Prove:

(Lemma (ANdominates 2))

$$\underset{n,\pi,x,y}{\forall}((\text{appAlloc}[n,x]\bigwedge \text{allocation}_n[y]\bigwedge(W[n,x,y]=\{1\})\bigwedge(L[n,x,y]=\{2\}))\bigwedge (\text{AN}[\pi,n]\bigwedge \text{powerfunction}[\pi,n])\Rightarrow(x_1>x_2\Rightarrow \text{dominates}[y,x,\pi,n]))$$

under the assumptions:

(Definition (dominates))

$$\underset{n,\pi,x,y}{\forall}(\text{dominates}[y,x,\pi,n]:\Leftrightarrow \pi[W[n,x,y],x]>\pi[L[n,x,y],x]),$$

(Lemma (AN all)) $\underset{n,\pi,x}{\forall}(\text{appAlloc}[n,x]\bigwedge \text{AN}[\pi,n]\Rightarrow (\pi[\{1\},x]=\pi[\{2\},\text{perm}[x,\sigma_{1,2}]])\bigwedge \text{allocation}_n[\text{perm}[x,\sigma_{1,2}]])$,

(Definition (SR))

$$\underset{n,\pi}{\forall}\left(\text{SR}[\pi,n]:\Leftrightarrow n\in\mathbb{N}\bigwedge \underset{C,x,y}{\forall}\left(\text{allocation}_n[x]\bigwedge \text{allocation}_n[y]\bigwedge(C\subseteq I[n]\bigwedge C\neq\{\})\Rightarrow \left(\underset{i}{\forall}(i\in C\Rightarrow y_i>x_i)\Rightarrow \pi[C,y]>\pi[C,x]\right)\right)\right)$$

(Lemma (2inI)) $\underset{n}{\forall}(\text{appropriateLength}[n]\Rightarrow\{2\}\subseteq I[n])$,

(Definition (powerfunction)) $\underset{n,\pi}{\forall}(\text{powerfunction}[\pi,n]:\Leftrightarrow \text{WC}[\pi,n]\bigwedge \text{WR}[\pi,n]\bigwedge \text{SR}[\pi,n])$,

(Lemma (perm swap)) $\underset{x}{\forall}(\text{perm}[x,\sigma_{1,2}]_2=x_1)$,

(Definition (appAlloc)) $\underset{n,x}{\forall}(\text{appAlloc}[n,x]:\Leftrightarrow \text{appropriateLength}[n]\bigwedge \text{allocation}_n[x])$.

We assume

(1)

$$\left(\text{appAlloc}[n_0,x_0]\bigwedge \text{allocation}_{n_0}[y_0]\bigwedge(W[n_0,x_0,y_0]=\{1\})\bigwedge(L[n_0,x_0,y_0]=\{2\})\right)\bigwedge (\text{AN}[\pi_0,n_0]\bigwedge \text{powerfunction}[\pi_0,n_0])$$

and show

(2) $x_{0_1}>x_{0_2}\Rightarrow \text{dominates}[y_0,x_0,\pi_0,n_0]$.

We prove (2) by the deduction rule.

We assume

(3) $x_{0_1} > x_{0_2}$

and show

(4) $\text{dominates}[y_0, x_0, \pi_0, n_0]$.

By modus ponens, from (1.1.1), (1.2.1) and an appropriate instance of (Lemma (AN all)) follows:

(5) $(\pi_0[\{1\}, x_0] = \pi_0[\{2\}, \text{perm}[x_0, \sigma_{1,2}]]) \wedge \text{allocation}_{n_0}[\text{perm}[x_0, \sigma_{1,2}]]$,

Formula (1.1.1), by (Definition (appAlloc)), implies:

$\text{appropriateLength}[n_0] \wedge \text{allocation}_{n_0}[x_0]$,

which, by (Lemma (2inI)), implies:

(6) $\{2\} \subseteq \text{I}[n_0] \wedge \text{allocation}_{n_0}[x_0]$.

Formula (1.2.2), by (Definition (powerfunction)), implies:

$\text{SR}[\pi_0, n_0] \wedge \text{WC}[\pi_0, n_0] \wedge \text{WR}[\pi_0, n_0]$,

which, by (Definition (SR)), implies:

(7) $\text{WC}[\pi_0, n_0] \bigwedge \text{WR}[\pi_0, n_0] \bigwedge n_0 \in \mathbb{N} \bigwedge$
$\underset{C,x,y}{\forall} \left(C \neq \{\} \wedge C \subseteq \text{I}[n_0] \wedge \text{allocation}_{n_0}[x] \wedge \text{allocation}_{n_0}[y] \Rightarrow \left(\underset{i}{\forall} (i \in C \Rightarrow y_i > x_i) \Rightarrow \pi_0[C, y] > \pi_0[C, x]\right)\right)$.

Formula (4), using (Definition (dominates)), is implied by:

$\pi_0[\text{W}[n_0, x_0, y_0], x_0] > \pi_0[\text{L}[n_0, x_0, y_0], x_0]$,

which, using (1.1.3), is implied by:

$\pi_0[\{1\}, x_0] > \pi_0[\text{L}[n_0, x_0, y_0], x_0]$,

which, using (5.1), is implied by:

$\pi_0[\{2\}, \text{perm}[x_0, \sigma_{1,2}]] > \pi_0[\text{L}[n_0, x_0, y_0], x_0]$,

which, using (1.1.4), is implied by:

$\pi_0[\{2\}, \text{perm}[x_0, \sigma_{1,2}]] > \pi_0[\{2\}, x_0]$,

which, using (7.4), is implied by:

(8) $\underset{i}{\forall} (i \in \{2\} \Rightarrow \text{perm}[x_0, \sigma_{1,2}]_i > x_{0_i}) \bigwedge \{2\} \neq \{\} \bigwedge$
$\{2\} \subseteq \text{I}[n_0] \bigwedge \text{allocation}_{n_0}[\text{perm}[x_0, \sigma_{1,2}]] \bigwedge \text{allocation}_{n_0}[x_0]$.

Using builtin simplification rules we simplify (8) to

(9) $\underset{i}{\forall}\,(i \in \{2\} \Rightarrow \mathrm{perm}[x_0,\ \sigma_{1,2}]_i > x_{0_i}) \bigwedge \mathrm{True} \bigwedge$
$\{2\} \subseteq \mathrm{I}[n_0] \bigwedge \mathrm{allocation}_{n_0}[\mathrm{perm}[x_0,\ \sigma_{1,2}]] \bigwedge \mathrm{allocation}_{n_0}[x_0]$.

We prove the individual conjunctive parts of (9):

Proof of (9.1) $\underset{i}{\forall}\,(i \in \{2\} \Rightarrow \mathrm{perm}[x_0,\ \sigma_{1,2}]_i > x_{0_i})$:

We assume

(10) $i_0 \in \{2\}$,

and show

(11) $\mathrm{perm}[x_0,\ \sigma_{1,2}]_{i_0} > x_{0_{i_0}}$.

From what we already know follows:

From (10) we can infer

(12) $i_0 = 2$.

Formula (11), using (12), is implied by:

$\mathrm{perm}[x_0,\ \sigma_{1,2}]_2 > x_{0_2}$,

which, using (Lemma (perm swap)), is implied by:

(15) $x_{0_1} > x_{0_2}$.

Formula (15) is true because it is identical to (3).

Proof of (9.2) True:

Formula (9.2) is true because it is the constant True.

Proof of (9.3) $\{2\} \subseteq \mathrm{I}[n_0]$:

Formula (9.3) is true because it is identical to (6.1).

Proof of (9.4) $\mathrm{allocation}_{n_0}[\mathrm{perm}[x_0,\ \sigma_{1,2}]]$:

Formula (9.4) is true because it is identical to (5.2).

Proof of (9.5) $\mathrm{allocation}_{n_0}[x_0]$:

Formula (9.5) is true because it is identical to (6.2).

□

Additional Proof Generation Information