Задание 9

5. $A = \begin{pmatrix} 3 & 0 & -1 & 0 & 1 \\ -2 & 0 & 0 & 3 & 0 \\ 0 & 4 & 0 & 0 & 1 \end{pmatrix}$ $b = \begin{pmatrix} 30 \\ -11 \\ 2 \end{pmatrix}$

$c = (5 \quad 0 \quad -4 \quad 15 \quad 5)$

$d_* = \begin{pmatrix} 2 \\ 0 \\ 1 \\ 0 \\ -1 \end{pmatrix}$ $d^* = \begin{pmatrix} 10 \\ 4 \\ 5 \\ 4 \\ 3 \end{pmatrix}$

Пусть $J_Б = \{1, 2, 3\}$

Итерация 1.

1. $(u_1 u_2 u_3)\begin{pmatrix} 3 & 0 & -1 \\ -2 & 0 & 0 \\ 0 & 4 & 0 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ -4 \end{pmatrix}^T$

$u = (4 \quad 3.5 \quad 0)$

2. $\Delta_4 = 15 - (4 \quad 3.5 \quad 0)\begin{pmatrix} 0 \\ 3 \\ 0 \end{pmatrix} = 15 - 10.5 = 4.5$

$\Delta_5 = 5 - (4 \quad 3.5 \quad 0)\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = 5 - 4 = 1$

3. $x_4 = 4$
$x_5 = 3$

Решаем: $\begin{pmatrix} 3 & 0 & -1 \\ -2 & 0 & 0 \\ 0 & 4 & 0 \end{pmatrix} \cdot x_Б =$

$= \begin{pmatrix} 30 \\ -11 \\ 2 \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ 3 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 3 \end{pmatrix}$

$x_1 = 11.5$
$x_2 = -0.25$
$x_3 = 7.5$

4. $\mathcal{H}_1$: $2 \le 11.5 \le 10$ => план не оптимален

5. $j^* = 2$

$$\begin{pmatrix} 3 & -2 & 0 \\ 0 & 0 & 4 \\ -1 & 0 & 4 \end{pmatrix} \cdot \begin{pmatrix} p_{41} \\ p_{42} \\ p_{43} \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$p_4 = (0 \quad 0 \quad 0.25)$

6. $p_{\delta 4} = -(0 \quad 3 \quad 0)\begin{pmatrix} 0 \\ 0 \\ 0.25 \end{pmatrix} = 0$

$p_{\delta 5} = -(1 \quad 0 \quad 1)\begin{pmatrix} 0 \\ 0 \\ 0.25 \end{pmatrix} = -0.25$

7. $p_{\delta 4} \cdot \Delta_4 = 0 \Rightarrow \sigma_4 = \infty$

$p_{\delta 5} \cdot \Delta_5 < 0 \Rightarrow \sigma_5 = 4$

$\sigma^* = 4 = \sigma_5$

8. $J_5 = \{1,2,3\} \setminus \{2 \cup 4 5\} = \{1,3,5\}$

Итерация 2

1. $(u_1 \quad u_2 \quad u_3)\begin{pmatrix} 3 & -1 & 1 \\ -2 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = (5 \quad -4 \quad 5)$

$u = (4 \quad 3.5 \quad 1)$

2. $\Delta_2 = 0 - (4 \; 3.5 \; 1)(0 \; 0 \; 4)^T = -4$

$\Delta_4 = 15 - (4 \; 3.5 \; 1)(0 \; 3 \; 0)^T = 4.5$

3. $\Delta_2 < 0 \Rightarrow \mathcal{H}_2 = 0$

$\Delta_4 > 0 \rightarrow x_4 = 4$

$$\begin{pmatrix} 3 & -1 & 1 \\ -2 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot x_б = \begin{pmatrix} 30 \\ -11 \\ 2 \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ 0 & 3 \\ 4 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 4 \end{pmatrix}$$

$x_1 = 11.5$

$x_3 = 6.5$

$x_5 = 2$

4. $x_1 : 2 \leq 11.5 \leq 10 \Rightarrow$ план не оптимален.

5. $j^* = 1$

$$\begin{pmatrix} 3 & -2 & 0 \\ -1 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} p_{41} \\ p_{42} \\ p_{43} \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$$

$p_4 = (0 \quad 0.5 \quad 0)$

6. $p_{б2} = (0 \; 0 \; -4) \cdot (0 \; 0.5 \; 0)^T = 0$

$p_{б4} = (0 \; -3 \; 0) \cdot (0 \; 0.5 \; 0)^T = -1.5$

7. $\sigma_2 = \infty$

$\sigma_4 = -\frac{\Delta_4}{p_{б4}} = \frac{-4.5}{-1.5} = 3$

$\sigma^\circ = 3 = \sigma_4$

8. $J_б = \{1, 3, 5\} \setminus \{1\} \cup \{4\} = \{3, 4, 5\}$

Итерация 3

1. $(u_1\ u_2\ u_3)\begin{pmatrix}-1 & 0 & 0\\ 0 & 3 & 0\\ -1 & 0 & 1\end{pmatrix} = \begin{pmatrix}-4\\ 15\\ 5\end{pmatrix}$

$u = (4\ \ 5\ \ 1)$

2. $\Delta_1 = 5 - (4\ 5\ 1)(3\ -2\ 0)^T = 3$

$\Delta_2 = 0 - (4\ 5\ 1)(0\ 0\ 4)^T = -4$

3. $\mathcal{H}_1 = 10$, т.к. $\Delta_1 > 0$

$\mathcal{H}_2 = 0$, т.к. $\Delta_2 < 0$

$$\begin{pmatrix}-1 & 0 & 1\\ 0 & 3 & 0\\ 0 & 0 & 1\end{pmatrix}\cdot\mathcal{H}_5 = \begin{pmatrix}30\\ -11\\ 2\end{pmatrix} - \begin{pmatrix}3 & 0\\ -2 & 0\\ 0 & 4\end{pmatrix}\cdot\begin{pmatrix}10\\ 0\end{pmatrix}$$

$\mathcal{H}_3 = 2$; $\mathcal{H}_4 = 3$; $\mathcal{H}_5 = 2$

4. $\mathcal{H}_3$: $1 \le 2 \le 5$

$\mathcal{H}_4$: $0 \le 3 \le 4$

$\mathcal{H}_5$: $-1 \le 2 \le 3$

Найдено оптимальное решение:

$$\left[\begin{array}{l} X = (10, 0, 2, 3, 2) \\ \varphi(X) = 97 \end{array}\right]$$