Homework 6 – Due Wednesday 11/2/2016

Problems (not review questions): **9.1, 9.5, 9.6, Exercise 9.5** (page 611), **Exercise 9.17** (page 648) – where $G_m = g_{m1,2}$, $R_o = r_{o2} || r_{o4}$) , **Exercise 9.20** (page 657)

Solutions for odd problems are on mycourses: Book_solutions.pdf,

Answer for 9.6: R_D= 4 kΩ, $(W/L)_{1,2}$ = 44.4, $(W/L)_3$ = 88.8, $(W/L)_4$ = 22.2, R = 12.5 kΩ, V_{CMmin} = -0.2 V, V_{CMmax} = 0.5 V

Answers for exercises (9.5, 9.17, 9.20) are given in the book in pages 622, 648, and 657.

EEEE381 HOMEWORK FORMAT GUIDELINES

GENERAL: NEATNESS AND ORGANIZATION WILL BE GRADED. THE SAME GUIDELINES SHOULD BE FOLLOWED FOR EXAMS.

- ALL HOMEWORK IS TO BE HANDED ON ENGINEERING GRAPH PAPER or PLAIN WHITE PAPER (8.5 inch x 11 inch).
- NUMBER EACH PROBLEM INCLUDING CHAPTER IT COMES FROM
- HIGHLIGHT EACH FINAL ANSWER WITH A BOX AND INCLUDE APPROPRIATE UNITS.
- INCLUDE YOUR NAME ON EVERY PAGE
- ALL PROBLEMS SHOULD BE TURNED IN, IN ORDER!
- IF WORK IS NOT LEGIBLE, IT WILL NOT BE GRADED
- CROSS OUTS ARE NOT ACCEPTABLE. USE A PENCIL AND ERASER OR PEN AND WHITE-OUT. (Green-out?)
- PRESENT SOLUTION IN A FORMAT THAT PROCEEDS FROM LEFT TO RIGHT, TOP TO BOTTOM. IF ORGANIZATION OF SOLUTION IS NOT CLEAR, PROBLEM WILL NOT BE GRADED.
- PROVIDE A CONCISE DESCRIPTION OF YOUR METHOD OF SOLUTION. IF NONE IS PROVIDED, NO PARTIAL CREDIT WILL BE AFFORDED.
- PROVIDE AN APPROPRIATELY LABELED CIRCUIT DIAGRAM. IF CIRCUIT IS MODIFIED, INCLUDE MODIFIED DIAGRAM(S).
- MAKE SURE ALL PAGES ARE ATTACHED TO EACH OTHER SECURELY. STAPLES ARE A CLASSIC WAY TO DO THIS.
- -REMEMBER- HOMEWORK ASSIGNMENTS ARE NOT JUST ABOUT LEARNING TO DO THE PROBLEM. IT IS ABOUT LEARNING TO PRESENT YOUR WORK SO OTHERS CAN UNDERSTAND WHAT YOU DO - A VALUABLE SKILL IN THE WORKPLACE.

Things to remember

1) Re-Draw the Circuit on your homework sheet.
2) Show all work.
3) Final answer should be in decimal form.
4) Final answers should be boxed.
5) Your name should be on every page.

Homework 6 – Solutions

9.1 Refer to Fig. 9.2.

(a) $\frac{I}{2} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_{1,2} V_{OV}^2$

$0.08 = \frac{1}{2} \times 0.4 \times 10 \times V_{OV}^2$

$\Rightarrow V_{OV} = 0.2$ V

$V_{GS} = V_{tn} + V_{OV} = 0.4 + 0.2 = 0.6$ V

(b) $V_{CM} = 0$

$V_S = 0 - V_{GS} = -0.6$ V

$I_{D1} = I_{D2} = \frac{I}{2} = 0.08$ mA

$V_{D1} = V_{D2} = V_{DD} - I_{D1,2}R_D$

$= 1 - 0.08 \times 5 = +0.6$ V

(c) $V_{CM} = +0.4$ V

$V_S = 0.4 - V_{GS} = 0.4 - 0.6 = -0.2$ V

$I_{D1} = I_{D2} = \frac{I}{2} = 0.08$ mA

$V_{D1} = V_{D2} = V_{DD} - I_{D1,2}R_D$

$= 1 - 0.08 \times 5 = +0.6$ V

Since $V_{CM} = 0.4$ V and $V_D = 0.6$ V, $V_{GD} = -0.2$ V, which is less than V_{tn} (0.4 V), indicating that our implicit assumption of saturation-mode operation is justified.

(d) $V_{CM} = -0.1$ V

$V_S = -0.1 - V_{GS} = -0.1 - 0.6 = -0.7$ V

$I_{D1} = I_{D2} = \frac{I}{2} = 0.08$ mA

$V_{D1} = V_{D2} = V_{DD} - I_{D1,2}R_D$

$= 1 - 0.08 \times 5 = +0.6$ V

(e) The highest value of V_{CM} for which Q_1 and Q_2 remain in saturation is

$V_{CM\max} = V_{D1,2} + V_{tn}$

$= 0.6 + 0.4 = 1.0$ V

(f) To maintain the current-source operating properly, we need to keep a minimum voltage of 0.2 V across it, thus

$V_{S\min} = -V_{SS} + V_{CS} = -1 + 0.2 = -0.8$ V

$V_{CM\min} = V_{S\min} + V_{GS}$

$= -0.8 + 0.6$

$= -0.2$ V

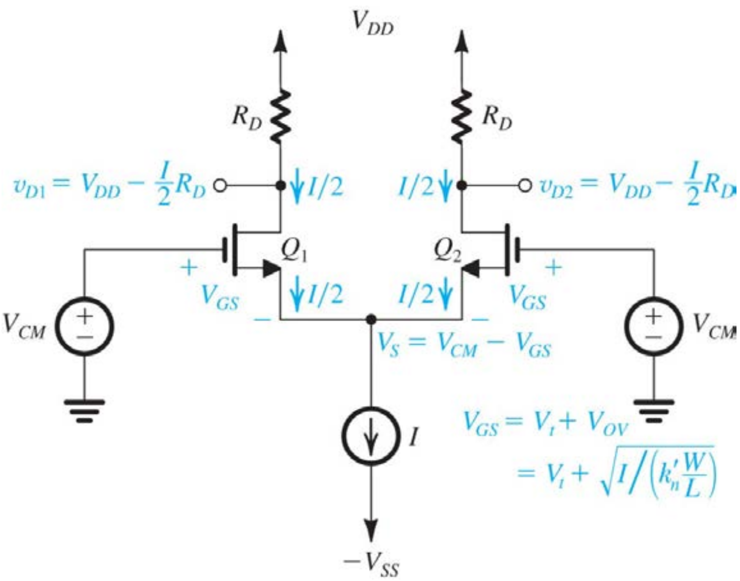
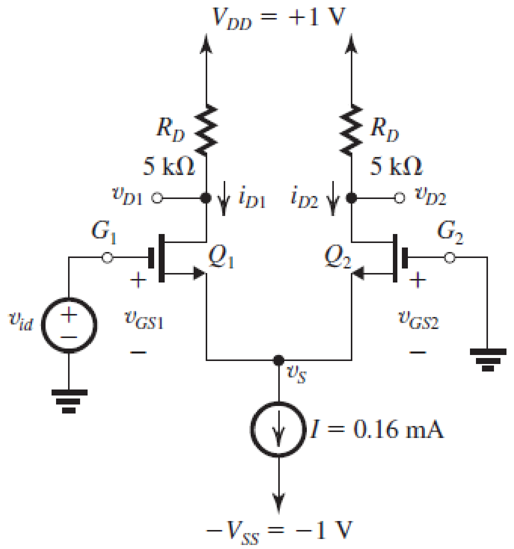


Figure 9.2 The MOS differential pair with a common-mode input voltage V_{CM}.

9.5



For $i_{D1} = 0.09$ mA and $i_{D2} = 0.07$ mA,

$$i_{D2} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)(v_{GS2} - V_{tn})^2$$

$$0.07 = \frac{1}{2} \times 0.4 \times 10(v_{GS2} - 0.4)^2$$

$$\Rightarrow v_{GS2} = 0.587 \text{ V}$$

and

$$v_S = -0.587 \text{ V}$$

$$i_{D1} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)(v_{GS1} - V_{tn})^2$$

$$0.09 = \frac{1}{2} \times 0.4 \times 10\,(v_{GS1} - 0.4)^2$$

$$\Rightarrow v_{GS1} = 0.612 \text{ V}$$

$$v_{id} = v_S + v_{GS1} = -0.587 + 0.612$$

$$= 0.025 \text{ V}$$

$$v_{D2} = V_{DD} - i_{D2}R_D$$

$$= 1 - 0.07 \times 5 = 0.65 \text{ V}$$

$$v_{D1} = 1 - 0.09 \times 5 = 0.55 \text{ V}$$

$$v_{D2} - v_{D1} = 0.65 - 0.55 = 0.10 \text{ V}$$

$$\text{Voltage gain} = \frac{v_{D2} - v_{D1}}{v_{id}} = \frac{0.10}{0.025} = 4 \text{ V/V}$$

To obtain the complementary split in current, that is, $i_{D1} = 0.07$ mA and $i_{D2} = 0.09$ mA,

$$v_{id} = -0.025 \text{ V}$$

9.6 Refer to the circuit in Fig. P9.6.

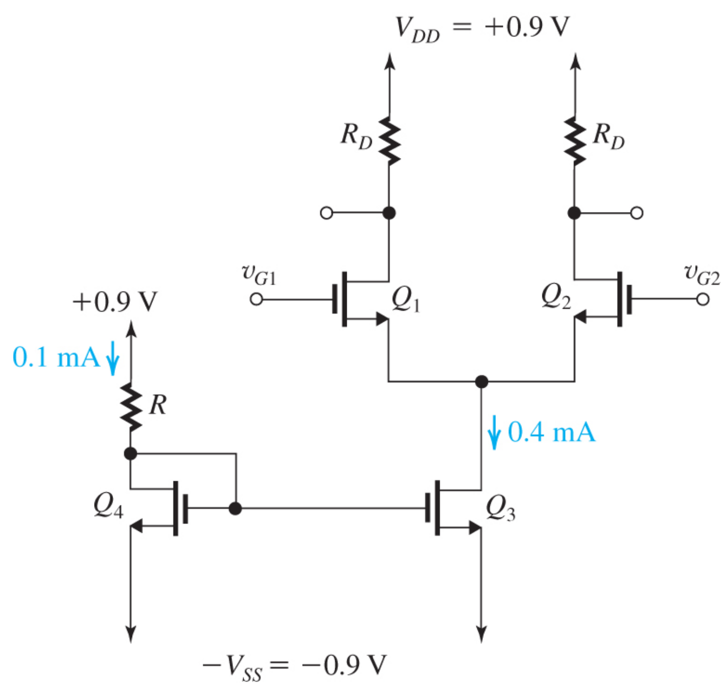


Figure P9.6

For $v_{G1} = v_{G2} = 0$ V,

$$I_{D1} = I_{D2} = \frac{0.4}{2} = 0.2 \text{ mA}$$

To obtain

$$V_{D1} = V_{D2} = +0.1 \text{ V}$$

$$V_{DD} - I_{D1,2}\, R_D = 0.1$$

$$0.9 - 0.2\, R_D = 0.1$$

$$\Rightarrow \; R_D = 4 \text{ k}\Omega$$

For Q_1 and Q_2,

$$I_{D1,2} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_{1,2} V_{OV}^2$$

$$0.2 = \frac{1}{2} \times 0.4\left(\frac{W}{L}\right)_{1,2} \times 0.15^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_{1,2} = 44.4$$

For Q_3,

$$0.4 = \frac{1}{2} \times 0.4 \times \left(\frac{W}{L}\right)_3 \times 0.15^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_3 = 88.8$$

Since Q_3 and Q_4 form a current mirror with $I_{D3} = 4I_{D4}$,

$$\left(\frac{W}{L}\right)_4 = \frac{1}{4}\left(\frac{W}{L}\right)_3 = 22.2$$

$$V_{GS4} = V_{GS3} = V_{tn} + V_{OV} = 0.4 + 0.15$$

$$= 0.55 \text{ V}$$

$$R = \frac{0.9 - (-0.9) - 0.55}{0.1}$$

$$= 12.5 \text{ k}\Omega$$

The lower limit on V_{CM} is determined by the need to keep Q_3 operating in saturation. For this to happen, the minimum value of V_{DS3} is $V_{OV} = 0.15$ V. Thus,

$$V_{ICM\,\min} = -V_{SS} + V_{OV3} + V_{GS1,2}$$

$$= -0.9 + 0.15 + 0.4 + 0.15$$

$$= -0.2 \text{ V}$$

The upper limit on V_{CM} is determined by the need to keep Q_1 and Q_2 in saturation, thus

$$V_{ICM\,\max} = V_{D1,2} + V_{tn}$$

$$= 0.1 + 0.4 = 0.5 \text{ V}$$

Thus,

$$-0.2 \text{ V} \leq V_{ICM} \leq +0.5 \text{ V}$$

Ex: 9.5 With $I = 200\ \mu A$, for all transistors,

$$I_D = \frac{I}{2} = \frac{200\ \mu A}{2} = 100\ \mu A$$

$$L = 2(0.18\ \mu m) = 0.36\ \mu m$$

$$r_{o1} = r_{o2} = r_{o3} = r_{o4} = \frac{|V'_A|\,L}{I_D}$$

$$= \frac{(10\ V/\ \mu m)\ (0.36\ \mu m)}{0.1\ mA} = 36\ k\Omega$$

Since $I_{D1} = I_{D2} = \frac{1}{2}\ \mu_n C_{ox}\left(\frac{W}{L}\right)V_{OV}^2$,

$$\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = \frac{2I_D}{\mu_n C_{ox} V_{OV}^2}$$

$$\frac{2(100\ \mu A)}{\left(400\ \mu A/V^2\right)(0.2\ V)^2} = 12.5$$

$$\left(\frac{W}{L}\right)_3 = \left(\frac{W}{L}\right)_4 = \frac{2I_D}{\mu_p C_{ox} |V_{OV}|^2}$$

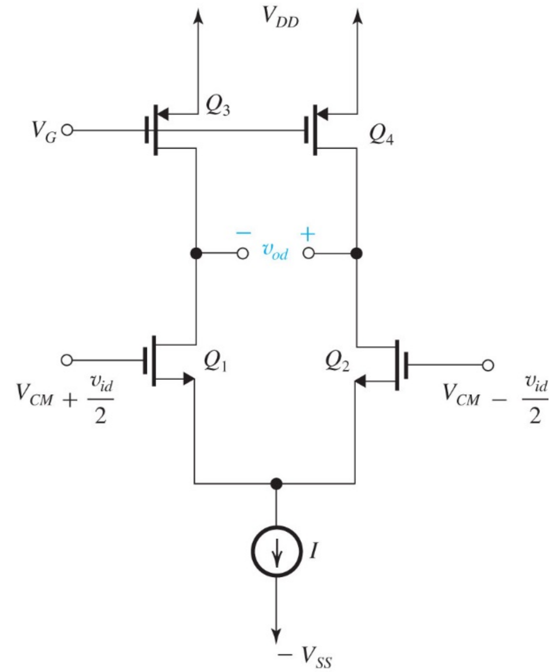
$$\frac{2(100\ \mu A)}{\left(100\ \mu A/V^2\right)(0.2)^2} = 50$$

$$g_m = \frac{I_D}{V_{OV}/2} = \frac{(100\ \mu A)\ (2)}{0.2\ V} = 1\ mA/V,$$

so

$$A_d = g_{m1}(r_{o1} \parallel r_{o3}) = 1(mA/V)\ (36\ k\Omega \parallel 36\ k\Omega)$$

$$= 18\ V/V$$



(a)

Ex: 9.17 $I_D = \frac{1}{2}I = 0.4 \text{ mA}$

$$I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_n V_{OV}^2$$

$$0.4 = \frac{1}{2} \times 0.2 \times 100 \times V_{OV}^2$$

$$\Rightarrow V_{OV} = 0.2 \text{ V}$$

$$g_{m1,2} = \frac{2I_D}{V_{OV}} = \frac{2 \times 0.4}{0.2} = 4 \text{ mA/V}$$

$$G_m = g_{m1,2} = 4 \text{ mA/V}$$

$$r_{o2} = \frac{V_{An}}{I_D} = \frac{20}{0.4} = 50 \text{ k}\Omega$$

$$r_{o4} = \frac{|V_{Ap}|}{I_D} = \frac{20}{0.4} = 50 \text{ k}\Omega$$

$$R_o = r_{o2} \parallel r_{o4} = 50 \parallel 50 = 25 \text{ k}\Omega$$

$$A_d = G_m R_o = 4 \times 25 = 100 \text{ V/V}$$

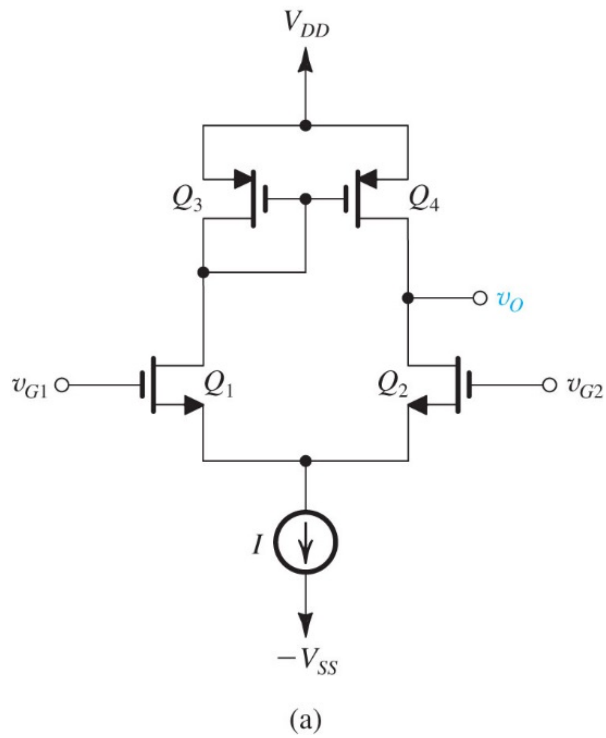


Figure 9.32 (a)

Ex: 9.20 From Exercise 9.17, we get

$$I_D = 0.4 \text{ mA}$$

$$V_{OV} = 0.2 \text{ V} \quad g_{m1,2} = 4 \text{ mA/V}$$

$$G_m = 4 \text{ mA/V} \quad A_d = 100 \text{ V/V}$$

Now,

$$R_{SS} = 25 \text{ k}\Omega$$

$$g_{m3} = \sqrt{2\mu_p C_{ox}\left(\frac{W}{L}\right)_p I_D}$$

$$= \sqrt{2 \times 0.1 \times 200 \times 0.4} = 4 \text{ mA/V}$$

$$|A_{cm}| = \frac{1}{2g_{m3}R_{SS}} = \frac{1}{2 \times 4 \times 25} = 0.005 \text{ V/V}$$

$$\text{CMRR} = \frac{|A_d|}{|A_{cm}|} = \frac{100}{0.005}$$

$$= 20{,}000 \text{ or } 20 \log 20{,}000 = 86 \text{ dB}$$