Spheres

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Abstract

Gravity is the dominant force shaping the spatial distribution of galaxies in the Universe. Under the assumption of homogeneity and isotropy of the Universe beyond a certain physical scale one can usually approximate the local kinematic evolution of galaxy by the matter distribution below that homogeneity scale. In this letter we show that if the matter distribution is composed by a discrete set of points then, due to statistical fluctuations, the influence of matter has a measurable effect at all scales. Our results are based on straightforward analytical considerations, monte-carlo realizations and the analysis of cosmological N-body simulations. We discuss the implications of these results in the interpretation of the peculiar motion of our galaxy. We suggest possible cosmological tests of these ideas for future observational facilities.

1. Of all the fundamental forces gravity plays the dominant role in defining the large scale structure of the Universe. From very homogeneous conditions gravitational instability drives the emergence of a web-like pattern. The influence of matter beyond that scale can be discarded on the grounds that the net gravitational force inside an homogeneous spherical shell is zero. [Inhomogeneity, Kirchoff's theorem] [...]

2. Large galaxy surveys help us to define different physical scales for homogeneity [...]

3. We consider first a simple phenomenological model where matter is homogeneously distributed over the surface of a sphere of radius R. This distribution consists of a set of N_p point masses with mass m. Under these conditions the total force, $F_{\rm T}$, at the center of that spherical distribution can be expressed in terms of the force $F_{\rm m}(R)$ produced by a single point mass located at a position R as follows:

$$F_{\rm T} = N_p^{1/2} F_{\rm m}(R), \tag{1}$$

a results that can be explained analytically [1, 2]. In Figure 2 we show the results of a simple Monte Carlo simulation where N_p particles are located over the surface sphere of radius R, the total force per unit mass on a test particle, $F_{\rm T}$, is then expressed as a multiple of $F_m(R) = Gm_p/R^2$. This shows how the analytical expectation is a good approximation to the results obtained through MonteCarlo simulations.

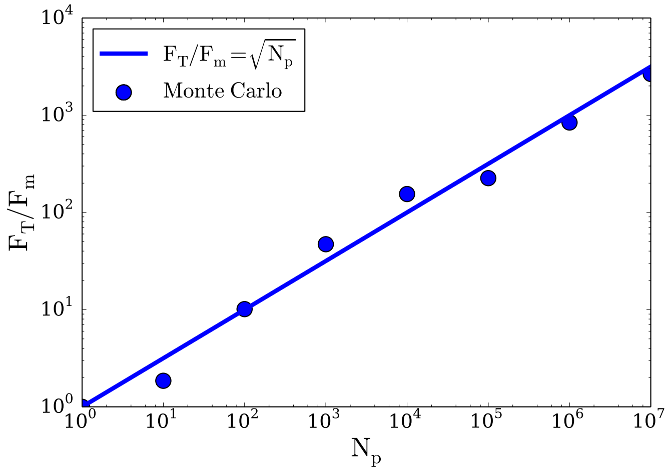

Figure 1: Norm of the total force per unit mass, F_T, produced by a distribution of N_p point masses randomly distributed over the surface of a sphere of radius R as a function of the number of point masses. The value of F_T is computed at the center of the sphere and is expressed as a multiple of the force per unit mass produced by a single point mass located at a distance R from the center.

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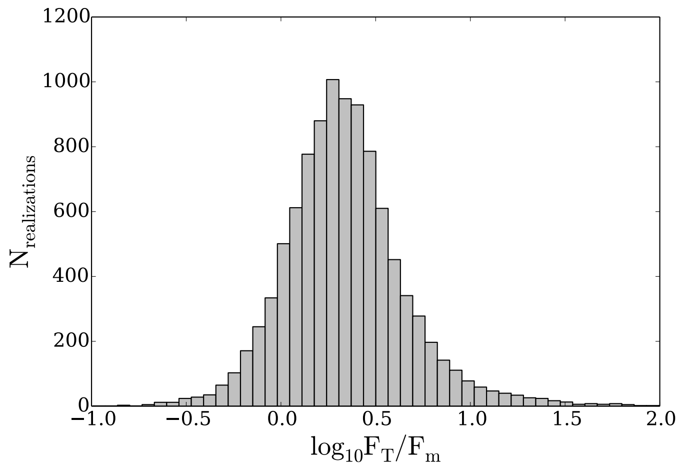

Figure 2: Frequency of the norm of the total force per unit mass produced by a set of identical N_p particles homogeneously distributed inside a sphere of radius R. This corresponds to 10^4 different realizations of $N_p = 10^4$ particles each. The values of F_T is normalized to the force per unit mass produced by a single particle located at a distance R_s equal to the average interparticle separation from the center. .

4. Conventional approximations consider that the net gravitational force inside an statistically homogeneous shell matter distribution is zero. However, by extending the result we have just derived, as long the matter distribution is discrete, this result does not hold. We can now extend this result for an statistically homogeneous distribution of points. We run again a simple Monte Carlo simulation to find that the total force per unit mass produced by the particle distribution is twice as the of a single mass m located at a distance R_s equal to the average interparticle separation, independent of the total number of particles.

$$F_\mathrm{T} = 2F_\mathrm{m}(R_\mathrm{s}). \tag{2}$$

Figure 2 presents the results of a MonteCarlo realization of this experiment. It shows that the peak of the distribution of values for $F_\mathrm{m}/F_\mathrm{m}(R_\mathrm{s})$ is located at values of ~ 2. Values two order of magnitude higher are possible due to a

5. In the case of the actual large scale matter distribution in the Universe, galaxies are found to have two distinct characteristics with respect our toy model. First, the galaxies are not randomly distributed, but clustered. Second, the galaxies span a wide range of masses, with less massive galaxies being more common than massive galaxies. In order to test the influence of these two carachteristics we use a large comological N-body simulation.[...]

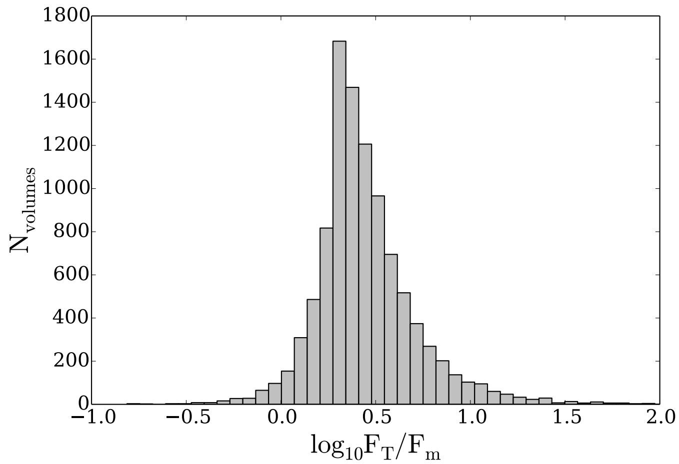

Figure 3: Frequency of the norm of the total force per unit mass produced by a halos inside a spherical volume from a cosmological N-body simulation. The radius of the spherical region is 300 h^{-1} Mpc. This corresponds to 10^4 different volumes The values of F_T are normalized to the force per unit mass produced by a single particle located at a distance R_s equal to the average interparticle separation from the center and a mass equal to the average mass of all halos in the spherical volumne. .

6. The results of the simulation [...]

7. We also consider the variation in the direction of the net force when the positions of the halos are perturbed within 1 h^{-1} which is the typical lnghtscale that these objects are expected to travel in the age of the Universe.

8. The fact that our simple analytical model describes the result obtained by simulations strenghtens our interpretation on the basis of fundamental point processes. [...]

9. The velocity of our galaxy in the rest-frame of the cosmic micriwave background is 627 km s^{-1}. [http://arxiv.org/pdf/1109.3856v1.pdf]. It has been inferred that the 382 km s^{-1} are induced by the mass distribution within $R = 30$ h^{-1} Mpc, we call this the local component. This gives a net result of 382 km s^{-1} that must be induced by the matter distribution from matter with positions beyond R, this is referred as the tidal component. Both components, local and tidal, are pointing in the same direction (??? SURE ??).[...]

10. The effects of the net force imposed by the matter distribution in the Universe is also in principle detectable in systems isolated from massive structures. The most interesting case would be the kinematic evolution of galaxies located in large scale voids. In these regions the dominant gravitational interaction would be provided by the tidal component and not by nearby structures.

[http://adsabs.harvard.edu/abs/2011IJMPS...1...41V]

11. Possible effects on detailed methods that seek to infer local density distributions from peculiar velocity measurements.

12. The physics of this effect are very simple to interpret.

References

[1] S. Chandrasekhar. Stochastic Problems in Physics and Astronomy. *Reviews of Modern Physics*, 15:1–89, January 1943.

[2] A. Carati, S. L. Cacciatori, and L. Galgani. Discrete matter, far fields, and dark matter. *EPL (Europhysics Letters)*, 83:59002, September 2008.