Practical - 7

* Sequencing problem, N - jobs through 2 Machines.

* The Algorithm

Step-1: List the jobs along with their processing times in a table as shown below:

Processing Time on Machine	Job numbers				
	1	2	3	----	n
M_1	t_{11}	t_{12}	t_{13}	----	t_{1n}
M_2	t_{21}	t_{22}	t_{23}	----	t_{2n}

Step-2: Examine the columns for processing times on machines M_1 and M_2, and find the smallest processing time in each column, ie. find out, $\min(t_{1j}, t_{2j})$ for all j.

Step-3(a): If the smallest processing time is for the first machine M_1, then place the corresponding job in the first available position in the sequence. If it is for the second machine, then place the corresponding job in the last available position in the sequence.

(b) If there is a tie in selecting the minimum of all the processing times, then there may be three situations:

(i) Minimum among all processing times is same for the machine.

ie $\min(t_{1j}, t_{2j}) = t_{1k} = t_{2r}$, then process the k^{th} job first and the r^{th} job last.

(ii) If the tie for the minimum occurs among processing times t_{1j} on machine M_1 only, then select the job corresponding to the smallest job subscript first.

(iii) If the tie for the minimum occurs among processing times t_{2j} on machine M_2, then select the job corresponding to the largest job corresponding to the largest job subscript last.

Step-4: Remove the assigned jobs from the table. If the table is empty, stop and go to step-5. otherwise, go to step-2.

Step-5: Calculate idle time for machines M_1 and M_2.

(a) Idle time for machine M_1 =

(Total elapsed time) - (Time when the last job in a Sequence finishes on Machine M_1)

(b) Idle time for machine M_2 =

Time at which the first job in a sequence finishes on Machine M_1 + $\sum_{j=2}^{n}$ {(Time when the j^{th} job in a Sequence starts on machine M_2) - (Time when the $(j-1)$th job in a sequence finishes on machine M_2)}.

Step-6: The total elapsed time to process all jobs through two machines is given by

Total elapsed time = Time when the n^{th} job in a Sequence finishes on machine M_2.

$$= \sum_{j=1}^{n} M_{2j} + \sum_{j=1}^{n} I_{2j}$$

where M_{2j} = Time required for processing j^{th} job on machine M_2.

I_{2j} = time for which machine M_2 remains idle after processing $(j-1)$th job and before starting work in j^{th} job.

Example: (1) Suppose we have five jobs, each of which has to be processed on two machines A & B in the order AB. Processing times are given in the following table.:

Job	Machine A	Machine B.
1	6	3
2	2	7
3	10	8
4	4	9
5	11	5

Determine an order in which these jobs should be processed so as to minimize the total processing time.

Solution: We can write the ~~follow~~ given problem in following table

Job	1	2	3	4	5
Machine A	6	2	10	4	11
Machine B	3	7	8	9	5

The smallest processing time between the two machine is 2 which corresponds to job 2 on machine A. Thus, job 2 will be processed first as shown below.

After the job 2 has been set for processing first we are left with 4 job and their processing time as given below.

Job	1	3	4	5
Machine A.	6	10	4	11
Machine B	3	8	9	5

The minimum processing time in this reduced problem is 3 which corresponds to job 1 on machine B

Thus, job 1 will be processed ~~first~~ last as shown below.

After the job 2 and job 1 has been set, we are left with 3 job and their processing time as ~~shown~~ given below.

Job	3	4	5
Machine A	10	4	11
Machine B	8	9	5

Similarly, doing previously we get the following table.

2	4	3	5	1

Here, we get this optimal job sequence

The minimum elapsed time for machines A and B is calculated as shown in following table.

Job	Machine A		Machine B		Idle time	
	In	out	In	out	A	B
2	0	0+2 = 2	2	2+7 = 9		2
4	2	2+4 = 6	9	9+9 = 18		
3	6	6+10 = 16	18	18+8 = 26		1
5	16	16+11 = 27	27	27+5 = 32		1
1	27	27+6 = 33	32	32+3 = 35	3	
					3	34

(i) Idle Time for machine A = 3 hours
Idle time for machine B = 4 hours
Total time for machines = 7 hours.

(ii) Total Elapsed time for machines A and B both to complete the work = 36 hours.

(2) Strong Book Binder has one printing machine, one binding machine, and the manuscripts of a number of different books. Processing times are given in the following table.

Book	Time in Hours	
	Printing	Binding
A	5	2
B	1	6
C	9	7
D	3	8
E	10	4

We wish to determine the order in which books should be processed on the machines, in order to minimize the total time required.

Sol^{n}: We can write the given problem in following table.

Book	A	B	C	D	E
Printing	5	1	9	3	10
Binding	2	6	7	8	4

First we find optimal book sequence. We get the following book sequence.

B	D	C	E	A

Printing Binding

The minimum elapsed time for Printing and Binding is calculated as shown in the following table.

Book	Printing		Binding		Idle time	
	In	out	In	out	Printing	Binding
B	0	0+1=1	1	1+6=7		1
D	1	1+3=4	7	7+8=15		
C	4	4+9=13	15	15+7=22		1
E	13	13+10=23	23	23+4=27		1
A	23	23+5=28	28	28+2=30	2	
					2	3

(i) Idle time for ~~machine~~ printing = 2 hours
Idle time for Binding = 3 hours
Total idle time = 5 hours

(ii) Total Elapsed time for printing and Binding to complete the work = 30 hours.

(3)

Projects	Andrew	Julie
A	4	2
B	3	5
C	5	1
D	7	3
E	8	6

Find total idle time and total elapsed time.

Soln: We can write the given problem in following table.

Projects	A	B	C	D	E
Andrew	4	3	5	7	8
Julie	2	5	1	3	6

First we find optimal project sequence. We get the following project sequence.

B	E	D	A	C

Andrew Julie

The minimum elapsed time for Andrew and Julie is calculated as shown in the following table

Projects	Andrew		Julie		Idle time	
	In	out	In	out	Andrew	Julie
B	0	0+3=3	3	3+5=8		3
E	3	3+8=11	11	11+6=17		1
D	11	11+7=18	18	18+3=21		1
A	18	18+4=22	22	22+2=24		3
C	22	22+5=27	27	27+1=28	1	
					1	8

(i) Idle time for Andrew = 1 hour
Idle time for Julie = 8 hours
Total idle time = 9 hours

(ii) Total Elapsed time for Andrew and Julie to complete the projects = 28 hours.

(4) There are nine jobs, each of which must go through two machines P and Q in the order PQ. the processing times (in hours) are given below.

Machine	Job (s)								
	A	B	C	D	E	F	G	H	I
P	2	5	4	9	6	8	7	5	4
Q	6	8	7	4	3	9	3	8	11

Find the sequence that minimizes the total elapsed time T. Also calculate the total idle time for the machines in this period.

First we find optimal job sequence.
We get the following job sequence.

A	C	I	B	H	F	D	G	E

P Q

or

A	C	I	H	B	F	D	G	E

P Q

The minimum elapsed time for P and Q is calculated as shown in the following table.

Jobs	Machine P		Machine Q		Idle time	
	In	out	In	out	P	Q
A	0	0+2 = 2	2	2+6 = 8		2
C	2	2+4 = 6	8	8+7 = 15		
I	6	6+4 = 10	15	15+11 = 26		
H	10	10+5 = 15	26	26+8 = 34		
B	15	15+5 = 20	34	34+8 = 42		
F	20	20+8 = 28	42	42+9 = 51		
D	28	28+9 = 37	51	51+4 = 55		
G	37	37+7 = 44	55	55+3 = 58		
E	44	44+6 = 50	58	58+3 = 61	11	

(i) Idle time for machine P = 11 hours
Idle time for machine Q = 2 hours
total idle time = 13 hours

(ii) Total elapsed time for machine P and machine Q to complete the job = 61 hours.

(5) Consider a single-workstation sequencing problem for 6 jobs. The job times and due dates are shown in following table. Find the sequence ~~using the~~ and total idle time and total elapsed time.

Job	A	B	C	D	E	F
Processing time	16	5	12	17	8	11
Due date time	24	12	55	46	16	36

First we find optimal job sequence.
We get the following job sequence.

B	E	F	C	A	D

Processing time — Due date

The minimum elapsed time for processing time and due date is calculated as shown in the following table.

Jobs	Processing time		Due date		Idle time	
	In	out	In	out	Processing	Duedate
B	0	0+5=5	5	5+12=17		5
E	5	5+8=13	17	17+16=33		
F	13	13+11=24	33	33+36=69		
C	24	24+12=36	69	69+55=124		
A	36	36+16=52	124	124+24=148		
D	52	52+17=69	148	148+46=194	125	

(i) Idle time for processing time = 125 hours
Idle time for due date time = 5 hours
Total idle time = 130 hours

(ii) Total elapsed time for processing time and due date time to complete the jobs
= 194 hours.