Prove:

(Lemma (ANdominates 1))

$$\underset{n,\pi,x,y}{\forall}\left(\left(\text{appAlloc}[n, x] \wedge \text{allocation}_n[y] \wedge (W[n, x, y] = \{1\}) \wedge (L[n, x, y] = \{2\})\right) \wedge (\text{AN}[\pi, n] \wedge \text{powerfunction}[\pi, n]) \Rightarrow (\text{dominates}[y, x, \pi, n] \Rightarrow x_1 > x_2)\right)$$

under the assumptions:

(Definition (dominates))

$$\underset{n,\pi,x,y}{\forall}\left(\text{dominates}[y, x, \pi, n] :\Leftrightarrow \pi[W[n, x, y], x] > \pi[L[n, x, y], x]\right),$$

(Lemma (AN all)) $\underset{n,\pi,x}{\forall}\left(\text{appAlloc}[n, x] \wedge \text{AN}[\pi, n] \Rightarrow (\pi[\{1\}, x] = \pi[\{2\}, \text{perm}[x, \sigma_{1,2}]]) \wedge \text{allocation}_n[\text{perm}[x, \sigma_{1,2}]]\right)$,

(Definition (WR contrapositive))

$$\underset{n,\pi}{\forall}\left(\text{WR}[\pi, n] :\Leftrightarrow n \in \mathbb{N} \bigwedge \underset{C,x,y}{\forall}\left(\text{allocation}_n[x] \wedge \text{allocation}_n[y] \wedge C \subseteq \text{I}[n] \Rightarrow \left(\pi[C, x] > \pi[C, y] \Rightarrow \underset{i}{\exists}(i \in C \wedge x_i > y_i)\right)\right)\right)$$

(Lemma (2inI)) $\underset{n}{\forall}\left(\text{appropriateLength}[n] \Rightarrow \{2\} \subseteq \text{I}[n]\right)$,

(Definition (powerfunction)) $\underset{n,\pi}{\forall}\left(\text{powerfunction}[\pi, n] :\Leftrightarrow \text{WC}[\pi, n] \wedge \text{WR}[\pi, n] \wedge \text{SR}[\pi, n]\right)$,

(Lemma (perm swap)) $\underset{x}{\forall}\left(\text{perm}[x, \sigma_{1,2}]_2 = x_1\right)$,

(Definition (appAlloc)) $\underset{n,x}{\forall}\left(\text{appAlloc}[n, x] :\Leftrightarrow \text{appropriateLength}[n] \wedge \text{allocation}_n[x]\right)$.

We assume

(1)

$$\left(\text{appAlloc}[n_0, x_0] \wedge \text{allocation}_{n_0}[y_0] \wedge (W[n_0, x_0, y_0] = \{1\}) \wedge (L[n_0, x_0, y_0] = \{2\})\right) \wedge (\text{AN}[\pi_0, n_0] \wedge \text{powerfunction}[\pi_0, n_0])$$

and show

(2) $\text{dominates}[y_0, x_0, \pi_0, n_0] \Rightarrow x_{0_1} > x_{0_2}$.

We prove (2) by the deduction rule.

We assume

(3) $\text{dominates}[y_0, x_0, \pi_0, n_0]$

and show

(4) $x_{0_1} > x_{0_2}$.

By modus ponens, from (1.1.1), (1.2.1) and an appropriate instance of (Lemma (AN all)) follows:

(5) $(\pi_0[\{1\}, x_0] = \pi_0[\{2\}, \text{perm}[x_0, \sigma_{1,2}]]) \wedge \text{allocation}_{n_0}[\text{perm}[x_0, \sigma_{1,2}]]$,

Formula (3), by (Definition (dominates)), implies:

$\pi_0[\text{W}[n_0, x_0, y_0], x_0] > \pi_0[\text{L}[n_0, x_0, y_0], x_0]$,

which, by (1.1.3), implies:

$\pi_0[\{1\}, x_0] > \pi_0[\text{L}[n_0, x_0, y_0], x_0]$,

which, by (5.1), implies:

$\pi_0[\{2\}, \text{perm}[x_0, \sigma_{1,2}]] > \pi_0[\text{L}[n_0, x_0, y_0], x_0]$,

which, by (1.1.4), implies:

(6) $\pi_0[\{2\}, \text{perm}[x_0, \sigma_{1,2}]] > \pi_0[\{2\}, x_0]$.

Formula (1.1.1), by (Definition (appAlloc)), implies:

$\text{appropriateLength}[n_0] \wedge \text{allocation}_{n_0}[x_0]$,

which, by (Lemma (2inI)), implies:

(7) $\{2\} \subseteq \text{I}[n_0] \wedge \text{allocation}_{n_0}[x_0]$.

Formula (1.2.2), by (Definition (powerfunction)), implies:

$\text{SR}[\pi_0, n_0] \wedge \text{WC}[\pi_0, n_0] \wedge \text{WR}[\pi_0, n_0]$,

which, by (Definition (WR contrapositive)), implies:

(8)

$$\text{SR}[\pi_0, n_0] \bigwedge \text{WC}[\pi_0, n_0] \bigwedge n_0 \in \mathbb{N} \bigwedge \underset{C,x,y}{\forall} \Big(C \subseteq \text{I}[n_0] \wedge \text{allocation}_{n_0}[x] \wedge \text{allocation}_{n_0}[y] \Rightarrow. \\ \Big(\pi_0[C, x] > \pi_0[C, y] \Rightarrow \underset{i}{\exists} (i \in C \wedge x_i > y_i)\Big)\Big)$$

Formula (6), by (8.4), implies:

(9) $\{2\} \subseteq \mathtt{I}[n_0] \bigwedge \mathtt{allocation}_{n_0}[\mathtt{perm}[x_0, \sigma_{1,2}]] \bigwedge \mathtt{allocation}_{n_0}[x_0] \Rightarrow \underset{i}{\exists} (i \in \{2\} \bigwedge \mathtt{perm}[x_0, \sigma_{1,2}]_i > x_{0_i})$.

Formula (7.1), by (9), implies:

(10) $\mathtt{allocation}_{n_0}[\mathtt{perm}[x_0, \sigma_{1,2}]] \bigwedge \mathtt{allocation}_{n_0}[x_0] \Rightarrow \underset{i}{\exists} (i \in \{2\} \bigwedge \mathtt{perm}[x_0, \sigma_{1,2}]_i > x_{0_i})$.

Formula (7.2), by (10), implies:

(11) $\mathtt{allocation}_{n_0}[\mathtt{perm}[x_0, \sigma_{1,2}]] \Rightarrow \underset{i}{\exists} (i \in \{2\} \bigwedge \mathtt{perm}[x_0, \sigma_{1,2}]_i > x_{0_i})$.

From (5.2) and (11) we obtain by modus ponens

(12) $\underset{i}{\exists} (i \in \{2\} \bigwedge \mathtt{perm}[x_0, \sigma_{1,2}]_i > x_{0_i})$.

By (12) we can take appropriate values such that:

(13) $i_0 \in \{2\} \bigwedge \mathtt{perm}[x_0, \sigma_{1,2}]_{i_0} > x_{0_{i_0}}$.

From what we already know follows:

From (13.1) we can infer

(14) $i_0 = 2$.

Formula (13.2), by (14), implies:

$\mathtt{perm}[x_0, \sigma_{1,2}]_2 > x_{0_2}$,

which, by (Lemma (perm swap)), implies:

(17) $x_{0_1} > x_{0_2}$.

Formula (4) is true because it is identical to (17).

□

■ Additional Proof Generation Information