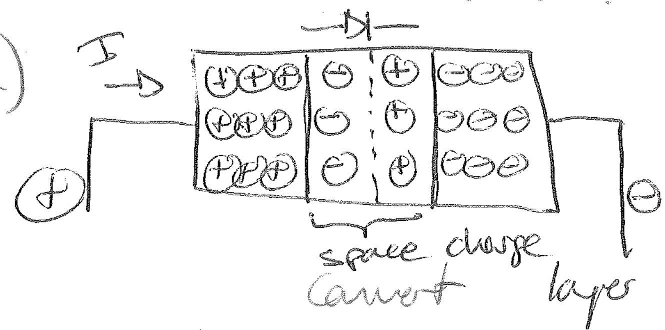
Chapter 4 - Diodes

(5)
lec 3 ↑
lec 4 ↓

→ Simplest and most fundamental nonlinear circuit element.

→ Two terminals. ~~(just like a)~~



(1) Ideal diode

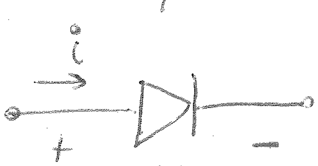
(2) Silicon junction diode.

→ ~~Applications: rectifying circuits (ac to dc)~~

Most common application is in rectifier circuits which convert ac to dc.

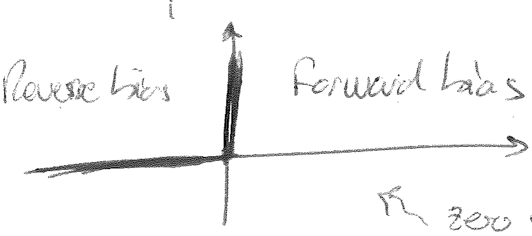
4.1.1 Current-voltage characteristics of the ideal diode

~~Ideal~~ Symbol



anode cathode

Ideal diode i-v characteristics

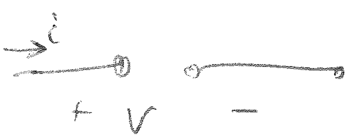


zero voltage drop, any current ~~short cir~~ diode ON

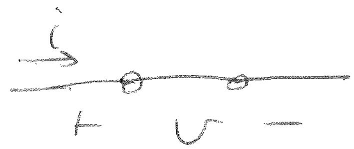
Zero current, diode is OFF

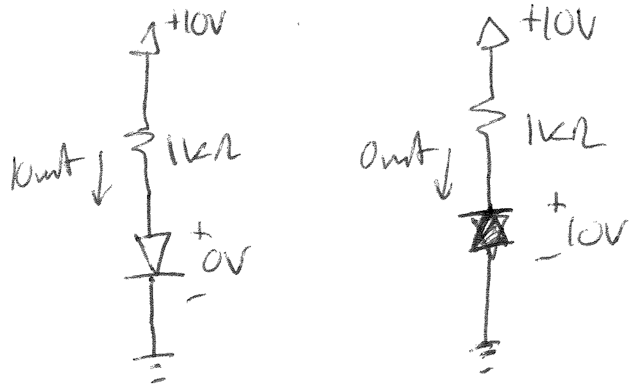
→ ~~External circuit needed to limit forward bias current and ^set reverse bias voltage.~~

$v < 0 \Rightarrow i = 0$



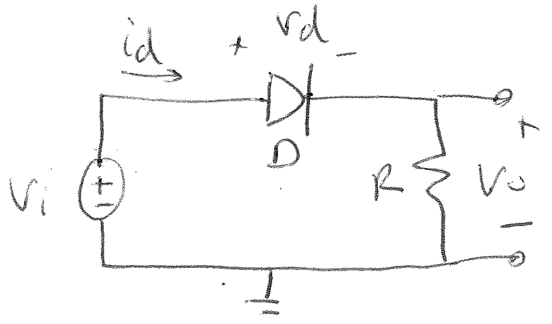
~~$v > 0$~~ = $v = 0$ $i > 0$

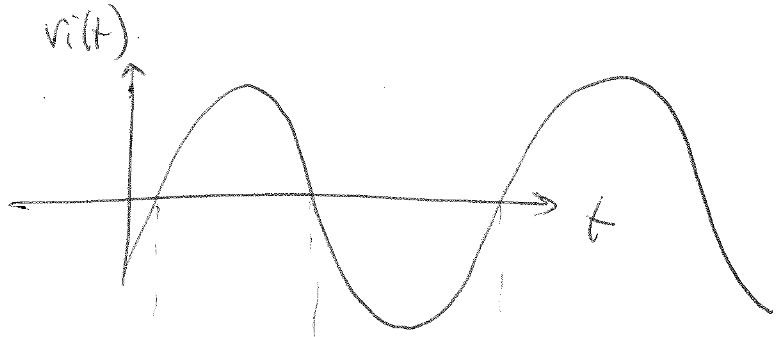


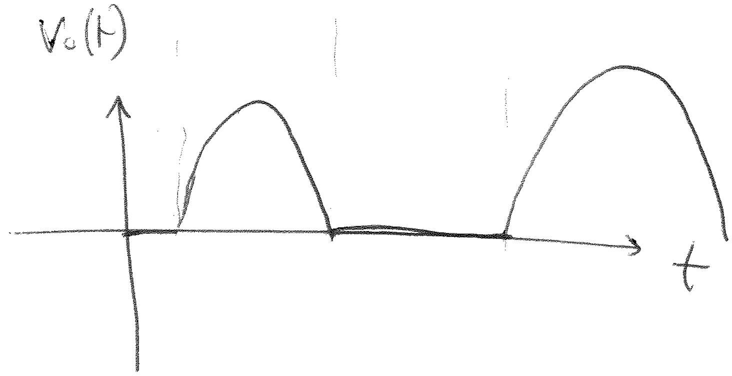


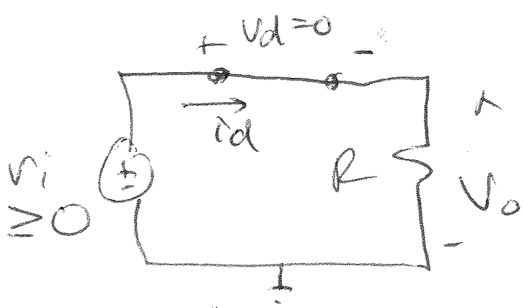
↑ external circuit need to limit forward bias current and to set reverse bias voltage.

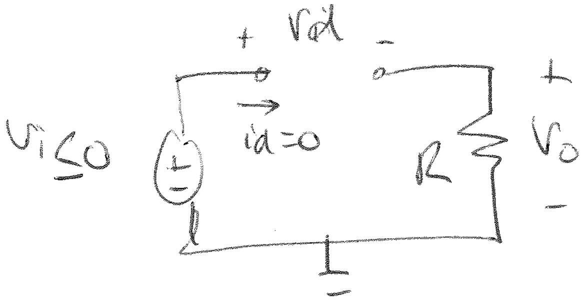
4.12. A simple application: The rectifier circuit.





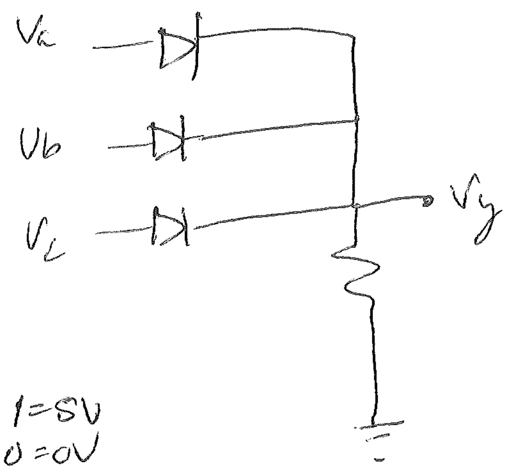






4.1.3 Diode logic gates:

OR Junction

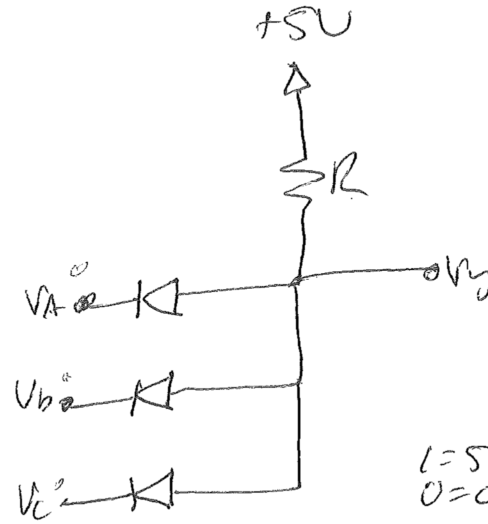


1=5V
0=0V

Va	Vb	Vc	Vy
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

$Y = A + B + C$

AND Junction



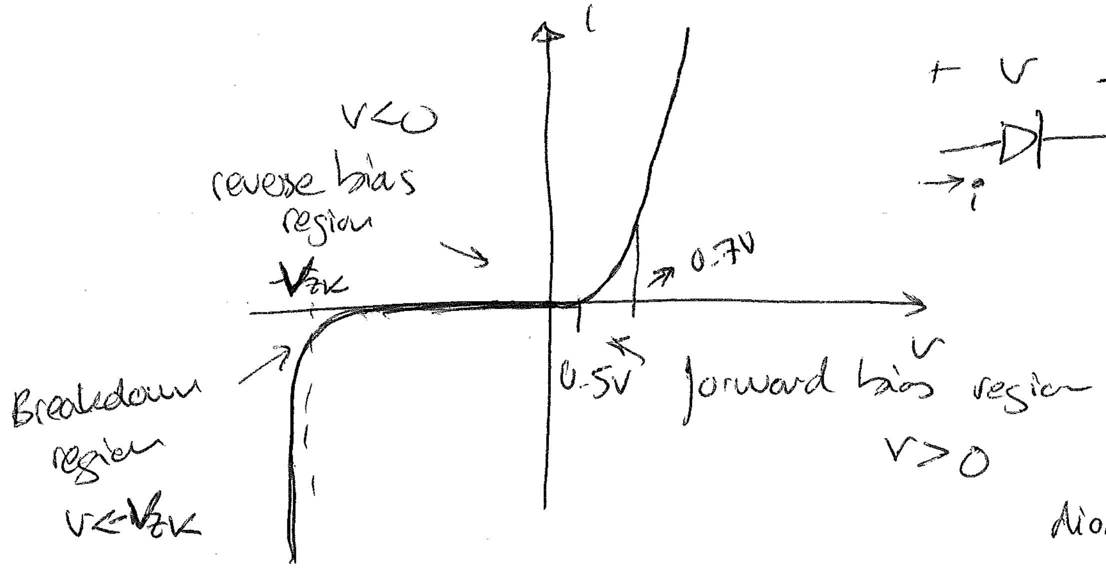
1=5V
0=0V

Va	Vb	Vc	Vy
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

$Y = A \cdot B \cdot C$

4.2 - Terminal characteristic of junction diodes

i-v characteristics of a silicon junction diode.



Diode ideality factor
n=1 from now on

4.2.1 Forward-bias region.

$v > 0$ $\quad \cancel{i = I_S(e^{v/V_T} - 1)}$ $\quad i = I_S\left(e^{v/nV_T} - 1\right)$

I_S - constant at a given temp. for a given diode.
→ called saturation or scaled current.

$$I_S = A \cdot q \cdot n_i^2 \left(\frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A} \right)$$

(10^{-18} to 10^{-12} A)

~~$\sim 10^{-15}$~~ $\sim 10^{-15}$ for small signal diodes.

V_T - thermal voltage

$$V_T = \frac{kT}{q}$$

k - boltzmann's constant $= 8.62 \times 10^{-5}$ eV/K
$= 1.38 \times 10^{-23}$ J/K

T - absolute temp (kelvin)

q → electronic charge $= 1.6 \times 10^{-19}$ coulomb

cont'd

1 cont'd

(at 20°C)

$V_T = 25.3\,mV$ → use $25\,mV$ at Room Temperature for quick analysis.

→ For $i \gg I_S$

$$i \simeq I_S e^{v/V_T}$$

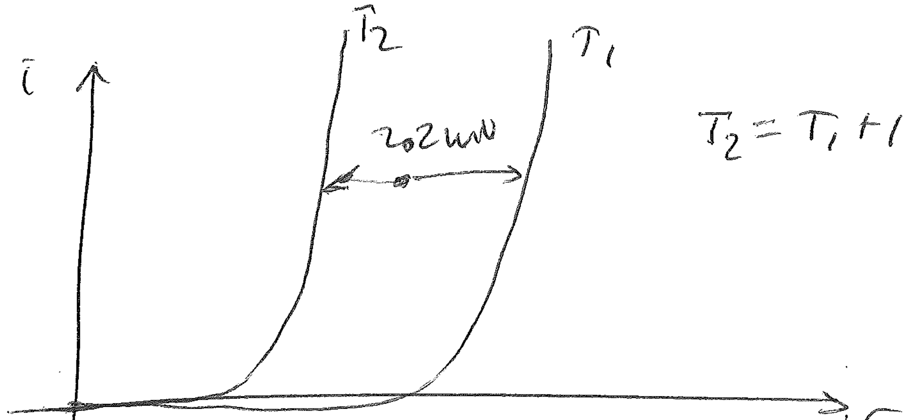
and $$v = V_T \ln \frac{i}{I_S}$$

→ slope of 60 mV/decade of current at $i \gg I_S$

→ Current is negligible for $v < 0.5V$.

→ TURN-ON voltage of diode ~ 0.7V.

→ At a constant current, the voltage drop across the diode decreases by ~2.2mV for every 1°C increase in temperature. (−2.2mV/°C)



$T_2 = T_1 + 1°C$

lec4 ↑
lec5 ↓

4.2.2.- The reverse-bias region.

when $v<0$

$$i = I_S\left(e^{v/V_T}-1\right)$$

$\hookrightarrow$ $i \simeq -I_S$

Current in reverse direction is constant and equal to I_S.
(reason to call it saturation current).

- → Real diodes $i(v<0)$ is larger and proportional to diode area.
- → Leakage current.
- → Leakage current doubles for every 10°C rise in temp.

4.2.3 Breakdown region

- → $v < -V_{ZK}$ Z → zener, K → knee (zener knee voltage)
- → Diode breakdown can be non-destructive if current limiting circuitry is used. (current limit on datasheet).
- → Used ~~as~~ in voltage regulation circuits

4.3 - Modeling the diode forward characteristics.

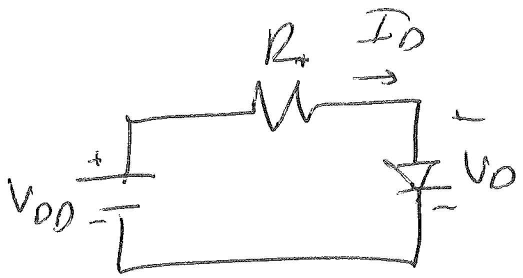
(1) Exponential model

(2) Ideal diode model

(3) ... (others)

4.3.1 Exponential model

Most accurate but most difficult to use.



Assume $V_{DD} > 0.5$

$\therefore\ i \gg I_S$

and

$$I_D = I_S e^{V_D/V_T} \quad \text{— (1)}$$

Also, KVL

$$-V_{DD} + I_D R + V_D = 0$$

$$I_D R = V_{DD} - V_D$$

$$I_D = \frac{V_{DD} - V_D}{R} = \frac{V_{DD} - V_D}{R} \quad \text{— (2)}$$

→ 2 equations / 2 unknowns (I_D, V_D)

if I_s is known ($V_T = 25\,mV$)

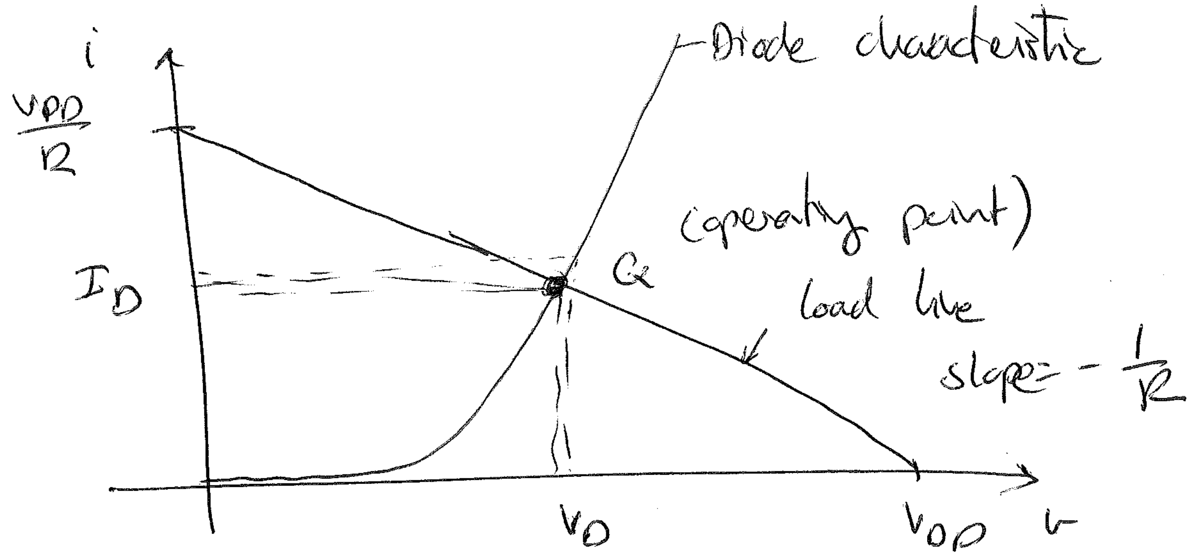
~~ex. $V_{DD} = 5V$ $R = 1k\Omega$ and 1mA at 0.7V.~~

→ If I_s is unknown → iterative solution.

4.3.2 ~~Iterative~~ ~~Graphical~~ Graphical analysis using exponential model.

~~If I_S is unknown~~

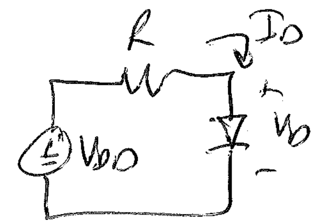
Plotting equations ① and ②.



4.3.3 Iterative analysis using exponential model.

Determine I_D and V_D with $V_{DD} = 5V$, $R = 1k\Omega$

Start by ~~And~~ assuming → diode has 1mA of current at 0.7V.



→ $I_D = \frac{V_{DD} - V_D}{R} = \frac{5 - 0.7}{1k\Omega} = 4.3\,mA$

→ Diode equation for better estimate of V_D.

$I_1 = I_S e^{V_1/V_T}$ $\qquad \frac{I_2}{I_1} = e^{(V_2 - V_1)/V_T}$

$I_2 = I_S e^{V_2/V_T}$

$V_2 - V_1 = V_T \ln \frac{I_2}{I_1}$

$\therefore\ V_D = V_1 + V_T \ln \frac{I_2}{I_1}$, $V_1 = 0.7V$, $V_T = 25mV$, $I_2 = 4\ A$, $I_1 = 1mA$

$V_D = 0.738V$

∂ cont'd

↳ cont'd

Second iteration.

$$I_D = \frac{5 - 0.738}{1k\Omega} = 4.262mA$$

$$V_D = 0.738 + 26mV \cdot \ln\left(\frac{4.262}{4.3}\right)$$

$V_D = 0.738V$ (same as before)

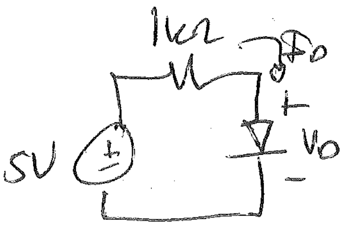
$\therefore$ Ans. $I_D = 4.262mA$, $V_D = 0.738V$

4.3.5 - The constant voltage drop method

Simplest and most widely used.

Assume $V_D = 0.7V$ when ON.

$= 0.7V$, $i > 0$



$$I_D = \frac{5V - 0.7V}{1k\Omega} = 4.3mA \quad \text{(close enough)}$$

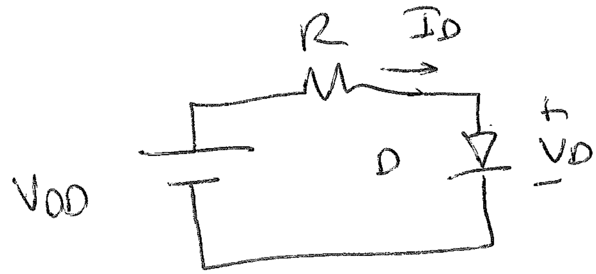
4.3.6 - Ideal-Diode Model

→ $V_D = 0V$, when V_{DD} is large.

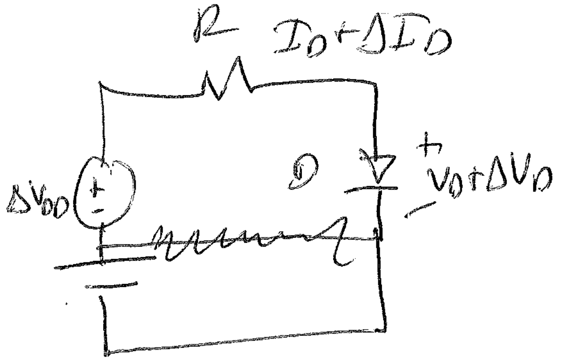
$$I_D = \frac{5 - 0V}{1k\Omega} = 5mA$$

→ Also help w/ rapid analysis to determine when diode is ON or OFF

4.3.7 The small-signal model

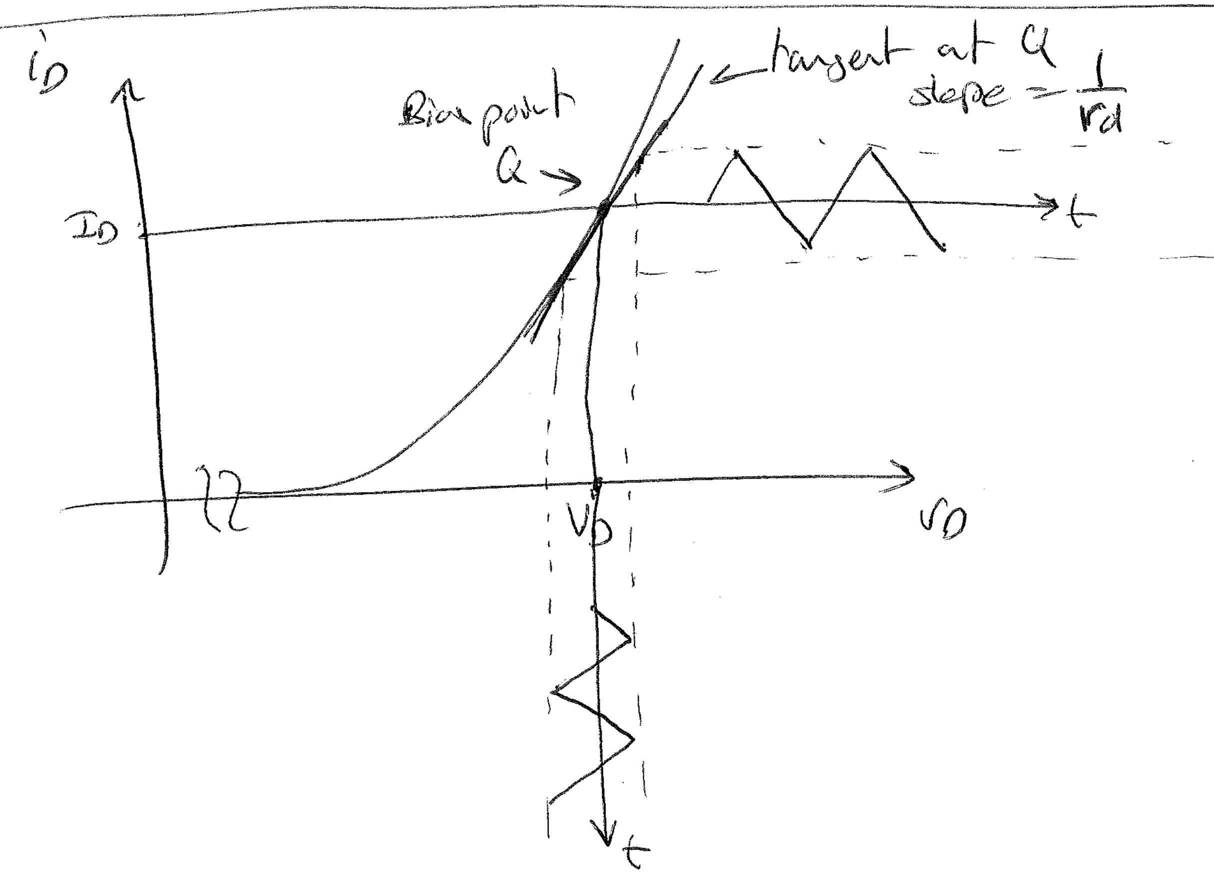


- Consider DC voltage V_{DD} and current I_D.
  - → I_D and V_D can be found using exponential or constant voltage drop model.



- V_{DD} undergoes a small change ΔV_{DD} I_D changes by ΔI_D and V_D by ΔV_D
  - → Need quick way to determine incremental values Δ.
  - → Small-signal model for the diode.

→ ΔV_{DD} can be time-varying
→ small as ΔV_D is kept sufficiently small.



Voltage across diode:

$$v_D(t) = V_D + v_d(t)$$

$$i_D(t) = I_S e^{v_D/V_T}$$

$$i_D(t) = I_S e^{(V_D + v_d(t))/V_T}$$

OR

$$i_D(t) = I_S e^{V_D/V_T} e^{v_d/V_T}$$

no small signal → $\cancel{i_D(t)} = I_S e^{V_D/V_T} = I_D$ (DC component)

$$\therefore \; i_D(t) = I_D e^{v_d/V_T}$$

If amplitude is $v_d(t)$ is small.

$$\frac{v_d}{V_T} \ll 1$$

then, series expansion approximation of exponent:

$$\boxed{i_D(t) \simeq I_D\left(1 + \frac{v_d}{V_T}\right)}$$

small-signal approximation for signals with amplitudes smaller than 5mV ($V_T = 25$mV)

then: $\cancel{i_D = I_D + i_d}$ $\quad i_D(t) = \underbrace{I_D}_{DC} + \underbrace{\frac{I_D}{V_T} v_d}_{\text{small signal current}}$

$$i_D = I_D + i_d$$

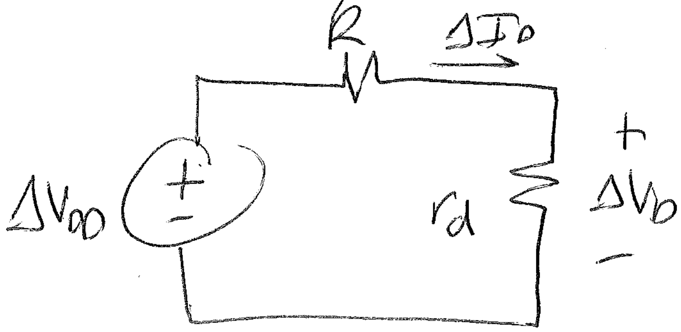
$$i_d = \frac{I_D}{V_T} v_d$$

→ $\frac{I_D}{V_T}$ is diode small-signal conductance

Also diode small-signal resistance
or incremental resistance $r_d = \frac{V_T}{I_D}$

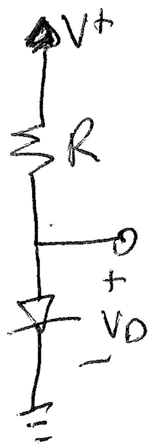
Small signal analysis is done independently of DC analysis by short-circuiting DC voltage sources and open-circuiting DC current sources and replacing the diode by its small-signal resistance

$$r_d = \frac{V_T}{I_D}$$



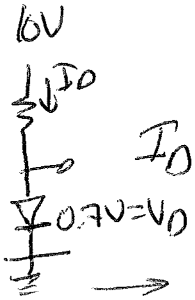
---

ex. 4.5



$R = 10k\Omega$
$V^+ = 10V\ DC + 60Hz$ sinusoid of 1-V peak.

→ Calculate DC voltage of diode and amplitude of the sine-wave signal appearing across it.
Assume diode has a 0.7V drop at 1-mA current.



$$I_D = \frac{10 - \cancel{0.7}}{10k\Omega} = 0.93mA \quad \text{(near 1-mA assumed current)}$$

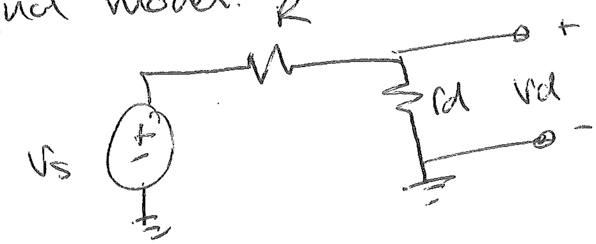
→ $$r_d = \frac{V_T}{I_D} = \frac{25mV}{0.93mA} = 26.9\Omega \quad \text{(diode incremental resistance)}$$

↳ Cont'd

8/31/2016 (11)

↑ cont'd

small signal model:



For 1V sine wave.

$$v_d(\text{peak}) = \hat{V}_s \frac{r_d}{R + r_d} = 1V \cdot \frac{26.9\Omega}{10k + 26.9\Omega}$$

$\boxed{v_d(\text{peak}) = 2.68mV}$ ~~≪ 25mV~~ < 5mV

lec 5 ↑
lec 6 ↓

4.3.8 Diode forward drop in voltage regulation.

Provide constant DC voltage between output terminals.

in spite of { a) changes in load current at output.
b) changes in DC power supply

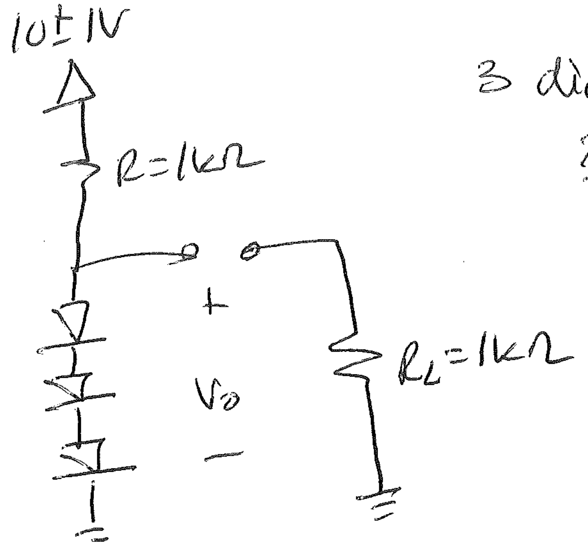
Diode remains at ~0.7V for large changes in current.

In previous example a 10V-dc w/ 2V peak-to-peak
±1V (±10%)

only ±2.7mV seen on diode.
(±0.4%)

---

ex.



3 diodes ≈ 2.1V constant.

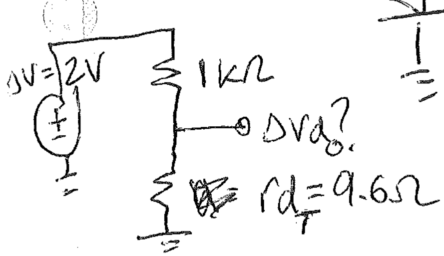
? a) ±10% voltage source.

$$I = \frac{10 - 2.1}{1k\Omega} = 7.9mA$$

$$r_d = \frac{V_T}{7.9mA} = \frac{25mV}{7.9mA} = 3.2\Omega$$

$\therefore r = 3 \times 3.2 = 9.6\Omega$ (total rd diodes)

small-signal model

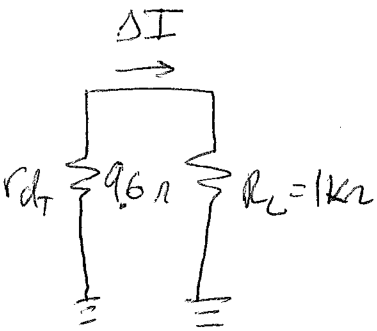


$$\Delta V_o = 2 \cdot \frac{r_{d_T}}{r_{d_T} + R} = 2 \frac{9.6}{9.6 + 10k} = 19mV \text{ peak-to-peak}$$

~1% total → ±0.5%.

b) Load resistor of $1k\Omega$ connected.

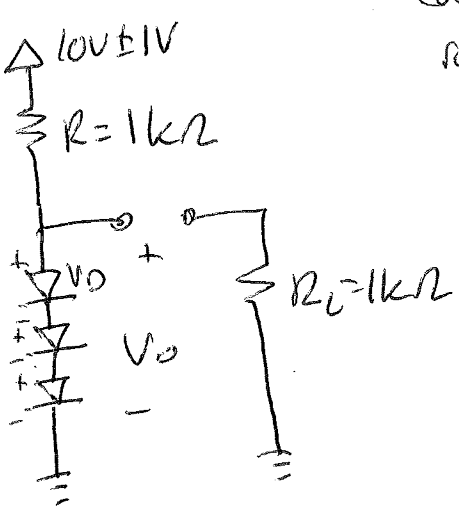
$$I_L = \frac{2.1V}{1k\Omega} = 2.1mA$$



$$\Delta V_o = 9.6\Omega(-2.1mA) = -20mV$$
$$r_{d_T} \cdot (\Delta I)$$

This implies that voltage across each diode decreases by ~6.7mV → small-signal model may not be entirely justified but is a good approximation. Detailed analysis yields $\Delta V_o = -23mV$.

Ex 3 detailed:



Calculate percentage change of regulated voltage V_o cased by
a) a ±10% change in power supply
b) connecting a $1k\Omega$ load resistance

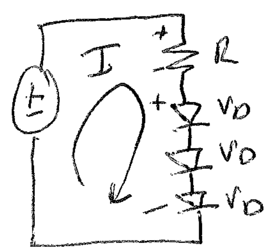
a) KVL w/ diodes. $3V_D = V_o = 2.1V$

$$-V_{SS} + IR + 3V_D = 0$$

$$IR = V_{SS} - 3V_D$$

$$I = \frac{V_{SS} - 3V_D}{R}$$

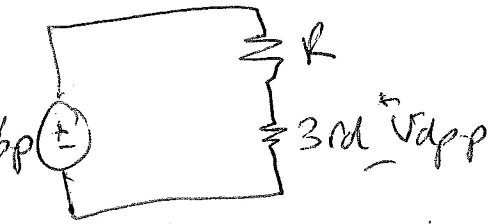
DC analysis



$$I_D = \frac{10 - 2.1}{10k\Omega} = 7.9mA$$

$$r_d = \frac{25mV}{7.9mA} = \frac{V_T}{I_D} = 3.2\Omega$$

$$3r_d = 9.6\Omega$$

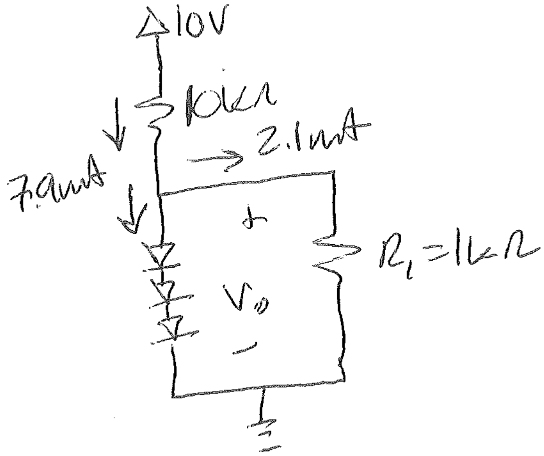


small signal.
turn off DC sources
keep small signal sources, replace diode with rd

$$V_{dpp} = 2V_{pp} \frac{9.6\Omega}{9.6\Omega + 10k\Omega}$$

$V_{dpp} = 19mV$ or ±0.5%.

b)



$\Delta I = -21mA$

$\therefore \Delta V_o = \Delta I \cdot 3rd$

$\Delta V_o = (-21mA)(9.6\Omega)$

$\boxed{\Delta V_o = -20mV}$

4.13 Find the value of the ~~diode signal~~ small-signal resistance r_d at bias currents of 0.1mA, 1mA and 10 mA.

$$r_d = \frac{V_T}{I_D}$$

$$r_{d_1} = \frac{25mV}{0.1mA} = 250\Omega$$

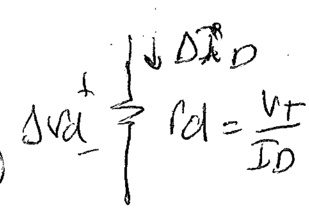
$$r_{d_2} = \frac{25mV}{1mA} = 25\Omega$$

$$r_{d_3} = \frac{25mV}{10mA} = 2.5\Omega$$

4.14 Consider a diode biased at 1 mA. Find the change in current as a result of changing the voltage by (a) -10mV, (b) -5 mV, (c) +5mV and (d) +10mV. In each case, do the calculations (i) using small-signal model and (ii) exponential model.

4.14 (ans)

For small-signal model $\Delta i_D = \dfrac{\Delta v_d}{r_d}$ → (1)



" exponential model $\cancel{i_D = I_S e^{v_d/V_T}}$ $\quad i_D = I_S e^{v/V_T}$

Need Δi_D not knowing I_S } and then → $\left\{ \begin{array}{l} i_{D1} = I_S e^{v_1/V_T} \\ i_{D2} = I_S e^{v_2/V_T} \end{array} \right\}$ → $\dfrac{i_{D2}}{i_{D1}} = \dfrac{I_S e^{v_2/V_T}}{I_S e^{v_1/V_T}}$

$\cancel{i_{D1} = i_{D2} e^{(v_1-v_2)/V_T}}$ $\qquad \dfrac{i_{D2}}{i_{D1}} = e^{(v_2-v_1)/V_T}$

$\Delta v_d = v_2 - v_1$

$\dfrac{i_{D2}}{i_{D1}} = e^{\Delta v_d/V_T}$

and $i_{D2} = i_{D2}\, e^{\Delta v_d/V_T}$

$$\Delta i_D = i_{D2} - i_{D1} = i_{D1} e^{\Delta v_d/V_T} - i_{D1} = i_{D1}\left(e^{\Delta v_d/V_T} - 1\right) \quad (2)$$

→ knowing $I_D = 1mA$, $V_T = 25mV$ and eq. (1) + (2)
$i_{D1} = 1mA$ → $r_d = 25\Omega$

$\Delta i_D = 1mA\left(e^{-10mV/25mV} - 1\right)$

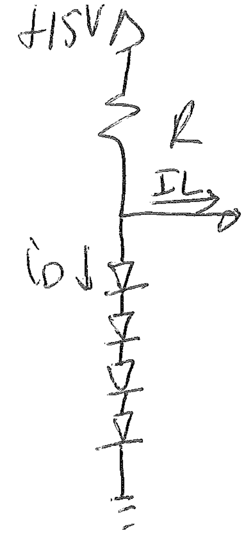
Δv_d (mV)	Δi_D (mA) small-signal	Δi_D (mA) exponential
−10 mV	−0.4 mA	~~0.491 mA~~ −0.329 mA
−5 mV	−0.2 mA	~~0.221~~ −0.181 mA
+5 mV	+0.2 mA	0.221 mA
+10 mV	+0.4 mA	0.491 mA

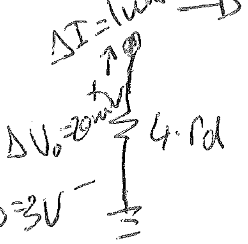
$\Delta i_D = 1\times10^{-3}\left(e^{\frac{-10\times10^{-3}}{25\times10^{-3}}} - 1\right)$

$\Delta i_D = 1\times10^{-3}\left(670.3\times10^{-3} - 1\right) = 1\times10^{-3}\left(-329.6\times10^{-3}\right)$

$\boxed{= -329.67\times10^{-6} A}$

O4.15 - Design circuit so that $V_0 = 3V$ when $I_L = 0$ and V_0 changes by 20 mV ~~ratio~~ per 1 mA of load current.





$\Delta V_0 = 20mV$ when $\Delta I = 1mA$

$\therefore \; r = \frac{20mV}{1mA} = 20\Omega$

4 diodes $r_d = \frac{20\Omega}{4} = 5\Omega$

And $r_d = \frac{V_T}{I_D} = \frac{25mV}{I_D} = 5\Omega$

$I_D = 5mA =$

$I_D = \frac{15V - 3V}{R} = 5mA$

lec 6 ↑
lec 7 ↓

$$\boxed{R = \frac{15-3}{5mA} = 2.4k\Omega}$$

(b) Value of I_s of each diode.

$V = \frac{3V}{4} = 0.75$

$i_D = I_S e^{V/V_T}$

$5mA = I_S e^{0.75/25mV}$

$I_S = \frac{5mA}{e^{0.75/25mV}} = 4.67 \times 10^{-16}$ Amps.

HW#2
4.2, 3, 16, 19, 41
56, 60, 61
extra: 4.66
(assign at end of zener diode lec.

(c) Use diode exponential model to find change in V_0 when current $I_L = 1mA$ is drawn from regulator.

$i_D = 5mA - 1mA = 4mA$

$i_D = I_S e^{V/V_T} \rightarrow 4mA = 4.67 \times 10^{-16}A \cdot e^{V/25mV}$

$V_D = V_T \ln\left(\frac{i_D}{I_S}\right)$

$e^{V/25mV} = \frac{4mA}{4.67 \times 10^{-16}A}$

$\rightarrow V_D = 25mV \cdot \ln\left(\frac{4mA}{4.67 \times 10^{-16}A}\right)$

$V_D = 0.744V$

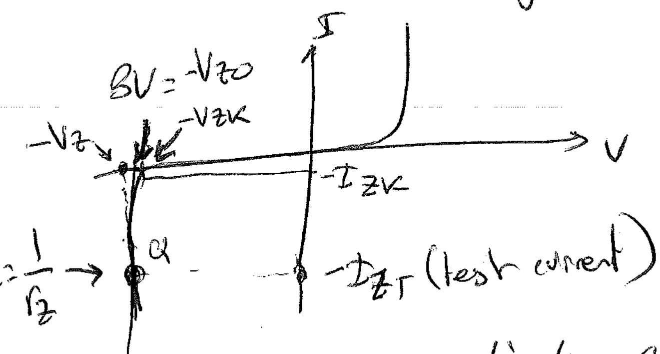
$V_0 = 4 \times 0.744 = 2.977$

$\Delta V = 3 - 2.977 = \cancel{22.1}$ ~~mV~~ $-23mV$

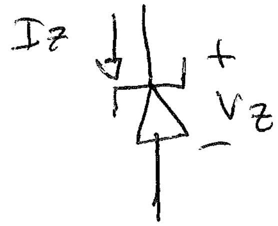
4.4 - Operation in the reverse breakdown region - Zener diodes

→ Diodes operating in the breakdown region can be used in the design of voltage regulators.

$\Delta V = r_Z \cdot \Delta I$



→ Breakdown diodes or zener diodes are designed to operate in BV condition.



Current flows into the cathode.
Cathode positive w.r.t. anode.
I_Z and V_Z have positive values

→ Datasheet indicates V_Z at certain I_{ZT}.
Q-point → operating parameters.
(i.e.) $V_Z = 6.8V$ zener diode at $I_{ZT} = 10mA$

→ Deviations from I_{ZT} result in changes in V_Z

$$\Delta V = r_Z \Delta I$$

r_Z → dynamic resistance of zener diode (datasheet)

Avoid running w/ low current settings as r_Z is larger.

9/1/2016 (14)

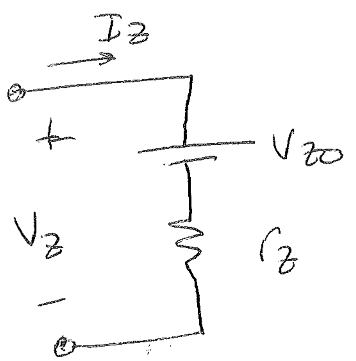
Zener diodes with voltages V_Z (few volts → few hundred)

spec sheet → V_Z (at I_{ZT}), r_Z, I_{ZK} and max. power.

i.e 0.5-W, 6.8-V zener diode

→ $I_{max} = 70\,mA$

→ Zener diode model:



V_{Z0} → point where 1/r_Z line intersects ~~y~~-axis ($\sim V_{ZK}$)

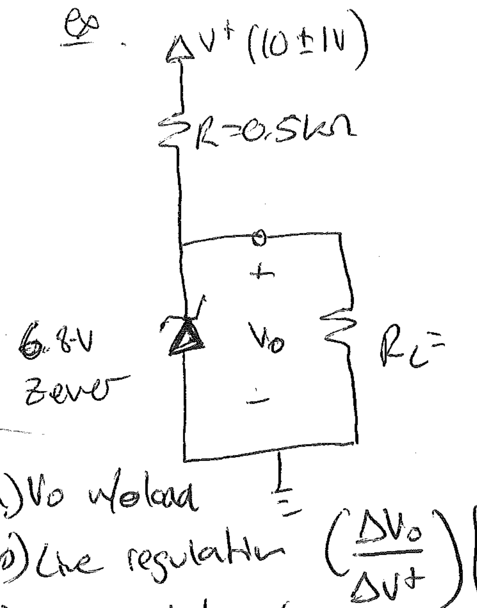
$V_Z = V_{Z0} + I_Z r_Z$ when $I_Z > I_{ZK}$, $V_Z > V_{Z0}$

4.4.2 Zener diode as shunt regulator

lec 7 ↑ / lec 8 ↓

→ Regulator circuit appears in parallel (shunt) with the load.

ex.



6.8-V zener → [$V_Z = 6.8V$ (at $I_Z = 5mA$)] $r_Z = 20\Omega$ and $I_{ZK} = 0.2mA$

a) Find V_o with no load at $V^+ = 10V$

~~b) Find line~~

~~→ Replace zener w/ model~~

~~$V_{Z0} = V_Z - I_Z r_Z = 6.8 - 5mA(20)$~~

~~$V_{Z0} = 6.7V$~~

~~Actual $I_Z = \frac{V^+ - V_{Z0}}{}$~~

breakdown region.

a) V_o w/o load

b) Line regulation $\left(\frac{\Delta V_o}{\Delta V^+}\right)\left[\frac{mV}{V}\right]$

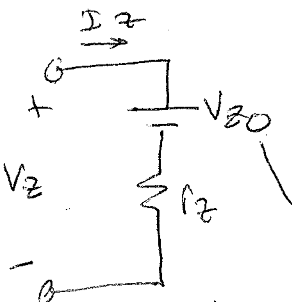
c) Load regulation $\left(\frac{\Delta V_o}{\Delta I_L}\right)\left[\frac{mV}{mA}\right]$

d) V_o w/ $R_L = 2k\Omega$

e) V_o w/ $R_L = 0.5k\Omega$

f) Min. R_L for diode in reverse.

→ First find actual V_{Z0} value for ~~set~~ current so that model can be used.



$$V_Z = V_{Z0} + r_Z I_Z$$

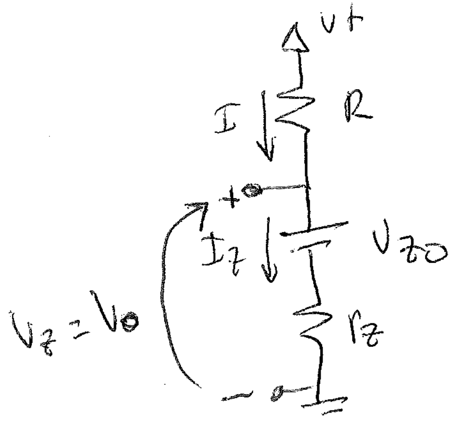
$$V_{Z0} = 6.8V - (20\Omega)(5mA) = 6.7V$$

→ Then find V_o with no load:

$$I = I_Z$$

$$-V^+ + IR + V_{Z0} + Ir_Z = 0$$

$$I(R + r_Z) = V^+ - V_{Z0}$$

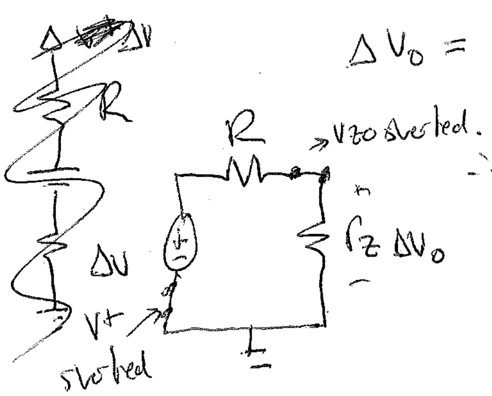


$$I = I_Z = \frac{V^+ - V_{Z0}}{R + r_Z}$$

$$I = \frac{10 - \cancel{0.67}\,6.7}{0.5k\Omega + 20\Omega} = 6.35mA.$$

thus: $V_Z = V_o = V_{Z0} + Ir_Z = 6.7V + (6.35mA)(20\Omega) = 6.83V$

b) For a ±1V change in V^+, find change in output voltage:

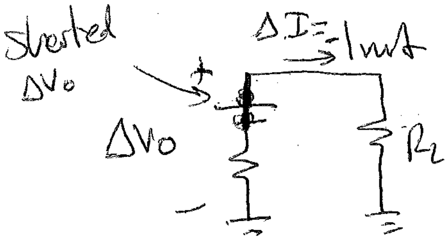


$$\Delta V_o = \Delta V^+ \frac{r_Z}{R + r_Z} = \pm 1V \cdot \frac{20}{500 + 20} = \pm 38.5mV$$

∴ Line regulation = 38.5mV/V

$$\hookrightarrow \frac{\Delta V_o}{\Delta V^+} \left(\frac{mV}{V}\right)$$

c) Find change in V_o resulting from connecting R_L that draws $I_L = 1mA$, and load regulation $\left(\frac{\Delta V_o}{\Delta I_L}\right)$ $\left[\frac{mV}{mA}\right]$



$$\Delta V_o = (\Delta I) \cdot r_Z$$

$$\Delta V_o = (-1mA)(20) = -20mV$$

$$\text{Load regulation} = \frac{\Delta V_o}{\Delta I_L} = -20mV/mA.$$

Assign HW #2: ✓ → Design

4.2, 3, 16, 19, 41, 56, 60, 61

extra 4.66 (Design problem)

9/7/2016

TS

---

(5) (Contd)

d) Find change in V_o when $R_L = 2k\Omega$

Load current becomes approx. $I_L \sim \frac{6.8V}{2k\Omega} = 3.4mA$

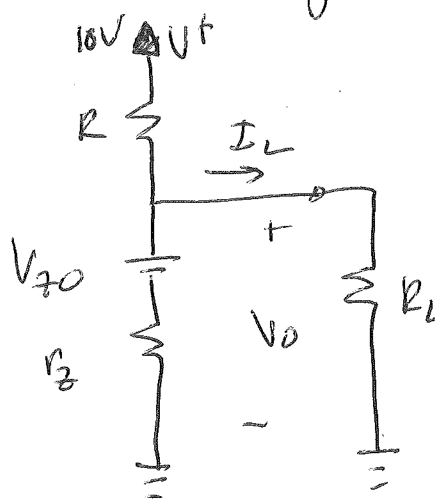
Zener current change $\Delta I_Z = -3.4mA$.

$\therefore \Delta V_o = r_z \Delta I_Z = 20\Omega(-3.4mA) = -68mV$

or from load regulation value of c)

$(-20mV/\cancel{mA}) \cdot (3.4\cancel{mA}) = -68mV$

e) Find value of V_o when $R_L = 0.5k\Omega$

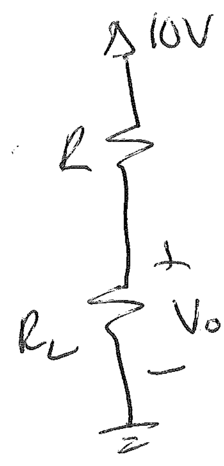


$I_L = \frac{6.8V}{0.5k\Omega} = 13.6mA$

but $I_R = 6.35mA$

$\therefore$ Zener is cut off (no current flows through zener diode).

→ new circuit.



$V_o = 10 \cdot \frac{R_L}{R_L + R}$

$V_o = 10 \cdot \frac{R}{R_L+R}$

$\boxed{V_o = 10 \cdot \frac{0.5}{0.5+0.5} = 5V}$

Also checks that $V_o < V_{Z0}$

j) what is minimum value of R_L for which diode still operates in breakdown region.

At edge of Breakdown region $\begin{cases} I_Z = I_{ZK} = 0.2\,mA \\ V_Z = V_{ZK} = 6.7V = V_{Z0} \end{cases}$

$\therefore$ Current supplied by R $\quad I = \dfrac{10-6.7}{0.5k\Omega} = 4.6\,mA$

Since $I_Z = 0.2\,mA$, then $I_L = 4.6 - 0.2 = 4.4\,mA$

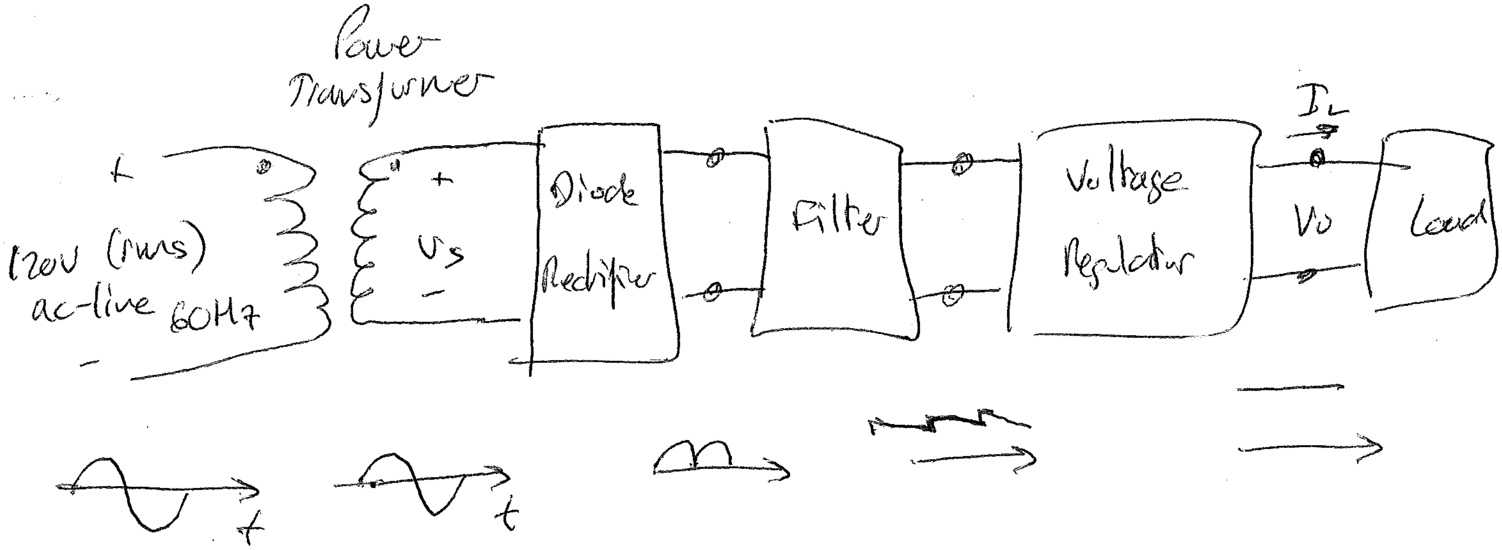
$\therefore \boxed{R_L = \dfrac{6.7V}{4.4\,mA} \approx 1.5k\Omega}$

---

lec 8 ↑
lec 9 ↓

4.5 - Rectifier circuits

Block diagram of a dc power supply

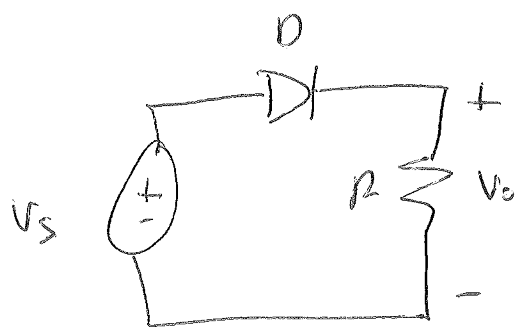


N-turns to step down voltage	Rectify signal	Filter signal	Regulate signal variation
	(1)	(2)	(3)

9/7/2016

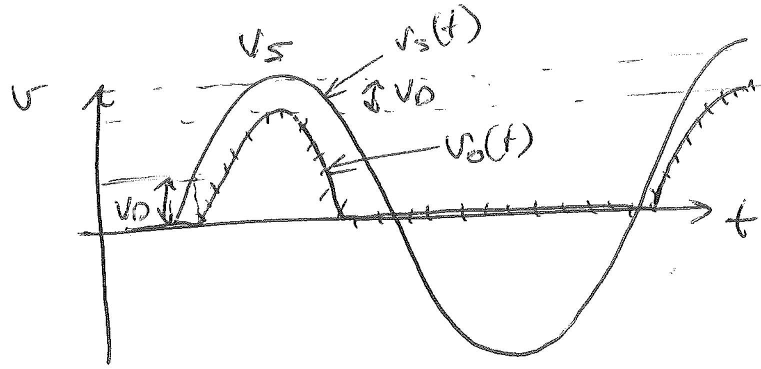
16

4.5.1 – The half-wave rectifier



Constant-voltage drop model: V_D

$$\begin{cases} v_o = 0 & v_s < V_D \\ v_o = v_s - V_D & v_s \geq V_D \end{cases}$$

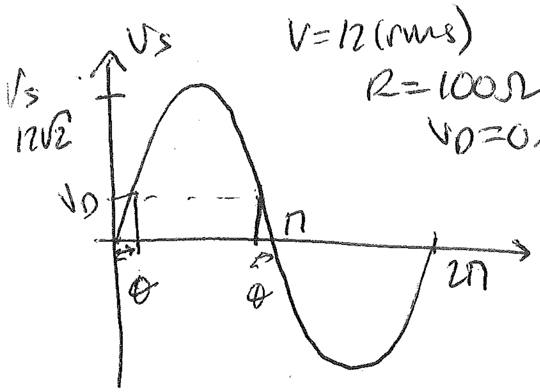


important: (i) current carrying rating of diode

(ii) Peak inverse voltage (PIV) capability of diode (i.e. $BV_{diode} > |V_s|$)

(iii) Not appropriate for small v_s.

ex. 4.19



$V = 12$ (rms)
$R = 100\Omega$
$V_D = 0.7V$

a) Diode starts conduction: $v_s = V_D = 0.7V$

$v_s = V_s \sin \omega t \qquad V_s = 12\sqrt{2} \qquad \omega t = \theta$

$v_s = 12\sqrt{2} \cdot \sin\theta = V_D = 0.7V$

$12\sqrt{2} \sin\theta = 0.7 \rightarrow \theta = \sin^{-1}\left(\frac{0.7}{12\sqrt{2}}\right) \simeq 2.4°$

∴ Conduction starts at 2.4°, ends at $(180-\theta) = 177.6°$

Total conduction angle = 175.2°

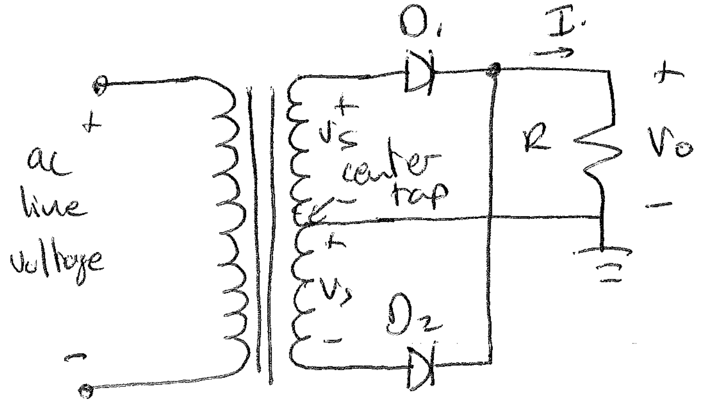
$$\therefore v_{o,avg} = \frac{1}{2\pi}\int_{\theta}^{(\pi-\theta)} (V_s \sin\phi - V_D)\, d\phi$$

$$= \frac{1}{2\pi}\left[-V_s\cos\phi - V_D\phi\right]_{\phi=\theta}^{\phi=\pi-\theta} = \frac{1}{2\pi}\left[V_s\cos\theta - V_s\cos(\pi-\theta) - V_D(\pi - 2\theta)\right]$$

$$\begin{cases} \cos\theta \simeq 1 \\ \cos(\pi-\theta) \simeq -1 \\ \pi - 2\theta \simeq \pi \end{cases}$$

$$\therefore v_{o,avg} \simeq \frac{2V_s}{2\pi} - \frac{V_D}{2} = \frac{V_s}{\pi} - \frac{V_D}{2} = \frac{12\sqrt{2}}{\pi} - \frac{0.7}{2} = 5.05V$$

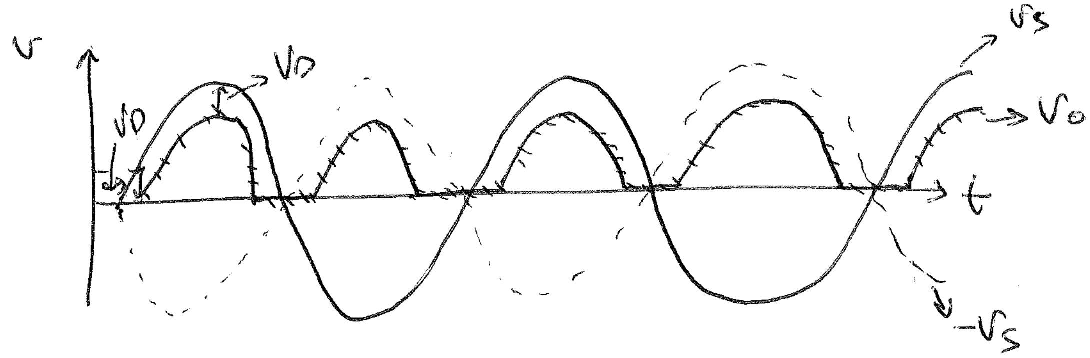
4.5.2 - The full-wave rectifier.



D_1 conducts during positive, + of cycle

D_2 conducts during negative part of cycle.

Current I always flows in same direction.



$PIV = 2V_S - V_D$ (twice that of half-wave circ. approx.)

---

ex. 4.19 (cont'd) Half-wave rectifier example continued...

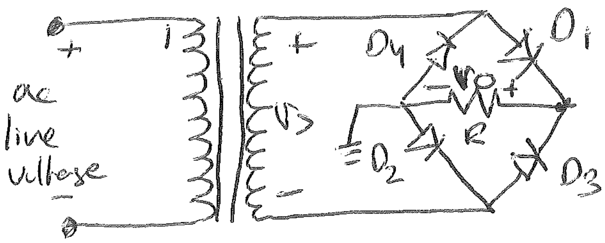
c) Peak current at peak diode voltage.

$$\therefore \hat{i}_D = \frac{V_S - V_D}{R} = \frac{12\sqrt{2} - 0.7}{100} = 163\,mA$$

$$PIV = V_S = 12\sqrt{2} = 17V$$

4.5.3 - The Bridge Rectifier

9/7/2016 17



(From Wheatstone bridge similarity)

→ Does not require a center-tap. (advantage over full-wave)

→ Diode bridge in single package.

→ Positive half-cycle: v_s is positive

→ current flows through D_1, R and D_2.

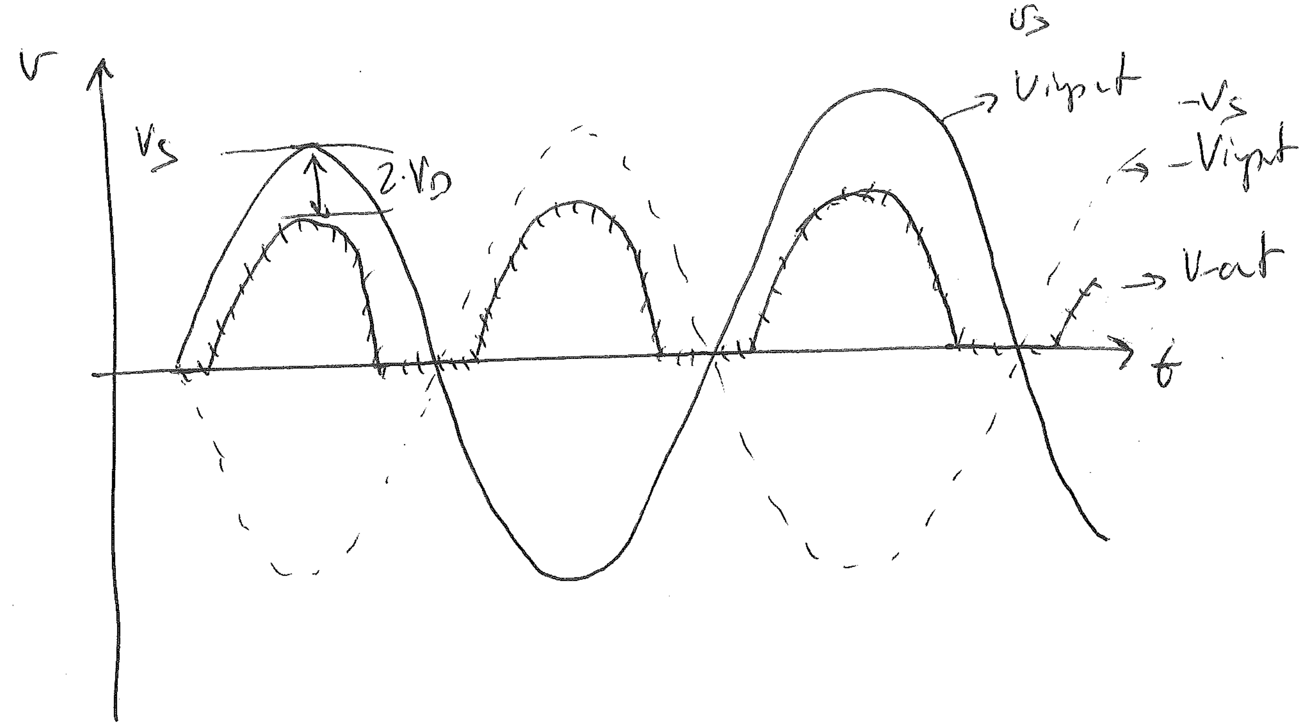
→ D_4 + D_3 → reversed biased

→ Negative half-cycle: v_s is negative [$(-v_s)$ is positive]

→ current flows thru D_3, R and D_4

→ D_1 + D_2 → reversed.

→ Current flows in same direction during both cycles

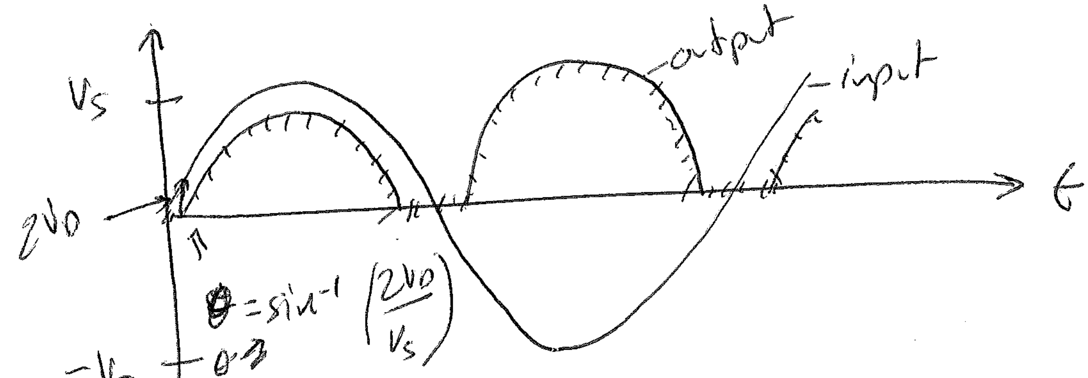


ex. 4.21 For bridge-rectifier circuit in previous page:

(a) Use constant-voltage-drop diode model to show that: (a) the average (or dc component) of the output voltage is $V_O \simeq \left(\frac{2}{\pi}\right) V_s - V_D$

(b) the peak diode current is $\frac{V_s - 2V_D}{R}$

→ Find numerical values for the quantities in (a) and (b) and the PIV for the case in which $v_s = 12\text{-V(rms)}$, $V_D \simeq 0.7V$, and $R = 100\Omega$.



$$\text{(a)}\quad V_{O,avg} = \frac{1}{2\pi}\int_{\phi=\theta}^{\pi-\theta} (V_s \sin\phi - 2V_D)\,d\phi = \frac{2}{2\pi}\left[-V_s\cos\phi - 2V_D\phi\right]_{\phi=\theta}^{\pi-\theta}$$

$$= \frac{1}{\pi}\left[V_s\cos\phi - V_s\cos(\pi-\theta) - 2V_D(\pi - 2\theta)\right], \text{ but } \cos\theta \simeq 1,$$

$\cos(\pi-\theta) \simeq -1$, $\pi - 2\theta \simeq \pi$.

Thus, $V_{O,avg} \simeq \frac{2V_s}{\pi} - 2V_D = \frac{2\times 12\sqrt{2}}{\pi} - 1.4 = \boxed{9.4V}$

(b) Peak diode current $= \frac{\text{Peak voltage}}{R} = \frac{V_s - 2V_D}{R} = \frac{12\sqrt{2} - 1.4}{100}$

$\boxed{= 156 \text{ mA}}$

$PIV = V_s - V_D = 12\sqrt{2} - 0.7 \boxed{= 16.3V}$

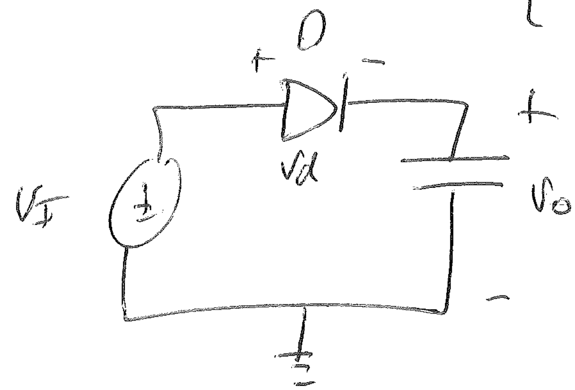
9/14/2016

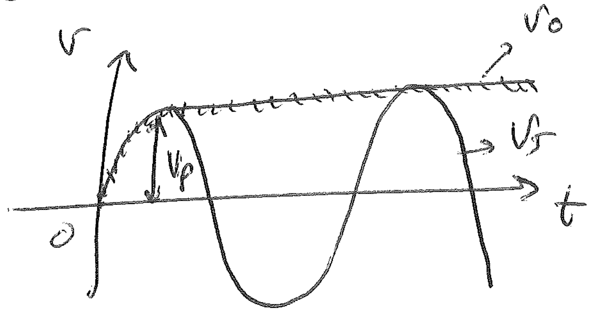
lec 9 ↑
lec 10 ↓

4.5.4 - The rectifier w/ a filter capacitor - The peak rectifier.

→ Place a capacitor across the load resistor.

→ First: simple example: { ideal capacitor only, ideal diode }

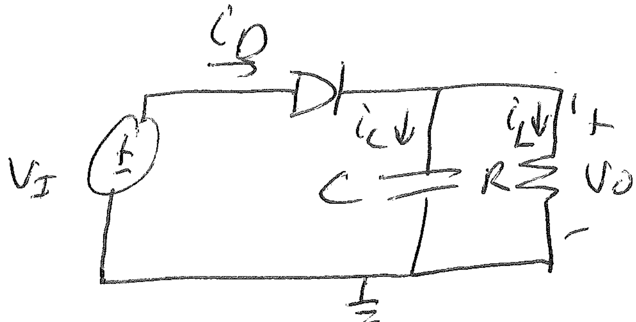


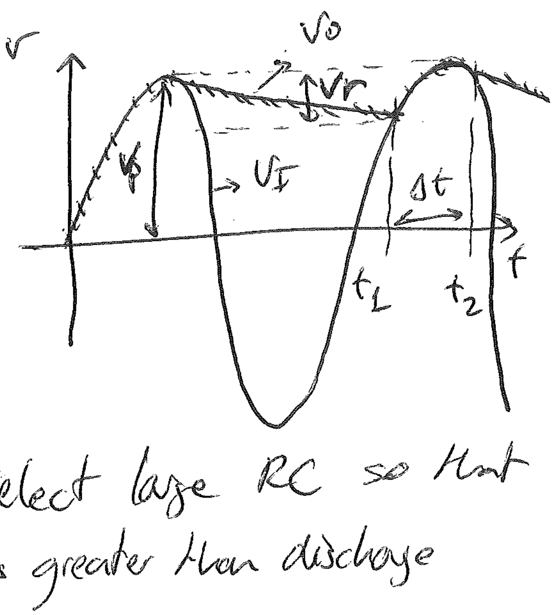


↑ Diode ON during positive cycle and $v_o \to V_p$.

Diode off as soon as $V_I < V_p$ since $v_D < 0$ when $V_o = V_p$.

→ More practical: load resistance across capacitor.
{ assume ideal diode for analysis.



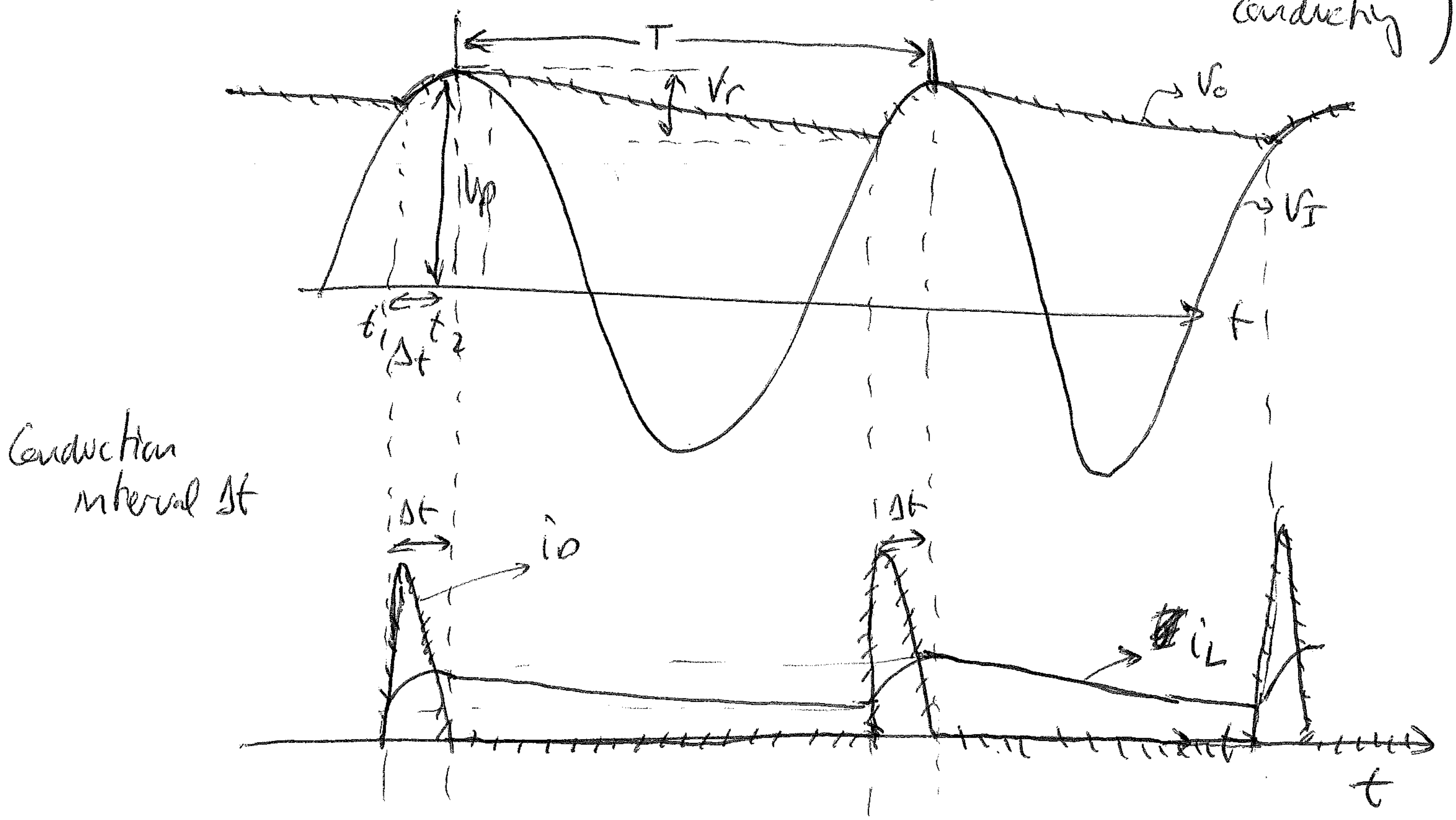


→ Select large RC so that time constant is greater than discharge interval.

Analyzing the circuit in detail:

→ Assume $CR \gg T$

$i_L = \frac{v_o}{R}$ $\quad i_D = i_C + i_L = C\frac{dv_I}{dt} + i_L$ (when diode conducting)



Observations: ① Diode conducts during time interval Δt and supplies charge equal to that lost during discharge.

② Diode conduction starts at t_1, when $v_I = v_o$. Conduction stops at t_2, shortly after peak of v_I (when $i_D = 0$).

③ During diode off interval, C discharges through R, and v_o decays exponentially with time constant RC. At end of discharge $v_o = V_p - V_r$, where V_r is ripple voltage. V_r is small when $RC \gg T$.

④ When V_r is small, $v_o \simeq V_p$. And i_L almost constant with DC $\quad I_L = \frac{V_p}{R}$

Cont'd ↷

$\hat{\text{c}}$ cont's

9/14/2016 (2)

$I_L \simeq \frac{V_p}{R}$ (for small V_r, ripple voltage)

and DC value of $V_{out} = V_p - \frac{1}{2}V_r$ ($V_p -$ average of ripple)

→ Derive expression for V_r and for average and peak values of diode current:

→ Diode-off interval:

$$v_o = V_p e^{-t/RC}$$

→ At end of discharge interval: $(V_p - V_r) \simeq V_p e^{-T/RC}$

Assuming $RC \gg T \rightarrow e^{-T/RC} \simeq 1 - T/RC$

$$\therefore V_p - V_r = V_p\left(1 - \frac{T}{RC}\right)$$

$$-V_r = V_p - V_p\frac{T}{RC} - V_p$$

$$V_r \simeq V_p\frac{T}{RC}$$ → select a large C to keep V_r small.

→ Expressing as frequency $f = \frac{1}{T}$

$$V_r = \frac{V_p}{fRC}$$, and knowing $I_L = \frac{V_p}{R}$

$$\rightarrow V_r = \frac{I_L}{fC}$$

→ Determine conduction interval: Δt.

→ assume conduction ends at peak $v_I = V_p$

$$V_p \cos(\omega \Delta t) = V_p - V_r \qquad \omega = 2\pi f = \frac{2\pi}{T}$$

→ assuming $\omega \Delta t$ to be small

$$\cos(\omega \Delta t) = 1 - \frac{1}{2}(\omega \Delta t)^2$$

$$\therefore V_p\left(1 - \frac{1}{2}(\omega \Delta t)^2\right) = V_p - V_p$$

$$\cancel{V_p} - \frac{V_p}{2}(\omega \Delta t)^2 = \cancel{V_p} - V_r$$

$$-\frac{V_p}{2}(\omega \Delta t)^2 = -V_r$$

$$(\omega \Delta t)^2 = 2\frac{V_r}{V_p}$$

$$\boxed{\omega \Delta t = \sqrt{2\frac{V_r}{V_p}}} \qquad \omega = \frac{2\pi}{T} = 2\pi f.$$

→ Determine average diode current during conduction i_{Dav}, by equating charge that diode supplies to capacitor: $Q_{supplied} = i_{Cav} \Delta t$

and $i_{Cav} = i_{Dav} - I_L$, and $Q_{lost} = CV_r$

$$\therefore i_{Dav} = I_L\left(1 + \pi\sqrt{\frac{2V_p}{V_r}}\right)$$

For half-wave rectifiers ↑

and $i_{Dmax} = I_L\left(1 + 2\pi\sqrt{\frac{2V_p}{V_r}}\right)$

For full-wave rectifiers: $V_r = \dfrac{V_p}{2fCR}$

$$i_{Dav} = I_L\left(1 + \pi\sqrt{\frac{V_p}{2V_r}}\right)$$

$$i_{Dmax} = I_L\left(1 + 2\pi\sqrt{\frac{V_p}{2V_r}}\right)$$

9/14/2016

(3)

example:

→ Peak rectifier fed by 60-Hz sinusoid w/ $V_p = 100V$. $R_L = 10k\Omega$. Find C for peak-to-peak ripple of 2V. Also fraction of cycle where diode conducts and average and peak values of diode current.

→ $$C = \frac{V_p}{V_r f R} = \frac{100}{(\underset{V_r}{2})(\underset{Hz}{60})(\underset{R_L}{10\times10^3})} = 83.3\,\mu F$$

→ $$\omega\Delta t \cong \sqrt{\frac{2V_r}{V_p}} = \sqrt{\frac{2\times2}{100}} = 0.2 \text{ rad.}$$

→ Diode conducts $\frac{0.2}{2\pi} \times 100 = 3.18\%$ of the cycle.

→ $$i_{Dav} = I_L\left(1 + \pi\sqrt{\frac{2V_p}{V_r}}\right) = 10mA\left(1+\pi\sqrt{\frac{2\times100}{2}}\right) = 324mA$$

$$I_L = \frac{100V}{10k\Omega} = 10mA$$

$$i_{Dmax} = I_L\left(1+2\pi\sqrt{\frac{2V_p}{V_r}}\right) = 10mA\left(1+2\pi\sqrt{\frac{2\times100}{2}}\right) = 638mA$$