EE 381 ELECTRONICS I (Madhu)
REVIEW OF CIRCUIT ANALYSIS

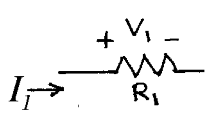
These notes are a review of the important concepts and procedures of circuit analysis, which are relevant to the analysis of electronic circuits. It is critically important that you study these notes carefully and use the information in solving the problems in *Problem Sheet 1*. If you have difficulty understanding any of the points made in these notes, do not hesitate to ask me for help. ***Students who do not do well in electronics are invariably those with an inadequate understanding of the principles and procedures in circuit analysis.***

We will be dealing only with resistors, dependent and independent voltage and current sources in the first course in electronics.

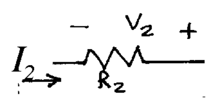
Resistors and Ohm's Law:

Note that ***in a real resistor, a positive current must flow from the terminal at the higher potential to that at the lower potential***. If, as occasionally happens, the voltage polarities are assigned to a resistor so that the current flows from a lower potential to the higher potential, then a negative sign has to be attached to the current so as to compensate for the wrong direction of current flow.

There are two versions of Ohm's law: one when the current is correctly taken as going from the "+" to the "–" terminal and the other when the current is arbitrarily taken as going the wrong way. These two versions are shown below. Note that a negative sign is necessary in the case where the current is assigned in the "–" to "+" direction in a resistor.



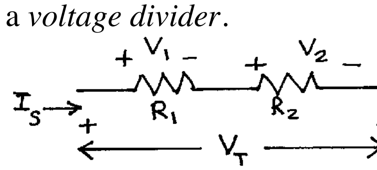
$V_1 = R_1 I_1$



$V_2 = -R_2 I_2$

Voltage Divider:

The series connection of *two* resistors is called a *voltage divider*.



Voltage Division:

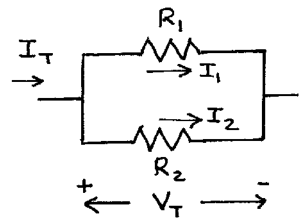
$$\frac{V_1}{V_T} = \frac{R_1}{R_1 + R_2} \quad \text{and} \quad \frac{V_2}{V_T} = \frac{R_2}{R_1 + R_2}$$

Remember that ***the larger resistance takes a larger share of the total voltage and the two voltages must add up to the total voltage.***

Current Divider:

The parallel connection of *two* resistors is called a *current divider*.

$$\frac{I_1}{I_S} = \frac{R_2}{R_1 + R_2} \quad \text{and} \quad \frac{I_2}{I_S} = \frac{R_1}{R_1 + R_2}$$



Remember that ***the larger resistance takes a smaller share of the total current and the two currents must add up to the total current.***

Thevenin equivalent of a circuit as seen from a specified terminal-pair:

Step 1: Open circuit the given terminal-pair. The usual convention in a diagram is that if the terminals are shown *extended* out, then the open circuiting has already been done. If the terminal-pair is marked on a component present in the given circuit, then you remove that component first to create an open circuit.

Step 2: Determine the voltage across the open circuit using any suitable circuit analysis procedure. Keep track of any negative sign that may occur in your final answer.

Step 3: Make the network relaxed by short-circuiting *independent* voltage sources and open-circuiting *independent* current sources. Do not touch the dependent sources: they lead a charmed life.

Step 4: Find the resistance looking into the relaxed network from the given terminal-pair using one of the following procedures.

(a) If the relaxed network *does not have any* dependent sources, then it is possible to find the Thevenin resistance by successive series and parallel combinations. Make sure that you start at the far end and work toward the given terminal-pair.

(b) If the relaxed network contains any dependent sources (or cannot be reduced by series/parallel combinations of resistors), you must apply a test source to the terminal pair. Assign a voltage V_T and a current I_T to the test source; it is ***absolutely essential*** that the current I_T be chosen as coming *out of the positive side* of V_T. In finding the ratio (V_T/I_T), the Thevenin resistance, it is convenient to choose $I_T = 1$ A.

Node Analysis:

Node analysis is the most efficient and most reliable tool in the analysis of electronic circuits. It is, therefore, extremely important to set up the equations *systematically* and *correctly*. If you scrupulously follow the steps outlined below then you can be sure of obtaining the correct set of equations with minimal effort. The key word is *systematic*; the setting up of node equations should be almost automatic without a great deal of thinking!

1. Choose a reference node. In most electronic circuits, a ground node is shown as part of the given circuit and automatically becomes the reference node. If a ground node is not shown, choose a convenient ground node. Do not write an equation for the ground node when you set up node equations. [If you do, you will have a set of equations that are not independent and cannot be solved.]

2. Label the other nodes with numbers.

3. *Writing the equation for a node:* As a first step, the left hand side will consist of all the currents meeting at the node and the right hand side will be zero. That is, each equation will be of the form:

Sum of the currents leaving the node through the resistors meeting at that node
= Net current entering the node from the sources meeting at that node

The left hand side term due to each resistor is written as:

$$\left[\frac{(Voltage\ of\ the\ node\ you\ are\ at)-(Voltage\ at\ the\ other\ end\ of\ the\ resistor)}{\text{Resistance}}\right]$$

Note that this rule makes the current in each resistor as a current *leaving* the given node.

The right hand side term due to each source is given by using a positive sign if a source current is entering the node and a negative sign if a source current is leaving the node. This rule applies to both dependent and independent sources.

In the case of dependent sources, you also need to write auxiliary equations for the variables controlling those sources.

Standard Form of the Node Equation:

In the standard form of a system of node equations, the coefficients must be combined appropriately and reduced to single numerical values.

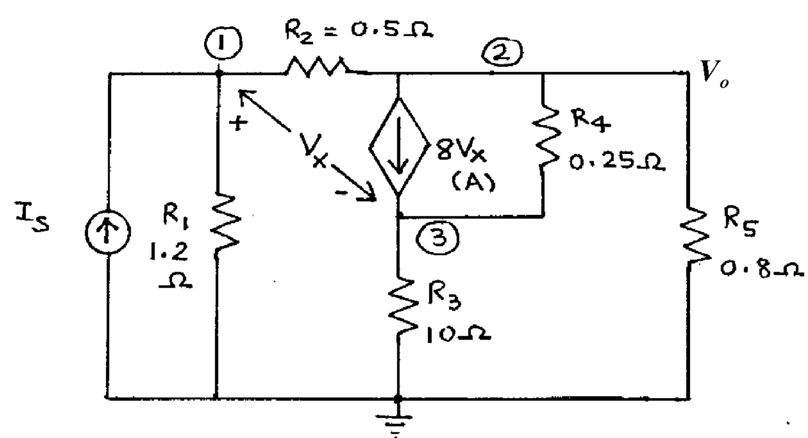
<u>**NOTE**</u>: Unless you write the final equations in standard form, I will not be able to check the correctness of your solution.

When you use a built-in solver in your calculator to solve the system of equations, you may assign a value of 1 V to the input voltage or 1 A to the input current.

NODE ANALYSIS EXAMPLES

Example 1:

Determine the ratio $\left(\frac{V_o}{I_S}\right)$ in the circuit shown below.



Node 1: $\frac{1}{R_1}V_1 + \frac{1}{R_2}(V_1 - V_2) - I_S = 0 \qquad \therefore \left(\frac{1}{R_1}+\frac{1}{R_2}\right)V_1 - \frac{1}{R_2}V_2 = I_S$

$$\therefore 2.833V_1 - 2V_2 = I_S \qquad (1)$$

Node 2: $\frac{1}{R_2}(V_2 - V_1) + \frac{1}{R_4}(V_2 - V_3) + \frac{1}{R_5}V_2 + 8V_X = 0$

But $V_X = V_1 - V_3$

$$\therefore \left(8-\frac{1}{R_2}\right)V_1+\left(\frac{1}{R_2}+\frac{1}{R_4}+\frac{1}{R_5}\right)V_2-8V_3=0$$

$$\therefore 6V_1+7.25V_2-12V_3=0 \qquad (2)$$

Node 3: $$\frac{1}{R_4}(V_3-V_2)+\frac{1}{R_3}V_3-8V_X=0$$

Again using $V_X = V_1 - V_3$, we get

$$-8V_1-\frac{1}{R_4}V_2+\left(\frac{1}{R_4}+\frac{1}{R_3}+8\right)V_3=0$$

$$\therefore -8V_1-4V_2+12.1V_3=0 \qquad (3)$$

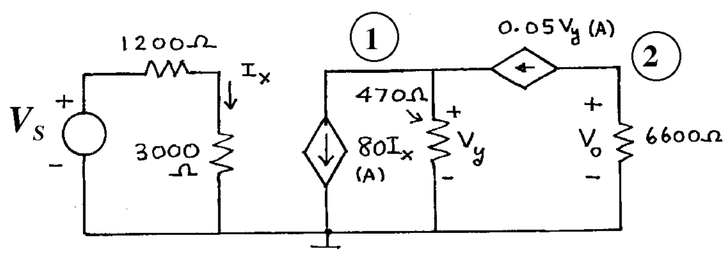
Solve the equations (1), (2), (3) using $I_S = 1$A. We get

$V_1 = 0.6043$ V; $V_2 = 0.3559$ V; $V_3 = 0.5172$ V.

Since $V_o = V_2$, we get

$$\left(\frac{V_o}{I_S}\right)=0.3559 \text{ Ohms}$$

Example 2: Determine the voltage ratio $\frac{V_o}{V_S}$ in the circuit shown below.



We have

$$I_X=\frac{V_S}{4200} = 2.381\times10^{-4}V_S \text{ (A)}$$

Node 1: $$\frac{1}{470}V_1+80I_X-0.05V_y=0$$

Using $V_y = V_1$ and $I_S = 2.381\times10^{-4}V_S$, the equation becomes

$$-4.787\times10^{-2}V_1=-1.905\times10^{-2}V_S \qquad (4)$$

Node 2: $$\frac{1}{6600}V_2+0.05V_y=0$$

Using $V_y = V_1$, we have

$$0.05V_1+1.515\times10^{-4}V_2=0 \qquad (5)$$

Solving Eqs (4) and (5), we get

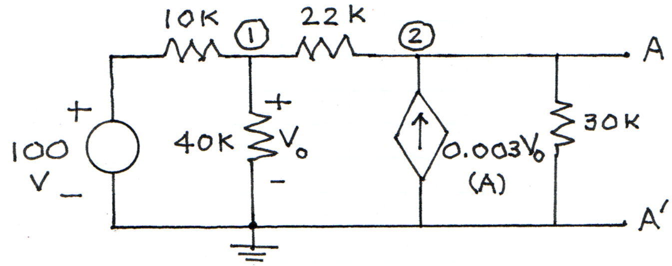
$V_1 = 0.3980V_S$; $V_2 = -131.3\ V_S$

$$\therefore \left(\frac{V_o}{V_S}\right)=-131.3$$

The negative sign simply means that when V_S increases, V_o will decrease; or, as you will soon learn in Circuits II, the output voltage is 180º out of phase with the input voltage. This occurs frequently in electronic circuits; in fact, you came across this in op amp circuits in Circuits I.

THEVENIN EQUIVALENT CIRCUIT EXAMPLE

Determine the Thevenin equivalent of the circuit shown below as seen from the terminals labeled **A–A'**.



Thevenin Voltage Calculation:

Node 1:
$$\frac{(V_1 - 100)}{10^4} + \frac{V_1}{40\times 10^3} + \frac{(V_1 - V_2)}{22\times 10^3} = 0$$
$$\therefore 1.704\times 10^{-4} V_1 - 4.545\times 10^{-5} V_2 = 0.01 \qquad (6)$$

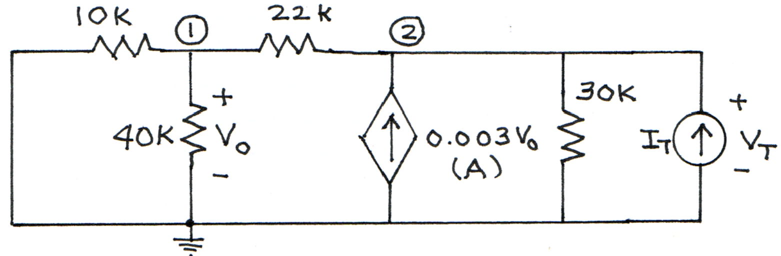
Node 2:
$$\frac{(V_2 - V_1)}{22\times 10^3} + \frac{V_2}{30\times 10^3} - 3\times 10^{-3} V_o = 0$$

Using $V_o = V_1$, we get
$$-3.045\times 10^{-3} V_1 + 7.879\times 10^{-5} V_2 = 0 \qquad (7)$$

Solving Eqs. (6) and (7):
$$V_1 = -6.305 \text{ V} \text{ and } V_2 = -243.6 \text{ V}$$
$$\therefore V_{Th} = -243.6 \text{ V}$$

Thevenin Resistance Calculation:

Replace the 100 V source by a short circuit and attach a test source to the terminals A-A'. Be sure to choose the current I_T in the test source as coming out of the + terminal of V_T.



Node 1:
$$\frac{V_1}{10^4} + \frac{V_1}{40\times 10^3} + \frac{(V_1 - V_2)}{22\times 10^3} = 0$$
$$\therefore 1.704\times 10^{-4} V_1 - 4.545\times 10^{-5} V_2 = 0 \qquad (8)$$

Node 2: $$\frac{\left(V_2 - V_1\right)}{22\times 10^3} + \frac{V_2}{30\times 10^3} - 3\times 10^{-3} V_o = I_T$$

$$\therefore -3.045\times 10^{-3} V_1 + 7.879\times 10^{-5} V_2 = I_T \qquad (9)$$

Solving Eqs. (8) and (9) with $I_T = 1\text{A}$:

$$V_1 = -363.7 \text{ V} \text{ and } V_2 = -1364 \text{ V}$$

$$\therefore R_{Th} = \frac{V_T}{I_T} = \frac{V_2}{I_T} = -1364\ \Omega$$

The Thevenin resistance is negative! What does a negative resistance mean? Whereas a positive resistor always consumes power provided by some other component(s), a positive resistance always delivers power to other components. This is analogous to the fire hydrant wetting the dog. Note that in electronic circuits, it is possible to obtain negative resistances and there are special configurations that actually do this. It is therefore, critically important to orient the voltage V_T and the current I_T in a test source correctly.