بسم الله الرحمن الرحیم

نظریه علوم کامپیوتر

نظریه علوم کامپیوتر - بهار ۱۴۰۰-۱۴۰۱ - جلسه چهارم: زبان‌های غیر مستقل از زمینه و ماشین تورینگ

Theory of computation - 002 - S04 - Non-CFG Languanges and Turing Machine

Review

Last time:

- Context free grammars (CFGs)
- Context free languages (CFLs)
- Pushdown automata (PDA)
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- Proving languages not Context Free
- Turing machines
- T-recognizable and T-decidable languages

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Recall Theorem: A is a CFL iff some PDA recognizes A

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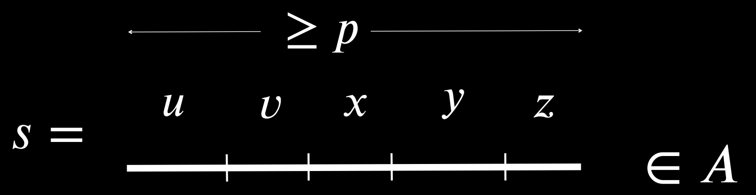

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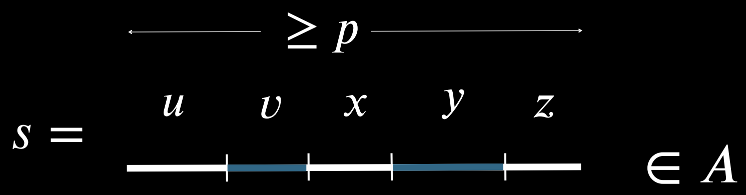

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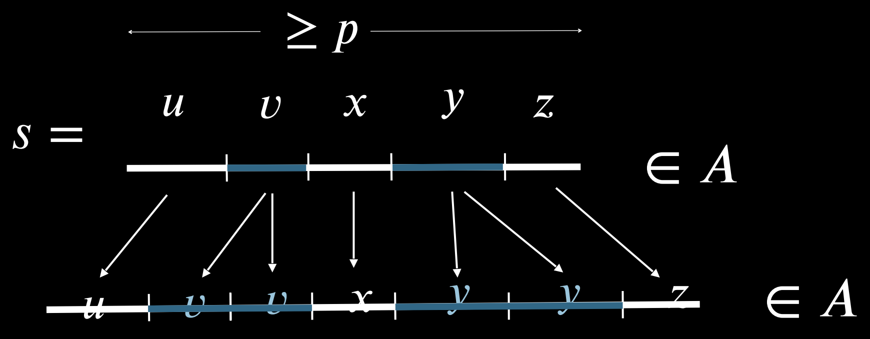

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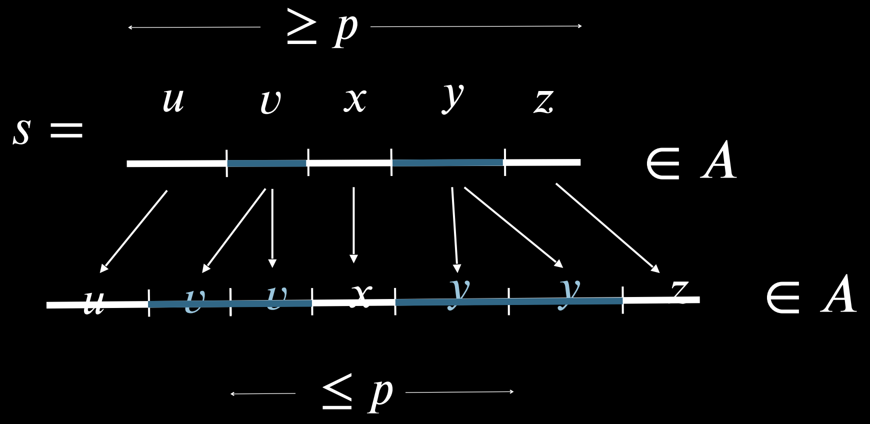

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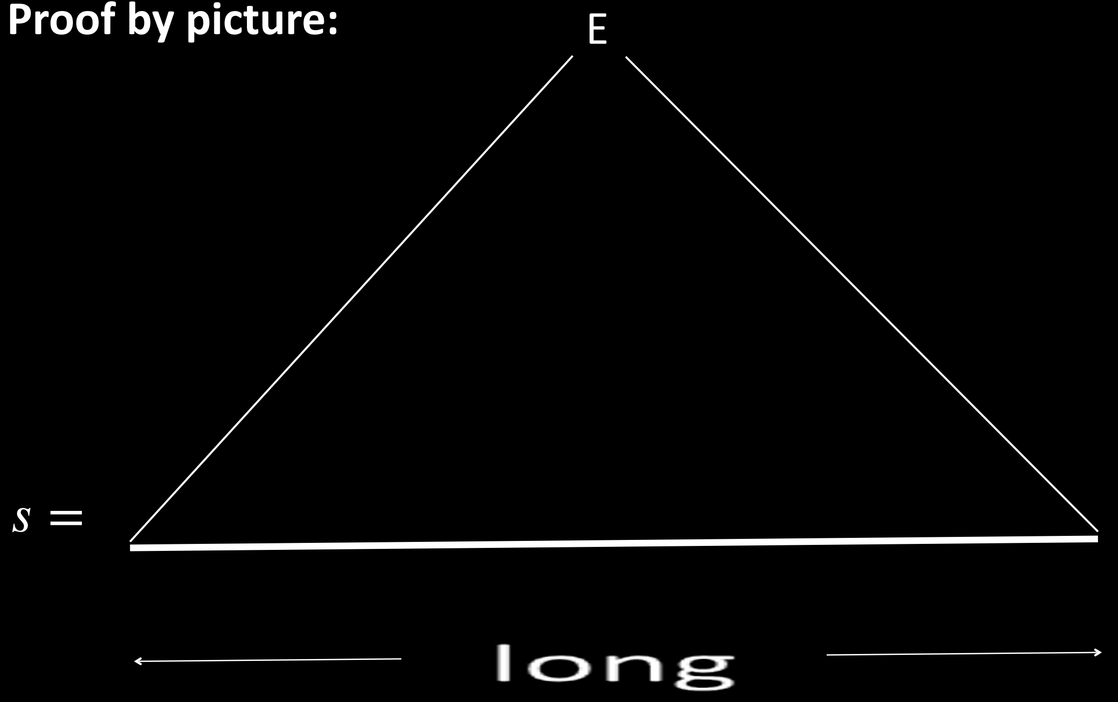

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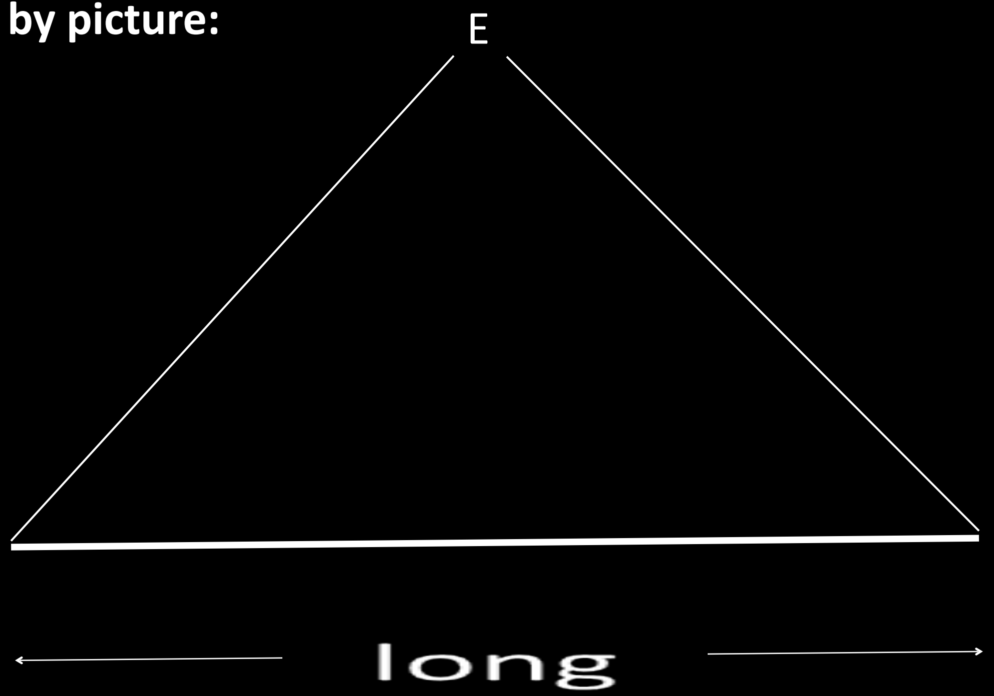

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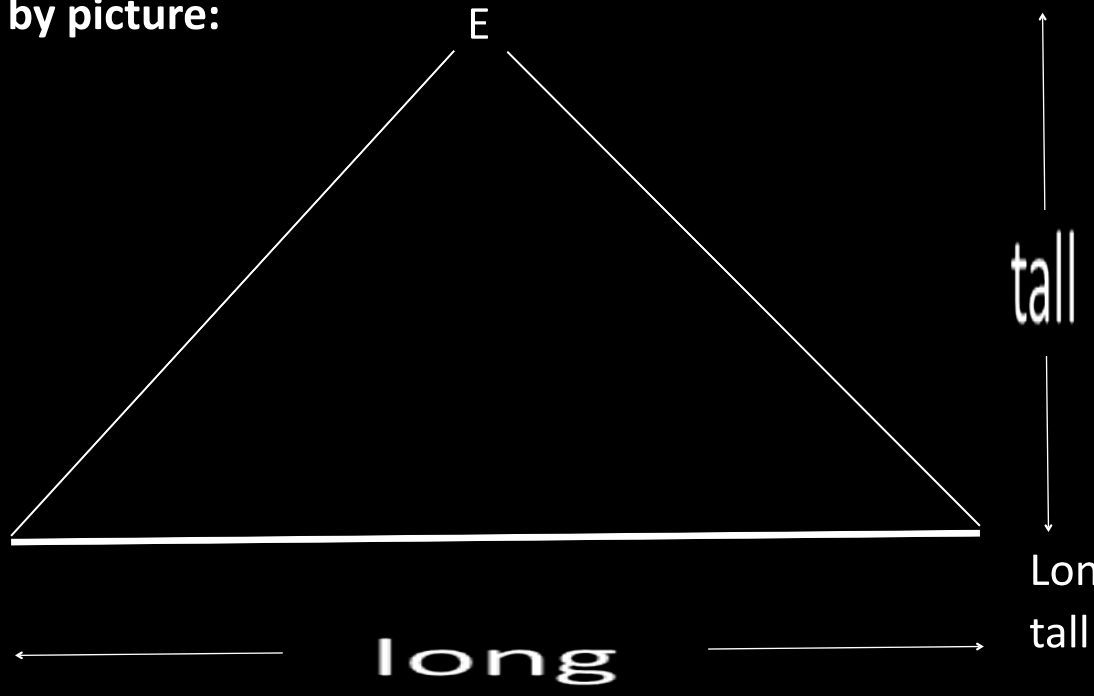

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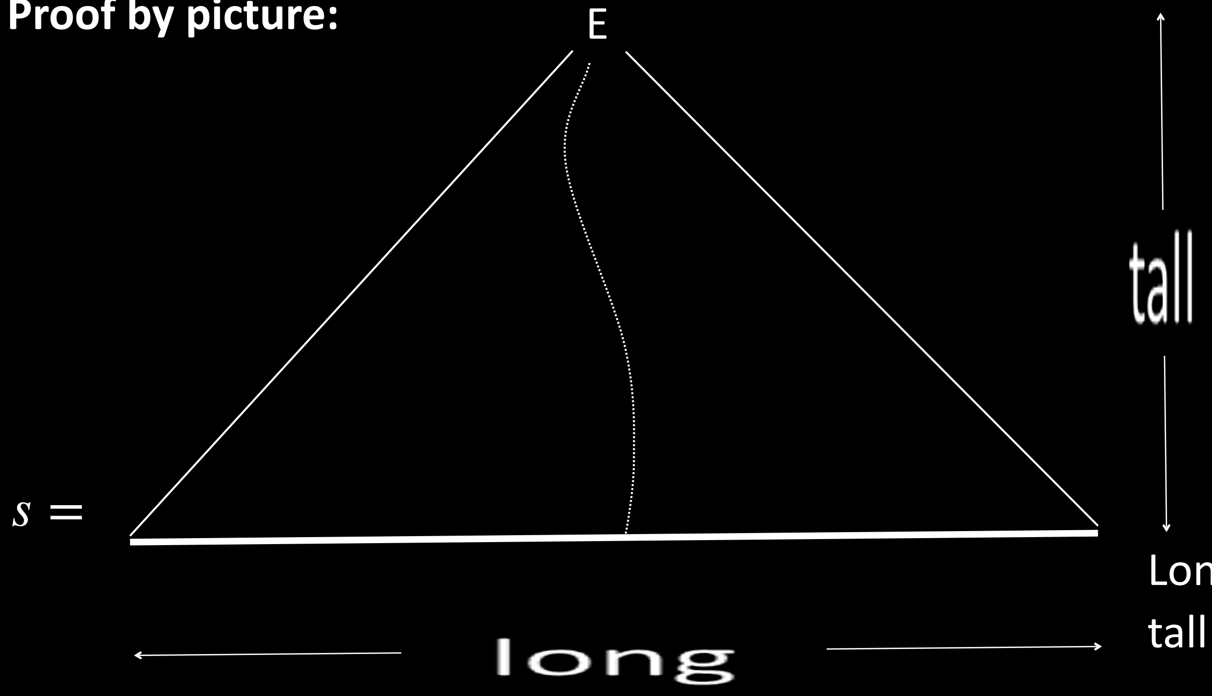

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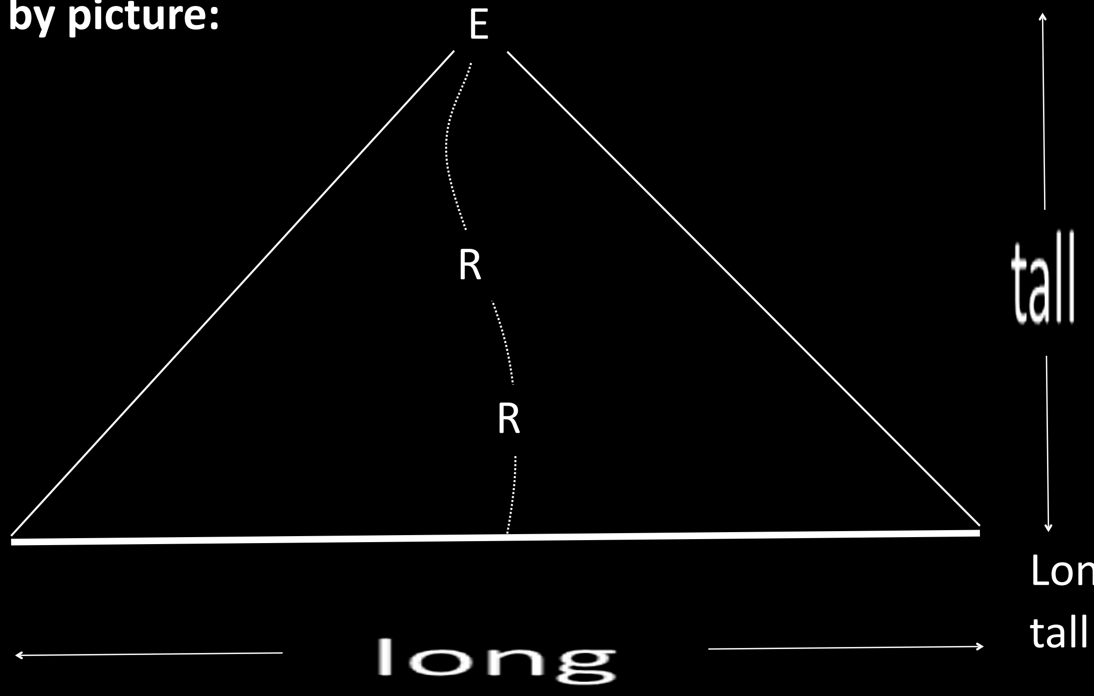

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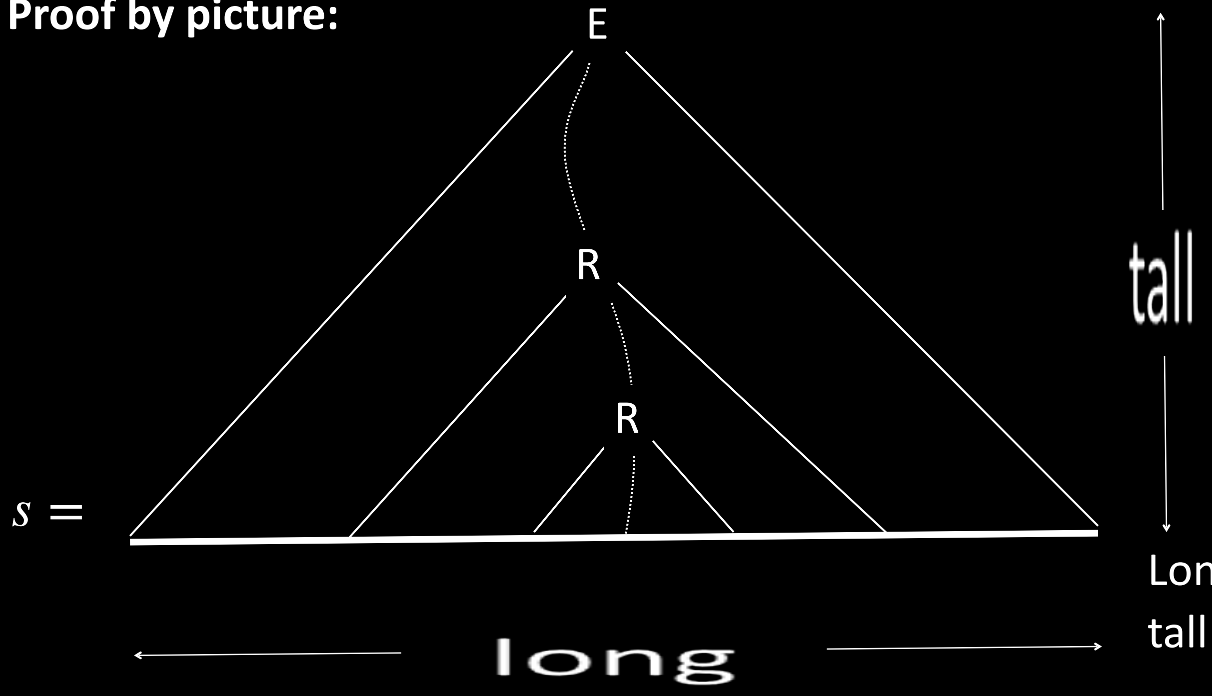

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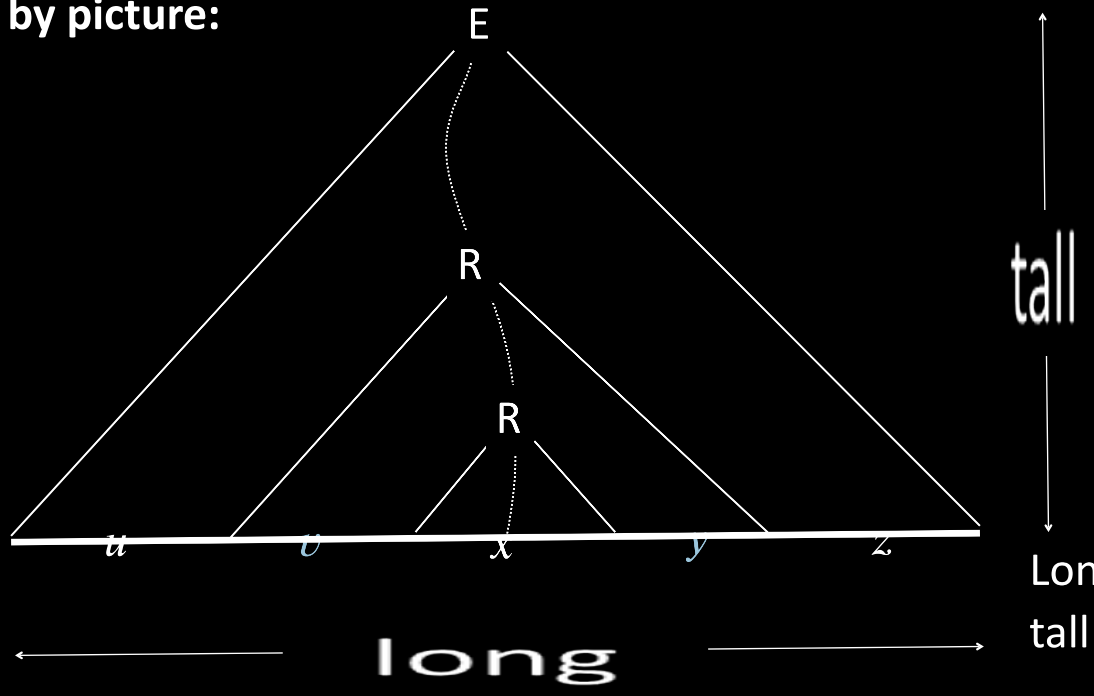

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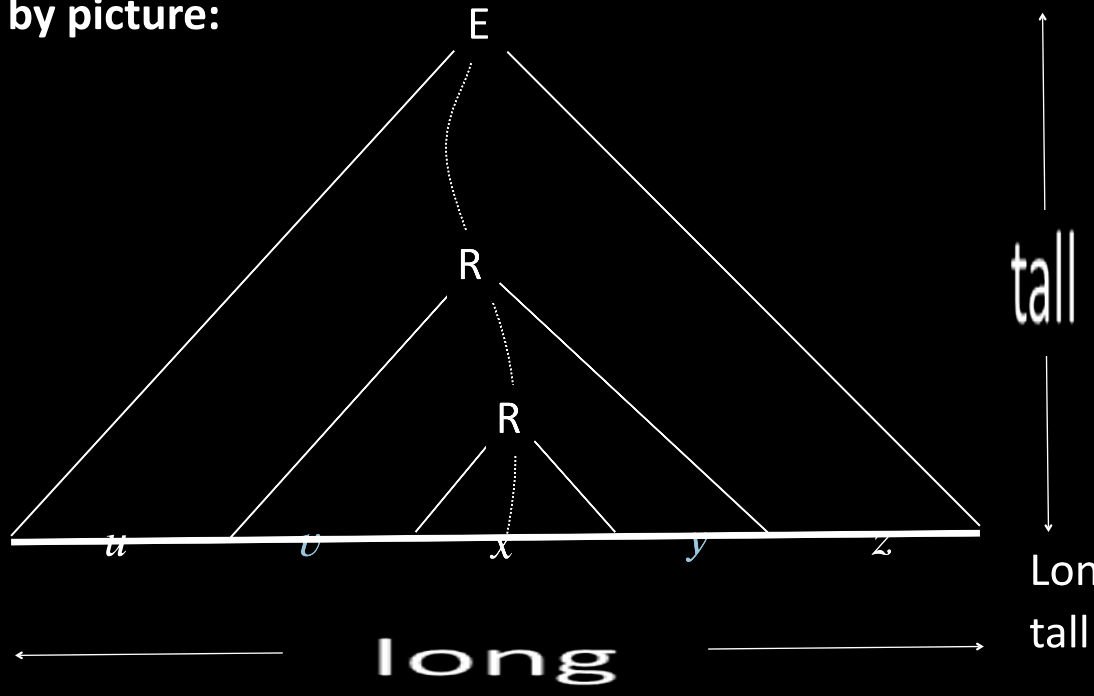

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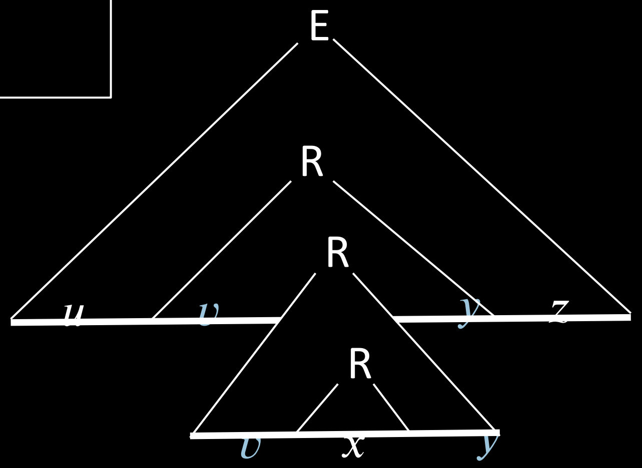

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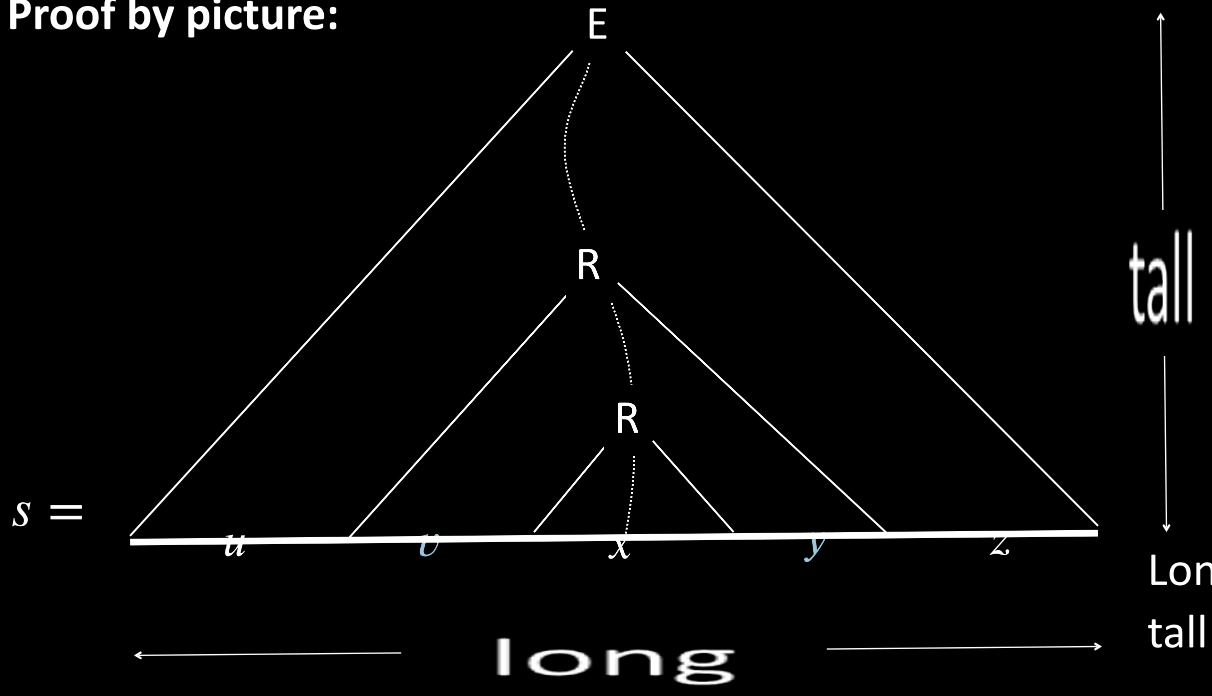

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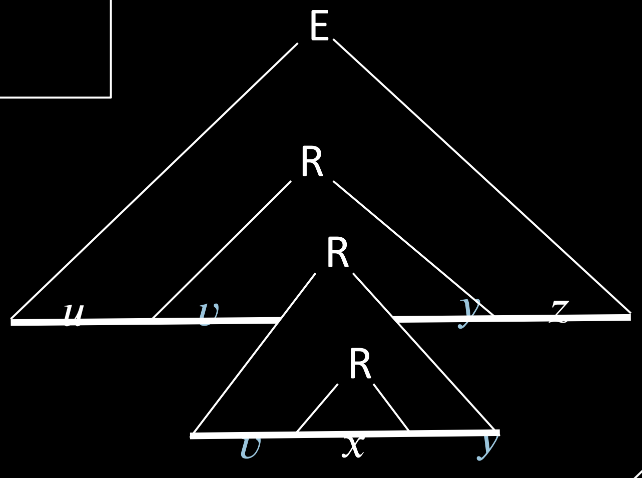

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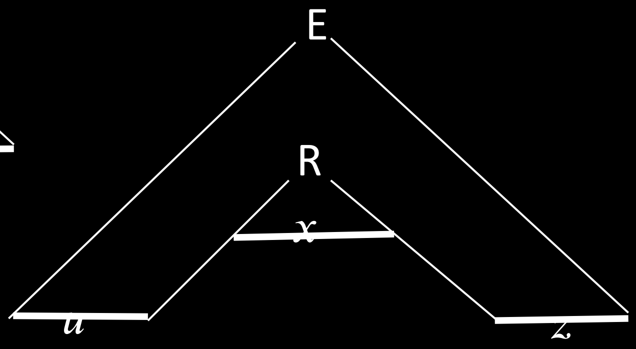

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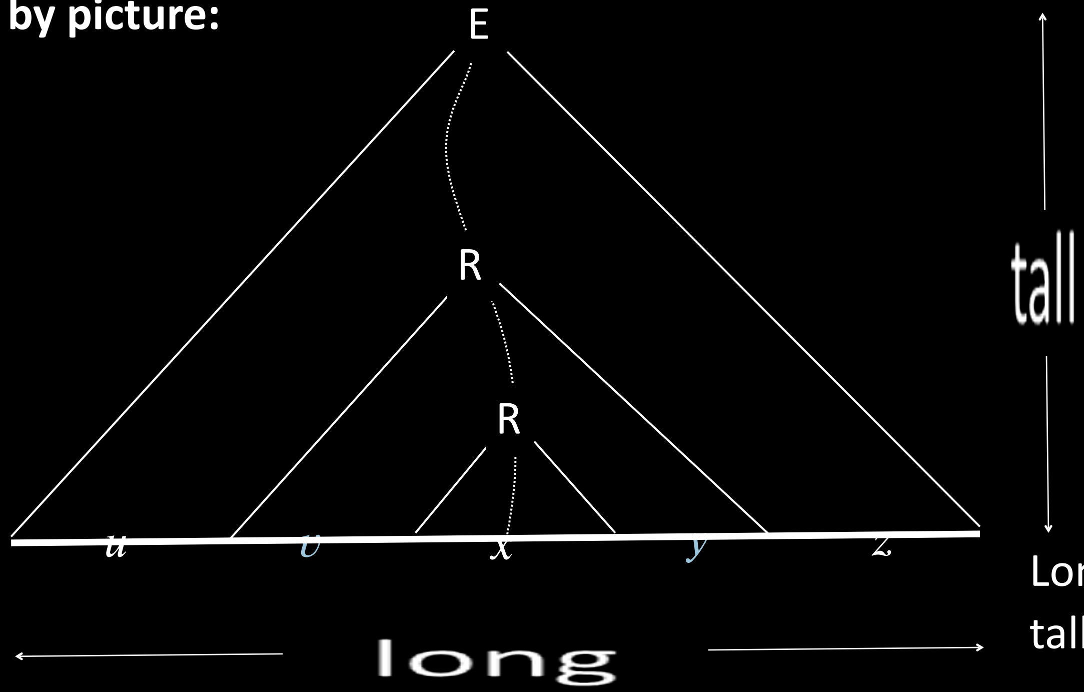
$s =$ 

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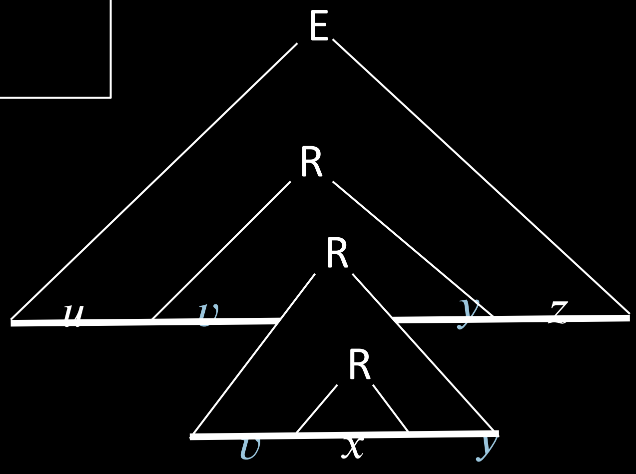

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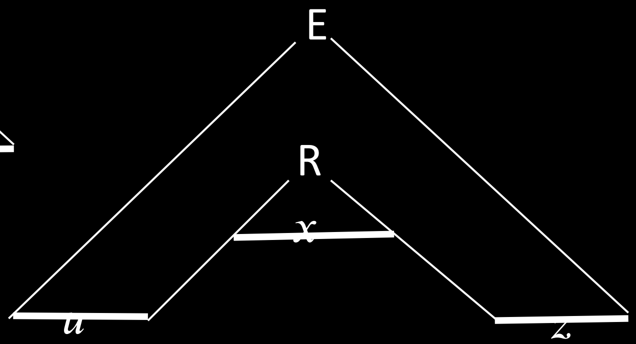

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"cutting and pasting" argument

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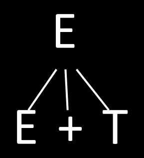

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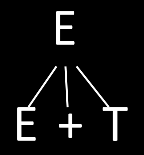

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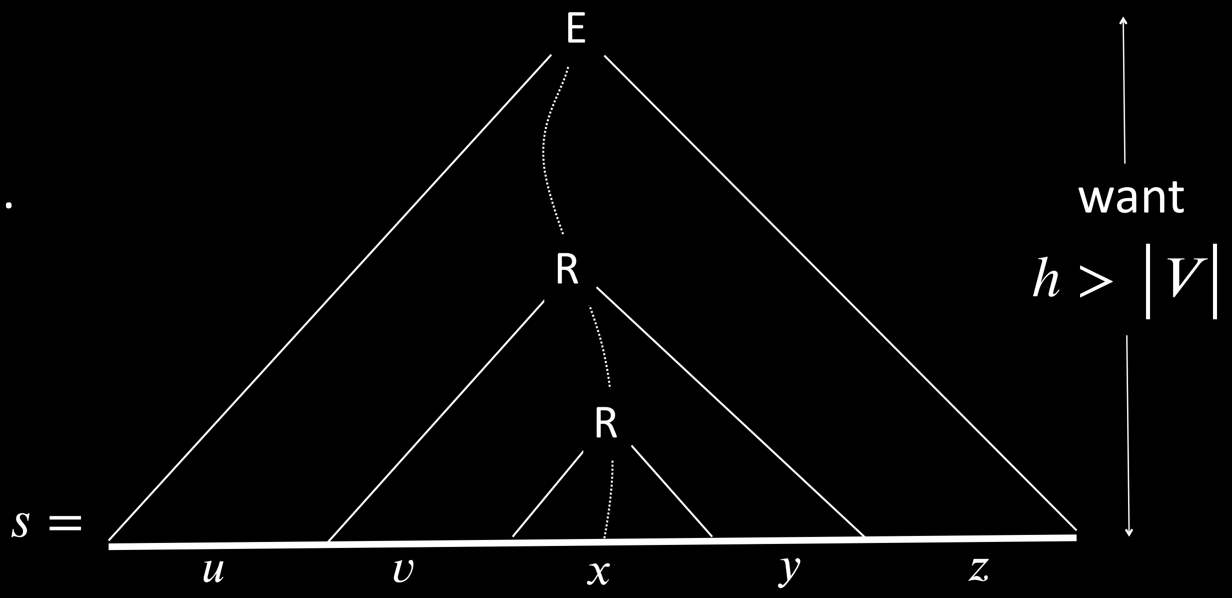

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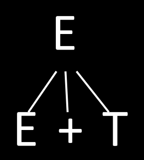

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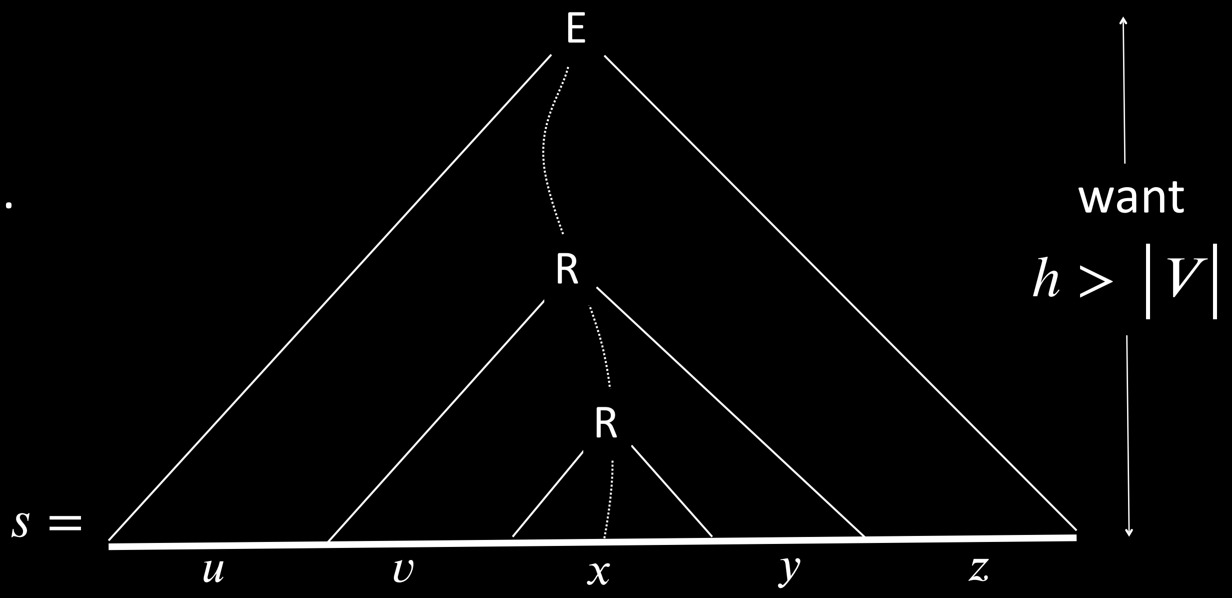

use $|s| > b^{|V|}$

set $p = b^{|V|} + 1$

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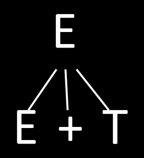

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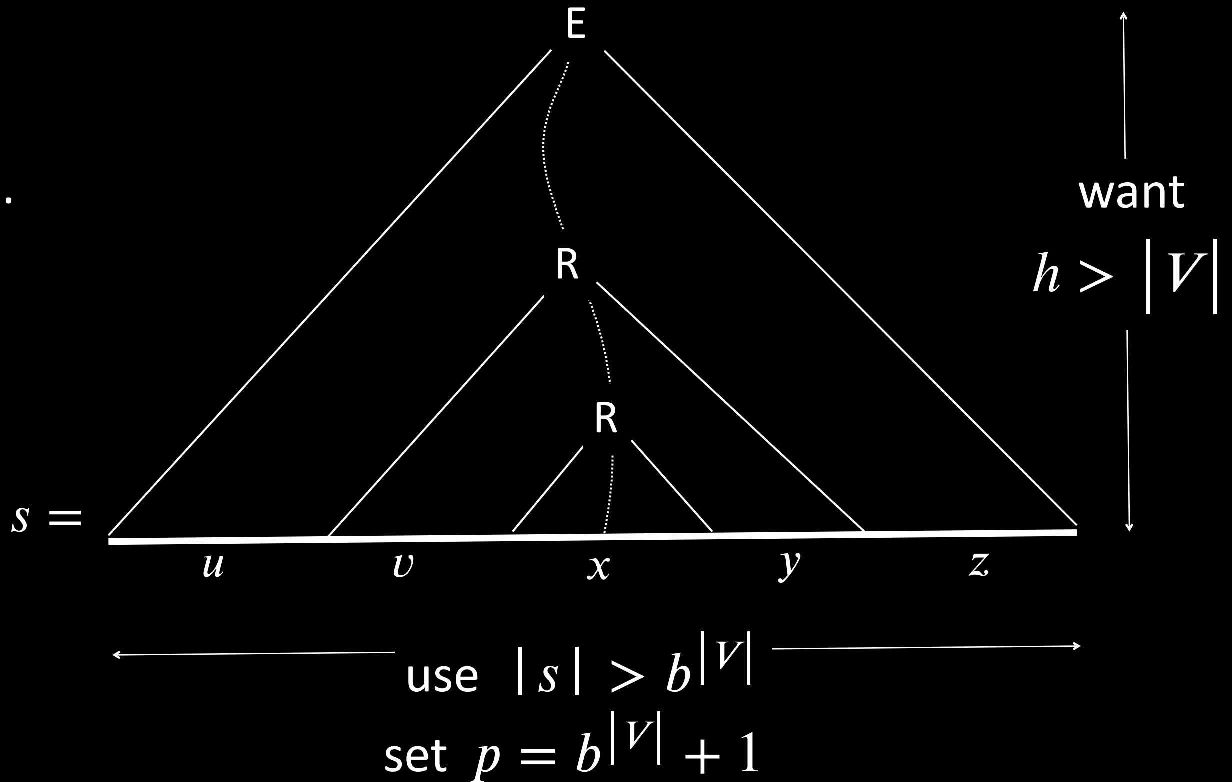

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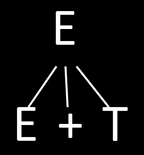

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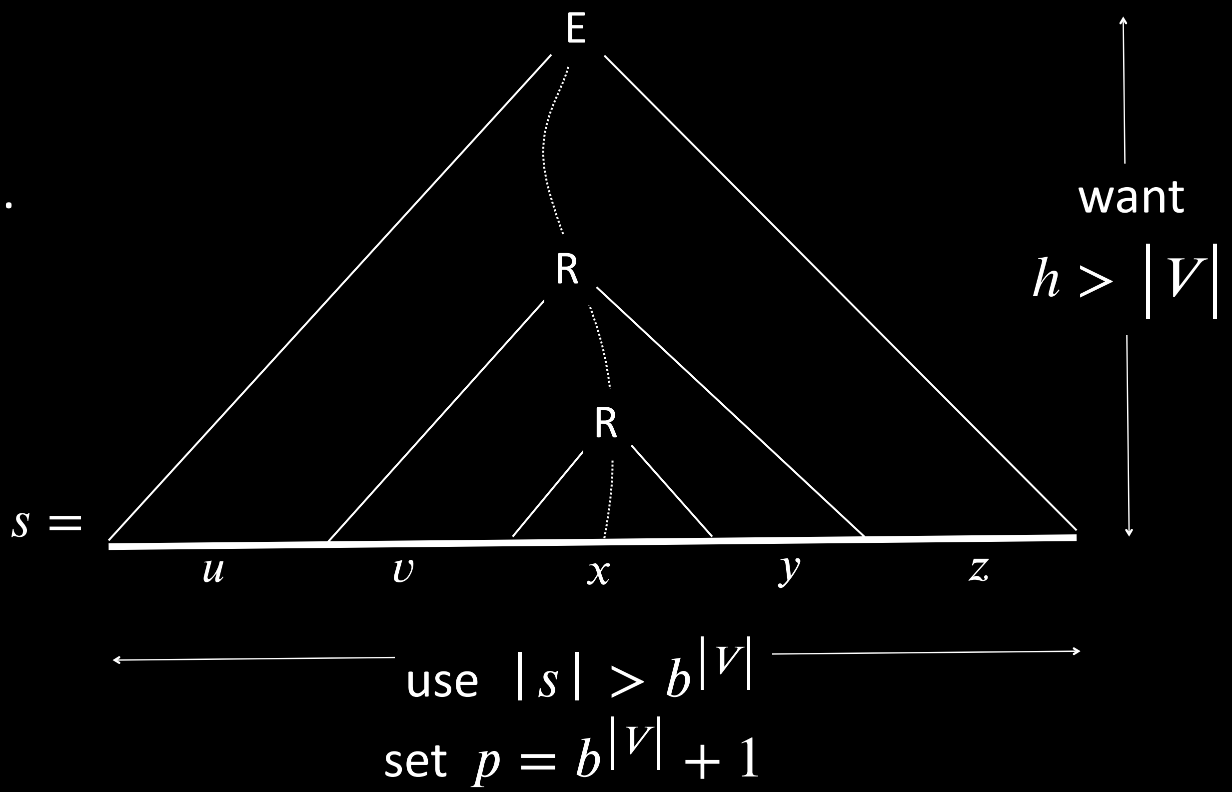
So if $|s| \geq p > b^{|V|}$ then $|s| > b^{|V|}$ and so $h > |V|$.



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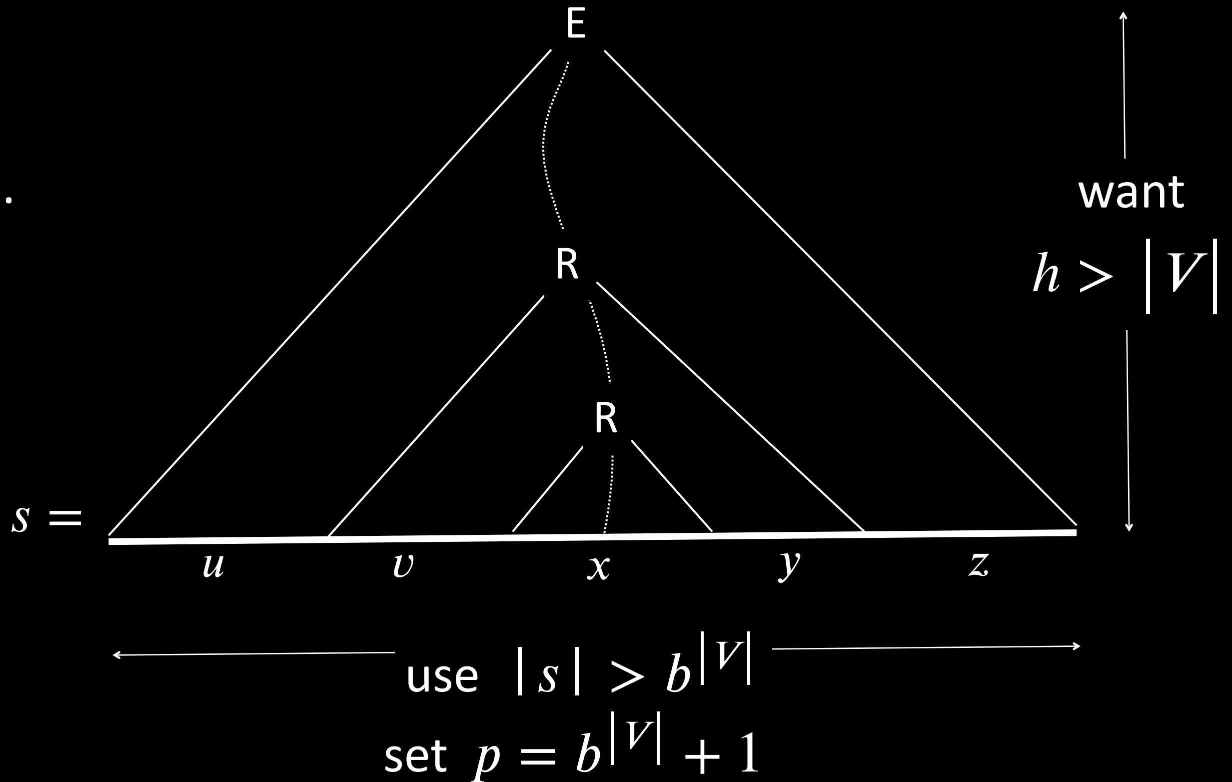

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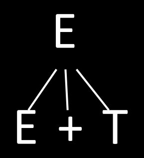

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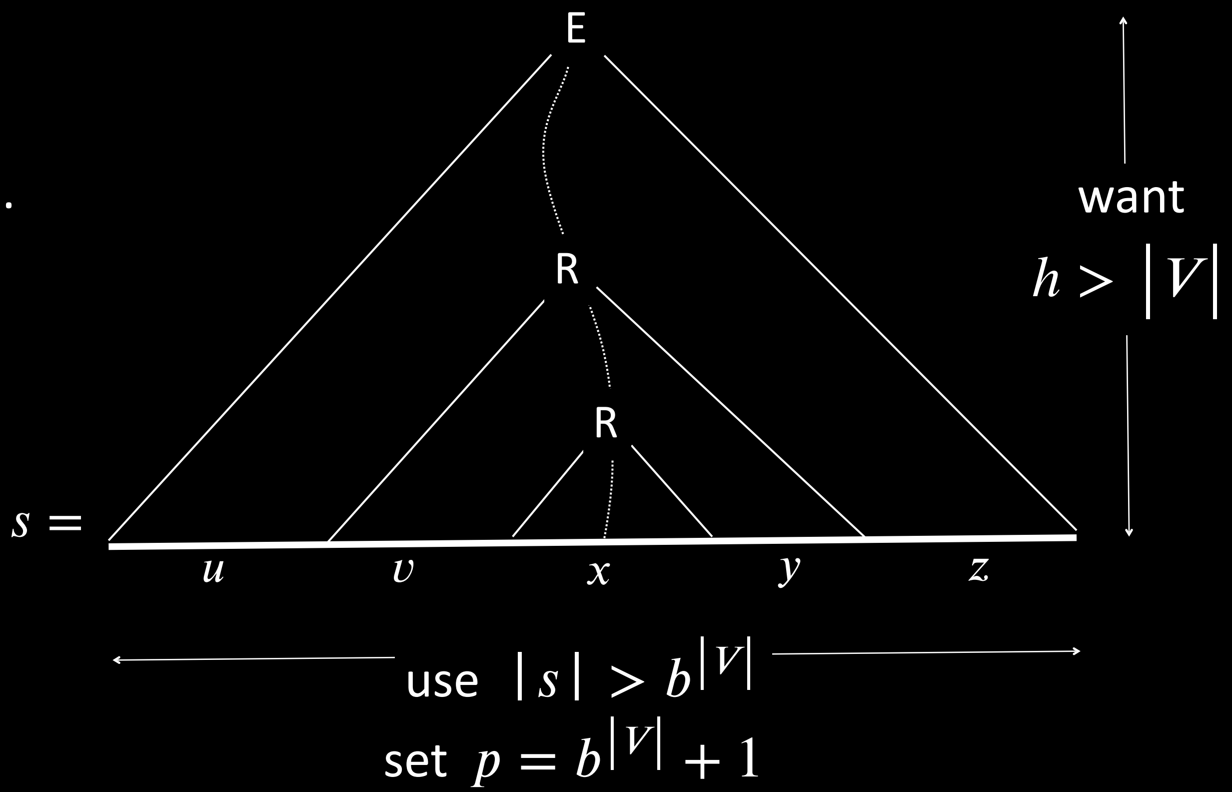

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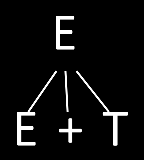

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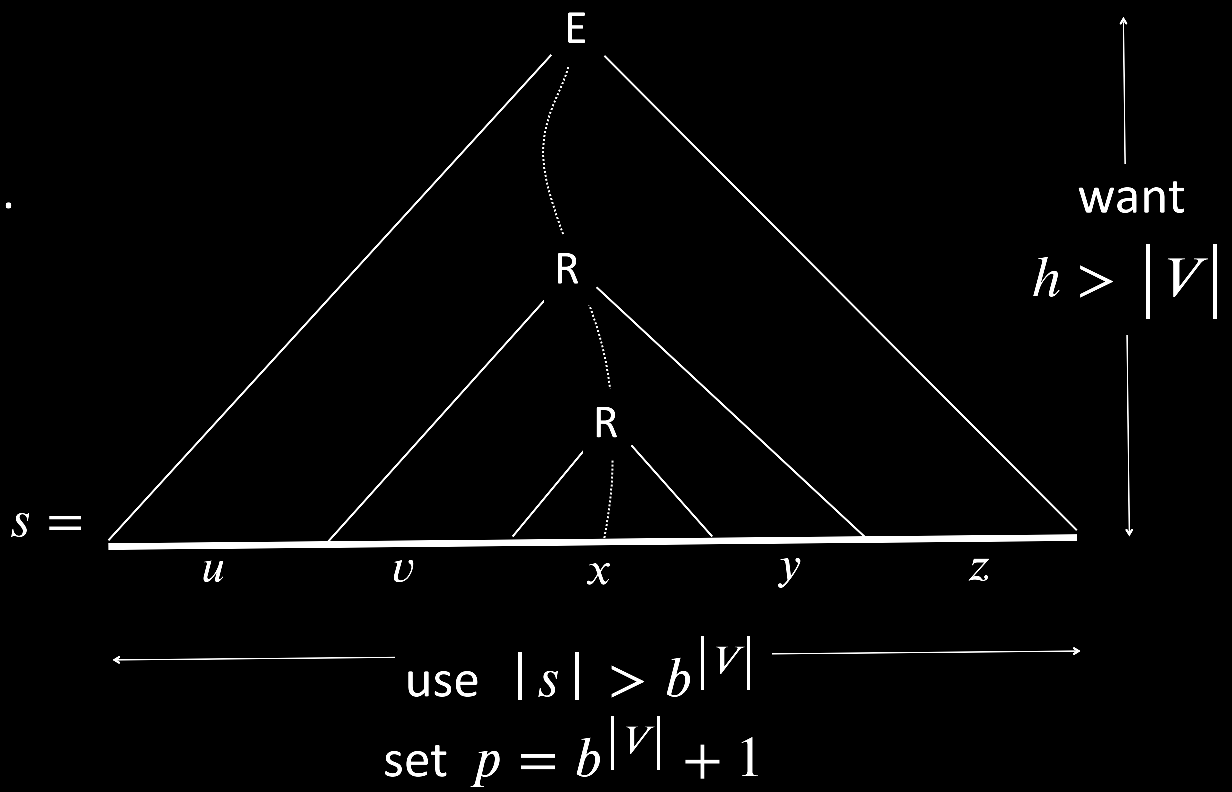

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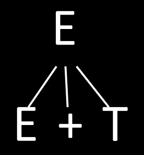

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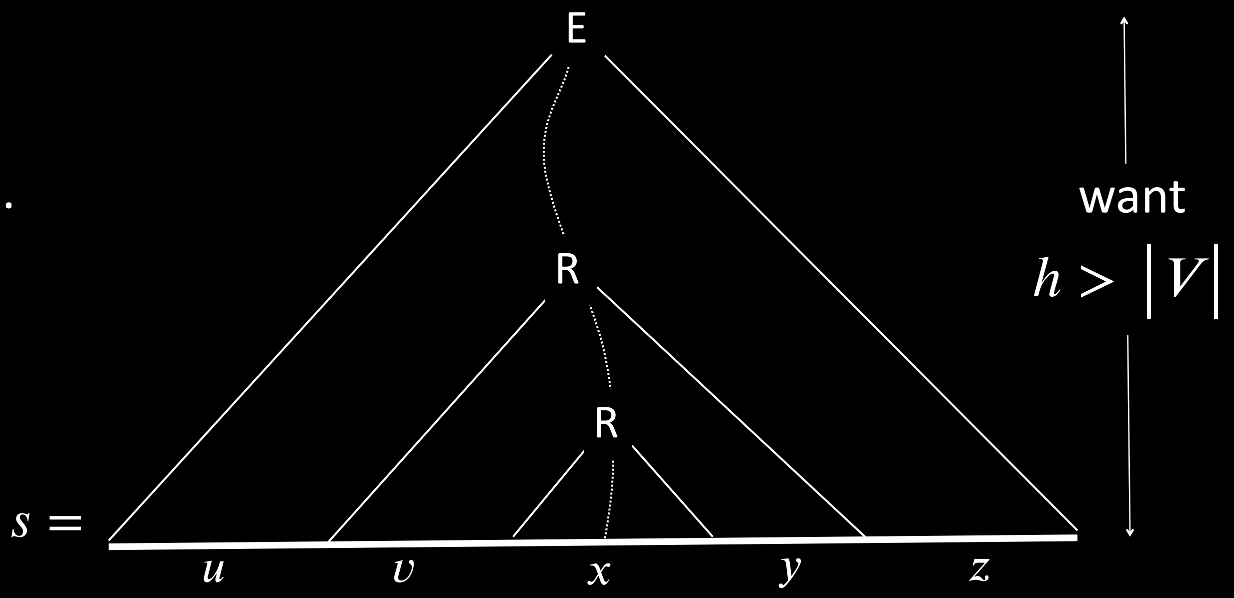

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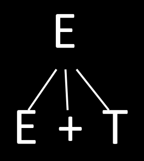

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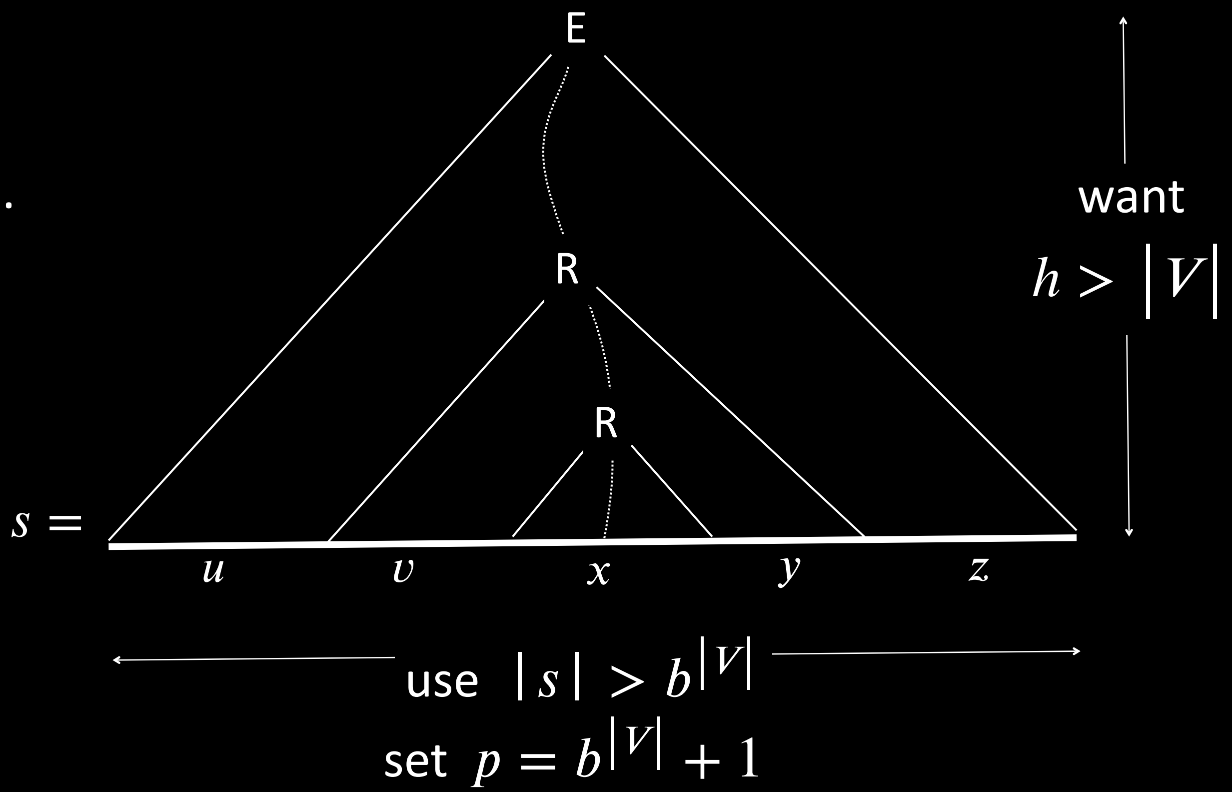

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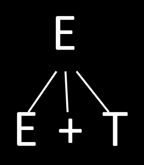

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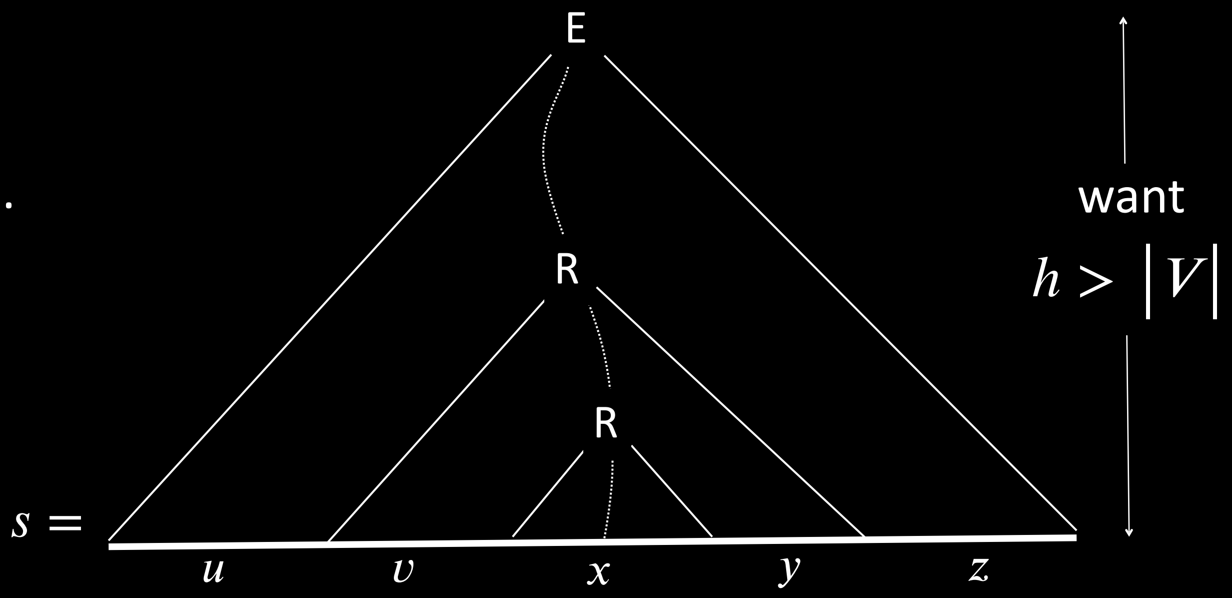

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Let $B = \{0^k1^k2^k \mid k \geq 0\}$

Show: B is not a CFL

Check-in 5.1

Let $A_1 = \{0^k1^k2^l \mid k, l \geq 0\}$ (equal #s of 0s and 1s)

Let $A_2 = \{0^l1^k2^k \mid k, l \geq 0\}$ (equal #s of 1s and 2s)

Observe that PDAs can recognize A_1 and A_2. What can we now conclude?

a) The class of CFLs is not closed under intersection.

b) The Pumping Lemma shows that $A_1 \cup A_2$ is not a CFL .

c) The class of CFLs is closed under complement.

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Let $F = \{ww \mid w \in \Sigma^*\}$. $\Sigma = \{0,1\}$.

Show: F is not a CFL.

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Show s_2 cannot be pumped $s_2 = uvxyz$ satisfying the 3 conditions.

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3) $|vxy| \leq p$

Let $F = \{ww \mid w \in \Sigma^*\}$. $\Sigma = \{0,1\}$.

Show: F is not a CFL.

Assume (for contradiction) that F is a CFL.

The CFL pumping lemma gives p as above. Need to choose $s \in F$. Which s?

Try $s_1 = 0^p10^p1 \in F$. But s_1 can be pumped and stay inside F. Bad choice of s.

Try $s_2 = 0^p1^p0^p1^p \in F$.

Show s_2 cannot be pumped $s_2 = uvxyz$ satisfying the 3 conditions.

$s_1 =$ $000\cdots001000\cdots001$

$u \mid v \mid x \mid y \mid z$

$\leftarrow \leq p \rightarrow$

$s_2 =$ $0\cdots01\cdots10\cdots01\cdots1$

$u \mid v \mid x \mid y \mid z$

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Example 2 of Proving Non-CF

Pumping Lemma for CFLs: For every CFL A, there is a p such that if $s \in A$ and $|s| \geq p$ then $s = uvxyz$ where

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Therefore, if uv^2xy^2z have at most 4 runs,
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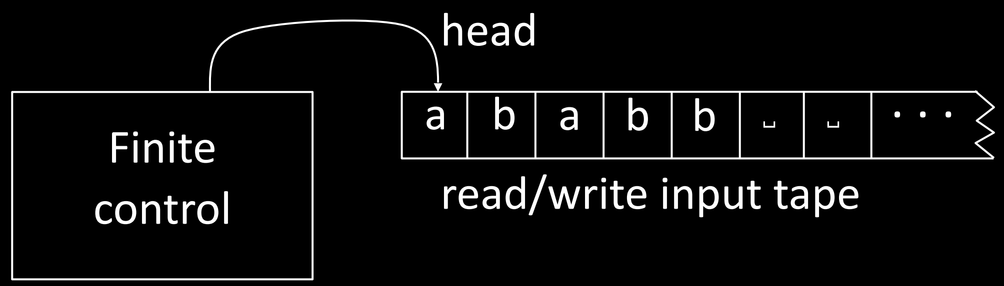
So $uv^2xy^2z \notin F$ violating Condition 1. Contradiction! Thus F is not a CFL.

Turing Machine

Turing Machines (TMs)



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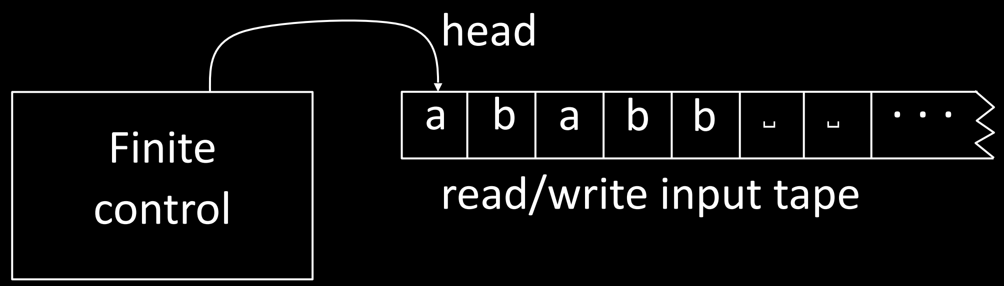

1) Head can read and write
2) Head is two way (can move left or right)
3) Tape is infinite (to the right)
4) Infinitely many blanks “␣“ follow input
5) Can accept or reject any time (not only at end of input)

TM – example

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TM recognizing $B = \left\{ \mathrm{a}^k \mathrm{b}^k \mathrm{c}^k \,\middle|\, k \geq 0 \right\}$

1) Scan right until ␣ while checking if input is in $\mathrm{a}^*\mathrm{b}^*\mathrm{c}^*$, *reject* if not.
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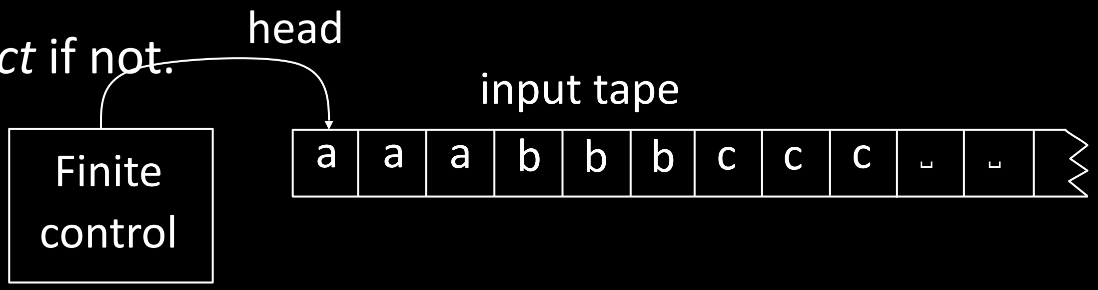

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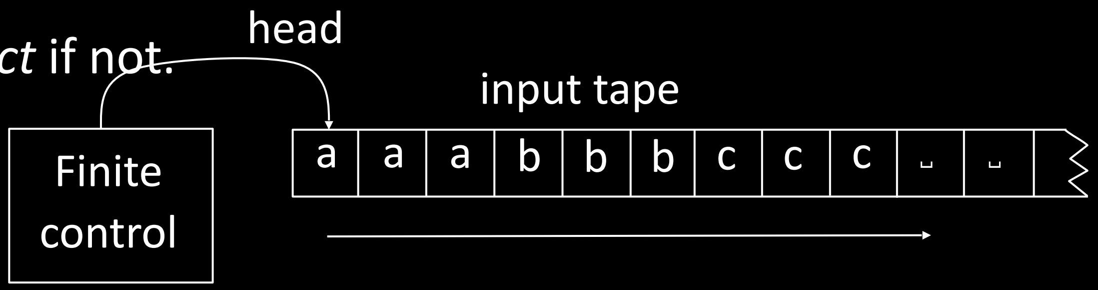

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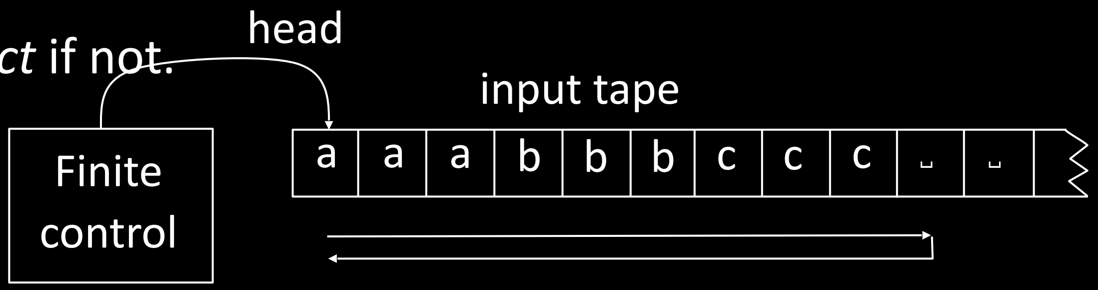

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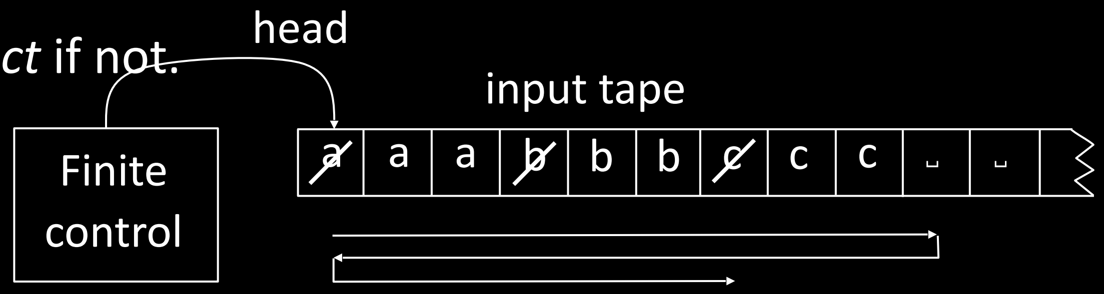

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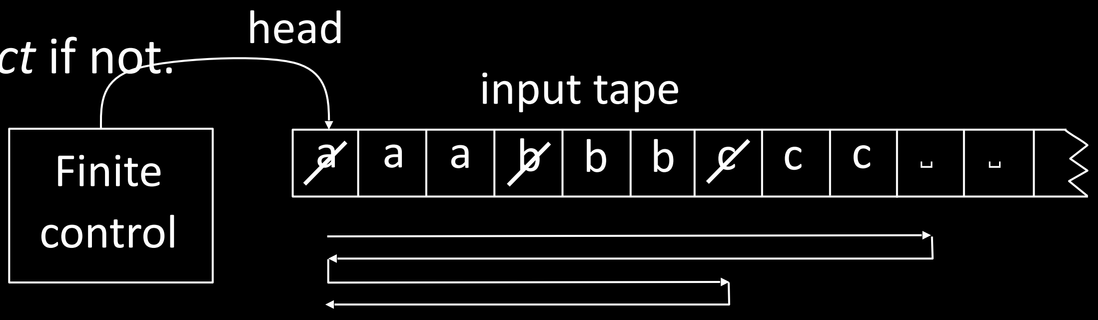

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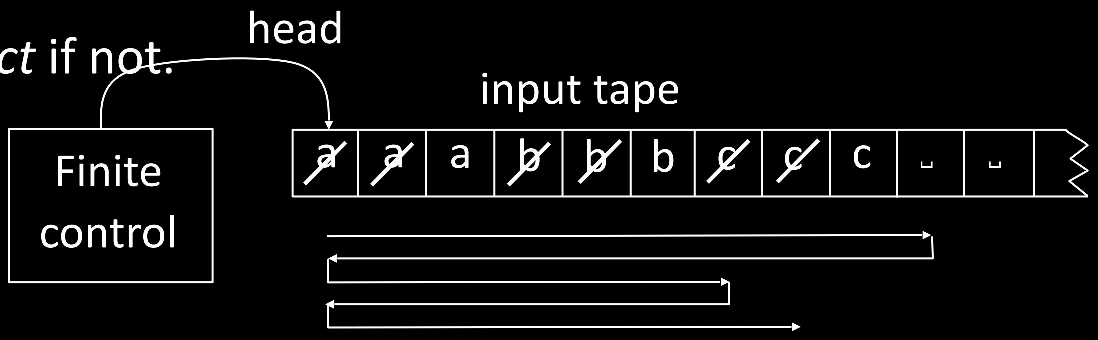

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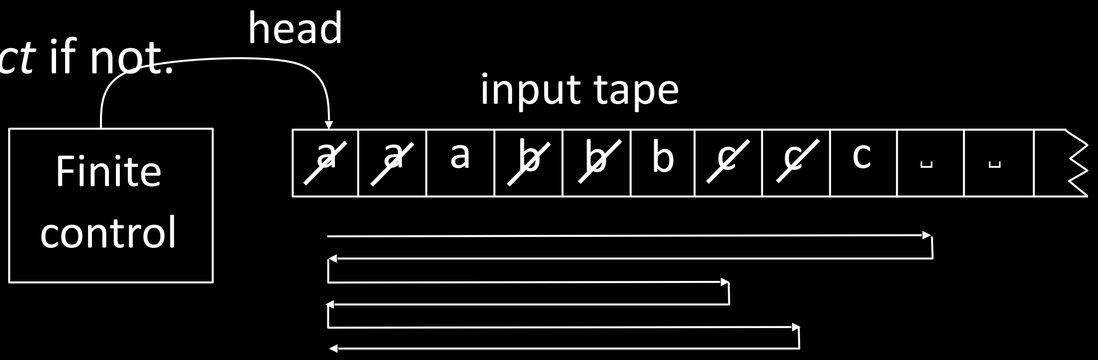

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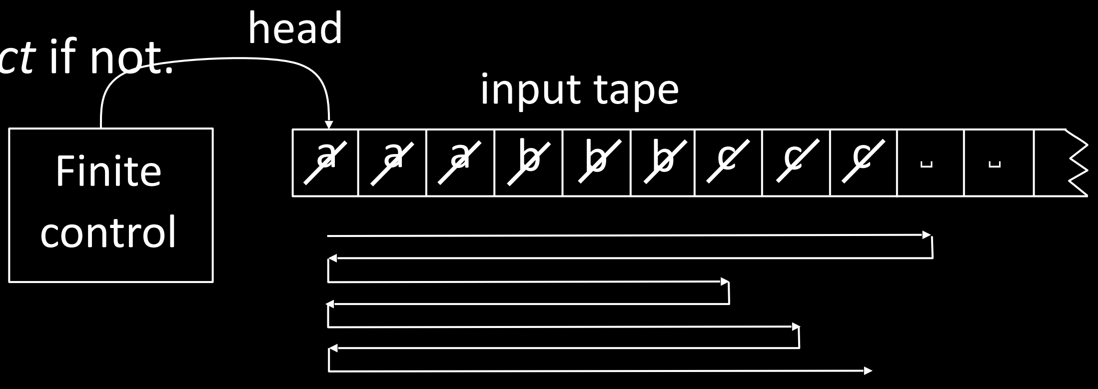

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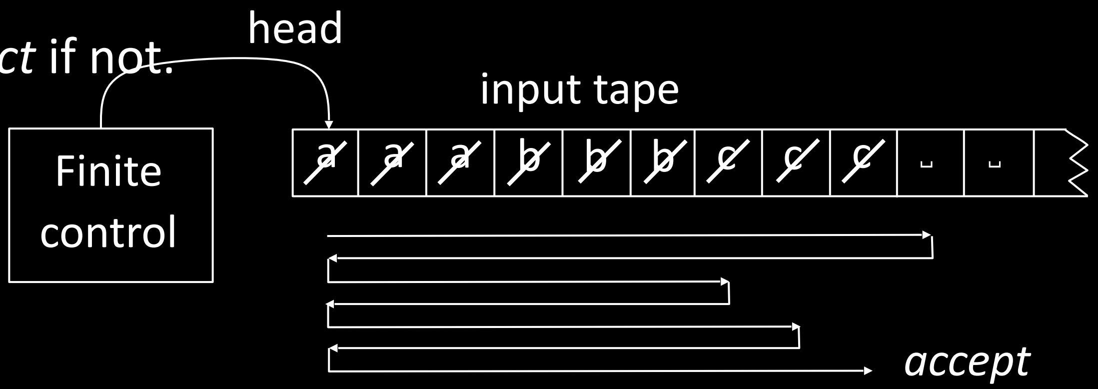

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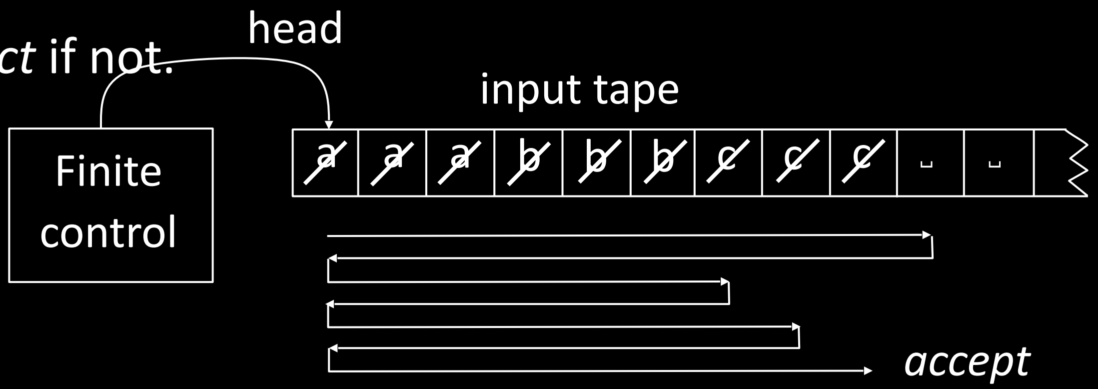

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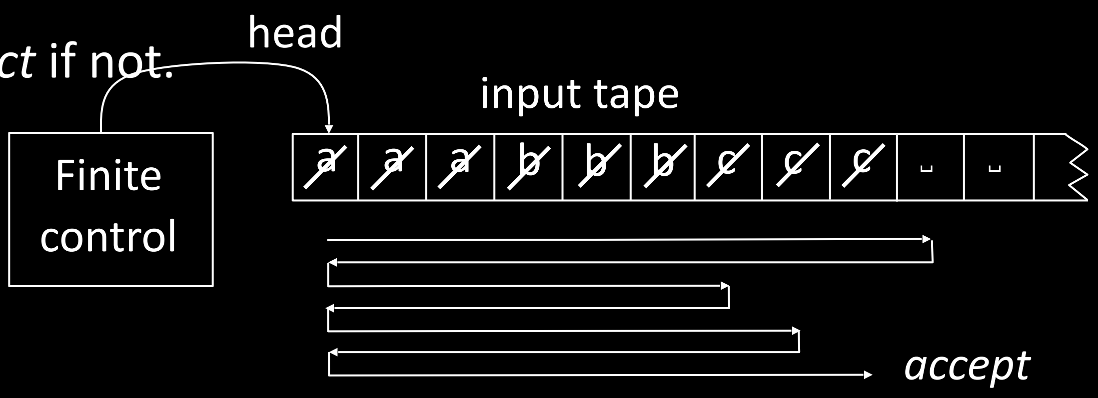

Check-in 5.2

How do we get the effect of “crossing off” with a Turing machine?

a) We add that feature to the model.

b) We use a tape alphabet $\Gamma = \{\mathrm{a}, \mathrm{b}, \mathrm{c}, \cancel{\mathrm{a}}, \cancel{\mathrm{b}}, \cancel{\mathrm{c}}, \sqcup\}$.

c) All Turing machines come with an eraser.

TM – Formal Definition

Defn: A Turing Machine (TM) is a 7-tuple

$(Q,\ \Sigma,\ \Gamma,\ \delta, q_0,\ q\text{acc}\ ,\ q\text{rej})$

Σ input alphabet

Γ tape alphabet ($\Sigma \subseteq \Gamma$)

δ: $\mathrm{Q} \times \Gamma \rightarrow Q \times \Gamma \times$ {L, R} (L = Left, R = Right)

$\delta(q, \mathrm{a}) = (r, \mathrm{b}, \ \mathrm{R})$

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On input w a TM M may halt (enter $q$acc or $q$rej)
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So M has 3 possible outcomes for each input w:

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Check-in 5.3

This Turing machine model is deterministic.
How would we change it to be nondeterministic?

a) Add a second transition function.

b) Change δ to be δ: $\mathrm{Q} \times \Gamma \rightarrow \mathscr{P}(Q \times \Gamma \times \{\mathrm{L}, \mathrm{R}\})$

c) Change the tape alphabet Γ to be infinite.

TM Recognizers and Deciders

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Let M be a TM. Then $L(M) = \{w \mid M \text{ accepts } w\}$.

Say that M recognizes A if $A = L(M)$.

Defn: A is Turing-recognizable if $A = L(M)$ for some TM M.

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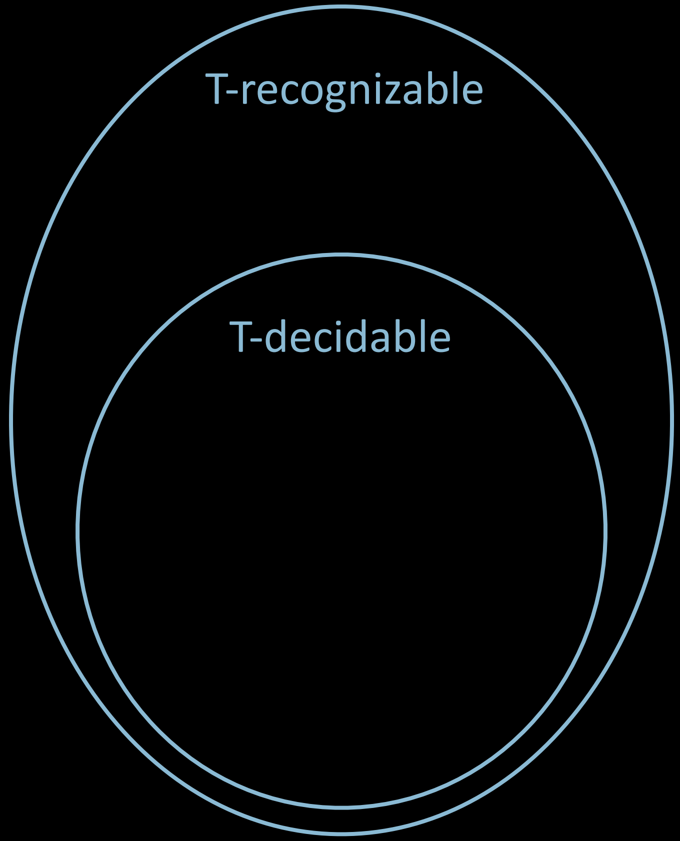

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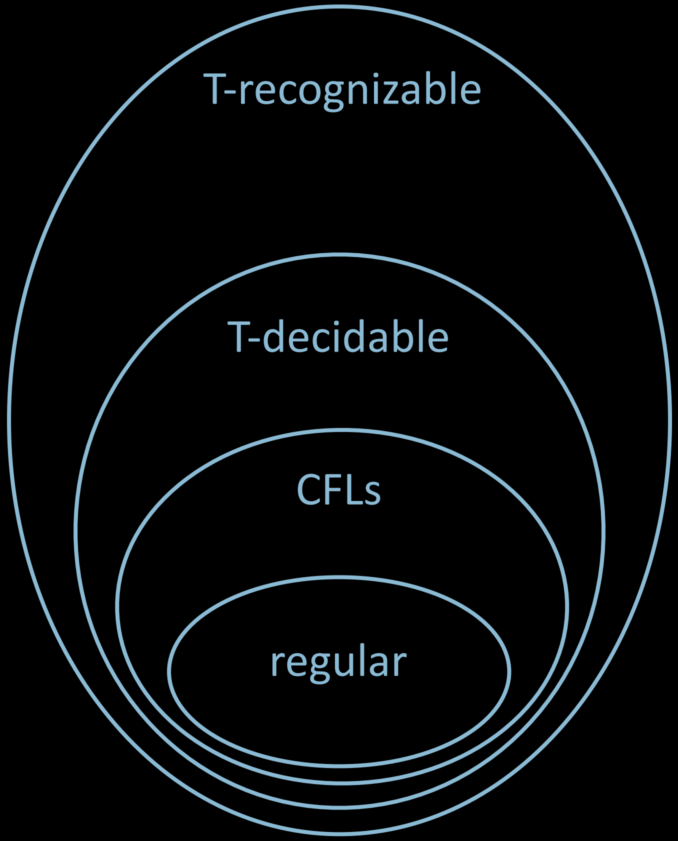
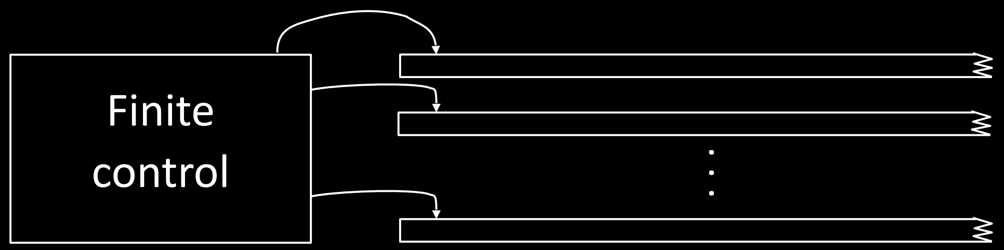


Multi-tape Turing machines



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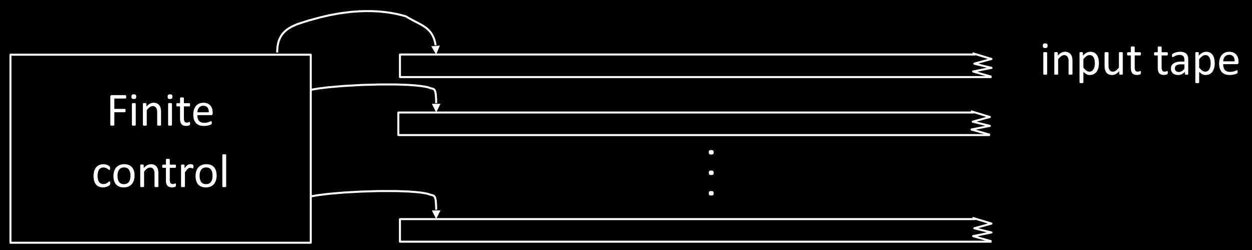

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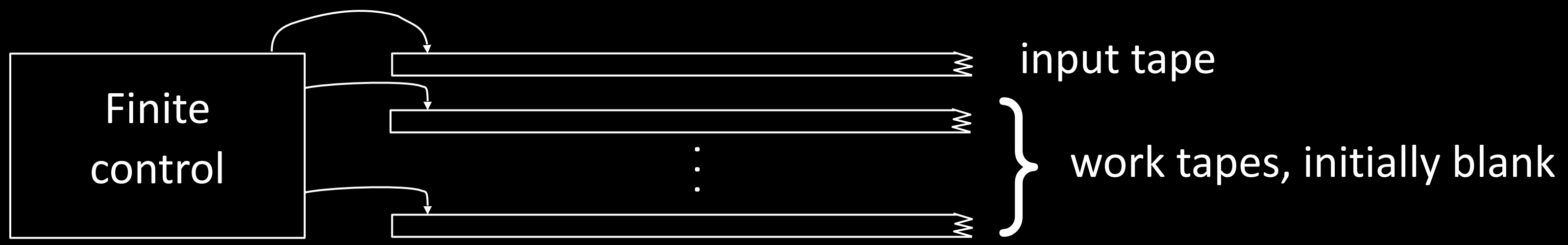

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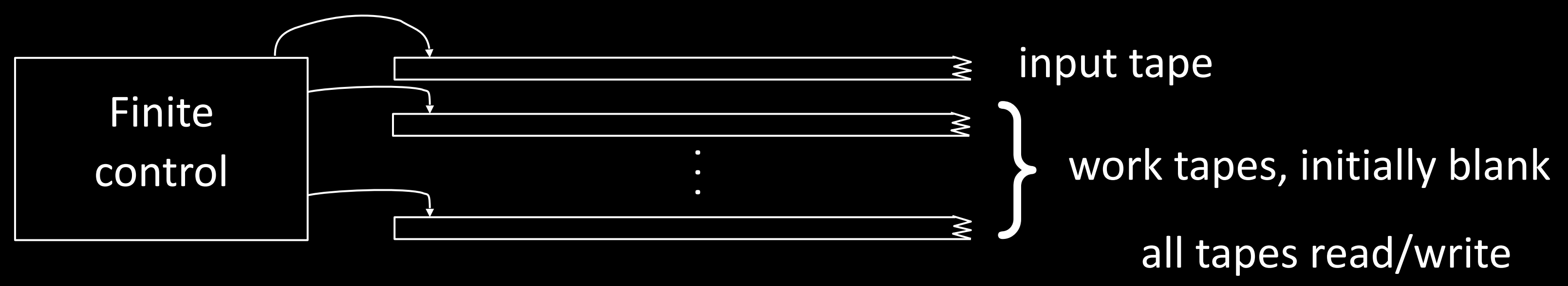

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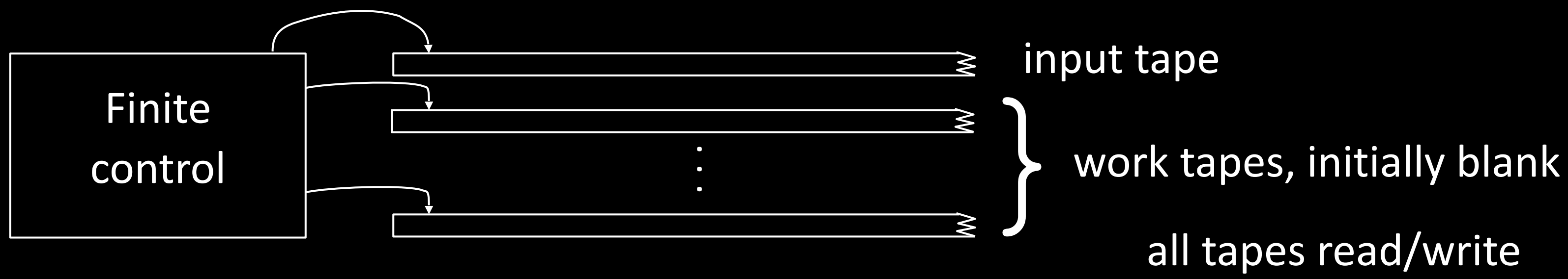

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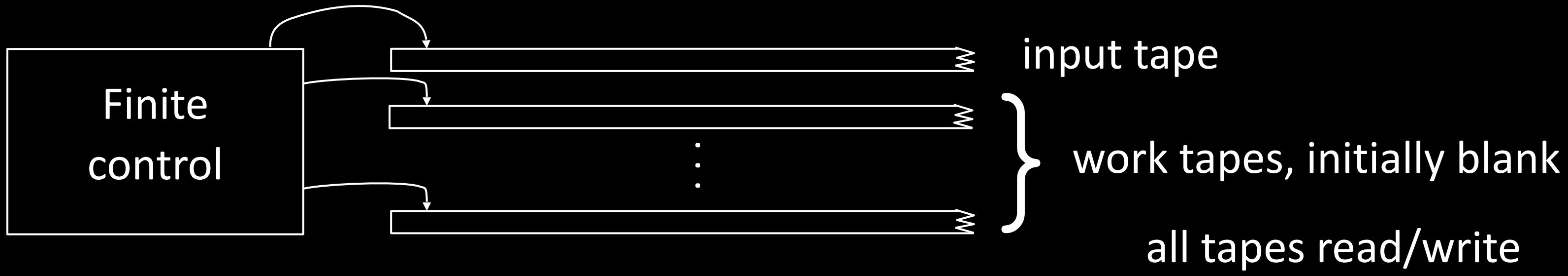

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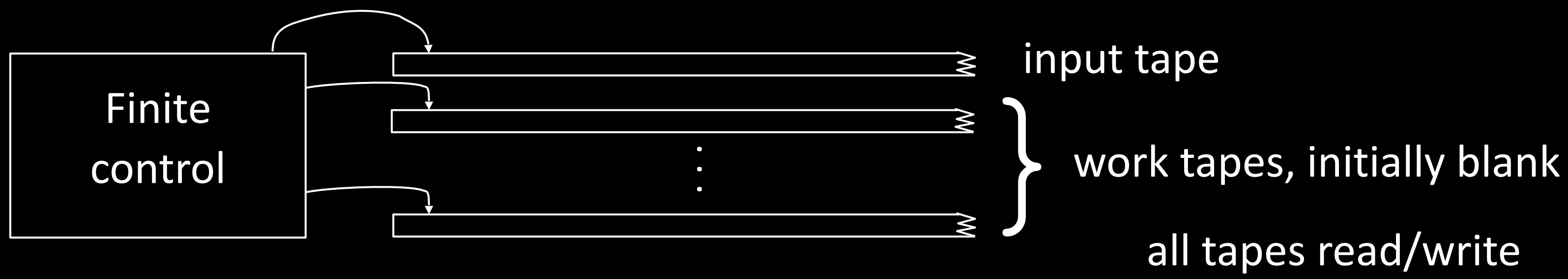

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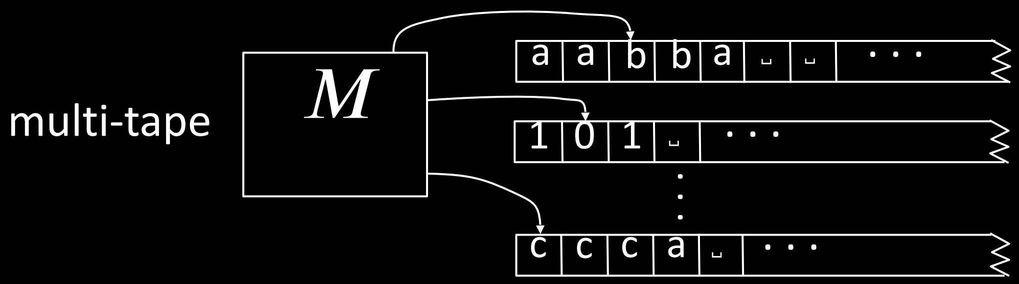
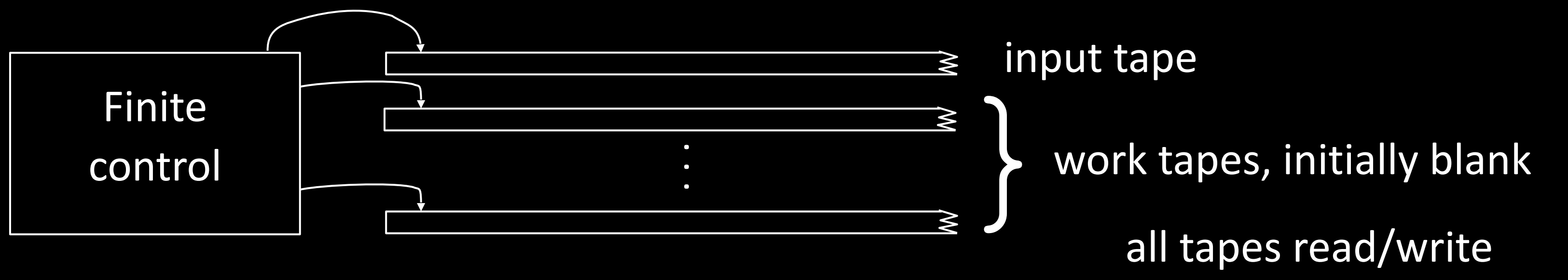
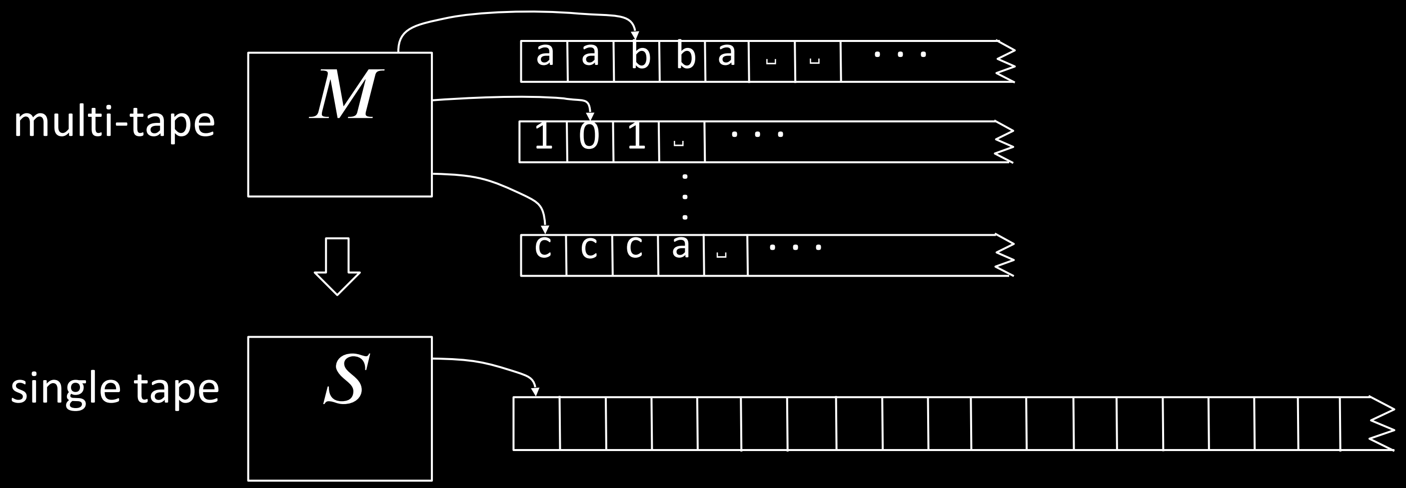


Multi-tape Turing machines

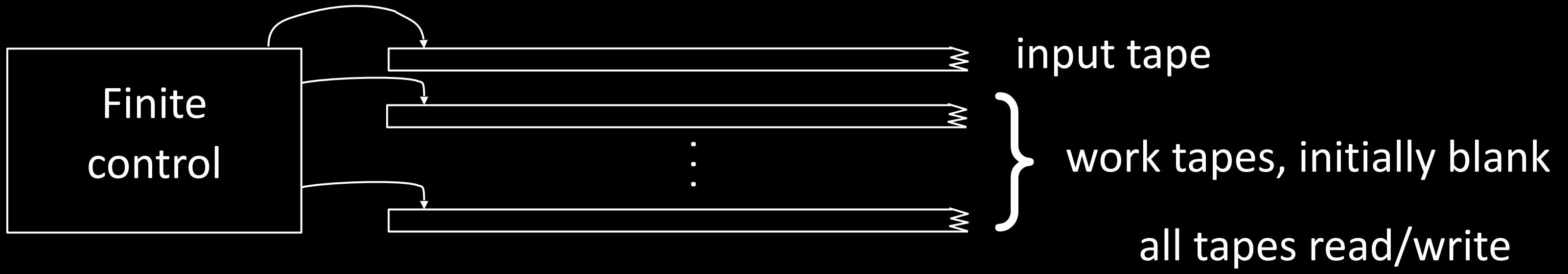


Theorem: A is T-recognizable iff some multi-tape TM recognizes A

Proof: $(\rightarrow)$ immediate. $(\leftarrow)$ convert multi-tape to single tape:



Multi-tape Turing machines



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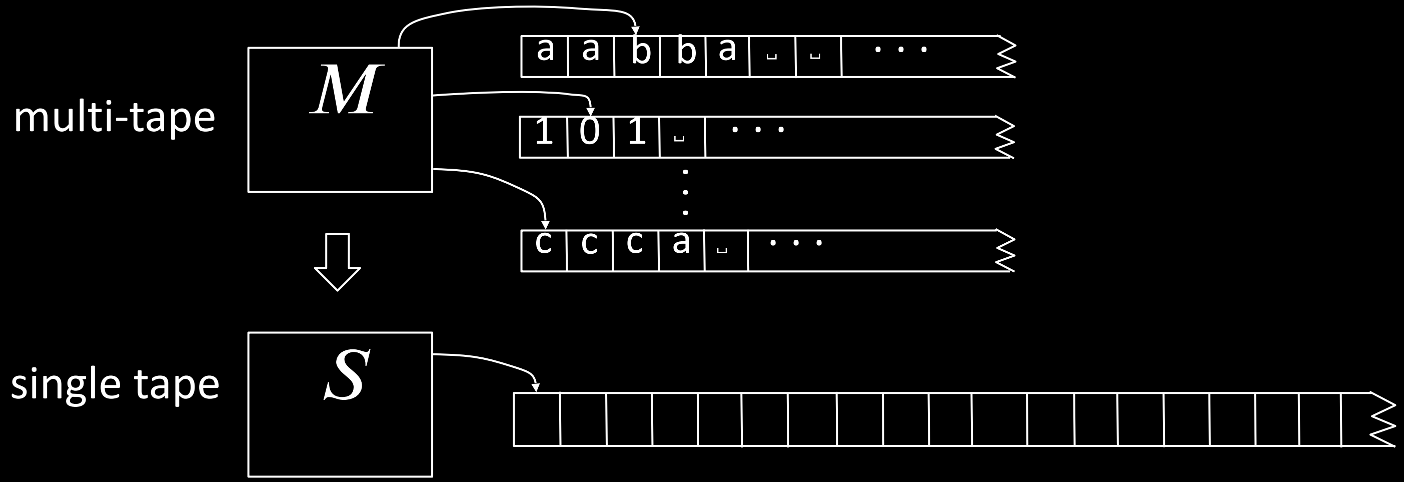
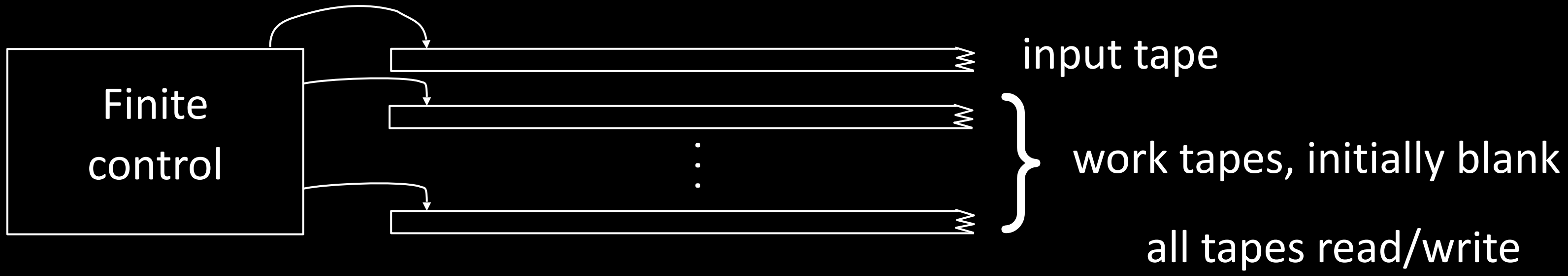
S simulates M by storing the contents of multiple tapes on a single tape in “blocks”.



Multi-tape Turing machines



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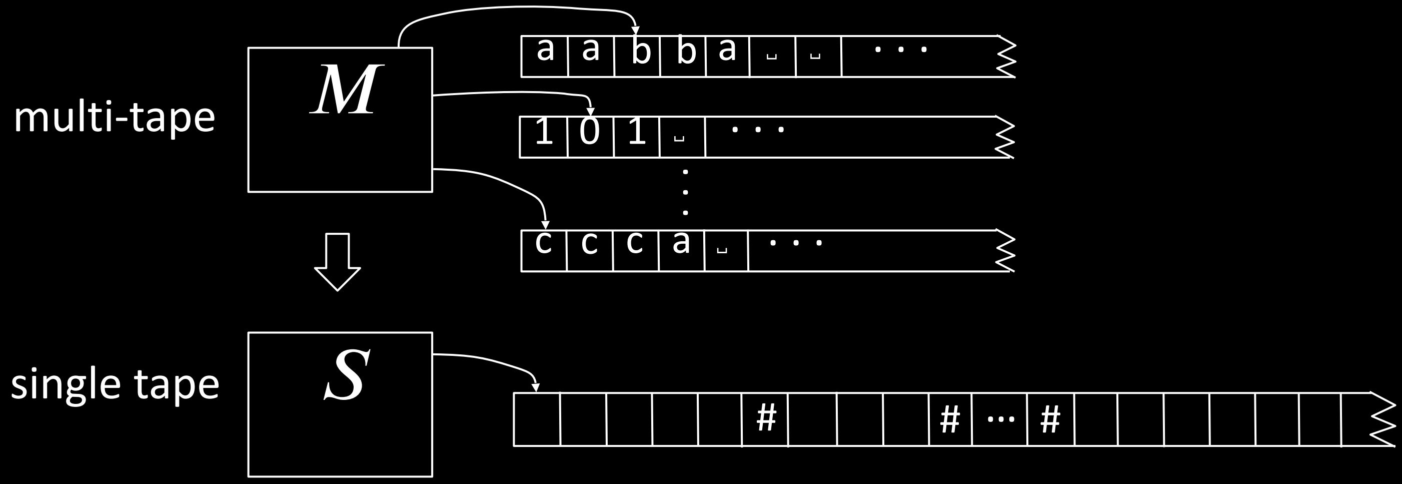

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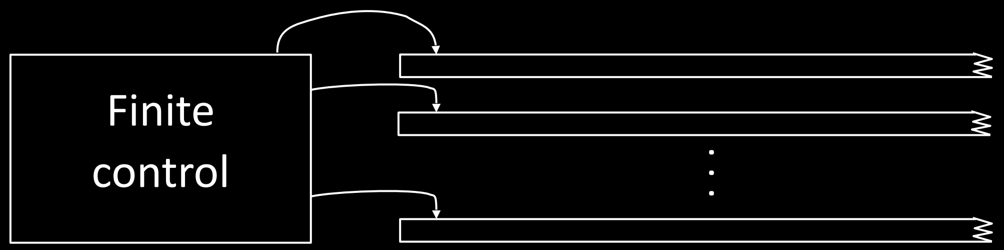
Multi-tape Turing machines



input tape

work tapes, initially blank

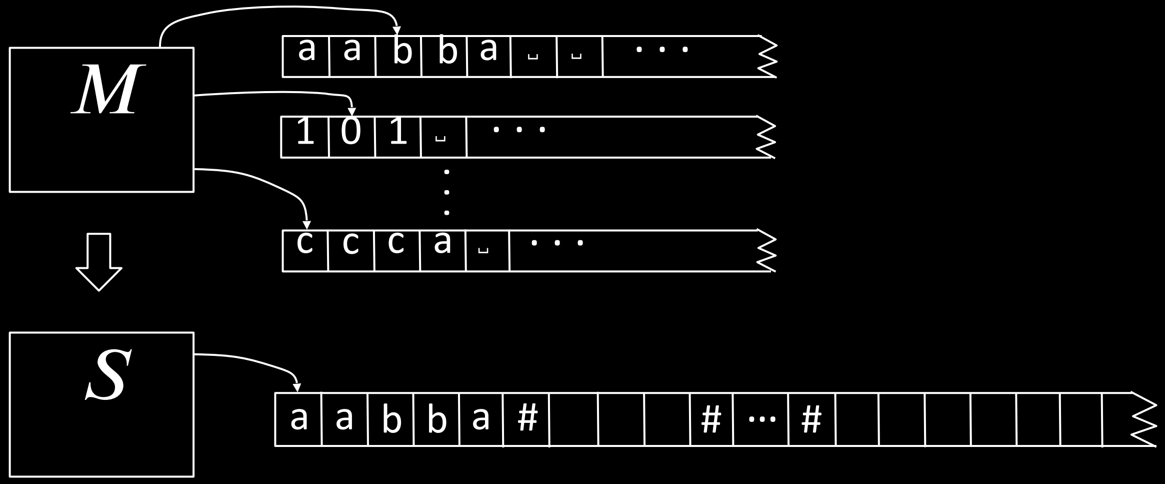
all tapes read/write

Theorem: A is T-recognizable iff some multi-tape TM recognizes A

Proof: $(\rightarrow)$ immediate. $(\leftarrow)$ convert multi-tape to single tape:

multi-tape

single tape



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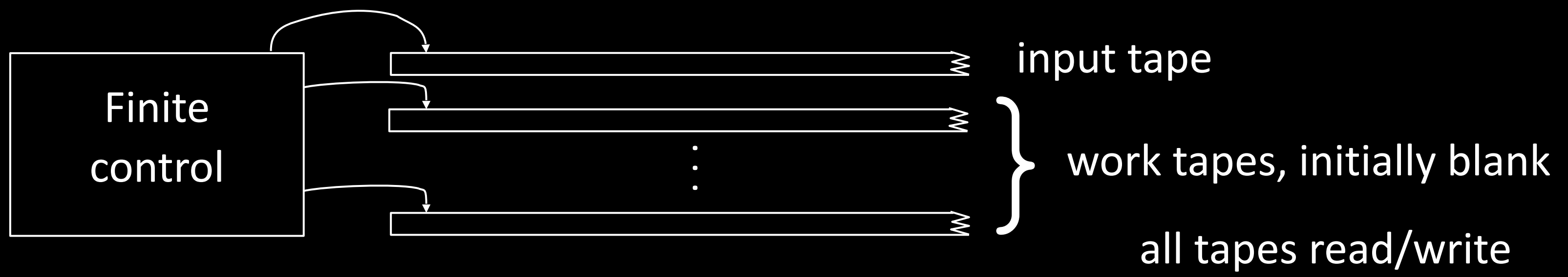
Multi-tape Turing machines



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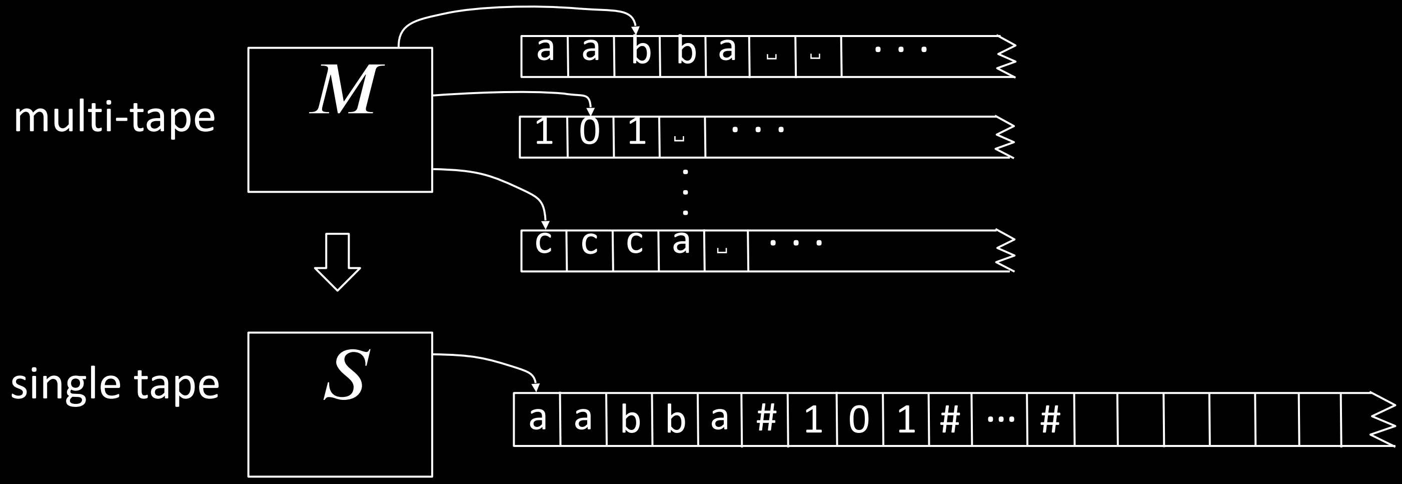
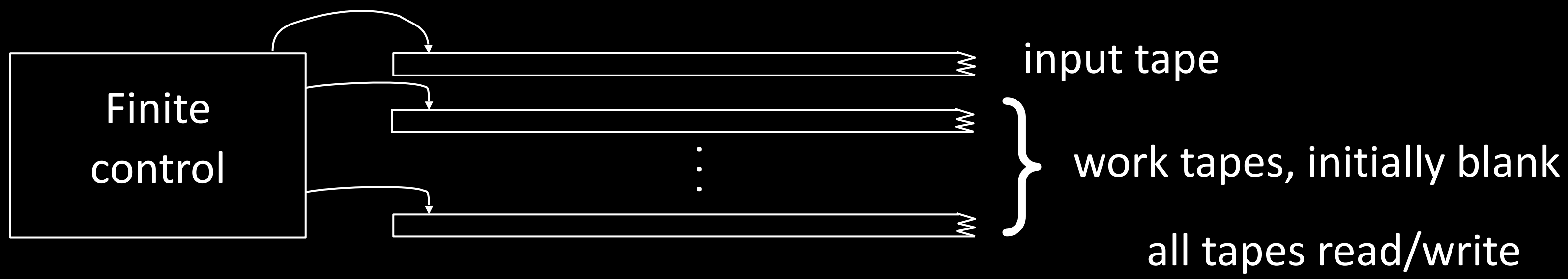


Multi-tape Turing machines



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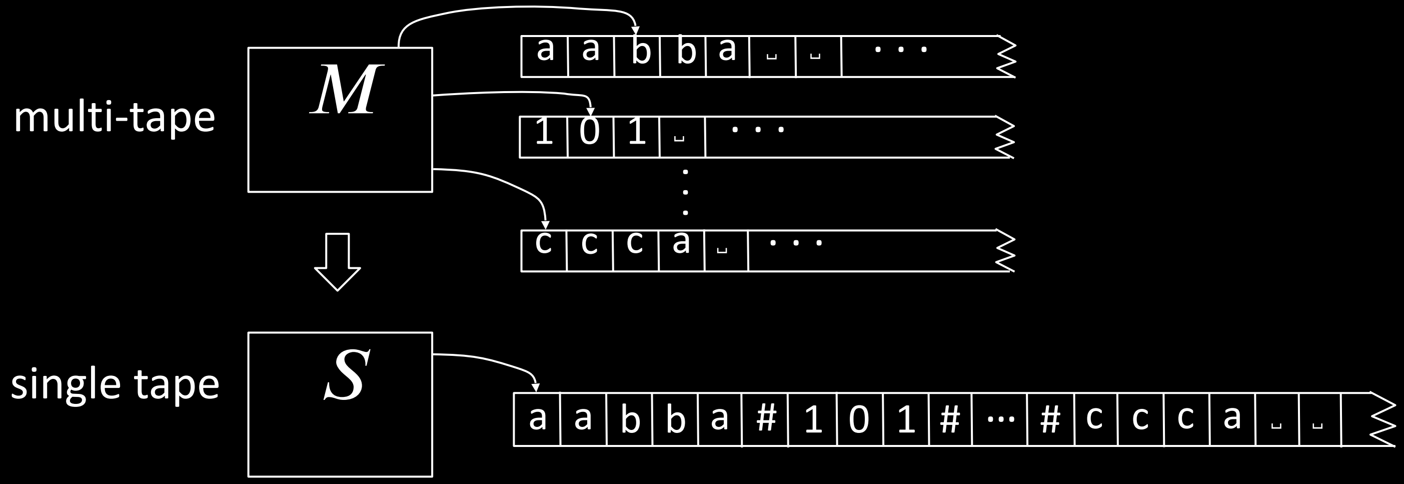

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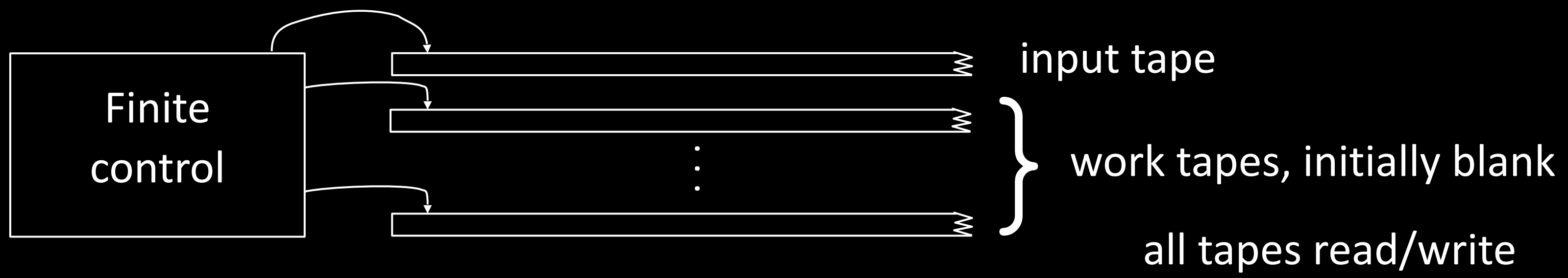
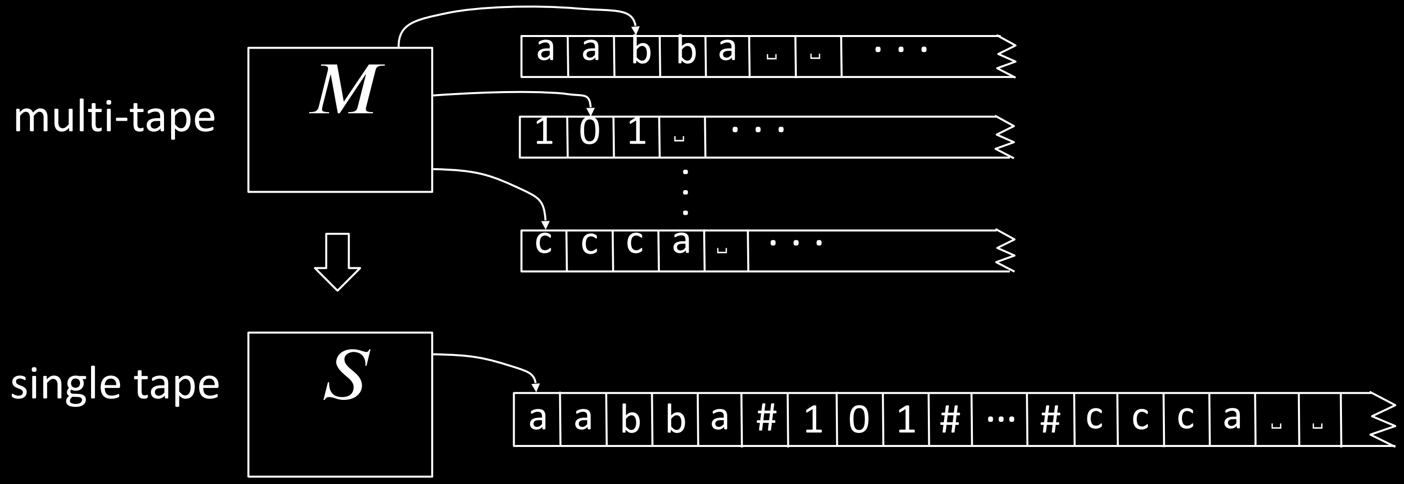
Multi-tape Turing machines



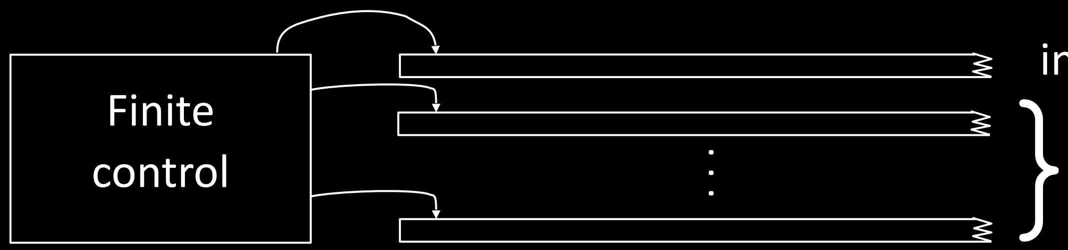
Theorem: A is T-recognizable iff some multi-tape TM recognizes A

Proof: $(\rightarrow)$ immediate. $(\leftarrow)$ convert multi-tape to single tape:

S simulates M by storing the contents of multiple tapes on a single tape in “blocks”. Record head positions with dotted symbols.



Multi-tape Turing machines



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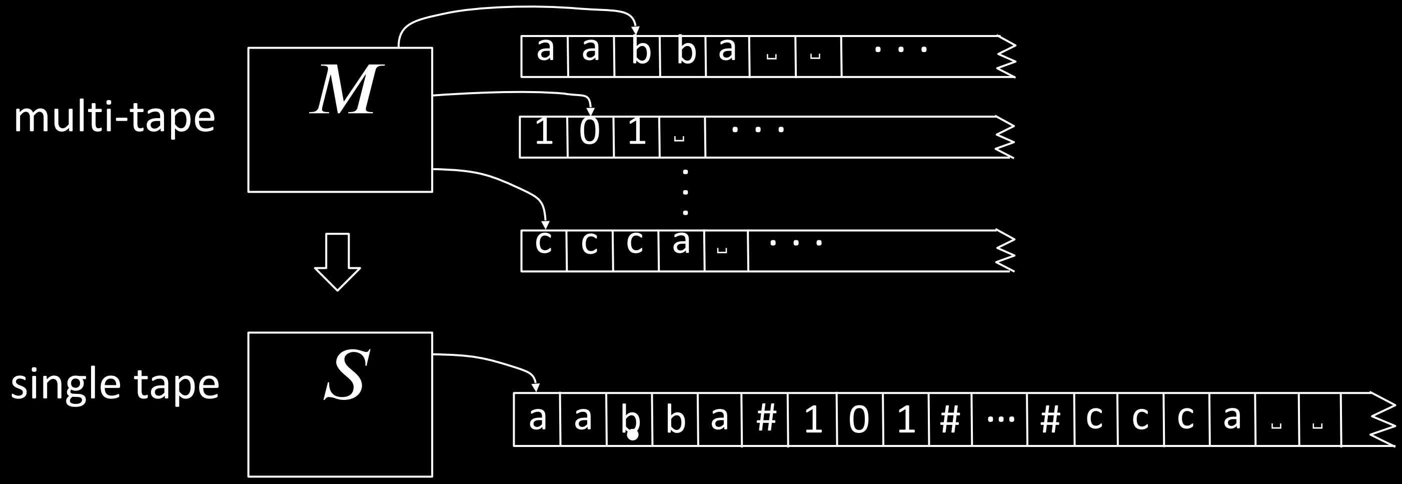
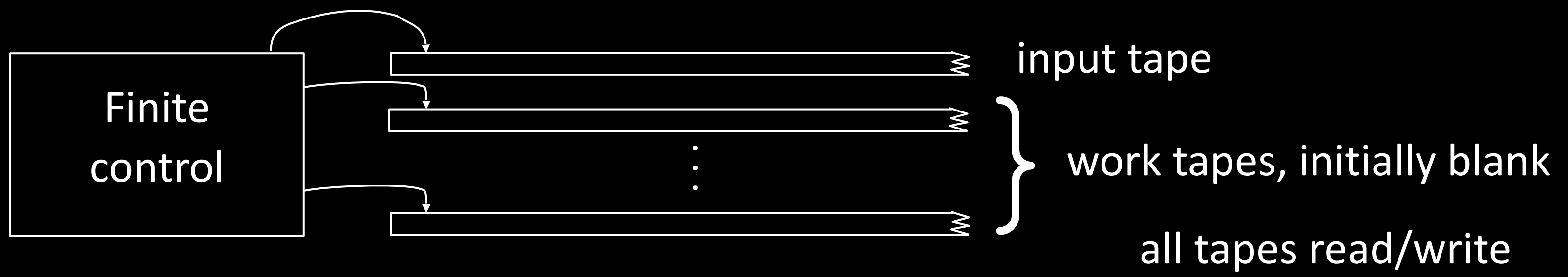


Multi-tape Turing machines



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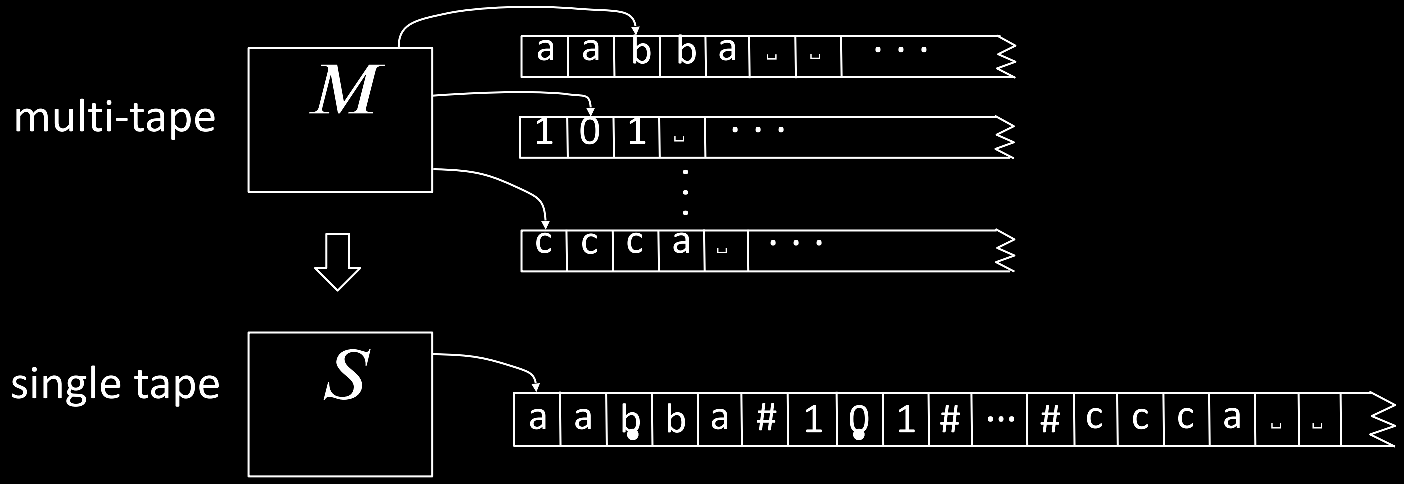
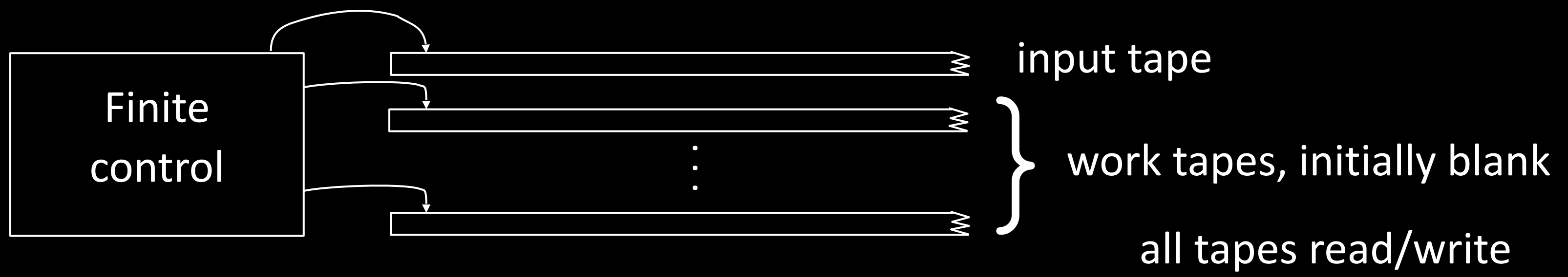


Multi-tape Turing machines



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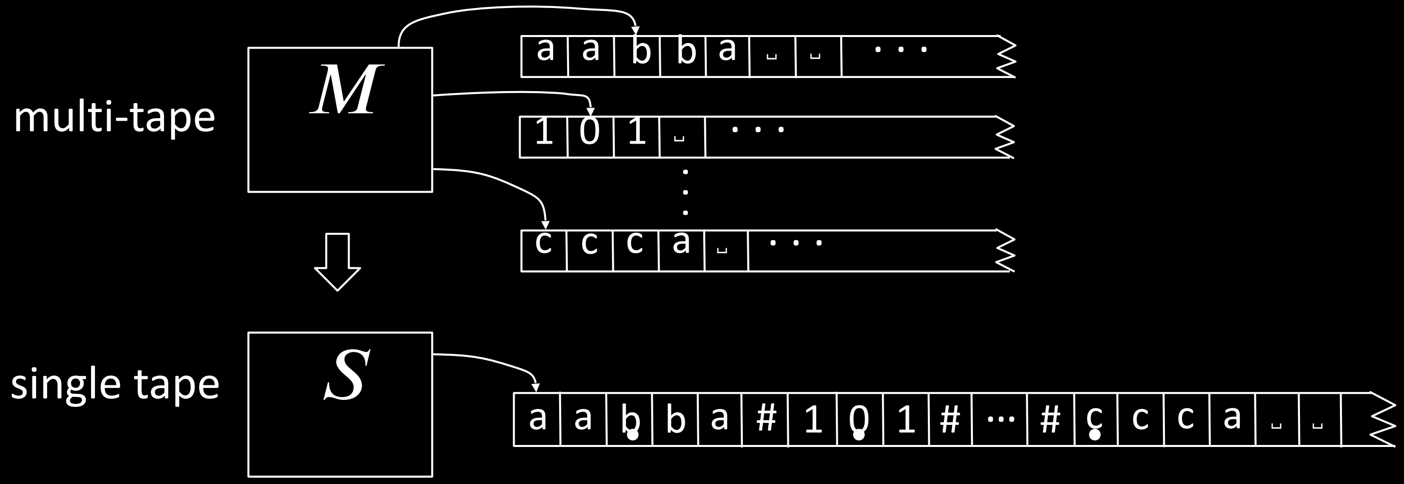
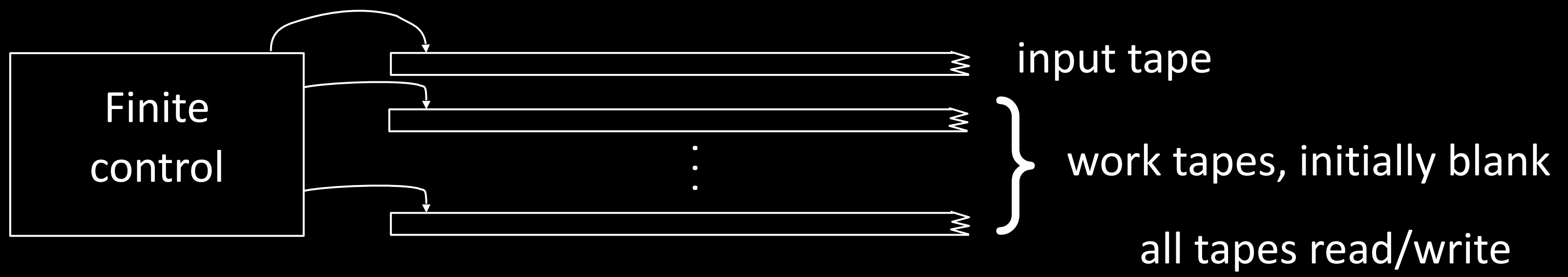


Multi-tape Turing machines



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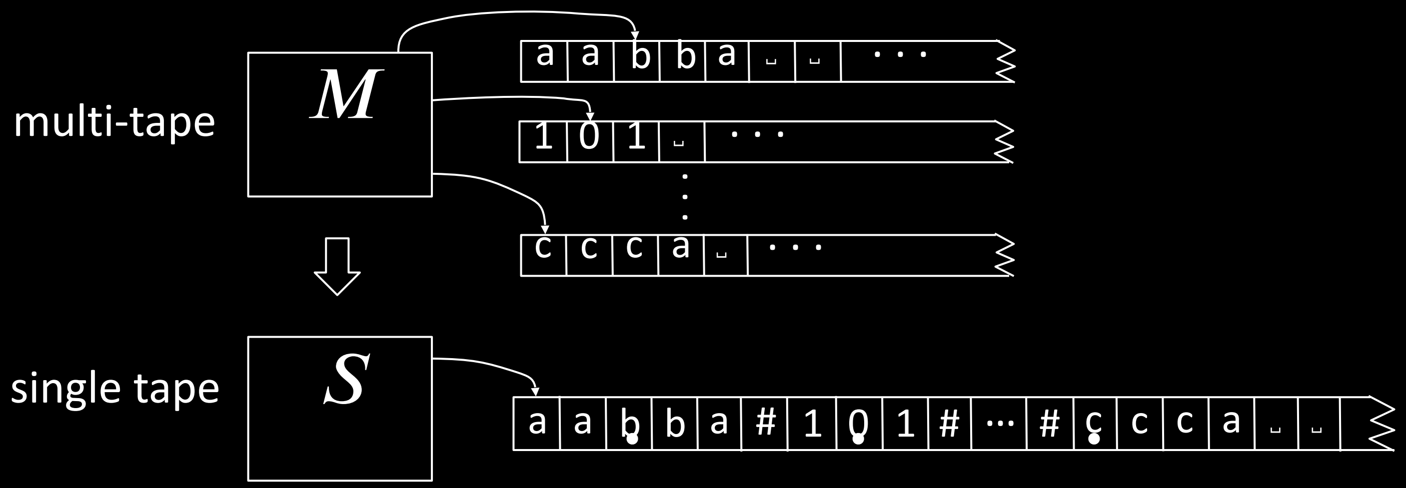
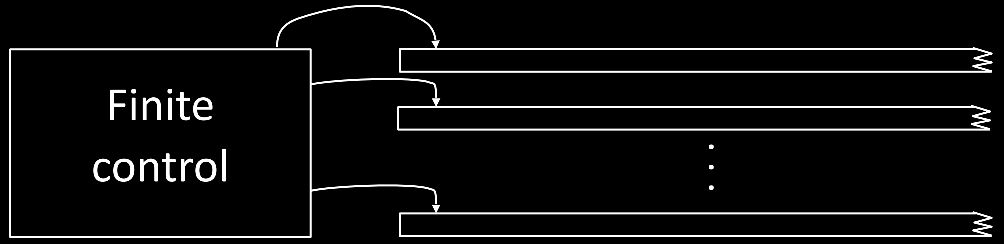


S simulates M by storing the contents of multiple tapes on a single tape in "blocks". Record head positions with dotted symbols.

Some details of S:

Multi-tape Turing machines



input tape

work tapes, initially blank

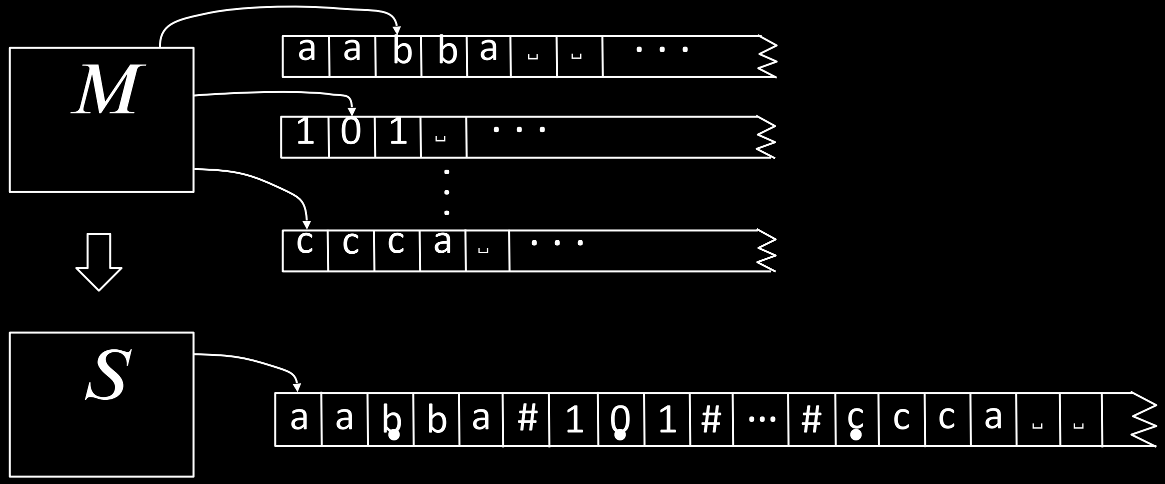
all tapes read/write

Theorem: A is T-recognizable iff some multi-tape TM recognizes A

Proof: ($\rightarrow$) immediate. ($\leftarrow$) convert multi-tape to single tape:

multi-tape

single tape

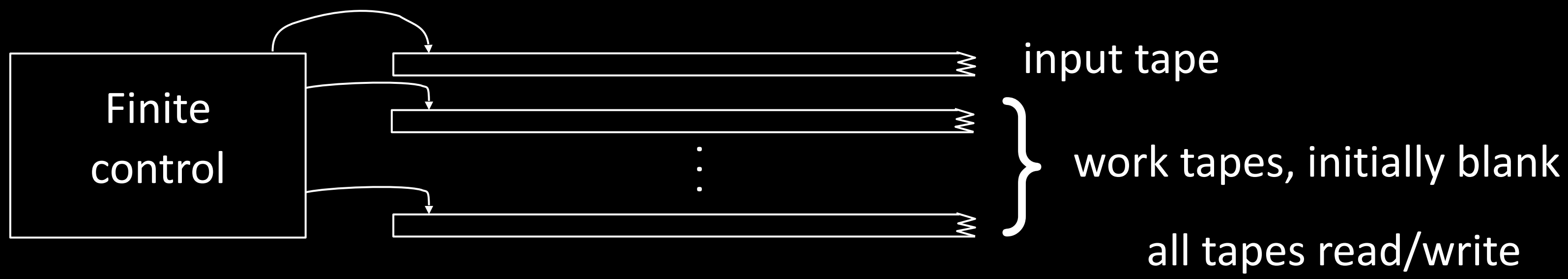


S simulates M by storing the contents of multiple tapes on a single tape in "blocks". Record head positions with dotted symbols.

Some details of S:

1) To simulate each of M's steps
   a. Scan entire tape to find dotted symbols.
   b. Scan again to update according to M's δ.

Multi-tape Turing machines



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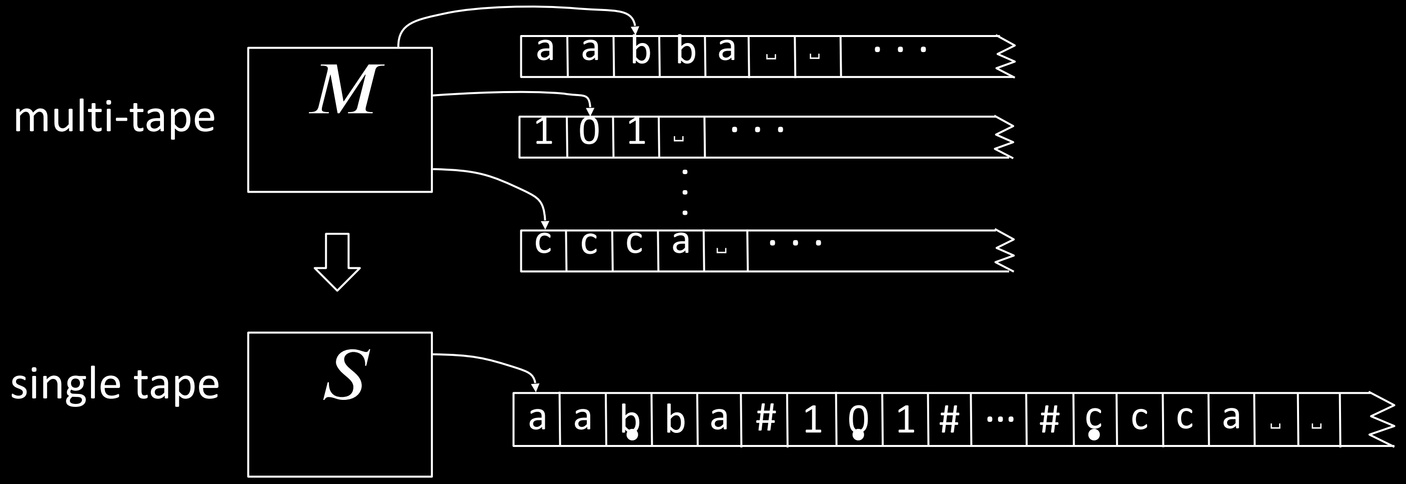

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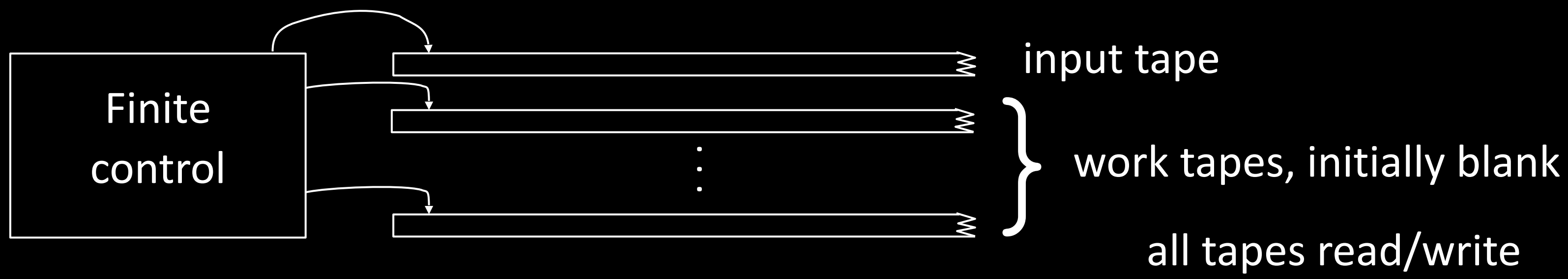
   c. Shift to add room as needed.

Multi-tape Turing machines



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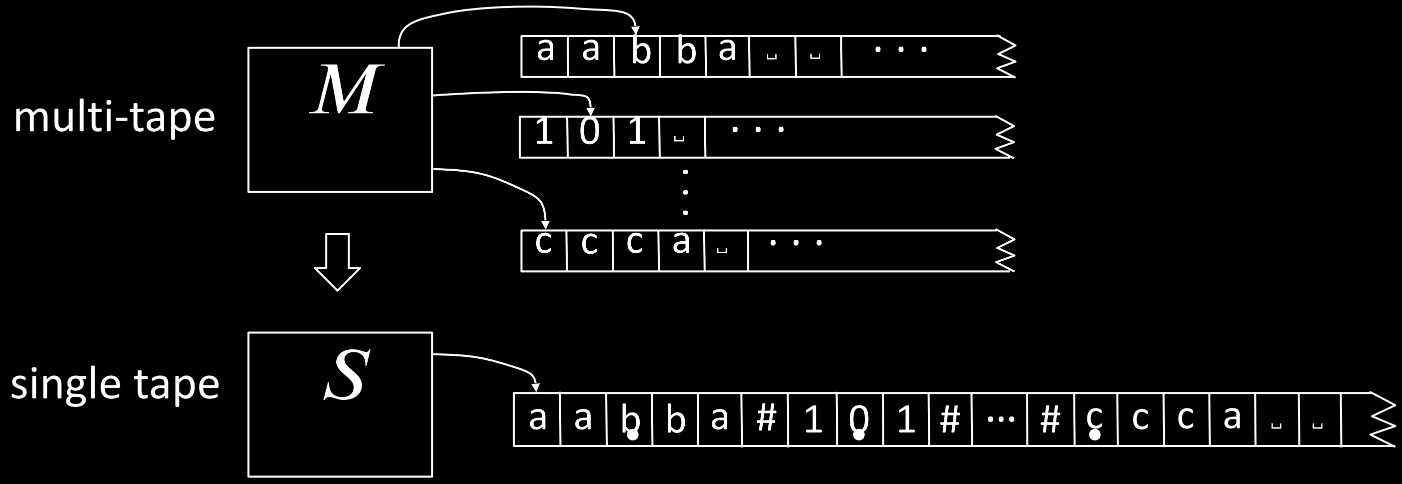

S simulates M by storing the contents of multiple tapes on a single tape in “blocks”. Record head positions with dotted symbols.

Some details of S:

1) To simulate each of M’s steps
   a. Scan entire tape to find dotted symbols.
   b. Scan again to update according to M’s δ.
   c. Shift to add room as needed.
2) Accept/reject if M does.

Nondeterministic Turing machines

NTM accepts if there is a series of proper actions leading to qacc

Nondeterministic Turing machines

A Nondeterministic TM (NTM) is similar to a Deterministic TM except for its transition function $\delta\colon \mathrm{Q} \times \Gamma \to \mathscr{P}(\, Q \times \Gamma \times$ {L, R}).

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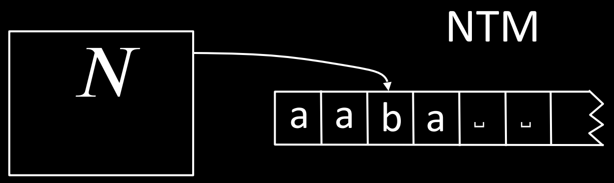

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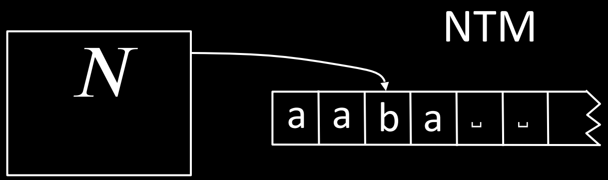
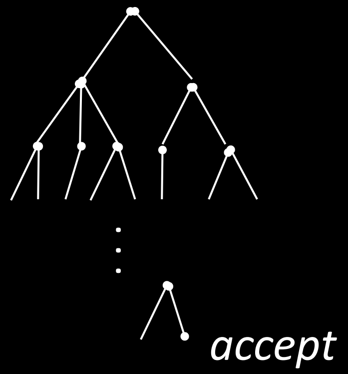


Nondeterministic computation tree for N on input w.



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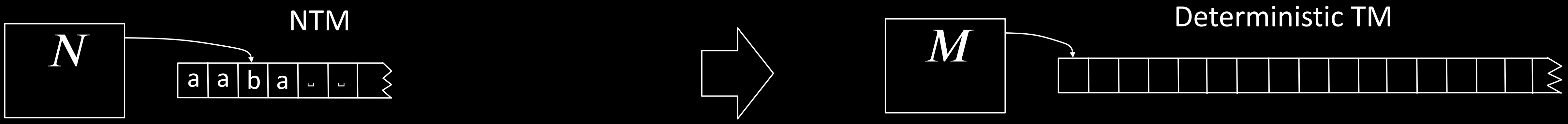

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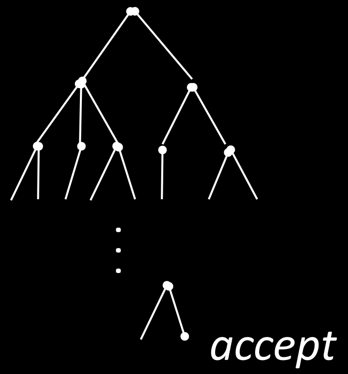

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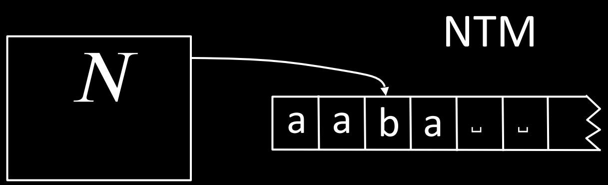

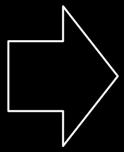

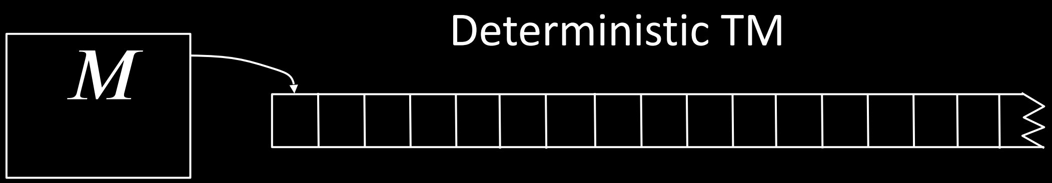

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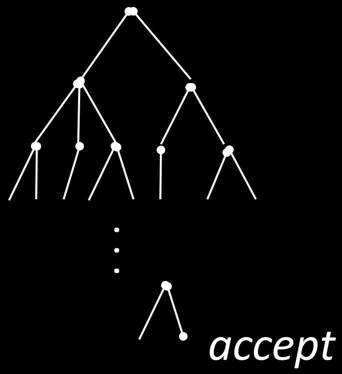

M simulates N by storing each thread's tape in a separate "block" on its tape.

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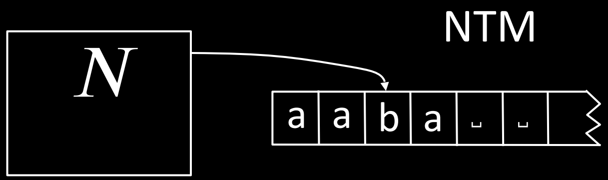
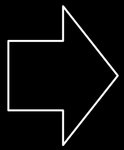
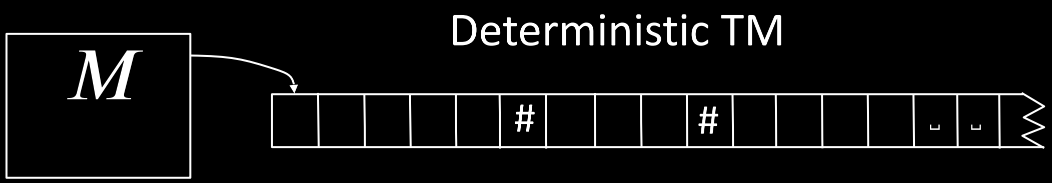
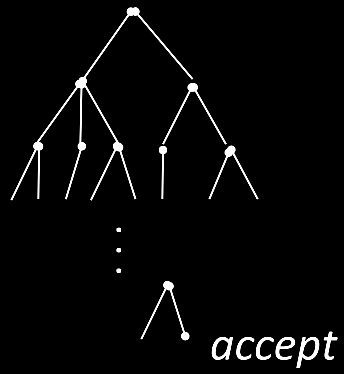






Nondeterministic computation tree for N on input w.



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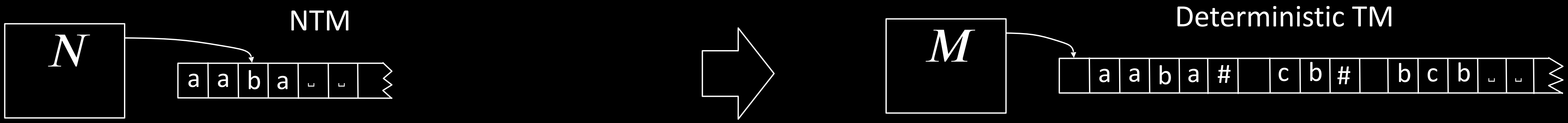

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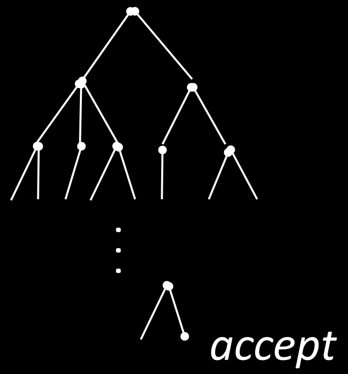

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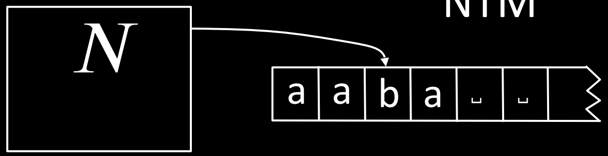
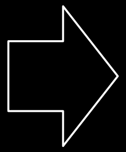
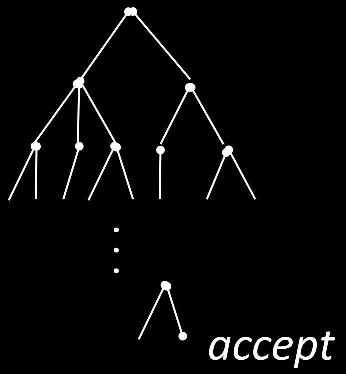
NTM





Deterministic TM

Nondeterministic computation tree for N on input w.



M simulates N by storing each thread's tape in a separate "block" on its tape.
Also need to store the head location,

Nondeterministic Turing machines

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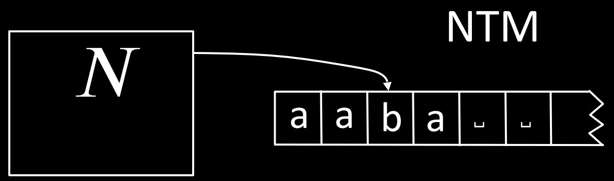
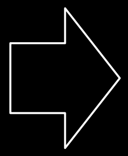
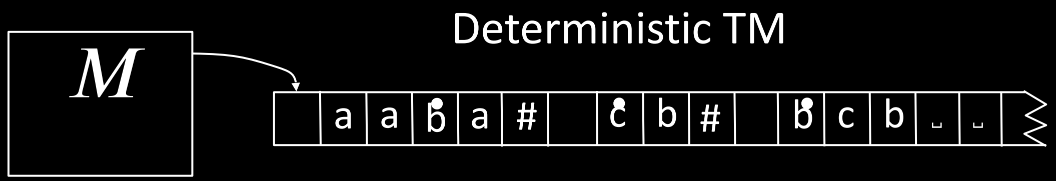
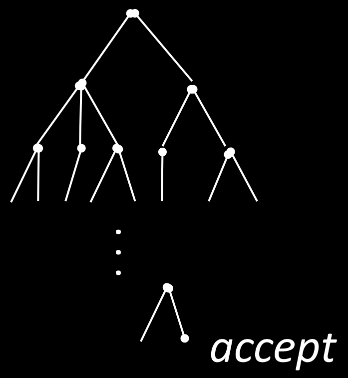






Nondeterministic computation tree for N on input w.



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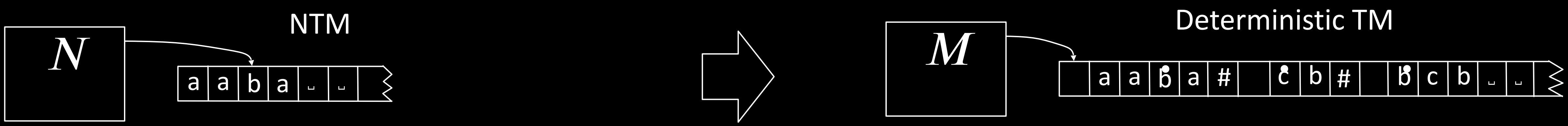
NTM accepts if there is a series of proper actions leading to qacc

Theorem: A is T-recognizable iff some NTM recognizes A

Proof: $(\rightarrow)$ immediate. $(\leftarrow)$ convert NTM to Deterministic TM.



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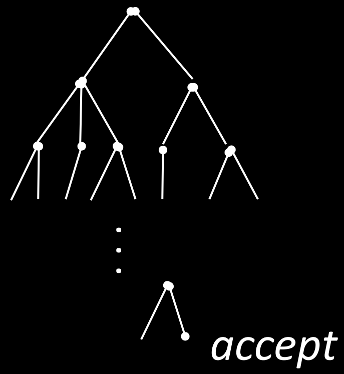

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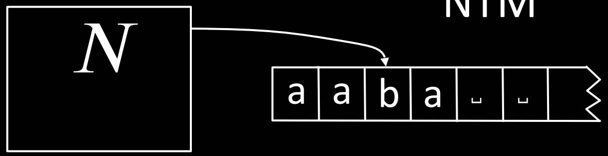
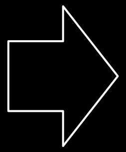
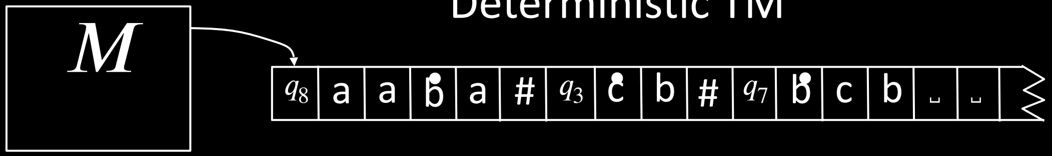




Deterministic TM



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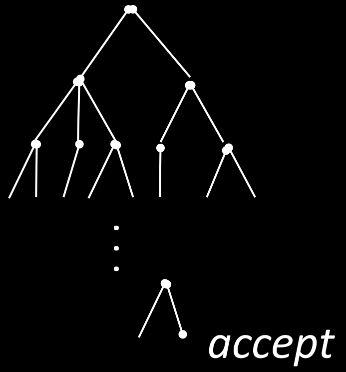

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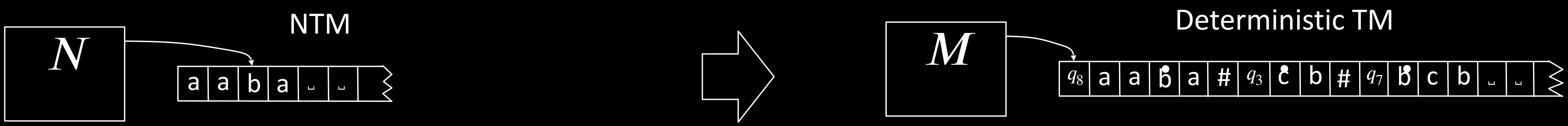

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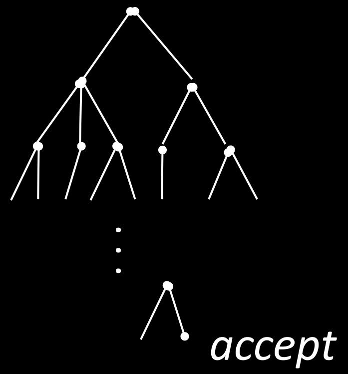

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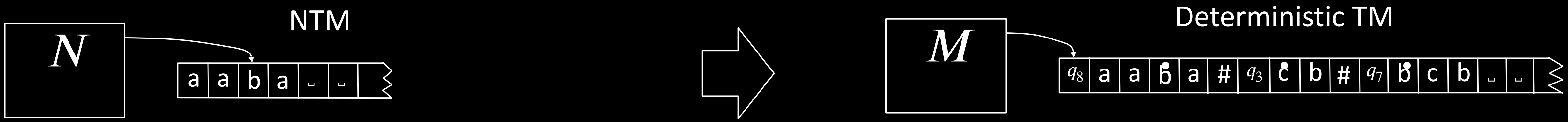

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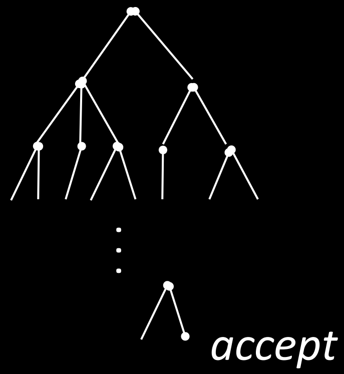

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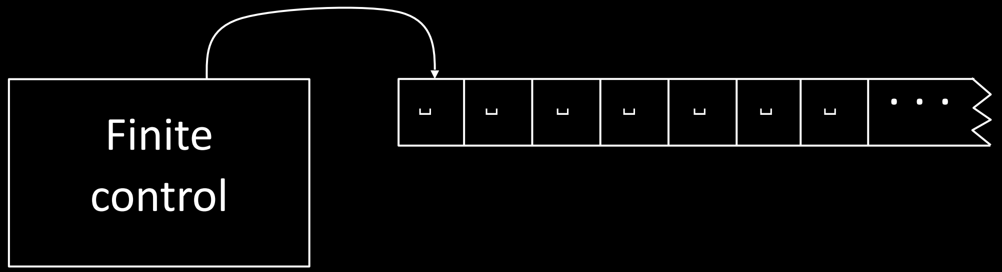
Turing Enumerators

Turing Enumerators



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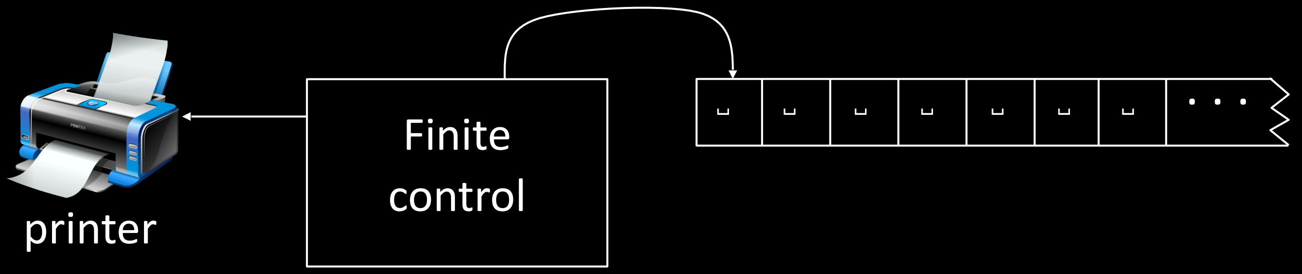

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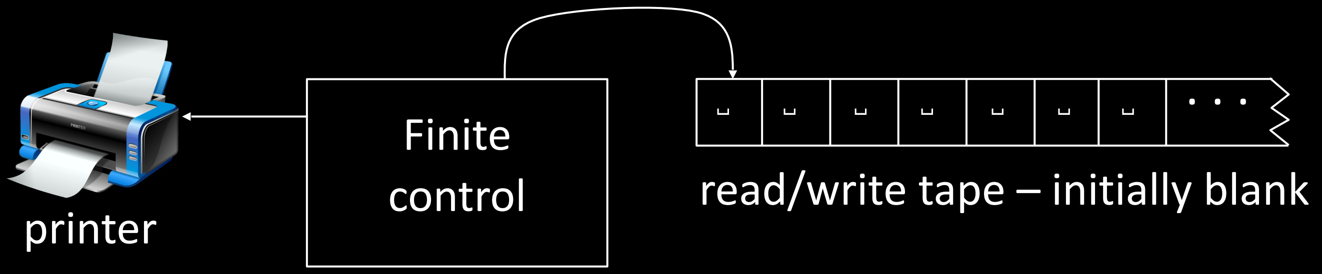

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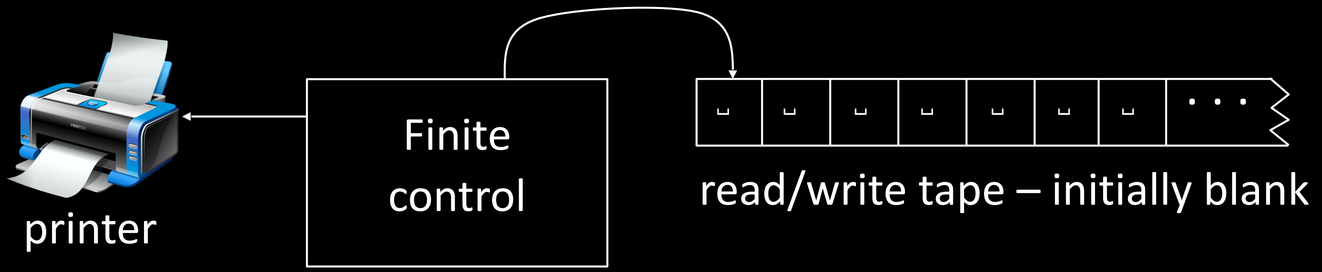

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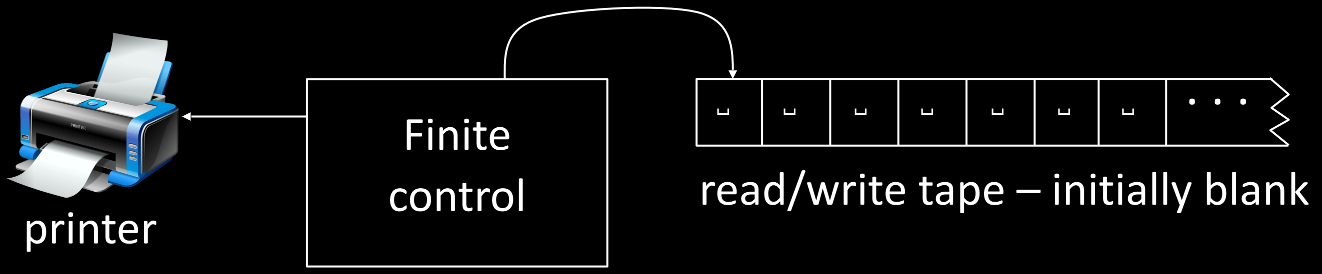

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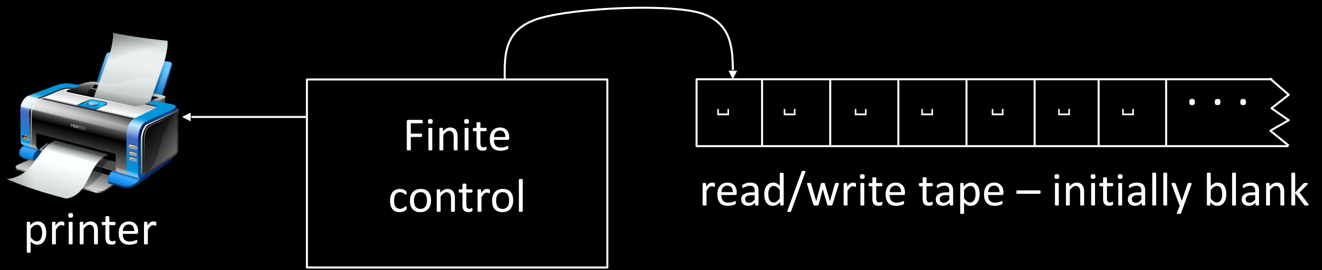

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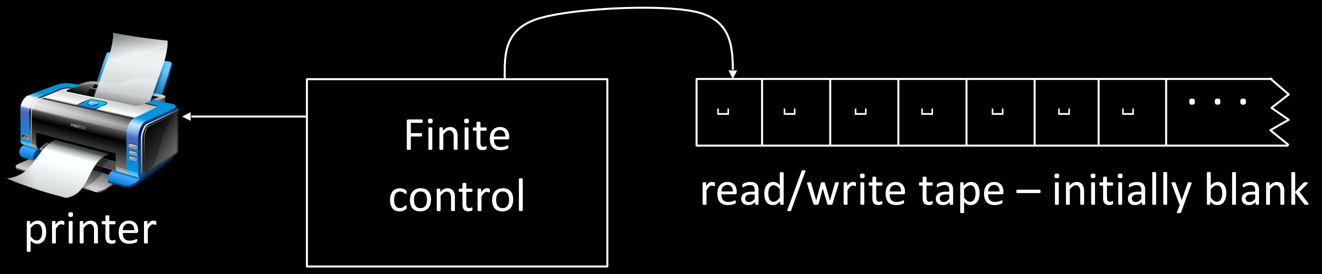

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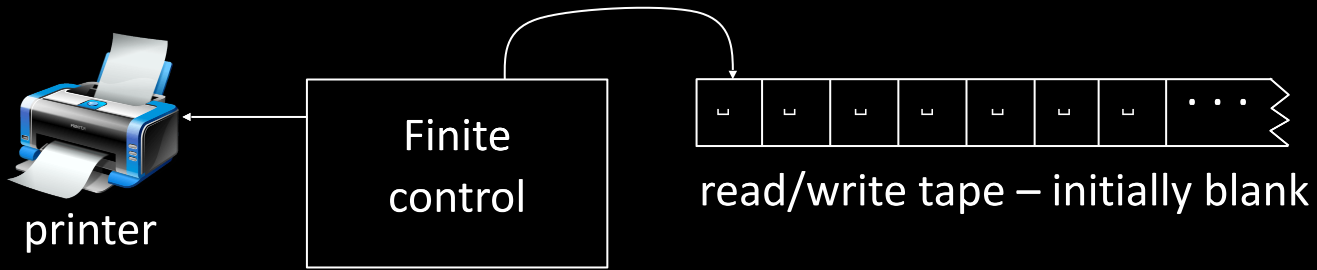

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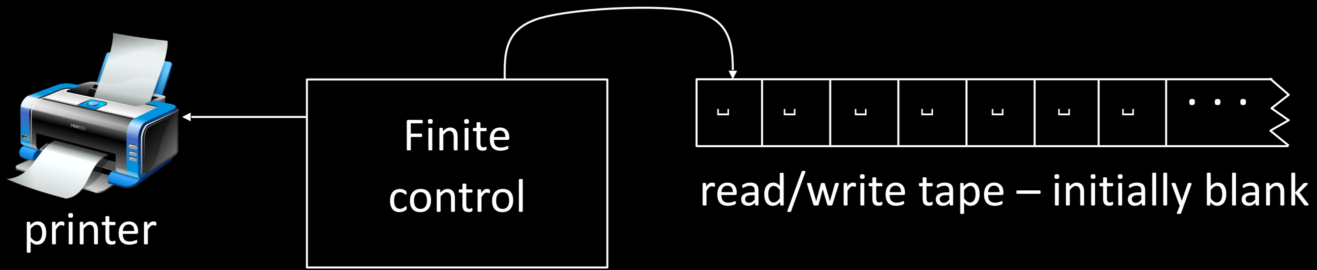

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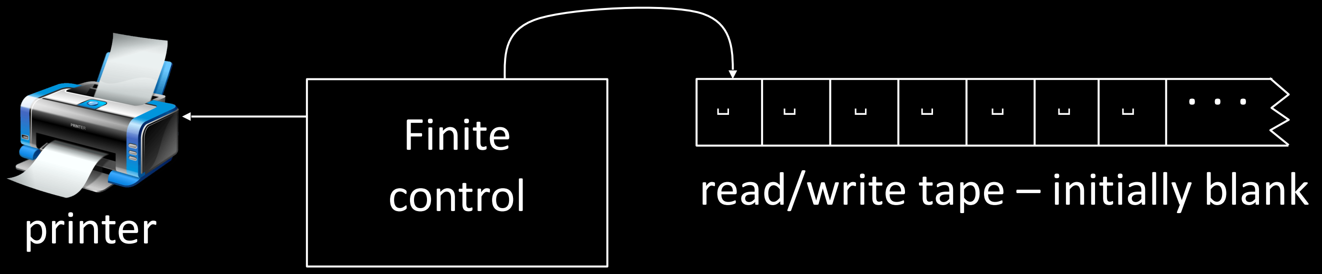

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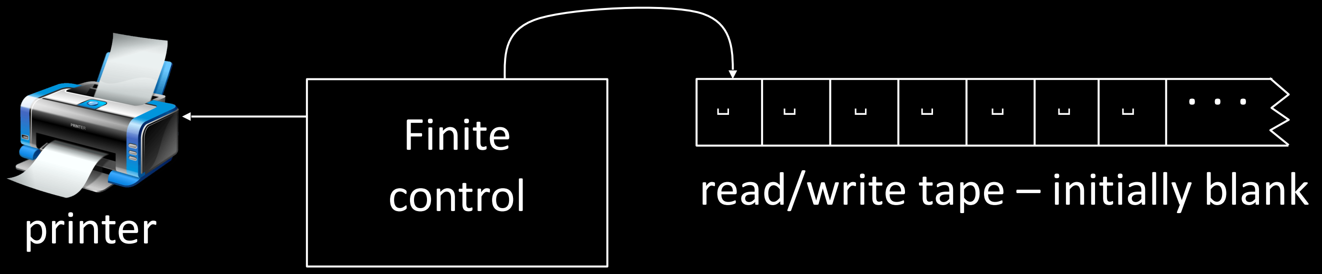

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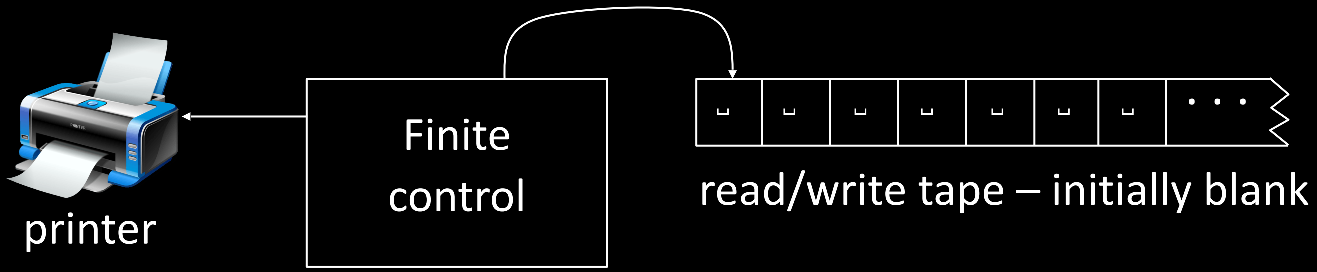

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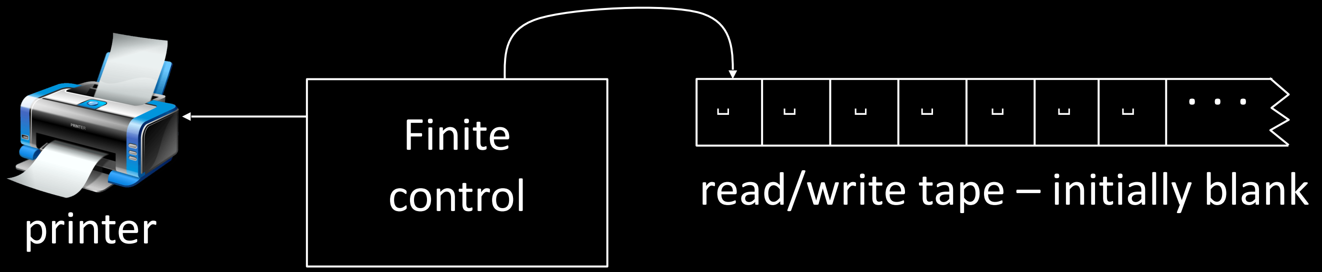

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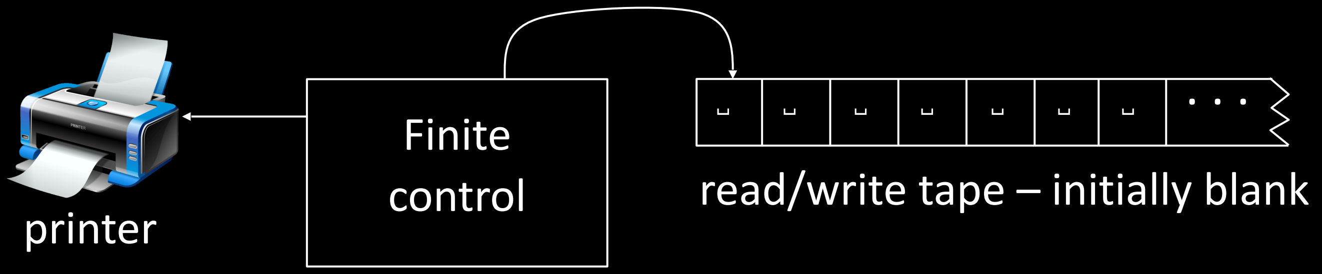

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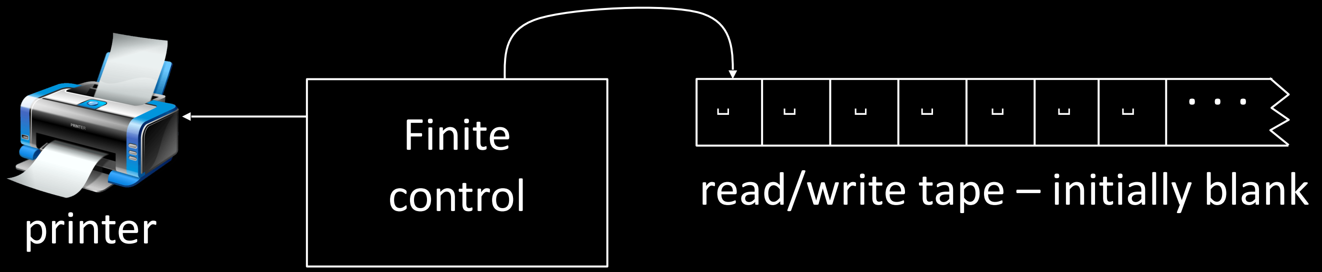

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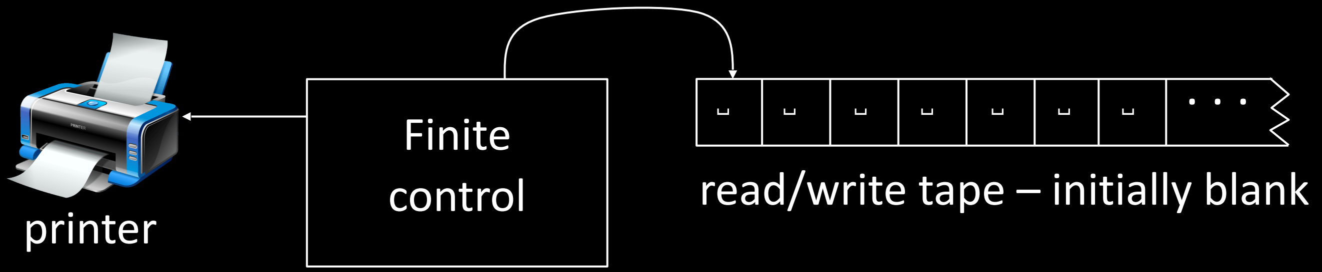

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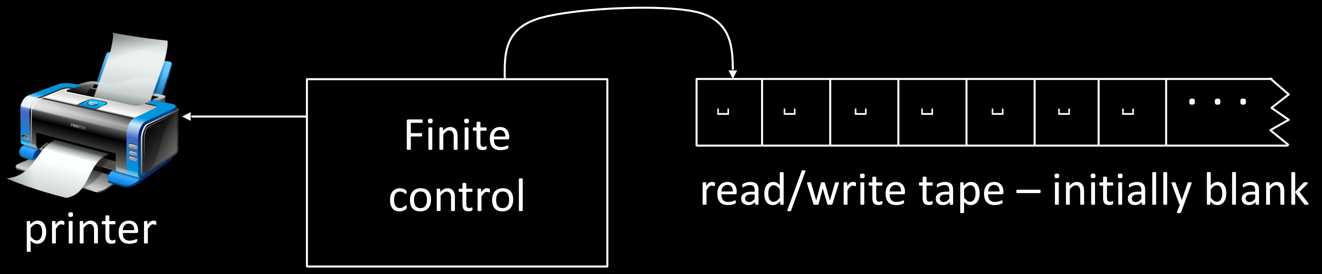

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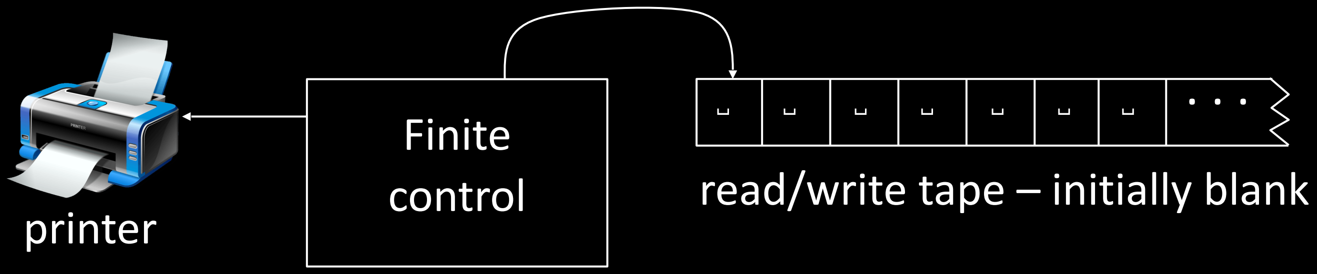

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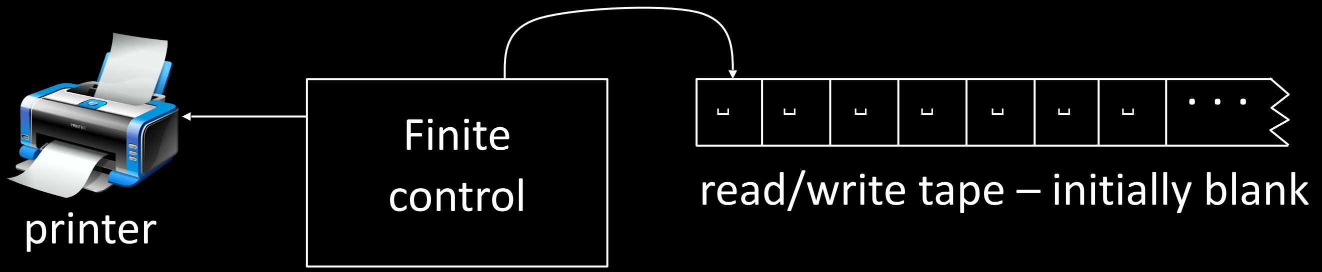

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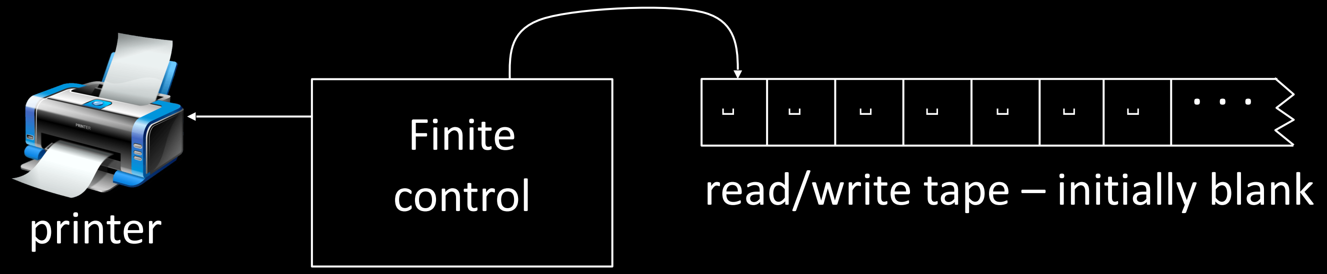

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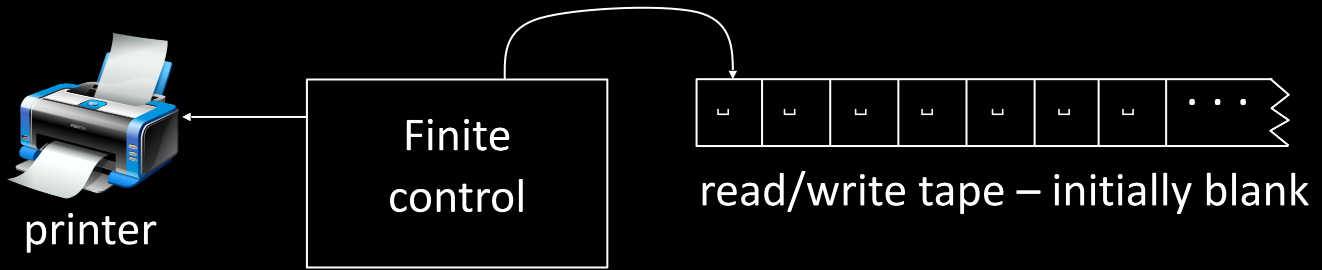
Check-in 6.1

Turing Enumerators



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M =

Check-in 6.1

When converting TM M to enumerator E,
does E always print the strings in ***string order***?

a) Yes.
b) No.

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Alonzo Church
1903–1995

Alan Turing
1912–1954

Church-Turing Thesis ~1936

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Algorithm

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Church-Turing Thesis ~1936

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1903–1995

Algorithm

Intuitive

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Algorithm	Turing machine
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Hilbert's 10th Problem

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Note: D is T-recognizable.

Notation for encodings and TMs

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Check-in 6.3

Notation for encodings and TMs

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If x and y are strings, would xy be a good choice for their encoding $\langle x, y\rangle$ into a single string?

a) Yes.

b) No.

Check-in 6.3

TM – example revisited

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TM M recognizing $B = \{\mathrm{a}^k\mathrm{b}^k\mathrm{c}^k \mid k \geq 0\}$

$M =$ "On input w

1. Check if $w \in \mathrm{a}^*\mathrm{b}^*\mathrm{c}^*$, *reject* if not.
2. Count the number of a's, b's, and c's in w.
3. *Accept* if all counts are equal; *reject* if not."

High-level description is ok.
You do not need to manage tapes, states, etc…

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$$C = \{\ x \mid\ \ \exists y \ \ \langle x, y \rangle \in D\ \} \qquad x, y \in \Sigma^*$$

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Think of D as a collection of pairs of strings.

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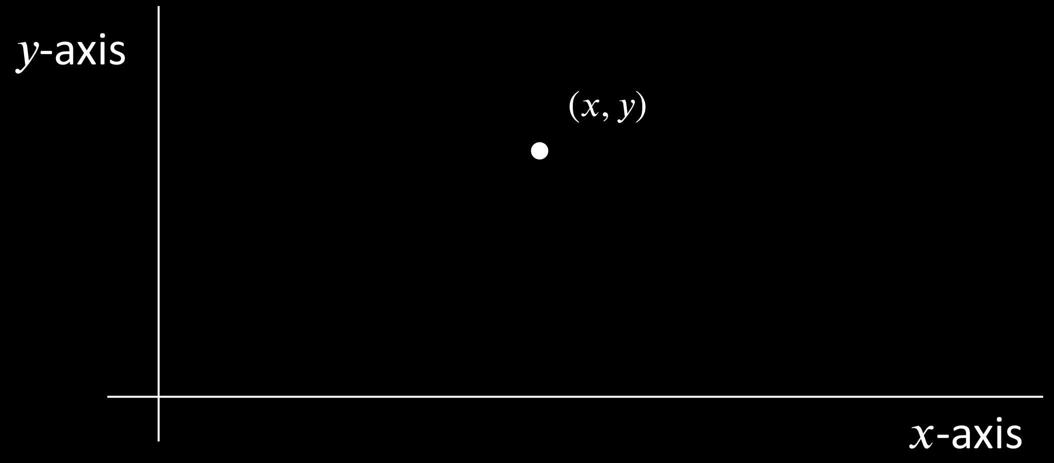

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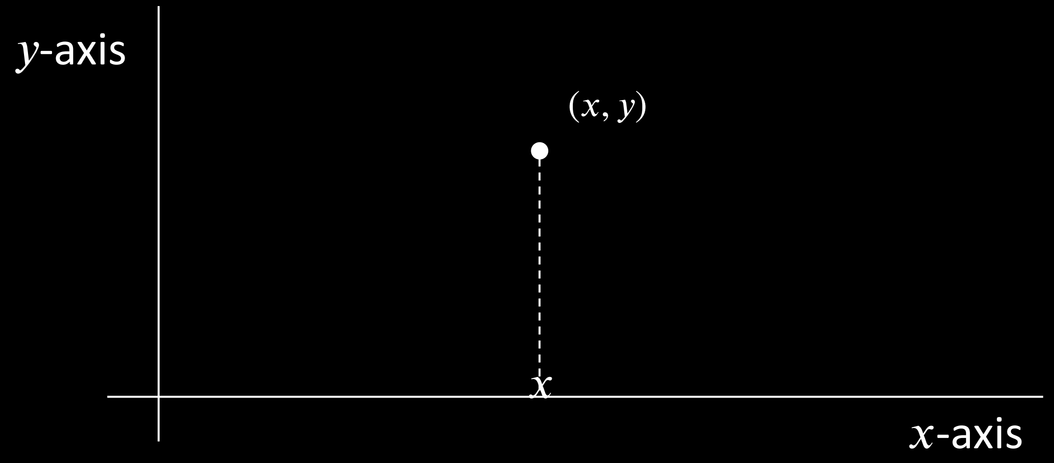

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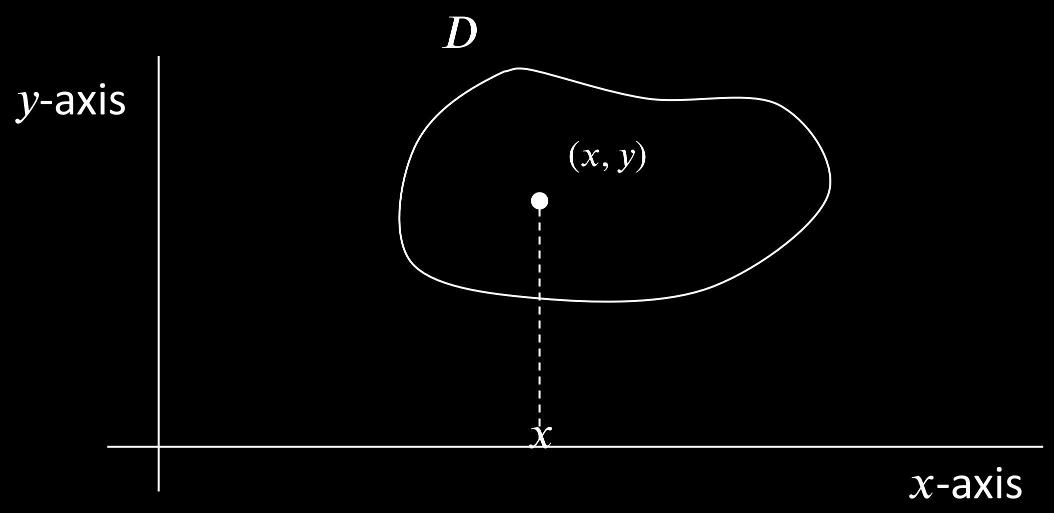

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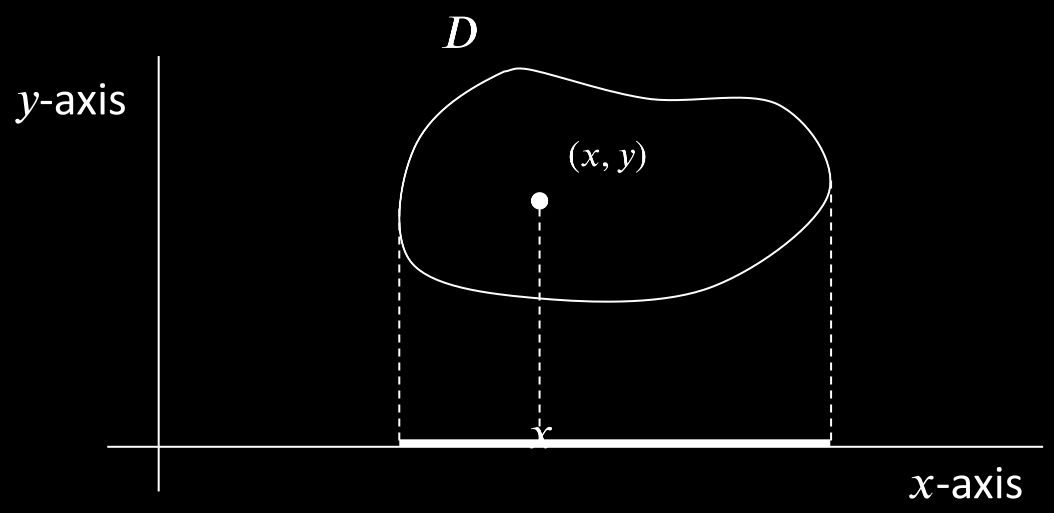

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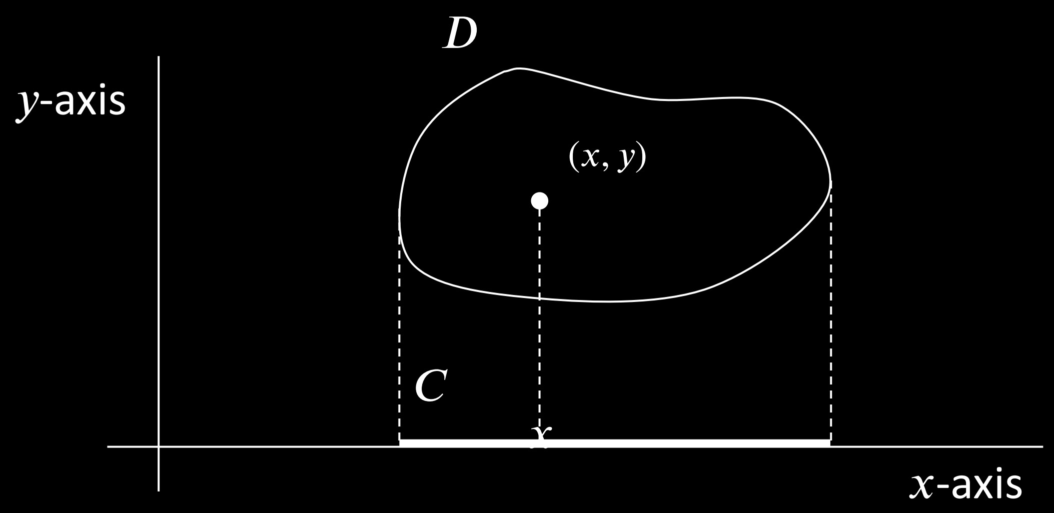

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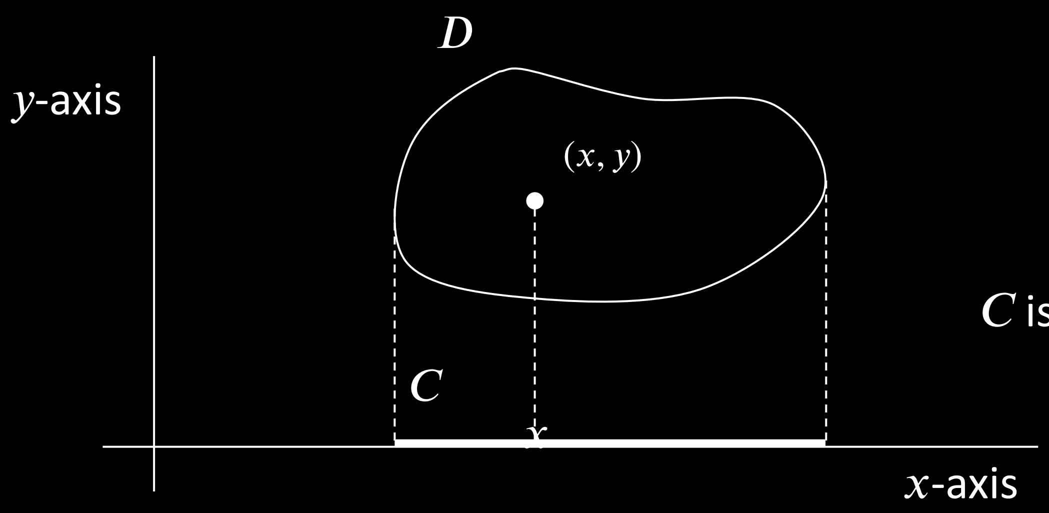

C is a "projection" of D

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