Outsurvival statistic

Method and application to sex differences in lifespan

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A question and a gut feeling

James W. Vaupel asked sometimes in the Fall of 2019

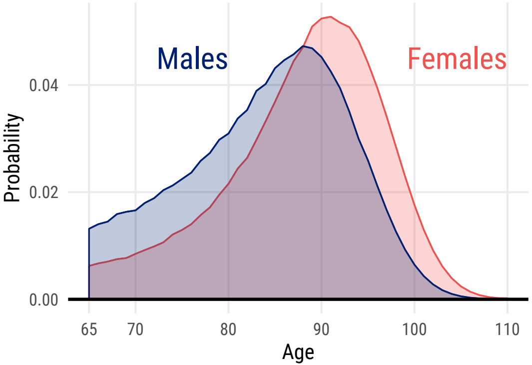
"What is the probability that a male outlive a female?"

"My gut feeling is 40%!"

Distance and overlap of two distributions

Outsurvival relates to the overlap between two lifespan distributions.

Figure 1: Life table age at death distributions from age 65 for French males and females, 2018



Source: Vaupel et al. 2021

Distance and overlap of two distributions

Other related studies in demography:

- **Edward and Tuljapurkar 2005**: Kullback-Leibler(KL) divergence.
- **Shi et al. 2022** : Measure of non-overlap as stratification index.

Method

Two angles, one approach

The probability that a random individuals from population A will outlive a random individual from population B:

- The life table perspective:

$$\phi = \int_0^\infty d^B(x) l^A(x) dx$$

- The generalized perspective:

$$\phi = \int_0^\infty f^A(x) F^B(x) dx$$

The outsurvival statistic

The latter equation relates to the first with $f^A(x) = d^A(x)$ and $F^B(x) = \int_0^x d^B(x)dx = D^B(x)$.

It can be shown that

$$\int_0^\infty d^B(x) l^A(x) dx = \int_0^\infty d^A(x) D^B(x) dx$$

Complement

$$\phi = \int_0^\infty d^B(x) l^A(x) dx = \int_0^\infty \mu^B(x) l^B(x) l^A(x) dx$$

$$1 - \phi = \int_0^\infty d^A(x) l^B(x) dx = \int_0^\infty \mu^A(x) l^A(x) l^B(x) dx$$

If $\mu^A(x) = \mu^B(x)$

$$\phi = \int_0^\infty d(x)l(x) = \int_0^\infty \mu(x)l^2(x) = 0.5$$

If $\mu^A(x) = \mu$ and $\mu^B(x) = \mu/2$

$e^A(0) = 1/\mu$ and $e^B(0) = 2/\mu$

$$\phi = \int_0^\infty \mu^B e^{-\mu^B x} e^{-\mu^A x} dx = \frac{\mu^B}{\mu^A + \mu^B} = \frac{1}{3}$$

Reinventing the wheel

Similar statistics have been developed in many other fields:

- Expected failure probability in a stress-strength interference model.
- Probability of superiority
- Mann-Withney U test
- etc.

Discrete approximation

$$\phi \approx \sum_{x=0}^{\omega} d^B(x-n)l^A(x) + \bar{d}$$

where $\bar{d}$ is the probability that individuals in populations A and B died in the same age group.

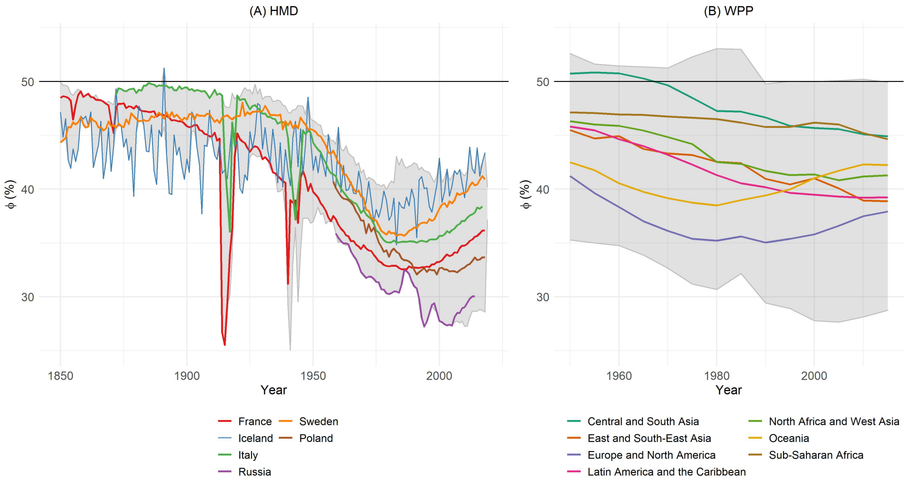
$\bar{d} = \frac{\sum_{x=0}^{\omega} d^A(x)d^B(x)}{2}$

Application to sex differences in lifespan

Data

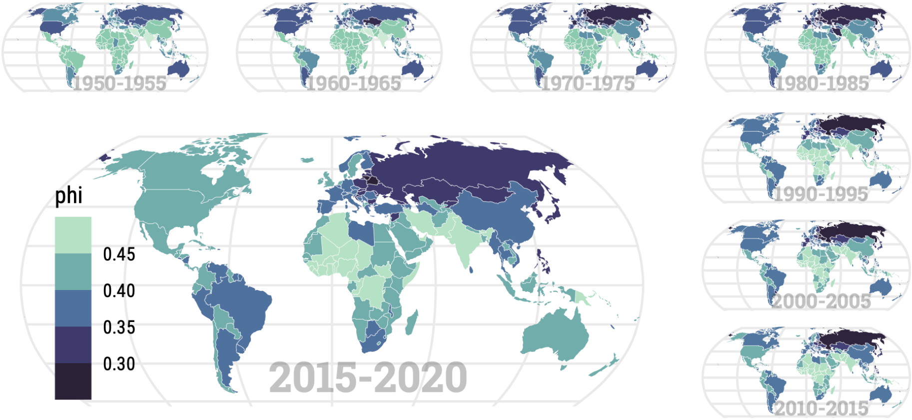
- Human Mortality Database (HMD), life tables by sex – 44 populations since 1751.
- World Population Prospects (WPP) 2019, abridged life tables by sex – 199 populations since 1950-1954.
- United States data by sex, marital status and education in 2015-2019:
  - Multiple Cause of Death Dataset (MCDD) from the National Vital Statistics System of the National Center for Health Statistics.
  - American Community Service (ACS) from the US Census Bureau.

Trends over time



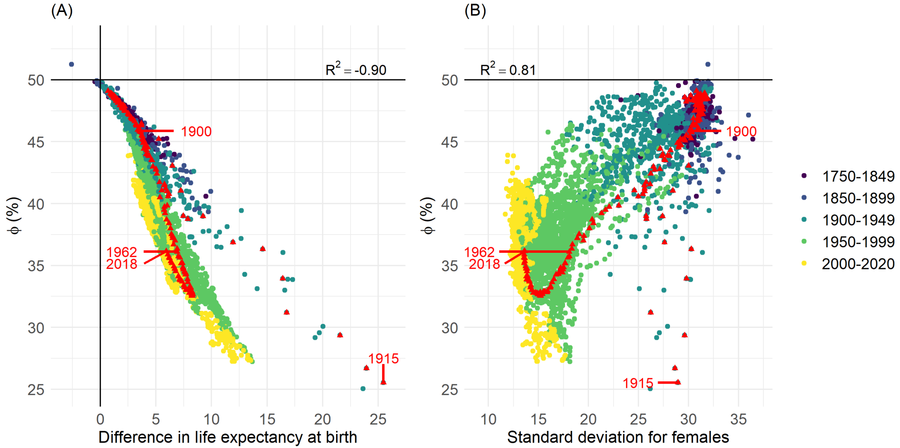
Source: Bergeron-Boucher et al. 2022

Across the world



Source: Bergeron-Boucher et al. 2022

Correlation with life expectancy and lifespan variation



Source: Bergeron-Boucher et al. 2022

Lesson from the expected failure probability

If the distributions of both populations follow a normal distribution with mean $\bar{x}_i$ and standard deviation s_i, the probability of failure is $P(Z)$ with:

$$Z = -\frac{\bar{x}_B - \bar{x}_A}{\sqrt{s_A^2 + s_B^2}}$$

Variation by marital status

		Female	
		Married	Unmarried
Male	Married	0.39	0.52
	Unmarried	0.26	0.37

Source: Bergeron-Boucher et al. 2022

Variation by education level

		Female		
		University	High school	< high school
Male	University	0.43	0.51	0.53
	High school	0.32	0.39	0.42
	< high school	0.30	0.37	0.39

Source: Bergeron-Boucher et al. 2022

Conclusive remarks

- How distinct are two lifespan distribution? vs
  How different are the mean lifespans?
- Overlap in lifespan distributions as results of their heterogeneity.
- Outsurvival statistics informative for public policies.

Possible extensions and future applications

- Outsurvival statistic for correlated/dependent populations.
- Outsurvival statistic for more than two populations.
- Applications to causes of death.
- Applications to income distribution, age-at-first-birth distribution, etc.
- And more...

Thank you!

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References

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