50 Years of Keyfitz's Population Momentum
25 Years since Preston / Guillot (1997)

ROLAND RAU[1,2]

[1] University of Rostock [2] Max Planck Institute for Demographic Research

Series: "My favorite formal demographic paper"
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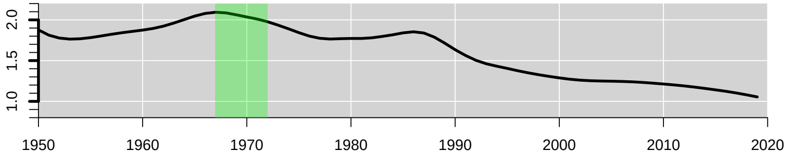
The era of:
· Paul Ehrlich (1968):
The Population Bomb

> *The battle to feed all of humanity is over. In the 1970s hundreds of millions of people will starve to death in spite of any crash programs embarked upon now. At this late date nothing can prevent a substantial increase in the world death rate . . .*

· Meadows et al. (1972):
The Limits to Growth

Starting point 50 years ago:

Nathan KEYFITZ (1971):
On the Momentum of Population Growth
Demography 8(1): 71–80



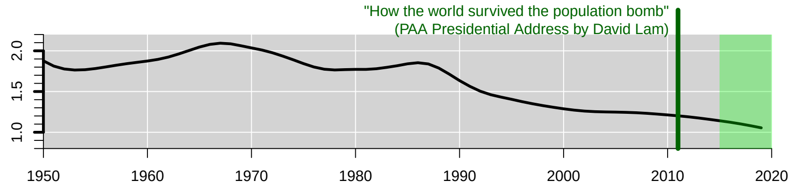
World Population Growth in %

...... fast forward 50 years

VIENNA YEARBOOK of Population Research 2023

The Vienna Yearbook of Population Research (VYPR) is seeking submissions for a Special Issue entitled

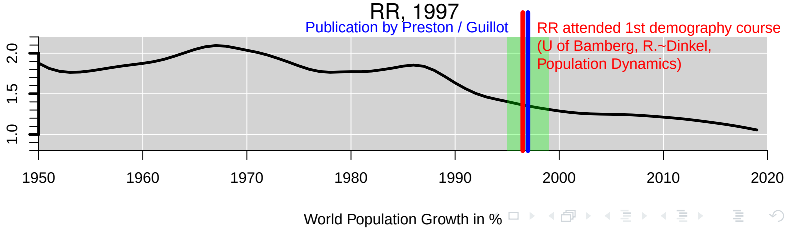
“The causes and consequences of depopulation”,



World Population Growth in %

Samuel H. PRESTON, Michel GUILLOT (1997):

Population dynamics in an age of declining fertility. *Genus*



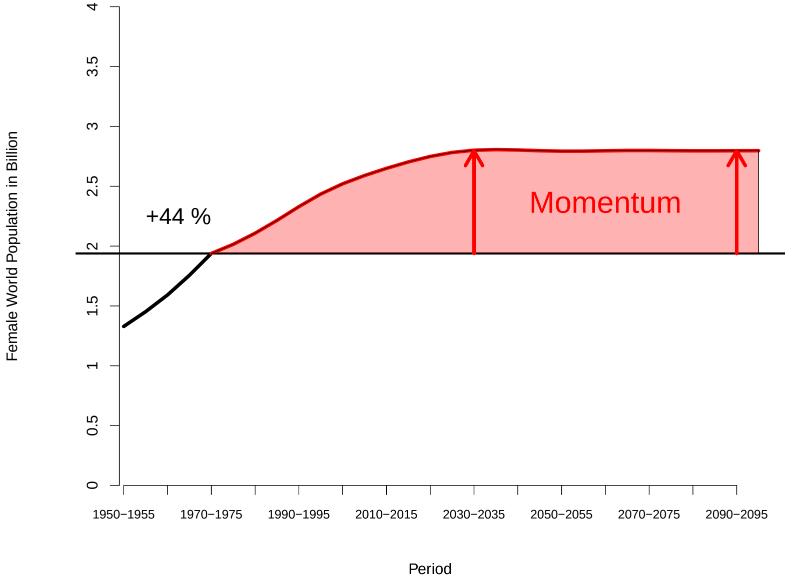
Assume a ***growing*** *population.*
Ask yourself this question:
What would happen to the size of this population if it was possible to instantly switch to a fertility regime that ensures stationarity?

continuous: $r > 0 \quad \rightarrow \quad r = 0$
discrete: $\lambda_1 > 1 \quad \rightarrow \quad \lambda_1 = 1$

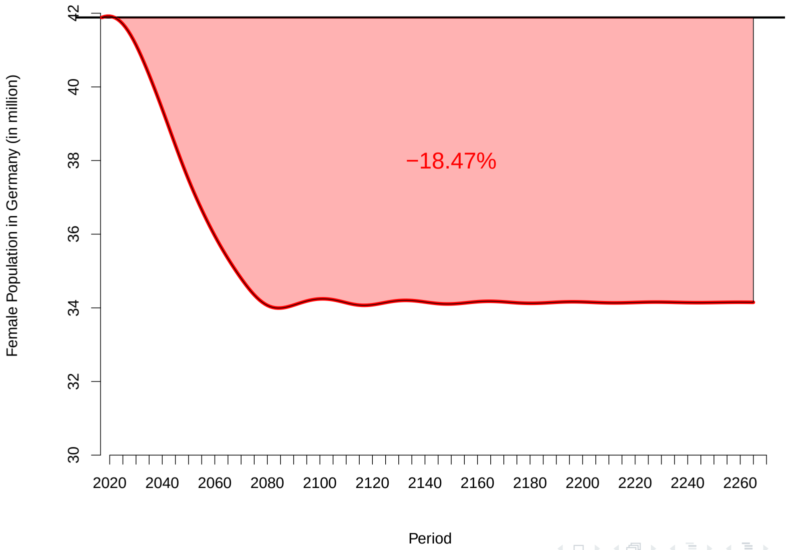
Or the opposite direction:
What would happen to the size of a ***shrinking*** *population if it was possible to instantly switch to a fertility regime that ensures stationarity?*

continuous: $r < 0 \quad \rightarrow \quad r = 0$
discrete: $\lambda_1 < 1 \quad \rightarrow \quad \lambda_1 = 1$

?


Female World Population in Billion
0
0.5
1
1.5
2
2.5
3
3.5
4
+44 %
Momentum
1950−1955
1970−1975
1990−1995
2010−2015
2030−2035
2050−2055
2070−2075
2090−2095
Period

. . . likewise for shrinking populations . . .



How can you estimate *Population Momentum*?

Obviously, via brute force . . . but is there an analytic way?

Keyfitz (1971) proposed such an equation:

net reproductive rate

life expectancy at birth

birth rate

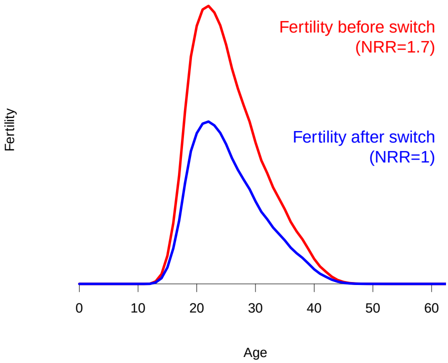
$$\left(\frac{b\ \mathring{e}_0}{r\ \mu}\right)\left(\frac{R_0 - 1}{R_0}\right)$$

growth rate

mean age at childbearing after fertility "switch"

I have to admit that I do not like the equation too much:

- it is complicated
- it requires the population to be already stable
- it assumes an instant fertility switch
- instant switch proportional at all ages
- does not provide more insight into the dynamics what's happening
- and I never managed to get meaningful numerical results.



I have to admit that I do not like the equation too much:

- it is complicated
- it requires the population to be already stable
- it assumes an instant switch
- instant switch proportional at all ages
- the components do not provide more insight into the dynamics what's happening (in my opinion)
- and I never managed to get meaningful numerical results.

But great research — as Keyfitz (1971) — does not close gaps.
It asks an interesting question & opens up new perspectives!
And this is definitely the case for Keyfitz' paper (see Kim and Schoen (1997), Li and Tuljapurkar (1999), Goldstein (2002) and many more!)

. . . then why Preston and Guillot (1997)?

$$\text{Momentum } M = \int_{0}^{\infty} \frac{c(a)}{c_s(a)} w(a)\, da$$

$c(a)$: Current age structure (before switch)
$c_s(a)$: Stationary age structure
$w(a)$: ???

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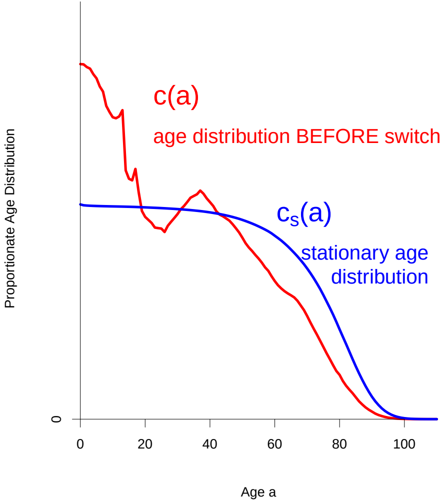

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$w(a)$: ???

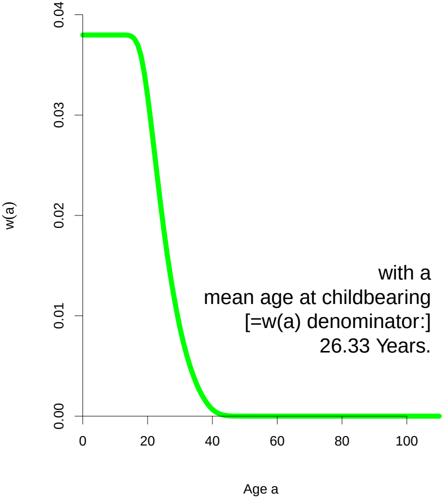
The third distribution [. . .], $w(a)$, is less familiar.

The numerator is the expected lifetime births that will occur above age a in the replacement-level fertility regime

$$\int_a^\infty p(y)\, m(y)\, dy$$

The denominator is [. . .] the mean age at birth in the stationary population.

Preston and Guillot (1997, p. 89)
formatting of quotation: RR



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Preston and Guillot (1997, p. 89)
formatting of quotation: RR

. . . and what is your contribution, Roland?

- My contribution: Implementation in R
- Discrete-time / matrix framework
- Caswell (2001, p. 104–105) provides also an equation for population momentum in a matrix framework. Unfortunately, I did not get any meaningful results (either I made a mistake or Caswell's equation also requires already a stable population before the switch(?!?)).
- Still assumes the instant switch to replacement-level fertility, proportional across all ages
- part of a function which estimates not only M but also
  - the long term growth rate (dominant eigenvalue, λ_1)
  - the long term age structure ($\lambda_1 \rightarrow$ right eigenvector w_1)
  - transient dynamics (period of oscillation, damping ratio, Keyfitz' Δ)
  - net reproductive rate (NRR, R_0)
  - generation time T
  - mean age at childbearing ($\bar{m}$, μ_1, $\bar{A}$)
  - life expectancy (at birth, remaining at age x)
  - reproductive value,
  - population momentum M
  - . . .

```
canada2020 <- stable.pop(country="CAN", year=2020, srb=1.07,
               HMD.user=my.HMD.user, HMD.pw=my.HMD.password,
               HFD.user=my.HFD.user, HFD.pw=my.HFD.password)
```

```
canada2020 <- stable.pop(country="CAN", year=2020, srb=1.07,
                 HMD.user=my.HMD.user, HMD.pw=my.HMD.password,
                 HFD.user=my.HFD.user, HFD.pw=my.HFD.password)
Error in stable.pop(country = "CAN", year = 2020, srb = 1.07, HMD.user
  No fertility data available for selected year.
>
```

```
canada2019 <- stable.pop(country="CAN", year=2019, srb=1.06,
                 HMD.user=my.HMD.user, HMD.pw=my.HMD.password,
                 HFD.user=my.HFD.user, HFD.pw=my.HFD.password)
```

```
> canada2019$annual.growth.rate
[1] 0.9889334
> canada2019$m.bar
[1] 31.17591
> canada2019$m.bar.HFD.official
[1] 31.18
>
```

```
canada2019 <- stable.pop(country="CAN", year=2019, srb=1.06,
                 HMD.user=my.HMD.user, HMD.pw=my.HMD.password,
                 HFD.user=my.HFD.user, HFD.pw=my.HFD.password)
```

```
> canada2019$Momentum.bruteforce
[1] 0.9698941
> canada2019$Momentum.PrestonGuillot
[1] 0.9698213
> canada2019$MomentumRecursive
[1] 0.9698213
>
```

not only an implementation for the HMD-/HFD-countries
but also for the 5×5 year-/age-data from the World Population Prospects (2019 Revision):

```
> world19901995 <- create.matrix(country=900, period="1990-1995")
> stable.world19901995 <- stable.pop(world19901995)
> stable.world19901995$Momentum.PrestonGuillot
[1] 1.436754
> stable.world19901995$Momentum.bruteforce
[1] 1.437138
> stable.world19901995$Momentum.recursive
[1] 1.436754
>
```

not only an implementation for the HMD-/HFD-countries but also for the 5×5 year-/age-data from the World Population Prospects (2019 Revision):

```
> Search.Code("World")
                    Country CountryCode
1                     WORLD         900
10 World Bank income groups        1802
> Search.Code("uNITed")
                         Country CountryCode
44   United Republic of Tanzania         834
106         United Arab Emirates         784
165 United States Virgin Islands         850
231               United Kingdom         826
255     United States of America         840
> Search.Country(276)
    Country CountryCode
249 Germany         276
>
```

not only an implementation for the HMD-/HFD-countries but also for the 5×5 year-/age-data from the World Population Prospects (2019 Revision):

```
> germany20152020 <- create.matrix(country=276, period="2015-2020")
> stable.germany20152020 <- stable.pop(germany20152020)
> stable.germany20152020$Momentum.PrestonGuillot
[1] 0.8150245
> stable.germany20152020$Momentum.bruteforce
[1] 0.813732
> stable.germany20152020$Momentum.recursive
[1] 0.8150244
>
```

Estimates for M for 2015–2020, WPP 2019

Continent	$\mathbf{M}$
Africa	1.56
Asia	1.22
Europe	0.87
Latin America & the Caribbean	1.29
Northern America	1.03
Oceania	1.22
World	**1.26**

Illustrative, preliminary (!) estimates

Estimates for M for 2015–2020, WPP 2019

Africa

Country	M
Africa	**1.56**
DR Congo	1.64
Nigeria	1.46
Rwanda	1.62
South Africa	1.26
Uganda	1.71

Asia

Country	M
Asia	**1.22**
Bangladesh	1.42
China	0.97
India	1.31
Saudi Arabia	1.41
Thailand	0.97
Turkey	1.29

Illustrative, preliminary (!) estimates

Europe

Country	M
Europe	**0.87**
Austria	0.87
Belgium	0.95
Czechia	0.83
Denmark	0.96
Germany	0.82
Italy	0.78
Netherlands	0.94
Spain	0.85
Sweden	0.99
Switzerland	0.92
United Kingdom	0.98

Country	M
Northern America	**1.03**
Canada	0.97
USA	1.04

Country	M
Latin America & the Caribbean	**1.29**
Argentina	1.25
Brazil	1.19
Ecuador	1.44
Mexico	1.34

Country	M
Oceania	**1.22**
Australia	1.09

Illustrative, preliminary (!) estimates

So why did you choose this topic and what should we take home?

- It is one of the topics, which ignited my love for population dynamics / demography. A love-relationship for 25 years!
- The concept of the Population Momentum is a nice combination of formal demography with policy relevance.
- I don't mind that the original idea does not assume a gradual transition but an instant switch: It illustrates how much a population would still grow / shrink despite an instant switch to the replacement level:

> *"In the absence of the rather unpredictable factor migration, the German population will shrink by 18% — even if it was possible to switch to replacement-level fertility from one year to the next!"*

- Weakness of my implementation I: The proportional shift I used Keyfitz (1971, as introduced by) is debatable.
- Weakness of my implementation II: My code and my estimates refer to female populations, i.e. the classic approach of stable population theory (unlike Preston and Guillot (1997) who show momentum for both sexes combined).
- Weakness of my implementation III: a lot more, probably
- Plan for February: Publish an R package with the code (either on CRAN or on a git repository).
  Please send me an email if you don't see it by the end of February!

Why formal demography?

"The study of quantitative demography or population analysis can be undertaken from two points of view or by two methods the empirical method and the rational or formal method.
The rational method is possible through the fact that between the various demographic characteristics there exist certain necessary relations, that is, relations imposed by the laws of physics or the laws of logic.
But the rational method is not only possible, it is indispensable if we wish to obtain an entirely satisfactory understanding of population phenomena. Undoubtedly the ideal process is to cultivate both methods side by side. According to our predilections, the empirical data will then be for us concrete illustrations of the abstract principles that mainly interest us; or, on the contrary, the formal relations will serve us as guides in the examination and interpretation of the empirical data which, in the case, will be our fundamental interest."

Alfred James Lotka (1938, p. 164)

Thank you very much!

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Contact

University of Rostock
Dept. Sociology & Demography
Chair of Demography
Ulmenstr. 69
18057 Rostock
Germany

Max Planck Institute for Demogr. Research
Lab. Stat. Demography
Konrad-Zuse-Str. 1
18057 Rostock
Germany

Tel.:	+49-381-498 4044	+49-381-2081 194
Email:	roland.rau@uni-rostock.de	rau@demogr.mpg.de

Personal Twitter-Account: www.twitter.com/Demographie