

# 某ZZJの表面模板

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#### 常用工具

```
#include <bits/stdc++.h>
using namespace std;
const double eps = 1e-10;
const double pi = 3.1415926535897932384626433832795;
const double eln = 2.718281828459045235360287471352;
typedef long long II;
typedef unsigned long long ull;
typedef vector<int> vi;
typedef pair<int,int> pii;
typedef pair<II,II> pII;
#define IN freopen("in.txt","r",stdin)
#define OUT freopen("out.txt","w",stdout)
#define pr(x) cout << #x << ": " << x << endl
#define lowbit(x) (x&(-x))
#define mp make_pair
#define pb push_back
#define fi first
#define se second
############//
//
                                                         愉悦的分割线
//
#############//
int main(){
   ios::sync_with_stdio(false);
   return 0;
}
inline void read(int& x){
   char ch = getchar();
   int flag = 1;
   while(ch < '0' || ch > '9'){}
       if(ch == '-') flag = -1;
       ch = getchar();
```

```
}
    x = 0:
    for(;ch \geq= '0' && ch \leq= '9';ch = getchar()) x=x*10+ch-'0';
    x *= flag;
}
inline void write(int x){
    if(x < 0){
         putchar('-');
         x *= -1:
    }
    char ch = x\%10+'0';
    if(x > 10) write(x/10);
    putchar(ch);
}
//无去重离散化,新的下标从1开始,最大下标为 cnt
 int idx[maxn],_cnt;
 inline int get_id(int x){
    return lower_bound(idx+1,idx+_cnt+1,x) - idx;
 }
 void discretize(int* f,int size){
    for(int i = 1;i \le size;i++) idx[i] = f[i];
    sort(idx+1,idx+size+1);
    _cnt = size;
 }
 //去重离散化,新的下标从1开始,最大下标为_cnt
 const int dis_maxn = 1e6+10;
 II idx[dis_maxn], tmp_id[dis_maxn];
 int _cnt;
 int get_id(ll x){
    return lower_bound(idx+1,idx+_cnt_1,x) - idx;
 }
 void discretize(II f, int size){
    _{cnt} = 0;
    for(int i = 1;i <= n;i++) tmp_id[i] = f[i];
    sort(tmp_id+1, tmp_id+size+1);
    for(int i = 1;i \le size;i++){
         if(i != 1 || tmp_id[i] != tmp_id[i-1]) idx[++_cnt] = tmp_id[i];
    }
}
II my_sqrt(II x){
    If ret = sqrt(x);
    while((ret+1)*(ret+1) \le x) + ret;
```

```
while( ret*ret > y) --ret;
return ret;
}
```

### 算法(LIS & 模拟退火)

```
//TODO:添加 lower bound 版本的支持一下操作的板子
/*
只返回结果值,需要区间求最大值线段树 + 离散化
可修改转移方程, 实现求最大上升序列和
*/
II LIS(int* f,int len){
   II ret = 0;
   //离散化,可以在仅主函数内进行一次
   discretize(f,len);
   memset(seg,0,sizeof(seg));
   for(int i = 1; i \le len; i++)
       //不是严格小于需要修改询问,方程可能需要修改
       If tmp = query(1,1, cnt,1,qet id(f[i])-1) + 1;
       add(1,1,_cnt,get_id(f[i]),tmp);
       ret = max(ret,tmp);
   }
   return ret;
}
在以上的基础上输出方案
由于线段树上的节点是一个pll,需修改线段树保证输出字典序最小方案
输出字典序最小方案, pair 里面的 second 存的是序号取负, 这样可以直接对 pair 取 max,
若是取 min 等,需修改
*/
int last[maxn];
vi LIS(int* f,int len){
   II ret = 0,mark;
   discretize(f,len);
   memset(seg,0,sizeof(seg));
   for(int i = 1; i \le len; i++)
       //不是严格小于需要修改询问, 方程可能需要修改
       pll tmp = query(1,1,\_cnt,1,get\_id(f[i])-1);
       II val = tmp.fi + 1;
       last[i] = -tmp.se;
       add(1,1,_cnt,get_id(f[i]),mp(val,-i));
       if(val > ret){
```

```
ret = val;
            mark = i;
        }
    }
    stack<int> s;
    while(mark != 0){
        s.push(mark);
        mark = last[mark];
    }
    vi retv;
    while(!s.empty()){
        retv.pb(s.top());
        s.pop();
    }
    return retv;
//TODO:以下模板还未测试过
/*
对数据进行处理之后,选取权值最大字典序最小的 x1 < x2, y1 < y2 的序列方案输出
若是 x1 <= x2, 可排序后归约为正常的 LIS
x和y需要是全局变量,大小为len,下标从1开始
*/
int ord[maxn],last[maxn];
bool cmp(int a,int b){
    return x[a] < x[b];
//线段树中的 first 是 val, second 是编号取负
vi LIS(int len){
    //x 取值可能是-1 时需要修改
    II ret = 0,mark,lastx = -1;
    for(int i = 1;i \le len;i++) ord[i] = i;
    sort(ord+1,ord+len+1,cmp);
    discretize(y,len);
    memset(seg,0,sizeof(seg));
    queue<pair<II,pII>>q;
    for(int i = 1; i <= len; i++){
        if(lastx != x[ord[i]]){
            while(!q.empty()){
                 add(1,1,_cnt,q.front().fi,q.front().se);
                 q.pop();
            }
            lastx = x[ord[i]];
        pll tmp = query(1,1,\_cnt,1,get\_id(y[ord[i]])-1);
```

```
II val = tmp.fi + 1;
         last[i] = -tmp.se;
         q.push(mp(get_id(y[ord[i]]), mp(val,-ord[i])));
         if(val > ret){
              ret = val;
              mark = ord[i];
         }
    }
    stack<int> s;
    while(mark != 0){
         s.push(mark);
         mark = last[mark];
    }
    vi retv;
    while(!s.empty()){
         retv.pb(s.top());
         s.pop();
    }
    return retv;
}
/*
上面那个占空间太大了,太复杂,写一个不输出方案只输出答案的版本
*/
int ord[maxn];
bool cmp(int a,int b){
    return x[a] < x[b];
}
II LIS(int len){
    //x 取值可能是-1 时需要修改
    II ret = 0, lastx = -1;
    for(int i = 1;i \le len;i++) ord[i] = i;
    sort(ord+1,ord+len+1,cmp);
    discretize(y,len);
    memset(seg,0,sizeof(seg));
    queue<pll> q;
    for(int i = 1; i <= len; i++){
         if(lastx != x[ord[i]]){
              while(!q.empty()){
                  add(1,1,_cnt,q.front().fi,q.front().se);
                  q.pop();
             }
              lastx = x[ord[i]];
         If tmp = query(1,1,\_cnt,1,get\_id(y[ord[i]])-1) + 1;
```

```
q.push(mp(get_id(y[ord[i]]),tmp));
          ret = max(ret,tmp);
    }
     return ret;
}
typedef long long II;
const II MOD = 1e9+7;
struct Mat{
     int size;
     vector<vector<II>> a;
     Mat(int size = 3):size(size){}
     void setZero(){
          for(int i = 0;i < size;i++){
               vector<II> tmp(size);
               for(int j = 0;j < size;j++){
                    tmp[j] = 0;
              }
               a.push_back(tmp);
          }
    }
     void setUnit(){
          setZero();
          for(int i = 0;i < size;i++) a[i][i] = 1;
    }
    vector<II>& operator[](size_t n){
          return a[n];
    }
     Mat operator*(Mat& tar){
          Mat ret(size);
          ret.setZero();
          for(int i = 0;i < size;i++){
               for(int j = 0;j < size;j++){
                    for(int k = 0; k < size; k++){
                         ret[i][j] = (ret[i][j] + a[i][k] * tar[k][j])%MOD;
                    }
               }
          return ret;
    }
};
Mat matPow(Mat a,ll b){
     Mat ret(a.size);
     ret.setUnit();
```

```
while(b){
       if(b & 1) ret = ret * a;
       a = a * a;
       b >>= 1;
   }
   return ret;
}
/*
模拟退火例题 1:
有n个重物,每个重物系在一条足够长的绳子上。每条绳子自上而下穿过桌面上的洞,然后
系在一起。图中X处就是公共的绳结。
假设绳子是完全弹性的(不会造成能量损失),桌子足够高(因而重物不会垂到地上),且忽
略所有的摩擦。
问绳结X最终平衡于何处。
可直接随机选点,每次往力的方向移动
也可根据评估函数,让每个物体的势能和最小
注意不同的题目,参数T的设置需要根据情况判断
*/
const double delta = 0.99;
Vector getNext(Point *p,Point now,int n){
   Vector ret = Point(0,0);
   for(int i = 0; i < n; i++){
       ret = ret + unit(p[i]-now)*w[i];
   }
   return ret;
Point work(Point* p,int n){
   double t = 1,min_dis = 1e18;
   Point now = p[0],ans = p[0];
   while(t > eps){}
       Vector dv = getNext(p,now,n);
       double I = Length(dv);
       if(min_dis > I){
          min_dis = I;
          ans = now;
       }
       now.x = now.x + dv.x * t;
       now.y = now.y + dv.y * t;
       t *= delta;
   }
   return ans;
```

}

```
/*
模拟退火例题 2:
最小球包含
精度不能达到很高
*/
const double delta = 0.98;
//寻找离当前圆心最远的点
int getPoint(Point *p,Point now,int n){
    int res = -1:
    double max_dis = 0,pre= 0;
    for(int i = 0; i < n; i++){
        max_dis = max(max_dis,Length(p[i]-now));
        if(max_dis != pre) res = i;
        pre = max_d;
   }
    return res;
//此处 t 的初始值设为 1 能过,设为 0.5 就 WA,目前还不是很懂
Point work(Point* p,int n){
    double t = 1, min_r = 1e18;
    Point now = p[0],ans = p[0];
    while(t > eps){
        int i = getPoint(p,now,n);
        now.x = now.x + (p[i].x - now.x) * t;
        now.y = now.y + (p[i].y - now.y) * t;
        now.z = now.z + (p[i].z - now.z) * t;
        if(min_dis > r){
            min_dis = r;
            ans = now;
        t *= delta;
   }
    return ans;
}
//TODO:添加 lower_bound 版本的支持一下操作的板子
只返回结果值,需要区间求最大值线段树 + 离散化
可修改转移方程, 实现求最大上升序列和
*/
II LIS(int* f,int len){
    II ret = 0;
    //离散化,可以在仅主函数内进行一次
    discretize(f,len);
    memset(seg,0,sizeof(seg));
```

```
for(int i = 1; i <= len; i++){
        //不是严格小于需要修改询问, 方程可能需要修改
        If tmp = query(1,1,\_cnt,1,get\_id(f[i])-1) + 1;
        add(1,1,_cnt,get_id(f[i]),tmp);
        ret = max(ret,tmp);
   }
    return ret;
}
/*
在以上的基础上输出方案
由于线段树上的节点是一个pll,需修改线段树保证输出字典序最小方案
输出字典序最小方案, pair 里面的 second 存的是序号取负, 这样可以直接对 pair 取 max,
若是取 min 等,需修改
*/
int last[maxn];
vi LIS(int* f,int len){
    II ret = 0,mark;
    discretize(f,len);
    memset(seg,0,sizeof(seg));
    for(int i = 1; i <= len; i++){
        //不是严格小于需要修改询问, 方程可能需要修改
        pll tmp = query(1,1,\_cnt,1,get\_id(f[i])-1);
        II val = tmp.fi + 1;
        last[i] = -tmp.se;
        add(1,1,_cnt,get_id(f[i]),mp(val,-i));
        if(val > ret){
            ret = val;
            mark = i;
        }
    }
    stack<int> s;
    while(mark != 0){
        s.push(mark);
        mark = last[mark];
   }
   vi retv;
    while(!s.empty()){
        retv.pb(s.top());
        s.pop();
   }
    return retv;
}
//TODO:以下模板还未测试过
/*
```

```
对数据进行处理之后,选取权值最大字典序最小的 x1 < x2, y1 < y2 的序列方案输出
若是 x1 <= x2, 可排序后归约为正常的 LIS
x和y需要是全局变量,大小为len,下标从1开始
*/
int ord[maxn],last[maxn];
bool cmp(int a,int b){
    return x[a] < x[b];
}
//线段树中的 first 是 val, second 是编号取负
vi LIS(int len){
    //x 取值可能是-1 时需要修改
    II ret = 0,mark,lastx = -1;
    for(int i = 1;i \le len;i++) ord[i] = i;
    sort(ord+1,ord+len+1,cmp);
    discretize(y,len);
    memset(seg,0,sizeof(seg));
    queue<pair<||,p||> > q;
    for(int i = 1; i <= len; i++){
         if(lastx != x[ord[i]]){
             while(!q.empty()){
                 add(1,1,_cnt,q.front().fi,q.front().se);
                 q.pop();
             }
             lastx = x[ord[i]];
        }
         pll tmp = query(1,1,\_cnt,1,get\_id(y[ord[i]])-1);
         II val = tmp.fi + 1;
         last[i] = -tmp.se;
         q.push(mp(get_id(y[ord[i]]), mp(val,-ord[i])));
         if(val > ret){}
             ret = val;
             mark = ord[i];
        }
    }
    stack<int> s;
    while(mark != 0){
        s.push(mark);
         mark = last[mark];
    }
    vi retv;
    while(!s.empty()){
         retv.pb(s.top());
        s.pop();
    }
```

```
return retv;
}
/*
上面那个占空间太大了,太复杂,写一个不输出方案只输出答案的版本
*/
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bool cmp(int a,int b){
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    //x 取值可能是-1 时需要修改
    II ret = 0, lastx = -1;
    for(int i = 1;i \le len;i++) ord[i] = i;
    sort(ord+1,ord+len+1,cmp);
    discretize(y,len);
    memset(seg,0,sizeof(seg));
    queue<pll> q;
    for(int i = 1; i <= len; i++){
         if(lastx != x[ord[i]]){
              while(!q.empty()){
                  add(1,1,_cnt,q.front().fi,q.front().se);
                  q.pop();
              lastx = x[ord[i]];
         }
         If tmp = query(1,1,\_cnt,1,get\_id(y[ord[i]])-1) + 1;
         q.push(mp(get_id(y[ord[i]]),tmp));
         ret = max(ret,tmp);
    }
    return ret;
}
typedef long long II;
const II MOD = 1e9+7;
struct Mat{
    int size;
    vector<vector<ll> > a;
    Mat(int size = 3):size(size){}
    void setZero(){
         for(int i = 0;i < size;i++){
              vector<II> tmp(size);
              for(int j = 0;j < size;j++){
                  tmp[j] = 0;
             }
```

```
a.push_back(tmp);
          }
    }
     void setUnit(){
          setZero();
          for(int i = 0; i < size; i++) a[i][i] = 1;
    vector<||>& operator[](size_t n){
          return a[n];
    }
     Mat operator*(Mat& tar){
          Mat ret(size);
          ret.setZero();
          for(int i = 0;i < size;i++){
               for(int j = 0;j < size;j++){
                    for(int k = 0; k < size; k++){
                         ret[i][j] = (ret[i][j] + a[i][k] * tar[k][j])%MOD;
                    }
               }
          return ret;
    }
};
Mat matPow(Mat a,ll b){
     Mat ret(a.size);
     ret.setUnit();
     while(b){
          if(b & 1) ret = ret * a;
          a = a * a;
          b >>= 1;
    }
     return ret;
}
/*
```

#### 模拟退火例题 1:

有 n 个重物,每个重物系在一条足够长的绳子上。每条绳子自上而下穿过桌面上的洞,然后 系在一起。图中X处就是公共的绳结。

假设绳子是完全弹性的(不会造成能量损失),桌子足够高(因而重物不会垂到地上),且忽 略所有的摩擦。

问绳结X最终平衡于何处。

可直接随机选点, 每次往力的方向移动 也可根据评估函数,让每个物体的势能和最小

```
注意不同的题目,参数T的设置需要根据情况判断
*/
const double delta = 0.99;
Vector getNext(Point *p,Point now,int n){
    Vector ret = Point(0,0);
    for(int i = 0;i < n;i++){
        ret = ret + unit(p[i]-now)*w[i];
    }
    return ret:
Point work(Point* p,int n){
    double t = 1,min_dis = 1e18;
    Point now = p[0],ans = p[0];
    while(t > eps){
        Vector dv = getNext(p,now,n);
        double I = Length(dv);
        if(min_dis > I){
            min dis = I;
            ans = now;
        now.x = now.x + dv.x * t;
        now.y = now.y + dv.y * t;
        t *= delta;
    }
    return ans;
}
/*
模拟退火例题 2:
最小球包含
精度不能达到很高
const double delta = 0.98;
//寻找离当前圆心最远的点
int getPoint(Point *p,Point now,int n){
    int res = -1;
    double max_dis = 0,pre= 0;
    for(int i = 0; i < n; i++){
        max_dis = max(max_dis,Length(p[i]-now));
        if(max_dis != pre) res = i;
        pre = max_d;
    }
    return res;
//此处 t 的初始值设为 1 能过,设为 0.5 就 WA,目前还不是很懂
```

```
Point work(Point* p,int n){
     double t = 1, min r = 1e18;
     Point now = p[0],ans = p[0];
    while(t > eps){
         int i = getPoint(p,now,n);
         now.x = now.x + (p[i].x - now.x) * t;
         now.y = now.y + (p[i].y - now.y) * t;
         now.z = now.z + (p[i].z - now.z) * t;
         if(min_dis > r){
              min_dis = r;
              ans = now;
         }
         t *= delta;
    }
    return ans;
}
```

#### 数据结构

```
struct hash_node{
    int val1,val2;
    hash_node* next;
};
const int MOD_hash = 4e6+7;
has_node* hash_table[MOD_hash];
//仅返回 key 值是否存在用于去重等操作,可修改为真正的 hash 表
bool check_and_add(int v1,int v2){
   //将 v1 范围控制在可以直接访问的范围
   v1 = v1 \% MOD_hash;
    hash_node** p = &hash_table[v1];
   while(*p){
        if((*p)->val1 == v1 \&\& (*p)->val2 == v2) return false;
        p = &((*p)->next);
   }
    *p = new node;
   (*p)->val1 = v1;
    (*p)->val2 = v2;
   (*p)->next = 0;
    return false;
}
顺便提供一些[1e6, 1e7]的大质数
```

```
1000003 1999993
2000003 2999999
3000017 3999971
4000037 4999999
5000011 5999993
6000011 6999997
7000003 7999993
8000009 8999993
9000011 9999991
*/
#include <bits/stdc++.h>
using namespace std;
typedef long long II;
const int maxn = 1e5+10;
II seg[maxn*4],lazy[maxn*4],dfn[maxn],size[maxn],son[maxn],fa[maxn],deep[maxn],top[maxn];
Il a[maxn],b[maxn];
II dfs clock = 0,n,m,MOD;
vector<int> G[maxn];
Il build(int o,int l,int r){
     lazy[o] = 0;
     if(I == r) return seg[o] = b[I] % MOD;
     int mid = (I + r) >> 1;
     build(o << 1, l, mid); build(o << 1|1, mid+1, r);
    seg[o] = (seg[o << 1] + seg[o << 1|1]) % MOD;
}
Il addLazy(int o,int l,int r,ll k){
     lazy[o] = (lazy[o] + k) % MOD;
    seg[o] = (seg[o] + k * (r - I + 1)) % MOD;
    return 0;
}
void push_down(int o,int l,int r){
     int mid = (I + r) >> 1;
     addLazy(o<<1,l,mid,lazy[o]);</pre>
     addLazy(o<<1|1,mid+1,r,lazy[o]);
    lazy[o] = 0;
}
Il add(int o,int l,int r,int L,int R,ll k){
     if(l >= L \&\& r <= R) return addLazy(o,l,r,k);
     if(lazy[o] != 0) push_down(o,l,r);
    int mid = (I + r) >> 1;
     if(L \le mid) add(o \le 1, I, mid, L, R, k);
     if(R > mid) add(o<<1|1,mid+1,r,L,R,k);
```

```
seg[o] = (seg[o << 1] + seg[o << 1|1]) % MOD;
}
Il query(int o,int l,int r,int L,int R){
     if(I >= L \&\& r <= R) return seg[o];
     if(lazy[o] != 0) push_down(o,l,r);
     int mid = (I + r) >> 1;
     II ans = 0;
     if(L \le mid) ans = (ans + query(o << 1,I,mid,L,R)) % MOD;
     if(R > mid) ans = (ans + query(o << 1|1, mid + 1, r, L, R)) % MOD;
     return ans;
}
void dfs1(int cur,int pre,int depth){
     size[cur] = 1,fa[cur] = pre,son[cur] = 0,deep[cur] = depth;
     for(auto nx : G[cur]){
          if(nx == pre) continue;
          dfs1(nx,cur,depth + 1);
          size[cur] += size[nx];
          if(size[nx] > size[son[cur]]) son[cur] = nx;
    }
}
void dfs2(int cur,int tp){
     dfn[cur] = ++dfs_clock;top[cur] = tp;
     if(son[cur]) dfs2(son[cur],tp);
     for(auto nx : G[cur]){
          if(nx == fa[cur] || nx == son[cur]) continue;
          dfs2(nx,nx);
    }
}
void modify(int x,int y,ll k){
     int t1 = top[x], t2 = top[y];
     while(t1 != t2){
          if(deep[t1] < deep[t2]){
               swap(x,y);swap(t1,t2);
          add(1,1,n,dfn[t1],dfn[x],k);
          x = fa[t1];t1 = top[x];
     if(deep[x] > deep[y]) swap(x,y);
     add(1,1,n,dfn[x],dfn[y],k);
Il queryTree(int x,int y){
     II ans = 0;
     int t1 = top[x], t2 = top[y];
     while(t1 != t2){
```

```
if(deep[t1] < deep[t2]){
              swap(x,y);swap(t1,t2);
         ans = (ans + query(1,1,n,dfn[t1],dfn[x])) % MOD;
         x = fa[t1];t1 = top[x];
    }
    if(deep[x] > deep[y]) swap(x,y);
     ans = (ans + query(1,1,n,dfn[x],dfn[y])) % MOD;
     return ans:
}
int main(){
    ios::sync_with_stdio(false);
     int root;
     cin>>n>>m>>root>>MOD;
     for(int i = 1; i <= n; i++) cin>>a[i];
     for(int i = 1; i \le n-1; i++){
         int u,v;
         cin>>u>>v;
         G[u].push_back(v);G[v].push_back(u);
    }
     dfs1(root,0,0);dfs2(root,root);
     for(int i = 1;i <= n;i++) b[dfn[i]] = a[i];
     build(1,1,n);
     for(int i = 1; i <= m; i++){
         II op,x,y,k;
         cin>>op;
         if(op == 1){}
              cin>>x>>y>>k;
              modify(x,y,k);
         } else if(op == 2){
              cin>>x>>y;
              cout<<queryTree(x,y)<<endl;</pre>
         } else if(op == 3){
              cin>>x>>k;
              add(1,1,n,dfn[x],dfn[x] + size[x] - 1,k);
         } else{
              cin>>x;
              cout < query(1,1,n,dfn[x],dfn[x] + size[x] - 1) < endl;
         }
    }
}
const int maxn = 5e5+10;
vector<int> G[maxn];//双向边
```

```
//子树大小, 重儿子, 深度, 父亲
int size[maxn],son[maxn],deep[maxn],fa[maxn];
int dfn[maxn],top[maxn];//DFS 序,所在重链的顶端
int dfs_clock;//记得初始化, dfn 计算需要
void dfs1(int cur, int father, int depth) {
    size[cur] = 1,son[cur] = 0,deep[cur] = depth,fa[cur] = father;
    for(auto nx: G[cur]) {
         if(nx != father) {
             dfs1(nx, cur, depth + 1);
             size[cur] += size[nx];
             if(size[nx] > size[son[cur]]) {
                  son[cur] = nx;
             }
         }
    }
}
void dfs2(int cur, int tp){
    top[cur] = tp,dfn[cur] = ++dfs_clock;
    if(son[cur]) dfs2(son[cur],tp);//优先遍历重儿子
    for(auto nx: G[cur]) {
         if(nx != fa[cur] && nx != son[cur]) {
             dfs2(nx, nx);
         }
    }
}
int lca(int x, int y) {
    int t1 = top[x], t2 = top[y];
    while(t1 != t2) {
         //不在一个重链上,将深度大的往上跳
         if(deep[t1] < deep[t2]) \{
             swap(x, y); swap(t1, t2);
         x = fa[t1], t1 = top[x];
    return deep[x] < deep[y] ? x : y;
//修改,对 u-v 这条链执行函数 f。查询也类似
void modify(int x, int y,void f(int,int)) {
    int t1 = top[x], t2 = top[y];
    while(t1 != t2) {
         if(deep[t1] < deep[t2]) \{
             swap(x, y); swap(t1, t2);
         }
```

```
f(dfn[t1], dfn[x]);
         x = fa[t1], t1 = top[x];
    }
    if(deep[x] > deep[y]) swap(x, y);
     f(dfn[x], dfn[y]);
}
#include <bits/stdc++.h>
using namespace std;
const int maxn = 1e5+10;
typedef long long II;
II lazy[maxn*4];
double seg[maxn*4],weight[maxn*4],num[maxn*4],len[maxn*4];
int _cnt;
double idx[maxn*4],tmp_id[maxn*4];
void discretize(double *f,int size){
     cnt = 0;
    int tmp = 0;
     for(int i = 1;i \le size;i++) tmp_id[++tmp] = f[i];
     sort(tmp_id+1,tmp_id+tmp+1);
     for(int i = 1; i \le tmp; i++){
          if(i == 1 || tmp_id[i] != tmp_id[i-1]) idx[++_cnt] = tmp_id[i];
    }
}
int get_id(double x){
     return lower_bound(idx+1,idx+_cnt+1,x) - idx;
}
Il build(int o,int l,int r){
     if(l == r) return weight[o] = len[l];
     int mid = (I + r) >> 1;
     build(o << 1, l, mid); build(o << 1|1, mid+1, r);
    weight[o] = weight[o << 1] + weight[o << 1|1];
}
Il addLazy(int,int,int,II);
void push_down(int o,int l,int r){
    int mid = (I + r) >> 1;
     addLazy(o<<1,l,mid,lazy[o]);
     addLazy(o<<1|1,mid+1,r,lazy[o]);</pre>
     lazy[o] = 0;
}
Il addLazy(int o,int l,int r,ll k){
     lazy[o] += k;
     if(lazy[o] > 0) return seg[o] = weight[o];
```

```
if(I == r) return seg[o] = 0;
     if(lazy[o] < 0) push_down(o,l,r);
     seg[o] = seg[o << 1] + seg[o << 1|1];
}
Il add(int o,int l,int r,int L,int R,II k){
     if(L > R) return 0;
     if(l >= L \&\& r <= R) return addLazy(o,l,r,k);
     if(lazy[o] != 0) push_down(o,l,r);
     int mid = (I + r) >> 1;
     if(L \le mid) add(o \le 1, I, mid, L, R, k);
     if(R > mid) add(o<<1|1,mid+1,r,L,R,k);
     seg[o] = seg[o << 1] + seg[o << 1|1];
}
double query(int o,int l,int r,int L,int R){
     if(I >= L \&\& r <= R) return seg[o];
     if(lazy[o]!= 0) push_down(o,l,r);
     int mid = (I + r) >> 1;
     double ans = 0;
     if(L \le mid) ans += query(o \le 1, I, mid, L, R);
     if(R > mid) ans += query(o << 1|1, mid + 1, r, L, R);
     return ans;
}
struct Line{
     double x,y1,y2;
     II k;
     bool operator<(const Line& tar)const{
          if(x != tar.x) return x < tar.x;
          return k < tar.k;
     }
}lines[maxn*2];
int main(){
     int n;
     while(cin>>n){
          if(n == 0) break;
          memset(lazy,0,sizeof(lazy));
          memset(seg,0,sizeof(seg));
          int tot = 0,cnt = 0;
          for(int i = 0; i < n; i++){
               double x1,y1,x2,y2;
               cin>>x1>>y1>>x2>>y2;
               num[++cnt] = y1, num[++cnt] = y2;
               lines[tot].x = x1, lines[tot].y1 = y1, lines[tot].y2 = y2, lines[tot++].k = 1;
               lines[tot].x = x2, lines[tot].y1 = y1, lines[tot].y2 = y2, lines[tot++].k = -1;
```

```
}
          discretize(num,cnt);
          for(int i = 1;i < \_cnt;i++) len[i] = idx[i+1] - idx[i];
          len[\_cnt] = 0;
          build(1,1,_cnt);
          sort(lines,lines+tot);
          double ans = 0, pre = 0;
          for(int i = 0;i < tot;i++){
               double tmp = query(1,1,_cnt,1,_cnt);
               ans += tmp * (lines[i].x - pre);
               add(1,1,_cnt,get_id(lines[i].y1),get_id(lines[i].y2)-1,lines[i].k);
               pre = lines[i].x;
         }
          printf("%.2f\n",ans);
    }
}
//扫描线, 例子为区间反转求1的个数
struct Line{
    II x,y1,y2;
     Line(){}
     Line(II x,II y1,II y2):x(x),y1(y1),y2(y2){}
}que[maxn];
bool cmp(Line a,Line b){
     return a.x < b.x;
}
int main(){
    ios::sync_with_stdio(false);
     return -1;
    int T,n,k;
     cin>>T;
    while(T){
          cin>>n>>k;
          memset(seg,0,sizeof(seg));
          memset(lazy,0,sizeof(lazy));
          int tot = 0;
          for(int i = 0; i < k; i++){
              int x1,y1,x2,y2;
              cin>>x1>>x2>>y1>>y2;
               que[tot++] = Line(x1-1,y1,y2);
               que[tot++] = Line(x2,y1,y2);
         }
         sort(que,que+tot,cmp);
          II last = 0,ans = 0;
          for(int i = 0;i < tot;i++){
```

```
If tmp = query(1,1,n,1,n);
          // cout<<tmp<<endl;
               if(que[i].x != 0) ans += (que[i].x - last) * query(1,1,n,1,n);
               add(1,1,n,que[i].y1,que[i].y2,1);
               last = que[i].x;
          }
          cout << ans << endl;
    }
}
/* TEST CODE
#include <bits/stdc++.h>
using namespace std;
typedef long long II;
const int maxn = 1e5+10;
int n,m,MOD;
Il seg[maxn*4],addl[maxn*4],mull[maxn*4],a[maxn];
Il build(int o,int l,int r);
Il addLazy(int o,int l,int r,ll k);
Il mulLazy(int o,int l,int r,ll k);
Il push_down(int o,int l,int r);
Il add(int o,int l,int r,int L,int R,ll k);
Il mul(int o,int l,int r,int L,int R,ll k);
Il query(int o,int l,int r,int L,int R);
int main(){
     ios::sync_with_stdio(false);
     cin > n > m > MOD;
     for(int i = 1; i <= n; i++) cin>>a[i];
     build(1,1,n);
     for(int i = 1; i <= m; i++){
          II op,x,y,k;
          cin>>op>>x>>y;
          if(op == 1){
               cin>>k;
               mul(1,1,n,x,y,k);
          } else if(op == 2){
               cin>>k;
               add(1,1,n,x,y,k);
          } else{
               cout<<query(1,1,n,x,y)<<endl;</pre>
          }
    }
```

```
}
*/
//seg addl mull a MOD
Il build(int o,int l,int r){
     \text{mull}[o] = 1; \text{addl}[o] = 0; // \text{init}
     if(I == r) return seg[o] = a[I] % MOD;
     int mid = (I + r) >> 1;
     build(o << 1, l, mid); build(o << 1, l, mid + 1, r);
     seg[o] = (seg[o << 1] + seg[o << 1|1]) % MOD;
Il addLazy(int o,int l,int r,ll k){
     addl[o] = (addl[o] + k) \% MOD;
     return seg[o] = (seg[o] + k * (r-l+1)) % MOD;
}
Il mulLazy(int o,int l,int r,ll k){
     mull[o] = (mull[o] * k) % MOD;
     addl[o] = (addl[o] * k) % MOD;
     return seg[o] = (seg[o] * k) % MOD;
Il push_down(int o,int l,int r){
     int mid = (I + r) >> 1;
     if(mull[o] != 1){
          mulLazy(o<<1,l,mid,mull[o]);
          mulLazy(o<<1|1,mid+1,r,mull[o]);
     if(addl[o] != 0){
          addLazy(o<<1,l,mid,addl[o]);
          addLazy(o<<1|1,mid+1,r,addl[o]);</pre>
     }
     addl[o] = 0; mull[o] = 1;
     return 0;
}
Il add(int o,int l,int r,int L,int R,ll k){
     if(I >= L \&\& r <= R) return addLazy(o,I,r,k);
     push_down(o,l,r);
     int mid = (I + r) >> 1;
     if(L \le mid) add(o \le 1, I, mid, L, R, k);
     if(R > mid) add(o << 1|1, mid + 1, r, L, R, k);
     seg[o] = (seg[o << 1] + seg[o << 1|1]) % MOD;
Il mul(int o,int l,int r,int L,int R,ll k){
     if(l >= L \&\& r <= R) return mulLazy(o,l,r,k);
     push_down(o,l,r);
     int mid = (I + r) >> 1;
```

```
if(L \le mid) mul(o \le 1, I, mid, L, R, k);
     if(R > mid) mul(o<<1|1,mid+1,r,L,R,k);
     seg[o] = (seg[o << 1] + seg[o << 1|1]) % MOD;
}
Il query(int o,int l,int r,int L,int R){
     if(I >= L \&\& r <= R) return seg[o];
     push_down(o,l,r);
     int mid = (I + r) >> 1;
     II ans = 0:
     if(L \le mid) ans = (ans + query(o << 1,I,mid,L,R)) % MOD;
     if(R > mid) ans = (ans + query(o << 1|1, mid + 1, r, L, R)) % MOD;
     return ans;
}
typedef long long II;
const int maxn = 1e6+10;
Il seg[maxn*4],lazy[maxn*4],a[maxn];
inline II merge(II a,II b){
     return a + b;
    //return max(a,b);
    //return min(a,b);
}
inline II addLazy(int o,int I,int r,II x){
     seg[o] += x * (r-l+1);
    //seg[o] += x;
    /*区间反转
     if(x == 1) seg[o] = (r-l+1) - seg[o];
     lazy[o] ^= x;
     lazy[o] += x;
}
inline void push_down(int o,int l,int r){
     int mid = (I + r) >> 1;
     addLazy(o<<1,l,mid,lazy[o]);</pre>
     addLazy(o<<1|1,mid+1,r,lazy[o]);</pre>
     lazy[o] = 0;
}
Il build(int o,int l,int r){
     if(I == r) return seg[o] = a[I];
     int mid = (I + r) >> 1;
     build(o << 1, l, mid); build(o << 1|1, mid+1, r);
     seg[o] = merge(seg[o << 1], seg[o << 1|1]);
Il add(int o,int l,int r,int L,int R,ll v){
```

```
if(l >= L \&\& r <= R) return addLazy(o,l,r,v);
    if(lazy[o]!= 0) push_down(o,l,r);
    int mid = (I + r) >> 1;
    if(L \le mid) add(o \le 1, I, mid, L, R, v);
    if(R > mid) add(o<<1|1,mid+1,r,L,R,v);
    seg[o] = merge(seg[o << 1], seg[o << 1|1]);
Il query(int o,int l,int r,int L,int R){
    if(L > R) return 0;
    if(I >= L \&\& r <= R) return seg[o];
    if(lazy[o]!= 0) push_down(o,l,r);
    int mid = (I + r) >> 1;
    II ans = 0;//or inf
    if(L \le mid) ans = merge(ans,query(o << 1,I,mid,L,R));
    if(R > mid) ans = merge(ans,query(o << 1|1,mid+1,r,L,R));
    return ans;
//可修改为每个节点为 bitset 的线段树,修改数据类型 + add 函数 + combine 函数即可
const int maxn = 1e5+10;
Il a[maxn],seg[maxn*4];
//合并函数, 取最大值、求和等等
inline II combine(II x,II y){
    return max(x,y);
    //return min(x,y);
    //return x+y;
}
Il build(int o,int l,int r){
    if(l == r) return seg[o] = a[l];
    int mid = (I + r) / 2;
    build(o*2,l,mid);build(o*2+1,mid+1,r);
    seg[o] = combine(seg[o*2],seg[o*2+1]);
}
Il add(int o,int l,int r,int x,ll v){
    //更新的这一行需要根据需求修改
    if(I == r) return seg[o] = combine(seg[o],v);
    int mid = (l + r) / 2;
    if(x \le mid) add(o*2,l,mid,x,v);
    else add(o*2+1,mid+1,r,x,v);
    seg[o] = combine(seg[o*2],seg[o*2+1]);
Il query(int o,int l,int r,int L,int R){
    if(L > R) return 0;
    if(I >= L \&\& r <= R) return seg[o];
    int mid = (I + r) / 2;
```

```
II ans = 0;
    if(L \le mid) ans = combine(ans,query(o*2,I,mid,L,R));
    if(R > mid) ans = combine(ans,query(o*2+1,mid+1,r,L,R));
    return ans;
}
//TODO: 这一个函数还没有测试过
Il query(int o,int l,int r,int x){
    if(l == r) return seg[o];
    int mid = (I + r) / 2;
    if(x \le mid) return query(o*2,I,mid,x);
    else return query(o*2+1,mid+1,r,x);
#include <bits/stdc++.h>
using namespace std;
const int maxn = 1e5+10;
const int MOD = 1e9+7;
typedef long long II;
II seg[maxn*4],mull[maxn*4],addl[maxn*4],mulsig[maxn*4],sigma[maxn*4],adds[maxn*4];
Il addLazy(int o,int l,int r,ll k){
    addl[o] = (addl[o] + k) % MOD;
    return seg[o] = (seg[o] + k * (r - l + 1)) % MOD;
}
Il mulLazy(int o,int l,int r,ll k){
    mull[o] = (mull[o] * k) % MOD;
    addl[o] = (addl[o] * k) % MOD;
    adds[o] = (adds[o] * k) % MOD;
    return seg[o] = (seg[o] * k) % MOD;
Il mulLazySig(int o,int l,int r,ll k){
    mulsig[o] = (mulsig[o] * k) % MOD;
    return sigma[o] = (sigma[o] * k) % MOD;
}
addLazySig(int o,int I,int r,ll k){
    adds[o] = (adds[o] + mulsig[o] * k) % MOD;
    return seg[o] = (seg[o] + sigma[o] * k) % MOD;
Il push_down(int o,int l,int r){
    int mid = (I + r) >> 1;
    if(mull[o] != 1){
         mulLazy(o<<1,l,mid,mull[o]);
         mulLazy(o<<1|1,mid+1,r,mull[o]);
    }
```

```
if(adds[o] != 0){
          addLazySig(o<<1,l,mid,adds[o]);
          addLazySig(o<<1|1,mid+1,r,adds[o]);</pre>
     }
     if(mulsig[o] != 1){
          mulLazySig(o<<1,l,mid,mulsig[o]);
          mulLazySig(o<<1|1,mid+1,r,mulsig[o]);
    }
     if(addl[o] != 0){
          addLazy(o<<1,l,mid,addl[o]);
          addLazy(o<<1|1,mid+1,r,addl[o]);
     addl[o] = adds[o] = 0; mull[o] = mulsig[o] = 1;
     return 0;
}
Il build(int o,int l,int r){
     mull[o] = mulsig[o] = 1;
     addl[o] = adds[o] = 0;
     if(l == r){}
          seg[o] = 0;
          sigma[o] = 10;
          return 0;
    }
     int mid = (I + r) >> 1;
     build(o<<1,1,mid);build(o<<1|1,mid+1,r);
     seg[o] = (seg[o << 1] + seg[o << 1|1]) % MOD;
     sigma[o] = (sigma[o << 1] + sigma[o << 1|1]) % MOD;
}
II add(int o,int l,int r,int L,int R,II d){
     if(I >= L \&\& r <= R) return addLazy(o,I,r,d);
     push_down(o,l,r);
     int mid = (I + r) >> 1;
     if(L \le mid) add(o \le 1, I, mid, L, R, d);
     if(R > mid) add(o << 1|1, mid + 1, r, L, R, d);
     seg[o] = (seg[o << 1] + seg[o << 1|1]) % MOD;
     sigma[o] = (sigma[o << 1] + sigma[o << 1|1]) % MOD;
}
Il mul(int o,int l,int r,int L,int R,ll d){
     if(I >= L \&\& r <= R) return mulLazy(o,I,r,d);
     push_down(o,l,r);
     int mid = (I + r) >> 1;
     if(L \le mid) mul(o \le 1, I, mid, L, R, d);
     if(R > mid) mul(o<<1|1,mid+1,r,L,R,d);
     seg[o] = (seg[o << 1] + seg[o << 1|1]) % MOD;
```

```
sigma[o] = (sigma[o << 1] + sigma[o << 1|1]) % MOD;
}
Il mulSig(int o,int l,int r,int L,int R,ll d){
     if(I >= L \&\& r <= R) return mulLazySig(o,I,r,d);
     push_down(o,l,r);
     int mid = (I + r) >> 1;
     if(L \le mid) mulSig(o \le 1, I, mid, L, R, d);
     if(R > mid) mulSig(o << 1|1, mid+1, r, L, R, d);
     seg[o] = (seg[o << 1] + seg[o << 1|1]) % MOD;
     sigma[o] = (sigma[o << 1] + sigma[o << 1|1]) % MOD;
}
Il addBySig(int o,int l,int r,int L,int R,ll d){
     if(I \ge L \&\& r \le R) return addLazySig(o,I,r,d);
     push_down(o,l,r);
     int mid = (I + r) >> 1;
     if(L \le mid) addBySig(o \le 1,l,mid,L,R,d);
     if(R > mid) addBySig(o<<1|1,mid+1,r,L,R,d);
     seg[o] = (seg[o << 1] + seg[o << 1|1]) % MOD;
     sigma[o] = (sigma[o << 1] + sigma[o << 1|1]) % MOD;
Il query(int o,int l,int r,int L,int R){
     if(I >= L \&\& r <= R) return seg[o];
     push_down(o,l,r);
     int mid = (I + r) >> 1;
     II ans = 0;
     if(L \le mid) ans = (ans + query(o << 1,I,mid,L,R)) % MOD;
     if(R > mid) ans = (ans + query(o < < 1|1, mid + 1, r, L, R)) % MOD;
     return ans;
}
int main(){
     int T,n,m;
     cin>>T;
     for(int cas = 1;cas <= T;cas++){
          cin>>n>>m;
          build(1,1,n);
          cout << "Case " << cas << ": " << endl;
          for(int i = 1; i \le m; i++){
               int x,y,d;string op;
               cin>>op>>x>>y;
               if(op[0] == 'w'){}
                    cin>>d;
                    mul(1,1,n,x,y,10);
                    add(1,1,n,x,y,d);
```

#### 图论

```
void dijkstra(int st,int n){
     priority_queue<pll> q;
     for(int i = 1;i <= n;i++) dist[i] = 1e18+10;
     dist[st] = 0;q.push(mp(0,st));
     while(!q.empty()){
          pll cur = q.top();q.pop();
          if(dist[cur.se] != -cur.fi) continue;
          for(pll p : G[cur.se]){
               if(dist[p.fi] > dist[cur.se] + p.se){
                    dist[p.fi] = dist[cur.se] + p.se;
                    q.push(mp(-dist[p.fi],p.fi));
               }
          }
     }
bool spfa(int st,int n){
     II cnt[maxn] = \{0\};
     queue<pll> q;
     for(int i = 1;i <= n;i++) dist[i] = 1e18+10;
     dist[st] = 0;q.push(mp(0,st));
          pll cur = q.front();q.pop();
          if(++cnt[cur.se] > n + 1) return false;
          for(pll p : G[cur.se]){
               if(dist[p.fi] > dist[cur.se] + p.se){
                    dist[p.fi] = dist[cur.se] + p.se;
                    q.push(mp(dist[p.fi],p.fi));
               }
          }
     }
     return true;
}
```

```
const int inf = 0x3f3f3f3f;
const int mm = 111111;
const int maxn = 999;
int node, src, dest, edge;
int ver[mm],flow[mm],cst[mm],nxt[mm];
int head[maxn],work[maxn],dis[maxn],q[maxn];
int tot_cost;
void prepare(int _node,int _src,int _dest){
     node=_node,src=_src,dest=_dest;
     for(int i=0; i<node; ++i)head[i]=-1;
     edge=0;
     tot_cost = 0;
}
void add_edge(int u,int v,int c,int cost){
    ver[edge]=v,flow[edge]=c,nxt[edge]=head[u],cst[edge]=cost,head[u]=edge++;
    ver[edge]=u,flow[edge]=0,nxt[edge]=head[v],cst[edge]=-cost,head[v]=edge++;
}
int ins[maxn];
int pre[maxn];
bool Dinic_spfa(){
     memset(ins,0,sizeof(ins));
     memset(dis,inf,sizeof(dis));
     memset(pre,-1,sizeof(pre));
     queue<int> Q;
     //int i,u,v,l,r=0;
     Q.push(src);
     dis[src] = 0,ins[src] = 1;
     pre[src] = -1;
     while(!Q.empty()){
         int u = Q.front();Q.pop();
         ins[u] = 0;
         for(int e = head[u];e != -1;e = nxt[e]){
              int v = ver[e];
              if(!flow[e]) continue;
              if(dis[v] > dis[u] + cst[e]){
                   dis[v] = dis[u] + cst[e];
                   pre[v] = e;
                   if(!ins[v]) ins[v] = 1,Q.push(v);
              }
         }
    }
     return dis[dest] < inf;
int Dinic_flow(){
```

```
int i,ret=0,delta=inf;
while(Dinic_spfa()){
    for(int i=pre[dest];i != -1;i = pre[ver[i^1]])
        delta = min(delta,flow[i]);
    for(int i=pre[dest];i != -1;i = pre[ver[i^1]])
        flow[i] -= delta,flow[i^1] += delta;
    ret += delta;
    tot_cost += dis[dest]*delta;
}
return ret;
}
```

#### Manacher

```
#include <bits/stdc++.h>
using namespace std;
const int maxn = 2e7+10;
//重要性质: 一个字符串最多只有 n 个本质不同的回文子串
int p[maxn*2];
int manacher(string s_source){
    //预处理左加#右加$, ##a#b#c#$
    string s = "##";
    for(int i = 0;i < s_source.size();<math>i++){
         s += s_source[i];
         s += '#';
    }
    s += '$';
    int max_len = -1, mx = 0, id;
    for(int i = 1; i < s.size() - 1; i++){
         p[i] = i < mx ? min(p[2 * id - i], mx - i) : 1;
         while(s[i - p[i]] == s[i + p[i]]) p[i]++;//在这一行修改可统计所有的回文子串
         if (mx < i + p[i]) mx = i + p[i], id = i;
         max_{len} = max(max_{len}, p[i] - 1);
    }
    return max_len;
}
int main(){
    string s;
    cin>>s;
```

```
cout<<manacher(s);
return 0;
}</pre>
```

#### Lucas

```
long long fac[400],inv[400];
const int MOD = 1e9+7;
long long pow_mod(long long x, long long n, long long mod){
    long long res=1;
    while(n>0){
         if(n&1)res=res*x%mod;
         x=x*x%mod;
         n>>=1;
    }
    return res;
}
void init(){
    fac[0] = 1;
    for(int i = 1;i <= 300;++i){
         fac[i] = fac[i-1]*i%MOD;
    }
    for(int i = 0;i <= 300;++i){
         inv[i] = pow_mod(fac[i],MOD-2,MOD);
    }
long long C(long long n,long long m){
    long long ans = fac[n];
    ans = ans * inv[m] % MOD;
    ans = ans * inv[n-m] % MOD;
    return ans;
}
```

#### SG 函数

```
//DFS 求 SG 函数,前提是不会有循环依赖(环) int DFS(int x){
    if(sg[x] != -1) return sg[x];
    set<int> s;
```

```
for(new status nx by x){
    if(!check(nx)) continue;
    s.insert(DFS(nx));
}
int tmp = 0;
while(s.count(tmp)) tmp++;
return sg[x] = tmp;
}
```

#### 数位 DP

```
//计算[1-x]有多少个数最多只有 3 个位置不是 0
int num[20],len;
Il cal(int x,bool limit,int last){
     if(last == 0 \parallel x \ge 1 = 1 = 1) return 1;
     if(limit\&num[x] == 0) return cal(x+1,limit,last);
     II ans = 0;
     if(limit){
          //1 - num[x]-1
          if(num[x] != 1) ans += (num[x]-1) * cal(x+1,false,last - 1);
          //0
          if(num[x] != 0) ans += cal(x+1,false,last);
          ans += cal(x+1,true,last-1);
    } else{
          ans += 9 * cal(x+1,false,last-1);
          ans += cal(x+1,false,last);
    }
     return ans;
II solve(II x){
     II tmp[20],ps = 0;
    while(x){
          tmp[ps++] = x\%10;
          x /= 10;
    }
     for(int i = 0; i < ps; i++){
          num[i] = tmp[ps-1-i];
    }
     len = ps;
     return cal(0,true,3);
}
```

#### 计算几何

```
struct Point{
    double x,y;ll id;
    Point(double _x = 0,double _y = 0):x(_x),y(_y){
};
bool operator < (const Point & a, const Point & b){
    return (a.x < b.x) \parallel (a.x == b.x && a.y < b.y);
}
int dcmp(double x){if (fabs(x)<eps)return 0;else return x<0?-1:1;}
Point operator + (Point a, Point b){return Point(a.x+b.x,a.y+b.y);}
Point operator - (Point a, Point b){return Point(a.x-b.x,a.y-b.y);}
Point operator * (Point a,double b){return Point(a.x*b,a.y*b);}
Point operator / (Point a,double b){return Point(a.x/b,a.y/b);}
double operator * (Point a,Point b){return a.x*b.y-a.y*b.x;}
bool operator == (Point a,Point b){return dcmp(a.x-b.x) == 0 && dcmp(a.y-b.y) == 0;}
double Dot(Point a,Point b){return a.x*b.x+a.y*b.y;} //点积
double Length(Point a){return sqrt(Dot(a,a));}
double Cross(Point a,Point b){return a.x*b.y-a.y*b.x;} //叉积
inline double Length(Point a){
    return sqrt(a.x*a.x + a.y*a.y);
}
inline double Length2(Point a){
    return a.x*a.x + a.y*a.y;
//单位化向量 , 若是零向量直接返回
Point unit(Point a){
    double I = Length(a);
    if(I < eps) return a;
    return Point(a.x/l,a.y/l);
}
//求向量 A 的左转法向量
Point normal(Point a){
    return Point(-a.y,a.x);
}
//求单位左转法向量, 调用前请保证 A 不是零向量
Point unitNormal(Point a){
    double I = Length(a);
    return Point(-a.y/l,a.x/l);
}
inline double Angle(Point a, Point b){
    return acos(a * b / Length(a) / Length(b));
```

```
}
//有向面积
double Area2(Point a,Point b,Point c){
    return Cross(b-a,c-a);
}
Point rotate(Point a, double rad){
    return Point(a.x*cos(rad)-a.y*sin(rad), a.x*sin(rad)+a.y*cos(rad));
}
//不损失精度判断线段规范相交(不含端点)
//若要判断线段是否有点在多边形内部,最好缩多边形,判任一公共点,
//或者把线段端点往里缩一下,同时取中点,check 一下这三个点是不是在多边形内部
bool isSegmentsIntersection(Point A,Point B,Point C,Point D){
    //跨立试验
    if(dcmp(Cross(C-A,D-A) * Cross(C-B,D-B)) >= 0) return false;
    if(dcmp(Cross(A-C,B-C) * Cross(A-D,B-D)) >= 0) return false;
    //快速排斥试验
    if(dcmp(min(max(A.x,B.x),max(C.x,D.x)) - max(min(A.x,B.x),min(C.x,D.x))) < 0) return false;
    if(dcmp(min(max(A.y,B.y),max(C.y,D.y)) - max(min(A.y,B.y),min(C.y,D.y))) < 0) return false;
    return true;
//点在线段上(//不含端点)
bool isPointOnSegment(Point P,Point a,Point b){
    if(P == a || P == b) return true;
    //if(p == a || p == b) return false;
    return dcmp(Cross(a-P,b-P)) == 0 && dcmp((a-P)*(b-P)) < 0;
}
//判断两条线段是否有公共点
bool isSegmentsCrash(Point A,Point B,Point C,Point D){
    if( isPointOnSegment(A,C,D) || isPointOnSegment(B,C,D) ||
        isPointOnSegment(C,A,B) || isPointOnSegment(D,A,B)) return true;
    if(dcmp(Cross(B-A,D-C)) == 0) return false;//共线
    return isSegmentsIntersection(A,B,C,D);//判断线段规范相交
Point midPoint(Point a,Point b){
    return Point((a.x+b.x)*0.5,(a.y+b.y)*0.5);
//-----线段相关内容------线段相关内容------
//有向直线
struct Line{
    Point p1,p2;//直线上两点,从 p1 到 p2, 左边是半平面
    double ang;//极角,从x正半轴转到 v 所需的角(弧度)
    Line(){}
    Line(Point p1, Point p2):p1(p1),p2(p2){
        ang = atan2(p2.y-p1.y, p2.x-p1.x);
```

```
}
    bool operator < (const Line& L) const{ //半平面交需要的排序函数
        return ang < L.ang;
    }
};
//直线相交,使用前保证有唯一交点, cross(v,w)非 0
Point getLineIntersection(Point A, Point B, Point C, Point D){
    Point u = A - C, v = B - A, w = D - C;
    double t = Cross(w, u) / Cross(v, w);
    return A + v * t;
}
Point getLineIntersection(Line L1, Line L2){
    Point u = L1.p1 - L2.p1, v = L1.p2 - L1.p1, w = L2.p2 - L2.p1;
    double t = Cross(w, u) / Cross(v, w);
    return L1.p1 + v * t;
}
//点到直线距离
double distanceToLine(Point P,Line L){
    Point v1 = L.p2 - L.p1, v2 = P - L.p1;
    return fabs(Cross(v1,v2)) / Length(v1);//不取绝对值就是有向距离
//-----多边形相关内容------
typedef vector<Point> polygon;
//调用时最好 zoom(poly, eps*1000) 或者 zoom(poly, sqrt(eps))
void zoom(polygon& poly, double rate){
    int n = poly.size();
    vector<Point> tmp;
    for(int i = 0;i < n;i++) tmp.push_back(unitNormal(poly[i] - poly[(i-1+n)\%n]));
    for(int i = 0;i < n;i++){
        poly[i] = poly[i] + ((tmp[i] + tmp[(i+1)%n]) * rate);
    }
}
struct Circle{
    Point o;
    double r;
    Circle(Point o,double r):o(o),r(r){}
    Point point(double rad){
        return Point(o.x+cos(rad)*r,o.y+sin(rad)*r);
    }
};
//给定两点作为直径获取圆
Circle getCircle(Point a,Point b){
    return Circle((a+b)*0.5,Length(a-b)*0.5);
```

```
}
//给予三个点, 求外接圆
Circle Getcir(Point A,Point B,Point C){
    double a = 2*(B.x - A.x);
    double b = 2*(B.y - A.y);
    double c = (B.x*B.x+B.y*B.y) - (A.x*A.x+A.y*A.y);
    double d = 2*(C.x-B.x);
    double e = 2*(C.y-B.y);
    double f = (C.x*C.x + C.y*C.y) - (B.x*B.x + B.y*B.y);
    double x = (b*f-e*c)/(b*d-e*a);
    double y = (d*c-a*f)/(b*d-e*a);
    double r = sqrt((x-A.x)*(x-A.x) + (y-A.y)*(y-A.y));
    Point ans(x,y);
    return Circle(ans,r);
}
//包含三个点的面积最小的圆(注意,不是外接圆)
Circle getMinCircle(Point a,Point b,Point c){
    if(dcmp(Cross(b-a,c-a)) == 0){
        //三点共线
        if (dcmp(Length(a-b)+Length(b-c)-Length(a-c))==0) return getCircle(a,c);
        if (dcmp(Length(b-a)+Length(a-c)-Length(b-c))==0) return getCircle(b,c);
        if (dcmp(Length(a-c)+Length(c-b)-Length(a-b))==0) return getCircle(a,b);
    } else{
        if((b-a)*(c-a) \le 0) return getCircle(b,c);
        if((a-b)*(c-b) \le 0) return getCircle(a,c);
        if((a-c)*(b-c) \le 0) return getCircle(a,b);
        Point m1 = midPoint(a,b), m2 = midPoint(a,c);
        Line L1 = Line(m1, m1 + normal(b-a));
        Line L2 = Line(m2,m2 + normal(c-a));
        Point o = getLineIntersection(L1,L2);
        return Circle(o,Length(a-o));
    }
}
//点在圆内(不含边界是<)
bool pointInCircle(Point a,Circle c){
    return dcmp(Length2(a-c.o)-c.r*c.r) <= 0;
}
/*
Andrew 算法基于水平序求凸包
输入点的数组 p,点的个数 n,布尔数组为 1 表示跳过该编号的点;返回凸包点的个数,凸
包的点存在 ch 数组里
直线上的点也要算的话,把两个<=改成<
精度要求高时建议使用三态函数
warning: 下标从 0 开始
```

```
warning: 如果允许计算直线上的多个点,同时可以会是退化多边形的话,用下面的另一个版
本
*/
int convexHull(Point* p,bool* check,int n,Point* ch,polygon& poly){
    sort(p,p+n);
    int m = 0;
    for(int i = 0;i < n;i++){
         if(check[p[i].id]) continue;
         while(m > 1 \&\& Cross(ch[m-1]-ch[m-2], p[i]-ch[m-2]) <= 0) m--;
         ch[m++] = p[i];
    }
    int k = m;
    for(int i = n-2;i >= 0;i--){
         if(check[p[i].id]) continue;
         while(m > k && Cross(ch[m-1]-ch[m-2], p[i]-ch[m-2]) <= 0) m--;
         ch[m++] = p[i];
    }
    if(m > 1) m--;
    poly.clear();
    for(int i = 0; i < m; i++) poly.push_back(ch[i]);
    return m;
}
/*
版本 2, 用于处理特殊情况。
特殊情况: 需要计算直线上的点, 同时可能会有退化成直线的多边形
int convexHull(Point* p,bool* check,int n,Point* ch,polygon& poly){
    sort(p,p+n);
    int m = 0, st = n;
    bool vis[n] = \{0\};
    for(int i = 0;i < n;i++){
         if(check[p[i].id]) continue;
         st = min(st,i);
         while(m > 1 \&\& Cross(ch[m-1]-ch[m-2], p[i]-ch[m-2]) <= 0) vis[ch[--m].id] = false;
         vis[p[i].id] = true;
        ch[m++] = p[i];
    }
    int k = m;
    for(int i = n-2;i >= 0;i--){
         if(check[p[i].id]) continue;
         if(i != st && vis[p[i].id]) continue;
         while(m > k && Cross(ch[m-1]-ch[m-2], p[i]-ch[m-2]) <= 0) m--;
         ch[m++] = p[i];
    }
```

```
if(m > 1) m--;
    poly.clear();
    for(int i = 0;i < m;i++) poly.push_back(ch[i]);</pre>
    return m;
}
//点 p 在直线 L 的左边(不包括线上) 修改 >=
bool onLeft(Point p,Line L){
    return Cross(L.p2, p-L.p1) > 0;
}
//半平面交,不能计算退化的多边形。但是可以将每个半平面略微扩大求得交为单点或者线
的情况
//返回半平面交后的多边形的顶点数,多边形存在 poly 种
int halfplaneIntersection(Line* L,int n,Polygon& poly){
                             //按极角排序
    sort(L,L+n);
                             //双端队列指针
    int first, last;
    Point *p = new Point[n]; //p[i]为 q[i]和 q[i+1]的交点
    Line *q = new Line[n];
                             //双端队列
    q[first=last=0] = L[0];
                             //双端队列初始化
    for(int i = 1; i < n; i++){
        while(first < last && !onLeft(p[last-1], L[i])) last--;</pre>
        while(first < last && !onLeft(p[first], L[i])) first++;</pre>
        q[++last] = L[i];
        if(fabs(Cross(q[last].v, q[last-1].v)) < eps){</pre>
            //两向量平行且同向, 取内侧的一个
            last--;
            if(onLeft(L[i].P, q[last])) q[last] = L[i];
        }
        if(first < last) p[last-1] = getLineIntersection(q[last-1],q[last]);</pre>
    while(first < last && !onLeft(p[last-1], q[first])) last--;</pre>
    //此处要注意,若可能会出现无界区域,应在运行前手动加入四个特殊半平面将区域框
起来
    //删除无用平面(*)
    if(last - first <= 1) return 0; //空集
    p[last] = getLineIntersection(q[last],q[first]); //计算首尾两个半平面的交点
    //从双端队列把答案复制到 poly 中
    poly.size = last - first + 1;
    poly.ps.clear();
    for(int i = first;i <= last;i++) poly.ps.pb(p[i]);</pre>
    return poly.size;
}
*/
//二维最小圆覆盖, 随机增量法
```

```
Circle minCircle(Point *p, int n){
     random_shuffle(p,p+n);
     Circle cur = Circle(p[0],0);
     for(int i = 1; i < n; i++){
          if(pointInCircle(p[i],cur)) continue;
          cur = Circle(p[i],0);
          for(int j = 0; j < i; j++){
               if(pointlnCircle(p[j],cur)) continue;
               cur = getCircle(p[i],p[j]);
               for(int k = 0; k < j; k++){
                    if(pointlnCircle(p[k],cur)) continue;
                    cur = getMinCircle(p[i],p[j],p[k]);
               }
          }
     }
     return cur;
}
struct Point{
     II x,y,id;
     Point(II _x = 0,II _y = 0):x(_x),y(_y){}
};
bool operator < (const Point & a, const Point & b){
     return (a.x < b.x) \parallel (a.x == b.x && a.y < b.y);
}
Point operator-(Point a, Point b){
     return Point(a.x-b.x, a.y-b.y);
}
Point operator+(Point a,Point b){
     return Point(a.x+b.x, a.y+b.y);
}
Il operator*(Point a,Point b){
     return a.x*b.x + a.y*b.y;
Point operator*(Point a,ll b){
     return Point(a.x*b, a.y*b);
inline II Cross(Point a, Point b){
     return a.x*b.y - a.y*b.x;
inline double Length(Point a){
     return sqrt((double)(a*a));
inline double Angle(Point a, Point b){
```

```
return acos((double)(a * b) / Length(a) / Length(b));
}
//不损失精度判断线段规范相交(不含端点)
/若要判断线段是否有点在多边形内部, 最好缩多边形, 判任一公共点,
//或者把线段端点往里缩一下,同时取中点,check 一下这三个点是不是在多边形内部
bool isSegmentsIntersection(Point A,Point B,Point C,Point D){
   //跨立试验
   if(Cross(C-A,D-A) * Cross(C-B,D-B) >= 0) return false;
   if(Cross(A-C,B-C) * Cross(A-D,B-D) >= 0) return false;
   //快速排斥试验
   if(min(max(A.x,B.x),max(C.x,D.x)) < max(min(A.x,B.x),min(C.x,D.x))) return false;
   if(min(max(A.y,B.y),max(C.y,D.y)) < max(min(A.y,B.y),min(C.y,D.y))) return false;
   return true;
}
//点在线段上(//不含端点)
bool isPointOnSegment(Point P,Point a,Point b){
   if(P == a || P == b) return true;
   //if(p == a || p == b) return false;
   return Cross(a-P,b-P) == 0 && (a-P)*(b-P) < 0;
}
//判断两条线段是否有公共点
bool isSegmengtsCrash(Point A,Point B,Point C,Point D){
   if( isPointOnSegment(A,C,D) || isPointOnSegment(B,C,D) ||
       isPointOnSegment(C,A,B) || isPointOnSegment(D,A,B)) return true;
   if(Cross(B-A,D-C) == 0) return false;//共线
   return isSegmentsIntersection(A,B,C,D);//判断线段规范相交
}
旋转卡壳
多边形内接最大三角形
圆相关的计算
分治半平面交
平面区域
立体几何
三维凸包
自适应辛普森积分
多圆面积并/面积交
维诺图
三角剖分
1.有多边形阻挡的情况下, 求两点是否能直接连线, 最好将多边形向内缩很小的距离. 然后
判断线段是否有任一公共点(不是判规范相交)
```