Newton-Raphson Method and Anti-derivatives

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Newton-Raphson Method

- Want to solve equations of the form $f(x) = 0$.

  1. Set initial value x_0
  2. Find x_1, where the tangent line to the function at x_0 crosses the x-axis:

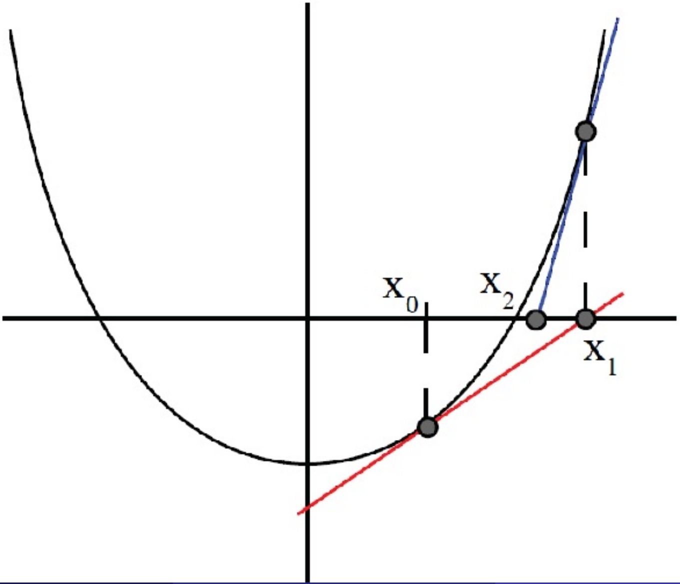
  $$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}.$$

  3. Repeat this process until x_k converge.

  $$x_k = x_{k-1} - \frac{f(x_{k-1})}{f'(x_{k-1})}$$

Ex) Solve $f(x) = 0$, for $f(x) = x^2 - 3$ with initial value $x_0 = 1$ or $x_0 = -1$.

Newton-Raphson Method



Introduction to integration

Definition

Given a function $f(x)$, an anti-derivative of $f(x)$ is any function $F(x)$ such that

$$F'(x) = f(x).$$

If $F(x)$ is any anti-derivative of $f(x)$ then an indefinite integral is given by

$$\int f(x)dx = F(x) + c,$$

where c is any constant.

- $\int$ is called integral symbol, $f(x)$ is called the integrand, x is called the integration variable.

Properties of the indefinite integrals

- If $F'(x) = f(x)$, and $G'(x) = f(x)$, then $G(x) - F(x) = c$ for some constant c,
- $\int cf(x)dx = c\int f(x)dx$, where c is any constant.
- $\int f(x) \pm g(x)dx = \int f(x)dx \pm \int g(x)dx$
- $\int f(x)g(x)dx \neq \int f(x)dx \int g(x)dx$
- $\int f(x)/g(x)dx \neq \int f(x)dx / \int g(x)dx$
- $\int f(x)G(x)dx = F(x)G(x) - \int F(x)g(x)dx$
- $\int f(x)dx = \int f(x(t))\frac{dx}{dt}dt$

Basic indefinite integrals

- $\int kdx = kx + c$
- $\int x^n dx = \frac{1}{n+1}x^{n+1} + c, \quad n \neq= 1.$
- $\int sinxdx = cosx + c$, $\int cosxdx = -sinx + c$.
- $\int e^x dx = e^x + c$, $\int a^x dx = a^x / loga + c$.
- $\int 1/xdx = log|x| + c$.
- $\int \frac{1}{1+x^2} dx = arctanx + c$, $\int \frac{1}{\sqrt{1-x^2}} dx = arcsinx + c$.

Examples

- $\int x^4 + 3x + 9dx$
- $\int \frac{x}{\sqrt{1+x^2}} dx$
- $\int e^{6x} dx$
- $\int xe^{-x^2} dx$
- $\int sinxcosxdx$
- $\int x^3(x^4 + 2)^5 dx$
- $\int \frac{1}{xlog`x} dx$, assume $x > 0$

Q & A

Thank You !