ENGG5402 Advanced Robotics
Homework 1: Spatial Descriptions, Forward Kinematics, Differential Kinematics

Spring 2018
Due Date: Feb 2, 2018

1. Consider the robot shown in Fig. 1 and Fig. 2, where the system has six degrees of freedom as $q = [q_1\ q_2\ q_3\ q_4\ q_5\ q_6]^T$.

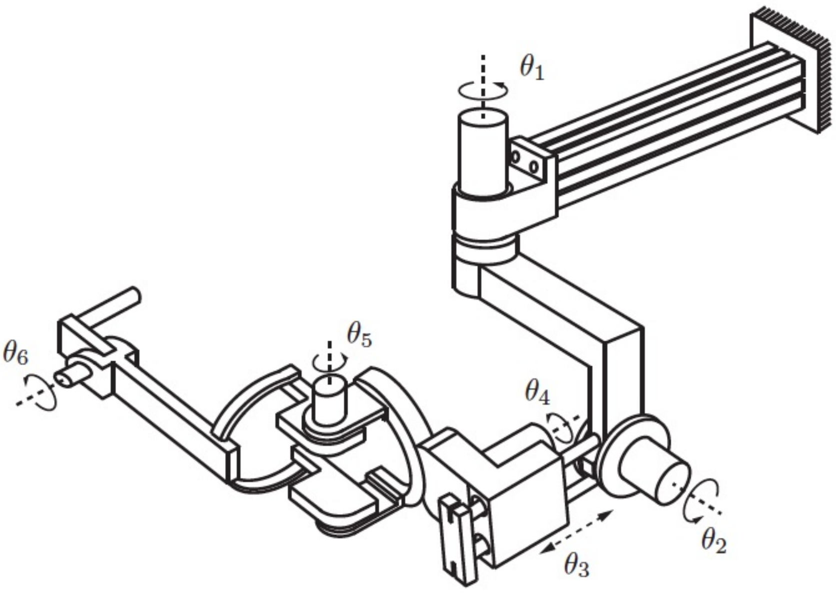


Figure 1: Rehabilitation Robot ARMinIII from ETH Zurich

(a) Determine the DH parameters ($a_{i-1}, \alpha_{i-1}, d_i$, and θ_i, for $i = 1, 2, ...$) for each frame transformation. In other words, create a DH table for this robot. (Please use the DH convention derived from Craig's Textbook)

(b) Determine the transformation matrix ($^{i-1}T_i$) for each frame ($i = 1, 2, ...$) using the DH approach.

(c) Write a Matlab script to compute the forward kinematics using the DH Method. Create some corner cases that can help you to verify the implementation of your Matlab code/script (Provide the explanation in the solutions). Simulate the forward kinematics for 5 seconds with a 1 Hz sinusoid movement for joint 2, q_2 (Amp = 30° degrees, Phase = 0) and a 1 Hz sinusoid movement at joint 3, q_3 (Amp = 0.1 meter, Phase = 0). Other joints can be set as any constant positions throughout the simulation (e.g. $q_1 = 0°$). Provide the plots of the simulation movement for each joint and the corresponding Cartesian tip trajectory. Think about how to represent the orientation trajectory in a figure.

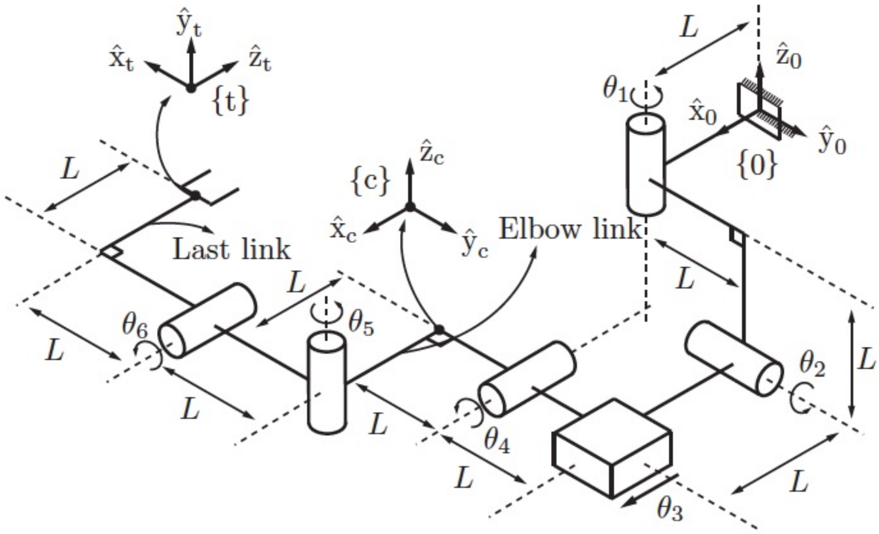


Figure 2: Kinematics model for the rehabilitation robot ARMinIII from ETH Zurich

(d) Denote the tip of the robot as point P. Derive the rotation matrices and linear translation vectors for each local frame up to the tip point P using basic geometric approach.

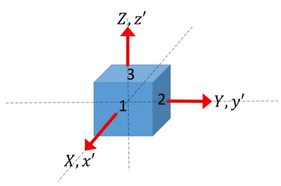
(e) If we denote the operational space as the position of point P in the inertial frame $X = [{}^0X_P \; {}^0Y_P \; {}^0Z_P]^T$, express the forward kinematics $X = f(q)$ with respect to the generalized coordinates q. Using the results from (d) to compute the forward kinematics.

(f) Derive the Jacobian matrix for the manipulator using the geometric approach described in the lecture. The tip velocity of the manipulator contains both the translational and angular velocity components.

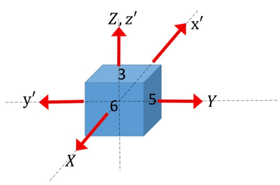
(g) Suppose $q_2 = 90°$ and all the other joint variables are fixed at zero, set the joint velocities to $(\dot{q}_1, \dot{q}_2, \dot{q}_3, \dot{q}_4, \dot{q}_5, \dot{q}_6) = (1, 0, 1, 0, 0, 1)$, find the Cartesian velocity at the manipulator tip.

(h) Is the above configuration a kinematic singularity? Explain your answer.

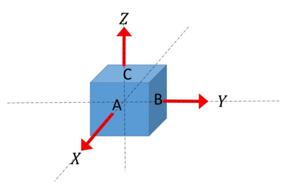
2. **Euler Angle** This problem is designed to build up our intuition about the conventions typically used to specify the orientation of a rigid body.



(a) Case 1



(b) Case 2



(c) Case 3

Figure 3: Frame Descriptions

The cube described in Figure 3 is derived based upon the notation of a standard American die. The number labeled on each face of the dice is arranged such that the sum of the number on the opposite face and itself is equal to 7. For example, 1 is opposite to 6, 2 is opposite to 5, ...etc. To illustrate this fact, let us define an inertial frame (fixed global frame) $oXYZ$ and a body frame $ox'y'z'$ attached onto the dice. Initially, they are both aligned with each other as shown in Figure 3(a). If we rotate the dice about the Z axis of the inertia frame with 180° degrees, you will see the face with number '6' aligning with the X axis of the inertia frame (Figure 3(b)).

If we start with the orientation of the dice as shown in Figure 3(a). Determine the new orientation of the dice (A=?, B=?, C=?) in Figure 3(c) after the following rotations:

(a) Perform an Euler angle rotation in the order of axes z', y', z' with angles -90°,+90°,-90°.

(b) Perform a fixed angle rotation in the order of axes Z, Y, Z with angles -90°,+90°,-90°.

(c) Are these two results identical? If not, can you suggest a set of rotation pattern for the fixed angle rotation that results the same orientation as shown in question 2(a).

3. **Rotational Matrix**

   (a) If a matrix R satisfies $R^T R = I$, then

      i. Show that det $R = \pm 1$

      ii. Show that det $R = +1$ if we restrict ourselves to right-handed coorindate frames

      iii. If R is a rotation matrix, please show that $+1$ is an eigenvalue of R. Let k be a unit eigenvactor corresponding to the eigenvalue $+1$. Give a physical interpretation of k.

   (b) The fixed frame and the body frame share the same origin. Given the rotational matrix $R \in \mathcal{R}^{3\times 3}$,

$$R = \begin{bmatrix} -\frac{\sqrt{3}}{4} & -\frac{1}{2} & \frac{3}{4} \\ \frac{1}{4} & -\frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{4} \\ \frac{\sqrt{3}}{2} & 0 & \frac{1}{2} \end{bmatrix}$$

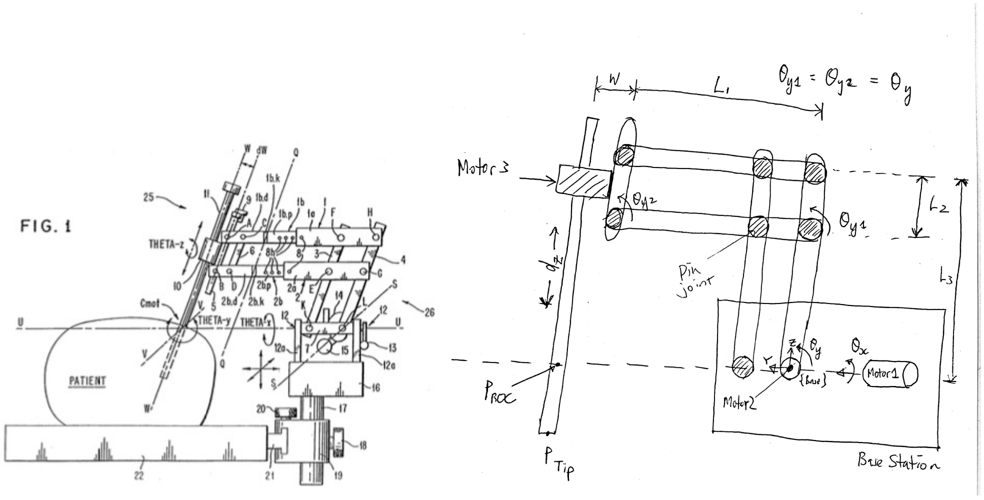
   a rigid body is rotating at the Euler angle rates of $\dot{\alpha} = \dot{\beta} = \dot{\gamma} = 1°/sec$. The Euler angles α, β and γ are defined as $R = Rot(z, \alpha)Rot(x, \beta)Rot(z, \gamma)$ and $\alpha, \beta, \gamma \in [0, \pi]$.

      i. Determine the Euler angles.

      ii. Determine the angular velocity of the rigid body.

4. **Remote Center of Motion Mechanism** This question is designed to build up your intuitions on Remote Center of Motion Mechanism which is a commonly used robotic mechanism in surgical robot design. In this mechanism, three motion axes intersect each other at a common point. The Black Falcon presented in the paper by Madhani et al [1] is one example. Figure 4(a) shows the original schematics of the Black Falcon system presented in the patent (US5397323). A schematics is shown to further illustrate the kinematic principle of the Black Falcon (Figure 4(b)). It is a 3 DoFs manipulator in which motors 1, 2, 3 create the movement of θ_x, θ_y, d_z correspondingly. (In the real robot, all the motors were installed in the base of the manipulator to reduce the overall inertia and the tip motion of the manipulator was created through actuating the cables.) For simplicity, in this cartoon picture, Motors 2 and 3 are intentionally described as collocated with the joints.

   (a) Describe the relationship between the point $P_{ROC} = [x_{ROC}\ y_{ROC}\ z_{ROC}]^T$ and P_{Base}. The center of the frame $Base$ is located at the center of the Motor 2. If any of the motors move, how will it affect the position of the point P_{ROC}?



(a) Original Schematics

(b) Simiplified Schematics

Figure 4: Black Falcon Robot

(b) Derive the forward kinematics between the tip position $P_{Tip} = [x_{Tip}\ y_{Tip}\ z_{Tip}]^T$ and the motion of the joints $\theta_{joint} = [\theta_x\ \theta_y\ d_z]^T$. Other than referencing P_{Tip} to the frame $Base$, please think carefully what other frame that you can use to simplify the result.

(c) Derive the Jacobian matrix for the given operational space with respect to the chosen reference frame.

(d) Determine the singularity conditions and provide the physical significant for the singularity of this particular system.

(e) Given the property of the point P_{ROC}, if we want to use this robot for surgical application such as eye surgery or adbominal surgery (Feel free to find more information online for this kind of surgery), describe how you can take advantage of the property of the point P_{ROC} to provide a safe operation.

(f) Considering Figure 4(a), the robot instrument goes into the body through an entry point. If we would have the robot to accommodate to a patient with wider and thicker body than what's desrcibed in the figure. How would you adjust or re-design the robot (i.e. what system parameter should we change?) to accomodate this situation? Feel free to use pictures to describe your proposed design.

(g) Motor 2 uses the parallelogram linkage to convert the motion at joint 2 (θ_y) into the rotation of the instrument. Instead of using the parallelogram linkage approach, propose a new manipulator design based a standard serial chain approach to achieve the same goal. If you need more than one motor in your new approach, how these motor motions relate to the overall pitch motion (or translation) of the instrument. By comparing your proposed approach and the existing parallelogram approach, which one is better? Why? After doing this exercise, please provide us the main advantage of using parallelogram linkage mechanism in this context?

References

[1] A. J. Madhani, G. Niemeyer, and J. K. Salisbury, Jr. The Black Falcon: a teleoperated surgical instrument for minimally invasive surgery. In Proceedings of the IEEE/RSJ International Conference on Intelligent Robots and Systems, pp. 936-944, 1998.