QUIZ-3

1. Suppose that an ordinary deck of 52 cards (which contains 4 aces) is randomly divided into 4 hands of 13 cards each. We are interested in determining p, the probability that each hand has an ace. Let E_i be the event that the *i*th hand has exactly one ace. Determine $p = \mathrm{P}(E_1E_2E_3E_4)$ by using the multiplication rule.

2. A fire station is to be located along a road of unit length—stretching from point 0 outward to 1. If a fire occurs at a point uniformly distributed in [0,1], where should the fire station be located so as to minimize the expected absolute distance between the fire and the fire station? That is, we want to minimize $\mathrm{E}[|X - a|]$, where X is uniformly distributed in [0,1], and a is the location of the fire station.

3. If X_1 and X_2 are independent exponential random variables with respective parameters λ_1 and λ_2. Compute $\mathrm{P}(X_1 < X_2)$.

4. The joint density function of X and Y is

$$\mathrm{f}(x,y) = \begin{cases} xy, & 0 < x < 1, 0 < y < 2 \\ 0, & \text{otherwise.} \end{cases}$$

Find $\mathrm{P}(X + Y < 1)$.

5. Suppose that X and Y are independent geometric random variables with the same parameter p. (a) Compute $\mathrm{E}(X)$ (b) Compute $\mathrm{P}(X = i|X + Y = n)$.