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B.Tech Degree Examination – DECEMBER 2022

Open Book Examination

18CSC301T – Formal Language and Automata

Answer key.

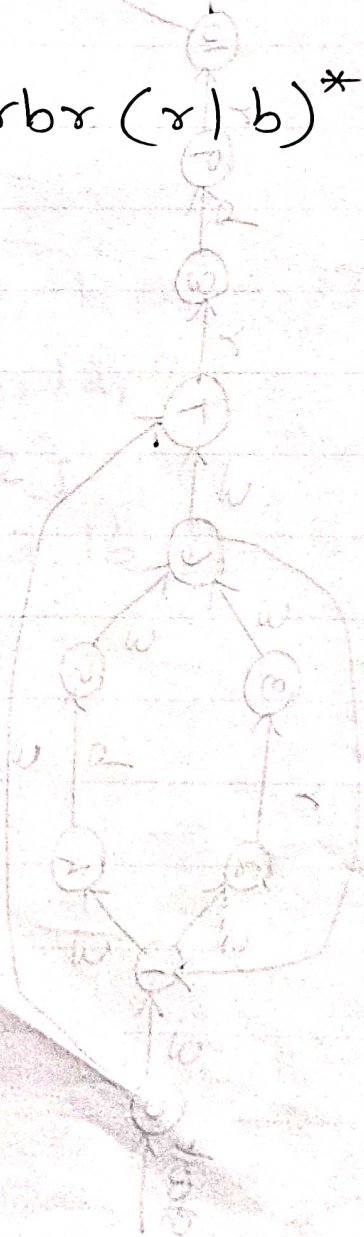
1. red ball and blue ball can be represented as 'r' and 'b' respectively.

(i) $RE = (r+b)^* rbr (r+b)^*$ (2 Marks)

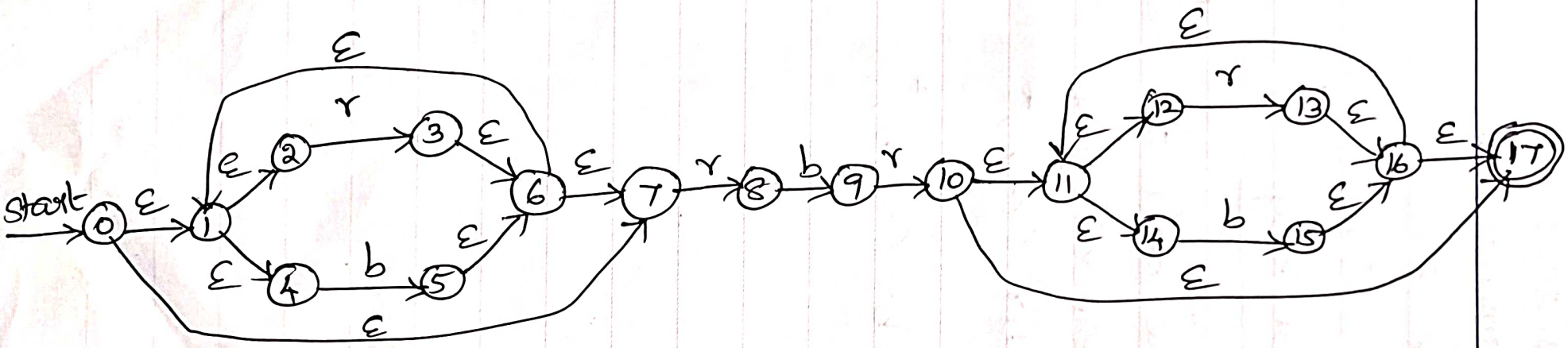
(or)

$RE = (r|b)^* rbr (r|b)^*$

(ii)



(ii) ε-NFA (4 Marks)



(iii) ECLOSURE (3 Marks)

ECLOSE(0) = {0, 1, 2, 4, 7}

ECLOSE(1) = {1, 2, 4}

ECLOSE(2) = {2}

ECLOSE(3) = {3, 6, 1, 7, 2, 4}

ECLOSE(4) = {4}

ECLOSE(5) = {5, 6, 1, 7, 2, 4}

ECLOSE(6) = {6, 1, 7, 2, 4}

" (7) = {7}

" (8) = {8}

" (9) = {9}

" (10) = {10, 11, 12, 14, 17}

" (11) = {11, 12, 14}

" (12) = {12}

" (13) = {13, 16, 11, 17, 12, 14}

" (14) = {14}

" (15) = {15, 16, 11, 17, 12, 14}

" (16) = {16, 11, 17, 12, 14}

" (17) = {17}

DFA .

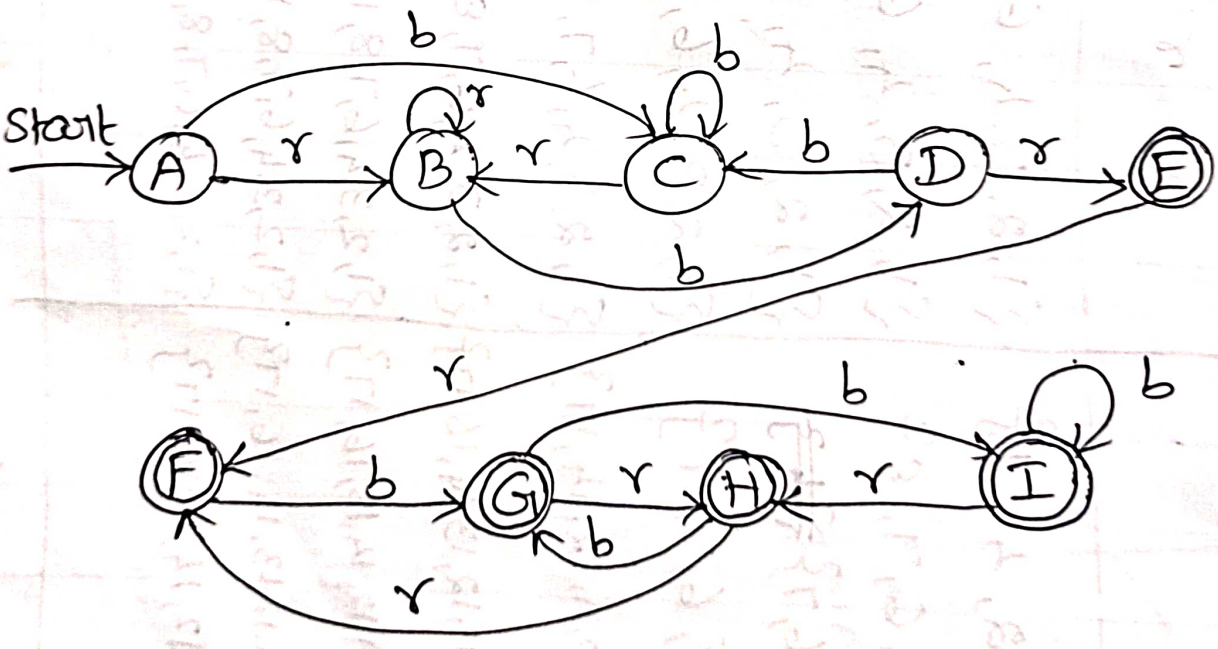
(5 Marks)

States	a	b
→ {0, 1, 2, 4, T}	{1, 2, 3, 4, 6, T, 8}	{1, 2, 4, 5, 6, T}
{1, 2, 3, 4, 6, T, 8}	{1, 2, 3, 4, 6, T, 8}	{1, 2, 4, 5, 6, T, 9}
{1, 2, 4, 5, 6, T}	{1, 2, 3, 4, 6, T, 8}	{1, 2, 4, 5, 6, T}
{1, 2, 4, 5, 6, T, 9}	{1, 2, 3, 4, 6, T, 8, 10, 11, 12, 14, 17}	{1, 2, 4, 5, 6, T}
*{1, 2, 3, 4, 6, T, 8, 10, 11, 12, 14, 17}	{1, 2, 3, 4, 6, T, 8, 11, 12, 13, 14, 16, 17}	{1, 2, 4, 5, 6, T, 9, 11, 12, 14, 15, 16, 17}
*{1, 2, 3, 4, 6, T, 8, 11, 12, 13, 14, 16, 17}	{1, 2, 3, 4, 6, T, 8, 11, 12, 13, 14, 16, 17}	{1, 2, 4, 5, 6, T, 9, 11, 12, 14, 15, 16, 17}
*{1, 2, 4, 5, 6, T, 9, 11, 12, 14, 15, 16, 17}	{1, 2, 3, 4, 6, T, 8, 10, 11, 12, 14, 17, 13, 16}	{1, 2, 4, 5, 6, T, 11, 12, 14, 15, 16, 17}
*{1, 2, 3, 4, 6, T, 8, 10, 11, 12, 13, 14, 16, 17}	{1, 2, 3, 4, 6, T, 8, 11, 12, 13, 14, 16, 17}	{1, 2, 4, 5, 6, T, 9, 11, 12, 14, 15, 16, 17}
*{1, 2, 4, 5, 6, T, 11, 12, 14, 15, 16, 17}	{1, 2, 3, 4, 6, T, 8, 11, 12, 13, 14, 16, 17}	{1, 2, 4, 5, 6, T, 11, 12, 14, 15, 16, 17}

iv) Minimize the DFA.

DFA

		r	b
→	A	B	C
	B	B	D
	C	B	C
	D	E	C
*	E	F	G
*	F	F	G
*	G	H	I
*	H	F	G
*	I	H	I



$\Pi_0 = \{\{A, B, C, D\}, \{E, F, G, H, I\}\}$

1 2

	A	B	C	D	E	F	G	H	I
r	1	1	1	2	2	2	2	2	2
b	1	1	1	1	2	2	2	2	2

$\Pi_1 = \{\{A, B, C\}, \{D\}, \{E, F, G, H, I\}\}$

3 4 2

	A	B	C	D	E	F	G	H	I
r	3	3	3	2	2	2	2	2	2
b	3	4	3	3	2	2	2	2	2

$\Pi_2 = \{\{A, C\}, \{B\}, \{D\}, \{E, F, G, H, I\}\}$

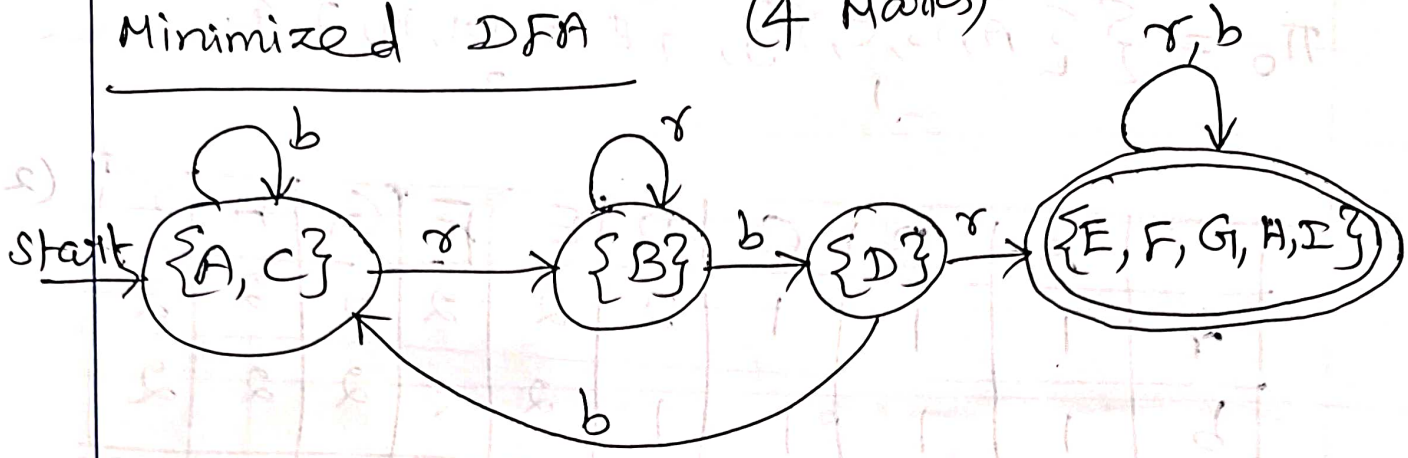
5 6 4 2

	A	B	C	D	E	F	G	H	I
r	6	6	6	2	2	2	2	2	2
b	5	4	5	5	2	2	2	2	2

$\Pi_3 = \{\{A, C\}, \{B\}, \{D\}, \{E, F, G, H, I\}\}$

Π_3 is Same as Π_2

Minimized DFA (4 Marks)



v) (B)

vi) (C)

2. Routes A, B, C and D are considered as a, b, c and d respectively.

(i) CFG

CFG, $G = (\{S, A\}, \{a, b, c, d\}, P, S)$ (1 M)

$P = \{ \; S \rightarrow aSd \mid aAd$ (2 M)

$A \rightarrow bAc \mid bc \; \}$

(ii) Language for the CFG. (3 Marks)

$L(G) = \{a^n b^m c^m d^n \mid n \geq 1, m \geq 1\}$

(iii) Any string can be chosen. Let us take string = "aabcdd"

Leftmost derivation (2 Marks)

$$S \underset{lm}{\Rightarrow} aSd \underset{lm}{\Rightarrow} aaAdd \underset{lm}{\Rightarrow} aabcdd$$

Rightmost derivation (2 Marks)

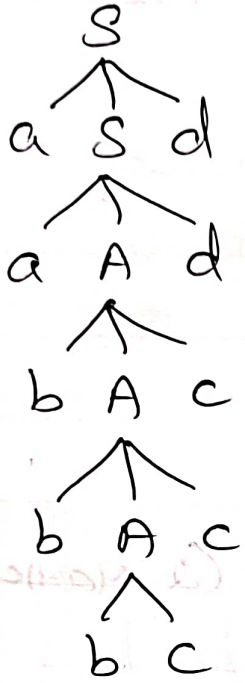
$$S \underset{rm}{\Rightarrow} aSd \underset{rm}{\Rightarrow} aaAdd \underset{rm}{\Rightarrow} aabcdd$$

Parse Tree (2 Marks)

(iv) Ambiguous or Not.

Let string = "aabbbcccdd"

(4 Marks)



(2 Marks)

We can construct only 1 parse tree for this string. Hence the grammar is unambiguous.

v) (C)

vi) (B)

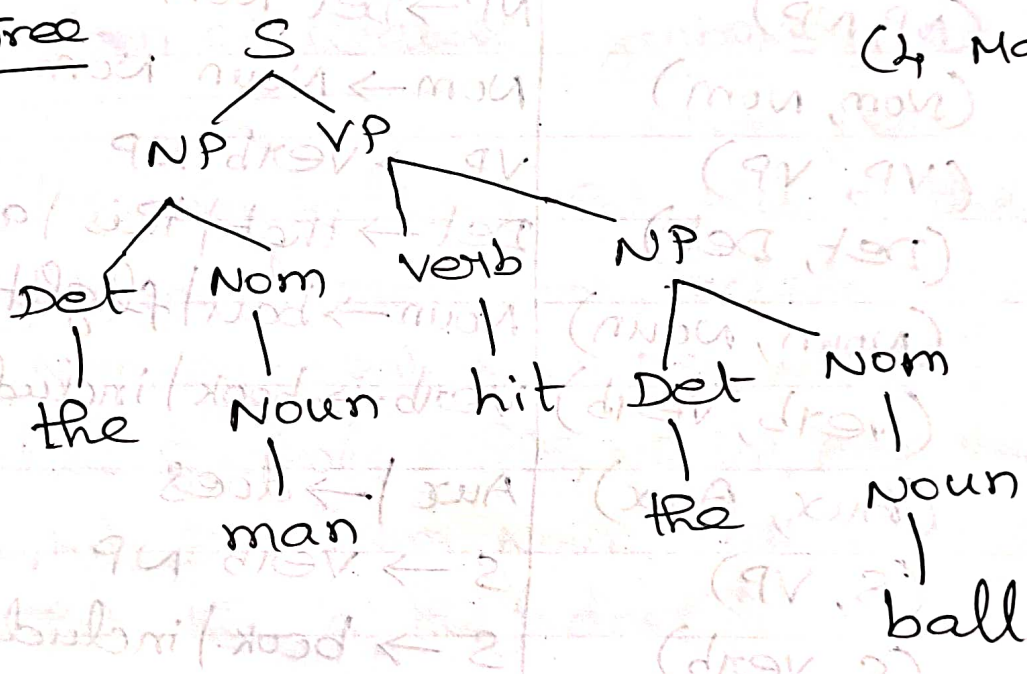
3.

(i) Terminals = { that, this, a, the, ball, flight, meal, man, book, include, hit, does } (1.5 Marks)

Non-terminals = { S, NP, VP, Nom, Det, Noun, Verb, Aux } (1.5 Marks)

(ii) Left-most derivation or rightmost derivation or parse tree can be used for proving. (4 Marks)

Parse Tree



(iii) Simplification of Grammar.

Elimination of ε-productions (1 Mark)

No ε-productions in this grammar.

Elimination of Unit productions (4 Marks)

Unit pairs	Not unit productions
(S, S)	S → NP VP | Aux NP VP
(NP, NP)	NP → Det Nom
(Nom, Nom)	Nom → Noun Nom
(VP, VP)	VP → Verb NP
(Det, Det)	Det → that | this | a | the
(Noun, Noun)	Noun → ball | flight | meal | man
(Verb, Verb)	Verb → book | include | hit
(Aux, Aux)	Aux → does
(S, VP)	S → Verb NP
(S, Verb)	S → book | include | hit
(Nom, Noun)	Nom → ball | flight | meal | man
(VP, Verb)	VP → book | include | hit

S → NP VP | Aux NP VP | Verb NP | book | include | hit

NP → Det Nom

Nom → Noun Nom | ball | flight | meal | man

VP → Verb NP | book | include | hit

Det → that | this | a | the

Noun → ball | flight | meal | man

Verb → book | include | hit

Aux → does

Elimination of useless Symbols: (2 Marks)

Generating Symbols = { that, this, a, the, ball, flight, meal, man, book, include, hit, does, VP, Nom, Aux, Verb, Noun, Det, NP, S }

Reachable Symbols = { S, NP, VP, Aux, Verb, book, include, hit, Det, Nom, Noun, ball, flight, meal, man, that, this, a, the, does }

There are no non-generating and non-reachable symbols. So there are no useless symbols.

iv) Chomsky Normal Form (4 Marks)

$C_1 \rightarrow NP\ VP$

$S \rightarrow NP\ VP \mid Aux\ C_1 \mid Verb\ NP \mid book \mid include \mid hit$

$NP \rightarrow Det\ Nom$

$Nom \rightarrow Noun\ Nom \mid ball \mid flight \mid meal \mid man$

$VP \rightarrow verb\ NP \mid book\ include \mid hit$

$Det \rightarrow that \mid this \mid a \mid the$

$Noun \rightarrow ball \mid flight \mid mean \mid man$

$Verb \rightarrow book \mid include \mid hit$

$Aux \rightarrow does$

v) (A)

vi) (C)

4. (i) case (i) Red, green, violet, yellow $\Rightarrow r, g, V, y$

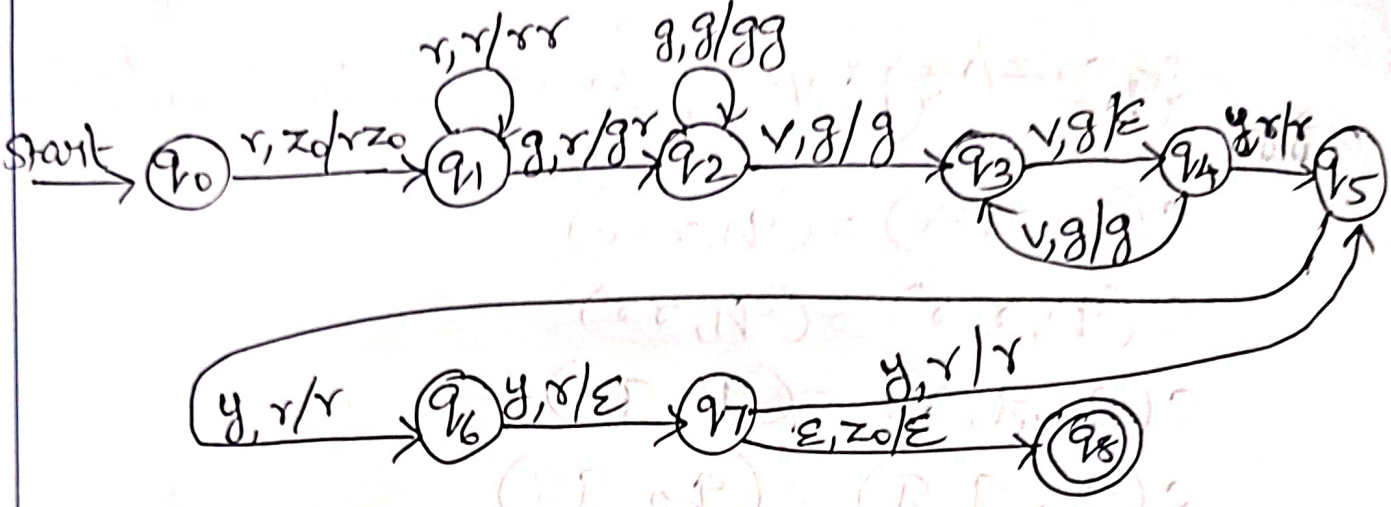
$L_1 = \{ r^n g^m V^{2m} y^{3n} \mid n \geq 1, m \geq 1 \}$ (3 Marks)

case (ii)

$L_2 = \{ r^n g^{4n} \mid n \geq 1 \}$ (2 Marks)

(ii) PDA, $P = (\{q_0, q_1, \ldots q_8\}, \{r, g, v, y\}, \{r, g, z_0\}, \delta, q_0, z_0, \{q_8\})$

(4 Marks)



$\delta(q_0, r, z_0) = (q_1, rz_0)$

$\delta(q_1, r, r) = (q_1, rr)$

$\delta(q_1, g, r) = (q_2, gr)$

$\delta(q_2, g, g) = (q_2, gg)$

$\delta(q_2, v, g) = (q_3, g)$

$\delta(q_3, v, g) = (q_4, \varepsilon)$

$\delta(q_4, v, g) = (q_3, g)$

$\delta(q_4, y, r) = (q_5, r)$

$\delta(q_5, y, r) = (q_6, r)$

$\delta(q_6, y, r) = (q_7, \varepsilon)$

$\delta(q_7, y, r) = (q_5, \varepsilon)$

$\delta(q_7, \varepsilon, z_0) = (q_8, \varepsilon)$

b) PDA, $P = (\{q_0, q_1, \ldots q_6\}, \{r, g\}, \{r, z_0\}, \delta, q_0, z_0, \{q_6\})$

(4 Marks)

$\delta(q_0, r, z_0) = (q_1, rz_0)$

$\delta(q_1, r, r) = (q_1, rr)$

$\delta(q_1, g, r) = (q_2, r)$

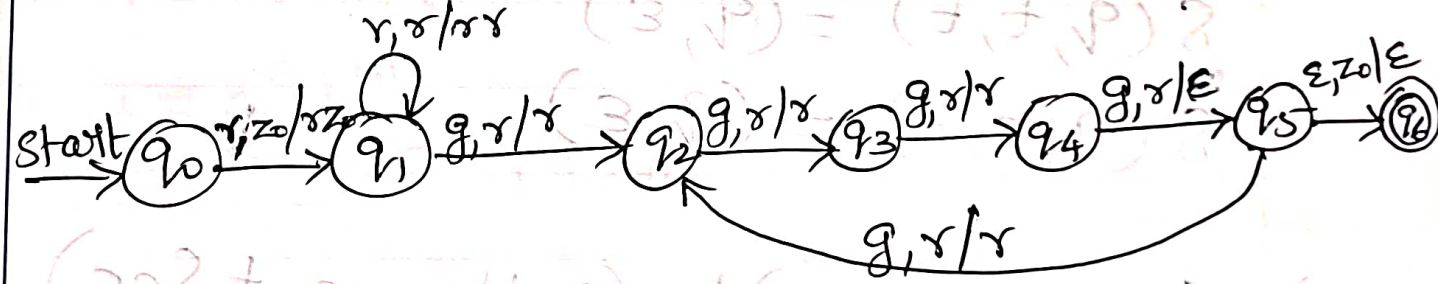
$\delta(q_2, g, r) = (q_3, r)$

$\delta(q_3, g, r) = (q_4, r)$

$\delta(q_4, g, r) = (q_5, \varepsilon)$

$\delta(q_5, g, r) = (q_2, r)$

$\delta(q_5, \varepsilon, z_0) = (q_6, \varepsilon)$



(iii) $(q_0, rgggg, z_0) \vdash (q_1, gggg, rz_0)$

$\vdash (q_2, ggg, rz_0)$

$\vdash (q_3, gg, rz_0)$

$\vdash (q_4, g, rz_0)$

$\vdash (q_5, \varepsilon, z_0)$

$\vdash (q_6, \varepsilon, \varepsilon)$

(5 Marks)

So 1 red flower followed by 4 green flowers are picked.

iv) (B)

v) (D)

5. Thread mill and cycle ⇒ t & c

(i) CFG, $G=(\{S\},\{t,c\},P,S)$ (2 Marks)

$P=\{S \rightarrow tScc \mid tcc\}$

(ii) PDA

PDA, $P=(\{q\},\{t,c\},\{S,t,c\},\delta,q,S)$ (2 Marks)

$\delta(q,\varepsilon,S)=\{(q,tScc),(q,tcc)\}$ (4 Marks)

$\delta(q,t,t)=(q,\varepsilon)$

$\delta(q,c,c)=(q,\varepsilon)$

(iii) $(q,ttcccc,S)\vdash(q,ttcccc,tScc)$ (4 Marks)

$\vdash(q,tcccc,Scc)$

$\vdash(q,tcccc,tcccc)$

$\vdash(q,cccc,cccc)$

$\vdash(q,ccc,ccc)$

$\vdash(q,cc,cc)$

$\vdash (q, c, c)$

$\vdash (q, \varepsilon, \varepsilon)$

iv) $L = \{a^n b^m c^n d^m \mid n, m \geq 1\}$ (6 Marks)

Proof:

* Let L be a CFL
* Let n be a constant
* Let $z = a^n b^m c^n d^m$, $|z| \geq n$
* Split $z = uvwxy$, such that
  (i) $|vwx| \leq n$
  (ii) $vx \neq \varepsilon$
  (iii) $\forall\ i \geq 0$, $uv^i wx^i y \in L$
* $u = a^n b^m$

  $vwx = c^n$ where $|vwx| \leq n$

  $vx = c^{n-p}$ where $|vx| \geq 1$

  $y = d^m$

$uv^i wx^i y = uvv^{i-1} wxx^{i-1} y$

$= uvwx\,(vx)^{i-1} y$

$= a^n b^m c^n (c^{n-p})^{i-1} d^m$

$$= a^n b^m c^n c^{ni-n-pi+p} d^m$$

$$= a^n b^m c^{\cancel{n}+ni-\cancel{n}-pi+p} d^m$$

$$= a^n b^m c^{ni-pi+p} d^m$$

Pick $i=0$, $a^n b^m c^p d^m \notin L$

Hence the language $L=\{a^n b^m c^n d^m \mid n, m \geq 1\}$ is not a CFL.

(v) (D)

(vi) (B)

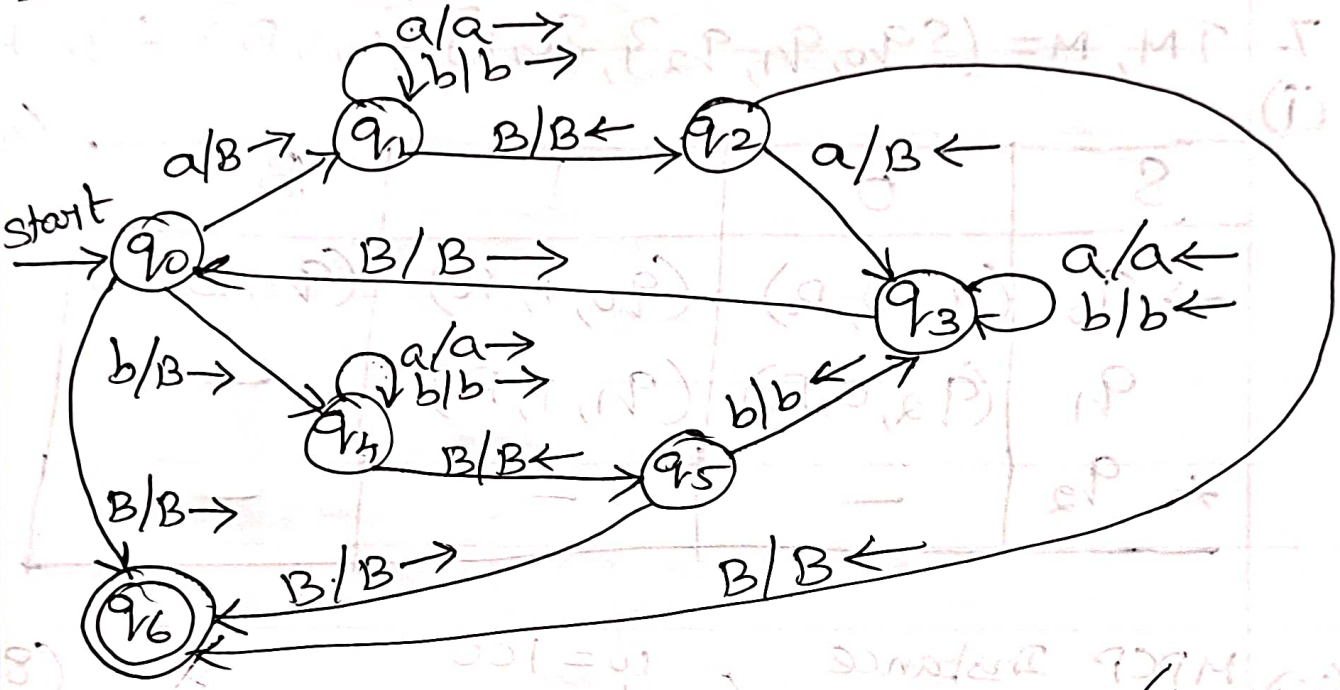
6.(i) Let us assume that the two symbols are a & b (8 Marks)

δ	a	b	B
$\rightarrow q_0$	(q_1, B, R)	(q_4, B, R)	(q_6, B, R)
q_1	(q_1, a, R)	(q_1, b, R)	(q_2, B, L)
q_2	(q_3, B, L)	—	(q_6, B, R)
q_3	(q_3, a, L)	(q_3, b, L)	(q_0, B, R)
q_4	(q_4, a, R)	(q_4, b, R)	(q_5, B, L)
q_5	—	(q_3, b, L)	(q_6, B, R)
$* q_6$	—	—	—

TM, $M = (\{q_0, q_1, \ldots q_6\}, \{a, b\}, \{a, b, B\}, \delta, q_0, B, \{q_6\})$

(or)

Transition Diagram



(10 MARKS)

(ii) TM; $M = (\{q_0, q_1, \ldots q_7\}, \{a, b\}, \{a, b, X, B\}, \delta, q_0, B, \{q_7\})$

δ	a	b	X	B
→ q_0	(q_1, B, R)	(q_6, B, R)	—	—
q_1	(q_1, a, R)	(q_2, b, R)	—	—
q_2	(q_3, X, R)	(q_5, b, L)	—	—
q_3	(q_3, a, R)	(q_3, b, R)	—	(q_4, a, L)
q_4	(q_4, a, L)	(q_4, b, L)	(q_2, X, R)	—
q_5	(q_5, a, L)	(q_5, b, L)	(q_5, a, L)	(q_0, B, R)
q_6	(q_6, B, R)	(q_7, b, R)	—	—
* q_7	—	—	—	—

(iii) (C)

(iv) (B)

7. TM, $M = (\{q_0, q_1, q_2\}, \{0,1\}, \{0,1,B\}, \delta, q_0, B, \{q_2\})$

(i) (4 Marks)

δ	0	1	B
$\rightarrow q_0$	$(q_0, 0, R)$	$(q_0, 1, R)$	(q_1, B, L)
q_1	$(q_2, 0, R)$	$(q_1, 1, R)$	—
$* q_2$	—	—	—

(ii) MPCP Instance, $\omega = 100$ (4 Marks)

Rule	List A	List B	Source
(1)	#	$\#q_0100\#$	
(2)	0	0	
	1	1	
	#	#	
(3)	q_00	$0q_0$	$\delta(q_0, 0) = (q_0, 0, R)$
	q_01	$1q_0$	$\delta(q_0, 1) = (q_0, 1, R)$
	$0q_0\#$	$q_10\#$	$\delta(q_0, B) = (q_1, B, L)$
	$1q_0\#$	$q_11\#$	$\delta(q_0, B) = (q_1, B, L)$

(4)	$q_1 0$ $q_1 1$	$0 q_2$ $1 q_1$	$\delta(q_1,0)=(q_2,0,R)$ $\delta(q_1,1)=(q_1,1,R)$
(5)	$0 q_2 0$ $0 q_2 1$ $1 q_2 0$ $1 q_2 1$ $0 q_2$ $1 q_2$ $q_2 0$ $q_2 1$	q_2 q_2 q_2 q_2 q_2 q_2 q_2 q_2	
(6)	$q_2 \# \#$	$\#$	

ID for 100 (2 Marks)

$q_0 100 \vdash 1 q_0 00 B$
$\vdash 10 q_0 0 B$
$\vdash 100 q_0 B$
$\vdash 10 q_1 0 B$
$\vdash 100 q_2 B$

TM halts. Since q_2 is a final state "100" is accepted.

A: #$q_0$100#1$q_0$00#10$q_0$0#100q_0#10$q_1$0#100q_2#10q_2#1q_2#q_2#

B: #$q_0$100#1$q_0$00#10$q_0$0#100q_0#10$q_1$0#100q_2#10q_2#1q_2#q_2#

∴ MPCP has solution.

(iii) Code for TM

$q_0 \Rightarrow q_1$
$q_1 \Rightarrow q_2$
$q_2 \Rightarrow q_3$

$0 \Rightarrow X_1$
$1 \Rightarrow X_2$
$B \Rightarrow X_3$

$L \Rightarrow D_1$
$R \Rightarrow D_2$

$\delta(q_1, x_1) = (q_1, x_1, D_2)$

$\delta(q_1, x_2) = (q_1, x_2, D_2)$

$\delta(q_1, x_3) = (q_2, x_3, D_1)$

$\delta(q_2, x_1) = (q_3, x_1, D_2)$

$\delta(q_2, x_2) = (q_2, x_2, D_2)$

$C_1 = 0101010100$

$C_2 = 0100101001 00$

$C_3 = 0100010010000 10$

$C_4 = 0010100001 0100$

$C_5 = 0010010010010 00$

(4 Marks)

Code for TM is

$C_1 11 C_2 11 C_3 11 C_4 11 C_5$

(2 Marks)

0101010100 11 0100101001 00 11 0100010010000 11

0010111001010001 0 1001100100100100100

(iv) (C)

(v) (D)