ENGG5402 Advanced Robotics

Homework 2:Inverse Kinematics, Statics, Singularity, Workspace, and Dexterity

Spring 2018

Due Date: Feb 16, 2018

1. **Manipulability.** Consider the manipulator shown in Fig. 1 . The system has three degrees of freedom where the joint variables are $q = [\theta_1\ \theta_2\ \theta_3]^T$. Assume all the link lengths are equal to 1m.

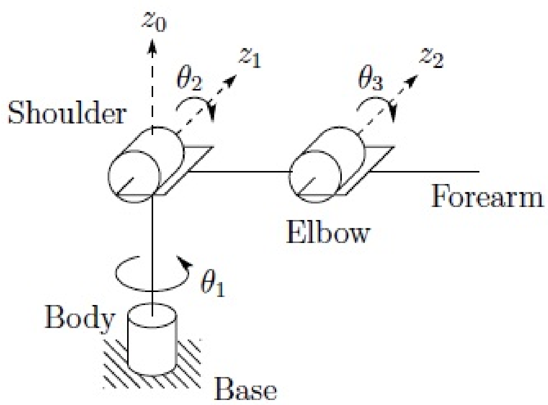

Figure 1: A 3 DoFs Robot

   (a) If the robot is located at $q = [-45^\circ\ 45^\circ\ 0]^T$ degrees (both the joint angles q_2 and q_3 start from the Cartesian X-axis) and the joint velocities are $\dot{q} = [180\ 90\ 0]^T$ degrees per second, what are the corresponding Cartesian endpoint velocity and manipulability? If the robot moves to the location where $q = [-45^\circ\ 0\ 45^\circ]^T$ degrees and $\dot{q} = [180\ 90\ 0]^T$ degrees per second. What will be the corresponding Cartesian end effector velocity and manipulability? If the results are different, provide a physical explanation for the observed phenomina.

   (b) If we want to use the robot for welding service, we will need the end-effector to push on the environment with a Cartesian force vector of $F = [4\ 1\ 3]^T$N. Compute the required joint torque vectors to create the given cartesian force vector for two sets of joint angle conditions listed in problem 1(a). At which joint angle position will require lesser joint torque to provide the same output cartesian force? Why?

   (c) Assume the joint angle q_1 is always constant and set at -45°. Create a Matlab simulation to generate a velocity manipulability ellipse plot as well as force manipulability ellipse plot (along the Cartesian

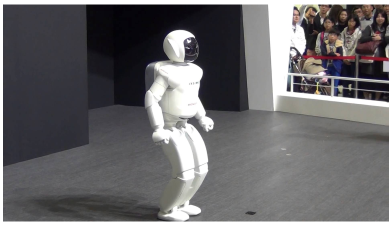

Figure 2: Honda Asimo robot.

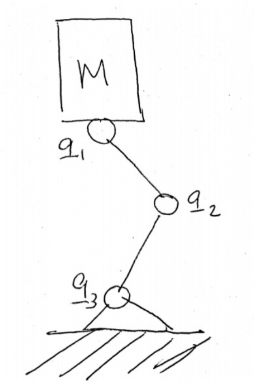

Figure 3: Honda Asimo robot. Conceptually, in the sagital plane, Honda robot can be roughly modeled as a 3 DoFs manipulator with joint angles $q = [q_1 \; q_2 \; q_3]^T$.

X-Z plane) for the robot at different joint postures. (Review the Figures 3.23 and 3.25 of Siciliano's Textbook for reference. Show only five representative joint postures in each plot)

i. Intuitively, for a given input joint torque, at which posture should the robot be to maximize the output cartesian force along the Z-direction? How will this configuration affect the robot's cartesian velocity performance?

ii. If the cartesian velocity of the robot is the user's main concern such as in the transportation or packaging applications, at which nominal position should the robot be to maximize both the cartesian velocities along x-axis and z-axis directions?

iii. Honda Asimo robot (See Figs. 2 and 3) is one of the most prominent walking robots in the world. The robot has a unique pattern of walking in which the knees of the robot remain bent during walking. On the contrary, human walks straight up to improve the efficiency of walking. Using the concept of the velocity and force manipulability of a robot, provide the explanations or motivations for this unique robotic walking pattern.

2. **Inverse Kinematics with Redundancy.**

In this question, we will study the performance of different types of Jacobian-based inverse kinematics algorithms using a simple 3R robot (Fig. 4). The operational space is set as the cartesian tip position $x = [x_p \; y_p]^T$ and the joint angles are $q = [\theta_1 \; \theta_2 \; \theta_3]^T$. In the simulation, assume all the link lengths are set to 1 meter.

(a) **Pseudoinverse** Create a simuation of the Jacobian-based inverse kinematics loop (Fig. 5) of the robot (Fig. 4) in Matlab. Use the simulation to track a circular trajectory in cartesian space defined as $x(t) = [0.25(1-\cos \pi t) \; 0.25(1-\sin \pi t)]^T$. To complete the simulation, you will need to implement

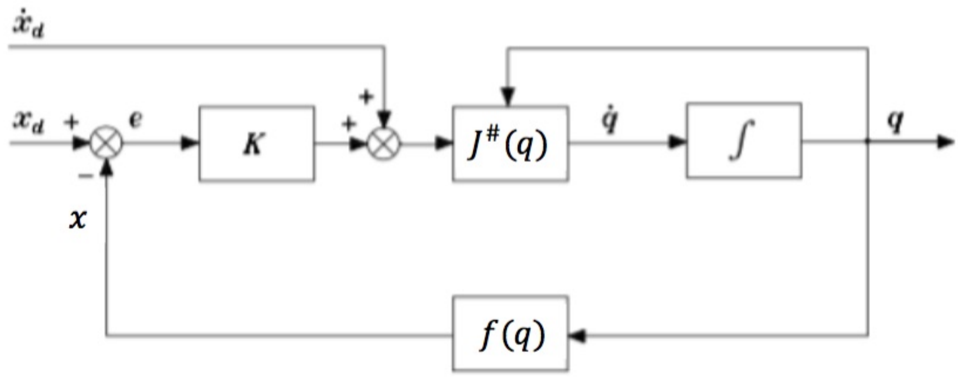

Figure 4: A block diagram of the Jacobian-based inverse kinematics algorithm

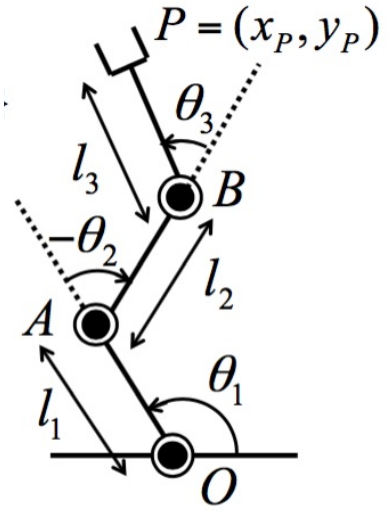

Figure 5: 3 DoFs Planar Robot

the forward kinematics, Jacobian, and pseudo-inverse of the Jacobian in Matlab. In particular, compute the pesudo-inverse of the Jacobian based on the SVD approach, instead of using the built pseudo-inverse function in Matlab. When running the simulation, set $\delta t = 1$ ms and the gain matrix $K = \text{diag}([500,\ 500])$. Also use the simple Euler method to integrate the joint positions from joint velocities such as $q[k+1] = q[k] + \delta q[k]\delta t$. Also decide a reasonable time span for the simulation based on the convergent rate and the period of the circular trajectory.

(b) **Weighted Pseudoinverse** Create a weighted matrix to minimize the velocity of the first joint of the robot. For example, the velocity ratio between the first joint and the second/third is about 0.5. To do that, you need to modify the pseudoinverse function as described in the lecture notes. The modified function can also support the previous simulation by setting the weighted matrix to an identity matrix.

(c) Vary the magnitude of the gain matrix by 50%, 100%, 500% and 1000 %, and check if the inverse kinematics loop still converges. Plot $||x_{error}||$ against t for each condition (run the simulation for 5 seconds). If the simulation does diverge when increasing the gain matrix, please provide an explanation for the obersvation.

(d) **Null Space and Redundancy** For a set of joint velocity vectors $\dot{q}$ that are non-zero and if the set of vectors will cause the tip velocity vector to be zero, i.e.,

$$\dot{x} = J\dot{q} = 0,$$

then we call this set of vectors the null space of the Jacobian. We, here define it as $\dot{\theta}_{Null}$. The following questions aim to help you to explore different ways to obtain the null space of the Jacobian or find this set of vectors $\dot{\theta}_{Null}$ computationally.

i. Compute the numerical value of the Jacobian and its pseudo-inverse when $q = [\pi \ \frac{-\pi}{2} \ \frac{\pi}{2}]^T$.

ii. Compute the tip velocity vector for each of the following input joint velocity conditions: (A) $\dot{q} = [\frac{\pi}{4} \ 0 \ 0]^T$, (B) $\dot{q} = [0 \ \frac{\pi}{4} \ 0]^T$, (C) $\dot{q} = [0 \ 0 \ \frac{\pi}{4}]^T$.

iii. Find the null space (matrix) of the robot using the pseudo-inverse approach. Based on the null space projection approach, compute the corresponding null space vectors ($\dot{q}_{Null}$) for each of the above conditions. Calculate the tip velocity vector for each of these null space vectors. Based on the results, what kind of implications can you draw about the relationship between the null space vectors and the tip velocity vectors?

iv. Find the null space (matrix) of the robot using the SVD approach.

v. Based on the concept of null space projection approach, expand the existing inverse kinematics simulation to support an additional task: Maximize the manipulability as defined in the lecture. Run a simulation with this objective function and compare the results of manipulability against the case of the standard minimum norm (Pseudoinverse) solution. Plot the manipulability ellipsoid at the tip of the manipulator to indicate the variation of the ellipsod along with the trajectory.

vi. Based on the concept of null space projection approach, expand the existing inverse kinematics simulation to support an additional task: ~~Minimze~~ the distance from the mechanical joint limits as defined in the lecture. Run a simulation with this objective function and compare the results against the case of the standard minimum norm (Pseudoinverse) solution. Choose a set of joint limits to create noticeable result. Plot the simulation result of joint angles against time (Please label the joint limit for each joint angle in the same plot as a reference).

3. **Statics and Null Space** Consider the manipulator shown in Fig. 4. The system has three degrees of freedom where the joint variables are $q = [\theta_1 \ \theta_2 \ \theta_3]^T$. Assume all the link lengths are equal to 1m.

(a) The robot is now located at the joint variables $q = [\pi \ \frac{-\pi}{2} \ \frac{\pi}{2}]^T$ and there is an external force applying at the tip of the manipulator F_{tip} forming a static equilibrium. If $F_{tip} = [1 \ 0]^T$, which joint will have zero torque? Please provide the corresponding explanations.

(b) Given the Jacobian matrix developed in question 2, compute the required torque for each joint numerically.

(c) Compute the null space matrix at the current configuration for static condition such that

$$\tau = J^T F_{tip} = 0.$$