element of X are mapped into distinct element of Y. In other words f is one-one if & only if $f(x_1) \neq f(x_2)$ whenever $x_1 \neq x_2$ and $f(x_1) = f(x_2)$ whenever $x_1 = x_2$.



(ii) Onto-function (surjective) :-

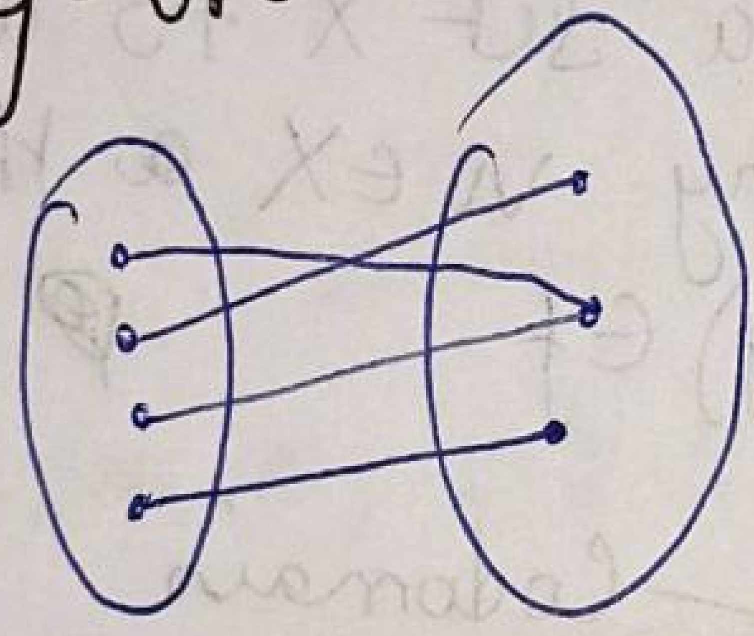
A function $f : X \to Y$ is called surjective if $R_f = Y$ otherwise it is called into function.

In other words f is said to be onto iff for every element $y \in Y$ there exists $x \in X$ such that $f(x) = y$.

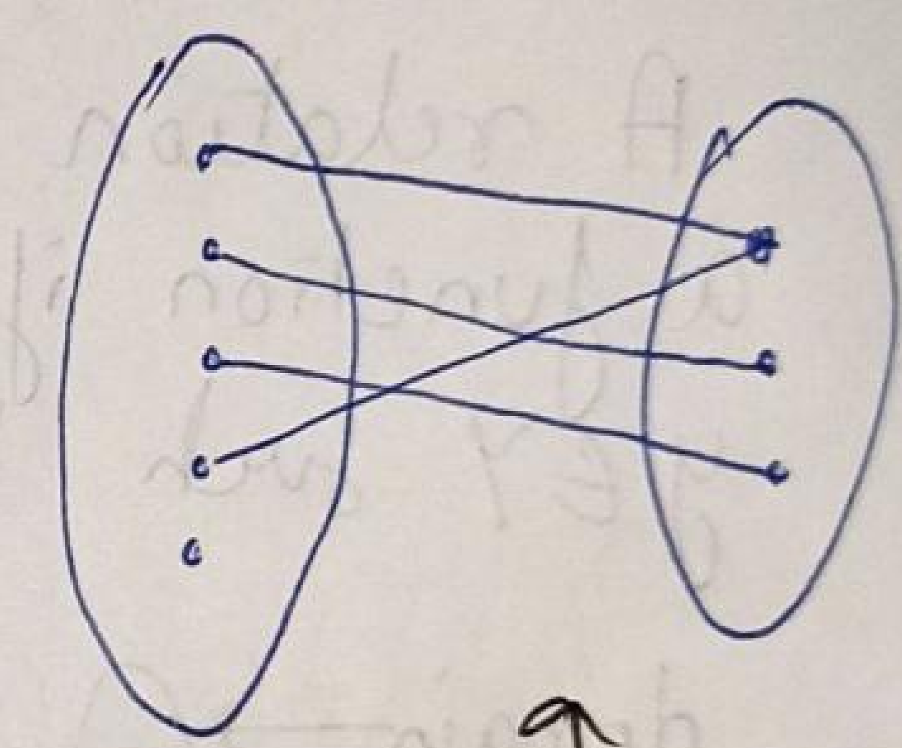


onto

(iii) Many-one :-

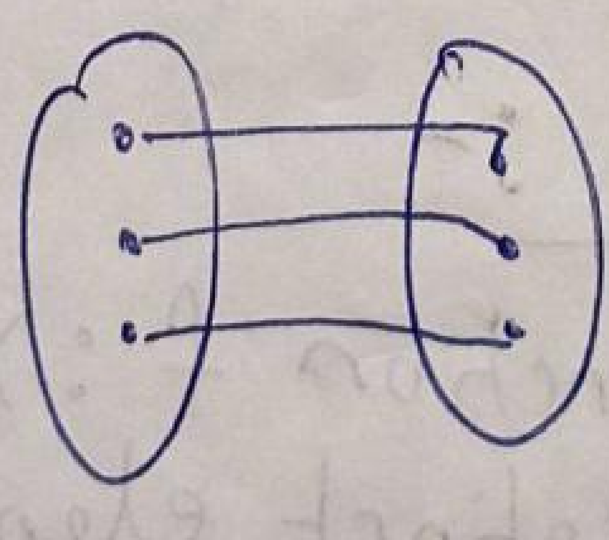


Many one

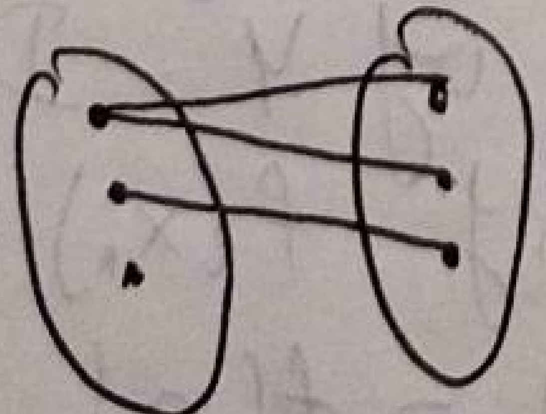


Not a function

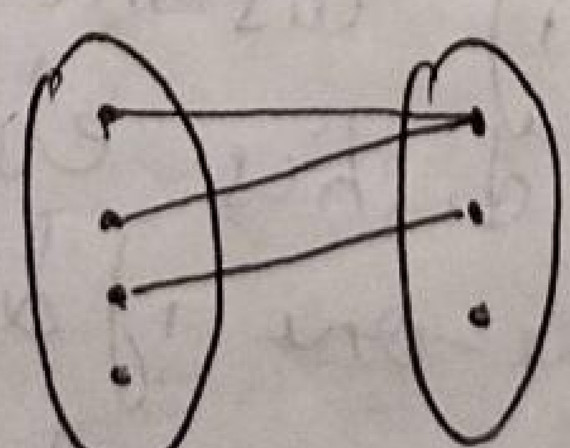
(iv) both one-one & onto (bijective) :-



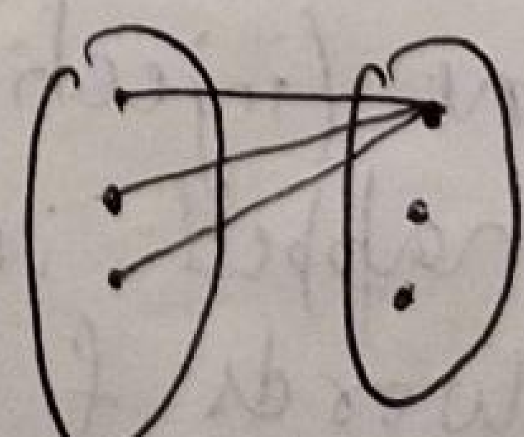
Q



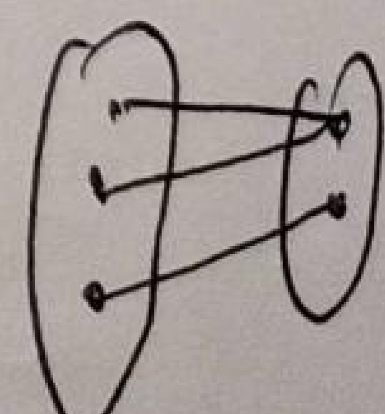
X



X



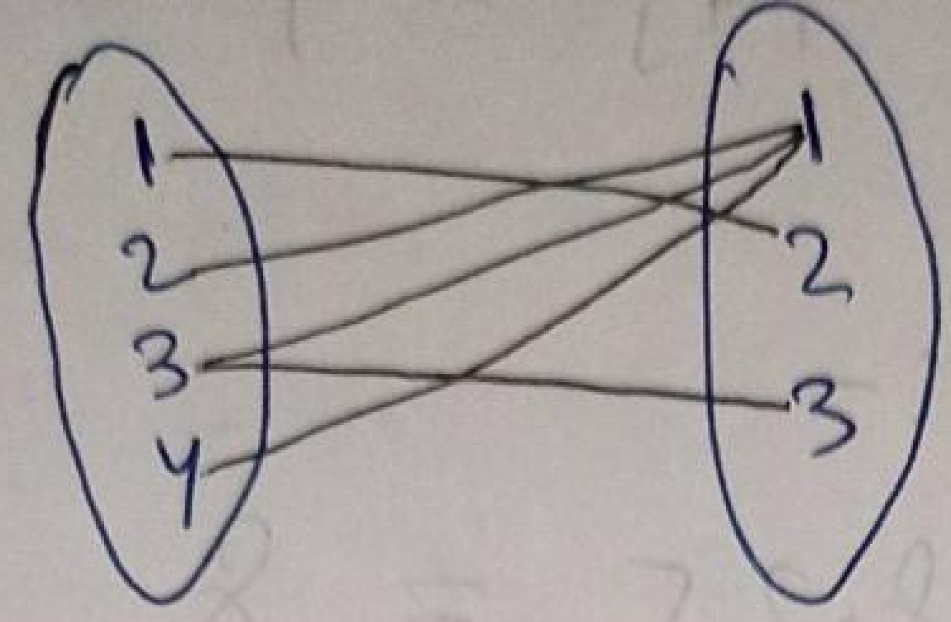
many one-into



many-one onto

Q. Determine whether the given relation is function or not. $R = \{(1,2), (2,1), (3,1), (4,1) (3,3)\}$

Sol →



⇒ Not a function.

Classification of function :-

Algebric function :- A function made up of finit terms and has operations $+, -, *, /$ is called algebric function.

Rational function :- $F(n) = \frac{f(n)}{g(n)}$ form.

Irrational function → Not rational one.

Transcendental function :- which are not algebric and has logarithmic, trigonometric and exponential terms.

$$f(n) = n^2 + \sin n$$

$$f(n) = n^2 + \log_e n + e^x.$$

Identity function :- function, $f: A \to A$ is called Identity function.

* Floor & Ceiling function :- floor $= \lfloor n \rfloor$, Ceiling $= \lceil x \rceil$

$$\lfloor x \rfloor = n \text{ for } n < n < n+1$$

$$\lceil x \rceil = n+1 \text{ for } n < n < n+1$$

E.g. $\lfloor 4.37 \rfloor = 4$ | $\lceil 4.37 \rceil = 5$
$\lfloor -4.37 \rfloor = -5$ | $\lceil -4.37 \rceil = -4$
$\lfloor 4 \rfloor = 4$ | $\lceil 4 \rceil = 4$

Integer value function :-
integer value of $8.25 = 8$
$-8.25 = -8$
$8 = 8$

Q Determine whether each of the following function is injective and/or surjective
(i) $f: Z^+ \to Z^+$ s.t $f(n) = n^2 + 2$

Sol → One-one :- ~~$f(x_1) = f(x_2)$~~
$\Rightarrow$ Let $f(x_1) = f(x_2)$
$\Rightarrow n_1^2 + 2 = n_2^2 + 2$
$\Rightarrow x_1^2 - x_2^2 = 0$
$\Rightarrow (x_1 - x_2)(x_1 + x_2) = 0$
$\Rightarrow (x_1 - x_2) = 0$ or $(x_1 + x_2) = 0$
Since $x_1 + x_2 = 0$ is not possible
So, $x_1 - x_2 = 0$
$\Rightarrow x_1 = x_2$
$\therefore f$ is one-one

Onto :- Let $y \in Y$, s.t $f(x) = y$
$x^2 + 2 = y$
$x = \sqrt{y-2} \notin Z^+$

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$\Rightarrow$ Not an onto function.

(ii) $f: R \to R$ s.t $f(x) = -4x^2 + 12x - 9$.

Sol $\to$ Let $f(x_1) = f(x_2)$

$\Rightarrow -4x_1^2 + 12x_1 - 9 = -4x_2^2 + 12x_2 - 9$

$\Rightarrow -(2x_1 - 3)^2 = -(2x_2 - 3)^2$

$f(1) = -1$

$f(2) = -1$

$f(1) = f(2)$

$1 \neq 2$

$\Rightarrow$ Not one-one

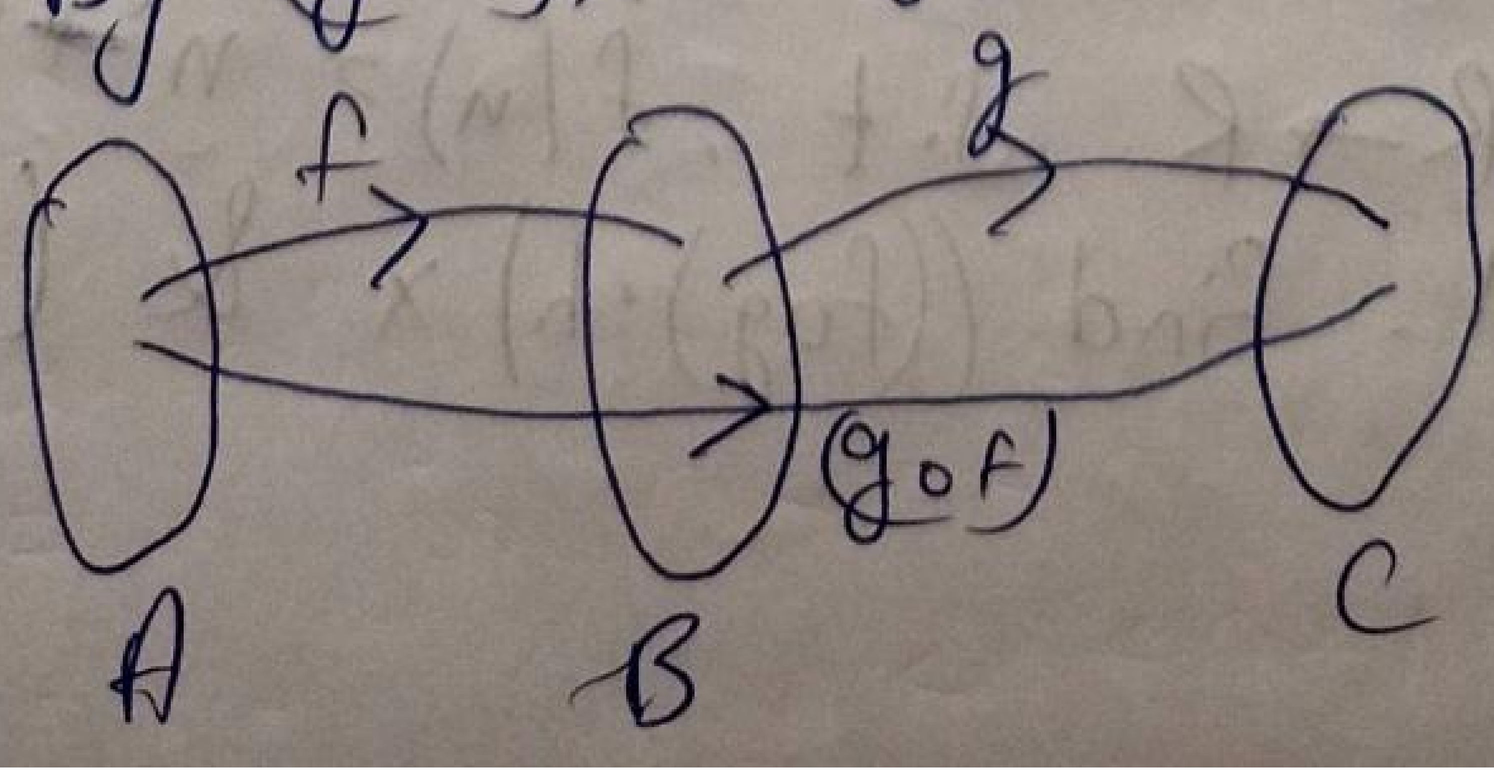
Onto :- $y = -(2x-3)^2$

$\Rightarrow x = \dfrac{\sqrt{-y} + 3}{2} \notin R$

$\Rightarrow$ Not Onto.

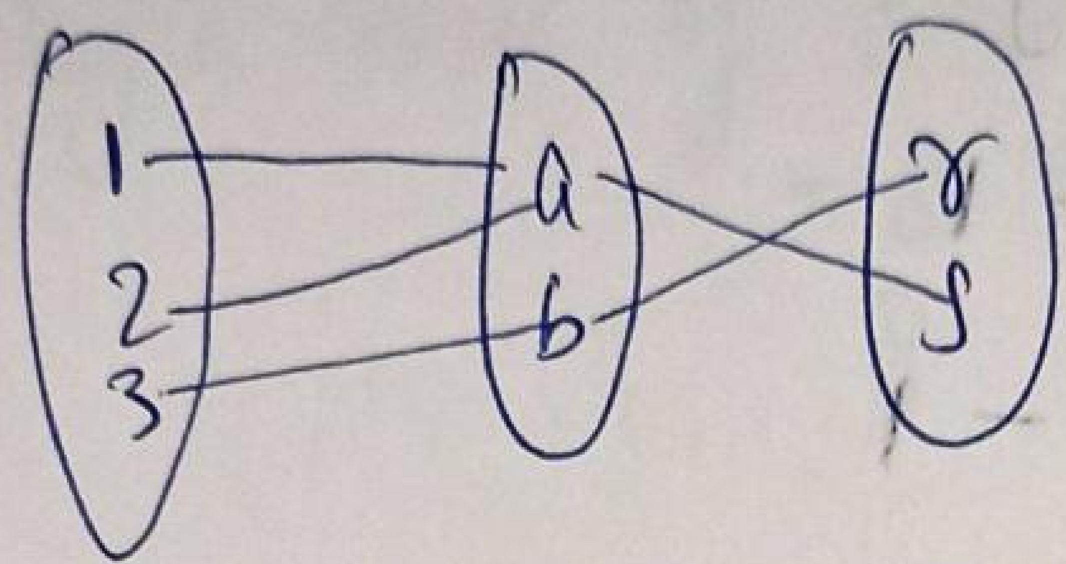
Composition of function :-

If $f: A \to B$ and $g: A \to C$, then the composition of f and g is the new function from $A \to C$ denoted by $(g \circ f)x = g(f(x))$



Q Let $A = \{1,2,3\}$, $B = \{a,b\}$, $C = \{r,s\}$ &
$f: A \to B$ defined by $f(1) = a$, $f(2) = a$, $f(3) = b$.
$g: B \to C$ defined by $g(a) = s$, $g(b) = r$.
$g \circ f: A \to C$. $g\circ f(1) = ?$ find $g\circ f(2)$, $g\circ f(3)$.

Sol →



$$g\circ f(1) = g[f(1)] = g[a] = s$$
$$g\circ f(2) = g[f(2)] = g[a] = s$$
$$g\circ f(3) = g[f(3)] = g[b] = r$$

Q. $f: R \to R$ & $g: R \to R$ is defined by $f(n) = x+2$ $\forall x \in R$ & $g(n) = n^2$ $\forall x \in R$. Then calculat $g\circ f$ and $f\circ g$.

Sol → $g\circ f = g[f(n)] = g[(n+1)] = (x+1)^2$

$f\circ g = f[g(n)] = f[n^2] = n^2 + 2$

$$g\circ f \neq f\circ g.$$

Q. $f, g, h: R \to R$ s.t. $f(n) = n^2 + 4n$, $g(n) = \frac{1}{x^2+1}$
$h(n) = n^4$. find $((f\circ g)\cdot h)x$ & $(f\circ(g\circ f))n$

Sol → $((f \circ g) \circ h) x = [f[g(x)]) \circ h]$

$f \circ g = f\left(\frac{1}{x^2+1}\right) \circ h$

$f \circ g = \left[\left(\frac{1}{x^2+1}\right)^3 - 4\left(\frac{1}{x^2+1}\right)\right]$

$((f \circ g) \circ h) = f \circ g (h(x))$

$= f \circ g (x^4)$

$= \left(\frac{1}{x^8+1}\right)^3 - 4\left(\frac{1}{x^8+1}\right)$

$(f \circ (g \circ h))_x$:-

$(g \circ h)_x = g(h(x)) = g(x^4)$

$= \frac{1}{x^8+1}$

$(f \circ (g \circ h)) = f \circ (g \circ h)$

$= f\left(\frac{1}{x^8+1}\right)$

$= \left(\frac{1}{x^8+1}\right)^3 - 4\left(\frac{1}{x^8+1}\right)$

$(f \circ g) \circ h = f \circ (g \circ h)$

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Properties :

1) Associative Law Property :-

Statement :- Composition of function is associative.

Viz: if $f : A \to B$, $g : B \to C$ and $h : C \to D$ are function then. $h \circ (g \circ f) = (h \circ g) \circ f$.

Proof :- Let $f : A \to B$ $g : B \to C$. $h : C \to D$

$g \circ f : A \to C$ $\quad h : C \to D$

$h \cdot (g \cdot f) = A \to D$.

$g : B \to C$ $\quad h : C \to D$

$h \cdot g : B \to D$. $\quad f : A \to B$.

$(h \cdot g) \cdot f = A \to D$.

Thus the domain & codomain of $h \cdot (g \cdot f)$ and those of $(h \cdot g) \cdot f$ are same.

Let $x \in A$, $y \in B$ and $z \in C$ s.t

$y = f(x)$. $z = g(y)$. $h(z) = a$.

LHS :- $\{h \cdot (g \cdot f)\}(x)$.

$$h \cdot (g \cdot f(x)) = h \cdot (g(f(x)))$$
$$= h \cdot (g(y)) = h(z)$$
$$= a \to ①$$

RHS : $\{(h \circ g) \cdot f\}(x)$

$\{h[g(x)] \circ f\}$

$\cancel{hf} \cdot (h \circ g)$

$$(h \circ g)\{f(x)\} = h(g\{f(x)\}\}$$
$$= h[g(y)]$$
$$= h(z) = a$$

LHS = RHS.

So. ~~Comparission~~ Composition of f^n is associative

2. When $f: A \to B$ and $g: B \to C$ are functions, then $g \circ f: A \to C$ is an injection, surjection or bijection according as f and g are injection, surjection or bijection.

Proof :- Injection :- Here we assume that $f: A \to B$ & $g: B \to C$ are injection & we have to prove that $g \circ f: A \to C$ is also injective.

$$[g \circ f](a_1) = [g \circ f](a_2) \quad a_1, a_2 \in A$$

$$g[f(a_1)] = g[f(a_2)]$$

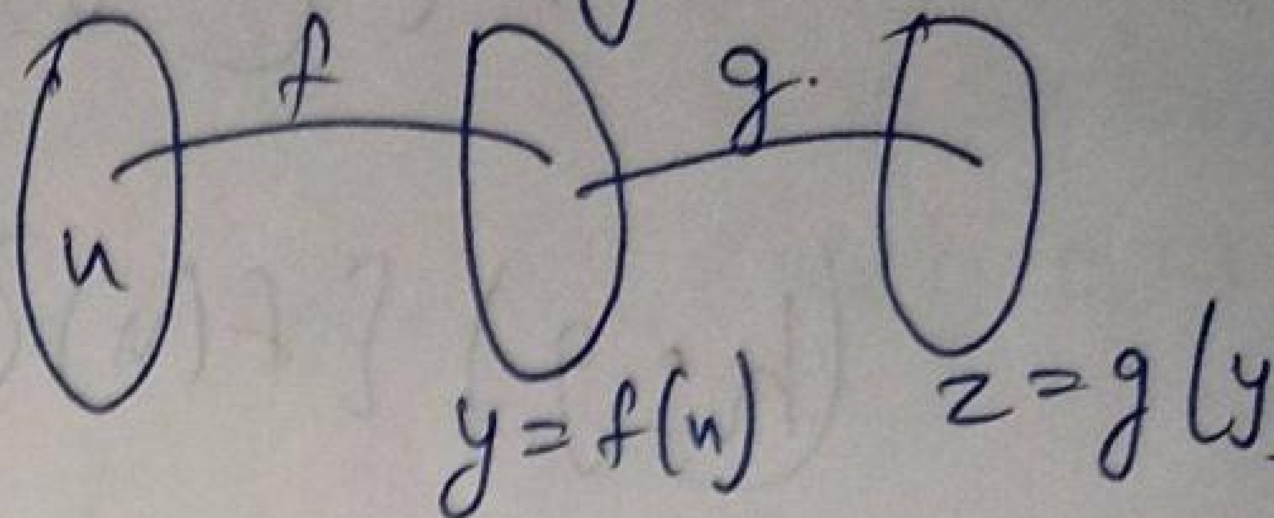
$$f(a_1) = f(a_2) \quad [\because g \text{ is injective}]$$

$$a_1 = a_2 \quad [\because f \text{ is injective}].$$

$\therefore g \circ f$ is injective

Onto :-

Here we assume that $f: A \to B$ & $g: B \to C$ are surjective & we have to prove that $g \circ f: A \to C$ is also surjective.



Since f is onto.

Let $x \in A$ s.t $f(x) = y$

Since g is onto. Let $y \in B$. s.t. $g(y) = z$.

$$(g \circ f)(x) = g[f(x))$$
$$= g(y)$$
$$= z.$$

$\therefore$ $g \circ f$ is onto.

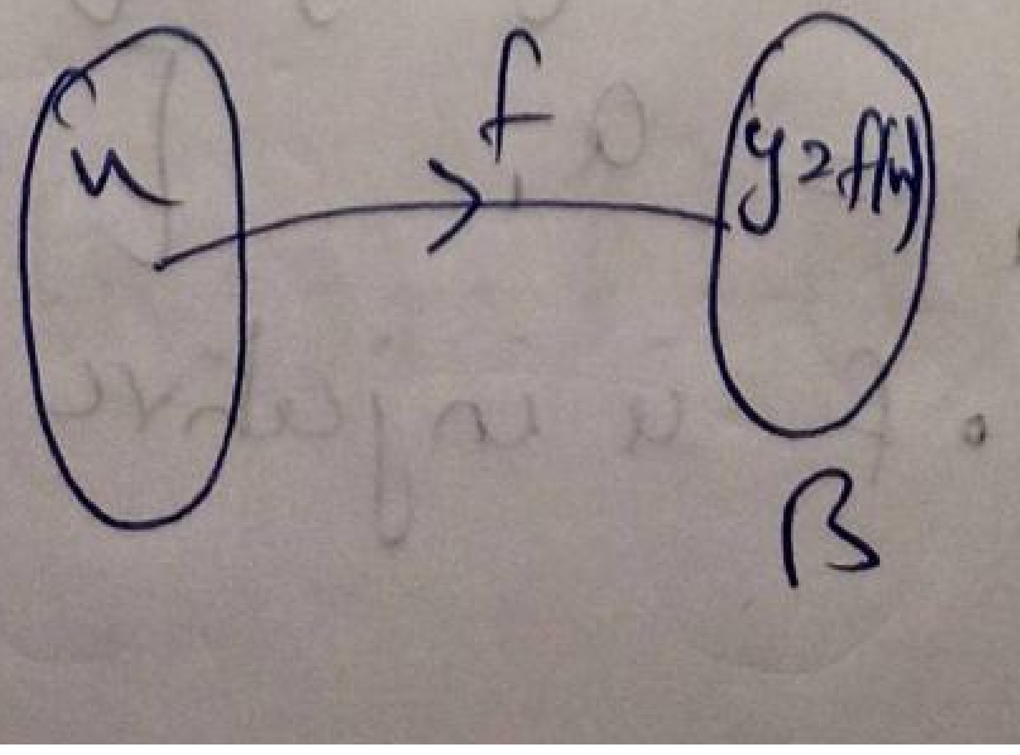
Since it is both injective & surjective. it is also bijective

3 Inverse of a function :-

If $f: A \to B$ & $g: B \to A$ then function g is called inverse of function f if.

$$g \circ f = I_A \text{ and } f \circ g = I_B.$$

Proof :- Let $x \in A$, & $y \in B$.



$(g \circ f)(x) = I_A(x)$.

$= g[f(x)] = x$ —— (1)

$(f \circ g)\{y\} = I_B\{y\}$

$f[g(y)] = y$ —— (2)

from (1) & (2), we observe that if $f(x) = y$, then $x = g(y)$ & vice versa.

Thus function $g : B \to A$ is called inverse of $f : A \to B$ if $x = g(y)$ whenever $y = f(x)$.

The inverse of f viz. g is denoted by f^{-1}.

Thus, if f^{-1} is the inverse of f then $x = f^{-1}(y)$ whenever $y = f(x)$. $\boxed{g^{-1}, \text{ the inverse of } g \text{ is } f.}$

4. The inverse of a function f, if exist is unique.

Proof :- Let g and h be two inverse of f then by definition $g \circ f = I_A$

$f \circ g = I_B$.

Also, $h \circ f = I_A$

$f \circ h = I_B$.

Now, $h = h \circ I_B = h \circ (f \circ g)$

$= (h \circ f) \circ g$

$= I_A \circ g$

$h = g$

which is the contradiction. So the inverse is unique.

5. If $f : A \to B$ and $g : B \to C$ are invertible functions, then $g \cdot f : A \to C$ is also invertible $(g \cdot f)^{-1} = f^{-1} \cdot g^{-1}$.

Proof :- Do same like (2) to prove invertible. After that we have to prove $(g \cdot f)^{-1} = \underline{f^{-1} \cdot g^{-1}}$.

Let $f : A \to B$ & $g : B \to C$.

$g \cdot f : A \to C$

$(g \cdot f)^{-1} : C \to A$

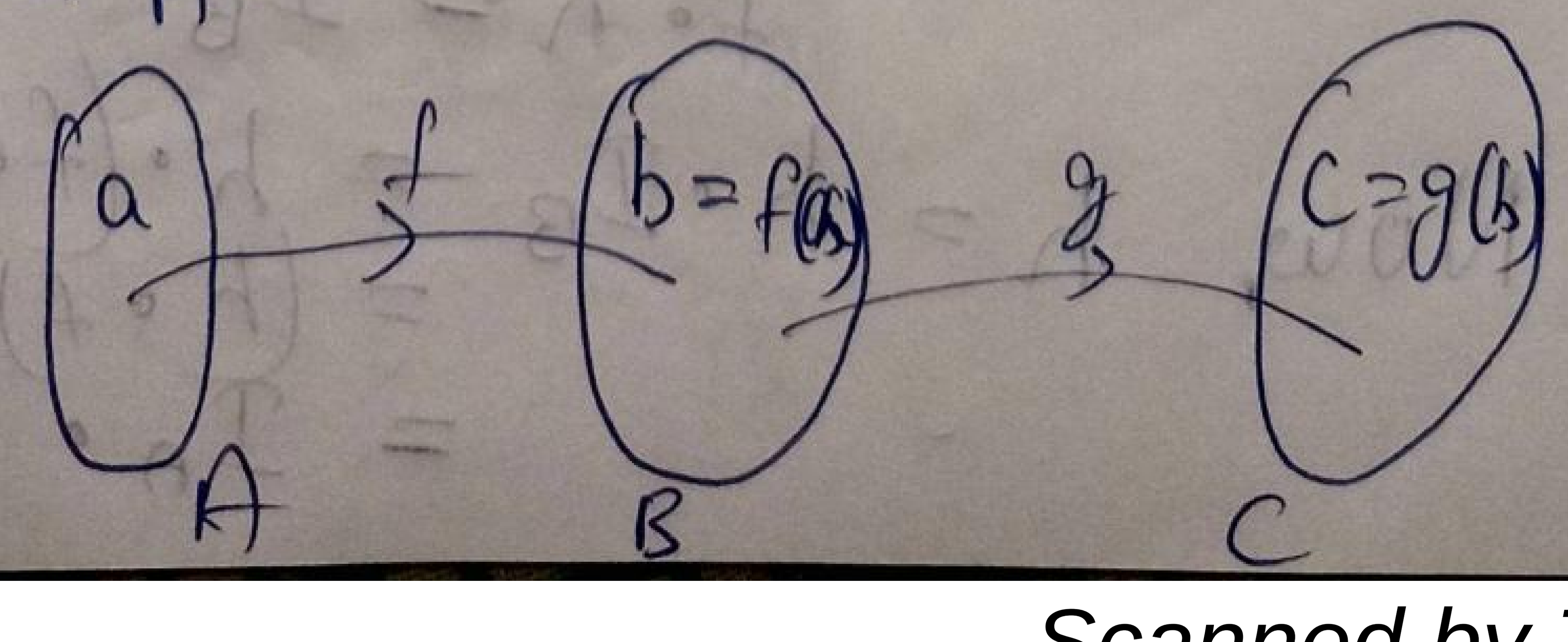
$f : A \to B \Rightarrow f^{-1} : B \to A$

$g : B \to C \Rightarrow g^{-1} : C \to B$.

$g^{-1} : C \to B \quad f^{-1} : B \to A$

$f^{-1} \cdot g^{-1} : C \to A$

Thus Both $(g \cdot f)^{-1}$ & $f^{-1} \cdot g^{-1}$ are function from $C \to A$.



Let $a \in A$, $b \in B$ & $c \in C$ s.t
$f(a) = b$ & $g(b) = c$.
$\Rightarrow f^{-1}(b) = a$ & $g^{-1}(c) = b$.

LHS $= (g \circ f)\{a\} = g(f(a)) = g(b) = c$.

$(g \circ f)a = c$.

$(g \circ f)^{-1}(c) = a$ -- (i)

RHS :- $f^{-1} \cdot g^{-1}\{c\} \Rightarrow f^{-1}\{g^{-1}(c)\}$

$= f^{-1}(b)$

$= a$ -- (2)

from (1) & (2), $(g \circ f)^{-1} = f^{-1} \cdot g^{-1}$.

Q. If $A = \{x \in R : x \neq \frac{1}{2}\}$ and $f : A \to R$ is defined by $f(x) = \frac{4x}{2x-1}$.

(i) find the range of f
(ii) Show that f is invertible
(iii) find dom(f^{-1}), range (f^{-1}) & formula for f^{-1}.

Sol $\rightarrow$ $f(x) = \frac{4x}{2x-1}$

$y = \frac{4x}{2x-1}$

$(2x-1)y = 4x$

$$2xy - y = 4x$$

$$2xy - 4x = y.$$

$$x = \frac{y}{2y-4} = \frac{1}{2-\frac{4}{y}}$$

If $y = 2$, $n = \infty$

$y \neq 2$, $n \neq \infty$

$$y = \{y \in R : y \neq 2\}.$$

(ii): One-one :- $f(x_1) = f(x_2)$

$$\frac{4x_1}{2x_1-1} = \frac{4x_2}{2x_2-1}.$$

$$\Rightarrow 8x_1x_2 - 4x_1 = 8x_1x_2 - 4x_2$$

$$\Rightarrow x_1 = x_2$$

$\Rightarrow$ One-one.

Onto :- $y = f(x).$

$$y = \frac{4x}{2x-1}. \quad x = \frac{4}{2y-4}$$

$$\Rightarrow x = \frac{1}{2-\frac{4}{y}}.$$

$\Rightarrow \{y \in R : y \neq 2\}$

$\Rightarrow$ onto.

$\Rightarrow$ Invertible

(iii) $y = f(n)$

$$f^{-1}(y) = n = \frac{y}{2y-4}$$

domain of f^{-1} = Range of $f = R - \{2\}$

~~do~~ Range of f^{-1} = Domain of f = $\{x \in R, n \neq \frac{1}{2}\}$

~~Q If A = {n ∈ R, n~~

Q. $f : Z \to N$ is defined by

$$f(n) = \begin{cases} 2n-1 & \text{if } n \geq 0 \\ -2x & \text{if } n \leq 0 \end{cases}$$

(a) Prove that f is one-one & onto.

(b) Determine f^{-1}.

Sol → For $f(n)$ is odd:

$$2x_1 - 1 = 2x_2 - 1$$

$$x_1 = x_2$$

$\Rightarrow$ one-one

For $f(n)$ is even:

$$-2x_1 = -2x_2$$

$$x_1 = x_2$$

$\Rightarrow$ one-one

For $f(n)$ is odd

$$y = 2x - 1$$

$$x = \frac{y+1}{2} \quad \Rightarrow y \in N$$

$\Rightarrow$ onto.

For $f(n)$ is even.

$y = -2x$

$x = \frac{-y}{2}$

$x \in R$.

$\Rightarrow$ onto.

So, f is both one-one & onto.

(ii) f^{-1} :- $n = \frac{y+1}{2}$ if y is odd

$n = \frac{-y}{2}$ if y is even.

* Partition of a Set :-

If S is non-empty set, a collection of disjoint non-empty subset of S whose union is S is called Partition.

In other words, the collection of subset A_i is called Partition of S if (a) $A_i \neq \phi$

(b) $A_i \cap A_j = \phi$ for $\forall i \neq 1$

(c) $\cup A_i = S$.