



SRM Institute of Science and Technology
Ramapuram Campus
Department of Mathematics
18MAB101T – Calculus And Linear Algebra

Year/Sem: I/I

Branch: Common to ALL B.Tech. except B.Tech. (Business Systems)

Unit – I

MATRICES

Part – A

1.	The sum of the eigen values of $A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$ is	1 Mark	
	(a) 2 (b) 4 (c) -3 (d) 0	Ans (a)	(CLO – 1Apply)
2.	The eigen values of A^{-1} , if $A = \begin{pmatrix} 1 & 5 & -1 \\ 0 & 3 & 2 \\ 0 & 0 & 4 \end{pmatrix}$ are	1 Mark	
	(a) 2, 3, 4 (b) 2, 5, -1 (c) 0, 0, 0 (d) $1, \frac{1}{3}, \frac{1}{4}$	Ans (d)	(CLO -1Apply)
3.	If two eigen values of $A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$ are 3 and 15, then the third eigen value is _____.	1 Mark	
	(a) 1 (b) 0 (c) 2 (d) 3	Ans (b)	(CLO -1 Apply)
4.	If -1, -1, 2 are the eigen values of a matrix $A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$, then the eigen values of A^T are	1 Mark	
	(a) -1, -1, 2 (b) 1, 1, 1/2 (c) 1,1,4 (d) -1,-1,-2	Ans (a)	(CLO - 1 Apply)

	(a) first (b) second (c) third (d) fourth	Ans (b)	(CLO - 1 Remember)
11.	A square matrix A is called orthogonal if		1 Mark
	(a) $A = A^2$ (b) $A = A^{-1}$ (c) $A^T = A^{-1}$ (d) $AA^{-1} = I$	Ans (c)	(CLO - 1 Remember)
12.	The sum of the squares of the eigen values $\begin{pmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{pmatrix}$ is		1 Mark
	(a) 10 (b) 38 (c) 45 (d) 20	Ans (b)	(CLO - 1 Apply)
13.	All the eigen values of a symmetric matrix with real elements are		1 Mark
	(a) distinct (b) real (c) equal (d) conjugate complex numbers	Ans (b)	(CLO - 1 Remember)
14.	If the sum of two eigen values and trace of a 3×3 matrix A are equal, then the value of $\det(A)$ is		1 Mark
	(a) 0 (b) 1 (c) -1 (d) 2	Ans (a)	(CLO - 1 Apply)
15.	If the canonical form of a quadratic form is $-y_1^2 + y_2^2 + 2y_3^2$, then the signature of the quadratic form is		1 Mark
	(a) 2 (b) 1 (c) 0 (d) 3	Ans (b)	(CLO - 1 Apply)
16.	Find the sum and product of the eigen values of $A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$		1 Mark

23.	Find the eigen values of A^2 if $A = \begin{pmatrix} 3 & 2 & 4 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{pmatrix}$.		1 Mark
	(a) 6, 4, 10 (b) 9, 4, 25 (c) 9, 2, 5 (d) 3, 2, 5	Ans (b)	(CLO - 1 Apply)
24.	Find the nature of the quadratic form $2x^2 + 3y^2 + 2z^2 + 2xy$		1 Mark
	(a) Positive definite (b) Negative definite (c) Positive semi-definite (d) Indefinite	Ans (a)	(CLO – 1 Apply)
25.	Find the eigen values of A^{10} if $A = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$		1 Mark
	(a) $1, 3^{10}$ (b) 1, 3 (c) $3^2, 1^{10}$ (d) 1, 10	Ans (a)	(CLO - 1 Apply)
26.	Find the eigen values of the matrix $A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$.		1 Mark
	(a) 1, -3 (b) 3, 1 (c) 2, 1 (d) 1, 2	Ans (b)	(CLO - 1 Apply)
27.	If the sum of two eigen values and trace of a 3×3 matrix A are equal, then the value of determinant of A is		1 Mark
	(a) 0 (b) 1 (c) -1 (d) 2	Ans (a)	(CLO - 1 Apply)
28.	Find the eigen values of the matrix $A^3 = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$.		1 Mark

	(a) 1, -3 (b) 3, 1 (c) 1, 27 (d) 1, -9	Ans (c)	(CLO - 1 Apply)
29.	The eigen values of the matrix $A^{-1} = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$		1 Mark
	(a) 1, 1/3 (b) 3, 1 (c) -2, 1 (d) 1, 2	Ans (a)	(CLO - 1 Apply)
30.	Find the sum and product of the eigen values of the matrix $A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$.		1 Mark
	(a) 4, 3 (b) 3, 1 (c) -2, 1 (d) 1, 2	Ans (a)	(CLO - 1 Apply)
31.	Find the eigen values of $A = \begin{pmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{pmatrix}$.		1 Mark
	(a) 1,3,-4 (b) 1,-3,-4 (c) 1,-3,4 (d) -1,3,-4	Ans (a)	(CLO - 1 Apply)
32.	Two eigen values of $A = \begin{pmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -5 & -2 \end{pmatrix}$ are equal and they are double the third. Find them.		1 Mark
	(a) 1, 2, 2 (b) 2, 1, 1 (c) 2, 0, 1 (d) 1, 2, 3	Ans (a)	(CLO - 1 Apply)
33.	The eigen values of a diagonal matrix are the _____ elements of the matrix.		1 Mark

	(a) diagonal (b) upper triangular (c) zero (d) unity	Ans (a)	(CLO - 1 Remember)
34.	Cayley-Hamilton theorem states that “Every _____ matrix satisfies its own characteristic equation”.		1 Mark
	(a) square (b) column (c) row (d) zero	Ans (a)	(CLO - 1 Remember)
35.	Find rank and index of the QF whose canonical form is $3x^2 - 3y^2$.		1 Mark
	(a) 2, 1 (b) 1, 2 (c) 0, 1 (d) 0, 2	Ans (a)	(CLO – 1 Apply)
36.	Write the Q.F. defined by the matrix $A = \begin{pmatrix} 6 & 1 & -7 \\ 1 & 2 & 0 \\ -7 & 0 & 1 \end{pmatrix}$		1 Mark
	(a) $6x^2 + 2y^2 + z^2 + 2xy - 14xz$ (b) $6x + y^2 + 6z^2 + xy - 7xz$ (c) $6x^2 + 2y^2 + z^2 + 2xy + 14xz$ (d) $6x + y^2 + 6z^2 + xy - 14xz$	Ans (a)	(CLO -1 Apply)



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Unit – I

MATRICES

Part – B

1. Find the characteristic equation of the matrix $A = \begin{pmatrix} 1 & 2 \\ 0 & 2 \end{pmatrix}$.

(A) $\lambda^2 - 3\lambda + 2 = 0$ (B) $\lambda^2 + 3\lambda + 2 = 0$

(C) $\lambda^2 - 3\lambda - 2 = 0$ (D) $\lambda^2 + 3\lambda - 2 = 0$

Solution: Let $A = \begin{pmatrix} 1 & 2 \\ 0 & 2 \end{pmatrix}$. Its characteristic equation is $\lambda^2 - S_1\lambda + S_2 = 0$ where $S_1 =$
sum of the main diagonal elements $= 1 + 2 = 3$,

$S_2 = \text{Determinant of } A = |A| = 1(2) - 2(0) = 2$

Therefore, the characteristic equation is $\lambda^2 - 3\lambda + 2 = 0$ (**Option A**)

2. Find the characteristic equation of $A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$.

(A) $\lambda^3 - 18\lambda^2 + 45\lambda = 0$ (B) $\lambda^3 - 28\lambda^2 + 45\lambda = 0$

(C) $\lambda^3 - 18\lambda^2 + 35\lambda = 0$ (D) $\lambda^3 - 18\lambda^2 - 45\lambda = 0$

Solution: Its characteristic equation is $\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$, where $S_1 =$
sum of the main diagonal elements $= 8 + 7 + 3 = 18$, $S_2 =$
Sum of the minors of the main diagonal elements $= \begin{vmatrix} 7 & -4 \\ -4 & 3 \end{vmatrix} + \begin{vmatrix} 8 & 2 \\ 2 & 3 \end{vmatrix} + \begin{vmatrix} 8 & -6 \\ -6 & 7 \end{vmatrix} = 5 +$
 $20 + 20 = 45$, $S_3 = \text{Determinant of } A = |A| = 8(5) + 6(-10) + 2(10) = 40 - 60 + 20 = 0$

Therefore, the characteristic equation is $\lambda^3 - 18\lambda^2 + 45\lambda = 0$ (**Option A**)

3. Find the eigen values of the matrix $A = \begin{pmatrix} 1 & 1 \\ 3 & -1 \end{pmatrix}$.

Solution: Let $A = \begin{pmatrix} 1 & 1 \\ 3 & -1 \end{pmatrix}$ which is a non-symmetric matrix.

The characteristic equation of A is $\lambda^2 - S_1\lambda + S_2 = 0$ where $S_1 = \text{sum of the main diagonal elements} = 1 - 1 = 0$,

$$S_2 = \text{Determinant of } A = |A| = 1(-1) - 1(3) = -4$$

Therefore, the characteristic equation is $\lambda^2 - 4 = 0$ i.e., $\lambda^2 = 4$ or $\lambda = \pm 2$

Therefore, the eigen values are 2, -2. **(Option A)**

4. Find the sum and product of the eigen values of the matrix $A = \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$.

- (A) $-3, 4$ (B) $-3, -4$
 (C) $3, 4$ (D) $-3, -4$

Solution: Sum of the eigen values = Sum of the main diagonal elements = -3

$$\text{Product of the eigen values} = |A| = -1(1-1) - 1(-1-1) + 1(1-(-1)) = 2 + 2 = 4$$

(Option A)

5. Find the sum and product of eigen values of the matrix A^T where $A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$.

Solution: Since matrix A is symmetric , A and A^T have same eigen values.

$$\text{Sum of Eigen value of } A^T = \text{trace}(A) = 8+7+3=18$$

Product of Eigen value of $A^T = |A| = 8(5) + 6(-10) + 2(10) = 40 - 60 + 20 = 0$ (**Option A**)

6. If 1, 1, 5 are the eigen values of $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$, find the eigen values of $5A$.

- (A) 5, 5, 2 (B) 5, 5, 25 (C) 2, 3, 2 (D) 7, 8, 7

Solution: By the property “If $\lambda_1, \lambda_2, \lambda_3$ are the eigen values of A , then $k\lambda_1, k\lambda_2, k\lambda_3$ are the eigen values of kA , the eigen values of $5A$ are 5(1), 5(1), 5(5) ie., 5,5,25. (Option B)

7. Find the eigen values of the matrix $2A^{-1}$ where $A = \begin{pmatrix} 3 & 8 & 9 \\ 0 & 1 & 3 \\ 0 & 0 & -2 \end{pmatrix}$.

- (A) $\frac{2}{3}, 2, -1$ (B) $\frac{1}{3}, 2, -4$ (C) $\frac{2}{3}, 2, 1$ (D) $\frac{2}{3}, 1, -2$

Solution: Since given matrix is triangular matrix, the Eigen values are its diagonal elements.

$$\therefore \lambda_1 = 3, \lambda_2 = 1, \lambda_3 = -2$$

Eigen values of $2A^{-1}$ are $\frac{2}{3}, 2, -1$ (Option A)

8. If the sum of two eigen values and trace of a 3×3 matrix A are equal, find the value of $|A|$.

- (A) 5 (B) 25 (C) 2 (D) 0

Solution: Sum of the eigen values $= \lambda_1 + \lambda_2 + \lambda_3 =$ sum of the diagonal elements

Given $\lambda_1 + \lambda_2 =$ trace of A .

i.e., $\lambda_1 + \lambda_2 = \lambda_1 + \lambda_2 + \lambda_3$

Therefore $\lambda_3 = 0$. Then $|A| =$ Product of Eigen values $= \lambda_1 \lambda_2 \lambda_3 = 0$ (Option D)

9. Write the matrix corresponding to the quadratic form $x^2 + 2yz$.

- (A) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 1 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 2 & 0 \end{bmatrix}$ (C) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ (D) $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 2 & 0 \end{bmatrix}$

Solution: Given $\mathbf{X}^T \mathbf{A} \mathbf{X} = x^2 + 2yz$

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \text{ (Option C)}$$

10. If 2, 3, -1 are the eigen values of 3×3 matrix, find rank, index and signature of the quadratic form.

- (A) 5, 5, 2 (B) 5, 5, 25 (C) 3, 2, 1 (D) 1, 2, 3

Solution: Rank (r) = number of non zero terms in canonical form = 3

Index (p) = Number of positive terms in canonical form = 2

Signature (s) = Difference between number of positive terms and negative terms

$$\begin{aligned} &= 2p - r \\ &= 4 - 3 \\ &= 1 \text{ (Option C)} \end{aligned}$$

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Year/Sem: I/I

Part-A

Branch: Common to All

Unit -II

Functions of several variables

1.	If u and v are functionally dependent then their Jacobian value is	1 Mark	
	a)zero b) one c) non-zero d)greater than zero	Ans (a)	(CLO-2 / Remember)
2.	If $rt - s^2 < 0$ then the point is	1 Mark	
	a)maximum point b) minimum point c) saddle point d) fixed point	Ans (c)	(CLO-2 / Remember)
3.	If $z = x^2 + y^2 + 3xy$ then $\frac{\partial z}{\partial x} =$	1 Mark	
	a)2y+3x b) 3y c) 2x+3y d) 2x	Ans (c)	(CLO-2 / Apply)
4.	If $u = \sin^{-1}\left(\frac{x^2 + y^2}{x - y}\right)$ is a homogeneous function of degree	1 Mark	
	a) 2 b) 3 c)1 d) 4	Ans (c)	(CLO-2 / Apply)
5.	If $f(x,y)$ is an implicit function then $\frac{dy}{dx} =$	1 Mark	

	a) $-\frac{\left(\frac{\partial f}{\partial x}\right)}{\left(\frac{\partial f}{\partial y}\right)}$ b) $\frac{\left(\frac{\partial f}{\partial x}\right)}{\left(\frac{\partial f}{\partial y}\right)}$ c) $\frac{\left(\frac{\partial f}{\partial y}\right)}{\left(\frac{\partial f}{\partial x}\right)}$ d) $-\frac{\left(\frac{\partial f}{\partial y}\right)}{\left(\frac{\partial f}{\partial x}\right)}$	Ans (a)	(CLO-2 / Remember)
6.	If $x = r \cos \theta$ and $y = r \sin \theta$ then $\frac{\partial(x, y)}{\partial(r, \theta)} =$		1 Mark
	a) r b) r^2 c) $2r$ d) $1/r$	Ans (a)	(CLO-2 / Apply)
7.	If u is a homogeneous function of degree n then $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} =$		1 Mark
	a) n b) nu c) u d) n^2u	Ans (b)	(CLO-2 / Remember)
8.	If $x = u^2 - v^2$ and $y = 2uv$ then $\frac{\partial(x, y)}{\partial(u, v)} =$		1 Mark
	a) $u^2 + v^2$ b) $2(u^2 + v^2)$ c) $4(u^2 + v^2)$ d) $4v^2$	Ans (c)	(CLO-2 / Apply)
9.	If $J_1 = J\left(\frac{x, y}{u, v}\right)$ and $J_2 = J\left(\frac{u, v}{x, y}\right)$ then $J_1 J_2 =$		1 Mark
	a) 0 b) -1 c) 2 d) 1	Ans (d)	(CLO-2 / Remember)
10.	The point (0,0) for $f(x, y) = x^3 + y^3 - 3axy$ is		1 Mark
	a) maximum point b) minimum point	Ans (c)	(CLO-2 / Apply)

	c) saddle point d) fixed point		
11.	If $f(x, y) = x^2 y + \sin y + e^x$ then $f_x(1, \pi)$ is	1 Mark	
	a) $2\pi - e$ b) 2π c) $2\pi + e$ d) 0	Ans (c)	(CLO-2 / Apply)
12.	The stationary points of $x^2 + y^2 + 6x + 12$ are	1 Mark	
	a) (-3,0) b) (0,3) c) (0, -3) d) (3,0)	Ans (a)	(CLO-2 / Apply)
13.	The stationary points for $f(x, y) = \sin x + \sin y + \sin(x + y)$ are	1 Mark	
	a) $\left(\frac{\pi}{2}, \frac{\pi}{3}\right)$ b) $\left(\frac{\pi}{3}, \frac{\pi}{3}\right)$ c) $\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$ d) $\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$	Ans (b)	(CLO-2 / Apply)
14.	If $u = x^2 - y^2$ and $v = 2xy$ then $J\begin{pmatrix} x, y \\ u, v \end{pmatrix} X J\begin{pmatrix} u, v \\ x, y \end{pmatrix} =$	1 Mark	
	a) 0 b) -1 c) 2 d) 1	Ans (d)	(CLO-2 / Apply)
15.	If $f(x, y) = e^x \cos y$ then $f_{xy}(0,0)$ is	1 Mark	
	a) 0 b) -1 c) 2 d) 1	Ans (a)	(CLO-2 / Apply)
16.	If $u = ax^2 + by^2 + 2hxy$ then $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} =$	1 Mark	

	a) u b) 2u c) 3u d) 4u	Ans (b)	(CLO-2 / Apply)
17.	If $x^y = y^x$, then $\frac{dy}{dx} =$		1 Mark
	a) $(x \log y - y)y/x(y \log x - x)$ b) $(x \log x - x)/x(y \log y - y)$ c) $(x \log x - y)y/(y \log x - x)$ d) Does not exists	Ans (a)	(CLO-2 / Apply)
18.	If $f(x, y) = e^{xy}$ then $f_{yyy}(1,1)$ is		1 Mark
	a) -e b) $\frac{1}{e}$ c) e d) $-\frac{1}{e}$	Ans (c)	(CLO-2 / Apply)
19.	If $z = \log(x^2 + y^2 + xy)$ then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} =$		1 Mark
	a) 1 b) 2 c) 0 d) 4	Ans (b)	(CLO-2 / Apply)
20.	If $f(x, y) = \tan^{-1}(y/x)$ then $f_x(1,1)$ is		1 Mark
	a) 1/2 b) -1/2 c) 2 d) 1	Ans (b)	(CLO-2 / Apply)
21.	If $V = x/y$, then $x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y} =$		1 Mark
	(a) 2V (b) 3V (c) 4V (d) 0V	Ans (d)	(CLO-2 / Apply)

22.	Saddle points are -----	1 Mark	
	(a) a minimum (b) a maximum (c) neither a minimum nor a maximum (d) None	Ans (c)	(CLO-2 / Remember)
23.	If $u = x+y/1-xy$, $v = \tan^{-1}x + \tan^{-1}y$ then the functional relationship between u and v is		1 Mark
	a) $u = \tan v$ (b) $v = \tan u$ (c) $x = \tan y$ (d) $y = \tan x$	Ans (a)	(CLO-2 / Apply)
24.	Lagrange's method of undetermined multipliers is to find the maximum or minimum value of a function of		1 Mark
	a) Two variables (b) Three or more variables (c) One variable (d) None	Ans (b)	(CLO-2 / Remember)
25.	The condition for a function $f(x,y)$ to have a maximum value is that	1 Mark	
	(a) $rt-s^2$ (b) $rt-s^2 > 0, r > 0$ or $s > 0$ (c) $rt-s^2 > 0, r < 0$ or $s < 0$ (d) $rt-s^2 = 0, r > 0$	Ans (C)	(CLO-2 / Remember)

	(a) maximum points (c) saddle points	(b) minimum points (d) none	Ans (C)	(CLO-2 / Remember)
32.	If $u = xe^y \sin x$ $v = xe^y \cos x$ $w = x^2 e^{2y}$ then the functional relationship is			1 Mark
	(a) $u^2 + w^2 = v$ (c) $x^2 + y^2 = u$	(b) $v^2 + w^2 = u$ (d) $u^2 + v^2 = w$	Ans (d)	(CLO-2 / Apply)
33.	In PDE, a real function depends			1 Mark
	(a) One independent variable independent variable (c) No independent variable	(b) More than one independent variable (d) None	Ans (b)	(CLO-2 / Remember)
34.	If $z = x^2 + y^2 + 2xy$ then $\frac{\partial z}{\partial x}$ is			1 Mark
	(a) $2x^2 + 2y$ (d) $2y$	(b) $2x + 2y$ (c) $2x - 2y$	Ans (b)	(CLO-2 / Apply)
35.	If $x = r \cos \theta$ $y = r \sin \theta$ then $\frac{\partial(r, \theta)}{\partial(x, y)}$			1 Mark
	(a) 0 (b) 1 (c) r (d) $1/r$		Ans (d)	(CLO-2 / Apply)
36.	If $u = \frac{x}{y} + \frac{y}{z} + \frac{z}{x}$, then $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} =$ _____.			1 Mark

	(a) 1	(b) 3 u	(c) -1	(d) 0	Ans (d)	(CLO-2 / Apply)
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Unit – II

FUNCTIONS OF SEVERAL VARIABLES

Part – B

- 1. If $u = (x - y)(y - z)(z - x)$, then find $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$.**
- (A) 0 (B) 1 (C) 2 (D) 3

Solution:

$$\text{Given } u = (x - y)(y - z)(z - x)$$

$$\begin{aligned}\frac{\partial u}{\partial x} &= (y - z)[(x - y)(-1) + (z - x)(1)] \\ &= -(x - y)(y - z) + (y - z)(z - x) \\ \frac{\partial u}{\partial y} &= (z - x)[(x - y)(1) + (y - z)(-1)] \\ &= (x - y)(z - x) - (y - z)(z - x) \\ \frac{\partial u}{\partial z} &= (x - y)[(y - z)(1) + (z - x)(-1)] \\ &= (x - y)(y - z) - (x - y)(z - x) \\ \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} &= 0 \quad (\text{Option A})\end{aligned}$$

- 2. If $x = r \cos \theta, y = r \sin \theta$ find $\frac{\partial x}{\partial r}, \frac{\partial y}{\partial \theta}$.**
- (A) $\cos \theta, \sin \theta$ (B) $\cos \theta, r \cos \theta$ (C) $r \cos \theta, \sin \theta$ (D) r, θ

Solution:

$$\begin{aligned}\frac{\partial x}{\partial r} &= \cos \theta \\ \frac{\partial y}{\partial \theta} &= r \cos \theta \quad (\text{Option B})\end{aligned}$$

3. If $f(x, y) = \sin\left(\frac{x}{y}\right)$, then find $x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y}$.
- (A) 0 (B) 1 (C) 2 (D) 3

Solution:

$$\frac{\partial f}{\partial x} = \cos\left(\frac{x}{y}\right) \frac{1}{y}, \quad \frac{\partial f}{\partial y} = \cos\left(\frac{x}{y}\right) \left(-\frac{x}{y^2}\right)$$

$$x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} = 0 \text{ (Option A)}$$

4. Find $\frac{dy}{dx}$ when $x^3 + y^3 = 3axy$.

$$(A) -\frac{x^2 - ay}{y^2 - ax} \quad (B) \frac{x^2 - ay}{y^2 - ax} \quad (C) \frac{y^2 - ax}{x^2 - ay} \quad (D) -\frac{y^2 - ax}{x^2 - ay}$$

Solution:

$$\text{Let } f(x, y) = x^3 + y^3 - 3axy$$

$$\begin{aligned} \frac{dy}{dx} &= -\frac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial y}} = -\frac{3x^2 - 3ay}{3y^2 - 3ax} \\ &= -\frac{x^2 - ay}{y^2 - ax} \text{ (Option A)} \end{aligned}$$

5. If $x = uv$, $y = \frac{u}{v}$, find $\frac{\partial(x, y)}{\partial(u, v)}$.

$$(A) \frac{-2u}{v} \quad (B) \frac{2u}{v} \quad (C) \frac{-2v}{u} \quad (D) \frac{2v}{u}$$

Solution:

$$\frac{\partial(x, y)}{\partial(u, v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} v & u \\ \frac{1}{v} & \frac{-u}{v^2} \end{vmatrix} = -\frac{2u}{v} \text{ (Option A)}$$

6. If $f(x, y) = e^x \sin y$, then find $f_{yy}(0, 0)$.

(A) 0 (B) 1 (C) 2 (D) 3

Solution:

$$f_y = e^x \cos y$$

$$f_{yy}(x, y) = e^x (-\sin y)$$

$$f_{yy}(0, 0) = 0 \text{ (Option A)}$$

7. If $x^y = y^x$, then find $\frac{dy}{dx}$.

(A) $\frac{yx^{y-1} - y^x \log y}{xy^{x-1} - x^y \log x}$

(B) $\frac{yx^{y-1} + y^x \log y}{xy^{x-1} - x^y \log x}$

(C) $\frac{yx^{y-1} + y^x \log y}{xy^{x-1} + x^y \log x}$

(D) $\frac{yx^{y-1} - y^x \log y}{xy^{x-1} + x^y \log x}$

Solution:

$$f(x, y) = x^y - y^x = 0$$

$$\frac{dy}{dx} = \frac{-f_x}{f_y} = \frac{yx^{y-1} - y^x \log y}{xy^{x-1} - x^y \log x} \quad (\text{Option A})$$

8. If $f(x, y) = \tan^{-1}\left(\frac{y}{x}\right)$, then find $f_x(x, y)$ at the point $(1, 1)$.

(A) -1/2

(B) 1

(C) 1/2

(D) 3

Solution:

$$f_x(x, y) = \frac{-y}{x^2 + y^2}$$

$$f_x(1, 1) = -\frac{1}{2} \quad (\text{Option A})$$

9. If $x = r \cos \theta$, $y = r \sin \theta$, then find $\frac{\partial(x, y)}{\partial(r, \theta)}$.

(A) r

(B) $1/r$

(C) $1/2$

(D) 1

Solution:

$$\text{Now } \frac{\partial(x, y)}{\partial(r, \theta)} = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{vmatrix} = \begin{vmatrix} \cos \theta & -r \sin \theta \\ \sin \theta & r \cos \theta \end{vmatrix} = r(\sin^2 \theta + \cos^2 \theta) = r(1) = r$$

(Option A)

10. If $u = 2xy$, $v = x^2 - y^2$, then find $\frac{\partial(u, v)}{\partial(x, y)}$.

- (A) $-4y^2 - 4x^2$ (B) $-4y^2 + 4x^2$
 (C) $4y^2 - 4x^2$ (D) $4y^2 + 4x^2$

$$\frac{\partial(u, v)}{\partial(x, y)} = \begin{vmatrix} 2y & 2x \\ 2x & -2y \end{vmatrix} = -4y^2 - 4x^2 \quad (\textbf{Option A})$$

* * * * *

Year/Sem: I/I

Part – A Branch: Common to ALL Branches

Unit – III Ordinary Differential Equations

1.	The order and degree of $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{-2x}$ are	1 Mark	
	a) 2, 1 b) 1, 2 c) 2, 2 d) 1, 1	Ans (a)	(CLO-3 Remember)
2.	The order and degree of $\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^2 + 2y = \sin 3x$ are	1 Mark	
	a) 1, 2 b) 2, 1 c) 2, 2 d) 1, 1	Ans (b)	(CLO-3 Remember)
3.	The order and degree of $\left(\frac{d^2y}{dx^2}\right)^2 + 2\frac{dy}{dx} + y = 5x$ are	1 Mark	
	a) 1, 2 b) 2, 1 c) 2, 2 d) 1, 1	Ans (c)	(CLO-3 Remember)
4.	The order and degree of $\frac{dy}{dx} + 3y = 5x$ are	1 Mark	
	a) 1, 2 b) 2, 1 c) 2, 2 d) 1, 1	Ans (d)	(CLO-3 Remember)
5.	The number of arbitrary constants in the solution of a differential equation is equal to the _____ of that differential equation.	1 Mark	
	a) degree b) number of variables c) order d) number of terms	Ans (b)	(CLO-3 Remember)
6.	The number of arbitrary constants in the most general solution of n^{th} order differential equation is _____	1 Mark	
	a) 1 b) $n - 1$ c) n d) $n + 1$	Ans (c)	(CLO-3 Remember)
7.	The solution of $(D^3 - D^2 + D - 1)y = 0$ is	1 Mark	
	a) $y = Ae^x + B \cos x + C \sin x$ b) $y = Ae^x + B \cos x - C \sin x$ c) $y = Ae^{-x} + B \cos x + C \sin x$ d) $y = Ae^x + B \cosh x + C \sinh x$	Ans (a)	(CLO-3 Remember)
8.	The complementary function of $(D^2 + D + 1)y = 0$ is	1 Mark	

	a) $e^{\frac{1}{2}x} \left(C_1 \cos \frac{\sqrt{3}}{2}x + C_2 \sin \frac{\sqrt{3}}{2}x \right)$ c) $e^{\frac{-1}{2}x} \left(C_1 \cos \frac{\sqrt{3}}{2}x + C_2 \sin \frac{\sqrt{3}}{2}x \right)$ b) -1, 2 d) $\cos x + i \sin x$	Ans (c)	(CLO-3 Remember)
9.	The complementary function of $\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 15y = 0$		1 Mark
	a) $C_1 e^{-5x} + C_2 e^{-3x}$ b) $C_1 e^{4x} + C_2 e^{4x}$ c) $C_1 e^{5x} + C_2 e^{3x}$ d) $C_1 e^{2x} + C_2 e^{6x}$	Ans (c)	(CLO-3 Remember)
10.	The complementary function of $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = 3e^{4x}$		1 Mark
	a) $C_1 e^{-3x} + C_2 e^{-3x}$ b) $C_1 e^{3x} + C_2 e^{3x}$ c) $(C_1 + C_2 x)e^{-3x}$ d) $(C_1 + C_2 x)e^{3x}$	Ans (c)	(CLO-3 Remember)
11.	The complementary function of $(D^2 + 4)y = x \sin x$ is		1 Mark
	a) $C_1 e^{-3x} + C_2 e^{-3x}$ b) $C_1 e^{3x} + C_2 e^{3x}$ c) $C_1 \cos 2x + C_2 \sin 2x$ d) $(C_1 + C_2 x)e^{3x}$	Ans (c)	(CLO-3 Remember)
12.	The particular integral of $(D^3 - D^2 + D - 1)y = 0$ is		1 Mark
	a) 0 b) $Ae^x + B\cos x - C\sin x$ c) $B\cos x + C\sin x$ d) $Ae^x + B\cosh x + C\sinh x$	Ans (a)	(CLO-3 Remember)
13.	The particular integral of $(D^2 + 2D + 1)y = 5$ is		1 Mark
	a) 0 b) 5 c) 2 d) 1	Ans (b)	(CLO-3 Remember)
14.	The particular integral of $(D^2 + 9)y = e^{-2x}$ is		1 Mark
	a) $\frac{e^{-2x}}{15}$ b) $\frac{e^{2x}}{15}$ c) $\frac{e^{-2x}}{13}$ d) $\frac{e^{-2x}}{14}$	Ans (c)	(CLO-3 Remember)
15.	The particular integral of $(D^2 + 16)y = e^{-4x}$ is		1 Mark
	a) $\frac{x}{32}e^{-4x}$ b) $\frac{1}{32}e^{-4x}$ c) $\frac{x}{16}e^{-4x}$ d) $\frac{1}{16}e^{-4x}$	Ans (b)	(CLO-3 Remember)
16.	The particular integral of $(D - 1)^2 y = e^x$ is		1 Mark
	a) $\frac{x}{32}e^{-4x}$ b) $\frac{x^2}{2}e^x$ c) $\frac{x}{16}e^{-4x}$ d) $\frac{1}{16}e^{-4x}$	Ans (b)	(CLO-3 Remember)

	The particular integral of $(D^2 + a^2)y = \cos ax$ is	1 Mark	
17.	a) $\frac{-x}{2a} \sin ax$ b) $\frac{-x}{2a} \cos ax$ c) $\frac{x}{2a} \cos ax$ d) $\frac{x}{2a} \sin ax$	Ans (d)	(CLO-3 Remember)
	The particular integral of $(D^2 + 4)y = \sin 2x$ is	1 Mark	
18.	a) $\frac{x}{2} \sin x$ b) $\frac{-x}{2} \sin x$ c) $\frac{-x}{4} \cos 2x$ d) $\frac{x}{4} \cos 2x$	Ans (c)	(CLO-3 Remember)
	The particular integral of $(D^2 + 2)y = x^2$ is	1 Mark	
19.	a) $\frac{1}{2}x^2$ b) $\frac{1}{2}(x^2 - 1)$ c) $\frac{1}{2}(x^2 + 1)$ d) $\frac{-1}{2}x^2$	Ans (b)	(CLO-3 Remember)
	The method of variation of parameters is used to find the particular integral of a second order differential equation whose _____ is known.	1 Mark	
20.	a) Complementary function b) constant c) variable d) degree	Ans (a)	(CLO-3 Remember)
	The order and degree of $\left(\frac{d^2y}{dx^2}\right)^2 + 3\frac{dy}{dx} + 2y = e^x$ are	1 Mark	
21.	a) 2, 1 b) 1, 2 c) 2, 2 d) 1, 1	Ans (c)	(CLO-3 Remember)
	The order and degree of $\left(\frac{d^2y}{dx^2}\right)^2 + 3\left(\frac{dy}{dx}\right)^4 + 2y = \sin 2x$ are	1 Mark	
22.	a) 2, 1 b) 1, 2 c) 2, 2 d) 1, 1	Ans (c)	(CLO-3 Remember)
	The particular integral of $(D^3 - 1)y = 0$ is	1 Mark	
23.	a) 0 b) $Ae^x + B\cosh x$ c) $A\cos x + B\sin x$ d) $Ae^x + B\cosh x + C\sinh x$	Ans (a)	(CLO-3 Remember)
	The particular integral of $(D^2 + 2D + 1)y = 1$ is	1 Mark	
24.	a) 0 b) 5 c) 2 d) 1	Ans (d)	(CLO-3 Remember)
	The particular integral of $(D^2 + 2)y = x$ is	1 Mark	
25.	a) $\frac{1}{2}x$ b) $\frac{1}{2}(x^2 - 1)$ c) $\frac{1}{2}(x^2 + 1)$ d) $\frac{-1}{2}x^2$	Ans (a)	(CLO-3 Remember)

26.	The particular integral of $(D^2 + 4)y = \cos 2x$ is	1 Mark	
	a) $\frac{x}{2}\sin x$ b) $\frac{-x}{2}\sin x$ c) $\frac{-x}{4}\cos 2x$ d) $\frac{x}{4}\sin 2x$	Ans (d)	(CLO-3 Remember)
27.	The particular integral of $(D^2 + 1)y = \cos 2x$ is	1 Mark	
	a) $\frac{x}{2}\sin x$ b) $\frac{1}{5}\cos 2x$ c) $\frac{-x}{4}\cos 2x$ d) $\frac{x}{4}\sin 2x$	Ans (b)	(CLO-3 Remember)
28.	The complementary function of $\frac{d^2y}{dx^2} + 8\frac{dy}{dx} + 15y = 0$	1 Mark	
	a) $C_1 e^{-5x} + C_2 e^{-3x}$ b) $C_1 e^{4x} + C_2 e^{4x}$ c) $C_1 e^{5x} + C_2 e^{3x}$ d) $C_1 e^{2x} + C_2 e^{6x}$	Ans (a)	(CLO-3 Remember)
29.	The complementary function of $(D^2 + 4)y = \sin x$ is	1 Mark	
	a) $C_1 e^{-3x} + C_2 e^{-3x}$ b) $C_1 e^{3x} + C_2 e^{3x}$ c) $C_1 \cos 2x + C_2 \sin 2x$ d) $(C_1 + C_2 x)e^{3x}$	Ans (c)	(CLO-3 Remember)
30.	The complementary function of $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = e^{4x}$	1 Mark	
	a) $C_1 e^{-3x} + C_2 e^{-3x}$ b) $C_1 e^{3x} + C_2 e^{3x}$ c) $(C_1 + C_2 x)e^{-3x}$ d) $(C_1 + C_2 x)e^{3x}$	Ans (d)	(CLO-3 Remember)
31.	The particular integral of $(D - 1)^2 y = e^{-x}$ is	1 Mark	
	a) $\frac{x}{32}e^{-4x}$ b) $\frac{x^2}{2}e^x$ c) $\frac{x}{16}e^{-4x}$ d) $\frac{1}{4}e^{-x}$	Ans (d)	(CLO-3 Remember)
32.	The particular integral of $(D - 3)^2 y = 3^x$ is	1 Mark	
	a) $\frac{3^x}{(\log 3 - 3)^2}$ b) $\frac{2^x}{(\log 3 - 3)^2}$ c) $\frac{3^x}{(\log 2 - 2)^2}$ d) $\frac{2^x}{(\log 2 - 2)^2}$	Ans (a)	(CLO-3 Remember)
33.	The complementary function of $(D - 1)^2 y = e^{-5x}$ is	1 Mark	
	a) $C_1 e^{-x} + C_2 e^{-x}$ b) $C_1 e^x + C_2 e^x$ c) $(C_1 + C_2 x)e^{-x}$ d) $(C_1 + C_2 x)e^{-x}$	Ans (c)	(CLO-3 Remember)
34.	The particular integral of $(D + 1)^2 y = e^{-5x}$ is	1 Mark	

	a) $\frac{1}{16}e^{-5x}$ b) $\frac{x^2}{2}e^x$ c) $\frac{x}{36}e^{-5x}$ d) $\frac{1}{4}e^{-x}$	Ans (a)	(CLO-3 Remember)
35.	The particular integral of $(D^2 + 1)y = \cos x$ is	1 Mark	
	a) $\frac{x}{2}\sin x$ b) $\frac{-x}{3}\cos 2x$ c) $\frac{-x}{4}\cos 2x$ d) $\frac{x}{4}\sin 2x$	Ans (a)	(CLO-3 Remember)
36.	The particular integral of $(D^2 + 9)y = \sin 3x$ is	1 Mark	
	a) $\frac{x}{2}\sin x$ b) $\frac{-x}{6}\cos 3x$ c) $\frac{-x}{4}\cos 2x$ d) $\frac{x}{4}\sin 2x$	Ans (b)	(CLO-3 Remember)



**SRM Institute of Science and Technology
Ramapuram Campus
Department of Mathematics
18MAB101T – Calculus And Linear Algebra**

Year/Sem: I/I

Branch: Common to ALL B.Tech. except B.Tech. (Business Systems)

Unit – III

ORDINARY DIFFERENTIAL EQUATIONS

Part – B

1. Solve $(D^2 - 7D + 12)y = 0$.

$$(a) y = Ae^{3x} + Be^{4x}$$

$$(b) y = Ae^{-3x} + Be^{4x}$$

$$(c) y = Ae^{3x} + Be^{-4x}$$

$$(d) y = Ae^{-3x} + Be^{-4x}$$

$$\begin{aligned} m^2 - 7m + 12 &= 0 \\ (m-3)(m-4) &= 0 \\ m &= 3, 4 \\ y &= Ae^{3x} + Be^{4x} \text{ (Option (a))} \end{aligned}$$

2. Find the particular integral of $(D^2 - 9)y = e^{-2x}$.

$$(a) PI = \frac{1}{13}e^{-2x}$$

$$(b) PI = -\frac{1}{5}e^{-2x}$$

$$(c) PI = \frac{x}{5}e^{-2x}$$

$$(d) PI = \frac{1}{5}e^{-2x}$$

$$\begin{aligned}
 PI &= \frac{1}{D^2 - 9} e^{-2x} \\
 &= \frac{1}{4-9} e^{-2x} \\
 &= -\frac{1}{5} e^{-2x} \text{ (Option (b))}
 \end{aligned}$$

3. Find the particular integral of $(D^2 + 3D + 2)y = e^{-2x}$.

$$(a) PI = -xe^{-2x}$$

$$(b) PI = xe^{-2x}$$

$$(c) PI = \frac{e^{-2x}}{12}$$

$$(d) PI = \frac{xe^{-2x}}{12}$$

$$\begin{aligned}
 PI &= \frac{1}{D^2 + 3D + 2} e^{-2x} \\
 &= \frac{1}{4-6+2} e^{-2x} \\
 &= x \cdot \frac{1}{2D+3} e^{-2x} \\
 &PI = -xe^{-2x} \text{ (Option (a))}
 \end{aligned}$$

4. Find the particular integral of $(D^2 + 4)y = \sin 2x$.

$$(a) PI = -\frac{x \cos 2x}{4}$$

$$(b) PI = -\frac{\sin 2x}{8}$$

$$(c) PI = \frac{x \sin 2x}{4}$$

$$(d) PI = \frac{\sin 2x}{8}$$

$$\begin{aligned}
 PI &= \frac{1}{D^2 + 4} \sin 2x \\
 &= x \cdot \frac{1}{2D} \sin 2x \\
 &= -x \cdot \frac{\cos 2x}{4} \text{ (Option (a))}
 \end{aligned}$$

5. Find the particular integral of $(D^2 + D + 1)y = 3x - 1$.

(a) $PI = 3x - 4$

(b) $PI = 3x$

(c) $PI = 3x - 1$

(d) $PI = 3x^2 - 4$

$$\begin{aligned}
 PI &= \frac{1}{D^2 + D + 1} (3x - 1) \\
 &= [1 + (D + D^2)]^{-1} (3x - 1) \\
 &= (3x - 1) - D(3x - 1) \\
 &PI = 3x - 4 \text{ (Option (a))}
 \end{aligned}$$

6. Find the particular integral of $(D^2 + D + 1)y = x$

(a) $PI = 3x - 4$

(b) $PI = 3x$

(c) $PI = x - 1$

(d) $PI = 3x^2 - 4$

$$\begin{aligned}
 PI &= \frac{1}{D^2 + D + 1} (x) \\
 &= [1 + (D + D^2)]^{-1} (x) \\
 &= [1 - (D + D^2)] (x) \\
 &= (x - D(x)) = x - 1 \\
 PI &= x - 1
 \end{aligned}$$

(Option C)

7. Solve $(D^3 - 6D^2 + 11D - 6)y = 0$

$$(a) y = Ae^x + Be^{2x} + Ce^{3x}$$

$$(b) y = Ae^x + Be^{-2x} + Ce^{3x}$$

$$(c) y = Ae^x + Be^{2x} + Ce^{-3x}$$

$$(d) y = Ae^x + Be^{-2x} + Ce^{-3x}$$

$$m^3 - 6m^2 + 11m - 6 = 0$$

$$(m-1)(m-2)(m-3) = 0$$

$$m = 1, 2, 3$$

$$C.F = Ae^x + Be^{2x} + Ce^{3x}$$

Hence

$$y = Ae^x + Be^{2x} + Ce^{3x}$$

(Option A)

8. Find the particular integral of $(D^2 + D - 2)y = \sin x$

$$(a) PI = \frac{-1}{10}(\cos x + 3\sin x)$$

$$(b) PI = \frac{1}{10}(\cos x + 3\sin x)$$

$$(c) PI = \frac{-1}{10}(\sin x + 3\cos x)$$

$$(d) PI = \frac{-1}{10}(\sin x - 3\cos x)$$

$$\text{P.I} = \frac{1}{D-3} \sin x = \frac{D+3}{D^2-9} \sin x, \text{ Rationalizing the denominator}$$

$$= \frac{(D+3) \sin x}{-10}, \text{ Putting } D^2 = -1$$

$$\therefore \text{P.I.} = \frac{-1}{10} (D \sin x + 3 \sin x)$$

$$= \frac{-1}{10} (\cos x + 3 \sin x)$$

(Option A)

9. Find the complementary function of $(D^2 + 1)y = \cos ec x$.

$$(a) CF = (A + Bx)e^x$$

$$(b) CF = (A + Bx)e^x$$

$$(c) CF = A \cos x + B \sin x$$

$$(d) CF = (A \cos x + B \sin x)e^x$$

$$m^2 + 1 = 0 \Rightarrow m = \pm i$$

$$CF = A \cos x + B \sin x \text{ (Option (c))}$$

10. Solve $(D^2 + 4D + 4)y = 0$.

$$(a) y = Ae^{-2z} + Be^{-2z}$$

$$(b) y = (A + Bx)e^{-2x}$$

$$(c) y = \frac{A}{x} + \frac{B}{x^2}$$

$$(d) y = Ax + Bx^2$$

$$m^2 + 4m + 4 = 0$$

$$m = -2, -2$$

$$y = (A + Bx)e^{-2x} \text{ (Option (B))}$$

* * * * *



Year/Sem: I/I

Part-A

Branch: Common to All

Unit – IV

Differential Calculus

6.	What is the curvature of a circle of radius 3?	1 Mark	
	(a) 3 (c) $\frac{1}{3}$	(b) -3 (d) $-\frac{1}{3}$	Ans (c) (CLO-4 Remember)
7.	In an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ the radius of curvature at the end of which axis is equal to the semi-latus rectum?	1 Mark	
	(a) minor (c) vertical	(b) major (d) horizontal	Ans (b) (CLO-4 Remember)
8.	Evolute of a curve is the envelope of _____ of that curve.	1 Mark	
	(a) tangent (c) parallel	(b) normal (d) locus	Ans (b) (CLO-4 Remember)
9.	The evolute of the cycloid $x = a(\theta - \sin \theta)$, $y = a(1 - \cos \theta)$ is	1 Mark	
	(a) astroid (c) cycloid	(b) parabola (d) circle	Ans (c) (CLO-4 Remember)
10.	A curve which touches each member of a family of the curves is called --- of that family	1 Mark	
	(a) Evolute (c) Circle of curvature	(b) Envelope (d) Radius of curvature	Ans (b) (CLO-4 Remember)
11.	Envelope of the curve $y = mx + \frac{a}{m}$ (where m is the parameter) is	1 Mark	
	(a) $x^2 + ay = 0$ (c) $y^2 - 4ax = 0$	(b) $x + 4ay = 0$ (d) $y^2 + 4ax = 0$	Ans (c) (CLO-4 Remember)

12.	If the radius of curvature and curvature of a curve at any point are ρ and k respectively, then	1 Mark	
	(a) $\rho = \frac{-1}{k}$ (b) $\rho = k$ (c) $\rho = -k$ (d) $\rho = \frac{1}{k}$	Ans (d)	(CLO-4 Remember)
13	The radius of curvature at the point $(0, c)$ of the curve $y = c \cosh \left(\frac{x}{c} \right)$ is	1 Mark	
	(a) $\rho = c$ (b) $\rho = c^2$ (c) $\rho = kc$ (d) $\rho = kc^2$	Ans (a)	(CLO-4 Remember)
14	The radius of curvature of the curve $y = e^x$ at $x=0$ is	1 Mark	
	(a) $2\sqrt{2}$ (b) $\sqrt{2}$ (c) 2 (d) 4	Ans (a)	(CLO-4 Remember)
15	The radius of curvature at the point (x, y) of the curve $y = c \log \sec \left(\frac{x}{c} \right)$ is	1 Mark	
	(a) $\rho = c \sec \left(\frac{x}{c} \right)$ (b) $\rho = c \cos \left(\frac{x}{c} \right)$ (c) $\rho = c \sin \left(\frac{x}{c} \right)$ (d) $\rho = c \tan \left(\frac{x}{c} \right)$	Ans (a)	(CLO-4 Remember)
16	The parametric form of the curve $y^2 = 4ax$ is	1 Mark	
	(a) $x = at^2; y = 2at$ (b) $x = at; y = 2at$ (c) $x = at^2; y = 2at^2$ (d) $x = 2at^2; y = 2at$	Ans (a)	(CLO-4 Remember)

17	<p>The envelope of the curve $y = mx + \frac{a}{m}$ where m is the parameter is</p> <p>(a) $y^2 - 4ax = 0$ (b) $y^2 + 4ax = 0$ (c) $x^2 + y^2 = 1$ (d) $xy = c^2$</p>	1 Mark	
	<p>(a) Ans (a) (b) (CLO-4 Remember)</p>		
18	<p>The radius of curvature of the curve $y = \log \sec x$ at any point on it is</p> <p>(a) $\sec x$ (b) $\tan x$ (c) $\cot x$ (d) $\operatorname{cosec} x$</p>	1 Mark	
	<p>(a) Ans (a) (b) (CLO-4 Remember)</p>		
19	<p>The radius of curvature of the curve $x = t^2$, $y = t$ at $t = 1$ is</p> <p>(a) $5 \frac{\sqrt{5}}{2}$ (b) $\frac{\sqrt{5}}{2}$ (c) $\frac{5}{2}$ (d) $\sqrt{5}$</p>	1 Mark	
	<p>(a) Ans (a) (b) (CLO-4 Remember)</p>		
20	<p>The radius of curvature of the parabola $y^2 = 12x$ at $(3, 6)$ is</p> <p>(a) $12\sqrt{2}$ (b) $2\sqrt{2}$ (c) $10\sqrt{2}$ (d) $\sqrt{2}$</p>	1 Mark	
	<p>(a) Ans (a) (b) (CLO-4 Remember)</p>		
21	<p>The radius of curvature of the curve $y = 4 \sin x$ at $x = \frac{\pi}{2}$ is</p> <p>(a) $\frac{1}{2}$ (b) $\frac{-1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{-1}{3}$</p>	1 Mark	
	<p>(a) Ans (c) (b) (CLO-4 Remember)</p>		
22	<p>The envelope of family of lines $y = m x + a m^2$ (where m is the parameter) is</p> <p>(a) $x^2 + 2ay = 0$ (b) $x^2 + 4ay = 0$ (c) $y^2 + 2ax = 0$ (d) $x^2 + 4ax = 0$</p>	1 Mark	
	<p>(a) Ans (b) (b) (CLO-4 Remember)</p>		

23	The envelope of the family of lines $\frac{x}{t} + yt = 2c$, t being the parameter is (a) $x^2 + y^2 = c^2$ (b) $xy = c^2$ (c) $x^2 - y^2 = c^2$ (d) $x^2 - y^2 = c^2$	1 Mark	
	Ans (b)	(CLO-4 Remember)	
24	The radius of curvature at any point on the curve $r = e^\theta$ is	1 Mark	
	(a) $\frac{\sqrt{2}}{r}$ (b) $\frac{r}{\sqrt{2}}$ (c) r (d) $\sqrt{2} r$	Ans (d)	(CLO-4 Remember)
25	The radius of curvature in Cartesian coordinates is	1 Mark	
	(a) $\rho = \frac{(1+y_1^2)^{\frac{3}{2}}}{y_2}$ (b) $\rho = \frac{(1-y_1^2)^{\frac{3}{2}}}{y_2}$ (c) $\rho = \frac{(1+y_1^2)^{\frac{2}{3}}}{y_2}$ (d) $\rho = \frac{(1+y_1^2)^{\frac{3}{2}}}{y_1}$	Ans (a)	(CLO-4 Remember)
26	The envelope of $ty - x = at^2$, t is the parameter is	1 Mark	
	(a) $x^2 = 4ay$ (b) $y^2 = 4ax$ (c) $x^2 + y^2 = 1$ (d) $x^2 - y^2 = 1$	Ans (b)	(CLO-4 Remember)
27	The radius of curvature in polar coordinates is	1 Mark	
	(a) $\rho = \frac{(r^2 + r'^2)^{\frac{3}{2}}}{r^2 - rr' + 2r'^2}$ (b) $\rho = \frac{(r^2 - r'^2)^{\frac{3}{2}}}{r^2 - rr' + 2r'^2}$ (c) $\rho = \frac{(r^2 - r''^2)^{\frac{3}{2}}}{r^2 - rr' + 2r'^2}$ (d) $\rho = \frac{(r^2 + r'^2)^{\frac{3}{2}}}{r^2 - rr'' + 2r'^2}$	Ans (d)	(CLO-4 Remember)



**SRM Institute of Science and Technology
Ramapuram Campus**

Department of Mathematics

18MAB101T - Calculus And Linear Algebra

Year/Sem: I/I

Branch: Common to ALL B.Tech. except B.Tech. (Business Systems)

Unit – IV

DIFFERENTIAL CALCULUS

Part – B

1. Envelope of the curve $y = mx + \frac{a}{m}$ (where m is the parameter) is

(A) $x^2 + a y = 0$ (B) $x + 4 a y = 0$

(C) $y^2 - 4 a x = 0$ (D) $y^2 + 4ax = 0$

Solution: Given: $y = mx + \frac{a}{m}$

$$y = \frac{m^2 x + a}{m}$$

$$m^2 x - y m + a = 0$$

The above equation is a quadratic equation in 'm'.

The discriminant is $b^2 - 4ac = 0$.

The envelope of the curve is $y^2 - 4 a x = 0$. (**Option C**)

2. The radius of curvature of the curve $y = e^x$ at $x = 0$ is

- | | |
|-----------------|----------------|
| (A) $2\sqrt{2}$ | (B) $\sqrt{2}$ |
| (C) 2 | (D) 4 |

Solution:

$$y_1 = e^x \text{ at } x = 0 \text{ is } 1$$

$$y_2 = e^x \text{ at } x = 0 \text{ is } 1$$

$$\rho = \frac{(1 + y_1^2)^{\frac{3}{2}}}{y_2}$$

$$\rho = 2\sqrt{2} \quad (\textbf{Option A})$$

3. The radius of curvature of the curve $y = 4 \sin x$ at $x = \frac{\pi}{2}$ is

- | | |
|-------------------|--------------------|
| (A) $\frac{1}{2}$ | (B) $-\frac{1}{2}$ |
| (C) $\frac{1}{4}$ | (D) $\frac{3}{4}$ |

Solution:

$$y_1 = 4 \cos x \text{ at } x = \frac{\pi}{2} \text{ is } 0$$

$$y_2 = -4 \sin x \text{ at } x = \frac{\pi}{2} \text{ is } -4$$

$$\rho = \frac{(1 + y_1^2)^{\frac{3}{2}}}{y_2}$$

$$|\rho| = \frac{1}{4} \quad (\textbf{Option C})$$

4. The envelope of family of lines $y = mx + am^2$ (where m is the parameter) is

- | | |
|---------------------|----------------------|
| (A) $x^2 + 2ay = 0$ | (B) $x^2 + 4a y = 0$ |
| (C) $y^2 + 2ax = 0$ | (D) $x^2 + 4a x = 0$ |

Solution:

The given equation is quadratic in ‘ m ’.

The discriminant is $b^2 - 4ac = 0$.

Envelope of the family of lines is $x^2 + 4ay = 0$. **(Option B)**

5. The envelope of the family of lines $\frac{x}{t} + y t = 2c$, t being the parameter is

- | | |
|-----------------------|-----------------------|
| (A) $x^2 + y^2 = c^2$ | (B) $xy = c^2$ |
| (C) $x^2 - y^2 = c^2$ | (D) $x^2 - y^2 = c^2$ |

Solution:

Simplifying the equation $\frac{x}{t} + y t = 2c$, we get $yt^2 - 2ct + x = 0$

The discriminant is $b^2 - 4ac = 0$.

Envelope of the family of lines is $xy = c^2$. **(Option B)**

6. The radius of curvature of the curve $r = e^\theta$ at any point on it is

- | | |
|-----------------|-----------------|
| (a) $2\sqrt{2}$ | (b) $\sqrt{2}r$ |
| (c) 2 | (d) 4 |

Solution:

$$r' = e^\theta$$

$$r'' = e^\theta$$

$$\rho = \frac{\left(r^2 + r'^2\right)^{\frac{3}{2}}}{r'^2 - rr'' + 2r'^2}$$

$$\rho = \sqrt{2}r$$

7. The radius of curvature at the point (3, 4) on the curve $x^2 + y^2 = 25$ is

- (A) 5 (B) 4 (C) 0 (D) 2

Solution:

We know that the radius of curvature of the circle is equal to its radius.

$$\rho = 5 \text{ (Option A)}$$

8. B (5/2, 1/2) = _____.

- | | |
|--------------|-----------|
| (A) 1 | (B) 4 |
| (C) $3\pi/8$ | (D) π |

Solution:

$$B(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$$

$$B(5/2, 1/2) = \frac{\Gamma(5/2)\Gamma(1/2)}{\Gamma(3)} = \frac{3\pi}{8} \quad \text{(Option C)}$$

9. $\Gamma(-5/2) = _____.$

- | | |
|-----------|-------------------------------|
| (A) 1 | (B) 4 |
| (C) $1/2$ | (D) $\frac{-8\sqrt{\pi}}{15}$ |

Solution:

$$\Gamma\left(-n + \frac{1}{2}\right) = \frac{(-2)^n}{1 \cdot 3 \cdot 5 \cdots (2n-1)} \sqrt{\pi}$$

$$\Gamma\left(-\frac{5}{2}\right) = \Gamma\left(\frac{-6+1}{2}\right)$$

$$= \Gamma\left(-3 + \frac{1}{2}\right) = \frac{-8}{15} \sqrt{\pi}$$

(Option D)

10. Evaluate $\int_0^{\infty} e^{-x} x^4 dx$.

Solution

$$\Gamma(n) = \int_0^\infty e^{-x} x^{n-1} dx$$

$$\int_0^\infty e^{-x} x^4 dx = \int_0^\infty e^{-x} x^{5-1} dx = \Gamma(5) = 4! = 24$$

(Option B)

* * * *

Year/Sem: I/I
Part – A
Branch: Common to All
Unit – V

Sequence and Series

1.	A sequence $\{a_n\}$ is said to be convergent if	1 Mark	
	(a) $\lim_{n \rightarrow \infty} a_n = \text{finite}$ (b) $\lim_{n \rightarrow \infty} a_n = \infty$ (c) $\lim_{n \rightarrow \infty} a_n = -\infty$ (d) $\lim_{n \rightarrow \infty} a_n = \text{infinite}$	Ans (a)	(CLO 5, Remember)
2.	The sequence $\{(-1)^n\}$ is	1 Mark	
	(a) oscillatory (b) monotonic (c) divergent to $-\infty$ (d) divergent to $+\infty$	Ans (a)	(CLO 5, Remember)
3.	A sequence which is monotonic and bounded is	1 Mark	
	(a) conditionally convergent (b) absolutely convergent (c) convergent (d) divergent	Ans (c)	(CLO 5, Remember)
4.	The necessary condition for the convergence of $\sum u_n$ is	1 Mark	
	(a) $\lim_{n \rightarrow \infty} u_n = 0$ (b) $\lim_{n \rightarrow \infty} u_n = \infty$ (c) $\lim_{n \rightarrow \infty} u_n = -\infty$ (d) $\lim_{n \rightarrow \infty} u_n \neq 0$	Ans (a)	(CLO 5, Remember)
5.	If $\{a_n\}$ and $\{b_n\}$ are two convergent sequences, then $\{a_n + b_n\}$ is	1 Mark	
	(a) convergent (b) divergent (c) oscillatory (d) neither convergent nor divergent	Ans (a)	(CLO 5, Remember)
6.	The geometric series $1 + x + x^2 + x^3 + \dots$ converges if	1 Mark	
	(a) $-1 < x < 1$ (b) $x < 1$ (c) $x > 1$ (d) $x \geq 1$	Ans (a)	(CLO 5, Remember)
7.	The series $\frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \dots$ converges if	1 Mark	
	(a) $p > 0$ (b) $p < 1$ (c) $p > 1$ (d) $p \leq 1$	Ans (c)	(CLO 5, Remember)

8.	The series $\frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \dots$ diverges if	1 Mark	
	(a) $p > 0$ (b) $p < 1$ (c) $p > 1$ (d) $p \leq 1$	Ans (d)	(CLO 5, Remember)
9.	If $\sum u_n$ is a convergent series, then $\lim_{n \rightarrow \infty} u_n =$	1 Mark	
	(a) 1 (b) ± 1 (c) 0 (d) ∞	Ans (c)	(CLO 5, Remember)
10.	According to D' Alembert's ratio test, if $\sum u_n$ is a series of positive terms and $\lim_{n \rightarrow \infty} \frac{u_{n+1}}{u_n} = l$, then $\sum u_n$ converges if	1 Mark	
	(a) $l < 1$ (b) $l > 1$ (c) $l = 1$ (d) $-1 < l < 1$	Ans (a)	(CLO 5, Remember)
11.	By Cauchy's root test, if $\sum u_n$ is a series of positive terms and $\lim_{n \rightarrow \infty} (u_n)^{\frac{1}{n}} = l$, then $\sum u_n$ converges if	1 Mark	
	(a) $l < 1$ (b) $l > 1$ (c) $l = 1$ (d) $-1 < l < 1$	Ans (a)	(CLO 5, Remember)
12.	By Raabe's test, if $\lim_{n \rightarrow \infty} n \left(\frac{u_n}{u_{n+1}} - 1 \right) = k$, then $\sum u_n$ diverges if	1 Mark	
	(a) $k < 1$ (b) $k > 1$ (c) $k = 1$ (d) $-1 < k < 1$	Ans (a)	(CLO 5, Remember)
13.	By Logarithmic test, if $\lim_{n \rightarrow \infty} n \log \frac{u_n}{u_{n+1}} = l$, then $\sum u_n$ converges if	1 Mark	
	(a) $l < 1$ (b) $l > 1$ (c) $l = 1$ (d) $-1 < l < 1$	Ans (b)	(CLO 5, Remember)
14.	The series $\sum u_n$ containing positive and negative terms is _____, if $\sum u_n $ is divergent but $\sum u_n$ is convergent.	1 Mark	
	(a) divergent (b) oscillating finitely (c) oscillating infinitely (d) conditionally convergent	Ans (d)	(CLO 5, Remember)
15.	The series $\sum u_n$ containing positive and negative terms is absolutely convergent, if $\sum u_n $ is	1 Mark	
	(a) convergent (b) divergent to $-\infty$ (c) divergent to $+\infty$ (d) oscillatory	Ans (a)	(CLO 5, Remember)

16.	Every absolutely convergent series is necessarily (a) divergent (b) convergent (c) oscillatory (d) conditionally convergent	1 Mark	
		Ans (b)	(CLO 5, Remember)
17.	The series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if (a) $p = 0$ (b) $p = 1$ (c) $p > 1$ (d) $p < 1$	1 Mark	
		Ans (c)	(CLO 5, Remember)
18.	As per D' Alembert's ratio test, if $\sum u_n$ is a series of positive terms and $\lim_{n \rightarrow \infty} \frac{u_{n+1}}{u_n} = l$, then the series is divergent if (a) $l = 0$ (b) $l = 1$ (c) $l > 1$ (d) $l < 1$	1 Mark	
		Ans (c)	(CLO 5, Remember)
19.	A series of positive terms cannot _____. (a) oscillate (b) absolutely converge (c) converge (d) diverge	1 Mark	
		Ans (a)	(CLO 5, Remember)
20.	The series $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$ is (a) divergent (b) conditionally convergent (c) oscillatory (d) neither convergent nor divergent	1 Mark	
		Ans (b)	(CLO 5, Apply)
21.	The series $1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots$ is (a) divergent (b) absolutely convergent (c) oscillatory (d) neither convergent nor divergent	1 Mark	
		Ans (b)	(CLO 5, Apply)
22.	By Raabe's test, if $\lim_{n \rightarrow \infty} n \left(\frac{u_n}{u_{n+1}} - 1 \right) = k$, then $\sum u_n$ converges if (a) $k < 1$ (b) $k > 1$ (c) $k = 1$ (d) $-1 < k < 1$	1 Mark	
		Ans (b)	(CLO 5, Remember)

23.	By Cauchy's root test, if $\sum u_n$ is a series of positive terms and $\lim_{n \rightarrow \infty} (u_n)^{\frac{1}{n}} = l$, then $\sum u_n$ diverges if	1 Mark	
	(a) $l < 1$ (b) $l > 1$ (c) $l = 1$ (d) $-1 < l < 1$	Ans (b)	(CLO 5, Remember)
24.	By Logarithmic test, if $\lim_{n \rightarrow \infty} n \log \frac{u_n}{u_{n+1}} = l$, then $\sum u_n$ diverges if	1 Mark	
	(a) $l < 1$ (b) $l > 1$ (c) $l = 1$ (d) $-1 < l < 1$	Ans (a)	(CLO 5, Remember)
25.	If $-1 < x < 1$, then the geometric series $1 + x + x^2 + x^3 + \dots$ converges to	1 Mark	
	(a) $\frac{1}{1-x}$ (b) $\frac{1}{1+x}$ (c) e^x (d) $\frac{1}{x!}$	Ans (a)	(CLO 5, Remember)
26.	The geometric series $1 + x + x^2 + x^3 + \dots$ oscillates finitely if	1 Mark	
	(a) $x = -1$ (b) $x < 1$ (c) $x > 1$ (d) $x \geq 1$	Ans (a)	(CLO 5, Remember)
27.	The geometric series $1 + x + x^2 + x^3 + \dots$ oscillates infinitely if	1 Mark	
	(a) $x < -1$ (b) $x < 1$ (c) $x > 1$ (d) $x \geq 1$	Ans (a)	(CLO 5, Remember)
28.	If $\sum u_n$ is convergent, then $\sum k u_n$ (where k is constant) is	1 Mark	
	(a) convergent (b) divergent (c) oscillatory (d) neither convergent nor divergent	Ans (a)	(CLO 5, Remember)
29.	The series $1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + \dots$ is	1 Mark	
	(a) divergent (b) neither convergent nor divergent (c) oscillatory (d) conditionally convergent	Ans (d)	(CLO 5, Apply)
30.	The convergence of the series $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$ is tested by	1 Mark	
	(a) Ratio test (b) Raabe's test (c) Leibnitz test (d) Cauchy Root test	Ans (c)	(CLO 5, Remember)

	A monotonic increasing sequence which is not bounded above is _____. 31. (a) oscillatory (b) convergent (c) divergent to $-\infty$ (d) divergent to $+\infty$	1 Mark Ans (d) (CLO 5, Remember)	
32.	A monotonic decreasing sequence which is not bounded below is _____. (a) oscillatory (b) convergent (c) divergent to $-\infty$ (d) divergent to $+\infty$	1 Mark Ans (c) (CLO 5, Remember)	
33.	The series $\sum u_n$ of positive terms is convergent if (a) $\lim_{n \rightarrow \infty} \frac{u_{n+1}}{u_n} < 1$ (b) $\lim_{n \rightarrow \infty} \frac{u_n}{u_{n+1}} = 1$ (c) $\lim_{n \rightarrow \infty} \frac{u_n}{u_{n+1}} \leq 1$ (d) $\lim_{n \rightarrow \infty} \frac{u_{n+1}}{u_n} > 1$	1 Mark Ans (a) (CLO 5, Remember)	
34.	The n th term of a series in Arithmetic Progression is (a) $t_n = a - (n-1)d$ (b) $t_n = a + (n+1)d$ (c) $t_n = a - (n+1)d$ (d) $t_n = a + (n-1)d$	1 Mark Ans (d) (CLO 5, Remember)	
35.	$\sum_1^{\infty} \frac{n^3}{3^n}$ is (a) oscillatory (b) convergent (c) divergent to $-\infty$ (d) divergent to $+\infty$	1 Mark Ans (b) (CLO 5, Apply)	
36.	The series $\sum \frac{1}{n} \sin\left(\frac{1}{n}\right)$ is (a) oscillatory (b) convergent (c) divergent (d) conditionally convergent	1 Mark Ans (b) (CLO 5, Apply)	
37.	If D'Alembert's ratio test fails, then use (a) Comparison test (b) Leibnitz's test (c) Cauchy's integral test (d) Raabe's test	1 Mark Ans (d) (CLO 5, Remember)	
38.	The series $\sum \frac{1}{n!}$ is (a) oscillatory (b) convergent (c) divergent (d) conditionally convergent	1 Mark Ans (b) (CLO 5, Apply)	



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Unit – V

SEQUENCE AND SERIES

Part – B

- 1. The sequence $\left\{ \frac{1}{n} \right\}$ converges to _____.**
- (A) 0 (B) 1 (C) $\frac{1}{2}$ (D) ∞

Solution:

$$a_n = \frac{1}{n}$$

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{1}{n} = 0$$

$\{a_n\}$ converges to 0. **(Option A)**

- 2. The sequence $\left\{ \frac{n+1}{2n+3} \right\}$ converges to _____.**
- (A) 0 (B) 1 (C) $\frac{1}{2}$ (D) ∞

Solution:

$$a_n = \frac{n+1}{2n+3}$$

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{n\left(1 + \frac{1}{n}\right)}{n\left(2 + \frac{3}{n}\right)} = \frac{1}{2}$$

$\{a_n\}$ converges to $\frac{1}{2}$. **(Option C)**

3. Test the convergence of the series $\sum \frac{1}{\sqrt{n+1}}$.

- | | |
|-------------------------|---------------------------|
| (A) converges | (B) diverges |
| (C) oscillates finitely | (D) oscillates infinitely |

Solution:

$$u_n = \frac{1}{\sqrt{n+1}} = \frac{1}{\sqrt{n\left(1 + \frac{1}{n}\right)}}$$

$$\text{Let } v_n = \frac{1}{\sqrt{n}}$$

$$\text{Now } \frac{u_n}{v_n} = \frac{1}{\sqrt{1 + \frac{1}{n}}}$$

$$\lim_{n \rightarrow \infty} \frac{u_n}{v_n} = 1$$

$$\sum v_n = \sum \frac{1}{\sqrt{n}} = \sum \frac{1}{n^{1/2}} \text{ is divergent.}$$

Hence by comparison test, $\sum u_n$ is divergent. (**Option B**)

4. Test the convergence of the series $1 + \frac{1}{3} + \frac{1}{5} + \dots$.

- | | |
|-------------------------|---------------------------|
| (A) converges | (B) diverges |
| (C) oscillates finitely | (D) oscillates infinitely |

Solution:

$$u_n = \frac{1}{2n-1} = \frac{1}{n\left(2 - \frac{1}{n}\right)}$$

$$\text{Let } v_n = \frac{1}{n}$$

$$\text{Now } \frac{u_n}{v_n} = \frac{1}{2 - \frac{1}{n}}$$

$$\lim_{n \rightarrow \infty} \frac{u_n}{v_n} = \frac{1}{2}$$

$$\sum v_n = \sum \frac{1}{n} \text{ is divergent.}$$

Hence by comparison test, $\sum u_n$ is divergent. (**Option B**)

5. Test the convergence of the series $\sum \frac{x^n}{n!}$ where $x > 0$.

- | | |
|--|---|
| (A) converges
(C) oscillates finitely | (B) diverges
(D) oscillates infinitely |
|--|---|

Solution:

$$u_n = \frac{x^n}{n!}, \quad u_{n+1} = \frac{x^{n+1}}{(n+1)!}$$

$$\text{Now } \frac{u_{n+1}}{u_n} = \frac{x}{n+1} = \frac{x}{n\left(1 + \frac{1}{n}\right)}$$

$$\lim_{n \rightarrow \infty} \frac{u_{n+1}}{u_n} = \lim_{n \rightarrow \infty} \frac{x}{n\left(1 + \frac{1}{n}\right)} = 0 < 1$$

Hence by Ratio test, $\sum u_n$ is convergent. (**Option A**)

6. Test the convergence of the series $\sum \frac{n!}{n^n}$.

- | | |
|--|---|
| (A) converges
(C) oscillates finitely | (B) diverges
(D) oscillates infinitely |
|--|---|

Solution:

$$u_n = \frac{n!}{n^n}, \quad u_{n+1} = \frac{(n+1)!}{(n+1)^{n+1}}$$

$$\text{Now } \frac{u_{n+1}}{u_n} = \frac{1}{\left(1 + \frac{1}{n}\right)^n}$$

$$\lim_{n \rightarrow \infty} \frac{u_{n+1}}{u_n} = \frac{1}{e} < 1$$

Hence by Ratio test, $\sum u_n$ is convergent. (**Option A**)

7. The series $1 + \frac{x}{1!} + \frac{x^2}{2!} + \dots$ is _____.

- | | |
|--|---|
| (A) absolutely convergent
(C) oscillates finitely | (B) diverges to $+\infty$
(D) oscillates infinitely |
|--|---|

Solution:

$$u_n = \frac{x^{n-1}}{(n-1)!}, \quad u_{n+1} = \frac{x^n}{n!}$$

$$\text{Now } \frac{u_{n+1}}{u_n} = \frac{x}{n}$$

$$\lim_{n \rightarrow \infty} \left| \frac{u_{n+1}}{u_n} \right| = 0 < 1$$

Hence the series is absolutely convergent. **(Option A)**

8. Test the convergence of the series $\sum \frac{n^3}{3^n}$.

- | | |
|-------------------------|---------------------------|
| (A) converges | (B) diverges |
| (C) oscillates finitely | (D) oscillates infinitely |

Solution:

$$u_n = \frac{n^3}{3^n}, \quad (u_n)^{1/n} = \frac{(n^{1/n})^3}{3}$$

$$\lim_{n \rightarrow \infty} (u_n)^{1/n} = \frac{1}{3} < 1$$

Hence by Root test, $\sum u_n$ is convergent. **(Option A)**

9. Test the convergence of the series $\sum \frac{3^n n!}{n^n}$.

- | | |
|-------------------------|---------------------------|
| (A) converges | (B) diverges |
| (C) oscillates finitely | (D) oscillates infinitely |

Solution:

$$u_n = \frac{3^n n!}{n^n}, \quad u_{n+1} = \frac{3^{n+1} (n+1)!}{(n+1)^{n+1}}$$

$$\text{Now } \frac{u_{n+1}}{u_n} = \frac{3}{\left(1 + \frac{1}{n}\right)^n}$$

$$\lim_{n \rightarrow \infty} \frac{u_{n+1}}{u_n} = \frac{3}{e} > 1$$

Hence by Ratio test, $\sum u_n$ is divergent. **(Option B)**

10. Test the convergence of the series $\sum \frac{1}{n^2}$.

- | | |
|-------------------------|---------------------------|
| (A) converges | (B) diverges |
| (C) oscillates finitely | (D) oscillates infinitely |

Solution:

By Harmonic Series test or p-test, $\sum \frac{1}{n^2}$ converges. **Option (A)**

* * * * *

Unit – I: Matrices

PART A

MULTIPLE CHOICE QUESTIONS

1. The matrix of the quadratic form $x_1^2 + 5x_2^2 + x_3^2 + 2x_1x_2 + 2x_2x_3 + 6x_3x_1$ is

✓(a) $\begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 2 & 3 \\ 1 & 5 & 2 \\ 3 & 2 & 1 \end{pmatrix}$ (c) $\begin{pmatrix} 1 & 4 & 4 \\ 4 & 5 & 3 \\ 4 & 3 & 1 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & 4 & 3 \\ 4 & 5 & 4 \\ 3 & 4 & 1 \end{pmatrix}$

2. The number of positive terms in the canonical form is called

- (a) Signature ✓(b) Index (c) Quadratic form (d) Positive definite

3. A homogeneous polynomial of second degree in any number of variables is

- (a) Canonical form ✓(b) Quadratic form (c) Orthogonal (d) Diagonal form

4. Find the eigen values of A^2 if $A = \begin{bmatrix} 3 & 2 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$

- (a) 6, 4, 10 ✓(b) 9, 4, 25 (c) 9, 2, 5 (d) 3, 2, 5

5. Find the sum and product of the eigen values of $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

- ✓(a) 5, 3 (b) 3, 5 (c) 2, 1 (d) 0, 1

6. The eigen values of an orthogonal matrix have the absolute value _____

- (a) 0 ✓(b) 1 (c) 2 (d) ± 1

7. All the eigen values of a symmetric matrix with real elements are

- (a) Distinct ✓(b) Real (c) Equal (d) Conjugate complex numbers

8. Find the nature of the quadratic form $2x^2 + 3y^2 + 2z^2 + 2xy$

- ✓(a) Positive definite (b) Negative definite (c) Positive semi-definite (d) Indefinite

9. Write the Q.F. defined by the matrix $A = \begin{pmatrix} 6 & 1 & -7 \\ 1 & 2 & 0 \\ -7 & 0 & 1 \end{pmatrix}$

- (a) $6x_1^2 + 2x_2^2 + x_3^2 + 2x_1x_2 - 14x_1x_3$ (b) $6x_1^2 + x_2^2 + 6x_3^2 + x_1x_2 - 7x_1x_3$
 (c) $6x_1^2 + 2x_2^2 + x_3^2 + 2x_1x_2 + 14x_1x_3$ (d) $6x_1^2 + x_2^2 + 6x_3^2 + x_1x_2 - 14x_1x_3$

10. Find the eigen values of the matrix $\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$

- (a) 1, 3 (b) 3, 1 (c) 2, 1 (d) 1, 2

11. Find the eigen values of A^{10} if $A = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$

- (a) 1, 3^{10} (b) 3, 1 (c) $3^2, 1^{10}$ (d) 0, 2

12. If the sum of two eigen values and trace of a 3×3 matrix A are equal, find the value of $|A|$

- (a) 0 (b) 1 (c) -1 (d) 2

13. Find the characteristic equation of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}$

- (a) $\lambda^3 + \lambda^2 - 18\lambda - 40$ (b) $\lambda^3 - \lambda^2 + 18\lambda - 40$
 (c) $\lambda^3 + \lambda^2 + 18\lambda + 40$ (d) $\lambda^3 + \lambda^2 - 18\lambda + 40$

14. Find the nature of the quadratic form $x_1^2 + 2x_2^2 + 3x_3^2 + 2x_1x_2 + 2x_2x_3 - 2x_3x_1$

- (a) Indefinite (b) Positive definite (c) Negative definite (d) Positive semidefinite

15. Find the eigen values of $A = \begin{bmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{bmatrix}$

- (a) 1, 3, -4 (b) 1, -3, -4 (c) 1, -3, 4 (d) -1, 3, -4

16. The matrix of the quadratic form $x^2 + xy$ is

- ✓ (a) $\begin{pmatrix} 1 & 1/2 \\ 1/2 & 0 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 2 \\ 2 & 0 \end{pmatrix}$ (c) $\begin{pmatrix} 0 & 1/2 \\ 1/2 & 1 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & 2 \\ 1 & 0 \end{pmatrix}$

17. Two eigen values of the matrix $A = \begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{pmatrix}$ are 1 and 2. Find the third eigen value.

- ✓ (a) 3 (b) b (c) 2 (d) 1

18. Two of the eigen values of 3×3 matrix A are 2, 1 and $|A| = 12$. Find the third eigen value

- ✓(a) 6 (b) 3 (c) 2 (d) 1

19. If A is an orthogonal matrix then

- (a) $|A| = 0$ (b) A is singular (c) $A^2 = I$ ✓(d) $A^T = A^{-1}$

20. Two eigen values of $A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -5 & -2 \end{bmatrix}$ are equal and they are double the third. Find them.

- ✓(a) 1, 2, 2 (b) 2, 1, 1 (c) 2, 0, 1 (d) 1, 2, 3

21. Find the inverse of the eigen values of the matrix if $A = \begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$

- ✓(a) $-1, 1/6$ (b) $1, 1/6$ (c) $1, -1/6$ (d) $-1, -1/6$

22. Find rank and index of the QF whose canonical form is $3y_2^2 - 3y_3^2$

- ✓(a) 2, 1 (b) 1, 2 (c) 0, 1 (d) 0, 2

23. Find signature of the QF whose canonical form is $2y_1^2 - y_2^2 - y_3^2$,

- (a) 1 ✓(b) -1 (c) 0 (d) 6

24. The eigen vectors corresponding to the distinct eigen values of a real symmetric matrix are

- (a) imaginary (b) non-orthogonal (c) real ✓(d) orthogonal

25. Find the characteristic equation of the matrix $A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$

- (a) $\lambda^2 - 3\lambda - 2 = 0$ ✓(b) $\lambda^2 + 3\lambda + 2 = 0$ (c) $\lambda^2 - 3\lambda + 3 = 0$ (d) $\lambda^2 - 6\lambda + 3 = 0$

UNIT-II-FUNCTIONS OF SEVERAL VARIABLES

1. If $Z = \underline{x}^2 + \underline{y}^2 + 3\underline{x}\underline{y}$ then what is $\frac{\partial z}{\partial x}$?

- (i) $2\underline{y} + 3\underline{x}$ (ii) $3\underline{y}$ (iii) $2\underline{x} + 3\underline{y}$ (iv) $2\underline{x}$

2. $u = \sin^{-1} \left(\frac{\underline{x}^2 + \underline{y}^2}{\underline{x} - \underline{y}} \right)$ is homogeneous function of degree

- (i) 2 (ii) 3 (iii) 1 (iv) 4

3. If $u = a\underline{x}^2 + 2h\underline{x}\underline{y} + b\underline{y}^2$ then using Euler's theorem find $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = ?$

- (i) u (ii) $2u$ (iii) $3u$ (iv) $n(n-1)$

4. If $f(x, y) = e^{\underline{x}\underline{y}}$ then what is $f_{yy}(1, 1)$?

- (i) -e (ii) $\frac{1}{e}$ (iii) e (iv) $-\frac{1}{e}$

5. If $z = \log(\underline{x}^2 + \underline{x}\underline{y} + \underline{y}^2)$ then what is $x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = ?$

- (i) 1 (ii) $\frac{2\underline{x} + \underline{y}}{\underline{x}^2 + \underline{x}\underline{y} + \underline{y}^2}$ (iii) 2 (iv) $\frac{\underline{x} + 2\underline{y}}{\underline{x}^2 + \underline{x}\underline{y} + \underline{y}^2}$

6. If $f(x, y)$ is an implicit function then $\frac{dy}{dx} = ?$

- (i) $-\frac{(\partial f / \partial x)}{(\partial f / \partial y)}$ (ii) $\frac{(\partial f / \partial x)}{(\partial f / \partial y)}$ (iii) $\frac{(\partial f / \partial y)}{(\partial f / \partial x)}$ (iv) $-\frac{(\partial f / \partial y)}{(\partial f / \partial x)}$

7. If $f(x, y) = e^x \cos y$ then what is $f_{xy}(0, 0)$?

- (i) 1 (ii) -1 (iii) 0 (iv) 2

8. If $f(x, y) = \cos x \cos y$ then $f_{yy}(0, 0) = ?$

- (i) 1 (ii) 0 (iii) -1 (iv) $\frac{1}{2}$

9. If $f(x, y) = \tan^{-1} \left(\frac{y}{x} \right)$ then $f_x(1, 1)$ is

- (i) $\frac{\pi}{4}$ (ii) $\frac{1}{2}$ (iii) $-\frac{1}{2}$ (iv) 0

10. If $rt - s^2 < 0$ at (a, b) then the point is

- (i) Maximum point (ii) minimum point (iii) saddle point (iv) none of these

11. The stationary points of $x^2 + y^2 + 6x + 12$ are

- (i) (-3, 0) (ii) (0, 3) (iii) (0, -3) (iv) (3, 0)

12. If $x=u^2 - v^2$ and $y=2uv$ then $J\left(\frac{x,y}{u,v}\right)$ is

- (i) $u^2 + v^2$ (ii) $2(u^2 + v^2)$ (iii) $4(u^2 + v^2)$ (iv) $4v^2$

13. If $x=r\cos\theta$ and $y=r\sin\theta$ Then what is $\frac{\partial(x,y)}{\partial(r,\theta)}$ = ?

- (i) r^2 (ii) r (iii) 2r (iv) 0

14. If $v=\tan^{-1}x+\tan^{-1}y$ then $\frac{\partial v}{\partial x}$ is

- (i) $1+y^2$ (ii) $\frac{1}{1+y^2}$ (iii) $\frac{1}{1+x^2}$ (iv) $1+x^2$

15. u and v are functionally dependent if their jacobian value is

- (i) zero (ii) one (iii) non-zero (iv) greater than zero

16. if $J_1 = J\left(\frac{x,y}{u,v}\right)$ and $J_2 = J\left(\frac{u,v}{x,y}\right)$ then $J_1 J_2 = ?$

- (i) 0 (ii) 1 (iii) -1 (iv) 2

17. The stationary points of $f(x,y)=\sin x + \sin y + \sin(x+y)$ are

- (i) $\left(\frac{\pi}{2}, \frac{\pi}{3}\right)$ (ii) $\left(\frac{\pi}{3}, \frac{\pi}{3}\right)$ (iii) $\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$ (iv) $\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$

18. The point (0,0) for $f(x,y)=x^3 + y^3 - 3axy$ is

- (i) a maximum point (ii) a minimum point (iii) a saddle point (iv) none of these

19. If $f(x,y)=x^2 + y^2$ where $x=r\cos\theta$ and $y=r\sin\theta$ then $\frac{\partial f}{\partial\theta}$ is

(i) r

(ii) r^2

(iii) 1

(iv) 0

20. If $f(x, y) = x^2y + \sin y + e^x$ then $f_x(1, \pi)$ is

(i) 2π -e

(ii) 2π

(iii) $2\pi + e$

(iv) 0

21. $u = \cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ is homogeneous function of degree

(i) $\frac{1}{2}$

(ii) 1

(iii) 2

(iv) 3

22. If $u = \tan^{-1} \left(\frac{x^3 + y^3}{x - y} \right)$ then $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = ?$

(i) sinu

(ii) cosu

(iii) $\sin 2u$

(iv) tanu

23. If $u = \frac{y^2}{x}$, $v = \frac{x^2}{y}$ then $\frac{\partial(x,y)}{\partial(u,v)} = ?$

(i) -3

(ii) 3

(iii) - $\frac{1}{3}$

(iv) $\frac{1}{3}$

$$24. \text{ if } x = r \cos \theta, y = r \sin \theta, z = z \text{ then } \frac{\partial(x, y, z)}{\partial(r, \theta, z)} = ?$$

(i) 2r

(ii) r^2

(iii) $\frac{1}{x}$

(iv)r

25. If $u = x^2 - 2y$ and $v = x + y$ then $\frac{\partial(u, v)}{\partial(x, y)} = ?$

(i) $2x$

ANSWERS

1.(iii) $2x+3y$

2. (iii) 1

3. (ii) $2u$

4.(iii)e

5. (iii) 2

6.i) $-\frac{(\partial f/\partial x)}{(\partial f/\partial y)}$

7. (iii) 0

8. (iii) - 1

9. (iii) - 1/2

10. (iii) saddle point

11. (i) $(-3, 0)$

12. (iii) $4(u^2 + v^2)$

13. (ii) r

14. (iii) $\frac{1}{1+x^2}$

15. (i) zero

16. (ii) 1

17. (ii) $\left(\frac{\pi}{3}, \frac{\pi}{3}\right)$

18. (iii) a saddle point

19. (iv) 0

20. (iii) $2\pi + e$

21. (i) $\frac{1}{2}$

22. (iii) $\sin 2u$

23. (iii) $-\frac{1}{3}$

24. (iv)r

25. (ii) $2x+2$



SRM UNIVERSITY

Unit-III Ordinary Differential Equations

Multiple Choice Questions

1. Which of the following is the general solution to $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - 10y = 0$
(a) $y = Ae^{2x} + Be^{-5x}$ (b) $y = Ae^{-2x} + Be^{5x}$ (c) $y = Ae^{-2x} + Be^{-5x}$ (d) $y = Ae^{2x} + Be^{5x}$
2. Solution of $(D^2 + 4)y = 0$ is
(a) $y = A \cos 2x + B \sin 2x$ (b) $y = Ae^{2x} + Be^{-2x}$ (c) $y = A \cos \sqrt{2}x + B \sin \sqrt{2}x$
(d) $y = (Ax + B)e^{2x}$
3. The P.I of $(D^2 + 4)y = \sin 2x$ is
(a) $\frac{-x}{4} \cos 2x$ (b) $\frac{x}{4} \cos 2x$ (c) $\frac{x}{2} \cos 2x$ (d) $\frac{-x}{2} \cos 2x$
4. The equation $(a_0 x^2 D^2 + a_1 x D + a_2) y = Q(x)$ is called, where $a_0, a_1, a_2 \in C$
(a) Cauchy's equation (b) Legendre's equation (c) Taylor's equation (d) Clairaut's equation
5. Use the transformation $z = \log x$, convert the D.E $x^2 y'' - xy' + y = x^2$ to an equation with constant coefficients
(a) $(\theta^2 - 2\theta + 1)y = e^{2z}$ (b) $(\theta^2 - 2\theta + 1)y = e^z$ (c) $(\theta^2 + 2\theta + 1)y = e^{2z}$
(d) $(\theta^2 + 2\theta + 1)y = e^z$
6. The solution of $(D^2 + 2D + 1)y = 7$ is
(a) $y = (Ax + B)e^{-x} + 7$ (b) $y = (Ax + B)e^{-x} - 7$ (c) $y = (Ax + B)e^x + 7$
(d) $y = (Ax + B)e^x - 7$
7. The P.I of $(D - 1)^2 y = e^x \sin x$ is
(a) $-e^x \cos x$ (b) $e^x \cos x$ (c) $e^x \sin x$ (d) $-e^x \sin x$
8. The P.I of $(D - 1)^2 y = x$ is
(a) $2 - x$ (b) $x + 2$ (c) x^2 (d) $-x^2$
9. If $1 \pm 2i$ are the roots of A.E of a differential equation $f(D)y = 0$ then the general solution is
(a) $e^{-2x} (A \cos x - B \sin x)$ (b) $Ae^x + Be^{-2x}$ (c) $e^x (A \cos 2x + B \sin 2x)$ (d) $Ae^t + Be^{2x}$
10. Convert the equation $(5 + 2x)^2 y'' - 6(5 + 2x)y' + 8y = 0$ to an equation with constant coefficient by using the transformation $z = \log(5 + 2x)$
(a) $(\theta^2 + 4\theta + 2)y = 0$ (b) $(\theta^2 - 4\theta + 2)y = 0$ (c) $(\theta^2 + 4\theta + 4)y = 0$ (d) $(\theta^2 + 4\theta - 2)y = 0$
11. The P. I of $(D^2 + 4)y = \sinh 2x$ is
(a) $y_p = \frac{\sinh 2x}{8}$ (b) $y_p = \frac{\sinh 2x}{4}$ (c) $y_p = \frac{-\sinh 2x}{8}$ (d) $y_p = \frac{-\sinh 2x}{4}$

12. The P.I of $(D^2 + 6D + 5)y = e^{-x}$ is
 (a) $y_p = \frac{xe^{-x}}{4}$ (b) $y_p = \frac{xe^{-x}}{2}$ (c) $y_p = \frac{e^{-x}}{2}$ (d) $y_p = \frac{e^{-x}}{4}$
13. The solution of $(D^2 - 2aD + a^2)y = 0$ is
 (a) $Ae^{ax} + Be^{bx}$ (b) $Ae^{ax} + Be^{-ax}$ (c) $(Ax + B)e^{ax}$ (d) $(Ax + B)e^{-ax}$
14. The P.I of $(D^2 + 16)y = \cos 4x$ is
 (a) $\frac{x}{2} \sin 2x$ (b) $\frac{x \sin 4x}{8}$ (c) $\frac{x}{2} \cos 2x$ (d) $\frac{x \cos 4x}{8}$
15. The C.F of $D^2y + y = \operatorname{cosecx}$ is
 (a) $Ae^{ax} + Be^{bx}$ (b) $A \cos x + B \sin x$ (c) $(Ax + B)e^{ax}$ (d) $(Ax + B)e^{-ax}$
16. If $y_1 = \cos ax, y_2 = \sin ax$ then the value of $y_1y_2' - y_2y_1'$ is
 (a) -a (b) 0 (c) 1 (d) a
17. Solve $(D^2 + 1)y = 0$ given $y(0) = 0, y'(0) = 1$
 (a) $y = \sin x$ (b) $y = \cos x$ (c) $y = A \cos x + B \sin x$ (d) $y = 0$
18. The P.I of $(D - 2)^2y = e^{2x}$ is
 (a) $\frac{x^2}{2} e^{2x}$ (b) $\frac{x}{4} e^{2x}$ (c) $\frac{x^2}{2} e^{-2x}$ (d) $\frac{x^2}{2} e^{-2x}$
19. The P.I of $(D^2 + 4)y = \sin(2x + 5)$ is
 (a) $-\frac{x}{2} \sin(2x + 5)$ (b) $\frac{x}{4} \sin(2x + 5)$ (c) $-\frac{x}{4} \cos(2x + 5)$ (d) $\frac{x}{2} \cos(2x + 5)$
20. Solve $(x^2D^2 + xD + 1)y = 0$ is
 (a) $Ae^{az} + Be^{bz}$ (b) $A \cos z + B \sin z$ (c) $(Az + B)e^{az}$ (d) $(Az + B)e^{-az}$
21. The roots of the auxiliary equation $(m^2 - 4) = 0$ are
 (a) ± 2 (b) $\pm 2i$ (c) $\pm\sqrt{2}$ (d) $1 \pm 2i$
22. The solution of $(x^2D^2 - 7xD + 12)y = 0$ is
 (a) $Ae^{-2z} + Be^{6z}$ (b) $Ae^{2z} + Be^{-6z}$ (c) $Ae^{2z} + Be^{6z}$ (d) $Ae^{-2z} + Be^{-6z}$
23. If $y_1 = \cos x, y_2 = \sin x$ then the value of $y_1y_2' - y_2y_1'$ is
 (a) -1 (b) 0 (c) 1 (d) $\frac{1}{2}$
24. If three roots of the auxiliary equation become equal to the real number a , then the corresponding C.F is
 (a) $(Ax^2 + Bx + C)e^{ax}$ (b) $Ae^{ax} + Be^{ax} + Ce^{ax}$ (c) $Ae^{ax} + (B \cos ax + C \sin ax)$ (d) a
25. The values of $\frac{e^{ax}}{D-a}$
 (a) xe^{ax} (b) e^{ax} (c) x^2e^{ax} (d) $\frac{x^2}{2}e^{ax}$

Answers:

1. a 2. a 3. a 4. a 5. a 6. a 7. d 8. b 9. c 10. b 11. a
 12. a 13. c 14. b 15. b 16. d 17. a 18. a 19. c 20. b 21. a
 22. c 23. c 24. a 25. a

**Unit-IV Geometrical Applications of Differential Calculus****Multiple Choice Questions**

1. If the radius of curvature and curvature of a curve at any point are ρ and κ respectively, then
(a) $\rho = \frac{1}{\kappa}$ (b) $\rho = \kappa$ (c) $\rho = -\kappa$ (d) $\rho = \frac{1}{\kappa}$
2. The locus of center of curvature is called
(a) Involute (b) Evolute (c) Radius of curvature (d) Envelope
3. The envelope of the family of curves $A\alpha^2 + B\alpha + C = 0$ (α is parameter) is
(a) $B^2 + 4AC = 0$ (b) $B^2 - AC = 0$ (c) $B^2 + AC = 0$ (d) $B^2 - 4AC = 0$
4. The curvature of the straight line is
(a) 1 (b) 2 (c) -1 (d) 0
5. The radius of curvature of the curve $y = 4 \sin x$ at $x = \frac{\pi}{2}$ is
(a) $\frac{1}{2}$ (b) $\frac{-1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{-1}{4}$
6. The envelope of $ty - x = at^2$, t is the parameter is
(a) $x^2 = 4ay$ (b) $y^2 = 4ax$ (c) $x^2 + y^2 = 1$ (d) $x^2 - y^2 = 1$
7. The curvature at any point of the circle is equal to $--$ of its radius
(a) Square (b) Same (c) Reciprocal (d) constant
8. What is the radius of curvature at $(4, 3)$ on the curve $x^2 + y^2 = 25$
(a) 5 (b) -5 (c) 25 (d) -25
9. What is the curvature of a circle of radius 3
(a) 3 (b) -3 (c) $\frac{1}{3}$ (d) $-\frac{1}{3}$
10. Find the envelope of the curve $y = mx + \frac{a}{m}$ where m is a parameter
(a) $y^2 - 4ax = 0$ (b) $y^2 + 4ax = 0$ (c) $x^2 + y^2 = 1$ (d) $xy = c^2$
11. The radius of curvature of $y = e^x$ at $x = 0$ is
(a) $2\sqrt{2}$ (b) $\frac{2}{\sqrt{2}}$ (c) $\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$
12. The radius of curvature of the curve $y = \log \sec x$ at any point of it is
(a) $\sec x$ (b) $\tan x$ (c) $\cot x$ (d) $\operatorname{cosec} x$
13. In an ellipse the radius of curvature at the end of which axis is equal to the semi latus rectum of the ellipse
(a) Minor (b) Major (c) Vertical (d) Horizontal
14. The radius of curvature of the curve $x = t^2$, $y = t$ at $t = 1$ is
(a) $5\frac{\sqrt{5}}{2}$ (b) $\frac{\sqrt{5}}{2}$ (c) $\frac{5}{2}$ (d) $\sqrt{5}$

15. Evolute of a curve is the envelope of —— of that curve
 (a) Tangent (b) Normal (c) Parallel (d) Locus
16. The evolute of the cycloid $x = a(\theta - \sin \theta)$, $y = a(1 - \cos \theta)$ is
 (a) Astroid (b) Parabola (c) Cycloid (d) Circle
17. A curve which touches each member of a family of the curves is called —— of that family
 (a) Evolute (b) Envelope (c) Circle of curvature (d) Radius of curvature
18. The envelope of family of lines $y = mx + am^2$ (*where m is the parameter*) is
 (a) $x^2 + 2ay = 0$ (b) $x^2 + 4ay = 0$ (c) $y^2 + 2ax = 0$ (d) $y^2 + 4ax = 0$
19. The envelope of the family of lines $\frac{x}{t} + yt = 2c$, t being the parameter is
 (a) $x^2 + y^2 = c^2$ (b) $xy = c^2$ (c) $x^2y^2 = c^2$ (d) $x^2 - y^2 = c^2$
20. The radius of curvature at any point on the curve $r = e^\theta$ is
 (a) $\frac{\sqrt{2}}{r}$ (b) $\frac{r}{\sqrt{2}}$ (c) r (d) $\sqrt{2}r$
21. The radius of curvature in Cartesian coordinates is
 (a) $\rho = \frac{(1+y_1^2)^{3/2}}{y_2}$ (b) $\rho = \frac{(1-y_1^2)^{3/2}}{y_2}$ (c) $\rho = \frac{(1+y_1^2)^{2/3}}{y_2}$ (d) $\rho = \frac{(1+y_2^2)^{3/2}}{y_1}$
22. The radius of curvature in polar coordinates is
 (a) $\rho = \frac{(r^2 + (r')^2)^{3/2}}{r^2 - rr' + 2(r')^2}$ (b) $\rho = \frac{(r^2 - (r')^2)^{3/2}}{r^2 - rr' + 2(r')^2}$ (c) $\rho = \frac{(r^2 - (r'')^2)^{3/2}}{r^2 - rr' + 2(r')^2}$
 (d) $\rho = \frac{(r^2 + (r')^2)^{3/2}}{r^2 - rr'' + 2(r')^2}$
23. The radius of curvature in parametric coordinates is
 (a) $\rho = \frac{((x')^2 + (y')^2)^{3/2}}{x'y'' - y'x''}$ (b) $\rho = \frac{((x')^2 + (y')^2)^{3/2}}{x'y'' + y'x''}$ (c) $\rho = \frac{((x')^2 - (y')^2)^{3/2}}{x'y'' - y'x''}$
 (d) $\rho = \frac{((x')^2 - (y')^2)^{3/2}}{x'y'' + y'x''}$
24. The equation of circle of curvature at any point (x, y) with center of curvature \bar{x}, \bar{y} and with radius of curvature ρ is
 (a) $(x + \bar{x})^2 + (y + \bar{y})^2 = \rho^2$ (b) $(x - \bar{x})^2 + (y - \bar{y})^2 = \rho^2$ (c) $(x - \bar{x})^2 - (y + \bar{y})^2 = \rho^2$
 (d) $(x + \bar{x})^2 + (y + \bar{y})^2 = \rho$

Answers:

1. d 2. b 3. d 4. d 5. c 6. b 7. c 8. a 9. c 10. b 11. a 12. a
 13. b 14. a 15. b 16. c 17. b 18. b 19. b 20. d 21. a 22. d 23.
 a 24. b



**SRM INSTITUTE OF SCIENCE & TECHNOLOGY
FACULTY OF ENGINEERING AND TECHNOLOGY
DEPARTMENT OF MATHEMATICS**

Unit –IV Geometrical Applications of Differential Calculus

(Beta ,Gamma Functions)

Multiple Choice Questions

1. The value of $\beta(4,4)$ is -----
 (a). $\frac{36}{7!}$ (b). $\frac{6!}{7!}$ (c). $\frac{4!4!}{8!}$ (d). $\frac{3!}{7!}$
2. The value of $\beta\left(\frac{5}{2}, \frac{3}{2}\right)$ is -----
 (a). $\frac{\pi}{8}$ (b). $\frac{\sqrt{\pi}}{8}$ (c). $\frac{\pi}{16}$ (d). $\frac{\pi^2}{16}$
3. $\beta(m,n)$ is equal to -----
 (a). $\frac{m!n!}{(m+n)!}$ (b). $\frac{m!n!}{(m-n)!}$ (c). $\frac{\Gamma m \Gamma n}{\Gamma(m+n)}$ (d). $\frac{\Gamma m \Gamma n}{\Gamma(m-n)}$
4. The value of $\Gamma\left(\frac{1}{2}\right)$ is -----
 (a). $\sqrt{\pi}$ (b). π^2 (c). π (d). 2π
5. $\Gamma n \Gamma(1-n)$ is equal to -----
 (a). $\int_0^\infty \frac{x^{1-n}}{1+x} dx$ (b). $\Gamma(1) \beta(n,1-n)$ (c). $\Gamma(1) \beta(1-n,1-n)$ (d). $\Gamma(1) \beta(1-n,n)$
6. $\int_0^{\frac{\pi}{2}} \sin^p \theta \cos^q \theta d\theta$ is equal to -----
 (a). $\frac{1}{2} \beta\left(\frac{p+1}{2}, \frac{q+1}{2}\right)$ (b). $\frac{1}{2} \beta\left(\frac{p-1}{2}, \frac{q-1}{2}\right)$ (c). $\frac{1}{2} \beta\left(\frac{p}{2}, \frac{q}{2}\right)$ (d). $\frac{1}{2} \beta\left(\frac{1}{2}, \frac{1}{2}\right)$
7. $\int_0^1 x^4 [\log(\frac{1}{x})]^3 dx$ is equal to -----
 (a). $\frac{6}{525}$ (b). $\frac{6}{625}$ (c). $\frac{6!}{5!}$ (d). $\frac{5!}{6!}$
8. The value of $\int_0^{\frac{\pi}{2}} \sqrt{\cot \theta} d\theta$ is -----
 (a). $\frac{\pi}{2}$ (b). $\frac{\pi}{\sqrt{2}}$ (c). $\frac{\sqrt{\pi}}{2}$ (d). $\sqrt{\frac{\pi}{2}}$

UNIT V

Sequence and Series

1. The series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if
 (a) $p=1$ (b) $p=0$ (c) $p>1$ (d) $p<1$
2. The series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is divergent if
 (a) $p>1$ (b) $p=0$ (c) $p\leq 1$ (d) $p<1$
3. If $\sum u_n$ is a series of positive term such that $\lim_{n \rightarrow \infty} (u_n)^{\frac{1}{n}} = l$ where $l > 1$, then the series $\sum u_n$ is
 (a) Convergent (b) Divergent (c) Oscillating (d) Monotonic
4. If $\sum u_n$ is a series of positive term such that $\lim_{n \rightarrow \infty} (u_n)^{\frac{1}{n}} = l$, then the series $\sum u_n$ is convergent
 if
 (a) $l < 1$ (b) $l = 0$ (c) $l > 1$ (d) $l = 1$
5. The series $\sum_{n=1}^{\infty} \frac{1}{n!}$ is
 (a) Convergent (b) Divergent (c) Oscillating (d) Monotonic
6. The series $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots \text{to } \infty$ is
 (a) Convergent (b) Divergent (c) Oscillating (d) Monotonic
7. By D'Alambert's Ratio test $\lim_{n \rightarrow \infty} \left(\frac{u_{n+1}}{u_n} \right) = l$, the series is convergent when
 (a) $l < 1$ (b) $l = 0$ (c) $l > 1$ (d) $l = 1$
8. By Raabe's test $\lim_{n \rightarrow \infty} \left[n \left(\frac{u_{n+1}}{u_n} - 1 \right) \right] = l$, the series is divergent when
 (a) $l < 1$ (b) $l = 0$ (c) $l > 1$ (d) $l = 1$
9. The series $\sum_{n=1}^{\infty} \frac{2^n}{n!}$ is
 (a) Convergent (b) Divergent (c) Oscillating (d) Monotonic
10. A series $\sum u_n$ is said absolutely convergent if the series
 (a) $\sum |u_n|$ is convergent (c) $\sum u_n$ is divergent
 (b) $\sum u_n$ is convergent (d) $\sum |u_n|$ is divergent
11. A series $\sum u_n$ is said conditionally convergent if the series

- (a) $\sum |u_n|$ is convergent (b) $\sum u_n$ is divergent & $\sum |u_n|$ is convergent
 (c) $\sum u_n$ is convergent & $\sum |u_n|$ is divergent (d) $\sum |u_n|$ is divergent
12. The series $1 - \frac{1}{2} + \frac{1}{2^2} - \frac{1}{2^3} + \dots$ is
 (a) Convergent (b) Divergent (c) Conditionally convergent (d) absolutely convergent
13. The series $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$ is
 (a) Convergent (b) Divergent (c) Conditionally convergent (d) absolutely convergent
14. The series $\sum \frac{1}{n \log n}$ is
 (a) Conditionally convergent (b) absolutely convergent (c) Convergent (d) Divergent
15. An absolutely convergent series is
 (a) Conditionally convergent (b) absolutely convergent (c) Convergent (d) Divergent
16. The series $\sum \frac{n^3}{3^n}$ is
 (a) Conditionally convergent (b) absolutely convergent (c) Convergent (d) Divergent
17. The series $\sum \frac{1}{(\log n)^n}$ is
 (a) Convergent (b) Conditionally convergent (c) absolutely convergent (d) Divergent

ANSWERS

- | | |
|-------|-------|
| 1. d | 11. c |
| 2. c | 12. d |
| 3. b | 13. c |
| 4. a | 14. d |
| 5. a | 15. c |
| 6. b | 16.c |
| 7. a | 17.a |
| 8. c | |
| 9. a | |
| 10. a | |