EXPERIMENT NO. 5

AIM: Simulation study of random processes. Find various statistical parameters of the random process.

OBJECTIVE: Write a program to study two random Processes:

- Auto correlation
- Cross correlation on MATLAB.

APPARATUS /SW: MATLAB Version _______.

THEORY:

Autocorrelation and cross-correlation are statistical measures used to analyze random processes. Autocorrelation quantifies the similarity between a random process and a time-shifted version of itself, while cross-correlation measures the similarity between two different random processes at different time points.

Autocorrelation:

- Definition:

Autocorrelation measures how well a random process correlates with itself at different time lags. It essentially checks how predictable a process is based on its past values.

Cross-correlation:

- Definition:

Cross-correlation measures the similarity between two different random processes at different time lags.

Mathematical Representation:

Autocorrelation:

For a random process $X(t)$, the autocorrelation function $R_{XX}(\tau)$ is defined as $R_{XX}(\tau) = E[X(t)X(t+\tau)]$, where E is the expected value and τ is the time lag.

Cross-correlation:

For two random processes $X(t)$ and $Y(t)$, the cross-correlation function $R_{XY}(\tau)$ is defined as $R_{XY}(\tau) = E[X(t)Y(t+\tau)]$, where E is the expected value and τ is the time lag.

Applications:

- Signal Processing: Analyzing audio signals, image processing, communication systems.
- Finance: Modeling stock prices, predicting market behavior, portfolio optimization.
- Economics: Understanding relationships between economic indicators.
- Engineering: Analyzing system behavior, detecting anomalies, control systems.

The autocorrelation function of a random signal describes the general dependence of the values of the samples at one time on the values of the samples at another time. Consider a random process x(t) (i.e. continuous-time), its autocorrelation function is written as:

$$R_{xx}(\tau) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x(t)x(t+\tau)dt \qquad (1)$$

Where T is the period of observation.

$R_{xx}(\tau)$ is always real-valued and an even function with a maximum value at $\tau = 0$.

For sampled signal (i.e. sampled signal), the autocorrelation is defined as either biased or unbiased defined as follows:

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$$R_{xx}(m) = \frac{1}{N-|m|} \sum_{n=1}^{N-m+1} x(n)x(n+m-1) \qquad \textit{[Biased Autocorrelation]}$$

(2)

$$R_{xx}(m) = \frac{1}{N} \sum_{n=1}^{N-m+1} x(n)x(n+m-1) \qquad \textit{[Uniased Autocorrelation]}$$

for m=1,2,…,M+1

where M is the number of lags.

Some of its properties are listed in table 1.1.

The *cross correlation* function however measures the dependence of the values of one signal on another signal. For two WSS (Wide Sense Stationary) processes x(t) and y(t) it is described by:

$$R_{xy}(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-T}^{T} x(t)y(t+\tau)dt \tag{3}$$

or

$$R_{yx}(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-T}^{T} y(t)x(t+\tau)dt \tag{4}$$

where T is the observation time.

For sampled signals, it is defined as:

$$R_{yx}(m) = \frac{1}{N} \sum_{n=1}^{N-m+1} y(n)x(n+m-1) \tag{5}$$

m=1,2,3,..,N+1

Where N is the record length (i.e. number of samples).

The properties of cross correlation function are listed in table 1.2.

Autocorrelation Properties

[1] Maximum Value:

The magnitude of the autocorrelation function of a wide sense stationary random process at lag m is upper bounded by its value at lag m=0:

$$R_{xx}(0) \geq |R_{xx}(k)| \text{ for } k \neq 0$$

[2] Periodicity:

If the autocorrelation function of a WSS random process is such that:

$R_{xx}(m_0) = R_{xx}(0)$ *for some* m_0, *then* $R_{xx}(m)$ *is periodic with period* m_0. *Furthermore* $E[|x(n) - x(n-m_0)|^2] = 0$ *and x(n) is said to be mean-square periodic.*

[3] The autocorrelation function of a periodic signal is also periodic:

Example: if $x(n) = A\sin(\omega_0 n + \varphi)$, *then,* $R_{xx}(m) = \frac{A^2}{2}\cos(\omega_0 m)$

Therefore if $\omega_0 = \frac{2\pi}{N}$, *then* $R_{xx}(m)$ *is periodic with periodic N and x(n) is mean-square periodic.*

[4] Symmetry:

The autocorrelation function of WSS process is a conjugate symmetric function of m:

$$R_{xx}(m) = R^*_{xx}(-m)$$

For a real process, the autocorrelation sunction is symmetric: $R_{xx}(m) = R_{xx}(-m)$

[5] Mean Square Value:

The autocorrelation function of a WSS process at lag, m=0, is equal to the mean-square value of the process:

$$R_{xx}(0) = E\left\{|x(n)|^2\right\} \geq 0$$

[6] If two random processes x(n) and y(n) are uncorrelated, then the autocorrelation of the sum x(n)=s(n)+w(n) is equal to the sum of the autocorrelations of s(n) and w(n):

$$R_{xx}(m) = R_{ss}(m) + R_{ww}(m)$$

[7] The mean value:

The mean or average value (or d.c.) value of a WSS process is given by:

$$mean, \bar{x} = \sqrt{R_{xx}(\infty)}$$

Properties of cross correlation function
[1] $R_{xy}(m)$ *is always a real valued function which may be positive or negative.*
[2] $R_{xy}(m)$ *may not necessarily have a maximum at m=0 nor* $R_{xy}(m)$ *an even function.*
[3] $R_{xy}(-m) = R_{yx}(m)$
[4] $\left\|R_{xy}(m)\right\|^2 \leq R_{xx}(0)R_{yy}(0)$
[5] $\left\|R_{xy}(m)\right\| \leq \frac{1}{2}\left[R_{xx}(0) + R_{yy}(0)\right]$
[6] When $R_{xy}(m) = 0$*, x(n) and y(n) are Sid to be 'uncorrelated' or they are said to be statistically independent (assuming they have zeros mean.)*

Flowchart: Draw on blank page

Algorithm: Draw on blank page

Program: Attach the Printout of Program

Input, Output, Result: Attach the printout of Input Output and Result.

Conclusion:

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Questions:

1. **Explain the subplot in MatLab?**
2. **Explain Grid and stem?**
3. **Explain difference between auto correlation and cross correlation?**
4. **Solve the Autocorrelation for [4 5 6 7] samples and crosscorelation for [2 1 4 6] manually.**

Timely submission (10)	Journal Presentation(10)	Performance(10)	Understanding(10)	Oral(10)	Total (50)
Sub Teacher Sign:					