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课程名称: <u>概率论与数理统计(B)</u> 学年学期: <u>2017—2018 学年第 1 学期</u> 课程编号: 73L187Q 开课学院: 威海 出题教师: 程

题 号	1	2	3	4	5	6	7	8	总分
得 分									
阅卷人									

备注:本试题闭卷。所有答案全部做在答题纸上,答题纸请勿使用反面。

1. Fill in the blanks (18 points)

- 1) Let A and B be events. If P(A) = a and P(B) = b, then the smallest possible value of $P(A \cap B)$ is ______, the greatest possible value of $P(A \cap B)$ is _____.
- 2) Let A and B be mutually independent events, $P(A \cup B) = 0.82$, $P(A \cap B) = 0.28$, $P(A) \ge P(B)$, then P(A) =______, P(B) =_____.
- 4) A team consists of 5 boys and 5 girls. Now a small working group of 4 people are to be selected at random from the team. The probability that there are more girls than boys in the working group is ______.
- 5) Let X be a random variable whose c.d.f. is F. The probability $P(1 \le X \le 2)$ expressed in F is ______. If F(0) = F(10) = 0.3, then P(X > 4) = _____.
- 6) Let X_1, X_2 be independent discrete random variables that both have the uniform distribution on integers 1,2,3. Let $Y = X_1 X_2$, then E(Y) =______, Var(Y) =

- 2. **(10 points)** Assume that the probability that any child born will be a girl is 1/2 and that all births are independent. A family has 3 children, determine the conditional probability that this family has at least one boy on the condition that this family has at least one girl.
- 3. **(10 points)** A random variable *X* has a continuous distribution with the following p.d.f.

$$f(x) = \begin{cases} \frac{c}{1+x^2}, & \text{for } x > 0\\ 0, & \text{otherwise} \end{cases}$$

- (a) Determine the value of c.
- (b) Let Y = ln(X). Determine the p.d.f. of Y.
- 4. (18 points) Suppose that random variables X and Y have the following joint p.d.f.:

$$f(x, y) = \begin{cases} 3y, & \text{for } 0 < x < y < 1 \\ 0, & \text{otherwise} \end{cases}$$

- (a) Evaluate P(X+Y<1).
- (b) Find E(X) and E(Y).
- (c) Find Cov(X,Y).
- 5. (10 points) Let X and Y be random variables such that Var(X)=Var(Y)=1, and $\rho(X,Y)=r$. Let U=aX+bY and V=aX-bY. Evaluate $\rho(U,V)$.

6. **(10 points)** Suppose that the joint p.d.f of two random variables *X* and *Y* is as follows:

$$f(x, y) = \begin{cases} c \sin(x), & \text{for } 0 \le x \le \frac{\pi}{2}, 0 \le y \le 1\\ 0, & \text{otherwise} \end{cases}$$

Determine the conditional p.d.f. of Y for every given value of X.

7. (9 points) Let *X* be a discrete random variable having the following p.f.:

$$f(x) = \begin{cases} p(1-p)^x, & \text{for } x = 0, 1, 2, \dots \\ 0, & \text{otherwise} \end{cases}$$

- (a) Find $P(X \ge k)$ for each integer k = 0, 1, 2, ...
- (b) Find $P(X \ge t + k \mid X \ge t)$ for each integer t, k = 0, 1, 2, ...
- 8. **(15 points)** Let X and Y be **independent** random variables that both have the uniform distribution on the interval [0,1]. Let $Z = \max(X^2, Y^2)$ and $W = \min(X^2, Y^2)$.
 - (a) Show that Z also has the uniform distribution on the interval [0,1].
 - (b) Find the p.d.f. of W.