## THE PLACE OF THE LOCAL GROUP IN THE COSMIC WEB

J. E. FORERO-ROMERO<sup>1</sup> AND R. GONZÁLEZ<sup>2,3</sup>

Departamento de Física, Universidad de los Andes, Cra. 1 No. 18A-10, Edificio Ip, Bogotá, Colombia
 Instituto de Astrofísica, Pontificia Universidad Católica, Av. Vicuña Mackenna 4860, Santiago, Chile
 Centro de Astro-Ingeniería, Pontificia Universidad Católica, Av. Vicuña Mackenna 4860, Santiago, Chile
 Submitted for publication in ApJL

#### ABSTRACT

We explore the characteristics of the Local Group (LG) location in the cosmic web using a cosmological simulation in the  $\Lambda$ CDM cosmology. We use the Hessian of the gravitational potential to classify regions in the cosmic web as a peak, sheet, filament or void. The LG is represented by two samples of halos pairs. The first is a general sample composed by halo pairs with similar masses and isolation criteria as observed in the LG. The second is a subset with additional kinematic constaints. We find that the pairs in the LG sample with all constraints are: i) Preferentially located in filaments and sheets, ii) located in in a narrow range of local overdensity  $0 < \delta < 4$ , web ellipticity 0.1 < e < 1.0 and prolateness -0.5 . iii) Clearly aligned with the cosmic web, in particular for pairs in filaments/sheets the orbital angular momentum tends to be perpendicular to the filament direction or the sheet plane. A stronger alignment is present for the vector linking the two halos, which lies along the filament or the sheet plane. We show that the first and second results are expected trends with the LG total mass. However, the strong alignment with the cosmic web cannot be explained by a simple mass dependency.

Subject headings: galaxies: Local Group — dark matter

#### 1. INTRODUCTION

The spatial and kinematic configuration of the Local Group (LG) galaxies is rare to find in the local Universe and in cosmological simulations. The LG is dominated by the two big spirals MW and M31, the next most-luminous galaxy is M33 which is  $\sim 10$  times less massive than M31, followed by several less luminous dwarf galaxies, up to a distance of  $\sim 3 \rm Mpc$ . The velocity vector of M31, with a low tangential velocity is consistent to a head-on collision orbit toward the MW (Cox & Loeb 2008; van der Marel et al. 2012b; Sohn et al. 2012).

Another feature of the Local group is the relatively low velocity dispersion of nearby galaxies up to  $\sim 8$  Mpc (Sandage & Tammann 1975; Aragon-Calvo et al. 2011, and references therein). Environment around the Local Group has density quite close to the average density of the universe (Klypin et al. 2003; Karachentsev 2005). In addition, the closest massive galaxy cluster, the Virgo Cluster, is  $\approx 16.5$  Mpc away (Mei et al. 2007).

All this combination of features make LG analogues rare to find. Using numerical simulatons González et al. (2013a) found less than 2% MW-sized halos reside in a pair similar to MW-M31 and in a similar environment. Furthermore, if we select pairs constrained within  $2\sigma$  error from current observational measurements of the velocity components and distance to M31, there are only 46 systems in a cubic volume of  $250~h^{-1}{\rm Mpc}$  side, giving a number density  $\sim 1.0 \times 10^{-6}{\rm Mpc}^3$ , comparable to the abundance of massive clusters.

Forero-Romero et al. (2013) also studied MW-M31 pairs in numerical simulations finding the typical quantities characterizing the orbital parameters of the LG are rare among typical pairs, but not enough to challenge the  $\Lambda {\rm CDM}$  model.

Another definition of LG analogues is made by Li & White (2008), but despite differences ocurr in the definitions and resulting fraction of LG analogues, all are in agreement with a low frequency of these pairs.

To better understand the properties of the LG and how this uncommon pair configuration fit in the cosmological context, an immediate question arise. What else can we say of the LG at larger scales?. In particular, which are the typical/prefered locations of these systems within the Cosmic Web?. To what extent is this an expected configuration in  $\Lambda$ CDM.

Looking the LG at larger scales we have it is located in a difuse and warped filament/wall conecting Virgo Cluster with Fornax Cluster, some nearby galaxies and groups members of this large structure are the Maffei group, NGC 6744, NGC 5128, M101, M81, NGC1023, Cen A group. At this scale, there is no evident alignment of the MW-M31 orbital plane with the local filament or the Virgo-Fornax direction. However, if we look in a smaller volume below scales of  $\sim 6$  Mpc, there is a clear alignment of the MW-M31 orbit with a local plane as shown by Figure 3 in Courtois et al. (2013).

In this paper we study the large scale environment of LG analogues in the context of  $\Lambda \text{CDM}$ . We use the Bolshoi simulation to explore in what structures they reside and if there is any correlation or alignment with the cosmic web. The large scale Environment is defined by the cosmic web components identified by Forero-Romero et al. (2009), and we use the LG analogues computed by González et al. (2013a). We pay spetial attention to quantify the kind of environment that hosts LG pairs and their alignments with respect to the prefered directions defined by the T-web.

This paper is organized as follows. In Section 2 we present the N-body cosmological simulation and the algorithm to define the cosmic web, next in 3 we describe

the sample of LG analogues extracted from the simulation. In Section 4 we presents our results to continue with a discussion about their implications for our Local Group in Section 5 to finally conclude in Section 6.

#### 2. SIMULATION AND WEB FINDING ALGORITHM

#### 2.1. The Bolshoi simulation

We use the Bolshoi simulation of  $\Lambda$ CDM cosmology:  $\Omega_{\rm m}=1-\Omega_{\Lambda}=0.27,\ H_0=70\,{\rm km/s/Mpc},\ \sigma_8=0.82,\ n_s=0.95$  (Klypin et al. 2011), compatible with the constraints from the WMAP satellite (Hinshaw et al. 2013). The simulation followed the evolution of dark matter in a  $250h^{-1}{\rm Mpc}$  box with spatial resolution of  $\approx 1h^{-1}$  kpc and mass resolution of  $m_{\rm p}=1.35\times10^8\,{\rm M_{\odot}}$ . Halos are identified with the BDM algorithm (Klypin & Holtzman 1997). The BDM algorithm is a spherical overdensity halo finding algorithm and is designed to identify both host halos and subhalos.

## 2.2. Cosmic web identification

The web finding algorithm is based on the tidal tensor computed as the Hessian of the gravitational potential field

$$T_{ij} = \frac{\partial^2 \phi}{\partial r_i \partial r_j},\tag{1}$$

where  $r_i$ , i=1,2,3 refers to the three spatial comoving coordiates and  $\phi$  is the gravitational potential renormalized to follow the Poisson equation  $\nabla^2 \phi = \delta$  where  $\delta$  is the matter overdensity.

This tensor is real and symmetric, which means that can be diagonalized. We note its eigenvalues as  $\lambda_1 \geq \lambda_2 \geq \lambda_3$  and their corresponding eigenvectors  $\hat{e}_1$ ,  $\hat{e}_2$  and  $\hat{e}_3$ . The web classification compares each one of the three eigenvalues to a threshold value  $\lambda_{\rm th}$ . If the three, two, one or zero eigenvalues are larger than this threshold the region is classified as peak, filament, sheet or void, respectively.

Forero-Romero et al. (2009) performed a detailed study for the topology of the cosmic web and its visual counterpart as a function of the parameter  $\lambda_{\rm th}$ . They found that reasonable results in terms of the volume fraction occupied by voids, the visual inspection and the halo populations in each web type can be reached by values of  $0.2 < \lambda_{\rm th} < 0.4$ . In this paper we choose the value of  $\lambda_{\rm th} = 0.3$  to proceed with our analysis. This is only relevant to the classification of the simulation into web elements. Other results are completely independent of this choice. Nevertheless we have checked that the main conclusions of this work do not depend on the choise of  $\lambda_{\rm th}$ .

The algorithm to compute the potential is grid based. First we interpolate the density into a cubic grid with a Cloud-In-Cell (CIC) scheme and smooth it with a gaussian kernel. Then we obtain the gravitational potential using FFT methods and use finite differences to compute the Hessian at every point in the grid. In our case we have used a grid size on and a gaussian smoothing with two times larger as the typical separation between the two halos in the Local Group. The purpose of this choice is to have both halos in the pair a common environment. In this paper we use a grid spacing of s=0.97

 $h^{-1}{\rm Mpc}$  , corresponding to a  $256^3$  grid in the Bolshoi volume. The scale for the gaussian smoothing uses the same value.

We use the matter overdensity, ellipticity and the prolatenes to further characterize the web. These quantities are defined in terms of the eigenvalues as follows

$$\delta = \lambda_1 + \lambda_2 + \lambda_3,\tag{2}$$

$$e = \frac{\lambda_3 - \lambda_1}{2(\lambda_1 + \lambda_2 + \lambda_3)},\tag{3}$$

$$p = \frac{\lambda_1 + \lambda_3 - 2\lambda_2}{2(\lambda_1 + \lambda_2 + \lambda_3)}. (4)$$

We also measure the alignment of the LG halos with respect to the cosmic web defined by the eigenvector. To this end we characterize each LG pair by two vectors. The first is  $\hat{n}$ , the vector marking the axis along the orbital angular momentum of the pair, normal to its orbital plane; the second is  $\hat{r}$ , the vector that connects the halos in the pair which can be related to the alignment of the radial velocities to the web. We quantify the alignment using the absolute value of the cosinus of the angle between the two vectors of interest  $\mu=|\hat{e}_i\cdot\hat{n}|$  or  $\mu=|\hat{e}_i\cdot\hat{r}|,$  where i=1,2,3.

The data of the BDM halos and the Tweb is publicly available through a database located at http://www.cosmosim.org/. A detailed description of the database structure was presented by Riebe et al. (2013).

## 3. LOCAL GROUP ANALOGUES

To construct a sample of the MW-M31 pairs at  $z \approx 0$ , we use a series of simulation snapshots at z < 0.1 (since the last  $\approx 1.3$  Gyr) spaced by  $\approx 150-250$  Myr. This is done because a particular configuration of MW and M31 is transient and corresponds to a relatively small number of systems at one snapshot. By using multiple snapshots we can increase the sample of systems in such configuration during a period of time in which secular cosmological evolution is small.

The LG analogues or General Sample (GS) in this paper are pairs selected in relative isolation, and in a wide range of masses from  $M_{200c} = 5 \times 10^{11} \, \mathrm{M}_{\odot}$  to  $5 \times 10^{13} \, \mathrm{M}_{\odot}$ . Isolation criteria includes a pair closer than 1.3Mpc, and with no massive neighbors within 5Mpc. In addition we require that pairs have no Virgo-like neighbor halo with mass  $M_{200c} > 1.5 \times 10^{14} \, \mathrm{M}_{\odot}$  within 12 Mpc. We have 5480 pairs under these general criteria. A full description of the selection criteria can be found in González et al. (2013a,b).

We also define two subsamples more closely related to the MW-M31 dynamics according to the tolerance in additional constraints. A sample named  $2\sigma$ , correspondig to LG analogues constrained by two times the observational errors in the orbital values (radial velocity, tangential velocity, and separation), and a more relaxed sample named  $3\sigma$  for LG analogues constrained by three times observational errors accordingly. The number of pairs in each sample is 46 and 120 respectively, notice we have less pairs than in González et al. (2013a) results, since we removed pairs which are too close at z=0, i.e. their virial radii overlaps, also we removed a couple pairs that

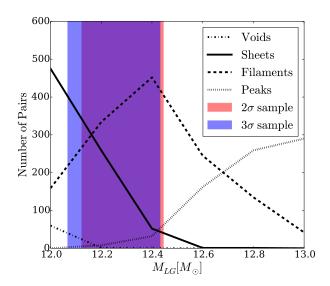


Fig. 1.— Mass distribution of pairs in the different environments for the general sample. The shaded regions show the mass ranges of  $2\sigma$  and  $3\sigma$  samples.

merged or change their mass more than 20% at present time since they were detected at z < 0.1.

#### 4. RESULTS

#### 4.1. The prefered environment for LGs

The first result we explore is the kind of environment occupied by our LGs. We find that the LGs in the general sample are located across all different evironment without any strong preferences; 1/3 are located in sheets, 1/4 in peaks, 1/4 in filaments and the remaning 1/6 in voids.

The situation in the restricted  $2\sigma$  and  $3\sigma$  samples is very different. By large the LGs in these samples are located in filaments and sheets. In both samples,  $\sim 50\%$  of the pairs can be found in filaments while  $\sim 40\%$  are in sheets. These absolute numbers in each environment for each sample are presented in Table 1.

This difference between the general and the restricted samples is due the mass ranges covered by each sample. In González et al. (2013a) the mass range covered by  $2\sigma$  and  $3\sigma$  is very narrow and it is used to constraint the LG mass. We show in table 1 that subset of the GS having a similar mass range to  $2\sigma$  and  $3\sigma$  reproduces similar environment fractions.

Figure 1 clearly shows the correlation between environment an total pair mass. Each line represents the mass distribution of pairs in the four different environments for the general sample. High mass pairs tend to be located in peaks and filaments while less massive ones in voids and sheets. The shaded regions represent the 68% confidence intervals of the mass distributions of  $2\sigma$  and  $3\sigma$  samples.

#### 4.2. Web Overdensity, Ellipticity and Anisotropy

We now describe the preferred place of the LG samples in terms of the web overdensity, ellipticity and anisotropy as defined in Section 2.

Figure 2 shows dependency of the web overdensity, ellipciticy, and prolateness on pair mass for the different

samples. GS is represented by the solid lines with the associated errors covered by the shaded region. The symbols represents the results for the  $2\sigma$  and  $3\sigma$  samples. In all cases it is immediatly clear that the range of values for the  $2\sigma$  and  $3\sigma$  samples are completely expected from its mass dependence.

Left panel shows the overdensity dependence on pair mass. Higher mass pairs are located in high density regions. The  $2\sigma$  and  $3\sigma$  samples having a narrower mass range as shown in previous figure, are consequently located within a narrower range of overdensities  $0.0 < \delta < 4.0$  peaking at  $\delta \sim 1$ . This is also consistent with the fact that these samples are mostly found in filaments and sheets. The average overdensity of  $2\sigma$  and  $3\sigma$  samples is expected from the values in the GS within the same mass range.

Middle and right panels show web ellipciticy and absolute prolateness dependence on mass. Again we noticed that within the same mass range, the  $2\sigma$  and  $3\sigma$  average ellipciticy and prolateness does not differ significantly from GS For the  $2\sigma$  and  $3\sigma$  samples most of the pairs are located in a narrow range for ellipticities in the range 0.1 < e < 1.0, and prolateness |p| < 0.5.

## 4.3. Alignments with the cosmic web

We now study different alignments of the LG with respect to the cosmic web.

**Orbital Angular Momentum**. Figure 3 shows the cumulative distribution of  $\mu \equiv \hat{e}_i \cdot \hat{n}$  for the three eigenvectors i=1,2,3. Lines in each panel correspond to different samples. The straight line across the diagonal shows the expected result for vectors with randomly distributed directions.

There are two important features in Figure 3. First the alignments themselves. There is a strong anti-alignment signature between  $\hat{n}$  and the third eigenvector. With respect to the second eigenvector the distribution is consistent with no alignment. For the first eigenvector there is a strong alignment, an expected result after considering the results with the other eigenvectors and keeping in mind that the eigenvectors are orthonormal. Second, the alignment strenght changes for the different samples. For the anti-alignment with  $\hat{e}_3$  the signal strenghtens as we move from the GS to the  $3\sigma$  into the  $2\sigma$  sample.

Quantitatively, the anti-alignment feature found with the  $\hat{e}_3$  vector means that for  $2\sigma$  sample,  $\sim 50\%$  pairs have  $|\mu| < 0.2$  (78 degrees angle), and  $\sim 75\%$  pairs have  $|\mu| < 0.4$  (66 degrees angle). These signals do not change significantly on different environments as has been already show in different alignment studies that similar (Libeskind et al. 2013) or identical (Forero-Romero et al. 2014) web finding techniques as ours. In particular these trends hold for pairs in filaments and walls. If we consider only pairs in filaments, we have that the pair orbital angular momentum tends to be perpendicular to the filament direction, in the case of sheets it tends to lie perpendicular to the sheet plane.

Can the increase of the alignment strenght in the different samples be explained as a consequence of different mass dependencies as the results in the previous section? No. In Figure 5 we show the median of  $\mu \equiv \hat{e}_i \cdot \hat{n}$  in different total mass bins. Lines show the median  $\mu$ -mass relation for the three eigenvectors in the GS, Dots represent the results for the  $2\sigma$ - $3\sigma$  samples. In a mass range

Sample	Peak	Filament	Sheet	Void
	n (%)	n (%)	n (%)	n (%)
$2\sigma$	4 (8.7)	24 (52.2)	17 (36.7)	1 (2.2)
$3\sigma$	10 (8.3)	58 (48.3)	47(39.2)	5(4.2)
General	1312 (23.9)	1472(26.9)	1769(32.3)	927(16.9)
General $(12.1 < \log_{10} M_{\rm LG}/{\rm M}_{\odot} < 12.3)$	8 (1.4)	334 (55.5)	259 (43.0)	1 (0.1)

#### TABLE 1

Number of pairs in the four different kinds of environments for each of the three samples presented in Section 3. In parethesis the same number as a percentage of the total population. The last line in the table corresponds to the general sample with an additional mass cut for the total pair mass.

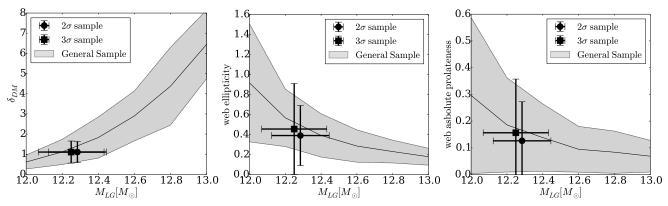


Fig. 2.— Mass dependency of the average dark matter overdensity (left), web ellipticity (middle) and web absolute value prolateness (right) at the pair location.

around  $1.5-2.0\times10^{12}~{\rm M}_{\odot}$  the median of  $\mu$  decreases as we increase the constraints on the pairs, not as a result of narrowing mass selection.

**Radial Vector**. Figure 4 presents the results for the eigenvectors alignments with respect to the vector connecting the two halos. In this case we find that the vector  $\hat{r}$  is strongly aligned along the direction defined by  $\hat{e}_3$  and anti-aligned along  $\hat{e}_1$ ; correspondingly the signal along  $\hat{e}_2$  is rather weak.

We also find a stronger signal as we move into more restrictive samples, although the signal from the GS is already very significant. Quantitatively, the alignment feature with  $\hat{e}_3$  means that for the  $2\sigma$  and  $3\sigma$  samples,  $\sim 50\%$  pairs have  $|\mu| > 0.8$  (36 degrees angle) and  $\sim 25\%$  pairs have  $|\mu| < 0.95$  (18 degrees angle).

Considering that the  $2\sigma$  and  $3\sigma$  samples move primarely along the radial direction, we can say that the motion of the LG halos is mostly done along the  $\hat{e}_3$  vectors, consistent with recent results that report a strong alignment of halo's peculiar velocities along that direction (Forero-Romero et al. 2014).

**Halo Spin**. We also explore the alignment of the angular momentum (spin) of each pair member  $\hat{j}_A$  and  $\hat{j}_B$  with each other, the orbital angular momentum and with the cosmic web.

Figure 6 shows the cumulative distribution of dot product between angular momentum of the two halos. We find a a slight alignment of spin vectors for the  $2\sigma$  sample with a median around  $|\mu| \sim 0.7$  (45 degrees) However, we found no significant alignment with the orbital angular momentum nor the cosmic web. This is also consistent with recent results that show that the spin alignment of halos of masses around  $10^{12} \rm M_{\odot}$  is very weak (Forero-Romero et al. 2014).

In the LG, the angle between MW and M31 spin is  $\sim$  60° ( $|\mu|=0.5$ ) and the angles between spins and orbital angular momentum are  $\sim$  33° ( $|\mu|=0.83$ ) and  $\sim$  76° ( $|\mu|=0.24$ ) for MW and M31 respectively (van der Marel et al. 2012a). Consistent with our results of no particular alignement.

# 5. DISCUSSION

The mass range of the LG pairs is tightly correlated with the properties of the web where they reside as shown in Table 1 and Figure 1. Recent analytical (Alonso et al. 2014) and numerical studies (Metuki et al. 2014) have shown that the local overdensity is the dominant web property that define the abundance and properties of halos. Quantities derived from the tidal tensor play a secondary role.

Therefore, in our case, the fact that the preferred LG total mass is around  $1-4\times12^{12}\mathrm{M}_{\odot}$ implies that the preferred environment are filaments and sheets with an overdensity close to the average value, with values for the ellipticity and prolateness also correlated with the overdensity.

However, there are alignment signals with the cosmic web that are not mass dependent and seem to be stronger for different LG samples. There is a clear anti-alignment between the third eigenvector and the orbital angular momentum vector, meaning that this vector is perpendicular to filaments and the sheets, depending on the particular web environment. We also found that the vector joining the LG halos are aligned with the third eigenvector; i.e. the pair is aligned with the filaments and lies on the sheets.

These alignment features are in agreement with the scenario that pairs created in-situ or falling into a filament/wall align their orbits with the large scale struc-

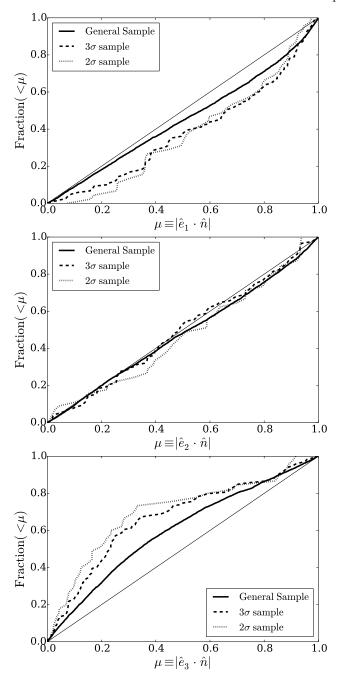


Fig. 3.— Cumulative distributions for the alignment between the normal vector to the pair orbital plane,  $\hat{n}$ , and the three eigenvectors in the Tweb.

ture in a relaxation process where pair members tend to moves along the slowest collapsing directions.

The most interesting feature is that these alignments are tighter as we narrow the kinematic conditions of the LG towards the observed values. A deeper understanding requires the study of the full formation history and coevolution with the cosmic web, a task beyond the scope of the work presented here.

# 6. CONCLUSIONS ACKNOWLEDGEMENTS

REG was supported by Proyecto Financiamiento Basal PFB-06 'Centro de Astronomia y Tecnologias Afines' and 0.8 0.8 0.8 0.8 0.9

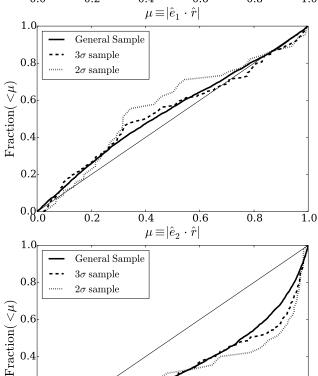


Fig. 4.— Cumulative distributions for the alignment between the vector linking the two halos in the pair,  $\hat{r}$ , and the three eigenvectors in the Tweb.

 $\mu \equiv |\hat{e}_3 \cdot \hat{r}|$ 

0.6

0.8

1.0

0.4

0.2

0.2

Proyecto Comité Mixto ESO 3312-013-82. The Geryon cluster at the Centro de Astro-Ingenieria UC was extensively used for the calculations performed in this paper. The Anillo ACT-86, FONDEQUIP AIC-57, and QUIMAL 130008 provided funding for several improvements to the Geryon cluster.

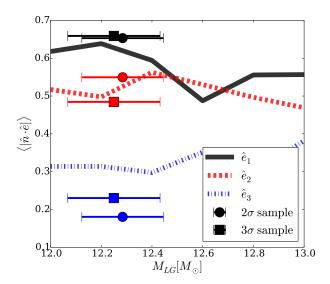
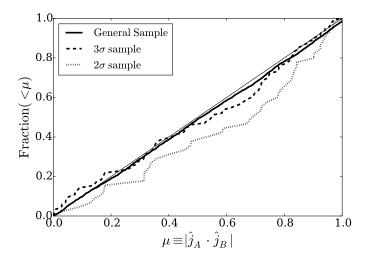


Fig. 5.— Mass dependency of the median value for the dot product between the normal vector  $\hat{n}$  and each one of the eigenvectors. The lines show the trends for the general sample.



 ${\rm Fig.~6.--}$  Alignment between the two angular momentum vectors of the two halos in the pair.

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