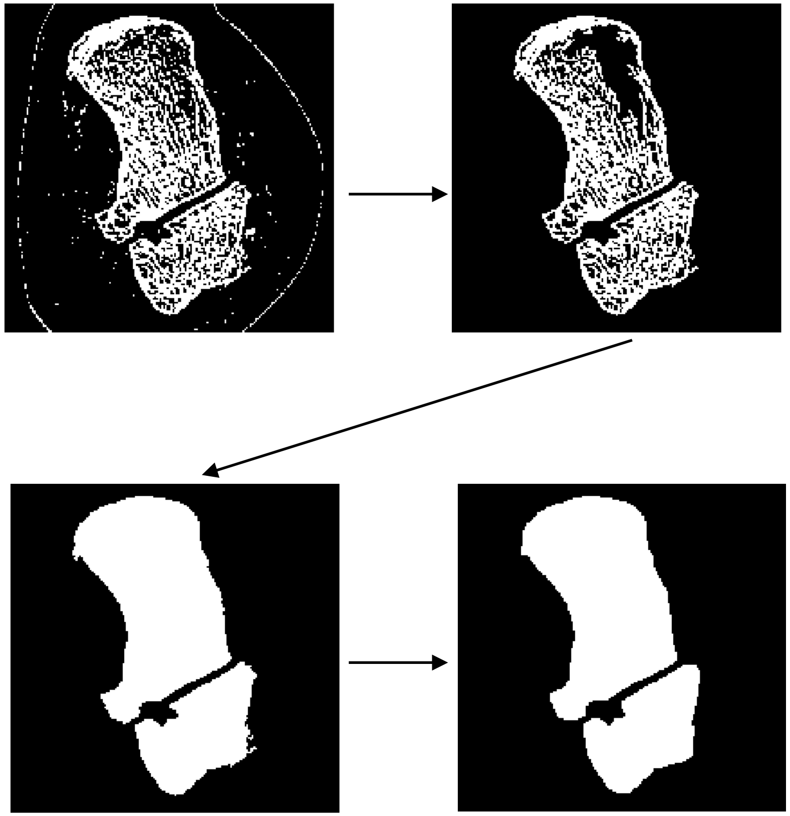
CSE 554

Lecture 2: Shape Analysis (Part I)

Fall 2018

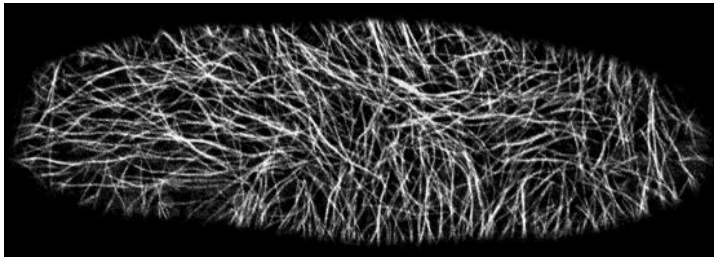
Review

- Binary pictures
  - Tresholding grayscale images
  - Basic operations
    - Connected component labeling
    - Morphological operators

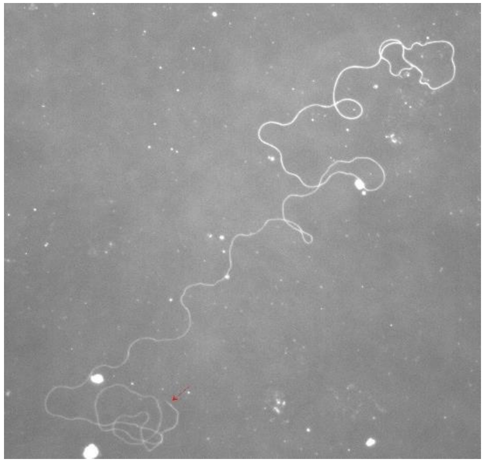


Shape analysis

- Questions about shapes:
  - Metrics: length? Width? orientation?
  - What are the parts?
  - How similar are two shapes?

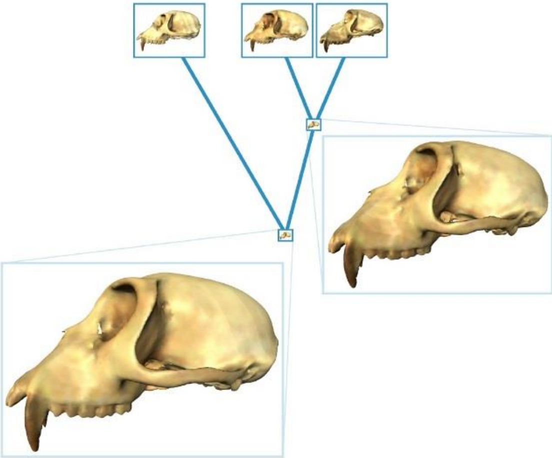


Microtubules on the cell surface



Sperms of fruit flies

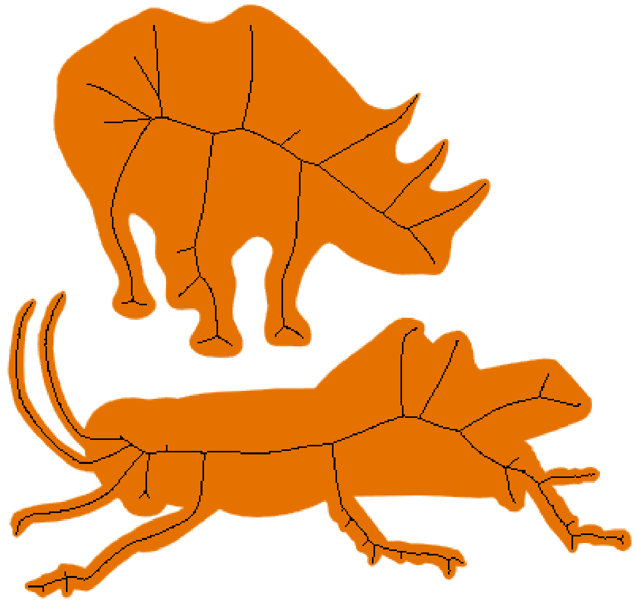
Cerebral artery aneurysms



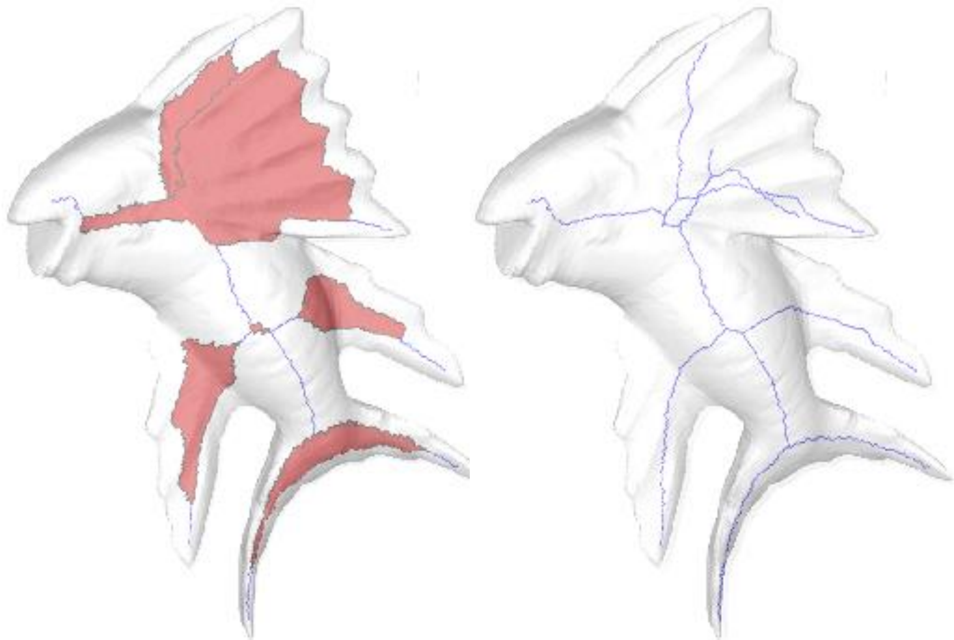
Monkey skulls

Skeletons

- Geometry at the *center* of the object
  - Compact, and capturing protruding shape parts



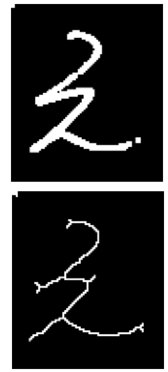
Skeleton of 2D shapes: 1D curves

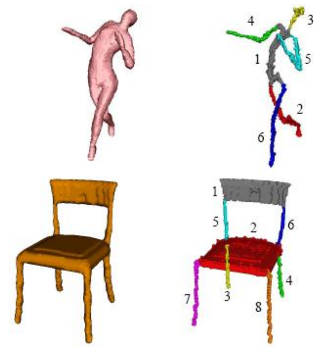


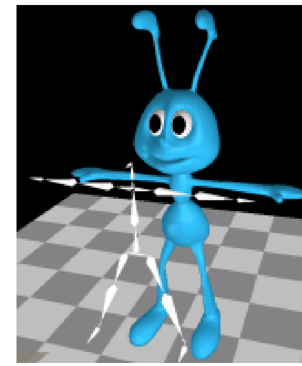
Skeleton of 3D shapes: 1D curves and 2D surfaces

Applications

- Computer graphics and vision
  - Optical character recognition (a)
  - Shape retrieval (b)
  - Animating articulated shapes (c)

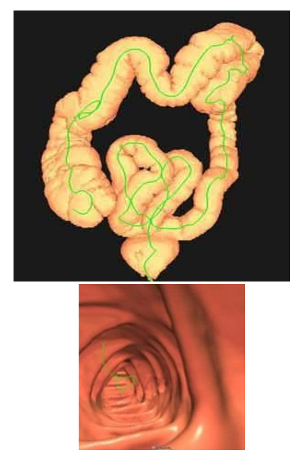

(a)

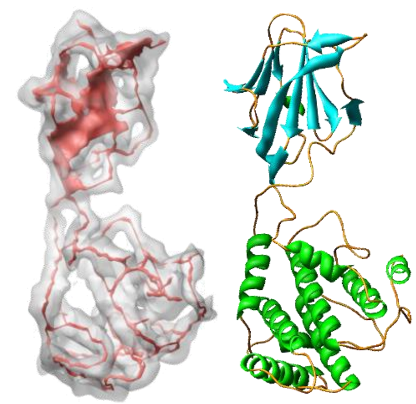

(b)


(c)

- Bio-medical image analysis
  - Vessel network analysis (d)
  - Virtual colonoscopy (e)
  - Protein modeling (f)

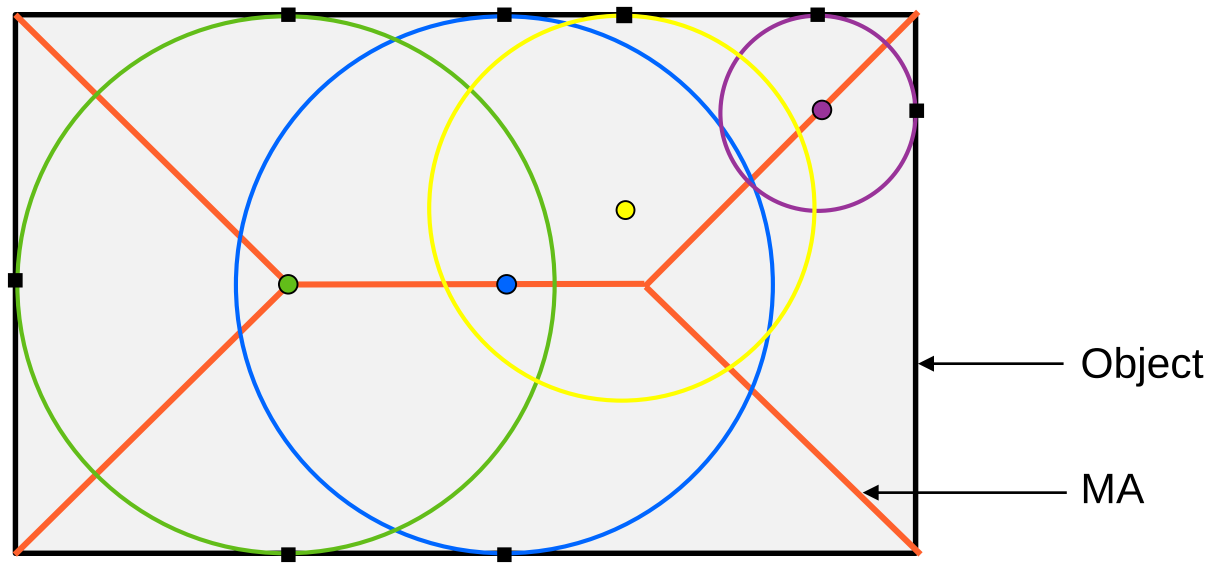

(d)


(e)


(f)

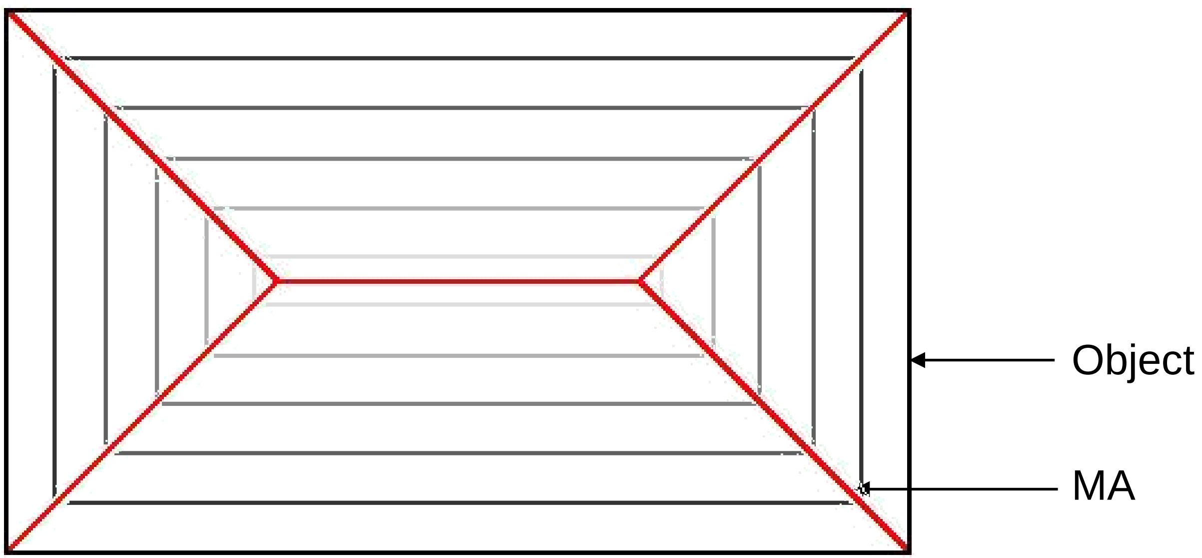
Medial Axes (MA)

- *Interior points with multiple closest points on the boundary*



Medial Axes (MA)

- Grassfire analogy:
  - Let the object represent a field of grass. A fire starts at the field boundary, and burns across the field at uniform speed.
  - MA are where the fire fronts meet.



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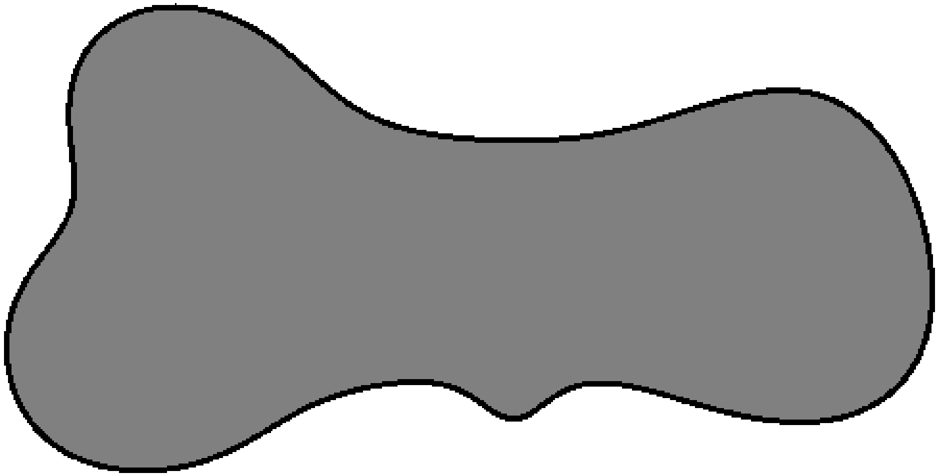
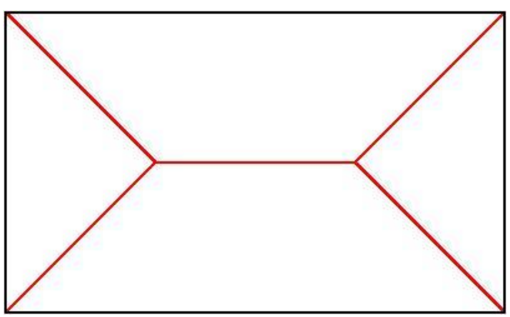
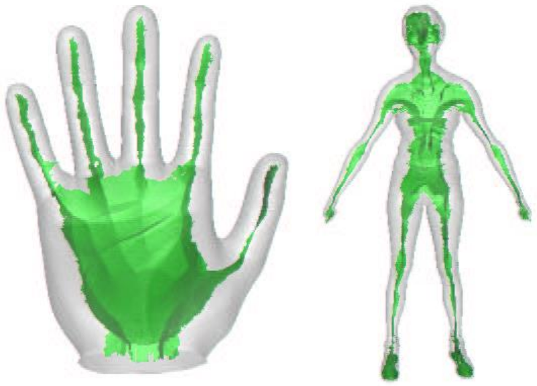


Medial Axes (MA)

- Properties
  - ✓ Thin
    - MA are curves (1D) in a 2D object, and surfaces (2D) in a 3D object.



2D MA

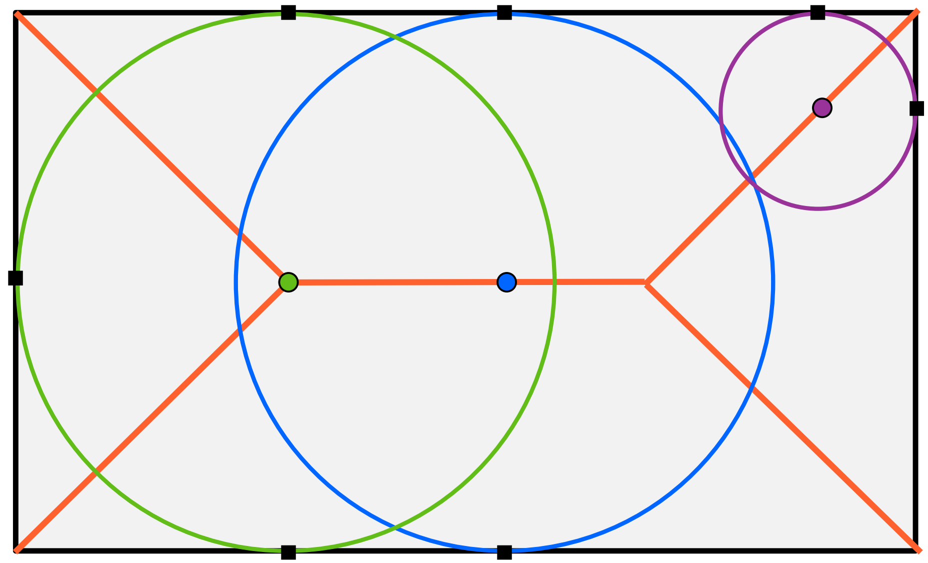


3D MA

Medial Axes (MA)

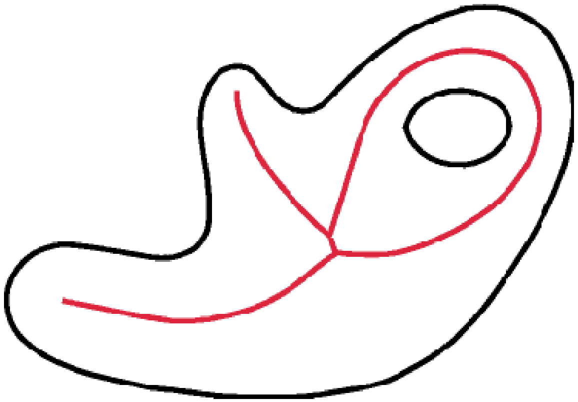
Washington University in St. Louis
School of Engineering & Applied Science

- Properties
  - ✓ Preserves object's shape
    - The object can be reconstructed from MA and its distances to the boundary

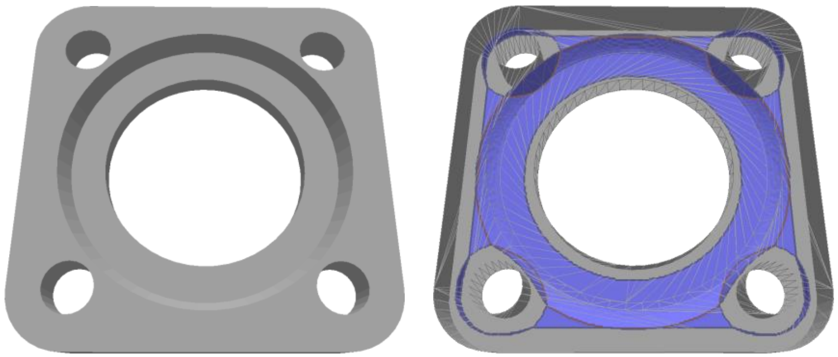


Medial Axes (MA)

- Properties
  - ✓ Preserves object's topology
    - 2D: # of connected components of object and background
    - 3D: # of connected components of object and background, and # of tunnels



A 2D shape with 1 object component and 2 background components

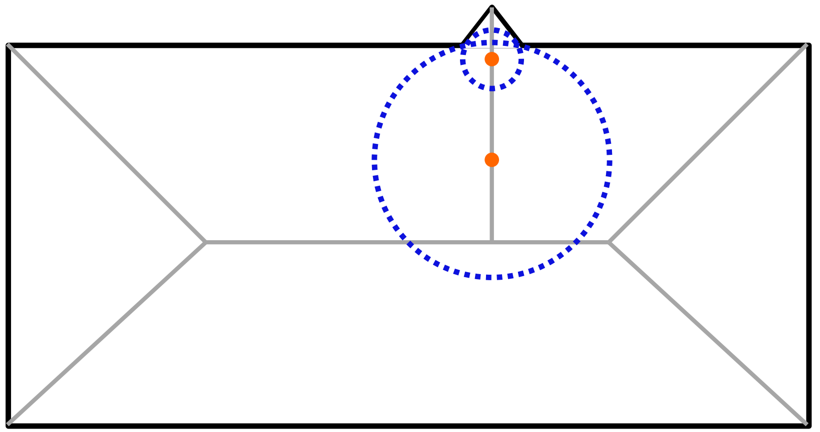


A 3D shape with 5 tunnels

Medial Axes (MA)

- Properties
  - Not stable under boundary perturbation

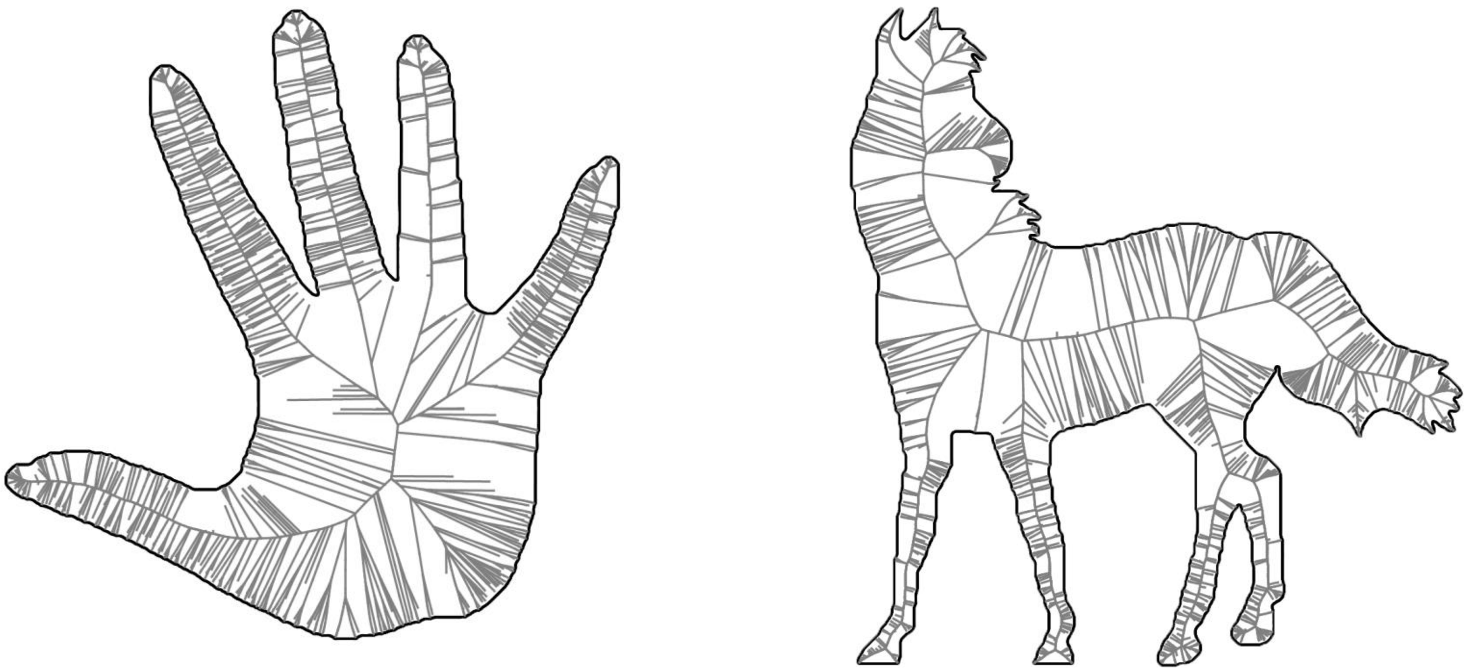
Original shape and medial axis



After adding a bump

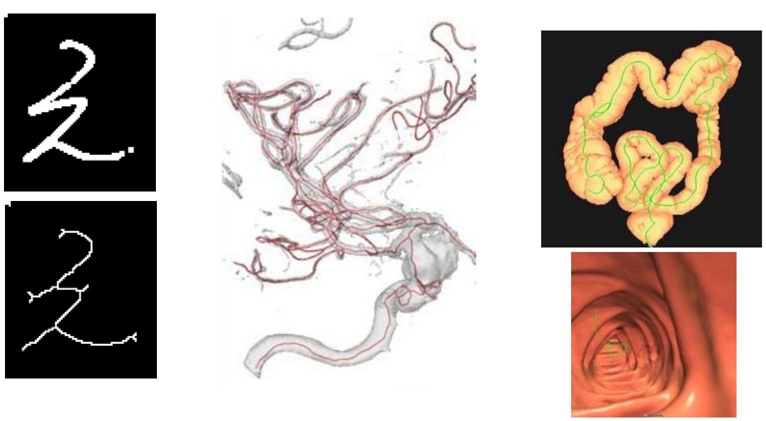
Medial Axes (MA)

- Properties
  - Not stable under boundary perturbation

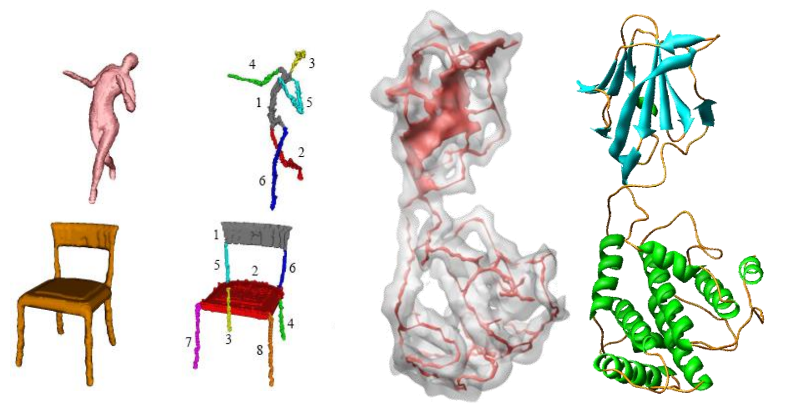


Skeletons

- Approximation of medial axes
  - Roughly corresponds to the stable parts of the medial axes
  - No unique or precise definition (e.g., application dependent)



Applications using curve skeletons



Applications using curve+surface skeletons

Computing Skeletons

- A classical method: thinning
  - Mimics the grassfire analogy
  - Can create curve or surface skeletons
- What we will cover:
  - Thinning on binary pictures (*this lecture*)
    - Simple to implement in 2D, but harder in 3D
    - Noise has to be dealt with separately
  - Thinning on cell complexes (*next lecture*) [Module 2]
    - Same implementation in any dimension
    - Noise removal as part of the algorithm

Medial Axes (MA)

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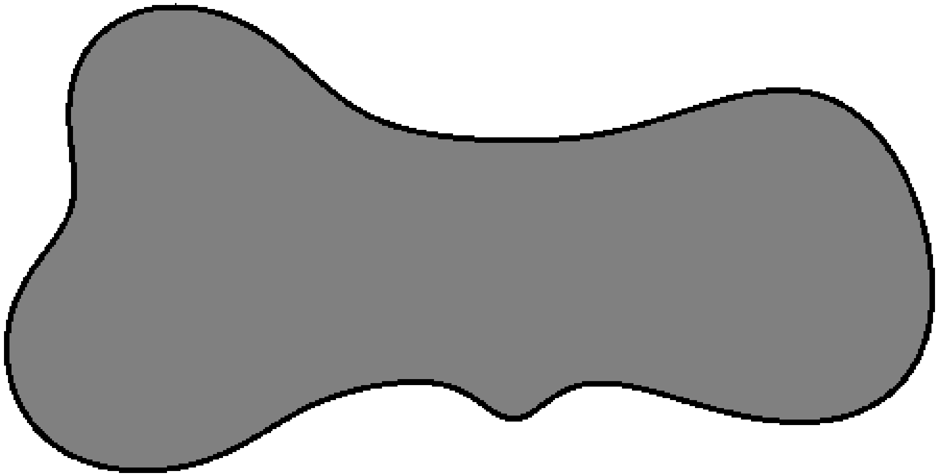
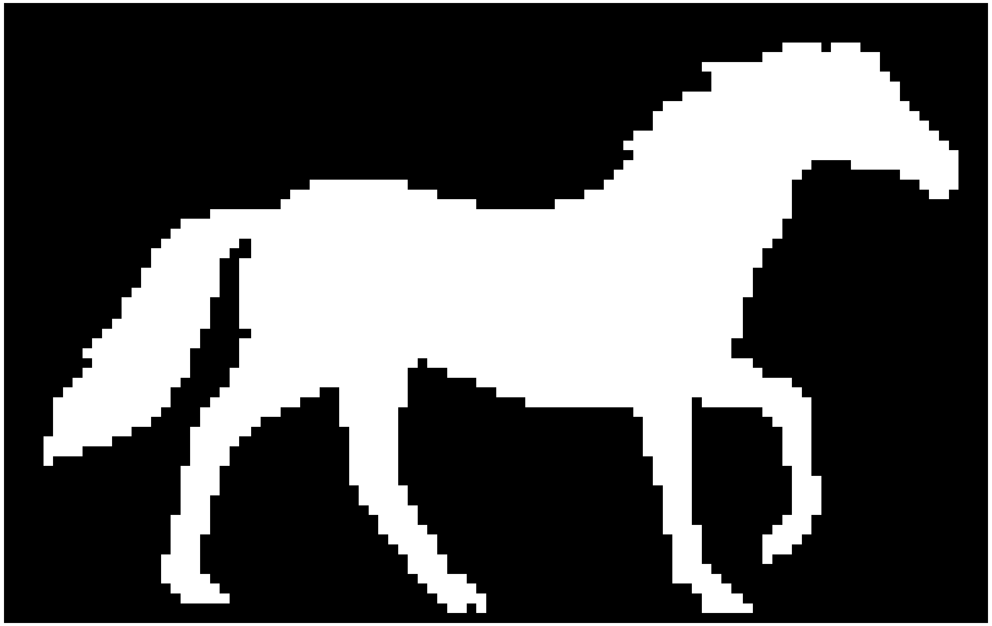


2D Thinning

- Discrete fire-burning on a binary picture
  - Repeated erosion while keeping track of where "erosion fronts" meet

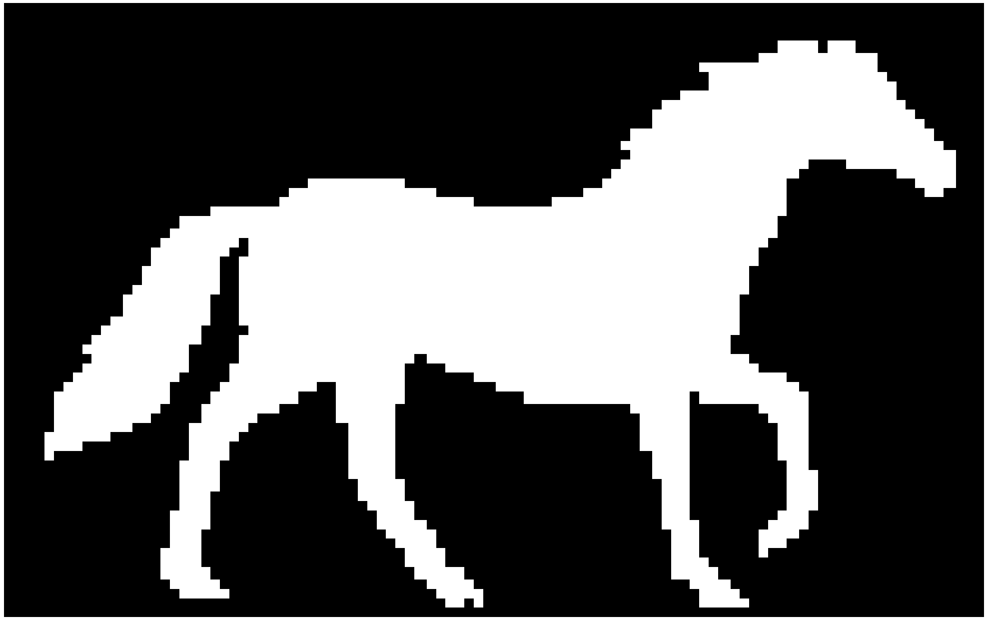
2D Thinning

- Repeated erosion eventually removes all object pixels



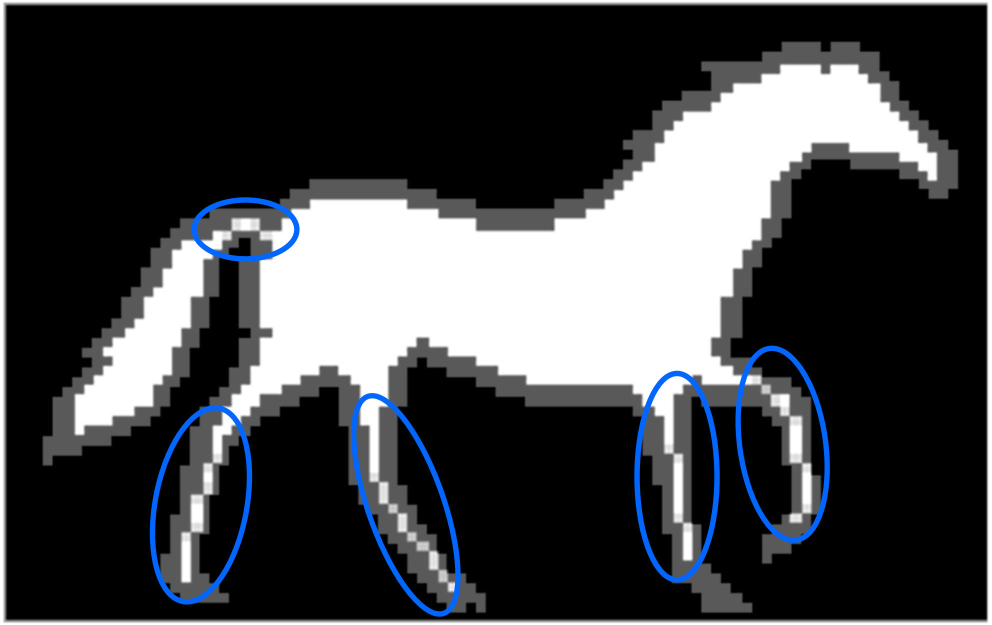
2D Thinning

- Repeated erosion eventually removes all object pixels
  - Need to identify and keep pixels where the (discrete) erosion fronts quench



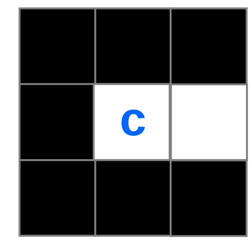
2D Thinning

- Repeated erosion eventually removes all object pixels
  - Need to identify and keep pixels where the (discrete) erosion fronts quench
  - These are object pixels that form digital curves (one-pixel-wide strands)



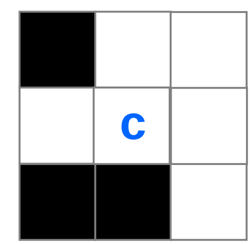
2D Thinning

- Identifying object pixels on digital curves
  - Curve-end pixel: connected to only one object pixel
    - Choose and fix the connectivity rule (4 or 8)



c is a curve-end pixel

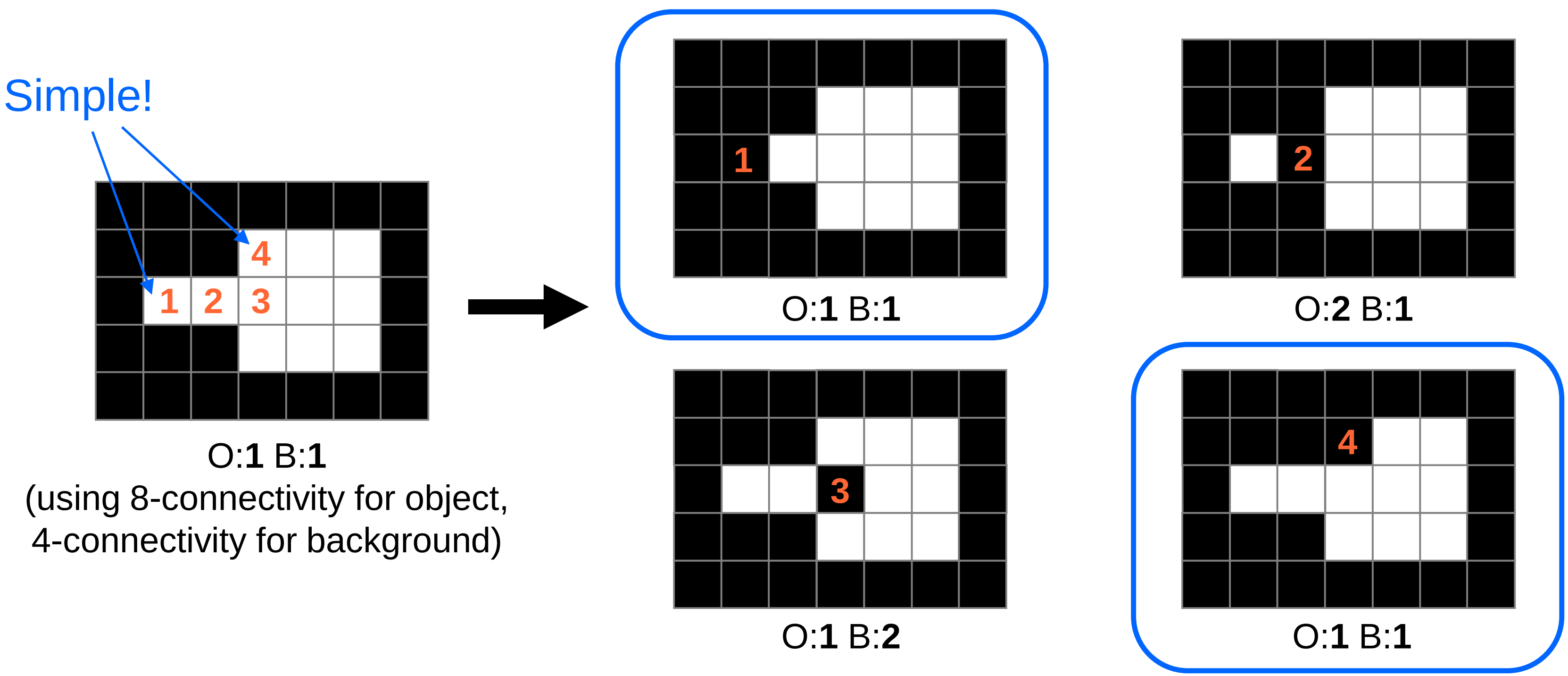
  - Pixels in the middle of a digital curve are harder to detect (ambiguity at curve junctions)
    - Instead, check to see if removal of the pixel changes the topology of the object



Is **c** on a digital curve?

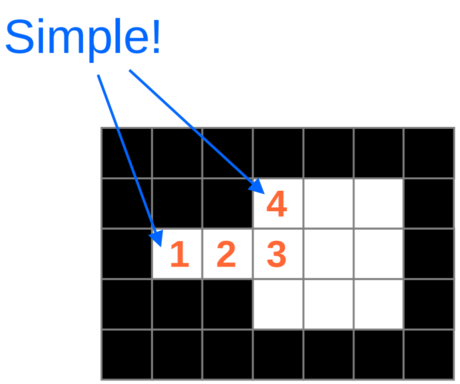
2D Thinning

- Simple pixels
  - Object pixels whose removal from the object does not change topology (i.e., # of components of object and background)

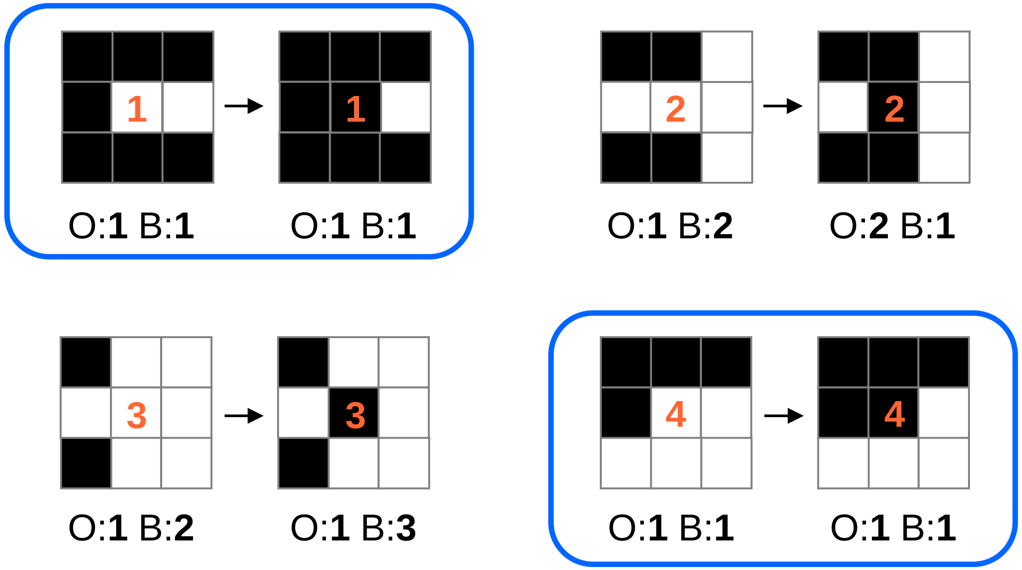


2D Thinning

- Simple pixels
  - Object pixels whose removal from the object does not change topology (i.e., # of components of object and background)
  - Sufficient to check # of components just in the 3x3 neighborhood!



(using 8-connectivity for object, 4-connectivity for background)



2D Thinning

- Simple pixels
  - Object pixels whose removal from the object does not change topology (i.e., # of components of object and background)
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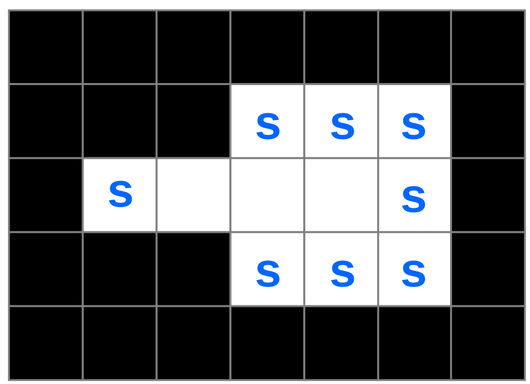
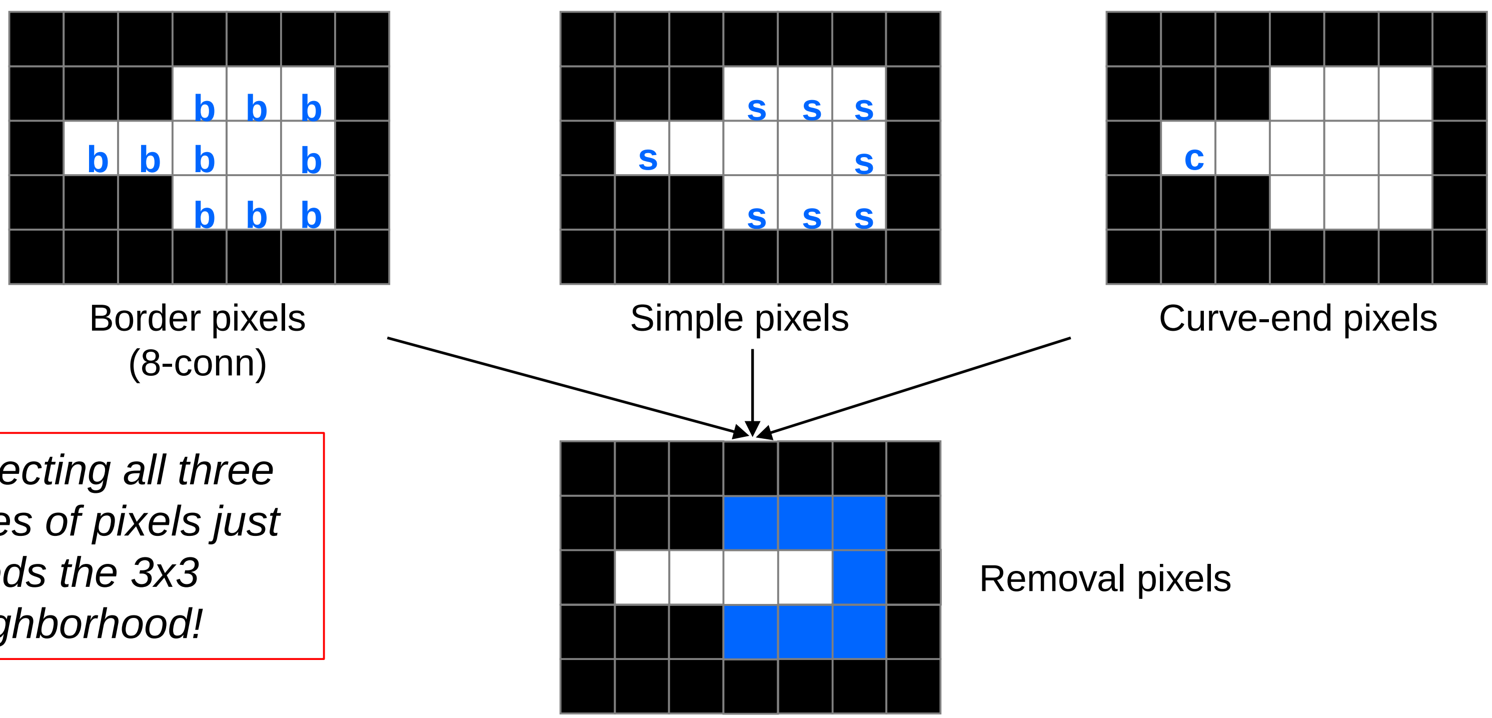


All simple pixels

2D Thinning

- Removable pixels during erosion
  - Border pixels (i.e., those connected to some background pixel) that are simple and not curve-end



Detecting all three types of pixels just needs the 3x3 neighborhood!

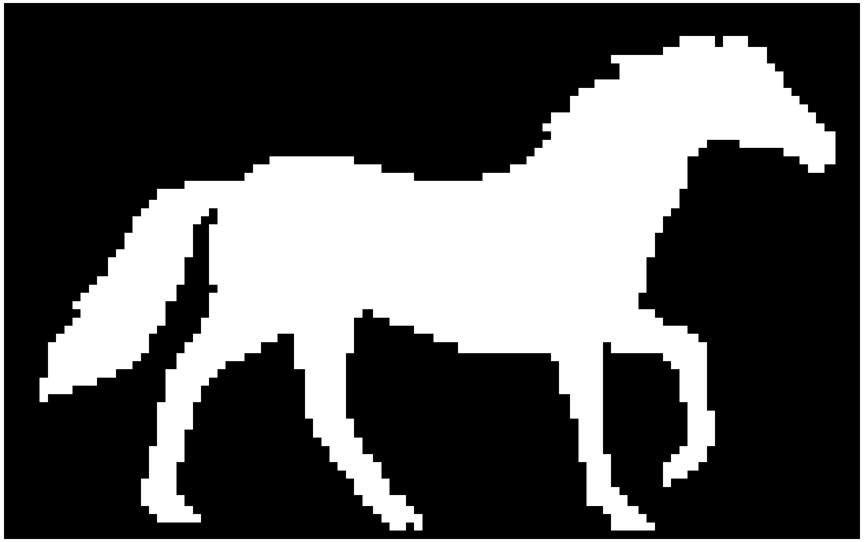
2D Thinning

- Algorithm (attempt) 1
  - Simultaneous removal of all removable points (“Parallel thinning”)

```
// Parallel thinning on a binary image I
1. Repeat:
    1. Collect all removable pixels as S
    2. If S is empty, Break.
    3. Set all pixels in S to be background in I
2. Output I
```

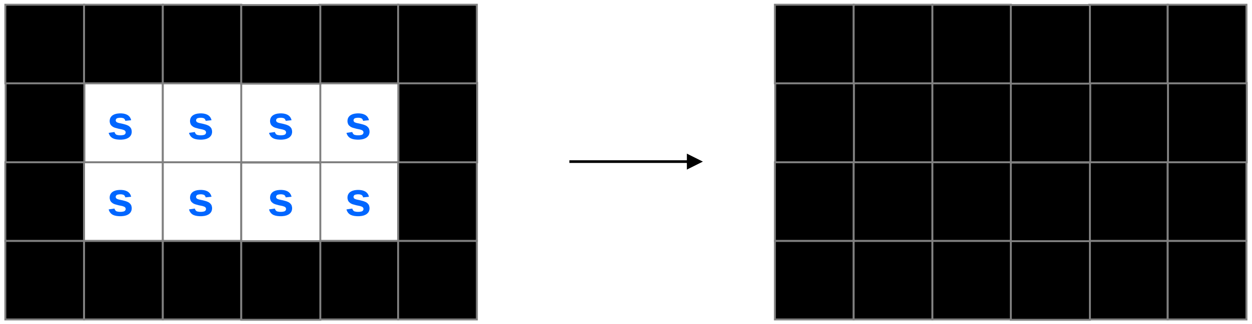
2D Thinning

- Algorithm (attempt) 1
  - Simultaneous removal of all removable points (“Parallel thinning”)



2D Thinning

- Why does parallel thinning breaks topology?
  - Simple pixels, when removed together, may change topology



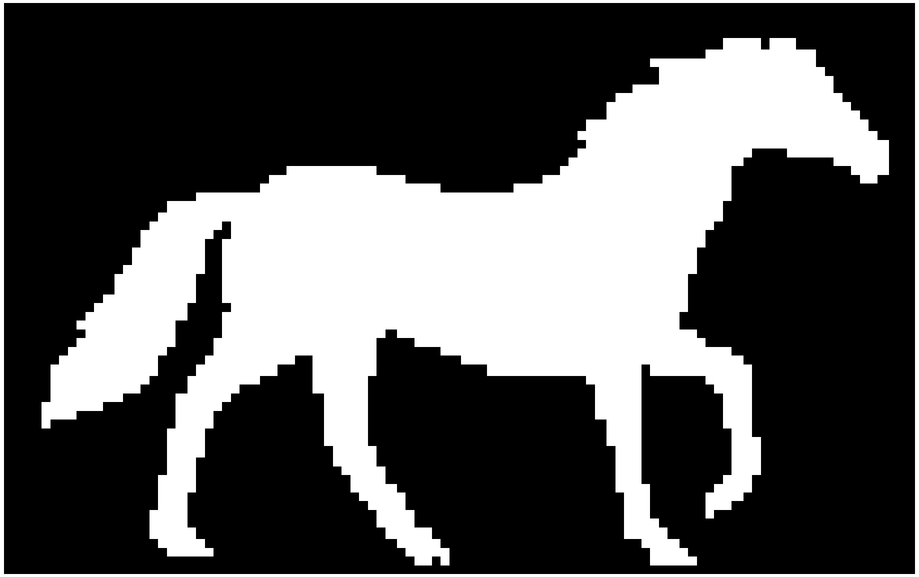
2D Thinning

- Algorithm 2
  - Sequentially visit each removable pixel and check its simple-ness before removing the pixel. (“Serial Thinning”)

```
// Serial thinning on a binary image I
1. Repeat:
     1. Collect all border pixels as S
     2. If S is empty, Break.
     3. Repeat for each pixel x in S (in certain order):
          1. If x is currently simple and not curve-end, set x to
             be background in I
2. Output I
```

2D Thinning

- Algorithm 2
  - Sequentially visit each removable pixel and check its simple-ness before removing the pixel. (“Serial Thinning”)



Serial thinning

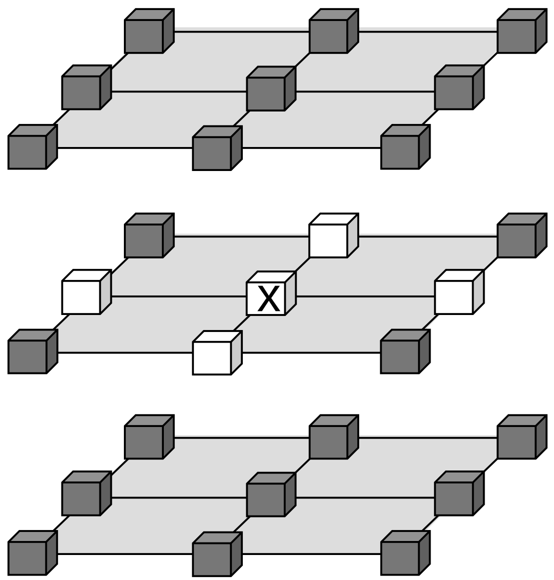
2D Thinning

- Algorithm 2
  - Sequentially visit each removable pixel and check its simple-ness before removing the pixel. (“Serial Thinning”)
  - Result is affected by the order that the border pixels are visited

Serial thinning using two different visiting orders of border pixels

3D Thinning

- Identifying removable voxels
  - Border voxels
    - Similar to 2D: object voxels connected to at least one background voxel
  - Simple voxels
    - Similar to 2D: only needs to check 3x3x3 neighborhood (but needs to count # of tunnels besides # components of obj/bg)
  - Curve-end and surface-end voxels
    - Curve-end criteria same as in 2D
    - Surface-end criteria are much harder to describe (e.g., requires a table look-up)

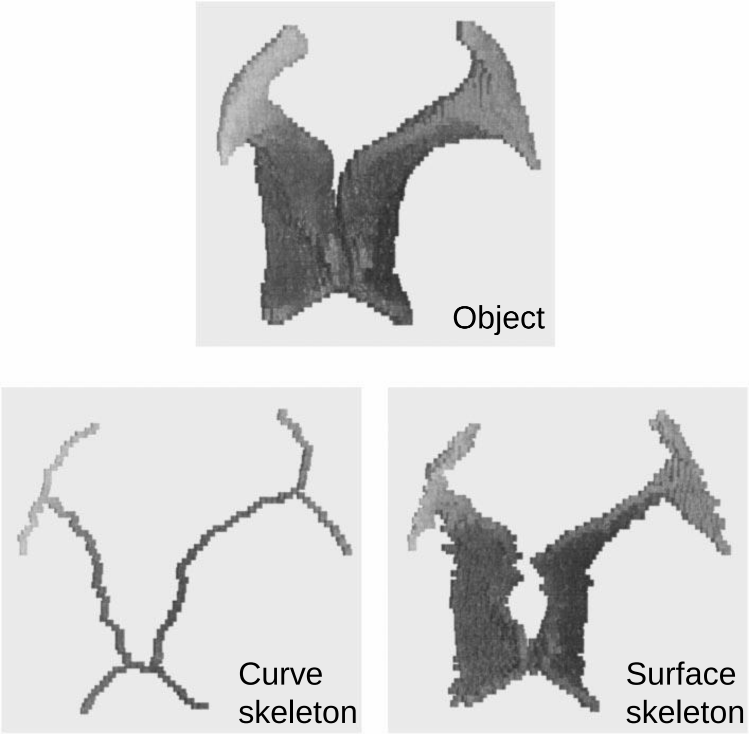


Setting voxel x to background creates a “tunnel” in the object (using 26-conn for object)

3D Thinning

- Two kinds of skeletons
  - Curve skeletons: only curve-end voxels are preserved during thinning
  - Surface skeletons: both curve-end and surface-end voxels are preserved

  (see further readings)



Method of [Palagyi and Kuba, 1999]

Skeleton Pruning

- Thinning is sensitive to boundary noise
  - Due to the instability of medial axes
- Skeleton pruning
  - During thinning
    - E.g., using more selective criteria for end pixels (voxels)
  - After thinning
    - E.g., based on branch length
  - See *Further Readings*

Object with boundary noise

Resulting skeleton

Further Readings on: Binary Pictures, MA and Thinning

- Books
  - "*Digital Geometry: geometric methods for digital picture analysis*", by Klette and Rosenfeld (2004)
  - "*Medial representations: mathematics, algorithms and applications*", by Siddiqi and Pizer (2008)
- Papers
  - "*Digital topology: introduction and survey*", by Kong and Rosenfeld (1989)
    - Theories of binary pictures
  - "*Thinning methodologies - a comprehensive survey*", by Lam et al. (1992)
    - A survey of 2D methods
  - "*A Parallel 3D 12-Subiteration Thinning Algorithm*", by Palagyi and Kuba (1999)
    - Includes a good survey of 3D thinning methods
  - "*Pruning medial axes*", by Shaked and Bruckstein (1998)
    - A survey of MA and skeleton pruning methods