Applied Algorithms, Summer 2017

Homework 1,

Due date: XX/7/17

1. **(16 pts.)** Use the Traditional Marriage Algorithm to find a stable pairing of the following instance with n=4. Boys={A,B,C,D,E}, Girls={V,W,X,Y,Z} and preferences list as follows. Show your work in a table.

V: C,D,E,A,B

W: A,B,C,D,E

X: E,D,A,C,B

Y: B,D,E,C,A

Z: A,B,C,D,E

A: V,W,X,Y,Z

B: Z,Y,X,W,V

C: Z,Y,W,X,V

D: X,Z,Y,V,W

E: X,Z,W,Y,V

1. **(24 pts)** Consider an input to the stable pairing problem with n men and n women. Out of the n men, there are k smart men and n-k stupid men. Also, there are k smart women and n-k stupid women (for some k between 1 and n-1). Everyone would rather marry any smart person than any stupid person (so the first k entries in any preference list are of smart people of the opposite gender in some order). Prove that in every stable pairing, every smart man is matched with a smart woman.

**3.a. (12 pts.)** The problem 1||ΣjCj is solved optimally on an instance of n jobs. The solutions' value is z. The processing time of each of the n jobs is increased by 1. The problem 1||ΣjCj is then solved on the resulting instance. What is the solution's value, as a function of n and z. Explain.

**3.b. (12 pts.)** For some instance of n jobs, and m=2 machines, the value of an optimal solution toP||Cmax is z. The processing time of each of the n jobs is increased by 1. Prove or give a counter example: The value of an optimal solution for P||Cmax on the resulting instance is at least z + ⎣n/2⎦.

**4.a. (16 pts.)** Find an optimal solution for P3||ΣjCj and the set of jobs of lengths {3,5,31,4,8,11,6,8,1,7}. What is Cmax in your schedule?

What is the makespan of the schedule produced for this set of jobs and machines by list-scheduling (i) when the jobs are processed from left to right (ii) from right to left (ii) in LPT order?

**4.b** **(20 pts.)** Let n=zm (n= number of jobs, m = number of machines). Prove that in the optimal solution to P||ΣjCj there are exactly z jobs on each machine.

**Note:** The algorithm SPT uses this property, so your proof cannot simply say ‘because this is what we get from SPT’).

**Directions:** Assume the claim is false. Therefore, there must be an optimal solution in which there is a machine M1 with z+x jobs (x≥1) and a machine M2 with z-y jobs (y≥1). Proceed using an exchange argument.

**Solutions**

1. The stable matching is V-A, W-C, X-E, Y-D, Z-B. TMA proceeds according to the following table:

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1. Assume toward contradiction that there is a pair (smart boy – stupid girl) in some stable matching. Since the number of smart boys and girls is identical, this implies that there is also at least one pair of (stupid boy- smart girl). Since in all the preference lists smart people come before stupid one, the smart couple from the above two pairs form a rouge couple, contradicting the stability of the matching.

**3**. a The optimal order remains the same (SPT). The k-th job completes k time units after its original completion time. All together the total completion time is increased by 1+2+..+n = n(n+1)/2, so the new value of the optimal solution is z+n(n+1)/2.

3.b False: Assume n=5, job lengths are {10,1,1,1,1}. We have z=10.

An optimal solution for {11,2,2,2,2} has value 11, which is less than 10+2.

**4.1.** An optimal schedule for P3||ΣjCj and jobs of lengths {3,5,31,4,8,11,6,8,1,7}is an SPT schedule:

M1: 1,5,8,31

M2: 3,6,8

M3: 4,7,11

The resulting ΣjCj is 132. The makespan is 45 (other solutions exist).

* List Scheduling left to right: Makespan=31

M1: 3,4,11,8

M2: 5,8,6,1,7

M3: 31

* List Scheduling right to left: Makespan=43

M1: 7,11,3

M2: 1,6,8,5

M3: 8,4,31

* LPT Schedule: Makespan=31

M1: 31

M2: 11,7,5,3,1

M3: 8,8,6,4,

**4.2** Assume the claim is false. Therefore, there must be an optimal solution in which there is a machine M1 with z+x jobs (x≥1) and a machine M2 with z-y jobs (y≥1). Consider the schedule in which, J1, the first job on M1, is moved to be first on M2.

The completion time of J1 does not change (remains p1). The completion time of each of the z-y jobs on M2 is increased by p1. The completion time of each of the z+x-1 jobs on M1 is decreased by p1. The completion times of jobs on other machines do not change. Summing up, ΣjC'j in the resulting schedule = ΣjCj  -(z+x-1+z-y)p1 = ΣjCj –(x+y-1)p1, which is less than ΣjCj  for any x+y-1>0, in particular for x≥1and y ≥1.

A contradiction to the assumption that the non-balanced schedule is optimal.