Statistical Mechanics for natural flocks of birds

Università
degli Studi
di Padova

Problem definition

Subject of study: large groups of animals, flocks of starlings

Phenomenon: emergence of a strong polarization at the macroscopic scale

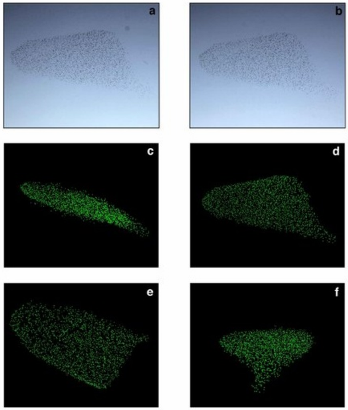
Can this collective behaviour be understood in the same way as collective behaviour are understood in physics?

- Construction of a maximum entropy model based on field data of flocks
- Show the effective interactions are local
- Show the interactions are ruled by topological rather than metric distance
- Show the model reproduces the scale invariance of long-range correlations among the fluctuations in flight direction

Data

Analyzed data were obtained from experiments on large flocks of starlings in the field. Using stereometric photography and computer vision techniques the individual 3D coordinates and velocities were measured.

- Number of flocks: 21
- Snapshot: 3D configuration of one flock at fixed instant of time
- Event: 40 consecutive 3D configurations of one flock at time intervals of $\frac{1}{10}$ s.



All events correspond to very polarized flocks

Global properties of the analyzed flocking events:

Event[a]	N	S	v_0 (m/s)	L (m)
17–06	552	0.935	9.4	51.8
21–06	717	0.973	11.8	32.1
25–08	1571	0.962	12.1	59.8
25–10	1047	0.991	12.5	33.5
25–11	1176	0.959	10.2	43.3
28–10	1246	0.982	11.1	36.5
29–03	440	0.963	10.4	37.1
31–01	2126	0.844	6.8	76.8
32–06	809	0.981	9.8	22.2
42–03	431	0.979	10.4	29.9
49–05	797	0.995	13.9	19.2
54–08	4268	0.966	19.1	78.7
57–03	3242	0.978	14.1	85.7
58–06	442	0.984	10.1	23.1
58–07	554	0.977	10.5	19.1
63–05	890	0.978	9.9	52.9
69–09	239	0.985	11.8	17.1
69–10	1129	0.987	11.9	47.3
69–19	803	0.975	13.8	26.4
72–02	122	0.992	13.2	10.6
77–07	186	0.978	9.3	9.1

- Configuration at a given instant = $\{\vec{s}_i\}$, where $\vec{s}_i = \frac{\vec{v}_i}{|\vec{v}_i|}$ is the normalized velocity
- Polarization of the flock $\vec{S} = S\vec{n} = \frac{1}{N}\sum_i \vec{s}_i$
- L = linear size of the flock
- v_0 =mean velocity of the group

Strategy

1. Estimate the correlations among interacting neighbors from experimental single snapshots of the birds' flight directions
2. Define maximum entropy distribution consistent with the directional correlations considering local, pairwise interactions
3. For each snapshot adjust the values of the parameters J and n_c (number of first nearest neighbors) in order to match the experimentally observed correlations
   - Computation with free boundary conditions
   - Computation with fixed boundary conditions

Strategy

4. Model prediction of:
   - two point and higher order local correlation functions
   - correlations across the full range of distances in order to test predictions on all length scales
   - individual flight direction and pairwise in order to investigate the propagation of order throughout the flock
5. Testing the mechanistic interpretation: simulations of a population of self-propelled particles in order to apply the model and compare the obtained parameters

Goal: construct the minimally structured distribution $P(\vec{x})$ for system variables $\vec{x}$, meaning the most random distribution consistent with the observed averages of computable functions of the system state $\{\langle f_\nu(\{\vec{x}\})\rangle_{exp}\}$.

Measure of randomness: **Shannon Entropy**

$$S[P] = -\sum_{\vec{x}} P(\vec{x}) lnP(\vec{x})$$

In order to maximize it and maintaining the consistency with the contraints, we impose:

$$0 = \frac{\partial S[P;\{\lambda_\nu\}]}{\partial P(\vec{x})},$$

where $$S[P;\{\lambda_\nu\}] = S[P] - \sum_{\mu}^{K} \lambda_\mu [\langle f_\mu(\vec{x})\rangle_P - \langle f_\mu(\vec{x})\rangle_{exp}]$$

Maximum entropy approach

Thus we obtain:

$$P(\{\vec{x}\}) = \frac{1}{Z(\lambda_\mu)} exp[-\sum_\mu \lambda_\mu f_\mu(\vec{x})]$$

$$Z(\lambda_\mu) = \sum_{\vec{x}} exp[-\sum_\mu \lambda_\mu f_\mu(\vec{x})]$$

Optimizing with respect to $\{\lambda_\mu\}$ gives us a set of K simultaneous equations of the form:

$$0 = \frac{\partial S[P;\{\lambda_\nu\}]}{\partial \lambda_\mu} = \langle f_\mu(\vec{x}) \rangle_{exp} - \langle f_\mu(\vec{x}) \rangle_P$$

Maximum entropy approach

Therefore:

$$S[P;\{\lambda_\nu\}] = -\langle log P(\vec{x})\rangle_{exp}$$

Thus the maximum entropy approach corresponds to maximizing the likelihood that the model generates the observed data.

In our case:

- System state $= \{\vec{s_i}\}$
- $\{f_\nu(\{\vec{x}\})\} = C_{int} = \frac{1}{N}\sum_i \frac{1}{n_c}\sum_j \langle \vec{s_i}\cdot\vec{s_j}\rangle$, it is a single scalar
- $\lambda = -J$, it is the only Lagrange multiplier

Maximum Entropy approach

$$\Downarrow$$

$$P(\{\vec{x}\}) = \frac{1}{Z(J,n_c)} exp[\frac{J}{2} \sum_i \sum_{j \in n_c} \vec{s}_i \cdot \vec{s}_j]$$

Similar to the Heisenberg model for magnetism with homogeneous interaction strength J. Effective energy is than:

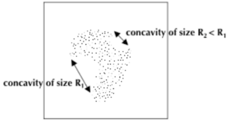
$$E(\{\vec{s}_i\}) = -\frac{J}{2} \sum_i \sum_{j \in n_c} \vec{s}_i \cdot \vec{s}_j$$

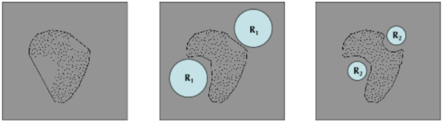
and an effective temperature $k_B T = 1$

Defining the border

α-shape algorithm:

the $3D$ space is carved out by spheres with $R = \alpha$ who can't have any point in their interior. When their surface hits three points these are added to the border.
The convex hull method is recovered for $\alpha \to \infty$.
The border for bird flocks must be redefined each time.





$R=\infty$
convex hull

$R=R_1$

The birds tend to align along a certain direction $\vec{n}$, analogously to spins aligning to the spontaneous magnetic field.

$$\vec{S} = \frac{1}{N}\sum_i \vec{s}_i = S\vec{n}$$

$\vec{S}$ is an order parameter. Individual orientations become

$$\vec{s}_i = s_i^L \vec{n} + \vec{\pi}_i$$

with $\sum_i \vec{\pi}_i = 0$ and $\sum_i \vec{s}_i^L = SN$.

$$Z(J_{ij}) = \int d^N\vec{s}\, e^{\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} J_{ij}\vec{s}_i\cdot\vec{s}_j}$$

$$\downarrow$$

$$\int d^N s^L d^N\vec{\pi} \prod_i \delta\left((s_i^L)^2 + |\vec{\pi}_i|^2 - 1\right)\delta\left(\sum_i \pi_i\right)\; e^{\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} J_{ij}(s_i^L s_j^L + \vec{\pi}_i\cdot\vec{\pi}_j)}$$

High polarization: $|\vec{\pi}_i| \ll 1 \rightarrow s_i^L \approx 1 - \frac{|\vec{\pi}_i|}{2}^2$

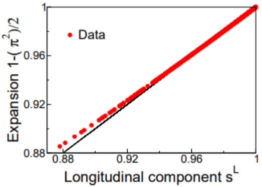


Figure: Longitudinal components of flight direction vs expansion value for all birds in the ...

Integrating over the longitudinal polarizations we get

$$Z(J_{ij}) = \int d^N\vec{\pi} \left[\prod_i \frac{1}{\sqrt{1-|\vec{\pi}_i|^2}}\right] \delta\left(\sum_i \vec{\pi}_i\right) e^{-\frac{1}{2}\sum_{i,j=1}^N A_{ij}\vec{\pi}_i\cdot\vec{\pi}_j+\frac{1}{2}\sum_{i,j=1}^N J_{ij}}$$

with $A_{ij} = \sum_k J_{ik}\delta_{ij} - J_{ij}$ with eigenvalues

$$A\,w^k = a_k w^k \qquad k = 1, \ldots, N$$

We can decompose $\vec{\pi} = (\vec{\pi}_1, \ldots, \vec{\pi}_N)$

$$\vec{\pi} = \sum_{k=1}^{N} \langle \vec{\pi}, w^k \rangle w^k$$

and rewrite

$$\sum_{i,j=1}^{N} A_{ij}\, \vec{\pi}_i \cdot \vec{\pi}_j = \vec{\pi}^T A\, \vec{\pi} = \sum_{k,l} \left(\langle \vec{\pi}, w^k \rangle \cdot \langle \vec{\pi}, w^l \rangle\, w_k^T A\, w_l \right) \qquad (*)$$

but $w_k^T A\, w_l = a_k w_k^T w_l = a_k \delta_l^k$.

With $\vec{\pi}'_k = \sum_{i=1}^{N} w_i^k \vec{\pi}_i$ $(*)$ becomes $\sum_k a_k |\vec{\pi}'_k|^2$.

Frame Title

The enforced condition is

$$\delta(\sum_i \vec{\pi}_i) = \delta\left(\vec{\pi}\cdot\frac{1}{\sqrt{N}}(1,\ldots,1)\right) = \delta(\vec{\pi}'_1)$$

The partition function can be written as

$$Z(J_{ij}) = \int d^N\vec{\pi'}\,\delta(\vec{\pi}'_1)e^{-\frac{1}{2}\sum_{k=1}^N a_k|\vec{\pi}'_k|^2+\frac{1}{2}\sum_{i,j=1}^N J_{ij}}$$

and integrating out the delta

$$Z(J_{ij}) = e^{\frac{1}{2}\sum_{i,j=1}^N J_{ij}}\int d^{N-1}\vec{\pi'}\,e^{-\frac{1}{2}\sum_{k>1}a_k|\vec{\pi}'_k|^2}$$

We can now take the logarithm of

$$Z(J_{ij}) = e^{\frac{1}{2}\sum_{i,j=1}^{N} J_{ij}} \prod_{k>1} \frac{2\pi}{a_k}$$

$$\log Z(J_{ij}) = \frac{1}{2} \sum_{i,j=1} J_{ij} - \sum_{k>1} \log a_k$$

Fixing J_{ij}

The parameters J_{ij} are fixed by requiring $\langle \vec{s}_i \cdot \vec{s}_j \rangle = \langle \vec{s}_i \cdot \vec{s}_j \rangle_{exp}$. Focusing on the perpendicular part of the correlation

$$\langle \vec{\pi}_i \cdot \vec{\pi}_j \rangle_{exp} = 2 \sum_{k>1} \frac{w_i^k w_j^k}{a_k}$$

To reconstruct J_{ij} I need to invert the LHS matrix, which must have $N-1$ nonzero eigenvalues (no zero mode, $k > 1$).

Average local correlations

Since J_{ij} depends on $|\vec{r}_i - \vec{r}_j|$ and I need to evaluate the experimental average over more independent samples than birds this approach is not feasible: the birds don't stay at fixed relative positions; experimental samples correspond to different networks and the J_{ij} cannot be averaged together.

This motivates the restriction to average local correlations (among pairs in a neighborhood of size n_c)

$$C_{int} = \frac{1}{N} \sum_i \frac{1}{n_c} \sum_{j \in n_c^i} \langle \vec{s}_i \cdot \vec{s}_j \rangle \approx \frac{1}{N} \sum_{i=1}^{N} \frac{1}{n_c} \sum_{j \in n_c^i} \vec{s}_i \cdot \vec{s}_j$$

which can be estimated from a single snapshot. (FORSE QUI VA SPIEGATO IL FATTO DEL RANGO che è la motivazione più precisa per usare Cint invece che quello globale)

Computation with free boundary

Birds move and change neighbors, thus interaction strength cannot depend directly on individual identities but only on some function of their relative positions:

$$J_{ij} = J n_{ij}$$

where

$$n_{ij} = \begin{cases} 1 & \text{if } j \in n_i^c \text{ and} i \in n_j^c \\ \frac{1}{2} & \text{if } j \in n_i^c \text{ and} i \notin n_j^c \text{ or viceversa} \\ 0 & \text{otherwise} \end{cases}$$

and we can also define

$$A_{ij} = J\widetilde{A}_{ij},$$

where $\widetilde{A}_{ij} = \delta_{ij} \sum_k n_{ik} - n_{ij}$

with eigenvalues $\lambda_k = \frac{a_k}{J}$ and eigenvectors $\vec{w}^k$

Interaction parameters inference

Now we can obtain:

$$\langle \vec{\pi}_i \cdot \vec{\pi}_j \rangle = \frac{2}{J} \sum_{k>1} \frac{w_i^k w_j^k}{\lambda_k} \quad \text{and} \quad \langle s_i^L s_j^L \rangle = 1 - \frac{1}{J} \sum_{k>1} \frac{(w_i^k)^2 + (w_j^k)^2}{\lambda_k}$$

How to calculate J and n_c?

$$C_{int}(J, n_c) = C_{int}^{exp} \quad \text{is true}$$

$$\Updownarrow$$

$$\langle logP(\{\vec{s}_i\}) \rangle = -logZ(J, n_c) + \frac{1}{2} J N n_c C_{int}^{exp} \quad \text{is maximum}$$

Interaction parameters inference

Solving the equation w.r.t. J brings to an explicit expression of J as a function of n_c

$$J(n_c) = \frac{2}{n_c} \frac{1}{(1 - C_{int}^{exp})}$$

Substituting back in the Likelihood the maximum can be found numerically

For each flock mean and standard deviation of the interaction parameters over time are then calculated

Computation with fixed boundaries

Birds on the boundary have a different kind of neighborhood and are exposed to external stimuli

$$\Downarrow$$

We take their velocities as given and try to predict order propagation throughout the flock.

Let $\mathcal{B}, \mathcal{I}$ be the subsets of external and internal border individuals.

Fixed border partition function

$$Z(J_{ij};\mathcal{B}) = \int d^{\mathcal{I}}\,\delta\left(\sum_i \vec{\pi}_i\right) e^{-H}$$

where

$$H = -\left[\frac{1}{2}\sum_{i,j\in\mathcal{I}} A_{ij}\vec{\pi}_i\cdot\vec{\pi}_j + \sum_{i\in\mathcal{I}}\vec{\pi}_i\cdot\vec{h}_i + \frac{1}{2}\sum_{i,j\in\mathcal{I}} J_{ij} + \frac{1}{2}\sum_{i\in\mathcal{I}} h_i^L + \frac{1}{2}\sum_{i,j\in\mathcal{B}} J_{ij}\vec{s}_i\cdot\vec{s}_j\right]$$

- $\vec{h}_i = \sum_{l\in\mathcal{B}} J_{il}\vec{s}_l = \sum_{l\in\mathcal{B}} J_{il}\left(s_l^L\vec{n} + \vec{\pi}_l\right) = h_i^L\vec{n} + \vec{h}_i^P$ is a "field" which aligns bird i with the border birds within n_c^i.
- $A_{ij} = \delta_{ij}\left(\sum_{k\in\mathcal{I}} J_{ik} + h_i^L\right) - J_{ij}\;\; i,j\in\mathcal{I}$ gets a diagonal contribution and is defined only for the internal part of the flock.

$\log Z$

$$\ln Z(\{J_{ij}\};\mathcal{B}) = \frac{1}{2}\sum_{ij\in\mathcal{I}}(A^{-1})_{ij}\vec{h}_i^P\vec{h}_j^P - \ln\det A - \ln\left[\sum_{ij\in\mathcal{I}}(A^{-1})_{ij}\right]$$

$$-\frac{1}{2}\frac{(A^{-1})_{ij}\vec{h}_i^P + \sum_{l\in\mathcal{B}}\vec{\pi}_l}{\sum_{ij\in\mathcal{I}}(A^{-1})_{ij}} + \frac{1}{2}\sum_{ij\in\mathcal{I}}J_{ij} + \sum_{i\in\mathcal{I}}h_i^L + \sum_{l,m\in\mathcal{B}}J_{l,m}\vec{s}_l\cdot\vec{s}_m$$

Now deriving wrt $\vec{h}_i^P$ gives

$$\langle\vec{\pi}_i\rangle = \sum_{j\in\mathcal{J}}(A^{-1})_{ij}\vec{h}_i^P - \mathit{se\ serve\ la\ scrivo}$$

Log likelihood setup

We are interested in

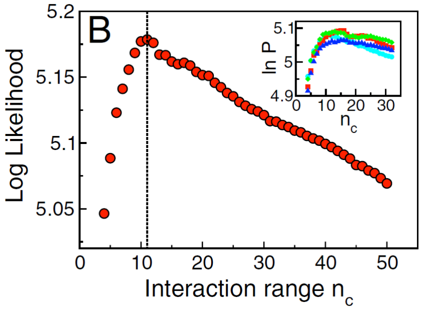
$$\langle \vec{\pi}_i \vec{\pi}_j \rangle = \langle \vec{\pi}_i \rangle \langle \vec{\pi}_j \rangle + 2 \left[(A^{-1})_{ij} - \frac{\sum_{k,n \in \mathcal{I}} (A^{-1})_{ik})(A^{-1})_{nj})}{\sum_{k,n \in \mathcal{I}} (A^{-1})_{kn})} \right]$$

since we can solve the previous maximum entropy model in the reduced case where $J_{ij} = Jn_{ij}$, by maximizing

$$\langle \log P(\{\vec{s}_i\}) \rangle_{exp} = - \log Z(J, n_c; \mathcal{B}) + \frac{1}{2} J n_c N C_{int}^{exp}$$

Maximizing the likelihood

Maximizing wrt J is equivalent to imposing $C_{int}(J, n_c; \mathcal{B}) = C_{int}^{exp}$, which gives $J(n_c; \mathcal{B})$.



The graph represents $\frac{\langle \log P(\{\vec{s}_i\})\rangle_{exp}}{N}$ vs n_c. The inset shows other snapshots (for the same flock).

Two boundary cases comparison

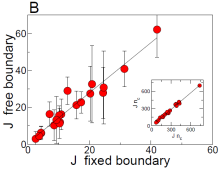
For each sample we have:

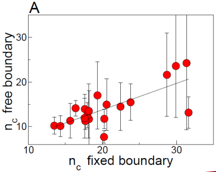
$$C_{int}^{exp}, J, n_c$$

and for each flocking event we can calculate:

$$\langle J \rangle, \ \langle n_c \rangle, \ \sigma_J, \ \sigma_{n_c}$$

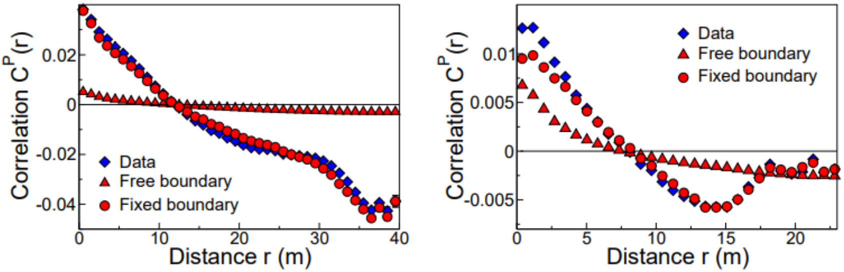
Thus we can compare the two boundary cases:





- both parameters are strong correlated
- n_c slightly smaller for free boundaries
- J slightly larger for free boundaries
- Jn_c is approximately the same

Two boundary cases comparison



$$C_{int} = C_{int}^{P} + S + (1 - \frac{1}{N}\sum_{i}\frac{1}{n_c}\sum_{j\in n_c}\langle|\vec{\pi}_i|^2\rangle)$$

The model with free boundaries underestimates the contribution on short scales from C_{int}^{P}, and compensates by overestimating the polarization terms.

Global model

Assumption: exist a unique value of n_c and J through time.

$\Downarrow$

The global likelihood corresponds to all the snapshots and it is optimized only once

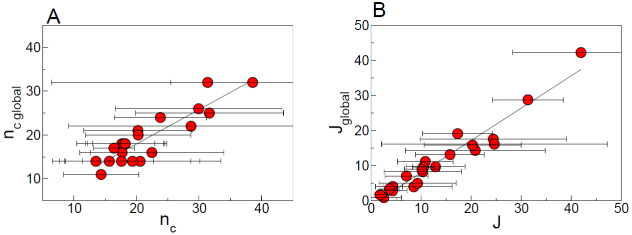
$$\Phi_{global}(J, n_c) = \frac{1}{N_{snap}} \sum_{\alpha} [-logZ(J, n_c; \mathcal{B}_\alpha) + \frac{1}{2} J N n_c C^{exp}_{int,\alpha}]$$

$$\Downarrow$$

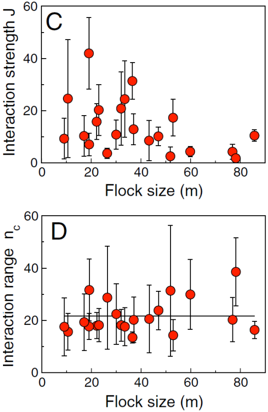
$$\frac{1}{J_{global}} = \frac{1}{N_{snap}} \sum_\alpha \frac{(N^\alpha_{snap} - 1)}{(N^{global}_{in} - 1)} \frac{1}{J_\alpha(n_c)}$$

Global model

Again substituting back in the global Log Likelihood we can obtain J^{global} and n_c^{global}



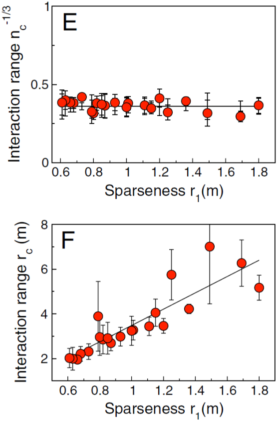
Metric dependence of model parameters



- Larger error bars for smaller flocks
- No significant trend with the flocks' linear dimensions

$\Downarrow$

No increase of n_c and J with the size of the flock means real interactions are not extended over long distances



- r_1 = typical distance between neighboring birds (closely related to the density)
- r_c = metric interaction range, meaning $J_{ij} = J$ if and only if birds i and j lie within r_c meters
- results are in contrast with interactions extended over some fixed metric distance r_0, if true $n^{\frac{1}{3}} \propto \frac{r_1}{r_0}$ and r_c = constant

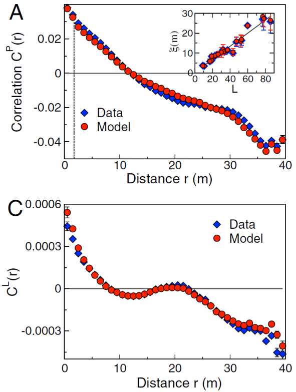
$$\Downarrow$$

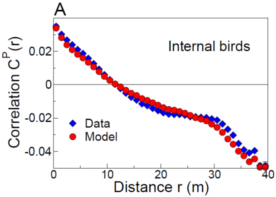
Birds interact with a fixed number of neighbors, rather than with all the birds within a fixed metric distance

Model prediction

Is the model able to describe correlations on all length scales?

$$C(r) = \frac{\sum_{ij} C_{ij}\delta(r_{ij} - r)}{\sum_{ij} \delta(r_{ij} - r)}, \qquad C(r) = C^L(r) + C^P(r)$$





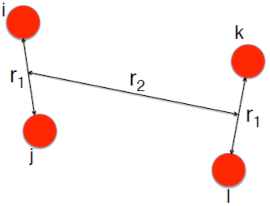
$\downarrow$

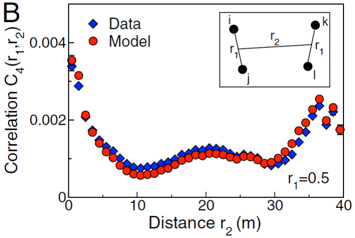
only taking in account the contributions from individuals inside the flock $(i, j \in \mathcal{I})$

Can the model predict correlations among quadruplets of birds?

$$C_4(r_1; r_2) = \frac{\sum_{ijkl} \langle (\vec{\pi}_i \cdot \vec{\pi}_j)(\vec{\pi}_i \cdot \vec{\pi}_j) \rangle \Delta_{ijkl}}{\sum_{ijkl} \Delta_{ijkl}},$$

$$\Delta_{ijkl} = \delta(r_{ij} - r_1)\delta(r_{kl} - r_1)\delta(r_{ij-kl} - r_2)$$

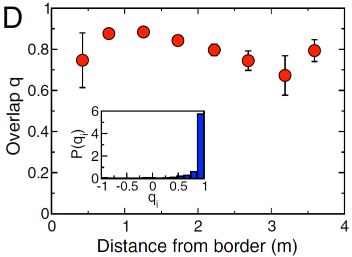


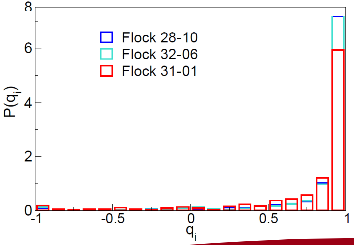


With $n_c \approx 20$ the field $\vec{h}_i$ generated by the border configuration is non-zero only in a small shell.

Can the model anyway predict the collective coherence?

$$q_i = \frac{\langle \vec{\pi}_i \rangle \cdot \vec{\pi}_i^{exp}}{\left|\langle \vec{\pi}_i \rangle\right| \left|\vec{\pi}_i^{exp}\right|}$$





Order Propagation

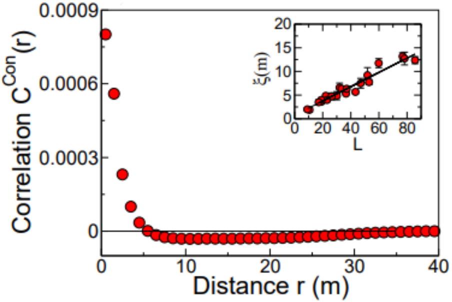
$$\langle \vec{\pi}_i \rangle = \sum_{j \in \mathcal{I}} C^{con}_{ij} \vec{h}^P_j - \frac{\sum_{j \in \mathcal{I}} (A^{-1})_{ij}}{\sum_{k,j \in \mathcal{I}} (A^{-1})_{kj}} \sum_{l \in \mathcal{B}} \vec{\pi}_l$$

$$\langle \vec{\pi}_i \cdot \vec{\pi}_j \rangle = C^{con}_{ij} + \langle \vec{\pi}_i \rangle \cdot \langle \vec{\pi}_j \rangle$$

$$C^{con}_{ij} = 2 \left[A^{-1}_{ij} - \frac{\sum_{k,n \in \mathcal{I}} (A^{-1})_{ik} (A^{-1})_{nj}}{\sum_{k,n \in \mathcal{I}} (A^{-1})_{kn}} \right]$$

The second term of the first equation is just there to ensure $\sum_{i \in \mathcal{I}} \langle \vec{\pi}_i \rangle + \sum_{l \in \mathcal{B}} \langle \vec{\pi}_l \rangle = 0$. Having $C_{ij} \neq 0$ between bird i and some bird j closer to the border ensures effective alignment.

Connected correlation function



We can see that C_{con} extends over a sufficiently large region to propagate order inside the flock. The inset shows that the correlation length grows linearly with the flock size L: this implies that the correlation is scale free. CAPIRE BENE QUESTA PARTE PER ORALE

Mechanistic Interpretation:

$$\frac{d\vec{s}_i}{dt} = -\frac{\partial H}{\partial \vec{s}_i} + \vec{\eta}_i(t) = \sum_{j=1}^{N} J_{ij}\vec{s}_j + \vec{\eta}_i(t)$$

Where:

- J_{ij} measures the strength of the force that tries to align the velocity of bird i along the direction defined by bird j
- The Hamiltonian is the one of Heisenberg model for spins $\vec{s}_i$:

$$H(\{\vec{s}_i\}) = -\frac{1}{2}\sum_{i,j} J_{ij}\vec{s}_i \cdot \vec{s}_j$$

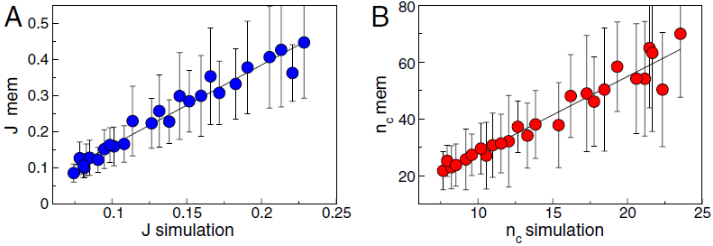
Numerical simulation

Simulation: population of self-propelled particles in three dimensions moving according to social forces that tend to align each particle with the average direction of its neighbors.

The strength of alignment is given by

$$J = \frac{v_0 \alpha}{n_c}$$

so it can be tuned varying n_c and α is chosen s.t. the system remains in a flock-like state, i.e. it does not crystalize and diffusion of individuals occurs throughout the group.



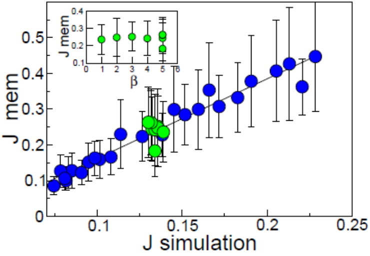
- Both J and n_c obtained by the model are proportional to the "microscopic" parameters used in the simulation
- n_c^{mem} is 3 $\times$ larger than n_c^{sim}

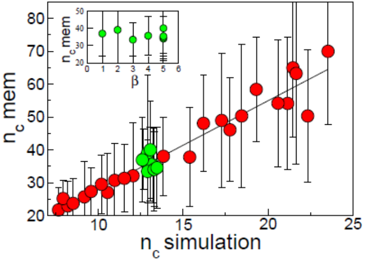
Possible explenation: the model, where interactions are static by construction, compensates the dynamical nature of the true interaction network by giving a larger effective value of n_c.

Numerical simulation

How much the metric dependency of the force affects the relationship between real and inferred parameters?

Additional simulations varying the parameters entering the distance dependent term:





Discussion with passion

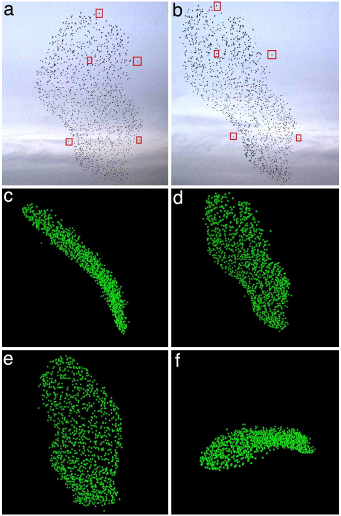
It is a complete theory for the propagation of directional order throughout the flock, with no free parameters. It is different from most of the theoretical previous studies of collective animal behavior because:

- It gives a quantitative description of collective motion thanks to methods used for statistical mechanics models.
- It is based on a general guiding principle overcoming the problem of choosing the right assumptions.

A possible implementation integrating the evolving positions.

Backup slides

3D space configuration reconstruction



(a), (b) photographs are a stereo pair, taken at the same instant of time but 25 m apart. To perform the 3D reconstruction, each bird's image on a must be matched to its corresponding image on b. Five matched pairs of birds are visualized by the red squares. (c), (e), (f) are 3D reconstruction of the flock under four different points of view. (f) is the reconstructed flock under the same perspective as in (b).

Self-propelled particles dynamics

Each particle moves according to the following equations:

$$\vec{v}_i(t+1) = v_0 \Theta[\alpha \sum_{j \in n_c^i} \vec{v}_j(t) + \beta \sum_{j \in n_c^i} \vec{f}_{ij} + n_c \vec{\eta}_i]$$

$$\vec{x}_i(t+1) = \vec{x}_i(t) + \vec{v}_i(t)$$

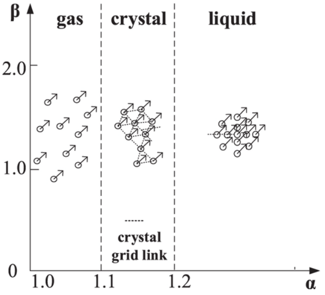
where the distance-dependent force is:

$$\vec{f}_{ij} = \begin{cases} -\infty \vec{e}_{ij} & \text{if } r_{ij} < r_b \\ \frac{1}{4} \cdot \frac{r_{ij} - r_e}{r_a - r_e} \vec{e}_{ij} & \text{if } \ r_b < r_{ij} < r_a \\ \vec{e}_{ij} & \text{if } r_a < r_{ij} < r_0 \end{cases}$$

Self-propelled particles dynamics

For the first simulation the parameters are fixed:

$$r_0 = 1, r_b = 0.2, r_e = 0.5, r_a = 0.8, \alpha = 35, \beta = 5, v_0 = 0.05, N = 512$$



The state of the population must be liquid, s.t. reproduces the bird flocks where the network is not static (property of crystal state) and birds are enough close to permit diffusion of individuals throughout the group (not possible in gas state)