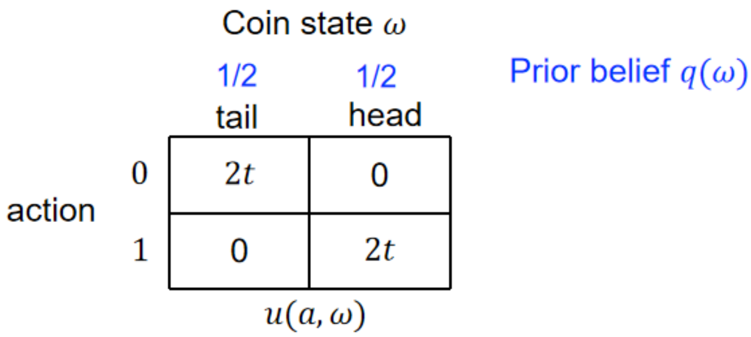
Lecture 11: The Valuation and Pricing of Information

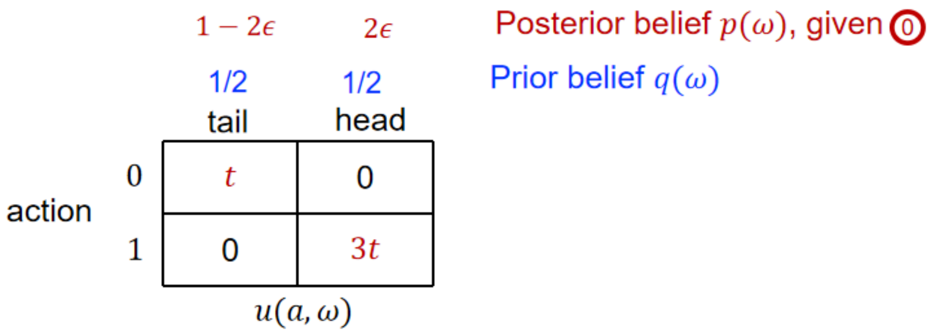
I.Economic Foundations of Value of Information

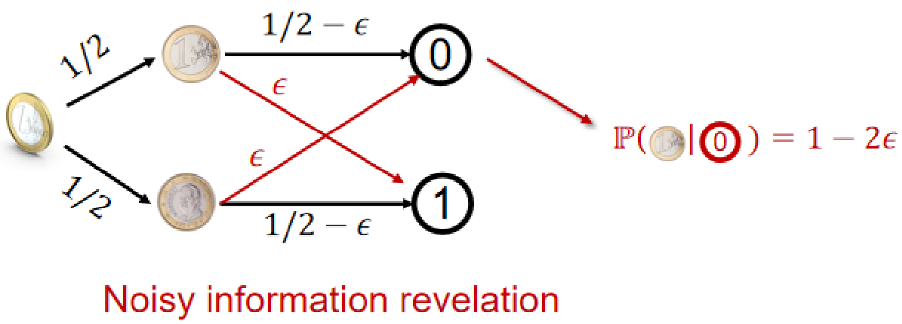


Without information, decision maker(DM) gets $\max_a \left[\mathbb{E}_{\omega\sim q} u(a,\omega)\right] = t$

With my (full) information, DM gets $\mathbb{E}_{\omega\sim q}[\max_a u(a,\omega)] = 2t$

Value of (full) info $= \mathbb{E}_{\omega\sim q}[\max_a u(a,\omega)] - \max_a \left[\mathbb{E}_{\omega\sim q} u(a,\omega)\right]$





Realistically, can think of 0 as noisy prediction of state ω (e.g., stock trend, purchase prob)

Question: What is the value of this noisy signal 0?

- Without knowing this signal, DM takes action 1
- With this signal 0, DM takes action 0(assuming ε very small)
- However, true distribution is the posterior p regardless

$$\text{Value of knowing } 0 = \mathbb{E}_{\omega \sim p}[u(0,\omega)] - \mathbb{E}_{\omega \sim p} u(1,\omega)$$

Definition(FK'19):Consider an arbitrary desicion making problem $u(a, \omega)$, suppose a signal updates the DM's belief about state ω from $q \in \Delta(\Omega)$ to $p \in \Delta(\Omega)$, the value of this signal is defined as

$$D^u(p;q) = \mathbb{E}_{\omega \sim p}[u(a^*(p),\omega)] - \mathbb{E}_{\omega \sim p} u(a^*(q),\omega)$$

Example 1:

- $a \in A = \Delta(\Omega) \to$ action is to pick a distribution over states
- $u(a,\omega) = \log a_\omega$
- Which action $a \in \Delta(\Omega)$ maximizes expected utility $\mathbb{E}_{\omega \sim p}[u(a,\omega)]$?

$$a^*(p) = p$$

$$D^u(p;q) = \sum_\omega p_\omega \log \frac{p_\omega}{q_\omega} \qquad \text{KL-divergence}$$

Example 2:

- $a \in A = \Delta(A) \to$ action is to pick a distribution over states
- $u(a,\omega) = -\left\| a - e_\omega \right\|^2$

$$a^*(p) = p$$

$$D^u(p;q) = \|p-q\|^2 \qquad \text{Squared distance}$$

Some obvious properties

- Non-negativity: $D^u(p;q) \geq 0$
- Null information has no value: $D^u(q;q) = 0$
- Order-invariant: if DM receives signal σ_1, σ_2, the order of receiving them does not affect final expected total value

In this case, we say $D(p;q)$ is a valid measure for value of information

Theorem 1(FK'19): Consider any $D(p;q)$ function. There exists a decision problem $u(a,\omega)$ such that

$$D^u(p;q) = \mathbb{E}_{\omega \sim p}[u(a^*(p),\omega)] - \mathbb{E}_{\omega \sim p} u(a^*(q),\omega)$$

if and only if $D(p;q)$ satisfies

- Non-negativity: $D^u(p;q) \geq 0$
- Null information has no value: $D^u(q;q) = 0$
- Order-invariant: if DM receives signal σ_1, σ_2, the order of receiving them does not affect final expected total value

Theorem 2(FK'19):

1. For any concave function H, its Bregman divergence is a valid measure for value of information.
2. Conversely, for any valid measure $D(p;q)$ for value of information,

   $$H(q) = \sum_\omega q^\omega D(e_\omega, q)$$

   is a concave function whosw Bregman divergence is $D(p;q)$

Why useful?

- Many functions - even natural ones like l_2 distance $\|p-q\|$- are not valid measures

- In fact, any metric is not valid, since metric cannot be a Bregman divergence
- There are efficient ways to tell whether a $D(p, q)$ is valid

II.Optimal Pricing of Information

III.Summary and Open Problems