On the adaptation of in-context learners for system identification

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SYSID 2024
17-18 July 2024, Boston, USA

Standard system identification/supervised machine learning

1. Collect dataset $\mathcal{D} = (u_{1:N}, y_{1:N})$ of input/outputs from system S.
2. Apply an algorithm to estimate a model $M(\hat{\theta})$ of S:

$$\hat{\theta} = \mathcal{A}(\mathcal{D}) \qquad \text{e.g. } \mathcal{A}(\mathcal{D}) = \arg\min_{\theta \in \Theta} \mathcal{L}(\mathcal{D}, M(\theta))$$

3. Make predictions/simulations using the model on new data:

$$\hat{y}^*_{1:M} = M(u^*_{1:M}; \hat{\theta})$$

Researchers keep on improving learning algorithms and model structures. Can we automate this process? Can we learn the learning algorithm itself?

Meta learning tries to answer this question.

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Our meta learning setting

- We have an infinite stream of datasets from a distribution $p(\mathcal{D})$:

$$\{\mathcal{D}^{(i)} = (u_{1:N}^{(i)}, y_{1:N}^{(i)}),\ i = 1, 2, \dots, \infty\}$$

- $\mathcal{D}^{(i)}$ generated by random system $S^{(i)}$ and input realization $u_{1:N}^{(i)}$
- Different but related to each other. There's a learnable structure!

Can we get better at identifying $S^{(i)}$ as we observe more datasets $\mathcal{D}^{(j)}$?

- $p(\mathcal{D})$ may be a physical simulator where we can change settings
- The learned algorithm could then be applied to real data

Meta learning from a finite collection would also be interesting...

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In-context learning

Many meta learning strategies around. Here focus on in-context learning.

- Transformers are expressive as a programming language
- We make Transformers behave like algorithms. We provide:
  - A context, namely an input/output sequence of a system
  - A task, like predicting the next output or simulating for more steps
- The Transformer must learn to identify the system to solve the task!

Context + task may be seen as a prompt to a Large Language Model, which can then continue the word sequence in an optimal way.

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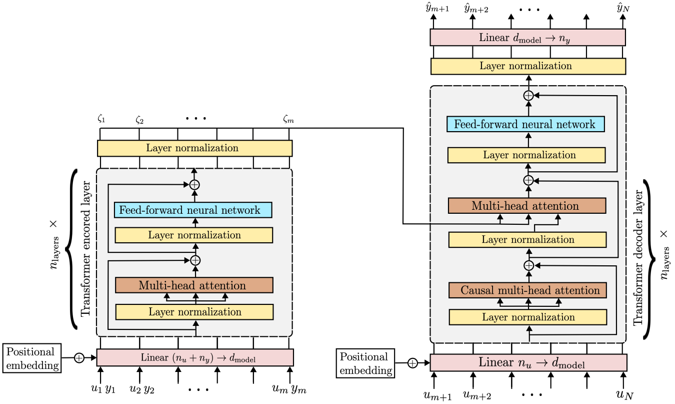
Multi-step simulation

Meta-model trained to predict $\hat{y}_{m+1:N} = \mathcal{M}_\phi(u_{1:m}, y_{1:m}, u_{m+1:N})$

- Full I/O sequence $(u_{1:m}, y_{1:m})$ characterizes the dynamics (context)
- Input squence $u_{m+1:N}$ defines the simulation objective (task)



- The Transformer $\mathcal{M}_\phi$ becomes a meta model of the system class!
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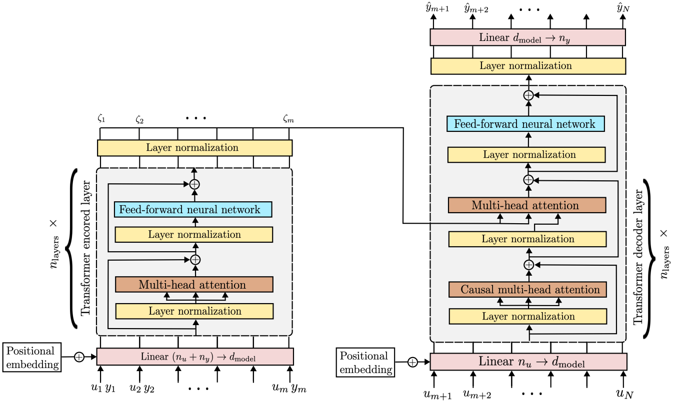

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Meta model training

Meta model trained in a standard supervised learning setting:

$$\hat{\phi} = \arg\min_{\phi} \mathcal{L}_{\rm sim}(\phi)$$

$$\begin{aligned}\mathcal{L}_{\rm sim}(\phi) &= \mathbb{E}_{p(\mathcal{D})}\left[\left\|y_{m+1:N} - \mathcal{M}_{\phi}(u_{1:m}, y_{1:m}, u_{m+1:N})\right\|^2\right] \\ &\approx \frac{1}{b}\sum_{i=1}^{b}\left\|y^{(i)}_{m+1:N} - \mathcal{M}_{\phi}(u^{(i)}_{1:m}, y^{(i)}_{1:m}, u^{(i)}_{m+1:N})\right\|^2\end{aligned}$$

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Previous experiments - System classes

One-step prediction and multi-step simulation on two system classes:

Linear Time Invariant (LTI):

In state-space form, order ≤ 10

$$x_{k+1} = Ax_k + Bu_k$$
$$y_{k+1} = Cx_k$$

- Random system matrices
- A constrained to be stable

Wiener-Hammerstein (WH):

$$\mathbf{u} \rightarrow \boxed{G(z)} \rightarrow \boxed{F(\cdot)} \rightarrow \boxed{G(z)} \rightarrow \mathbf{y}$$

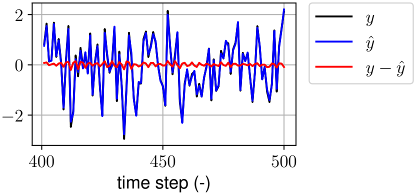
- Sequential LTI $\rightarrow F(\cdot) \rightarrow$ LTI
- Random LTI, order ≤ 5
- $F(\cdot)$: random feedforward NN.

- For both classes, input $u_{1:N}$ is a white Gaussian noise sequence.
- This defines a $p(\mathcal{D})$. We can generate infinite datasets!
- Each dataset from a different input/system realization!

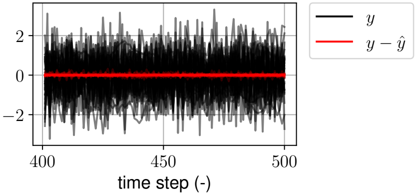
M. Forgione, F. Pura, D. Piga. In-context learning for model-free system identification. IEEE Control Systems Letters, 2023

Previous experiments - multi-step simulation results

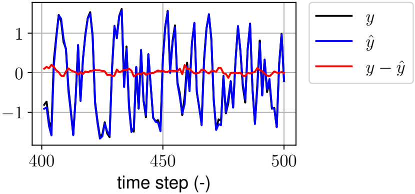
LTI: one sequence



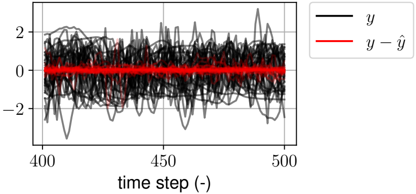
LTI: 256 sequences



WH: one sequence



WH: 32 sequences



New experiments - Generalization and adaptation

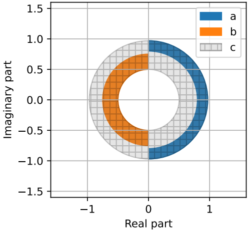
In the current contribution, we investigate:

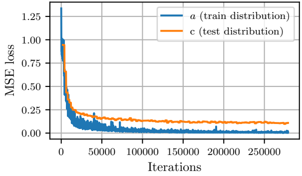
1. Generalization of a trained meta model on a new system class
2. Adaptation of a meta model to:
   1. A specific instance within the system class (specialization)
   2. A system instance outside of the system class
   3. New tasks. From 100- to 1000-step-ahead simulation

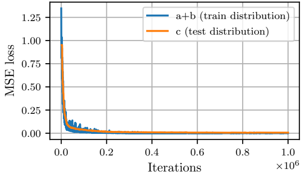
Generalization

Three LTI system classes. Order < 10, eigs with mag/phase in ranges:

- a $(0.8, 0.97)/(-\pi/2, \pi/2)$
- b $(0.5, 0.75)/(\pi/2, 3/4\pi)$
- c $(0.5, 0.97)/(-\pi, \pi)$







- Training only on a or b leads to a large generalization error on c.
- Training on a+b leads to a small generalization error on c
- Good generalization to unseen regions (gray area)

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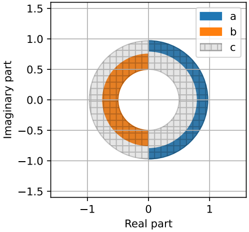

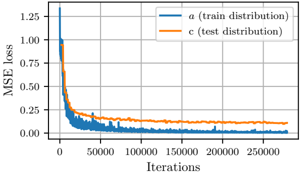

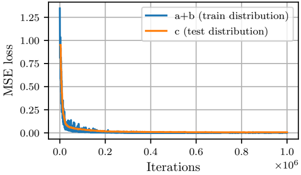

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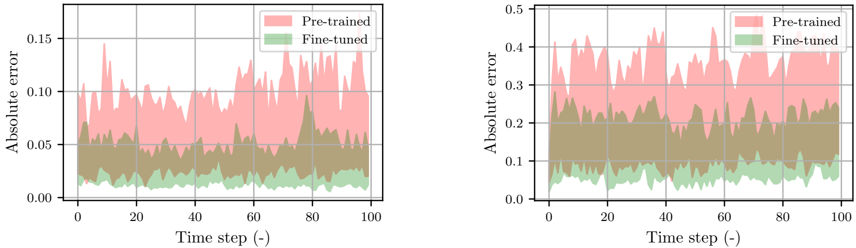
Adaptation to specific system instances

With just 140 sequences and $\approx$ 5 minutes of fine-tuning we can adapt, the WH meta model:

- To a specific instance of the WH class (left)
- To a PWH system instance, which is out of the WH class (right)



NOTE: training the WH meta model from scratch requires instead $\approx$ 1 day, 1M iterations, and 32M training instances!

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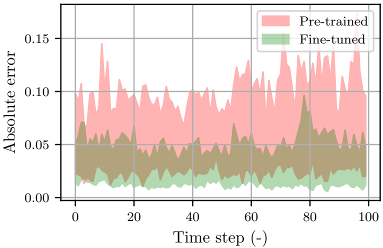

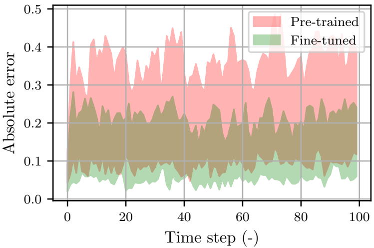

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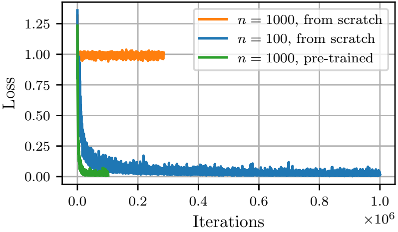
Adaptation to new tasks

We try to learn a 1000-step-ahead meta model for the WH class.



- Learning the 1000-step model from scratch (orange) seems hard. Loss is stuck at a high value
- Learning a 100-step model (blue) is possible. We also did it in our previous work...
- Starting from the 100-step model, the optimization of the 1000-step model converges very quickly (green line)!

Conclusions

- While training of a meta-model from scratch is computationally intensive, adaptation to new tasks and systems is fast and efficient.
- This paves the way to the development of foundation models for system identification.

Many possible research directions and applications including:

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Questions?

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