学生学院________ 班级________ 学号________ 姓名________ 任课教师________

装 订 线

北 京 交 通 大 学 考 试 试 题(期中)

课程名称：概率论与数理统计(B)　学年学期：2017—2018 学年第 1 学期

课程编号：　73L187Q　开课学院：　威海　出题教师：　程

题　号	1	2	3	4	5	6	7	8	总分
得　分									
阅卷人									

备注：本试题闭卷。所有答案全部做在答题纸上，答题纸请勿使用反面。

1. Fill in the blanks (18 points)

1) Let A and B be events. If $P(A)=a$ and $P(B)=b$, then the smallest possible value of $P(A\cap B)$ is ________________, the greatest possible value of $P(A\cap B)$ is ______________.

2) Let A and B be mutually independent events, $P(A\cup B)=0.82$, $P(A\cap B)=0.28$, $P(A)\ge P(B)$, then $P(A)=$ _________, $P(B)=$_________.

4) A team consists of 5 boys and 5 girls. Now a small working group of 4 people are to be selected at random from the team. The probability that there are more girls than boys in the working group is ____________________.

5) Let X be a random variable whose c.d.f. is F. The probability $P(1\le X\le 2)$ expressed in F is _________________________. If $F(0)=F(10)=0.3$, then $P(X>4)=$________________.

6) Let X_1, X_2 be independent discrete random variables that both have the uniform distribution on integers 1,2,3. Let $Y=X_1-X_2$, then $E(Y)=$______________, $\mathrm{Var}(Y)=$_______________.

2. **(10 points)** Assume that the probability that any child born will be a girl is 1/2 and that all births are independent. A family has 3 children, determine the conditional probability that this family has at least one boy on the condition that this family has at least one girl.

3. **(10 points)** A random variable X has a continuous distribution with the following p.d.f.

$$f(x) = \begin{cases} \dfrac{c}{1+x^2}, & \text{for } x > 0 \\ 0, & \text{otherwise} \end{cases}$$

   (a) Determine the value of c.

   (b) Let $Y = \ln(X)$. Determine the p.d.f. of Y.

4. **(18 points)** Suppose that random variables X and Y have the following joint p.d.f.:

$$f(x, y) = \begin{cases} 3y, & \text{for } 0 < x < y < 1 \\ 0, & \text{otherwise} \end{cases}$$

   (a) Evaluate $P(X + Y < 1)$.

   (b) Find $E(X)$ and $E(Y)$.

   (c) Find $\mathrm{Cov}(X, Y)$.

5. **(10 points)** Let X and Y be random variables such that Var(X)=Var(Y)=1, and $\rho(X, Y) = r$. Let $U = aX + bY$ and $V = aX - bY$. Evaluate $\rho(U, V)$.

6. **(10 points)** Suppose that the joint p.d.f of two random variables X and Y is as follows:

$$f(x,y)=\begin{cases} c\sin(x), & \text{for } 0 \le x \le \frac{\pi}{2}, 0 \le y \le 1 \\ 0, & \text{otherwise} \end{cases}$$

Determine the conditional p.d.f. of Y for every given value of X.

7. **(9 points)** Let X be a discrete random variable having the following p.f. :

$$f(x)=\begin{cases} p(1-p)^x, & \text{for } x = 0,1,2,\ldots \\ 0, & \text{otherwise} \end{cases}$$

(a) Find $P(X \ge k)$ for each integer $k = 0,1,2,\ldots$

(b) Find $P(X \ge t+k \mid X \ge t)$ for each integer $t,k = 0,1,2,\ldots$

8. **(15 points)** Let X and Y be **independent** random variables that both have the uniform distribution on the interval $[0,1]$. Let $Z = \max(X^2, Y^2)$ and $W = \min(X^2, Y^2)$.

(a) Show that Z also has the uniform distribution on the interval $[0,1]$.

(b) Find the p.d.f. of W.

Lancaster University College
at Beijing Jiaotong University

2019/20 Examinations

Course code: WB73L004Q

Course name: Probability Theory and Mathematical Statistics (B)

Midterm examination (November)

INSTRUCTIONS TO STUDENTS

1) Duration of the exam: 120 minutes
2) This paper contains 3 pages. There are 8 questions.
3) You must answer all questions.
4) This is a closed book exam. No books or notes may be brought into the exam room.
5) A scientific calculator is allowed in the examination. Other electronic devices are not allowed in the exam room.
6) Some values of CDF of the standard Normal distribution:

$$\Phi(1) = 0.8413, \qquad \Phi(1.645) = 0.95,$$

$$\Phi(1.96) = 0.975, \qquad \Phi(2) = 0.9772, \qquad \Phi(3) = 0.9987 \ .$$

1. (20pt)

Use the random events A, B, C to express the events in (1) and (2).

(1) The event that A occurs but neither B nor C occurs is

___________________________________.

(2) The event that at least one of A, B, C occurs is

______________________________________.

(3) An urn contains ten balls numbered from 1 to 10. If three balls are taken, the probability that the maximum number of them is 5 is __________________.

(4) A and B are two random events. $P(A) = 0.4, P(B) = 0.3, P(A \cup B) = 0.6$, then $P(A\overline{B}) =$_________________.

(5) A day's production of 850 manufactured parts contains 50 parts that do not meet customer requirements. Two parts are selected randomly without replacement from the batch. The probability that the second part is defective given that the first part is defective is _______________.

(6) If the random variable $X \sim N(1,2), Y = 2X + 3$, then $Y \sim$________________.

(7) If random variables $X \sim N(1,2), Y \sim N(3,4)$, X and Y are independent, then $2X + 3Y \sim$________________________.

(8) Suppose that X follows the Poisson distribution, and $P(X = 1) = P(X = 2)$. Then, $P(X = 3) =$_________________.

(9) The CDF of random variable X is $F(x) = \frac{1}{1+2^{-x}}$. Then $P(0 < X < 1) =$

__________________.

(10) Two people independently decode a message. The probabilities that they succeed are $\frac{1}{5}, \frac{1}{4}$, respectively. The probability that they can decode the message is ______

______ ______.

2. (10pt) Let X be a continuous random variable with the following PDF:

$$f(x) = \begin{cases} 2x, & 0 \leq x \leq 1, \\ 0, & otherwise. \end{cases}$$

Find the PDF of $Y = \sqrt{1 - x^2}$.

3. (10pt) The height of a population of woman has the normal distribution with mean 64 inches and standard deviation 2 inches.

(1) What is the probability that a randomly selected woman from this population is between 58 inches and 70 inches?

(2) Determine the height that defeats (i.e. is taller than) 95% of the population.

(3) What is the probability that five women selected independently at random from this population are all taller than 68 inches?

4. (10pt) An inspector working for a manufacturing company has a 0.99 chance of identifying a defective item if it is defective, and a 0.005 chance of incorrectly classifying a good item as defective. We have known that 0.01 of the items produced by the company are defective.

(1) What is the probability that an item selected for inspection is classified as defective?

(2) If an item selected at random is classified as good, what is the probability that it is indeed good?

5. (12pt) Two cards are drawn from a deck of 52 cards. Let X and Y be the number of kings and queens drawn (so that $X + Y \leq 2$), respectively.

(1) Determine the joint PF of X and Y (in table form).

(2) Are X and Y independent?

6. (12pt) The joint PDF of X and Y is given by

$$f(x,y) = \begin{cases} kx^2y, & x^2 \leq y \leq 1, \\ 0 \quad , & otherwise. \end{cases}$$

Find (1) The constant k. (2) $P(X \geq Y)$.

7. (14pt) Suppose that X and Y have the following joint PDF:

$$f(x,y) = \begin{cases} 2(x+y), & 0 < x < y < 1, \\ 0 \quad , & otherwise. \end{cases}$$

Determine (1) the marginal PDF of X;

(2) the conditional PDF of Y given that $X = x$.

8. (12pt) The joint PDF of X and Y is given by

$$f(x,y) = \begin{cases} \frac{1}{2}(x+y)e^{-(x+y)}, & x \geq 0, y \geq 0, \\ 0, & otherwise. \end{cases}$$

Find the PDF of $Z = X + Y$.

Lancaster University College at Beijing Jiaotong University

2020/21 Examinations

Course code: WB73L004Q

Course name: Probability Theory and Mathematical Statistics (B)

Midterm examination (November)

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4) This is a closed book exam. No books or notes may be brought into the exam room.
5) A scientific calculator is allowed in the examination. Other electronic devices are not allowed in the exam room.
6) Some values that might be useful:

$$\Phi(0.2) = 0.57926, \qquad \Phi(1) = 0.8413,$$

$$\Phi(1.4142) = 0.92135, \qquad \Phi(2.8284) = 0.99766$$

1. **(20pt)**

(1) Two events A and B are independent, and $P(A) = 0.4$, $P(B) = 0.7$, then $P(A\overline{B}) =$__________________.

(2) A box contains 3 left shoes and 3 right shoes. If two shoes are randomly chosen from the box, then the probability that they are a pair (i.e. a left shoe and a right shoe) is __________________.

(3) Let the CDF of random variable X be $F(\mathrm{x}) = \frac{1}{1+2^{-x}}$. Then $P(X < 1|X > 0) =$__________________.

(4) Let the PDF of random variable X be $f(\mathrm{x})$. $Y = 2X$. Then the PDF of Y is $g(\mathrm{y}) =$____________________.

(5) Suppose that $X \sim \mathrm{N}(1, 5)$. $Y = 3X - 2$. Then $Y \sim$____________________.

(6) Suppose that X has the uniform distribution on the interval $[2, 5]$. Then $P(3 < X \le 4) =$__________________.

(7) A neighborhood, which has a gymnasium in it, has 1000 residents. Every morning each resident independently has 0.05 chance to go to the gymnasium. Then the number of gymnasium goers every morning approximately has the _______________ distribution with parameter $\lambda =$_____________.

(8) Let the joint CDF of X and Y be $F(\mathrm{x}, \mathrm{y}) = \frac{1}{(1+e^{-x})(1+3^{-y})}$. Then the CDF of X is __________________ and the PDF of X is _______________________.

2. **(10pt)** A company produces products at three different factories A, B, and C. Of the company's total volume, factory A produces 20%, factory B produces 50%, and factory C produces the rest. The product defective rates at the factories are 5% at factory A, 2% at factory B, and 10% at factory C. If you buy this product and it turns out to be defective, what is the probability that it was produced at factory A?

3. **(10pt)** An airline sells 200 tickets for a certain flight on an airplane that has only 198 seats because, on the average, 1% of purchasers of airline tickets do not appear for the departure of their flight. Determine the probability that everyone who appears for the departure of this flight will have a seat.

4. **(12pt)** In a population, the men's weight (in kilograms) has the distribution N(60, 25) and the women's weight (in kilograms) has the distribution N(50, 25). Randomly and independently pick a man and a woman from the population.
   (1) What is the probability that the two persons' total weight exceeds 130 kilograms?
   (2) What is the probability that the difference between the two persons' weights is smaller than 10 kilograms?

5. **(14pt)** The joint PDF of X and Y is given by

$$f(x,y) = \begin{cases} 1, & 0 < x < 1, 0 < y < 2x, \\ 0, & \text{otherwise.} \end{cases}$$

   (1) Find the marginal density $f_X(x)$ and the conditional PDF $f_{Y|X}(y|x)$.
   (2) Evaluate $P(Y < \frac{1}{2} | X < \frac{1}{2})$.

6. **(10pt)** Suppose that a fair coin is tossed repeatedly until two consecutive heads or two consecutive tails appear. Let X be the number of tosses required. Let $Y = 1$ if it is two consecutive heads and $Y = 0$ if it is two consecutive tails. Determine the joint PF of *X* and *Y*.

7. **(12pt)** Let random variables X and W be independent and distributed uniformly on the interval $[0,1]$. Let $Y = -W$.
   (1) What is the distribution of *Y*?
   (2) Are *X* and *Y* independent?
   (3) Find the PDF of $Z = X + Y = X - W$.

8. **(12pt)** Let random variables X and Y be independent and $X \sim \text{Exp}(\lambda_1)$ and $Y \sim \text{Exp}(\lambda_2)$.
   (1) Give the CDF of *X* and the CDF of *Y*.
   (2) Let $Z = \min(X, Y)$. Find the CDF and PDF of Z.

Lancaster University College at Beijing Jiaotong University

2021/22 Examinations

Course code: WB73L004Q

Course name: Probability Theory and Mathematical Statistics (B)

Midterm examination (November)

INSTRUCTIONS TO STUDENTS

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2) This paper contains 3 pages. There are 8 questions.
3) You must answer all questions.
4) This is a closed book exam. No books or notes may be brought into the exam room.
5) A scientific calculator is allowed in the examination. Other electronic devices are not allowed in the exam room.
6) Some values that might be useful:

$$\Phi(0.2) = 0.57926, \qquad \Phi(1) = 0.8413,$$

$$\Phi(1.4142) = 0.92135, \qquad \Phi(2) = 0.97725$$

$$\Phi(2.8284) = 0.99766$$

1. (20pt)

(1) If events A and B are disjoint, C is the complement of A, then the relationship between B and C is ________________.

(2) Let the joint cumulative distribution function of random variables X and Y be $F(x, y)$. Then $P(1 < X \leq 2,\ Y \leq 3) =$____________________.

(3) Let random variables $X,\ Y$, and Z be independent. Which of the following sets of random variables are guaranteed independent? Your answer: __________

a) $X^2 - Z,\ Y,\ Z$

b) $\cos(X),\ e^{Y+Z}$

c) $Z^2 + Y^2,\ \sin(X + Z)$

d) $X + 3Y,\ Y - 2Z$

(4) Of the following three functions: CDF, PF, and PDF, the function that is applicable to all random variables is __________ and the function that **NOT** necessarily takes value between 0 and 1 is __________.

(5) The PDF of random variable X is $f(x) = \begin{cases} ax^{-2}, & if\ x > 1 \\ 0, & otherwise. \end{cases}$ Then $a =$____________.

(6) A box contains 6 red balls and 4 white balls. Three balls are chosen without replacement, then the probability that there is exactly one red ball is __________.

(7) If $X \sim B(49, 0.2)$, then its variance is ______________.

(8) If $X \sim N(1, 4)$, then $2X + 3 \sim$ ___________________.

(9) Let $\mathrm{E}(X) = 1$ and $\mathrm{Var}(X) = 2$. Then $\mathrm{E}[X(X - 1)] =$ ___________________.

2. **(10pt)** There are five children in a family. In this family, each child, independent from each other, has 1/4 chance to have blue eyes. If it is known that at least one of the children has blue eyes, then what is the probability that all the five children have blue eyes?

3. **(12pt)** Random variable X has the following PDF

$$f(x) = \begin{cases} 3x^2, & if\ 0 < x < 1, \\ 0, & \text{otherwise.} \end{cases}$$

Let $Y = \mathrm{e}^X$. Find the PDF $g(y)$ of Y.

4. **(14pt)** Random variables X and Y have the following joint PDF

$$f(x,y) = \begin{cases} 6xy, & if\ 0 < x < 1\ and\ 0 < y < \sqrt{x} \\ 0, & \text{otherwise.} \end{cases}$$

(1) Find $P(X \leq Y)$.
(2) Find marginal density $f_X(x)$.
(3) Find marginal density $f_Y(y)$.
(4) Check if X and Y are independent.

5. **(12pt)** Let $X \sim U[-3,5]$, and let

$$Y = \begin{cases} 2, & if\ X > 0, \\ -2, & \text{otherwise.} \end{cases}$$

(1) Is Y continuous or discrete? If Y is continuous, give its PDF; and if it is discrete, give its PF.
(2) Find $E(Y)$ and $Var(Y)$.

6. **(10pt)** Tom and Jerry agree to meet up in a station. They arrive at the station uniformly between 9:00 AM and 10:00 AM. Suppose that after arriving, each waits 20 minutes for the other person before leaving. What is the probability that they will meet?

7. **(10pt)** The length in cm (X) of some bolts is normally distributed with a mean of 10.05 and a standard deviation of 0.06. A bolt is qualified if its length is between 10.05 ± 0.12 cm. Find the probability that a bolt is defective.

8. **(12pt)** Random variables X and Y have the following joint PDF

$$f(x,y) = \begin{cases} 1, & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0, & \text{otherwise.} \end{cases}$$

Find the PDF of $Z = X + Y$.

Probability and Statistics
Model Answer of 2022 Midterm Exam
BJTU Lancaster University College

Yiping Cheng

November 12, 2022

1. (20pt) Fill in the blanks:

(1) Events A and B are disjoint and independent, $P(A) = 0.5$, then $P(A) + P(B) =$ ___.

(2) Out of the students in a class, 50% are geniuses, 70% love chocolate. Let p be the conditional probability that a randomly selected student is a chocolate lover given that he/she is a genius. Then the theoretically lowest value of p is ___

(3) $2n$ students are assigned randomly to 2 classes each having n students. A couple of boyfriend and girlfriend are in the $2n$ students. The probability that this couple are in the same class is ____. (or ______)

(4) When it rains, there is a 40% chance that students play football on that day. On the other hand, when it does not rain there is a 90% chance that they play. The probability that it will rain tomorrow is 0.2. The probability that students play football tomorrow is ___

(5) Let $X \sim B(20, 0.8)$, then the most likely value of X is ___

(6) The random variable X has PDF $f(x) = Ce^{-3|x|}$. Then $C =$ _ and $P(|X| < 1) =$ ______

(7) Let $X \sim U(0,1)$, $Y \sim U(0,1)$, and X and Y be independent. Then $P(X = Y) = \underline{\mathbf{0}}$ and $P((X - 1/2)^2 + (Y - 1/2)^2 < 1/4) =$ _.

(8) The joint CDF of X and Y is $F(x, y) = \begin{cases} \Phi(x)(1 - e^{-y}), & \text{for } y > 0 \\ 0, & \text{otherwise.} \end{cases}$ Then the joint PDF of X and Y is $f(x, y) =$

____

2. (10pt) Approximately 0.04% of human have liver cancer. A person with liver cancer has a 95% chance of a positive test, while a person without liver cancer has a 2% chance of a false positive result. What is the probability a person has liver cancer given that the person just had a positive test?

3. (10pt) Suppose that random variable X and X itself are independent. Show that there must exist a constant C such that $P(X = C) = 1$. (Hint: consider its CDF and use the properties of CDFs)

4. (10pt) A fair coin is tossed two times independently. Let

$$H_i = \begin{cases} 1, & \textbf{if head is obtained at the } i\textbf{th toss,} \\ 0, & \textbf{otherwise.} \end{cases}$$

Let $X = H_1 + H_2$, $Y = |H_1 - H_2|$. Give the joint PMF of X and Y in table form.

5. (10pt) A book has 300 pages. For each page, whether it contains typing error has Bernoulli distribution $B(1, 0.02)$. The events for each page to contain typing error are independent. What is (approximately) the probability that this book contains at most 3 pages with typing error?

6. (12pt) Scores on a certain standardized test, IQ scores, are approximately normally distributed with mean $\mu = 100$ and standard deviation $\sigma = 15$. Here we are referring to the distribution of scores over a very large population, and we approximate that discrete cumulative distribution function by a normal continuous cumulative distribution function.
(a) An individual is selected at random. What is the probability that his score X satisfies $120 < X < 130$?
(b) Two individuals are selected at random, independently. What is the probability that their scores X and Y satisfy $\max(X, Y) > 120$?

7. (14pt) Suppose that the joint distribution of X and Y is uniform over the region in the xy-plane bounded by the four lines $x = -1$, $x = 1$, $y = x + 1$, and $y = x - 1$. Determine
(a) $P(XY > 0)$
(b) the joint PDF of X and Y
(c) the PDF of X
(d) the conditional PDF of Y given that $X = x$
(e) $P(Y > 0|X = 1/2)$.

8. (14pt) Let X and Y be random variables for which the joint PDF is as follows:

$$f(x,y) = \begin{cases} 8xy & \textbf{for } 0 < x < y < 1, \\ 0 & \textbf{otherwise.} \end{cases}$$

Find the PDF of $Z = (X - Y)^2$.