Applied Algorithms, Summer 2017

Homework 1,

Due date: XX/7/17

1. **(16 pts.)** Use the Traditional Marriage Algorithm to find a stable pairing of the following instance with n=4. Boys={A,B,C,D,E}, Girls={V,W,X,Y,Z} and preferences list as follows. Show your work in a table.

V: C,D,E,A,B

W: A,B,C,D,E

X: E,D,A,C,B

Y: B,D,E,C,A

Z: A,B,C,D,E

A: V,W,X,Y,Z

B: Z,Y,X,W,V

C: Z,Y,W,X,V

D: X,Z,Y,V,W

E: X,Z,W,Y,V

1. **(24 pts)** Consider an input to the stable pairing problem with n men and n women. Out of the n men, there are k smart men and n-k stupid men. Also, there are k smart women and n-k stupid women (for some k between 1 and n-1). Everyone would rather marry any smart person than any stupid person (so the first k entries in any preference list are of smart people of the opposite gender in some order). Prove that in every stable pairing, every smart man is matched with a smart woman.

**3.a. (12 pts.)** The problem 1||SjCj is solved optimally on an instance of n jobs. The solutions' value is z. The processing time of each of the n jobs is increased by 1. The problem 1||SjCj is then solved on the resulting instance. What is the solution's value, as a function of n and z. Explain.

**3.b. (12 pts.)** For some instance of n jobs, and m=2 machines, the value of an optimal solution toP||Cmax is z. The processing time of each of the n jobs is increased by 1. Prove or give a counter example: The value of an optimal solution for P||Cmax on the resulting instance is at least z + ën/2û.

**4.a. (16 pts.)** Find an optimal solution for P3||SjCj and the set of jobs of lengths {3,5,31,4,8,11,6,8,1,7}. What is Cmax in your schedule?

What is the makespan of the schedule produced for this set of jobs and machines by list-scheduling (i) when the jobs are processed from left to right (ii) from right to left (ii) in LPT order?

**4.b** **(20 pts.)** Let n=zm (n= number of jobs, m = number of machines). Prove that in the optimal solution to P||SjCj there are exactly z jobs on each machine.

**Note:** The algorithm SPT uses this property, so your proof cannot simply say ‘because this is what we get from SPT’).

**Directions:** Assume the claim is false. Therefore, there must be an optimal solution in which there is a machine M1 with z+x jobs (x³1) and a machine M2 with z-y jobs (y³1). Proceed using an exchange argument.