UNIT 1

GRAPHS

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1.0 INTRODUCTION

As a physics student, you will be involved in observing some physical phenomena and measuring physical quantities such as length, mass, time, temperature and current of electricity. These are called fundamental physical quantities.
You should also note that such fundamental quantities are used to obtain derived quantities such as force, velocity, pressure, density etc.

Through such measurements we are able to learn more about nature. We are able to measure some constants about nature for example acceleration due to gravity, the resistance of a wire, and specific heat capacity of a substance. We

determine these constants by identifying how two variables are related through the use of graphs.
Thus in your practical sessions in the laboratory, you will be developing practical skills in measuring physical quantities and the show the relationship between the two physical quantities to be measured through the use of graphs.

In this unit therefore, you will be introduced to the various types of graphs and how to use such graphs to obtain the physical constants required.

2.0 OBJECTIVES

At the end of this unit you should be able to:

- identify the importance of graphs in the study of physics;
- identify the two variables as physical quantities to be measured,
- show how the two variables are related either linearly or non-linearly,
- identify linear relationship from non-linear relationship;
- translate non-linear relationships to linear relationships,
- use graphs to determine physical constants through the use of slopes (gradients) or intercepts.

3.0 GRAPHS

In physics, we are always interested in knowing how two variables are related to each other.

There are two types of variables

- independent variable
- dependent variable

For example, if the value of quantity P depends on the value of quantity Q, then, Q is the independent variable while P is the dependent variable.

For example, the change in position of an object with respect to time. Time (t) is regarded as the independent variable while the change in position (x) is regarded as the dependent variable.

If you therefore measure time (t) of a moving object and its corresponding change in position, it is possible to show the relationship between x and t. That is it is possible to show how x relates with t by means of a graph.

A graph, therefore gives a vivid picture of how two physical quantities are related. The following are the advantages of graphs:

- if t is plotted against x the relationship between the two variables is shown I in a pictorial form,

- the corresponding values of x and t other than those actually observed or measured can be read from the graph,

- it is possible to verify a known or expected relationship between x and t and to determine the numerical values of constants occurring in it,

- in many cases, the relationship between x and t is of a simple form and is not known or is suspected before hand, a graph enables us to discover this form of relationship.

3.1 TYPES OF GRAPHS

The following are the types of graphs you will come across.

3.1.1 LINEAR GRAPH THROUGH THE ORIGIN

Consider the equation

$$y = mx$$

where, x is the independent variable
y is the dependent variable
m is a fixed number

When x=0 then y=0
When x = I then y = m
When x = 2 then y = 2m
When x = 3 then y = 3m and so on.

If we therefore plot the graph of y on the vertical axis and x on the horizontal axis we would obtain a graph as shown in fig. 3.1.

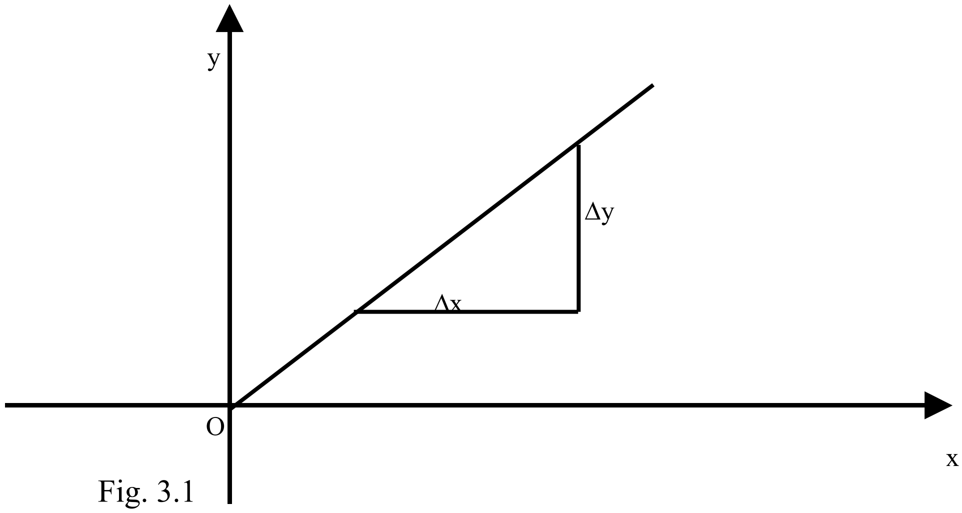


Fig. 3.1

We would have a straight line graph passing through the origin.
You will observe that the ratio of $\frac{y}{x}$ = m = a constant that is y is directly proportional to x.

The gradient or the slope of the graph is obtained from this ratio:

Increase in y - $\frac{\text{Increase in y}}{\text{Increase in x}} = m = \frac{\Delta y}{\Delta x}$

Thus a straight line through the origin shows that the quantity y is directly proportional to x. The constant of proportionality, m is given by the slope or the gradient of the graph.

3.1.2 LINEAR GRAPH NOT PASSING THROUGH THE ORIGIN

Consider another equation
$y= mx+b$
where, x = independent variable
y = dependent variable m and b are constants
When x = 0 then y = b
When x = 1 then y = m + b
When x=2 then y=2m+b
When x = 3 then y = 3m +band soon.
Again if we plot the graph of y on the vertical axis against x on the horizontal axis we also obtain a straight line. It will be noted that this time the graph does not pass through the origin as shown in fig. 3.2.

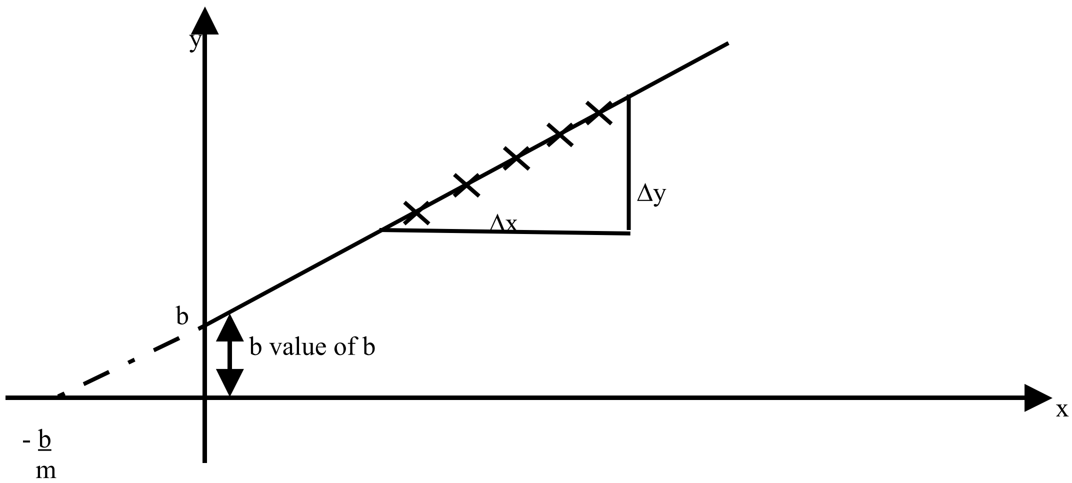


Fig. 3.2

You will also observe that when x = 0, y = b gives the intercept of the graph on the y-axis.

When y = 0 then $x = -\frac{b}{m}$ gives the intercept of the graph on the x-axis.

The relationship between y and x is also linear. It is however to be noted that this is not a direct relationship.

The slope or the gradient of the graph is given by

$\frac{\text{Increase in y}}{\text{Increase in x}} = \frac{\Delta y}{\Delta x} =$ while b is the intercept on the axes

If the line slopes downwards as shown in fig. 3.3, then the graph is said to have a negative slope.

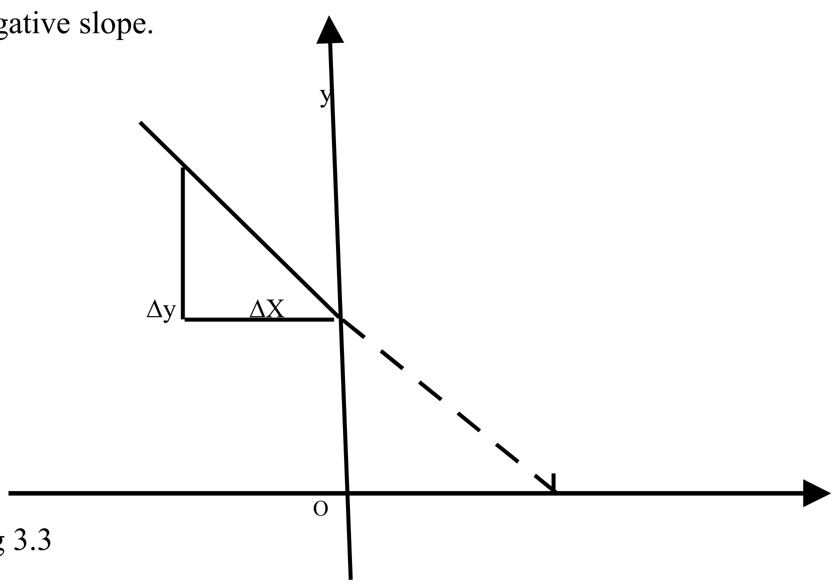


Fig 3.3

Then y = -mx + b .
In conclusion, therefore, whenever the graph of y against x is a straight line, this shows that the relationship between y and x is linear and can be expressed as

$$y=mx+b$$

If the graph passes through the origin when b = 0 then y is directly proportional to x.

$$\text{The slope or gradient} = \frac{\text{Increase in y}}{\text{Increase in x}} = m$$

3.1.3 REDUCING NON-LINEAR EQUATION TO LINEAR EQUATION

Consider the expression that relates the period of oscillation of a simple pendulum (T) with the length (l) of the pendulum.

$$T = 2\pi\, g \sqrt{\frac{\ell}{g}}$$

If T is plotted against $\sqrt{\ell}$ there will be no linear relationship between T and $\sqrt{\ell}$ But on squaring both sides of the equation we would obtain this expression

$$T^2 = 4\pi^2 \frac{\ell}{g}$$

$$\therefore T^2 = \frac{4\pi^2}{g} \ell$$

Plotting the graph of T^2 on the vertical axis and ℓ on the horizontal axis will give us a straight line passing through the origin as shown in fig. 3.4.

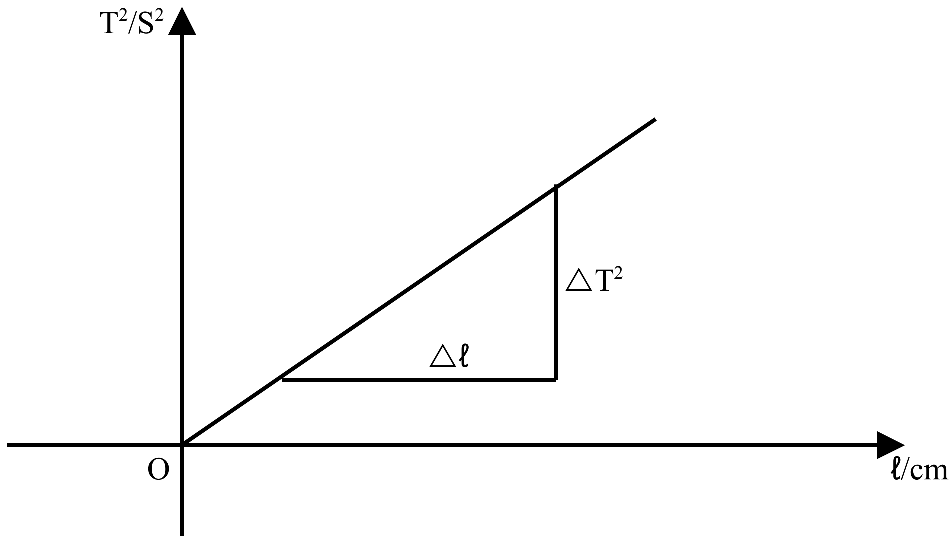


Fig. 3.4

The slope or the gradient of the graph is given as

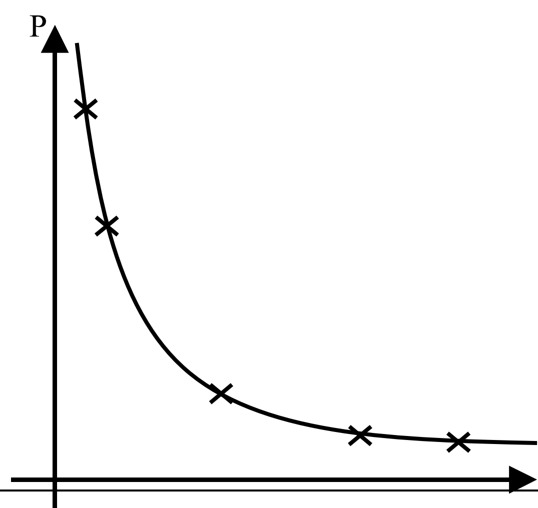
$$\frac{\Delta T^2}{\Delta \ell} = \frac{\text{Increase in } T^2}{\text{Increase in } \ell} = m = \frac{4\pi^2}{g}$$

The value of g, acceleration due to gravity may then be obtained, by substitution of values of known variables in the relation.

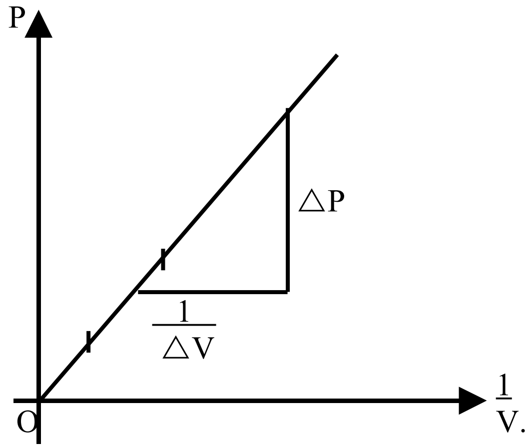
Thus we have reduced a non-linear relationship to a linear one. Another example of this reduction may be found in Boyle's law. Boyle's law shows the relationship between the pressure exerted on a given mass of gas (P) and its volume V provided the temperature of the gas is kept constant.

$$PV = K$$

If the values of P are plotted against the corresponding values of V, we do not obtain a linear relationship as shown in fig. 3.5 (i).



(i) O V.



(ii)

Fig. 3.5

By rearranging the expression we would obtain this

$$P = k \frac{1}{V}$$

When P is therefore plotted against $\frac{1}{V}$, the reciprocal of V, then we obtain a straight line passing through the origin as shown in fig. 3.5 (ii).
The gradient or the slope m of the graph is expressed as

$$\frac{\text{Increase in pressure}}{\text{Increase in the reciprocal of volume}} = \frac{\Delta P}{\frac{1}{\Delta V}}$$
$$= m$$
$$= K \text{ a constant}$$

The oscillation of a weighted spiral spring is another example.
There is a relationship between the period of oscillation T, the loaded mass M and the effective mass of the spiral spring m.

$$T = 2\pi \sqrt{\frac{M + m}{k}}$$

where, k is the spring constant.

We then reduce the expression to a linear one by squaring both sides of the equation.
T2

$$T^2 = 4\pi^2 \frac{(M+m)}{k}$$

$$\therefore T^2 = \frac{4\pi^2}{k} M + \frac{4\pi^2}{k} m$$ compare this equation with

$y = ax + b$ where $y \cong T^2$, $\frac{4\pi^2}{k} M \cong ax$ and $\frac{4\pi^2}{k} m = b$

By plotting the values of T^2 against the corresponding values of M, we would obtain a straight line graph which does not pass through the origin as shown in fig. 3.6.

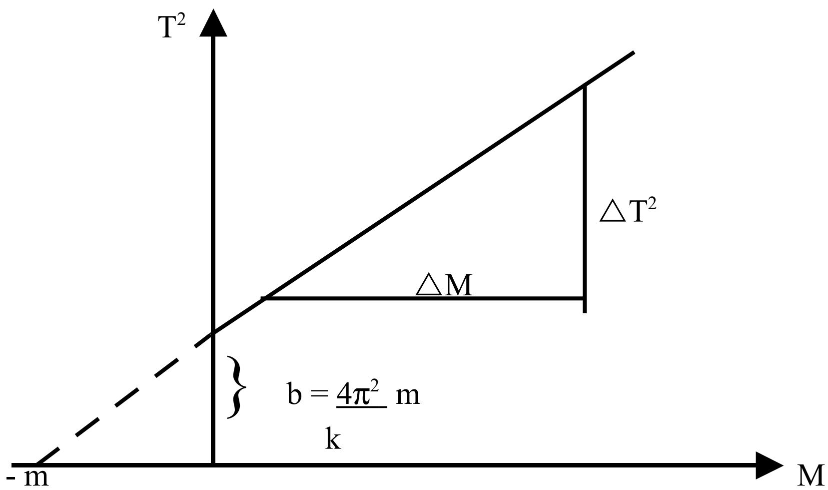


Fig. 3.6

The gradient or the slope of the graph is expressed as

$$\frac{\text{Increase in } T^2}{\text{Increase in } M} = \frac{\triangle T^2}{\triangle M} = \frac{4\pi^2}{k}$$

The intercept on the vertical axis is obtained when M = 0.

$$\therefore T^2 = \frac{4\pi^2}{k} m = b$$

The intercept on the horizontal axis is obtained when $T^2 = 0$.

$\therefore M = - m$

which gives us the effective mass of the spiral spring. Thus, through the knowledge of m, k, could be determined.

3.1.4 REDUCING TO A LINEAR EQUATION FROM UNKNOWN RELATIONSHIP

Suppose two physical quantities P and Q are related as

$P = kQ^n$

where the values of k and n are not known we can reduce the expression to a linear one by taking the logarithm of both sides of the equation.

$\log P = \log k + n \log Q$ or

$\log P = n \log Q + \log k$

compare it with

$y = nx + b$ where $n \log Q \cong nx$ and $\log k \cong b$.

Therefore if we plot log P on the vertical axis against log Q on the horizontal axis, we shall obtain a straight line graph as shown in fig. 3.7.

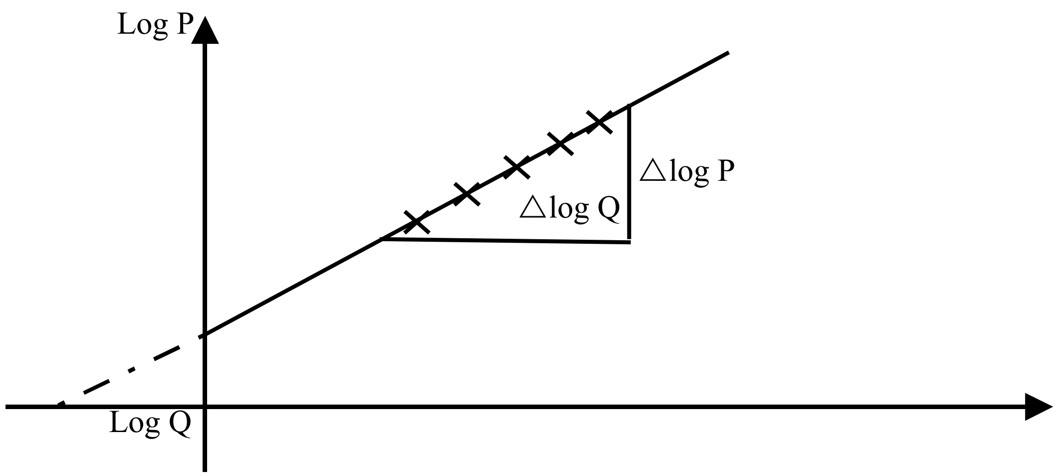


Fig. .3.7

The slope or the gradient of the graph is expressed as

$$\frac{\text{Increase in } \log P}{\text{Increase in } \log Q} \quad \frac{\triangle \log P}{\triangle \log Q} = n$$

When $\log Q = 0$ gives the intercept of the graph on the vertical axis

$\therefore \log P = \log k$

Thus the value of k is obtained.
You need to be careful when working with logarithm.

- If the value of P or Q is greater than l, log P is taken as positive.

For example, log 9 = 0.9542
log 90 = 1.9542
log 900= 2.9542 etc

- If the value of P or Q is less than 1 then you need to sensitise yourself on what value of log P or log Q you have to use. For example,

log 0.5 = $\bar{1}$.6990 = -1 + 0.6990 = -0.3010
log 0.006 = $\bar{3}$.7781 = -3 + 0.7781 = -2.22

If the intercept is -2.73
-2.73=-3+x
x=3-2.73=0.27
$\therefore$ -2.73 = $\bar{3}$ + 0.2700

The antilog of 0.2700 = 1.86
$\therefore$ Antilog of -2.73 = 1.86 x 10^{-3}

3.2 DETERMINATION OF GRADIENT OF NON-LINEAR GRAPH AT A POINT

From the equations
y = mx
y = mx +b
the highest power to which x is raised is 1 hence when y is plotted against x, we obtain a straight line graph.

But for equations such as
$y = ax^2$
y = ax + bx
$y = ax^2 + c$
they produce non-linear graphs because when you plot the values of y against x, we do not produce a straight line graph. You will observe that the graphs produced are usually parabolic because they are quadratic functions. The equations are described as being quadratic because the highest power to which x is raised is 2. Fig. 3 (i) – (vi) are

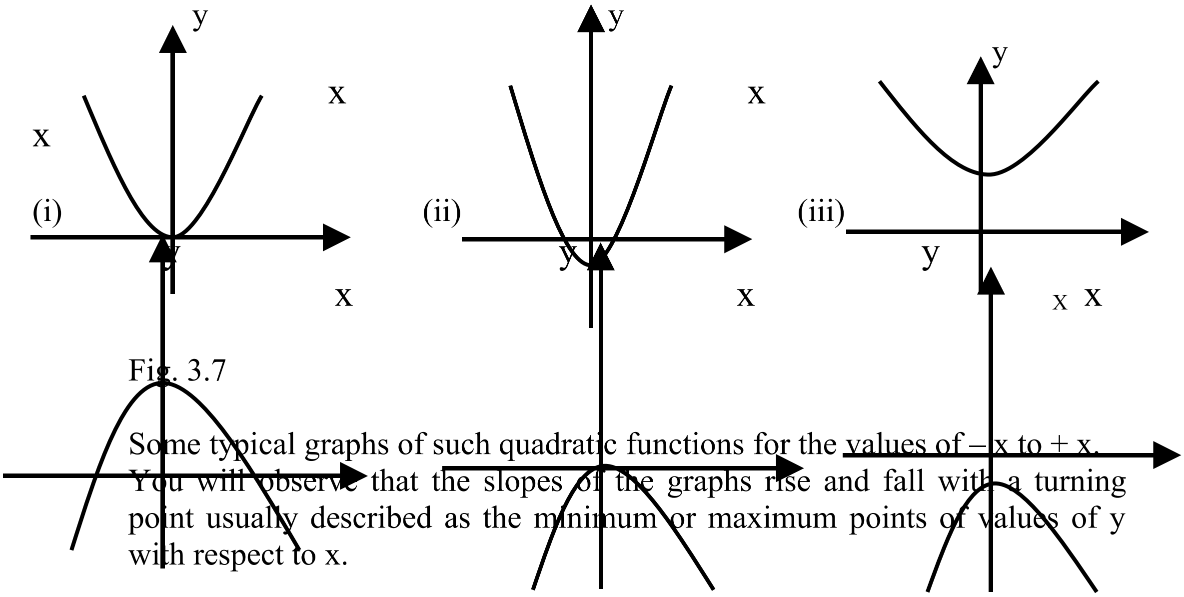


Fig. 3.7

Some typical graphs of such quadratic functions for the values of – x to + x. You will observe that the slopes of the graphs rise and fall with a turning point usually described as the minimum or maximum points of values of y with respect to x.

The slopes or gradients vary from one point of x to the other. At the minimum or maximum point, the gradient at that point is zero.

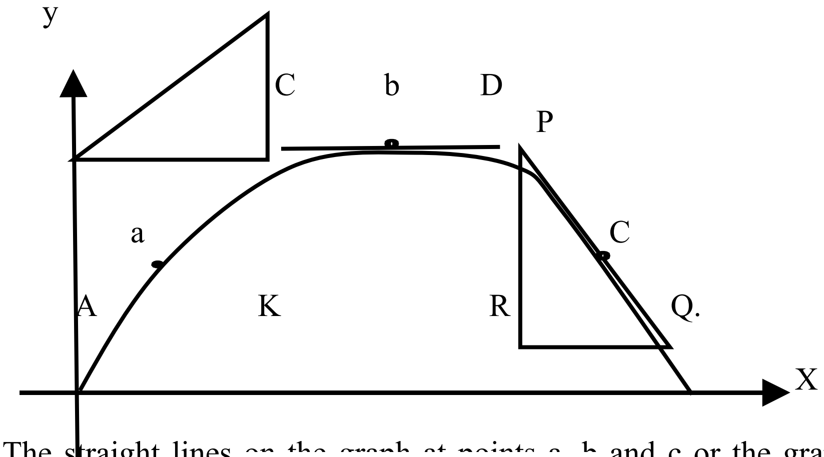
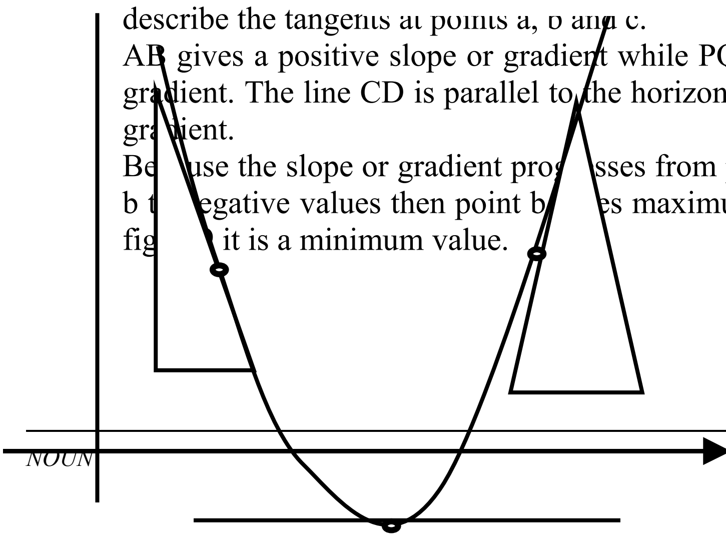


Fig. 3.8

The straight lines on the graph at points a, b and c or the graph in fig. 3.8 define the slopes [illegible] s. The lines AB, CD and PQ describe the tangents at points a, b and c.
AB gives a positive slope or gradient while PQ produces a negative slope or gradient. The line CD is parallel to the horizontal axis to produce no slope or gradient.
Because the slope or gradient progresses from positive values through zero at b to negative values then point b gives maximum value of y. Otherwise as in fig [illegible] it is a minimum value.



B Q

a C

K. A

P R

C D

b

Fig. 3.9

To obtain the gradient at a, we then produce a right-angled triangle on the tangent AB
such that AB forms the hypotenuse. Hence the gradient at point a is given as

$$\frac{BK}{AK}$$

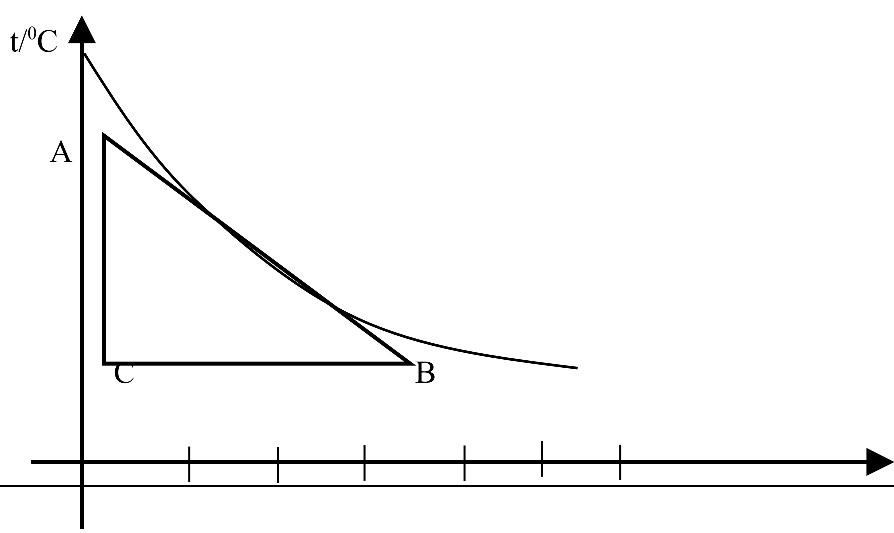
which must be interpreted to scale.

Displacement-time, velocity-time and Newton's cooling curves are examples of nonlinear graphs, which enable us to measure the rate of change of displacement, velocity and other quantities such as temperature with time.

Example

The table below shows the temperature of a cooling calorimeter at different times. Use the data to determine the rate of fall of temperature at 2 min.

Time in minutes	0	1	2	3	4	5	6
Temperature in °C	45.0	33.0	26.1	21.7	19.1	17.4	16.5



t/min

1 2 3 4 5 6

Fig 3.10

The gradient or slope is obtained from the tangent drawn on the graph at time t = 2 min.
The gradient is defined as $\frac{AC}{BC}$= negative which is about 5.6 °C per minute.
The negative sign shows that the temperature is falling.

Self Assessment Question

Discuss the advantages to be gained by a graphical representation of asset of physical measurements.

SOLUTION

See Text

4.0 CONCLUSION

Graphs are pictorial ways of showing how two physical measurements are related. Such physical measurements are regarded as variables. If the graph produced is a straight line, then the relationship is described as being linear. Then the equation that relates the two variables such as y and x are expressed as

$$y = mx \quad \text{or} \tag{1}$$

$$y = mx + c \tag{2}$$

where, x = independent variable
y = dependent variable
m = the slope or gradient of the slope
c = intercept on the y-axis when $x = 0$

The first equation tells us that the line passes through the origin while the second equation does not.

It is possible to reduce a linear graph from unknown relationship by taking the logarithm of the equation expressing the unknown relationship.

5.0 SUMMARY

At the end of this unit you have learnt,

- the importance of graphs in physics practicals
- that a straight line graph shows that there is a linear relationship between the two variables in question
- non-linear relationship can be reduced to a linear one by squaring the equation expressing the relationship between the two variables such as

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

- the slope of the graph is determined by the ratio of $\frac{\text{Increase of the Dependent Variable}}{\text{Increase of the Independent Variable}}$
- how to determine the slope of a non-linear graph at a given point
- non-linear graphs are usually in form of quadratic equations

6.0 TUTOR MARKED ASSIGNMENTS

EXERCISE 1

Plot the displacement-time graph for the following motion of an object and determine the velocity after 3 seconds of its motion.

Time/s	0	1	2	3	4	5
Displacement /m	0.0	4.0	12.0	36.0	64.0	100.0

EXERCISE 2

Plot the velocity-time graph for the following motion of an object and find the acceleration at t = 2 seconds.

Time in seconds	0	1	2	3	4
Velocity in cm/s	15	29.5	36	38	35

EXERCISE 3

The following values were obtained for the period of vertical oscillation for a spiral spring carrying different loads.

Load M/g	25	50	75	100	125
Period T/s	0.96	1.14	1.28	1.41	1.54

Suppose the formula relating T and M is

$$T = 2\pi\sqrt{\frac{M+m}{k}}$$

where m = effective mass of the spring and k = spring constant
Determine the values of m and k.

7.0 REFERENCES AND OTHER RESOURCES

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