

Probability and Statistics – Lancaster_BJTU

Final Exam 2017 Paper B

1 Fill in the blanks (20 points)

1. In a simple probability space that has 50 outcomes, the probability of an event containing 5 outcomes is _____.
2. If X has the uniform distribution on interval $[1, 5]$, then $\text{Var}(X) = \underline{\hspace{2cm}}$. If X has the binomial distribution with parameter $n = 5$ and $p = 0.4$, then $\text{Var}(X) = \underline{\hspace{2cm}}$.
3. Of the following six families of distributions: binomial, Poisson, geometric, uniform, normal, and exponential, the family of distributions for which the mean and the variance are always equal are the _____ distributions; the family of distributions for which the variance is always the square of the mean are the _____ distributions.
4. Suppose that X and Y are two random variables and $\text{Var}(X) > 0$. If $Y = -7X$, then their correlation coefficient $\rho(X, Y) = \underline{\hspace{2cm}}$; if X and Y are independent, then $\rho(X, Y) = \underline{\hspace{2cm}}$.
5. A sampling distribution is the distribution of a _____. For a sequence of estimators to be practically usable, it must be _____, which means that it must converge _____ to the parameter being estimated.

2

(10 points) Suppose that A , B , and C are events such that A and B are independent, $P(A \cap B \cap C) = 0.04$, $P(C|A \cap B) = 0.25$, and $P(B) = 4P(A)$. Evaluate $P(A \cup B)$.

3

(8 points) Two people are playing a game by rolling dice. The rule of the game is as follows:

- One of the people is designated as player A and the other as player B.
- Player A rolls the die first. If a 5 or a 6 appears on the die, then player A wins and the game is over, otherwise the game proceeds.
- Now player B rolls the die. If the number that appears on his die is equal to or greater than the number that appeared on player A's die, then player B wins, otherwise player A wins.

Determine the probability for player A to win the game and decide whether either player has any advantage.

4

(14 points) Suppose that random variables X and Y have the following joint p.d.f.

$$f(x, y) = \begin{cases} 2(x + y) & \text{for } 0 < x < y < 1 \\ 0 & \text{otherwise.} \end{cases}$$

Determine

- (a) the marginal p.d.f. of X ;
- (b) $P(X < 1/2)$;
- (c) the conditional p.d.f. of Y given that $X = x$.

5

(12 points) Suppose that X_1, \dots, X_n form a random sample of size n from a continuous distribution with the following p.d.f.

$$f(x) = \begin{cases} 2x & \text{for } 0 < x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

Let $Y = \max(X_1, \dots, X_n)$. Evaluate $E(Y)$.

6

(9 points) In a small town there are 500 people and there is a shop. Each hour, each person in the town has 0.01 probability to go to the shop and they go independently of one another. Approximately evaluate the probability that in a given hour, less than 3 people go to the shop.

7

(11 points) In a building there are 400 lights, and they are on or off independently of one another. At night, each light has 0.7 probability to be on. Using correction for continuity, approximately evaluate the probability that in the building there are between 270 (inclusive) and 290 (inclusive) that are on.

8

(16 points) Suppose that X_1, \dots, X_n form a random sample from a distribution for which the p.d.f. is as follows:

$$f(x; \theta) = \begin{cases} \theta x^{-(\theta+1)} & \text{for } x > 1, \\ 0 & \text{otherwise.} \end{cases}$$

where $\theta > 1$ is the parameter to be estimated.

- (a) Derive the method-of-moments estimator of θ .
- (b) Derive the maximum likelihood estimator of θ .

Table of the Standard Normal Distribution Function

$$\Phi(z) = \int_{-\infty}^z \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{u^2}{2}\right) du$$

<i>z</i>	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998