ENGG5402 Advanced Robotics
Homework 3:Trajectory Generation

Spring 2018
Due Date: March 15, 2018

1. (25%) **Point-to-point motion planning** This question is a replication of the simulations in Sec 4.2.1 in Sicilion's textbook (Figures 4.1 and 4.3) or Sec 7.3 in Craig's textbook (Examples 7.1 and 7.3) to help you understand how we can use cubic polynominal and blended polynominal for point-to-point interpolation.

   (a) If we want to move a single joint robot from $\theta = 15°$ to $75°$ in 3 secs. Create a simulation to generate a trajectory using the cubic polynominal that allows the single joint robot to start from a rest position, accomplish the motion, and finally end at rest at the goal position. Plot the position, velocity, acceleration of the joint as a function of time.

   (b) This time, we would like to move the robot with a longer travel range at a faster pace. Using the simulation developed in (a), move the robot from $\theta = 10°$ to $100°$ in 1 secs. Plot the position, velocity, acceleration of the joint as a function of time. In order to compare the difference between (a) and (b), please also plot the results of (a) in the same figures.

   (c) Repeat (a) with the linear function with parabolic blends approach, Plot the position, velocity, acceleration of the joint as a function of time. As usual, plot the results of (a) in the same figures for comparison. Also use two different accelerations setting to generate similar results as shown in the Example 7.3 of Craig's textbook.

   (d) Repeat (a) with the linear function with parabolic blends approach. Assume you are given a motor from Maxon, inc (EC-powermax 30, ϕ30mm, brushless, 200 Watt). You can download the specification for this motor from the same item (The model number should be 305014). We also install a gearhead with the motor and with a reduction ratio of $R = 10$. If we want to reach the max. velocity of the motor in the linear region, (i) verify if the motor has the capability to reach the required acceleration suggested by the interpolation method.You will need to compare the results obtained by the interpolation method against the max. acceleration computed based on the inertial effect of the motor and gearhead. If not possible, which specification of the desired trajectory (either the final angle or total travel time) do you want to adjust to satisfy the max. acceleration requirement of the motor? If you do so, please also specify how much adjustment do you need for the variable that you proposed?

2. (25%) **Linear function with parabolic blends for a path with via points** This question is a replication of the simulations in Sec 4.2.2 in Sicilion's textbook (Figure 4.9) or Sec 7.3 in Craig's textbook (Example 7.4) to help you understand how we can create a path moton with a sequence of points by interpolating the linear function with parabolic blends.

If we want to create a path motion for a single joint robot in which the path points are: 0, 2π, $\pi/2$, π radians. The duration of these three segments should be 2, 1, 2 seconds. The magnitude of the default acceleration to use are 15, 40, 30, and 5 rad/s^2, respectively. Create a simulation to generate a trajectory interpolating these points with the linear function with parabolic blends approach. Plot the position, velocity, acceleration of the joint as a function of time.

3. (25%)**Individual Joint Control:Optimal Transmission/Impedance Matching** Consider a common d-c motor ball screw system in Fig. 3. A motor with inertia or inertia mass J is connected to a standard ball screw which converts the rotary motion of the motor into linear motion of the load (in this context, the load is the moving mass M). The motor behaves as a torque source which can apply torque T in addition to the motor inertia. The transmission ratio, r is defined as the ratio of the transmission output velocity to input velocity, which is opposite to the definition of the gear reduction ratio. For a ball screw system, r is the ratio of output translational velocity (m/s) to input angular velocity (rad/s).

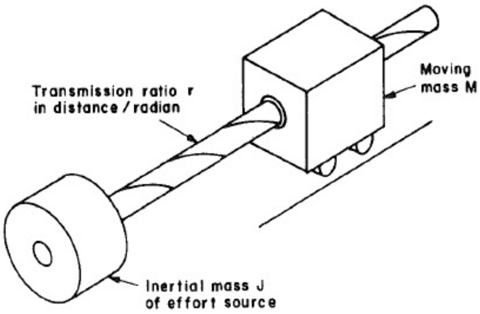

Figure 1: Standard ball screw transmission system

(a) If x and θ are the linear velocity of the mass m and angular velocity of the motor, derive the equation of motion for the ball screw system, given the parameters listed above. The transmission friction is negligible. Formulate the equation into the following linear form:

$$T = f(r, J, M)a$$

, where a is the acceleration of the linear mass ($\ddot{x}$).

(b) If we have a chance to pick a ball screw, i.e. we can adjust the transmission ratio r , what is the optimal transmission ratio that we should use in order to maximize the acceleration a of the linear mass? And with the optimal transmission, what is the maximum acceleration that we can transmit to the linear mass M? Given the optimal transmission ratio and maximum motor angular velocity ω_{max}, what is the maximum output linear velocity? (Hints: consider the partial differentiation of the $f(r, J, M)$ with respect to r.)

(c) In practice, it is hard to buy a ballscrew with a transmission ratio exactly matching to the optimal one. We need to investigate the performance degradation (or Sensitivity of the Transmission) when choosing a ballscrew deviated from the optimal version. To do that, we need to normalize the transmission ratio r and acceleration a by the optimal transmission ratio r_{opt} and optimal acceleration a_{opt}, respectively. Please write a Matlab script to plot the normalized transmission ratio r^* against the normalized acceleration a^*, where $r^* = \frac{r}{r_{opt}}$ and $a^* = \frac{a}{a_{opt}}$. In the plot, the range for the normalized transmission ratio r^* should be from 0 to 5. In the plot, you should see that a^* is at maximum when $r^* = 1$.

(d) In real life, it is normal to have the payload M different from what we expected during the transmission design process. The question is how it would affect the normalized acceleration a^*. Please plot the normalized transmission ratio r^* against the normalized acceleration with $M_{new} = kM$, where $k = 1.25, 1.5, 1.75$. From the plot, if we want to design a more "robust" solution, do you want to choose a transmission ratio larger than or smaller than the optimal one? Provide corresponding explanations to support your answer.

(e) **Determining optimal transmission ratio for velocity limited systems** Consider of using the trapezoidal velocity profile for the movement of the ball screw system (Fig. 2). This trajectory type allows the motor to apply constant torque (or acceleration) but limits the top speed ω_{max} of the system. As the top motor speed ω_{max} is bounded, the previous results on the optimal transmission ratio selection will not apply in this situation. A new optimal transmission ratio needs to be derived based on the travel time equaton of the motor.

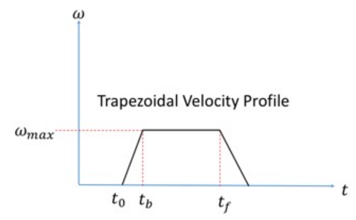

Figure 2: Trapezoidal Velocity Profile

i. Derive the travel time (t_l) of the motor to reach the peak velocity as follows:

$$t_l = \frac{\omega_{max}(J + Mr^2)}{T} + \frac{d}{r\omega_{max}}$$

where d is the total move distance after reaching the peak velocity. Then, re-derive the optimal transmission ratio based on the travel time t_l. Given the new optimal transmission ratio, derive the total distance travel d_m, which is the area under the Trapezoidal Velocity Profile.

4. (25%)**Individual Joint Control** Consider the following nonlinear system:

$$\tau = 5\theta\dot{\theta} + 2\ddot{\theta} - 13\dot{\theta}^3 + 5$$

(a) Design a nonlinear controller such that the closed-loop system is always critically damped with $k_{CL} = 10$. If the disturbance torque (t_{dist}) is equal to 0.1 Nm, what will be the steady state error for the closed loop system?

(b) Write a simulation in Matlab to show that the step response of the system is critically damped (Without a disturbance input). Remark: Unless you use simulink, I believe in your matlab script, you may need to create separate functions for the dynamics and controller computation. Feel free to use the built-in ODE45 function to do the numerical integration.

(c) Using the above simulation script, inject a "step" disturbance into the closed-loop system such as

$$\tau = \tau_{cmd} + \tau_d = 5\theta\dot{\theta} + 2\ddot{\theta} - 13\dot{\theta}^3 + 5$$

, where τ_{cmd} and τ_d are the commanded torque and disturbance torque, respectively. The commanded torque τ_{cmd} is created by the proposed controller. Set the disturbance value to 0.1Nm and plot the step response of the closed loop system to verify the behavior of the steady state error.