Experiment 3: Static Equilibrium

OBJECTIVES

When all the external forces acting on object do not accelerate the object, the object is in a state of mechanical equilibrium. If the object is also at rest, the object is in a state of static equilibrium. In this experiment, you arrange sets of forces to put an object into static equilibrium, measure the vector quantities of these forces, and calculate the net force acting upon an object in equilibrium. The objectives of this experiment are as follows:

1. To measure vector quantities for forces using the force table
2. To calculate the net force on an object using vector addition
3. To test the hypothesis that an object in equilibrium has no net force acting upon it

THEORY

According to Newton's second law of motion, an object accelerates in direct proportion to the net force acting on it. An object in static equilibrium is not moving, so has an acceleration of zero, and the net force on the object is also zero. Therefore, the necessary condition for equilibrium is that the vector sum of all external forces acting on the object is zero, as shown in equation 3.1.

Equilibrium $$F_{total} = \sum_i F_i = 0 \tag{3.1}$$

In this experiment, you apply forces to an object in two dimensions until it is in static equilibrium, measure the vector forces, and calculate the vector sum. Because all the forces are applied in the plane (two dimensions) then one can project the forces on the *x*- and *y*-axes, as shown in equations 3.2.

2-D Equilibrium $$\begin{aligned} F_{total,x} &= \sum_i F_{i,x} = 0 \\ F_{total,y} &= \sum_i F_{i,y} = 0 \end{aligned} \tag{3.2}$$

To decompose a force vector into its x and y components, it is convenient to choose the x-axis along the direction $\phi = 0^\circ$ and the y-axis along the direction $\phi = 90^\circ$. Then the components of the forces are shown in the pair of equations 3.3.

Component Forces $$F_{i,x} = F_i \cos\phi_i, \qquad F_{i,y} = F_i \sin\phi_i \tag{3.3}$$

ACCEPTED VALUES

The accepted value for the sum of the forces on a particle in equilibrium is 0.

APPARATUS

- Horizontal force table
- four pulleys
- one metal ring
- four cords
- four weight hangers
- a degree scale
- Assortment of known weights.

THE FORCE TABLE

A force table consists of a circular platform supported by a heavy tripod base. The circular platform has a graduated degree scale around its rim and a small peg located directly in the center. Four cords are attached to a metal ring placed over a peg in the center of the platform and the cords are connected over pulleys to weight hangers, as shown in Figure 3.1.

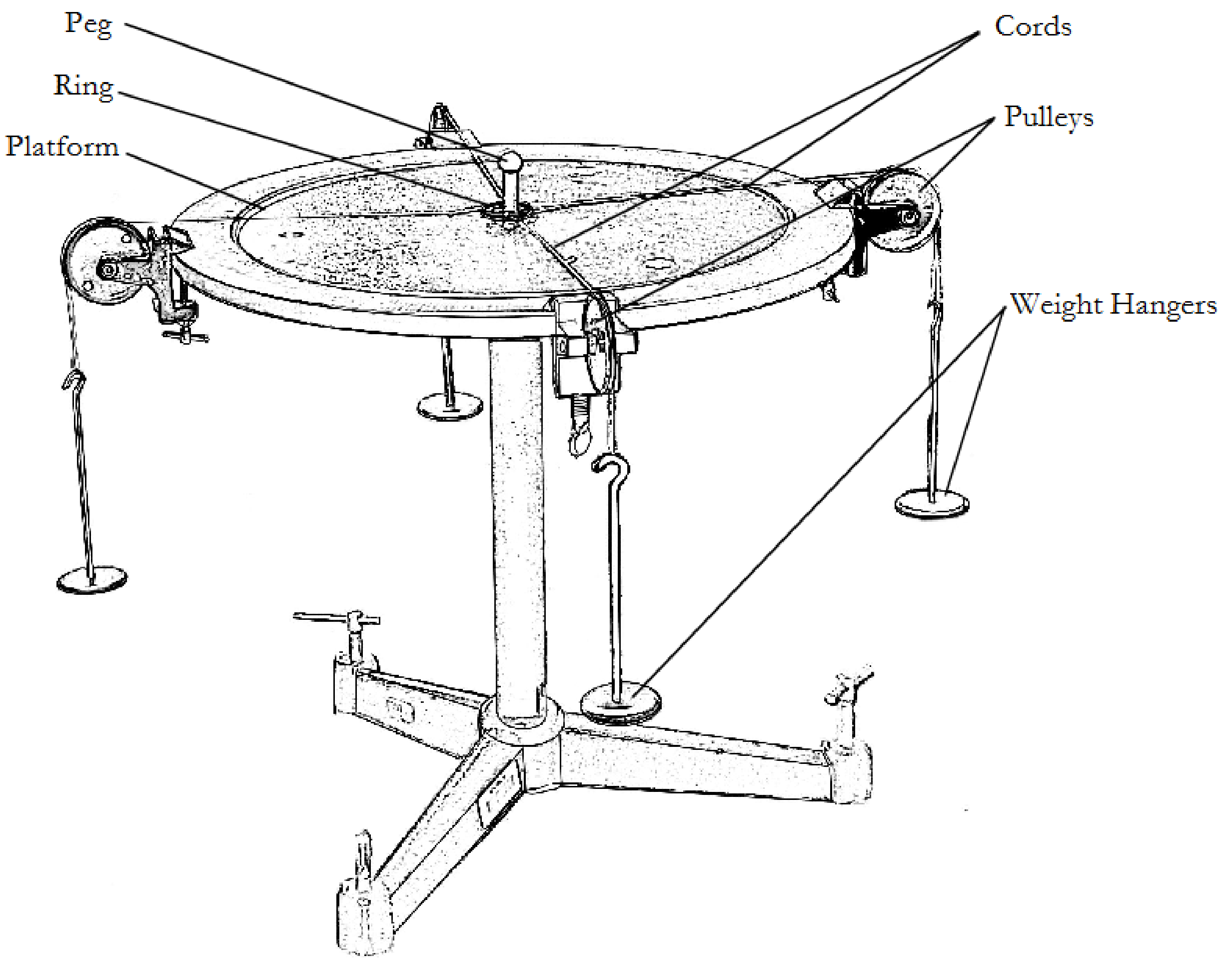

Figure 3.1 An assembled force table

Tension forces are applied to the ring by varying the total mass on each weight hanger and moving the pulleys to change the direction in which each force acts. The ring is in a state of static equilibrium when it is over the peg but not touching the peg, as shown in figure 3.2.

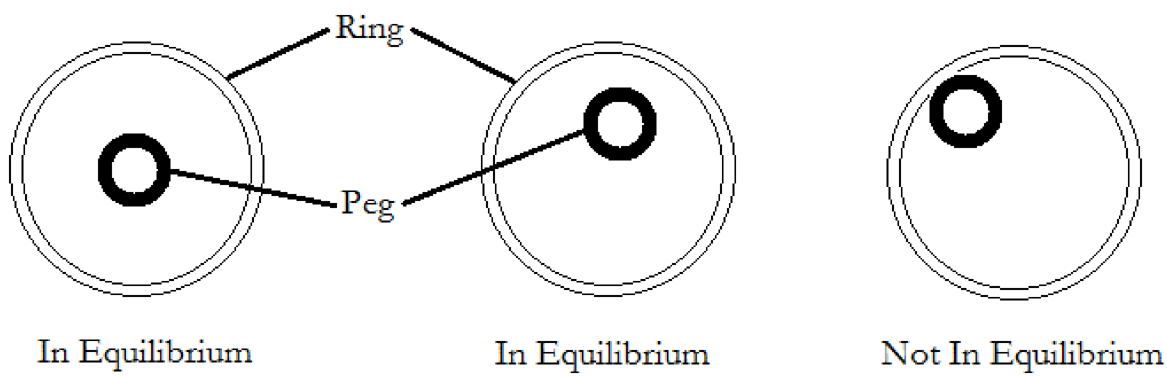

Figure 3.2 Overhead view of ring and peg positions for system in and not in equilibrium

PROCEDURE

1. Mount a pulley at 0° and attach 250 g to the cord running over it. Remember that the holder is part of the mass.
2. Mount a second pulley at 60° with a load of 350 g.
3. Holding a third cord in your hand, find the direction in which a third force should act in order to balance the system. Set the cord on a pulley in the proper position and add weights to the holder until the system is in static equilibrium, as shown in figure 3.2. It may be necessary to adjust the position of the weight holder to achieve equilibrium.
4. Record masses and angles in the data sheet labeled Trial 1.
5. Repeat step 1 through 4 using a 45° angle between the two loads. Record your results in the data sheet labeled Trial 2.
6. Set up four pulleys and suspend unequal loads on the cords running over them. Arrange the system so that it is in equilibrium, and record the masses and angles. Do not have any two cords form an angle of 180°. Record your results in the data sheet labeled Trial 3.
7. Suppose you pace a mass, m = 300 g, at ϕ = 210° mark. Compute the masses m_a and m_b you would place at 0° and 90° to balance mass m. Try it, and see if your solution is correct. Report what masses you had to place at 0° and 90° to balance the mass at 210°.

DATA

Trial 1

i	m_i (grams)	ϕ_i (°)
1		0.0
2		60.0
3		

Trial 2

i	m_i (grams)	ϕ_i (°)
1		
2		
3		

Trial 3

i	m_i (grams)	ϕ_i (°)
1		
2		
3		
4		

CALCULATION AND ANALYSIS

All the forces in this experiment are gravity forces and they have the form $F_i = m_i g$. Therefore, you can factor the gravitational acceleration, g, out of all calculations and compare forces in mass units, grams. For example, factoring out g does not change the result of equation 3.4 because g in the numerator and the denominator cancel.

1. Draw a free body diagram for Trial 1. Be sure to label each vector's direction and magnitude.
2. Calculate the component forces for each force vector, $F_{i,x}$ and $F_{i,y}$, using equation 3.3.
3. Calculate $|F_{total}|$ and $\sum_i |F_i|$ using equations 3.5 and 3.6

4. Calculate the % discrepancy of the force calculations using equation 3.4.
5. Repeat steps 2 through 4 using data from trials 2 and 3.
6. In procedure 7, was the system in equilibrium? If not, suggest some possible reasons.
7. What are the sources of experimental error in this experiment? Do any of these factors help the ring achieve static equilibrium?
8. Following from the logic of question 7, why is the uncertainty of this experiment functionally impossible to measure?

CALCULATING % ERROR WHEN THE ACCEPTED VALUE IS ZERO

In this experiment, the accepted value of the total force is zero. If you could measure all the forces on the ring with perfect precision, you would find that the net force vanishes. If zero is inserted into our initial % Error equation 1.7, the result is undefined because the denominator is zero. Due to experimental errors and measurement uncertainties, the calculated net force isn't zero. A useful way to characterize the accuracy of our measurements is to divide the magnitude of net force, $|F_{total}|$, by the sum of the magnitudes of all the component forces, $\sum_i |F_i|$, multiplied by 100% as shown in equation 3.4.

$$\%\text{ Discrepancy} = \frac{|F_{total}|}{\sum_i |F_i|} \times 100\% \tag{3.4}$$

Here the components of the equation are comprised of the calculated quantities shown in equations 3.5 and 3.6.

$$\text{Magnitude of Net Force, 2-D} \quad |F_{total}| = \sqrt{F_{x,total}^2 + F_{y,total}^2} \approx 0 \tag{3.5}$$

$$\text{Component Magnitude Sum} \quad \sum_i |F_i| = \sum_i \sqrt{F_{x,i}^2 + F_{y,i}^2} \neq 0 \tag{3.6}$$

$$= \sqrt{F_{x,1}^2 + F_{y,1}^2} + \sqrt{F_{x,2}^2 + F_{y,2}^2} + \ldots + \sqrt{F_{x,n}^2 + F_{y,n}^2}$$