



Lancaster University College
at Beijing Jiaotong University

2021/22 Examinations

Course code: [WB73L004Q](#)

Course name: [Probability Theory and Mathematical Statistics \(B\)](#)

Midterm examination (November)

INSTRUCTIONS TO STUDENTS

- 1) Duration of the exam: [120 minutes](#)
- 2) This paper contains [3](#) pages. There are [8](#) questions.
- 3) You must answer all questions.
- 4) This is a closed book exam. No books or notes may be brought into the exam room.
- 5) A scientific calculator is allowed in the examination. Other electronic devices are not allowed in the exam room.
- 6) Some values that might be useful:

$$\Phi(0.2) = 0.57926, \quad \Phi(1) = 0.8413,$$

$$\Phi(1.4142) = 0.92135, \quad \Phi(2) = 0.97725$$

$$\Phi(2.8284) = 0.99766$$

1. (20pt)

- (1) If events A and B are disjoint, C is the complement of A , then the relationship between B and C is _____.
- (2) Let the joint cumulative distribution function of random variables X and Y be $F(x, y)$. Then $P(1 < X \leq 2, Y \leq 3) =$ _____.
- (3) Let random variables X , Y , and Z be independent. Which of the following sets of random variables are guaranteed independent? Your answer: _____
- a) $X^2 - Z, Y, Z$
- b) $\cos(X), e^{Y+Z}$
- c) $Z^2 + Y^2, \sin(X + Z)$
- d) $X + 3Y, Y - 2Z$
- (4) Of the following three functions: CDF, PF, and PDF, the function that is applicable to all random variables is _____ and the function that **NOT** necessarily takes value between 0 and 1 is _____.
- (5) The PDF of random variable X is $f(x) = \begin{cases} ax^{-2}, & \text{if } x > 1 \\ 0, & \text{otherwise.} \end{cases}$ Then $a =$ _____.
- (6) A box contains 6 red balls and 4 white balls. Three balls are chosen without replacement, then the probability that there is exactly one red ball is _____.
- (7) If $X \sim B(49, 0.2)$, then its variance is _____.
- (8) If $X \sim N(1, 4)$, then $2X + 3 \sim$ _____.
- (9) Let $E(X) = 1$ and $\text{Var}(X) = 2$. Then $E[X(X - 1)] =$ _____.

2. **(10pt)** There are five children in a family. In this family, each child, independent from each other, has 1/4 chance to have blue eyes. If it is known that at least one of the children has blue eyes, then what is the probability that all the five children have blue eyes?

3. **(12pt)** Random variable X has the following PDF

$$f(x) = \begin{cases} 3x^2, & \text{if } 0 < x < 1, \\ 0, & \text{otherwise.} \end{cases}$$

Let $Y = e^X$. Find the PDF $g(y)$ of Y .

4. **(14pt)** Random variables X and Y have the following joint PDF

$$f(x, y) = \begin{cases} 6xy, & \text{if } 0 < x < 1 \text{ and } 0 < y < \sqrt{x} \\ 0, & \text{otherwise.} \end{cases}$$

- (1) Find $P(X \leq Y)$.
- (2) Find marginal density $f_X(x)$.
- (3) Find marginal density $f_Y(y)$.
- (4) Check if X and Y are independent.

5. **(12pt)** Let $X \sim U[-3, 5]$, and let

$$Y = \begin{cases} 2, & \text{if } X > 0, \\ -2, & \text{otherwise.} \end{cases}$$

- (1) Is Y continuous or discrete? If Y is continuous, give its PDF; and if it is discrete, give its PF.
- (2) Find $E(Y)$ and $Var(Y)$.

6. **(10pt)** Tom and Jerry agree to meet up in a station. They arrive at the station uniformly between 9:00 AM and 10:00 AM. Suppose that after arriving, each waits 20 minutes for the other person before leaving. What is the probability that they will meet?
7. **(10pt)** The length in cm (X) of some bolts is normally distributed with a mean of 10.05 and a standard deviation of 0.06. A bolt is qualified if its length is between 10.05 ± 0.12 cm. Find the probability that a bolt is defective.

8. **(12pt)** Random variables X and Y have the following joint PDF

$$f(x, y) = \begin{cases} 1, & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0, & \text{otherwise.} \end{cases}$$

Find the PDF of $Z = X + Y$.