The influence of discreteness and inhomogeneity on cosmic flows

Jaime E. Forero-Romero

September 26, 2013

Abstract

Gravity is the dominant force shaping the spatial distribution of galaxies in the Universe. Under the assumption of homogeneity and isotropy of the Universe beyond a certain physical scale one can usually approximate the local kinematic evolution of galaxy by the matter distribution below that homogeneity scale. In this letter we show that if the matter distribution is composed by a discrete set of points then, due to statistical fluctuations, the influence of matter has a measurable effect at all scales. Our results are based on straightforward analytical considerations, monte-carlo realizations and the analysis of cosmological N-body simulations. We discuss the implications of these results in the interpretation of the peculiar motion of our galaxy. We suggest possible cosmological tests of these ideas for future observational facilities.

1. Of all the fundamental forces gravity plays the dominant role in defining the large scale structure of the Universe. From very homogeneous conditions gravitational instability drives the emergence of a web-like pattern. The influence of matter beyond that scale can be discarded on the grounds that the net gravitational force inside an homogeneous spherical shell is zero. In classical newtonian mechanics it is possible to show that the net force inside an homogeneous and continus shell of matter is exactly zero. The same is valid in general relativity. That result receives the name of the Birkhoff's theorem.

2a. The observational advances in the last two decades have allowed us to measure the distribution of galaxies on very large scales. The spatial pattern they follow is highly structured. It resembles a large network of interconnected filaments and sheets crossing large underdense regions. This is why the community referes to this structure as the cosmic web. At the same time the concept of statistical homogeneity is a fundamental assumption to various theoretical aspects in cosmology.

2b. Quantitative tests of homogeneity seek to measure the indipendent volumes of the Universe and test whether they have similar mean densities of matter. Therefore, ine commonon way to estimate this scale is based on counting the number of massive galaxies inside a sphere of radius R. Also the variations of number density for different values of the radius R is used as a quantitative test to measure homogeneity [1]. Similar comoslogical tests aim at using measurements of large scale structure to estimate the peculiar acceleration of the local group that is induced by the surrounding matter [2, 3]. These two measurements can be then compared against the predictions of theories of large scale structure evolution and serve as a cosmological probe.

Once the scale of statistical homogeneinity is well determined one might be tempted to make the approximation that, due to Birkhoff's theorem, it becomes, a good approximation to neglect the influcene of matter beyond that scale on the dynamical influence of matter located in a sphere centerd in a sphere of radius equal to the homogeneity scale. In this letter we show how the effects of discretenes on a statistically homogeneous matter distribution can have a dynamical effect. We extend these results to the case of the large scale strucure of the universe by means of N-body simulations that presenting the effect of matter distributions beyond the assumed homogeneity scale of $70 - 200h^{-1}$Mpc .

3. We consider first a simple phenomenological model where matter is homogeneously distributed over the surface of a sphere of radius R. This distribution consists of a set of N_p point masses with mass m. Under these conditions the total force, F_{T}, at the center of that spherical distribution can be expressed in terms of the force $F_{\mathrm{m}}(R)$ produced by a single point mass located at a position R as follows:

$$F_{\mathrm{T}} = N_p^{1/2} F_{\mathrm{m}}(R), \tag{1}$$

a result that can be explained analytically [4, 5]. In Figure 2 we show the results of a simple Monte Carlo simulation where N_p particles are located over the surface sphere of radius R, the total force per unit mass on a test particle, F_{T}, is then expressed as a multiple of $F_m(R) = Gm_p/R^2$. This shows how the analytical expectation is a good approximation to the results obtained through MonteCarlo simulations.

4. We can now extend this result for an statistically homogeneous distribution of points. We run again a simple Monte Carlo simulation to find that the total force per unit mass produced by the particle distribution is twice as the of a single mass m located at a distance R_{s} equal to the average interparticle separation, independent of the total number of particles.

$$F_{\mathrm{T}} = 2F_{\mathrm{m}}(R_{\mathrm{s}}). \tag{2}$$

Figure 2 presents the results of a MonteCarlo realization of this experiment. It shows that the peak of the distribution of values for $F_{\mathrm{T}}/F_{\mathrm{m}}(R_{\mathrm{s}})$ is located at values of ~ 2. The distribution is broad and allows values one order of magnitude above/below the maximum.

5. In the case of the actual large scale matter distribution in the Universe, galaxies are found to have two distinct characteristics with respect our toy model. First, the galaxies are not randomly distributed, but clustered. Second, the galaxies span a wide range of masses, with less massive galaxies being more common than massive galaxies. In order to test the influence of these two carachteristics we use a large comological N-body simulation. We have used a

simulations that allows us to build spheres of $500h^{-1}$Mpc in radius and include halos with masses spanning a range from Milky Way to galaxy clusters.

6. We define a fiducial scale of homogeneity of $R_{\rm in} = 50h^{-1}$Mpc to measure the fluctuation of the force due to matter in a sphere below that scale ($F_{\rm in}$) and the force of the matter distribution outside that region ($F_{\rm out}$) in a spherical region up to $R_{\rm out} = 500h^{-1}$Mpc . Figure 3 shows the ratio $F_{\rm out}/F_{\rm in}$ as we travel in a straight line along trhough the cosmological simulation on steps of $1h^{-1}$Mpc . This figure shows that order of magnitude fluctuations are possible on scales of $\sim 10h^{-1}$Mpc . It also makes clear that the force produced from matter on scales $500h^{-1}$Mpc is usually ten times the force produced by the matter on scales below $50h^{-1}$Mpc .

7. These different values for the force ration also hints towards different peculiar acceleration growth histories as matter is integrated into spheres of growing radii. For instance for low values of $F_{\rm out}/F_{\rm in} \sim 1.0$ the peculiar acceleration induced by matter inside a sphere of fixed radis R barely changes as this sphere grows from from $R = 50h^{-1}$Mpc to $R = 500h^{-1}$Mpc . However the most common case seems to be $F_{\rm out}/F_{\rm in} \sim 10.0$. Figure 4 shows the growth of the force reation as the external radius increases, this result is measured for a random halo in the simulation with a typical value for $F_{\rm out}/F_{\rm in}$.

8. However until now we have focused on statistics derived over the simulation volume, but not on the halos from the simulation. Figure 5 showd the distribution of $F_{\rm out}/F_{rmin}$ for spheres centered on 1000 random halos in the simulation. Due to the mass distribution in cosmological volumes most of them should correspond to halos with masses in the Milky Way mass range. This shows that the most common expectation in the LCDM cosmological model is that the force produced by the halo mass distribution out to scales of $500h^{-1}$Mpc is ~ 5 times that of the matter distribution inside a scale of $50h^{-1}$Mpc .

9. The velocity of our galaxy in the rest-frame of the cosmic micriwave background is 627 km s^{-1}. [http://arxiv.org/pdf/1109.3856v1.pdf]. It has been inferred that the 382 km s^{-1} are induced by the mass distribution within $R = 30$ h^{-1} Mpc, we call this the local component. This gives a net result of 382 km s^{-1} that must be induced by the matter distribution from matter with positions beyond R, this is referred as the tidal component. Both components, local and tidal, are pointing in the same direction (??? SURE ??).[...]

11. Possible effects on detailed methods that seek to infer local density distributions from peculiar velocity measurements [like Cosmic Flows (Trade Mark)].

12. Possible effects on large scale structure measurements that want to sued redshift distortions to deduce the statistics of the underlying matter distribution.

13. The effects of the net force imposed by the matter distribution in the Universe is also in principle detectable in systems isolated from massive structures. The most interesting case would be the kinematic evolution of galaxies located in large scale voids. In these regions the dominant gravitational interaction would be provided by the tidal component and not by nearby structures. [http://adsabs.harvard.edu/abs/2011IJMPS...1...41V]

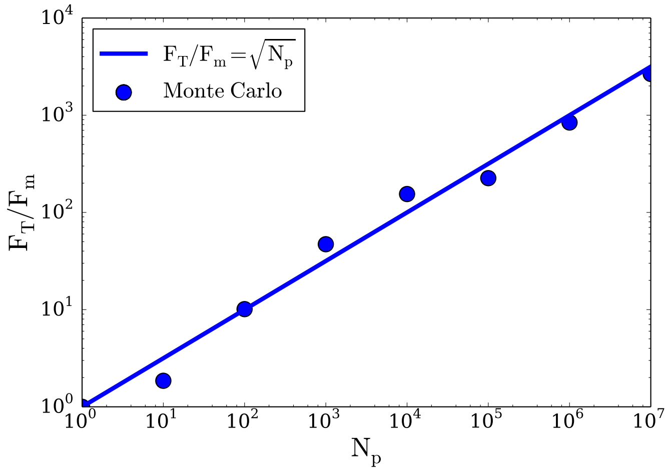

Figure 1: Norm of the total force per unit mass, F_T, produced by a distribution of N_p point masses randomly distributed over the surface of a sphere of radius R. The value of F_T is computed at the center of the sphere and is expressed as a multiple of the force per unit mass produced by a single point mass located at a distance R from the center.

.

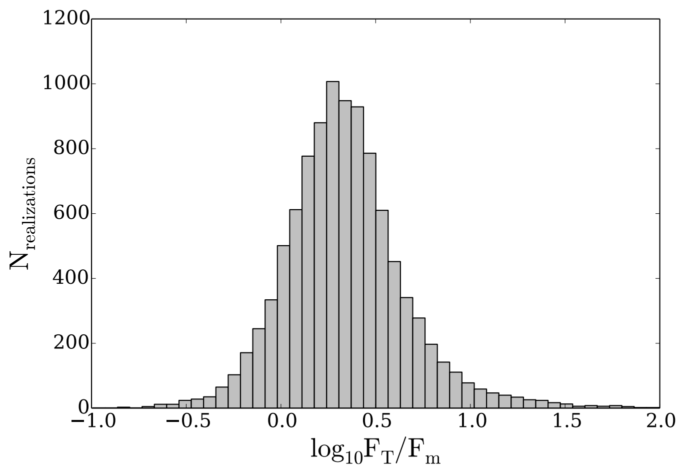

Figure 2: Frequency of the norm of the total force per unit mass produced by a set of identical N_p particles homogeneously distributed inside a sphere of radius R. This corresponds to 10^4 different realizations of $N_p = 10^4$ particles. The values of F_T is normalized to the force per unit mass produced by a single particle located at a distance R_s from the center equal to the average interparticle separation. .

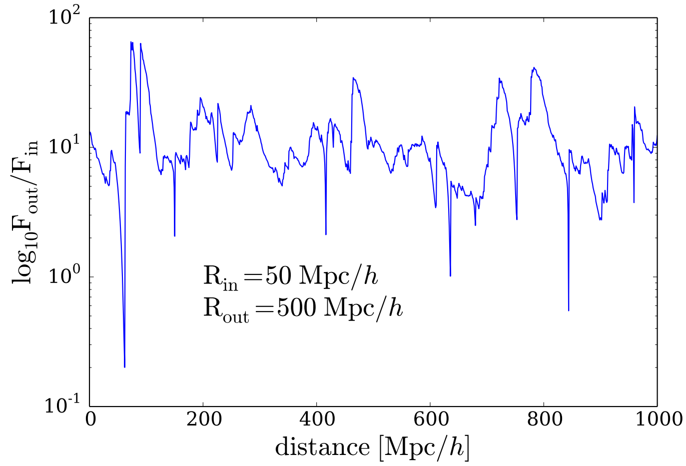

Figure 3: . Forces produced by dark matter halos in a cosmological simulation. The figure shows the ratio between the force produced by all halos within a spherical shell of halos with radial distances $50h^{-1}\mathrm{Mpc} < R < 500h^{-1}\mathrm{Mpc}$ and the halos in the inner core with $R < 50h^{-1}\mathrm{Mpc}$. This is presented as a function of the distance along a line randomly traced in the cosmological box.

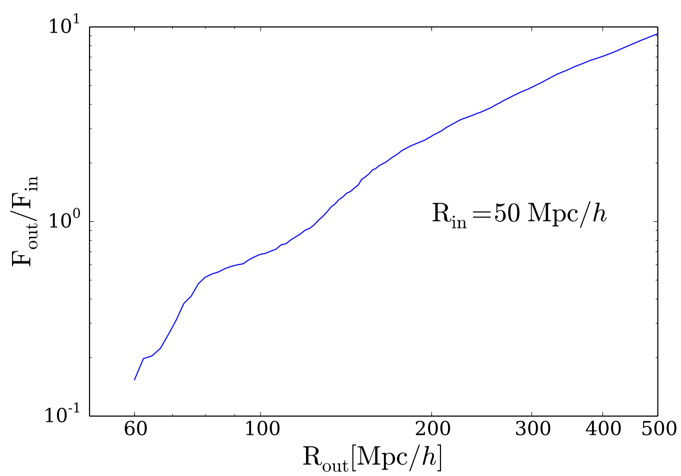

Figure 4: . Forces produced by dark matter halos over a random dark matter halo in a simulation. This figures presents the ration between the forces by all halos within a spherical shell with radial distances $50h^{-1}\mathrm{Mpc} < R < R_{\mathrm{out}}$ and he halos within a spherical shell with $R < 50h^{-1}\mathrm{Mpc}$.

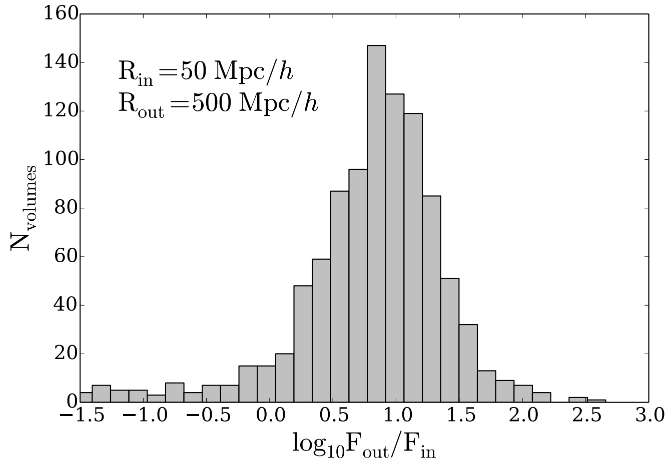

Figure 5: Frequency of the force ratio between the halos inside a spherical shell with radial distances $50h^{-1}$Mpc $< R < 500h^{-1}$Mpc and halos within a sphere of $R < 50h^{-1}$Mpc. The forces are measured at the positions of 1000 dark matter halos randomly selected in the simulated volume.

References

[1] D. W. Hogg, D. J. Eisenstein, M. R. Blanton, N. A. Bahcall, J. Brinkmann, J. E. Gunn, and D. P. Schneider. Cosmic Homogeneity Demonstrated with Luminous Red Galaxies. *ApJ*, 624:54–58, May 2005.

[2] C. Gibelyou and D. Huterer. Dipoles in the sky. *MNRAS*, 427:1994–2021, December 2012.

[3] M. Bilicki. Motion of the Local Group as a cosmological probe. *ArXiv e-prints*, May 2012.

[4] S. Chandrasekhar. Stochastic Problems in Physics and Astronomy. *Reviews of Modern Physics*, 15:1–89, January 1943.

[5] A. Carati, S. L. Cacciatori, and L. Galgani. Discrete matter, far fields, and dark matter. *EPL (Europhysics Letters)*, 83:59002, September 2008.