بسم الله الرحمن الرحیم

نظریه علوم کامپیوتر

نظریه علوم کامپیوتر - بهار ۱۴۰۰-۱۴۰۱ - جلسه دوازدهم: پیچیدگی حافظه

Theory of computation - 002 - S12 - space complexity

SPACE Complexity

Defn: Let $f\colon \mathbb{N} \to \mathbb{N}$ where $f(n) \geq n$. Say TM M runs in space $f(n)$ if M always halts and uses at most $f(n)$ tape cells on all inputs of length n.

An NTM M runs in space $f(n)$ if all branches halt and each branch uses at most $f(n)$ tape cells on all inputs of length n.

Defn: SPACE$\big(f(n)\big) = \{B \mid$ some deterministic 1-tape TM M decides B and M runs in space $O\big(f(n)\big)\}$

NSPACE$\big(f(n)\big) = \{B \mid$ some nondeterministic 1-tape TM M decides B and M runs in space $O\big(f(n)\big)\}$

PSPACE $= \bigcup_k$ SPACE(n^k) "polynomial space"

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Check-in 17.1

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Check-in 17.1

We define space complexity for multi-tape TMs by taking the sum of the cells used on all tapes.

Do we get the same class PSPACE for multi-tape TMs?

(a) No.

(b) Yes, converting a multi-tape TM to single-tape only squares the amount of space used.

(c) Yes, converting a multi-tape TM to single-tape only increases the amount of space used by a constant factor.

Relationships between Time and SPACE Complexity

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Theorem: NP $\subseteq$ PSPACE [next slide]

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Theorem: NP $\subseteq$ PSPACE [next slide]

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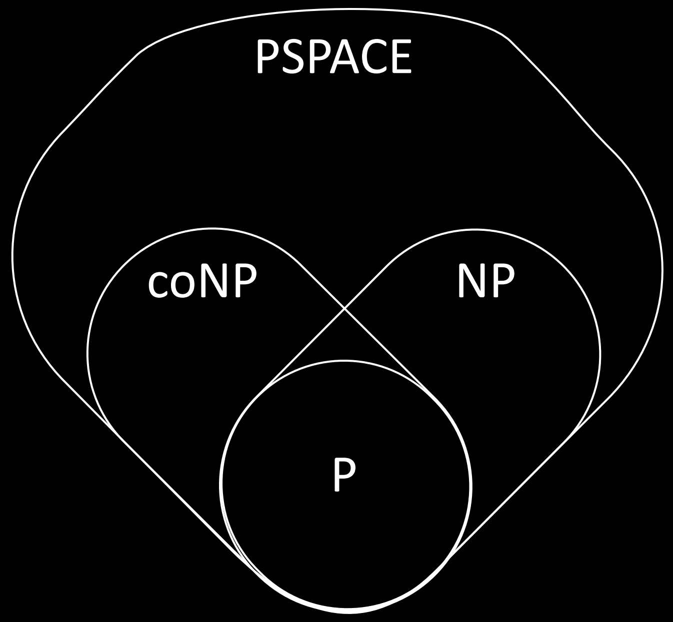

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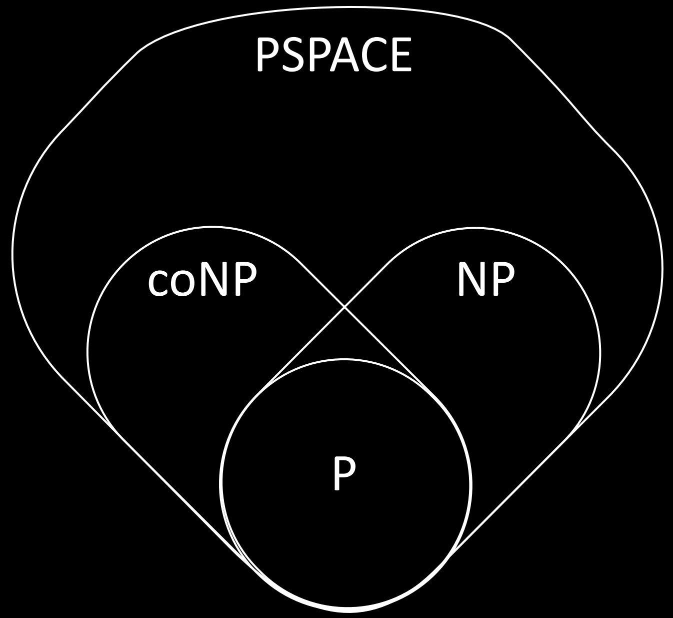

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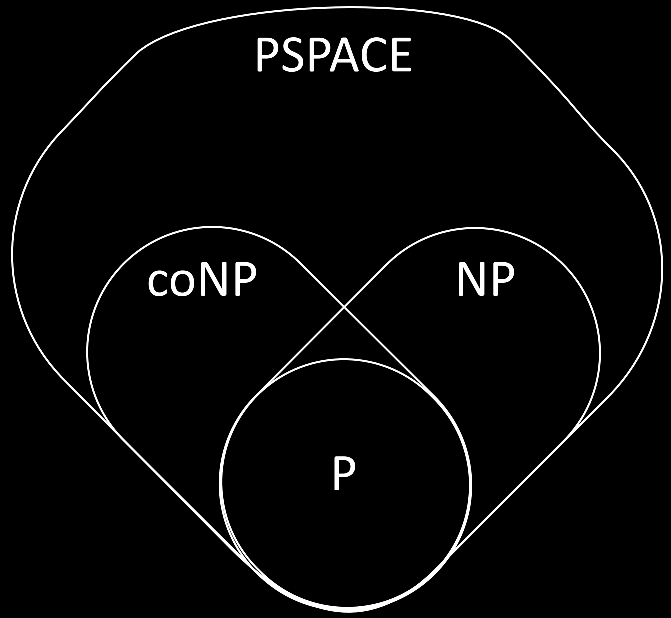

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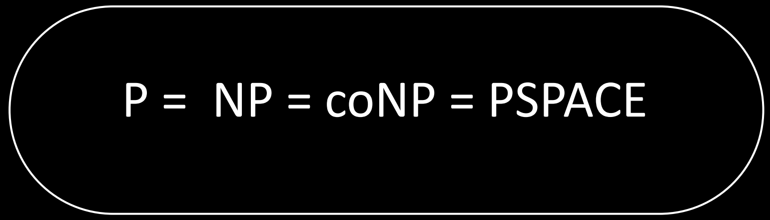

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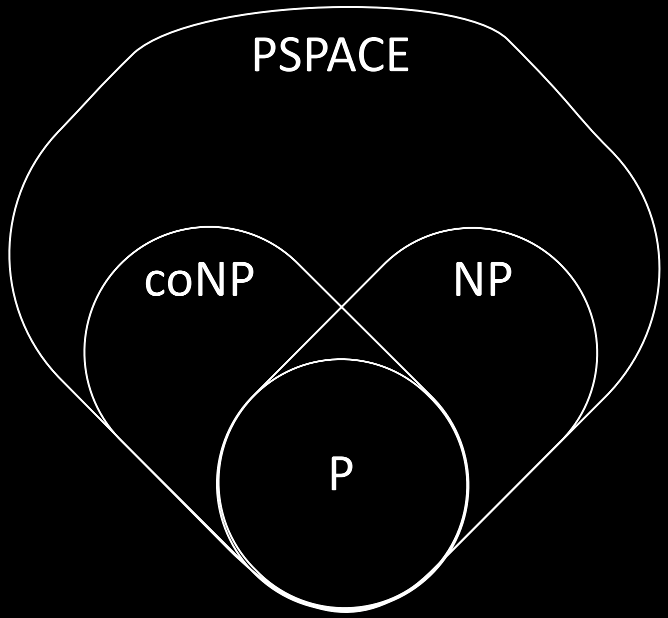

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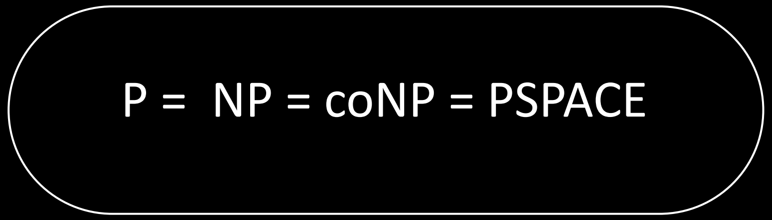

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Check-in 17.2

How is SAT a special case of $TQBF$?

(a) Remove all quantifiers.

(b) Add $\exists$ and $\forall$ quantifiers.

(c) Add only $\exists$ quantifiers.

(d) Add only $\forall$ quantifiers.

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Each recursive level uses constant space (to record the x value).

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A ladder is a sequence of strings of a common length where consecutive strings differ in a single symbol.

A word ladder for English is a ladder of English words.

Let A be a language. A ladder in A is a ladder of strings in A.

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Defn: $LADDER_{\mathrm{DFA}} = \Big\{\langle B, u, v\rangle \Big|$ B is a DFA and $L(B)$ contains a ladder $y_1,\ y_2,\ \ldots,\ y_k$ where $y_1 = u$ and $y_k = v\}$.

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Theorem: $LADDER_{\mathbf{DFA}} \in$ NPSPACE

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Theorem: $LADDER$**DFA** $\in$ NPSPACE

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$LADDER_{\mathrm{DFA}} \in \mathrm{NPSPACE}$

Theorem: $LADDER_{\mathrm{DFA}} \in \mathrm{NPSPACE}$

$LADDER_{\text{DFA}} \in$ NPSPACE

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Proof idea: Nondeterministically guess the sequence from u to v.

$LADDER_{\text{DFA}} \in$ NPSPACE

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$\leq t$

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$LADDER_{\text{DFA}} \in$ NPSPACE

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Careful- (a) cannot store sequence, (b) must terminate.

$\leq t$

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$LADDER$DFA $\in$ NPSPACE

Theorem: $LADDER$DFA $\in$ NPSPACE

Proof idea: Nondeterministically guess the sequence from u to v.

Careful- (a) cannot store sequence, (b) must terminate.

Proof: "On input $\langle B, u, v \rangle$

$\leq t$

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$LADDER_{\text{DFA}} \in \text{NPSPACE}$

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Proof idea: Nondeterministically guess the sequence from u to v.

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Proof: “On input $\langle B, u, v \rangle$

1. Let $y = u$ and let $m = |u|$.

$\leq t$

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Proof: "On input $\langle B, u, v \rangle$

1. Let $y = u$ and let $m = |u|$.
2. Repeat at most t times where $t = |\Sigma|^m$.

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Proof: "On input $\langle B, u, v \rangle$

1. Let $y = u$ and let $m = |u|$.
2. Repeat at most t times where $t = |\Sigma|^m$.
3. Nondeterministically change one symbol in y.

$\leq t$

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1. Let $y = u$ and let $m = |u|$.
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4. *Reject* if $y \notin L(B)$.

$\le t$

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5. *Accept* if $y = v$.

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Space used is for storing y and t.

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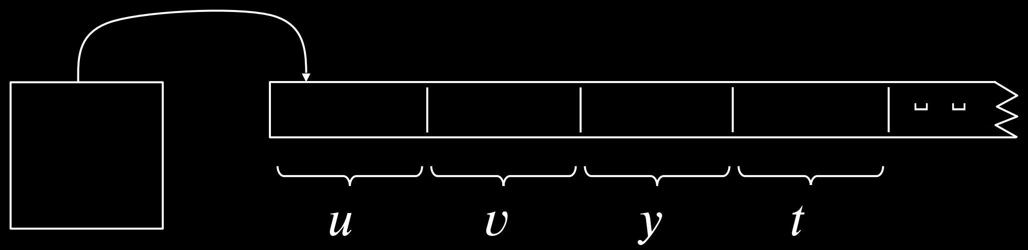

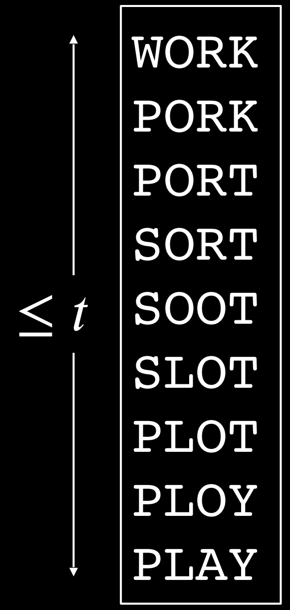

$LADDER_{\text{DFA}} \in \text{NPSPACE}$

Theorem: $LADDER_{\text{DFA}} \in \text{NPSPACE}$

Proof idea: Nondeterministically guess the sequence from u to v.

Careful- (a) cannot store sequence, (b) must terminate.

Proof: "On input $\langle B, u, v \rangle$

1. Let $y = u$ and let $m = |u|$.
2. Repeat at most t times where $t = |\Sigma|^m$.
3. Nondeterministically change one symbol in y.
4. *Reject* if $y \notin L(B)$.
5. *Accept* if $y = v$.
6. *Reject* [exceeded t steps].

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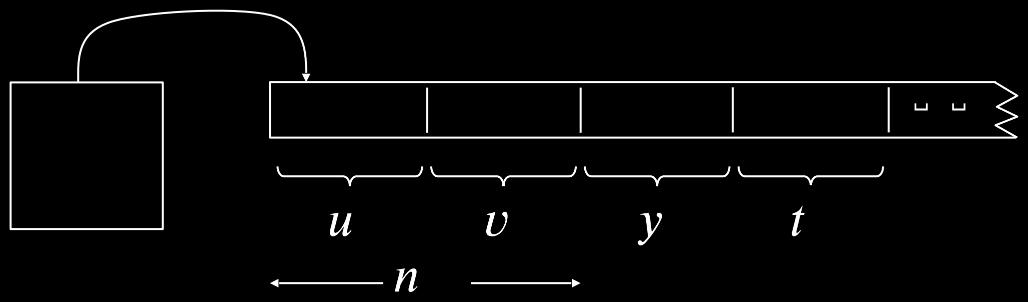

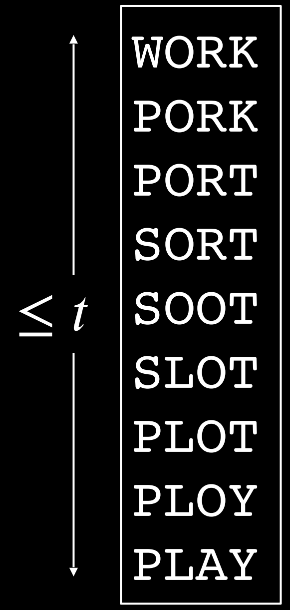

$LADDER$DFA $\in$ NPSPACE

Theorem: $LADDER$DFA $\in$ NPSPACE

Proof idea: Nondeterministically guess the sequence from u to v.

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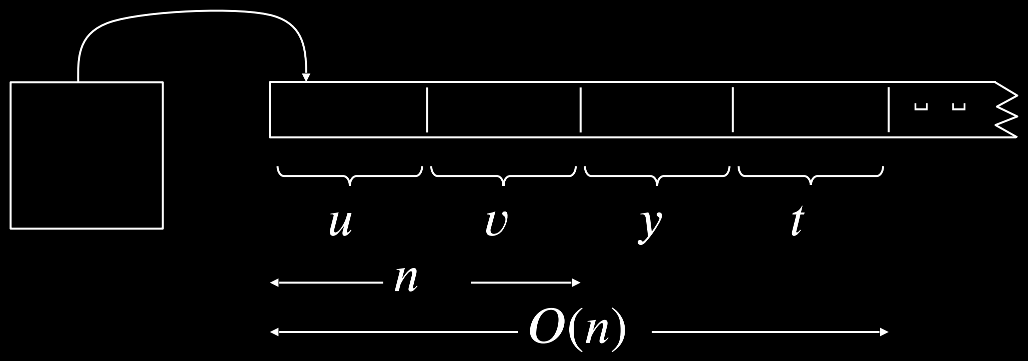

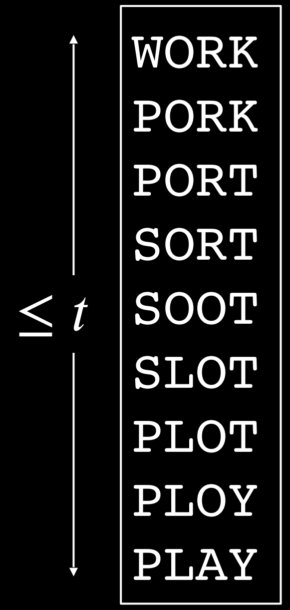

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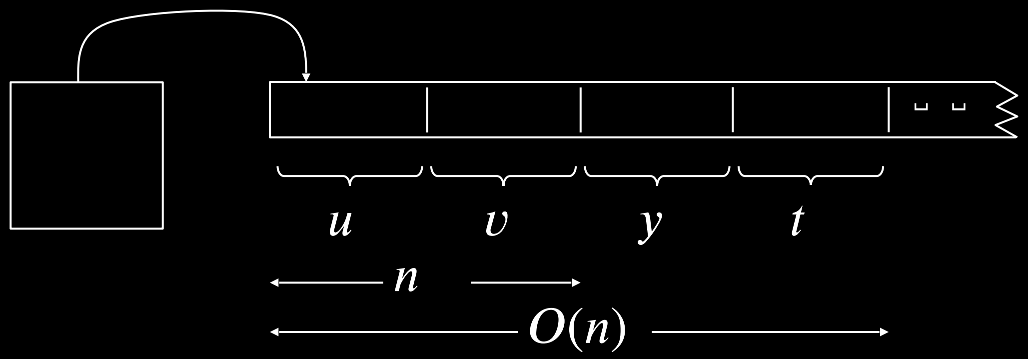

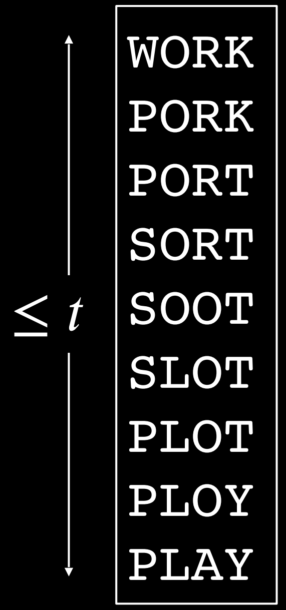

Space used is for storing y and t.

$LADDER$DFA $\in$ NSPACE(n).

$LADDER_{\text{DFA}} \in$ NPSPACE

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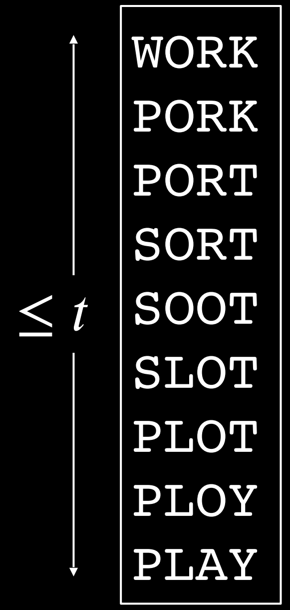
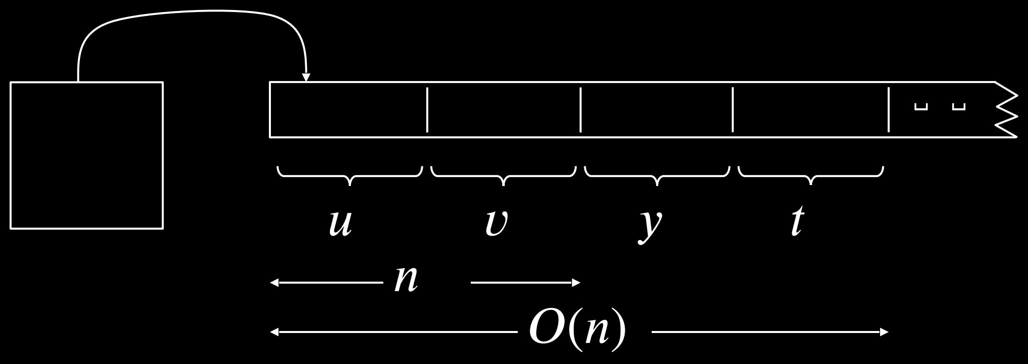


Space used is for storing y and t.

$LADDER_{\text{DFA}} \in \text{NSPACE}(n)$.



Theorem: $LADDER_{\text{DFA}} \in$ PSPACE (!)

$LADDER$DFA $\in$ NPSPACE

Theorem: $LADDER$DFA $\in$ NPSPACE

Proof idea: Nondeterministically guess the sequence from u to v.

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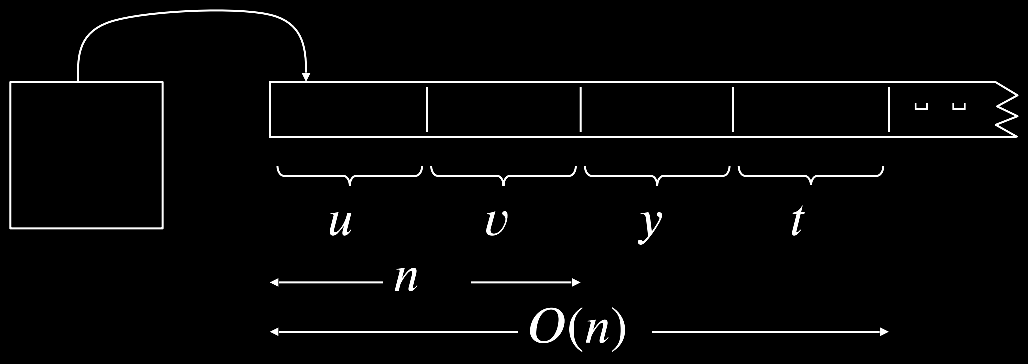

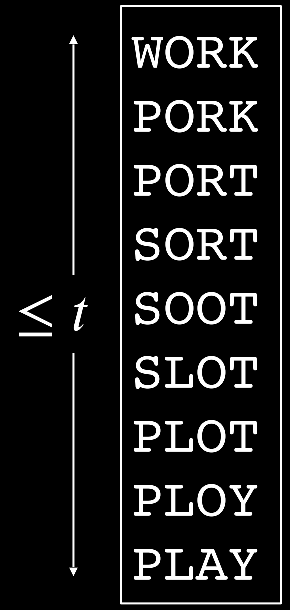

Space used is for storing y and t.

$LADDER$DFA $\in$ NSPACE(n).

Theorem: $LADDER$DFA $\in$ PSPACE (!)

$LADDER_{\mathrm{DFA}} \in \mathrm{PSPACE}$

$LADDER_{\text{DFA}} \in \text{PSPACE}$

Theorem: $LADDER_{\text{DFA}} \in \text{SPACE}(n^2)$

$LADDER_{\text{DFA}} \in$ PSPACE

Theorem: $LADDER_{\text{DFA}} \in$ SPACE(n^2)

Proof: Write $u \xrightarrow{b} v$ if there's a ladder from u to v of length $\leq b$.

$LADDER$DFA $\in$ PSPACE

Theorem: $LADDER$DFA $\in$ SPACE(n^2)

Proof: Write $u \xrightarrow{b} v$ if there's a ladder from u to v of length $\leq b$.
Here's a recursive procedure to solve the bounded DFA ladder problem:

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$LADDER_{\mathrm{DFA}} \in$ PSPACE

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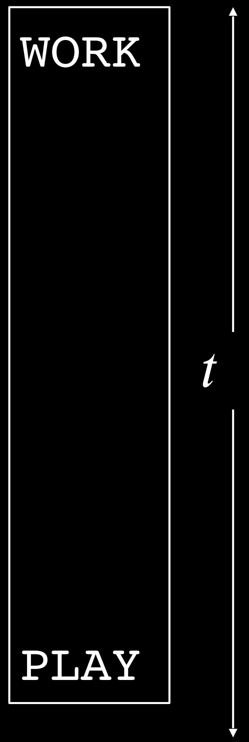

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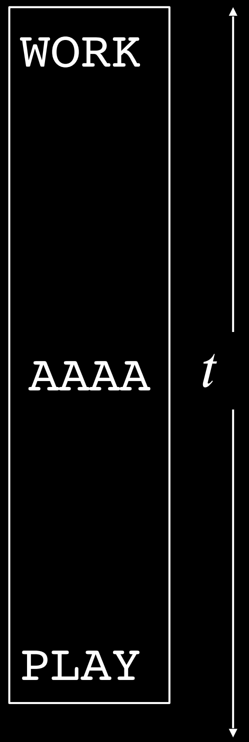

$LADDER$DFA $\in$ PSPACE

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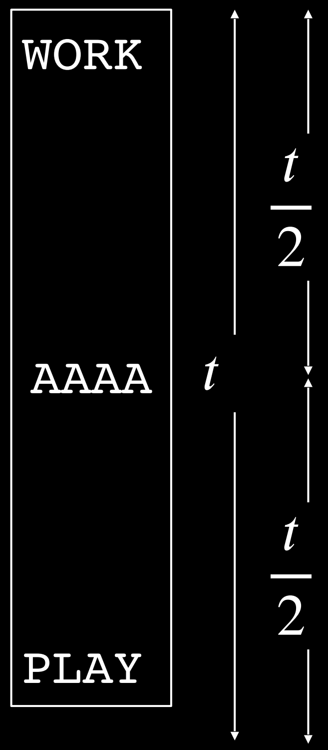

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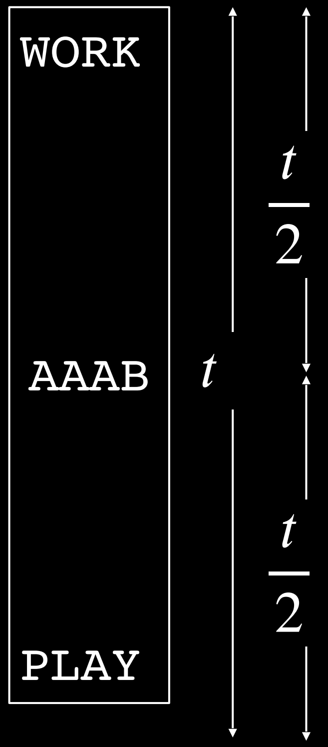

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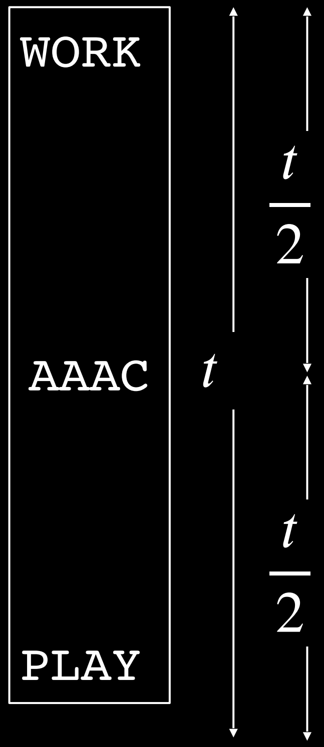

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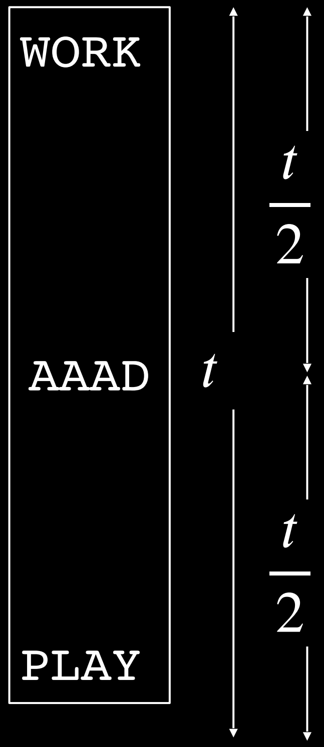

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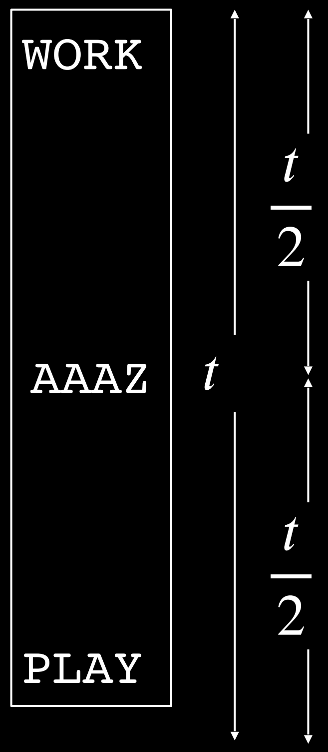

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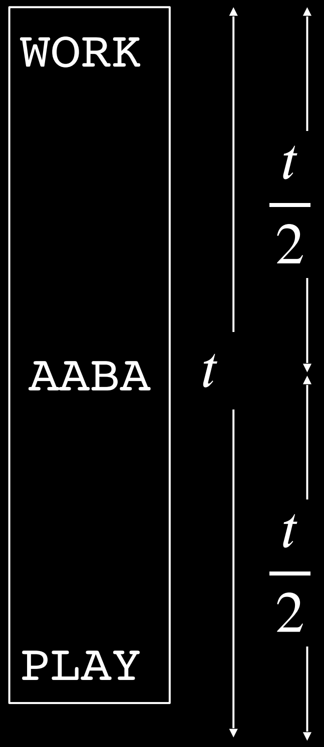

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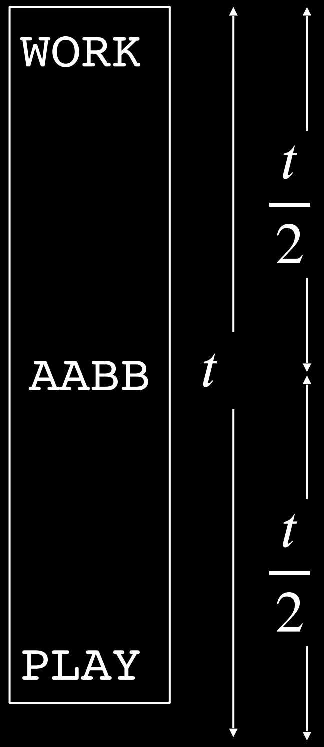

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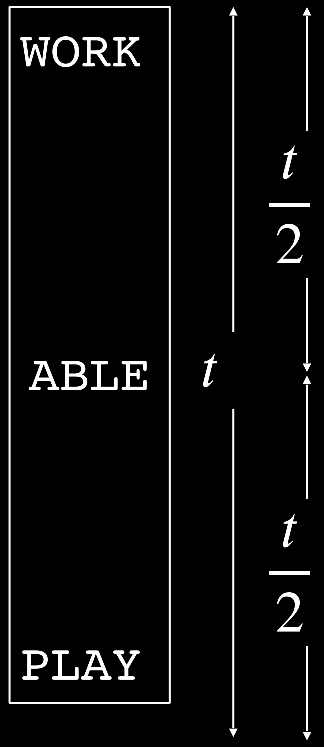

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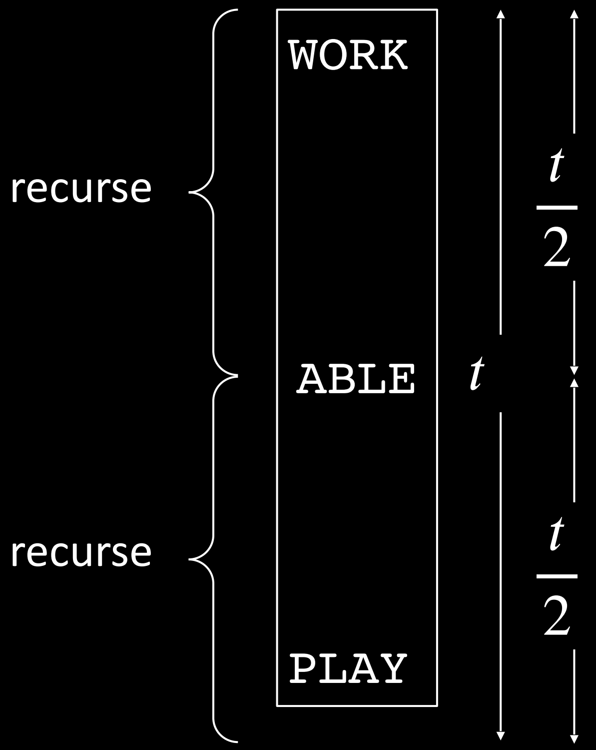

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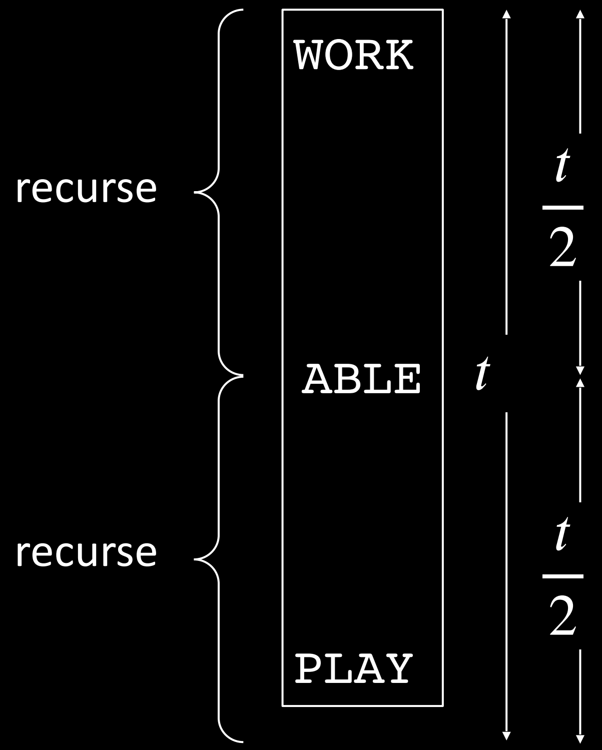

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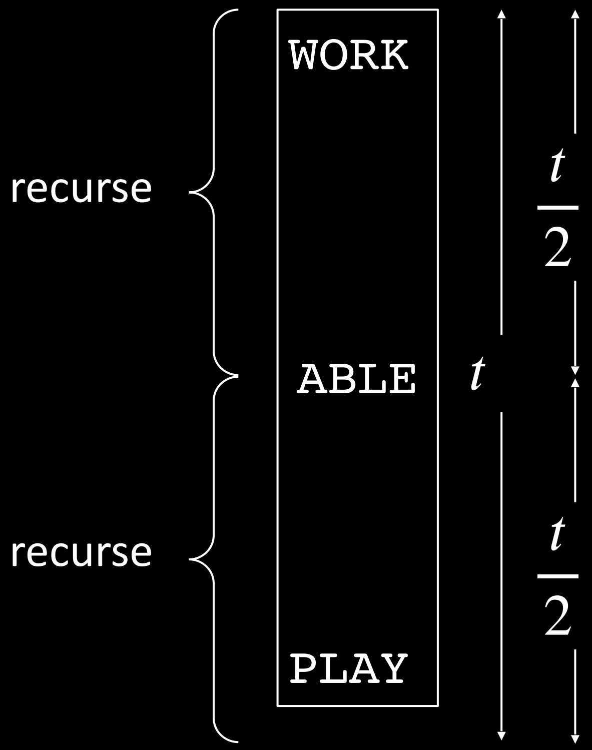

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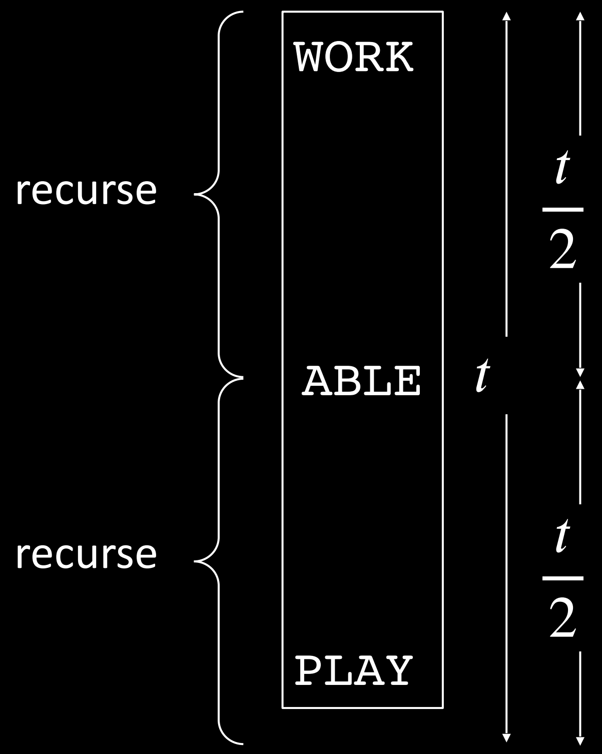

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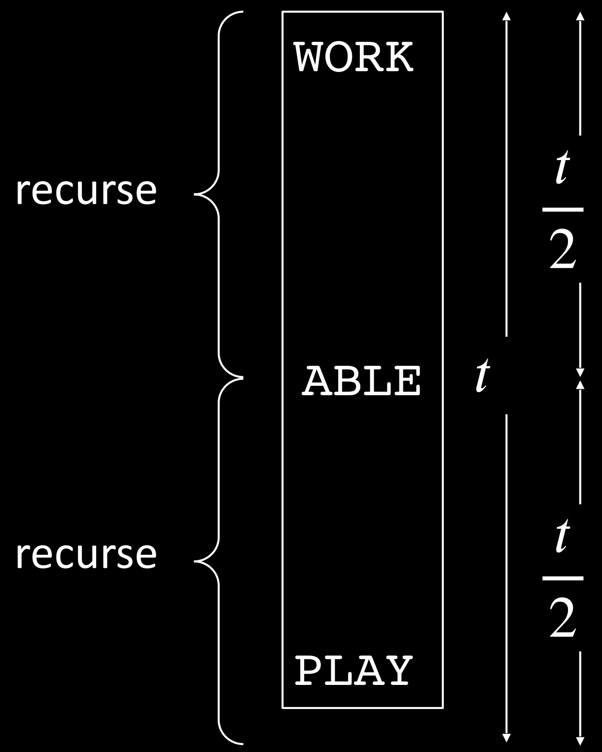

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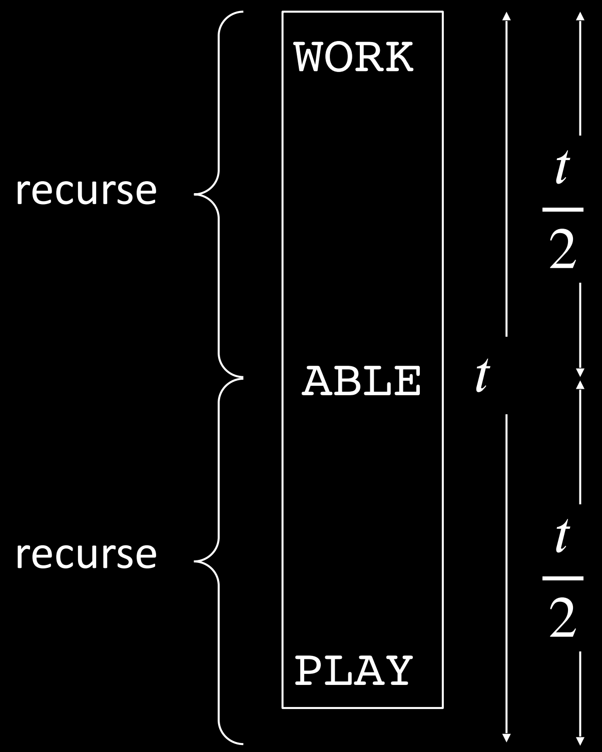

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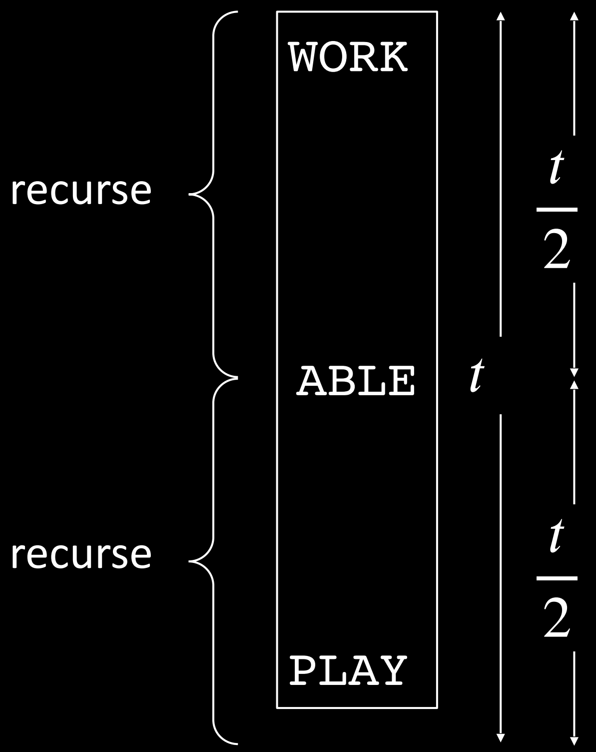

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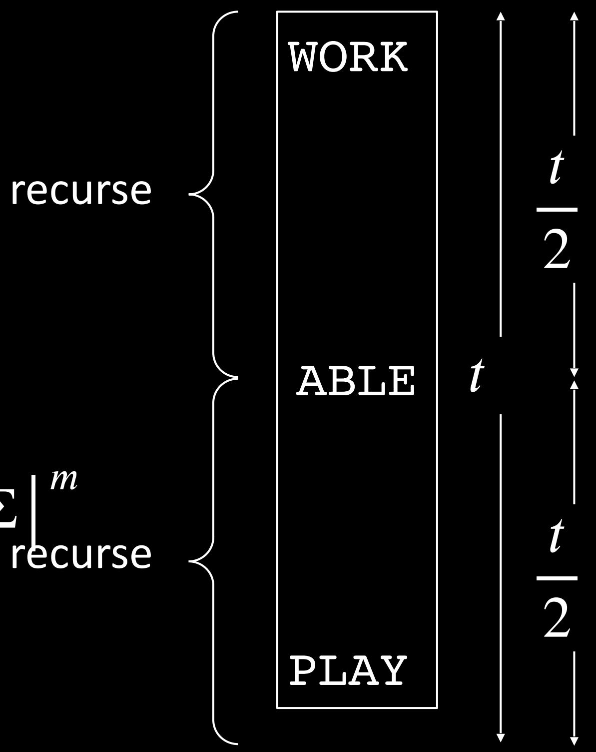

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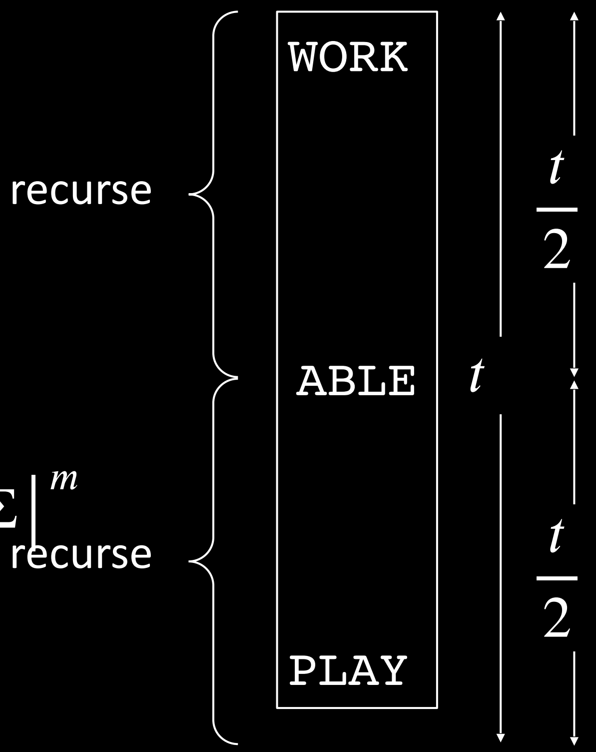

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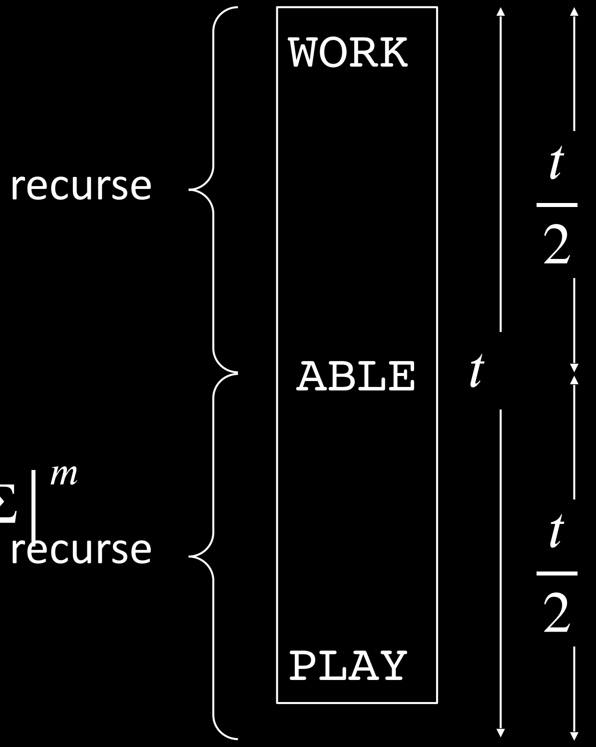

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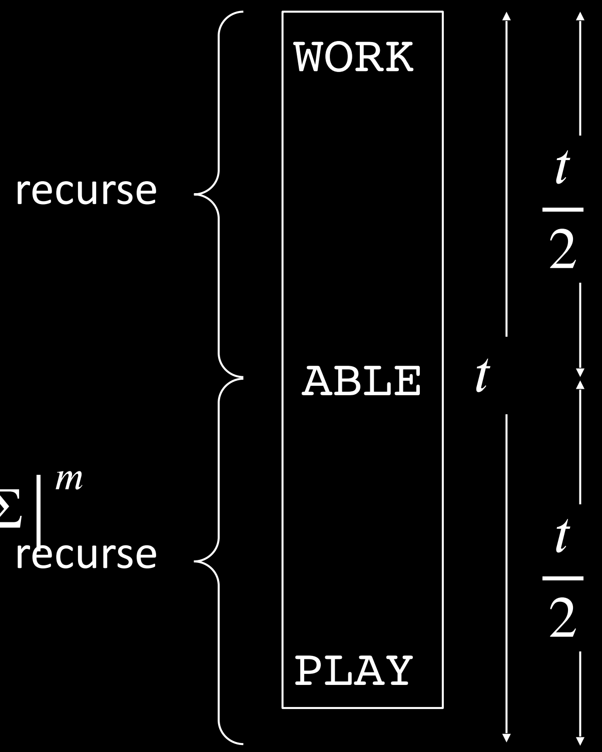

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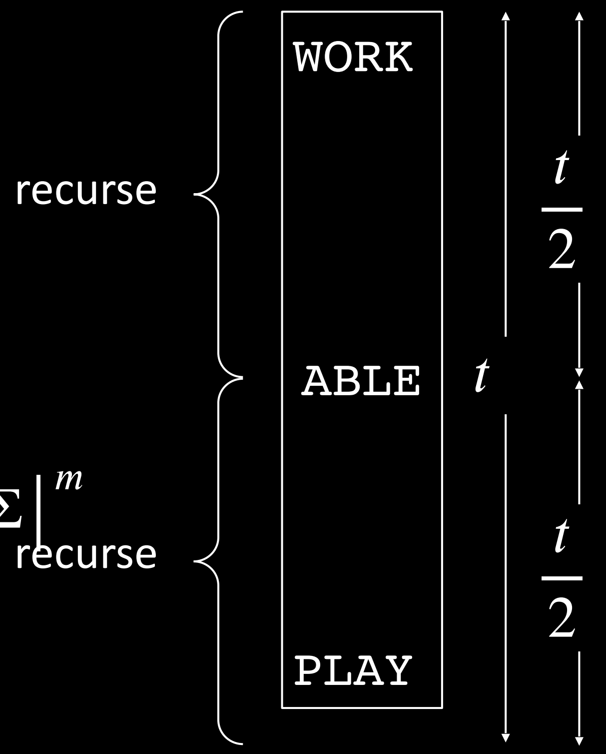

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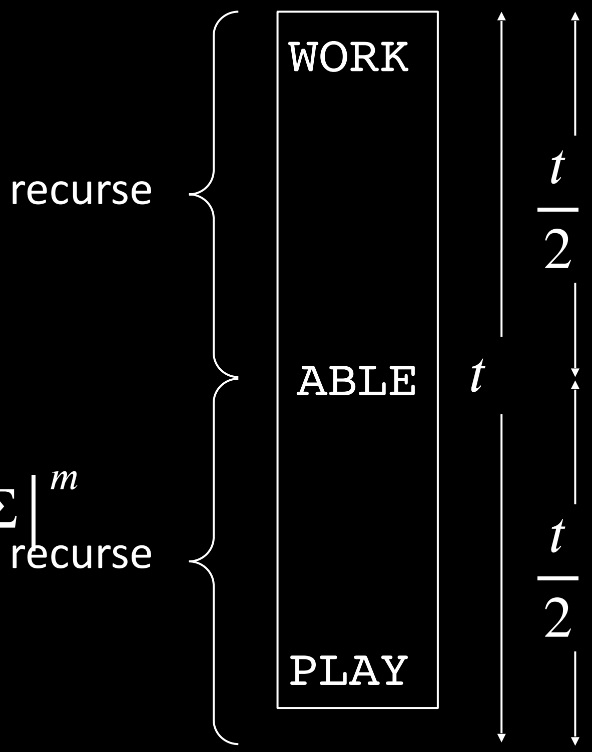

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Check-in 17.3

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Check-in 17.3

Find an English word ladder connecting MUST and VOTE.

(a) Already did it.

(b) I will.

Check-in 17.3

Review: SPACE Complexity

Defn: Let $f\colon \mathbb{N} \to \mathbb{N}$ where $f(n) \geq n$. Say TM M runs in space $f(n)$ if M always halts and uses at most $f(n)$ tape cells on all inputs of length n.

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$\text{PSPACE} = \bigcup_k \text{SPACE}(n^k)$ "polynomial space"

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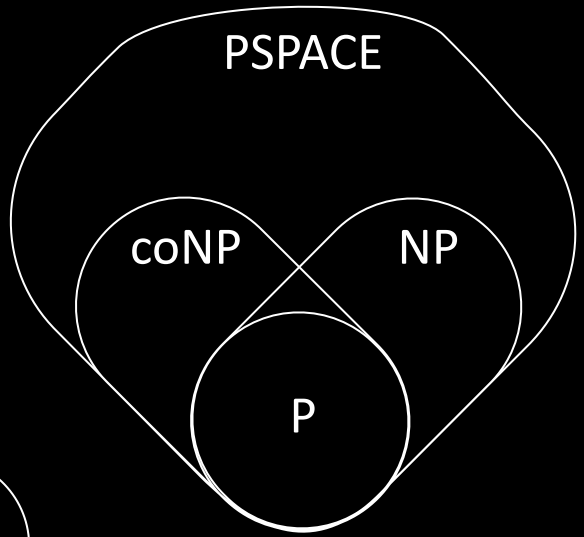
Or possibly: P = NP = coNP = PSPACE



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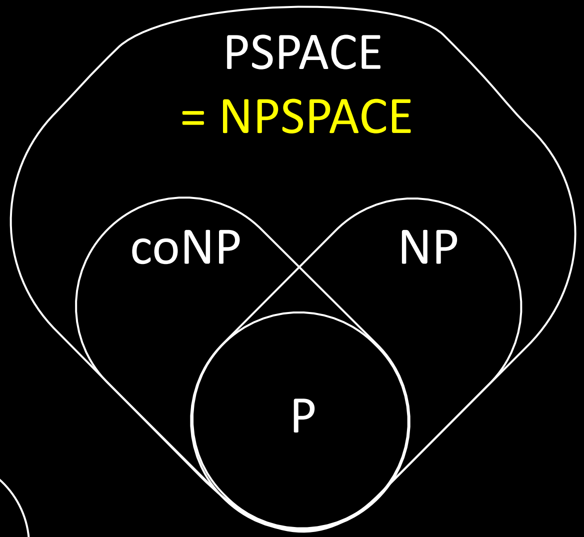

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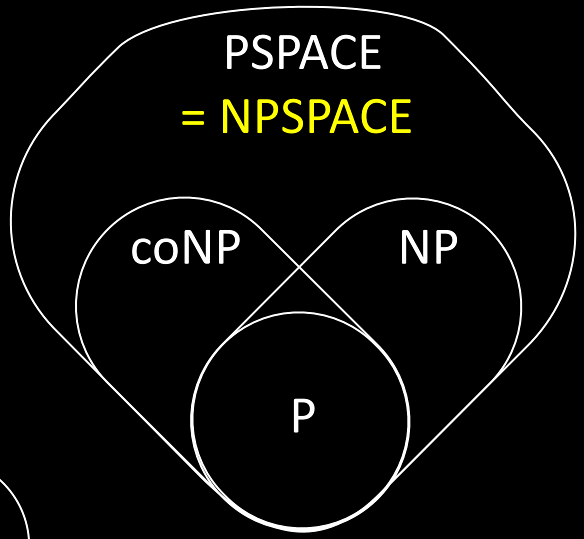

Today: PSPACE = NPSPACE

Review: $LADDER_{\text{DFA}} \in \text{PSPACE}$

Theorem: $LADDER_{\text{DFA}} \in \text{SPACE}(n^2)$

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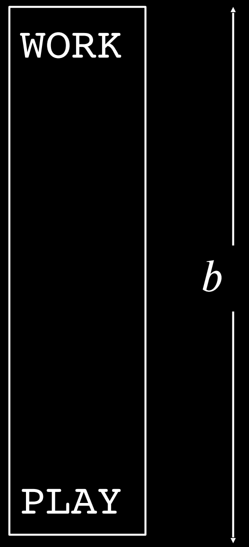

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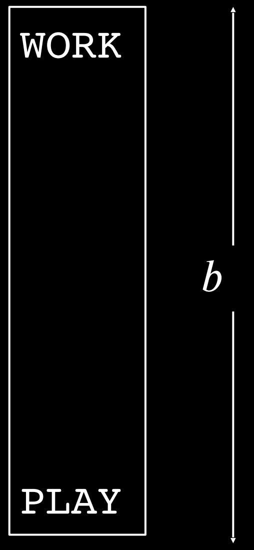

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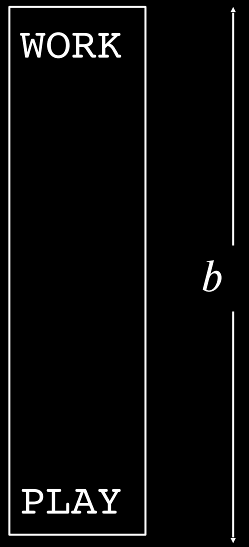

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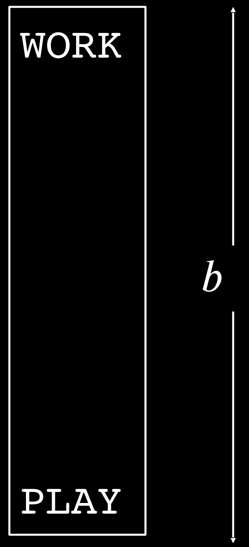

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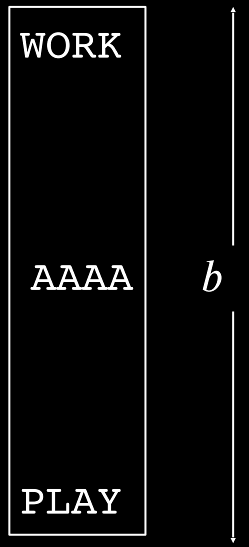

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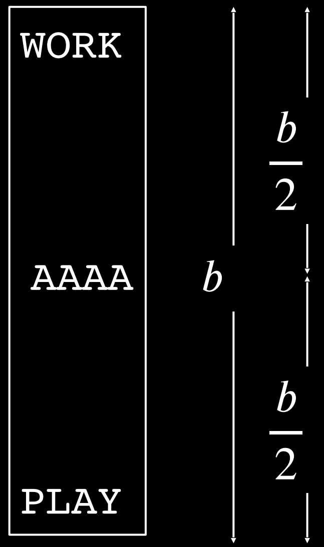

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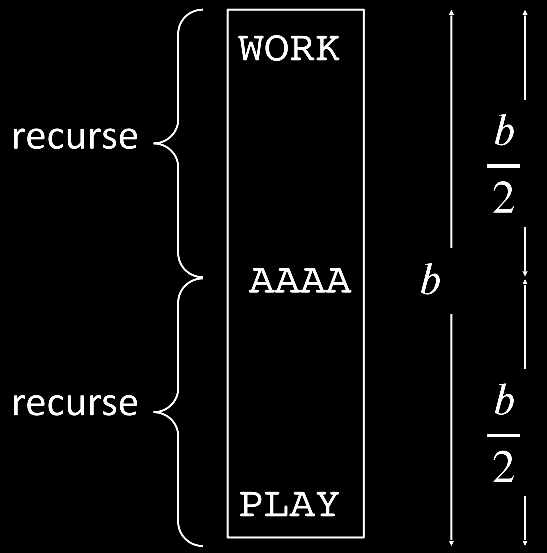

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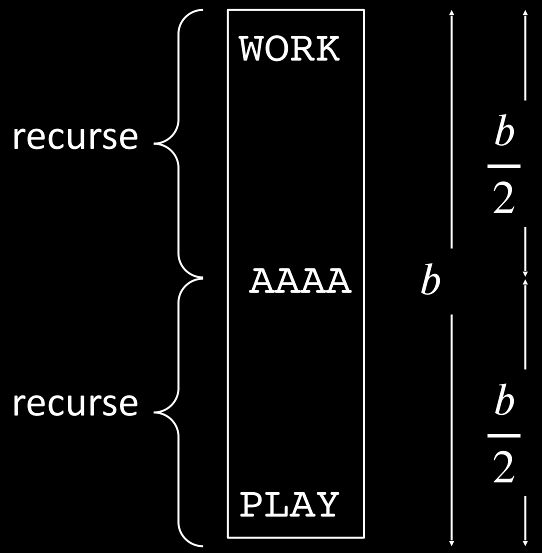

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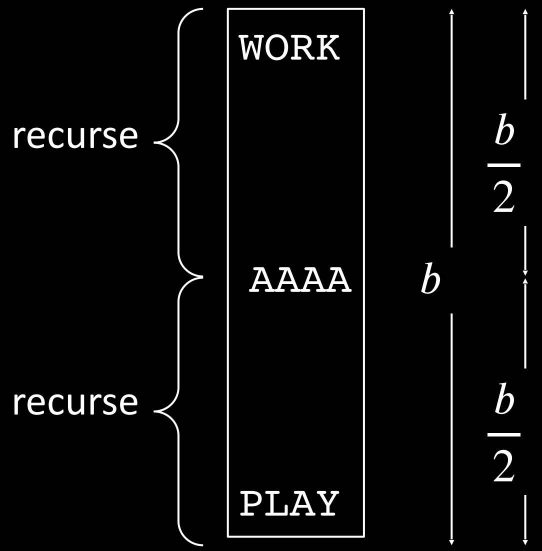

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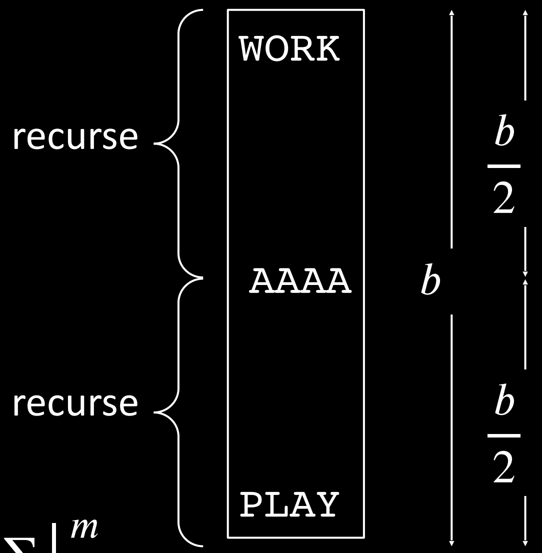

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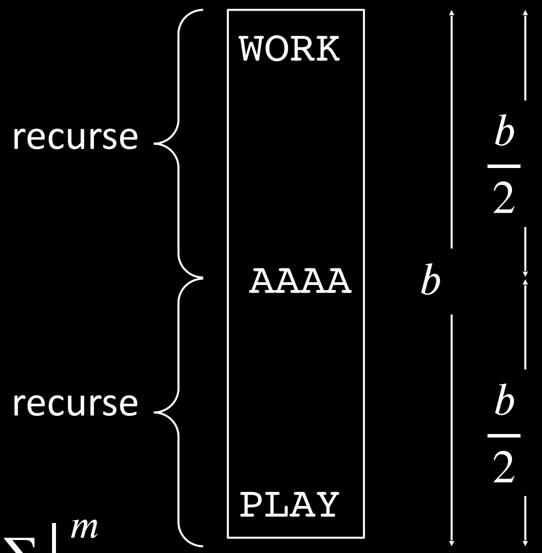

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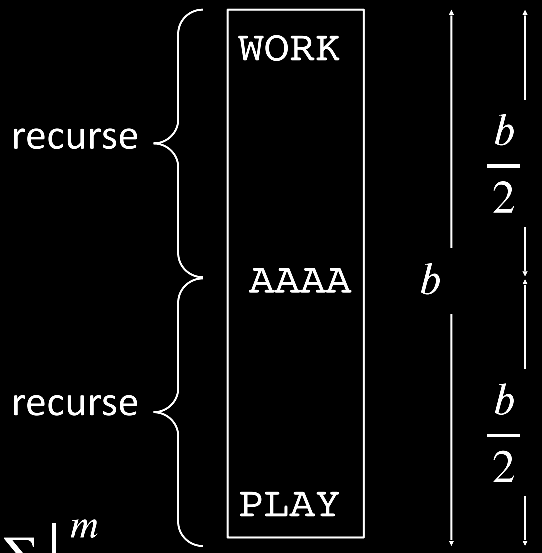

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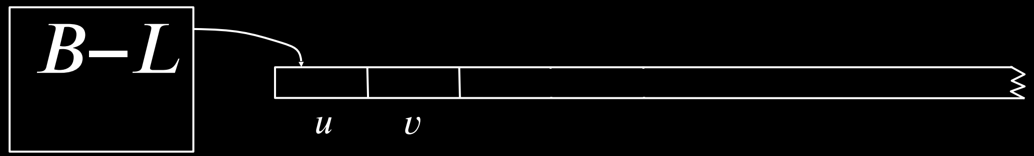

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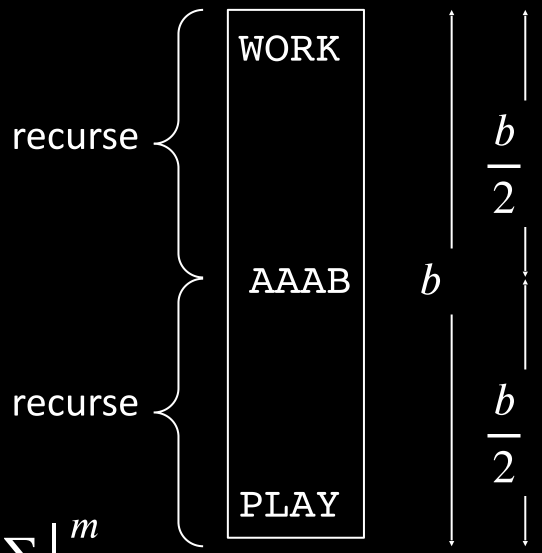

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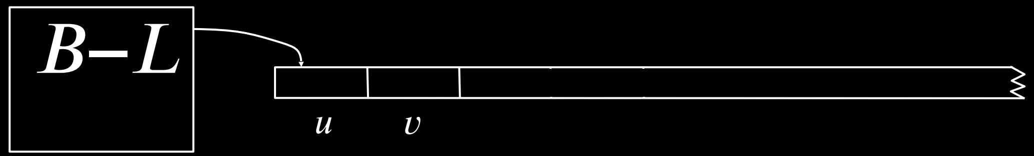

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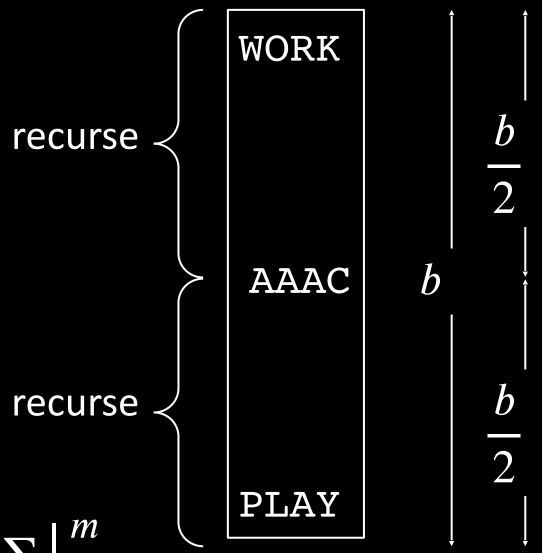

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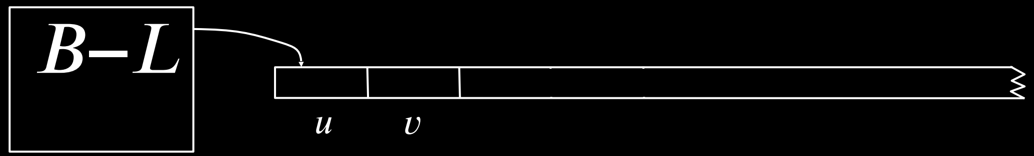

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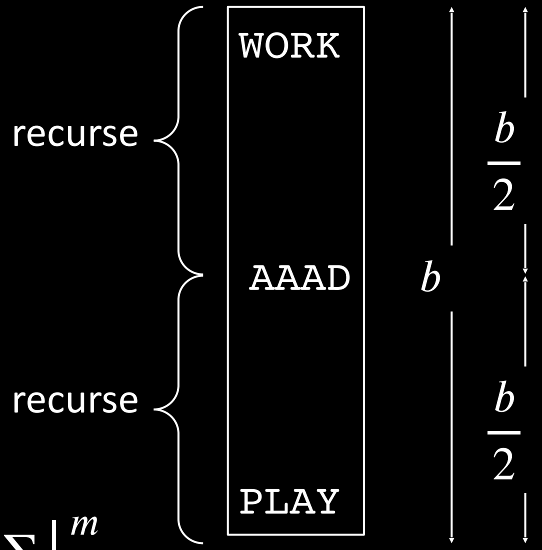

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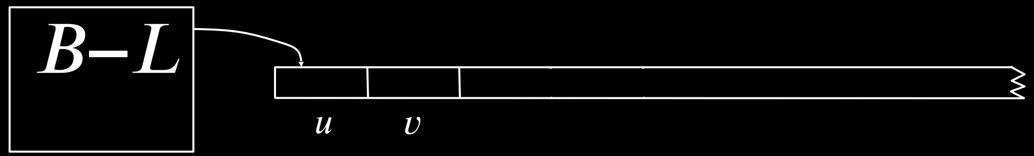

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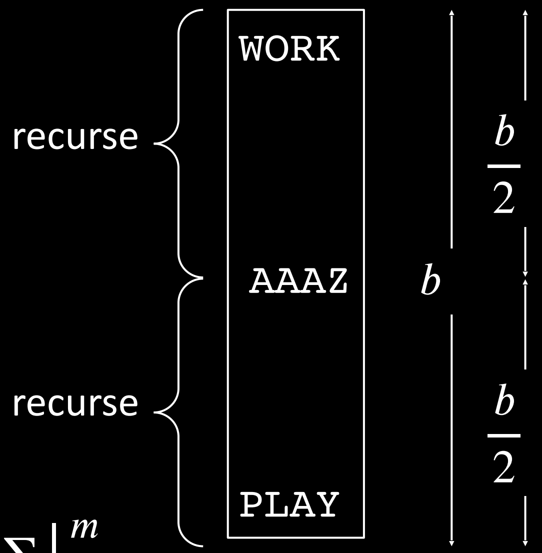

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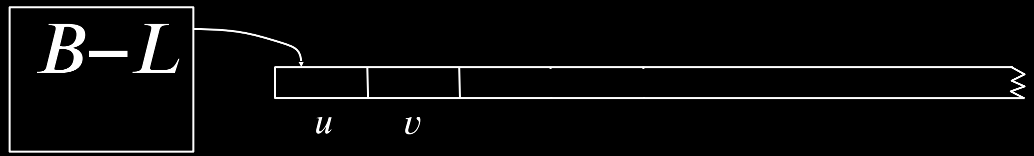

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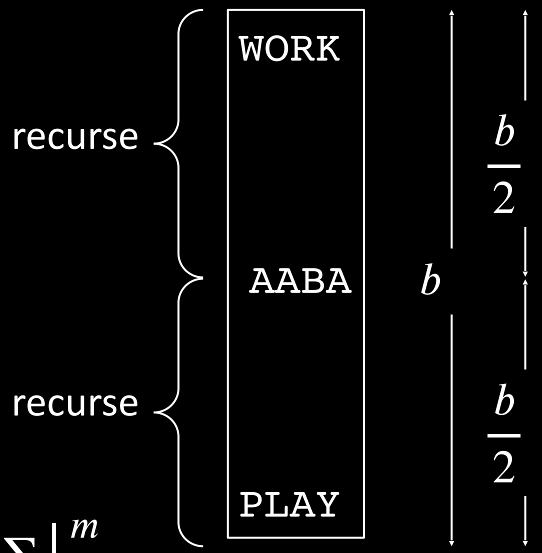

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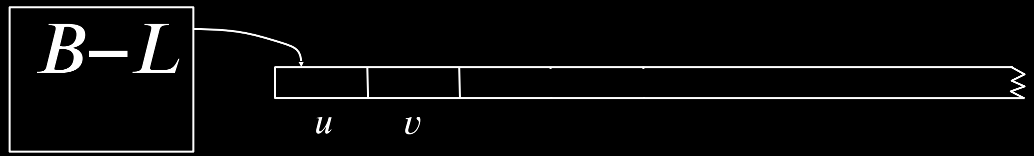

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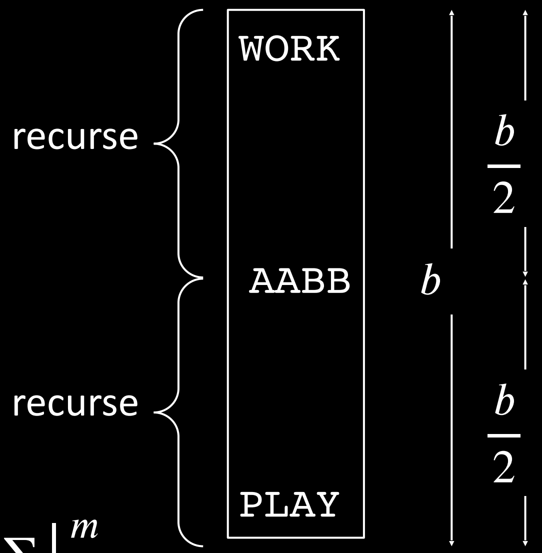

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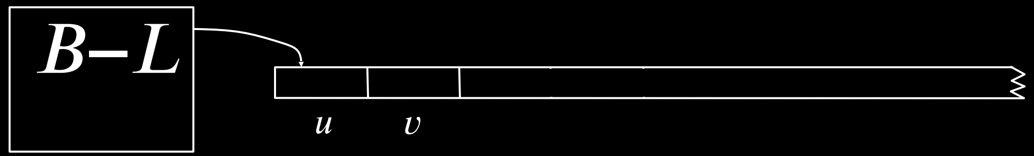

Review: $LADDER$DFA $\in$ PSPACE

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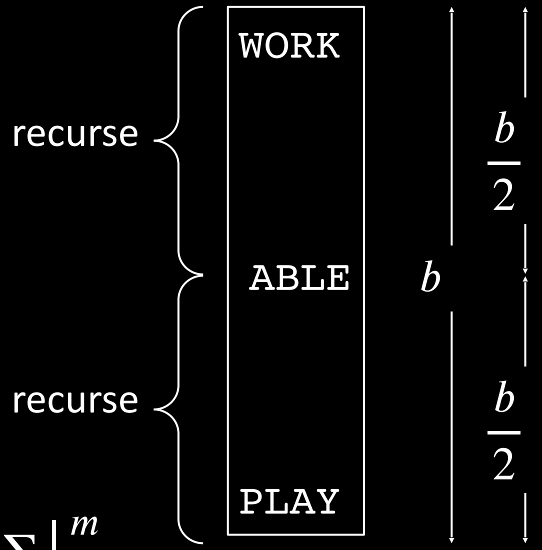

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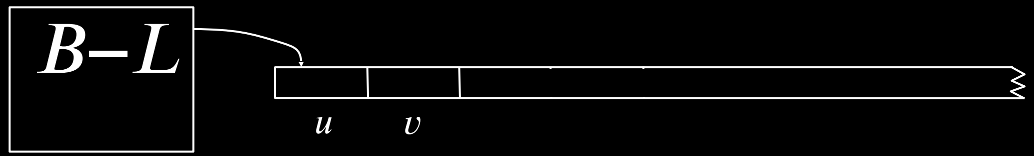

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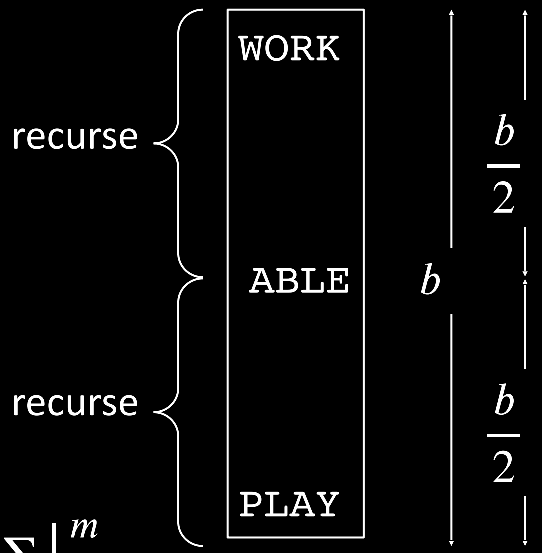

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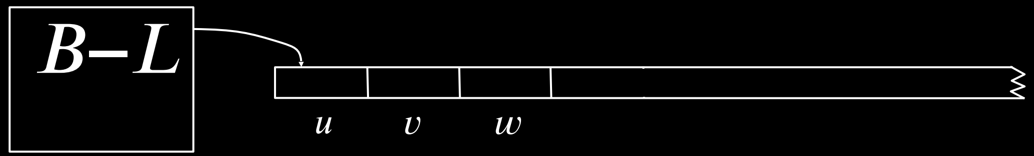

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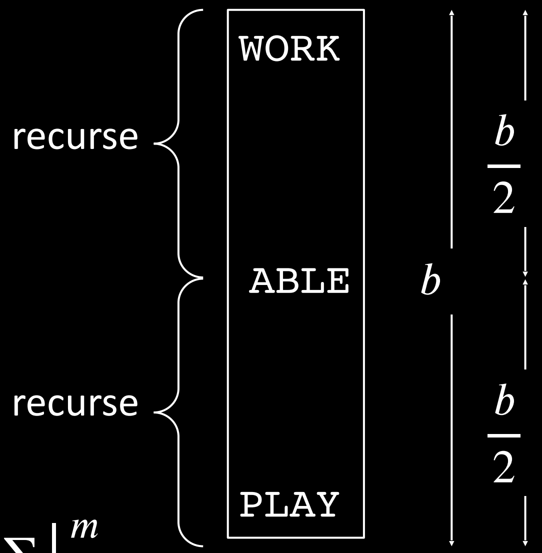

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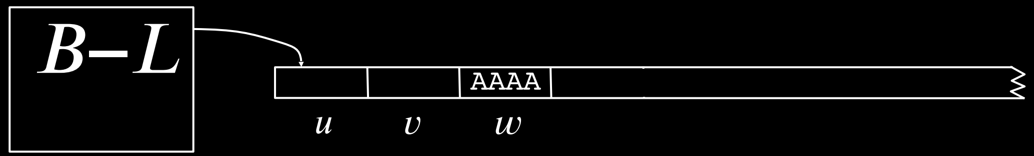

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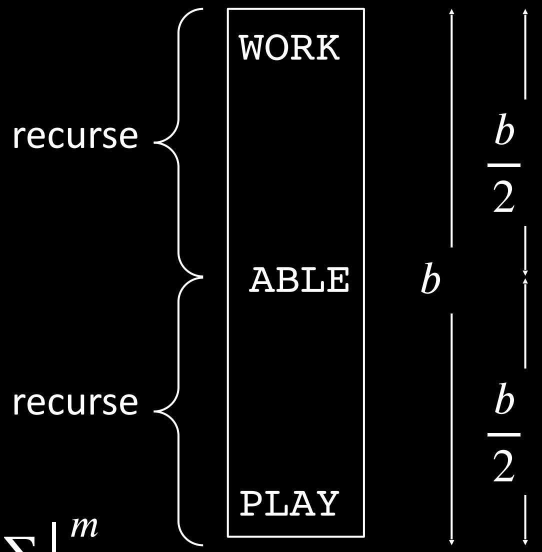

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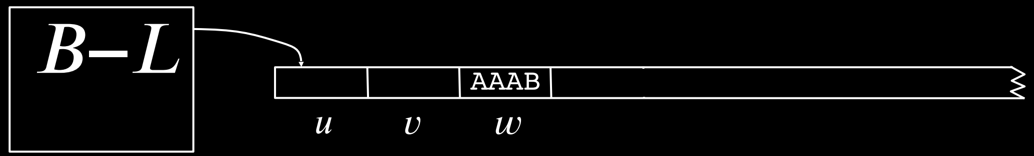

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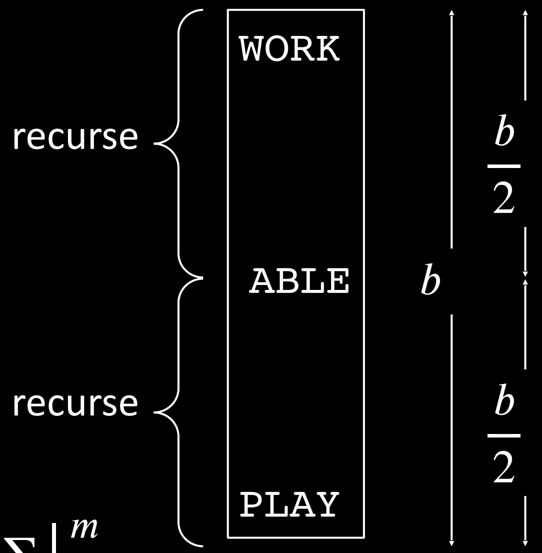

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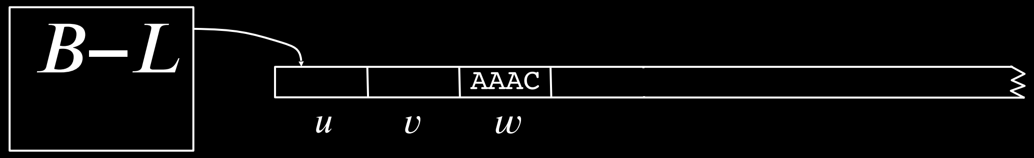

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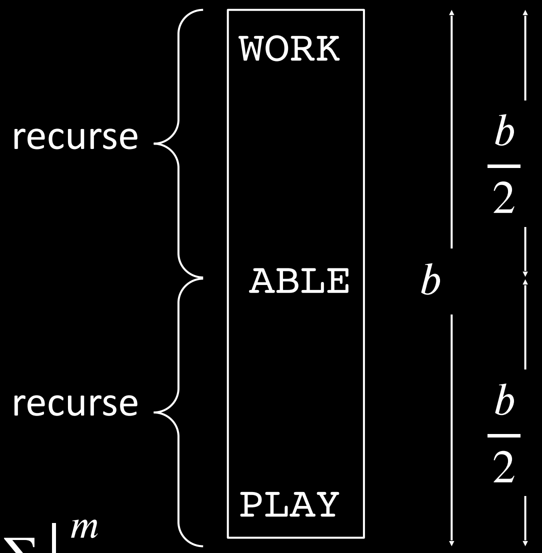

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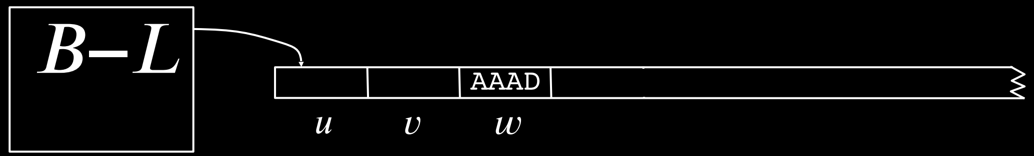

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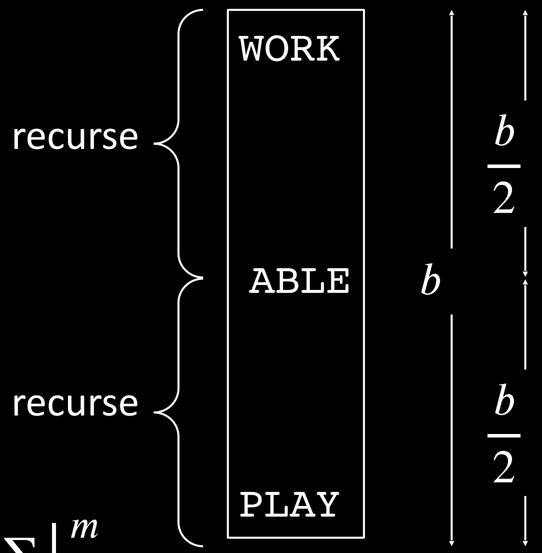

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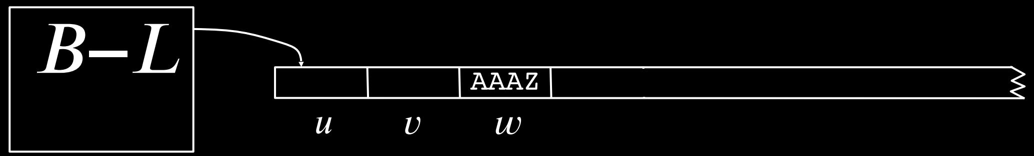

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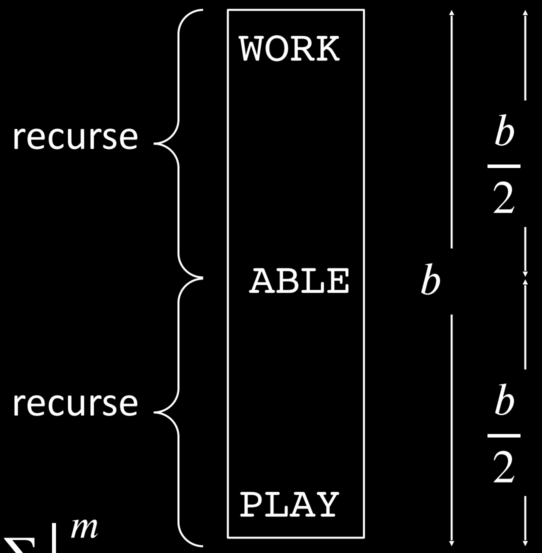

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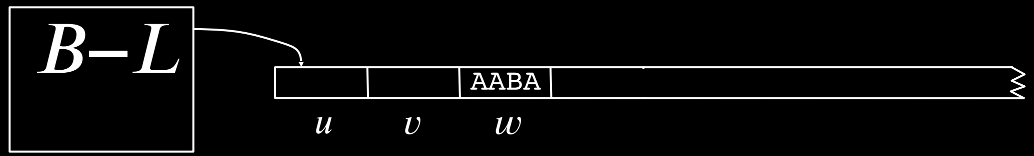

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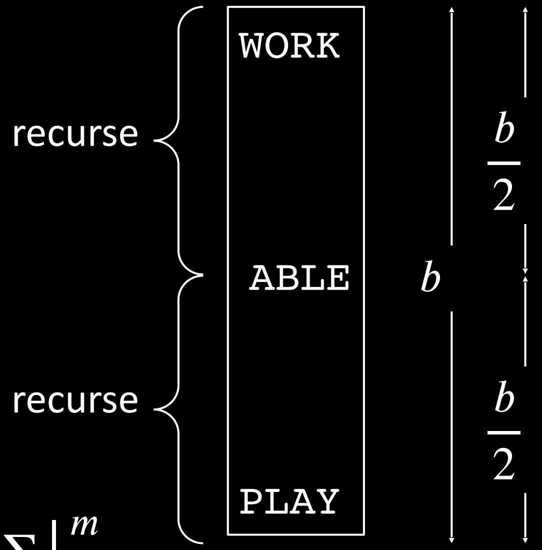

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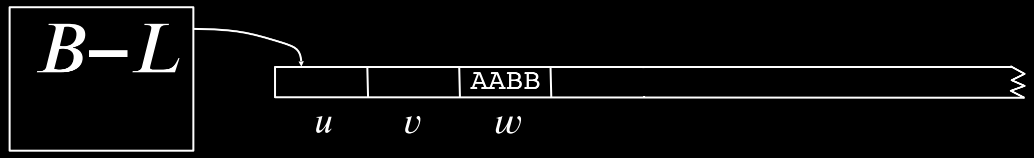

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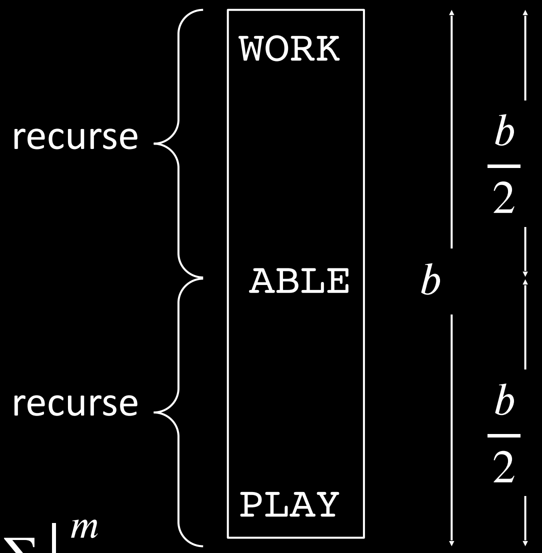

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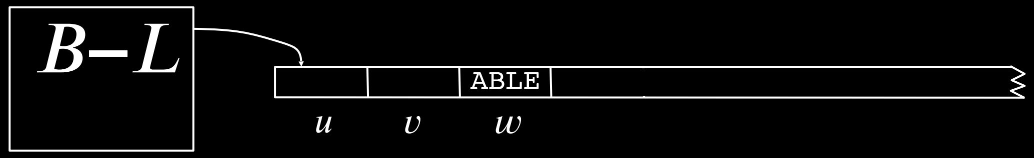

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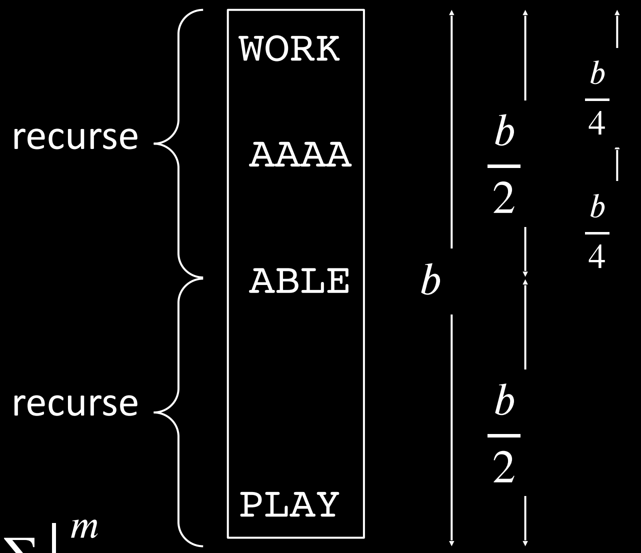

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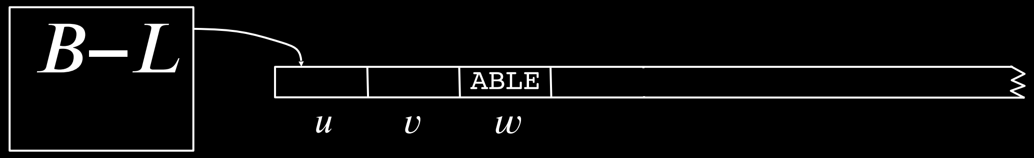

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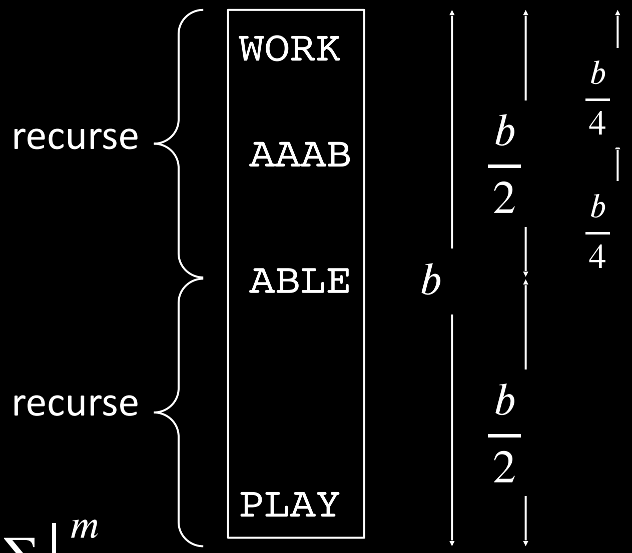

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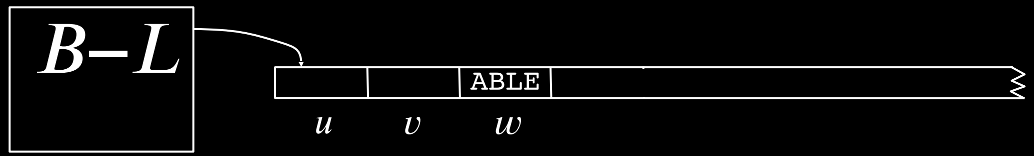

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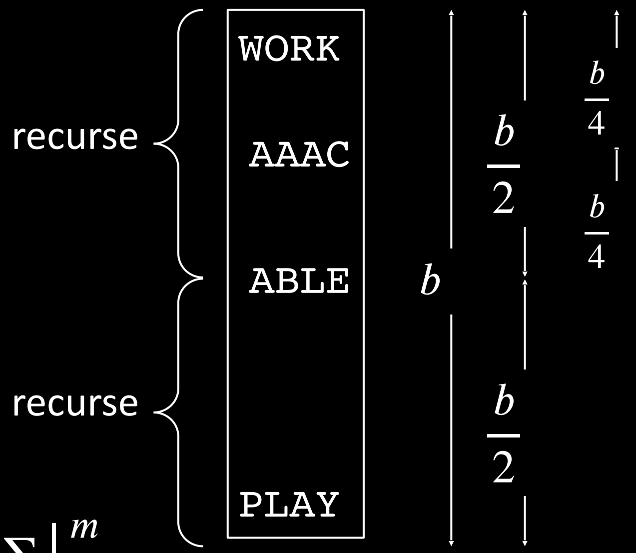

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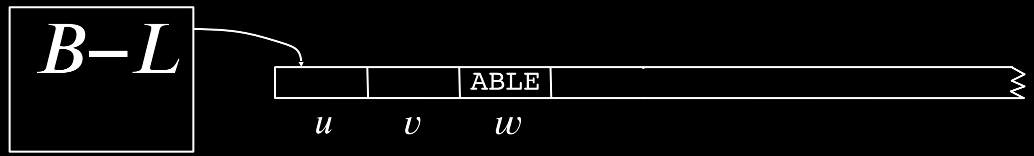

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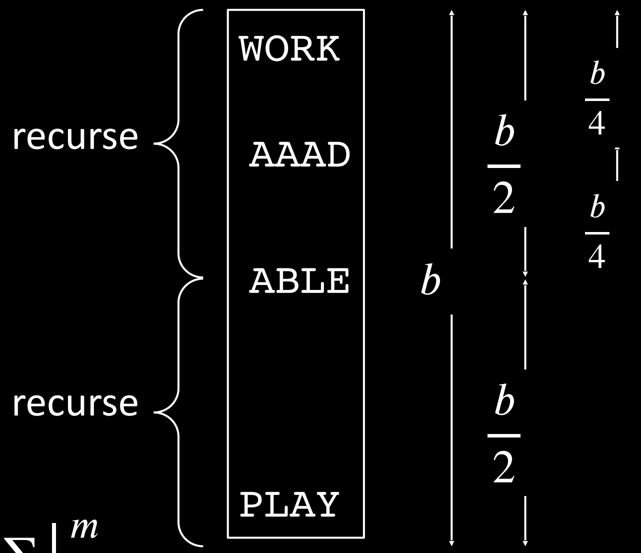

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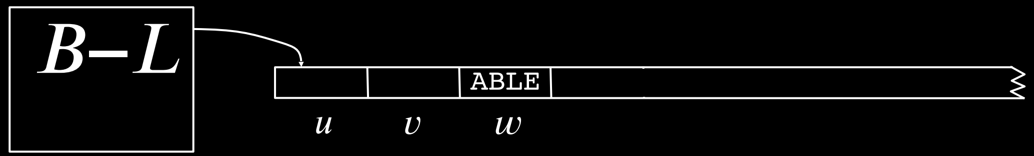

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Theorem: $LADDER$DFA $\in$ SPACE(n^2)

Proof: Write $u \xrightarrow{b} v$ if there's a ladder from u to v of length $\leq b$.
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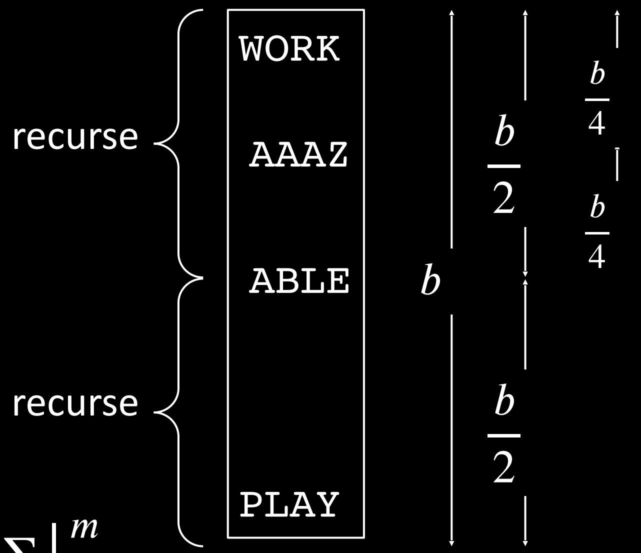

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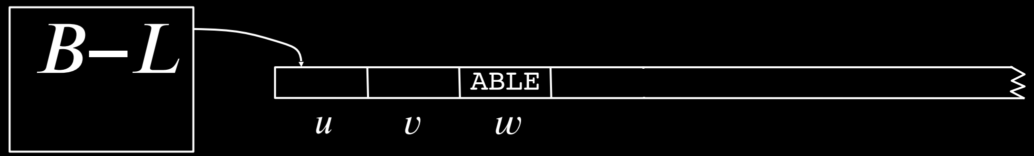
Space analysis:



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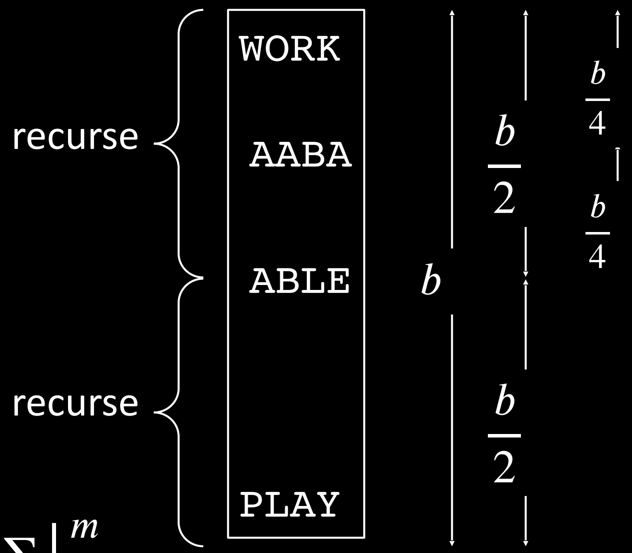

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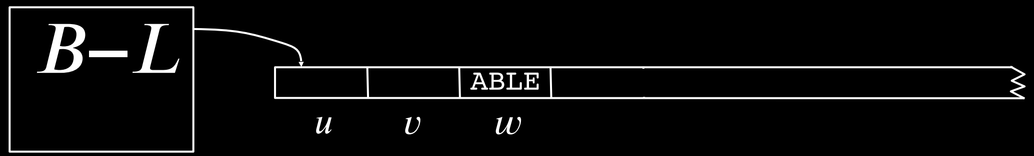

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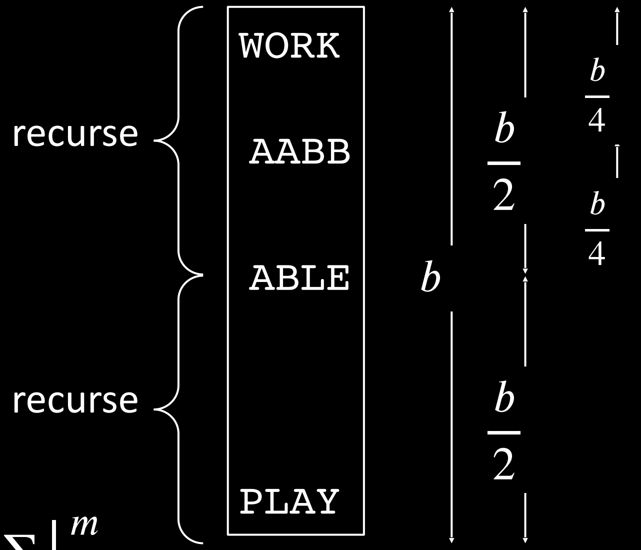

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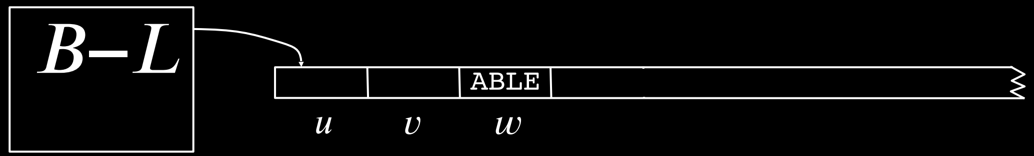

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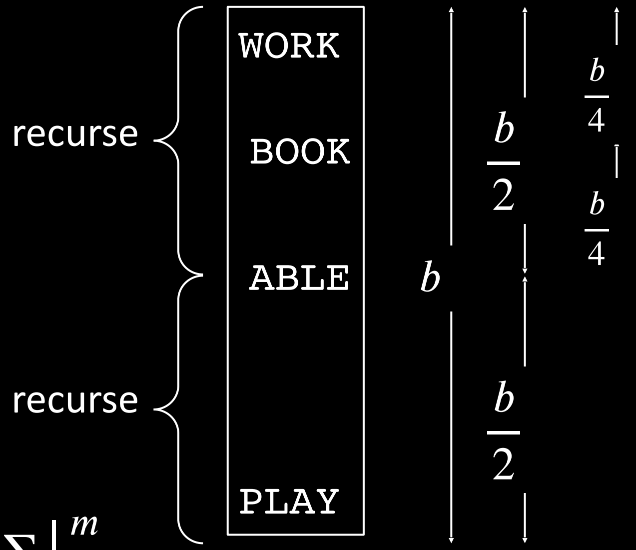

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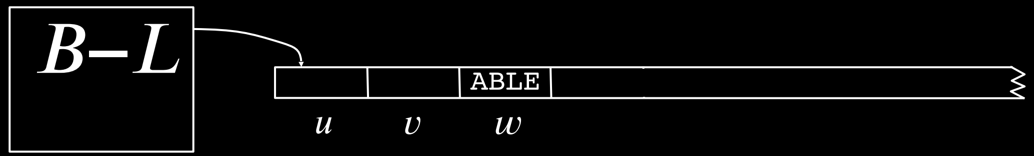

Review: $LADDER_{DFA} \in$ PSPACE

Theorem: $LADDER_{DFA} \in$ SPACE(n^2)

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Here's a recursive procedure to solve the bounded DFA ladder problem:

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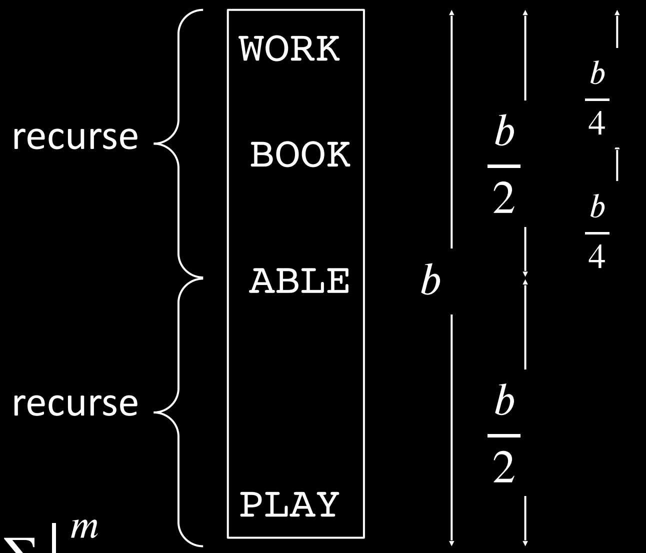

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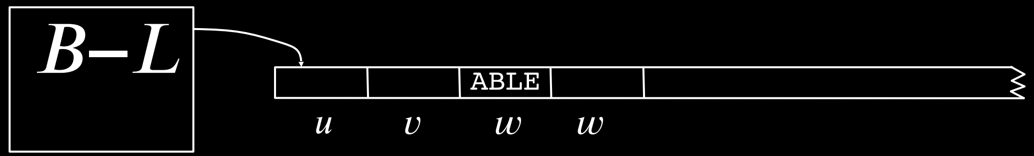

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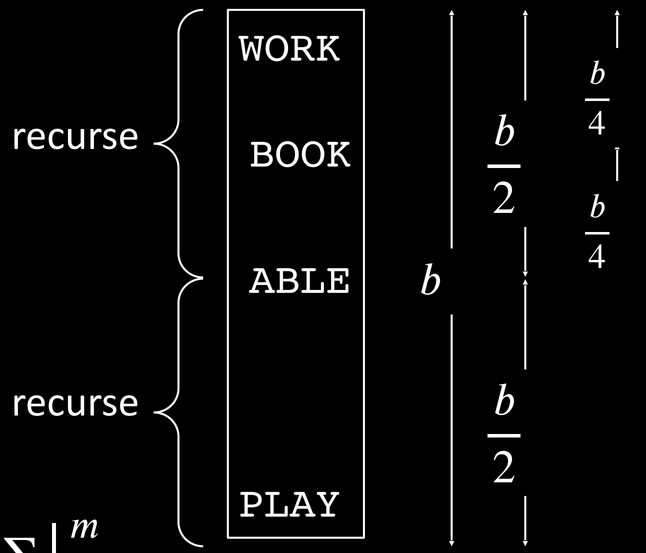

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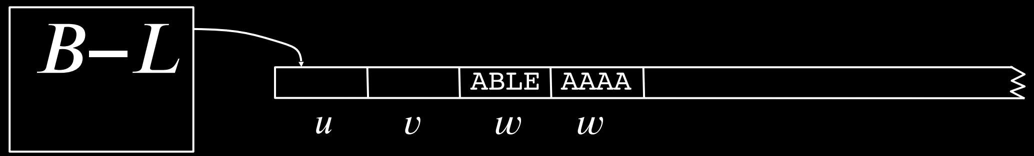

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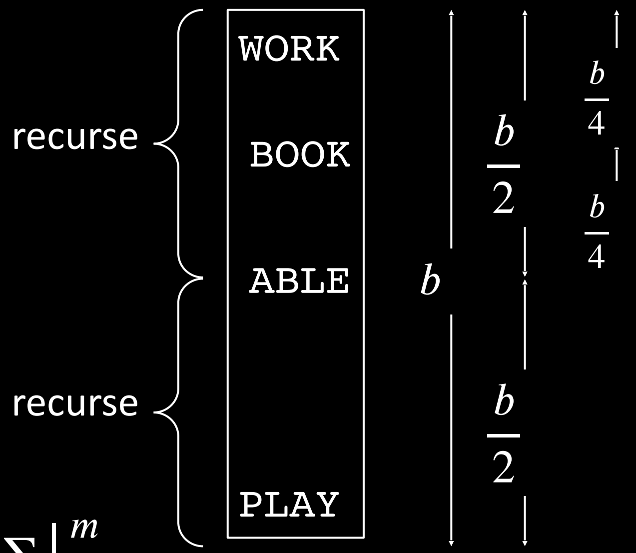

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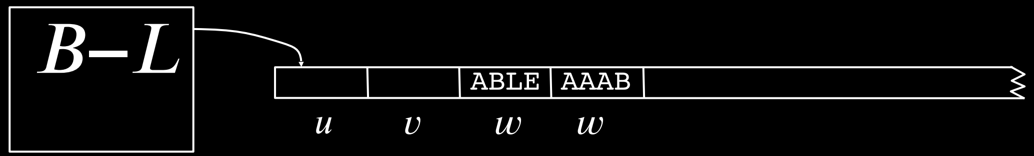

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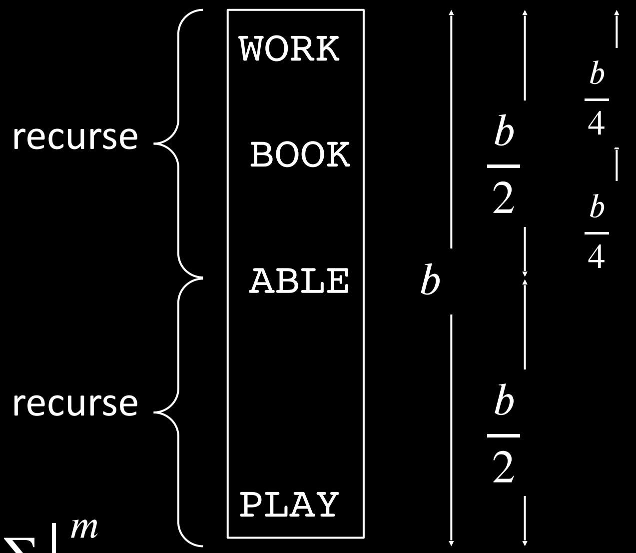

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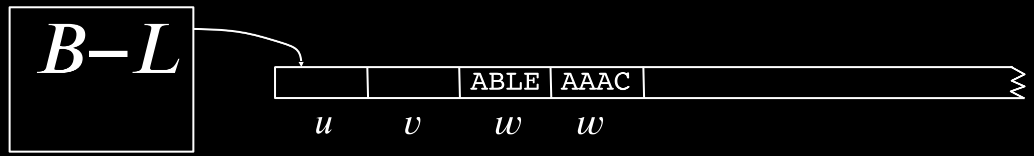

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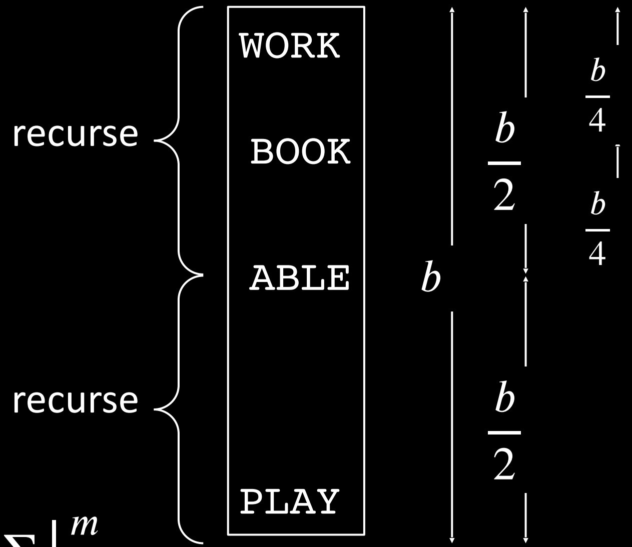

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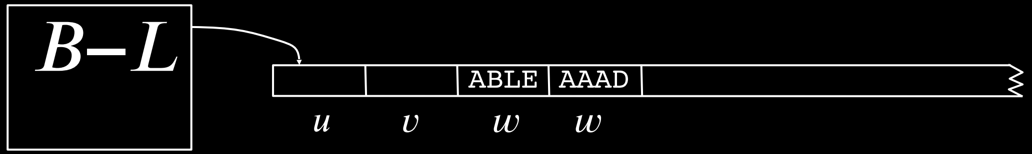

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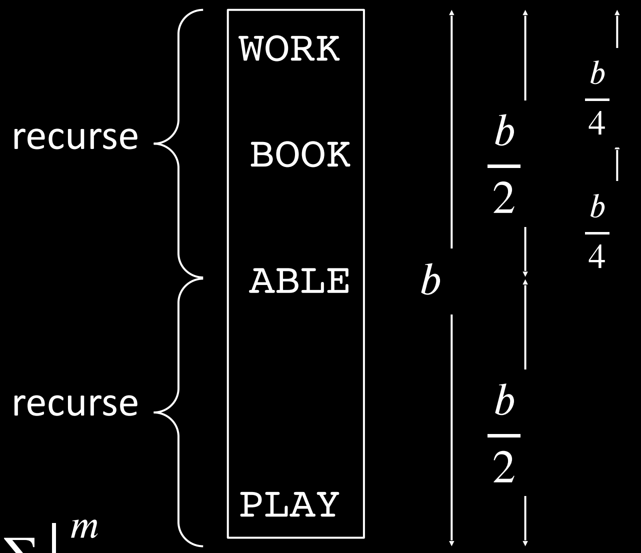

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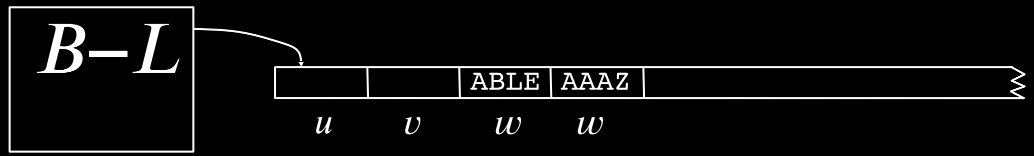

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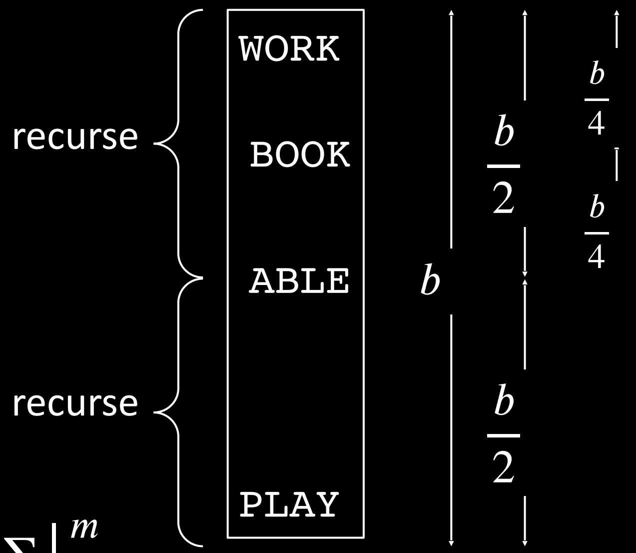

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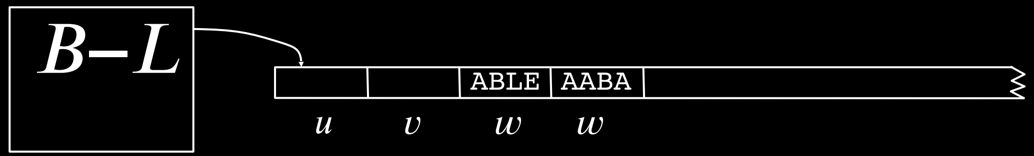

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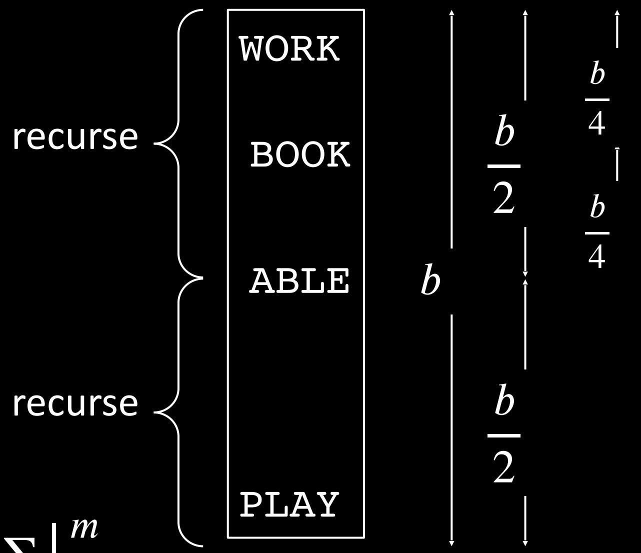

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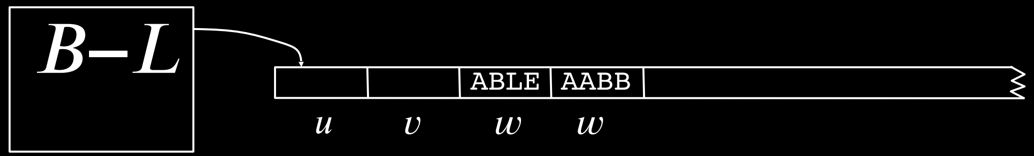

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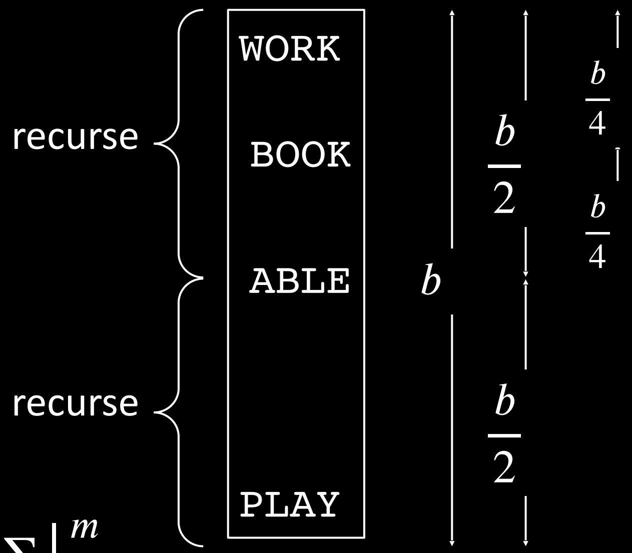

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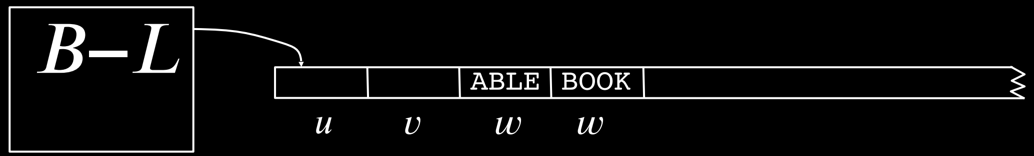

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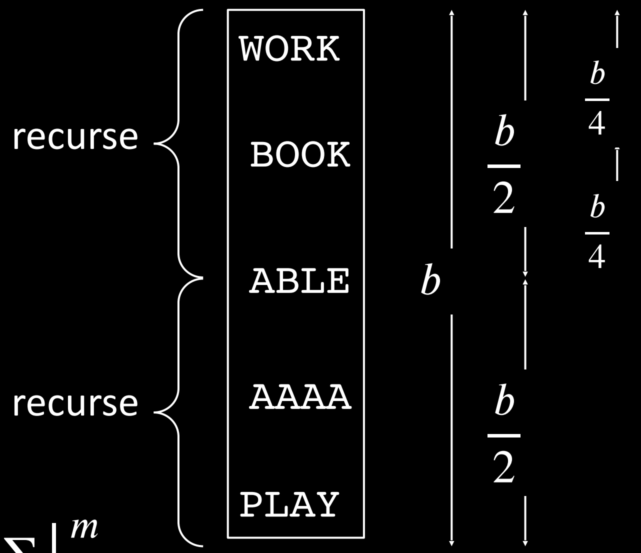

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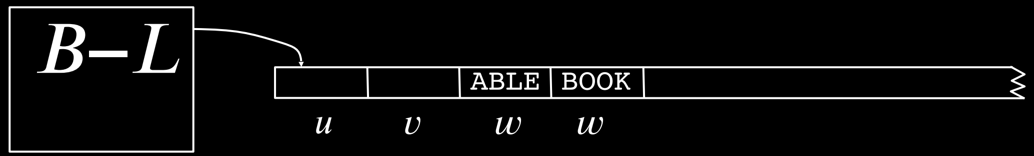

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4. *Accept* both accept.
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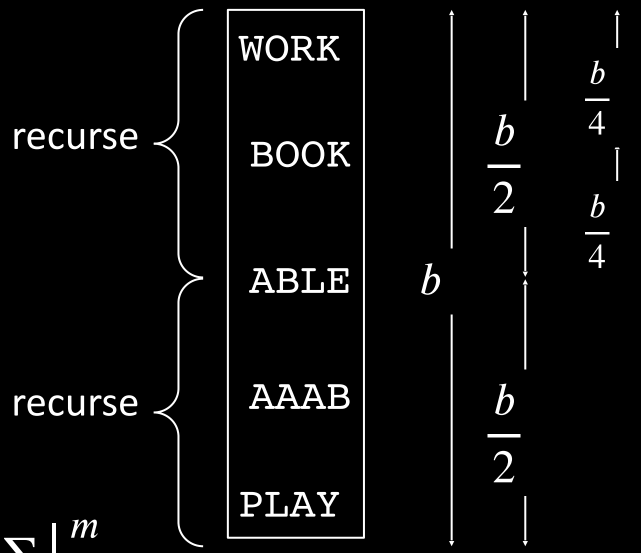
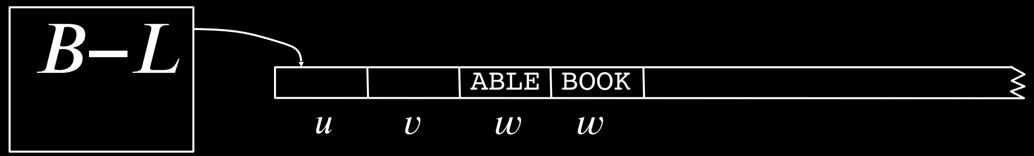


Test $\langle B, u, v\rangle \in LADDER$DFA with $B\text{-}L$ procedure on input $\langle B, u, v, t\rangle$ for $t = |\Sigma|^m$

Space analysis:



Review: $LADDER$DFA $\in$ PSPACE

Theorem: $LADDER$DFA $\in$ SPACE(n^2)

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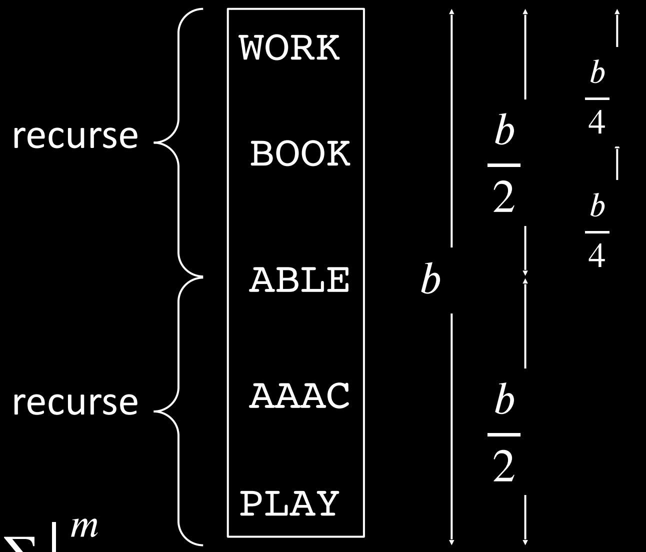

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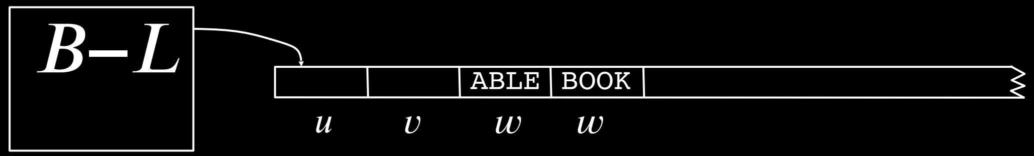

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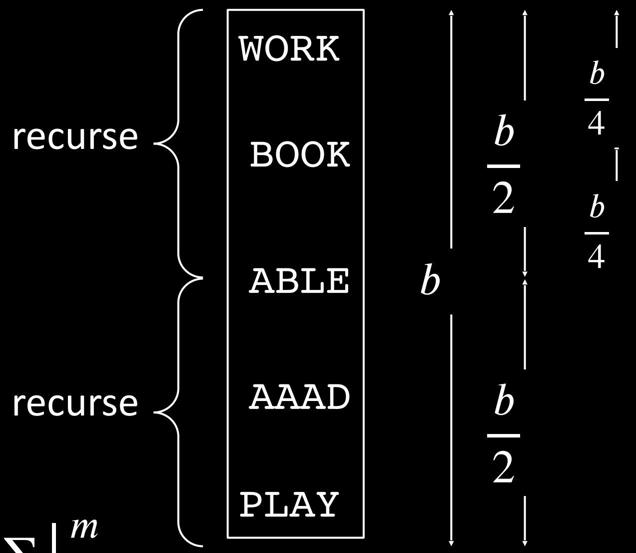

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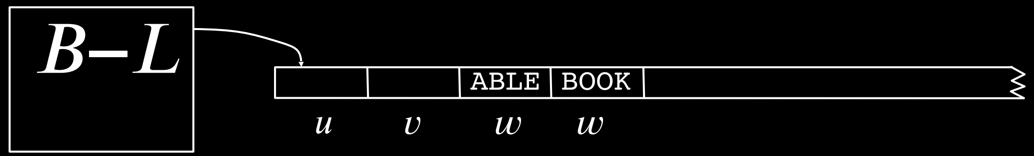

Review: $LADDER_{DFA} \in$ PSPACE

Theorem: $LADDER_{DFA} \in$ SPACE(n^2)

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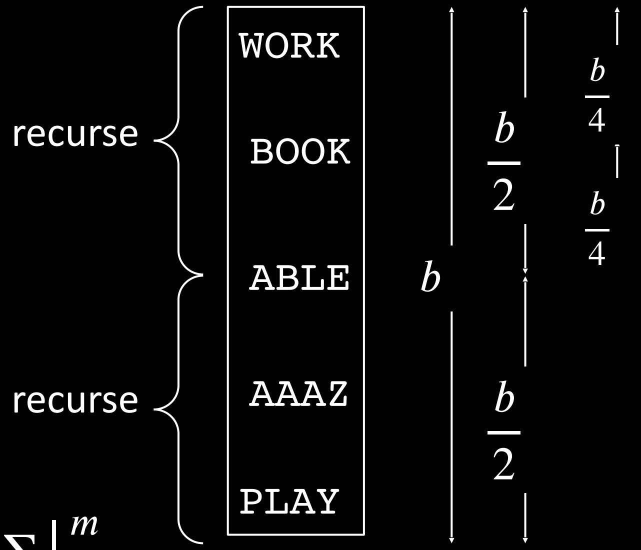

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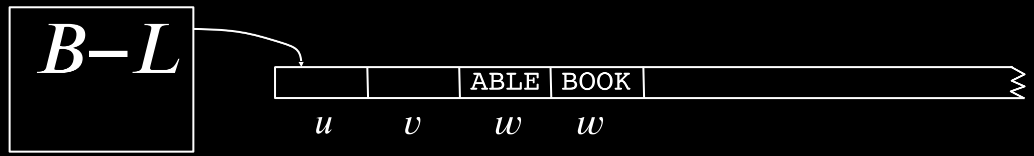

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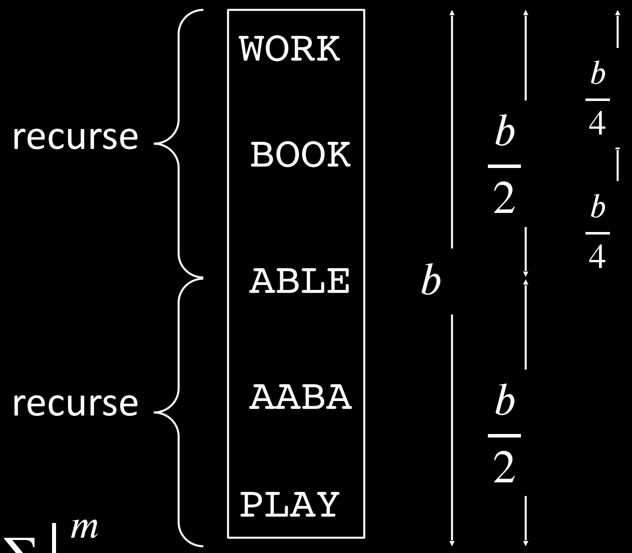

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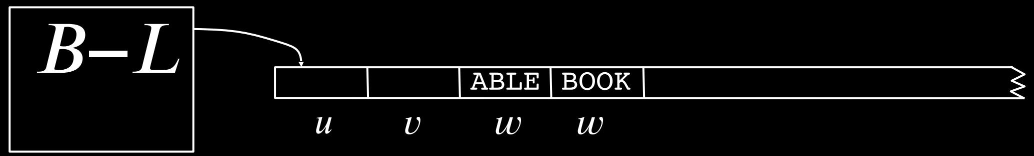

Review: $LADDER_{\text{DFA}} \in$ PSPACE

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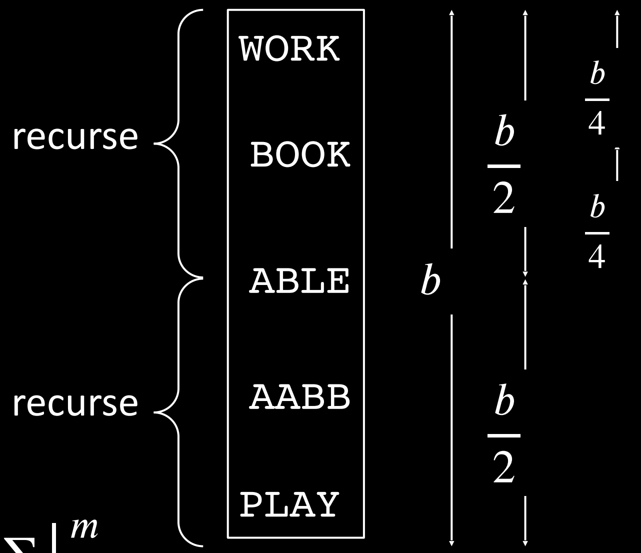

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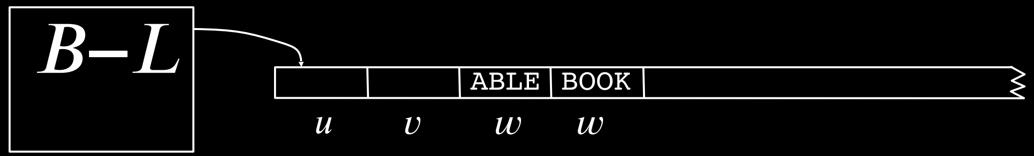

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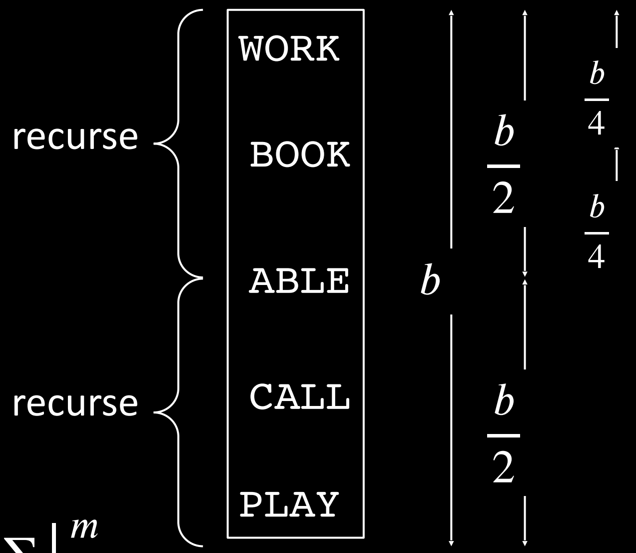

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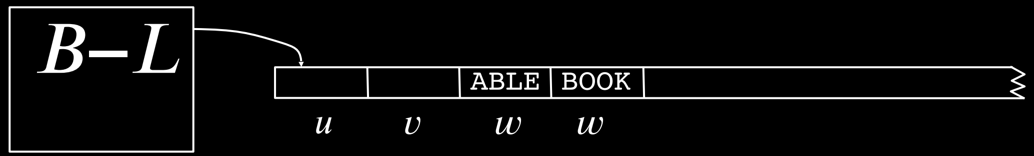

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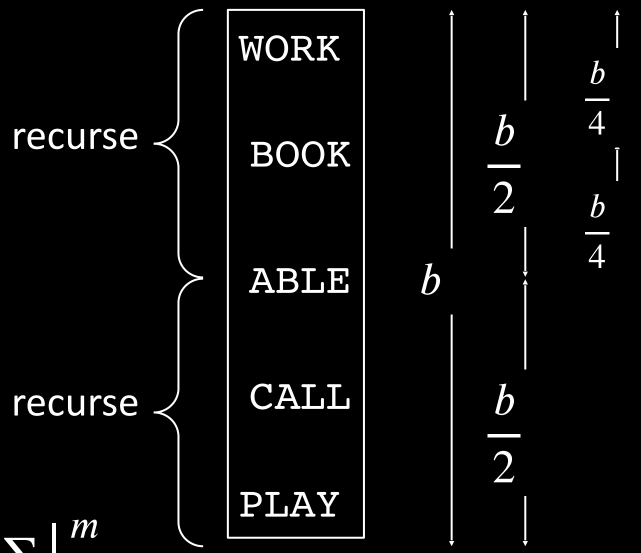

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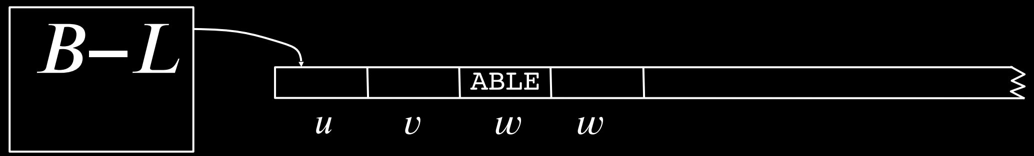

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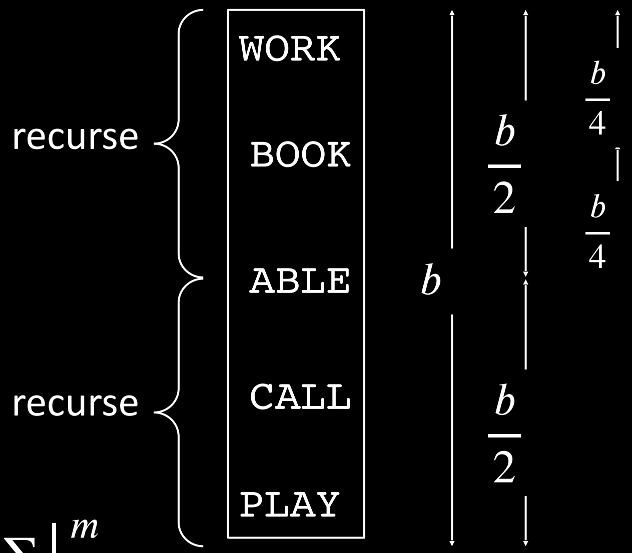

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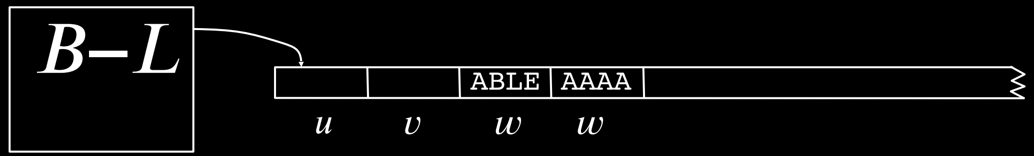

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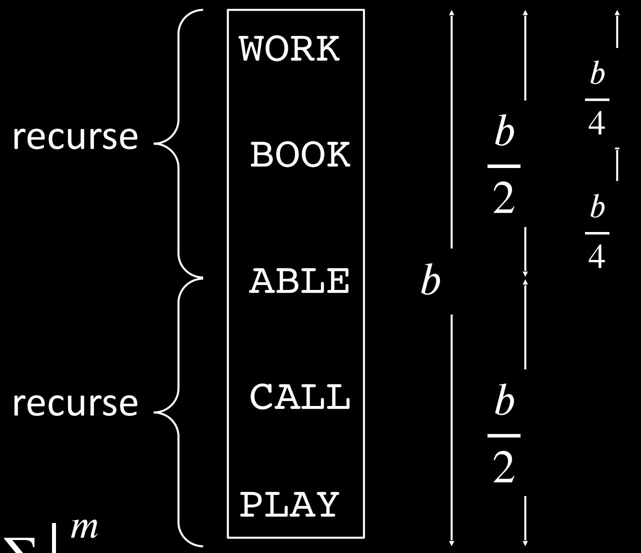

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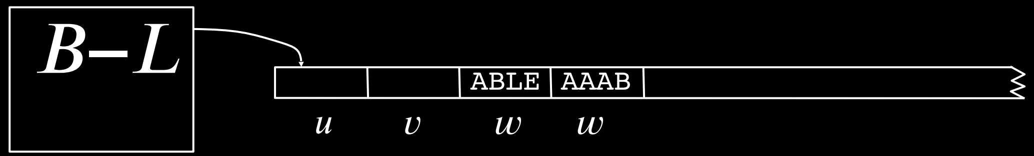

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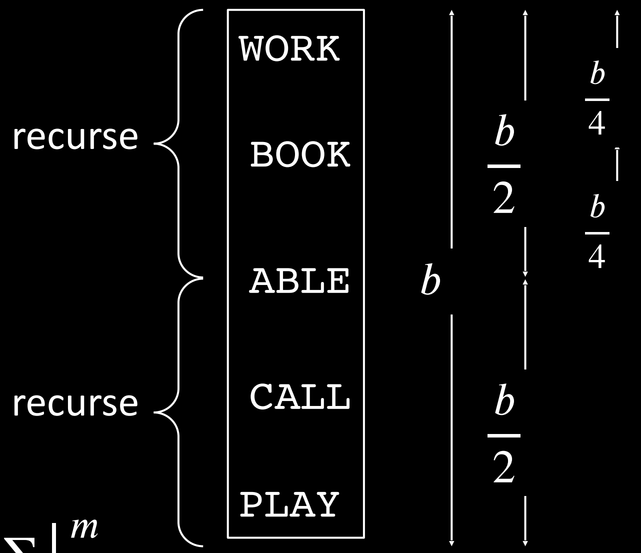

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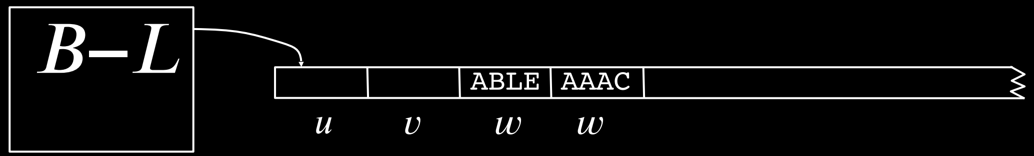

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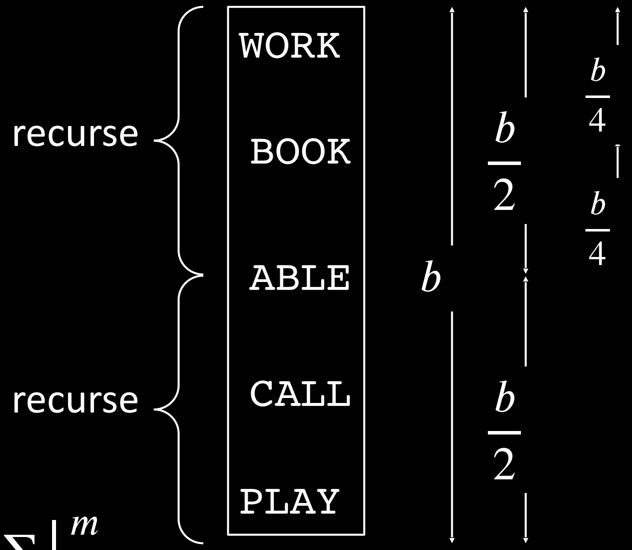

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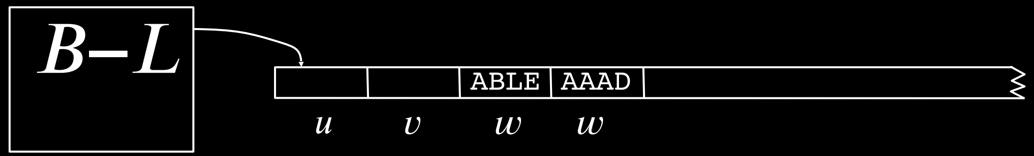

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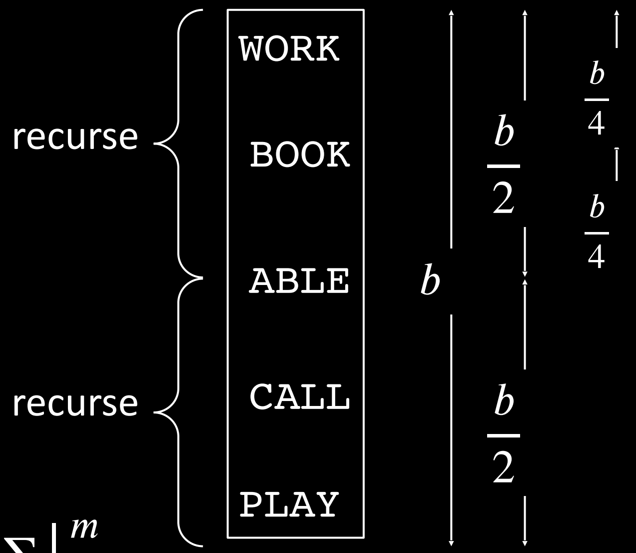

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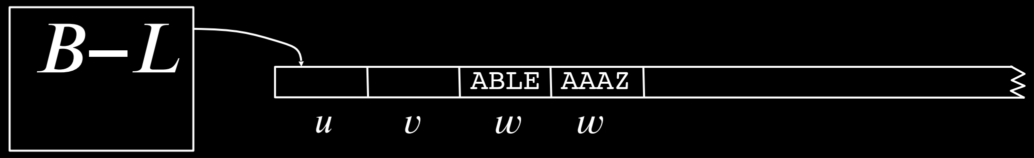

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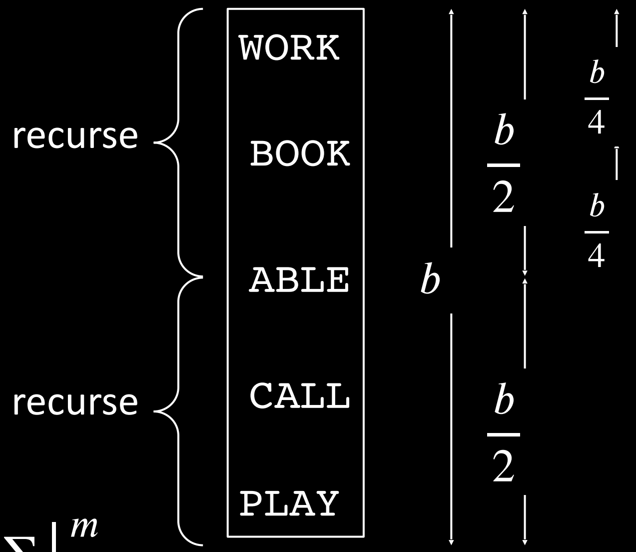

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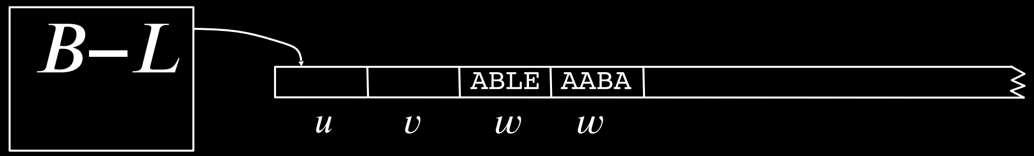

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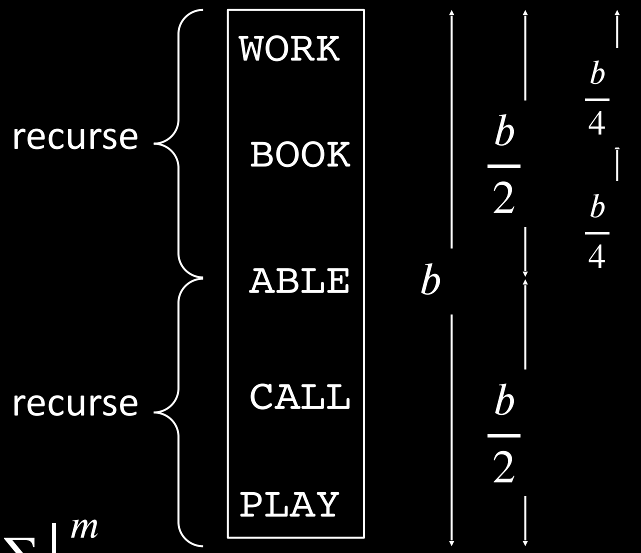

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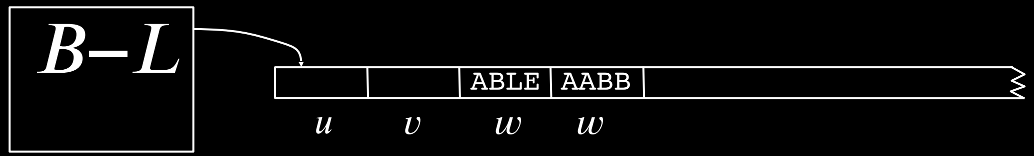

Review: $LADDER$DFA $\in$ PSPACE

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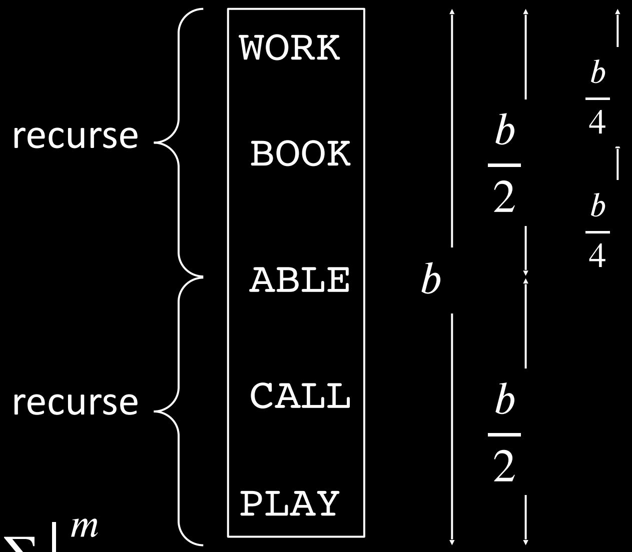

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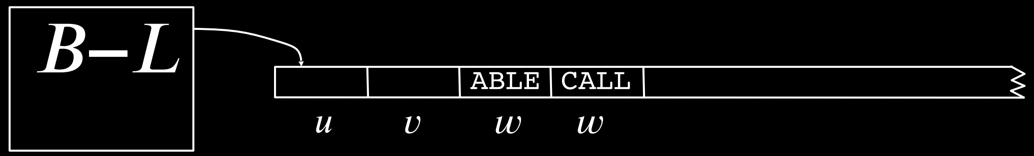

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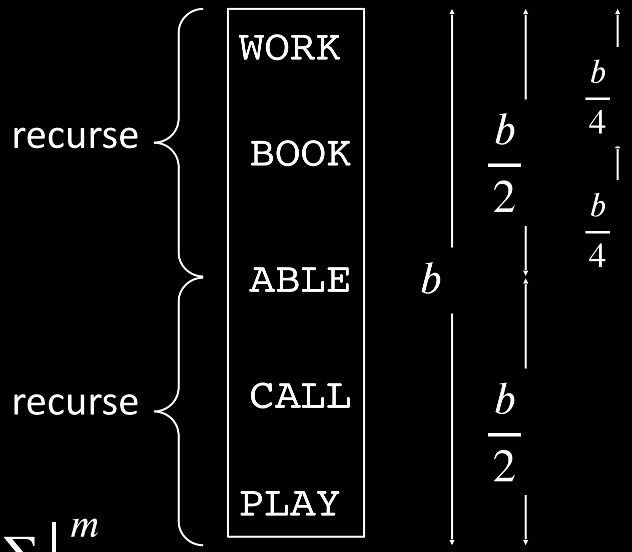

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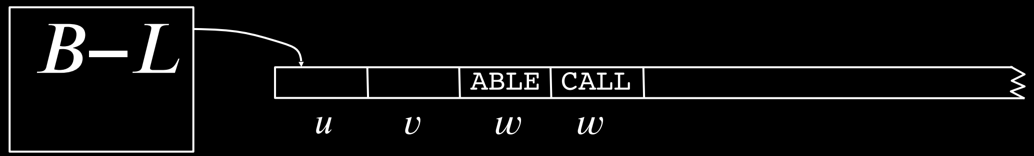

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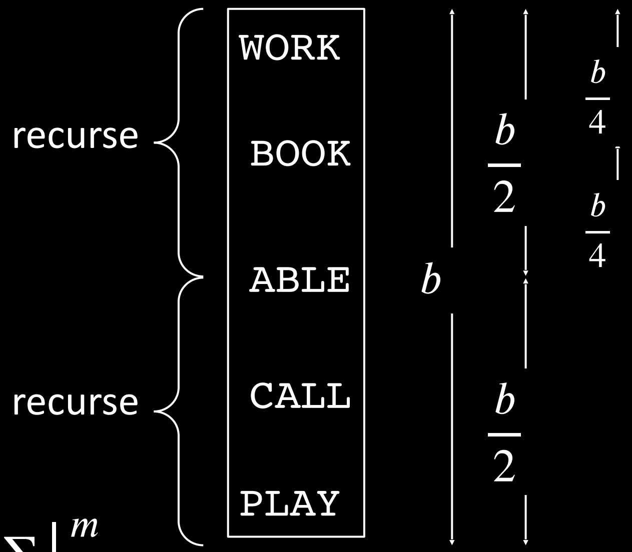

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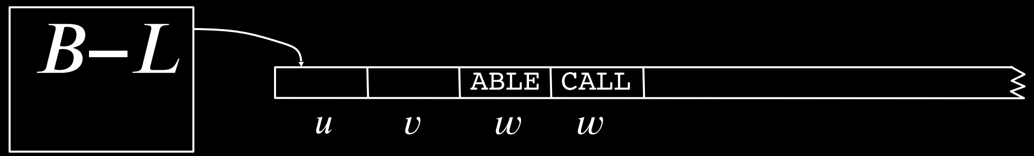

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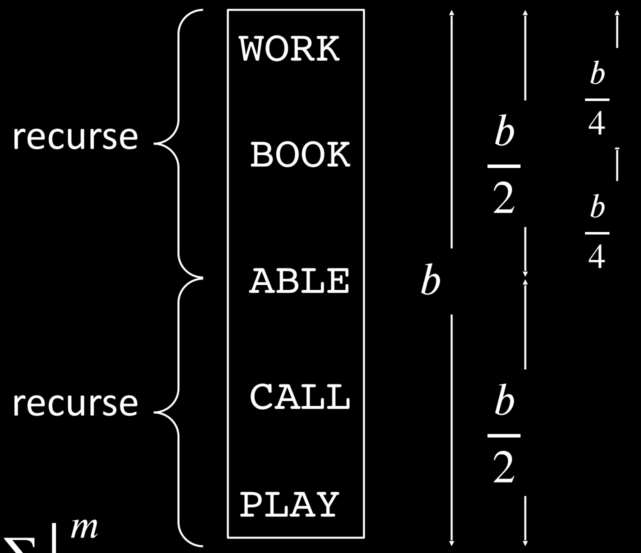

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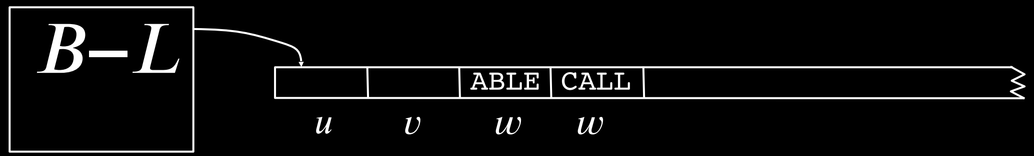

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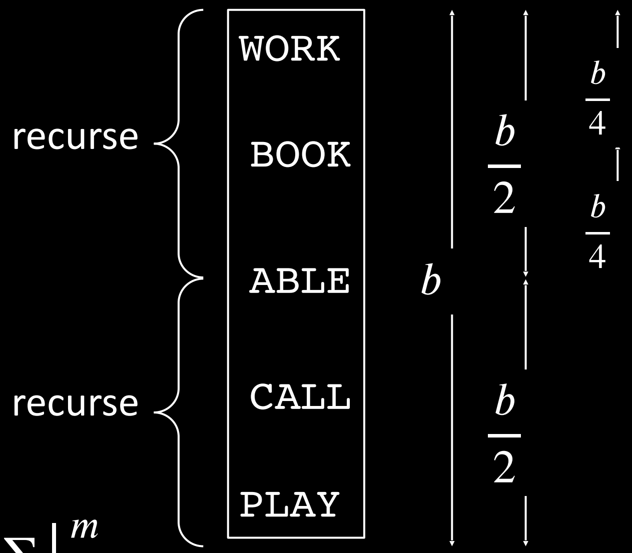

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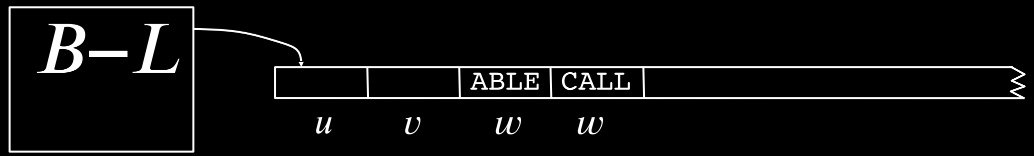

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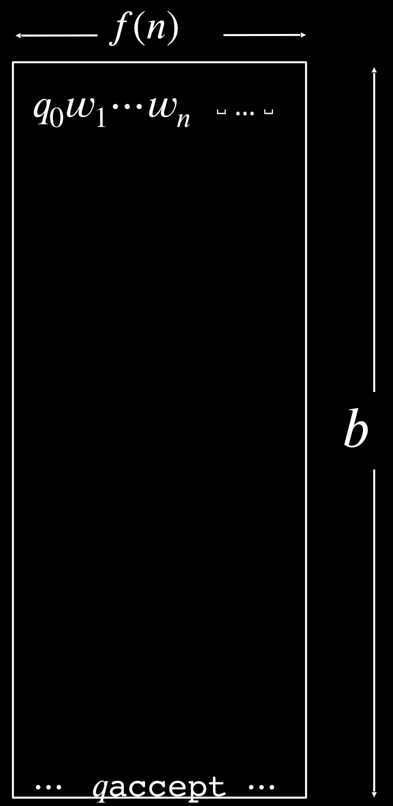

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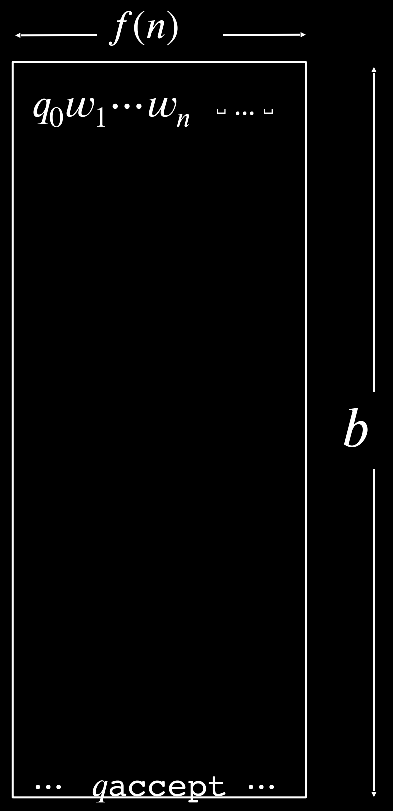

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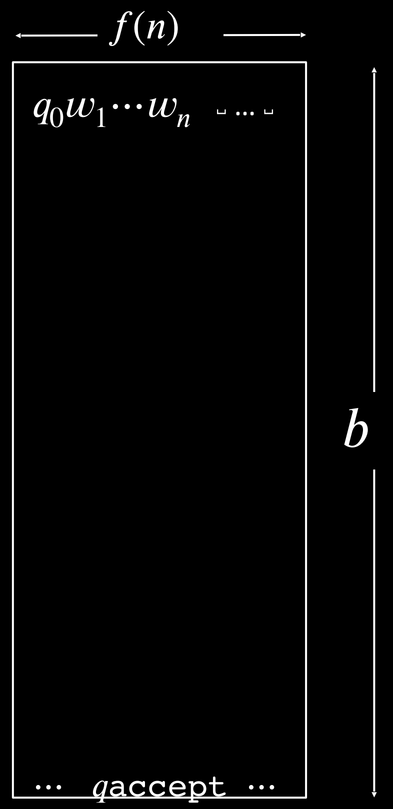

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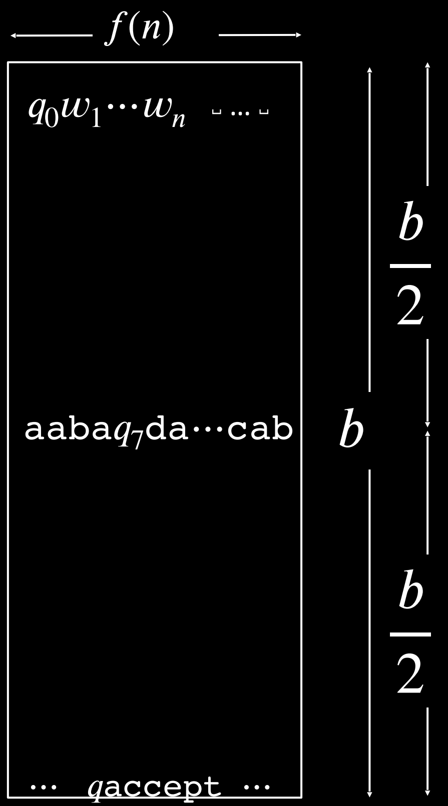

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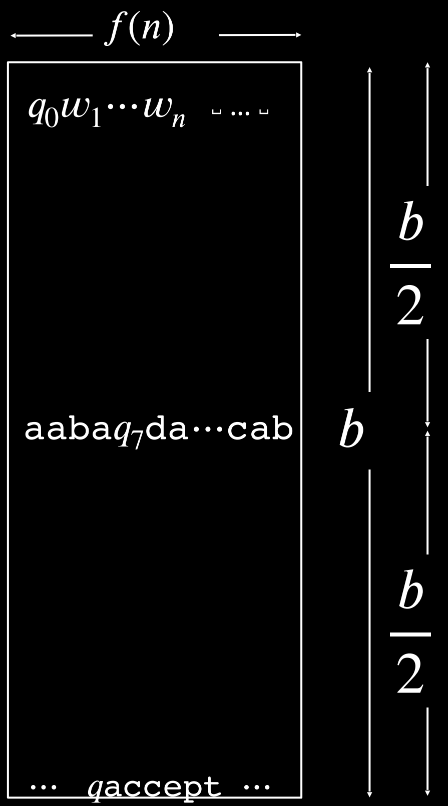

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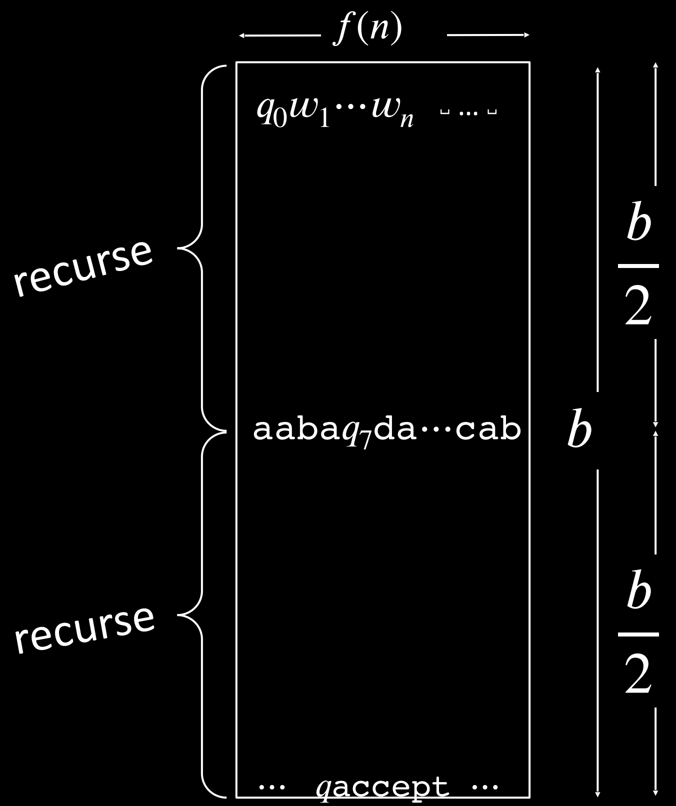

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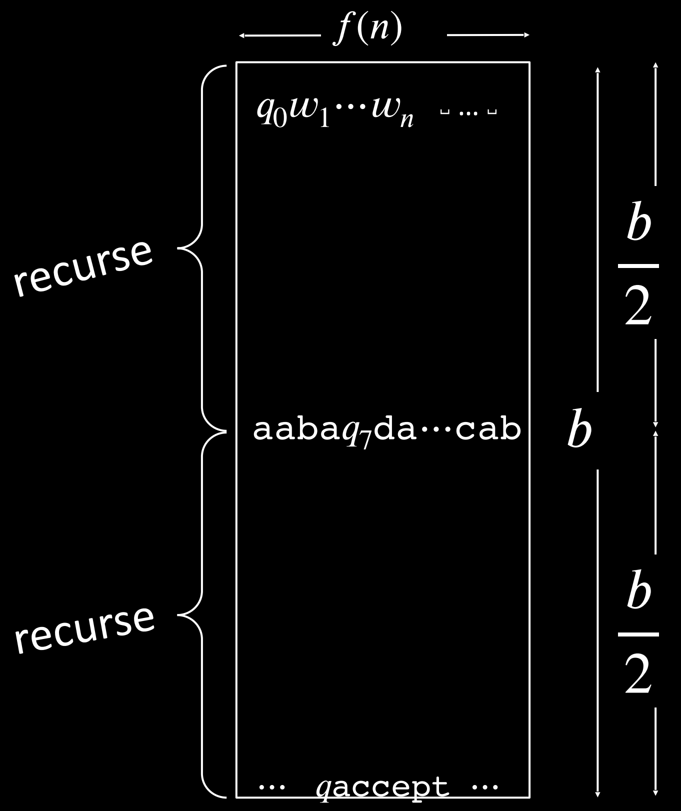

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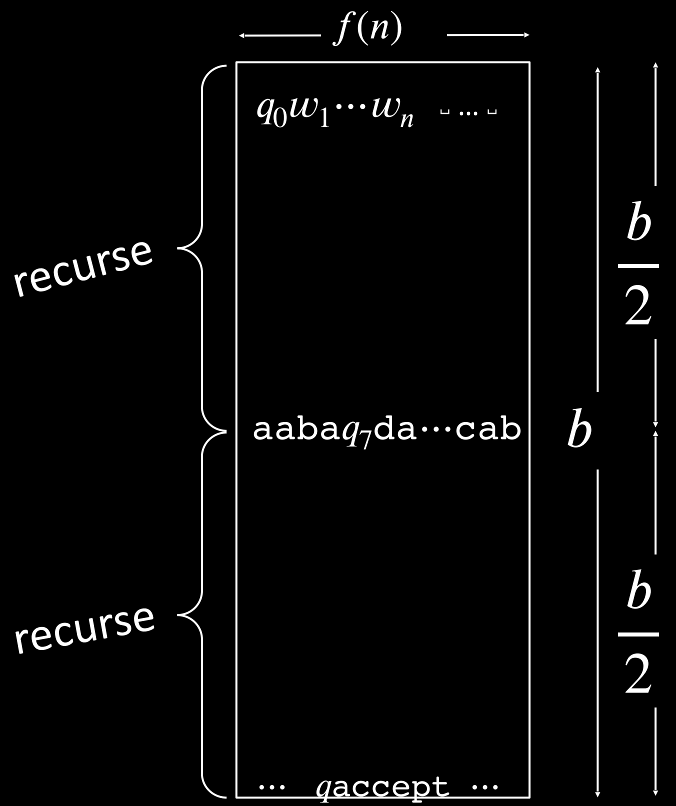

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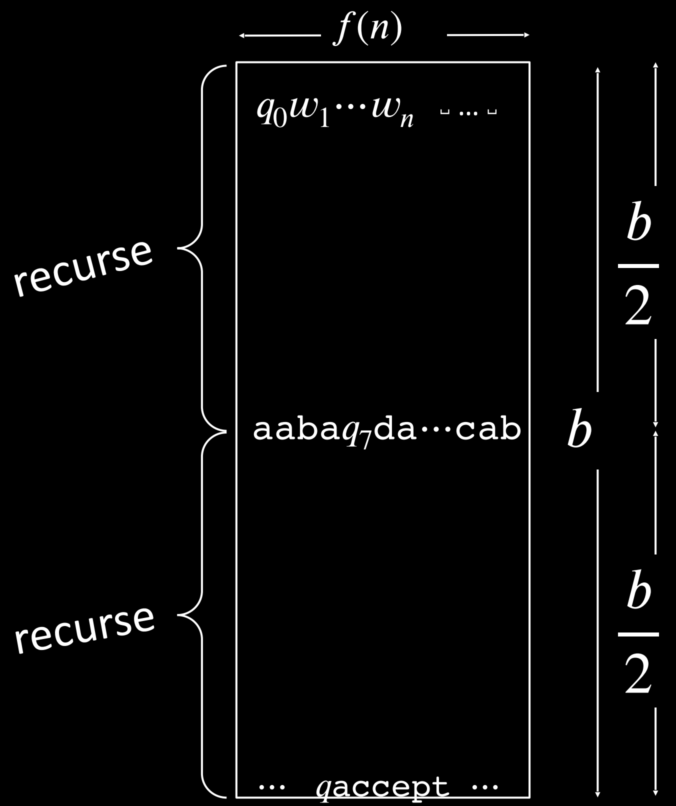

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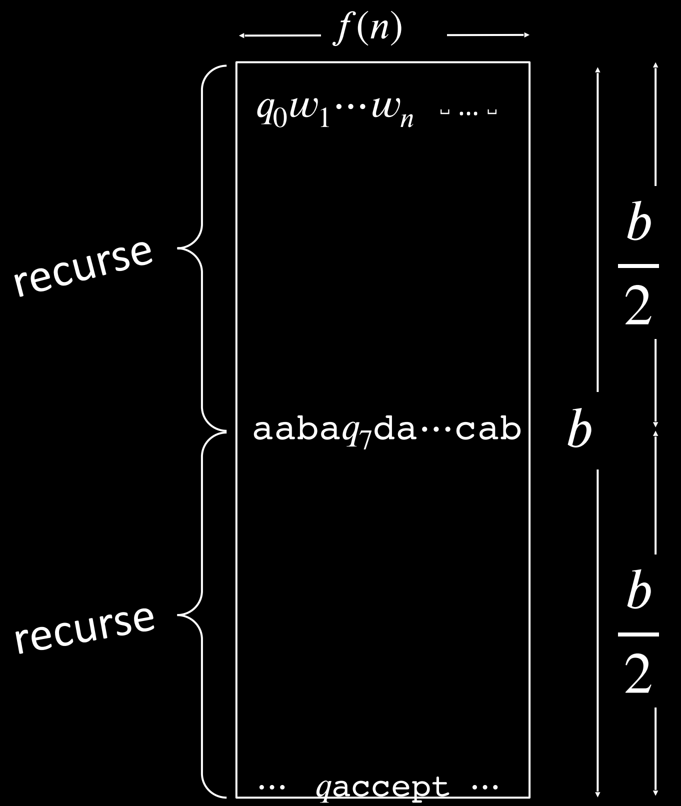

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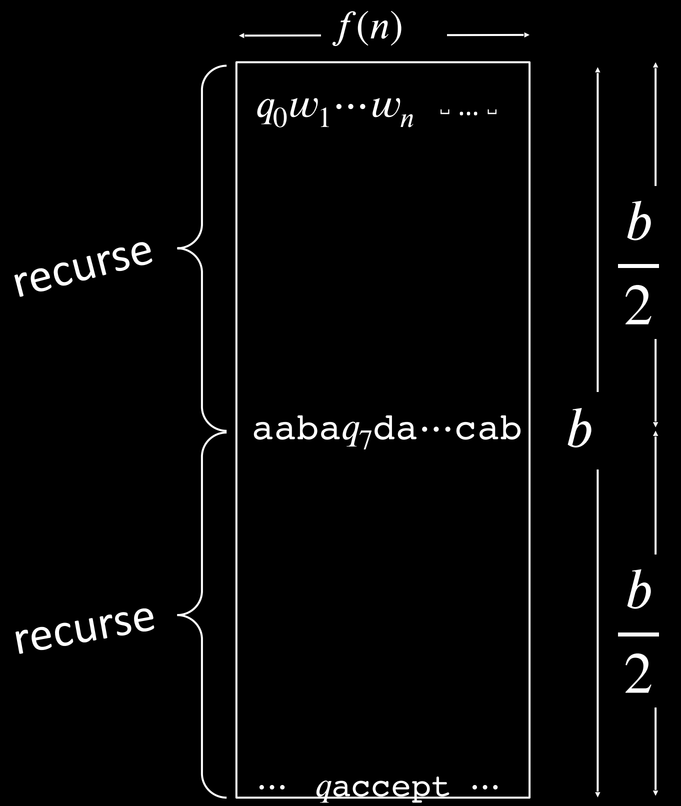


PSPACE = NPSPACE

Savitch's Theorem: For $f(n) \geq n$, $\text{NSPACE}\big(f(n)\big) \subseteq \text{SPACE}\big(f^2(n)\big)$

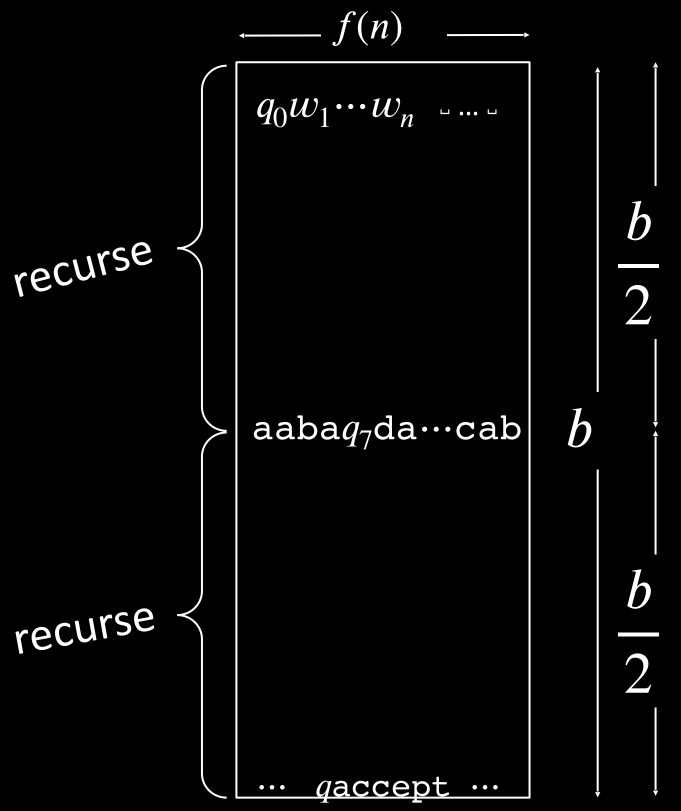
Proof: Convert NTM N to equivalent TM M, only squaring the space used.

For configurations c_i and c_j of N, write $c_i \xrightarrow{b} c_j$ if can get from c_i to c_j in $\leq b$ steps.

Give recursive algorithm to test $c_i \xrightarrow{b} c_j$:

M = "On input $c_i,\ c_j,\ b$ [goal is to check $c_i \xrightarrow{b} c_j$]

1. If $b = 1$, check directly by using N's program and answer accordingly.
2. If $b > 1$, repeat for all configurations c_{mid} that use $f(n)$ space.
3. Recursively test $c_i \xrightarrow{b/2} c_{\text{mid}}$ and $c_{\text{mid}} \xrightarrow{b/2} c_j$
4. If both are true, *accept*. If not, continue.
5. *Reject* if haven't yet accepted."



Test if N accepts w by testing $c_{\text{start}} \xrightarrow{t} c_{\text{accept}}$ where t = number of configurations

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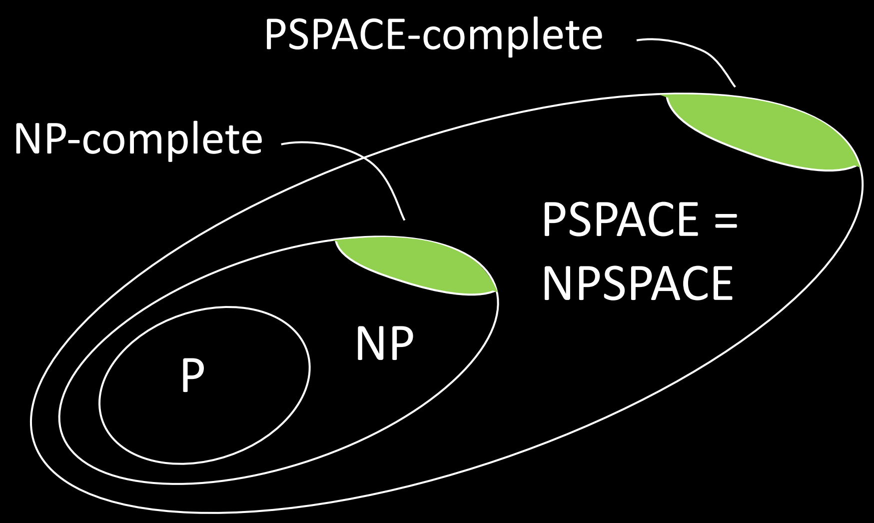

Think of complete problems as the “hardest” in their associated class.

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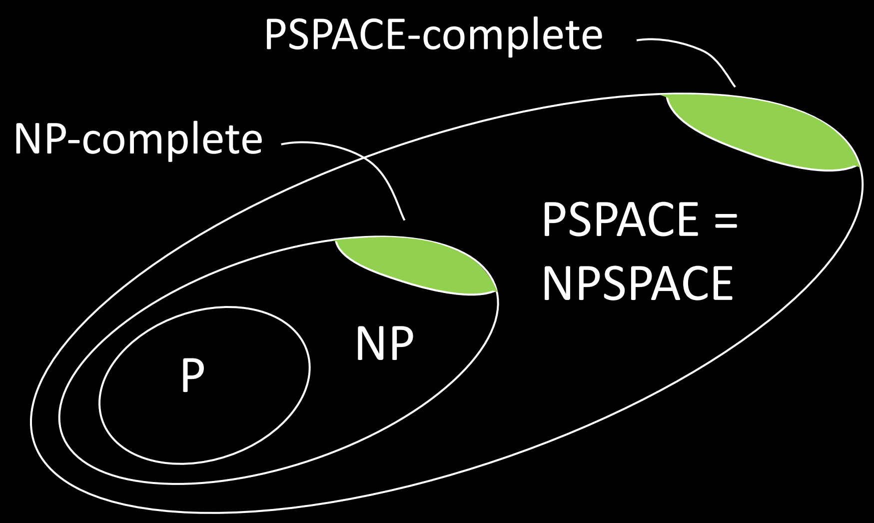

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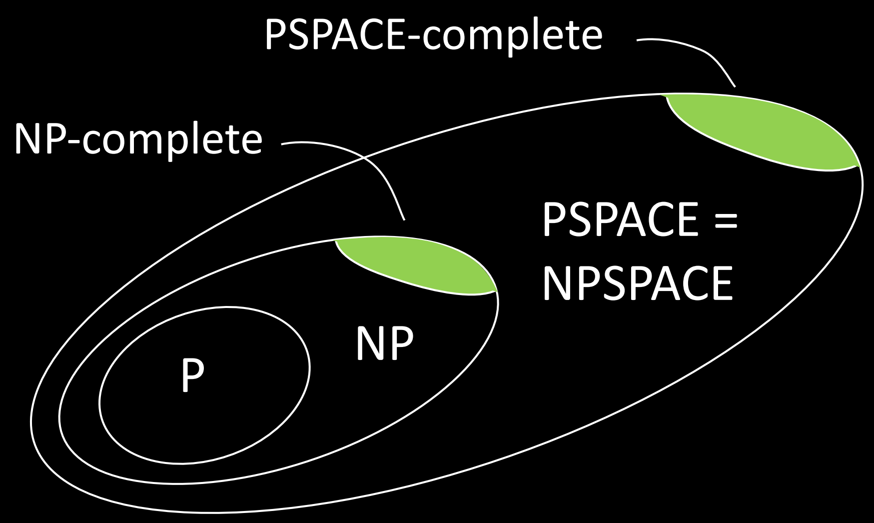

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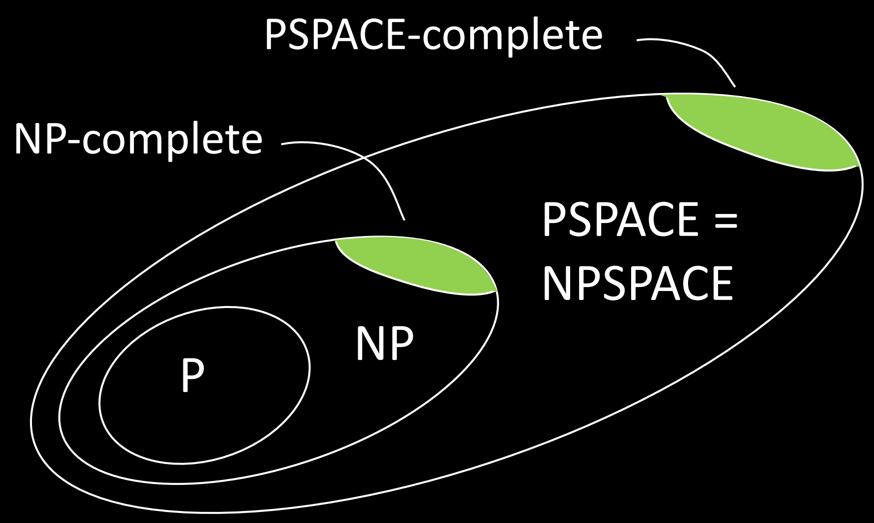

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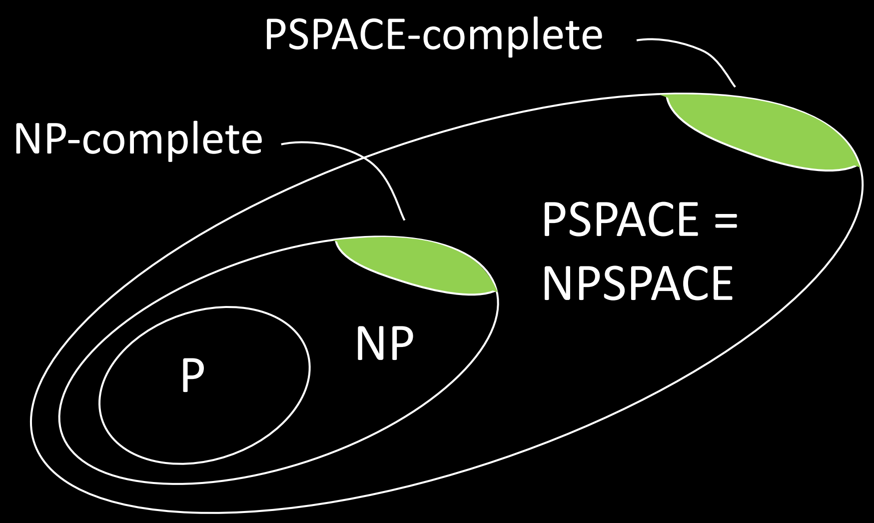

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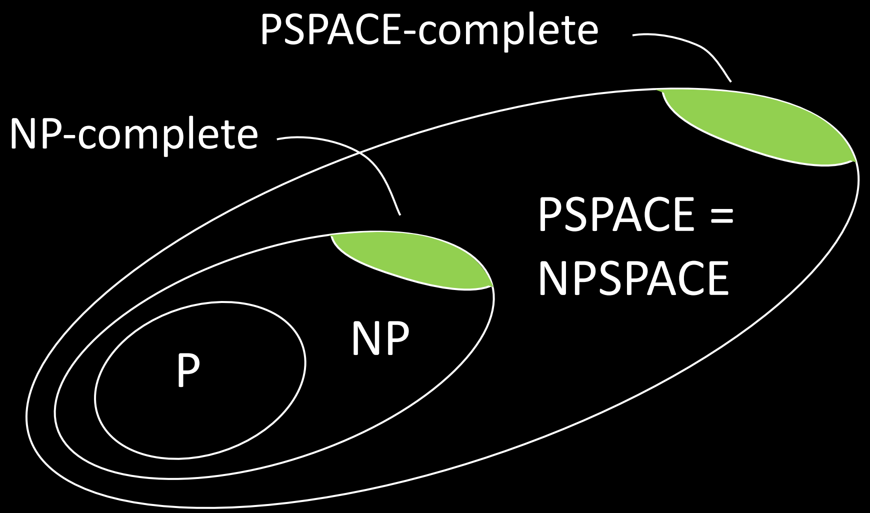

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Check-in 18.1

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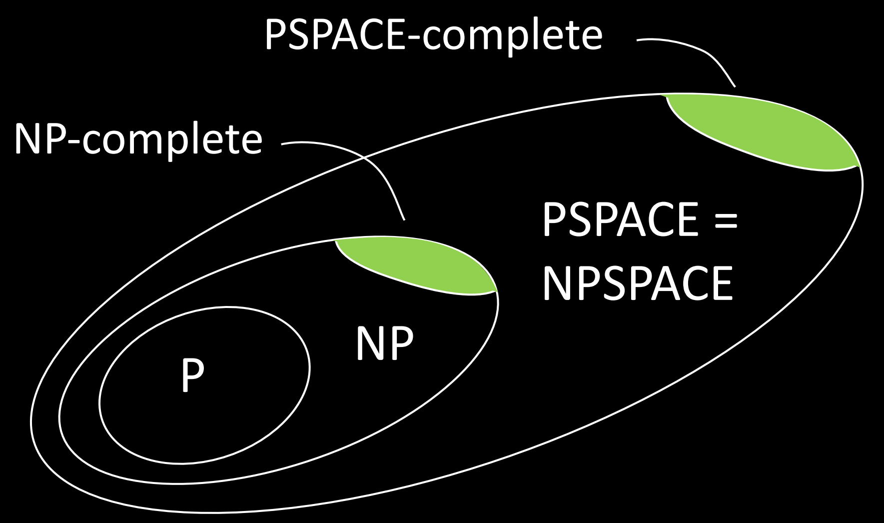
Knowing that $TQBF$ is PSPACE-complete, what can we conclude if $TQBF \in$ NP? Check all that apply.

(a) P = PSPACE

(b) NP = PSPACE

(c) P = NP

(d) NP = coNP



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$TQBF$ is PSPACE-complete

Recall: $TQBF = \left\{ \langle \phi \rangle \,\middle|\, \phi \text{ is a QBF that is } \textsc{True} \right\}$

Examples: $\phi_1 = \forall x \; \exists y \left[\left(x \vee y \right) \wedge \left(\overline{x} \vee \overline{y} \right) \right] \in TQBF$ [TRUE]

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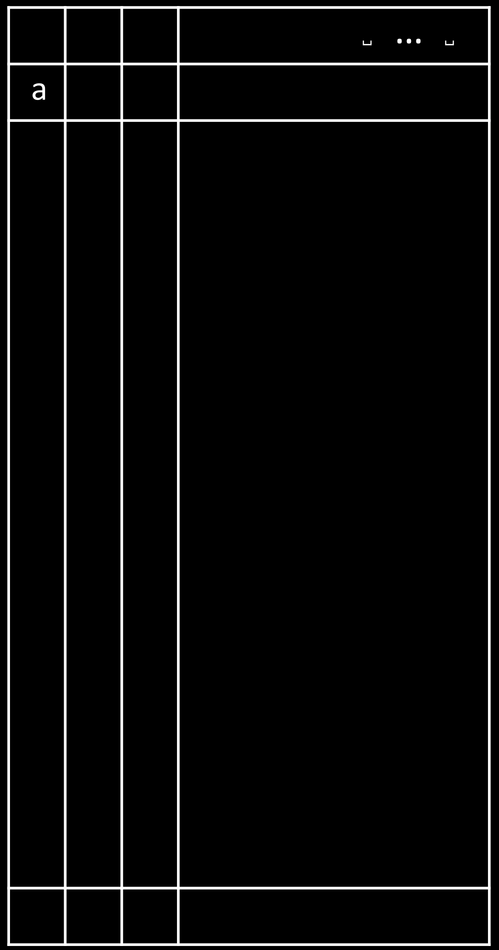
Tableau for M on w



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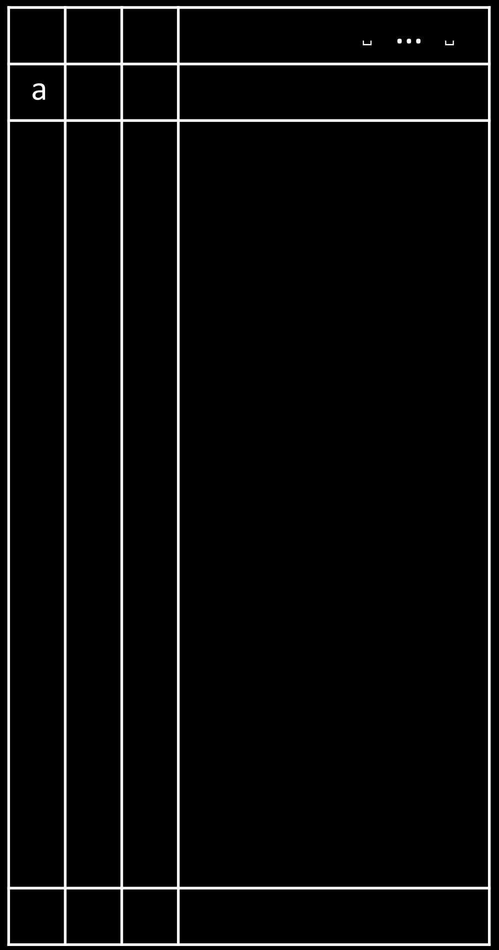

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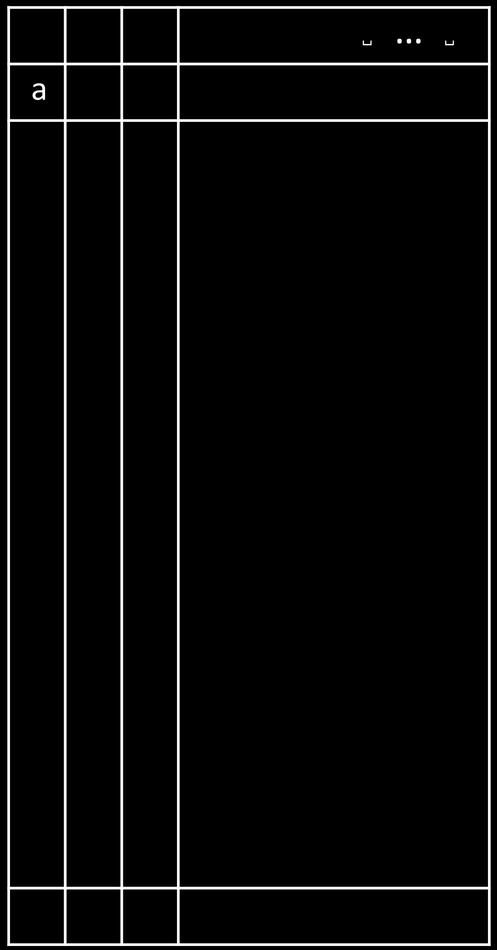

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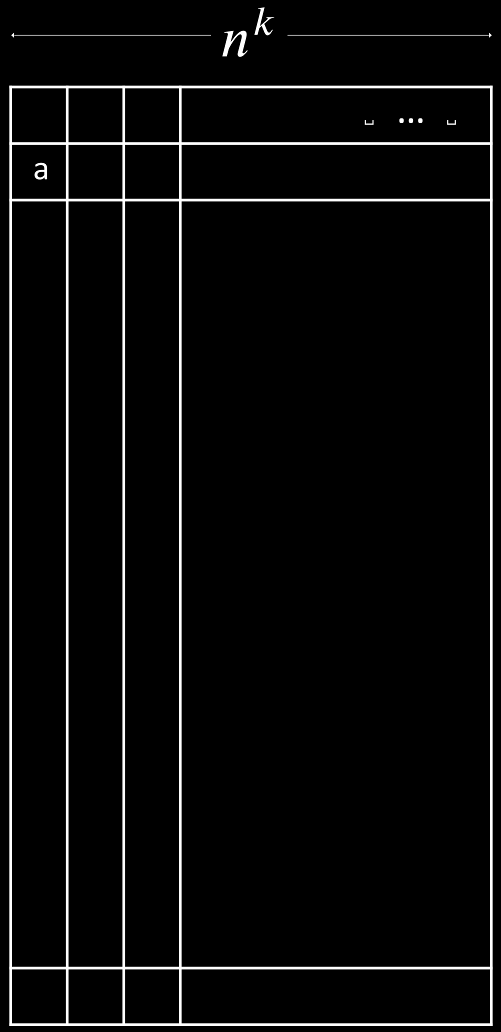

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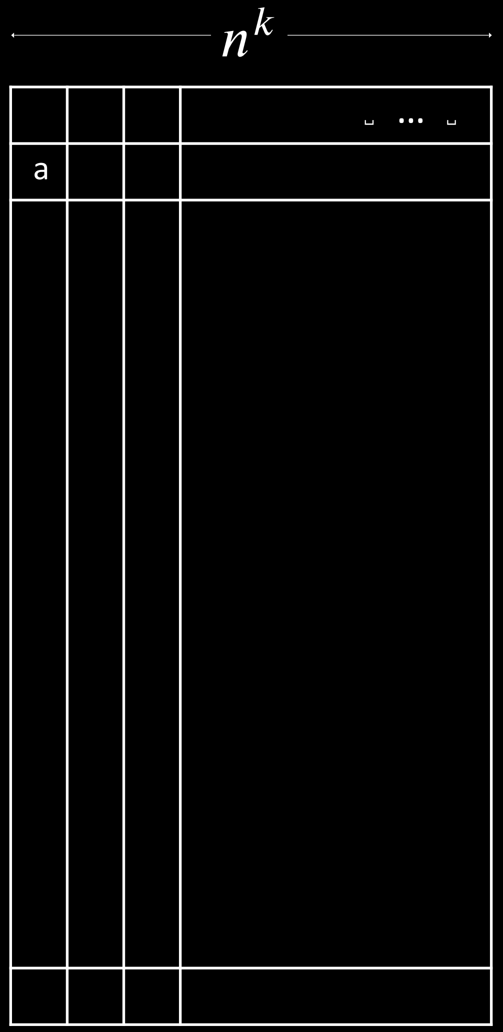

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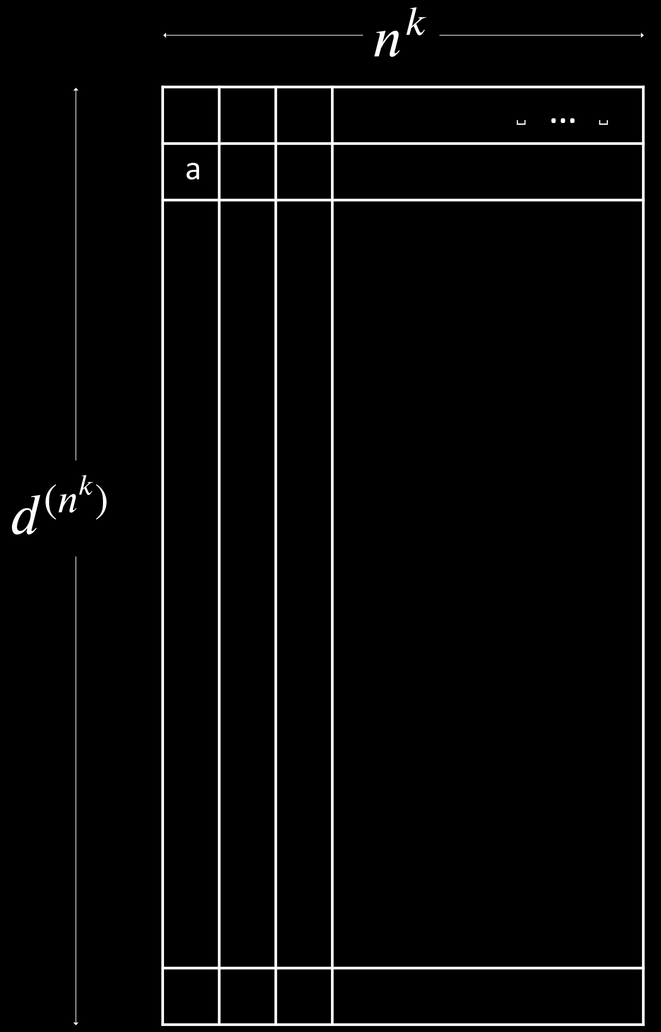

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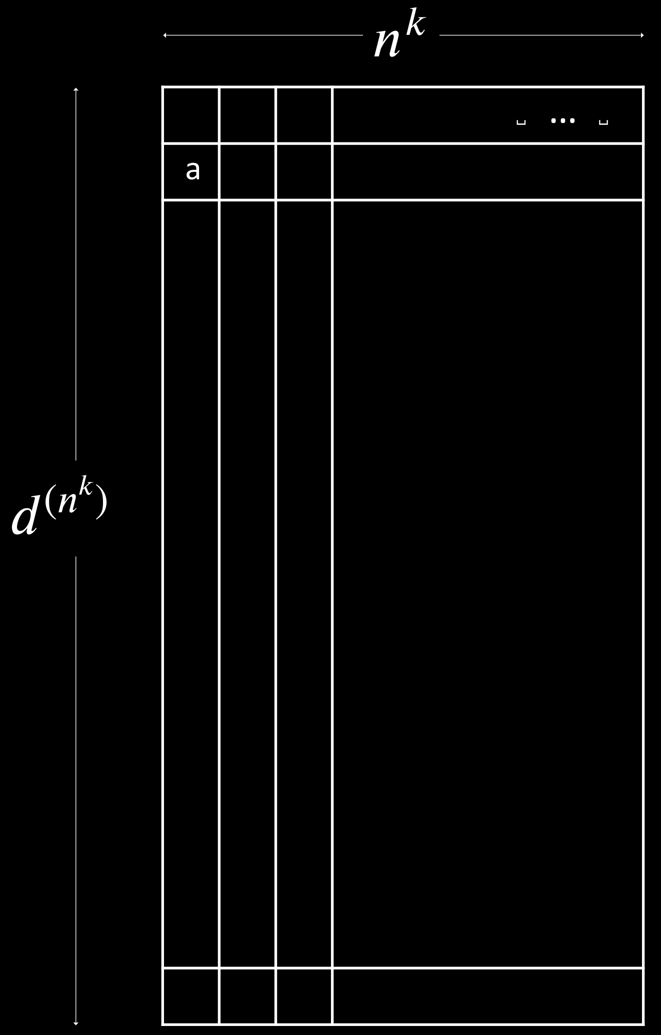

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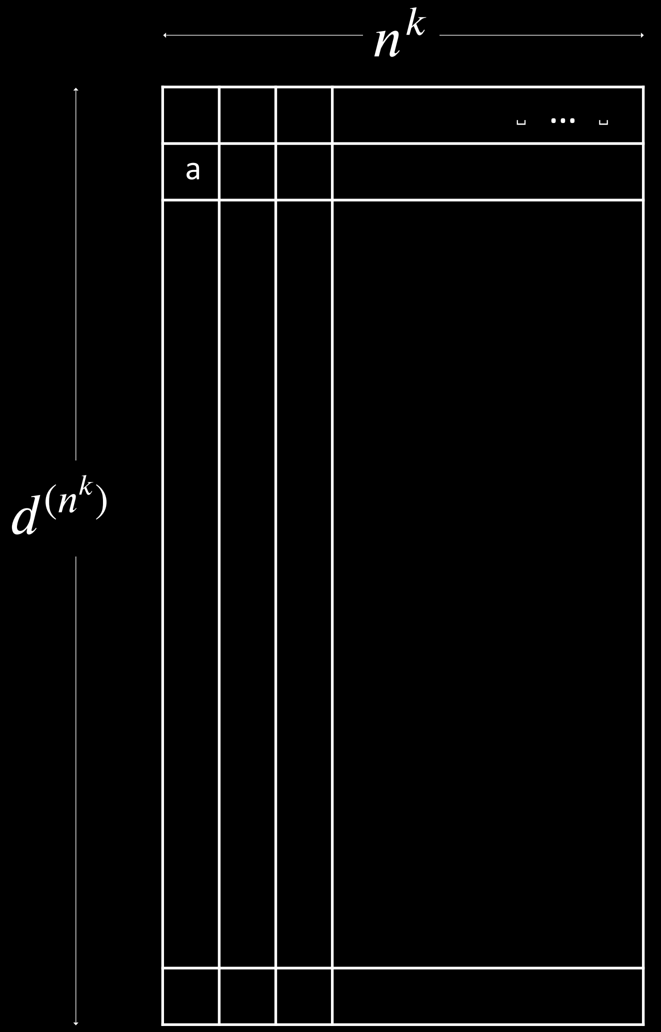

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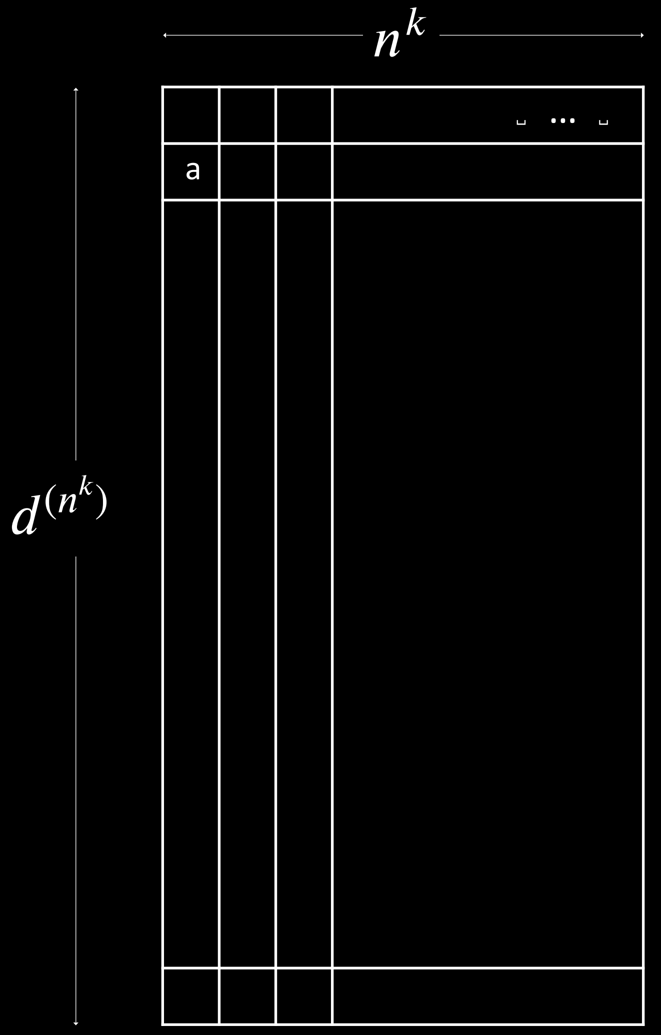

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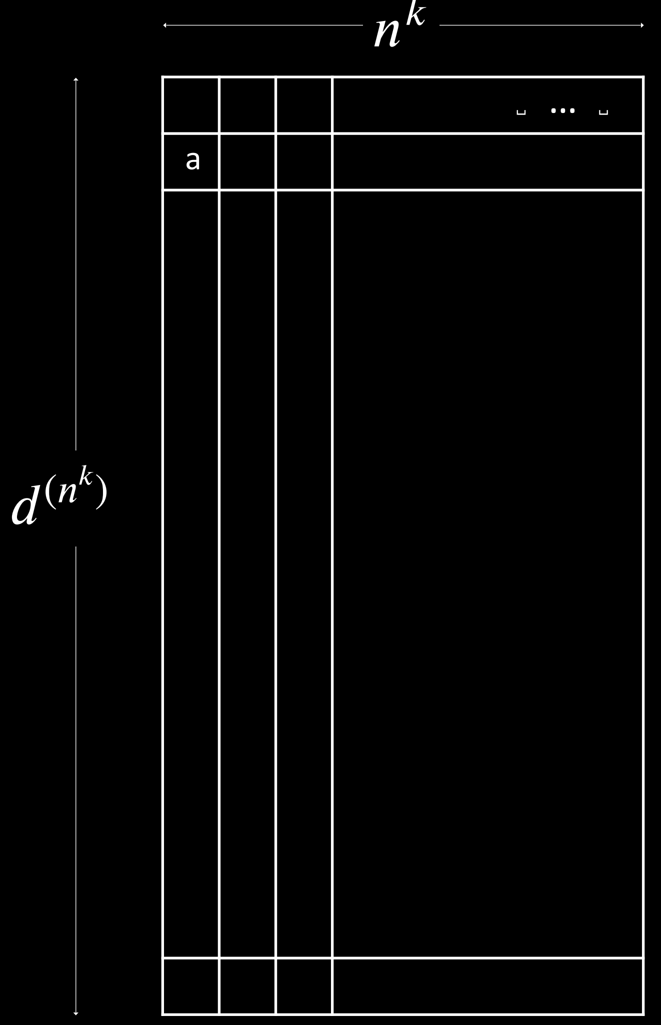

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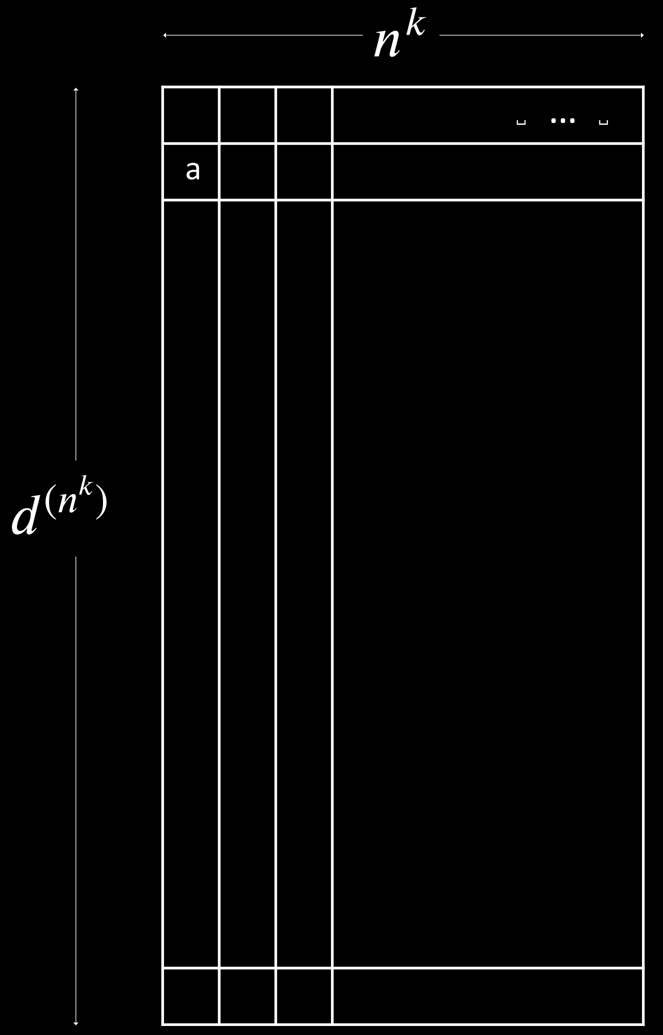

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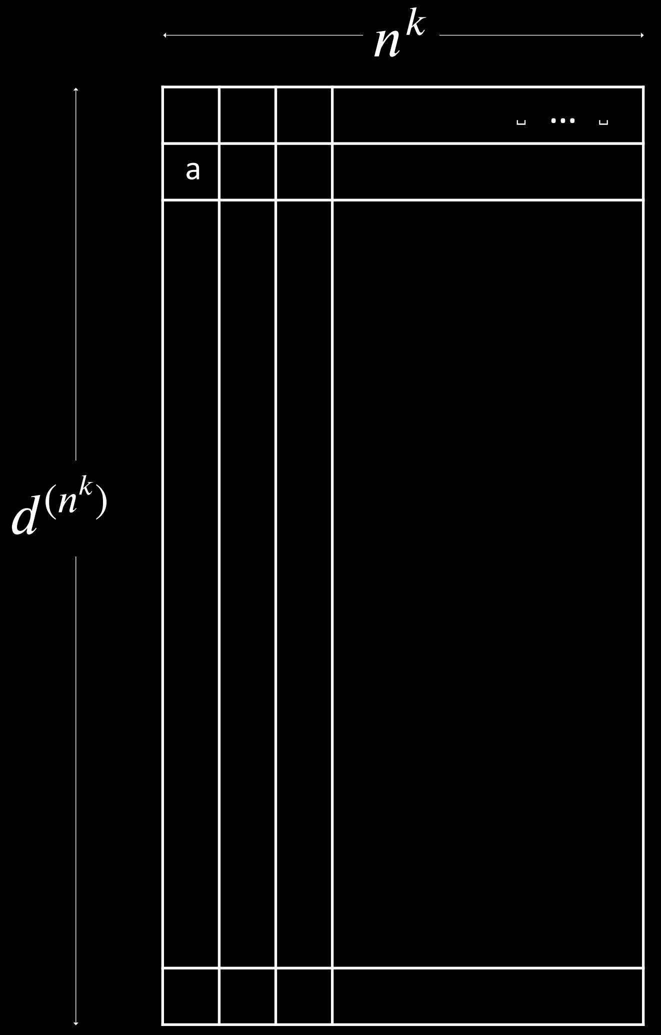

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hide →

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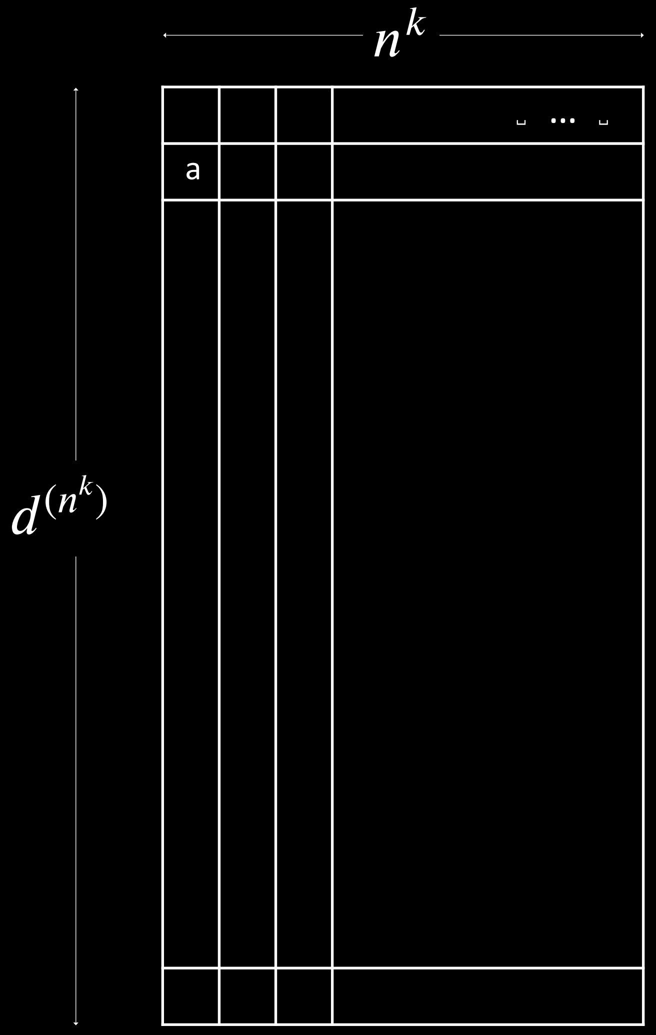

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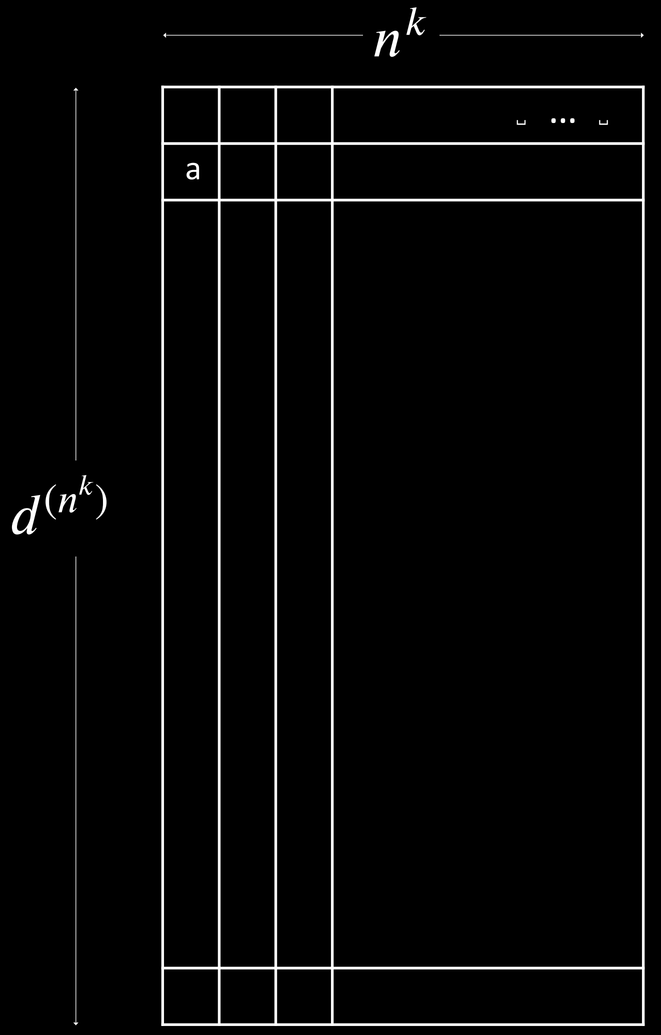

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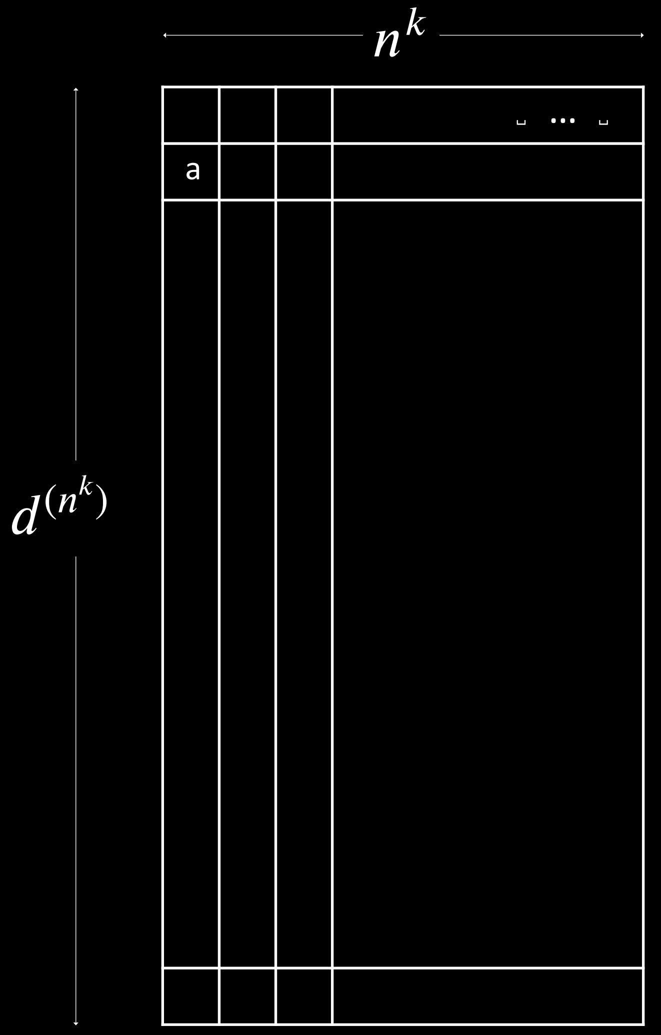

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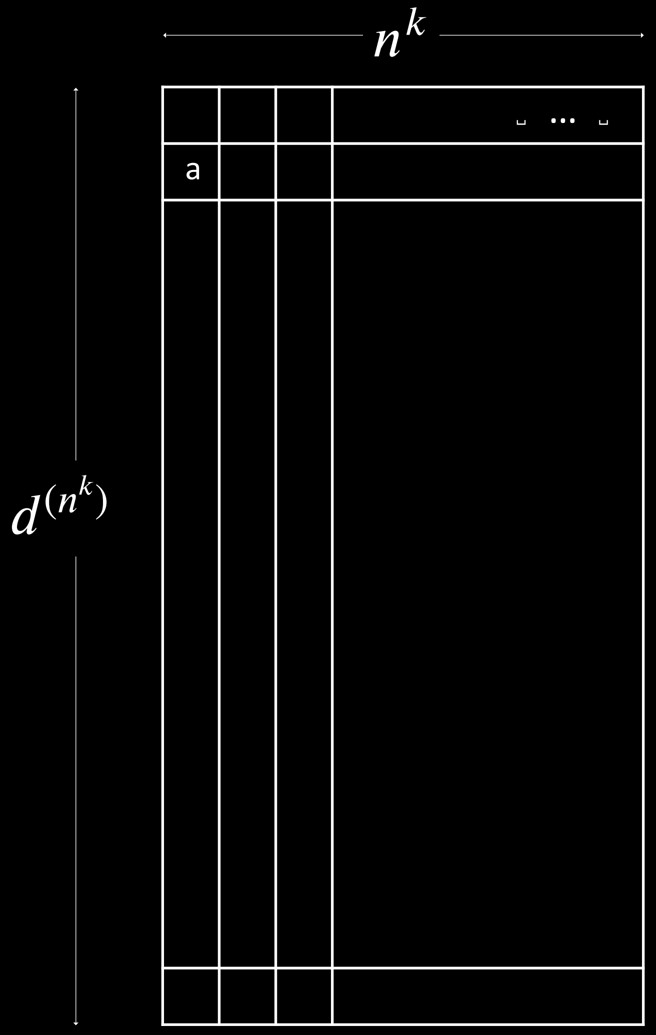

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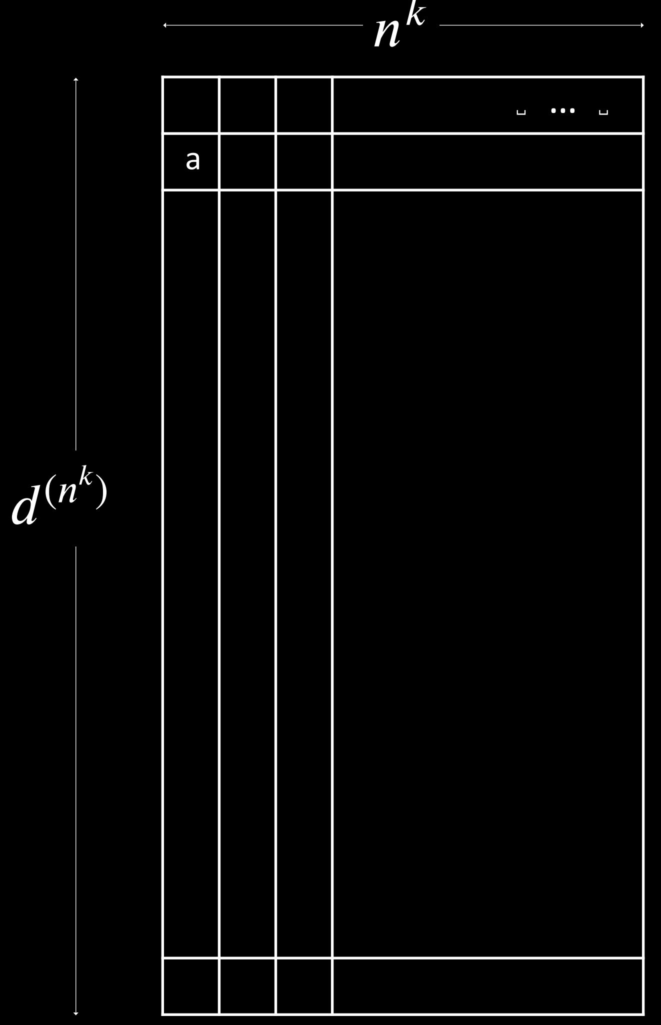

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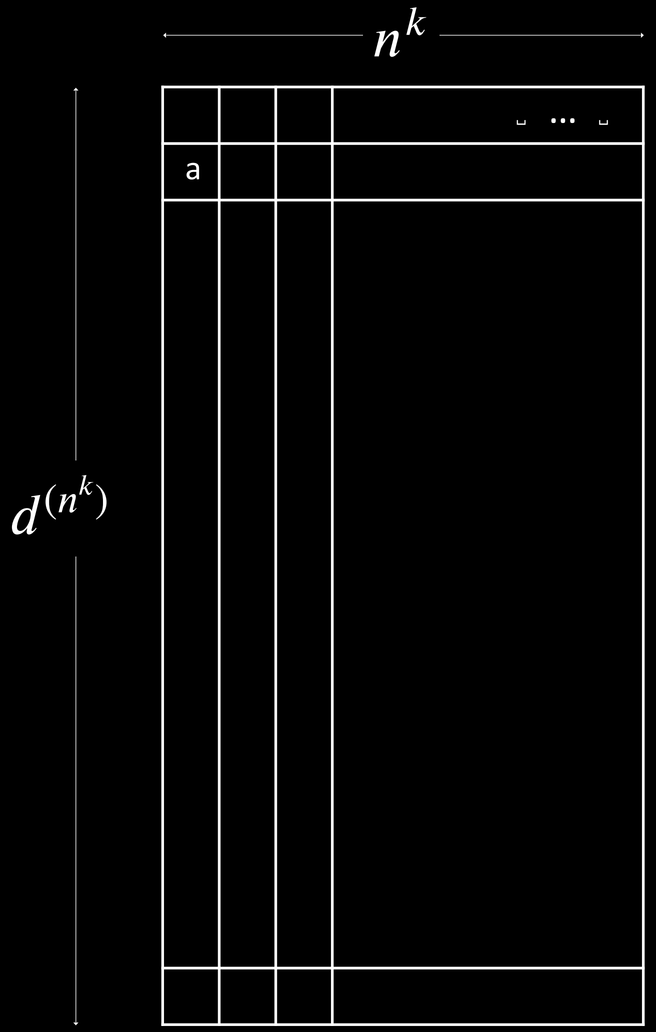
Constructing $\phi_{M,w}$: 2nd try

Tableau for M on w



For configs c_i and c_j construct $\phi_{c_i,\ c_j,\ b}$ which "says" $c_i \xrightarrow{b} c_j$ recursively.

$$\phi_{c_i,\ c_j,\ b} = \underbrace{\exists c_{\mathrm{mid}}}_{\boxed{\exists x_1, x_2, \cdots, c_l \text{ as in Cook-Levin}}} \left[\phi_{c_i,\ c_{\mathrm{mid}},\ b/2} \wedge \phi_{c_{\mathrm{mid}},\ c_j,\ b/2} \right]$$

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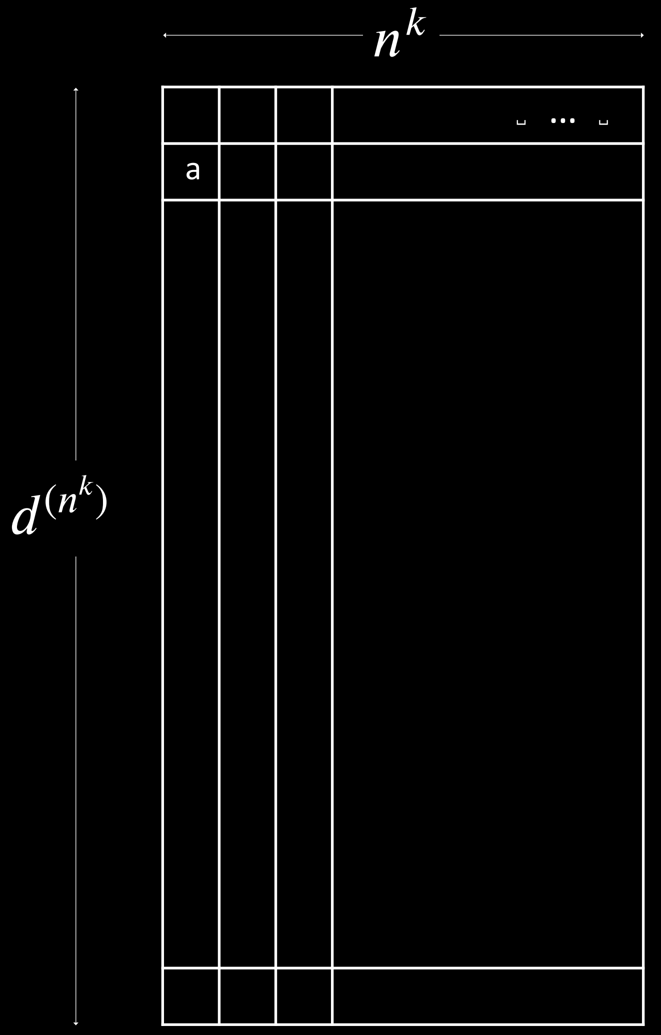

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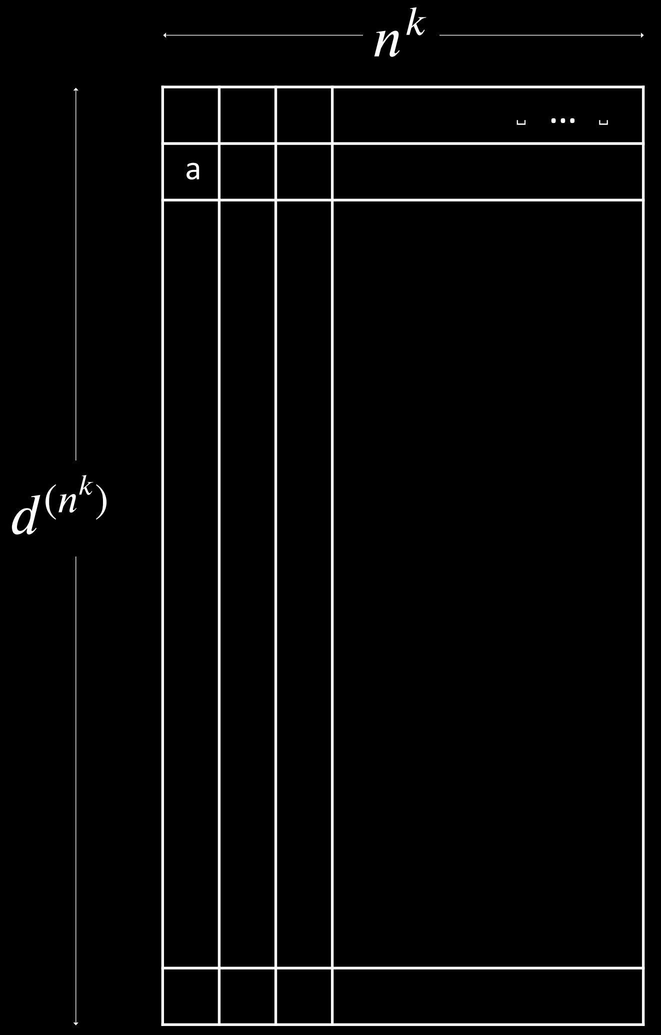

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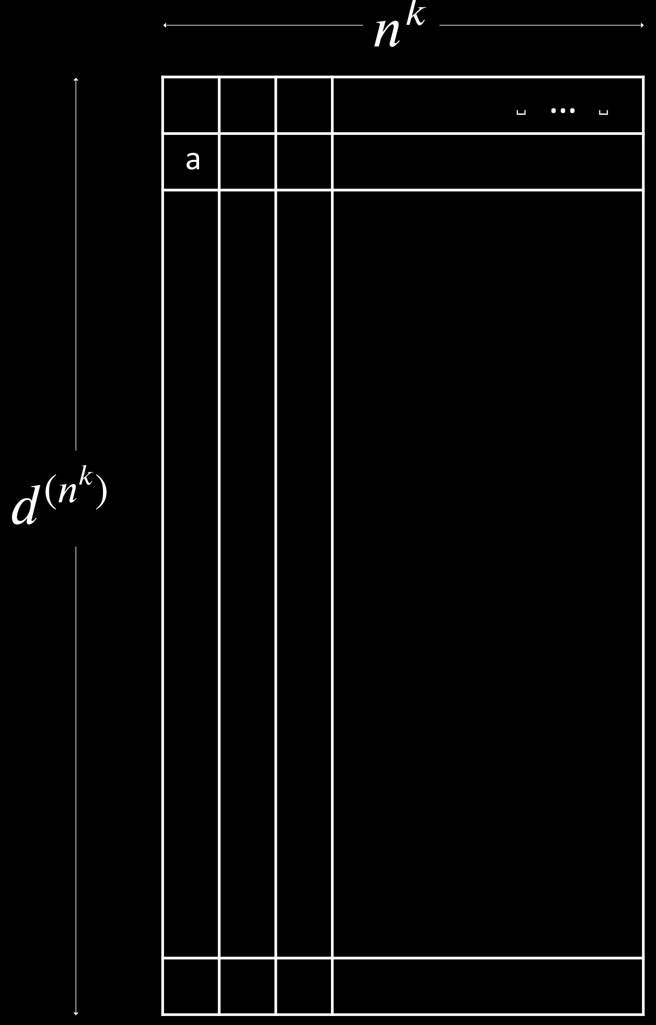

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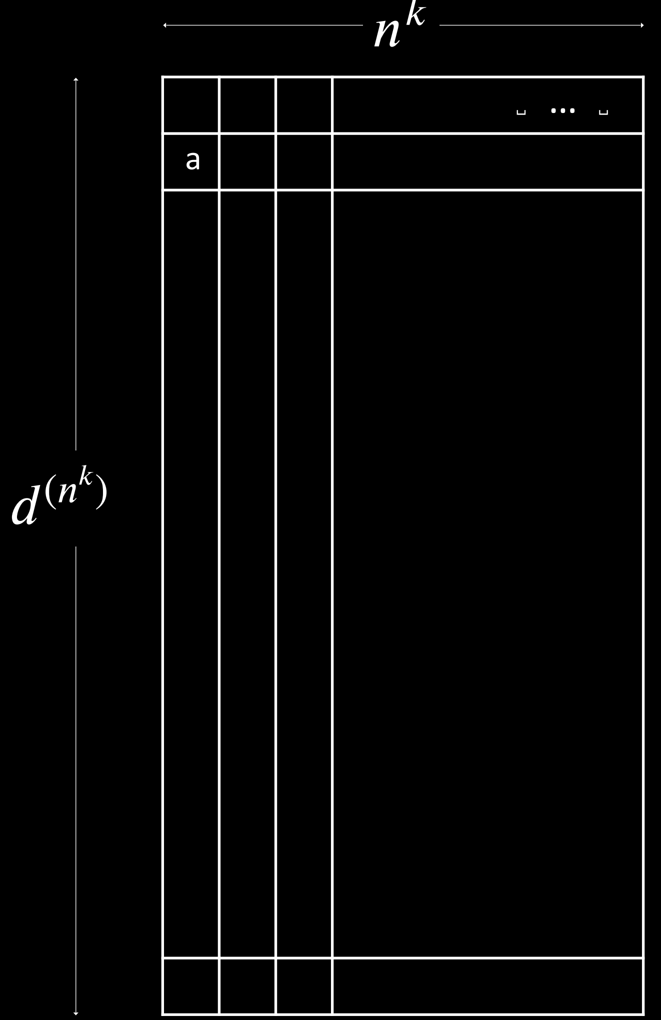

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Constructing $\phi_{M,w}$: 2nd try

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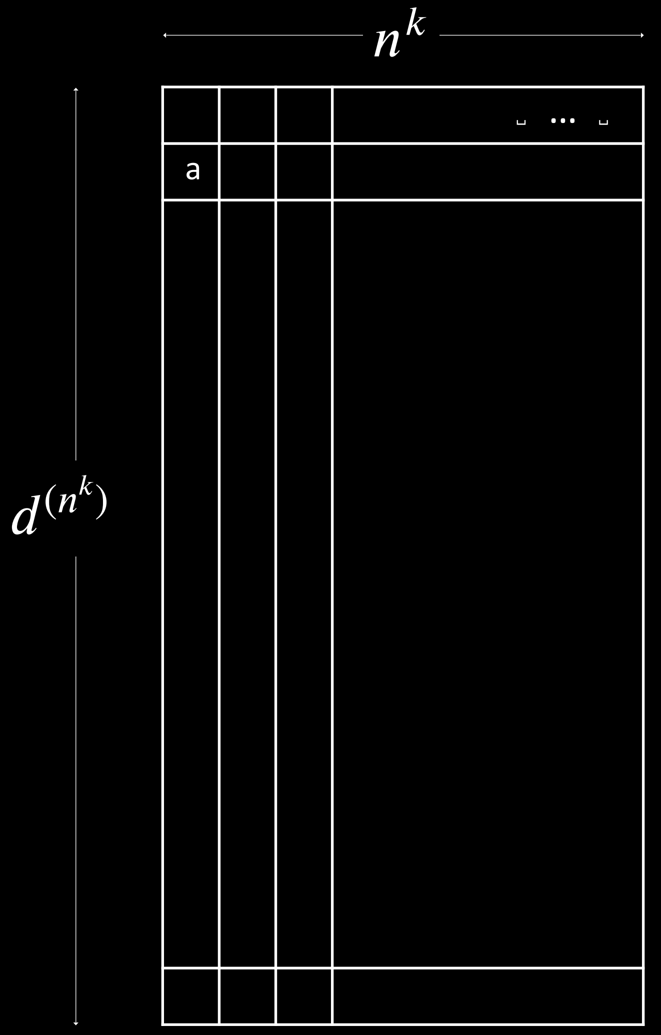

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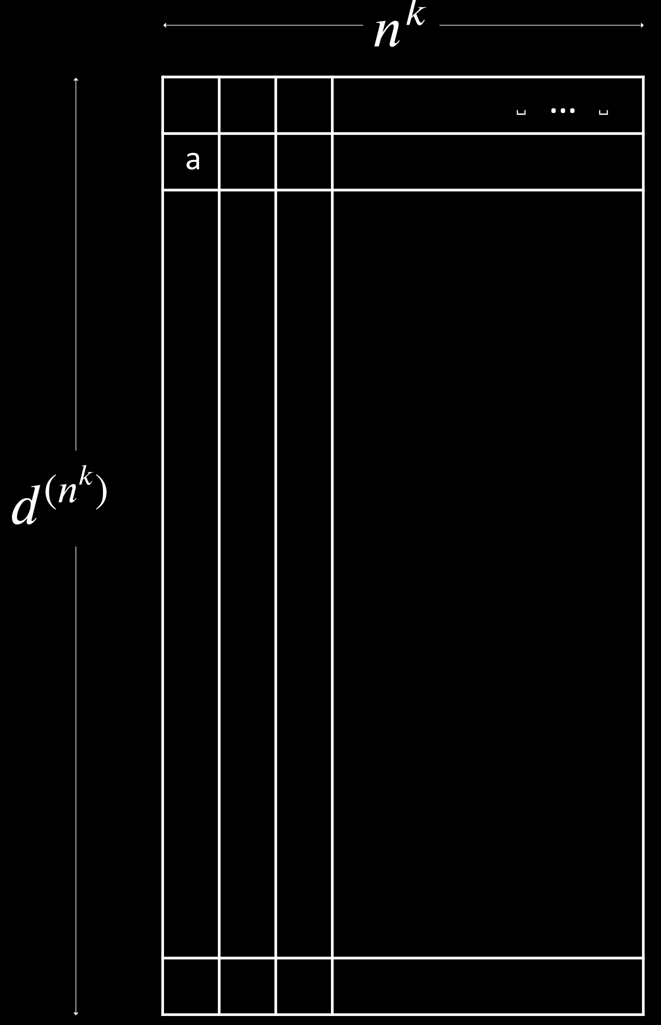
$\phi_{\ ,\ ,\ 1}$ defined as in Cook-Levin

$$\boxed{\begin{array}{c} \phi_{M,w} = \phi_{c_{\text{start}},\ c_{\text{accept}},\ t} \\ t = d^{(n^k)} \end{array}}$$

Constructing $\phi_{M,w}$: 2nd try

Tableau for M on w



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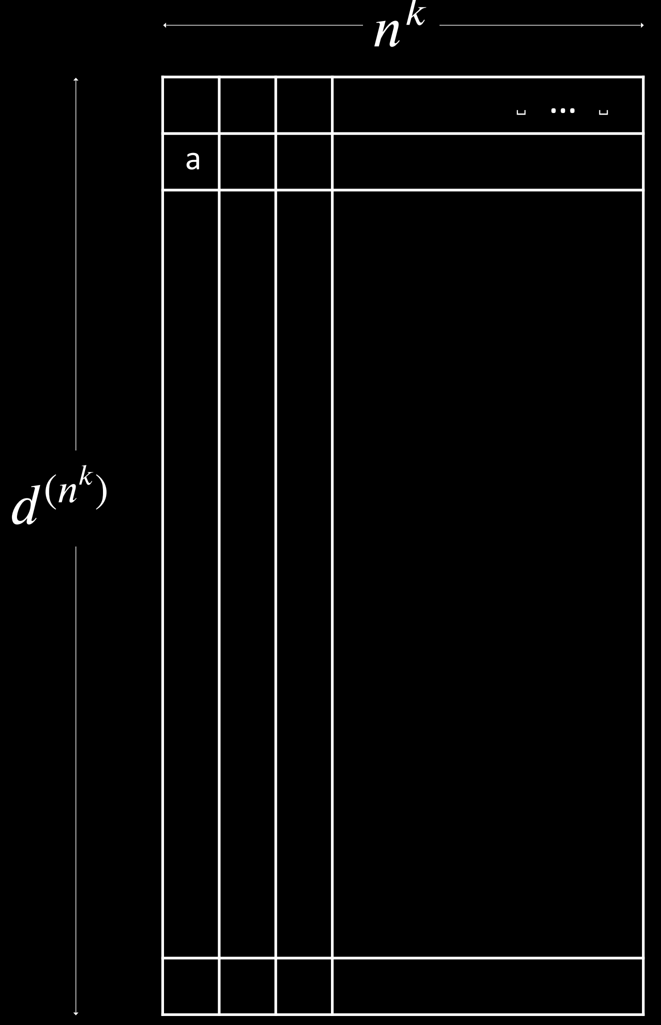
Size analysis:

Each recursive level doubles number of QBFs.

Constructing $\phi_{M,w}$: 2nd try

Tableau for M on w



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$$\phi_{M,w} = \phi_{c_{\text{start}},\ \ c_{\text{accept}},\ \ t}$$
$$t = d^{(n^k)}$$

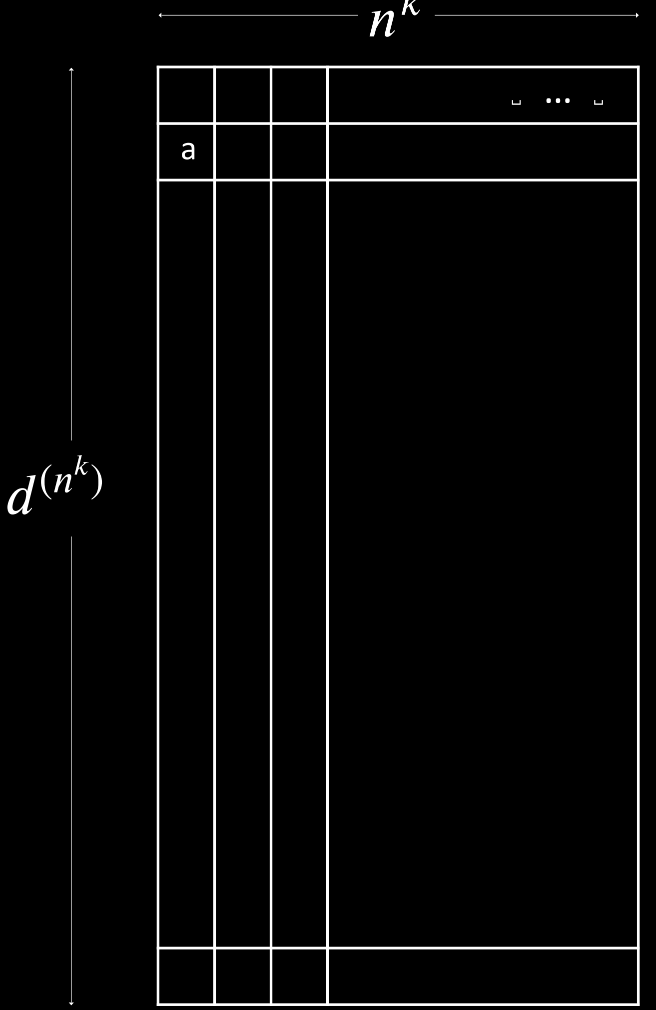
Size analysis:

Each recursive level doubles number of QBFs.

Number of levels is $\log d^{(n^k)} = O(n^k)$.

Constructing $\phi_{M,w}$: 2nd try

Tableau for M on w



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$\phi_{\ ,\ ,\ 1}$ defined as in Cook-Levin

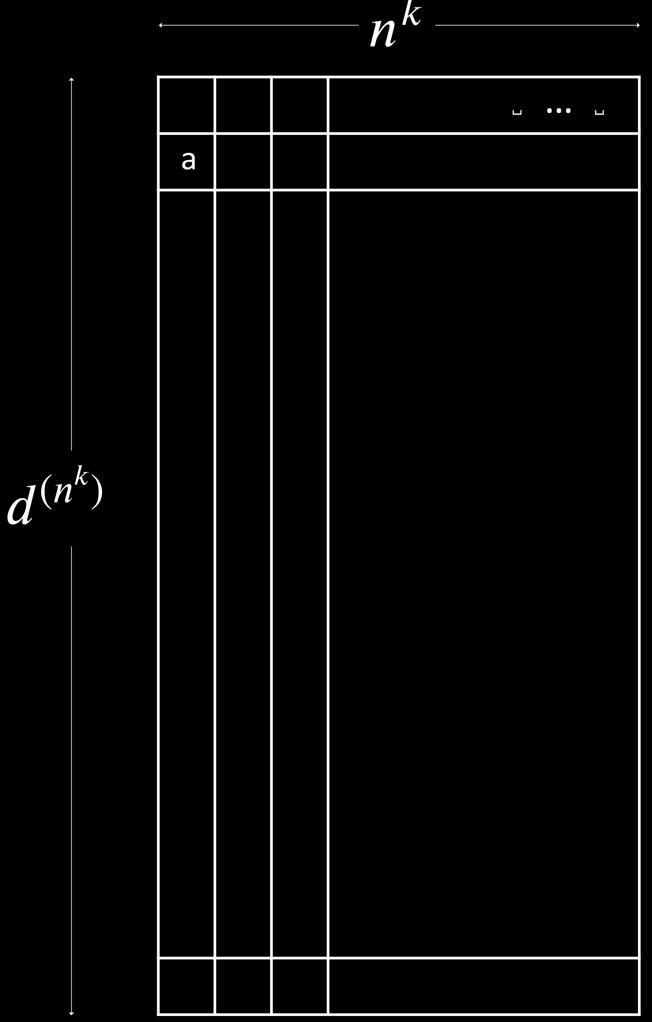
$$\boxed{\begin{array}{c} \phi_{M,w} = \phi_{c_{\text{start}},\ c_{\text{accept}},\ t} \\ t = d^{(n^k)} \end{array}}$$

Size analysis:
Each recursive level doubles number of QBFs.
Number of levels is $\log d^{(n^k)} = O(n^k)$.

→ Size is exponential. ☹

Constructing $\phi_{M,w}$: 2nd try

Tableau for M on w



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$\phi_{\ ,\ ,\ 1}$ defined as in Cook-Levin

$$\boxed{\phi_{M,w} = \phi_{c_{\text{start}},\ c_{\text{accept}},\ t} \qquad t = d^{(n^k)}}$$

Size analysis:
Each recursive level doubles number of QBFs.
Number of levels is $\log d^{(n^k)} = O(n^k)$.
→ Size is exponential. ☹

Check-in 18.2

Constructing $\phi_{M,w}$: 2nd try

configs c_i and c_j construct $\phi_{c_i,\ c_j,\ b}$ which "says" $c_i \xrightarrow{b} c_j$ recursively.

$$\phi_{c_i,\ c_j,\ b} = \underbrace{\exists c_{\text{mid}}}\left[\ \phi_{c_i,\, c_{\text{mid}},\ b/2} \ \wedge\ \phi_{c_{\text{mid}},\ c_j,\ b/2}\ \right]$$

$\exists x_1, x_2, \cdots, c_l$
as in Cook-Levin

$\exists c_{\text{mid}}\left[\phi_{\ ,\ ,\ b/4} \ \wedge\ \phi_{\ ,\ ,\ b/4}\right]$ $\quad$ $\exists c_{\text{mid}}\left[\phi_{\ ,\ ,\ b/4} \ \wedge\ \phi_{\ ,\ ,\ b/4}\right]$

$\vdots$ $\quad$ $\vdots$ $\quad$ $\exists c_{\text{mid}}[\phi_{\ ,\ ,\ b/8} \cdots]$

$\phi_{\ ,\ ,\ 1}$ defined as in Cook-Levin

$\phi_{M,w} = \phi_{c_{\text{start}},\ c_{\text{accept}},\ t}$

$t = d^{(n^k)}$

Size analysis:
Each recursive level doubles number of QBFs.
Number of levels is $\log d^{(n^k)} = O(n^k)$.

→ Size is exponential. ☹

Check-in 18.2

Check-in 18.2

Why shouldn't we be surprised that this construction fails?

(a) We can't define a QBF by using recursion.

(b) It doesn't use ∀ anywhere.

(c) We know that $TQBF \notin$ P.

Constructing $\phi_{M,w}$: 3rd try

$$\phi_{c_i,\ c_j,\ b} = \exists c_{\text{mid}} \left[\phi_{c_i,\, c_{\text{mid}},\ b/2} \ \wedge \ \phi_{c_{\text{mid}},\ c_j,\ b/2} \right]$$

Constructing $\phi_{M,w}$: 3rd try

$$\phi_{c_i,\ c_j,\ b} = \exists c_{\text{mid}} \underbrace{\left[\phi_{c_i,\, c_{\text{mid}},\ b/2} \ \wedge \ \phi_{c_{\text{mid}},\ c_j,\ b/2} \right]}_{\forall (c_g,\ c_h) \in \{ (c_i,\ c_{\text{mid}}),\ (c_{\text{mid}},\ c_j) \} [\phi_{c_g,\, c_h,\ b/2}]}$$

Constructing $\phi_{M,w}$: 3rd try

$$\phi_{c_i,\ c_j,\ b} = \exists c_{\text{mid}} \underbrace{\left[\phi_{c_i,\, c_{\text{mid}},\ b/2} \ \wedge\ \phi_{c_{\text{mid}},\ c_j,\ b/2} \right]}_{\forall \left(c_g,\ c_h\right) \in \left\{ \left(c_i,\ c_{\text{mid}}\right),\ \left(c_{\text{mid}},\ c_j\right) \right\} \left[\phi_{c_g,\, c_h,\ b/2} \right]}$$

$$\vdots$$

Constructing $\phi_{M,w}$: 3rd try

$$\phi_{c_i,\ c_j,\ b} = \exists c_{\text{mid}} \underbrace{\left[\ \phi_{c_i,\ c_{\text{mid}},\ b/2} \ \wedge\ \phi_{c_{\text{mid}},\ c_j,\ b/2}\ \right]}_{\forall (c_g,\ c_h) \in \left\{ (c_i,\ c_{\text{mid}}),\ (c_{\text{mid}},\ c_j) \right\} \left[\phi_{c_g,\ c_h,\ b/2}\right]}$$

$$\vdots$$

$\phi_{\ ,\ ,\ 1}$ defined as in Cook-Levin

Constructing $\phi_{M,w}$: 3rd try

$$\phi_{c_i,\ c_j,\ b} = \exists c_{\text{mid}} \underbrace{\left[\ \phi_{c_i,\, c_{\text{mid}},\ b/2} \ \wedge\ \phi_{c_{\text{mid}},\ c_j,\ b/2}\ \right]}$$

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$\phi_{\ ,\ ,\ 1}$ defined as in Cook-Levin

$\forall (x \in S)\ [\ \psi\ ]$
is equivalent to
$\forall x\ [(x \in S) \longrightarrow \psi]$

Constructing $\phi_{M,w}$: 3rd try

$$\phi_{c_i,\ c_j,\ b} = \exists c_{\text{mid}} \underbrace{\left[\ \phi_{c_i,\, c_{\text{mid}},\ b/2} \ \land\ \phi_{c_{\text{mid}},\ c_j,\ b/2}\ \right]}$$

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$\vdots$

$\phi_{M,w} = \phi_{c_{\text{start}},\ c_{\text{accept}},\ t}$

$t = d^{(n^k)}$

$\phi_{\ ,\ ,\ 1}$ defined as in Cook-Levin

Constructing $\phi_{M,w}$: 3[rd] try

$$\phi_{c_i,\ c_j,\ b} = \exists c_{\mathrm{mid}} \underbrace{\left[\phi_{c_i,\, c_{\mathrm{mid}},\ b/2} \ \wedge\ \phi_{c_{\mathrm{mid}},\ c_j,\ b/2} \right]}_{\displaystyle \forall \left(c_g,\ c_h\right) \in \left\{ \left(c_i,\ c_{\mathrm{mid}}\right),\ \left(c_{\mathrm{mid}},\ c_j\right) \right\} \left[\phi_{c_g,\, c_h,\ b/2} \right]}$$

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$\phi_{\ ,\ ,\ 1}$ defined as in Cook-Levin

$\phi_{M,w} = \phi_{c_{\mathrm{start}},\ c_{\mathrm{accept}},\ t}$

$t = d^{(n^k)}$

Size analysis:

Each recursive level <u>adds</u> $O(n^k)$ to the QBF.

Number of levels is $\log d^{(n^k)} = O(n^k)$.

→ Size is $O(n^k \times n^k) = O(n^{2k})$ ☺

Constructing $\phi_{M,w}$: 3rd try

$$\phi_{c_i,\ c_j,\ b} = \exists c_{\text{mid}} \underbrace{\left[\ \phi_{c_i,\, c_{\text{mid}},\ b/2}\ \wedge\ \phi_{c_{\text{mid}},\ c_j,\ b/2}\ \right]}$$

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$\phi_{\ ,\ ,\ 1}$ defined as in Cook-Levin

$$\phi_{M,w} = \phi_{c_{\text{start}},\ c_{\text{accept}},\ t}$$

$$t = d^{\left(n^k\right)}$$

Size analysis:

Each recursive level <u>adds</u> $O(n^k)$ to the QBF.

Number of levels is $\log d^{\left(n^k\right)} = O\left(n^k\right)$.

→ Size is $O\left(n^k \times n^k\right) = O\left(n^{2k}\right)$ ☺

Check-in 18.3

Constructing $\phi_{M,w}$: 3rd try

$$\phi_{c_i,\ c_j,\ b} = \exists c_{\text{mid}} \underbrace{\left[\phi_{c_i,\, c_{\text{mid}},\ b/2} \ \wedge \ \phi_{c_{\text{mid}},\ c_j,\ b/2} \right]}_{\forall (c_g,\ c_h) \in \{ (c_i,\ c_{\text{mid}}),\ (c_{\text{mid}},\ c_j) \} \left[\phi_{c_g,\, c_h,\ b/2} \right]}$$

$\forall (x \in S)\ [\psi]$
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$\forall x\ [(x \in S) \ \longrightarrow \ \psi]$

$\vdots$

$$\phi_{M,w} = \phi_{c_{\text{start}},\ c_{\text{accept}},\ t}$$
$$t = d^{(n^k)}$$

$\phi_{\ ,\ ,\ 1}$ defined as in Cook-Levin

Size analysis:

Each recursive level adds $O(n^k)$ to the QBF.

Number of levels is $\log d^{(n^k)} = O(n^k)$.

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Check-in 18.3

Would this construction still work if M were nondeterministic?

(a) Yes.

(b) No.

Check-in 18.3

Quick review of today

1. Space complexity
2. $\text{SPACE}\big(f(n)\big)$, $\text{NSPACE}\big(f(n)\big)$
3. PSPACE, NPSPACE
4. Relationship with TIME classes
5. $TQBF \in$ PSPACE
6. $LADDER_{\text{DFA}} \in \text{NSPACE}(n)$
7. $LADDER_{\text{DFA}} \in \text{SPACE}(n^2)$
8. Savitch's Theorem: $\text{NSPACE}\big(f(n)\big) \subseteq \text{SPACE}\big(f^2(n)\big)$
9. $TQBF$ is PSPACE-complete

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