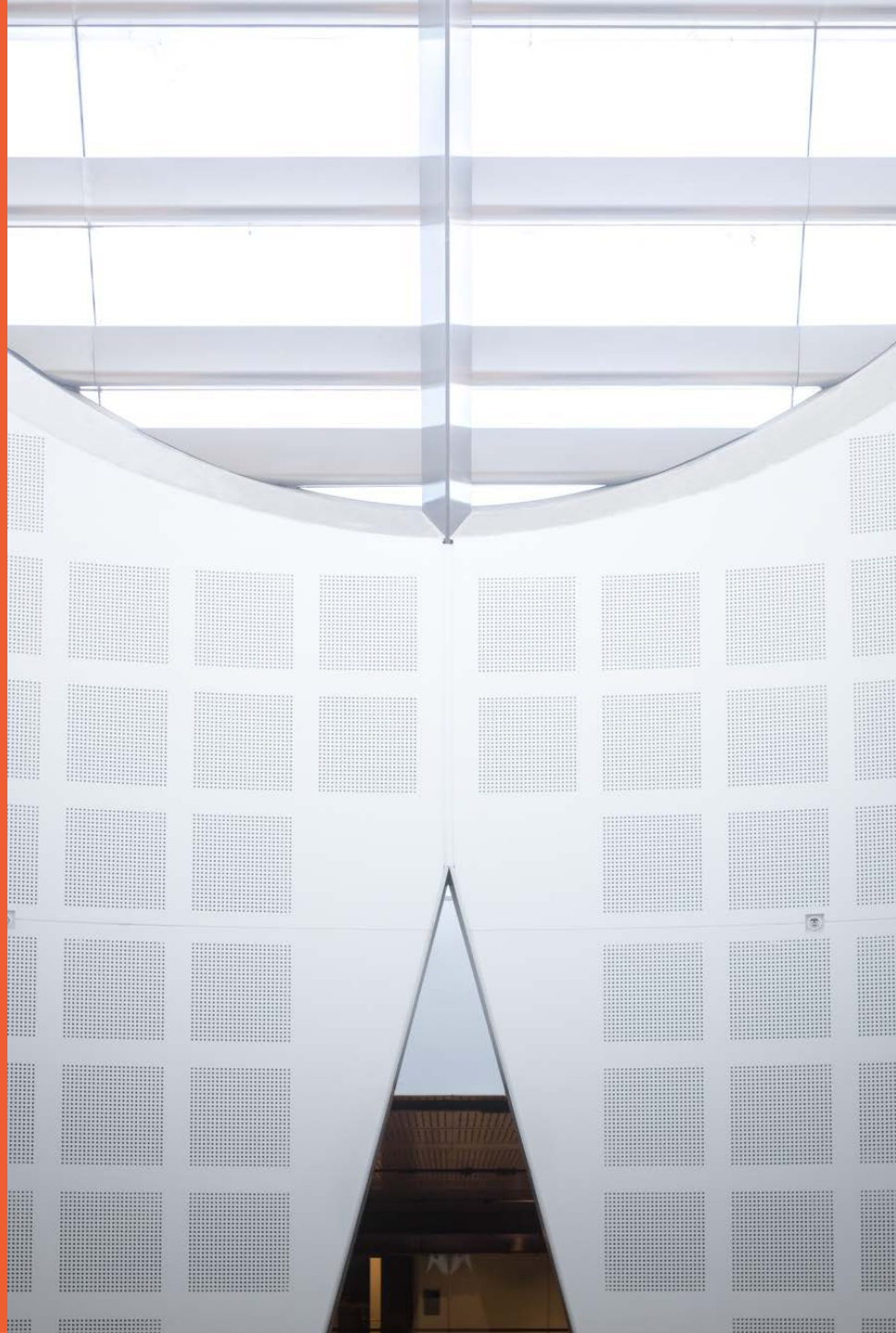
COMP5048
Visual Analytics

Week 2: Tree Visualisation

Professor Seokhee Hong
School of Information Technologies

THE UNIVERSITY OF SYDNEY



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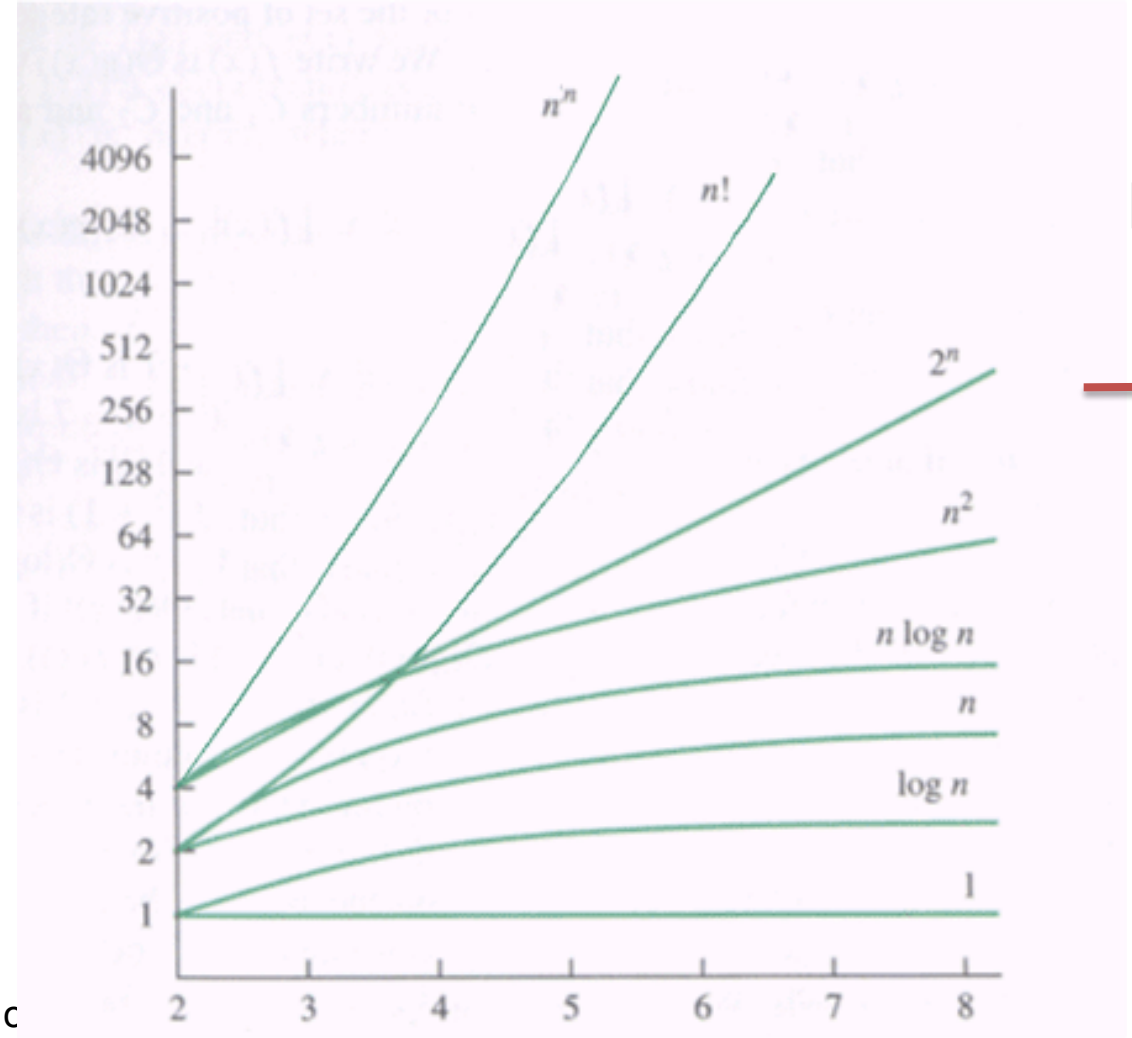
(1) Tree Drawing Algorithm

1. Terminology
2. Layered Drawing
3. Radial Drawing
4. HV-Drawing
5. Inclusion Drawing

Big O Notation

- Measure Time complexity (Efficiency) of algorithm: run-time
- Asymptotic: eg. 5n+10 => O(n), 10000n => O(n), n: size of input data



Exponential time

Polynomial time

Algorithm: Terminology

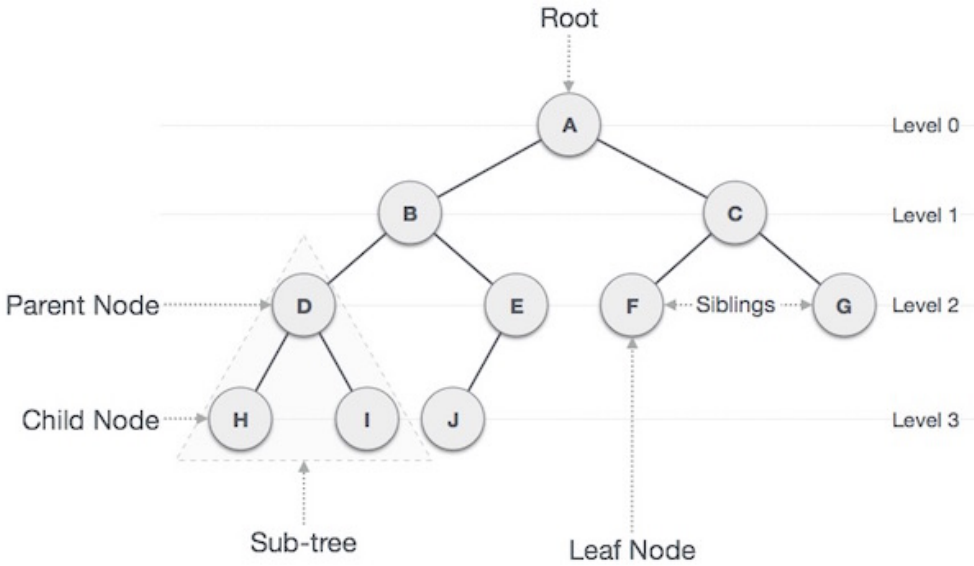
- **Divide and Conquer algorithm**
  - Divide into smaller subproblems/instances recursively
  - Solve solutions for subproblems 
  - Merge solutions to obtain a solution to the original problem

- **Dynamic Programming algorithm**
  - breaking it down into a collection of simpler subproblems
  - solve each subproblem, and store the solution in a table.
  - look up previously solved subproblems and use the solutions to compute the solution for the original problem.

- **Recursion:**
  - a method where the solution to a problem depends on solutions to smaller instances of the same problem

1. Tree: terminology

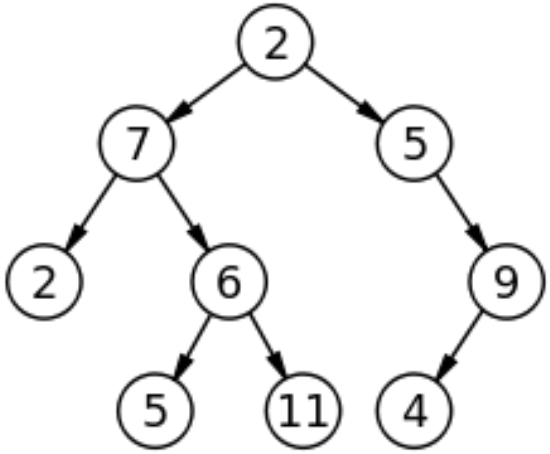
- Data structure to represent hierarchical information (no cycle)
- Rooted tree
  - Root: a distinguished vertex in a tree
  - directed edge *u*->*v* (*u* : *parent* of *v*, *v* : *child* of *u*)
  - Leaf node: no child
- Subtree rooted at *v* : subgraph induced by all “descendants” of *v*
- Depth (level) of a vertex v: number of edges from v to the root
- Height of a tree T: maximum depth
- Ordered tree: rooted tree with a fixed ordering for children of each vertex



Binary Tree

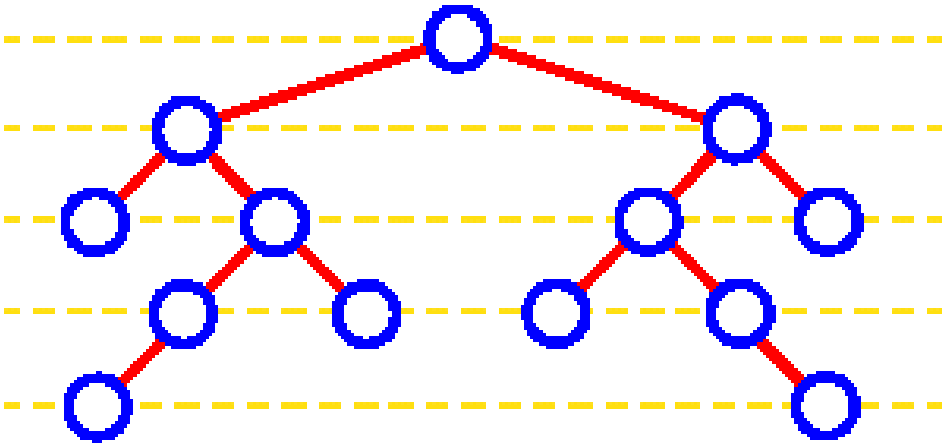
- Binary tree: rooted tree with every node has at most two children
  - Children: Left child and Right child
  - Subtrees: Left subtree and Right subtree

- Binary Tree Traversal
  - Inorder Traversal: Left subtree-Root-Right subtree
  - Preorder Traversal: Root-Left subtree-Right subtree
  - Postorder Traversal: Left subtree-Right-Root subtree



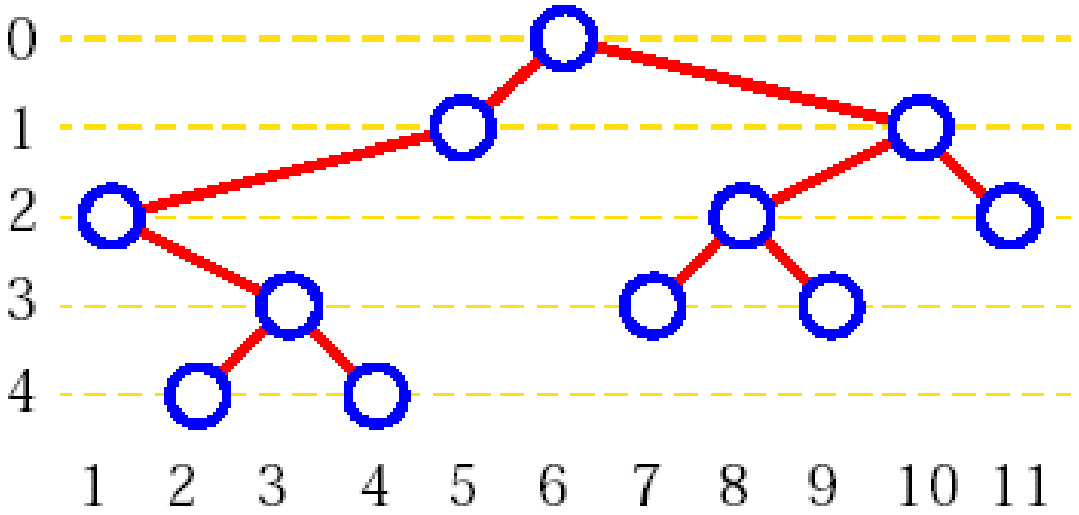
2. Layered Drawing

- rooted (binary) tree T
- assign layer according to the depth
  -> y-coordinates: *y(v) = depth of v*
- how to compute x-coordinates?



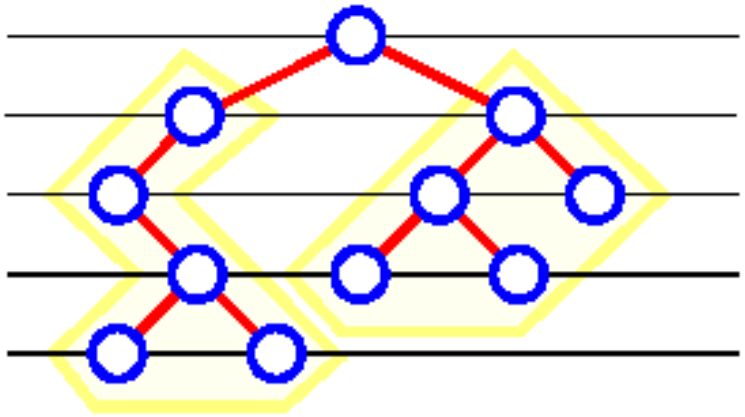
Simple Method

- Inorder Tree Traversal algorithm
  - layered grid drawing
  - two drawback:
    - too wide: *width n-1*
    - parent vertex is not centered with respect to the children



[Reingold-Tilford 81] Tidier Drawing Algorithm

- Divide: recursively apply the algorithm to draw the left and right subtrees of T.
- Conquer
  - move the drawings of subtrees until their horizontal distance equals 2.
  - place the root r vertically one level above and horizontally half way between its children.
  - If there is only one child, place the root at horizontal distance 1 from the child.

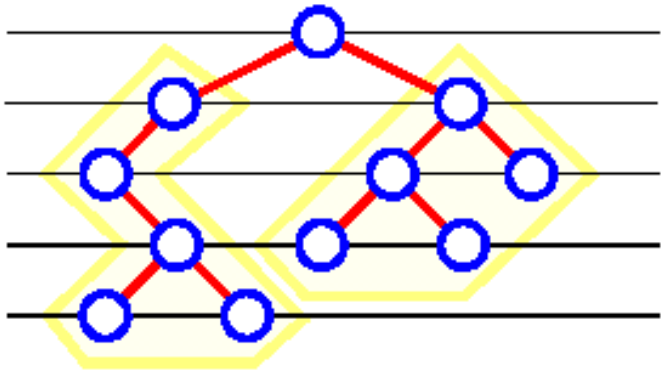


Tidier Drawing Algorithm

- Two traversals:
  - *Step 1. Postorder traversal*

    For each vertex v, recursively computes the horizontal displacement of the left & right children of v with respect to v.
  - *Step 2. Preorder traversal*

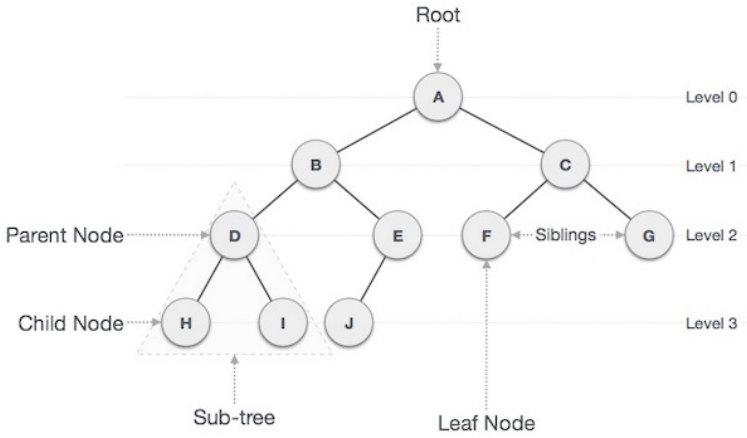
    Computes x-coordinates of the vertices by accumulating the displacements on the path from each vertex to the root.



Left/Right Contours for Postorder Traversal

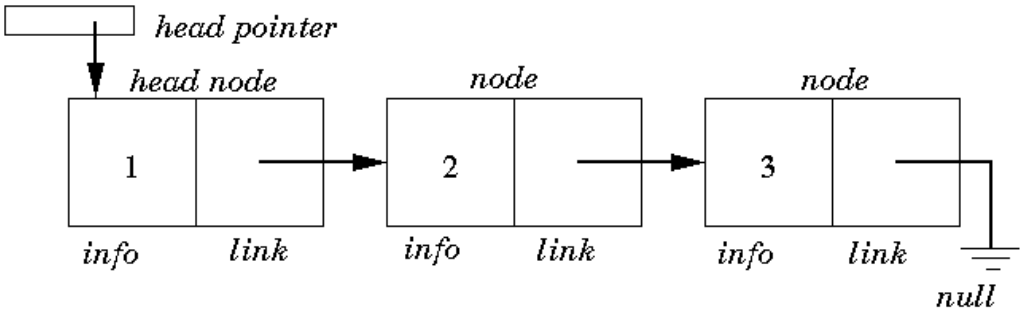
Left (Right) Contour:

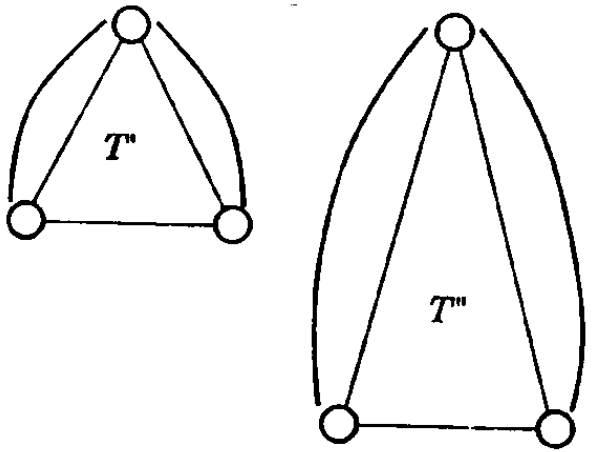
- sequence of vertices v_i such that v_i is the Leftmost (Rightmost) vertex of T at level i.
- store in linked list for each v.



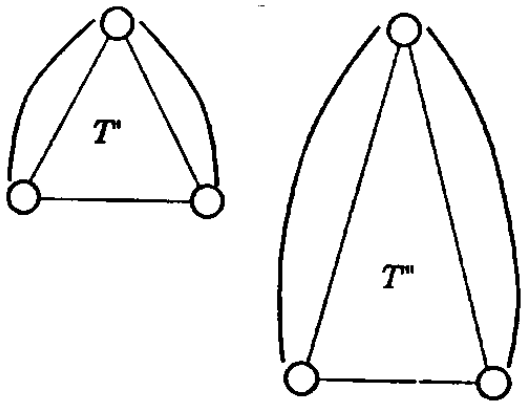
Compute contours of v

- scan the Right Contour of the Left Subtree and the Left Contour of the Right Subtree (follow the linked list).
- accumulate displacements of vertices on the contours
- keep track of the maximum cumulative displacement at each level.





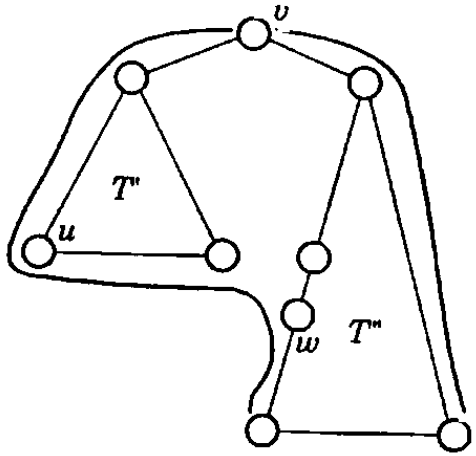
- *T: subtree rooted at v*
- *T' (T''): left (right) subtree of T*
- *L(T)* (*R(T)*): left (right) contours of *T*



How to Construct L(T) (R(T)) ?

- *Case 1: height(T') = height(T'')*
  - L(T) = v + L(T')
  - R(T) = v + R(T'')

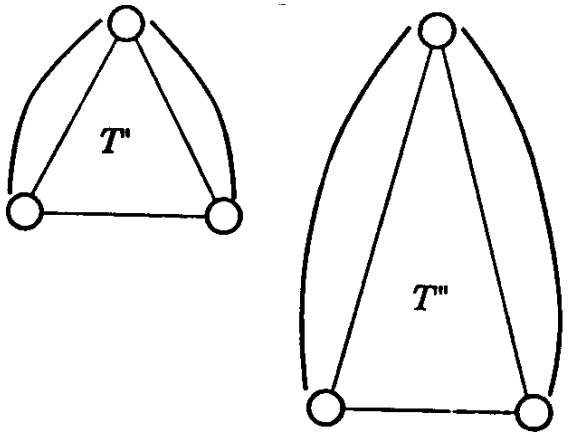
- *Case 2: height(T') < height(T'')*
  - R(T) = v + R(T'')
  - L(T) = v + L(T') + {part of L(T'') starting from w}
    - h': height of T'
    - u: vertex of L(T') with depth h' (bottom-most)
    - w: vertex on L(T'') with depth = h'+1

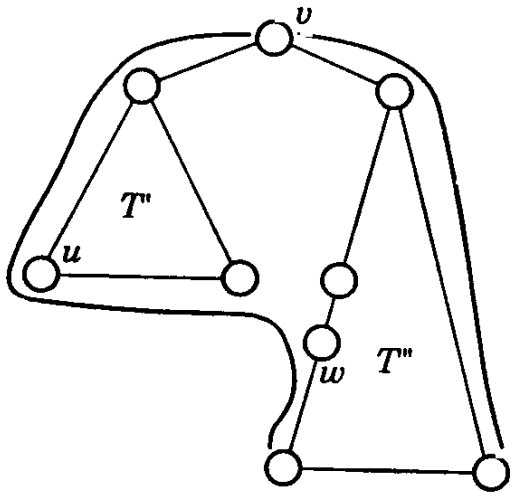


- *Case 3: height(T') > height(T'')* : similar to case2

Implementing Postorder Traversal in Linear Time

- It is necessary to travel down the contours of two subtrees T' and T'' only as far as the height of the subtree of *lesser height*.
- the time spent processing vertex v is proportional to the *minimum heights of T' and T''.*
- The sum over all vertices v of the minimum height of the subtrees of v is no more than the number of vertices of the tree.

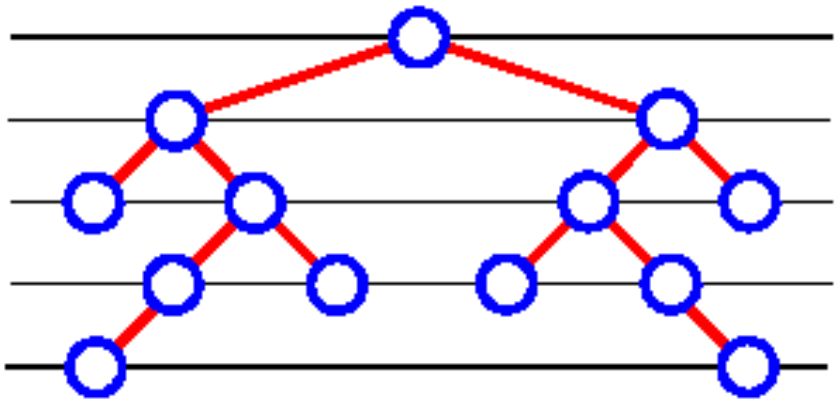




[Theorem]

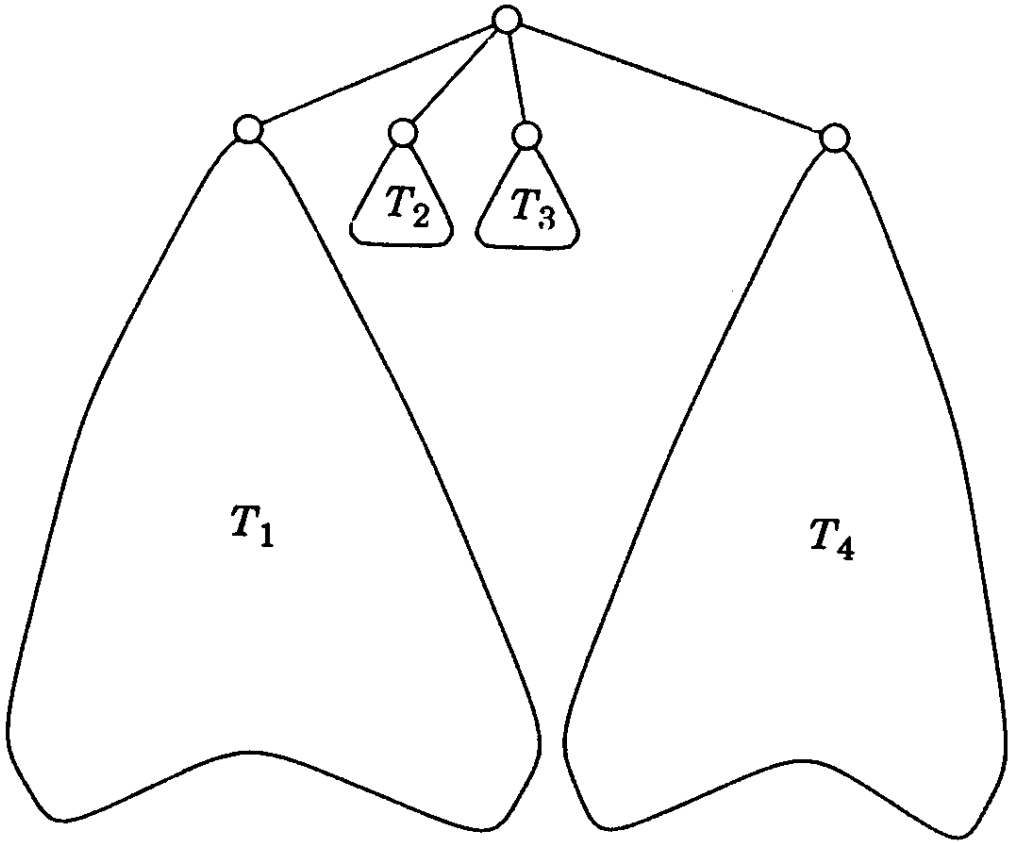
Tidier Drawing Algorithm constructs a drawing of a binary tree T in linear time such that the drawing is

- layered, planar, straight-line and strictly downward
- $O(n^2)$ area
- two vertices are at horizontal & vertical distance at least 1
- parent vertex is centered with respect to its children
- isomorphic subtrees have congruent drawing up to a translation
- axially isomorphic subtrees have congruent drawings, up to a translation & a reflection in y-axis



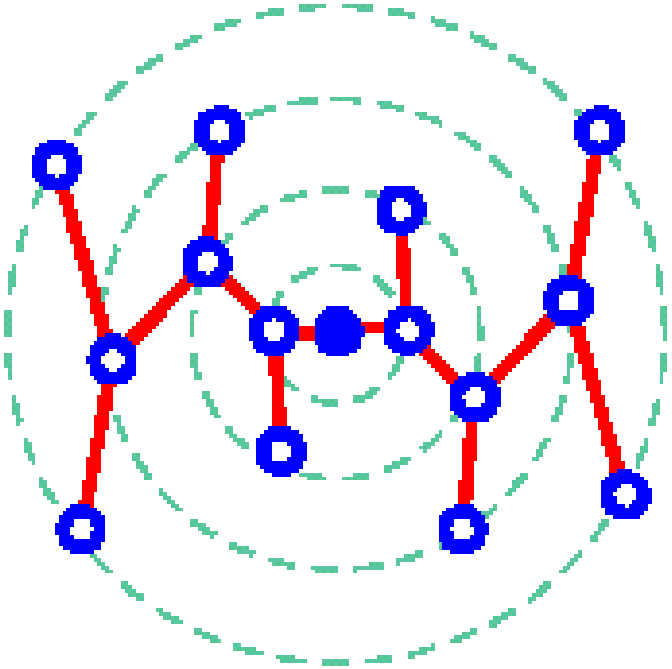
Generalization to Rooted Trees

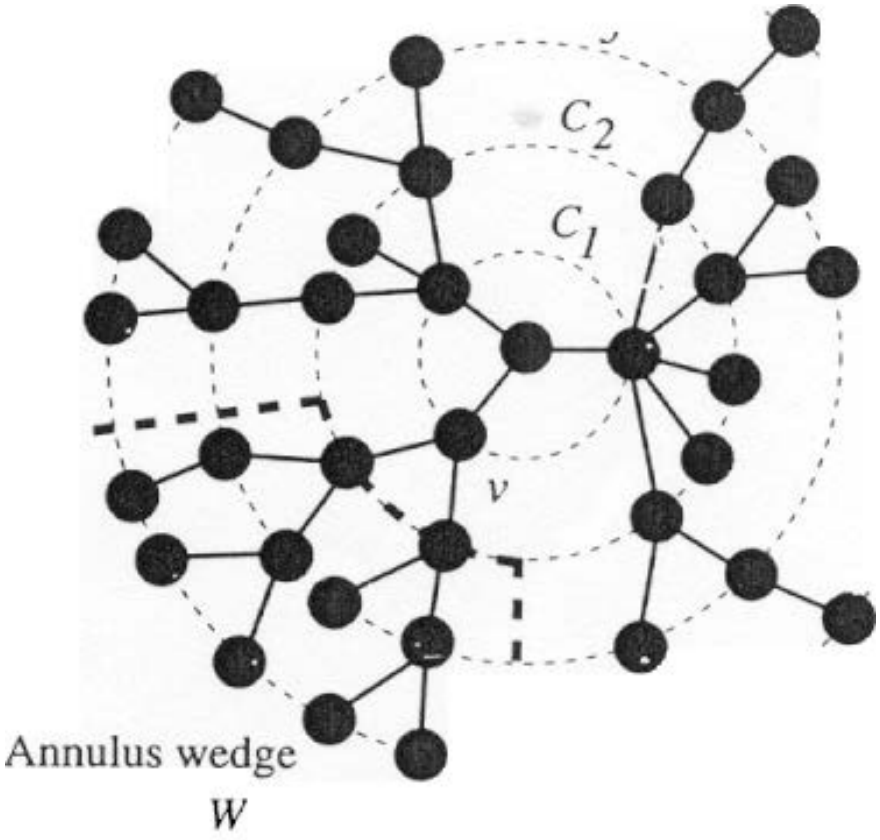
- root is placed at the average x-coordinates of its children
- small imbalance problem: T2 and T3 much closer to T1 than T4.
- Modify conquer step or postprocessing
  - [Walker90]: layered drawing for trees



3. Radial Drawing

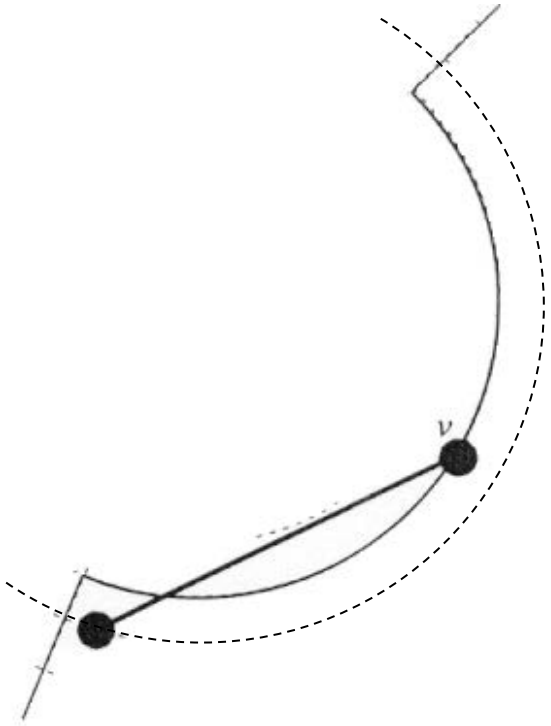
- Layers are represented as concentric circles.
- Draw each subtree in annulus wedge W.
- Angle of wedge: proportional to # of leaves of each subtree

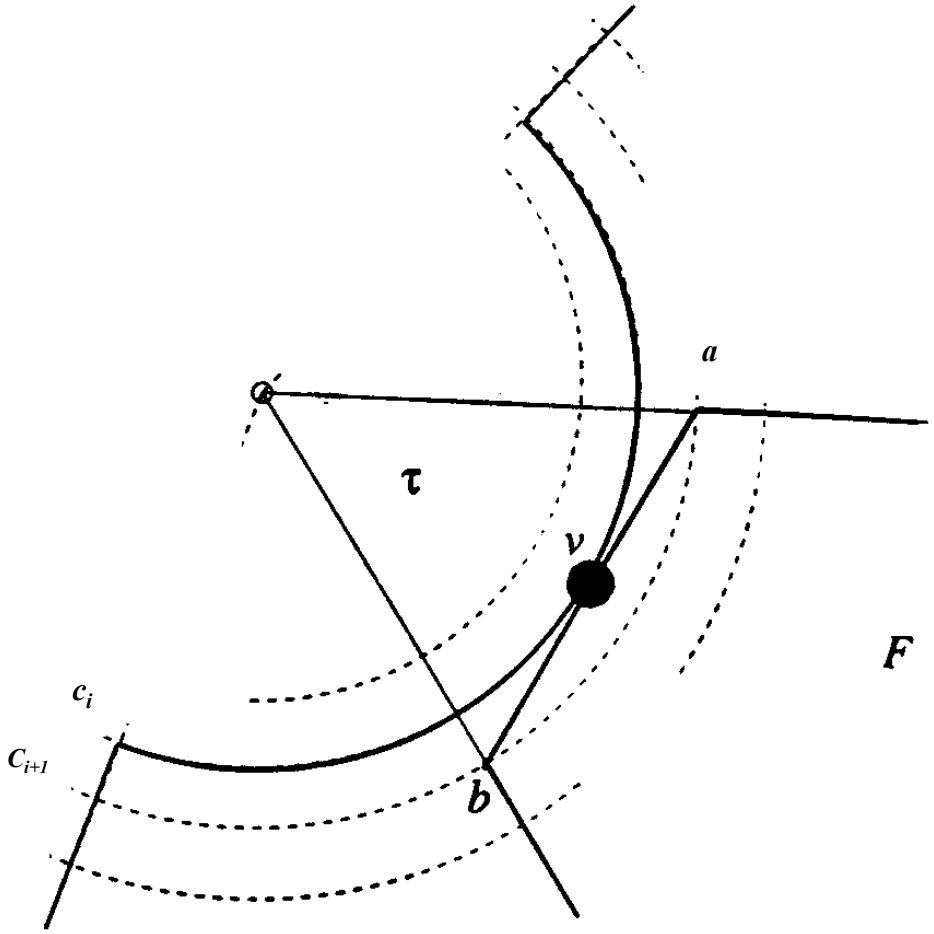




Planar Drawing

- to guarantee planarity, define convex subset F of the wedge.
- Draw a subtree inside F





Radial Drawing

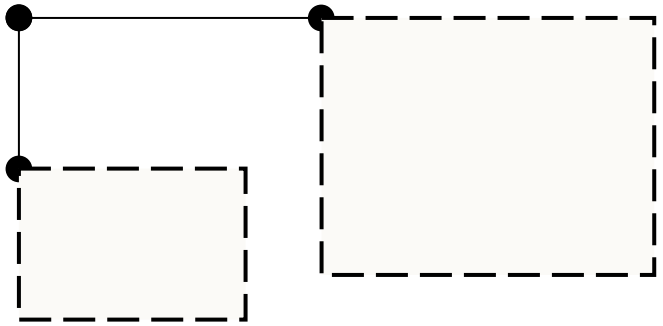
- Running time: linear
- used for free trees
  - select the center as a root
- can be used to display symmetry
  - Symmetry detection: [Manning, Atallah 88]
- Variations: [Eades 92], [Bernard 81], [Esposito 88]
  - choice of root
  - radii of the circles
  - how to determine the size of the wedge

4. hv-Drawing

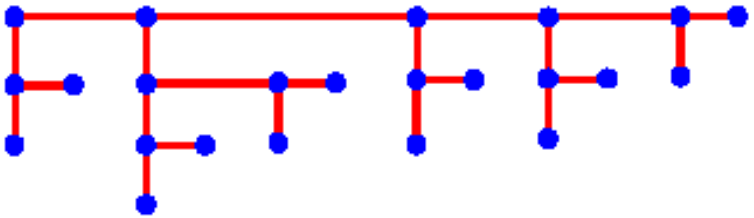
- hv-drawing of a binary tree T:

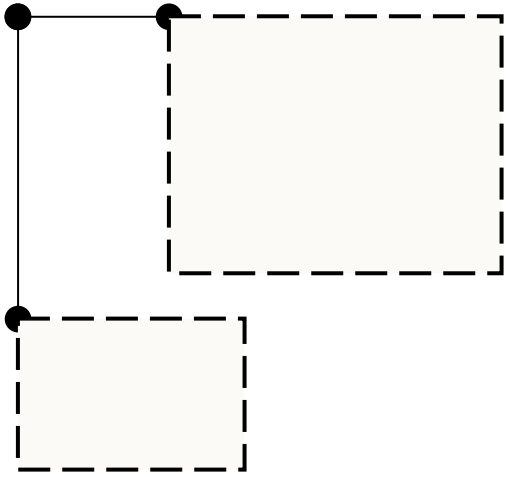
straight-line grid drawing such that for each u, a child of u is either

  - horizontally aligned to the right of u, or
  - vertically aligned below u
  - bounding rectangles of the subtrees of u do not intersect



- planar, straight-line, orthogonal, and downward

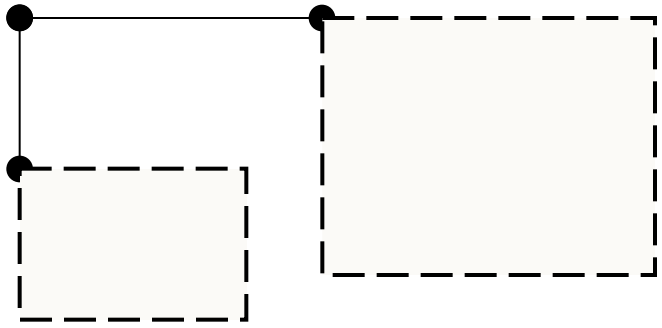




Divide & Conquer Algorithm

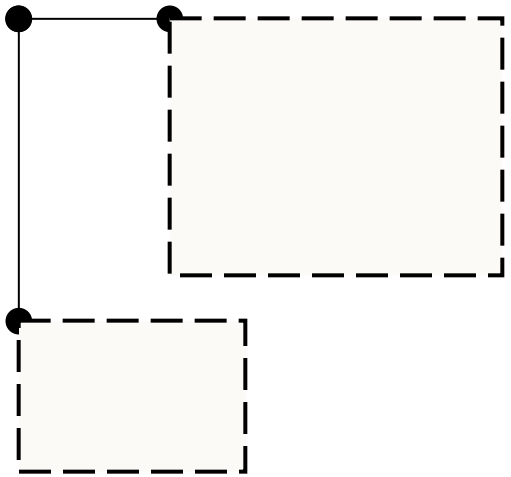
- Divide:

recursively construct hv-drawings for the left & right subtrees



- Conquer:

perform either *horizontal* combination or a *vertical* combination

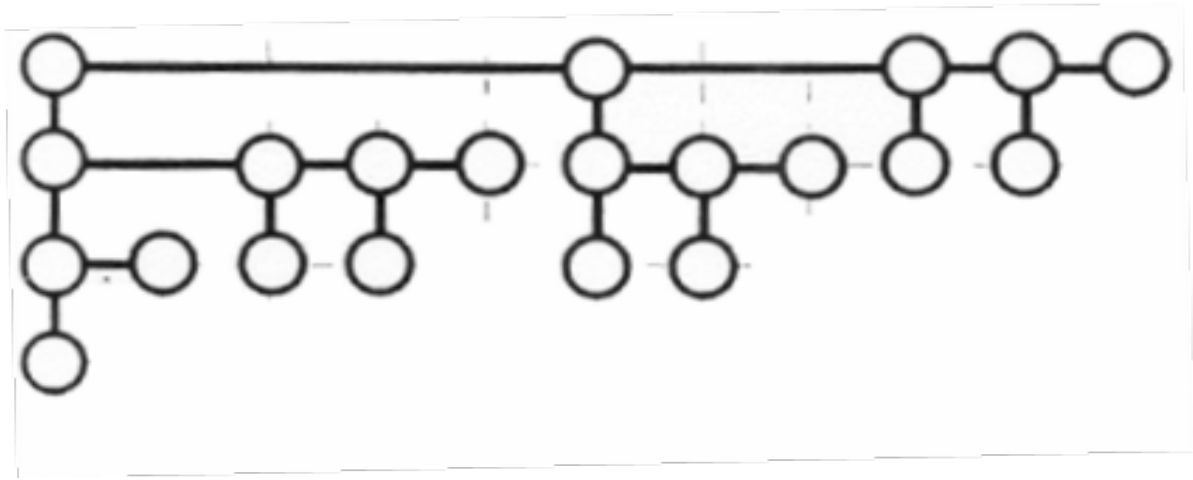


- The height & width are each at most n-1

Algorithm Right-Heavy-HV-Tree-Draw

1. Recursively construct drawing of the left & right subtrees.

2. Using only ***horizontal*** combination, place the subtree with the ***largest*** number of vertices to the ***right*** of the other one.



Algorithm Right-Heavy-HV-Tree-Draw

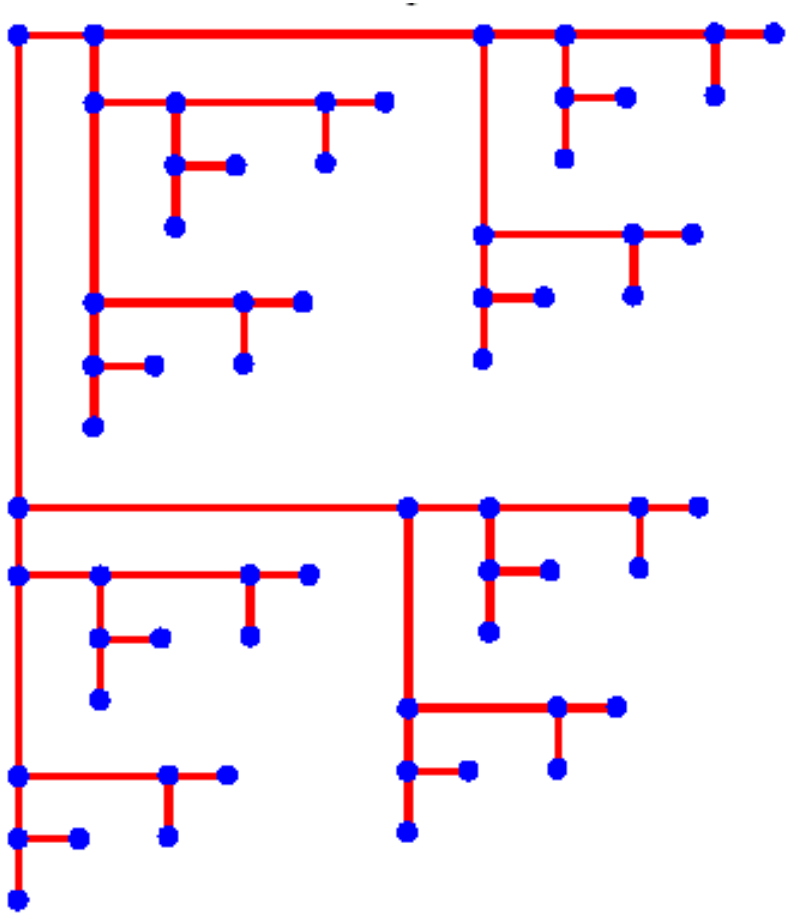
[Theorem]

Algorithm **Right-Heavy-HV-Tree-Draw** construct a drawing of binary tree T with n vertices such that the drawing is

- hv-drawing (downward, planar, grid, straight-line and orthogonal)
- area **O(nlogn)**
- width is at most n-1
- height is at most logn
- simply and axially isomorphic subtrees have congruent drawings, up to a translation

Algorithm Right-Heavy-HV-Tree-Draw

- Good area bound, but bad aspect ratio
- Better aspect ratio: use both horizontal/vertical combinations for odd/even depth.
- **Complete binary tree**: O(n) area and constant aspect ratio.



General Binary Tree

[Eades, Lin, Lin 92] [Eades, Lin, Lin 93]

It is possible to construct an **hv-drawing** of a general binary tree that is optimal with respect to "area" or "perimeter" in $O(n^2)$ time.

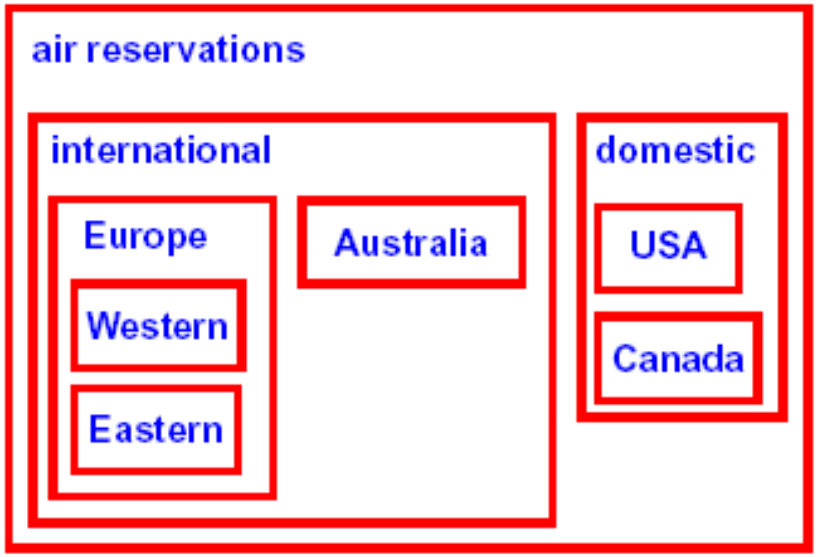
•use dynamic programming algorithm.

5. Inclusion Tree Drawing

[Eades, Lin, Lin 93] ***Inclusion Drawing*** of Rooted Trees

Display the parent-child relationship by the inclusion between isothetic rectangles.

- Minimization of area (perimeter, width, height)
  - NP-hard for general trees
  - Polynomial time algorithm for balanced trees



- used for *compounds graphs* (union of a graph and a tree)
- allow better fit the drawing in a prescribed region

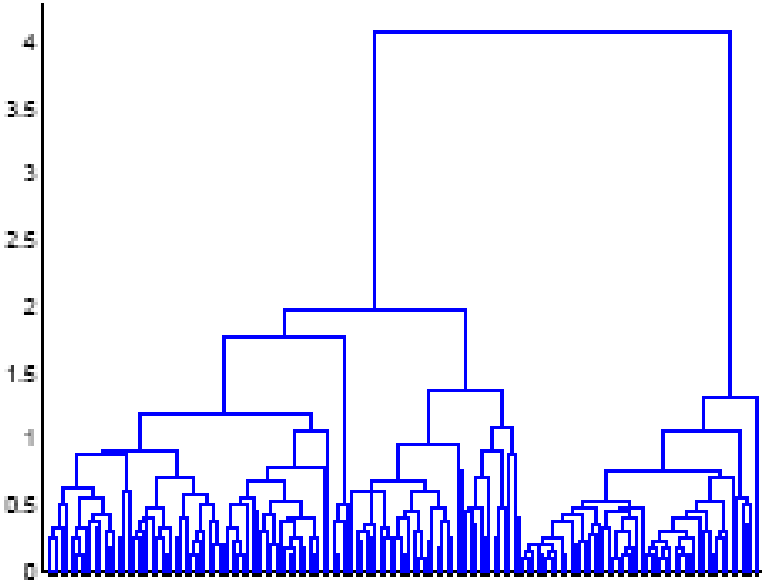
Summary: Tree Drawing Algorithm

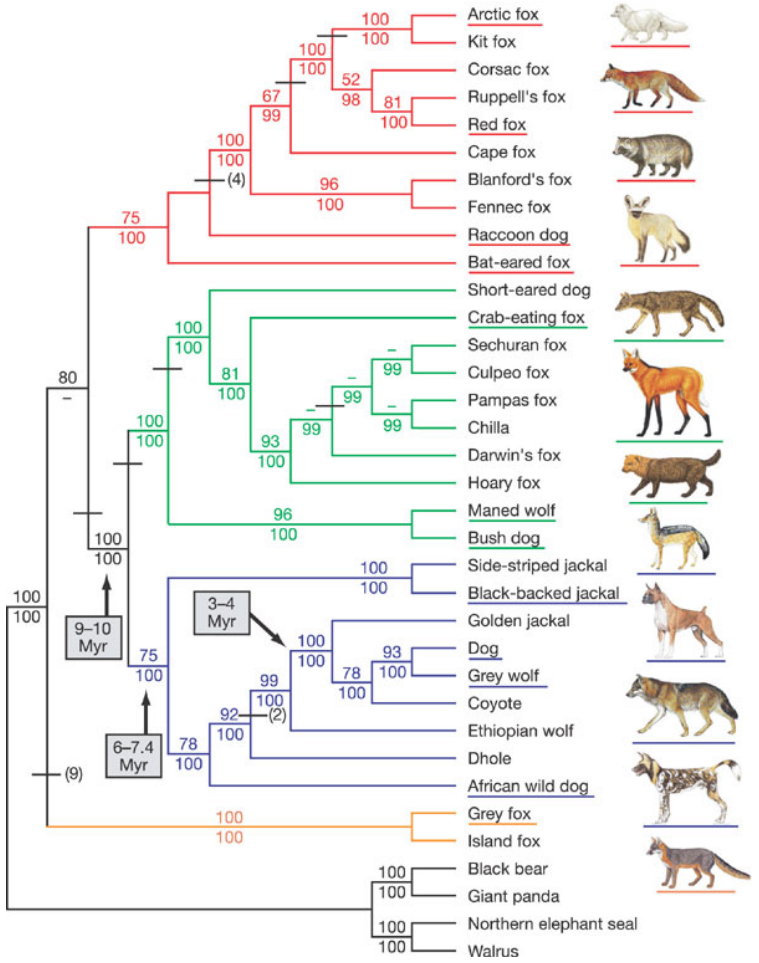
- There are many variations based on
  - edge representation
  - criteria
- Divide and Conquer algorithm
- mostly run in linear time (O(n) time)
- most popular methods
  - level drawing (tidy tree drawing)
  - radial drawing

(2) Tree Visualisation Methods

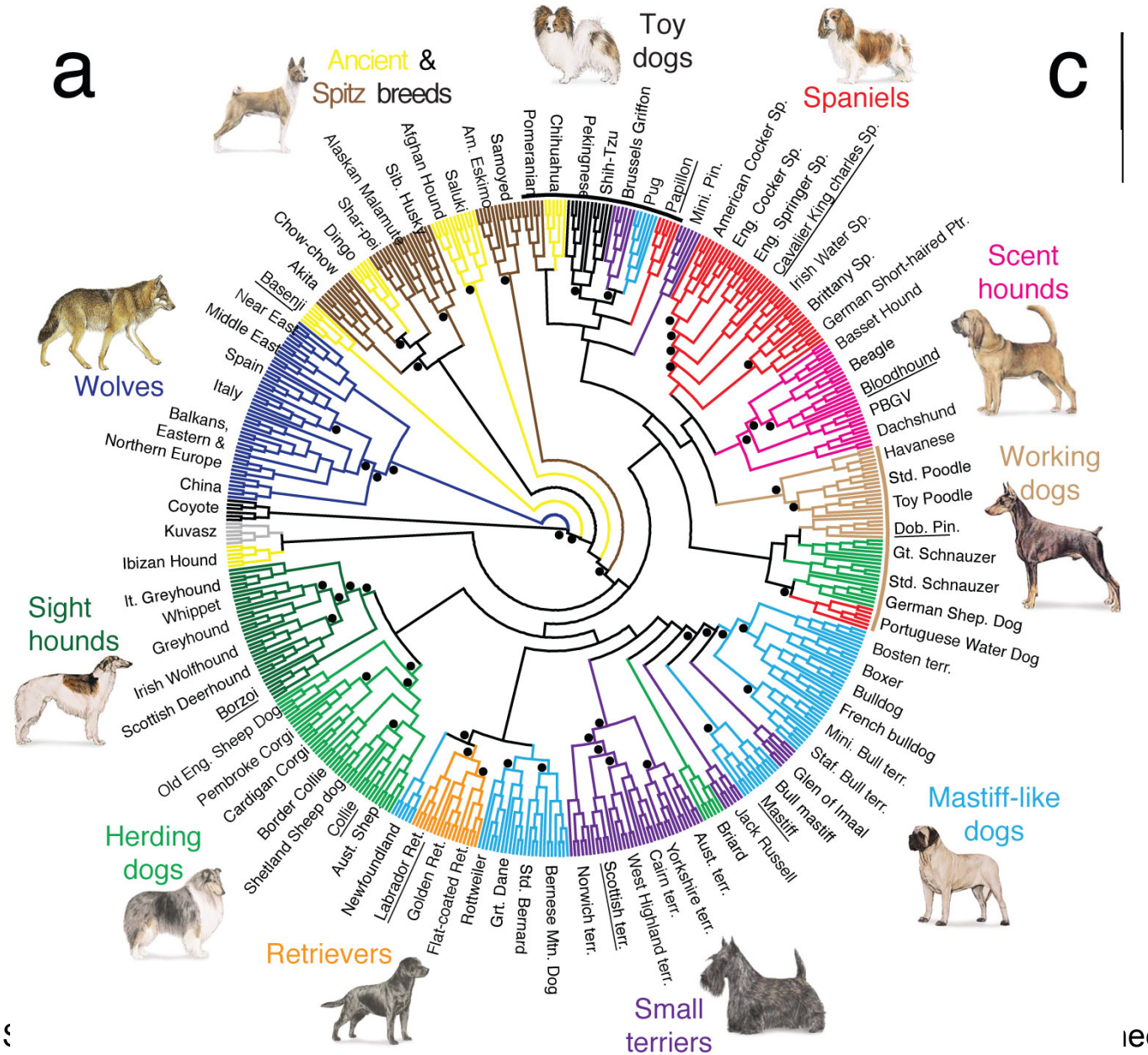
Dendrogram

- layered drawing with bended orthogonal edges.
- all the leaves are on the same layer.
- good for drawing large trees in small area.
- used in bioinformatics to represent
  - hierarchical clustering
  - phylogenetic trees

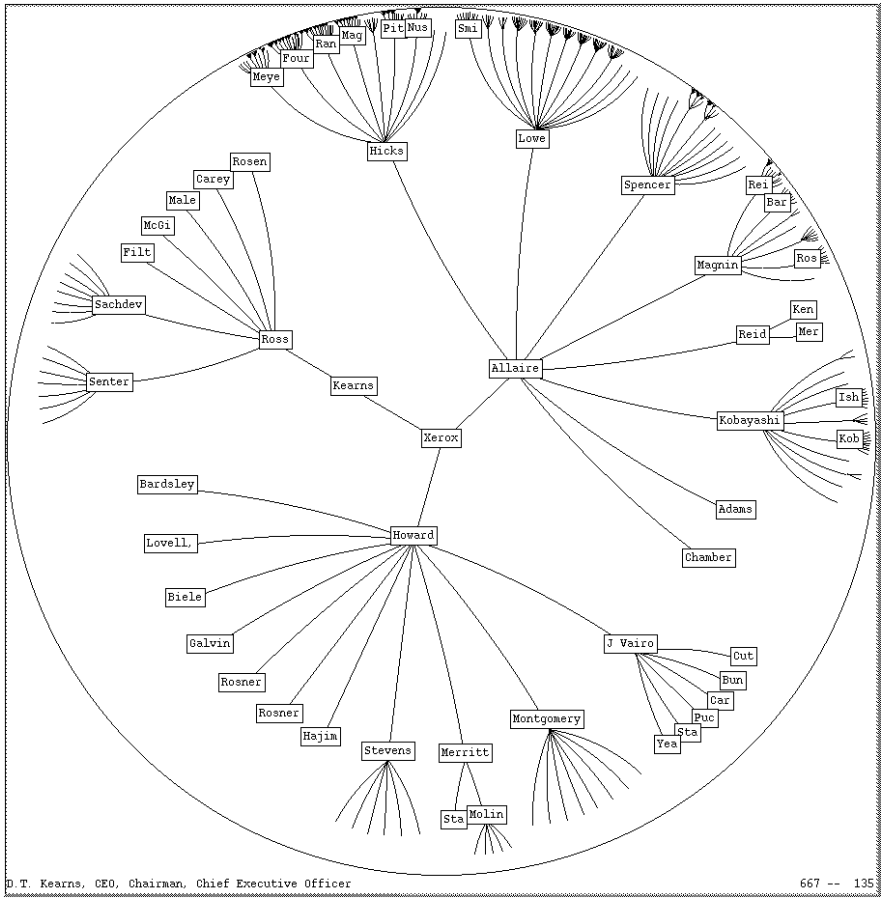


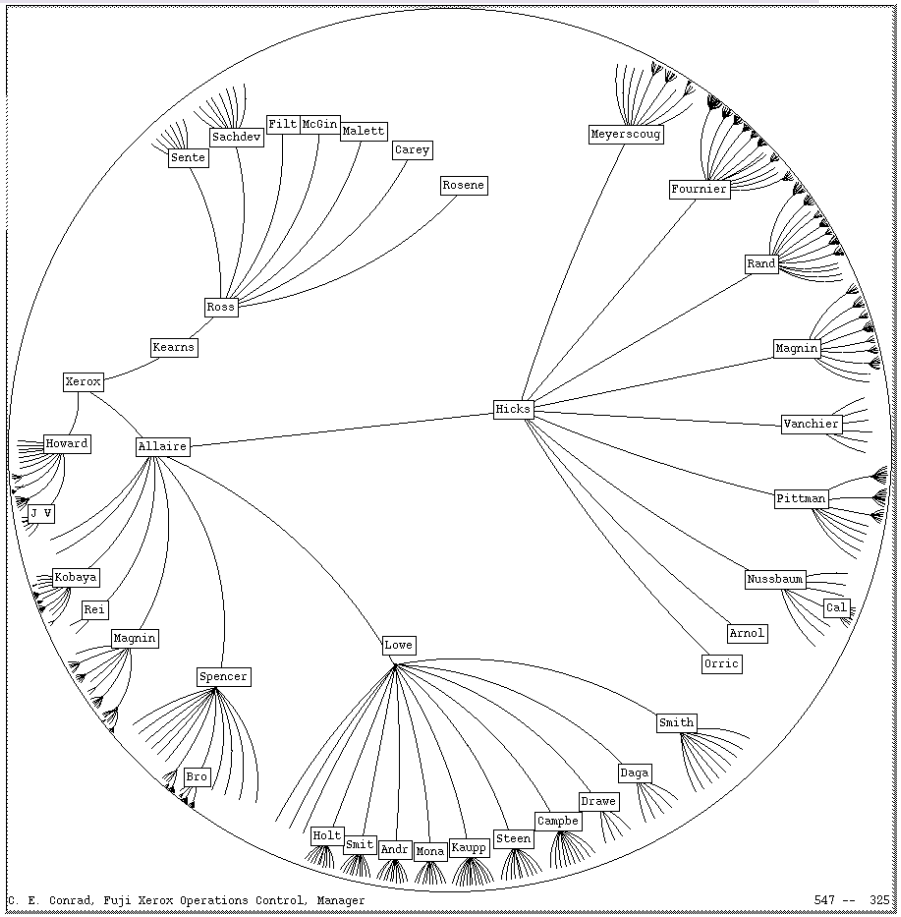


Radial Dendrogram



Hyperbolic Tree Browser

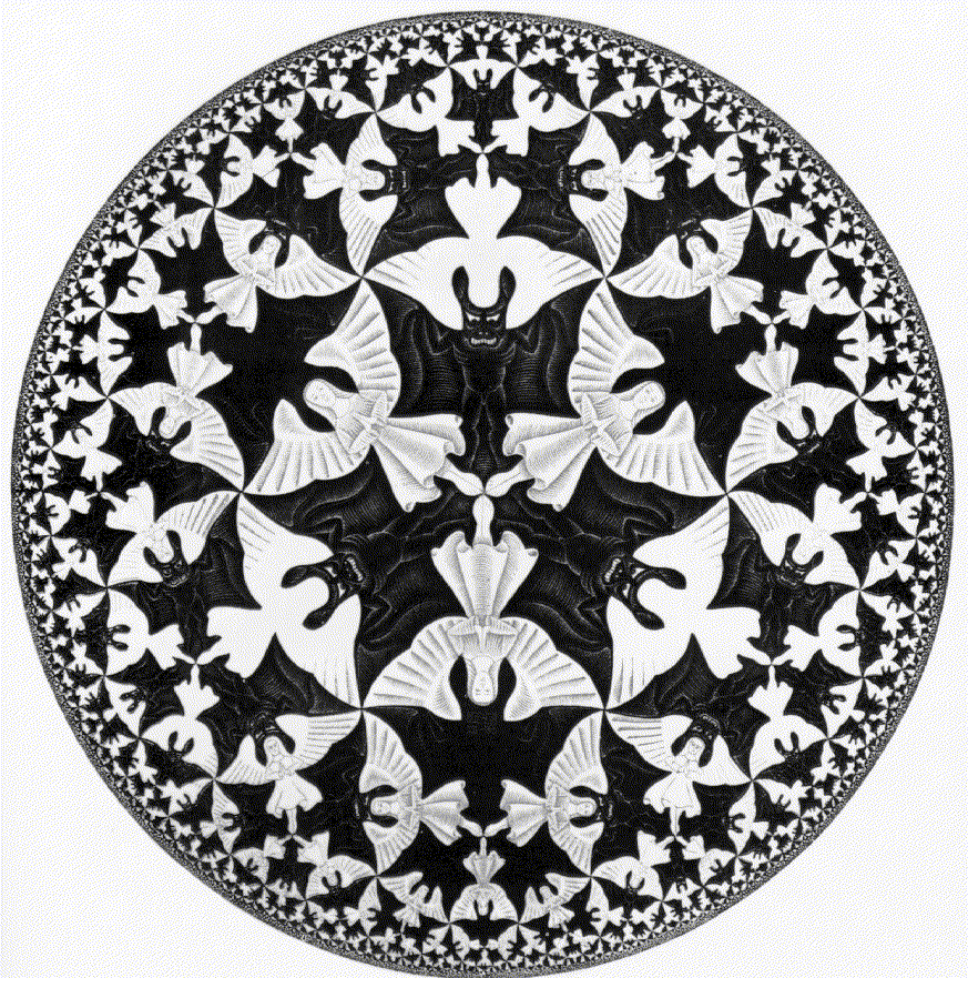


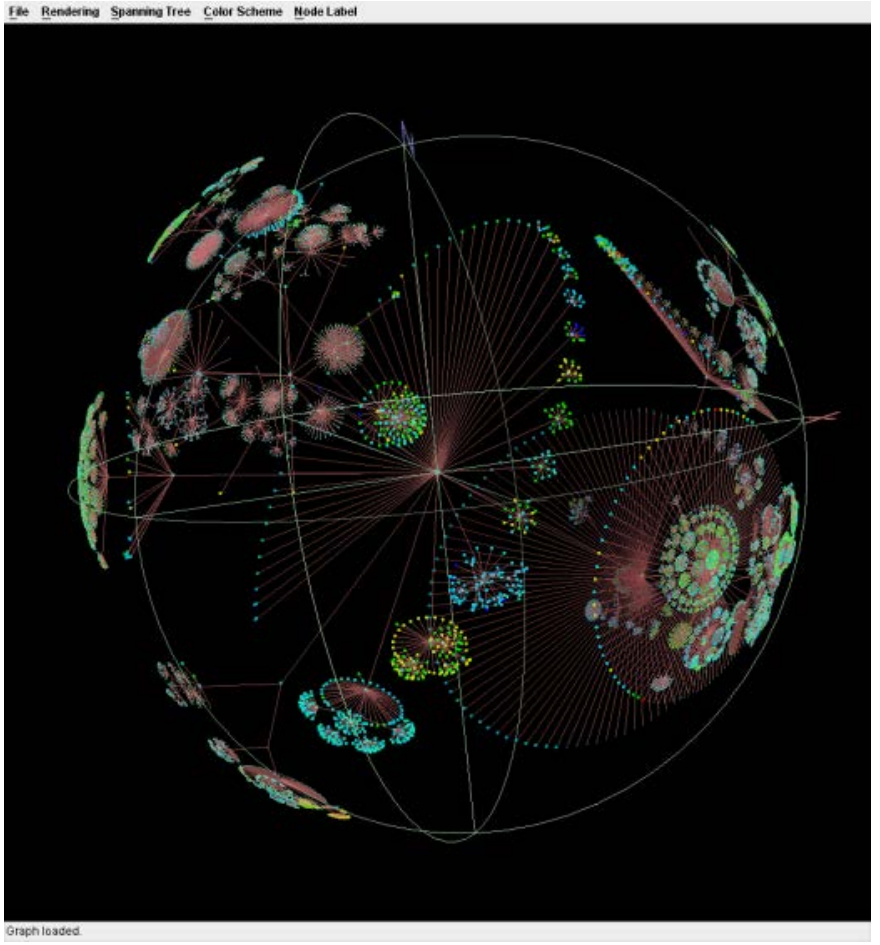


[Lamping, Rao, Pirolli 95]

- A Focus+Context Technique Based on Hyperbolic Geometry
- Distortion effect of fisheye lens
- Interaction method

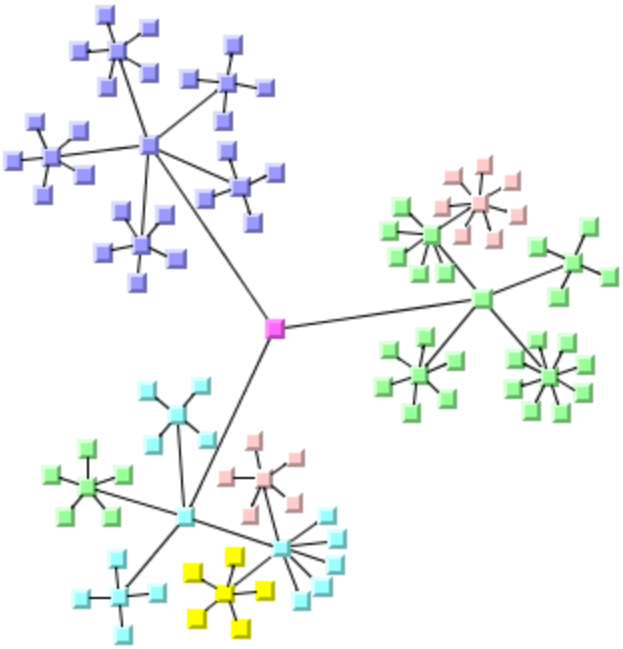
- M.C. Escher, Circle Limit IV (Heaven and Hell)
- 3D hyperbolic tree [Munzner]
  - projecting a graph on hyperbolic sphere
  - produces a distortion effect

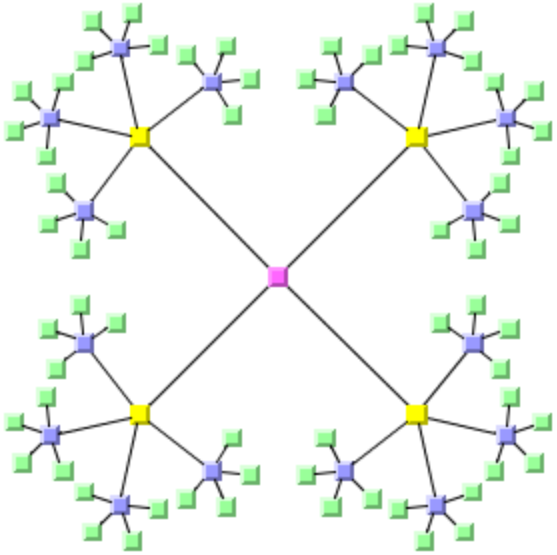


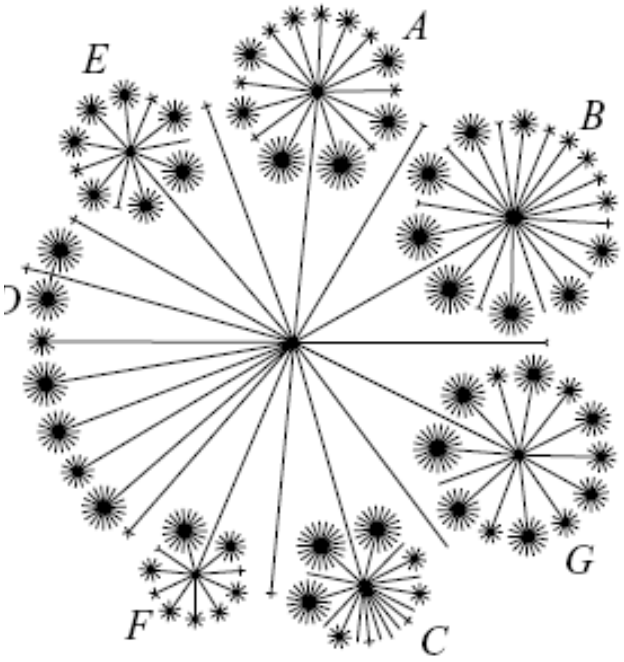


Balloon Tree Drawing

- A variation of radial layout.
- Children are drawn in a circle centered at their parents.
- [Yen 05] Deciding a good ordering: NP-hard problem



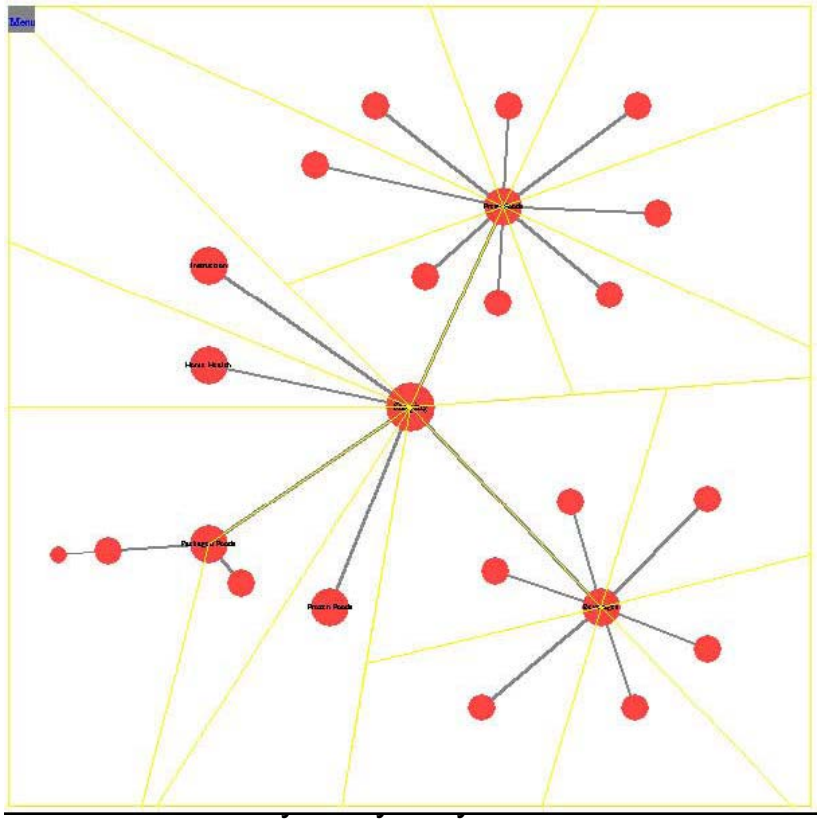


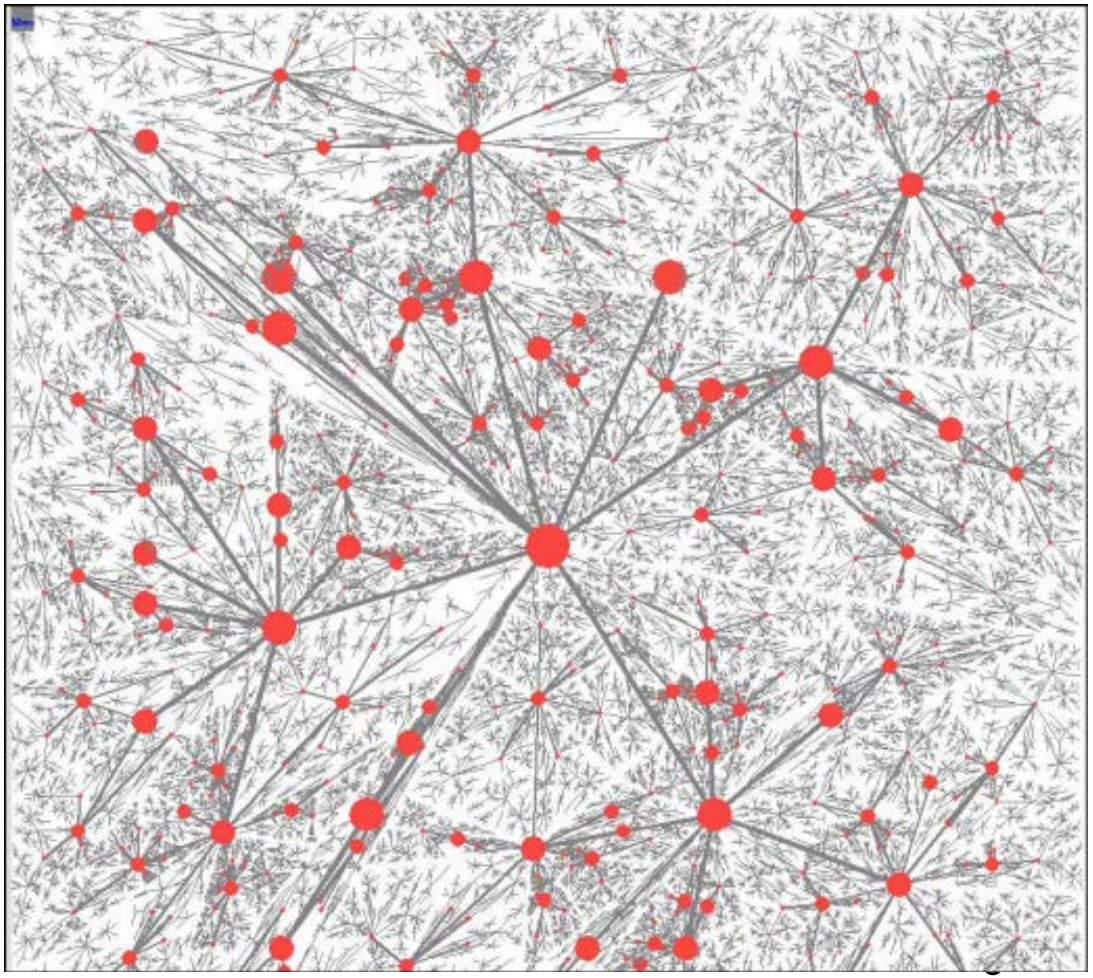


Space-filling Tree Layout

[Huang 00]

- Layout a tree according to the recursive partition of the screen space
  - area allocated to a subtree is proportional to its size
- example: 55000 nodes (use all the screen space)

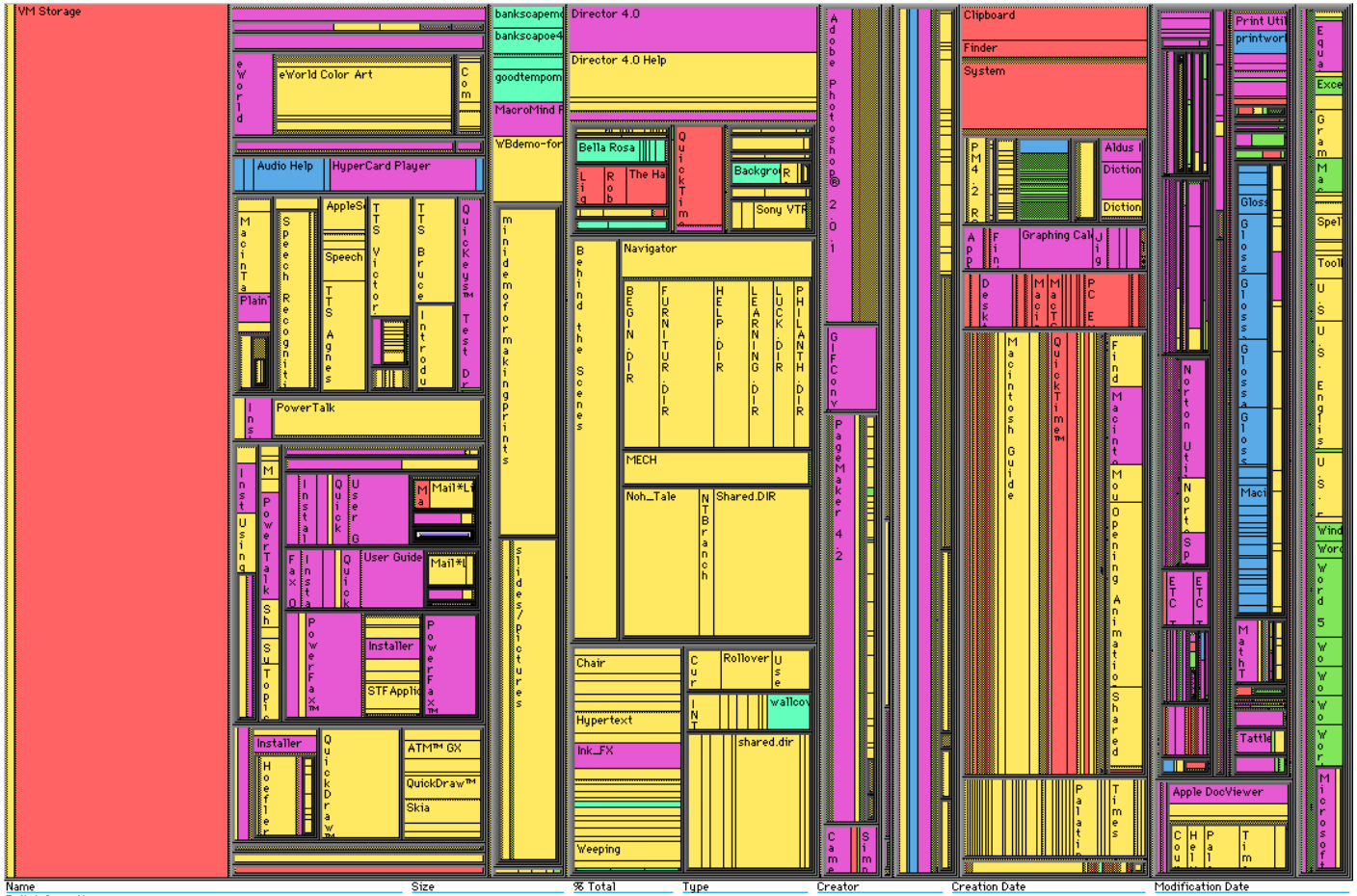




Treemap

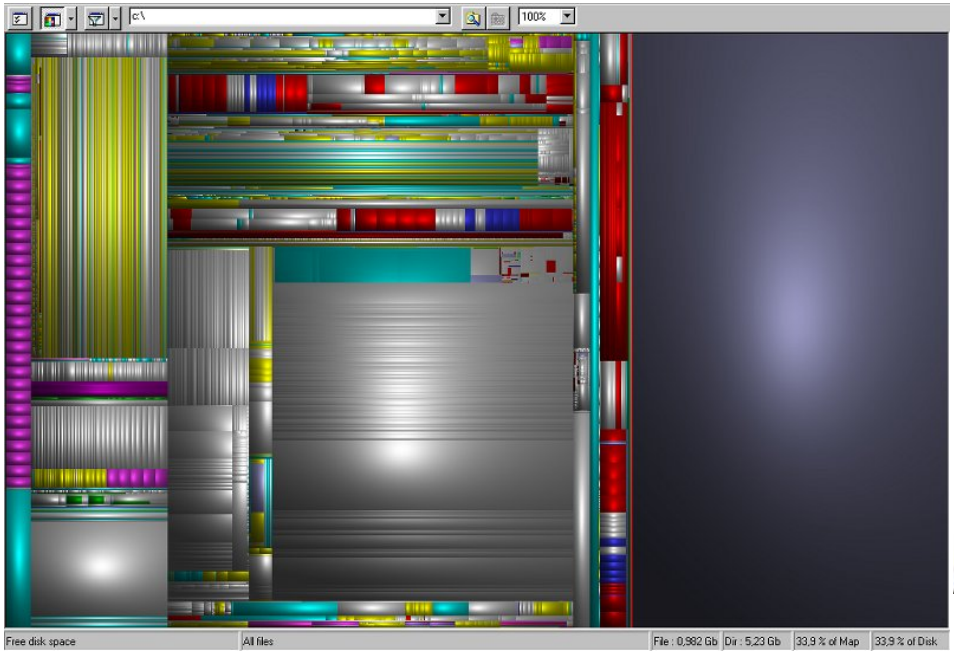
[Shneiderman 92] use containment to show the hierarchy.

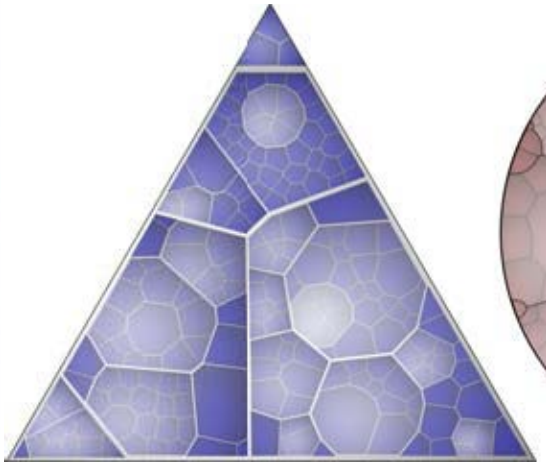
- partition the space recursively according to the size of subtrees.
- space-efficient compare to node-link diagram.
- difficult to follow parent-child relationship.

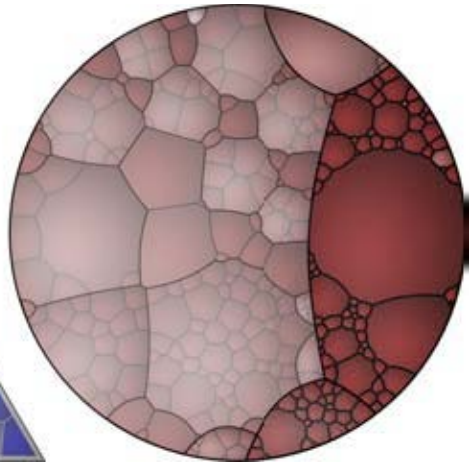


Variations of Treemap

- Cushion treemap [Wijk 00]
  - uses shading to help identify the levels in a treemap.
- Voronoi treemap
  - uses voronoi diagram as partition.



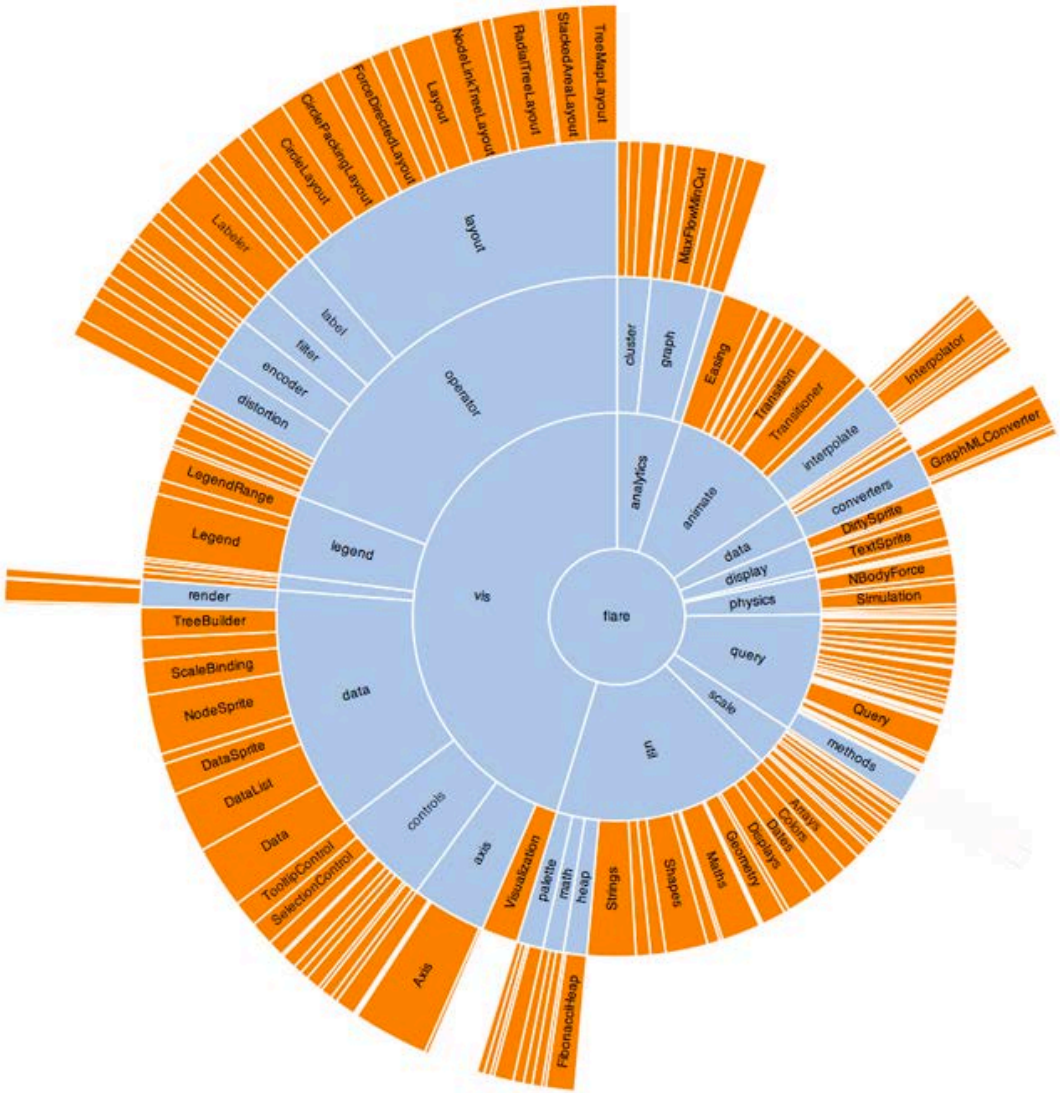




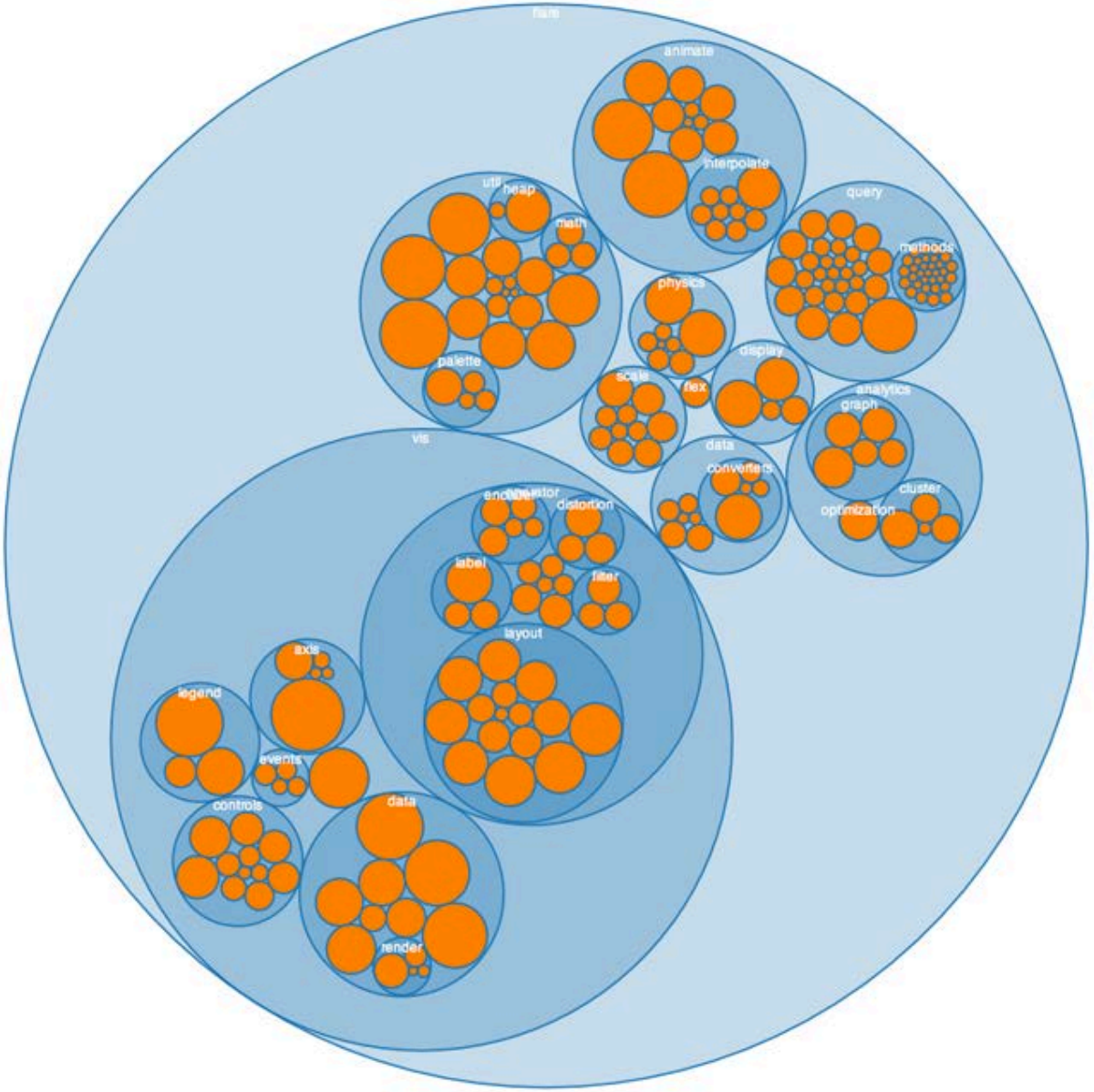
Sunburst Diagrams

[Stasko 00] space-filling visualization method

– radial version of tree map



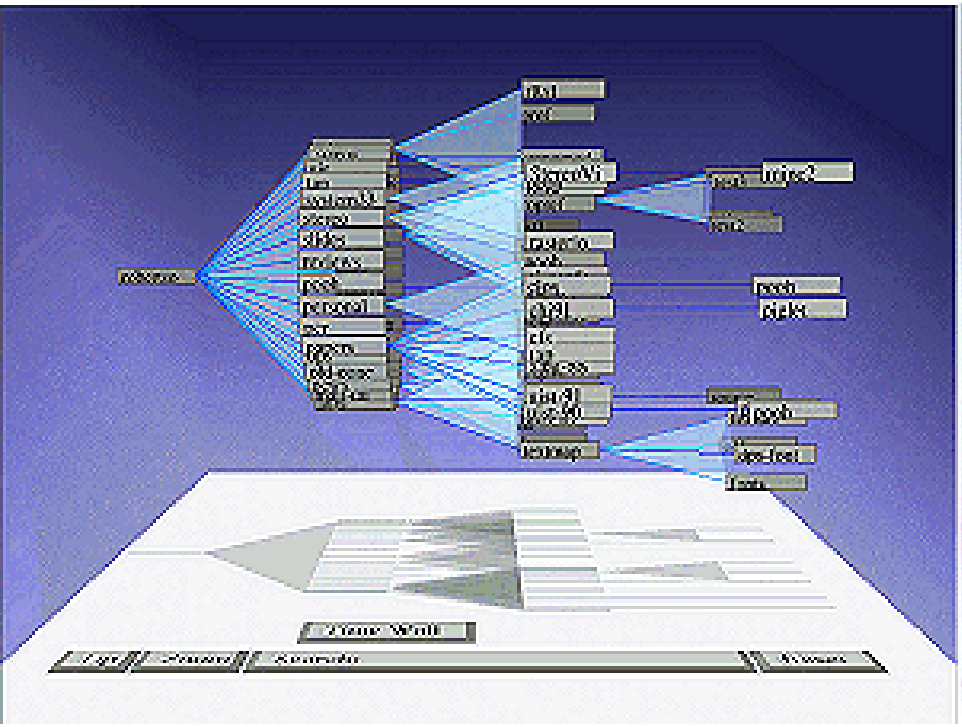
Space-filling by Circle Packing

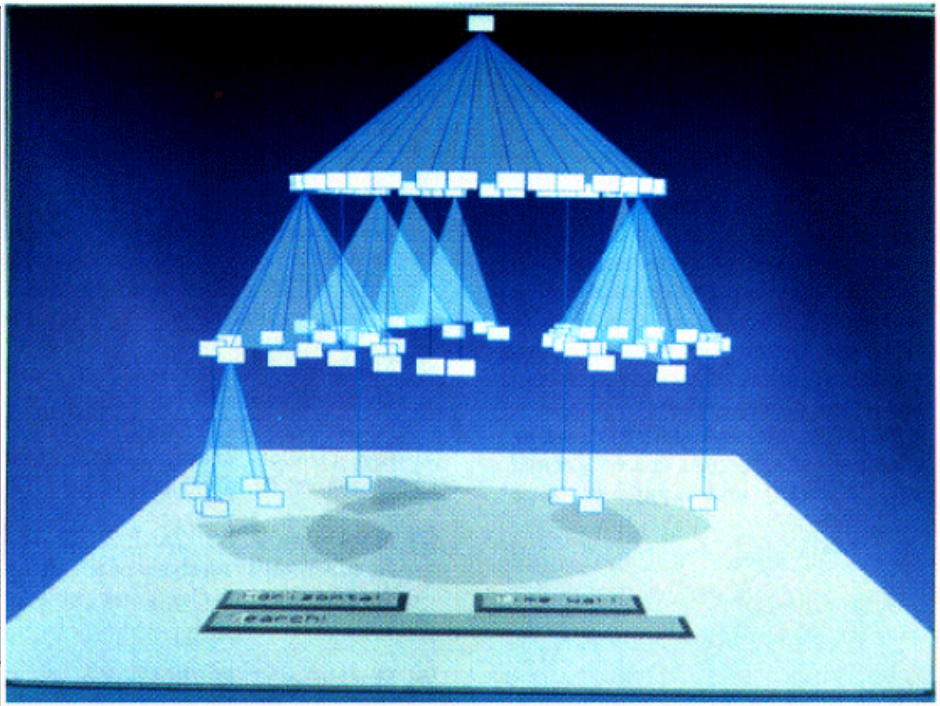


Cone Tree

[Roberston et al. 91]

- 3D extension of the 2D layered tree drawing method.
- The extension to 3D does not necessarily means more information can be displayed
  - occlusion problem
  - interaction is essential

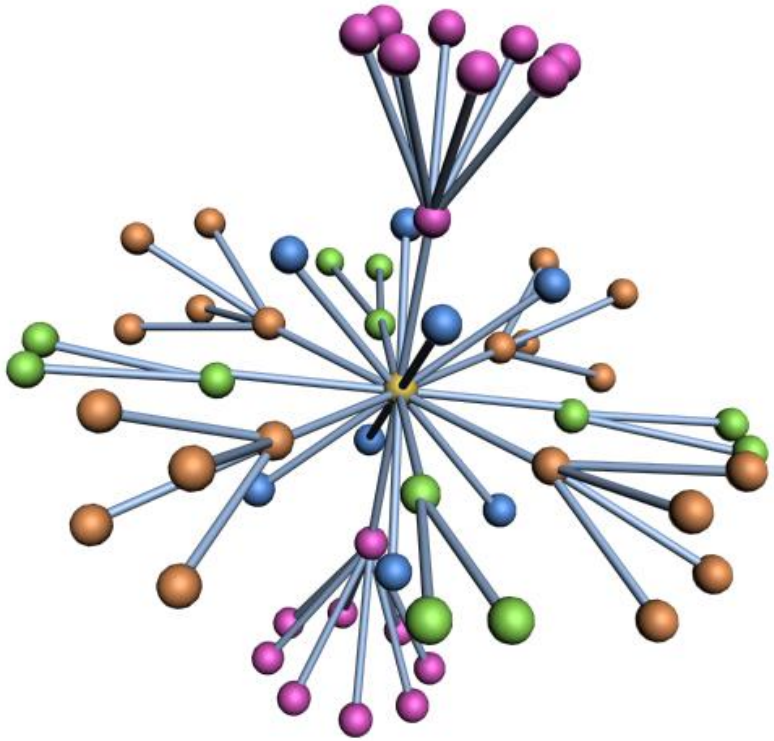


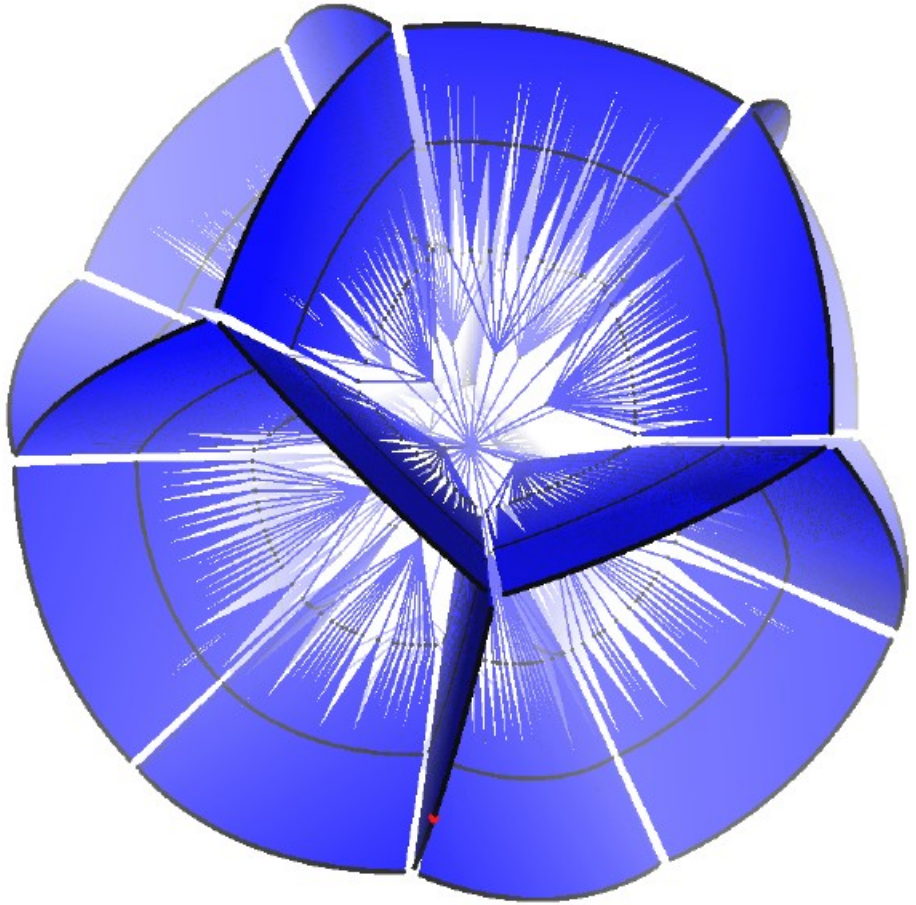


Robertson Plate 1

Polyplane

- [Hong 04] 2.5D tree visualisation
  - Place subtrees on 2D planes
  - Divide & Conquer algorithm
  - Arrange these planes in 3D to reduce occlusion using Platonic solids





Phyllo Tree

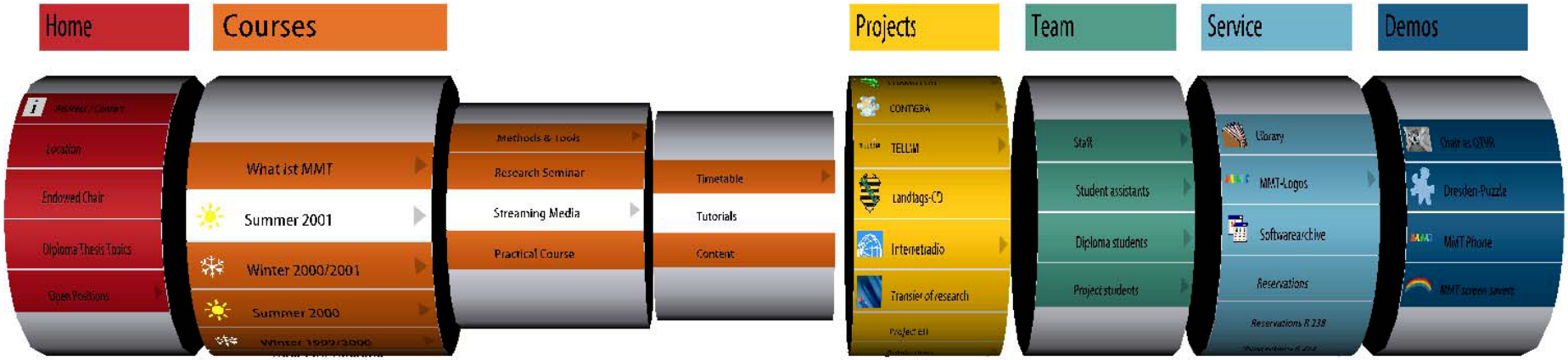
[Carpendale 06]

- use of nature's phyllotactic patterns
- optimal packing effect

Collapsible Cylindrical Trees

- Telescope metaphor: a set of nested cylinders
  - A cylinder is constructed for the children of a node with a smaller radius.
  - It can be pulled out to the right of the parent cylinder or collapsed.

- Only one path of the hierarchy is visible at once



Botanical Tree

- Resembles botanical trees [Wijk]
  - The root is the tree stem.
  - Non-leave nodes are branches .
  - Leave nodes are “bulbs” at the end of branches.

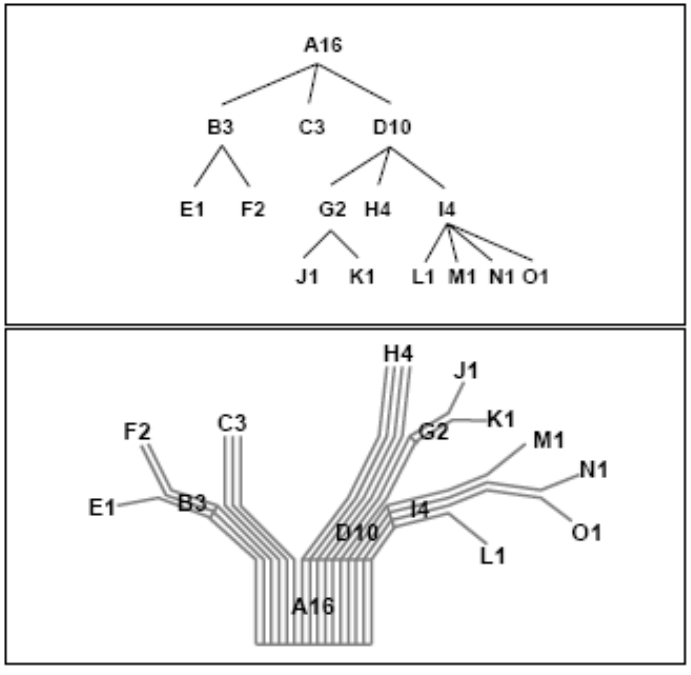
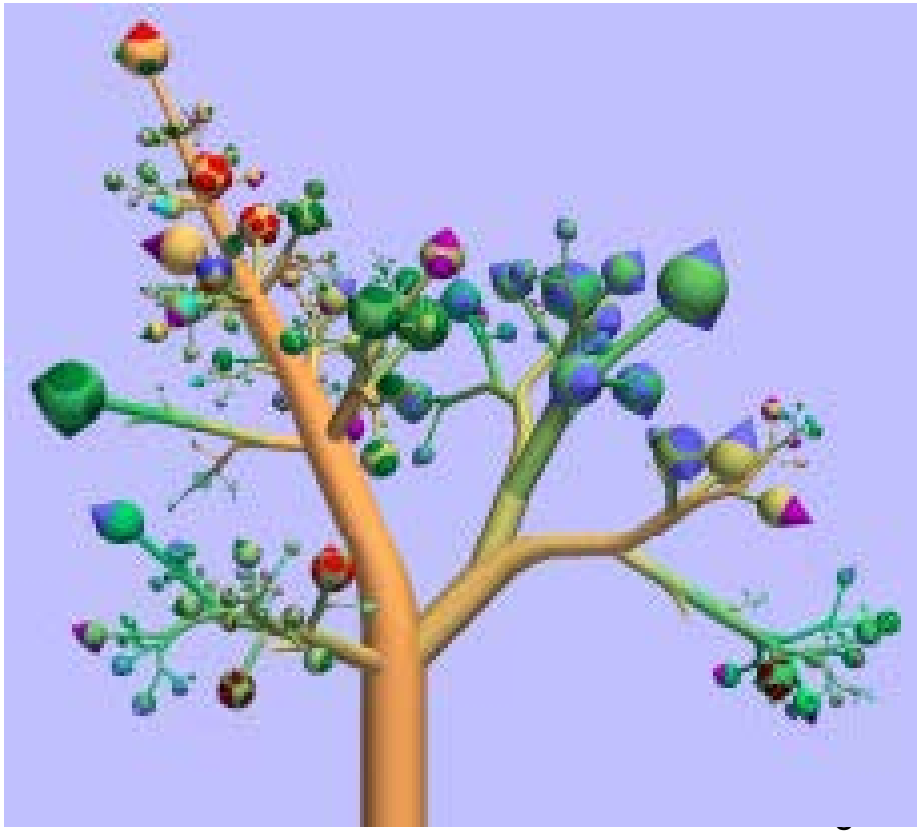


Figure 2. Node and link diagram (t) and corresponding strands model (d).



Summary: Tree Visualization Methods

- There are many variations based on
  - edge representation
  - criteria
- Divide and Conquer algorithm
- mostly run in linear time (O(n) time)

- most popular methods
  - treemap
  - dendrogram

A Visual *Survey* of *Tree Visualization*

http://vcg.informatik.uni-rostock.de/~hs162/treeposter/poster.html

Homework

- Tutorial (D3)

- Assumed Knowledge

basic terminology on graphs

  – degree
  – path
  – cycle
  – shortest path