ENGG5402 Advanced Robotics
Homework 4: Dynamics

Spring 2018
Due Date: March 30, 2018

Some tips for doing the ENGG5402 problem sets:

- Feel free to use abbreviations for trigonometric functions (e.g. $c\theta$ for $\cos(\theta)$ or $s\theta_1$ for $\sin(\theta_1)$)
- If you give a vector as an answer, make sure that you specify what frame it is given in and in what frame it references to. The same applies to the rotational and transformation matrices.

1. **Inertia Tensor**

   (a) Derive a formula that transforms an inertia tensor given in the body frame $\{B\}$ into a frame $\{A\}$. The frame $\{A\}$ can differ from frame $\{B\}$ by both translation and rotation.

   (b) Consider a uniform density box as shown in Fig. 1b. It has mass $= 12\ kg$ and dimensions $8 \times 5 \times 2$. Frame $\{B\}$ is located at the center of the solid box and the coordinate axes are ligned up with the principal axes of the box. Y_B is aligned with the long axis of the box. Compute the inertia tensor of the box in frame.

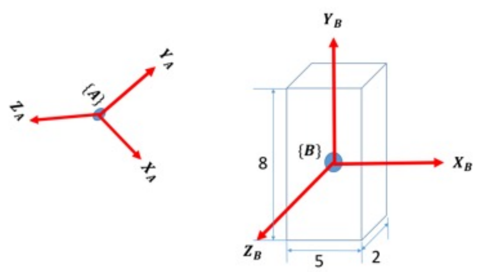

Figure 1: A rectangular solid with uniform mass density and corrdinate frame attached a the center of mass

   (c) Given the transformation from frame $\{B\}$ to $\{A\}$:

$$ {}^{A}T_{B} = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

   Compute the inertia tensor of the box in frame $\{A\}$.

2. Consider a 2R rotational inverted pendulum as shown in Fig. 2. Assuming that the mass of each link is concentrated at the tip and neglecting its thickness, the robot can be modeled as shown in the right-hand figure. Assume that $m_1 = m_2 = 2kg, L_1 = L_2 = 1m$, $g = 10ms^{-2}$, and the link inertias I_1 and I_2 (expressed in their respective link frames $\{b_1\}$ and $\{b_2\}$) are

$$I_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}, I_2 = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

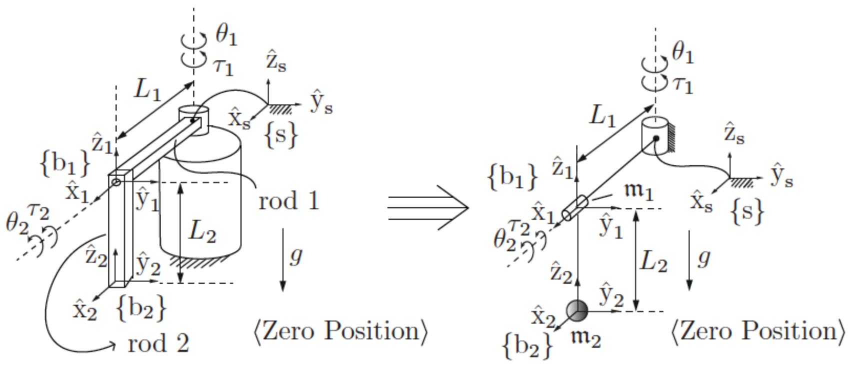

Figure 2: Diagram for a 2R rotational inverted pendulum

(a) Derive the dynamic equations for this robot.

(b) Perform a simulation of the robot for 5 seconds when initially $\theta_2 = 90$ degs and $\theta_1 = 0$ deg and all the torques are set to zero. Create plots for $\theta_i, \dot{\theta}_i$ for $i = 1, 2$ against time for each joint. Remark: please use the subplot function in Matlab to create all the plots for each joint in a column format. During the simulation, we need to integrate $\ddot{q}(t_k)$ to compute $\dot{q}(t_{k+1})$ and $q(t_{k+1})$ at the instant $t_{k+1} = t_k + \delta t$. Please compare the simulation results between the Runge-Kutta (ODE 45) and standard Euler integration approaches (with step size $\delta t = 0.01$ second). If they are different, please provide any corresponding explanations.

(c) Derive the dynamic manipulability ellipsoid of this robot (Review Section 7.9 of the Siciliano's book).

(d) Using the above result, draw the ellipsoid when $\theta_1 = \theta_2 = \pi/2$ and joint velocities are all zero. Indicate how the ellipsoid will change if the gravity is equal to zero.

3. Consider a three link cartesian robot depicted in Fig. 3.

(a) Compute the inertia tensor J_i for each link $i = 1, 2, 3$ assuming that the links are uniform rectangular solids of length=1, width= $\frac{1}{4}$, and height=$\frac{1}{4}$, and mass=$1kg$. (Remark: when submitting the homework, please also submit Fig. 3 in which you may need to label the dimension for each rectangular solid of the manipulator to illustrate your own definition of height, length, and width.)

(b) Compute the 3×3 inertia matrix $D(q)$ for this robot.

(c) Show that the Christoffel symbols c_{ijk} are all zero for this robot. What are the implications and interpretations from the dynamic perspective?

(d) Derive the equations of motion in matrix form: $D(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = u$

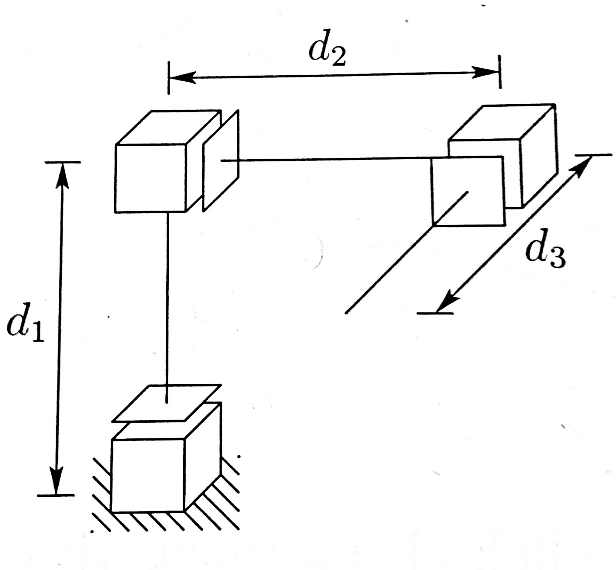

Figure 3: Diagram for a three link cartesian robot

4. Consider a two DoFs system with a revoluate and a linear joint (without gravity) as shown in Fig.4. Define the robot joint variables to be $\{\theta_1, d_2\}$ while the robot operational space variables are defined as $\{x_1, y_1\}$. The length of OB is l_1 with a total mass of m_1. The distance betweent point B and the line of movement of m_2 is small as compared to the length OB. The moment of inertia about an axis through the center of mass of the first link parallel to the z_1-axis is I_1. m_2 can be considered as a point mass. The generalized torques for joints 1 and 2 are defined as τ_1 and τ_2, respectively.

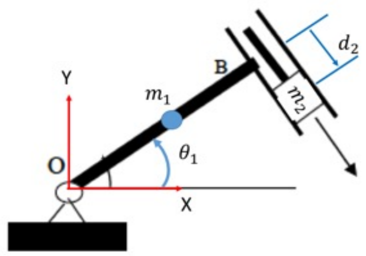

Figure 4: Two degrees of freedom revolute and linear joint robot

(a) Derive the equations of motion for the robot in joint space.

(b) Joint 1 is desired to move in a sinuoidal motion $\theta_1 = \sin(2\pi t)$ while joint 2 is desired to move in accordance with the function $d(t) = -t^2 + 5t$. These motions will also define the velocity and acceleration for each joint. The m_1 and m_2 are set to be 1 kg and 2kg while 1_1 will be set to be 1 meter long. Use Matlab to compute the necessary torque for each joint in order to create the suggested movement.

(c) If only joint 2 will move in accordance with the function $d(t) = -t^2 + 5t$ and all the torques are set to zero, we would like to run a dynamic simulation in Matlab (5 seconds) to see how joint 1 will behave. Create plots for $\theta, \dot{\theta}, \ddot{\theta}, \tau$ against time for each joint. Remark: please use the subplot function in Matlab to create all the plots for each joint in a column format. Use numercal method Runge-Kutta (or ODE 45 or 25 in Matlab) with integration step δt to compute $\dot{q}(t_{k+1})$ and $q(t_{k+1})$ from $\ddot{q}(t_k)$ at the instant $t_{k+1} = t_k + \delta t$.

(d) Express the equations of motion in operational space. Repeat Part (c) and plot the operational space variables against time.

(e) If friction is nonnegligible, it exists as viscous friction with coefficients of $\{b_1, b_2\}$. Let us set the friction as $b_1 = 0.1kgm^2s^{-1}, b_2 = 0.5kgs^{-1}$. Repeat Part (c) with the suggested frictions. Compare the changes in the operation variables against the non friction case.