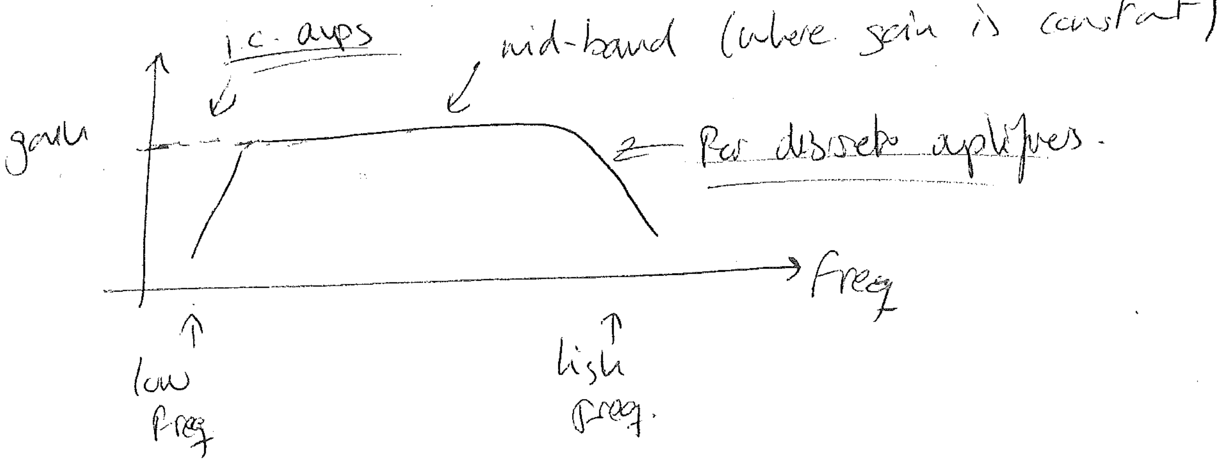
10/28/2016

FREQUENCY RESPONSE OF AMPLIFIERS

→ So far gain has been constant.

→ In real apps. the gain varies w/ frequency.



Is this good or bad?

→ Good/bad depending on application

- Hearing aid
- RF (cell phones)
- radio tuning
- Audio equipment

10/28/2016

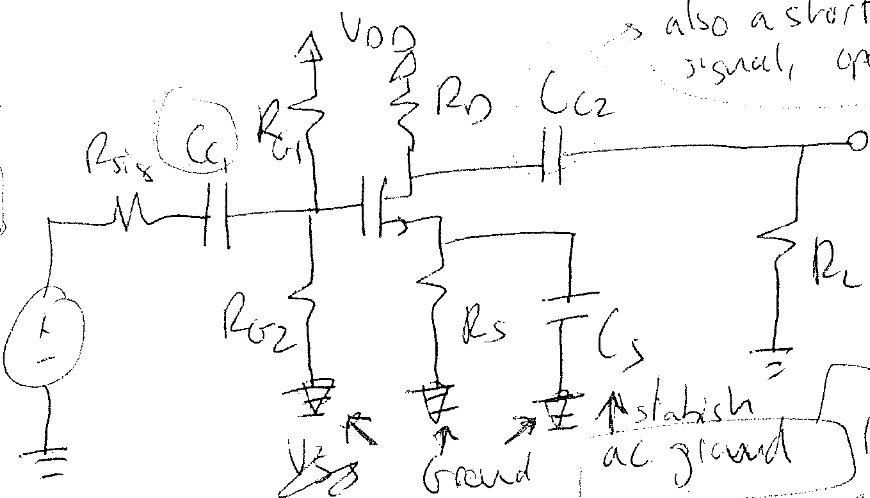
7.5.1 → The discrete CS Amplifier

(2) → Capacitors used to couple the signal source to the input of the amplifier. To couple the output to the load. And to establish a signal ground ~~at~~ at transistor terminals. → large values.

→ Capacitors block DC → do not affect DC analysis.

→ also a short for signal, open for DC.

(does not change DC bias point but ac signal goes thru.)



$R_{in} = R_{G1} \| R_{G2}$

$A_v = -\frac{R_{in}}{R_{in}+R_{sig}} g_m (R_D \| R_L \| r_o)$

↑ establish ac ground

Ground

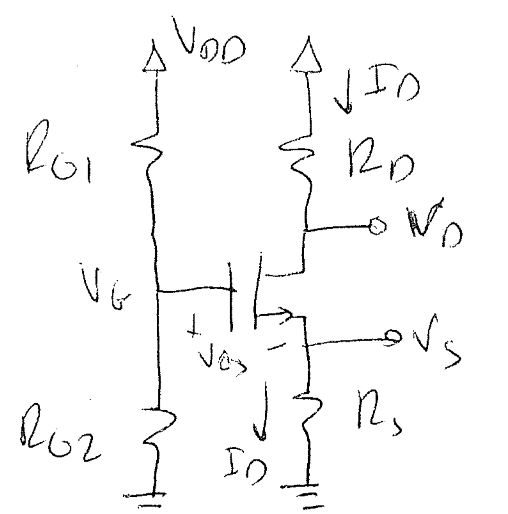
C → large
low impedance at freq. of interest
$Z = \frac{1}{C\omega}$ → signal current bypasses R_S.

→ $R_{G1} + R_{G2}$ large to keep high input impedance.

→ ~~R~~.

... here:

(1) →



$R_{G1} + R_{G2}$ → large

R_D → large for gain but in sat.

Discrete CS w/ biasing resistors.

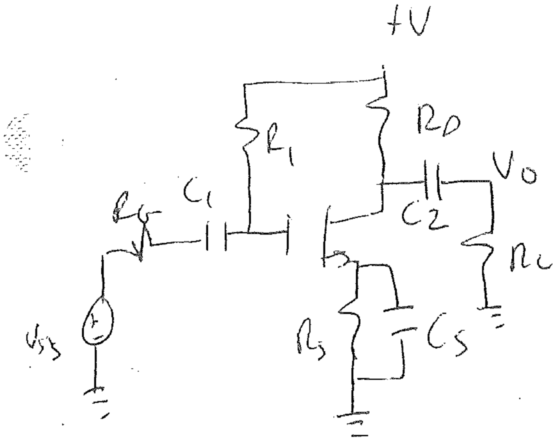
R_S provides negative feedback. $V_G = V_{GS} + I_D R_D$

→ $I \uparrow$ V_G constant, when I_D increases V_{GS}

10/28/2016

2

C.S example



→ Capacitances can no longer be considered as $Z \to 0$.

$$Z_C = \frac{1}{WC}$$

→ At low freq.

Cannot be ignored: C_1, C_2, C_3

→ $C_1 = \frac{1}{WC}$ less v_{sig} set to Vgate.

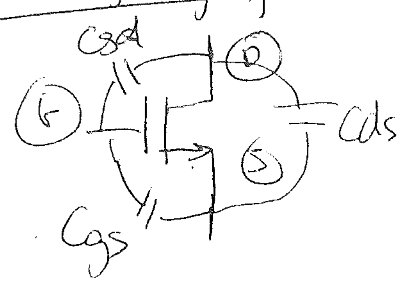
→ $C_2 \longrightarrow$ less v_D sets to load V_o.

→ C_S → effective impedance combines w/ R_s.

↳ originally to remove ac and stabilize Q-point.

At low freq some a.c. shows up through R_s → affects Q-point, gain.

→ At high frequencies



At low frequencies: Cgd, Cgs, Cds have high impedance and act like open circuit, don't exist.

As frequency increases → high freq.

Cgs, Cgd, Cds → low impedance and bridge between terminals.

10/20/2016

(3)

→ Simplify further by

→ Solving for each cap and determining which one is dominant for low freq + high freq.

Freq. Response of amplifiers contd

3 regions: (1) mid-band (purely resistance)

large C → short circuit

small C → open circuit)

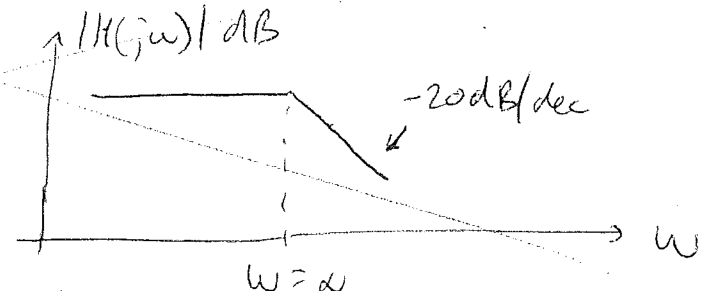
(2) low freq: bypass caps.

(3) high freq: device caps.

☆ Bode Plots.

Transfer functions $H(s)$ with negative real poles + zeros.

(1) $H(s) = \frac{1}{s+\alpha}$



Impedance of elements → transfer function.

s = complex frequency $s = j\omega$, trans. function $H(s) = \frac{V_o(s)}{V_i(s)}$ or $\frac{I_o}{I_i}$ or $\frac{V_o}{I_i}$ or $\frac{I_o}{V_i}$

$j\omega L$

$\frac{1}{j\omega C}$

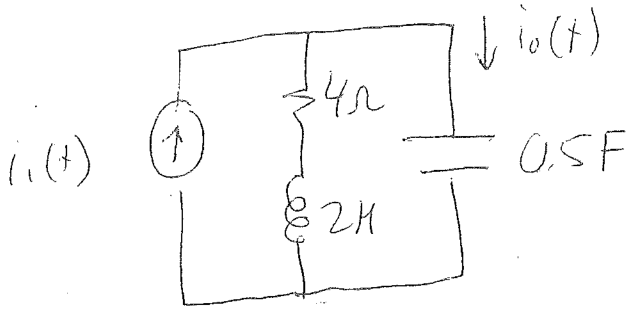
Transfer functions have poles and zeros.

Poles → values (of s) at which the function goes to infinity.

Zeros → " (of s) " " " " " " zero.

→ Only need poles + zeros to recreate transfer function.

10/31/2016



Calculate transfer function: $H(s) = \frac{I_o(s)}{I_i(s)}$

$$I_o(\omega) = \frac{4 + j2\omega}{4 + j2\omega + \frac{1}{j0.5\omega}} I_i(\omega)$$

$$\frac{I_o(\omega)}{I_i(\omega)} = \frac{j0.5\omega(4 + j2\omega)}{1 + j2\omega + (j\omega)^2} = \frac{s(s+2)}{s^2 + 2s + 1} \qquad s = j\omega$$

→ zeros $s(s+2) = 0 \Rightarrow z_1 = 0,\ z_2 = -2$

→ the poles are at
$$s^2 + 2s + 1 = (s+1)^2 = 0$$

double pole at $p = -1$

10/31/2016
Madhu

Frequency response

$$H(s)\Big|_{s=j\omega} \qquad F(s)\big|_{j\omega} = \cancel{F(j\omega)} \approx 10^6 \frac{j\omega(j\omega+1500)}{(j\omega+500)(j\omega+100)}$$

↑ s-space ↑ "real" $j\omega$ space.

$F(s)$: Gain function.

$20 \cdot \log |F(j\omega)| =$ Gain in dB. ← this is what we want.

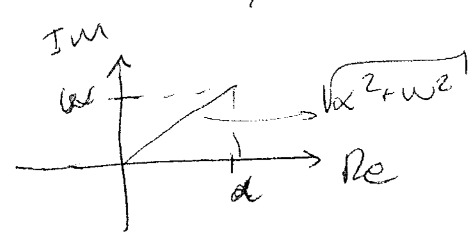
→ Want freq. response to find unstable frequencies and re-design or limit if necessary.

Instead of analyzing function by plugging in all frequencies →

Dr. Hendrick W. Bode:

Developed approach using asymptotic frequency response plots. → using straight lines.

ex. $H(s) = \dfrac{1}{s+\alpha}$ → single pole at $s = -\alpha$ (α → a number 5, 2000 (whatever))



$s = j\omega$

$H(j\omega) = \dfrac{1}{\alpha + j\omega}$

Magnitude: $|H(j\omega)| = \dfrac{1}{\sqrt{\alpha^2+\omega^2}}$

Convert to dB → $20 \log |H(j\omega)| = 20 \log \dfrac{1}{\sqrt{\alpha^2+\omega^2}}$

$= -20 \log \sqrt{\alpha^2+\omega^2} = -10 \log(\alpha^2+\omega^2)$

10/31/2016
(2)
Madhu.

slope: -20 dB/decade.
when $\omega = \omega_x$: $-20 \log \omega_x$

$\omega = 10\omega_x$: $-20 \log (10\omega_x)$

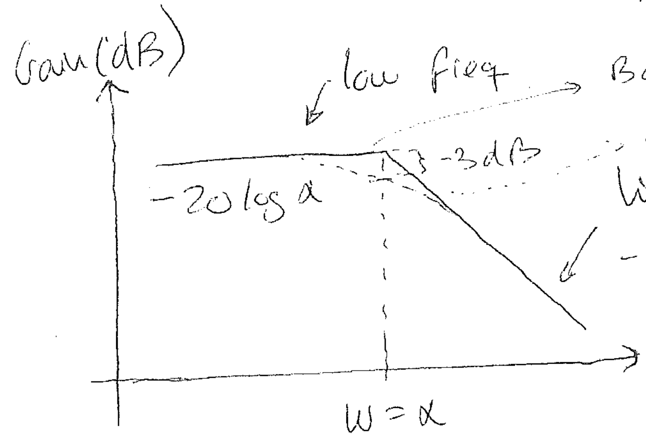
$= \underbrace{-20 \log 10}_{-20 \text{ dB/decade.}} - 20 \log \omega_x$

also: -6 dB/octave
↳ frequency factor of 2.

$$|H(j\omega)| = \frac{1}{\sqrt{\alpha^2 + \omega^2}}$$

→ Bode's method can represent this function by two-straight lines.

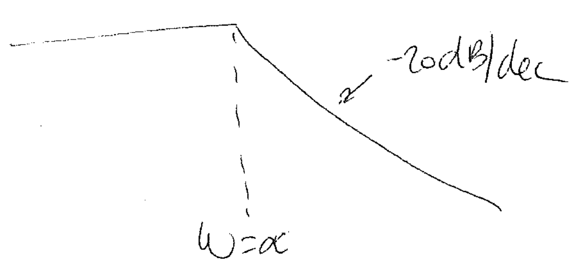
Bode-plot of Single Pole function



→ Bode approx.
actual behaviour (-3dB difference)
high freq.
$-20 \log \omega$

→ plot as log to cover many frequencies and also equal spacing.

First-case:
→ Single poles $s = -\alpha$ $H(s) = \frac{1}{s + \alpha}$



names: { cut-off frequency, break frequency
corner frequency, 3dB frequency

11/2/2016
Madhu

Bode Plots

Magnitude plots.

|Gain|dB vs. log ω.

obtain gain, convert to dB

critical frequencies

① Make a list of poles and zeros in ascending order.

② • If s=0 is not a critical frequency, then start w/ a horizontal line

• If s=0 is a pole → start with an initial slope −20 dB/dec

• If s=0 is a zero → start with an initial positive slope +20 dB/dec

③ If you hit a zero, slope changes by +20 dB/dec

If you hit a pole, slope changes by −20 dB/dec

→ Second order poles / zeros change slope by 2x. (± 40dB/dec)

→ Third order poles / zeros change slope by 3x and so on. (±60 dB/dec)

2 11/2/2010
Madhu

ex. pick vale between 100 and 5000

$\omega = 1987 = \cancel{\omega}$
$0 < \omega \rightarrow$ keep s
$5000 > \omega \rightarrow$ keep α
$100 < \omega \rightarrow$ keep s
$10^6 > \omega \rightarrow$ keep α

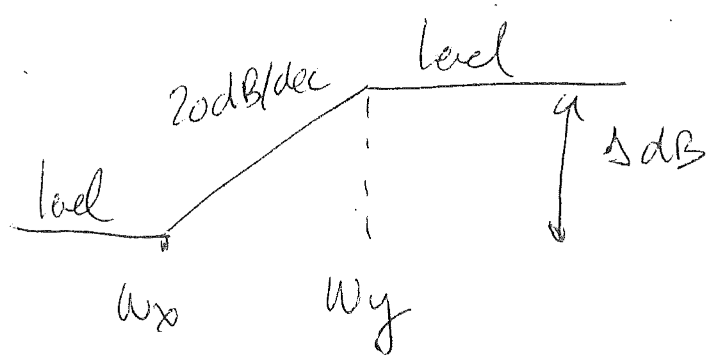
$$10^6 \frac{s(5000)}{s(10^6)} = 5000$$

$$20 \log 5000 = \boxed{73.98 dB}$$

* Method 2: To find the dB value of a level segment.

→ Use known value at other part of plot and calculate value based on slopes.

→ If you know the dB value at some ω_x



$$\Delta dB = 20 \log \left(\frac{\omega_y}{\omega_x} \right)$$

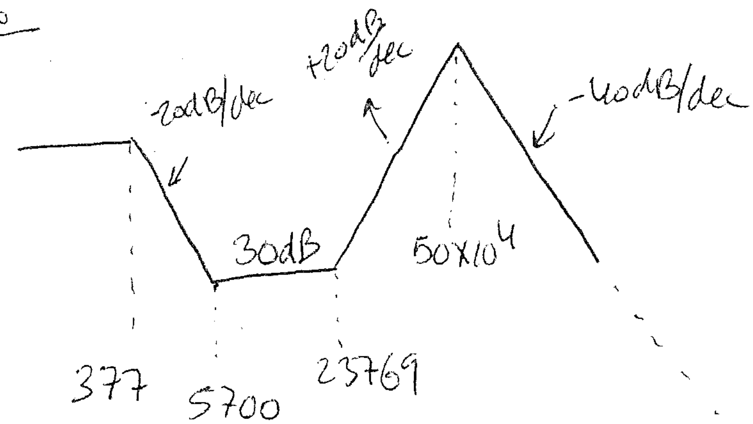
ex. start at 120dB at 10^6 rad/sec
Find dB at 5000.

$$dB_{5000} + 20 \cdot \log \left(\frac{10^6}{5000} \right) = 120$$

$$\boxed{dB_{5000} = 73.98 dB}$$

11/2/2016
3 Madhu

ex



r/s

377 → pole
5700 → zero
23769 → zero
50×10^4 → pole (cubed)
3x slope.
-20dB/dec

$$\rightarrow H(s) = K\,\frac{(s+5700)(s+23769)}{(s+377)(s+50\times10^4)^3}$$

30dB = 20 log H $\quad \frac{30}{20} = \log H \quad H = 10^{30/20}$

$\rightarrow$ 30dB → antilog $\left(\frac{30}{20}\right) = 10^{(30/20)} = 31.62$

$\rightarrow$ Pick ω between 5700 and 23769

$$\rightarrow H(s) = K\,\frac{\cancel{s}\,(23769)}{\cancel{s}\,(50\times10^4)^3} = 31.62$$

$\therefore \boxed{K = 1.663\times10^{14}}$

↳ K by itself has no meaning.
Not DC gain.

$\rightarrow$ Find $H(s)\big|_{\omega\to0} = 1.663\times10^{14}\,\frac{(5700)(23769)}{(377)(50\times10^4)^3} =$

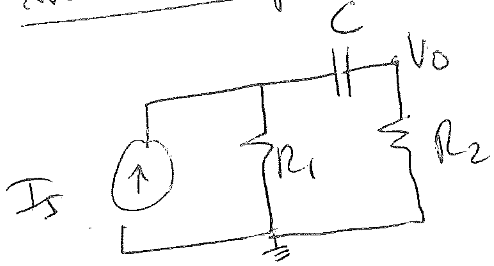
or ~~30dB~~ -

$x - 20\log\left(\frac{5700}{377}\right) = 30\text{dB}$ $\qquad x = 30 - 23.59$

$H(s)| = 6.41\text{dB}$

11/4/2016
Madhu

→ Another example for amplifier discussion:



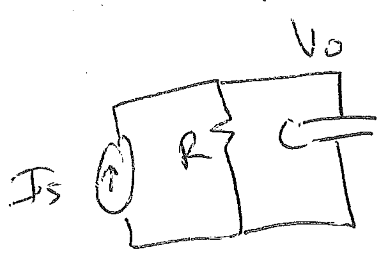
<u>DC</u>: $V_o = 0V$ (cap is open circuit)

<u>$\omega \to \infty$</u>: $V_o = I_s(R_1 \| R_2)$

Trans. funct → $\frac{V_o}{I_s}$

$$\omega = \frac{1}{\underbrace{(R_1 \| R_2)C}_{\tau = R_{TH} C}}$$

→ Another circuit for amplifier discussion:



<u>DC</u>: $\frac{V_o}{I_s} = R$

<u>$\omega \to \infty$</u>: $\frac{V_o}{I_s} = 0$

20 log (DC gain)

$\omega = \frac{1}{RC}$

low pass filter: LPF

11/4/2016

② Madhu

Lo cont's

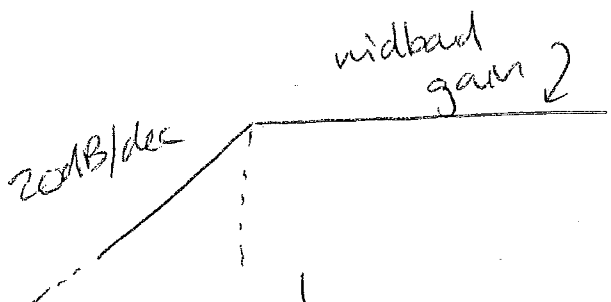
Only $\cancel{\frac{V_{gs}}{V_{gs}}}$ $\frac{V_{gs}}{V_{sig}}$ varies w/ freq.

$$\frac{V_{gs}}{V_{sig}} = \frac{R_{in}}{R_{in} + R_\sigma + \frac{1}{C_1 s}} = \frac{R_{in} C_1 s}{(R_{in} + R_\sigma) C_1 s + 1}$$

$$= \left(\frac{R_{in}}{R_{in} + R_\sigma}\right) \frac{s}{s + \frac{1}{(R_{in} + R_\sigma) C_1}}$$

$s = 0 \to$ zero

$s = -\frac{1}{(R_{in} + R_\sigma) C_1} \to$ pole



$$\omega = \underbrace{\frac{1}{(R_{in} + R_\sigma) C_1}}_{\tau = RC \text{ where } R \Rightarrow R_{TH}.}$$

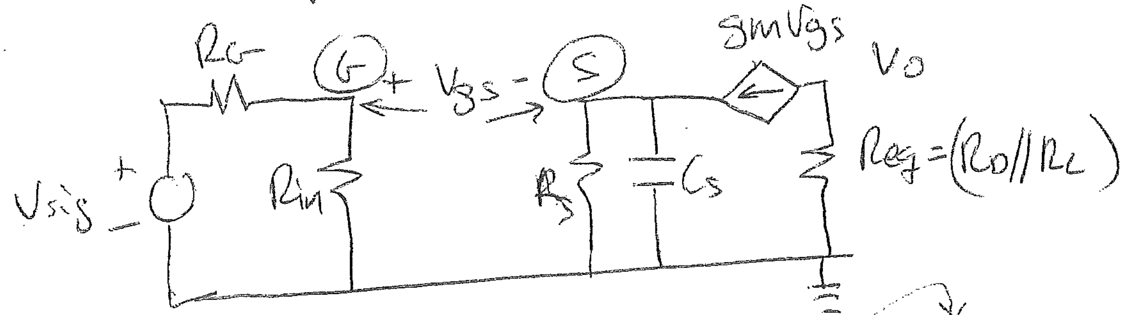
Lower cut-off frequency due to C_1 acting alone $\Big\} = \frac{1}{(R_{in} + R_\sigma) C_1}$

11/4/2[redacted]

③ Mach[redacted].

3:d → Assume C_s is dominant.

Keep C_s and short $C_1 + C_2$



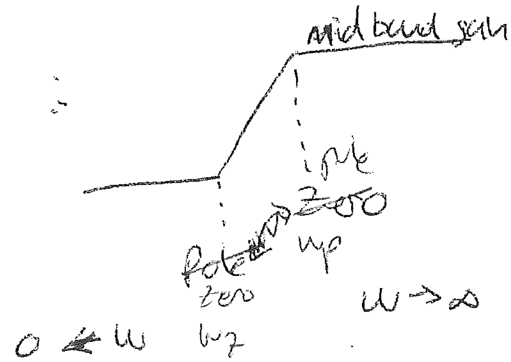
$$\frac{V_o}{V_{sig}} = \underbrace{\frac{V_o}{V_{gs}}}\ \underbrace{\frac{V_{gs}}{V_{sig}}}$$

both vary w/ frequency

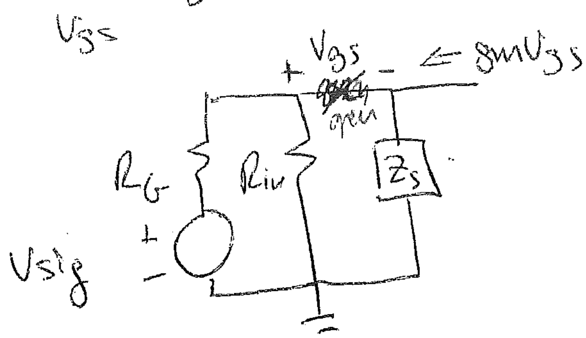
DC $\omega \to 0$ $\quad V_o = -g_m V_{gs} R_{eq}$

$\omega \to \infty$ $\quad V_o = -g_m V_{gs} R_{eq}$.

different V_{gs} but larger.



$$\frac{V_o}{V_{gs}} = -g_m R_{eq}$$



$$V_{gs} = V_g - \overbrace{g_m V_{gs} Z_s}^{V_s}$$

$$V_{gs}(1 + g_m Z_s) = V_g = \frac{R_G}{R_{in} + R_G} V_{sig}$$

$$\frac{V_{gs}}{V_{sig}} = \frac{R_G}{(R_G + R_{in})} \frac{1}{1 + g_m Z_s}$$

$$\boxed{\frac{V_{gs}}{V_g} = \frac{1}{1 + g_m Z_s}}$$ → Function of ω

$$\frac{V_g}{V_{sig}} = \frac{R_{in}}{R_G + R_{in}}$$

↳ finish next time.

11/7/2016

Madhu

↳ cont'd

Lower cut-off frequency:

① Due to C_1 acting alone

$$= \frac{1}{C_1(R_G + R_{in})}$$

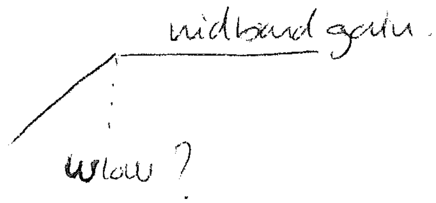
② Due to C_2 acting alone

$$= \frac{1}{C_2(R_D + R_L)}$$

③ Due to C_S acting alone.

$$\omega_z = \frac{1}{R_S C_S} \qquad \omega_p = \frac{1}{C_S(R_S \| \frac{1}{g_m})}$$

Know:



→ use above equations in design.

→ when analyzing circuit then:

→ Case 1: There is a dominant pole.

One of ω_{C1}, ω_{C2}, ω_p $\underbrace{\text{much higher}}_{\text{Factor of 10 or more.}}$ than other two.

→ Case 2: No dominant pole.

→ Plot all ω_{C1}, ω_{C2}, ω_z + ω_p → and study it.

step 1 → Gain function: $A(s) = \dfrac{s^2(s+\omega_z)}{(s+\omega_{C1})(s+\omega_{C2})(s+\omega_p)}$

step 2 → → when $s \to$ large $A(s) = 1$ ← reference midband gain.

step 3 → At ω_{Low}: $|A| = 0.707$

[3dB below 1] then

$$s = j\omega \longrightarrow |A(j\omega)| = \left| \frac{(j\omega)^2(j\omega + \omega_z)}{(j\omega + \omega_{C1})(j\omega + \omega_{C2})(j\omega + \omega_p)} \right| = 0.707$$

② 11/7/2016
Madhu

Using previous example.
Want low frequency cut off = 25 Hz

$\rightarrow \omega_{low} = 2\pi f = 2\pi(25) = 50\pi$

$$\omega_p = 50\pi = \frac{1}{\left(R_s \| \frac{1}{g_m}\right) C_s} = \frac{1}{511.6 \cdot C_s}$$

$$\boxed{C_s = 12.46\,\mu F}$$

$$\rightarrow \omega_z = \frac{1}{R_s C_s} = \frac{1}{(2200)(12.46 \times 10^{-6})} = 36.48\ r/s$$

$$\rightarrow C1: \quad \omega_{C_1} = 36.48\ r/s = \frac{1}{C_1 (R_G + R_{in})} = \frac{1}{C_1 (120k + 220k)}$$

$$\boxed{C_1 = 80.6\,nF}$$

$$\rightarrow C2: \quad \omega_{C_2} = \frac{1}{10}\omega_{low} = 5\pi = \frac{1}{C_2 (R_O + R_L)} = \frac{1}{C_2 (6.6k + 22k)}$$

$$\boxed{C_2 = 2.220\,\mu F}$$