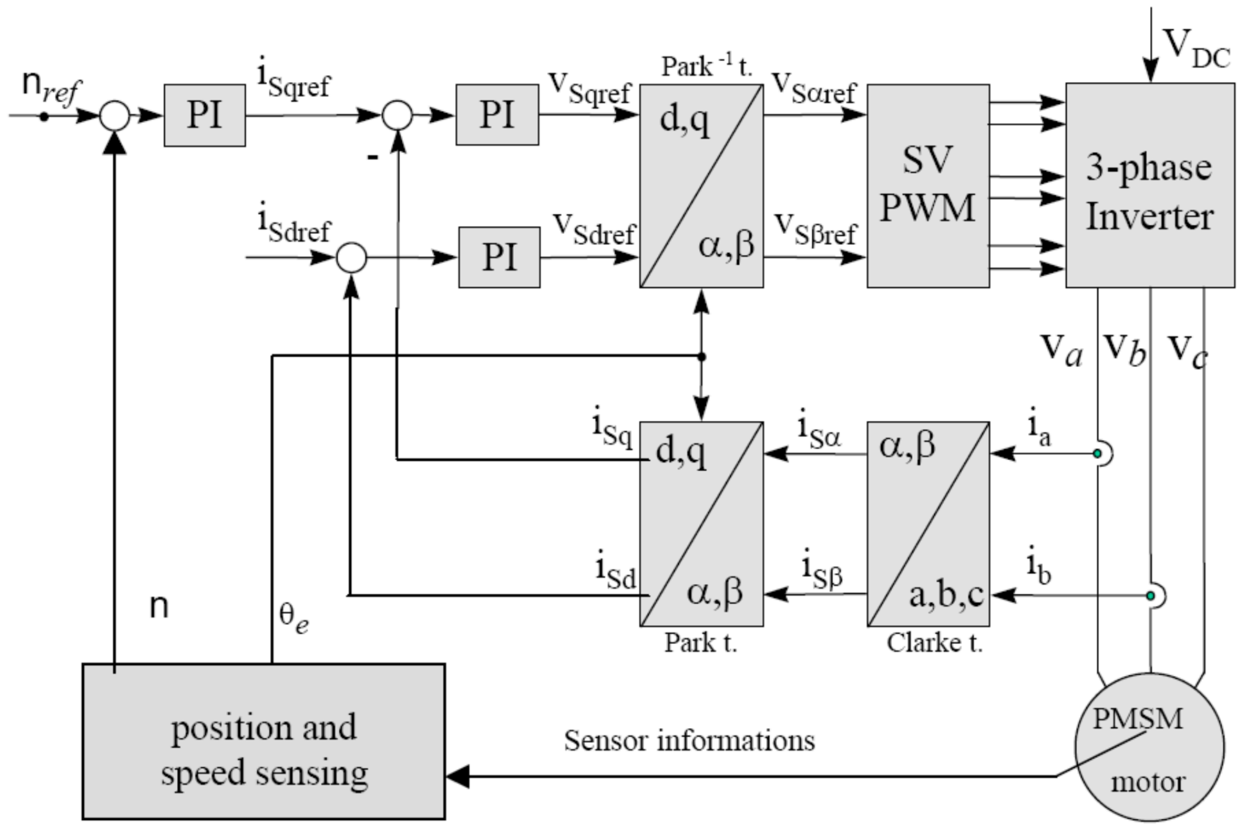
# Permanent Magnet Motor Field Oriented Control

# PMSM FOC | Topology

## PMSM FOC

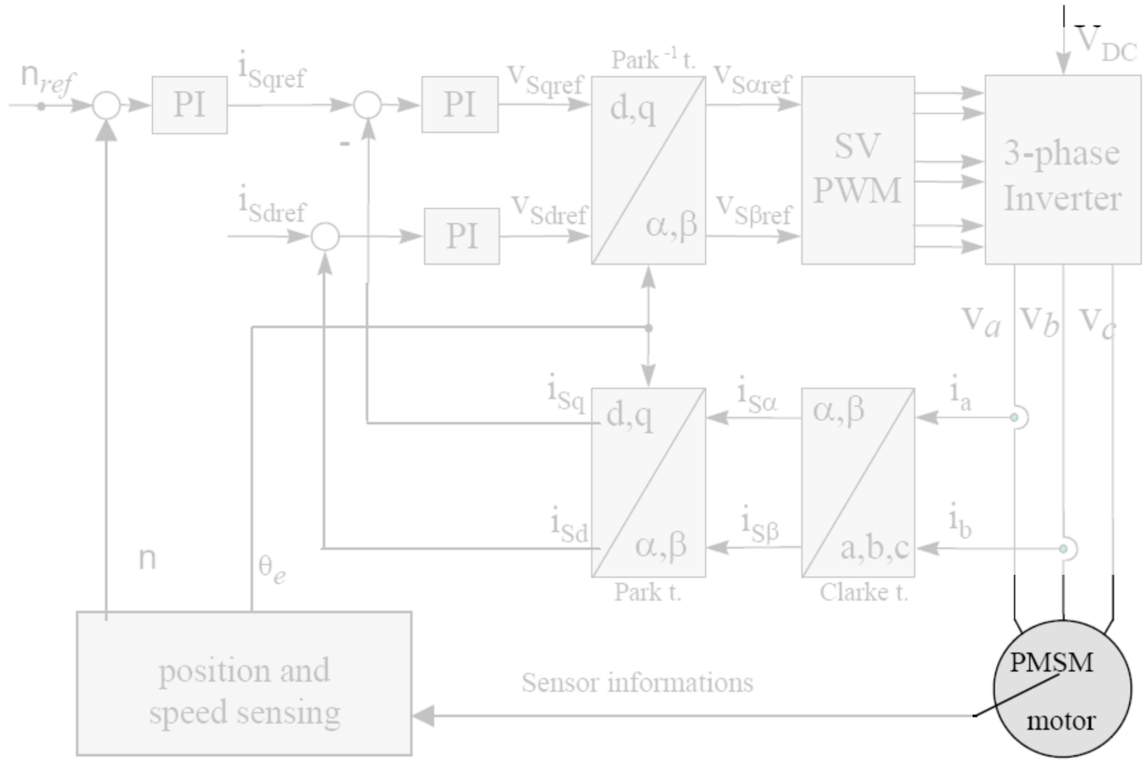
- Architecture
  - Permanent magnet synchronous motor
  - Field oriented control
  - Field weakening
- Feedback Loop
  - Rotor Position
  - Current
  - Speed
- anti-saturation
- anti-windup
- forwardfeed

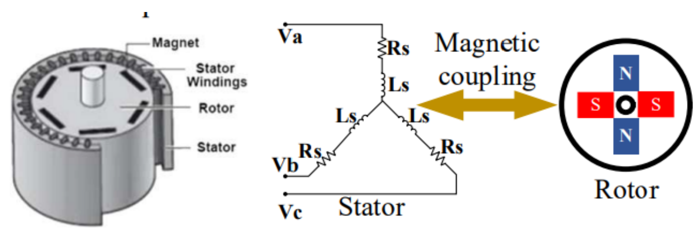


# PMSM FOC | Permanent magnet synchronous motor (PMSM) model

### 3 Phase Motor

- Stator
  - Stator core
  - 3 Phase winding
- Rotor
  - rotor core
  - magnet
  - shaft
  - bearing
- Position sensor
- Housing
- End plate
- Temperature sensor

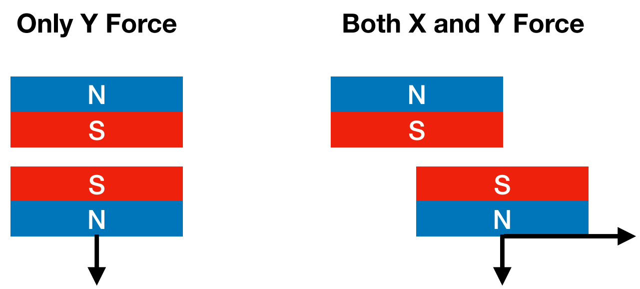


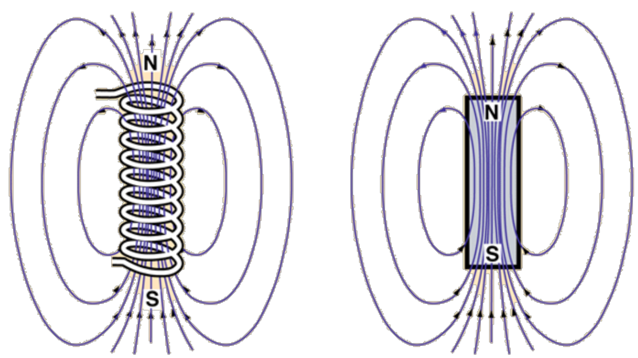
# **PMSM FOC** | Permanent Magnet Motor Fundamental

## Topic

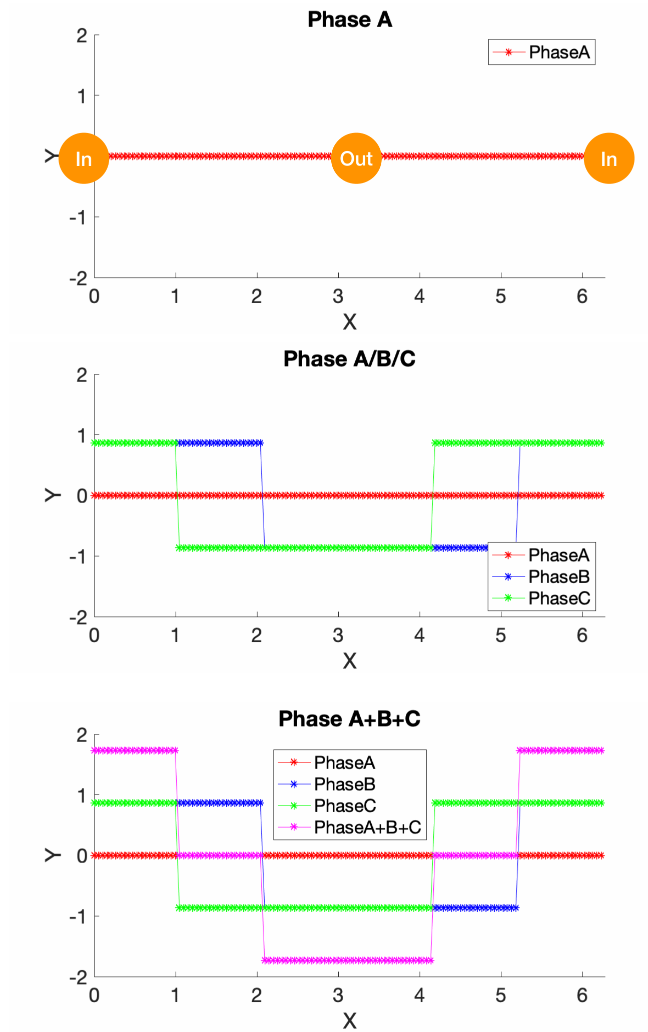
- Magnetic Force / Torque
  - Keep constant distance allows constant force
  - X Force x Radius = Torque
- Magnet field can be generate by both permanent magnet and coil
- Coil magnetic field
  - Single coil just generates oscillation magnetic field
  - 3 phase coil field combination allows traveling magnetic field




**Magnet field**




http://hyperphysics.phy-astr.gsu.edu/hbase/magnetic/elemag.html



# **PMSM FOC** | Permanent magnet synchronous motor (PMSM) model

## Model

- From ABC to DQ
  - The voltage/current/ inductance/pm flux is a function of time in ABC domain
  - DQ domain is constant value
- Control
  - Control Id/Iq to control Torque
  - Control voltage to control Id/Iq

### ABC Voltage/Current

$$v_a = R_s i_a + L_s \frac{d}{dt} i_a - \omega_r \lambda_{pm} \sin(\theta_r)$$

$$v_b = R_s i_b + L_s \frac{d}{dt} i_b - \omega_r \lambda_{pm} \sin(\theta_r - 2\pi/3)$$

$$v_c = R_s i_c + L_s \frac{d}{dt} i_c - \omega_r \lambda_{pm} \sin(\theta_r + 2\pi/3)$$

Park Tranform

### DQ Voltage/Current

$$v_d = R_s i_d - \omega_r \lambda_q + \frac{d}{dt}\lambda_d = R_s i_d + L_d \frac{d}{dt} i_d - \omega_r L_q i_q + \frac{d}{dt}\lambda_{pm}$$

$$v_q = R_s i_q + \omega_r \lambda_d + \frac{d}{dt}\lambda_q = R_s i_q + L_q \frac{d}{dt} i_q + \omega_r L_d i_d + \omega_r \lambda_{pm}$$

### Torque/Rotation Dynamic

$$T_e = \left(\frac{3}{2}\right)\left(\frac{P}{2}\right)\left(\lambda_d i_q - \lambda_q i_d\right)$$

$$= \left(\frac{3}{2}\right)\left(\frac{P}{2}\right)\left(\lambda_{pm} i_q + \left(L_d - L_q\right) i_d i_q\right)$$

$$\omega_m = \int \left(\frac{T_e - T_L - B\omega_m}{J}\right) dt$$

$$\omega_r = \frac{d}{dt}\theta_r = \left(\frac{P}{2}\right)\omega_m \Rightarrow \theta_m = \theta_r \left(\frac{2}{P}\right)$$

**D Axis** **Q Axis**


**Adjust Magnetic Field** **Generate Torque**

Mathematical Model of Permanent Magnet Synchronous Motor, 2021 Information Processing Society of Japan

# PMSM FOC | Park/Inverse Park Transform

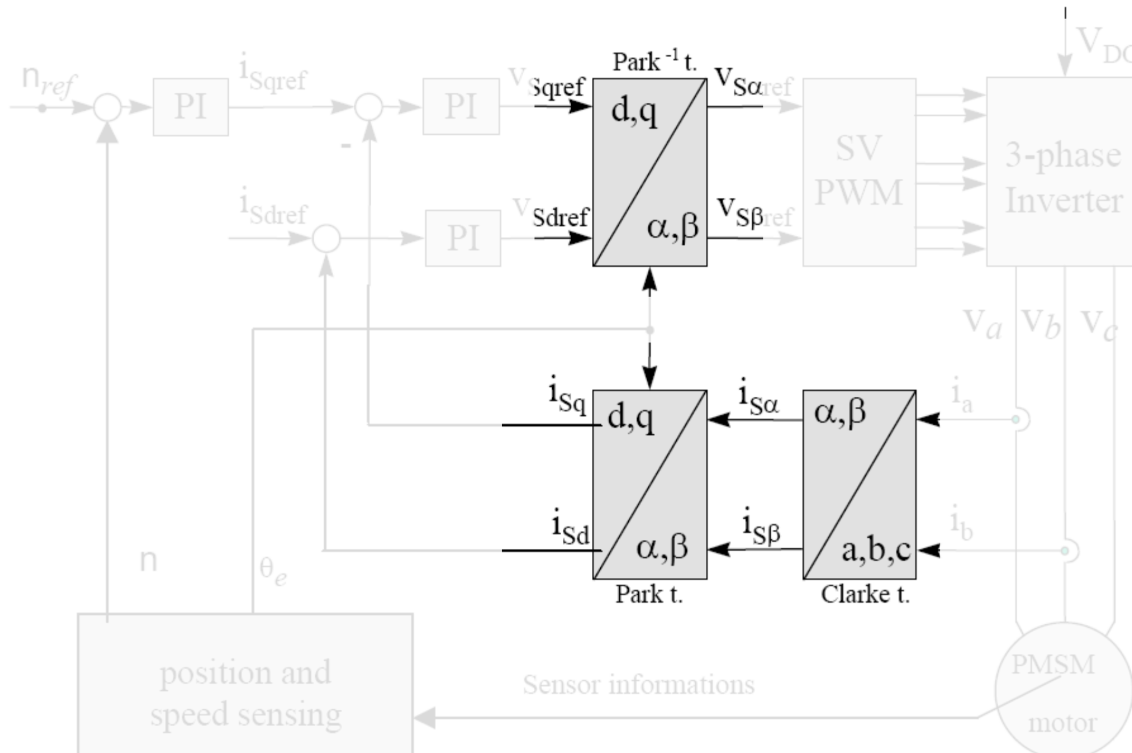
## Topic

- Clarke Tranform
  - space vector modulation

$$i_{\alpha\beta\gamma}(t) = T i_{abc}(t) = \frac{2}{3}\begin{bmatrix}1 & -\frac{1}{2} & -\frac{1}{2}\\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2}\\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2}\end{bmatrix}\begin{bmatrix}i_a(t)\\ i_b(t)\\ i_c(t)\end{bmatrix}$$

- Park Transform
  - Id/Iq PI control control

$$\begin{bmatrix}u_d\\ u_q\\ u_0\end{bmatrix} = \begin{bmatrix}\cos(\omega t) & \sin(\omega t) & 0\\ -\sin(\omega t) & \cos(\omega t) & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}u_a\\ u_\beta\\ u_0\end{bmatrix}$$



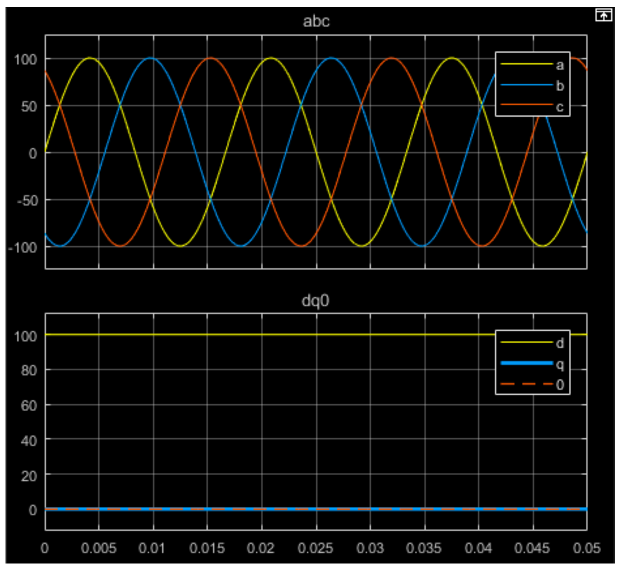
# PMSM FOC | Park/Inverse Park Transform

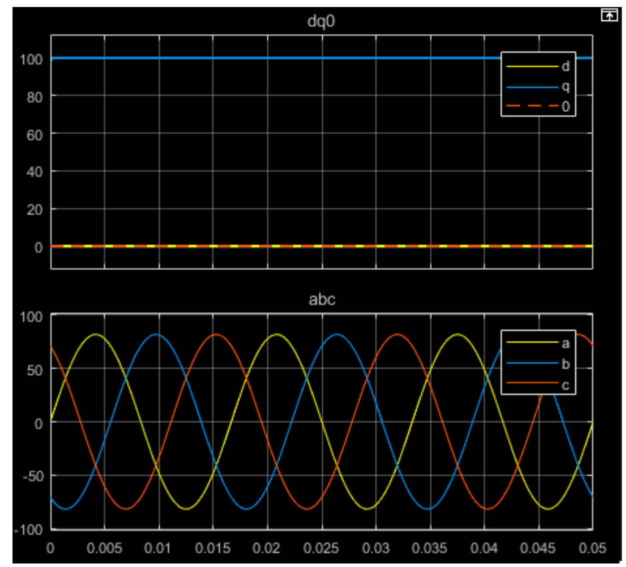
## Topic

- ABC to DQ frame
  - DQ frame allows constant value for
    - Voltage
    - Current
    - PM flux linkage
    - Inductance
- Easy to control

$$\begin{bmatrix} d \\ q \\ 0 \end{bmatrix} = \frac{2}{3} \begin{bmatrix} \sin(\theta) & \sin(\theta - \frac{2\pi}{3}) & \sin(\theta + \frac{2\pi}{3}) \\ \cos(\theta) & \cos(\theta - \frac{2\pi}{3}) & \cos(\theta + \frac{2\pi}{3}) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \sin(\theta) & \cos(\theta) & 1 \\ \sin(\theta - \frac{2\pi}{3}) & \cos(\theta - \frac{2\pi}{3}) & 1 \\ \sin(\theta + \frac{2\pi}{3}) & \cos(\theta + \frac{2\pi}{3}) & 1 \end{bmatrix} \begin{bmatrix} d \\ q \\ 0 \end{bmatrix}$$



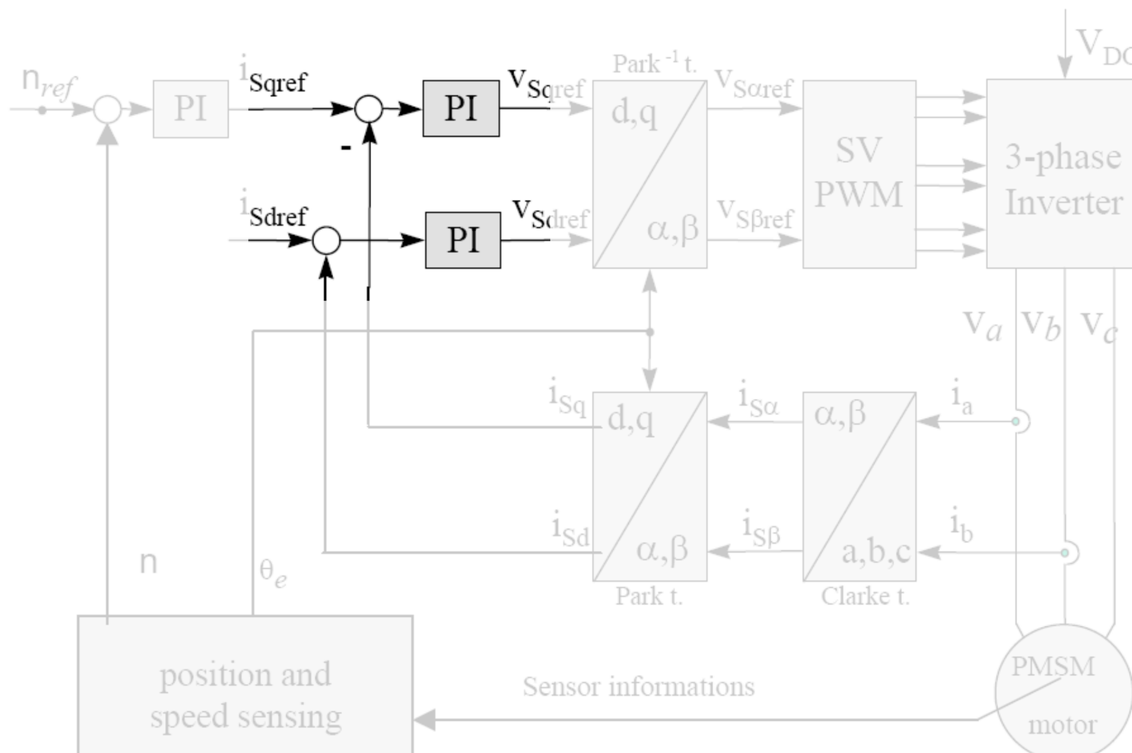




https://www.mathworks.com/help/physmod/sps/ref/parktransform.html

# **PMSM FOC** | Current Controller

### Topic

- Current controller
  - PI control
  - First order
  - avoid current overshoot
- Field weakening
  - Compare Vd/Vq with Vdc
  - Adjust Id for field weakening



# **PMSM FOC** | Current Controller

### Topic

- Current controller
  - PI control
  - First order transfer function, avoid overshoot current
- Cutoff frequency needs to be able to filter out switching noise in the current. 1/10~1/20 of switching frequency is popular
- Udq can be calculated from Idq

**DQ Voltage/Current**

$$U_d = R_s I_d + pL_d I_d - \omega L_q I_q$$

$$U_q = R_s I_q + pL_q I_q + \omega L_d I_d + \omega \Psi_m$$

**PI Control**

$$\left(I_q^* - I_q\right)\left(K_{piq} + \frac{K_{iiq}}{s}\right) + \omega L_d I_d + \omega \Psi_m = U_q^* \approx U_q = R_s I_q + pL_q I_q + \omega L_d I_d + \omega \Psi_m$$

$$\left(I_d^* - I_d\right)\left(K_{pid} + \frac{K_{iid}}{s}\right) - \omega L_q I_q = U_d^* \approx U_d = R_s I_d + pL_d I_d - \omega L_q I_q$$

**Transfer Function**

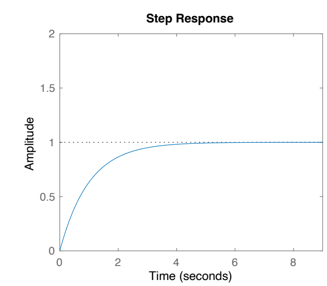
$$\frac{I_q}{I_q^*} = \frac{\frac{K_{piq}}{L_q}\left(s + \frac{K_{iiq}}{K_{piq}}\right)}{s^2 + \left(\frac{R_s + K_{piq}}{L_q}\right)s + \frac{K_{iiq}}{L_q}} \qquad \frac{I_d}{I_d^*} = \frac{\frac{K_{pid}}{L_d}\left(s + \frac{K_{iid}}{K_{pid}}\right)}{s^2 + \left(\frac{R_s + K_{pid}}{L_d}\right)s + \frac{K_{iid}}{L_q}}$$

**Pole Replacement Zero cancelling**

$$K_{piq} = L_q \omega_c$$
$$K_{pid} = L_d \omega_c$$
$$K_{iid} = R_s \omega_c$$
$$K_{iiq} = R_s \omega_c$$

$$\frac{I_d}{I_d^*} = \frac{\omega_c}{s + \omega_c}$$

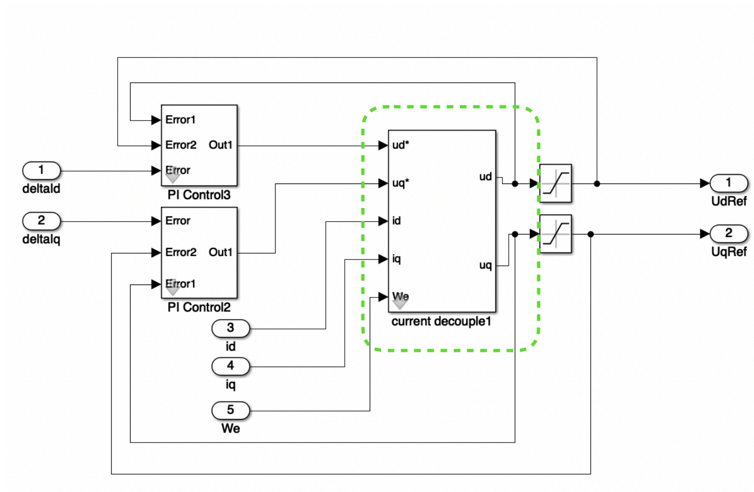
$$\frac{I_q}{I_q^*} = \frac{\omega_c}{s + \omega_c}$$

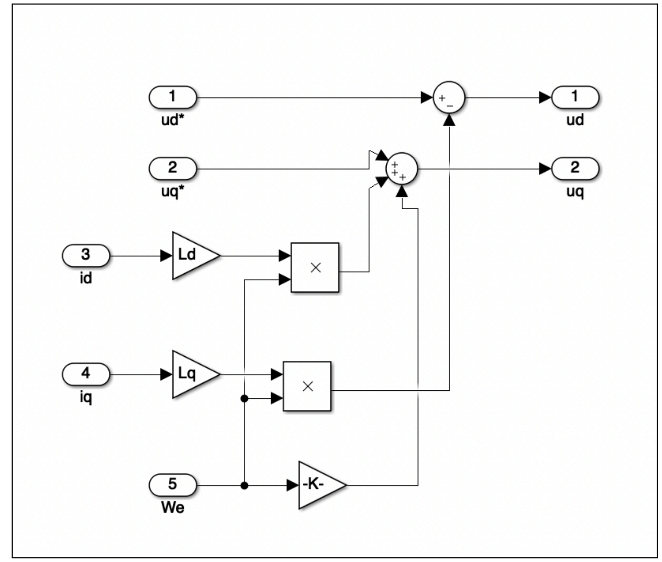


# PMSM FOC | Current Control and Decoupling

## Topic

- PI controller
- Anti-windup
- Voltage Limitation
- Feedforward and current decoupling




$$Ud = Ud' - \omega_e L_q I_q \left(Ud' = RsI_d\right)$$

$$Uq = Uq' + \omega_e L_d I_d + \omega_e \psi \left(Uq' = RsI_q\right)$$

# **PMSM FOC** | Current Reference Generator

## MTPA

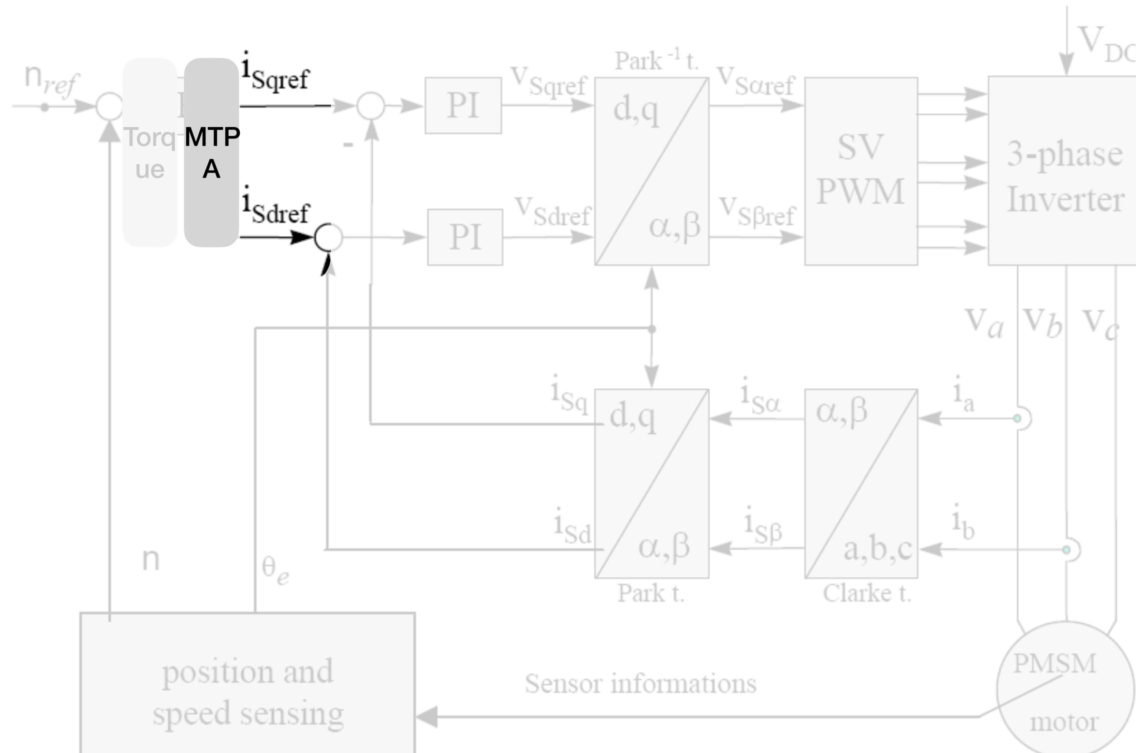
- Maximum Torque Per Ampe (MTPA)
  - Get $\beta$ from $\psi$,Ld,Lq and current
  - Then calculate id and iq

$$\begin{aligned} T &= \frac{3P}{4}[\psi_m i_q^e - (L_q - L_d) i_d^e i_q^e] \\ &= \frac{3P}{4}[\psi_m I_s \cos\beta + \frac{1}{2}(L_q - L_d) I_s^2 \sin 2\beta] . \end{aligned}$$

$$\frac{\partial T}{\partial \beta} = \frac{3P}{4}[-\psi_m I \sin\beta + (L_q - L_d) I^2 \cos 2\beta] = 0 .$$

$$\beta = \sin^{-1}\left[\frac{-\psi_m + \sqrt{\psi_m^2 + 8(L_q - L_d)^2 I^2}}{4(L_q - L_d) I}\right] .$$

$$i_d^e = \frac{1}{4(L_q - L_d)}\left(\psi_m - \sqrt{\psi_m^2 + 8 I^2 (L_q - L_d)^2}\right) .$$



# PMSM FOC | MTPA Control

## MTPA

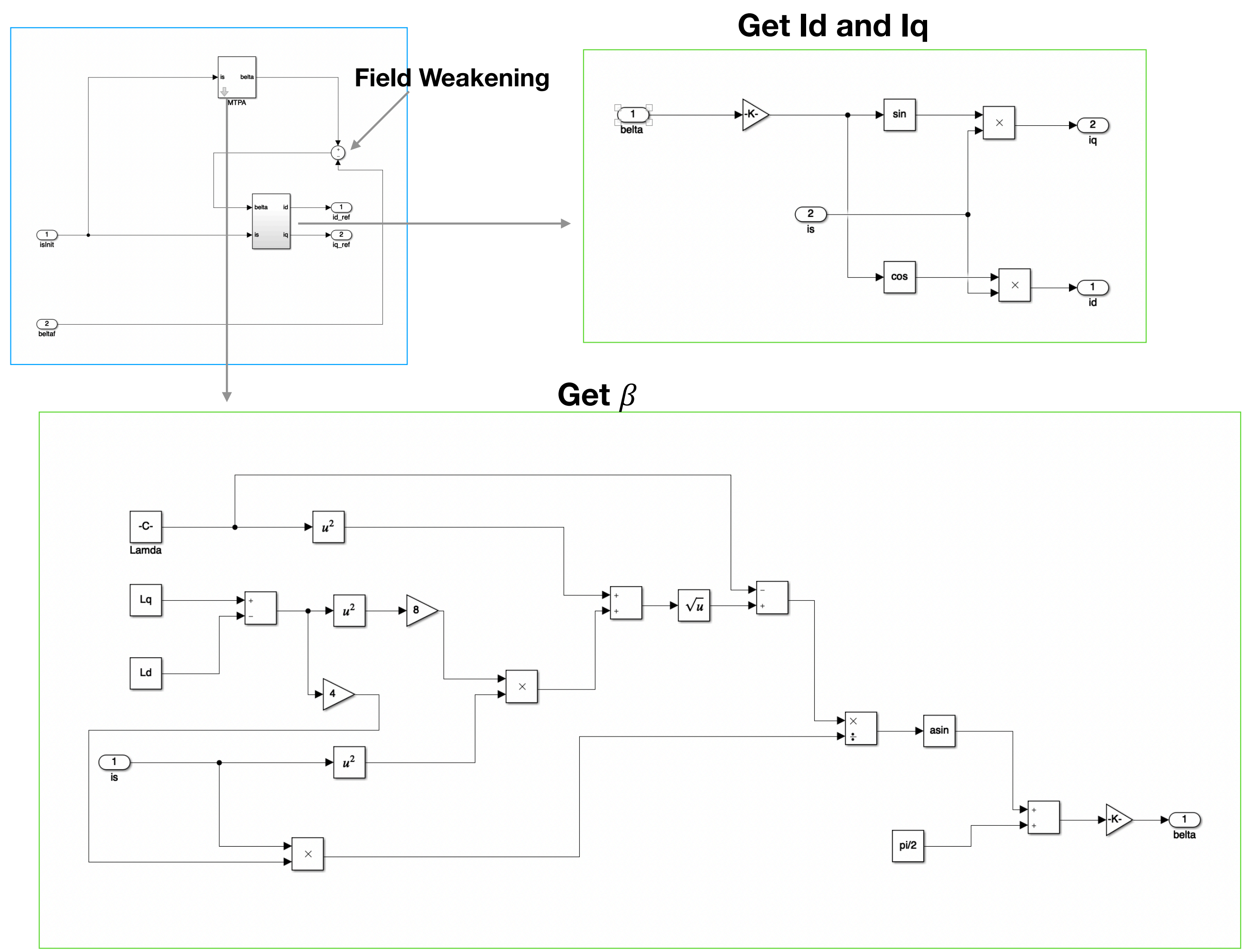
- Maximum Torque Per Ampe (MTPA)
  - Get $\beta$ from $\psi$,Ld,Lq and current
  - Then calculate id and iq

$$
\begin{aligned}
T &= \frac{3P}{4}[\psi_m i_q^e - (L_q - L_d) i_d^e i_q^e] \\
&= \frac{3P}{4}[\psi_m I_s \cos\beta + \frac{1}{2}(L_q - L_d) I_s^2 \sin 2\beta] .
\end{aligned}
$$

$$\frac{\partial T}{\partial \beta} = \frac{3P}{4}[-\psi_m I \sin\beta + (L_q - L_d) I^2 \cos 2\beta] = 0 .$$

$$\beta = \sin^{-1}\left[\frac{-\psi_m + \sqrt{\psi_m^2 + 8(L_q - L_d)^2 I^2}}{4(L_q - L_d) I}\right] .$$

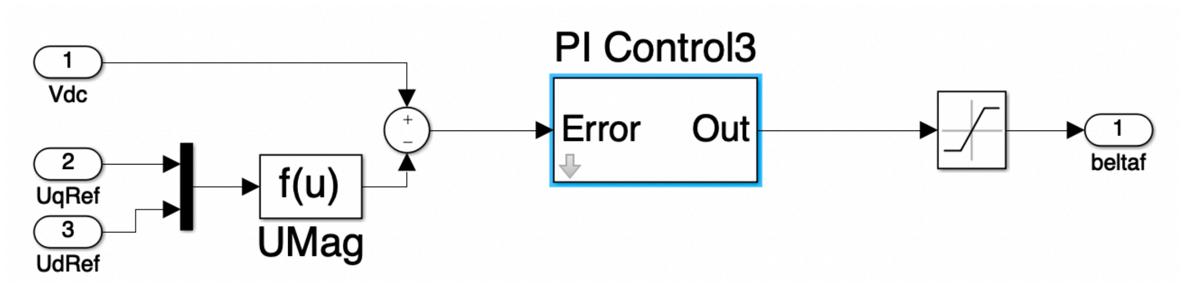
$$i_d^e = \frac{1}{4(L_q - L_d)}\left(\psi_m - \sqrt{\psi_m^2 + 8I^2(L_q - L_d)^2}\right) .$$

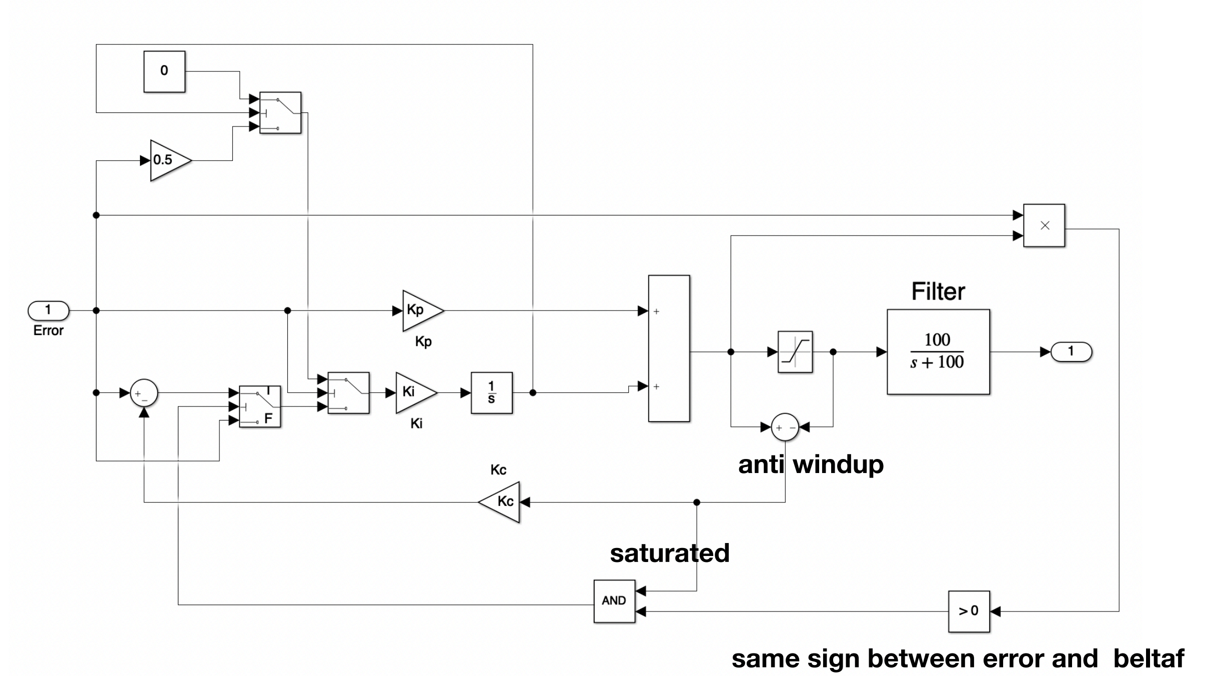


# PMSM FOC | Field Weakening

## Topic

- Boundary
  - Ud^2+Uq^2<=(Udc/1.7)^2
  - Id^2 + Iq^2<=Is^2
- Compare Ud^2+Uq^2 and (Udc/1.7)^2 to control beltaf to increase id for field weakening
- Anti windup

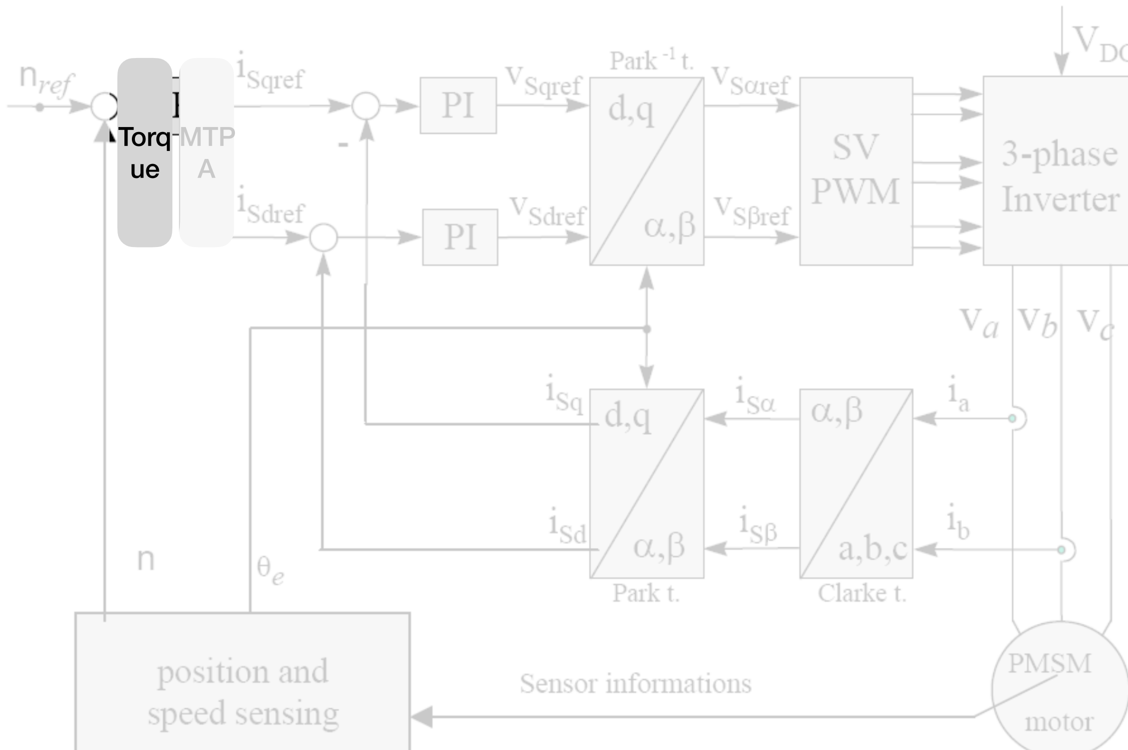


# PMSM FOC | Speed (Torque) Controller

## Topic

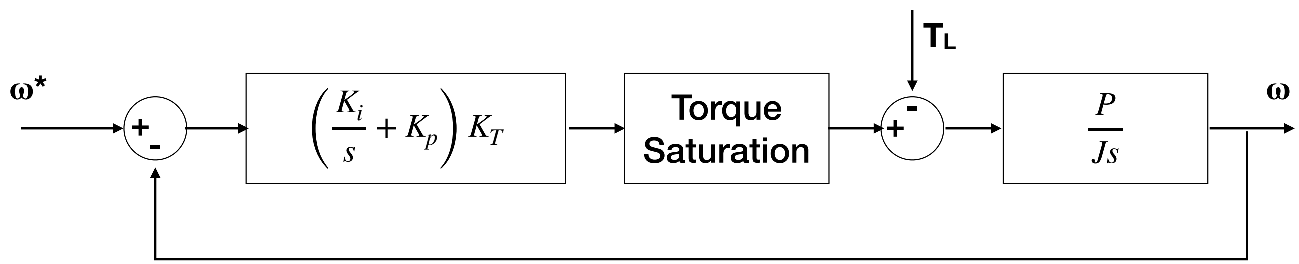
- Speed controller
  - Based on application
  - Over dumped
  - Critical dumped
  - Under dumped



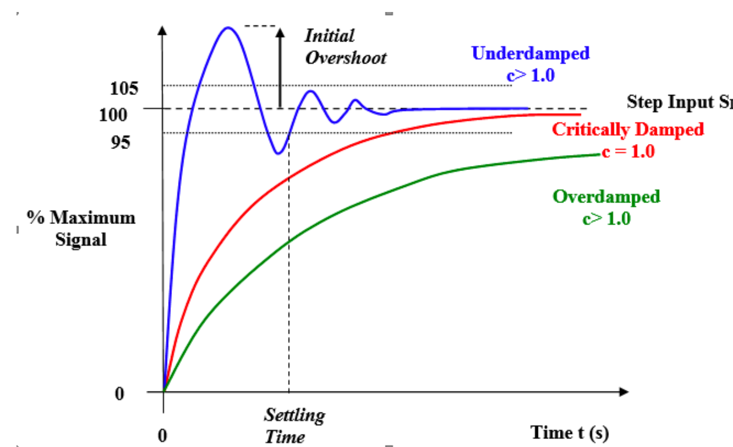
# **PMSM FOC** | Speed Controller

## Topic

- Parameter
  - $T_L$: Load Torque
  - J: Inertia
  - P: pole Pair
  - $\omega$: speed rad/s
  - $K_T$: Torque Constant
- Consideration
  - Torque saturation avoid over current
  - Natural frequency shall be much small than cutoff frequency




$$\frac{\omega}{\omega^*} = \frac{\left(\frac{P}{J}K_pK_T\right)s + \frac{PK_TK_i}{J}}{s^2 + \left(\frac{P}{J}K_pK_T\right)s + \frac{PK_TK_i}{J}}$$

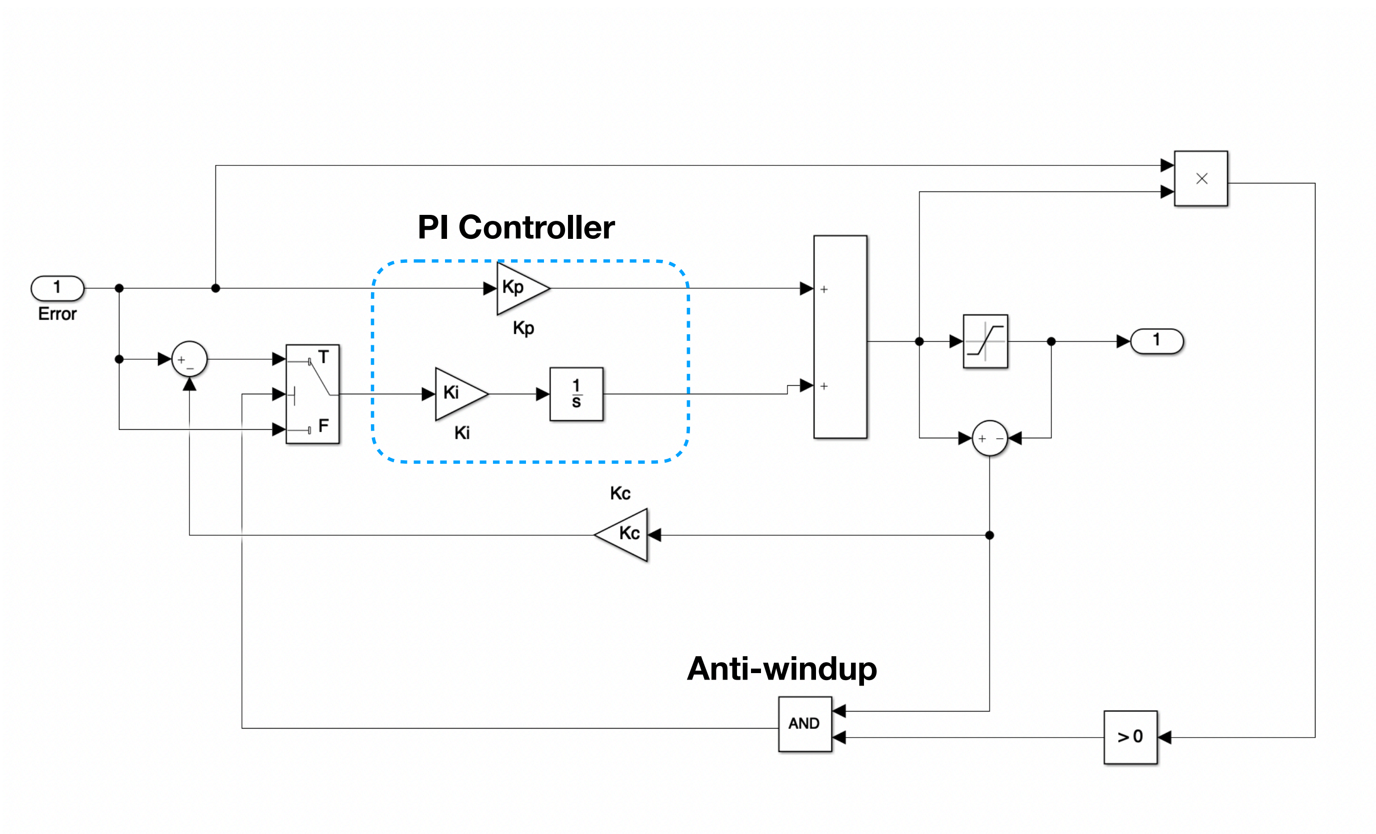



https://www.eng-tips.com/viewthread.cfm?qid=434790

# **PMSM FOC** | Speed Controller

## Topic

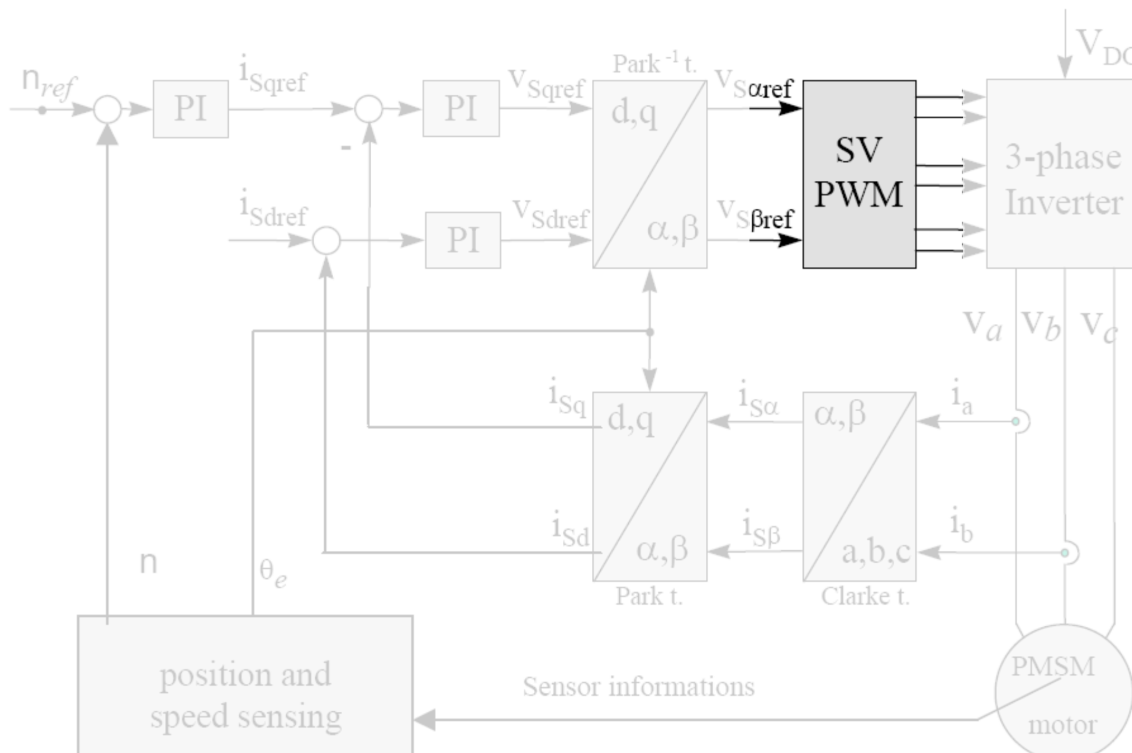
- standard PI controller + anti-windup



# PMSM FOC | PWM Generator

### Topic

- Speed controller
  - Based on application
  - Over dumped
  - Critical dumped
  - Under dumped

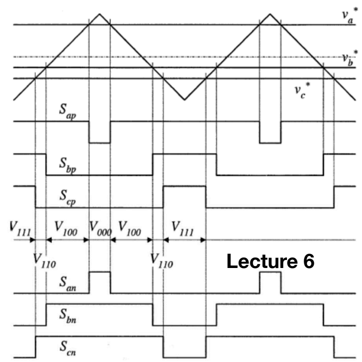


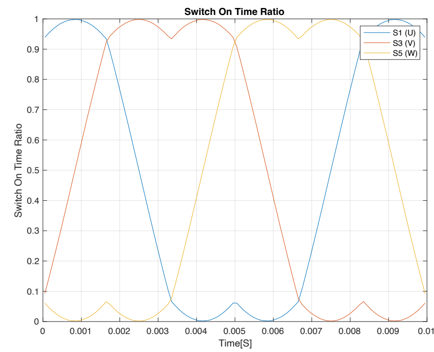
# PMSM FOC | PWM Generator

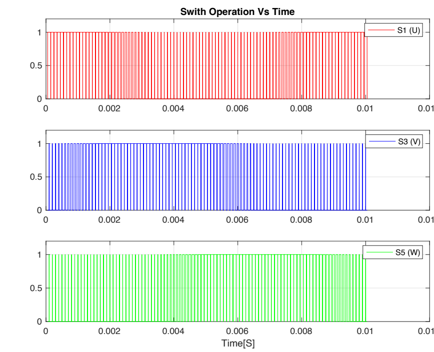
## Demo

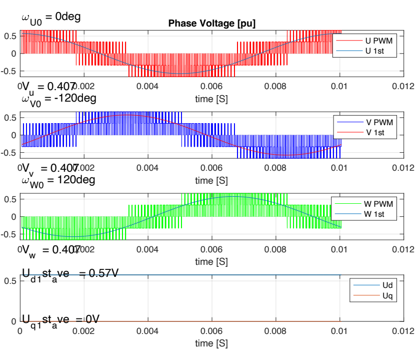
- Waveform compare
  - Fill timer in micro controller
- Switching duty

PWM Generator

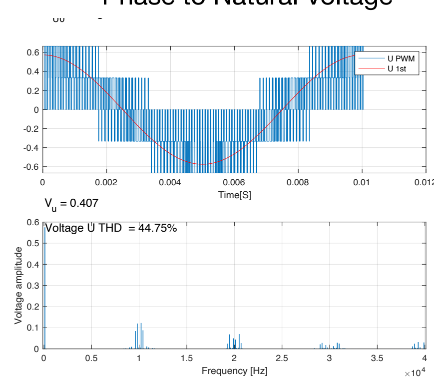


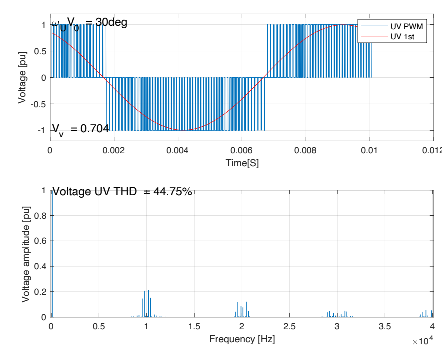




Phase to Natural Voltage




Phase to Phase Voltage



# PMSM FOC | Simulation case

## Speed Control Demo

- Speed Command
  - 1500RPM at 0 Sec
  - 3000RPM at 0.05 Sec