1. Let $X \sim U[0,1]$. Find the distribution of $Y = -2\ln(X)$.

2.

2.

The joint p.d.f. of **x** and **y** is defined as

$$f_{xy}(x, y) = \begin{cases} 6x & x \geq 0, y \geq 0, x + y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Define $\mathbf{z} = \mathbf{x} - \mathbf{y}$. Find the p.d.f. of **z**.

3.

x and **y** are independent uniformly distributed random variables in (0, 1). Let

$$\mathbf{w} = \max(\mathbf{x}, \mathbf{y}) \qquad \mathbf{z} = \min(\mathbf{x}, \mathbf{y})$$

Find the p.d.f. of (*a*) $\mathbf{r} = \mathbf{w} - \mathbf{z}$, (*b*) $\mathbf{s} = \mathbf{w} + \mathbf{z}$.

4.

Let **x** and **y** be independent Poisson random variables with parameters λ_1 and λ_2, respectively. Show that the conditional density function of **x** given $\mathbf{x} + \mathbf{y}$ is binomial.

QUIZ-2

1. A man bought 3 boxes of apples, 5 boxes of bananas and 2 boxes of oranges. When he received them, it was found that one box had been lost, but he was clueless as there was no sign of the fruit type on the boxes. Then he opened 2 boxes randomly from the received 9 boxes, and it was found that they were both bananas. Using the latest information, find the probability that the lost box was also bananas.

2. In statistical physics, the absolute value X of molecular motion velocity obeys Maxwell distribution:

$$f(x) = \begin{cases} \sqrt{\frac{2}{\pi}} x^2 e^{-\frac{x^2}{2}}, & x > 0 \\ 0, & \text{otherwise.} \end{cases}$$

Find $P(X < 1)$. Note: $\Phi(1) = 0.8413$

3. The joint PDF of X and Y is given by

$$f(x, y) = \begin{cases} 1, & 0 < x < 1, -x < y < x \\ 0, & \text{otherwise.} \end{cases}$$

Find the two conditional PDFs.

4. Let the PDF of X be

$$f(x) = \begin{cases} \frac{x+1}{4}, & -1 < x < 1 \\ \frac{1}{2}, & 1 \le x \le 2 \\ 0, & \text{otherwise.} \end{cases}$$

Find the distribution of $Y = X^2$+1.

QUIZ-3

1. Suppose that an ordinary deck of 52 cards (which contains 4 aces) is randomly divided into 4 hands of 13 cards each. We are interested in determining p, the probability that each hand has an ace. Let E_i be the event that the ith hand has exactly one ace. Determine $p = \mathrm{P}(E_1E_2E_3E_4)$ by using the multiplication rule.

2. A fire station is to be located along a road of unit length—stretching from point 0 outward to 1. If a fire occurs at a point uniformly distributed in [0,1], where should the fire station be located so as to minimize the expected absolute distance between the fire and the fire station? That is, we want to minimize $\mathrm{E}[|X - a|]$, where X is uniformly distributed in [0,1], and a is the location of the fire station.

3. If X_1 and X_2 are independent exponential random variables with respective parameters λ_1 and λ_2. Compute $\mathrm{P}(X_1 < X_2)$.

4. The joint density function of X and Y is

$$\mathrm{f}(x,y) = \begin{cases} xy, & 0 < x < 1, 0 < y < 2 \\ 0, & \text{otherwise.} \end{cases}$$

Find $\mathrm{P}(X + Y < 1)$.

5. Suppose that X and Y are independent geometric random variables with the same parameter p. (a) Compute $\mathrm{E}(X)$ (b) Compute $\mathrm{P}(X = i|X + Y = n)$.

QUIZ-4

1. Suppose that the PDF of *X* is f(*x*). $Y = |X|$. Show that the PDF of *Y* is

$$g(y) = \begin{cases} f(y) + f(-y), & y > 0 \\ 0, & \text{otherwise.} \end{cases}$$

2. The probability of hitting an aircraft is 0.001 for each shot. Using Poisson approximation, how many shots should be fired so that the probability of hitting with two or more shots is above 0.95?

3. You and three other people are to place bids for an object, with the high bid winning. You plan to bid 10 thousand dollars. The bids of the others can be regarded as being independent and uniformly distributed between 7 and 11 thousand dollars. Estimate the probability that you win the bid.

4. Jill's bowling scores are approximately normally distributed with mean 170 and standard deviation 20, while Jack's scores are approximately normally distributed with mean 160 and standard deviation 15. If Jack and Jill each bowl one game, then assuming that their scores are independent random variables, approximate the probability that (a) Jack's score is higher; (b) the total of their scores is above 350.

5. The joint density function of *X* and *Y* is given by

$$f(x, y) = \begin{cases} xe^{-xy}, & x > 0, y > 1 \\ 0, & \text{otherwise.} \end{cases}$$

Find $P(X<1|Y=2)$.