Example: Ordinary Kriging with Three Points

Data Points

- Known data points: $Z(x_1)$, $Z(x_2)$, $Z(x_3)$
- Unsampled location: x_0

Weights

Weights to be determined: λ_1, λ_2, λ_3 such that the estimate $\hat{Z}(x_0)$ at location x_0 is a weighted sum of the known values:

$$\hat{Z}(x_0) = \lambda_1 Z(x_1) + \lambda_2 Z(x_2) + \lambda_3 Z(x_3)$$

Function to Minimize

In ordinary kriging, the goal is to minimize the estimation variance, which can be expressed as:

$$\mathrm{Var}(\hat{Z}(x_0)) = \sum_{i=1}^{3} \sum_{j=1}^{3} \lambda_i \lambda_j \gamma(x_i, x_j) + \sum_{i=1}^{3} \lambda_i \gamma(x_i, x_0)$$

Constraint

The constraint is that the sum of the weights must equal 1:

$$\sum_{i=1}^{3} \lambda_i = 1$$

Lagrangian Function

The Lagrangian function for ordinary kriging is:

$$L(\lambda_1, \lambda_2, \lambda_3, \mu) = \sum_{i=1}^{3} \sum_{j=1}^{3} \lambda_i \lambda_j \gamma(x_i, x_j) + \sum_{i=1}^{3} \lambda_i \gamma(x_i, x_0) + \mu \left(\sum_{i=1}^{3} \lambda_i - 1 \right)$$

where $\gamma(x_i, x_j)$ is the semivariance between points x_i and x_j, and μ is the Lagrange multiplier.

Partial Derivatives

Taking the partial derivatives with respect to λ_i and μ:

$$\frac{\partial L}{\partial \lambda_i} = \sum_{j=1}^{3} \lambda_j \gamma(x_i, x_j) + \gamma(x_i, x_0) + \mu = 0 \quad \text{for } i = 1, 2, 3$$

$$\frac{\partial L}{\partial \mu} = \sum_{i=1}^{3} \lambda_i - 1 = 0$$

System of Equations

Rearranging the partial derivatives, we get the following system of equations:

$$\begin{cases} \lambda_1 \gamma(x_1, x_1) + \lambda_2 \gamma(x_1, x_2) + \lambda_3 \gamma(x_1, x_3) + \mu = \gamma(x_1, x_0) \\ \lambda_1 \gamma(x_2, x_1) + \lambda_2 \gamma(x_2, x_2) + \lambda_3 \gamma(x_2, x_3) + \mu = \gamma(x_2, x_0) \\ \lambda_1 \gamma(x_3, x_1) + \lambda_2 \gamma(x_3, x_2) + \lambda_3 \gamma(x_3, x_3) + \mu = \gamma(x_3, x_0) \\ \lambda_1 + \lambda_2 + \lambda_3 = 1 \end{cases}$$

By solving this system of equations, we can determine the weights λ_1, λ_2, and λ_3 that minimize the estimation variance while ensuring the sum of the weights is one.