1. **(20pt)**

(1) $\boldsymbol{P(A) = 0.8, P(B) = 0.4, P(B|A) = 0.25, P(A|B) =}$ ________________.

(2) Let A, B, C be mutually independent events. If $P(A) = 0.8$, $P(AB) = 0.4$, $P(BC) = 0.3$, then $P(ABC) =$ ________________.

(3) Tom and Jerry are in group of 10 people. A team of 5 people are to be selected from the group. The probability that both Tom and Jerry are selected into the team is ________________.

(4) A fair coin is tossed 5 times. The probability of obtaining 4 or more heads is ________________.

(5) Suppose that $\boldsymbol{P(X = k) = \frac{c\lambda^k}{k!}, (k = 1, 2, \cdots, \lambda > 0)}$ is a probability function, then $\boldsymbol{c} =$________________.

(6) Suppose that random vector $\boldsymbol{(X, Y) \sim N(0, 1; 0, 1; -0.5)}$, $\boldsymbol{aX + Y}$ and $\boldsymbol{Y}$ are independent, then $\boldsymbol{a} =$_______________.

(7) Suppose that $\boldsymbol{X \sim Exp(\lambda)}$, $\boldsymbol{P(X \geq 1) = e^{-2}}$, then $\boldsymbol{E(X)} =$_____________, $\boldsymbol{E(X^2)} =$________________.

(8) Given observed data $(6, -7, 4, -8, 10)$ of a random sample from a normal distribution $\mathrm{N}(\mu, \sigma^2)$. Then the maximum likelihood estimate of μ is ______________ and the maximum likelihood estimate of σ^2 is ________________.

2. (**12pt**) The CDF of X is given by

$$F(x) = \begin{cases} A + Be^{-2x}, & x > 0, \\ 0, & x \leq 0. \end{cases}$$

Find

(1) (**5pt**) A and B;

(2) (**3pt**) $P(-1 < X < 1)$;

(3) (**4pt**) PDF $f(x)$ of X.

3. (**10pt**) The diameter of the dot produced by a printer is normally distributed with a mean diameter of 2 micro-inch and a standard deviation of 0.4 micro-inch.

(1) (**5pt**) What is the probability that the diameter of a dot exceeds 2.6 micro-inch?

(2) (**5pt**) What is the probability that a diameter is between 1.4 and 2.6 micro-inch?

4. (**10pt**) Suppose that $X \sim \mathrm{P}(2)$, evaluate $\mathrm{E}[(X + 3)^2]$.

5. (**14pt**) The joint PDF of X and Y is given by

$$f(x,y) = \begin{cases} e^{-x}, & 0 < y < x, \\ 0, & \text{otherwise.} \end{cases}$$

Find

(1) **(4pt)** the marginal PDF $f_Y(y)$;

(2) **(4pt)** the conditional PDF $f_{X|Y}(x|y)$;

(3) (**6pt**) $P(X \le 1 | Y \le 1)$.

6. (**12pt**) The joint PDF of X and Y is given by

$$f(x,y) = \begin{cases} \frac{1}{4}(x+y), & 0 < y < x < 2, \\ 0, & \text{otherwise.} \end{cases}$$

(1) (**6pt**) Evaluate $E(X)$ and $E(Y)$.
(2) **(6pt)** Evaluate $Cov(X,Y)$.

7. (**12pt**) A company has 260 telephone extensions. In any time, each extension, independently from each other, has 4% probability to request an external communication channel. Using the central limit theorem, approximately estimate how many external communication channels should be equipped, so that the probability of channel request being satisfied exceeds 95%.

8. (**10pt**) Suppose $X_1, X_2, \cdots, X_n$ is a random sample from population X, where the PDF of X is

$$f(x) = \begin{cases} \frac{1}{\theta} e^{-\frac{x}{\theta}}, & x \ge 0, \\ 0, & \text{otherwise,} \end{cases}$$

where $\theta > 0$ is unknown parameter. Derive the maximum likelihood estimator of the parameter θ.

Some values that might be useful:

$$\Phi(0.6) = 0.72575, \qquad \Phi(0.9487) = 0.8286$$

$$\Phi(1) = 0.8413, \qquad \Phi(1.5) = 0.93319,$$

$$\Phi(1.645) = 0.95$$