

# Parsing:

## Bottom-Up Parsing

### LR(1)

# A Powerful Parser: LR(1)

- Bottom-up predictive parsing with
  - **L**: Left-to-right scan
  - **R**: Rightmost derivation
  - (**1**): One token lookahead
- *Substantially* more powerful than the other methods we've covered so far (more on that later).
- Tries to more intelligently find handles by using a lookahead **token** at each step.

# Lookahead

- Recall: Follow set.

# Lookahead

- Recall: Follow set.

*Idea:*

- Track series of productions.
- Use the information to find handles more efficiently.

# Lookahead

- Recall: Follow set.

*Idea:*

- Track series of productions.
- Use the information to find handles more efficiently.
- local follows or **lookahead**: i.e. what we expect follows **after** reduction.

# LR(1) Parsing: The Intuition

**S**  $\rightarrow$  **E**

**E**  $\rightarrow$  **T**

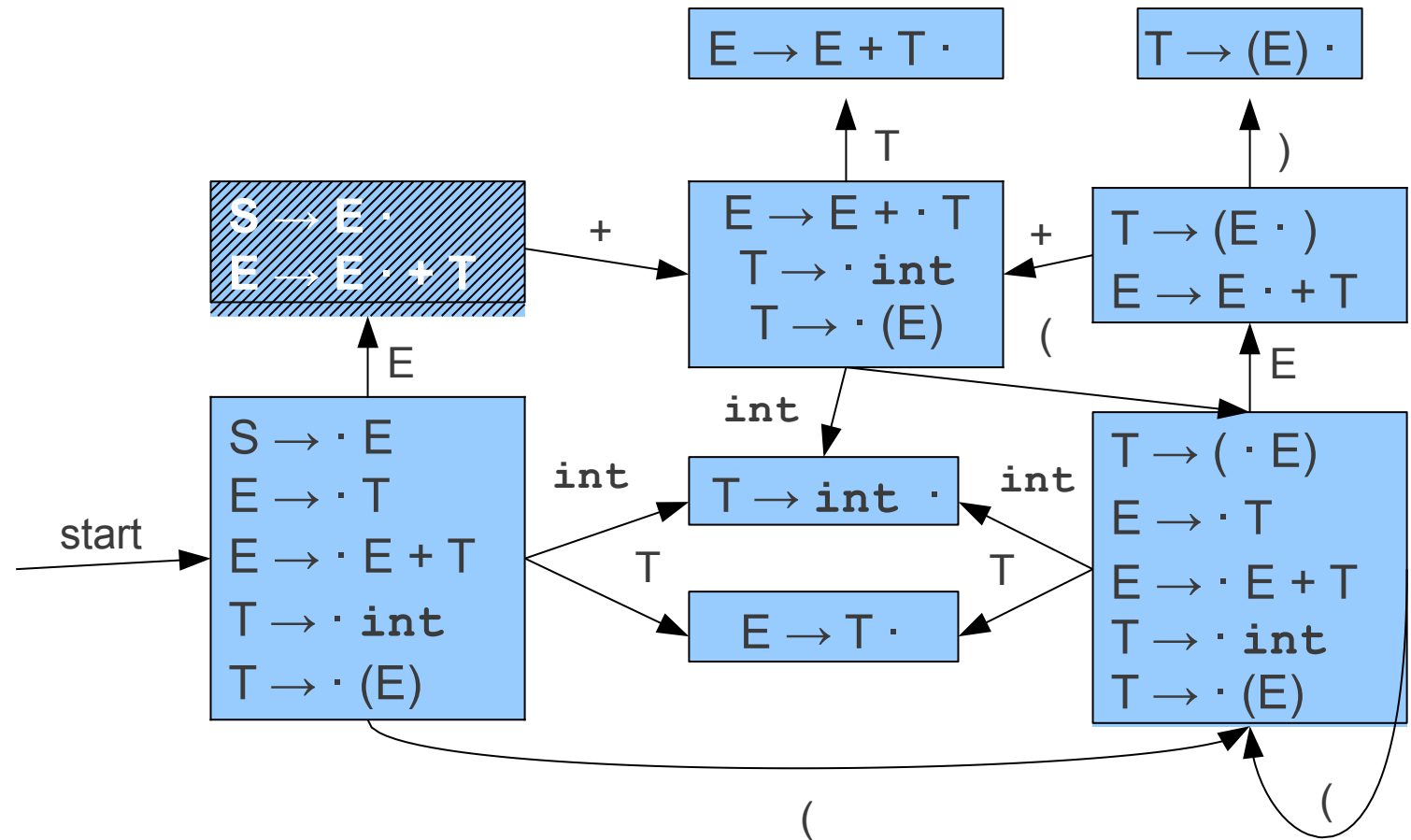
**E**  $\rightarrow$  **E** + **T**

**T**  $\rightarrow$  *int*

**T**  $\rightarrow$  (**E**)

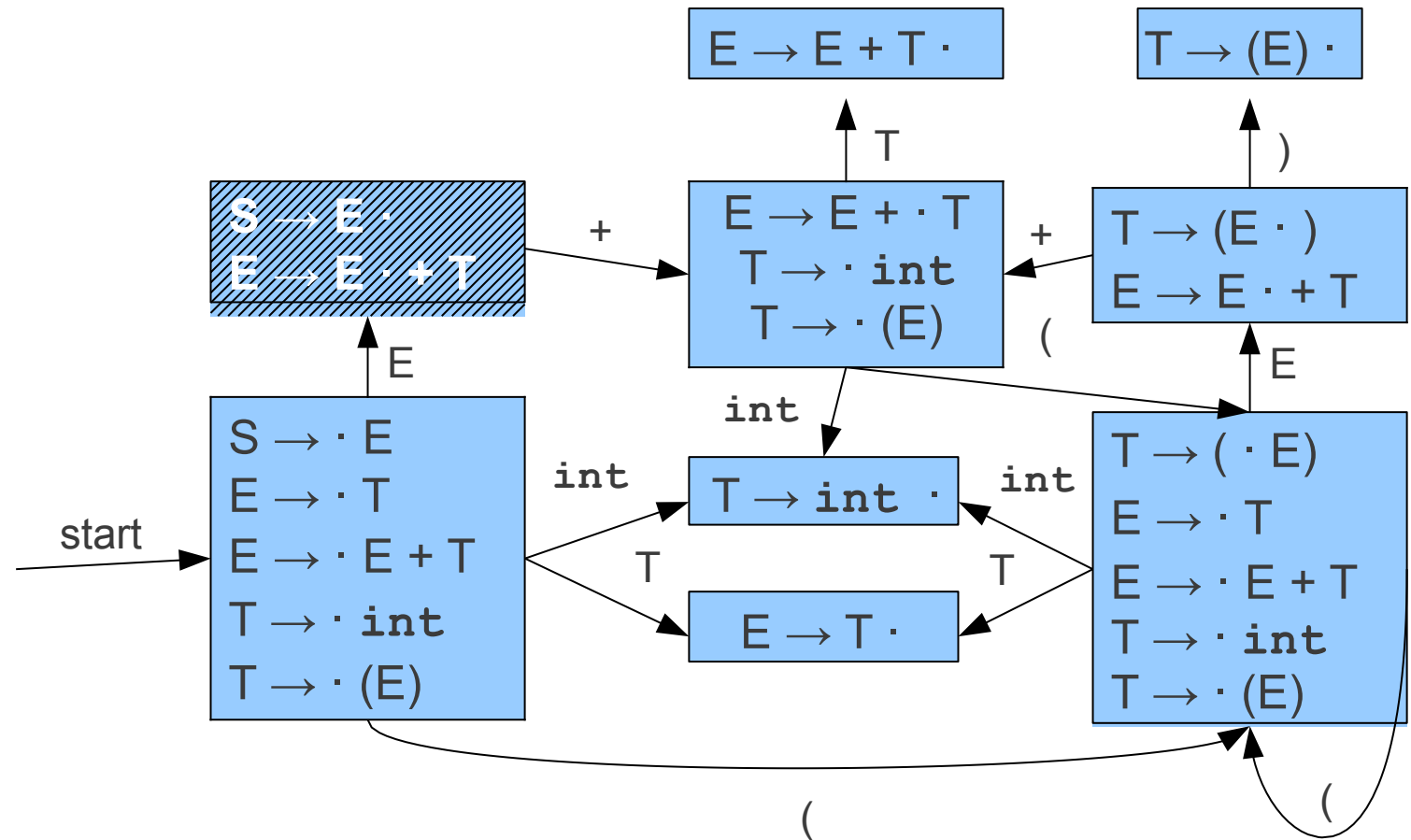
# LR(1) Parsing: The Intuition

$S \rightarrow E$   
 $E \rightarrow T$   
 $E \rightarrow E + T$   
 $T \rightarrow \text{int}$   
 $T \rightarrow (E)$



# LR(1) Parsing: The Intuition

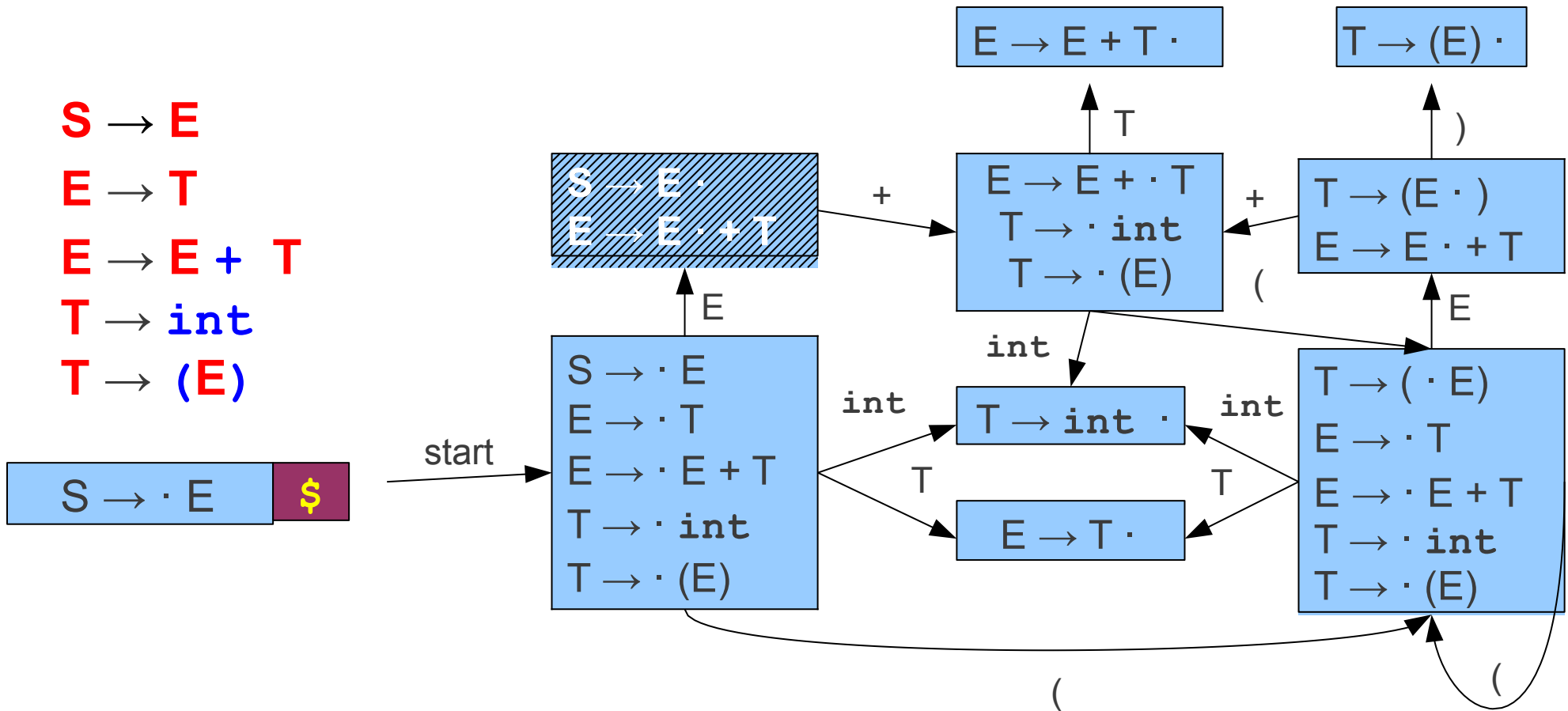
$S \rightarrow E$   
 $E \rightarrow T$   
 $E \rightarrow E + T$   
 $T \rightarrow \text{int}$   
 $T \rightarrow (E)$



int	+	(	int	+	int	+	int	)	\$
-----	---	---	-----	---	-----	---	-----	---	----



# LR(1) Parsing: The Intuition



int + ( int + int + int ) \$

# LR(1) Parsing: The Intuition

$S \rightarrow E$

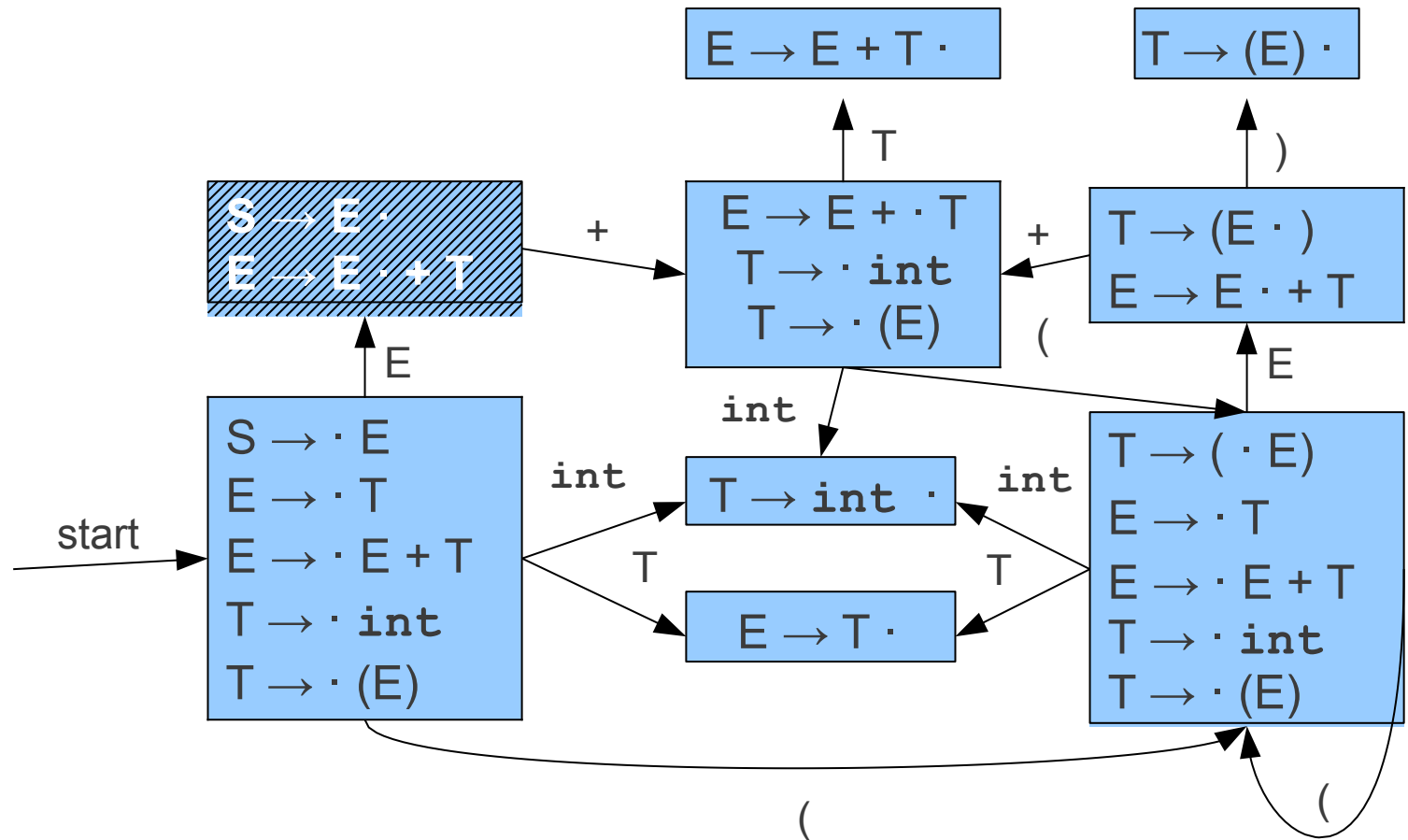
$E \rightarrow T$

$E \rightarrow E + T$

$T \rightarrow \text{int}$

$T \rightarrow (E)$

$S \rightarrow \cdot E$	\$
$E \rightarrow \cdot E + T$	\$

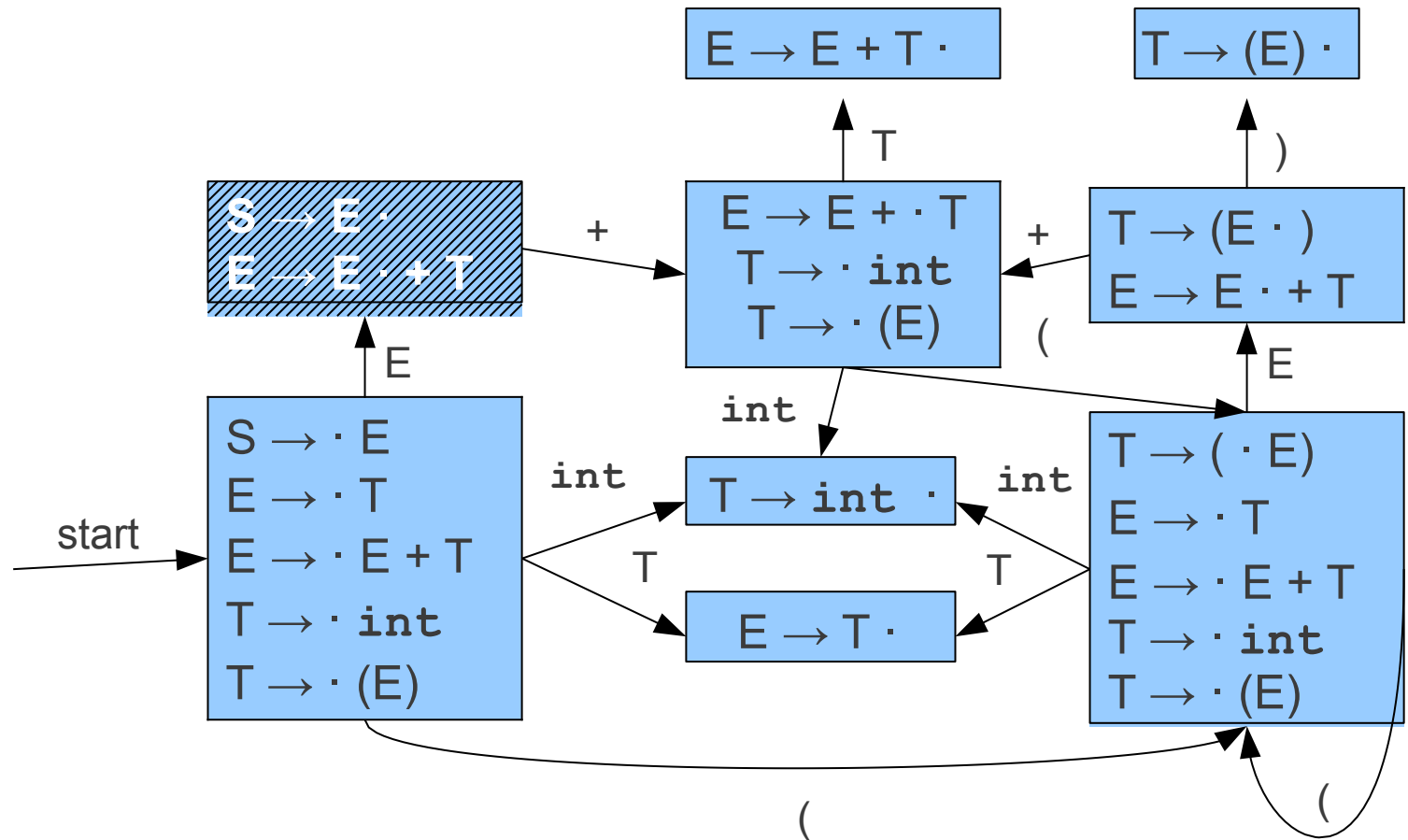


int	+	(	int	+	int	+	int	)	\$
-----	---	---	-----	---	-----	---	-----	---	----

# LR(1) Parsing: The Intuition

$S \rightarrow E$   
 $E \rightarrow T$   
 $E \rightarrow E + T$   
 $T \rightarrow \text{int}$   
 $T \rightarrow (E)$

$S \rightarrow \cdot E$	\$
$E \rightarrow \cdot E + T$	\$
$E \rightarrow \cdot T$	+

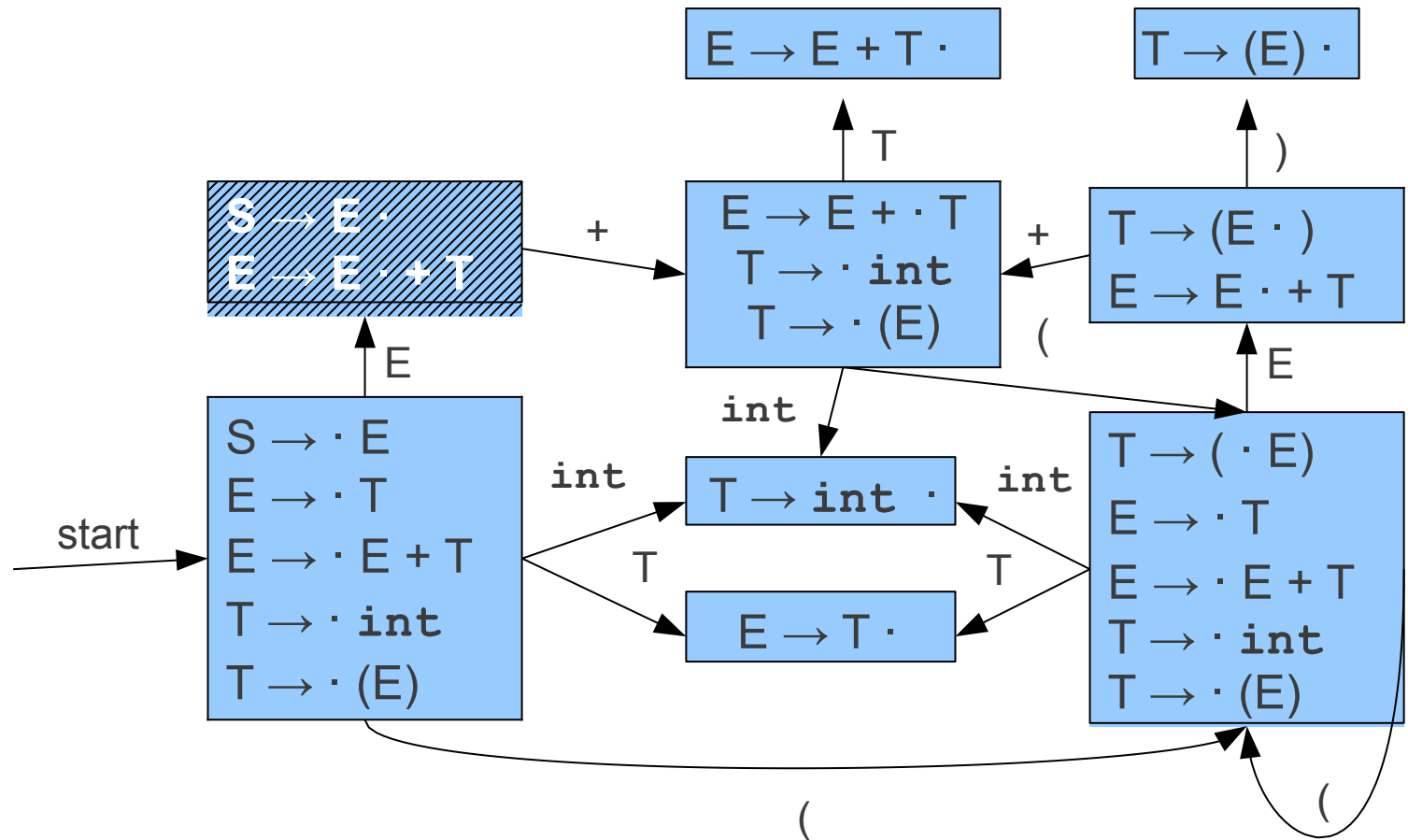


int	+	(	int	+	int	+	int	)	\$
-----	---	---	-----	---	-----	---	-----	---	----

# LR(1) Parsing: The Intuition

$S \rightarrow E$   
 $E \rightarrow T$   
 $E \rightarrow E + T$   
 $T \rightarrow \text{int}$   
 $T \rightarrow (E)$

$S \rightarrow \cdot E$	\$
$E \rightarrow \cdot E + T$	\$
$E \rightarrow \cdot T$	+
$T \rightarrow \cdot \text{int}$	+

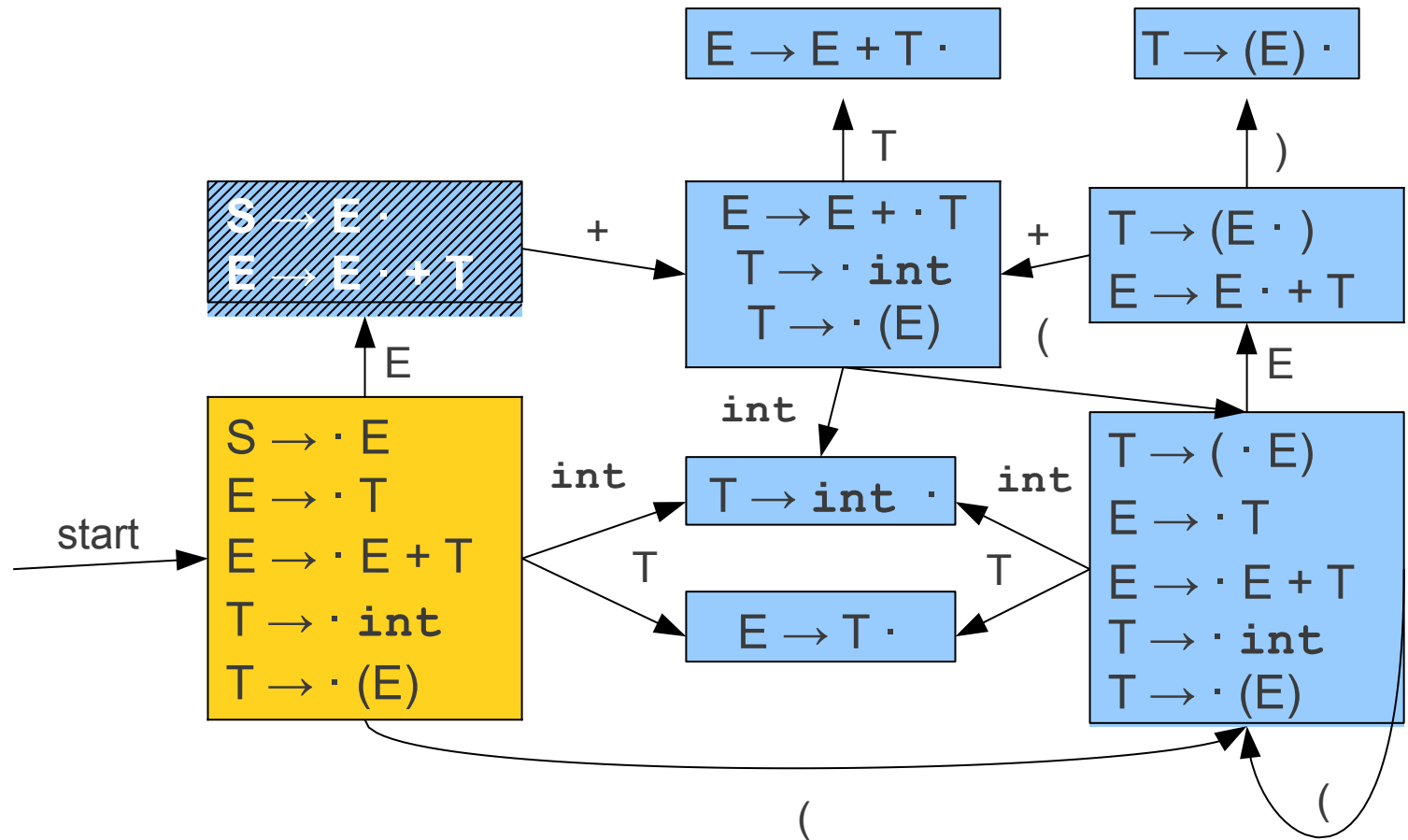


int	+	(	int	+	int	+	int	)	\$
-----	---	---	-----	---	-----	---	-----	---	----

# LR(1) Parsing: The Intuition

$S \rightarrow E$   
 $E \rightarrow T$   
 $E \rightarrow E + T$   
 $T \rightarrow \text{int}$   
 $T \rightarrow (E)$

$S \rightarrow \cdot E$	\$
$E \rightarrow \cdot E + T$	\$
$E \rightarrow \cdot T$	+
$T \rightarrow \cdot \text{int}$	+

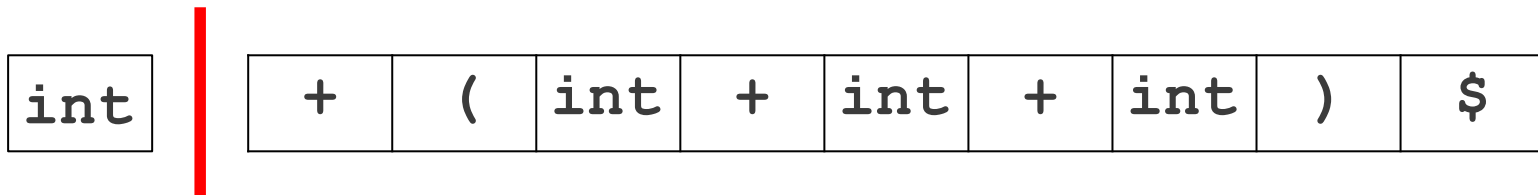
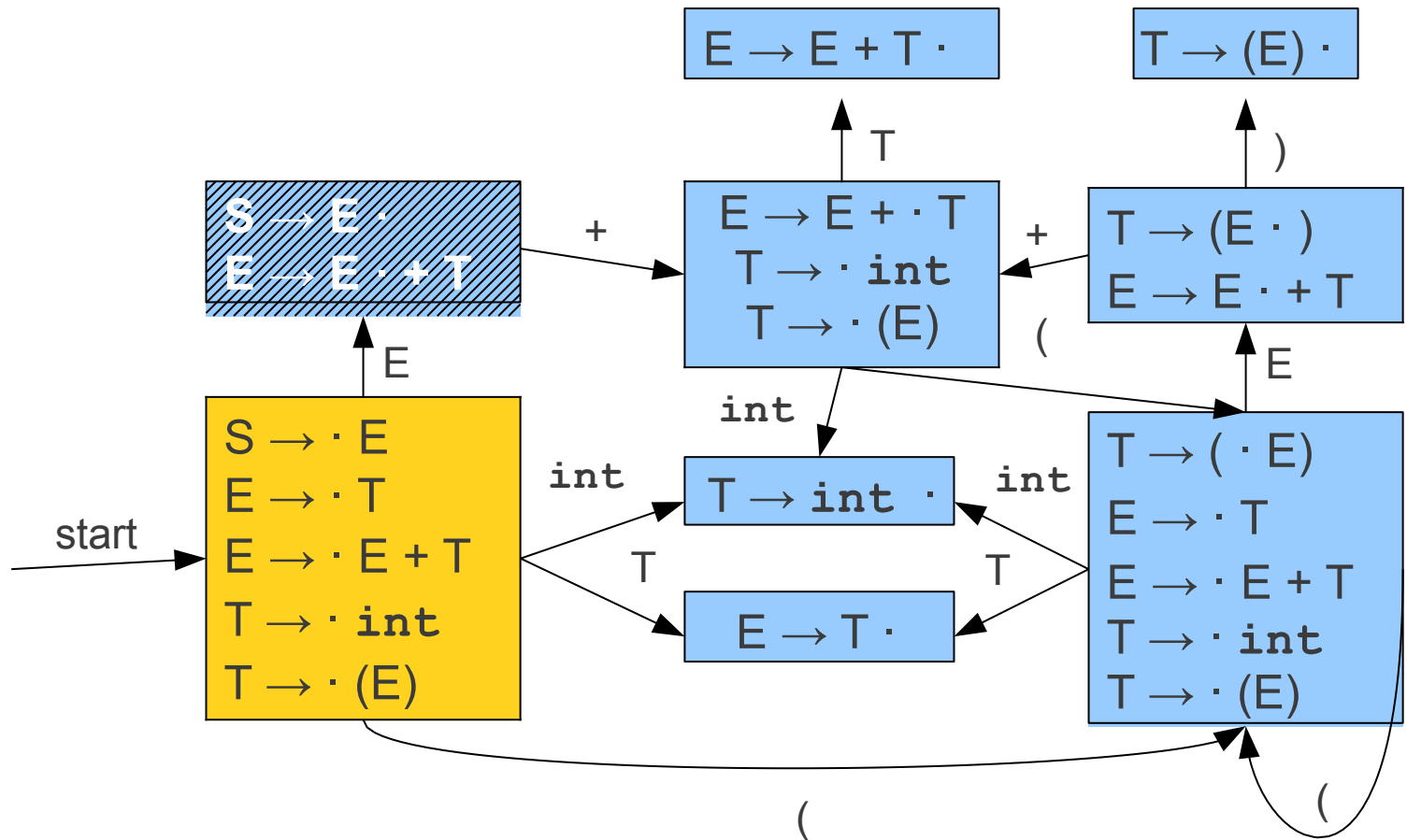


int	+	(	int	+	int	+	int	)	\$
-----	---	---	-----	---	-----	---	-----	---	----

# LR(1) Parsing: The Intuition

$S \rightarrow E$   
 $E \rightarrow T$   
 $E \rightarrow E + T$   
 $T \rightarrow \text{int}$   
 $T \rightarrow (E)$

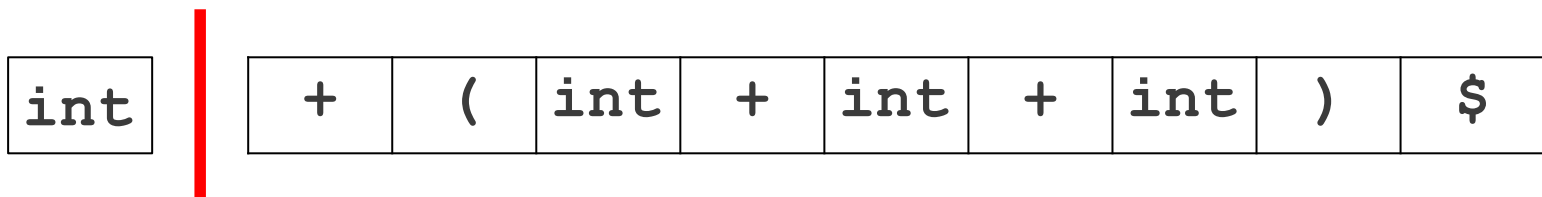
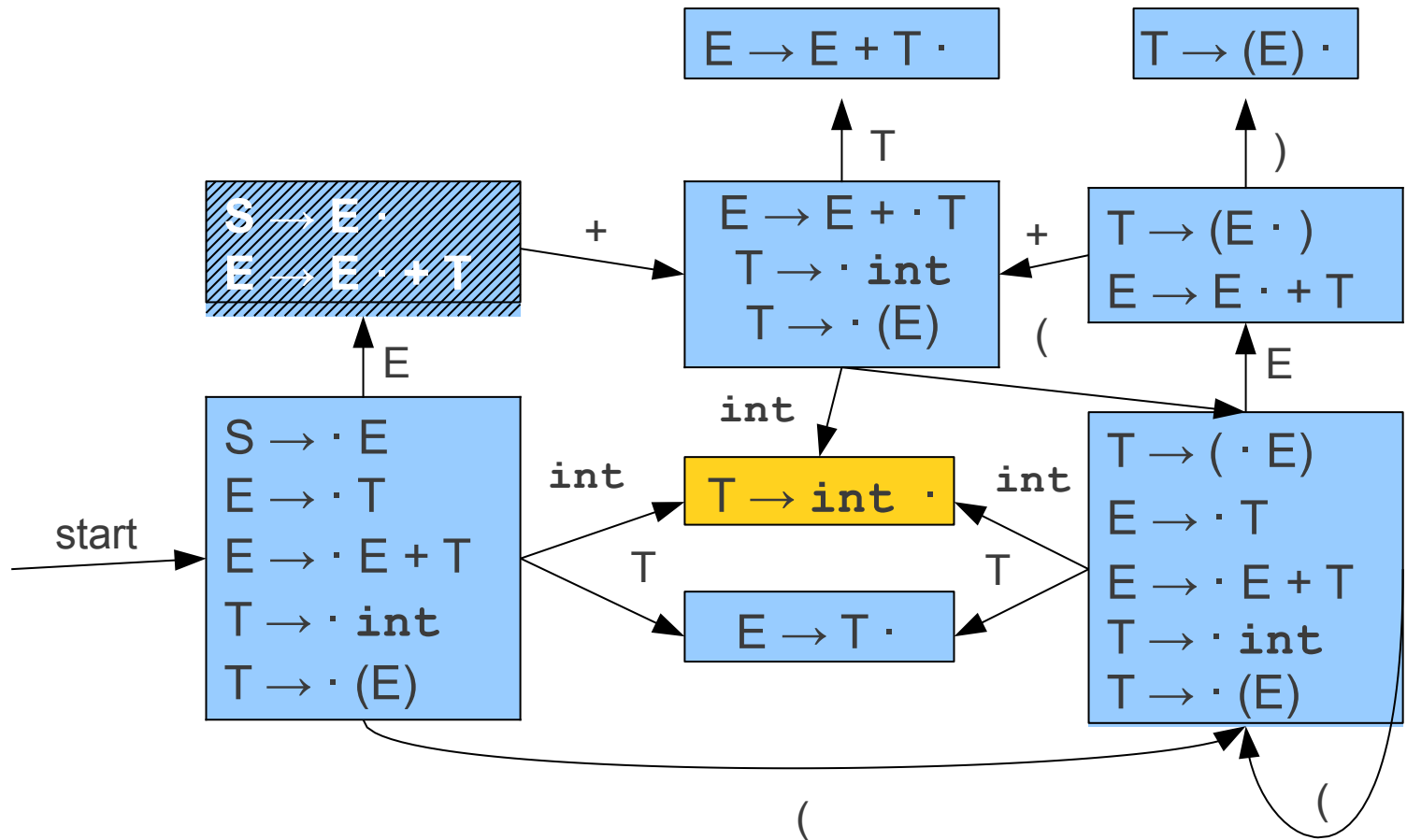
$S \rightarrow \cdot E$	\$
$E \rightarrow \cdot E + T$	\$
$E \rightarrow \cdot T$	+
$T \rightarrow \cdot \text{int}$	+



# LR(1) Parsing: The Intuition

$S \rightarrow E$   
 $E \rightarrow T$   
 $E \rightarrow E + T$   
 $T \rightarrow \text{int}$   
 $T \rightarrow (E)$

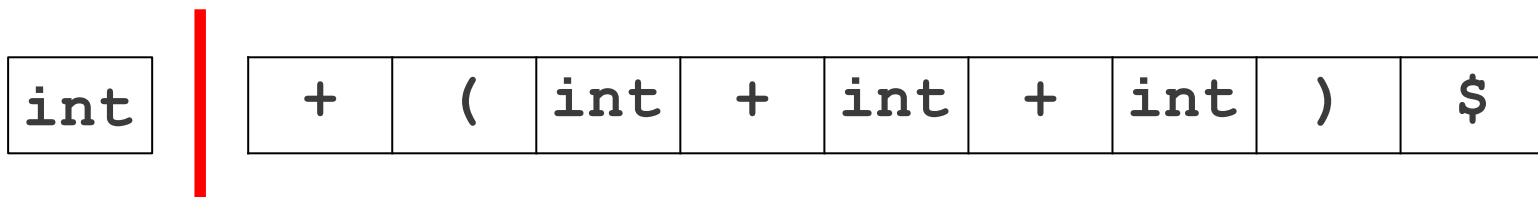
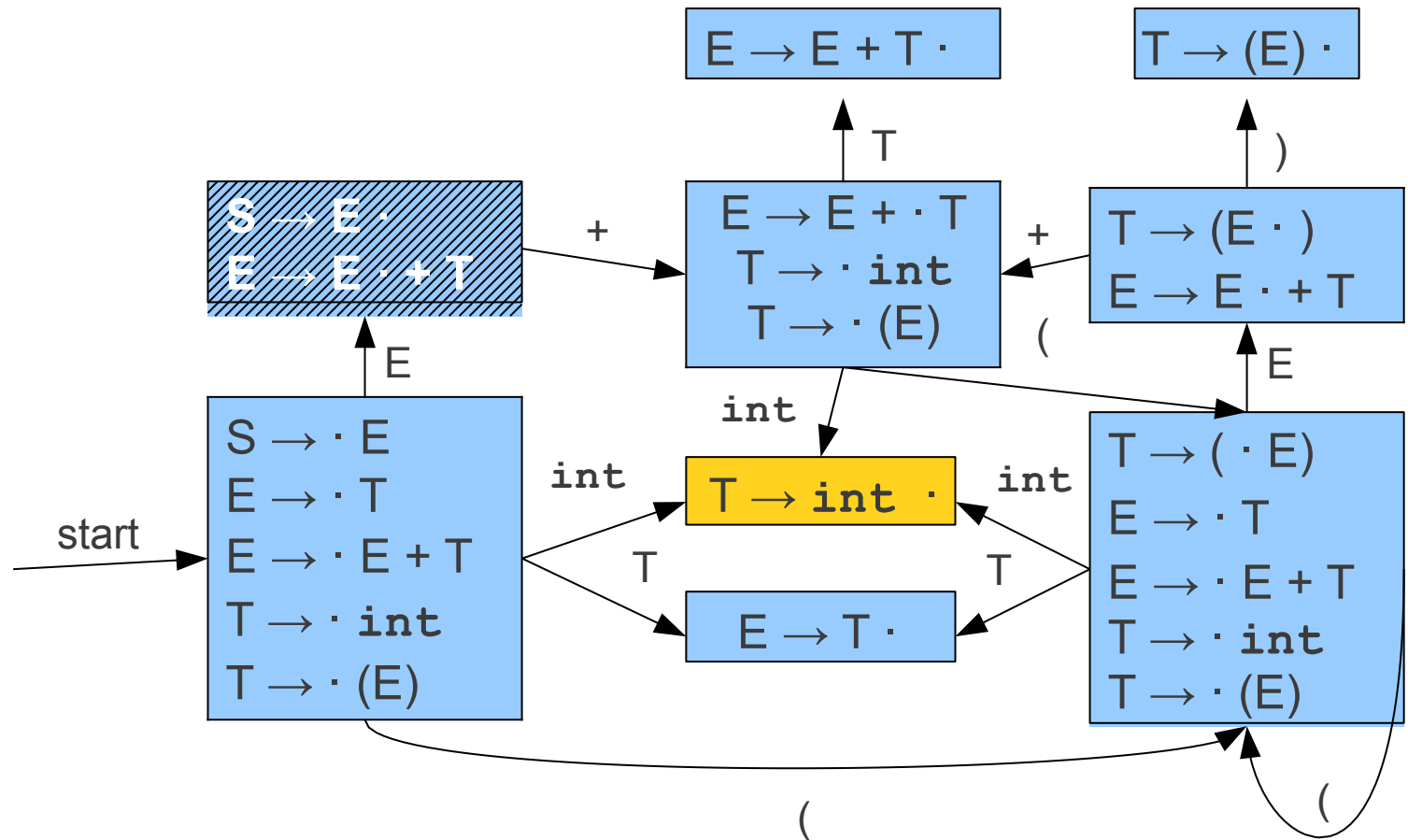
$S \rightarrow \cdot E$	\$
$E \rightarrow \cdot E + T$	\$
$E \rightarrow \cdot T$	+
$T \rightarrow \cdot \text{int}$	+



# LR(1) Parsing: The Intuition

$S \rightarrow E$   
 $E \rightarrow T$   
 $E \rightarrow E + T$   
 $T \rightarrow \text{int}$   
 $T \rightarrow (E)$

$S \rightarrow \cdot E$	\$
$E \rightarrow \cdot E + T$	\$
$E \rightarrow \cdot T$	+
$T \rightarrow \text{int} \cdot$	+

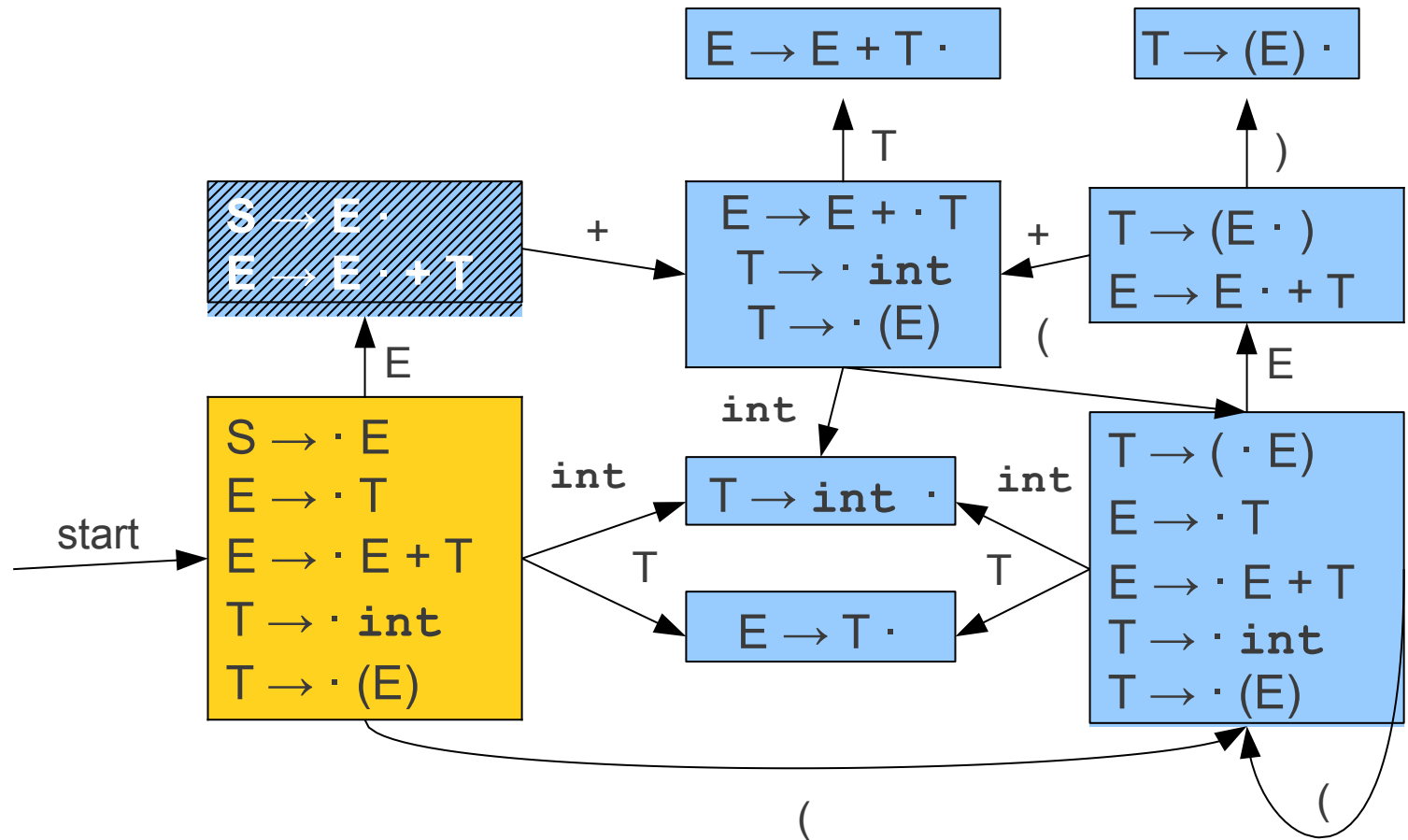




# LR(1) Parsing: The Intuition

$S \rightarrow E$   
 $E \rightarrow T$   
 $E \rightarrow E + T$   
 $T \rightarrow \text{int}$   
 $T \rightarrow (E)$

$S \rightarrow \cdot E$	\$
$E \rightarrow \cdot E + T$	\$
$E \rightarrow \cdot T$	+

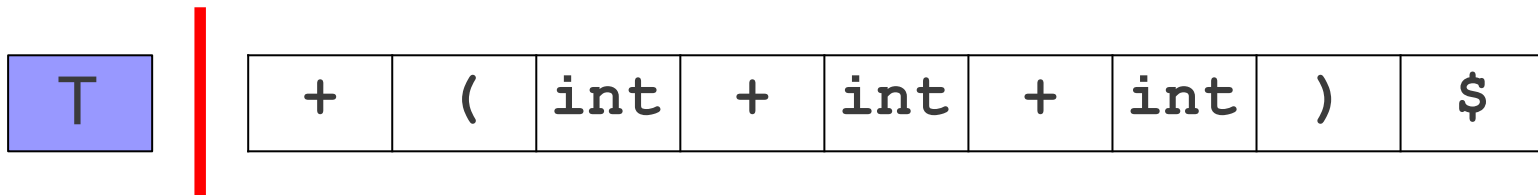
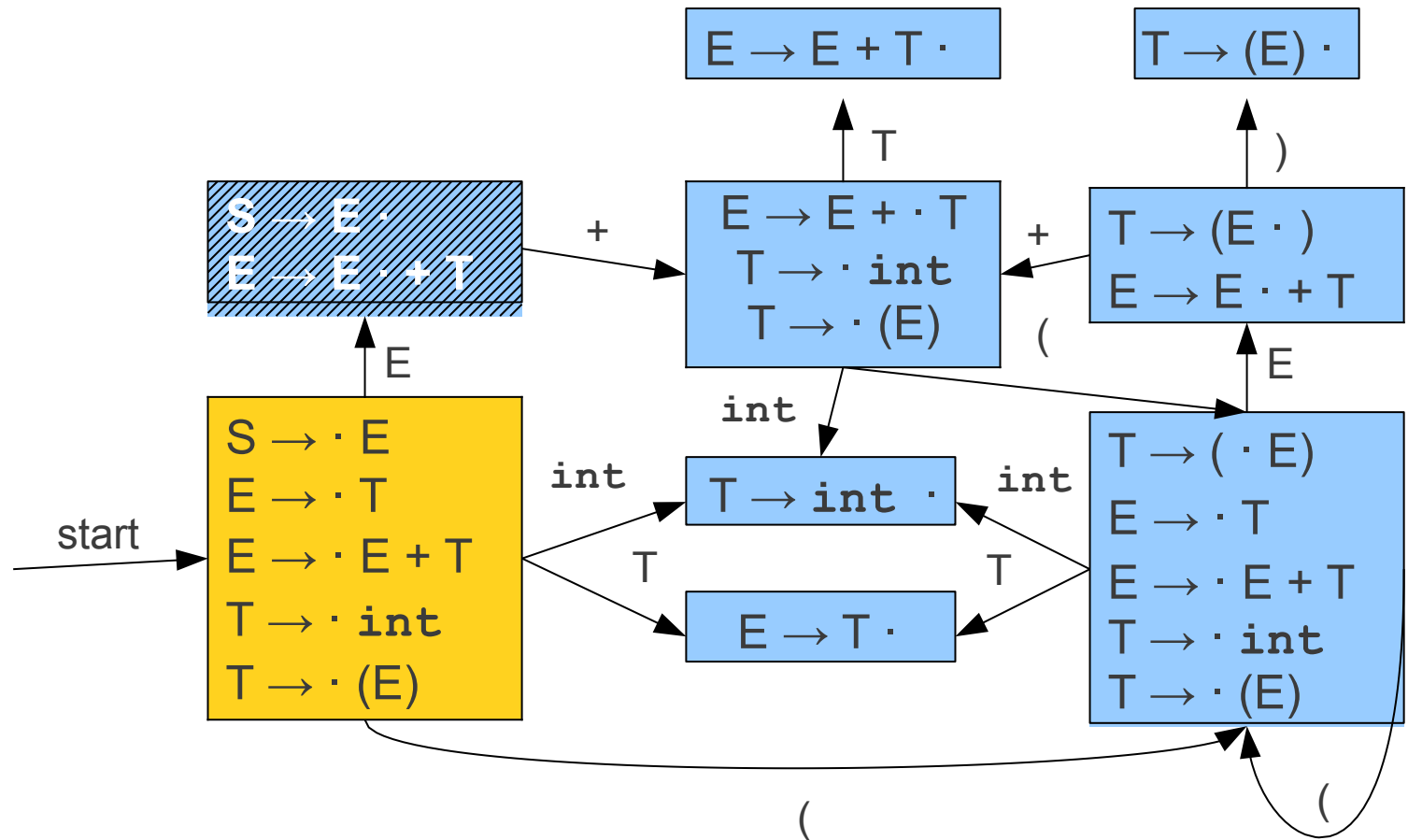


+	(	int	+	int	+	int	)	\$
---	---	-----	---	-----	---	-----	---	----

# LR(1) Parsing: The Intuition

$S \rightarrow E$   
 $E \rightarrow T$   
 $E \rightarrow E + T$   
 $T \rightarrow \text{int}$   
 $T \rightarrow (E)$

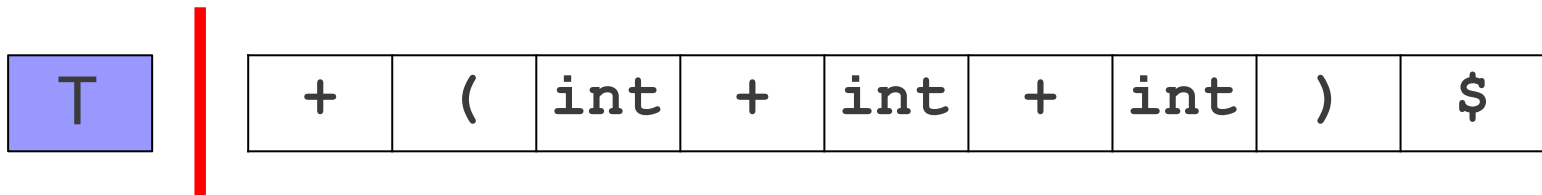
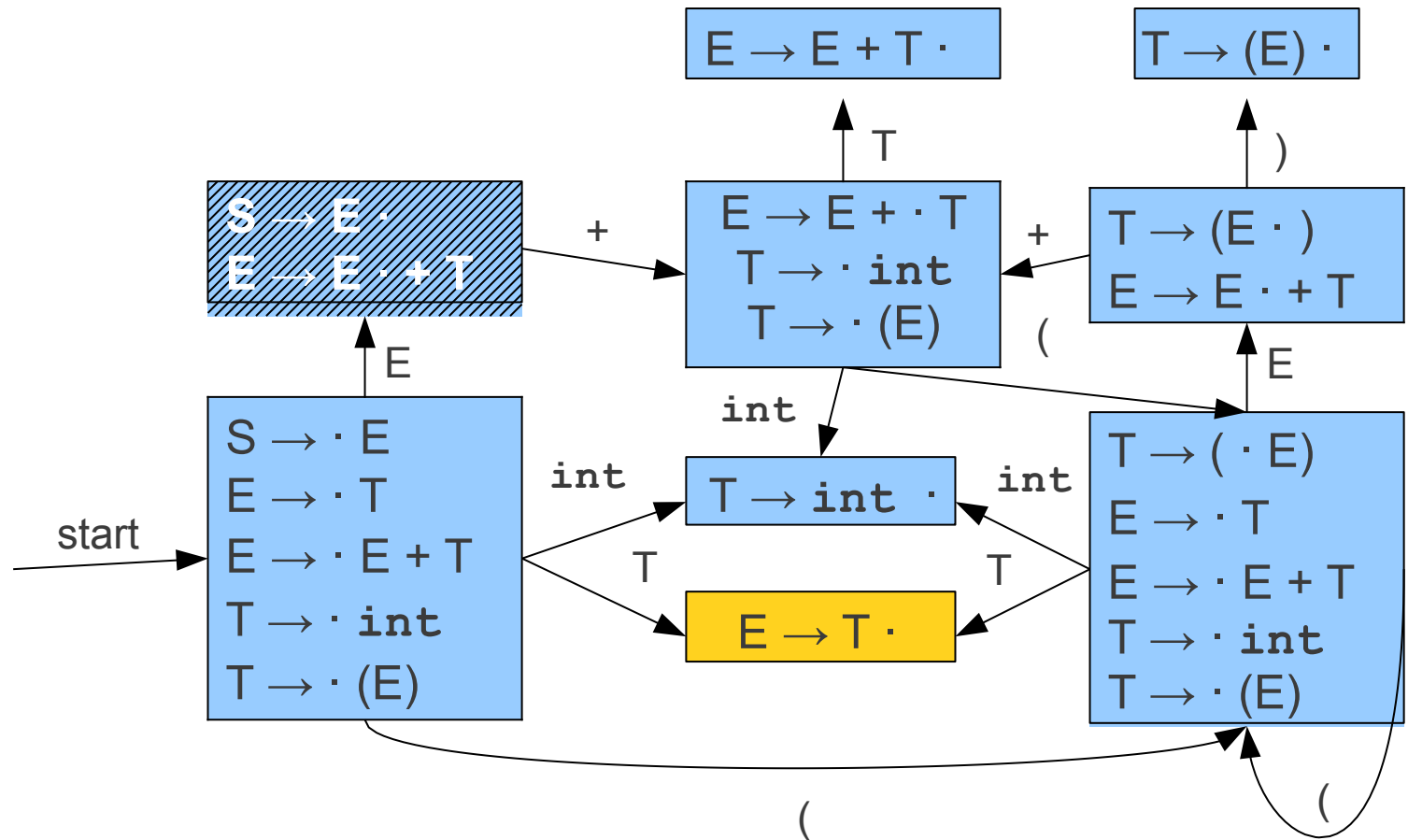
$S \rightarrow \cdot E$	\$
$E \rightarrow \cdot E + T$	\$
$E \rightarrow \cdot T$	+



# LR(1) Parsing: The Intuition

$S \rightarrow E$   
 $E \rightarrow T$   
 $E \rightarrow E + T$   
 $T \rightarrow \text{int}$   
 $T \rightarrow (E)$

$S \rightarrow \cdot E$	\$
$E \rightarrow \cdot E + T$	\$
$E \rightarrow T \cdot$	+



# LR(1) Parsing: The Intuition

$S \rightarrow E$

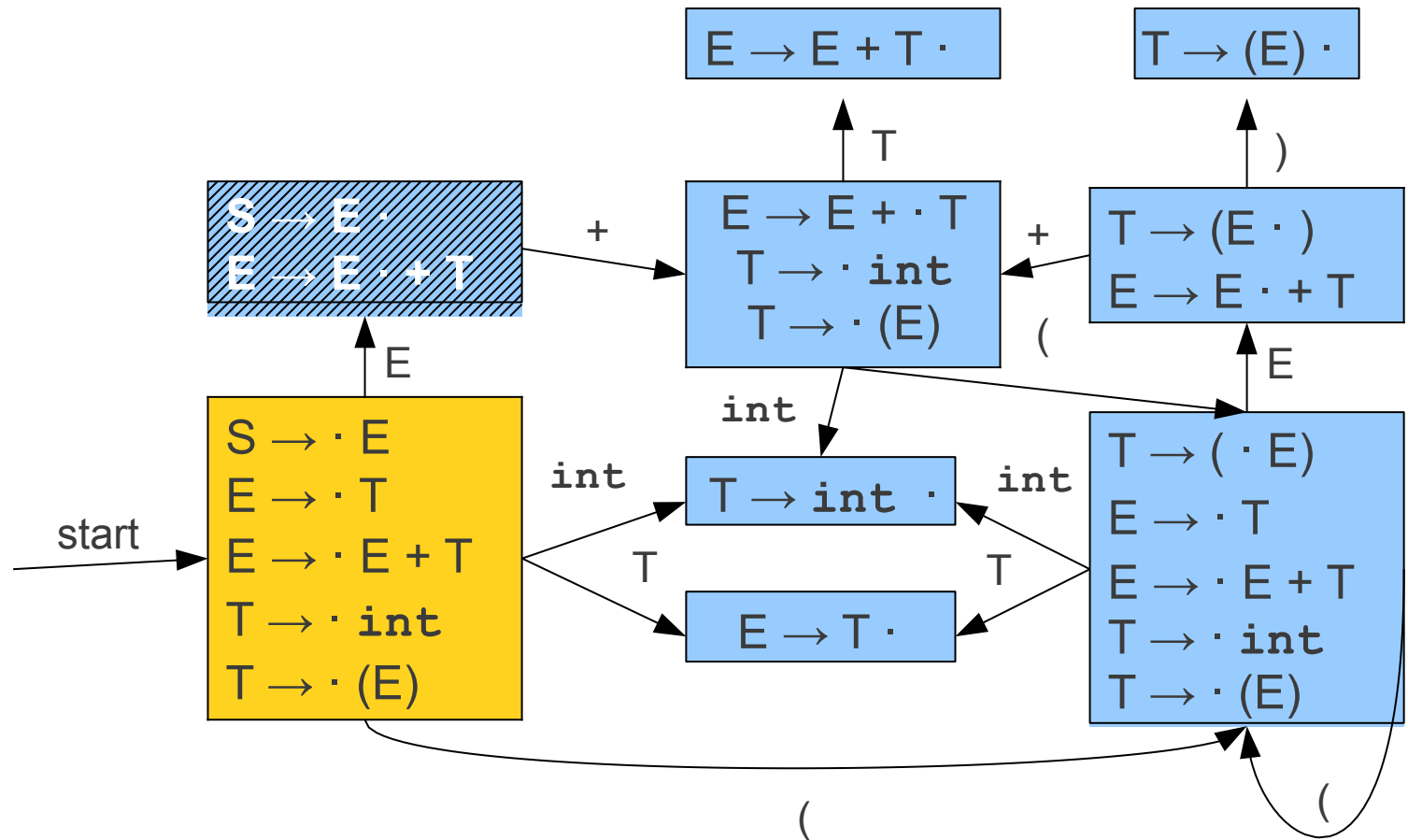
$E \rightarrow T$

$E \rightarrow E + T$

$T \rightarrow \text{int}$

$T \rightarrow (E)$

$S \rightarrow \cdot E$	\$
$E \rightarrow \cdot E + T$	\$

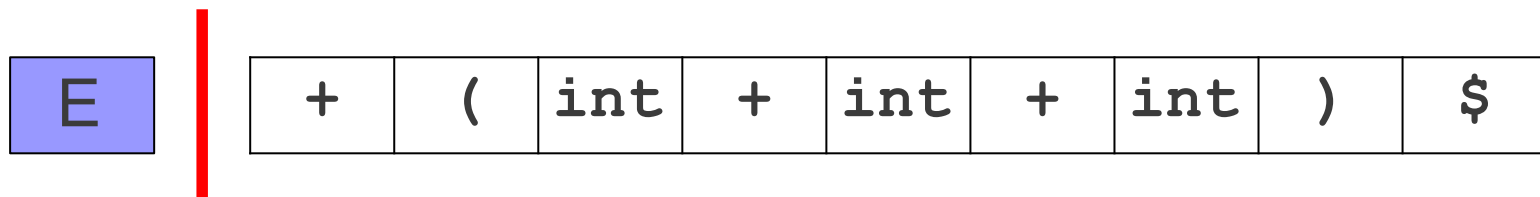
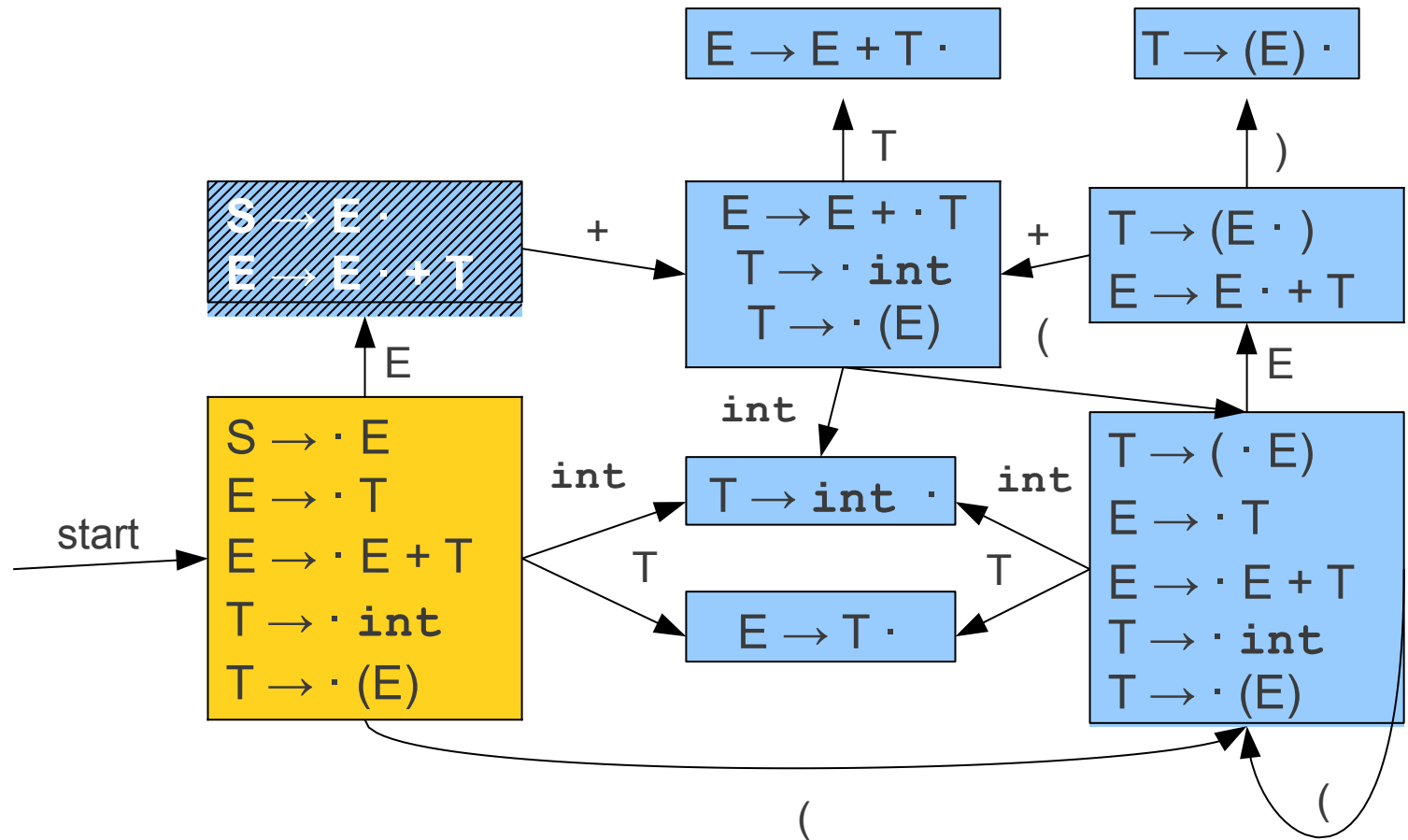


+	(	int	+	int	+	int	)	\$
---	---	-----	---	-----	---	-----	---	----

# LR(1) Parsing: The Intuition

$S \rightarrow E$   
 $E \rightarrow T$   
 $E \rightarrow E + T$   
 $T \rightarrow \text{int}$   
 $T \rightarrow (E)$

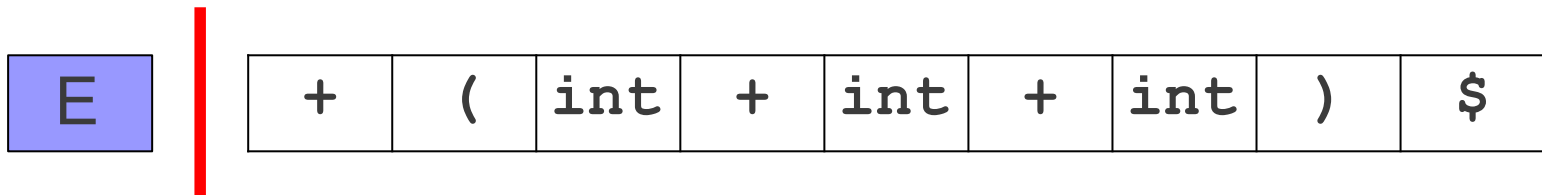
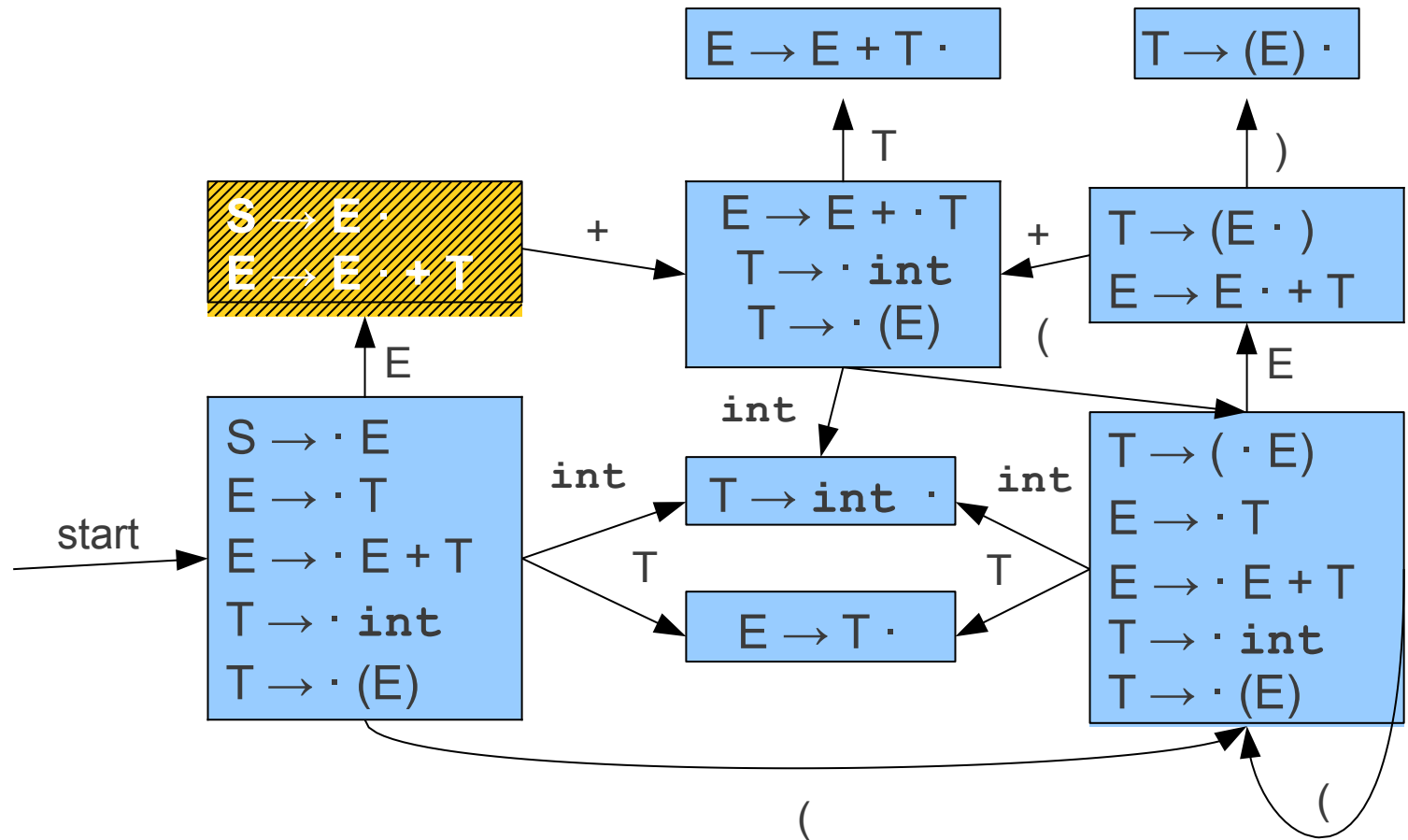
$S \rightarrow \cdot E$	\$
$E \rightarrow \cdot E + T$	\$



# LR(1) Parsing: The Intuition

$S \rightarrow E$   
 $E \rightarrow T$   
 $E \rightarrow E + T$   
 $T \rightarrow \text{int}$   
 $T \rightarrow (E)$

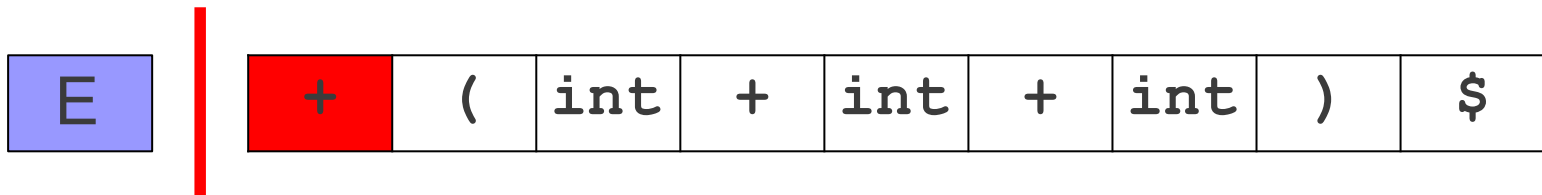
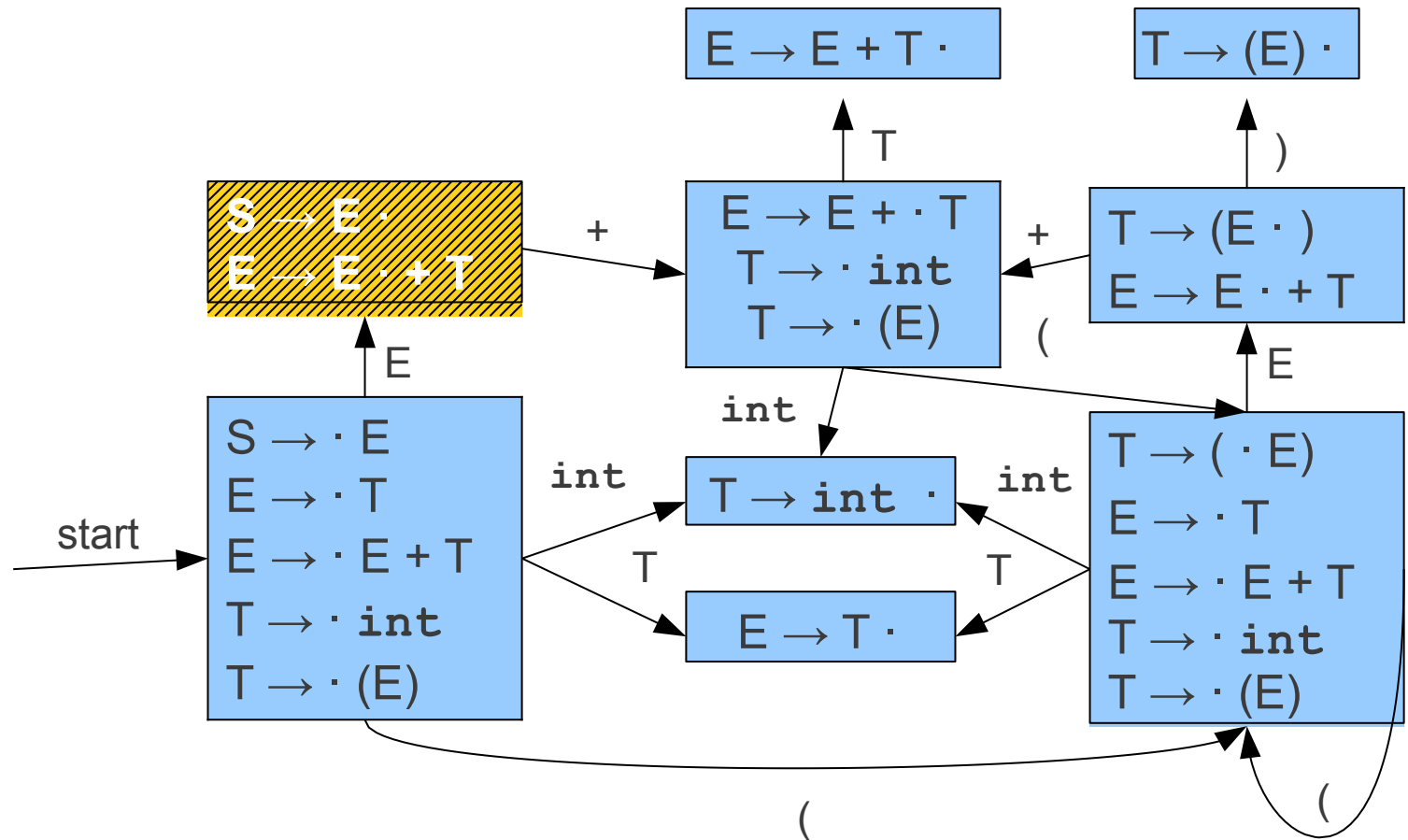
$S \rightarrow \cdot E$	\$
$E \rightarrow E \cdot + T$	\$



# LR(1) Parsing: The Intuition

$S \rightarrow E$   
 $E \rightarrow T$   
 $E \rightarrow E + T$   
 $T \rightarrow \text{int}$   
 $T \rightarrow (E)$

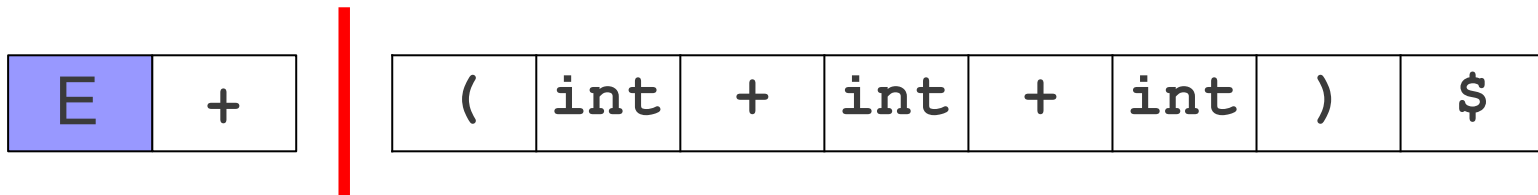
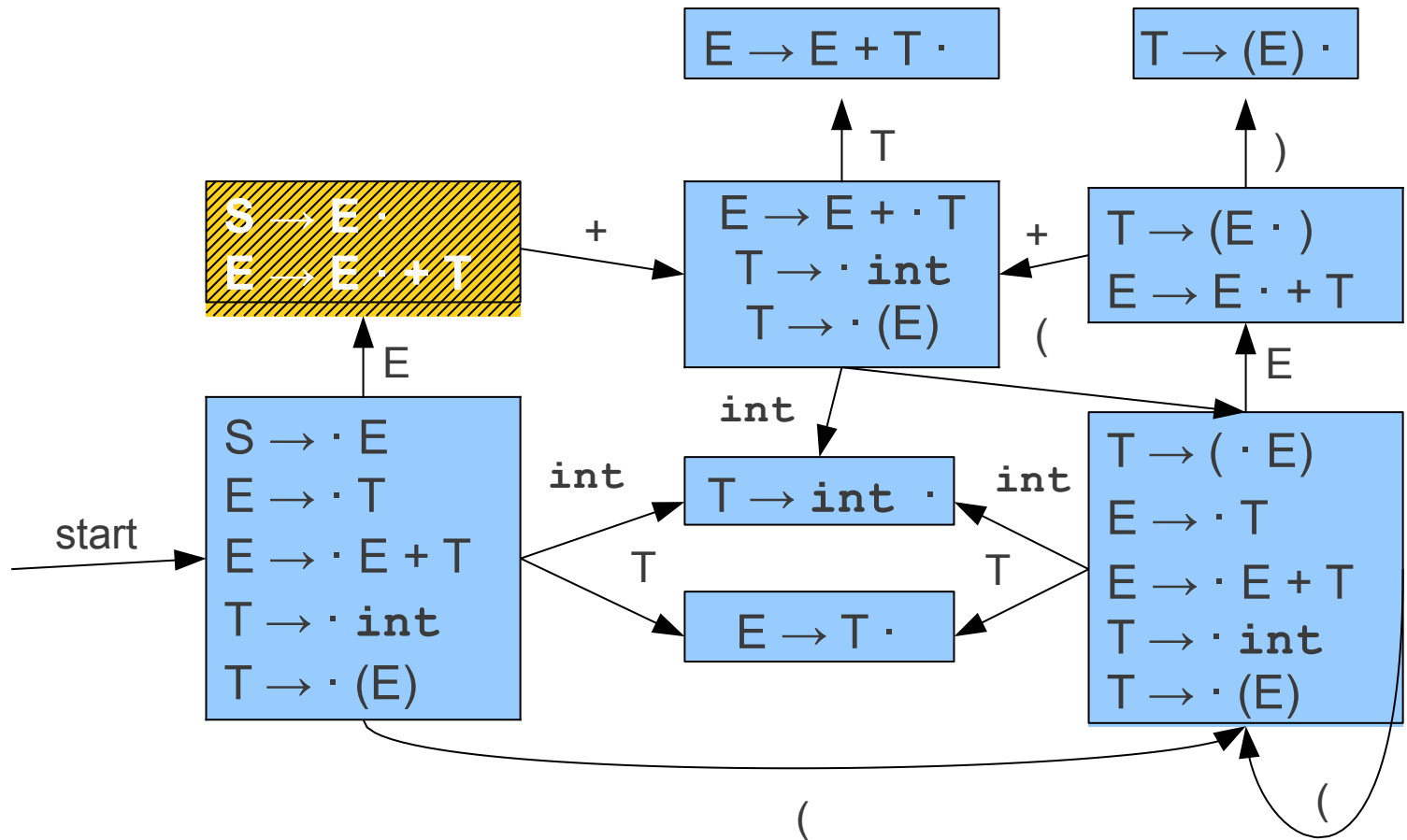
$S \rightarrow \cdot E$	\$
$E \rightarrow E \cdot + T$	\$



# LR(1) Parsing: The Intuition

$S \rightarrow E$   
 $E \rightarrow T$   
 $E \rightarrow E + T$   
 $T \rightarrow \text{int}$   
 $T \rightarrow (E)$

$S \rightarrow \cdot E$	\$
$E \rightarrow E \cdot + T$	\$

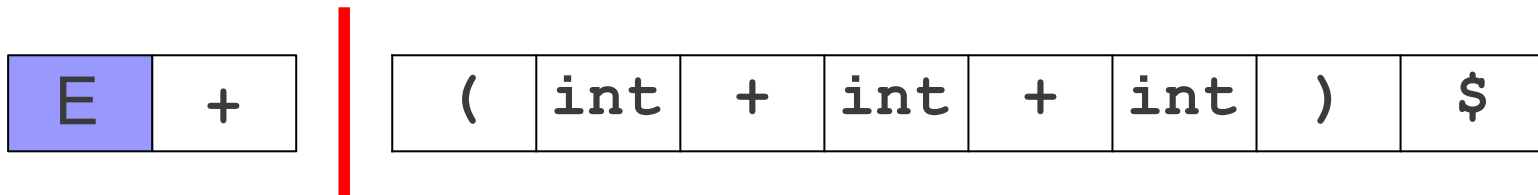
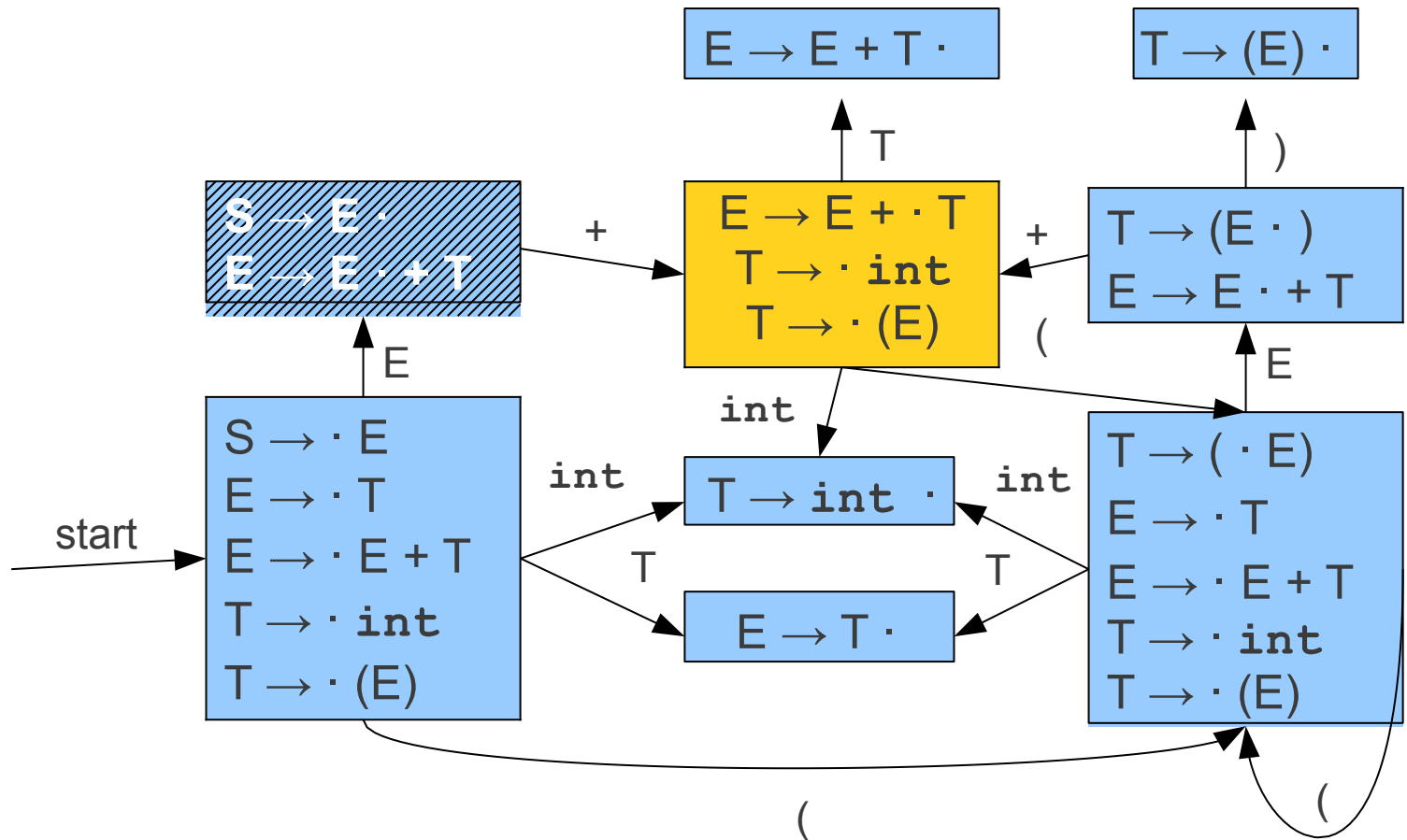




# LR(1) Parsing: The Intuition

$S \rightarrow E$   
 $E \rightarrow T$   
 $E \rightarrow E + T$   
 $T \rightarrow \text{int}$   
 $T \rightarrow (E)$

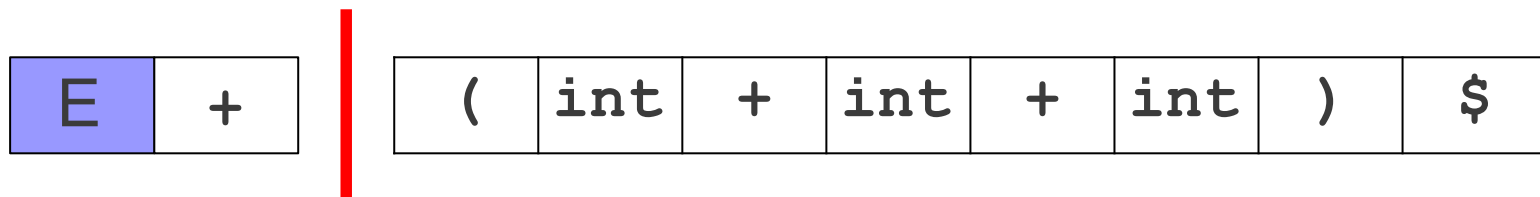
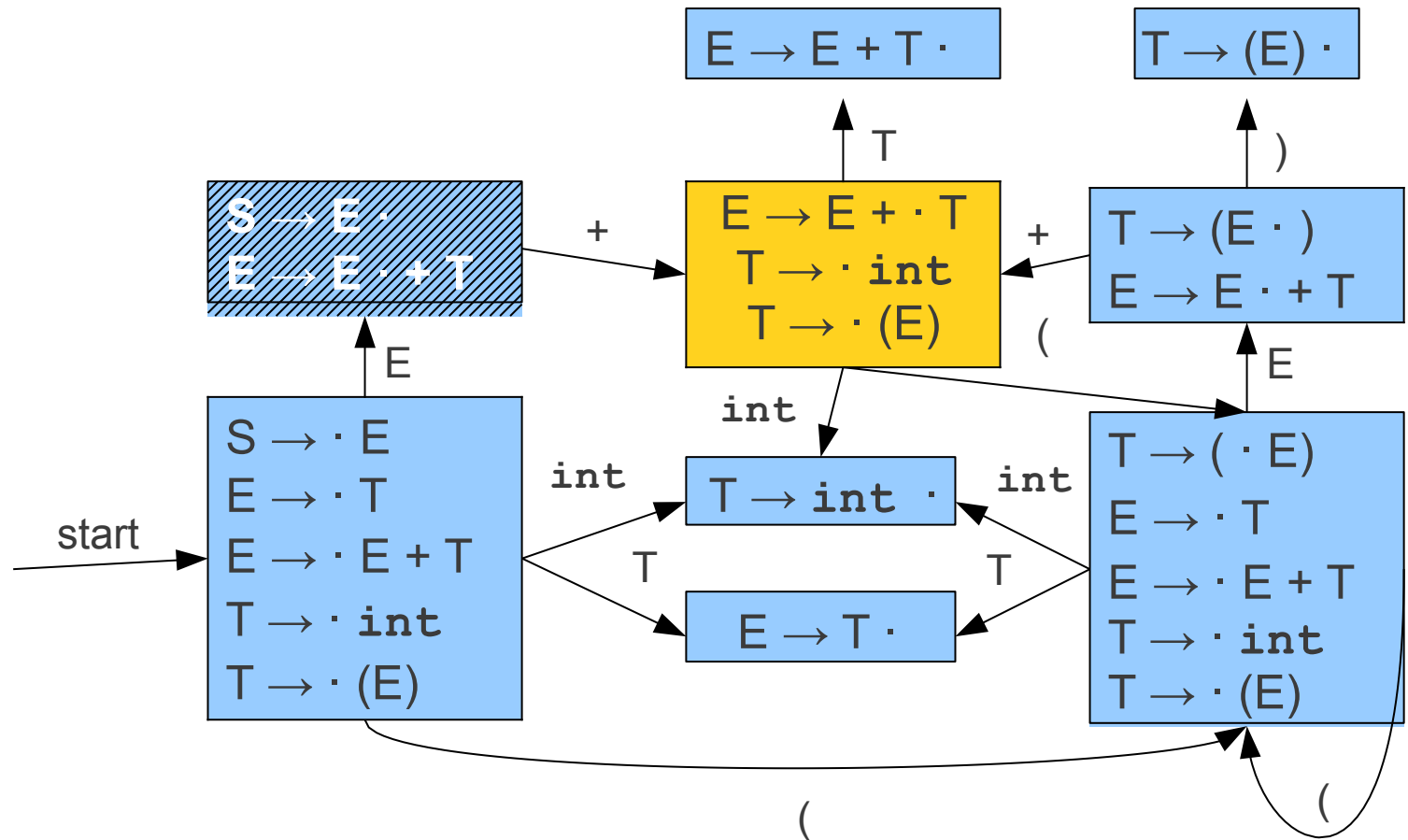
$S \rightarrow \cdot E$	\$
$E \rightarrow E + \cdot T$	\$



# LR(1) Parsing: The Intuition

$S \rightarrow E$   
 $E \rightarrow T$   
 $E \rightarrow E + T$   
 $T \rightarrow \text{int}$   
 $T \rightarrow (E)$

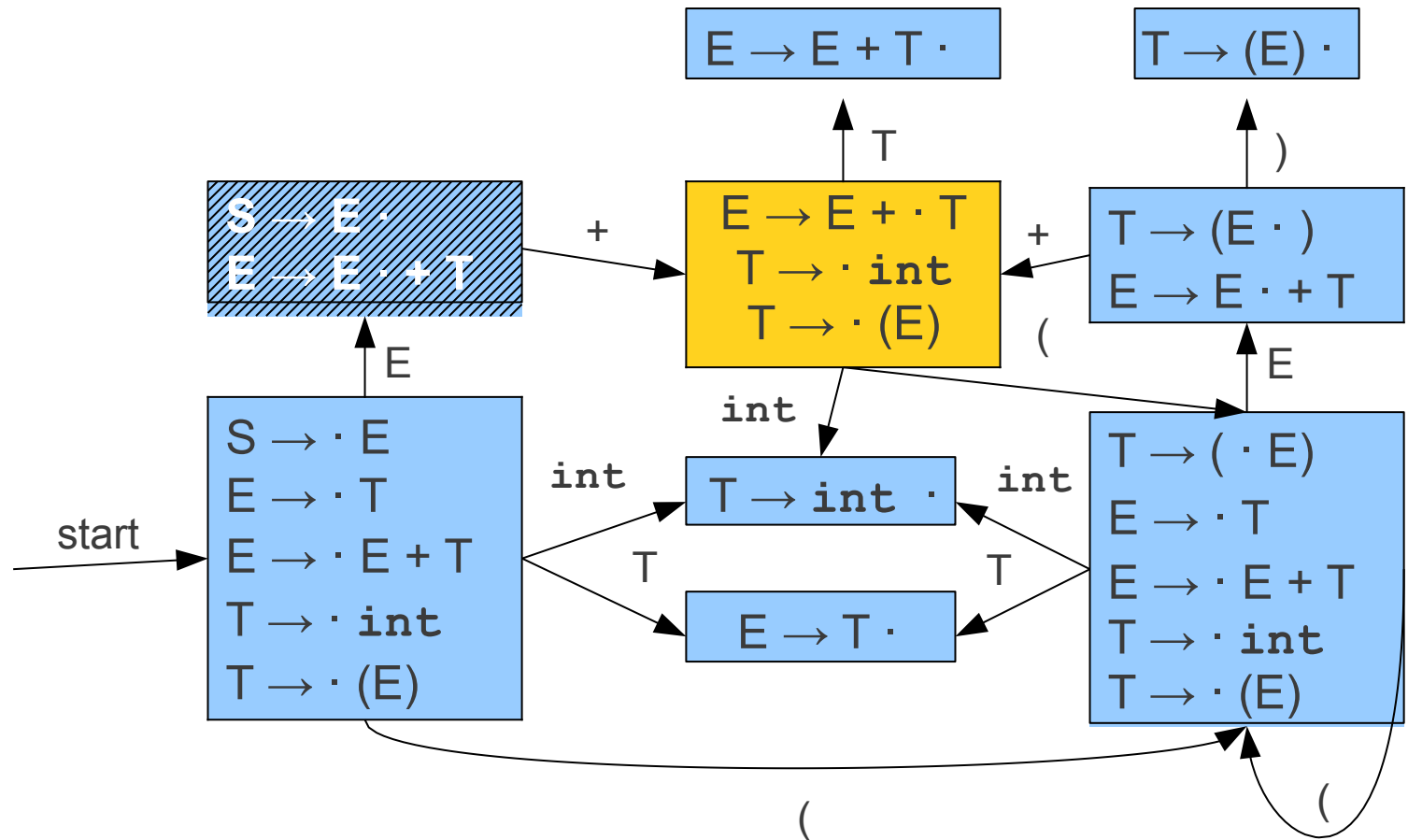
$S \rightarrow \cdot E$	\$
$E \rightarrow E + \cdot T$	\$
$T \rightarrow \cdot (E)$	\$



# LR(1) Parsing: The Intuition

$S \rightarrow E$   
 $E \rightarrow T$   
 $E \rightarrow E + T$   
 $T \rightarrow \text{int}$   
 $T \rightarrow (E)$

$S \rightarrow \cdot E$	\$
$E \rightarrow E + \cdot T$	\$
$T \rightarrow \cdot (E)$	\$



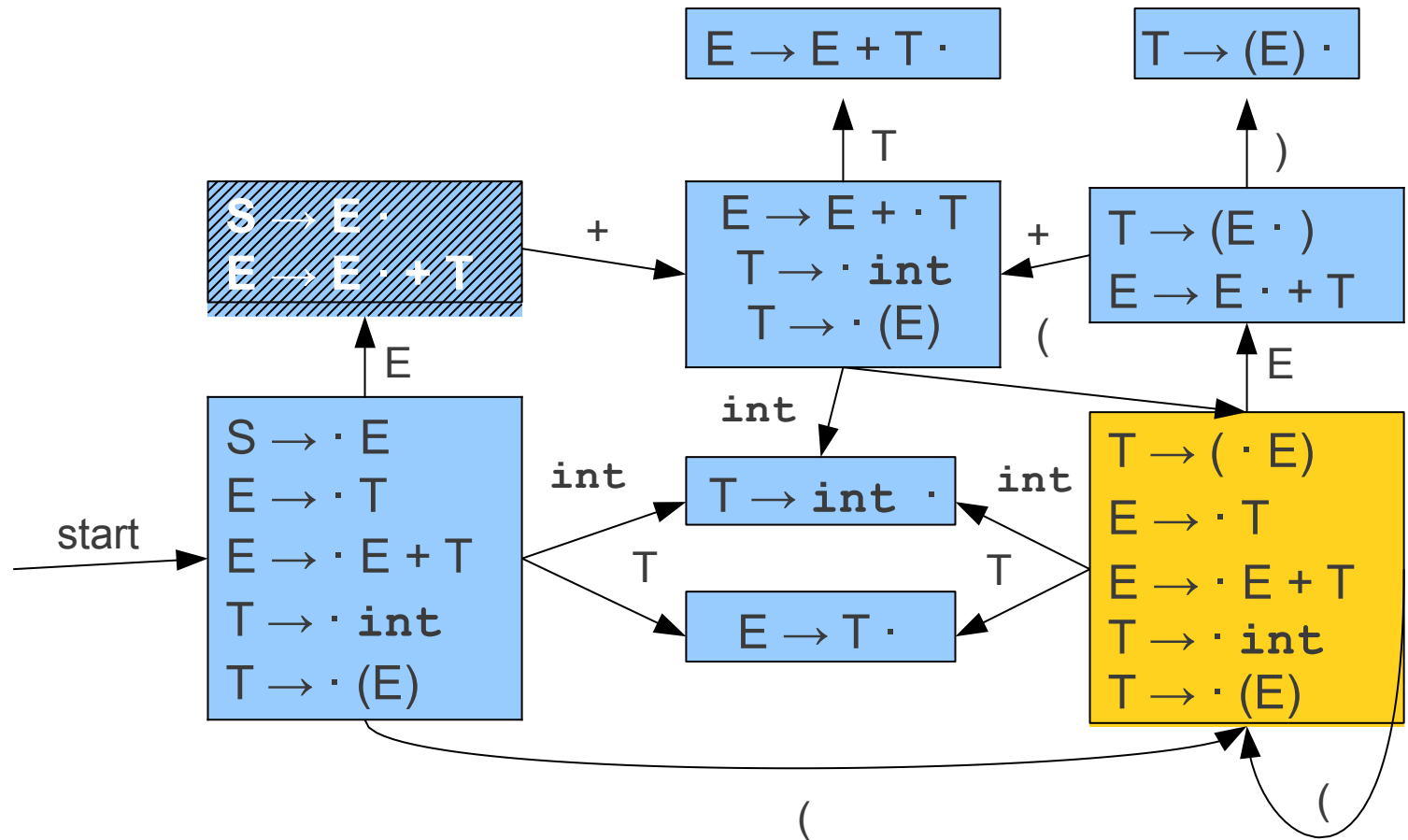
E	+	(
---	---	---

int	+	int	+	int	)	\$
-----	---	-----	---	-----	---	----

# LR(1) Parsing: The Intuition

$S \rightarrow E$   
 $E \rightarrow T$   
 $E \rightarrow E + T$   
 $T \rightarrow \text{int}$   
 $T \rightarrow (E)$

$S \rightarrow \cdot E$	\$
$E \rightarrow E + \cdot T$	\$
$T \rightarrow (\cdot E)$	\$



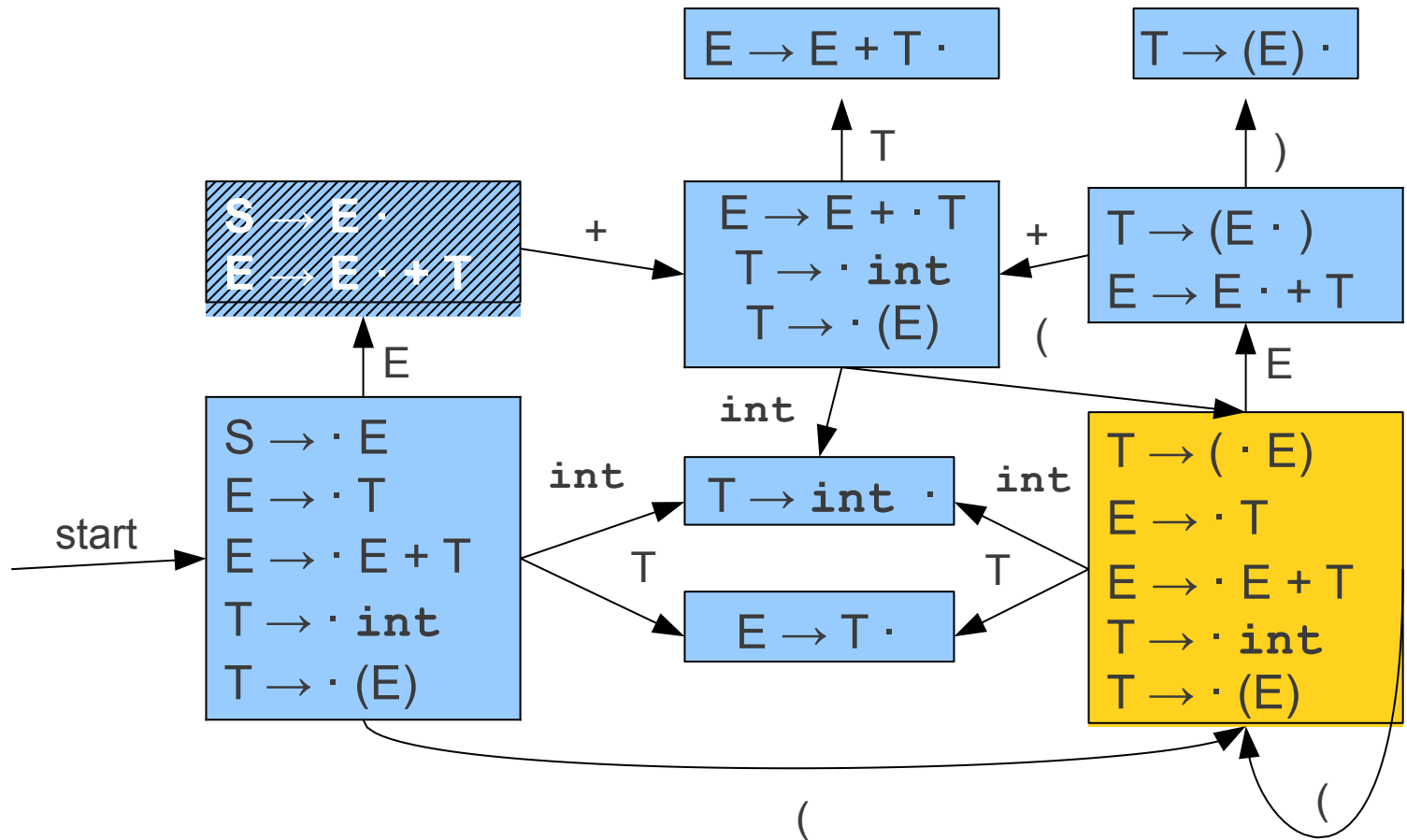
E	+	(
---	---	---

int	+	int	+	int	)	\$
-----	---	-----	---	-----	---	----

# LR(1) Parsing: The Intuition

$S \rightarrow E$   
 $E \rightarrow T$   
 $E \rightarrow E + T$   
 $T \rightarrow \text{int}$   
 $T \rightarrow (E)$

$S \rightarrow \cdot E$	\$
$E \rightarrow E + \cdot T$	\$
$T \rightarrow (\cdot E)$	\$
$E \rightarrow \cdot E + T$	)



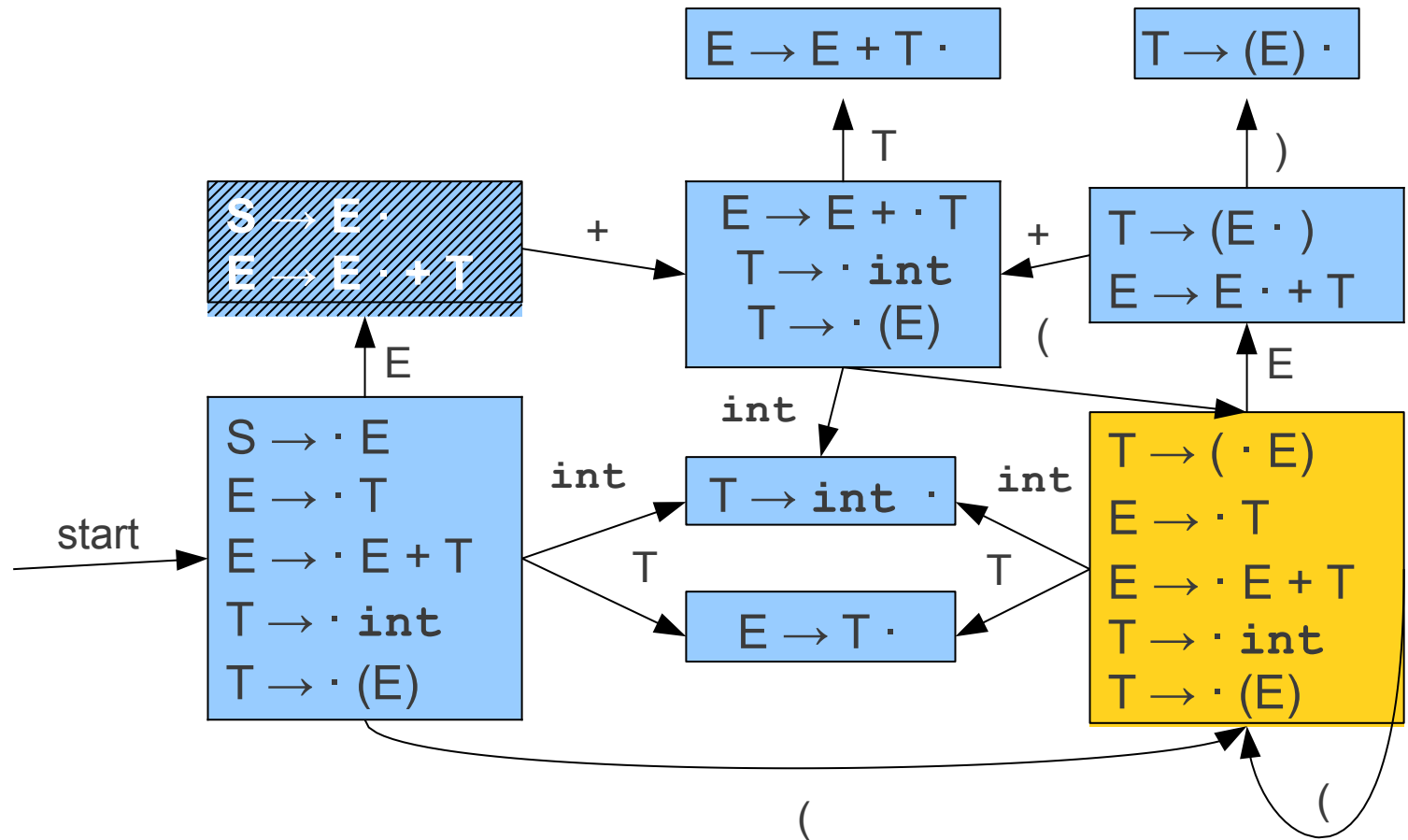
E	+	(
---	---	---

int	+	int	+	int	)	\$
-----	---	-----	---	-----	---	----

# LR(1) Parsing: The Intuition

$S \rightarrow E$   
 $E \rightarrow T$   
 $E \rightarrow E + T$   
 $T \rightarrow \text{int}$   
 $T \rightarrow (E)$

$S \rightarrow \cdot E$	\$
$E \rightarrow E + \cdot T$	\$
$T \rightarrow (\cdot E)$	\$
$E \rightarrow \cdot E + T$	)
$E \rightarrow \cdot T$	+



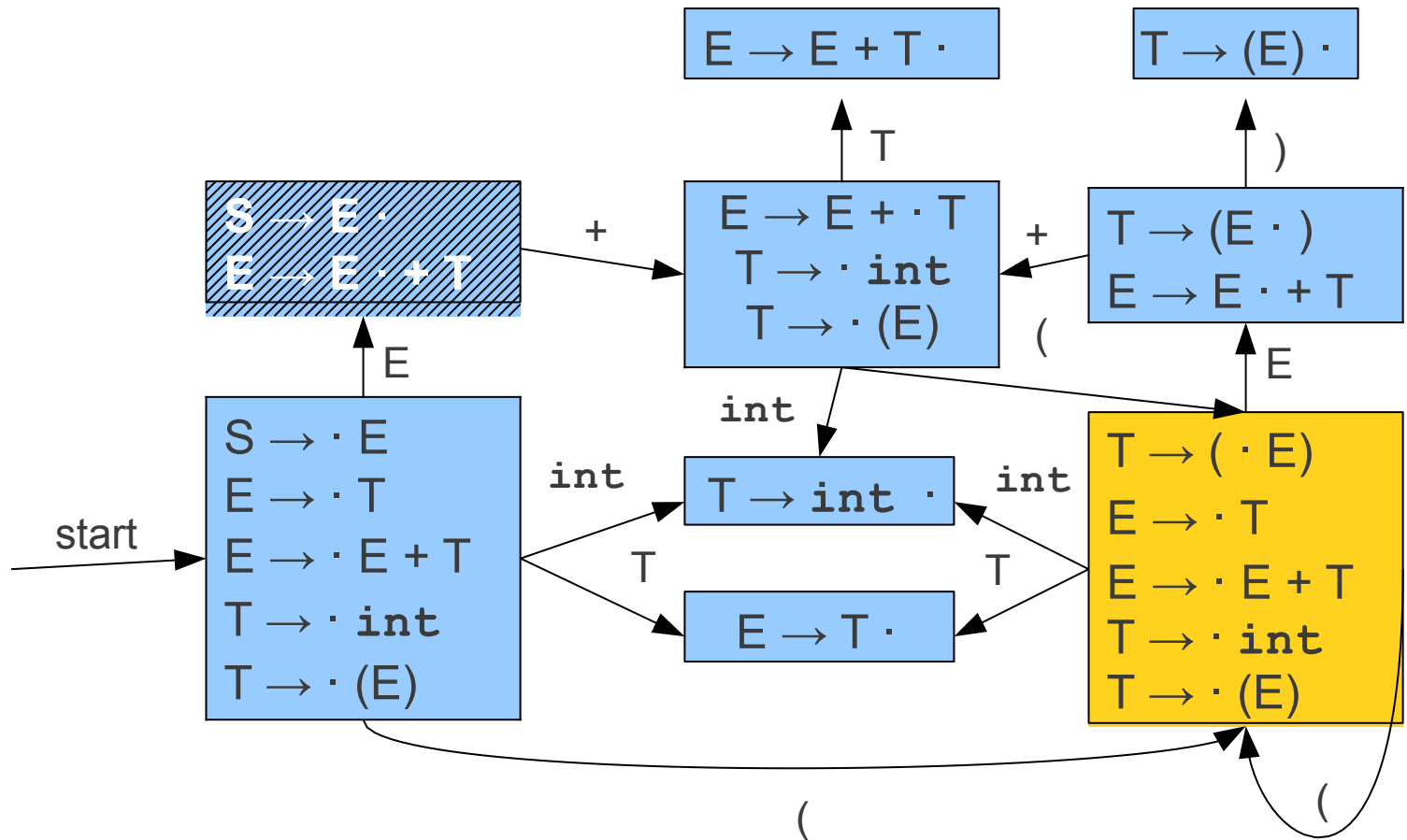
E + (

int + int + int ) \$

# LR(1) Parsing: The Intuition

$S \rightarrow E$   
 $E \rightarrow T$   
 $E \rightarrow E + T$   
 $T \rightarrow \text{int}$   
 $T \rightarrow (E)$

$S \rightarrow \cdot E$	\$
$E \rightarrow E + \cdot T$	\$
$T \rightarrow (\cdot E)$	\$
$E \rightarrow \cdot E + T$	)
$E \rightarrow \cdot T$	+
$T \rightarrow \cdot \text{int}$	+



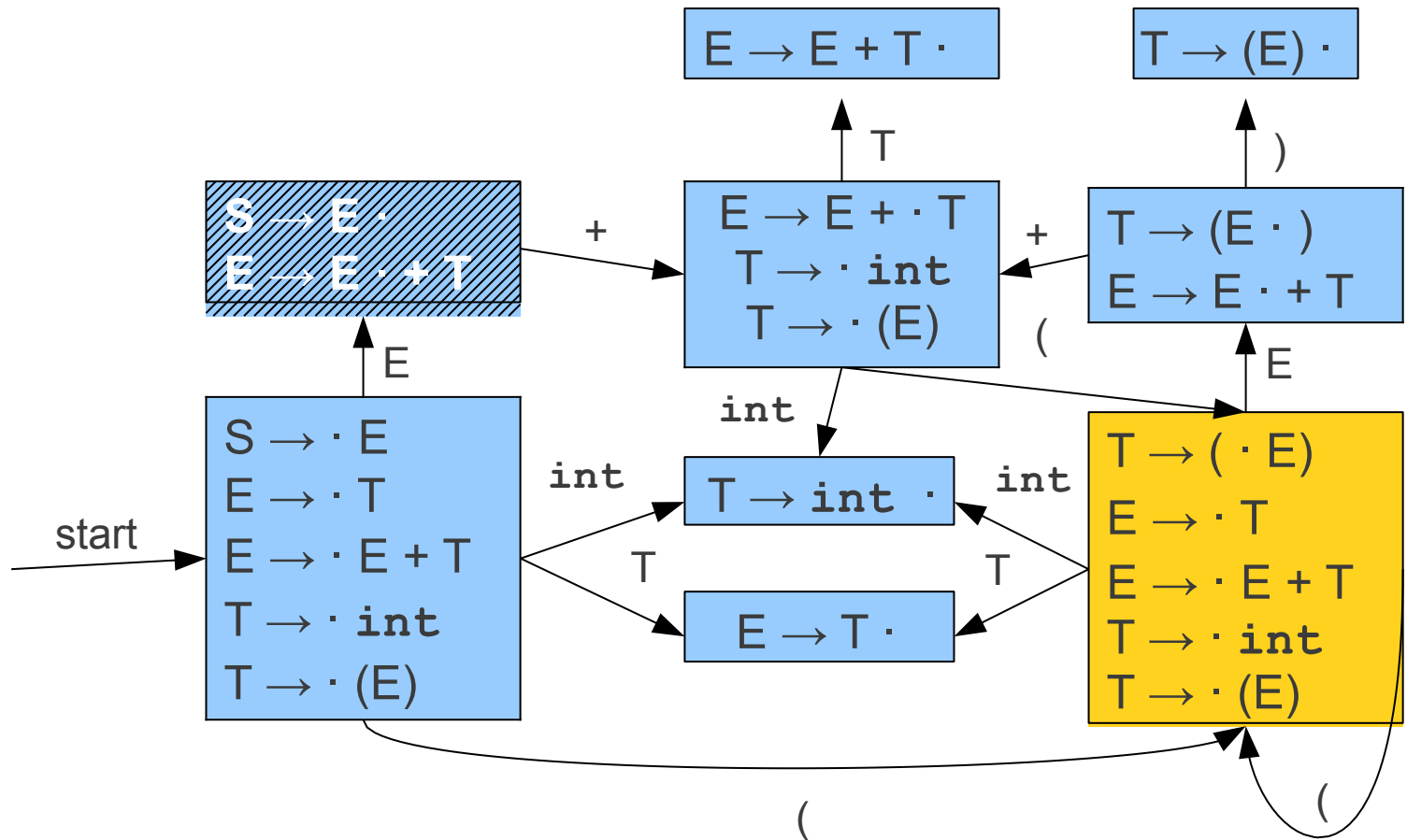
E	+	(
---	---	---

int	+	int	+	int	)	\$
-----	---	-----	---	-----	---	----

# LR(1) Parsing: The Intuition

$S \rightarrow E$   
 $E \rightarrow T$   
 $E \rightarrow E + T$   
 $T \rightarrow \text{int}$   
 $T \rightarrow (E)$

$S \rightarrow \cdot E$	\$
$E \rightarrow E + \cdot T$	\$
$T \rightarrow (\cdot E)$	\$
$E \rightarrow \cdot E + T$	)
$E \rightarrow \cdot T$	+
$T \rightarrow \cdot \text{int}$	+



E	+	(	int
---	---	---	-----

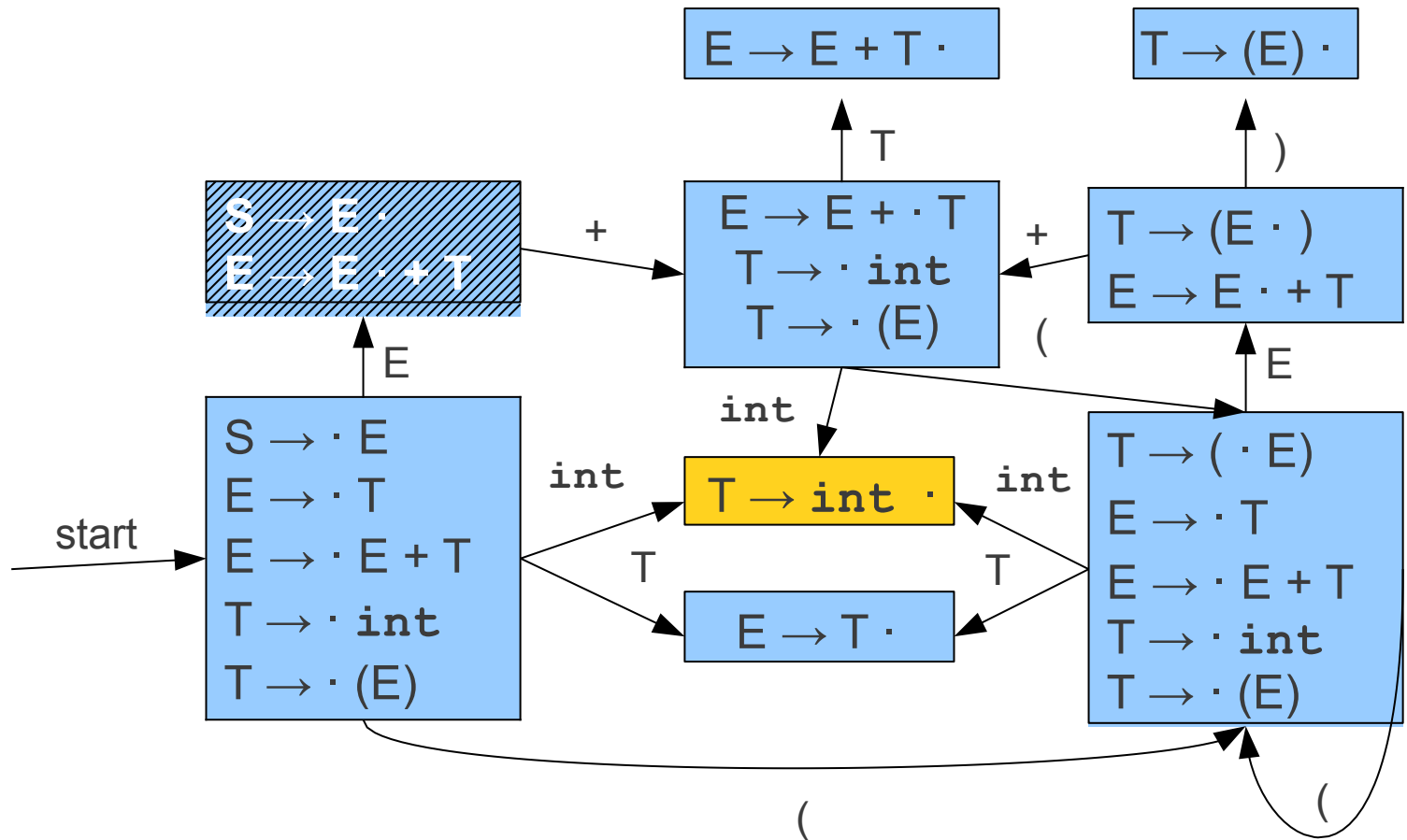
+	int	+	int	)	\$
---	-----	---	-----	---	----



# LR(1) Parsing: The Intuition

$S \rightarrow E$   
 $E \rightarrow T$   
 $E \rightarrow E + T$   
 $T \rightarrow \text{int}$   
 $T \rightarrow (E)$

$S \rightarrow \cdot E$	\$
$E \rightarrow E + \cdot T$	\$
$T \rightarrow (\cdot E)$	\$
$E \rightarrow \cdot E + T$	)
$E \rightarrow \cdot T$	+
$T \rightarrow \text{int} \cdot$	+



E + ( int

+ int + int ) \$

# The Intuition behind LR(1)

- Guess which series of productions we are reversing.
- Use this information to maintain what lookahead to expect.
- When deciding whether to shift or reduce, use lookahead to disambiguate.
- Construct an automaton to track lookaheads!

# Tracking Lookaheads

- How do we know what lookahead to expect at each state?
- Observation:
  - There are only finitely many productions we can be in at any point.
  - There are only finitely many positions we can be in each production.
  - **There are only finitely many lookahead sets** at each point.

Construct an automaton to track lookaheads!

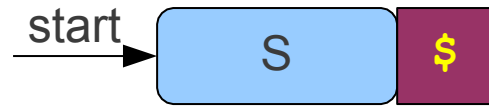
# Constructing LR(1) Automata

**S** → **E**  
**E** → **T**  
**E** → **E** + **T**  
**T** → *int*  
**T** → (**E**)

# Constructing LR(1) Automata

- Begin with a state **S** [**\$**].

# Constructing LR(1) Automata



**S** → **E**  
**E** → **T**  
**E** → **E** + **T**  
**T** → **int**  
**T** → (**E**)

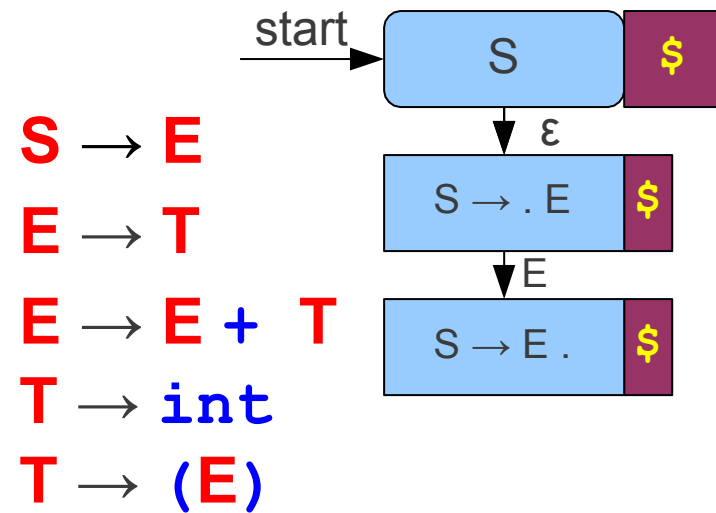
# Constructing LR(1) Automata

Begin with a state **S** [**\$**].

For each state **A** [**t**], for each production **A**  $\rightarrow \gamma$ :

1. Construct states **A**  $\rightarrow \alpha \cdot \omega$  [**t**] for all possible ways of splitting  $\gamma = \alpha\omega$ .
2. Add an  $\epsilon$ -transition from **A** [**t**] to each **A**  $\rightarrow \cdot \omega$  [**t**].
3. Add transitions on **x** between **A**  $\rightarrow \alpha \cdot x\omega$  [**t**] and **A**  $\rightarrow \alpha x \cdot \omega$  [**t**].

# Constructing LR(1) Automata





# Constructing LR(1) Automata

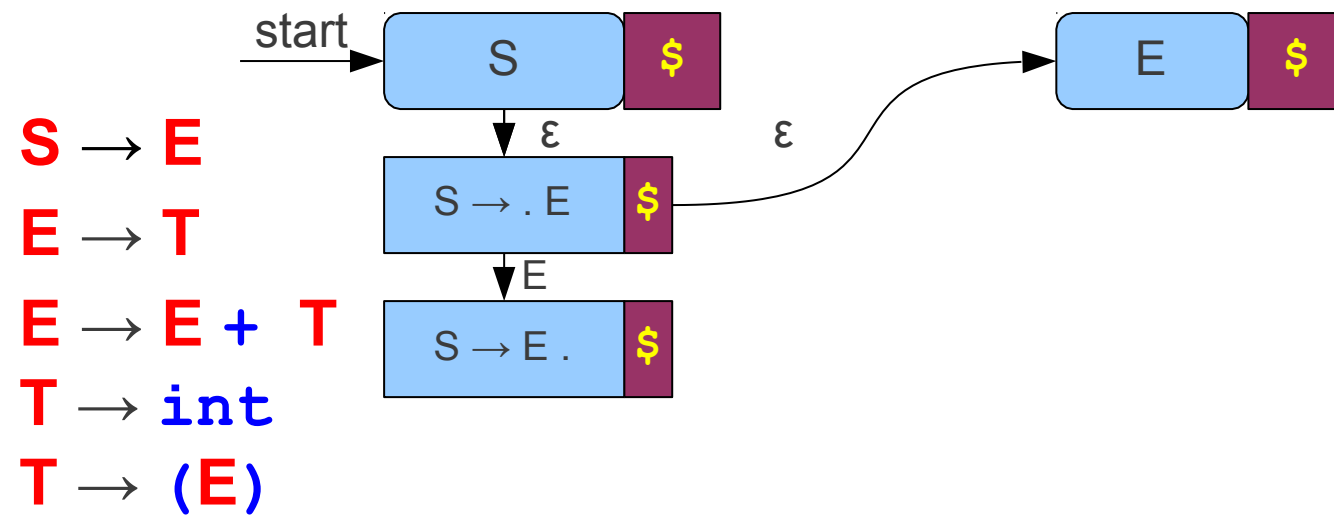
Begin with a state **S** [**\$**].

For each state **A** [**t**], for each production **A**  $\rightarrow$   $\gamma$ :

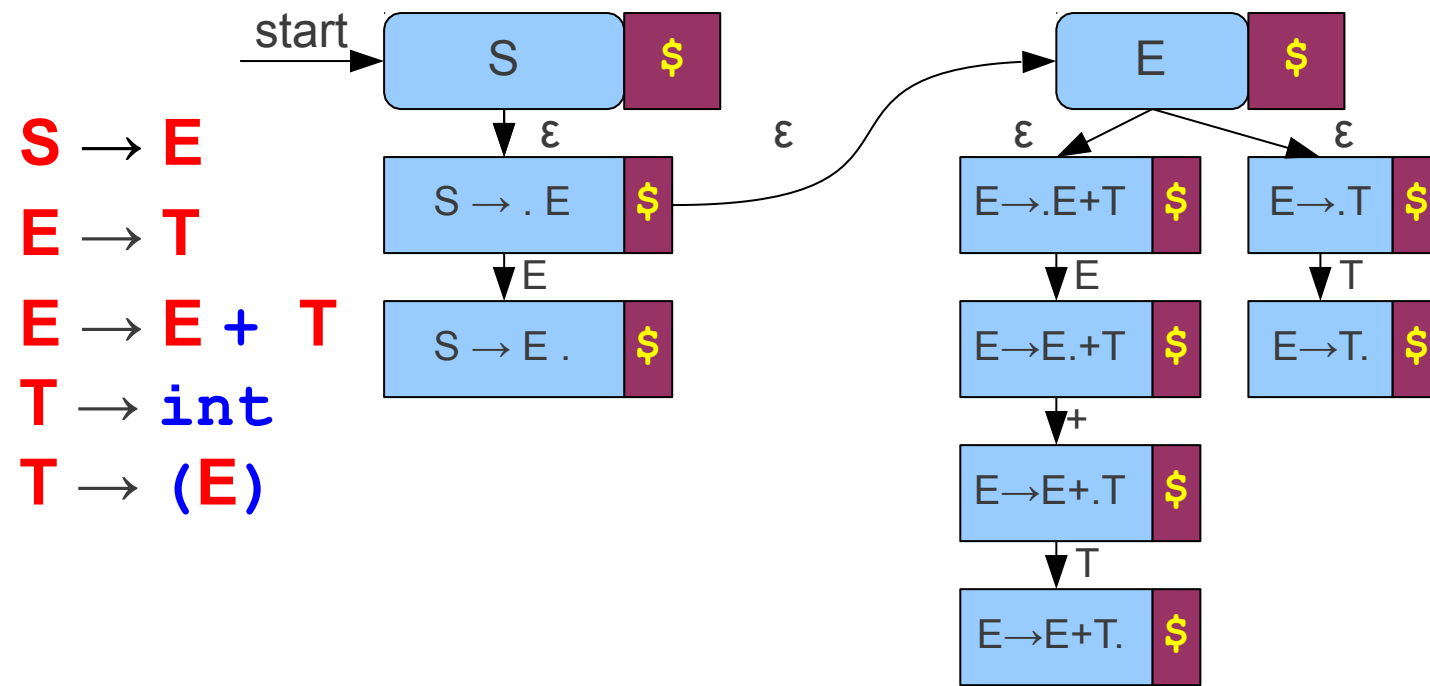
1. Construct states **A**  $\rightarrow$   $\alpha \cdot \omega$  [**t**] for all possible ways of splitting  $\gamma = \alpha\omega$ .
2. Add an  $\epsilon$ -transition from **A** [**t**] to each **A**  $\rightarrow$   $\cdot \omega$  [**t**].
3. Add transitions on **x** between **A**  $\rightarrow$   $\alpha \cdot \mathbf{x}\omega$  [**t**] and **A**  $\rightarrow$   $\alpha\mathbf{x} \cdot \omega$  [**t**].

For each state **A**  $\rightarrow$   $\alpha \cdot \mathbf{B}\omega$  [**t**], add an  $\epsilon$ -transition from **A**  $\rightarrow$   $\alpha \cdot \mathbf{B}\omega$  [**t**] to **B** [**r**] for each terminal **r**  $\in \text{FIRST}^*(\omega\mathbf{t})$ .

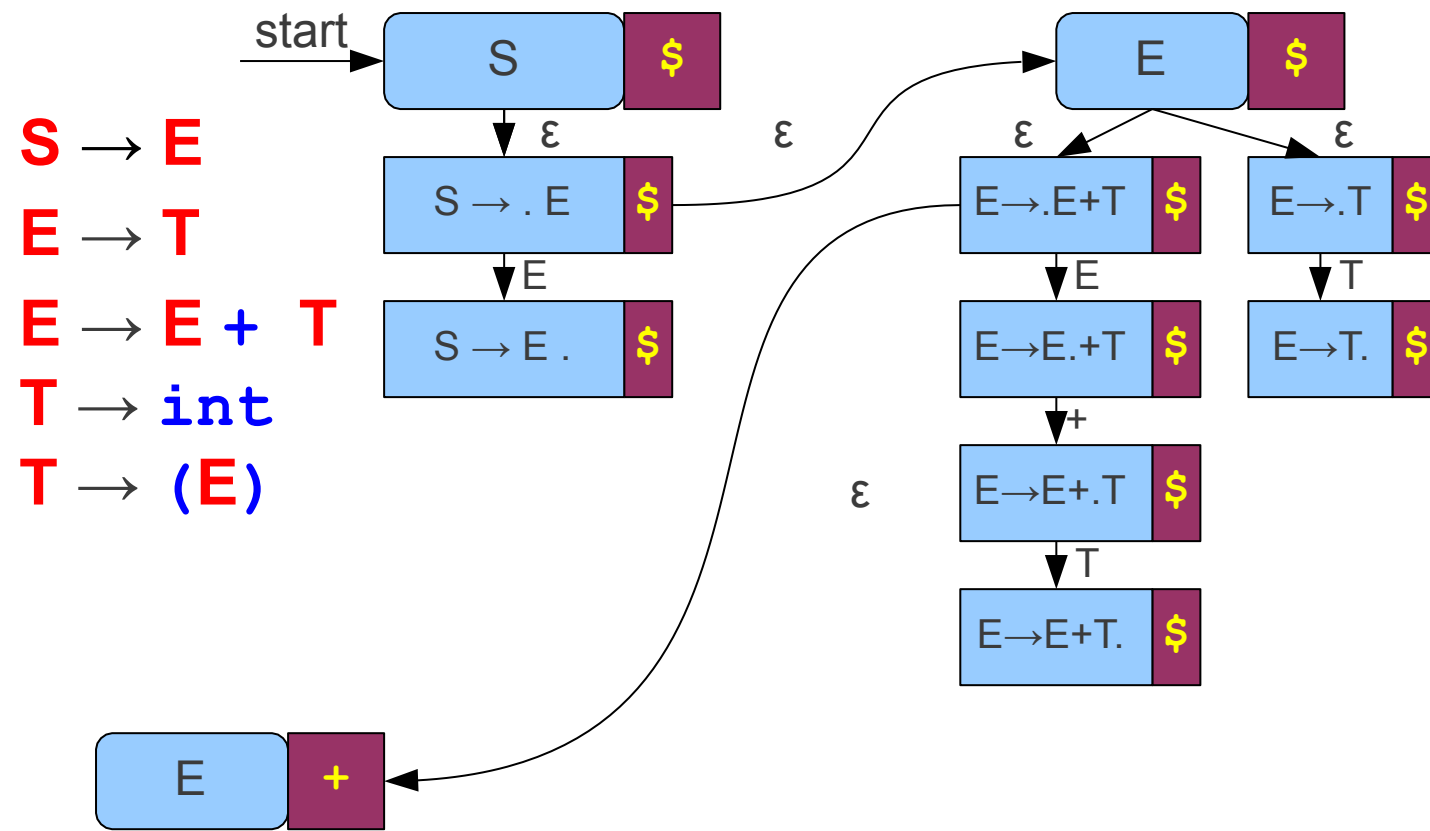
# Constructing LR(1) Automata



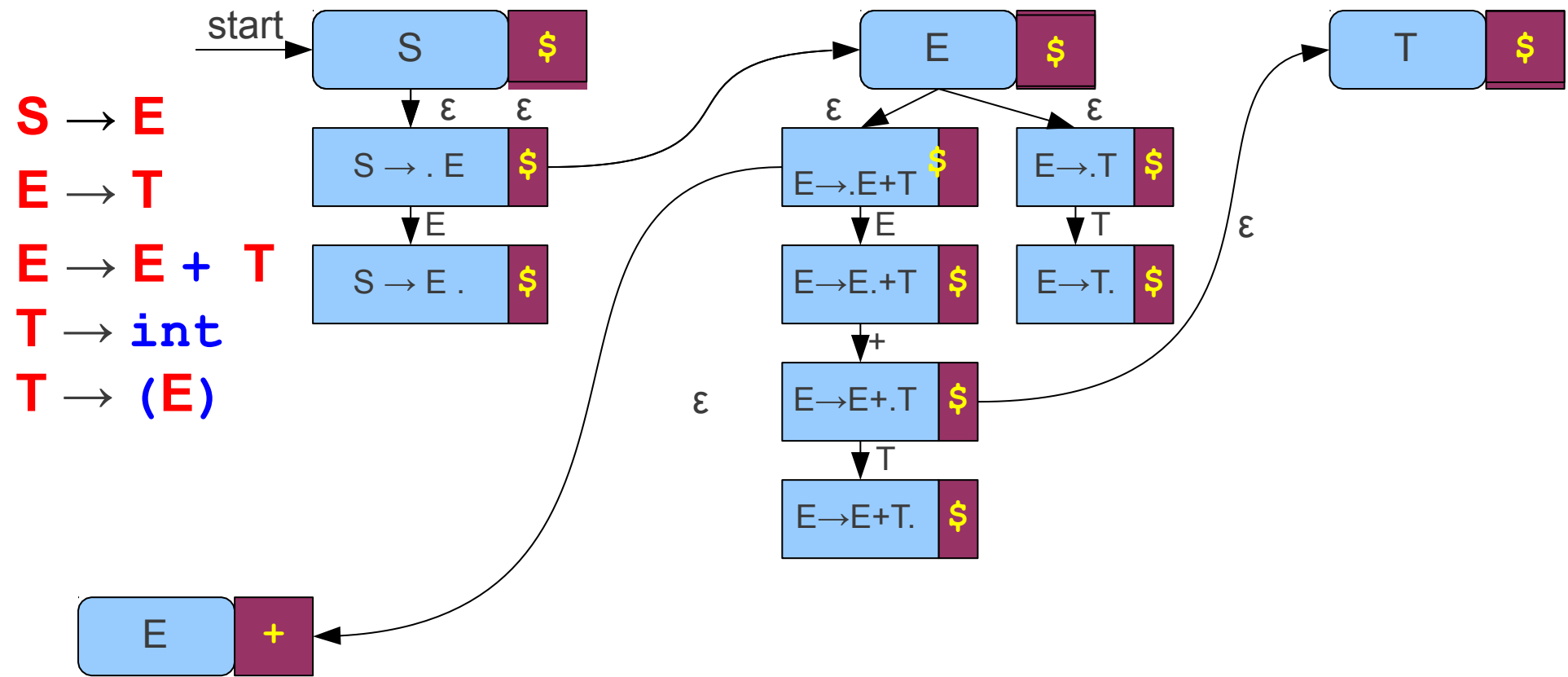
# Constructing LR(1) Automata



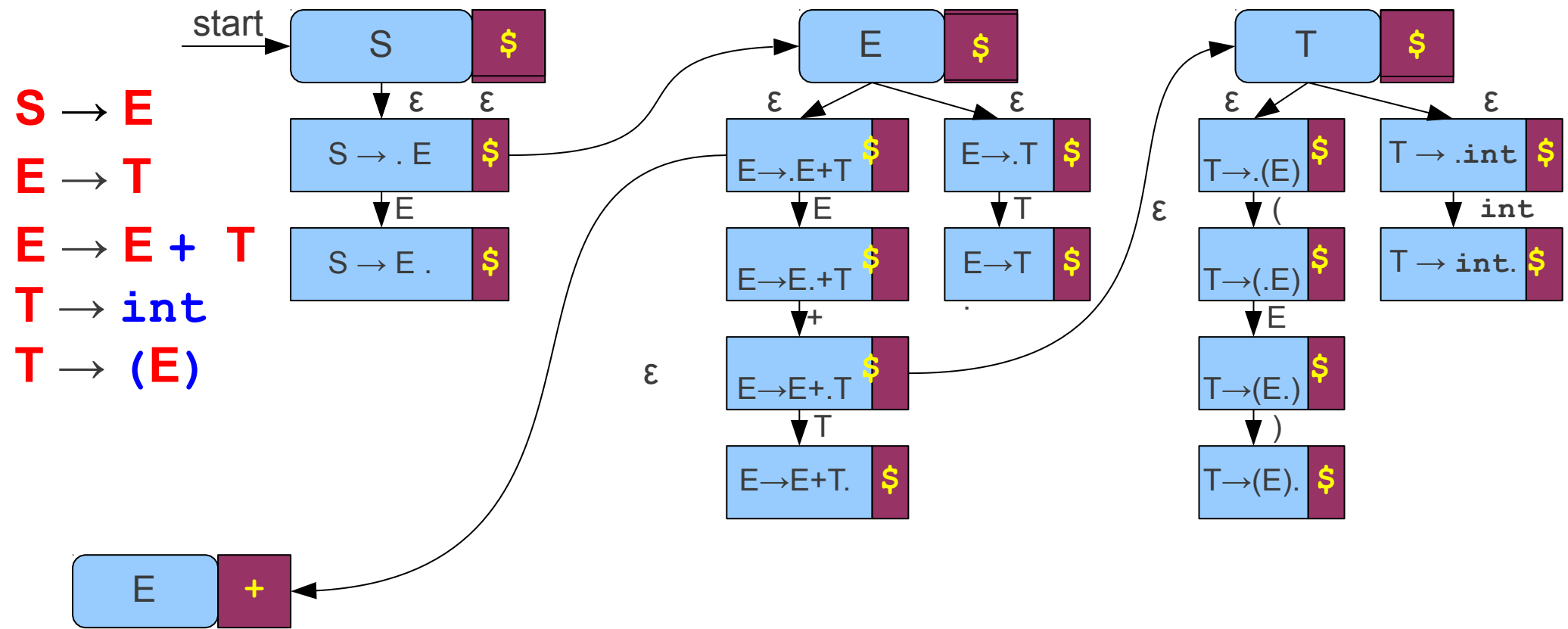
# Constructing LR(1) Automata



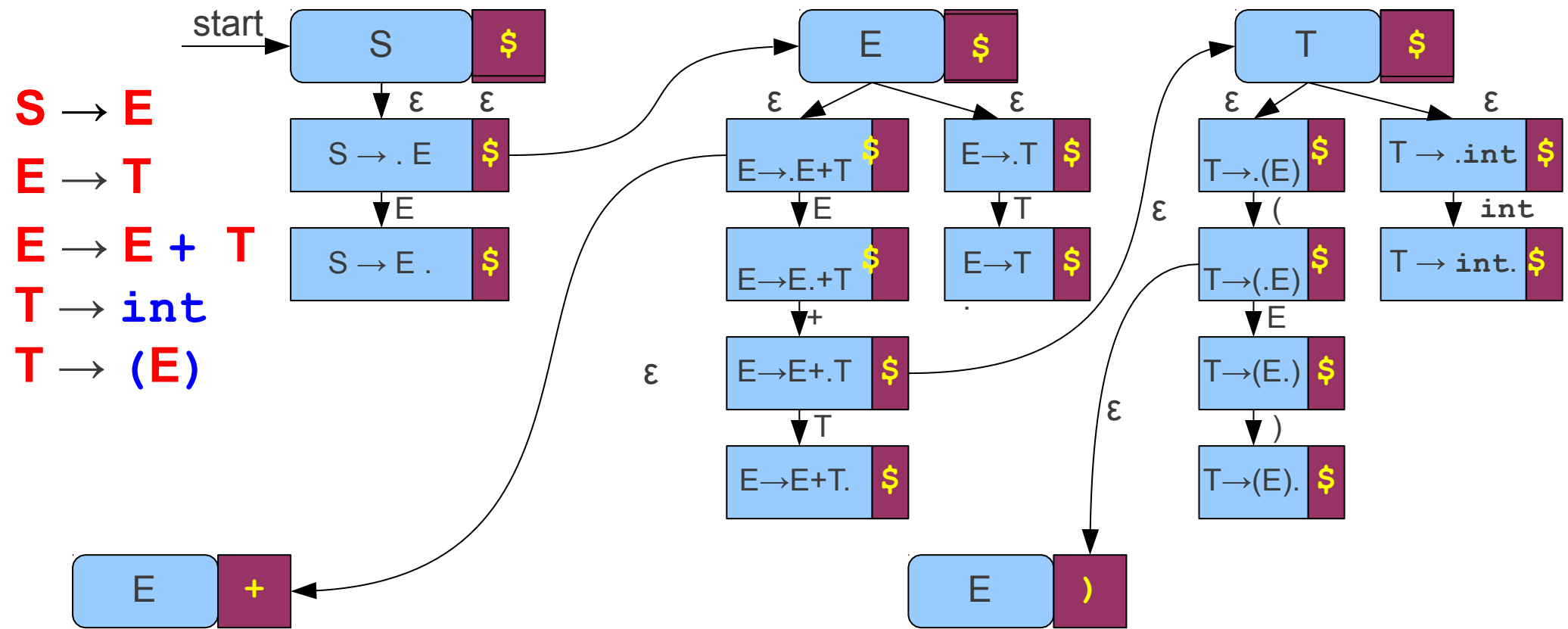
# Constructing LR(1) Automata



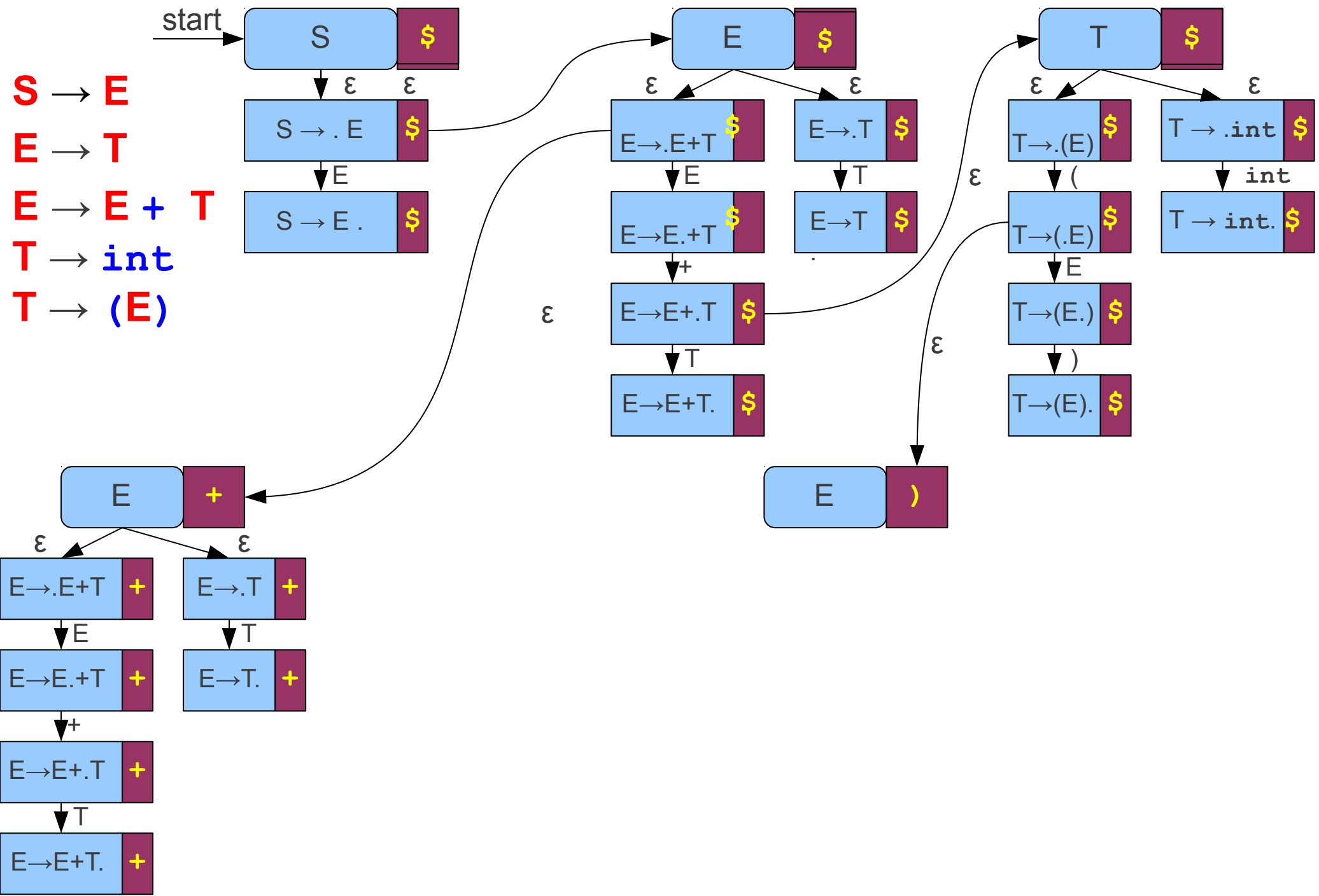
# Constructing LR(1) Automata



# Constructing LR(1) Automata

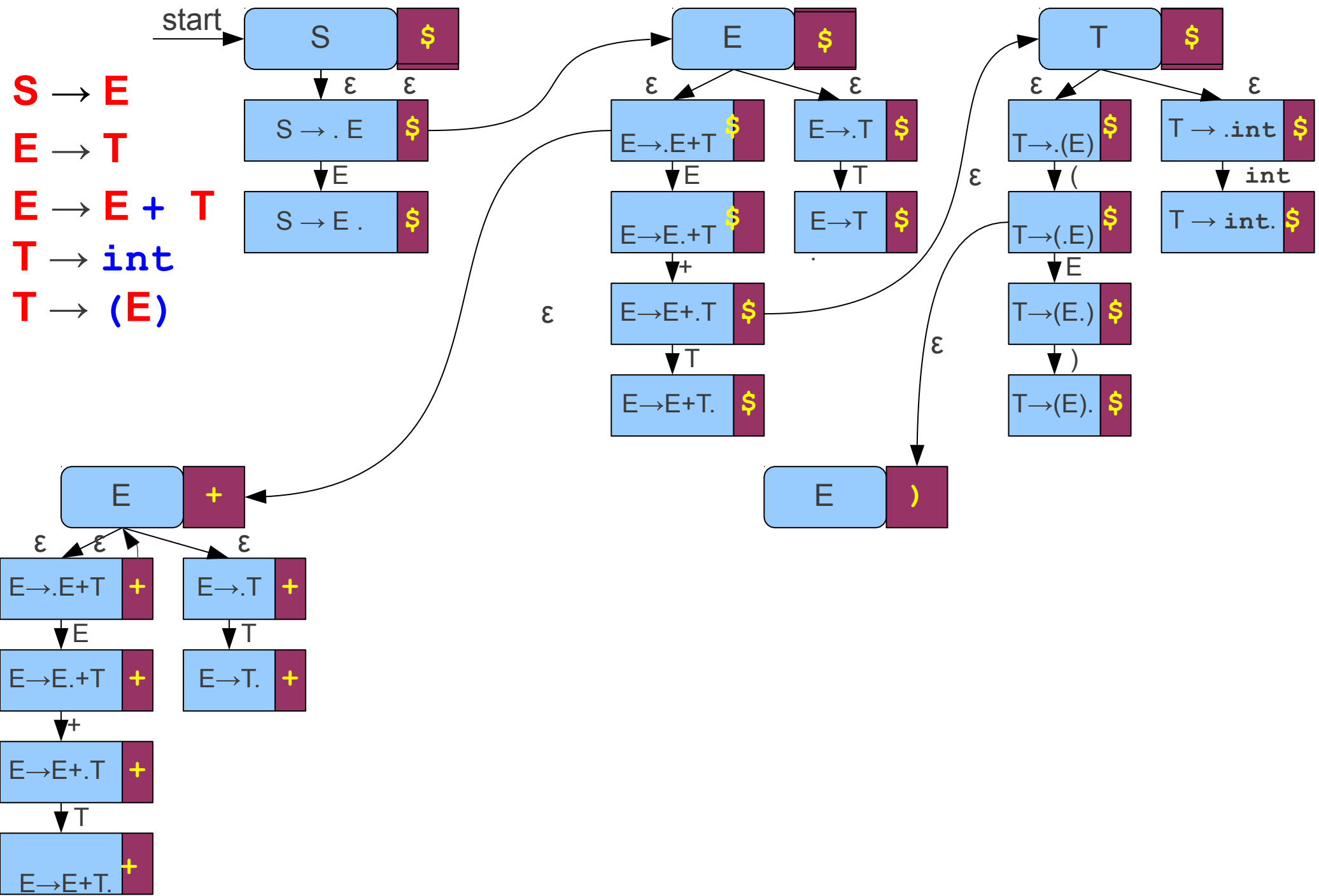


# Constructing LR(1) Automata

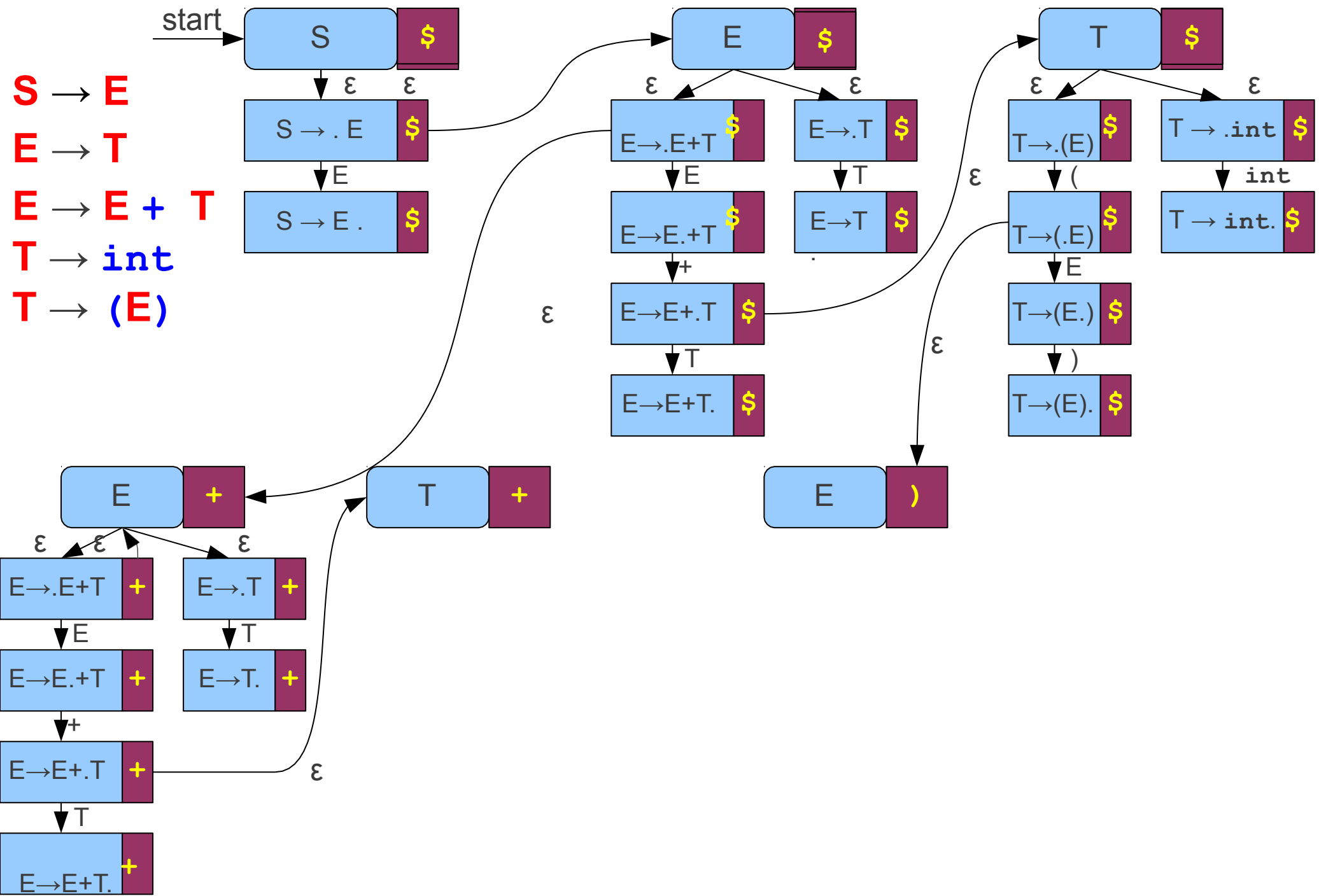




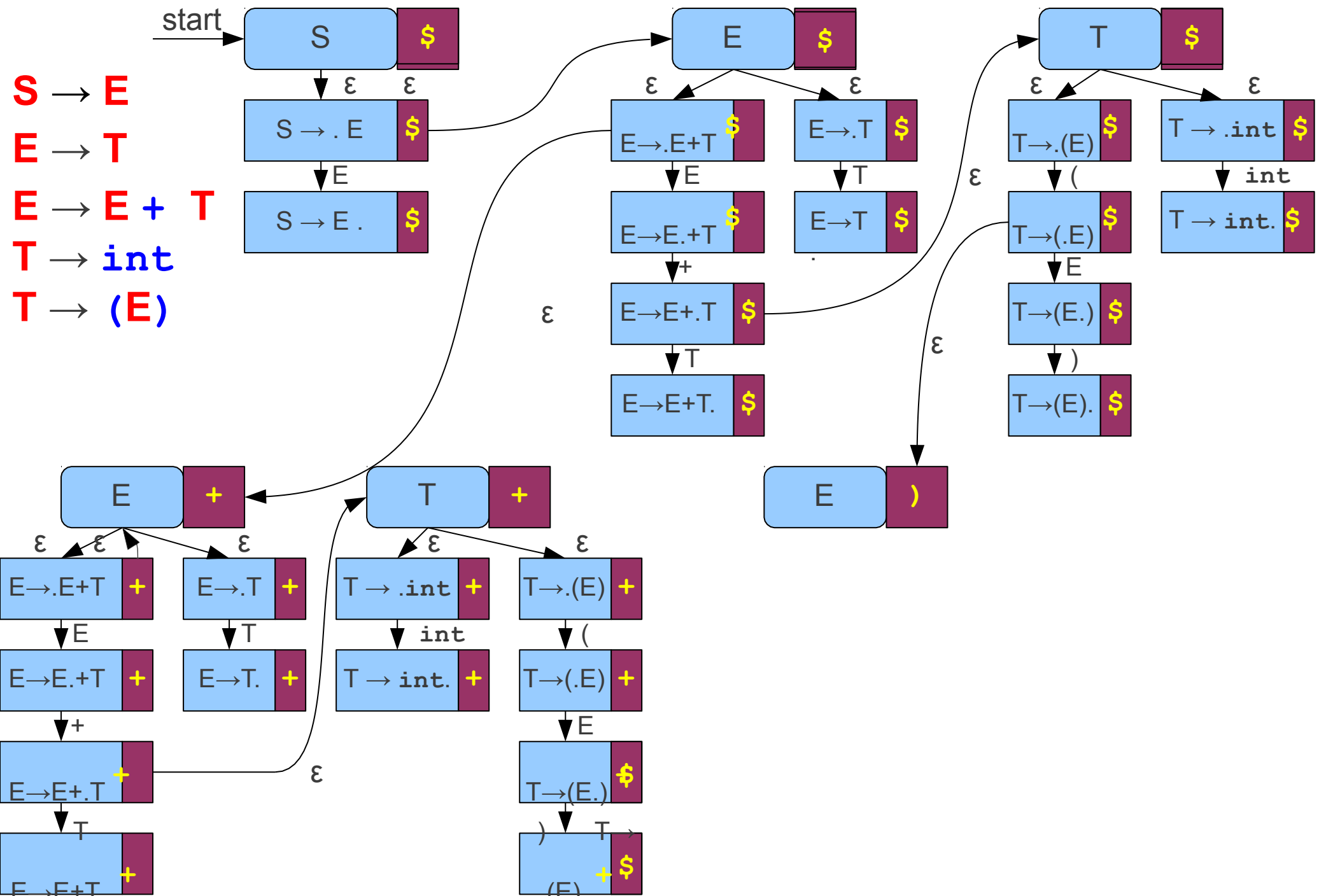
# Constructing LR(1) Automata



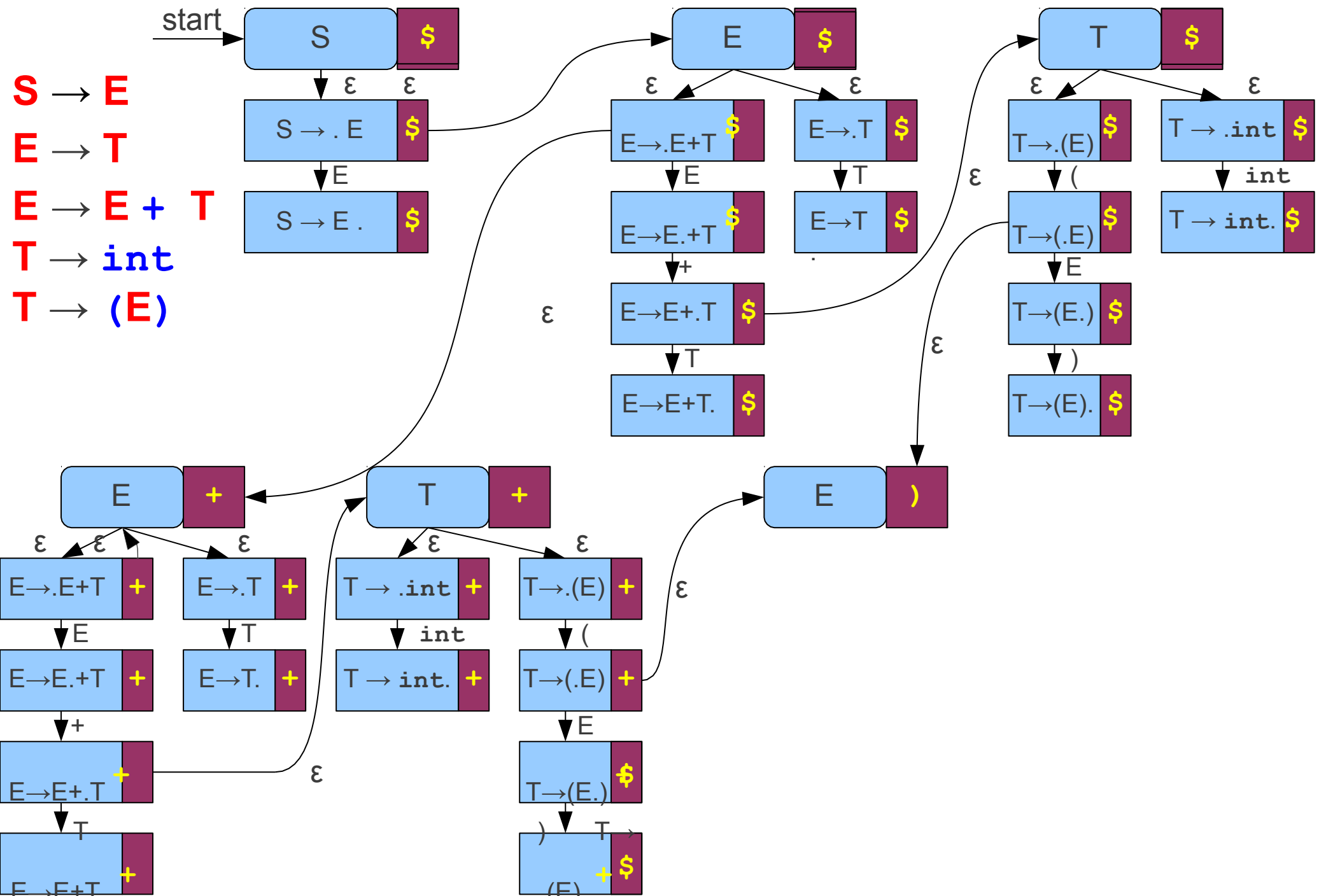
# Constructing LR(1) Automata



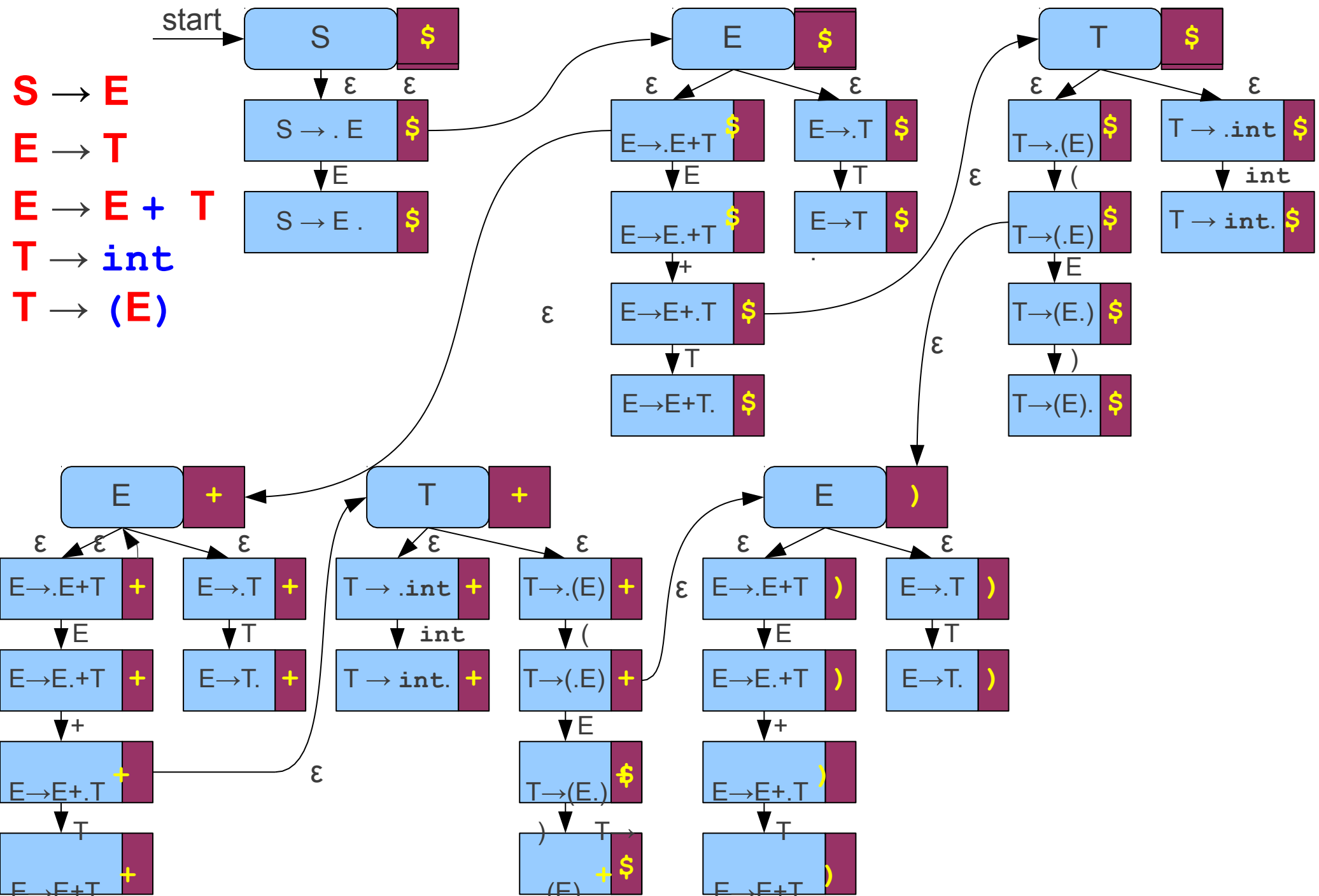
# Constructing LR(1) Automata



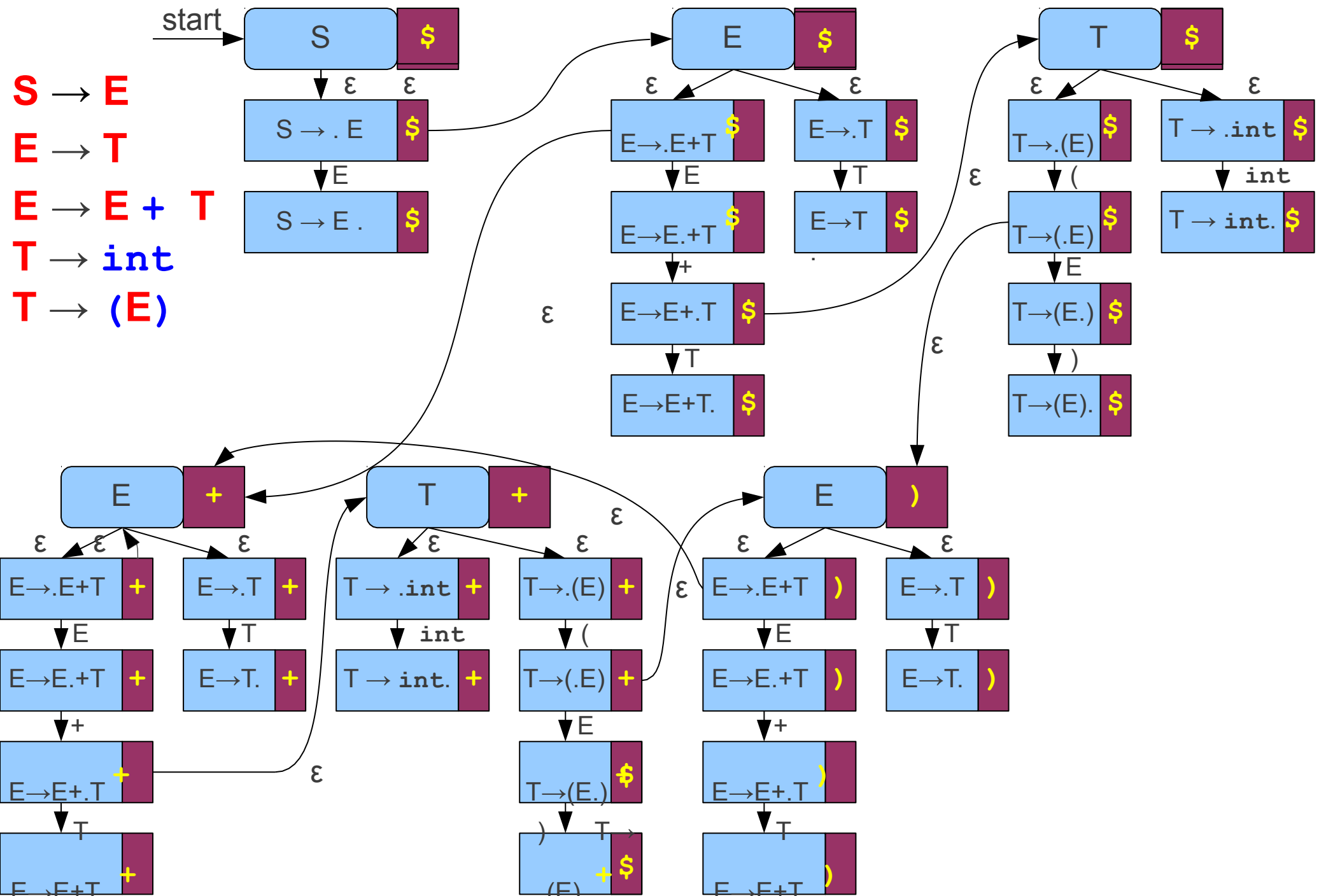
# Constructing LR(1) Automata



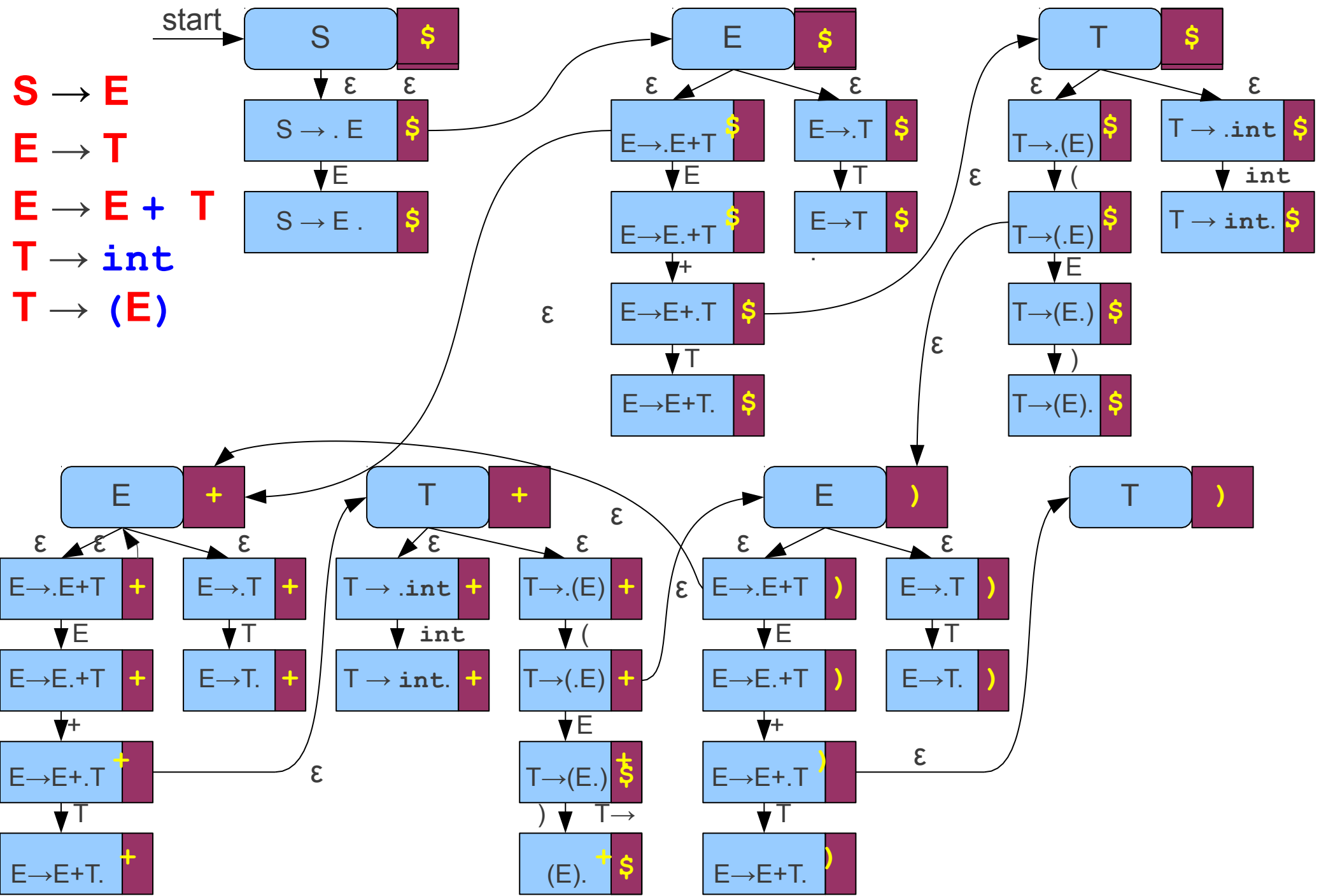
# Constructing LR(1) Automata



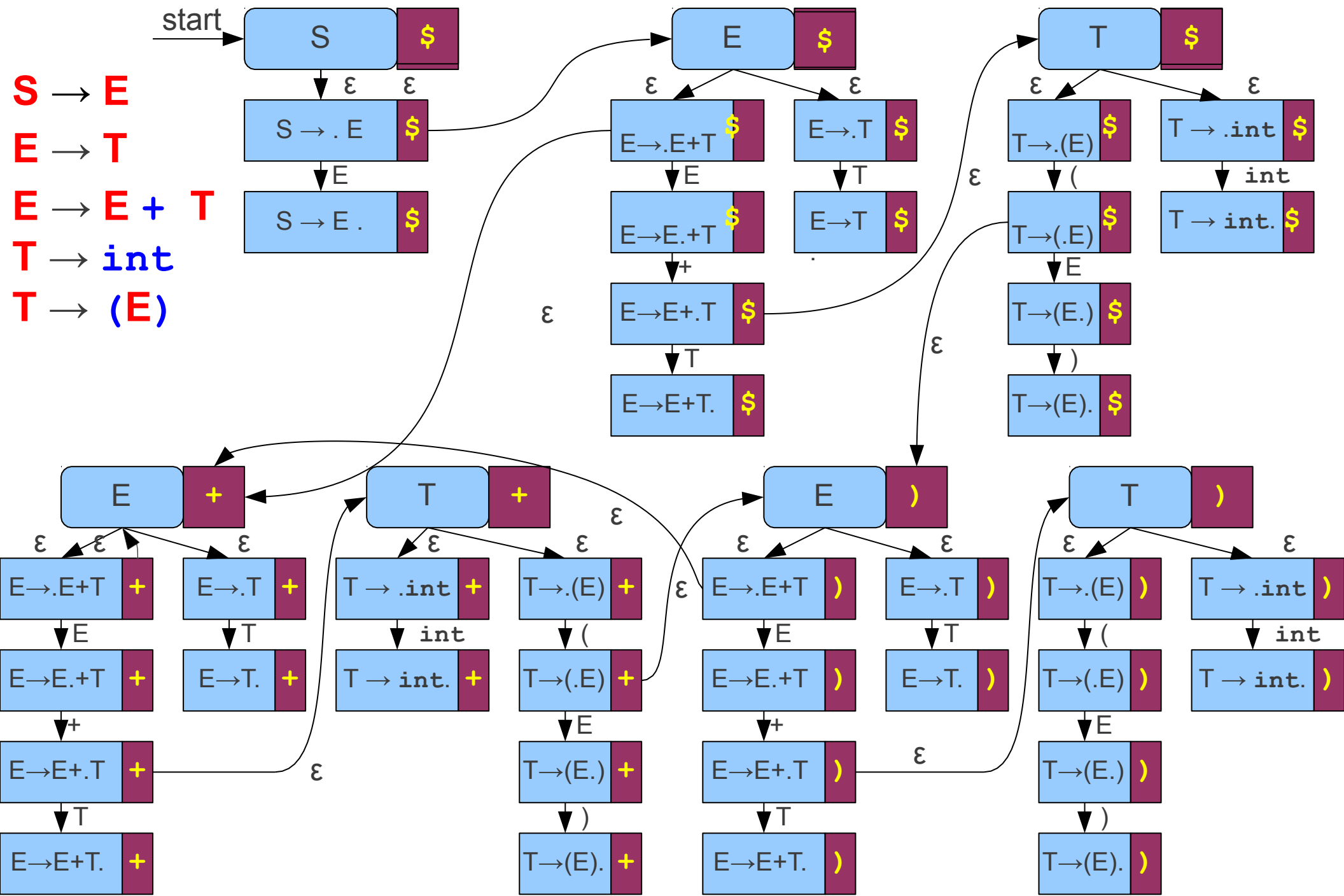
# Constructing LR(1) Automata



# Constructing LR(1) Automata

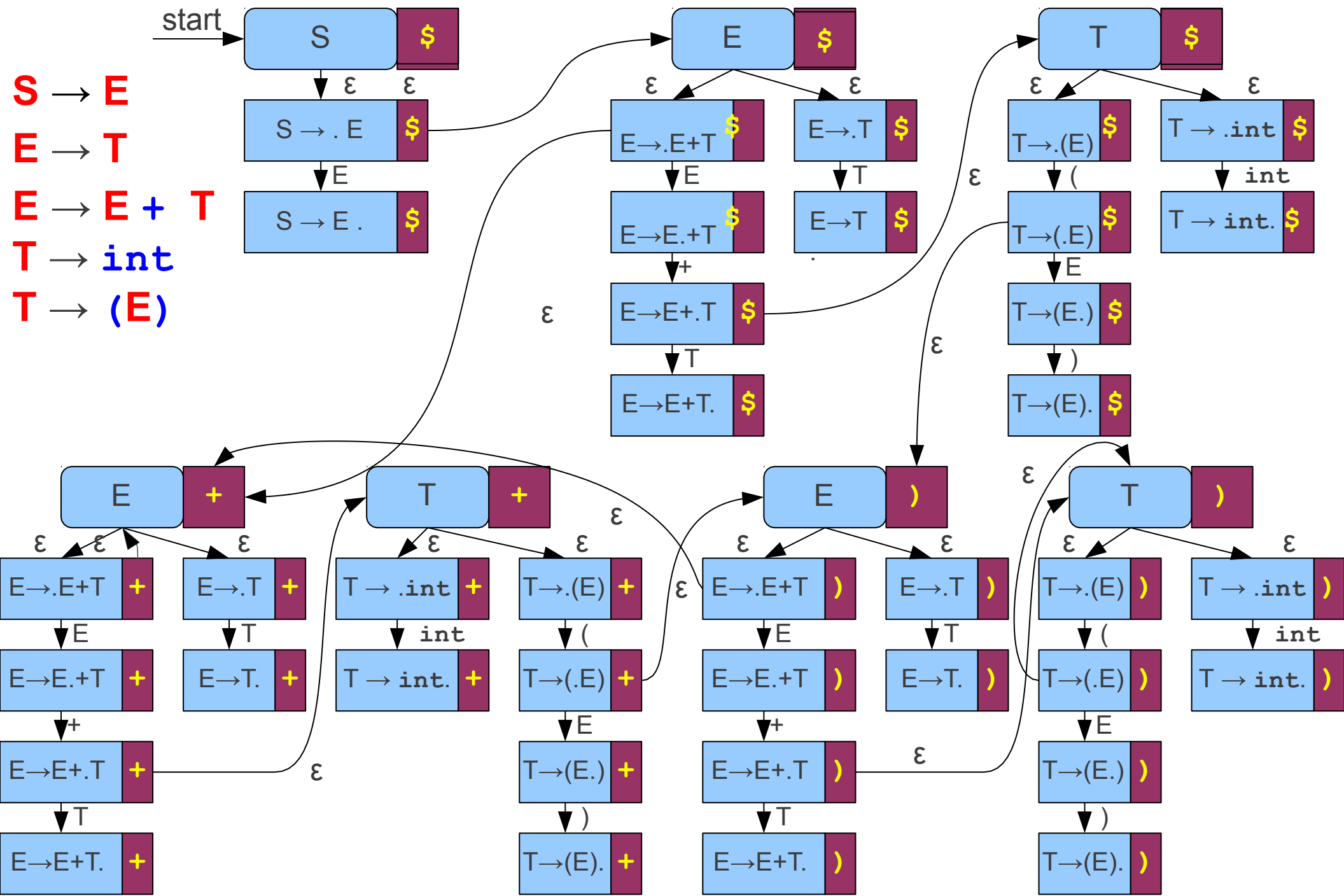


# Constructing LR(1) Automata

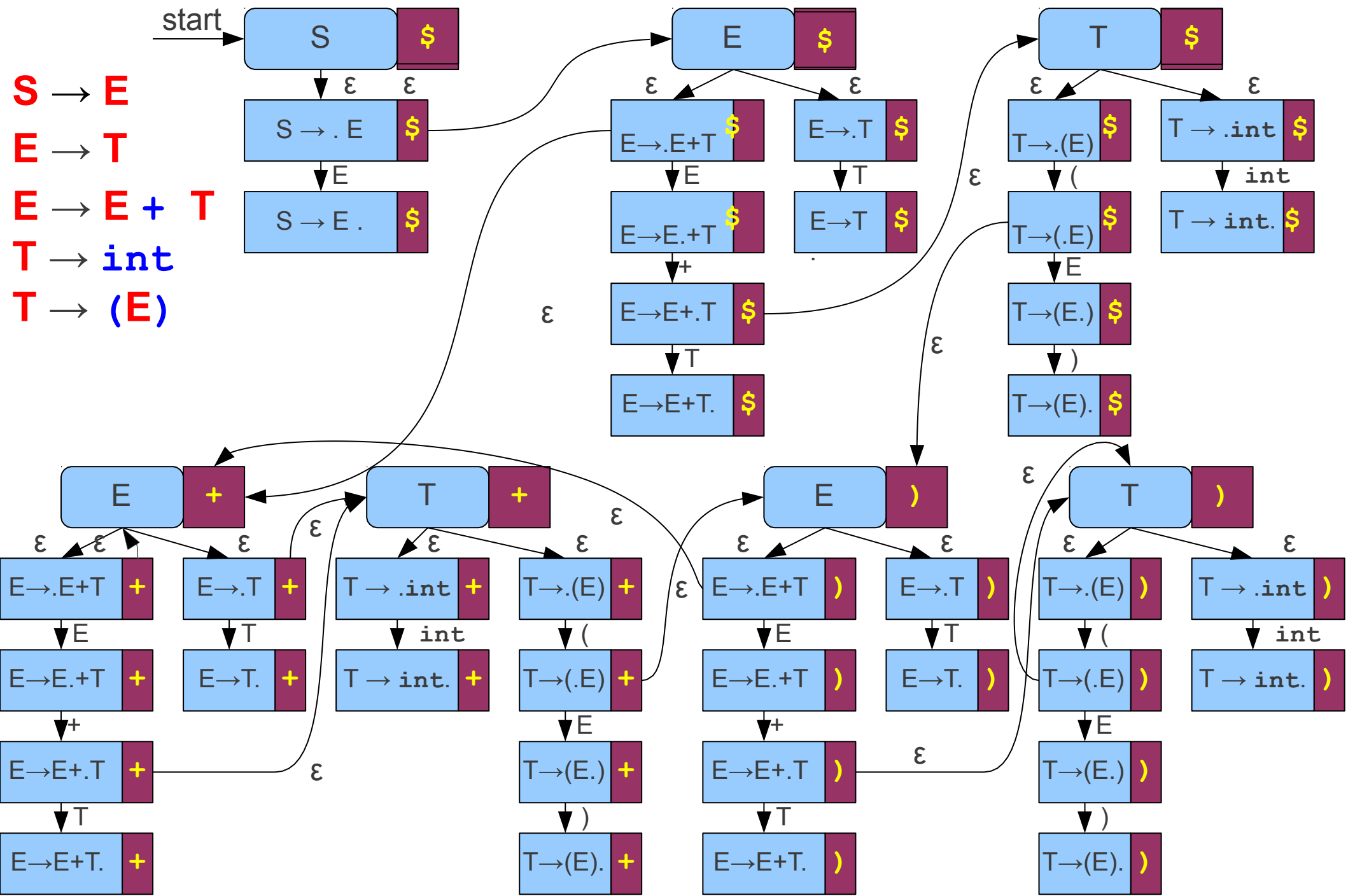




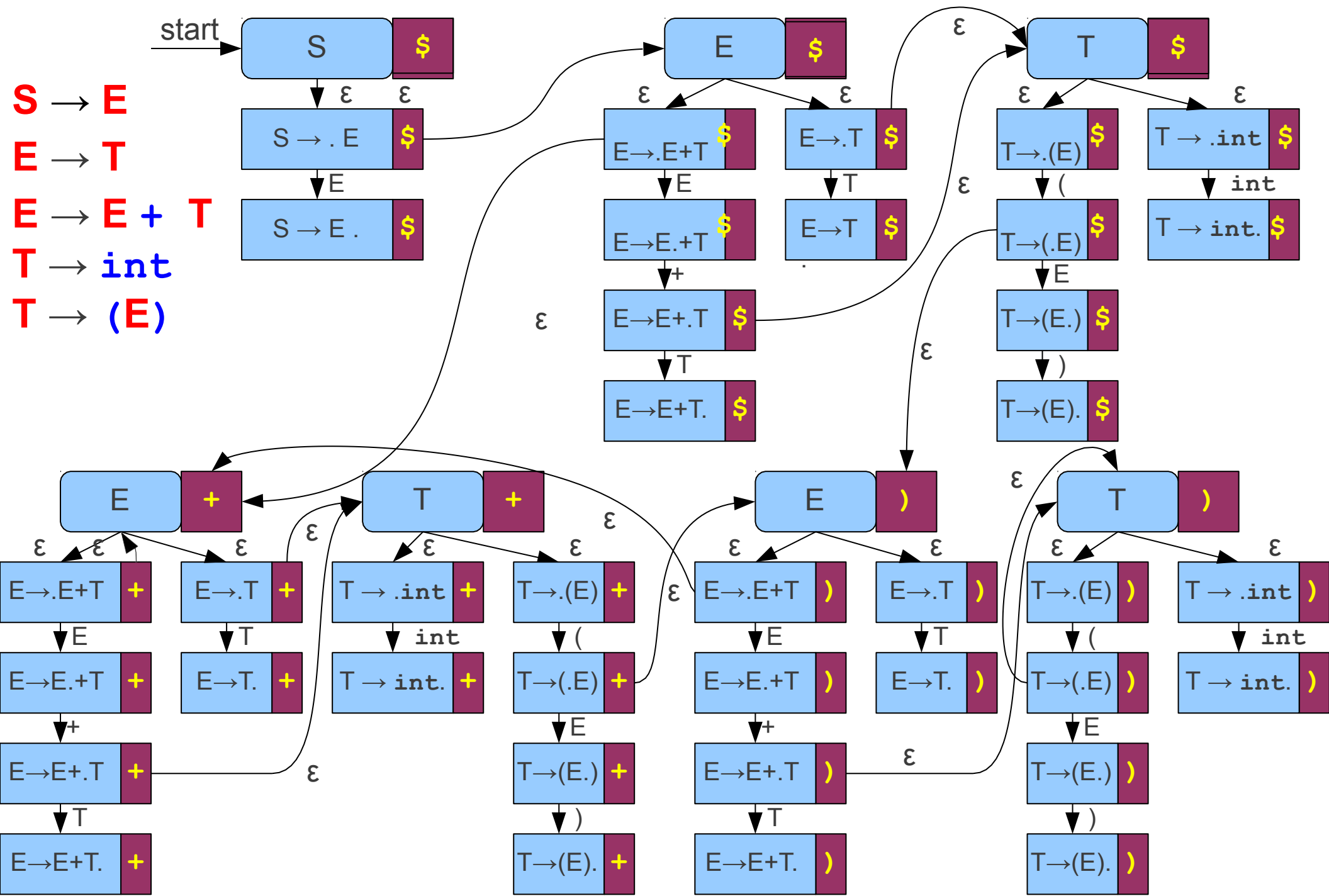
# Constructing LR(1) Automata



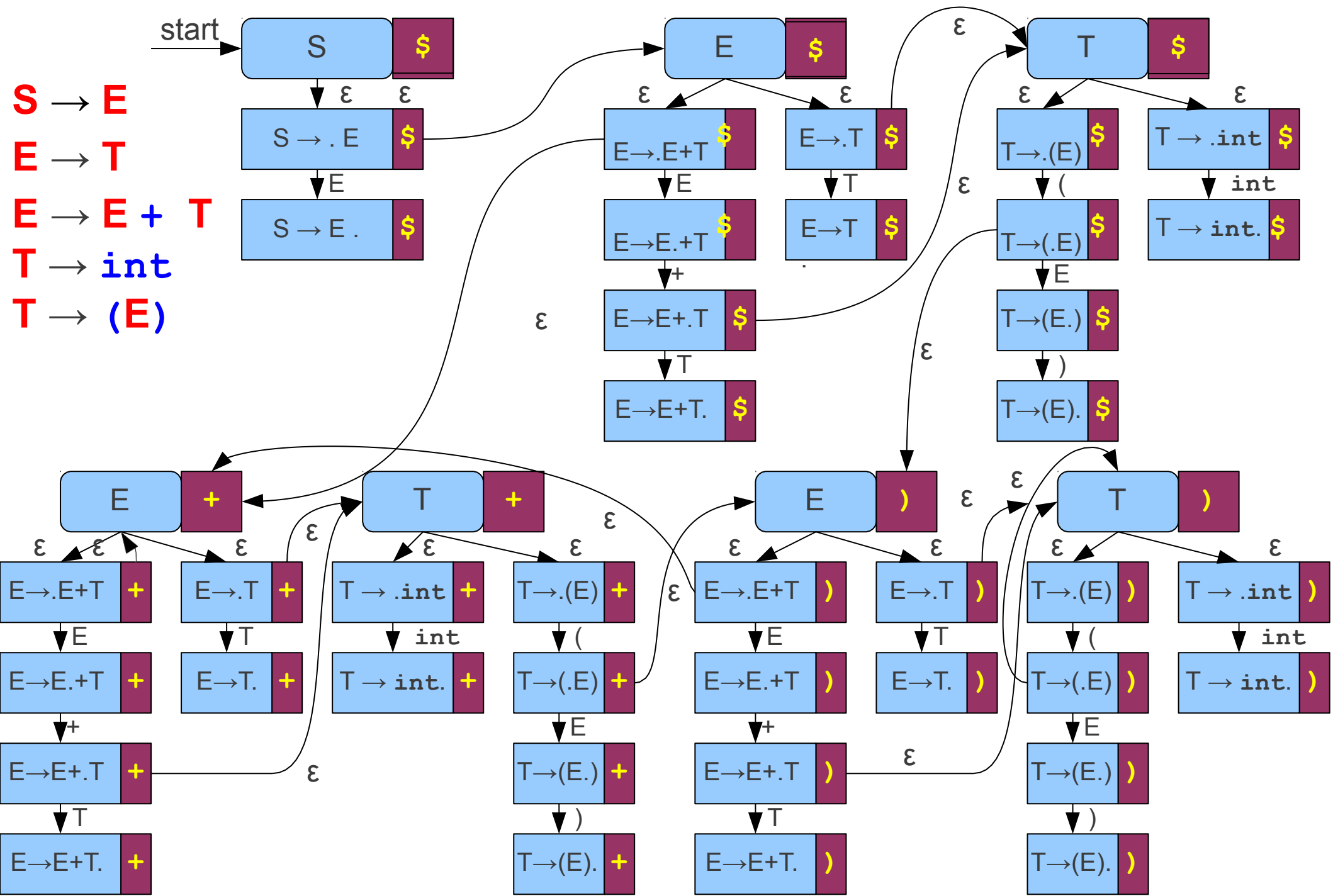
# Constructing LR(1) Automata



# Constructing LR(1) Automata

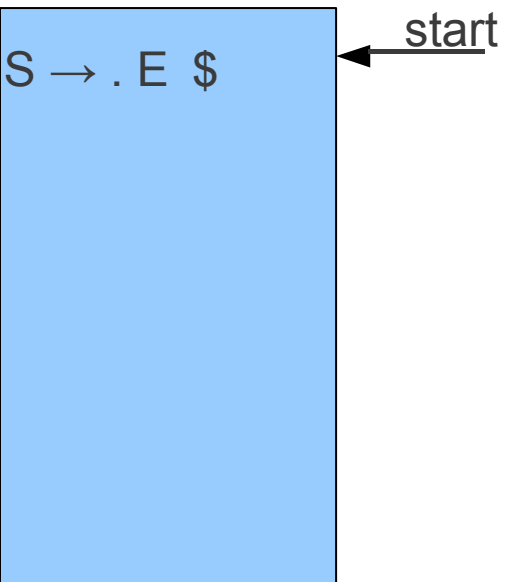


# Constructing LR(1) Automata

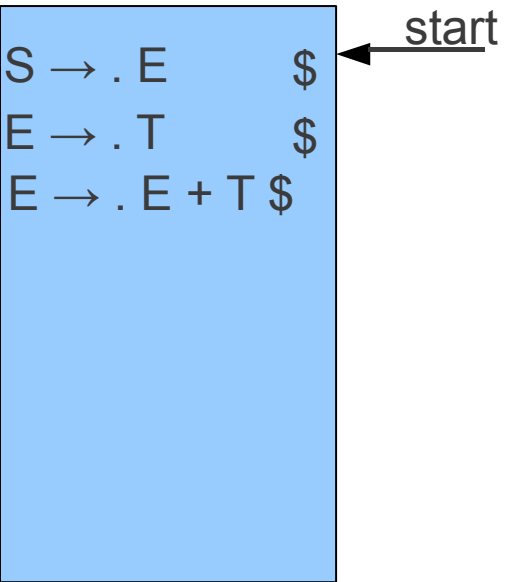


# Deterministic LR(1) Automata

# Deterministic LR(1) Automata



# Deterministic LR(1) Automata



# Deterministic LR(1) Automata

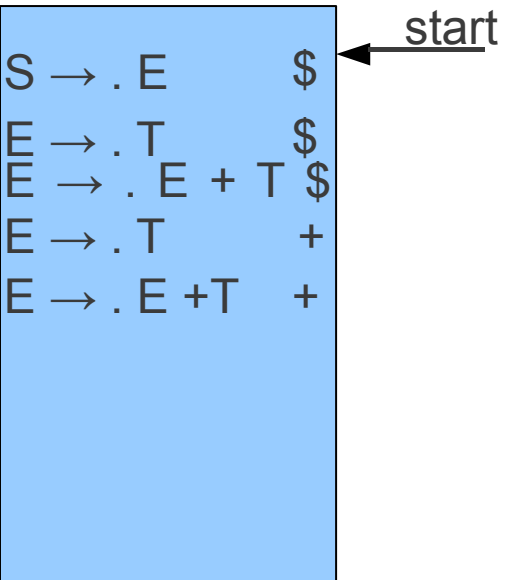



Diagram illustrating the initial state of a Deterministic LR(1) Automaton. The state is represented by a blue box containing five LR(1) items. An arrow labeled "start" points to the first item, indicating it is the initial state.


$S \rightarrow . E$	$\$$
$E \rightarrow . T$	$\$$
$E \rightarrow . E + T$	$\$$
$E \rightarrow . T$	$+$
$E \rightarrow . E + T$	$+$




# Deterministic LR(1) Automata

$S \rightarrow . E$	\$	 <u>start</u>
$E \rightarrow . T$	\$	
$E \rightarrow . E + T$	\$	
$E \rightarrow . T$	+	
$E \rightarrow . E + T$	+	
$T \rightarrow . \text{int}$	\$	
$T \rightarrow . (E)$	\$	

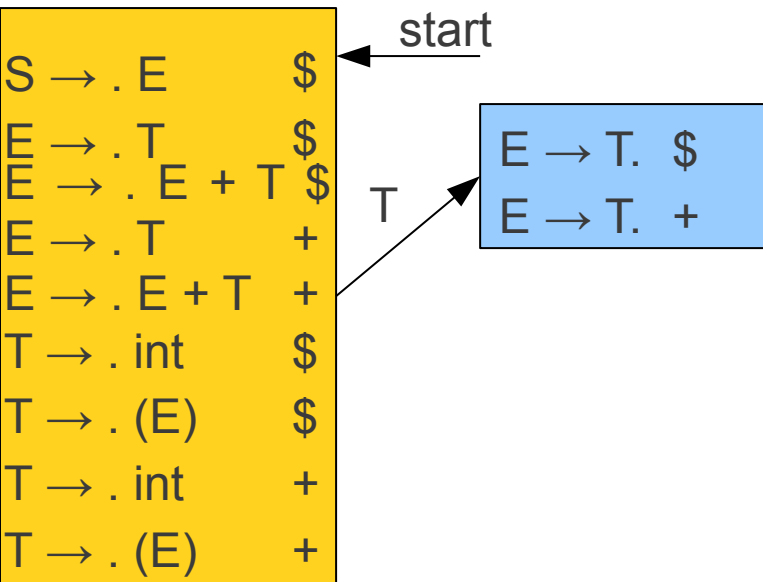
# Deterministic LR(1) Automata

$S \rightarrow . E$	\$	 <u>start</u>
$E \rightarrow . T$	\$	
$E \rightarrow . E + T$	\$	
$E \rightarrow . T$	+	
$E \rightarrow . E + T$	+	
$T \rightarrow . \text{int}$	\$	
$T \rightarrow . (E)$	\$	
$T \rightarrow . \text{int}$	+	
$T \rightarrow . (E)$	+	

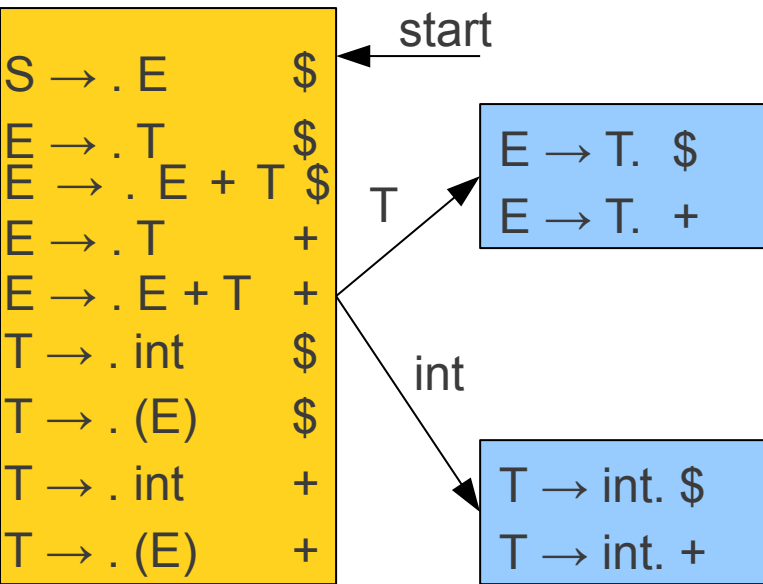
# Deterministic LR(1) Automata

$S \rightarrow . E$	\$	 <u>start</u>
$E \rightarrow . T$	\$	
$E \rightarrow . E + T$	\$	
$E \rightarrow . T$	+	
$E \rightarrow . E + T$	+	
$T \rightarrow . \text{int}$	\$	
$T \rightarrow . (E)$	\$	
$T \rightarrow . \text{int}$	+	
$T \rightarrow . (E)$	+	

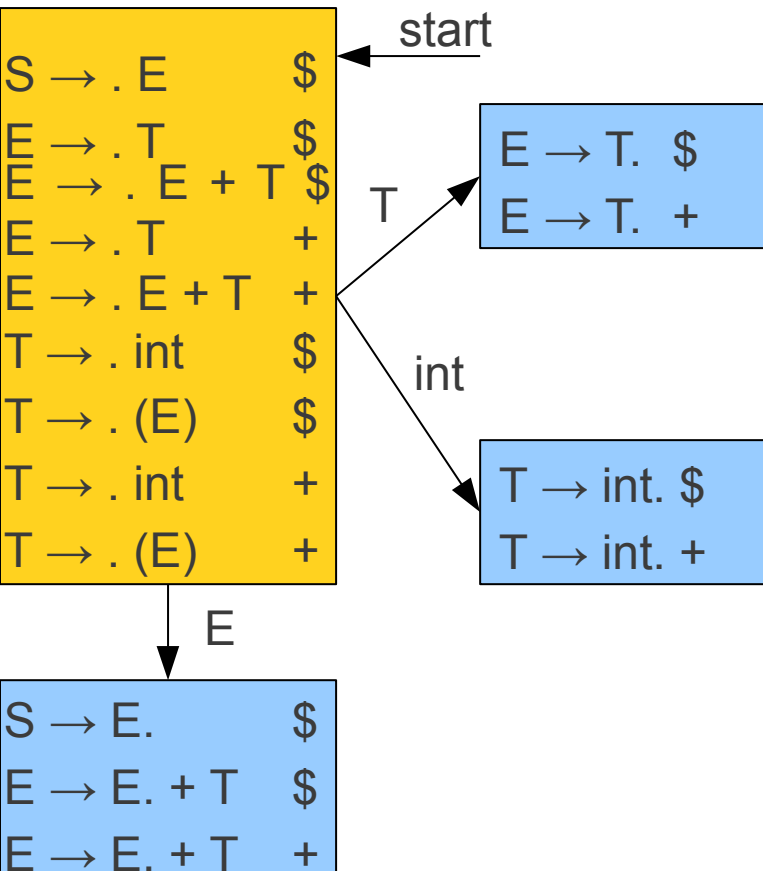
# Deterministic LR(1) Automata



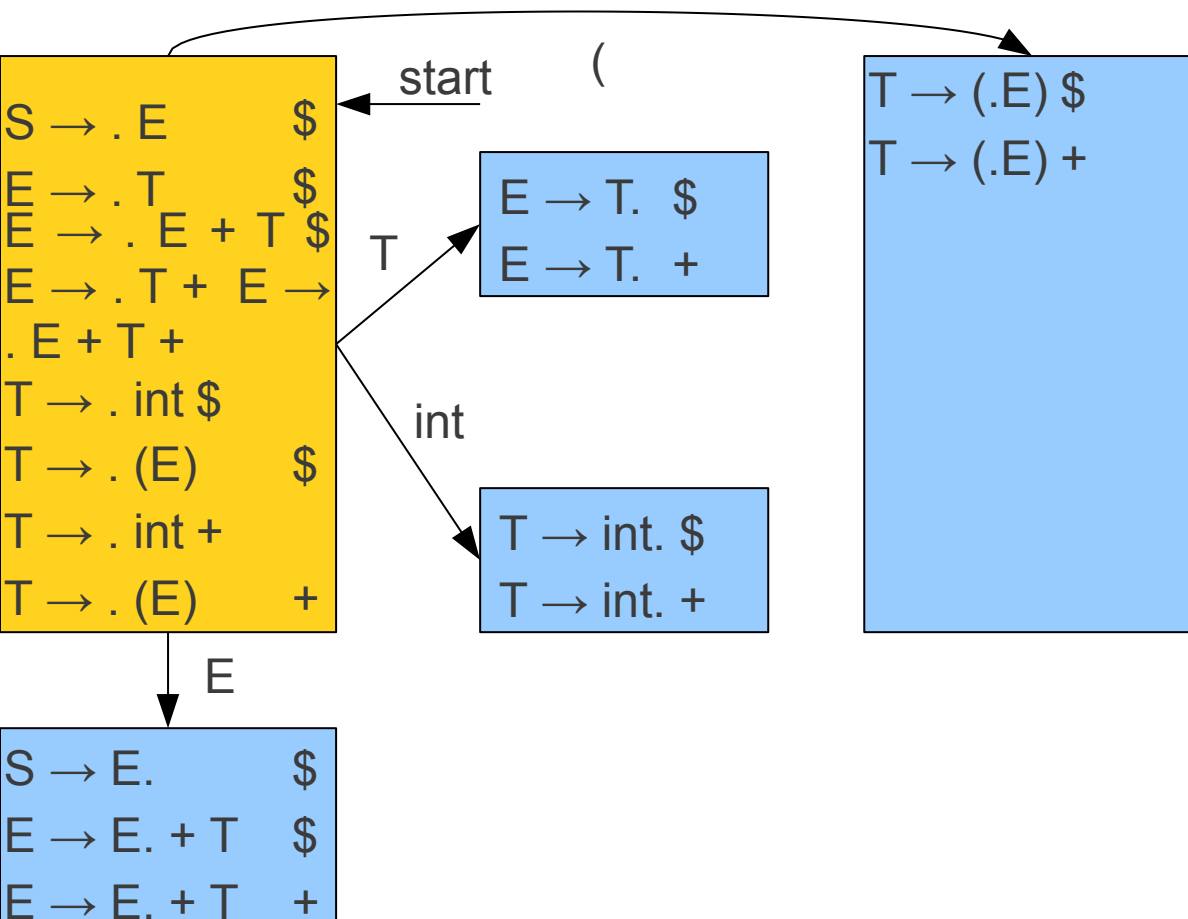
# Deterministic LR(1) Automata



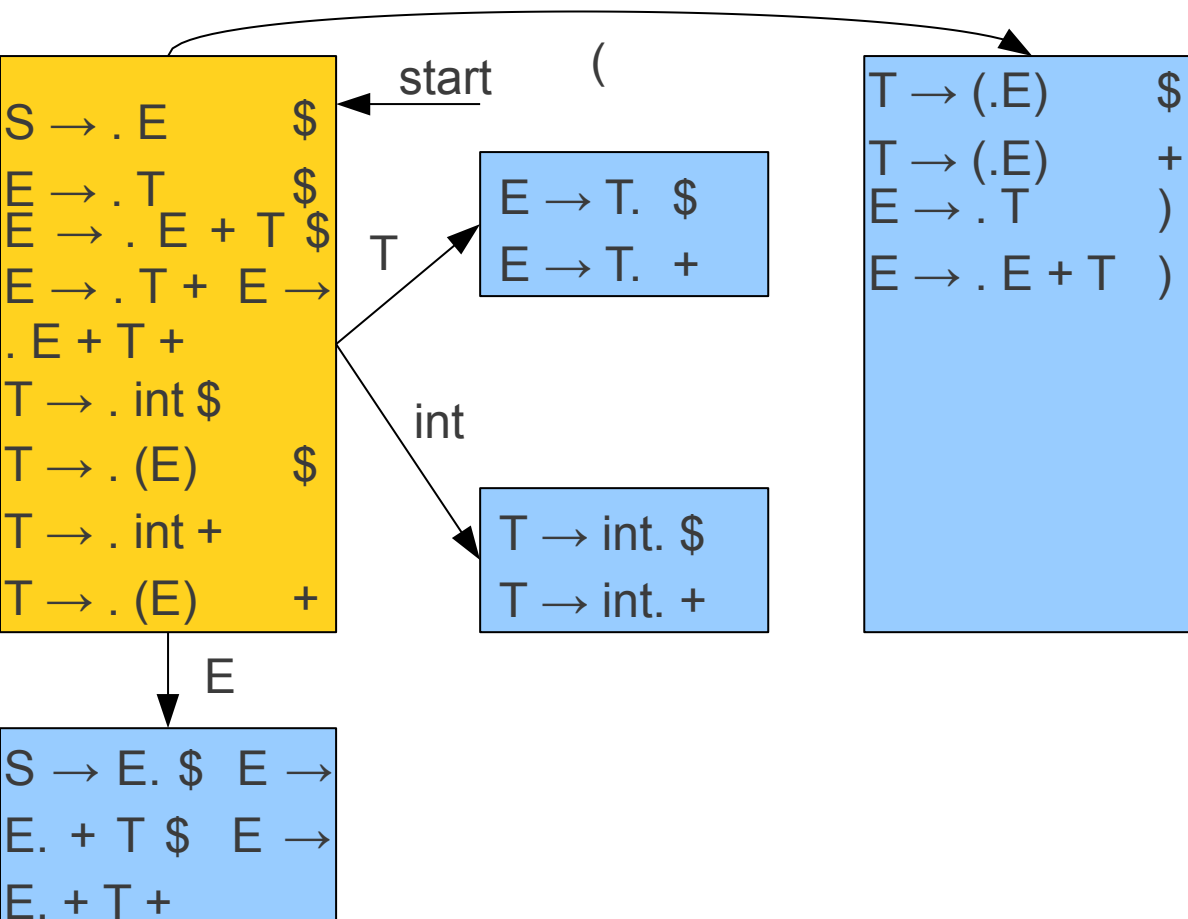
# Deterministic LR(1) Automata



# Deterministic LR(1) Automata

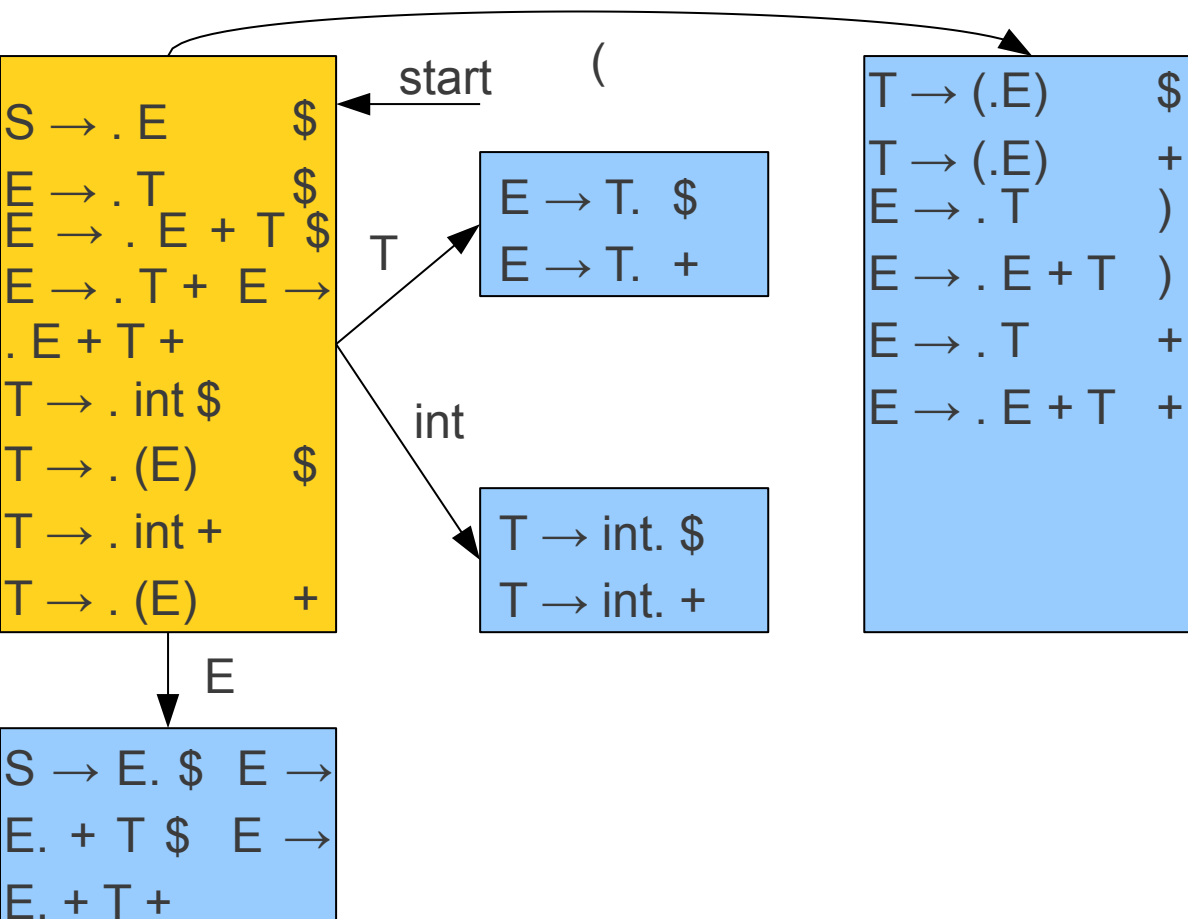


# Deterministic LR(1) Automata

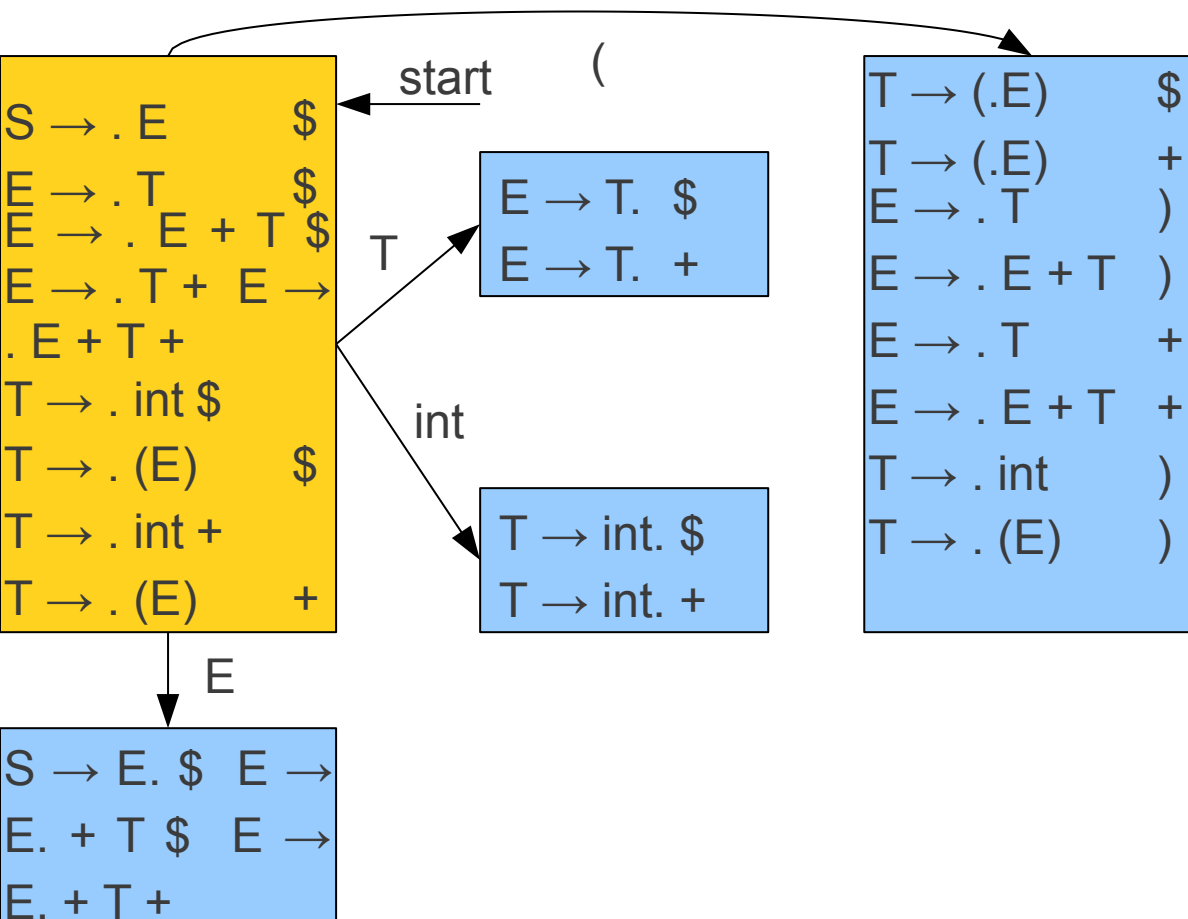




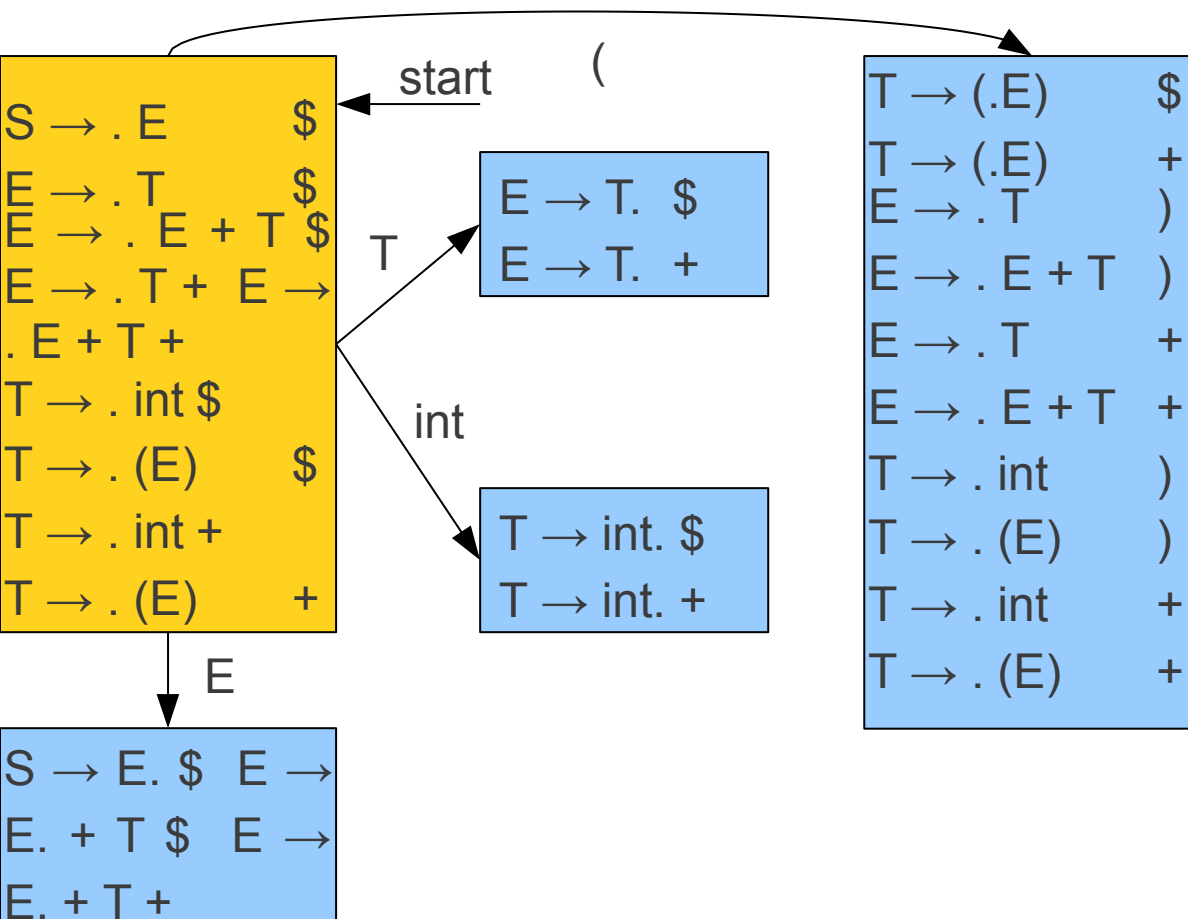
# Deterministic LR(1) Automata



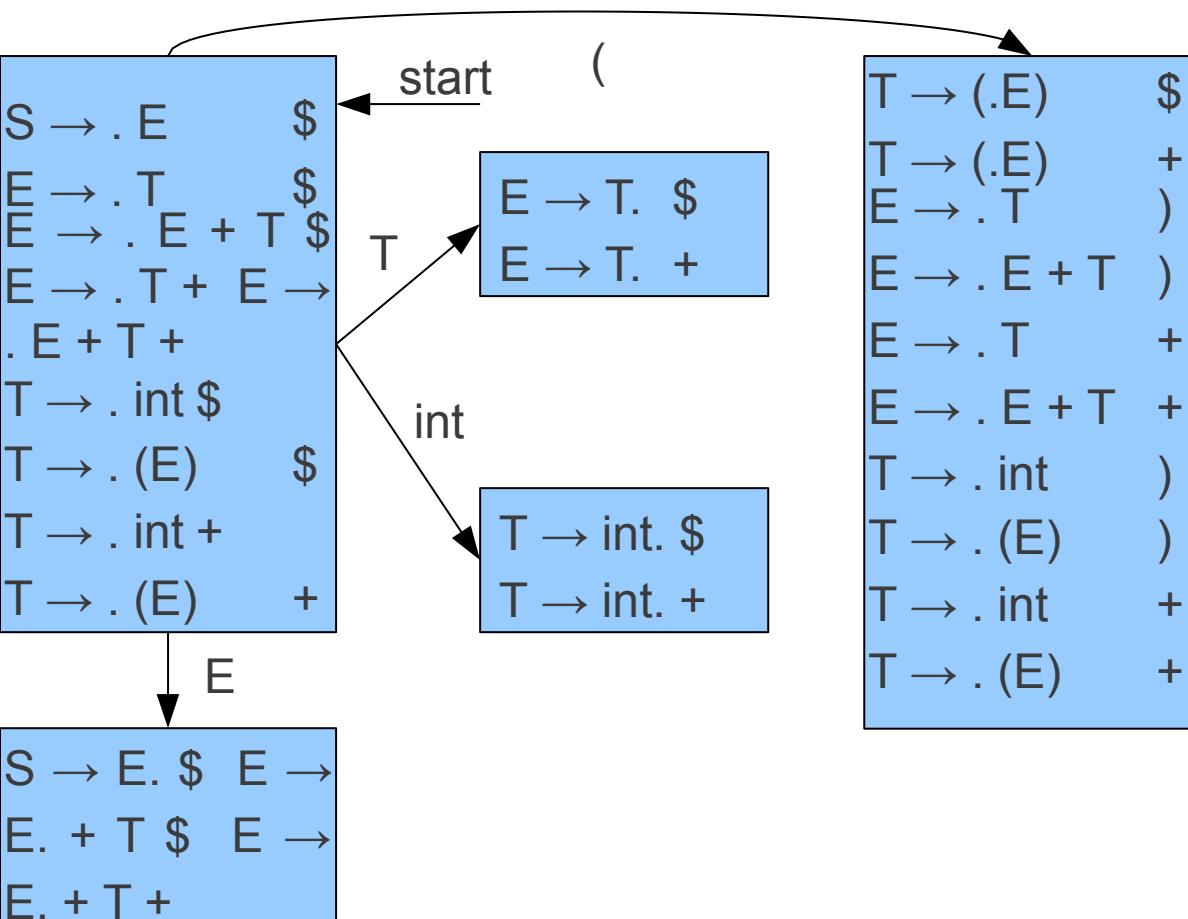
# Deterministic LR(1) Automata



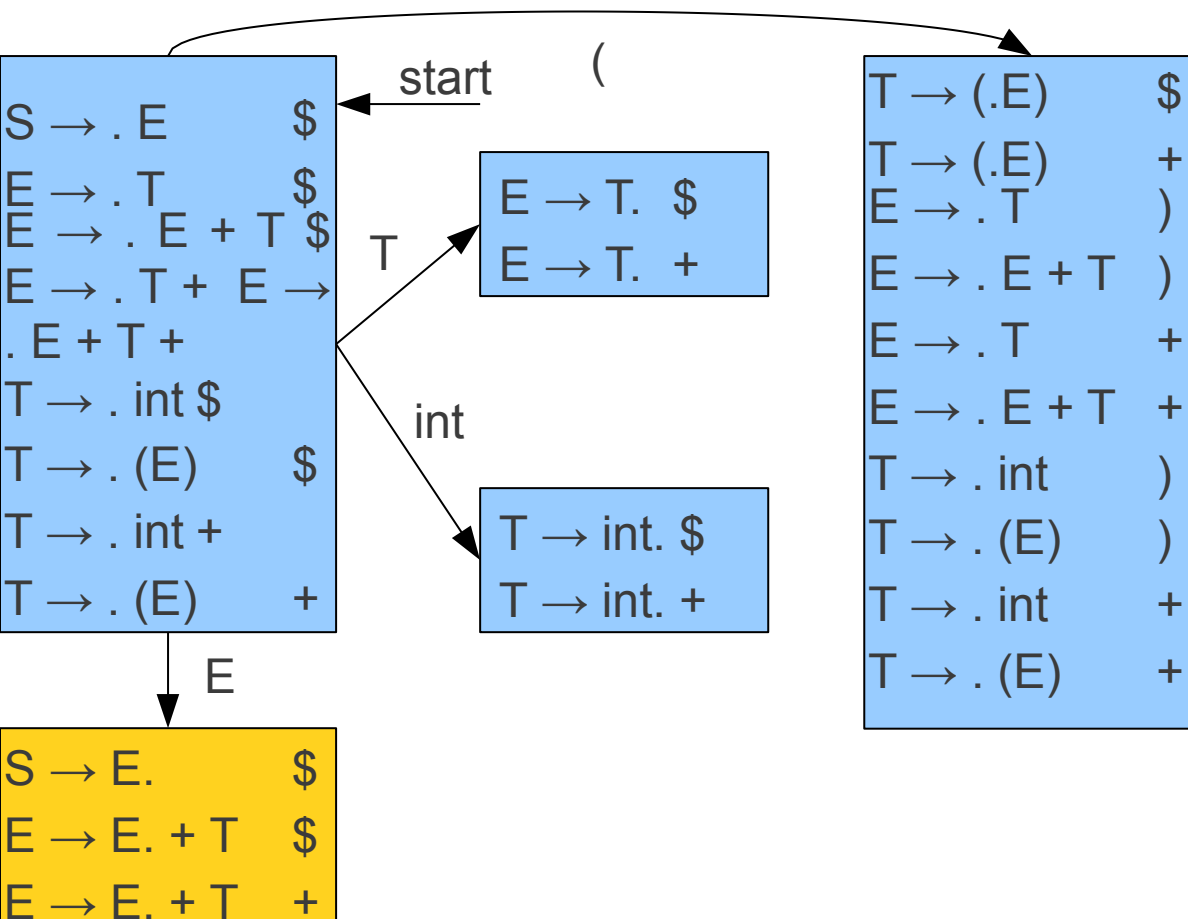
# Deterministic LR(1) Automata



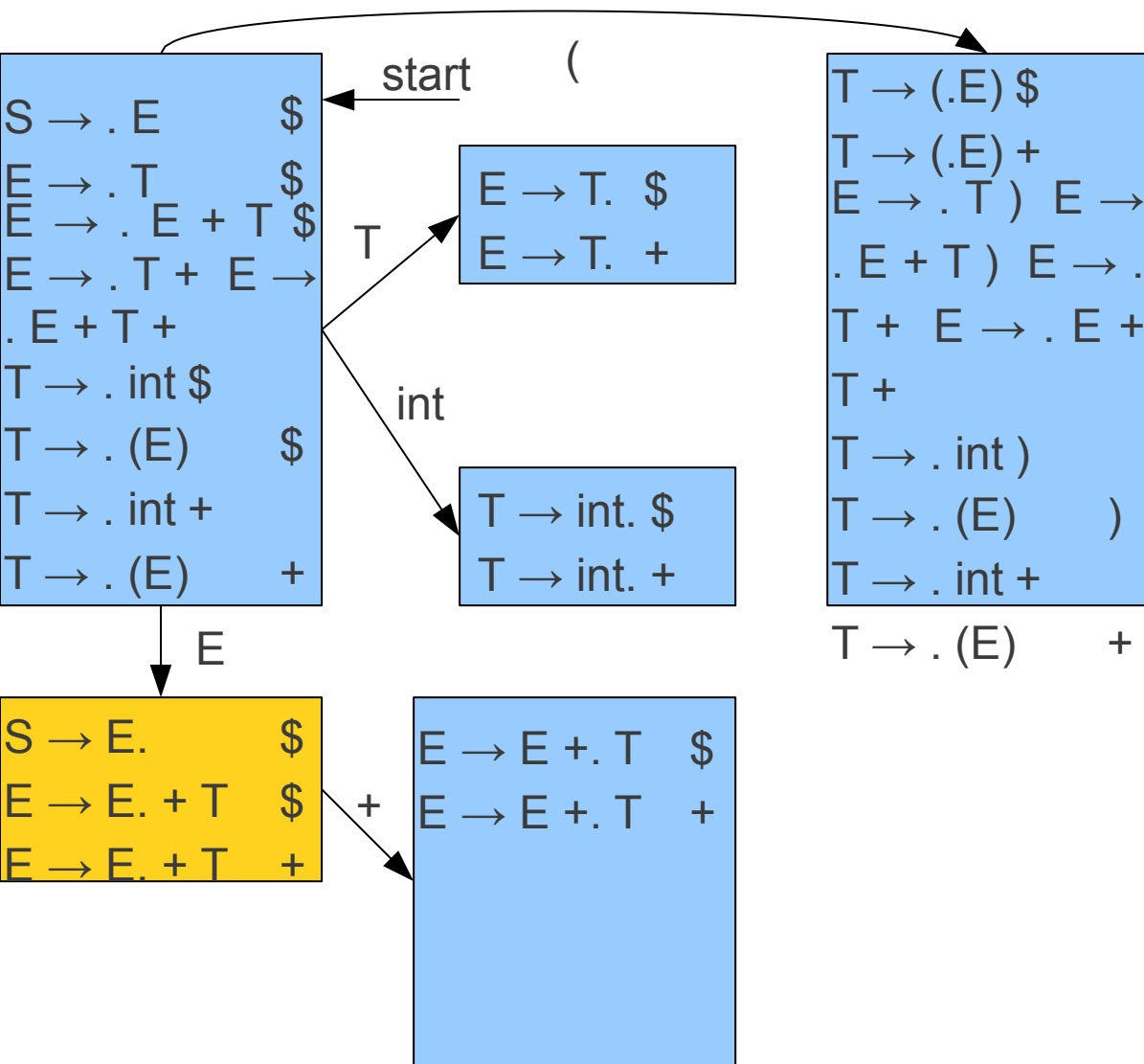
# Deterministic LR(1) Automata



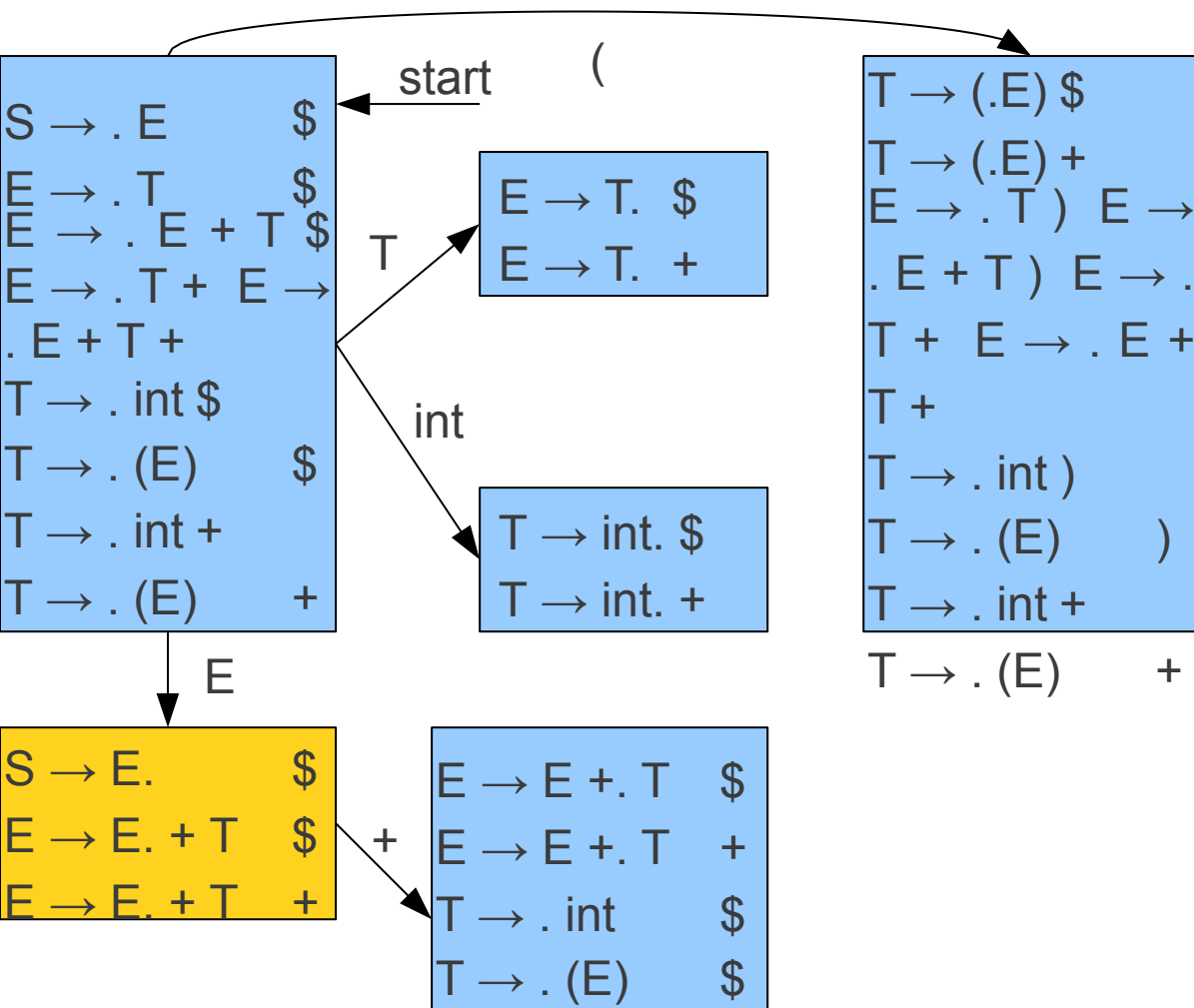
# Deterministic LR(1) Automata



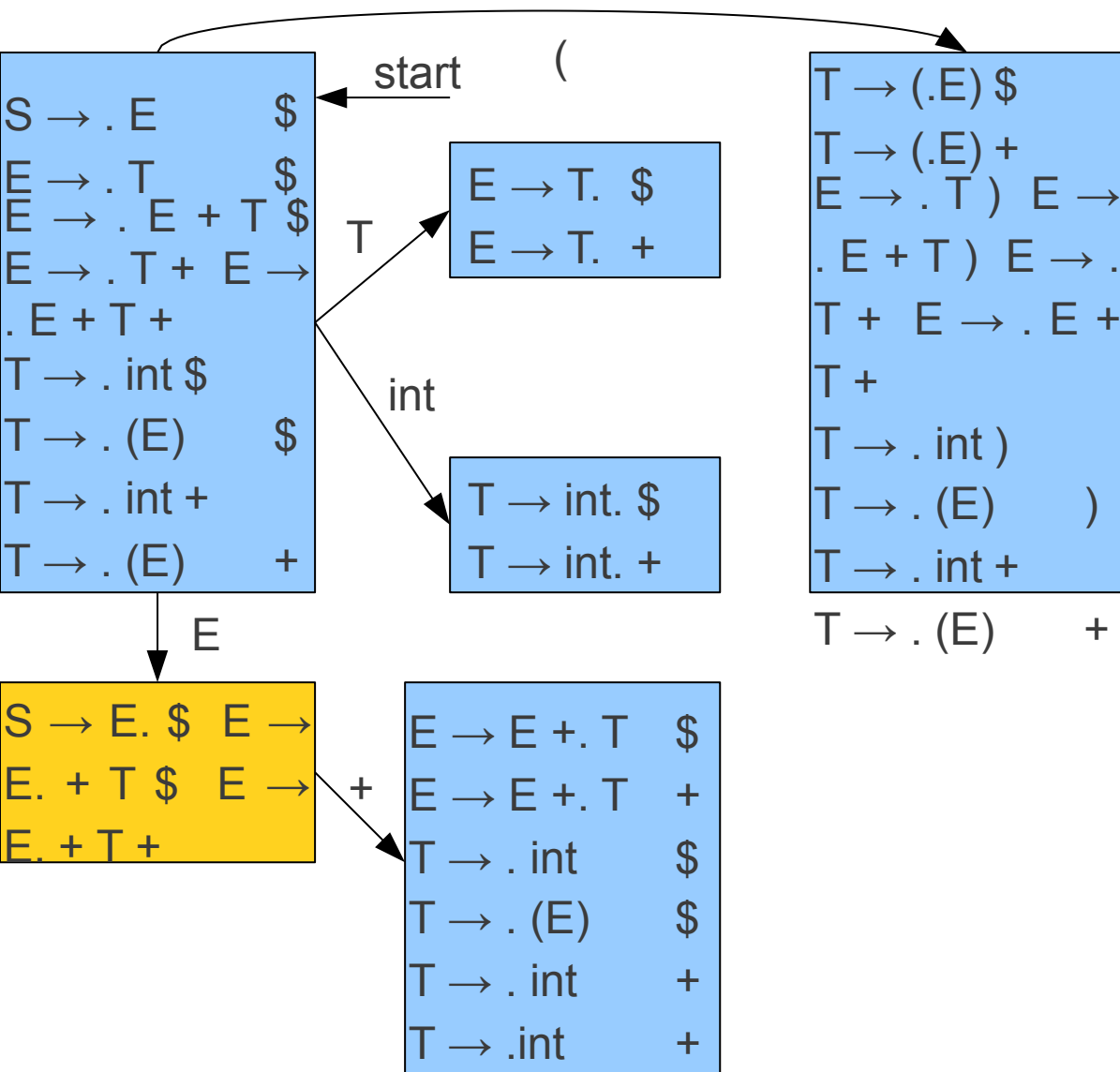
# Deterministic LR(1) Automata



# Deterministic LR(1) Automata

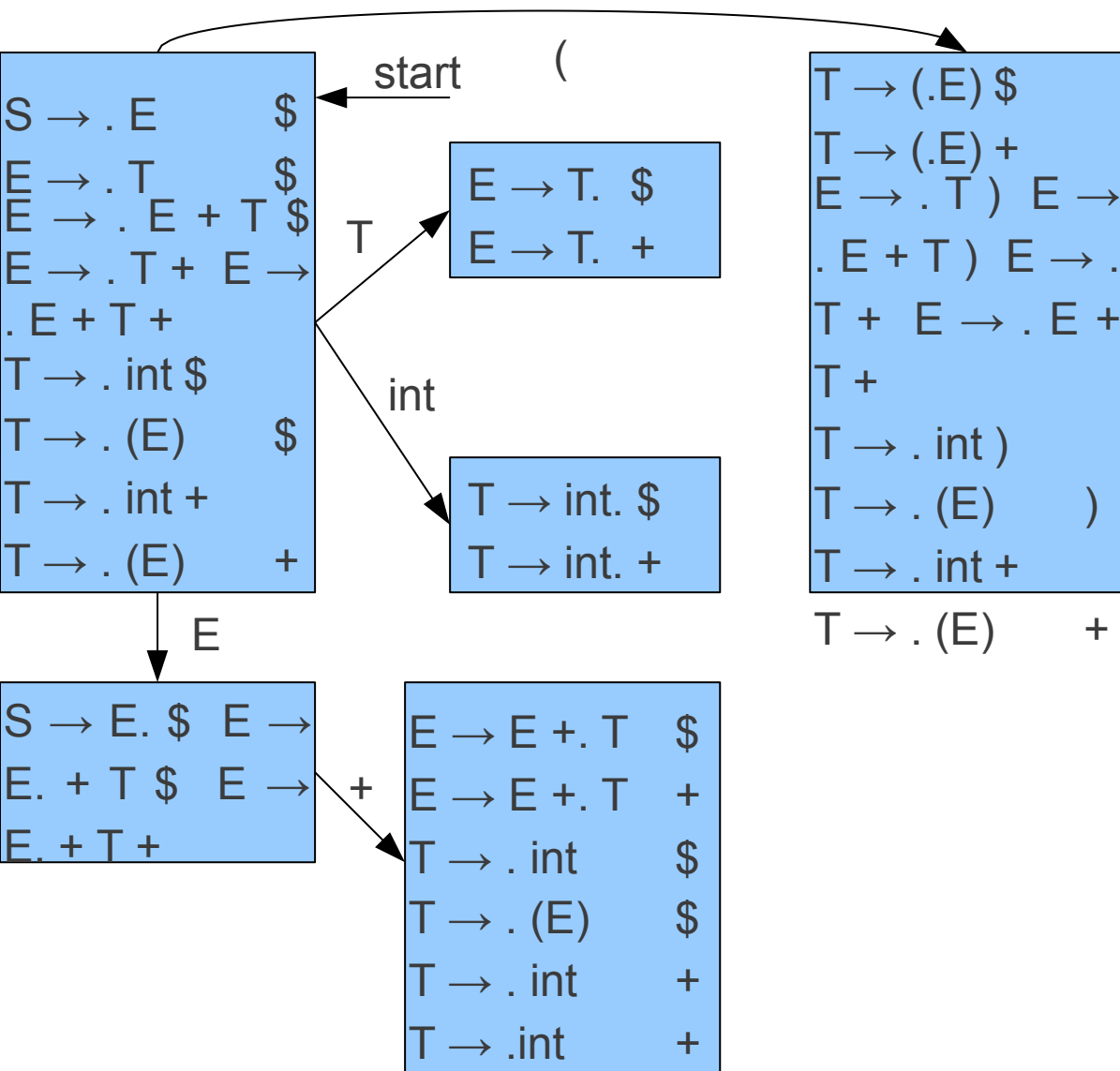


# Deterministic LR(1) Automata

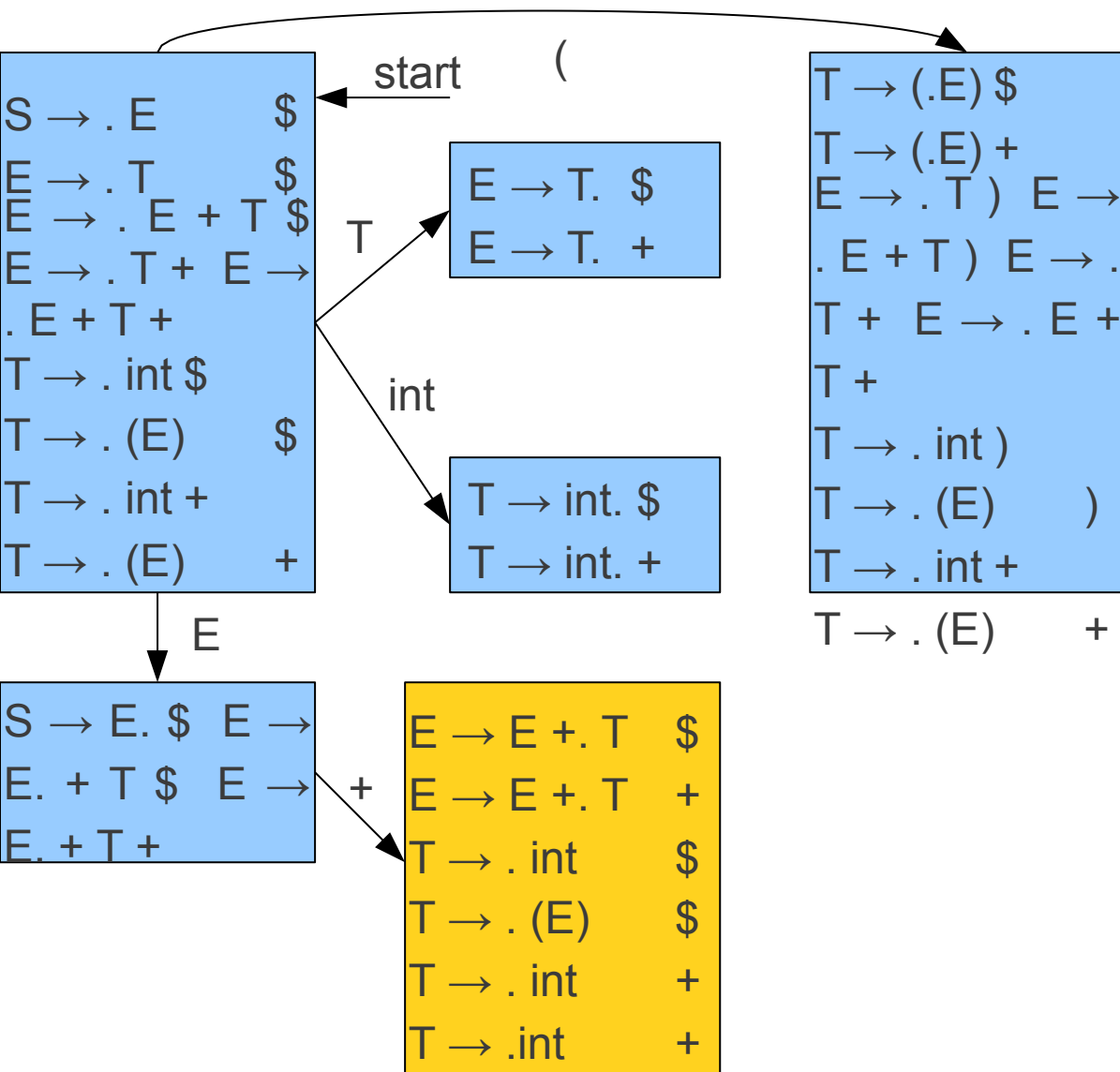




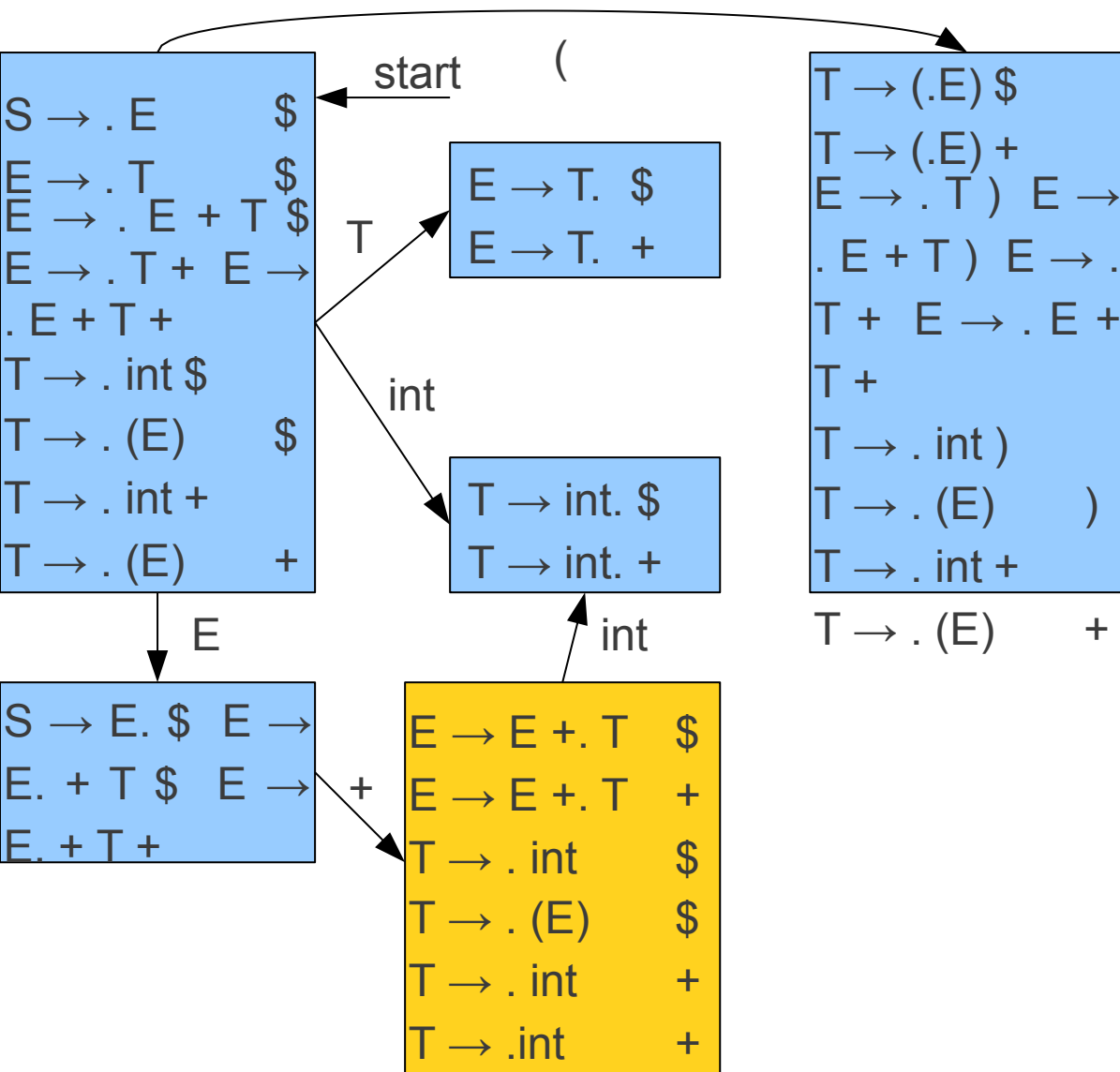
# Deterministic LR(1) Automata



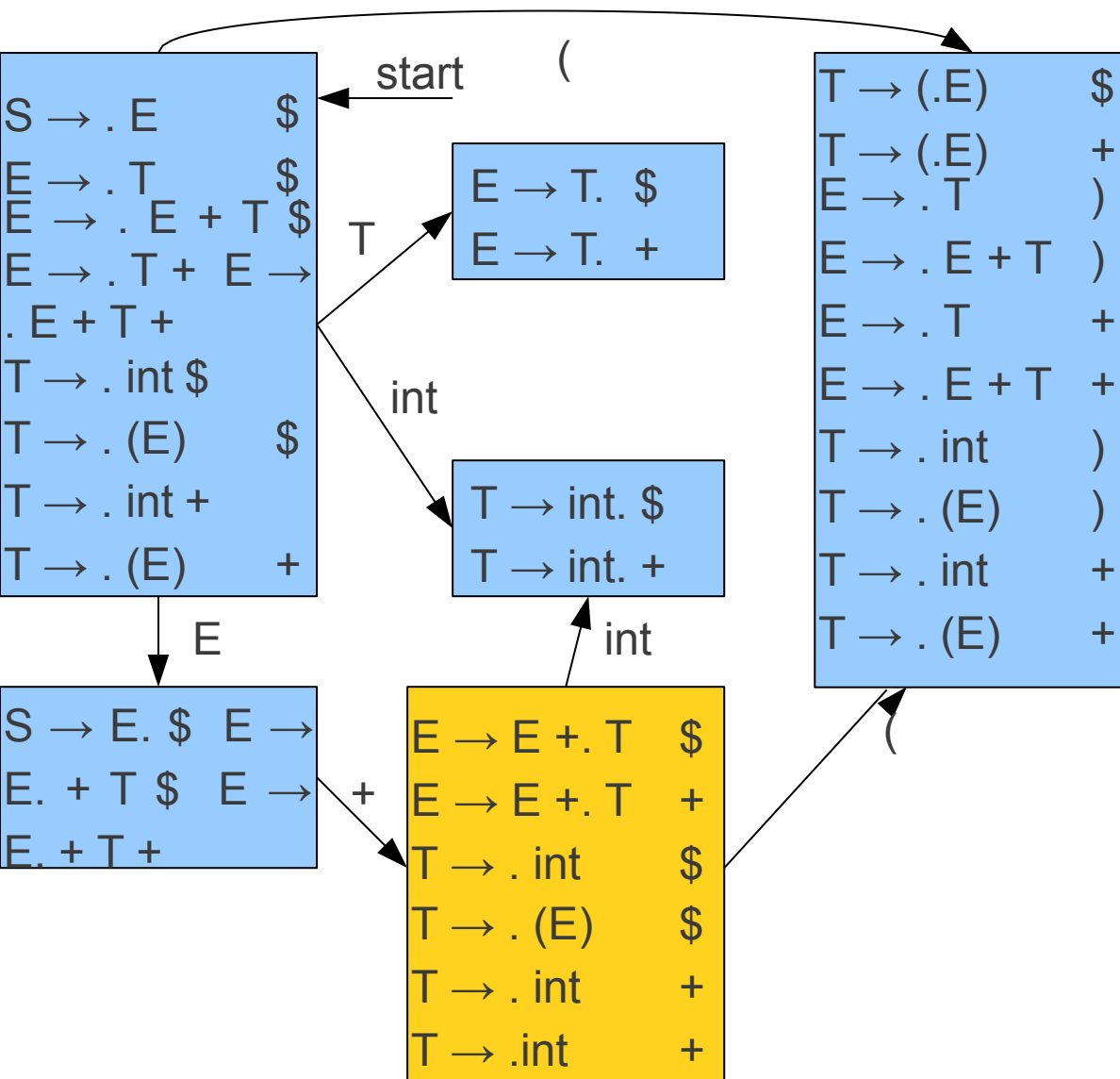
# Deterministic LR(1) Automata



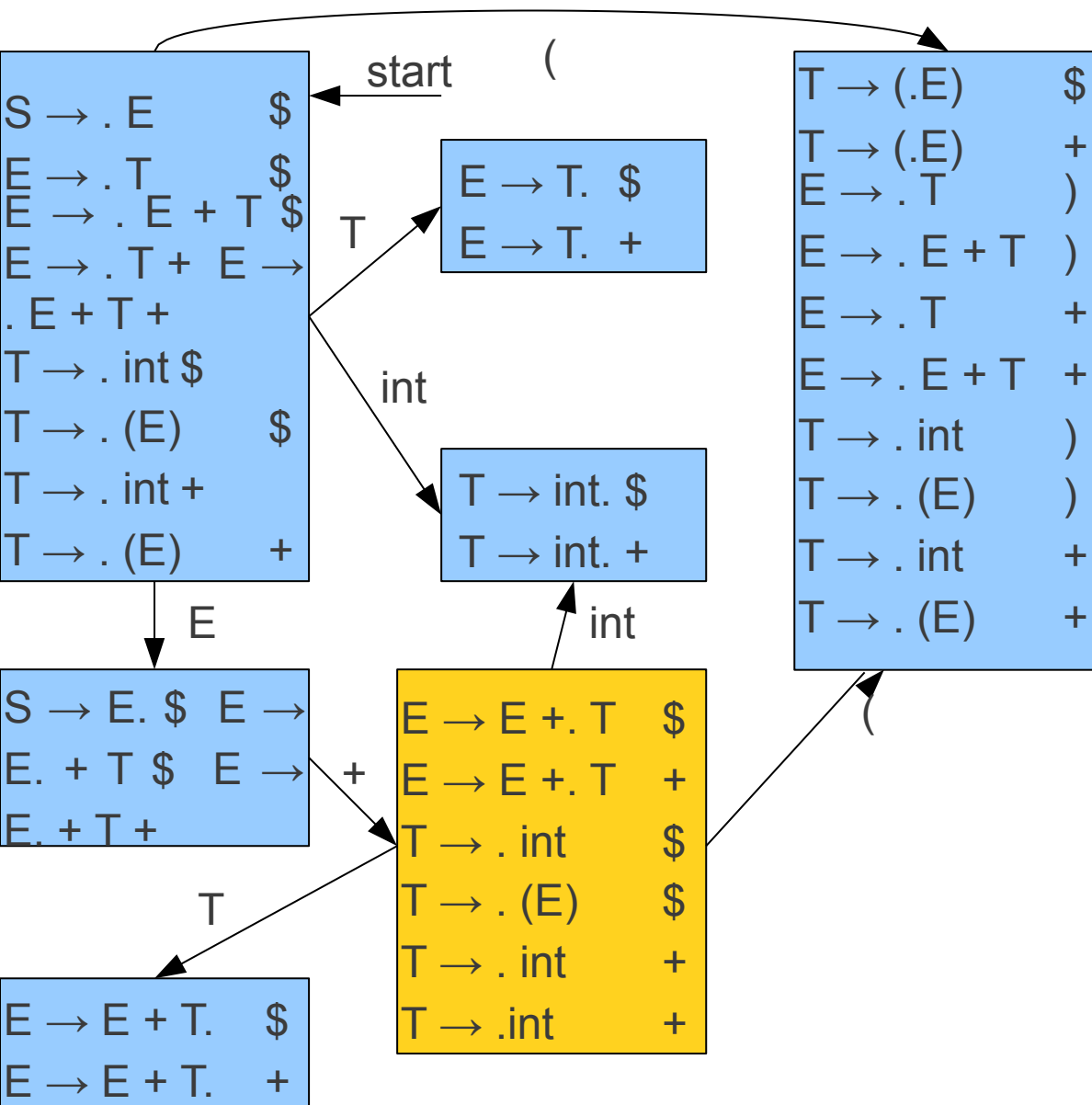
# Deterministic LR(1) Automata



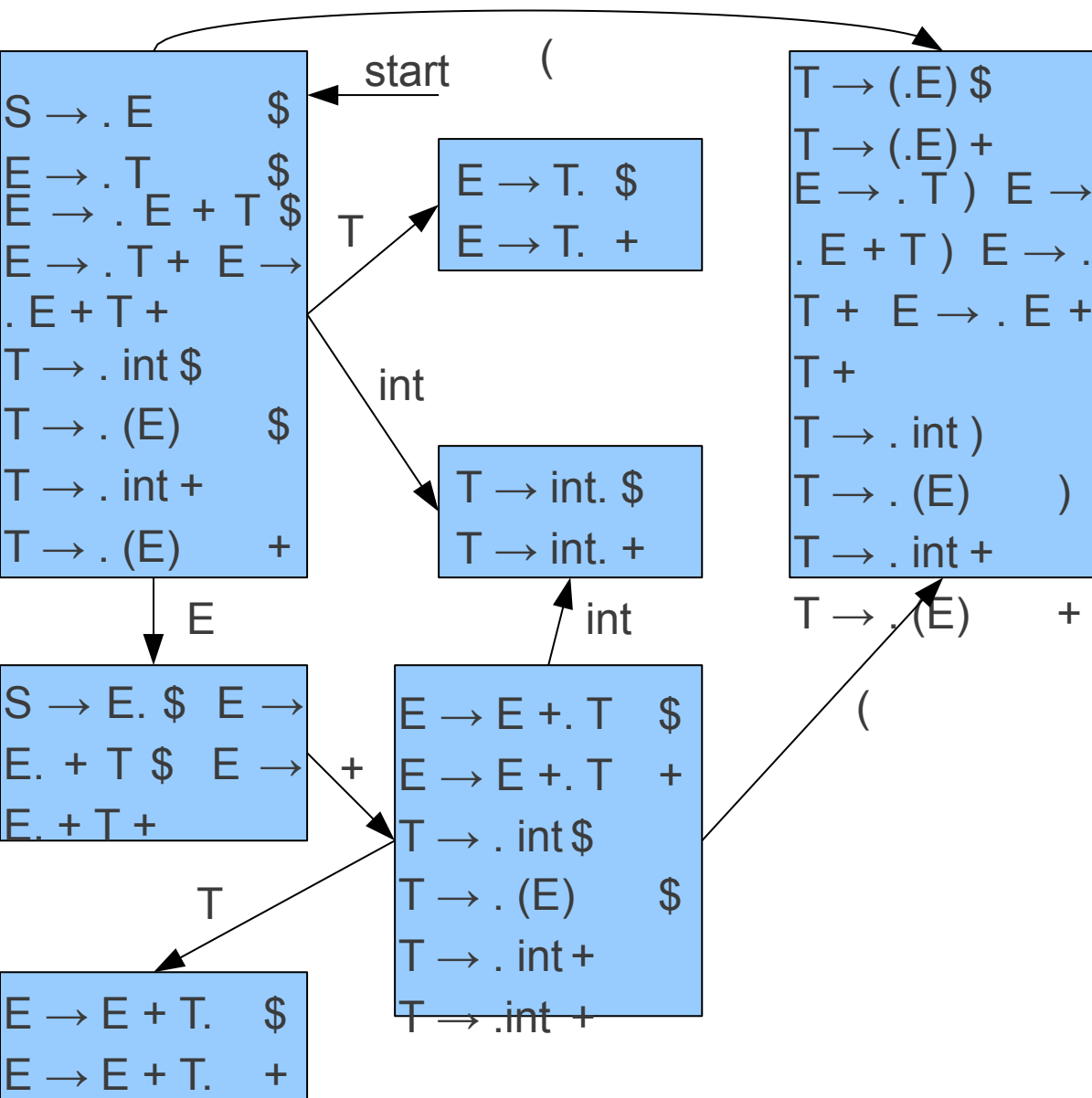
# Deterministic LR(1) Automata



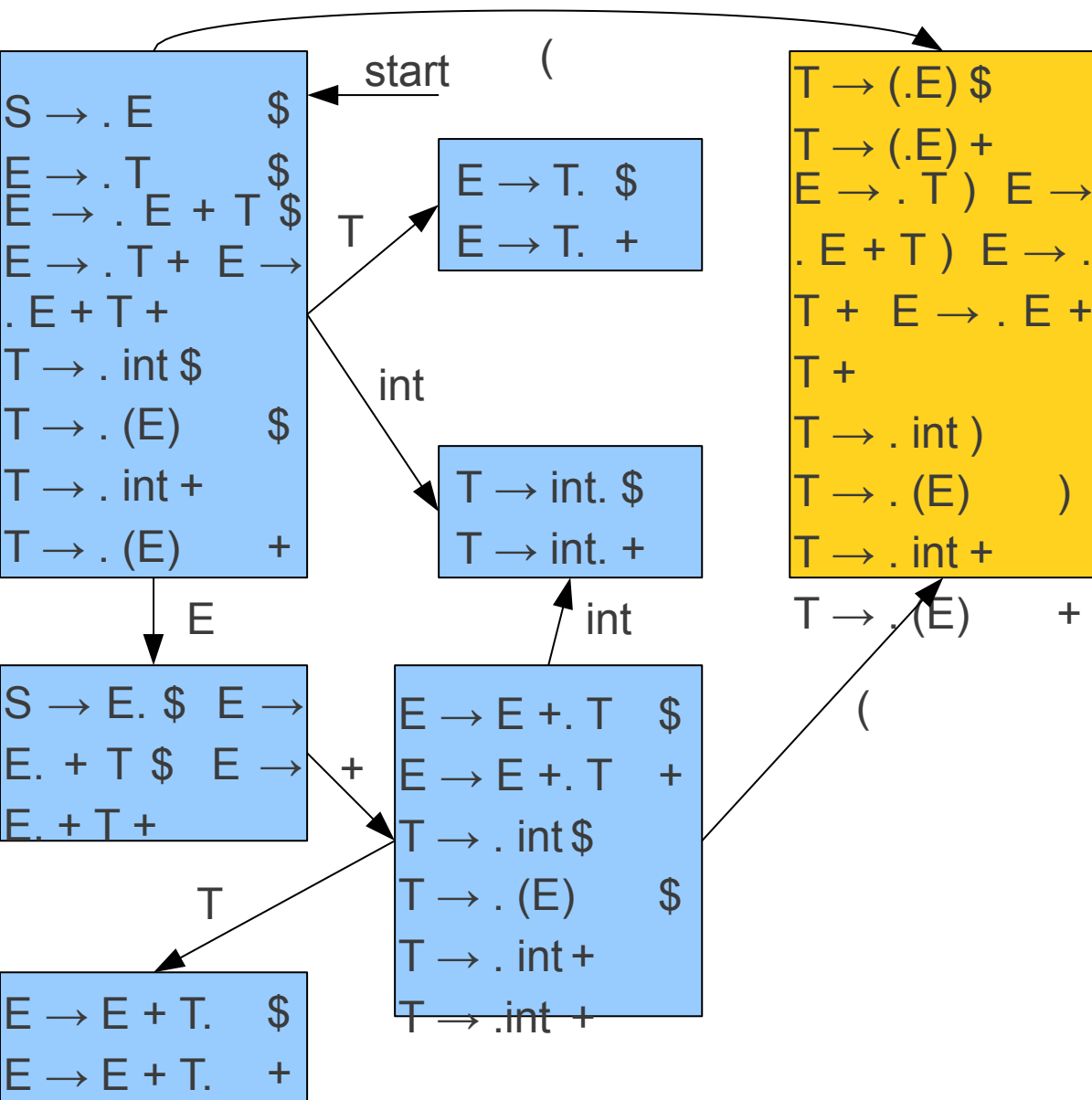
# Deterministic LR(1) Automata



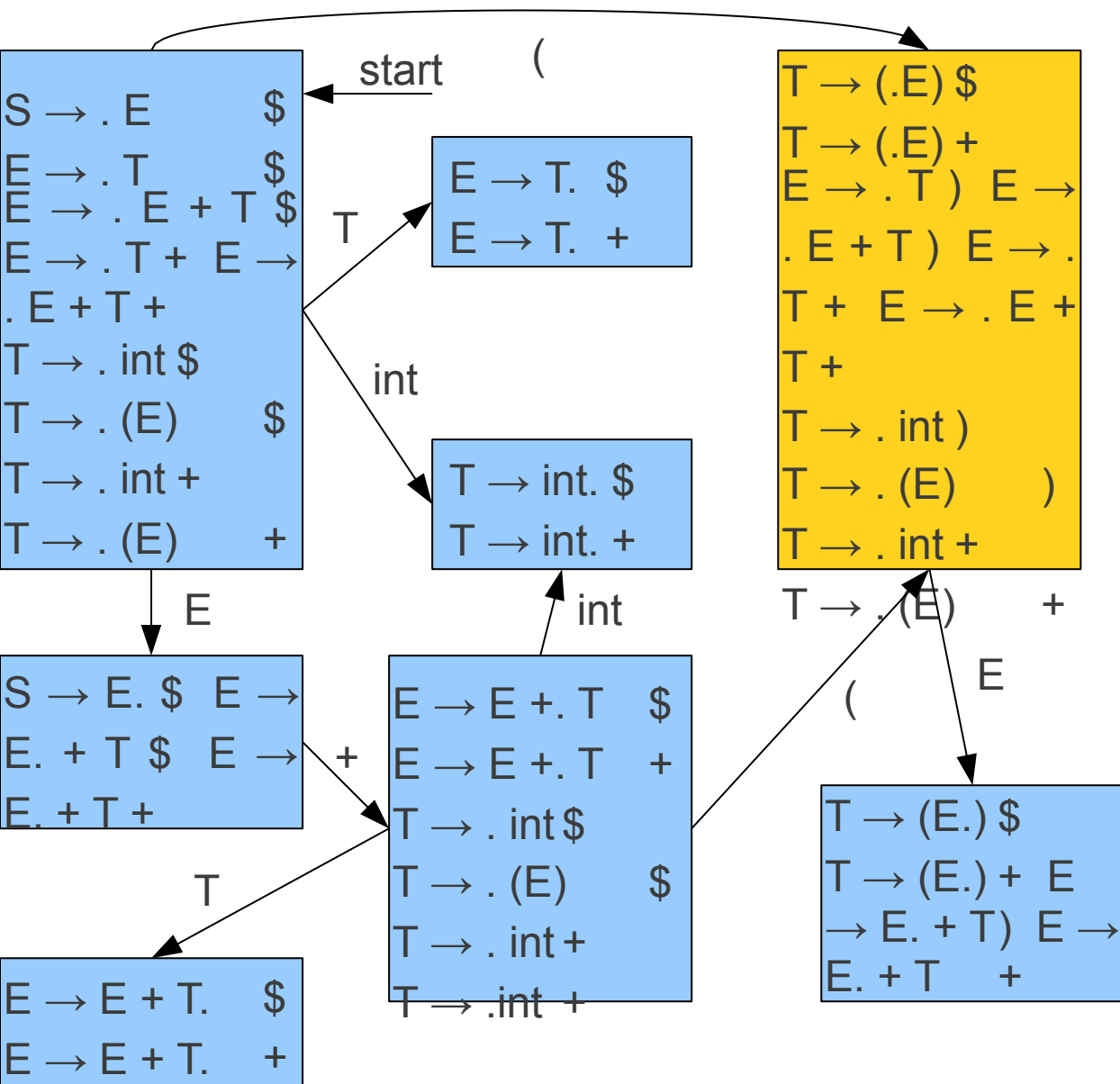
# Deterministic LR(1) Automata



# Deterministic LR(1) Automata

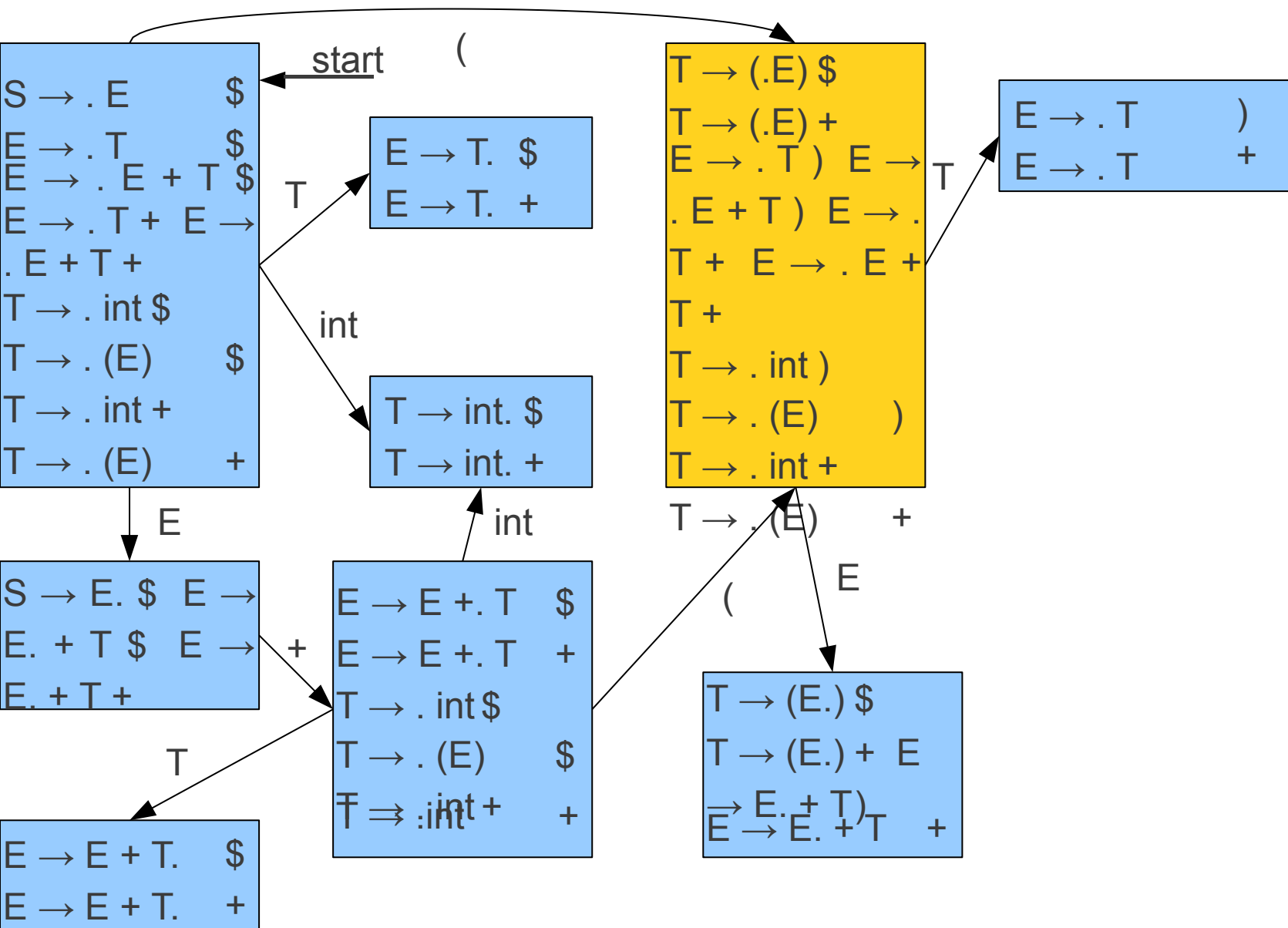


# Deterministic LR(1) Automata

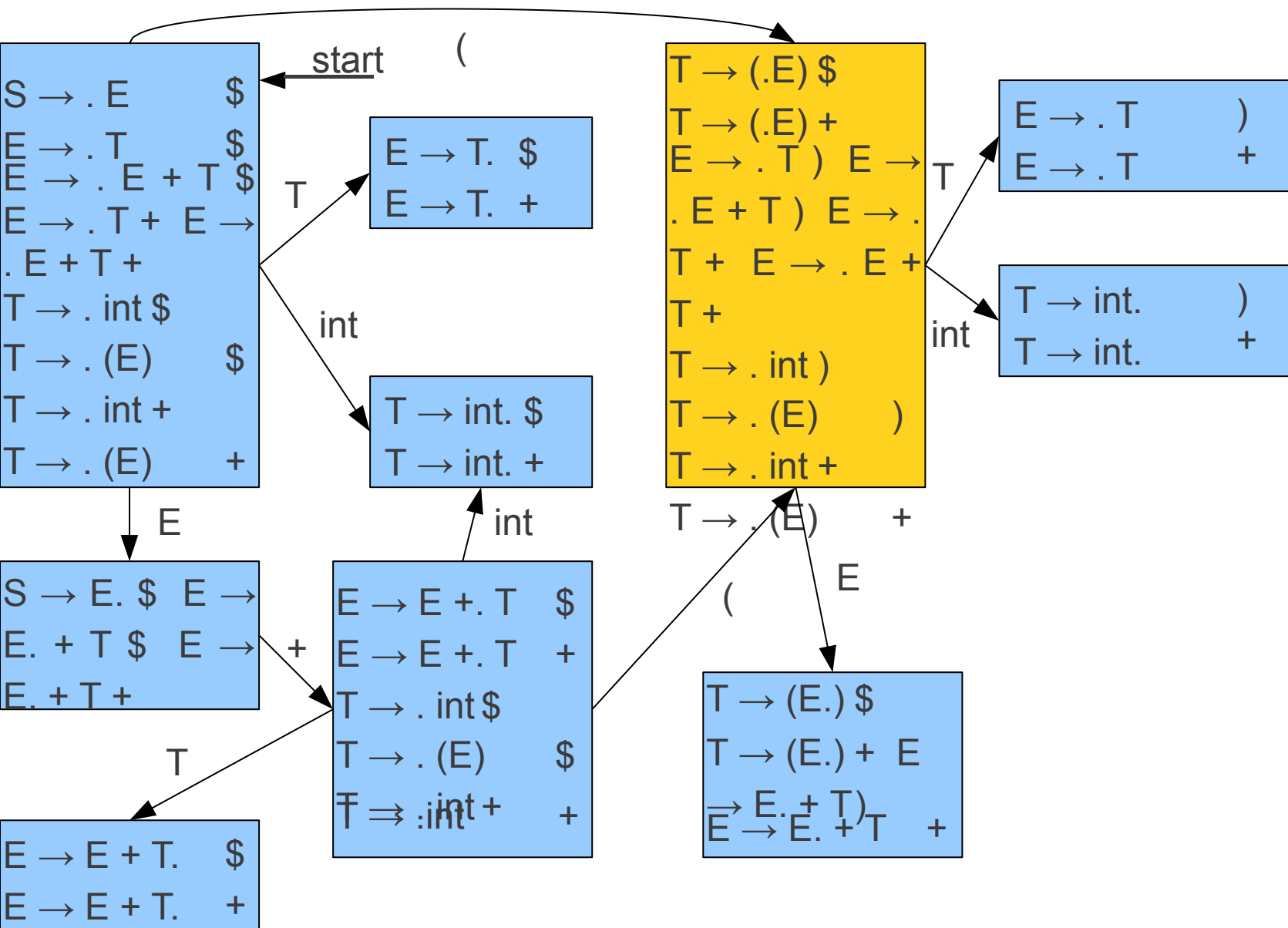




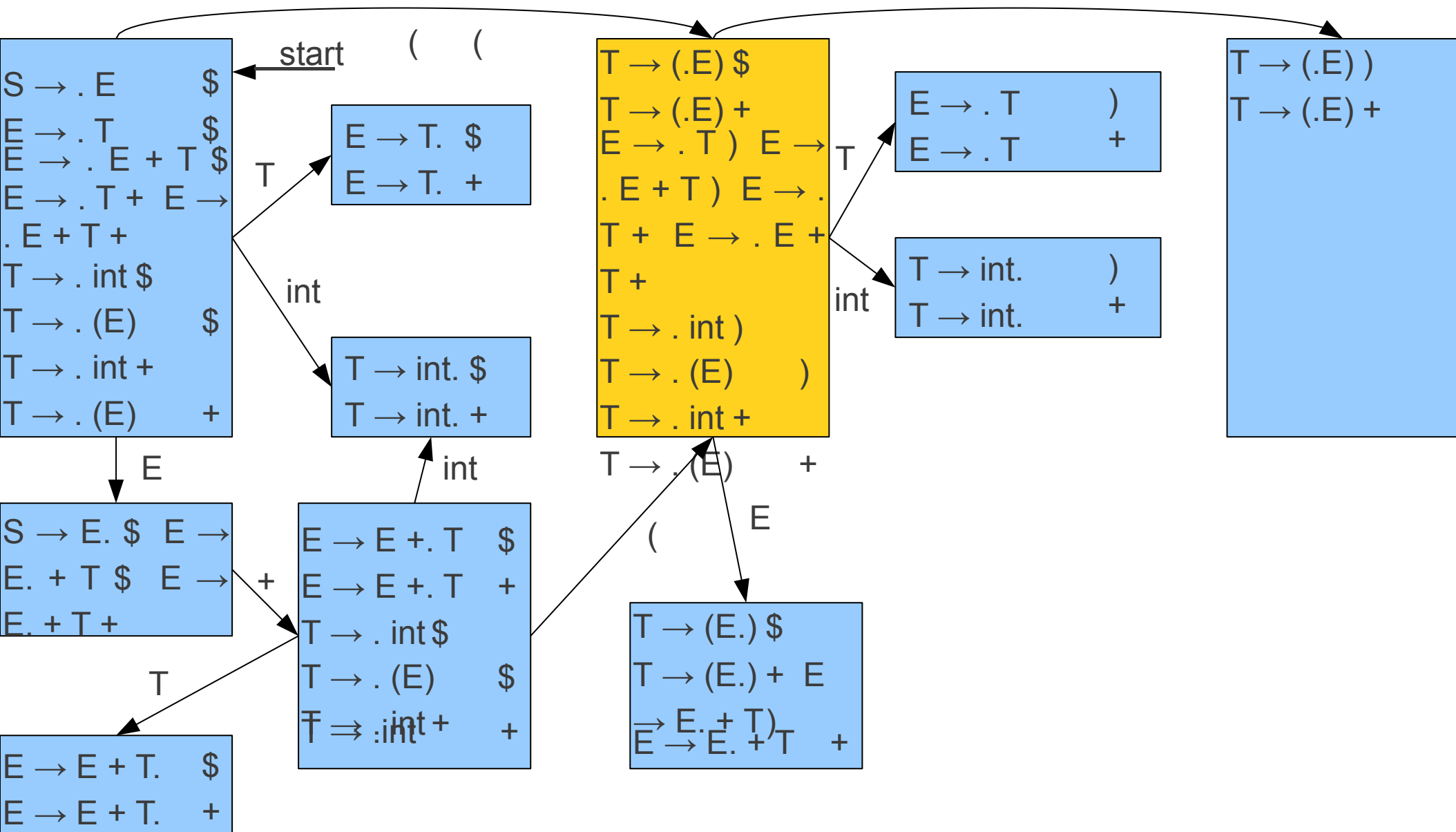
# Deterministic LR(1) Automata



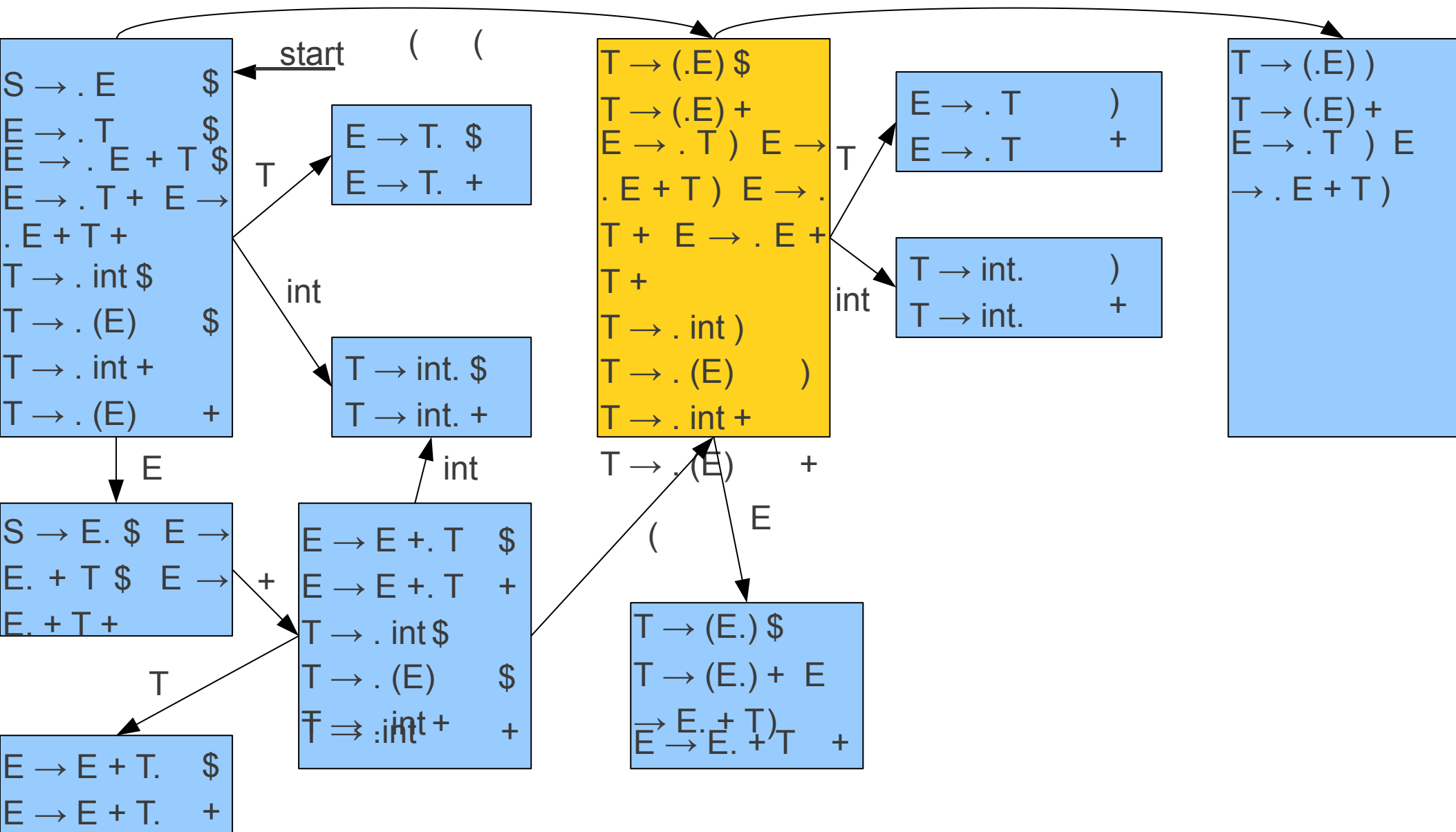
# Deterministic LR(1) Automata



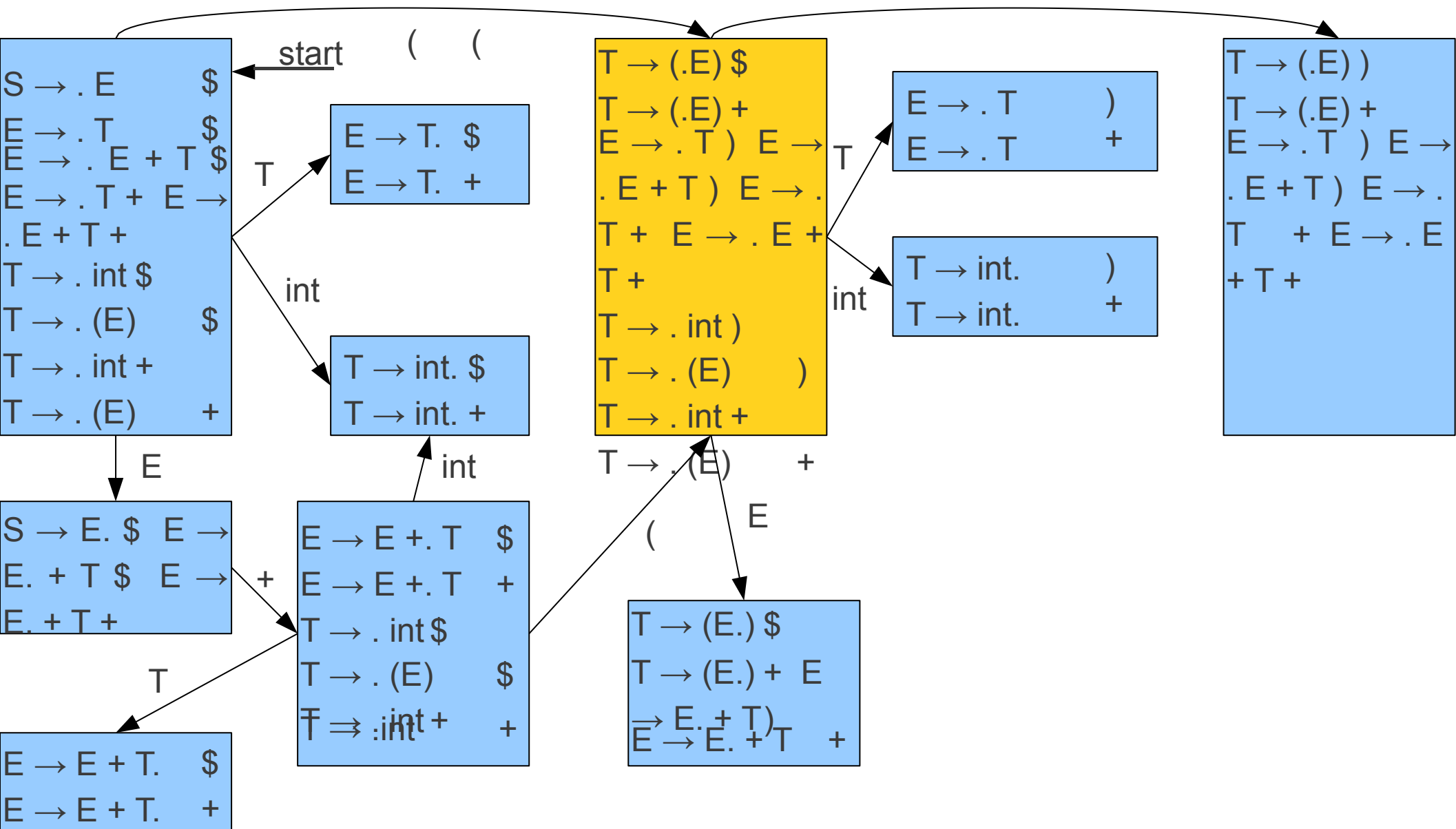
# Deterministic LR(1) Automata



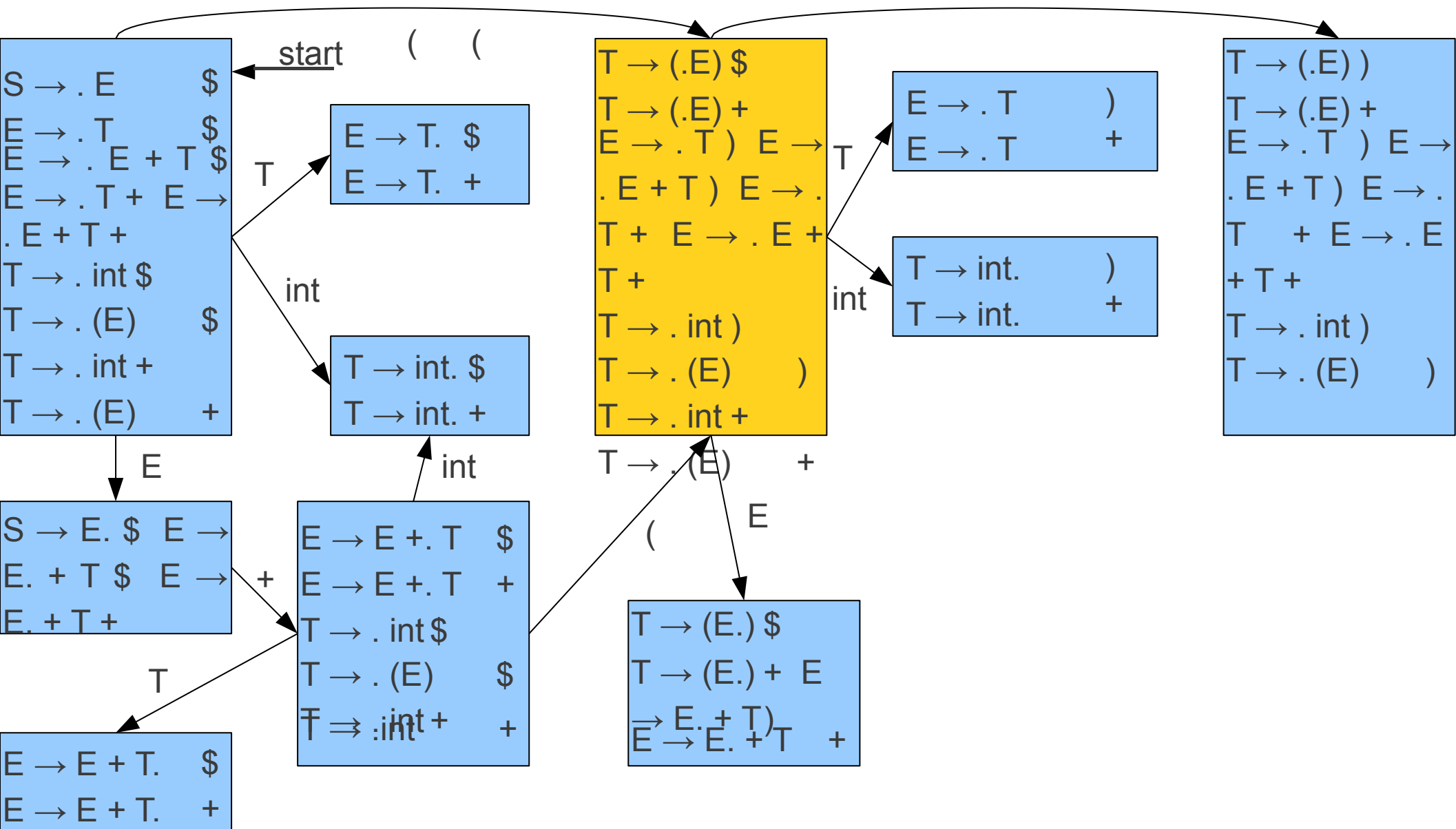
# Deterministic LR(1) Automata



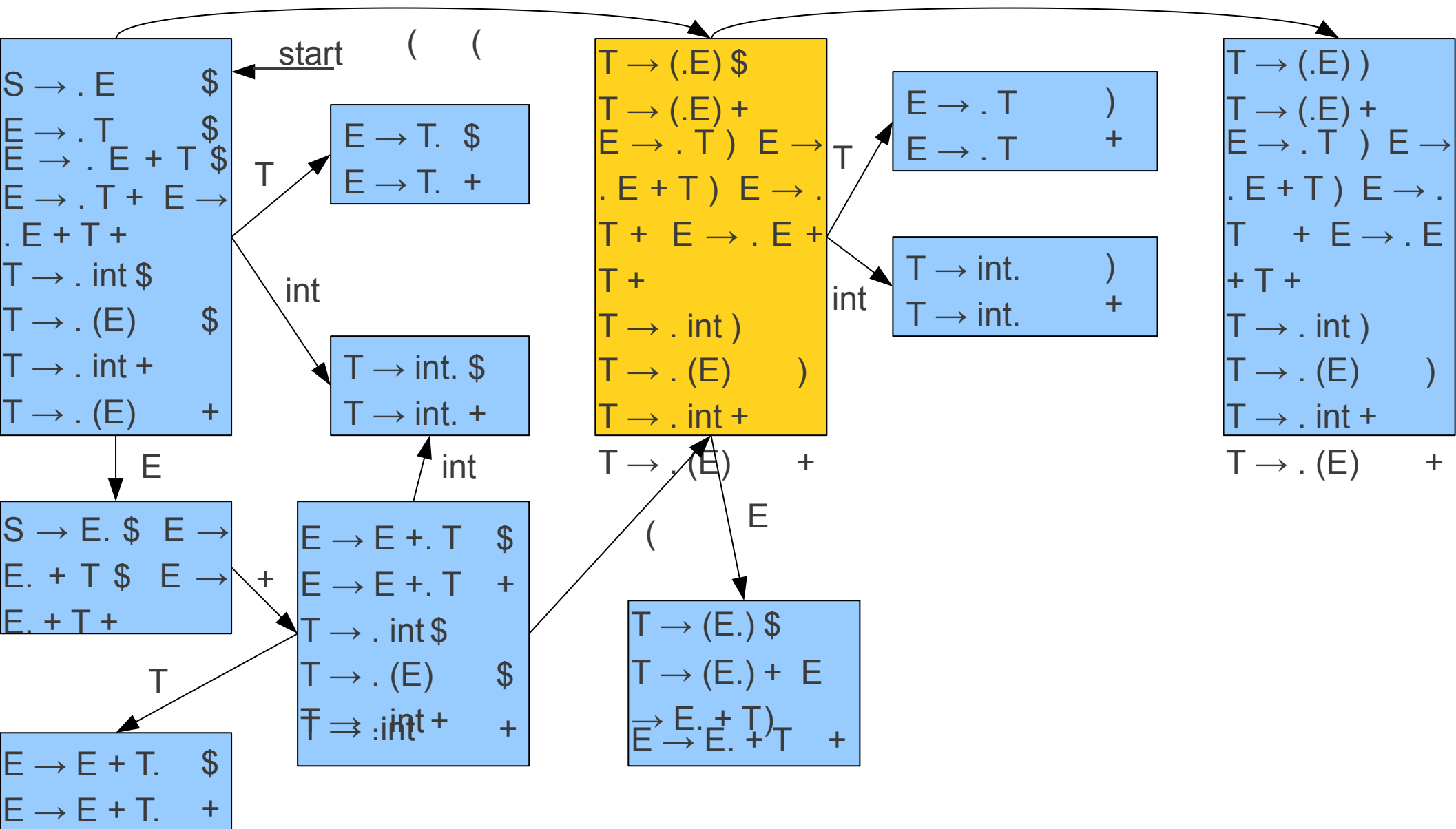
# Deterministic LR(1) Automata



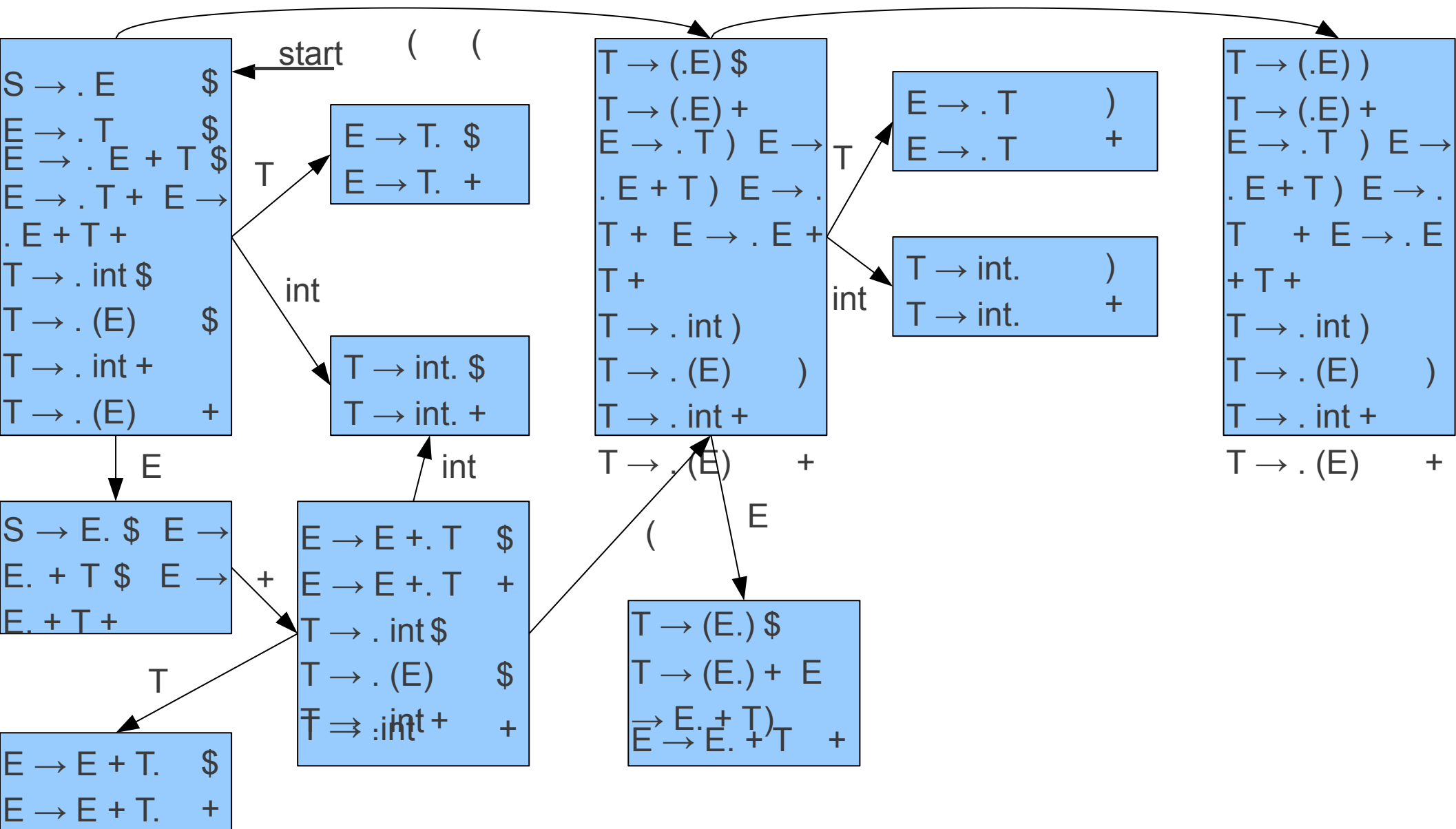
# Deterministic LR(1) Automata



# Deterministic LR(1) Automata

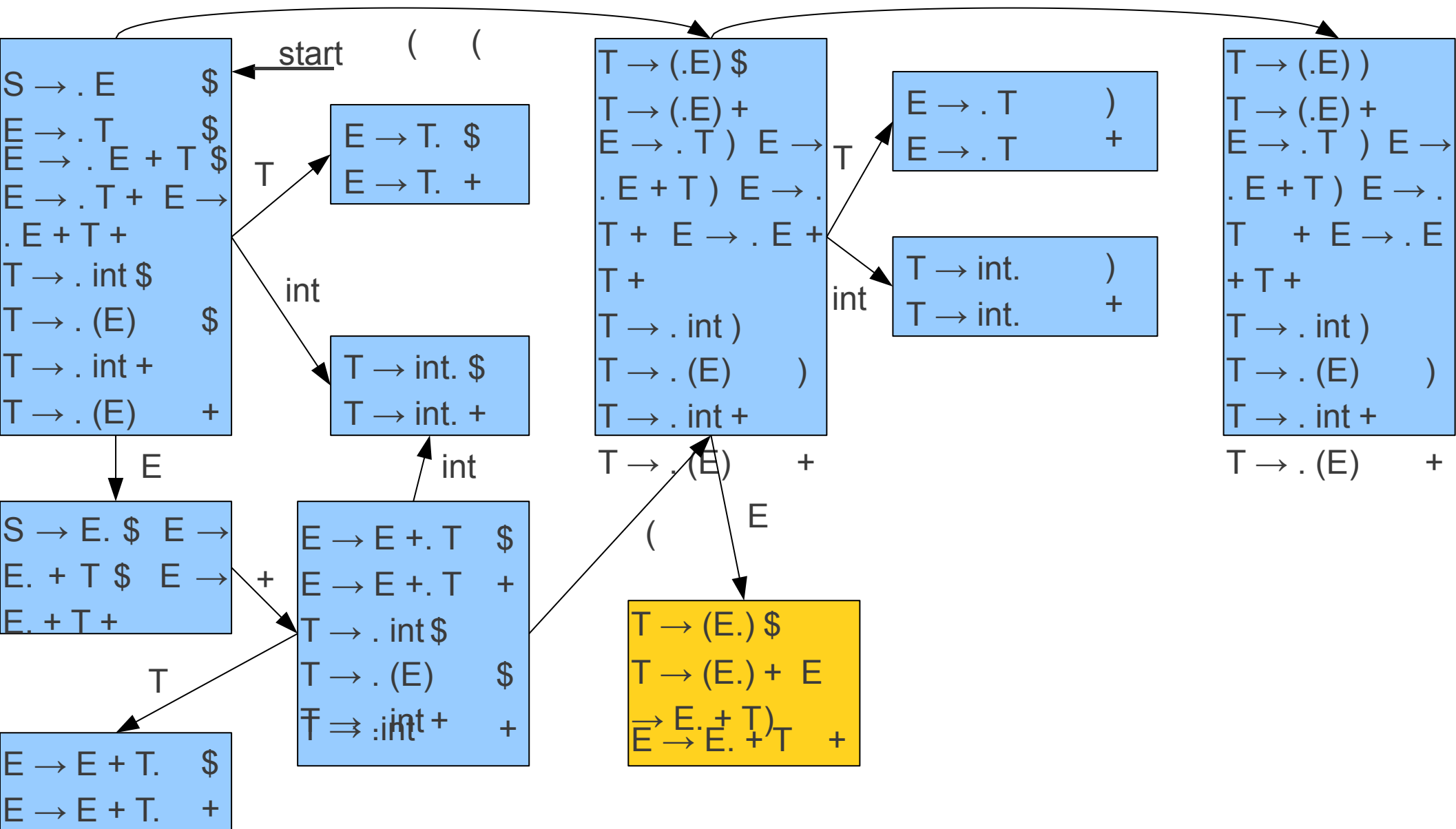


# Deterministic LR(1) Automata

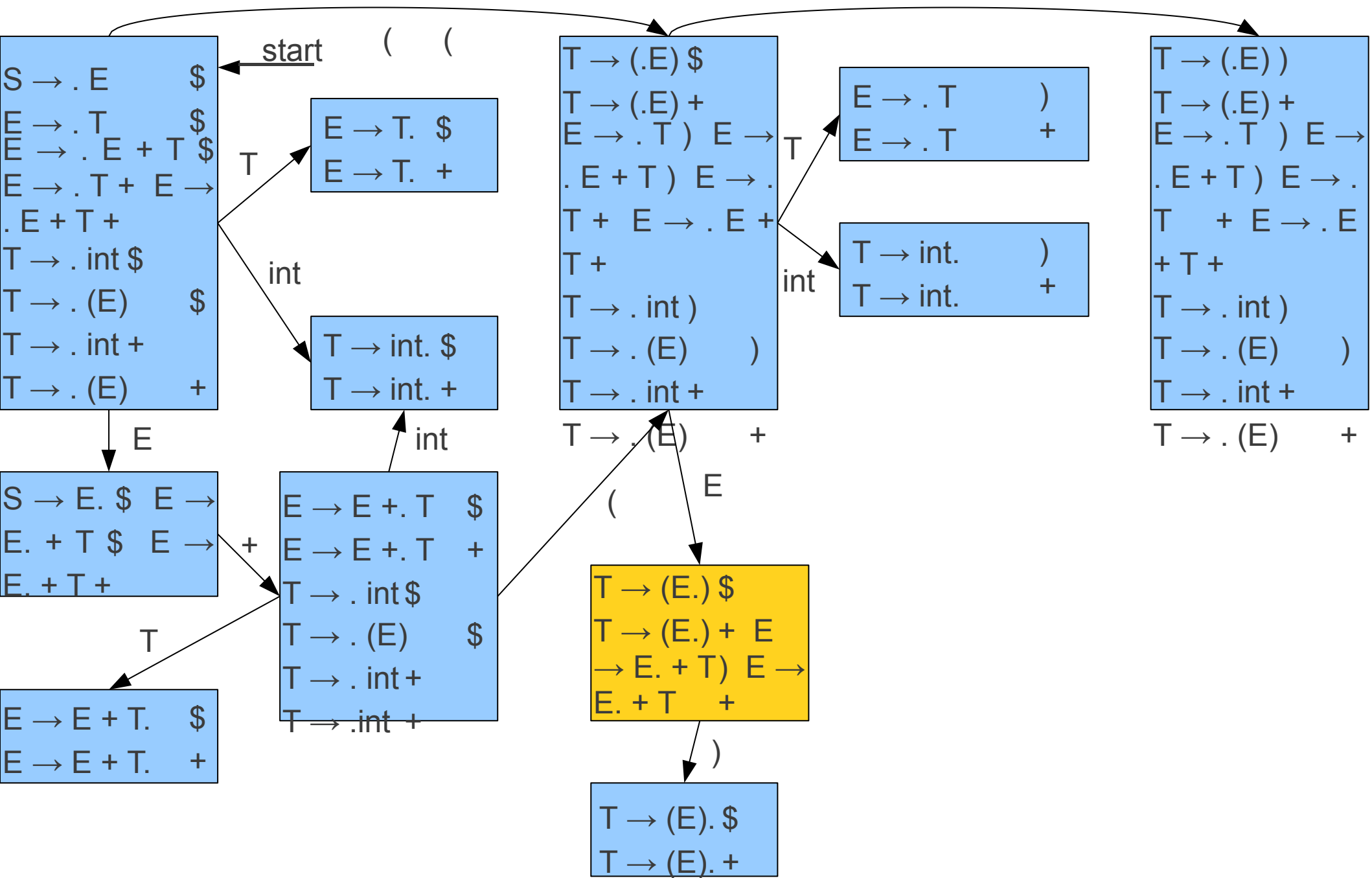




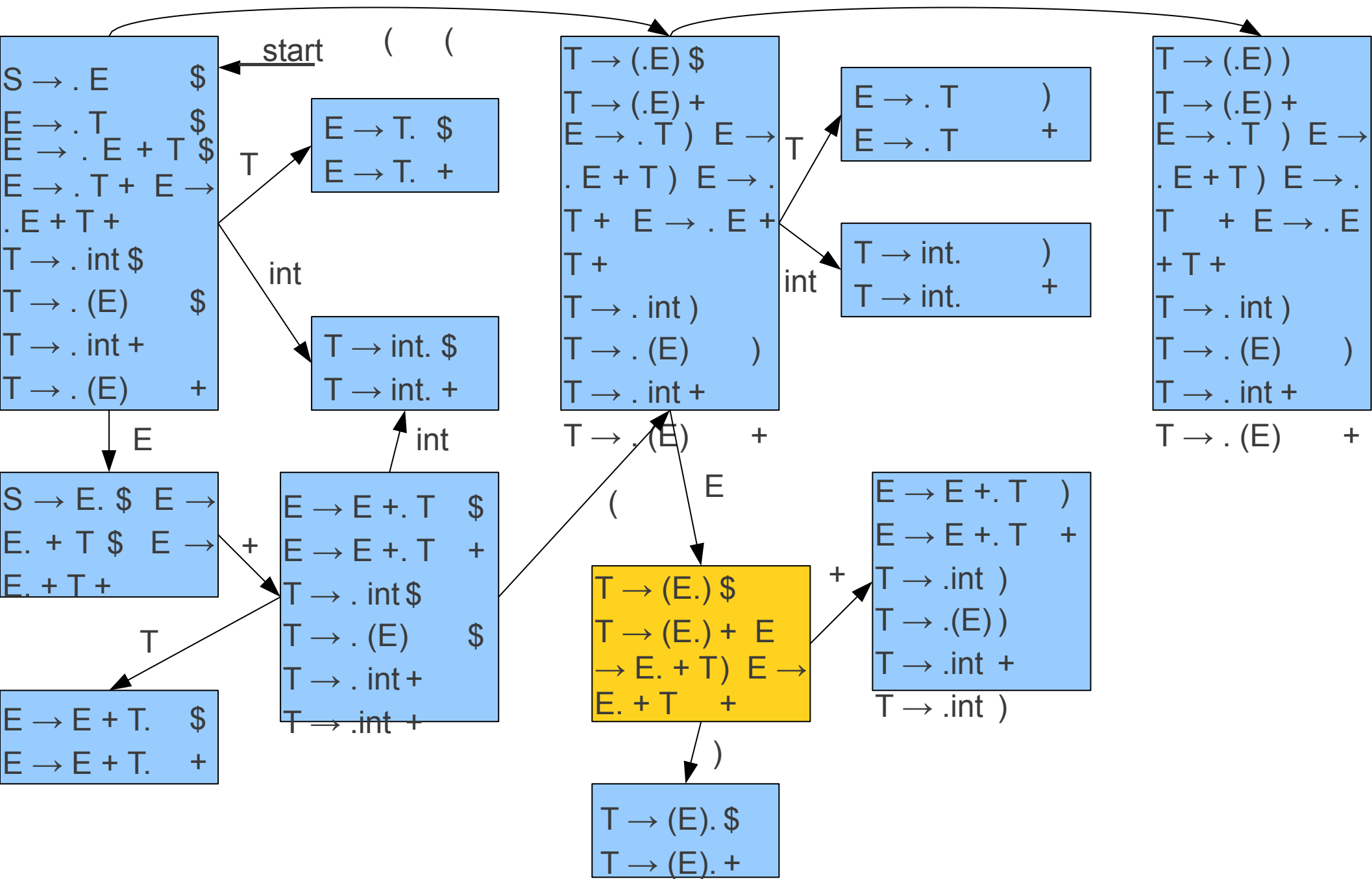
# Deterministic LR(1) Automata



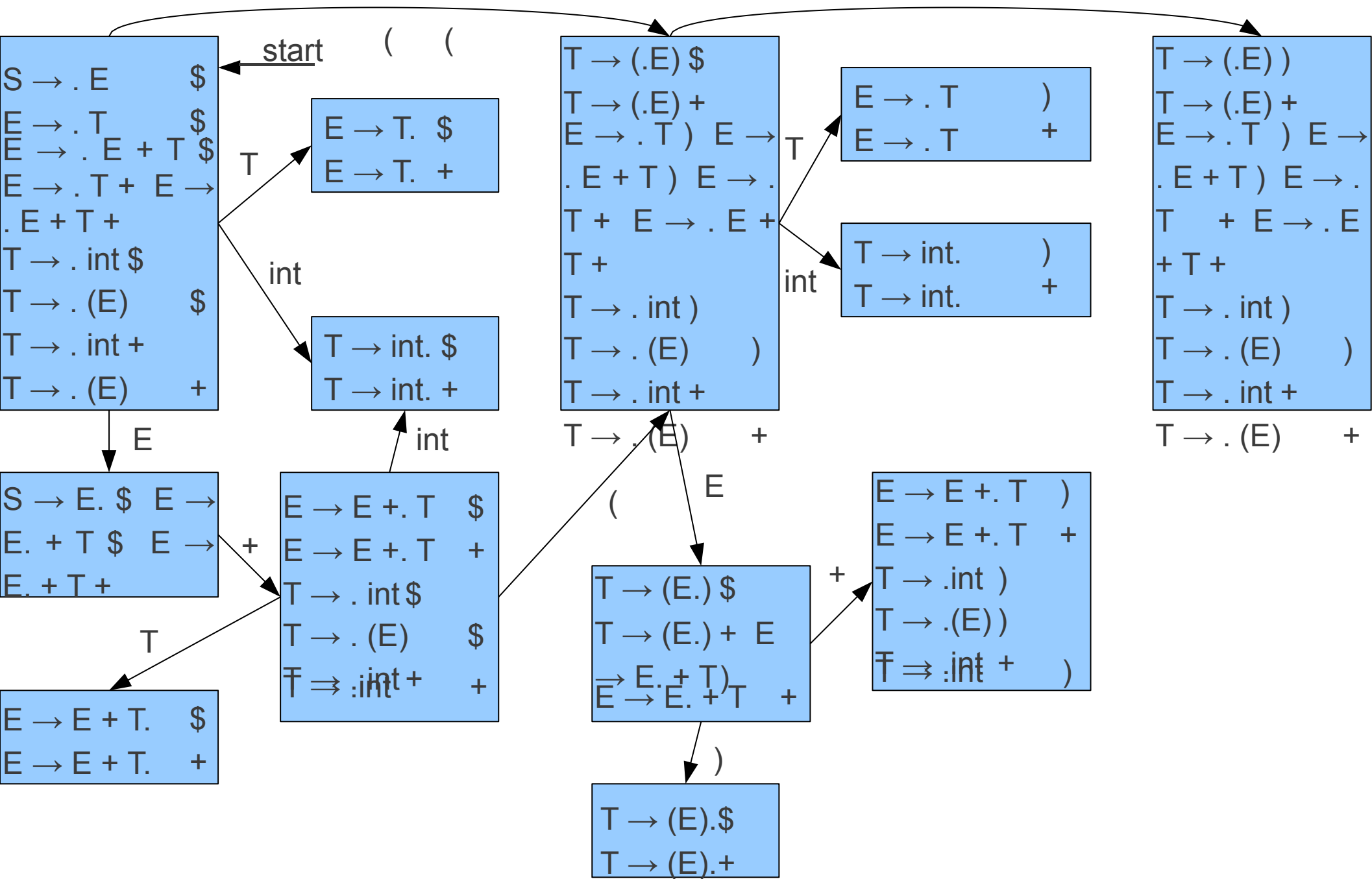
# Deterministic LR(1) Automata



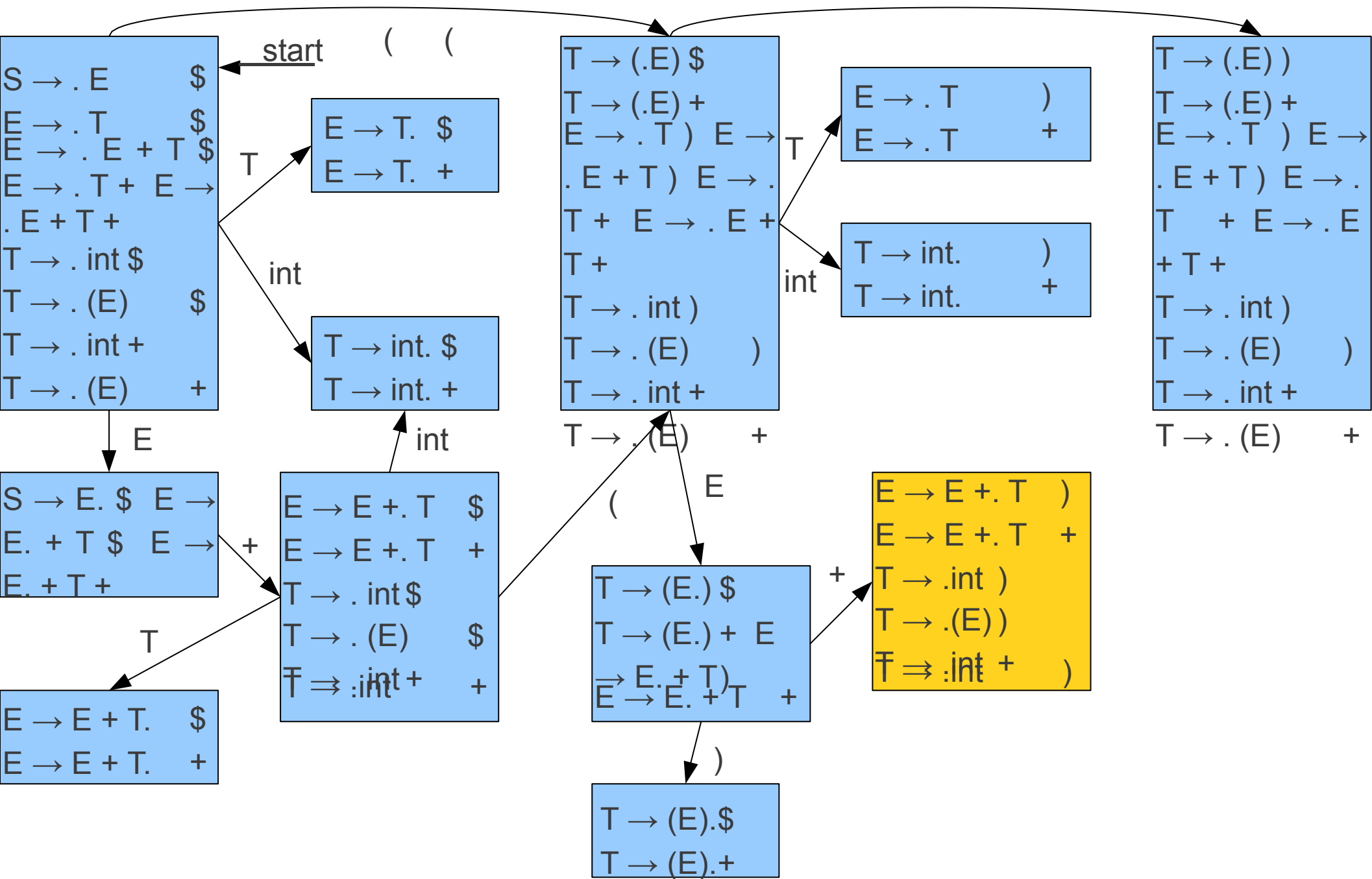
# Deterministic LR(1) Automata



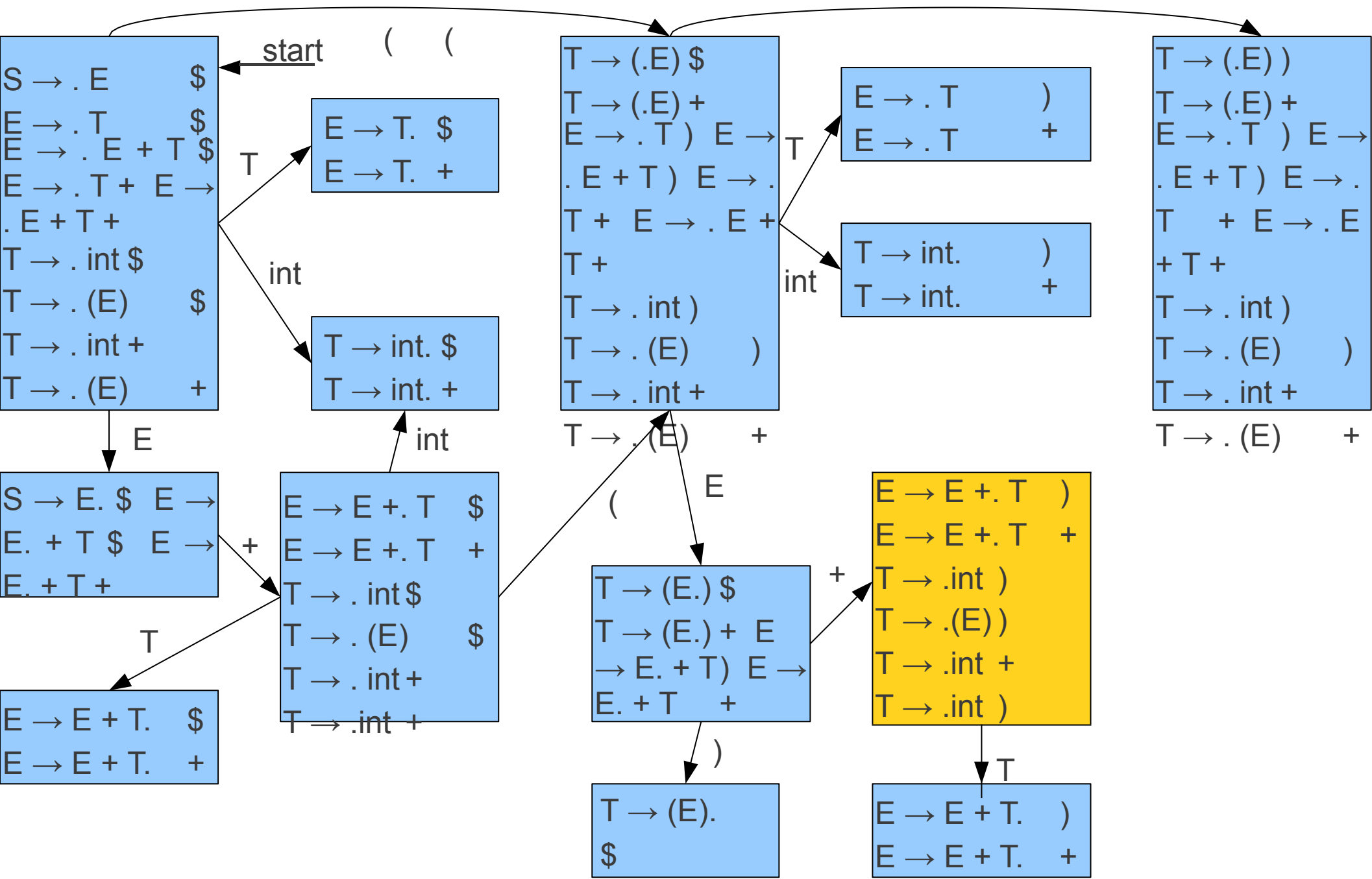
# Deterministic LR(1) Automata



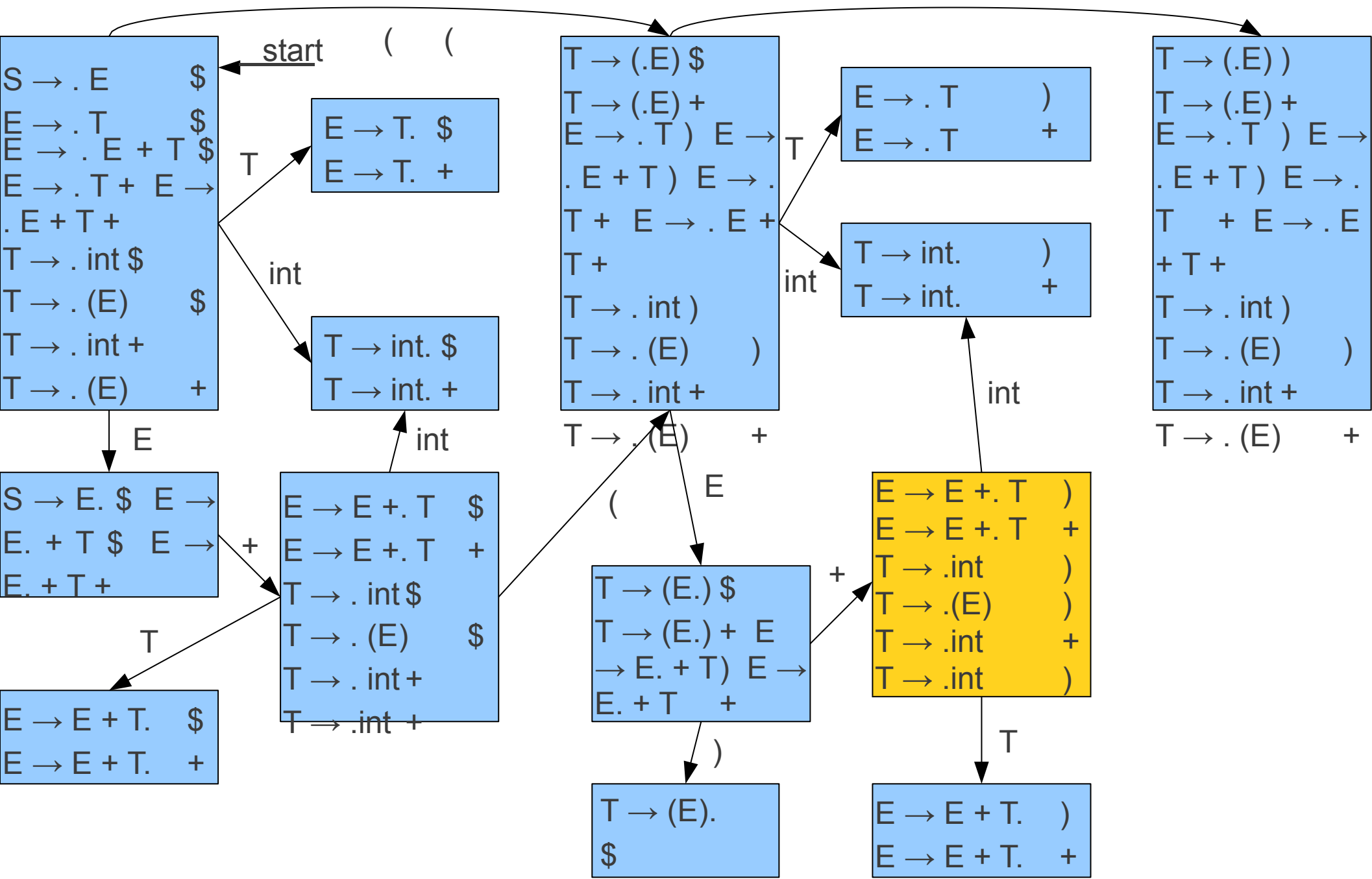
# Deterministic LR(1) Automata



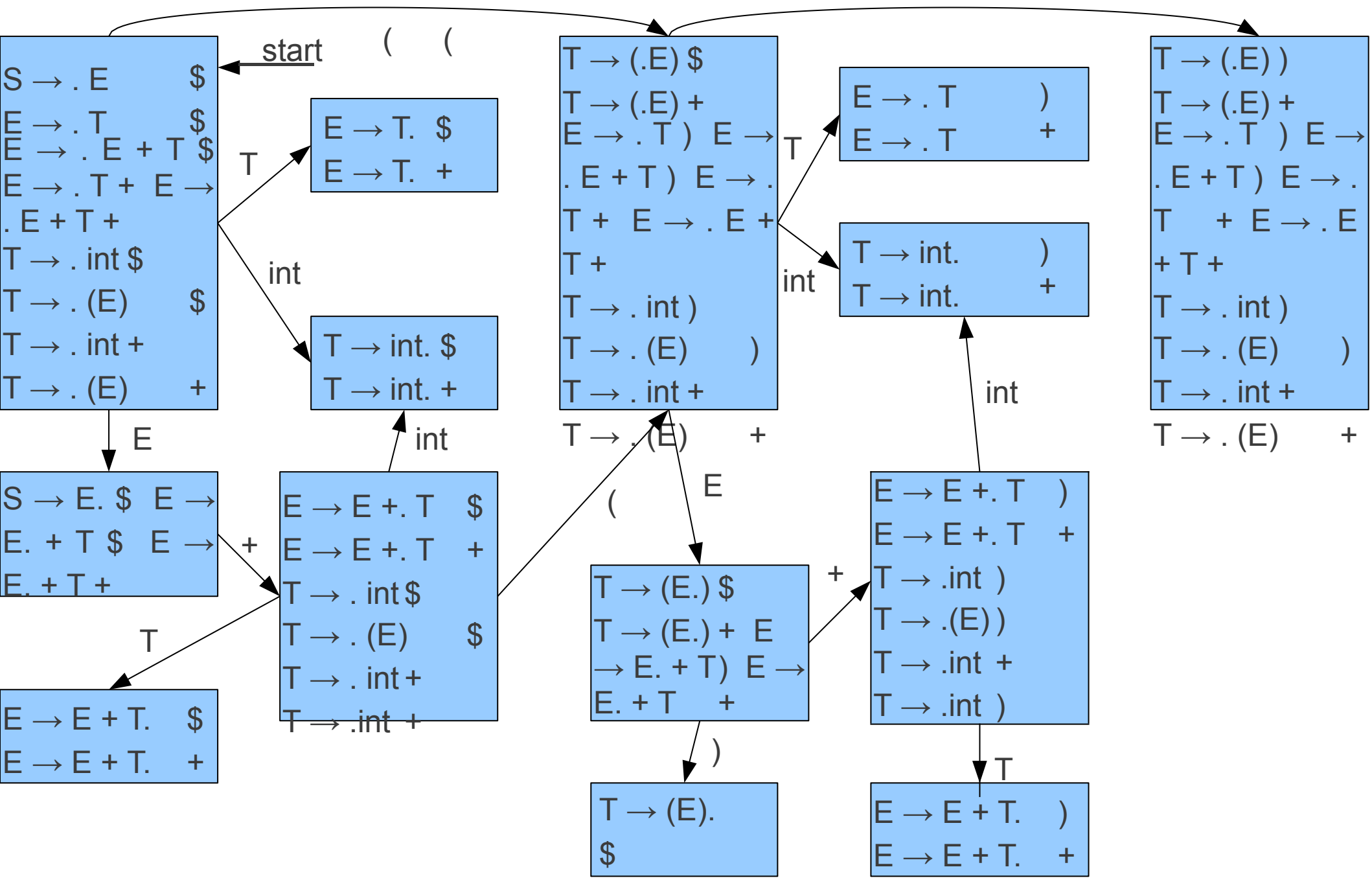
# Deterministic LR(1) Automata



# Deterministic LR(1) Automata

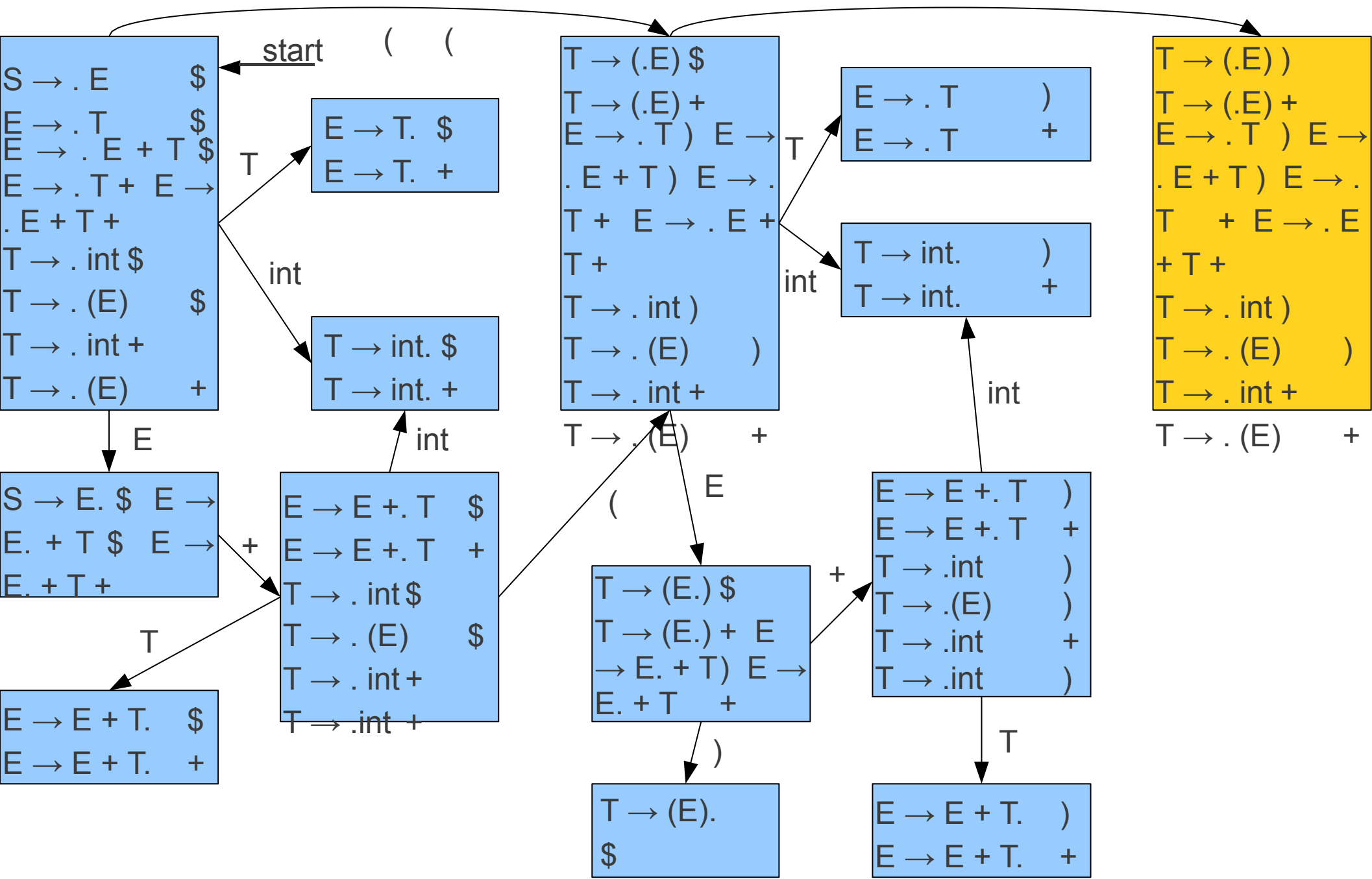


# Deterministic LR(1) Automata

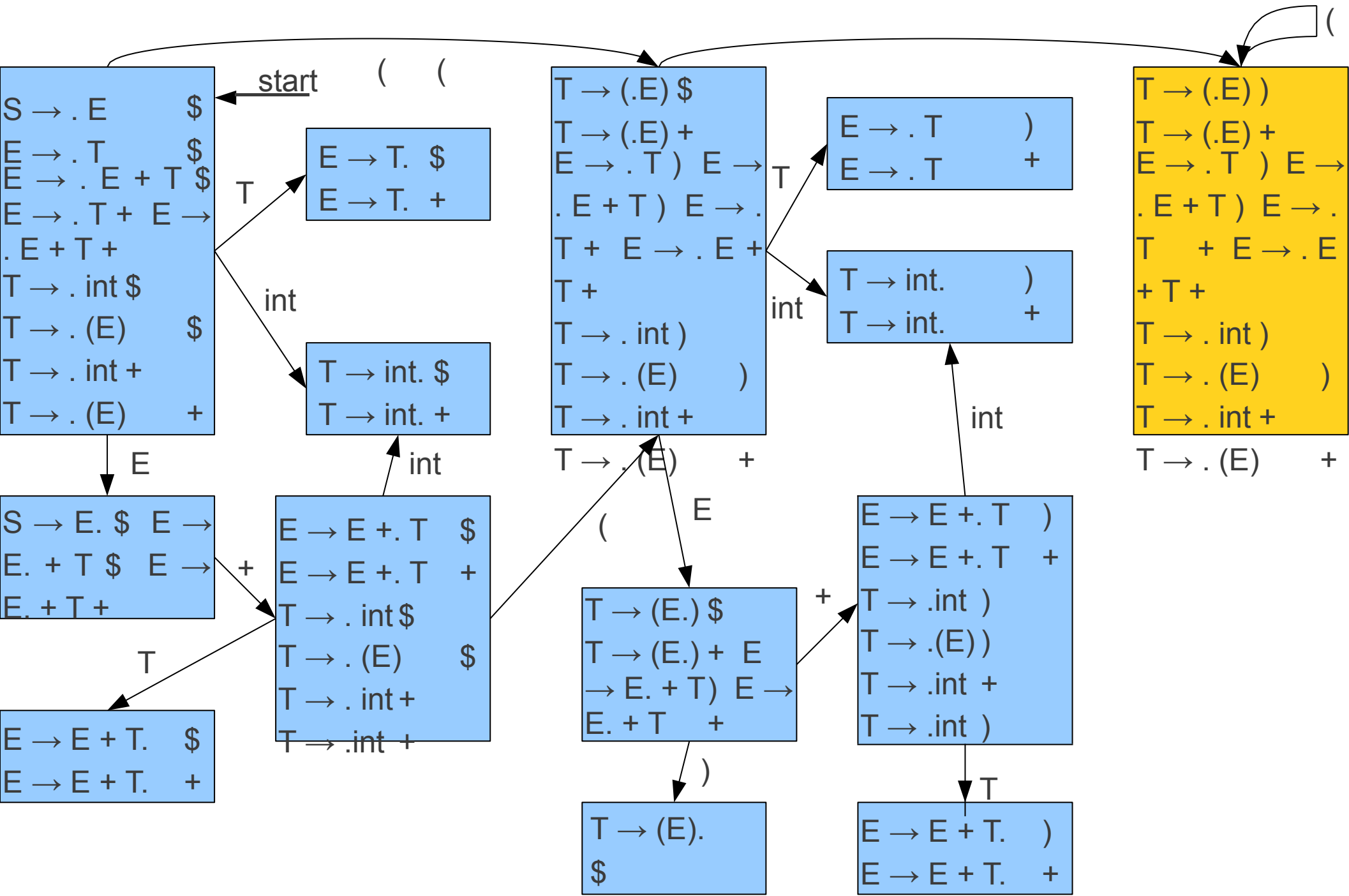




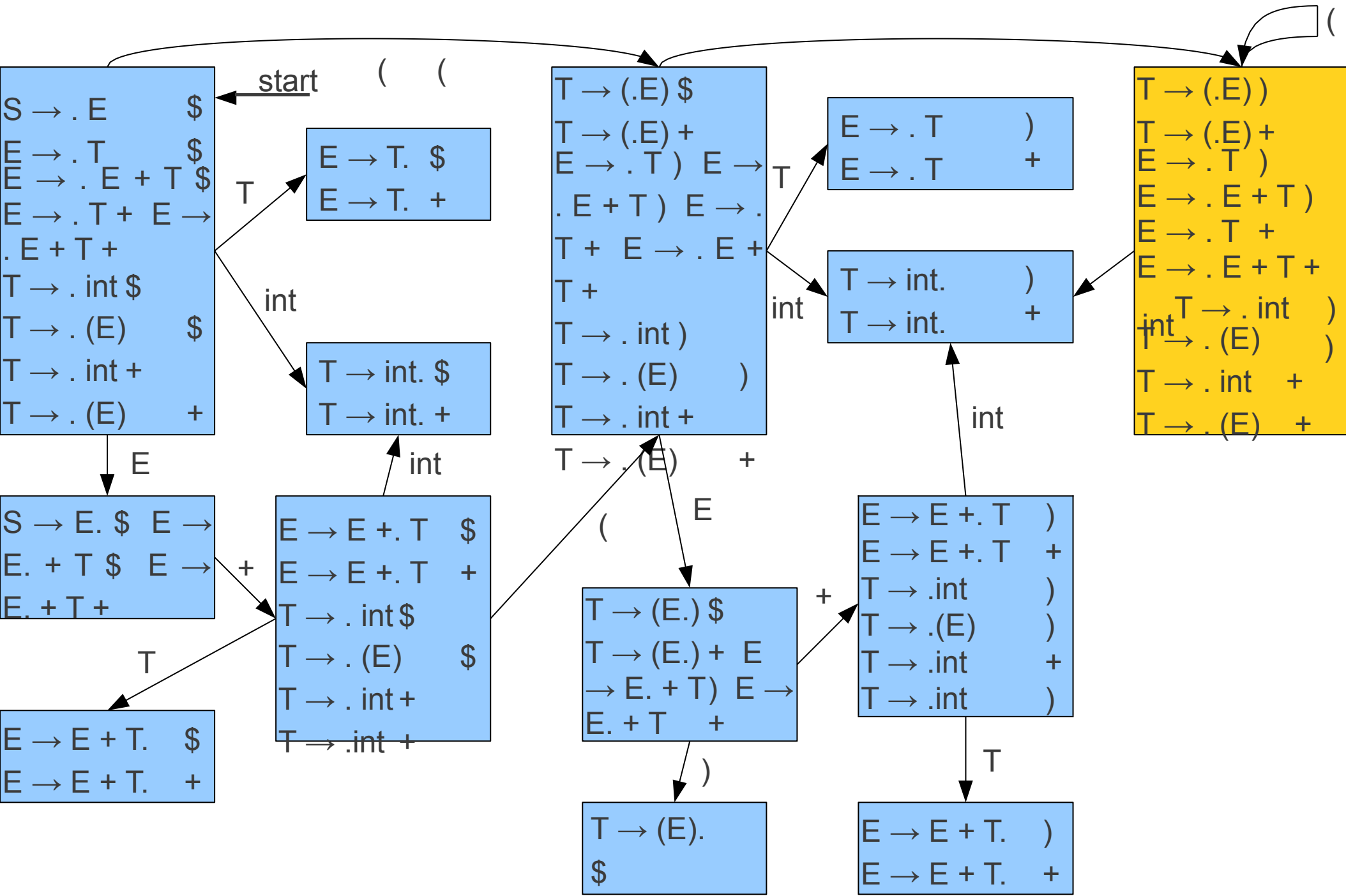
# Deterministic LR(1) Automata



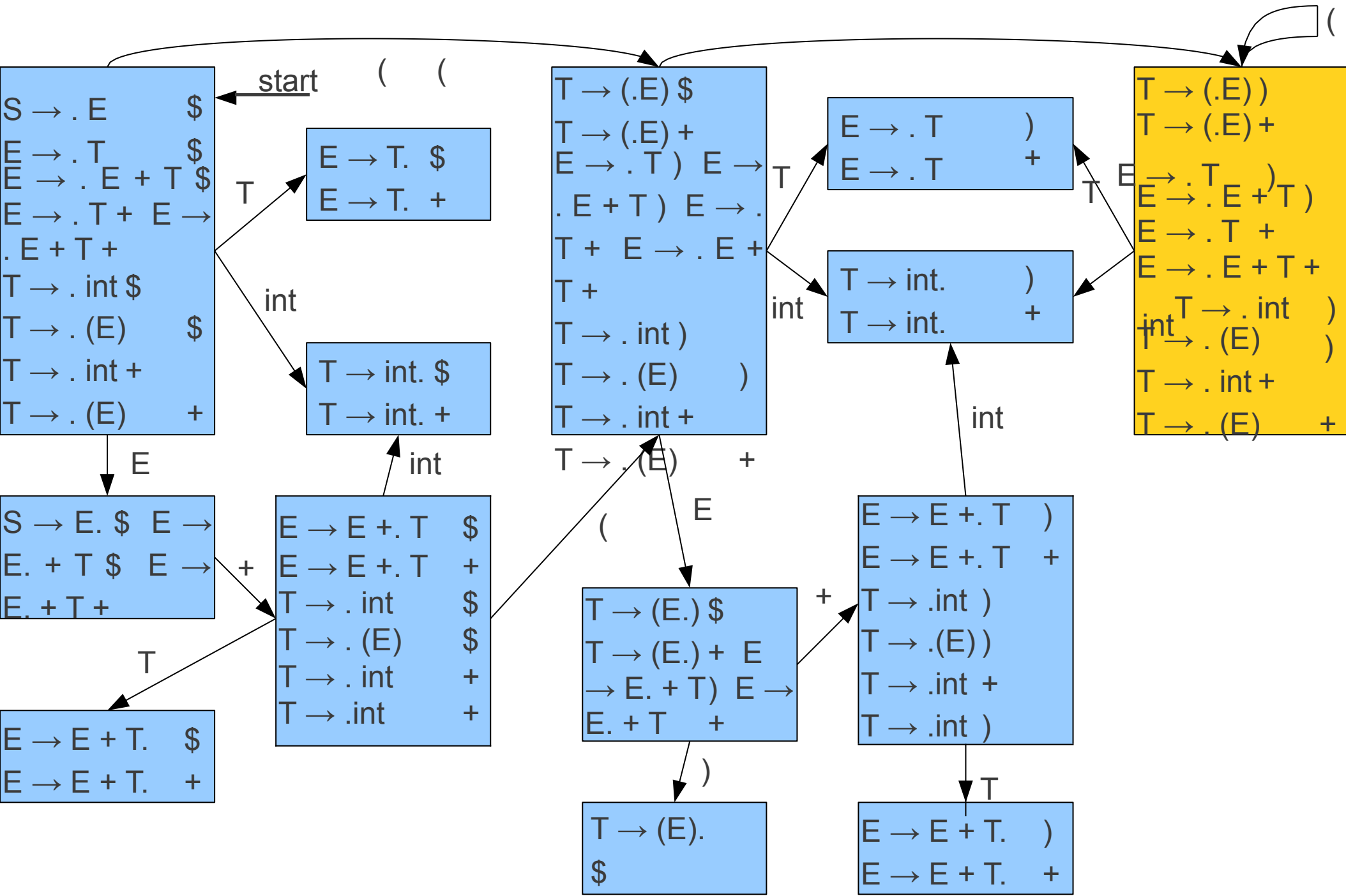
# Deterministic LR(1) Automata



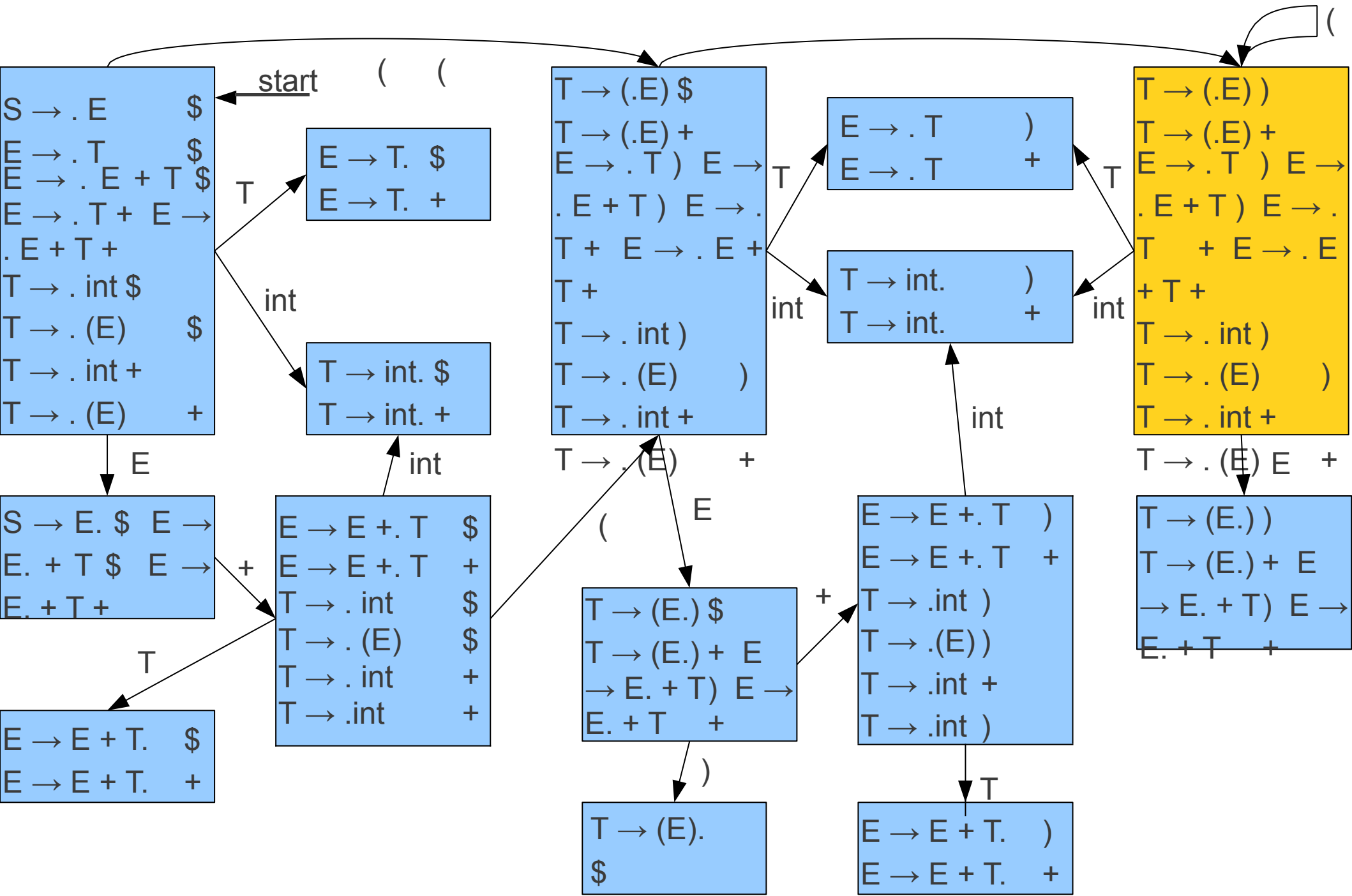
# Deterministic LR(1) Automata



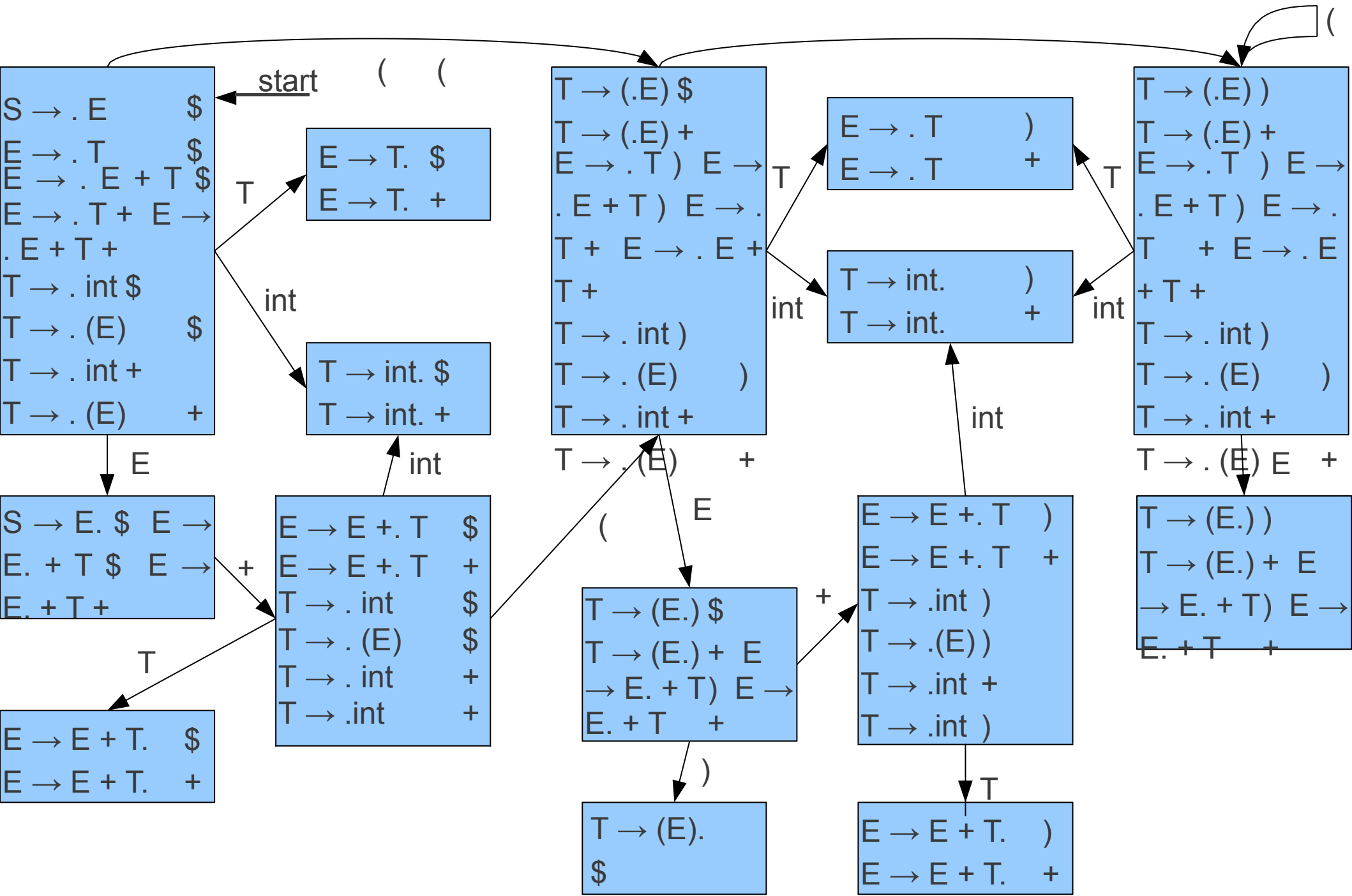
# Deterministic LR(1) Automata



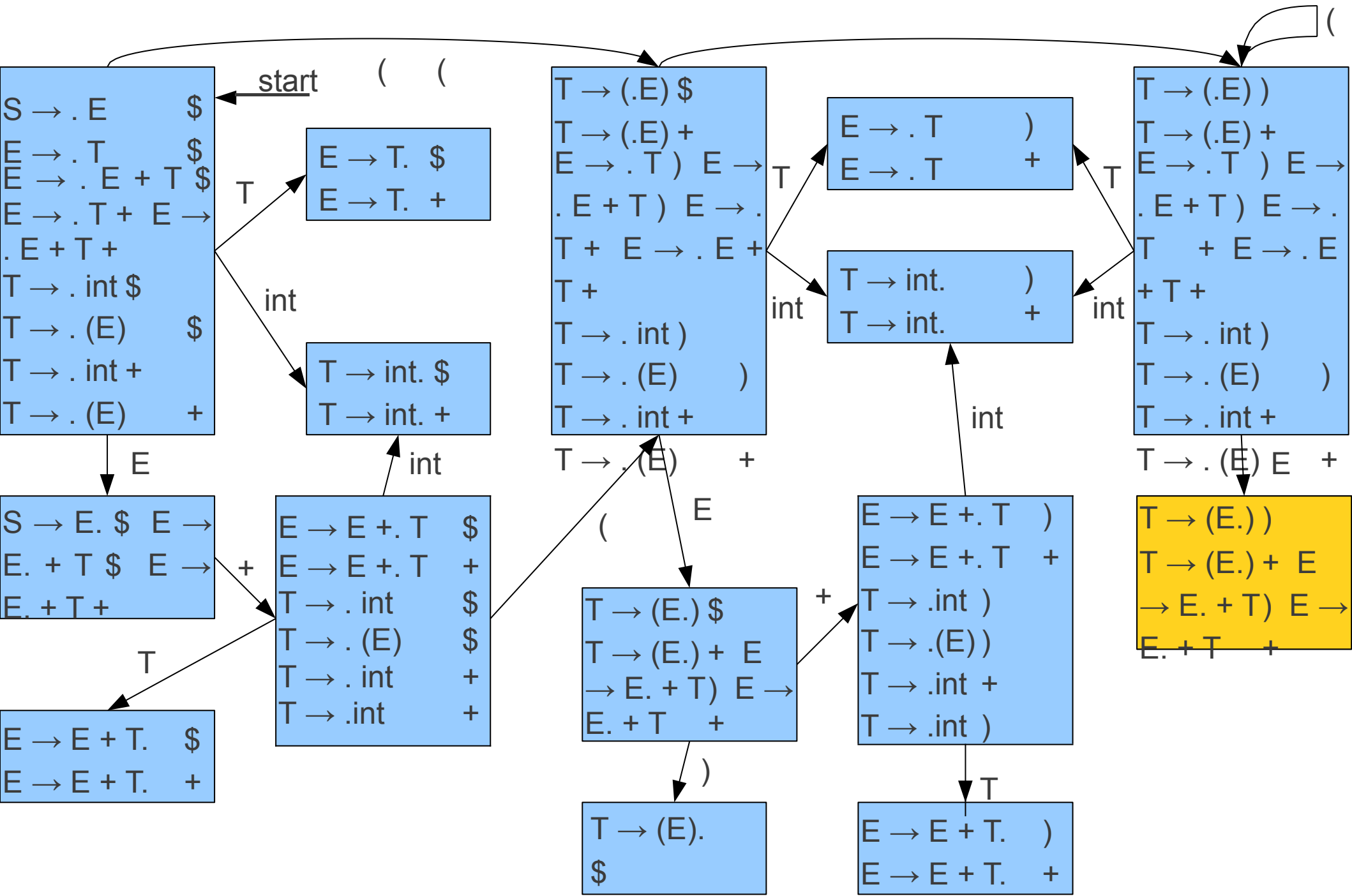
# Deterministic LR(1) Automata



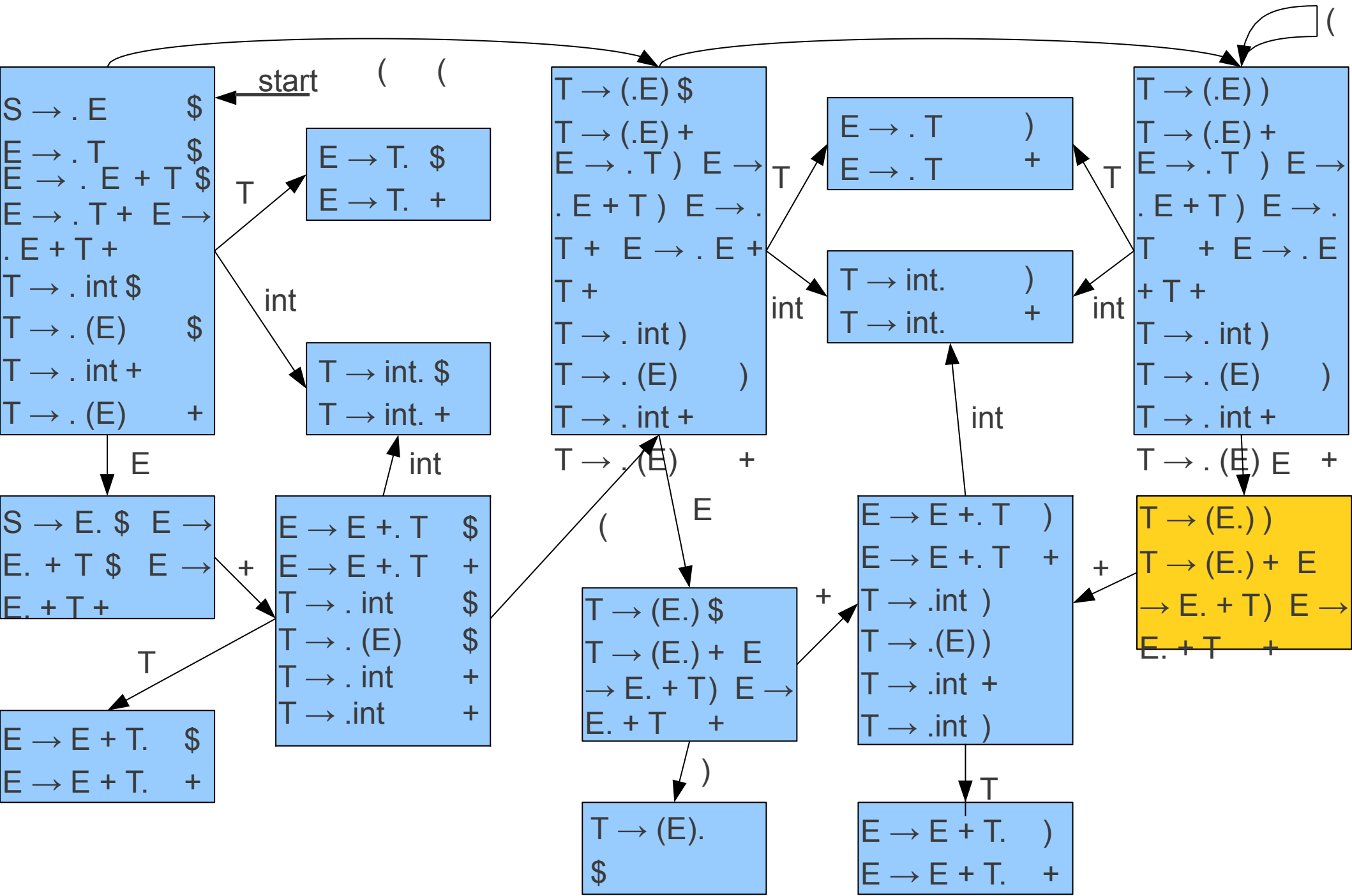
# Deterministic LR(1) Automata



# Deterministic LR(1) Automata

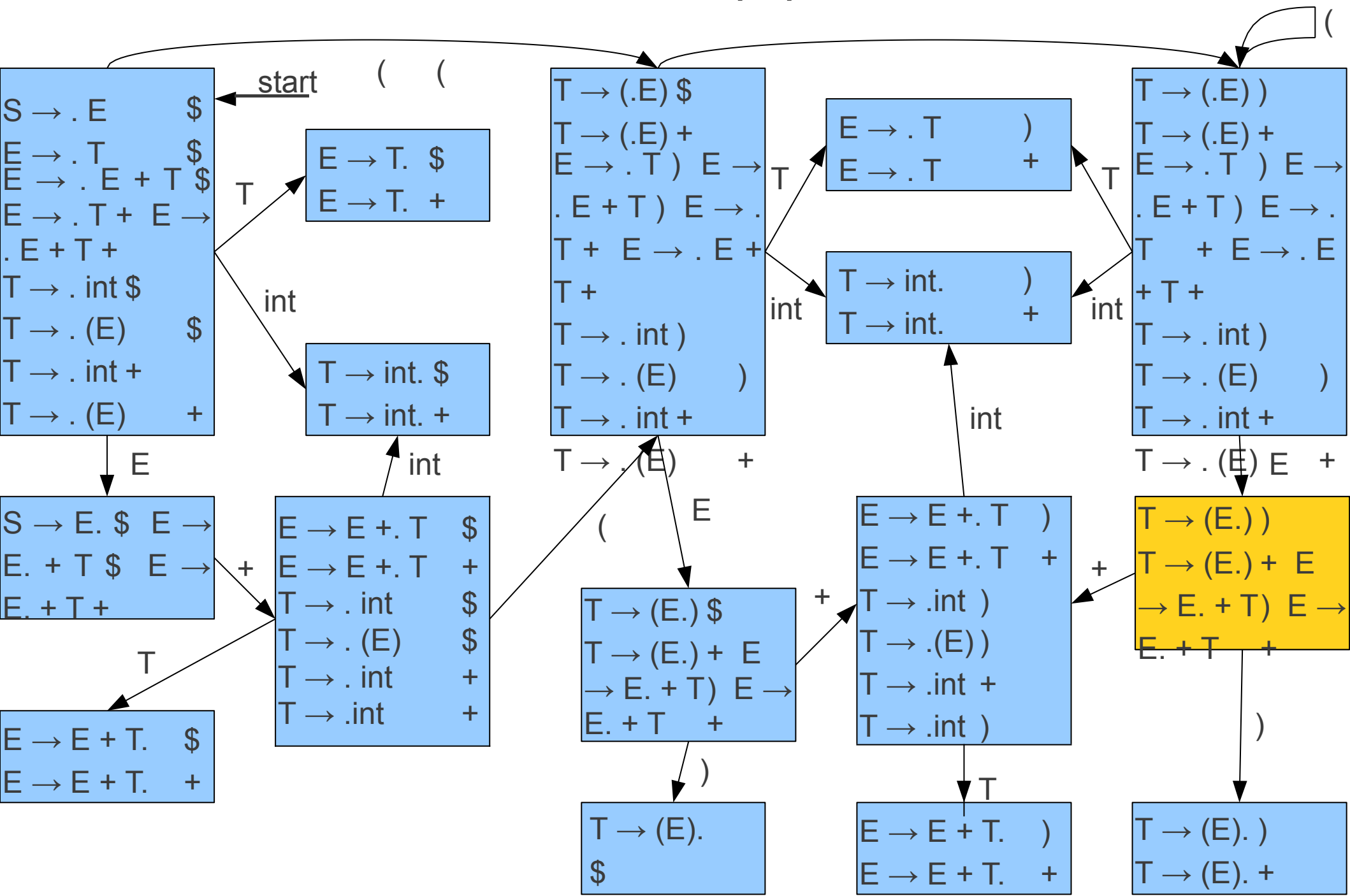


# Deterministic LR(1) Automata

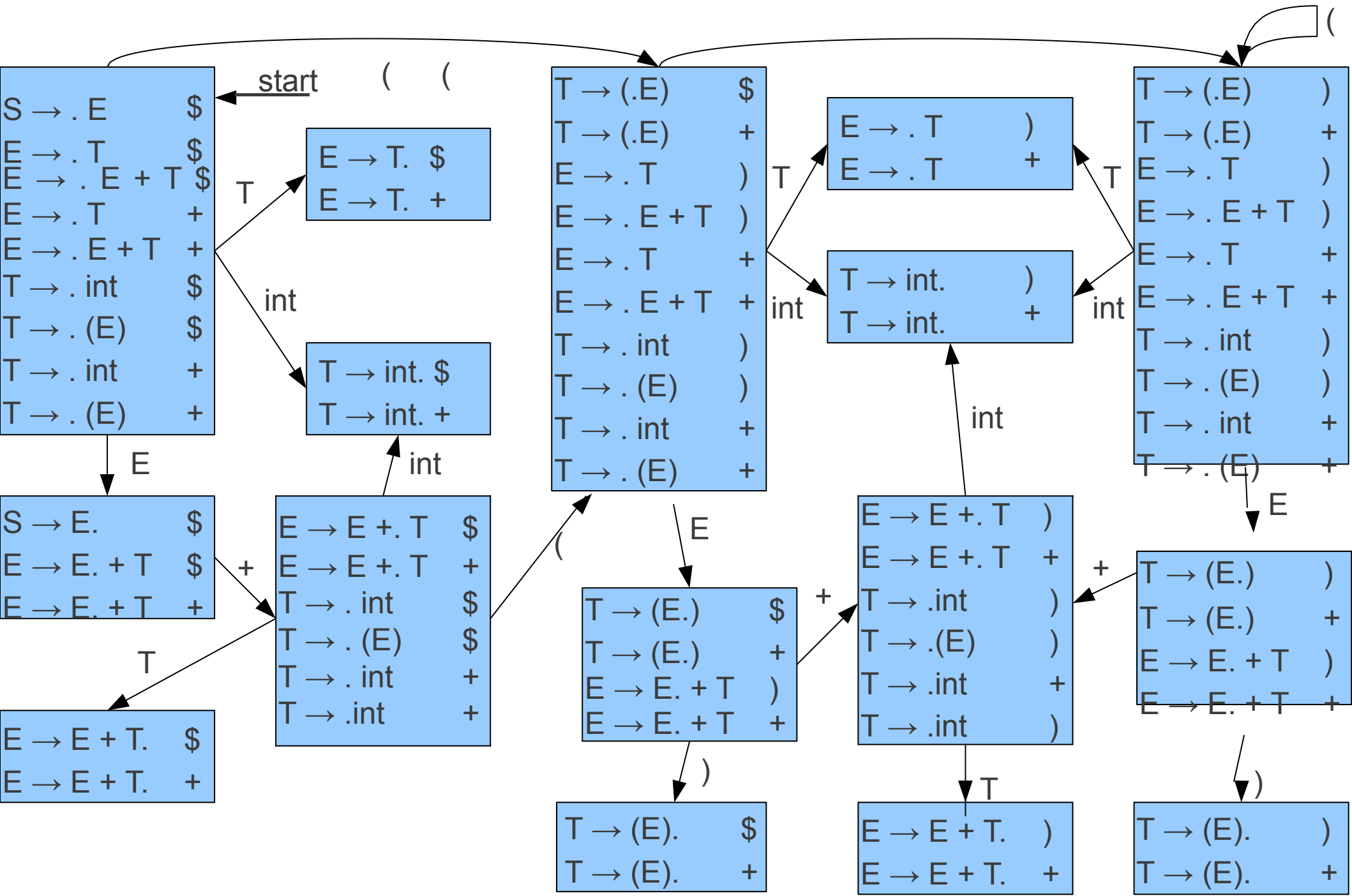




# Deterministic LR(1) Automata



# Deterministic LR(1) Automata



# Constructing LR(1) Automata II

- Begin in a state containing  $\mathbf{S} \rightarrow \cdot \mathbf{E} [\mathbf{\$}]$ ,  
where  $\mathbf{S}$  is the start symbol.

# Constructing LR(1) Automata II

- Begin in a state containing  $S \rightarrow \cdot E [\$]$ , where  $S$  is the start symbol.
- Compute the **closure** of the state:
  - If  $A \rightarrow \alpha \cdot B\omega [t]$  is in the state, add  $B \rightarrow \cdot \gamma [t]$  to the state for each production  $B \rightarrow \gamma$  and for each terminal  $t \in \text{FIRST}^*(\omega t)$

# Constructing LR(1) Automata II

- Begin in a state containing  $S \rightarrow \cdot E [\$]$ , where  $S$  is the start symbol.
- Compute the **closure** of the state:
  - If  $A \rightarrow \alpha \cdot B\omega [t]$  is in the state, add  $B \rightarrow \cdot \gamma [t]$  to the state for each production  $B \rightarrow \gamma$  and for each terminal  $t \in \text{FIRST}^*(\omega t)$
- Repeat until no new states are added:
  - If a state contains a production  $A \rightarrow \alpha \cdot x\omega [t]$ , add a transition on  $x$  from that state to the state containing the closure of  $A \rightarrow \alpha x \cdot \omega [t]$ .

# Structure of LR(1) Automata

- Every LR(1) automaton simulates two processes simultaneously:
  - An **LR(0) automaton** for finding handles.
  - A **lookahead tracker** for determining what the lookahead is.
- Removing the lookaheads from an LR(1) automaton results in a (much larger) LR(0) automaton for the same grammar.

# Representing LR(1) Automata

As with LR(0), use **action** and **goto** tables.

**goto** table defined as before; encodes transition table as map from (state, token) to states.

**action** table maps pairs (state, lookahead) to actions.

Commonly combined into a single **action/goto** table.

**S** → **E**  
**E** → **T**  
**E** → **E** + **T**  
**T** → **int**  
**T** → (**E**)

(1)  
(2)  
(3)  
(4)  
(5)

	int	(	)	+	\$	T	E
1	s5					g4	g2
2				s6	ACCEPT		
3				r3	r3		
4				r2	r2		
5				r5	r5		
6	s5	s7				g3	
7	s10	s14				g10	g8
8			s9	s12			
9				r5	r5		
10			r2	r2			
11			r4	r4			
12	s11					g13	
13			r3	r3			
14	s11		s14			g10	g15
15			s16	s12			
16			r5	r5			



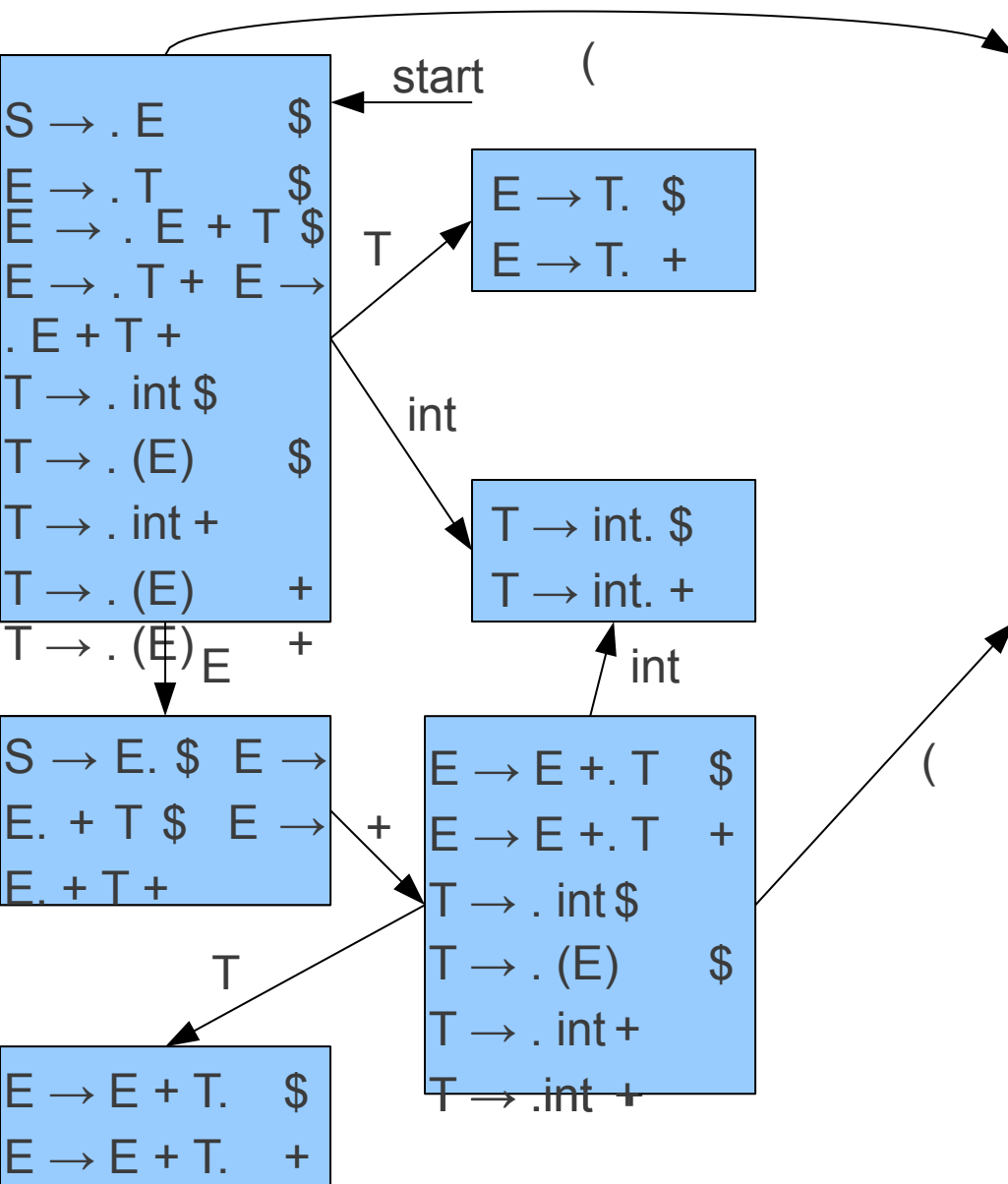
# Constructing LR(1) Parse Tables

- For each state  $X$ :
  - If there is a production  $A \rightarrow \omega \cdot [t]$ , set **action** $[X, t] = \text{reduce } A \rightarrow \omega$ .
  - If there is the special production  $S \rightarrow E \cdot [\$]$ , where  $S$  is the start symbol, set **action** $[X, t] = \text{accept}$ .
  - If there is a transition out of  $s$  on symbol  $t$ , set **action** $[X, t] = \text{shift}$ .
- Set all other actions to **error**.
- If any table entry contains two or more actions, the grammar is not LR(1).

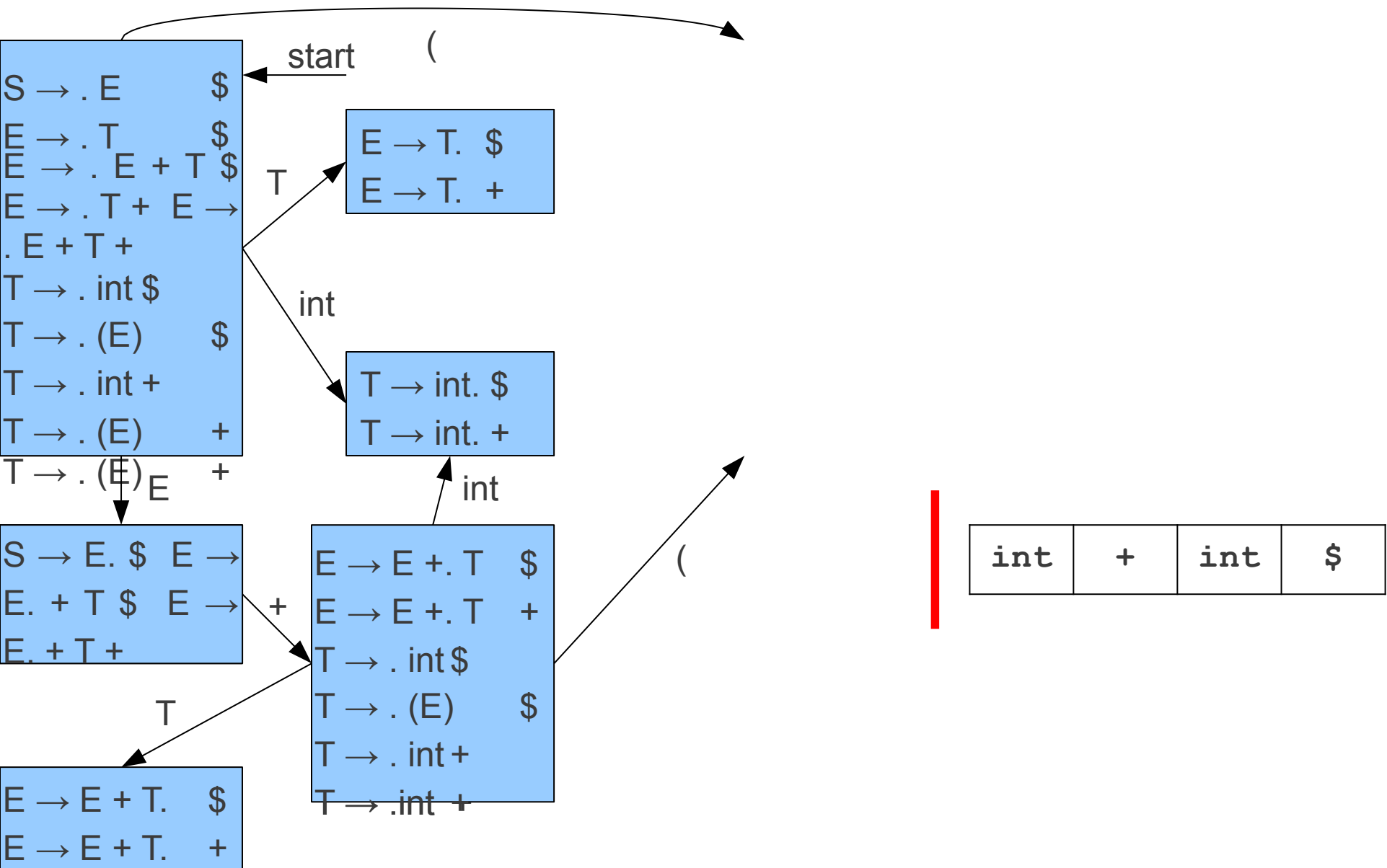
# The LR(1) Parsing Algorithm

- Begin with an empty stack and the input set to  $w\$,$  where  $w$  is the string to parse. Set **state** to the initial state.
- Repeat the following:
  - Let the next symbol of input be  $t$ .
  - If **action**[state,  $t$ ] is **shift**, then shift the input and set **state** = **goto**[state,  $t$ ].
  - If **action**[state,  $t$ ] is **reduce**  $A \rightarrow w$ :
    - \_ Pop  $|w|$  symbols off the stack; replace them with  $A$ . Let the
    - \_ state atop the stack be **top-state**.
    - Set **state** = **goto**[top-state,  $A$ ]
  - If **action**[state,  $t$ ] is **accept**, then the parse is done. If
  - **action**[state,  $t$ ] is **error**, report an error.

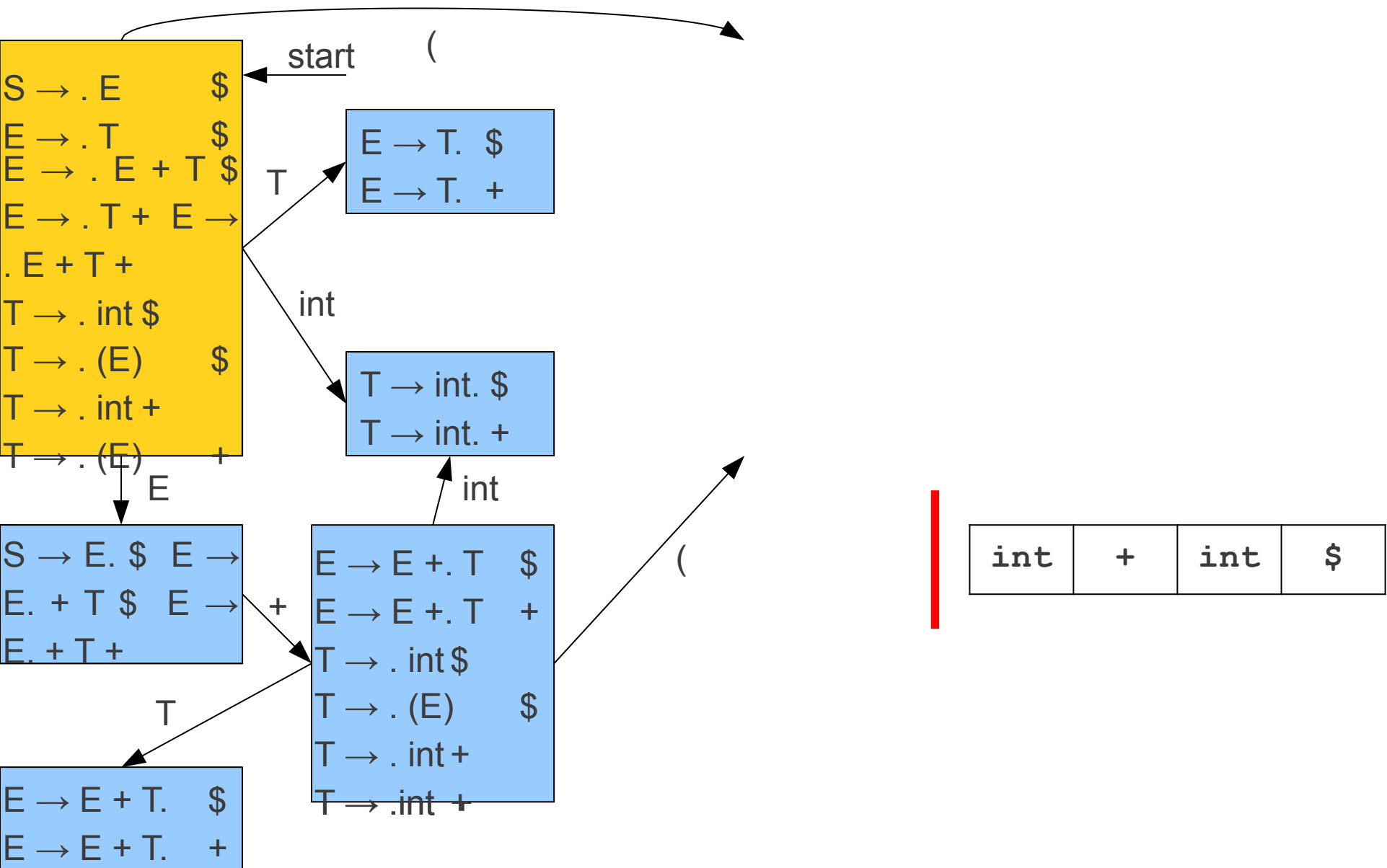
# Deterministic LR(1) Automata



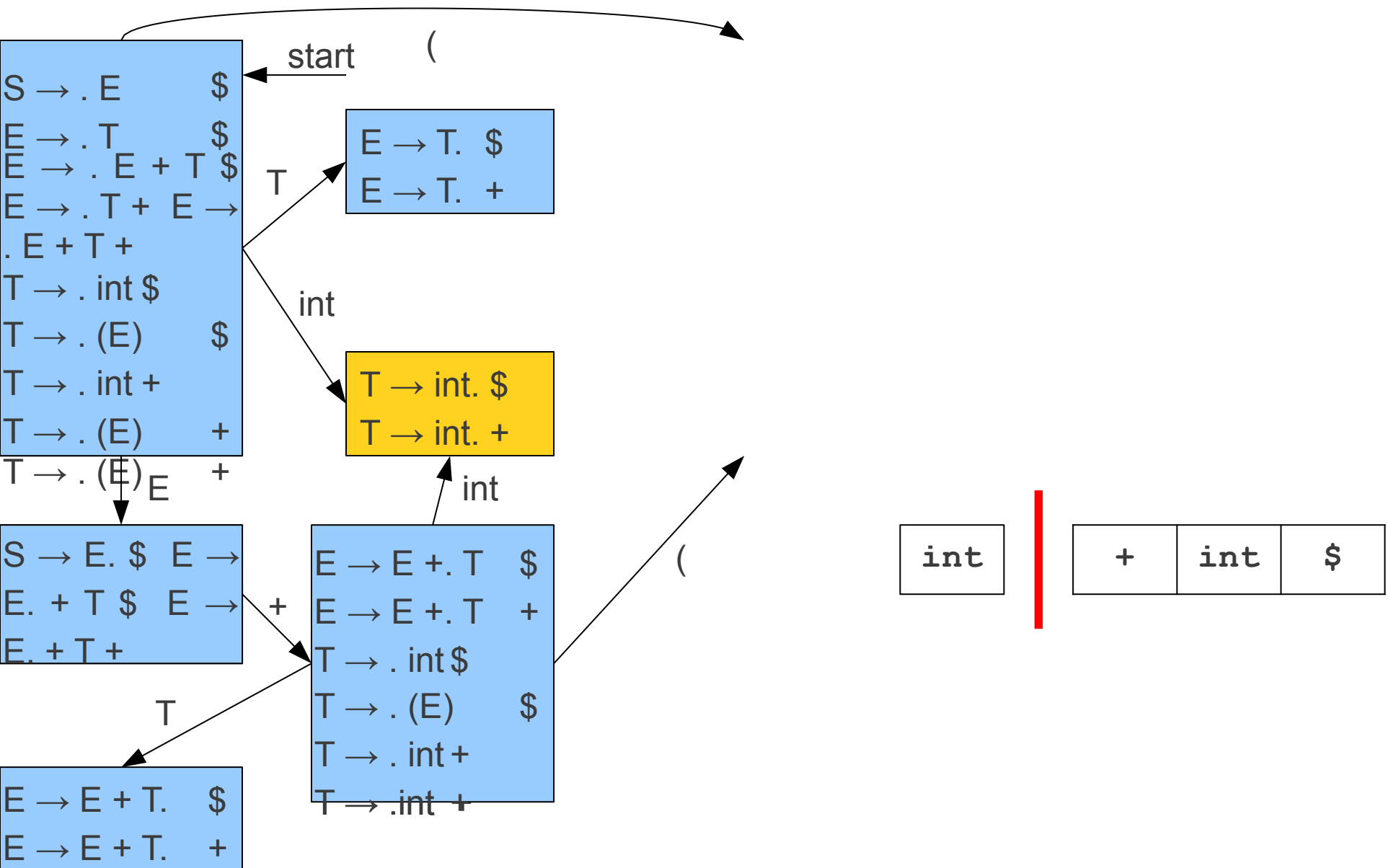
# Deterministic LR(1) Automata



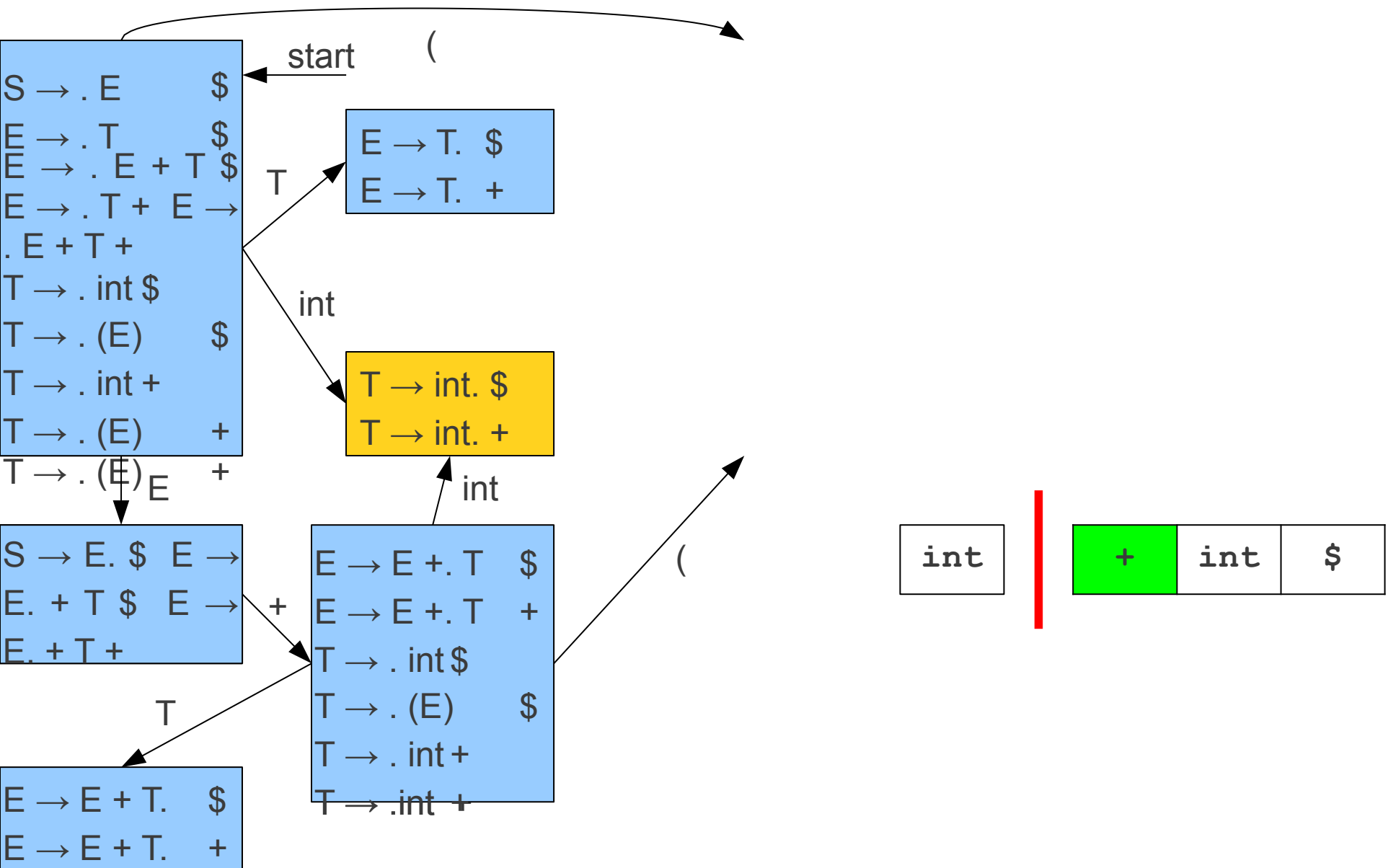
# Deterministic LR(1) Automata



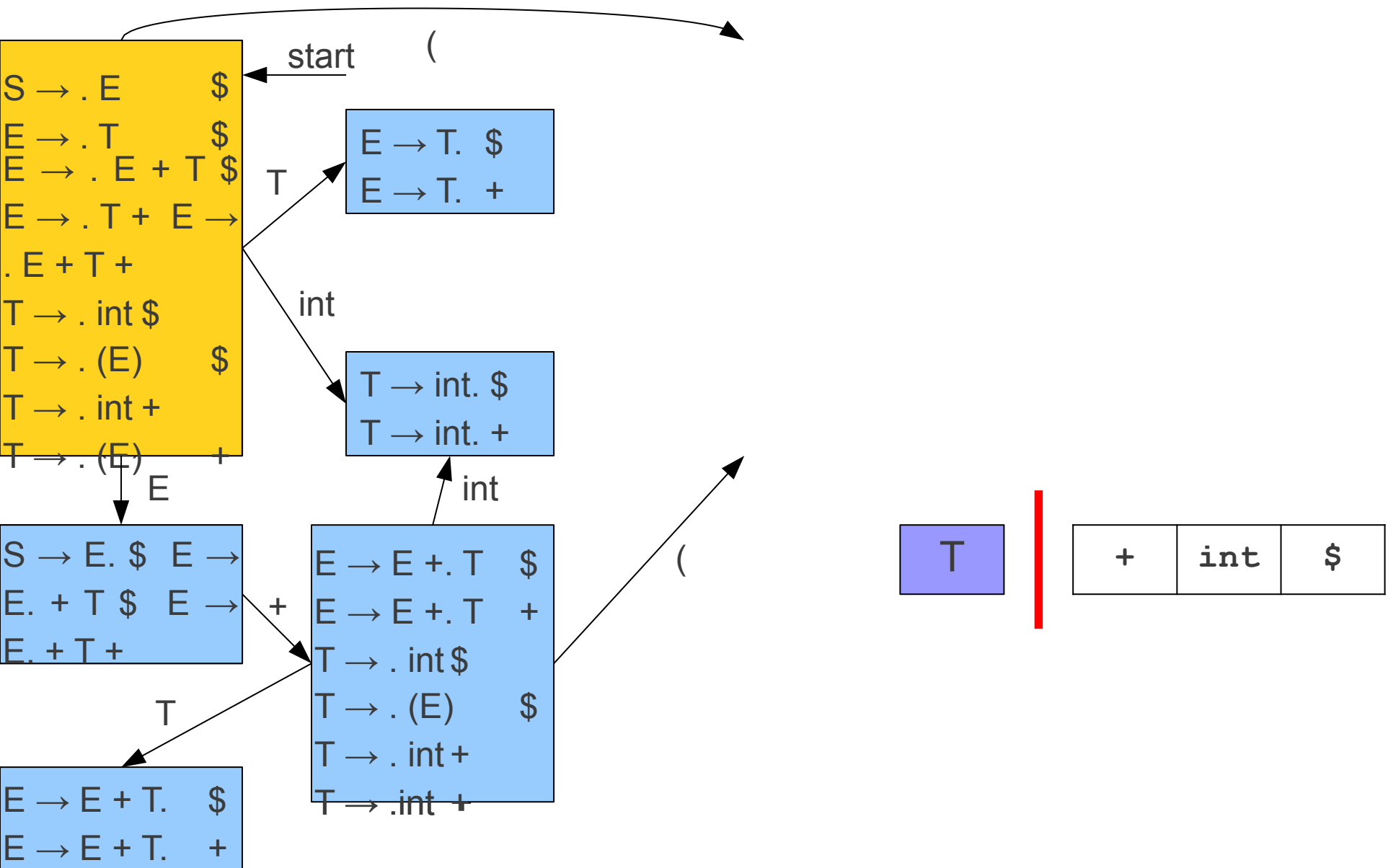
# Deterministic LR(1) Automata



# Deterministic LR(1) Automata

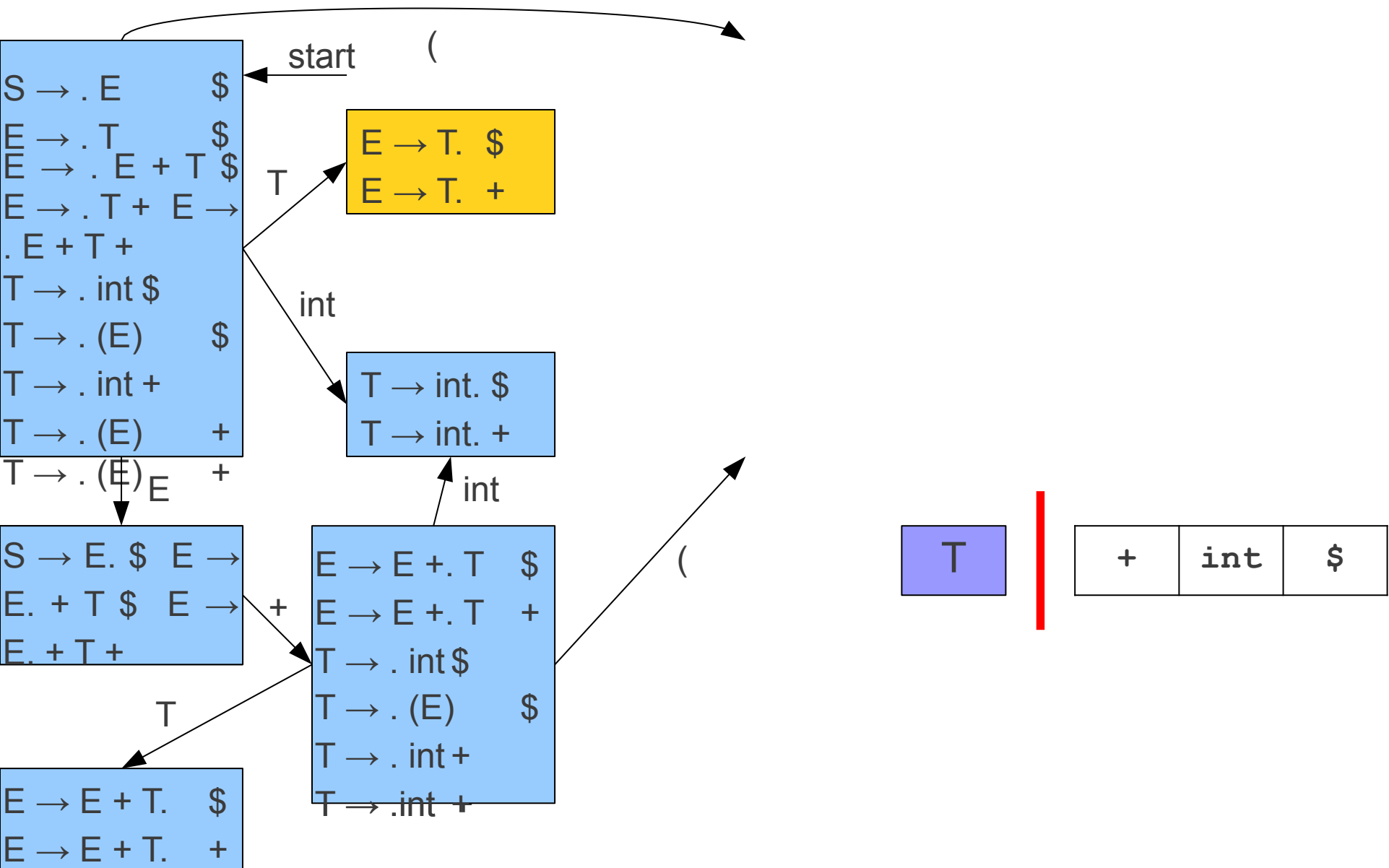


# Deterministic LR(1) Automata

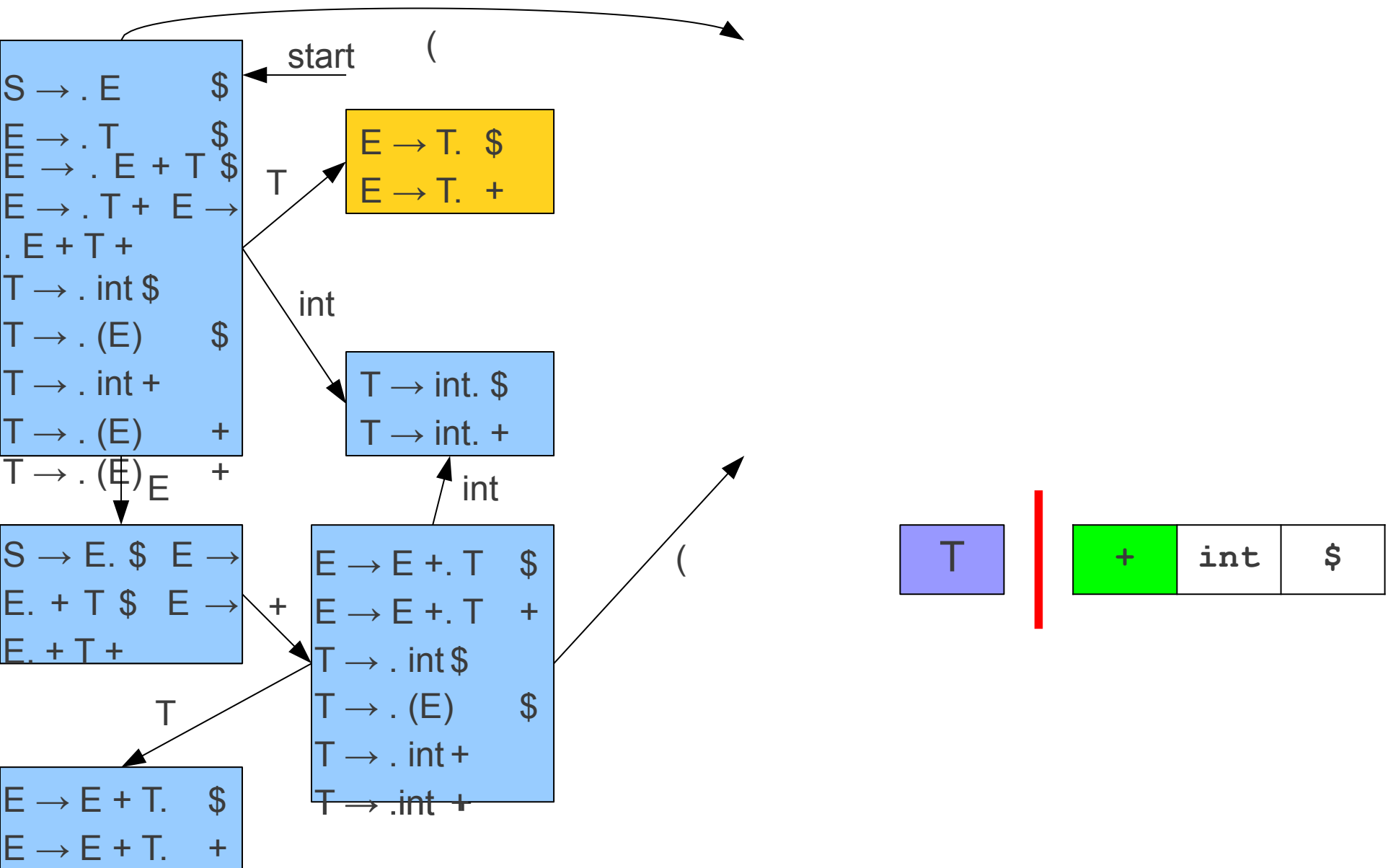




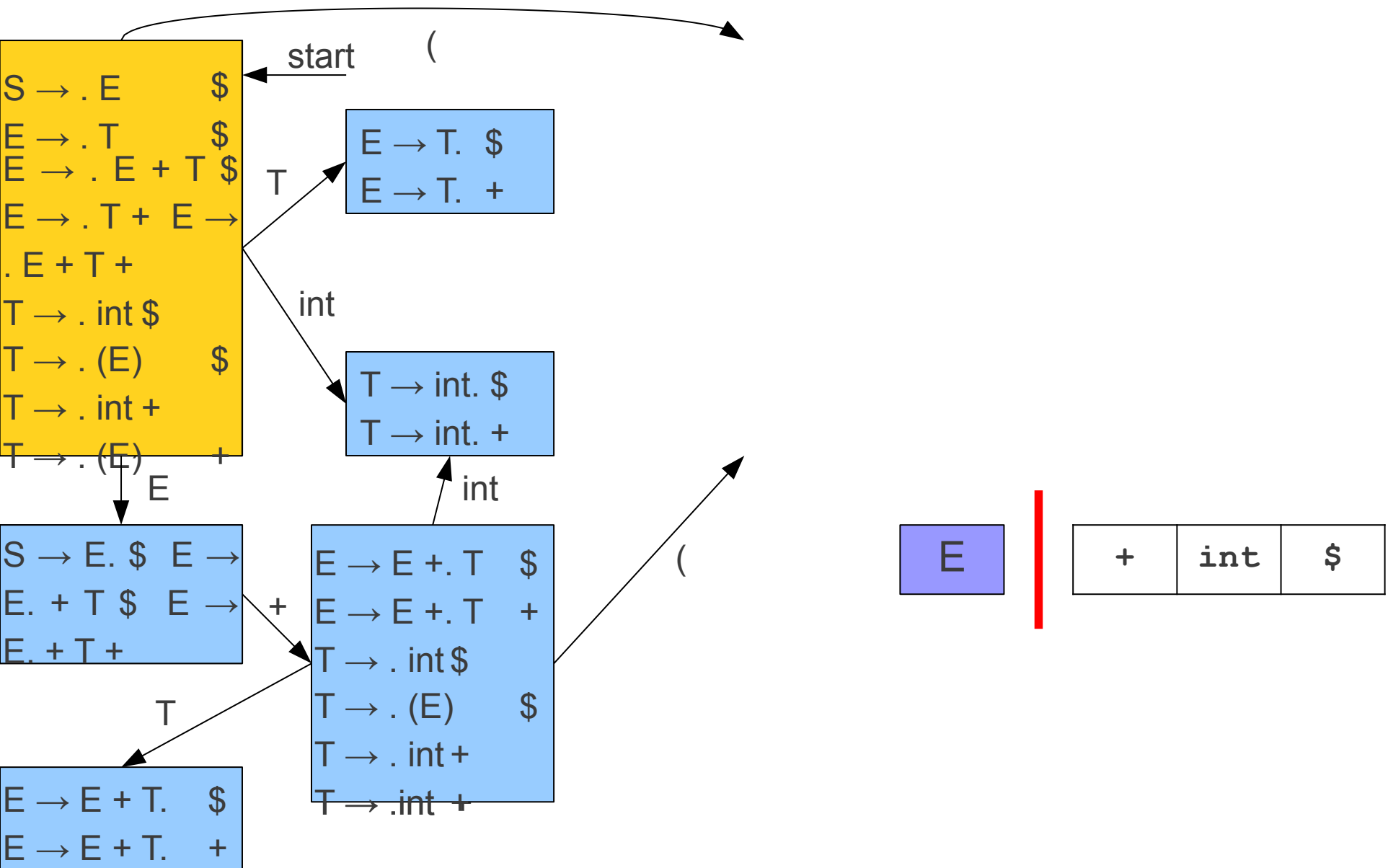
# Deterministic LR(1) Automata



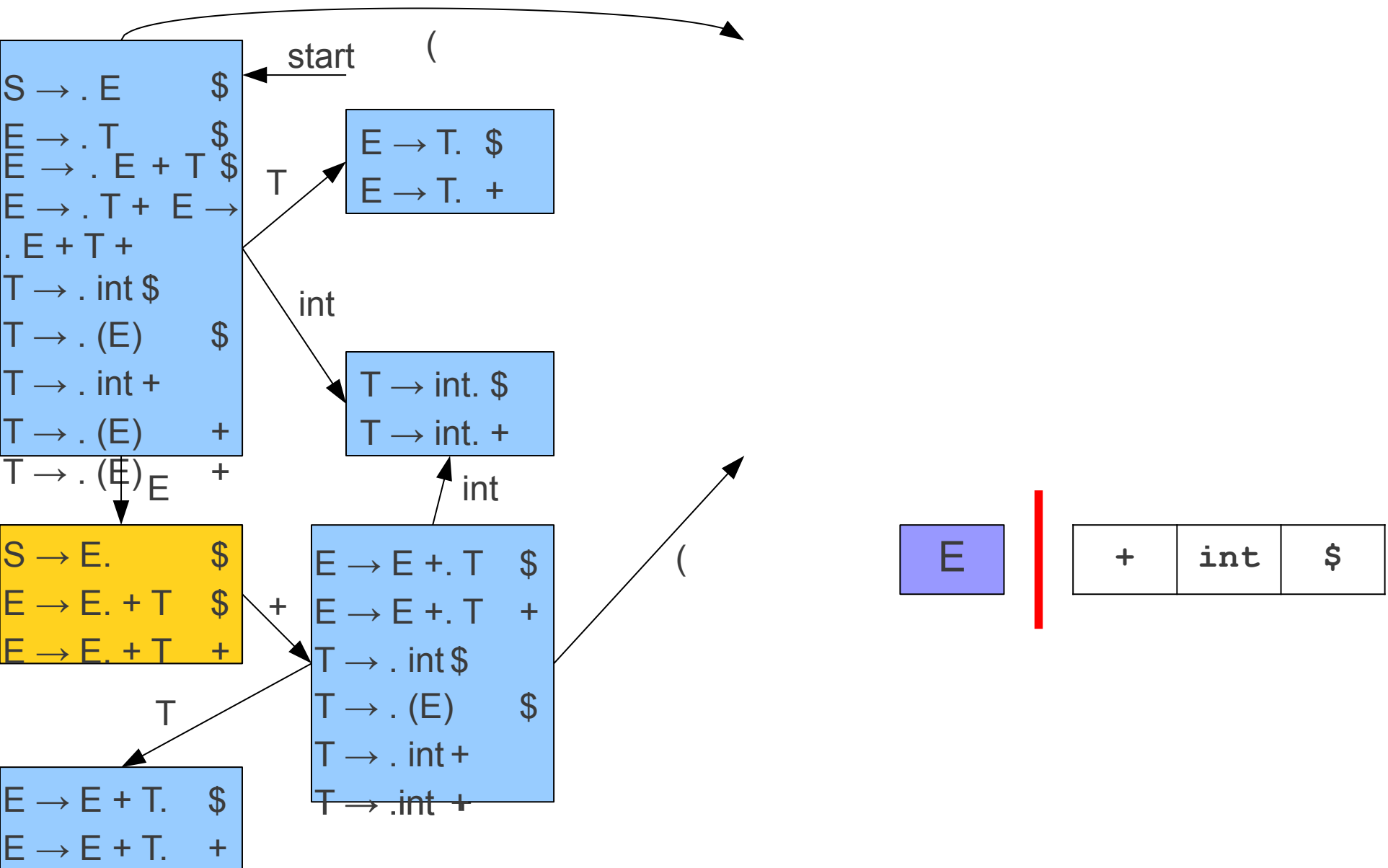
# Deterministic LR(1) Automata



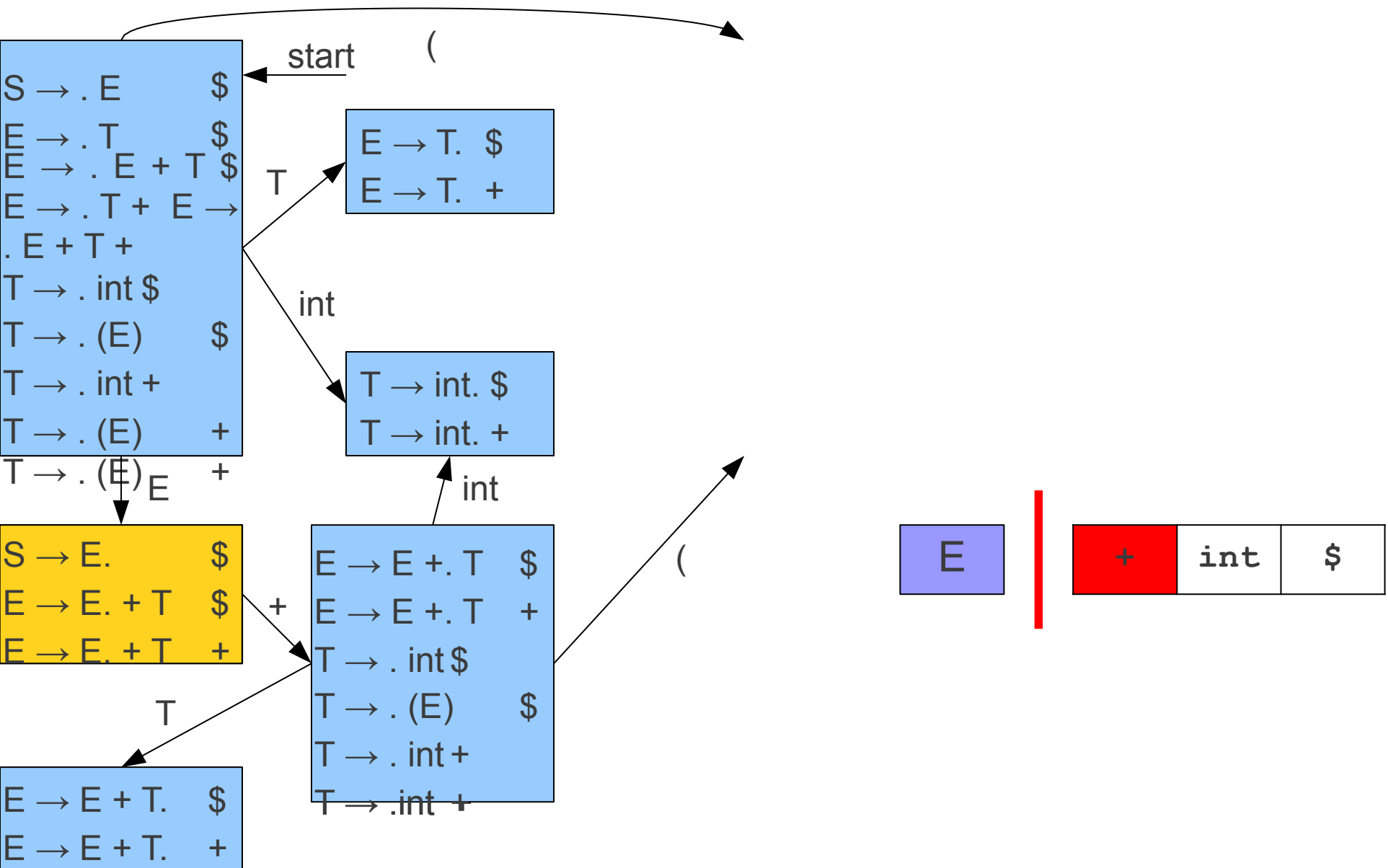
# Deterministic LR(1) Automata



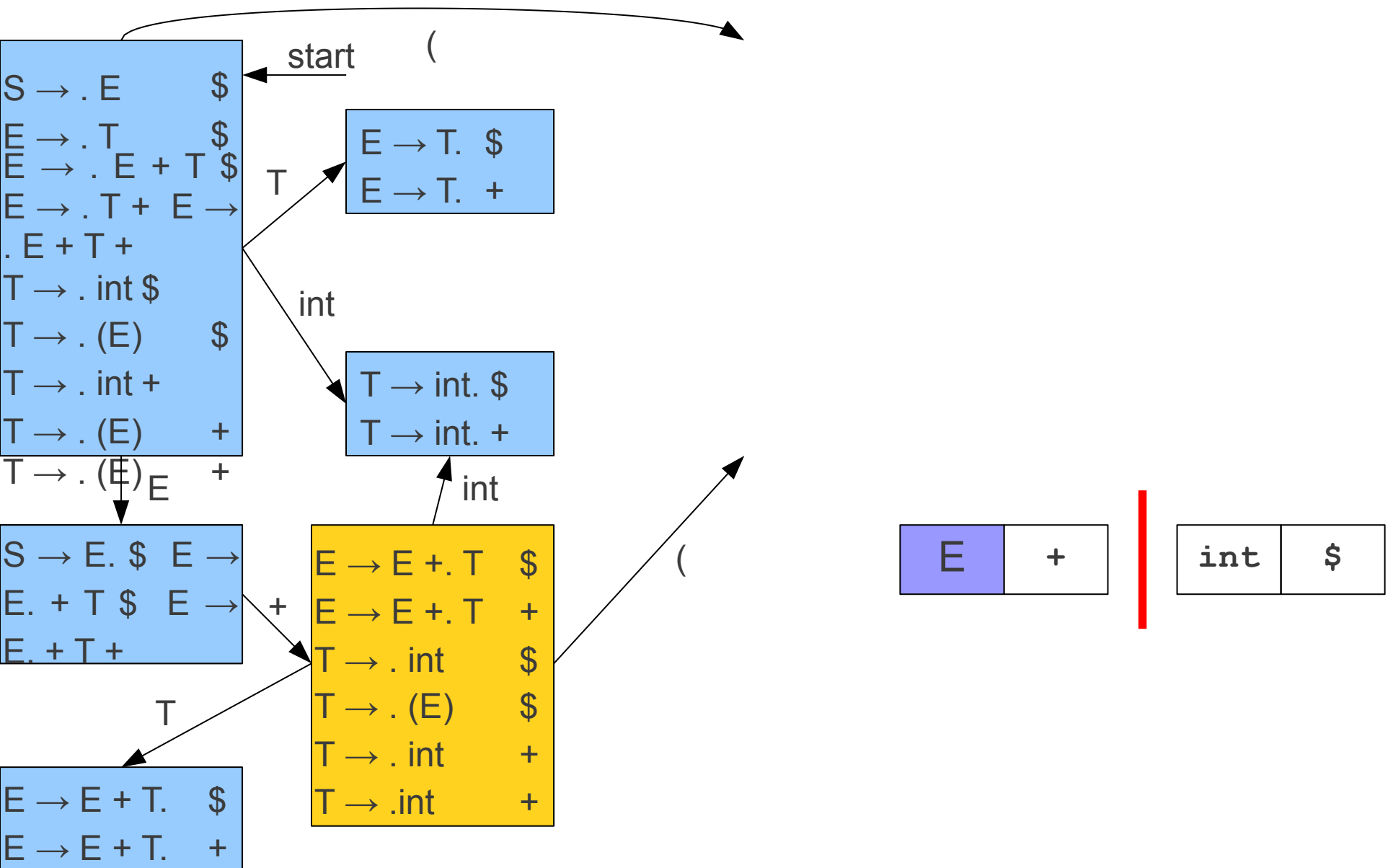
# Deterministic LR(1) Automata



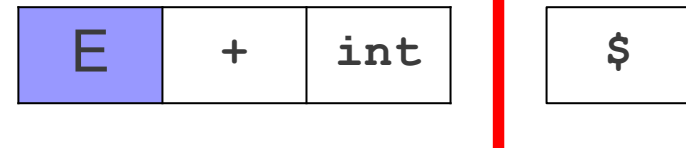
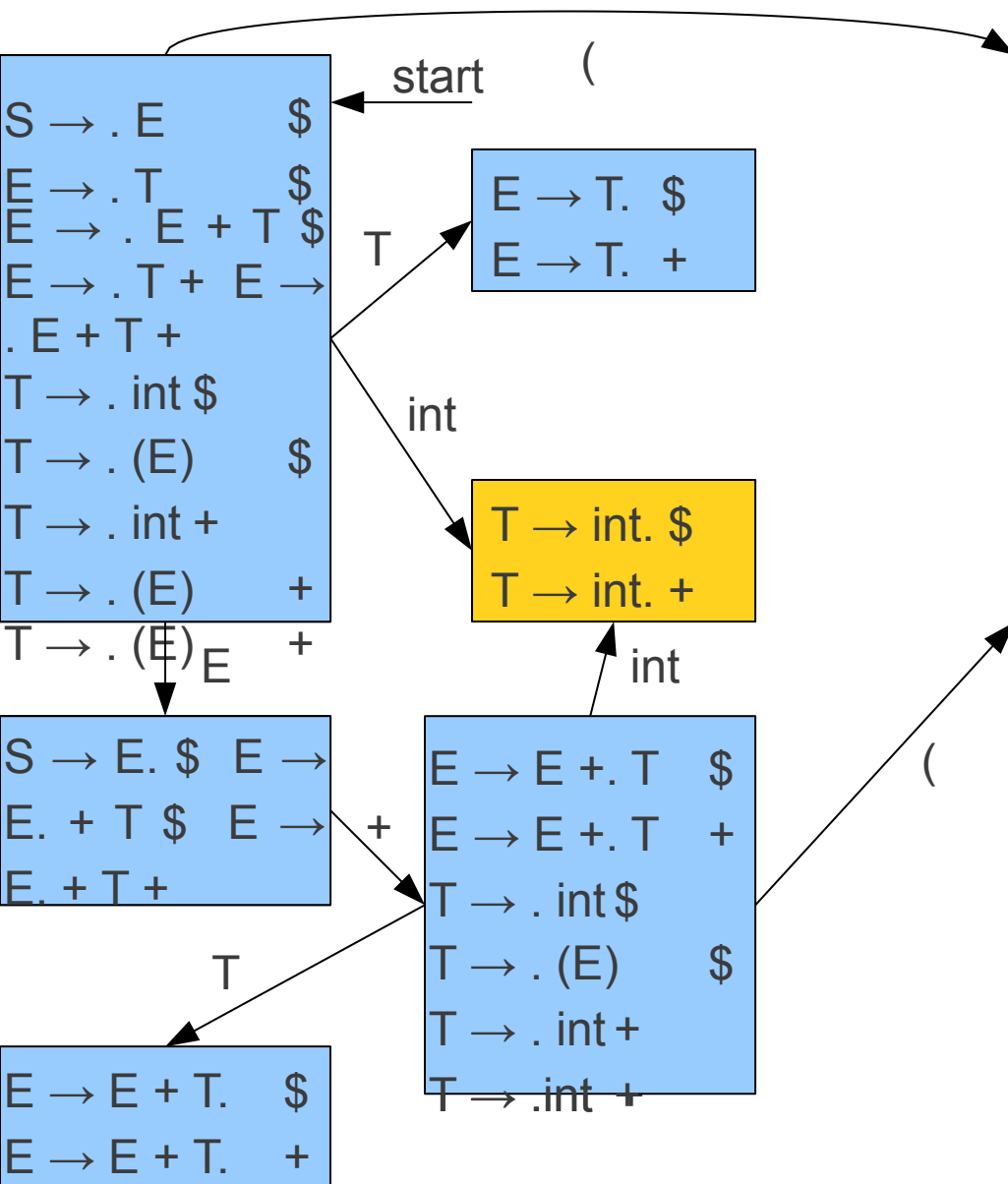
# Deterministic LR(1) Automata



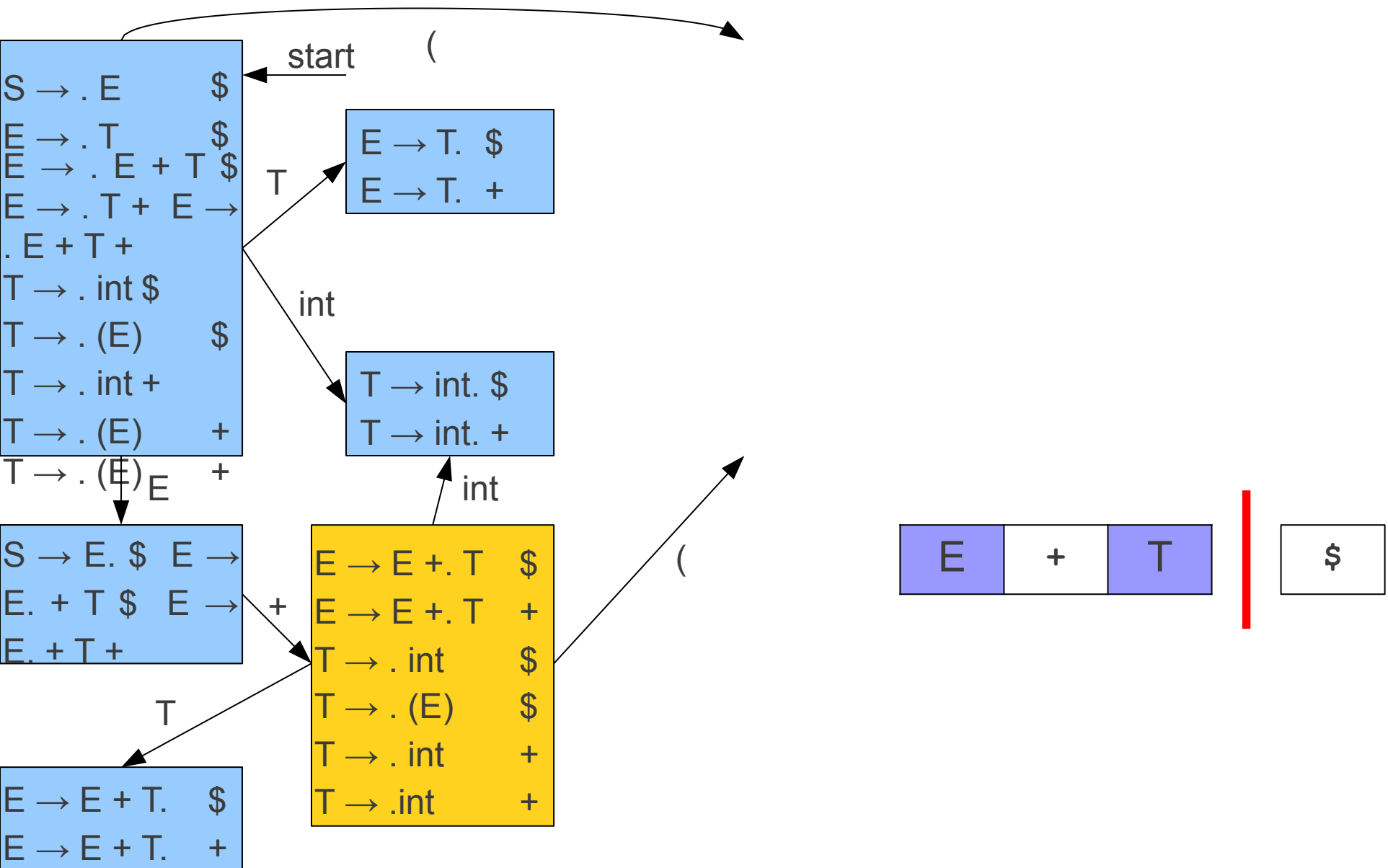
# Deterministic LR(1) Automata



# Deterministic LR(1) Automata

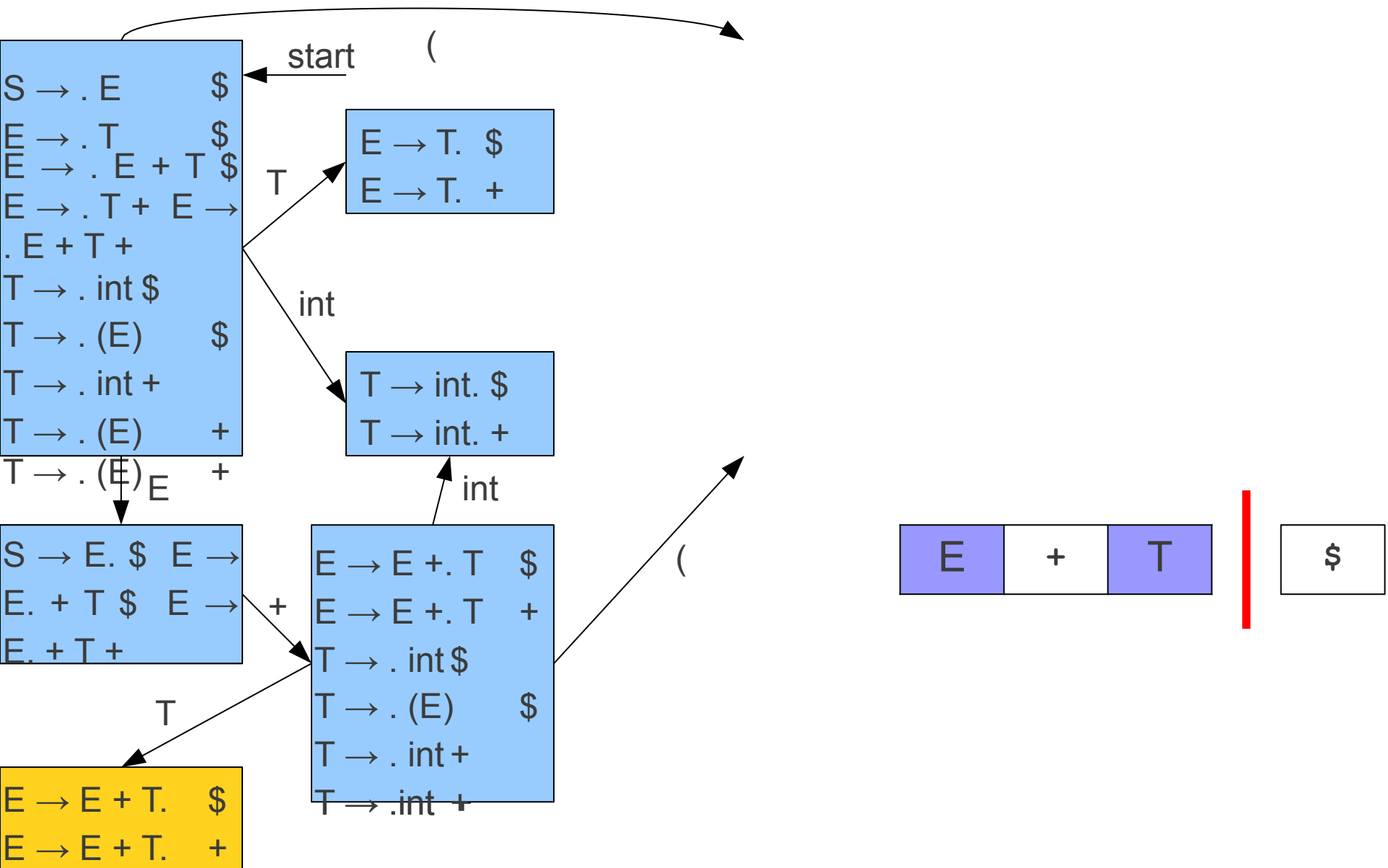


# Deterministic LR(1) Automata

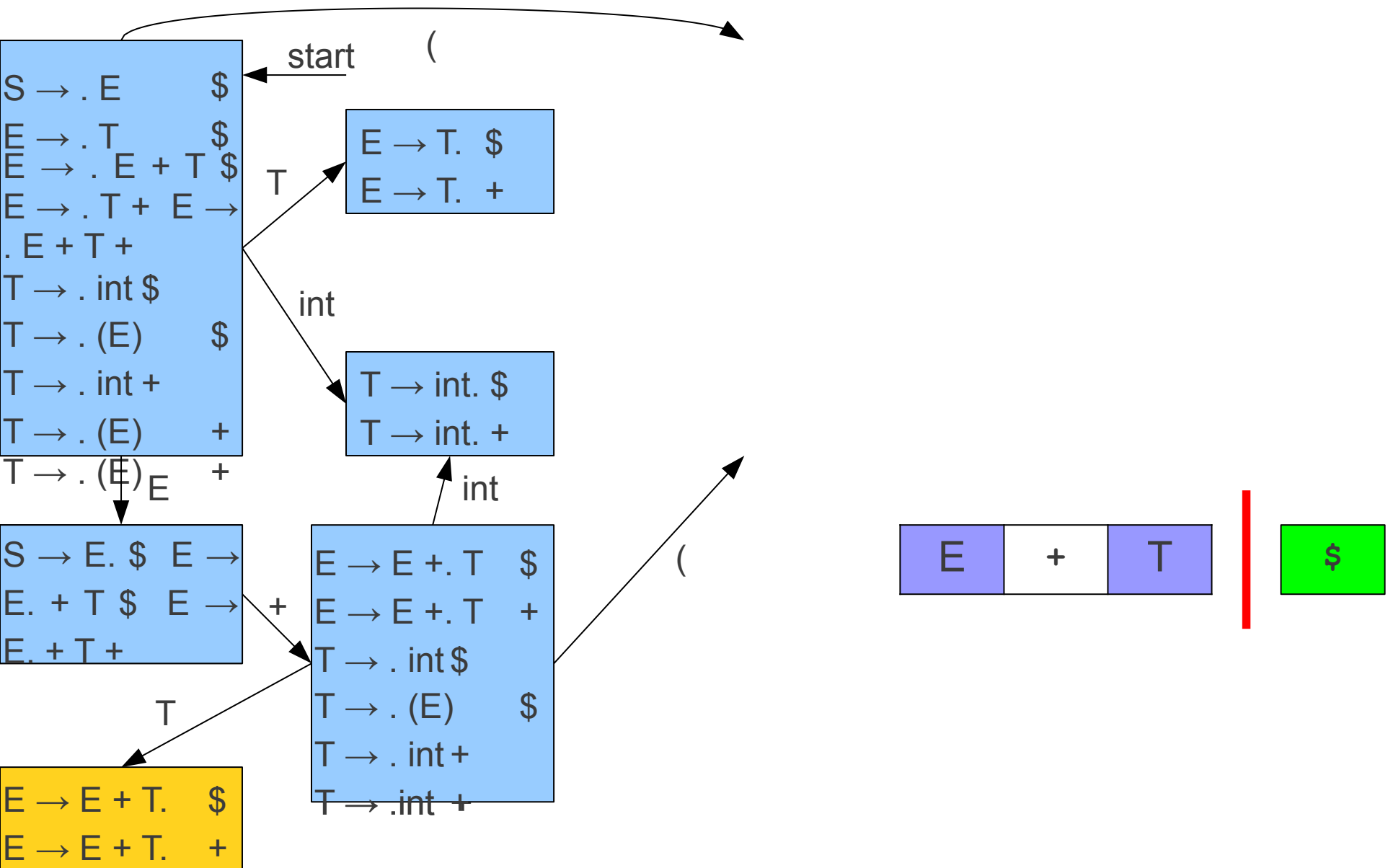




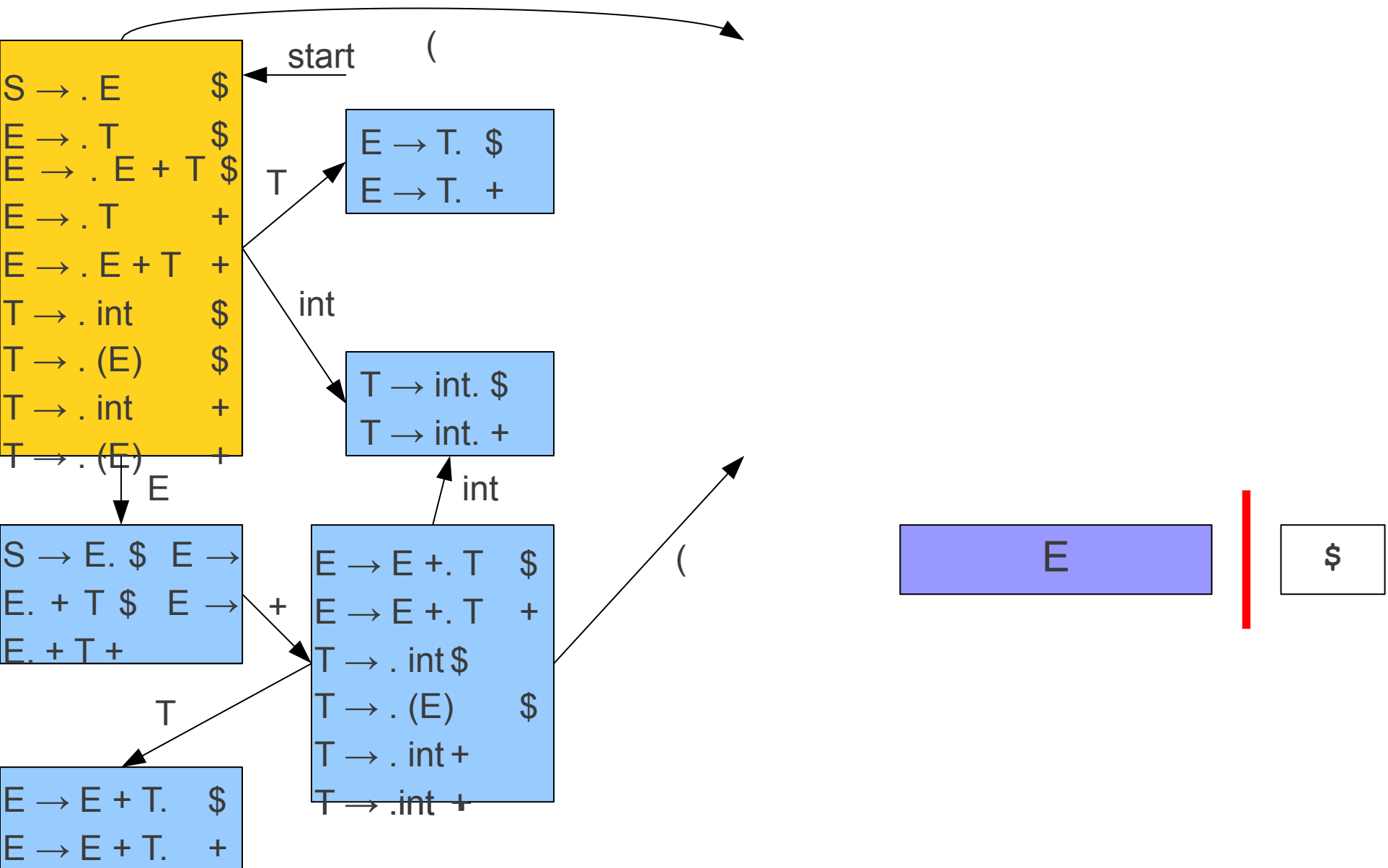
# Deterministic LR(1) Automata



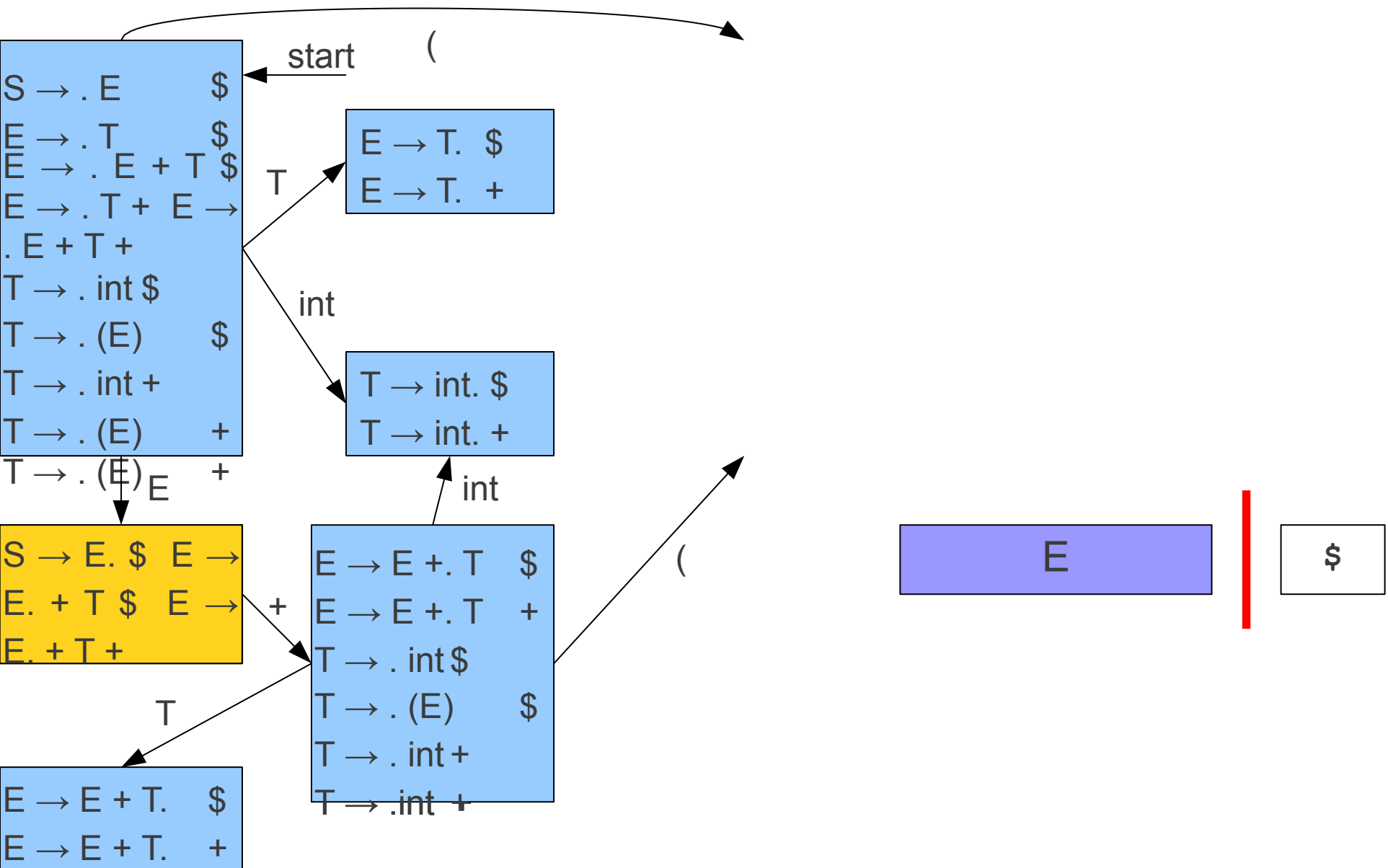
# Deterministic LR(1) Automata



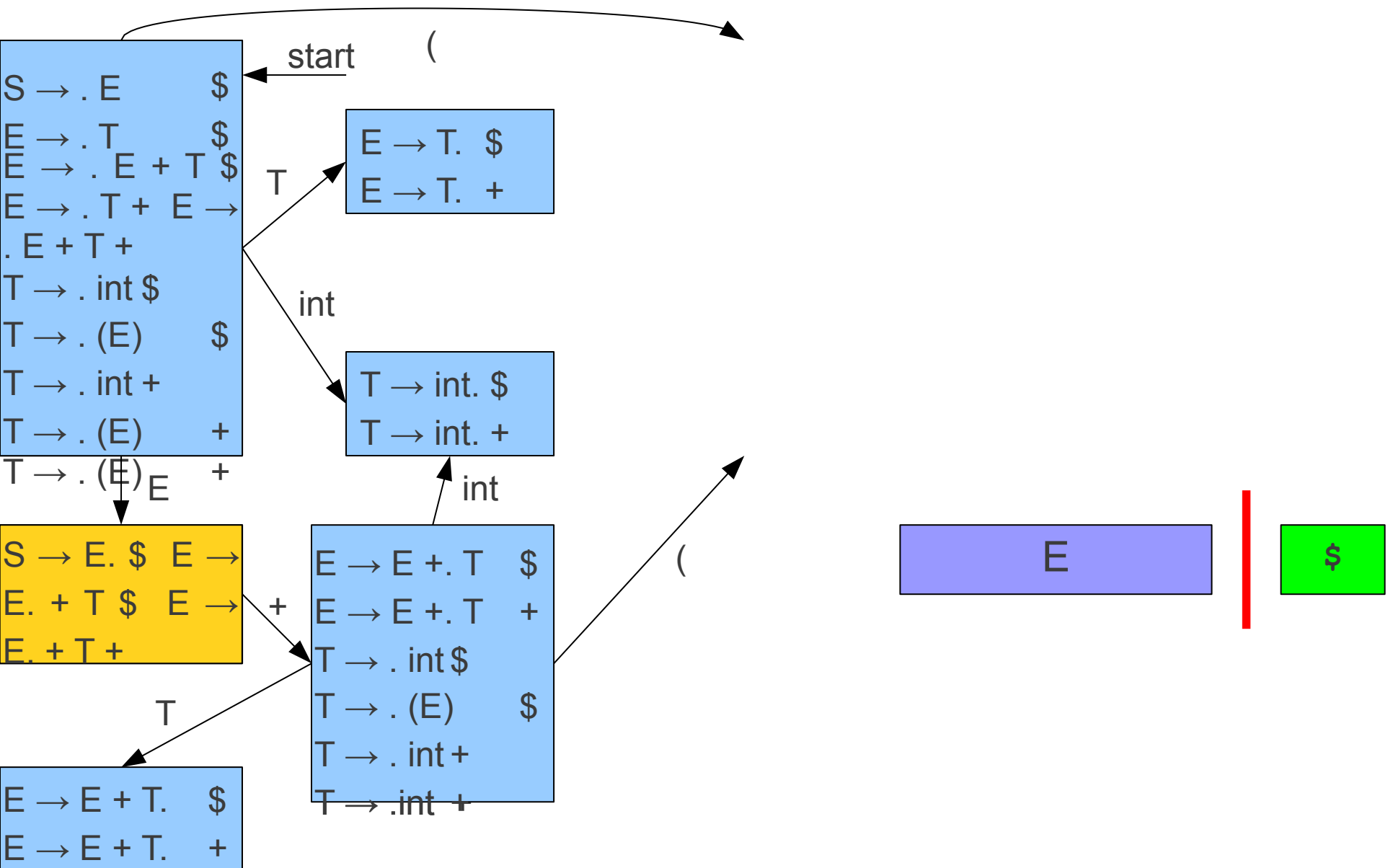
# Deterministic LR(1) Automata



# Deterministic LR(1) Automata



# Deterministic LR(1) Automata



# Representing LR(1) Automata

- As with LR(0), use **action** and **goto** tables.
- **goto** table defined as before; encodes transition table as map from (state, token) to states.
- **action** table maps pairs (state, lookahead) to actions.
- Commonly combined into a single **action/goto** table.

# The Power of LR(1)

- Any LR(0) grammar is LR(1).
- Any LL(1) grammar is LR(1).
- Any deterministic CFL (a CFL parseable by a *deterministic pushdown automaton*) has an LR(k) grammar.
- Any LL(k) *language* is LR(1), though individual LL(k) *grammars* might not be.
- Any LR(k) *language* is LR(1), though individual LR(k) *grammars* might not be.

# LR(1) Automata are **Huge**

- In a grammar with  $n$  terminals,
  - $\rightarrow$   $n$  times larger of non-deterministic automata
  - $\rightarrow 2^N$  vs  $2^{nN}$
- LR(1) tables for practical programming languages can have hundreds of thousands or even *millions* of states.
- Consequently, LR(1) parsers are rarely used in practice.



Is there a way to get the power of LR(1) without the huge table size?

# Why is LR(1) so powerful?

- Intuitively, for two reasons:
- **Lookahead makes handle-finding easier.**
  - The LR(0) automaton says whether there could be a handle later on based on no right context.
  - The LR(1) automaton can predict whether it needs to reduce based on more information.
- **More states encode more information.**
  - LR(1) lookaheads are very good because there's a greater number of states to be in.
- **Goal:** Incorporate lookahead without increasing the number of states.

# Revisiting Shift/Reduce Conflicts

A shift/reduce conflict is a state that looks like this:

$$\mathbf{A} \rightarrow \omega \cdot$$

$$\mathbf{B} \rightarrow \alpha \cdot \beta$$

In LR(0), this is simply not allowed.

In LR(1), this can be avoided by using lookahead to determine whether to shift or reduce.

Can we get some of the lookahead power of LR(1) without the huge tables?

# A Surprisingly Powerful Idea

In an LR(1) automaton, we have multiple states with the same core but different lookahead.

What if we merge all these states together?

This is called **LALR(1)**

**Lookahead(1) LR(0)**

# LR(1) $\rightarrow$ LALR

Merge states if the set of “cores of items” are equal

$E \rightarrow . E + T \{ \$+ \}$	$E \rightarrow . E + T \{ )+ \}$
$E \rightarrow . T \{ \$+ \}$	$E \rightarrow . T \{ )+ \}$

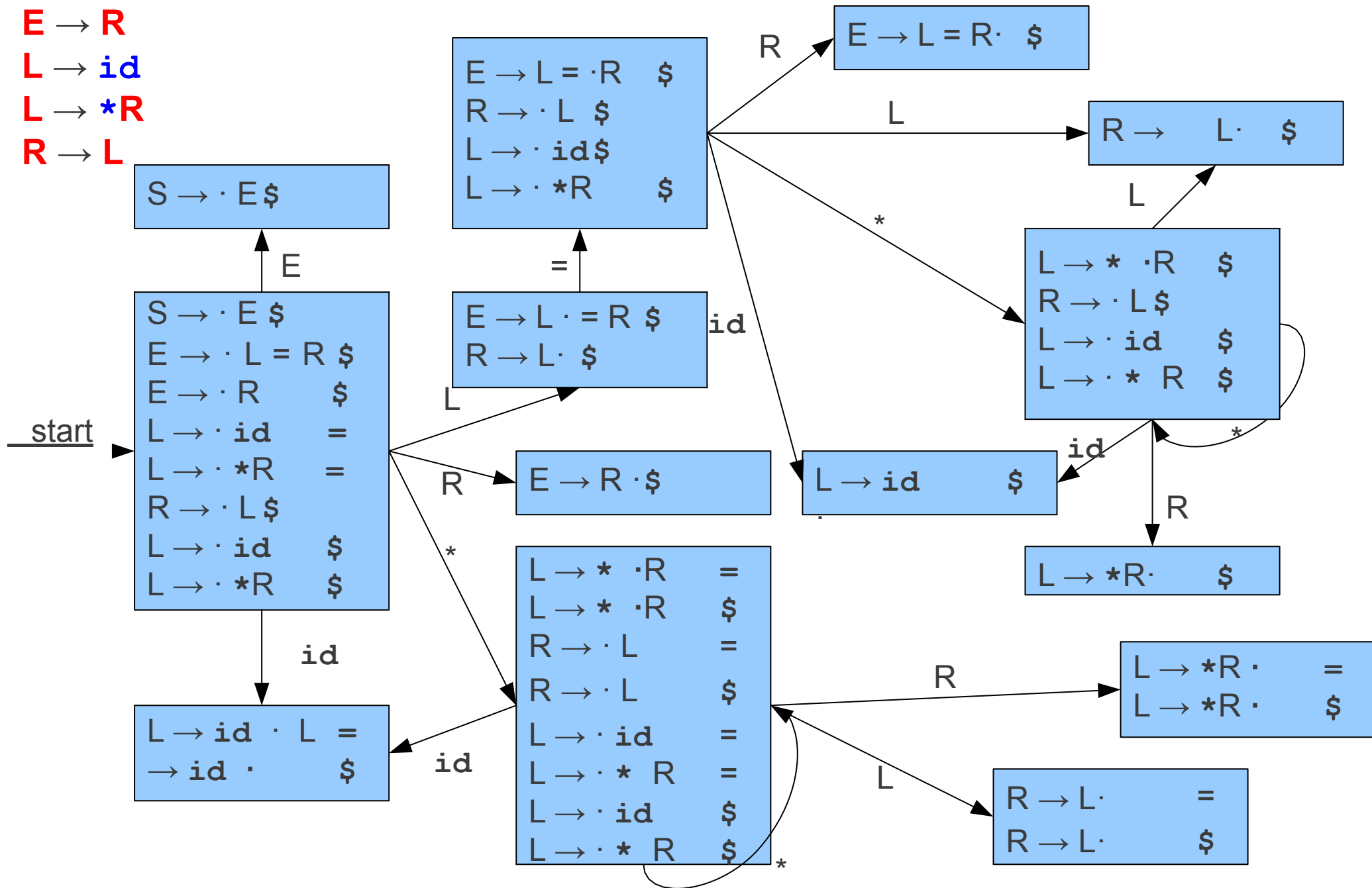
$E \rightarrow . E + T \{ \$)+ \}$
$E \rightarrow . T \{ \$)+ \}$

# From LR(1) to LALR(1)

$S \rightarrow E$   
 $E \rightarrow L = R$   
 $E \rightarrow R$   
 $L \rightarrow id$   
 $L \rightarrow *R$   
 $R \rightarrow L$

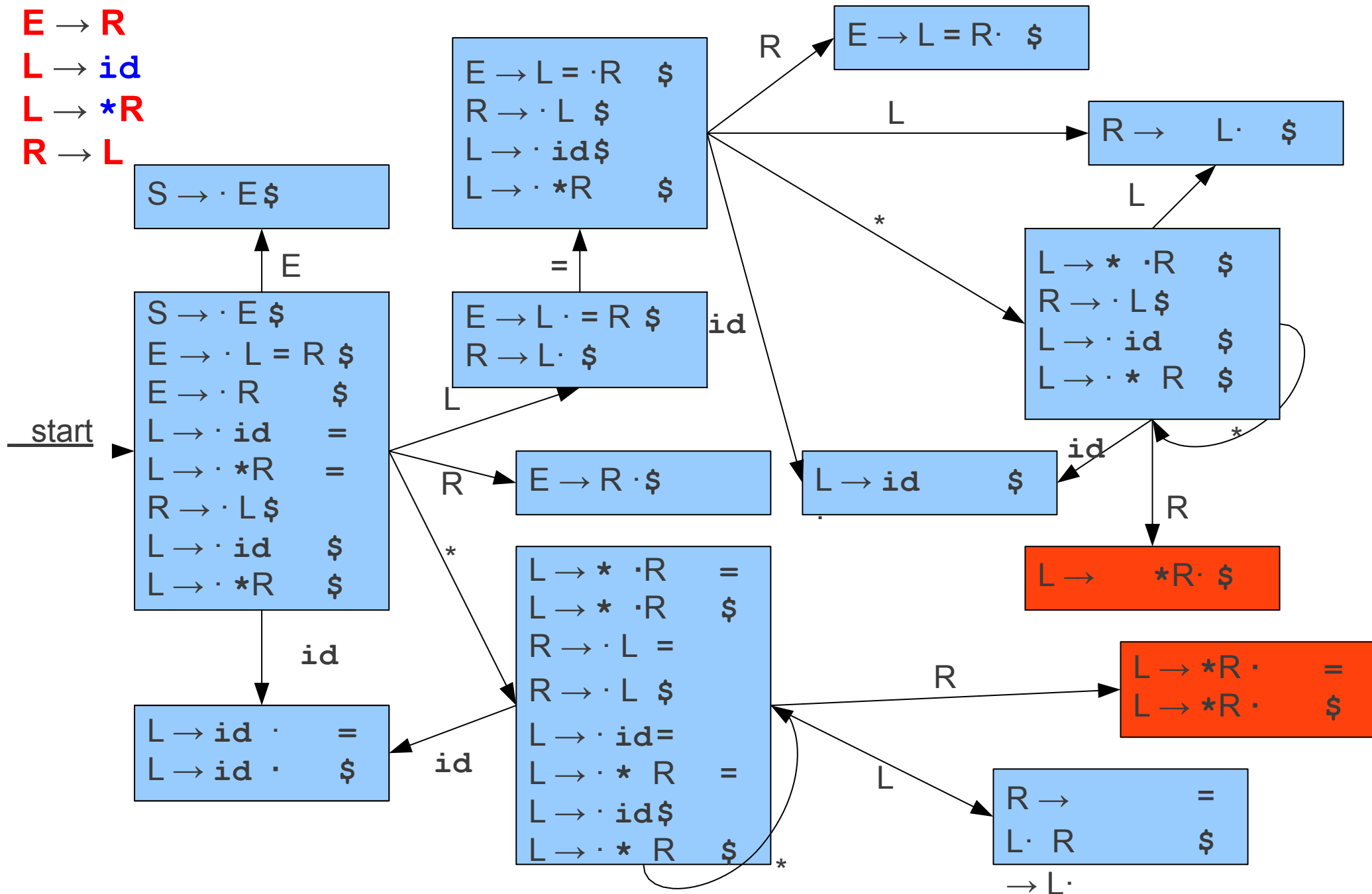
# From LR(1) to LALR(1)

$S \rightarrow E$   
 $E \rightarrow L = R$   
 $E \rightarrow R$   
 $L \rightarrow id$   
 $L \rightarrow *R$   
 $R \rightarrow L$



# From LR(1) to LALR(1)

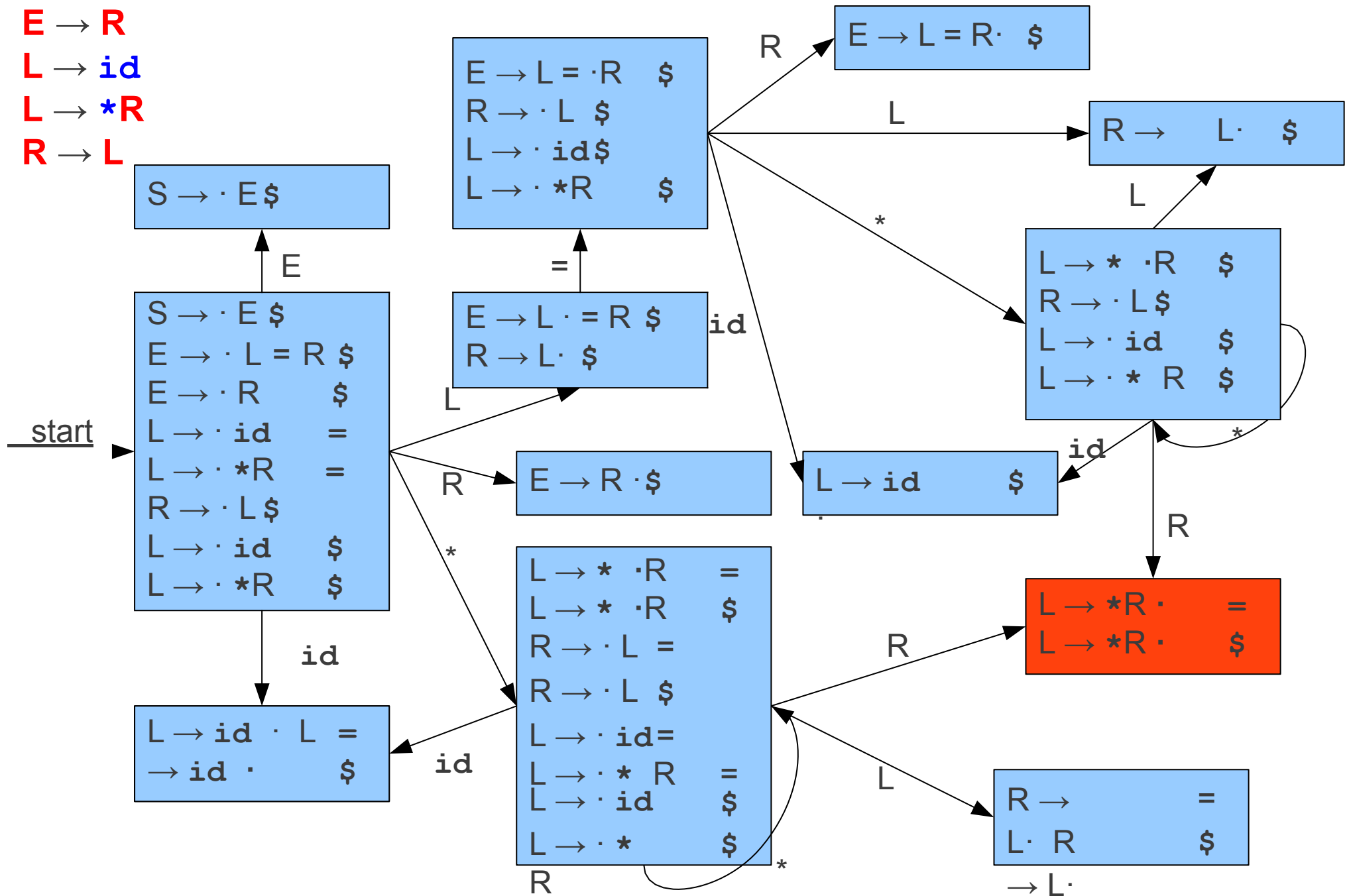
$S \rightarrow E$   
 $E \rightarrow L = R$   
 $E \rightarrow R$   
 $L \rightarrow id$   
 $L \rightarrow *R$   
 $R \rightarrow L$





# From LR(1) to LALR(1)

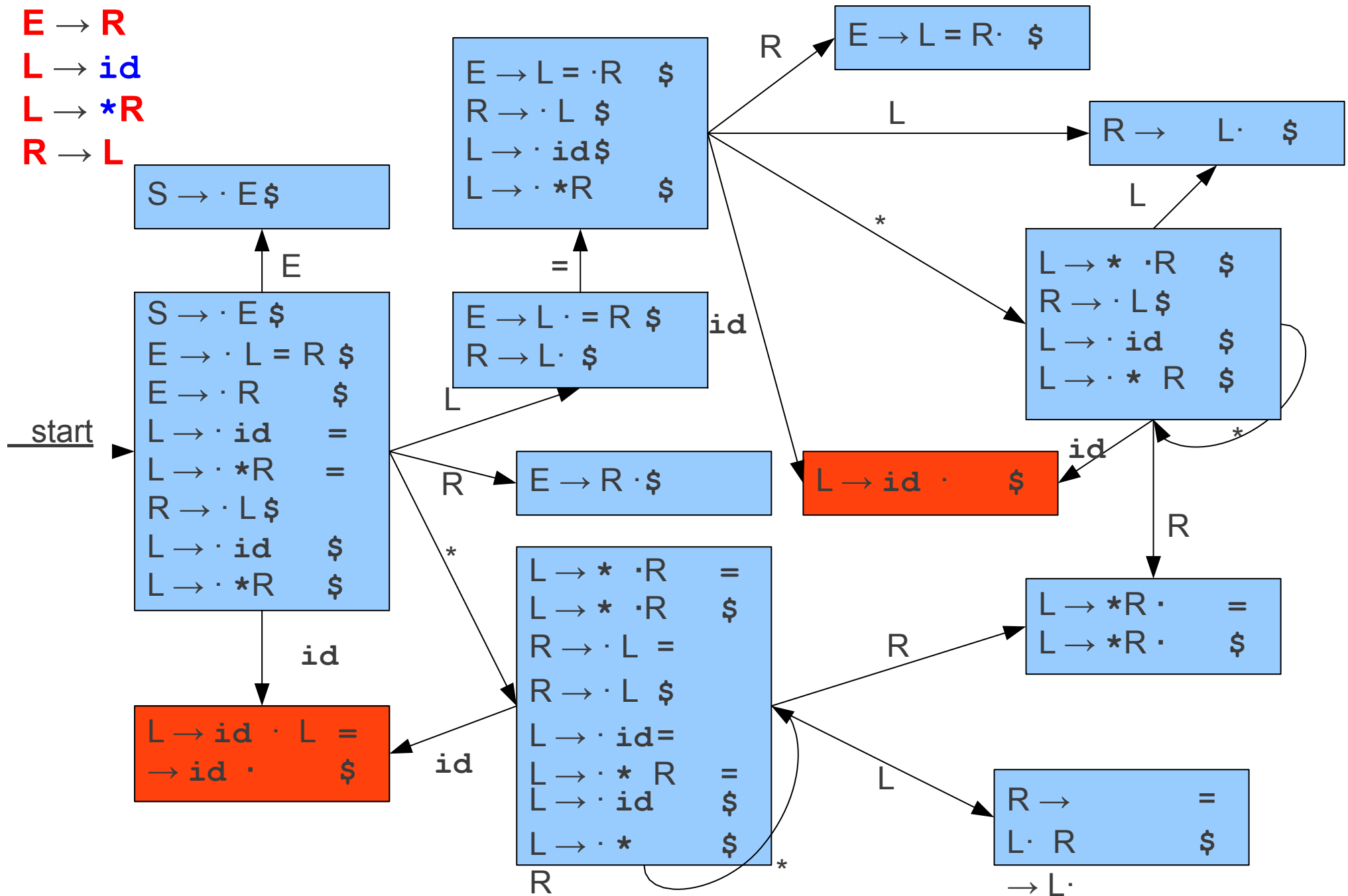
$S \rightarrow E$   
 $E \rightarrow L = R$   
 $E \rightarrow R$   
 $L \rightarrow id$   
 $L \rightarrow *R$   
 $R \rightarrow L$





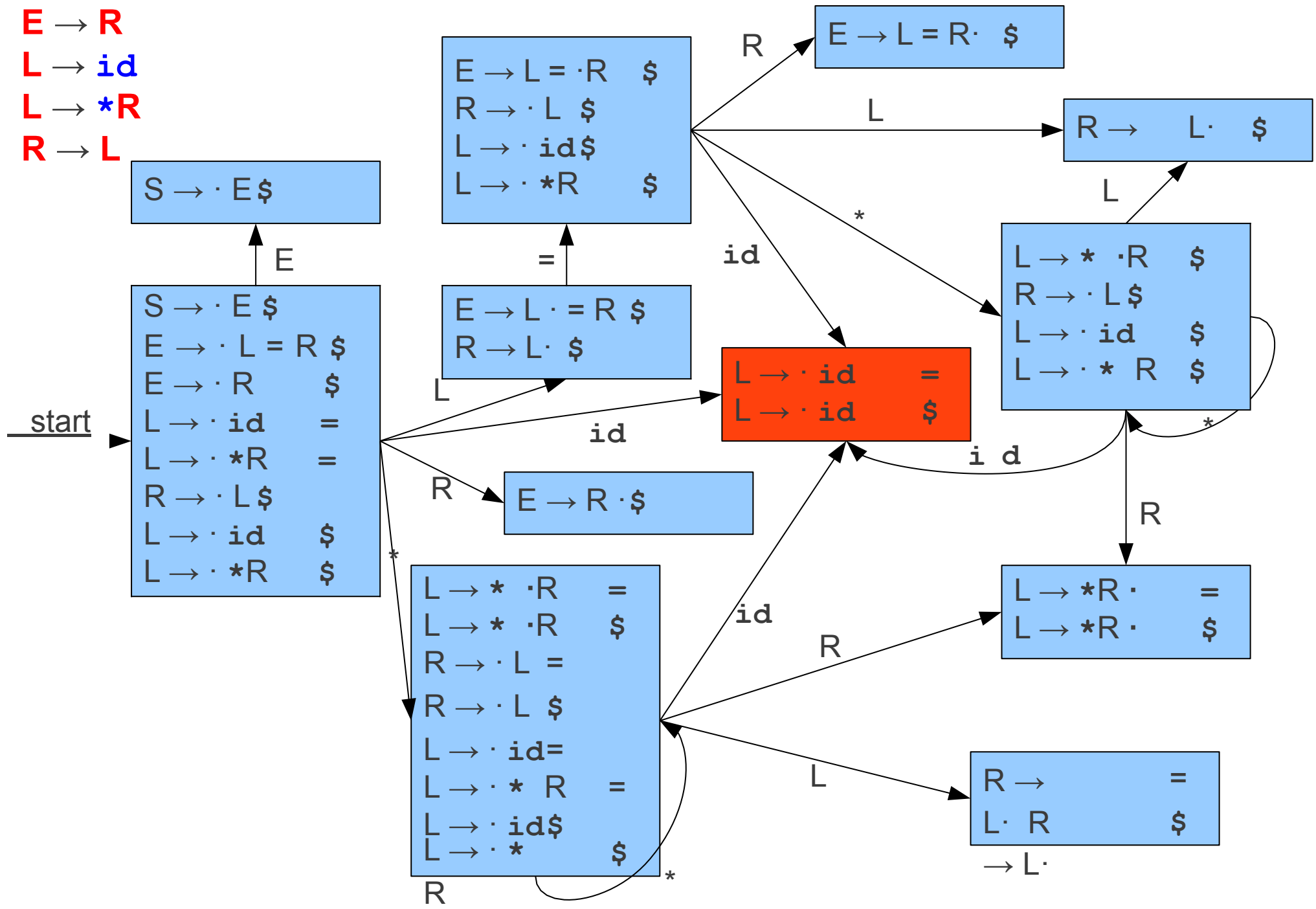
# From LR(1) to LALR(1)

$S \rightarrow E$   
 $E \rightarrow L = R$   
 $E \rightarrow R$   
 $L \rightarrow id$   
 $L \rightarrow *R$   
 $R \rightarrow L$



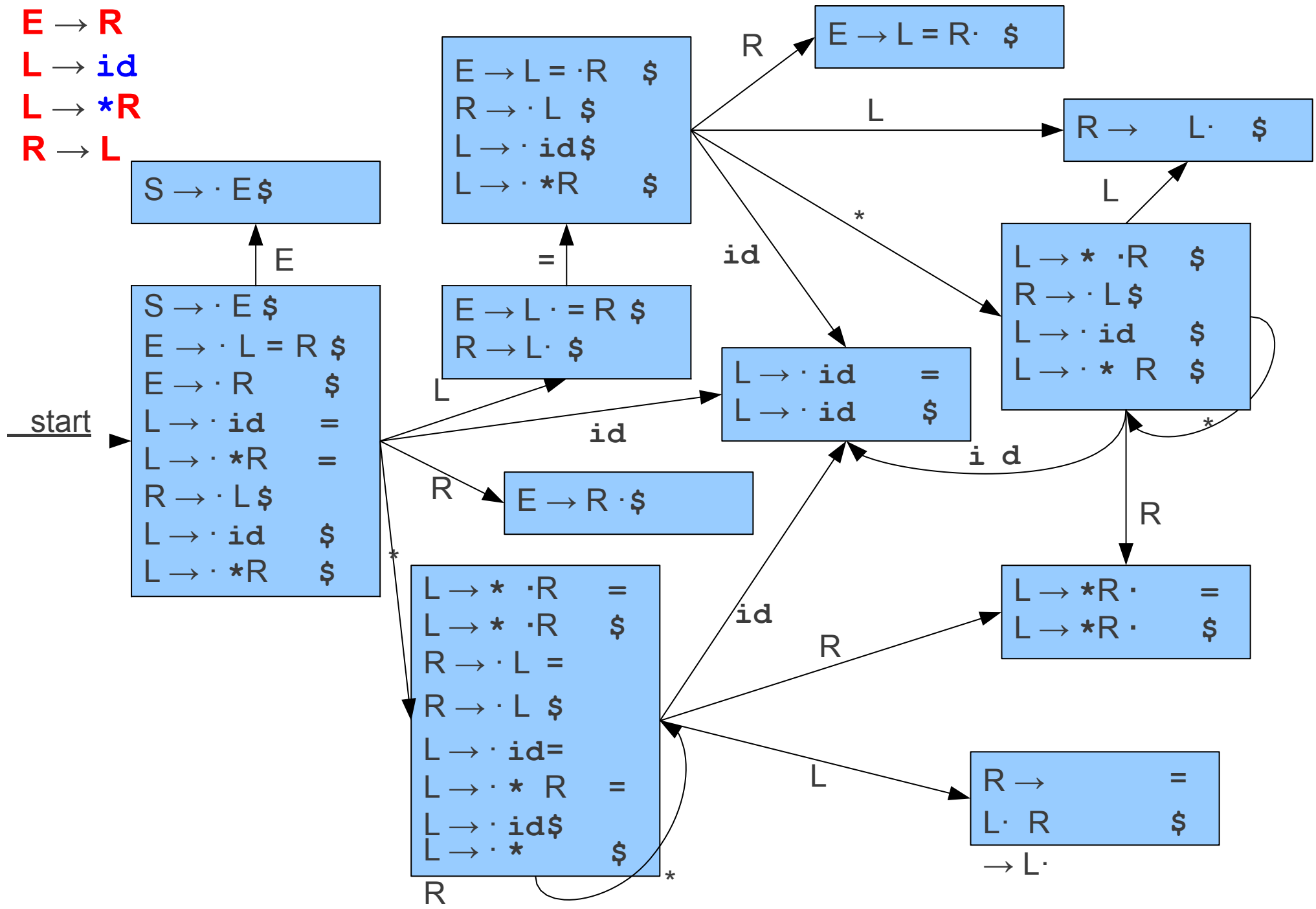
# From LR(1) to LALR(1)

$S \rightarrow E$   
 $E \rightarrow L = R$   
 $E \rightarrow R$   
 $L \rightarrow id$   
 $L \rightarrow *R$   
 $R \rightarrow L$



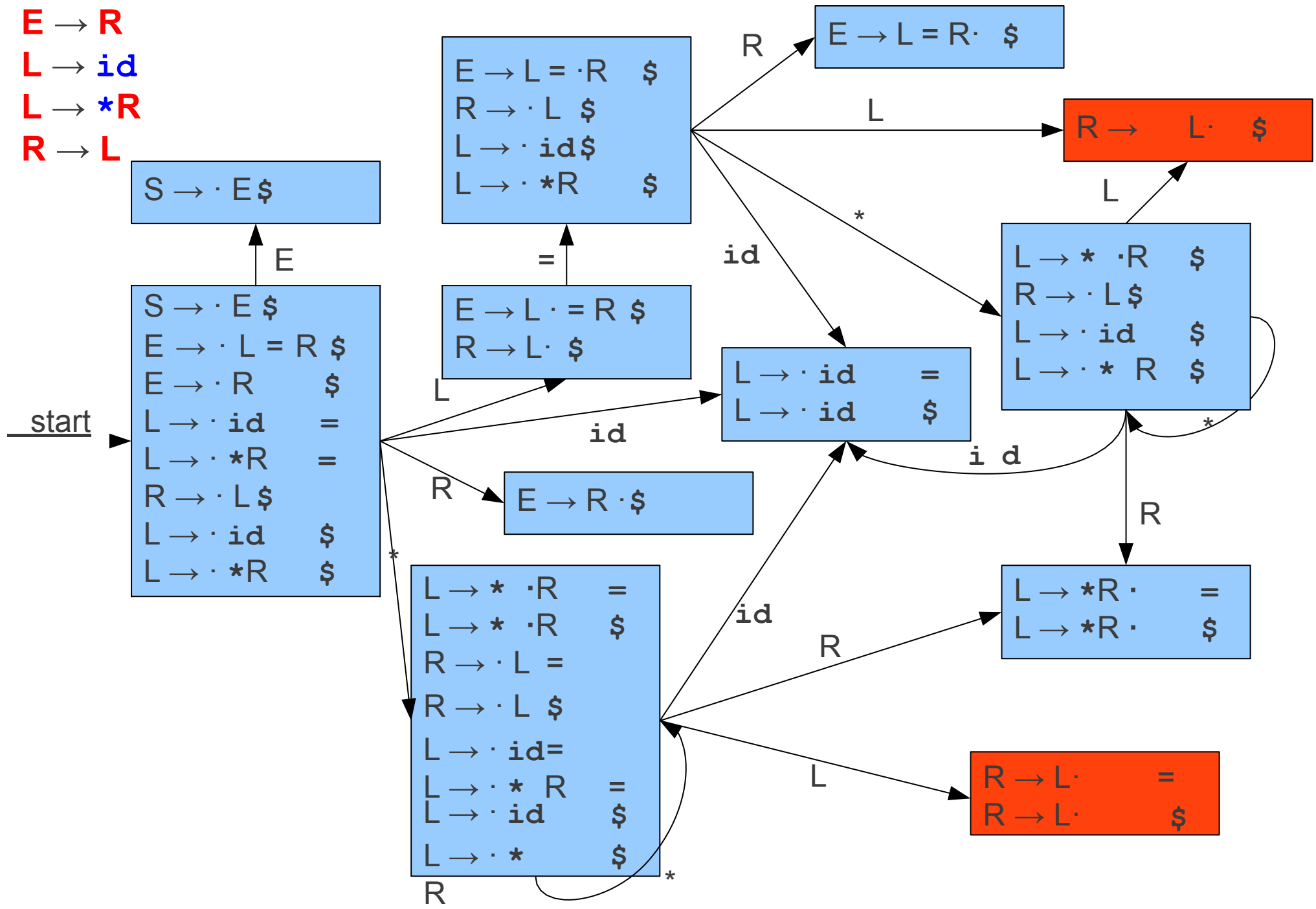
# From LR(1) to LALR(1)

$S \rightarrow E$   
 $E \rightarrow L = R$   
 $E \rightarrow R$   
 $L \rightarrow id$   
 $L \rightarrow *R$   
 $R \rightarrow L$



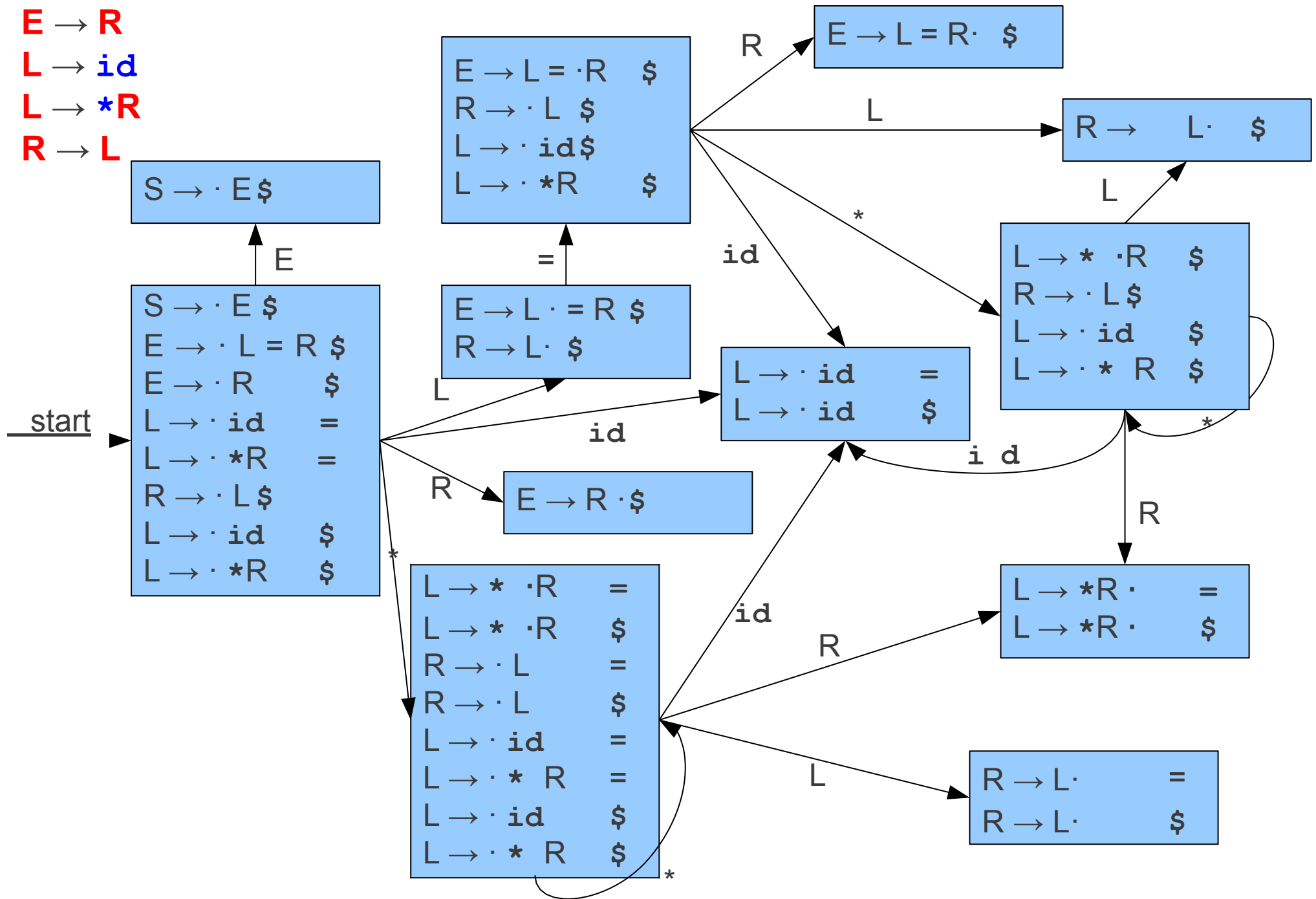
# From LR(1) to LALR(1)

$S \rightarrow E$   
 $E \rightarrow L = R$   
 $E \rightarrow R$   
 $L \rightarrow id$   
 $L \rightarrow *R$   
 $R \rightarrow L$



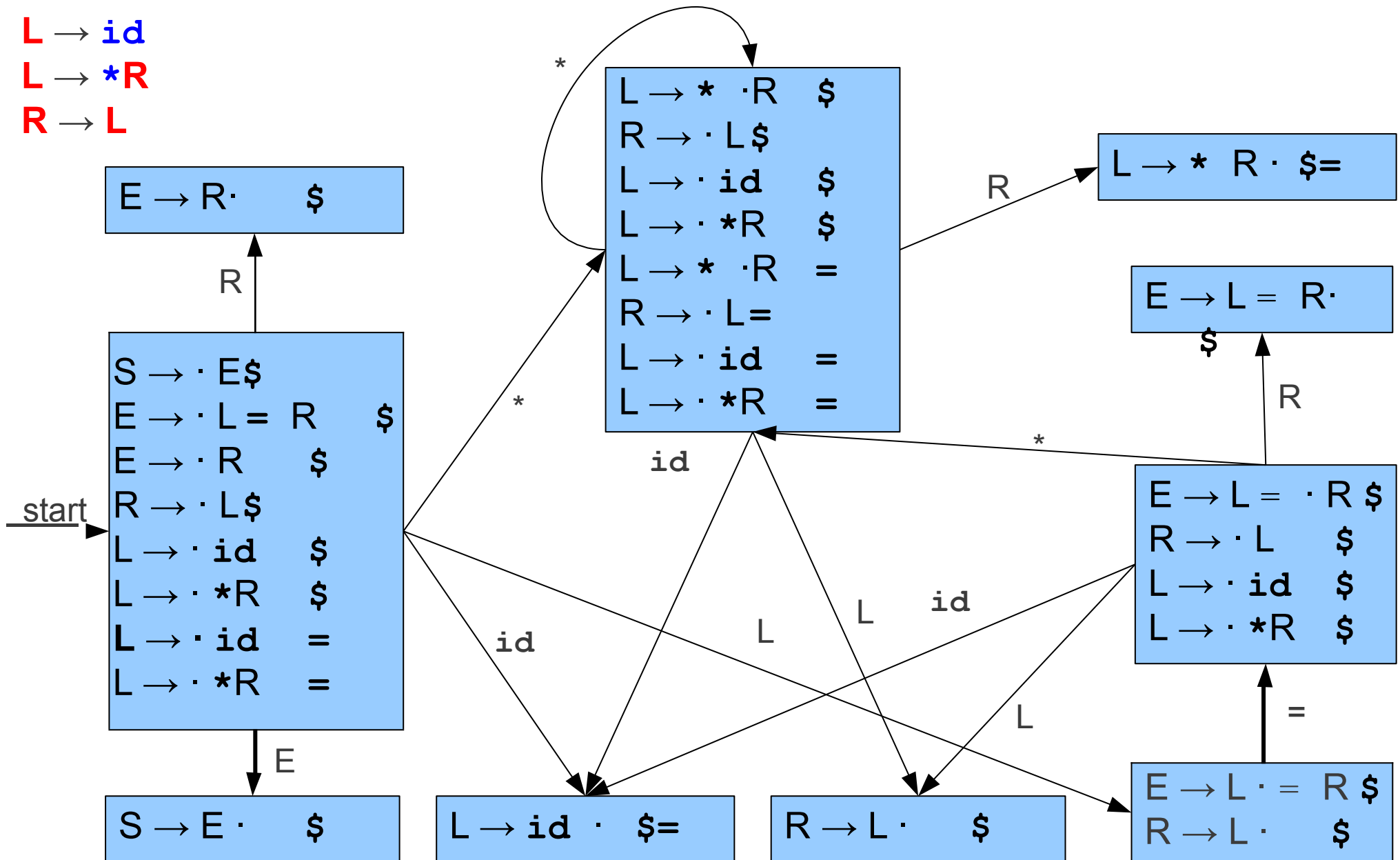
# From LR(1) to LALR(1)

$S \rightarrow E$   
 $E \rightarrow L = R$   
 $E \rightarrow R$   
 $L \rightarrow id$   
 $L \rightarrow *R$   
 $R \rightarrow L$



# From LR(1) to LALR(1)

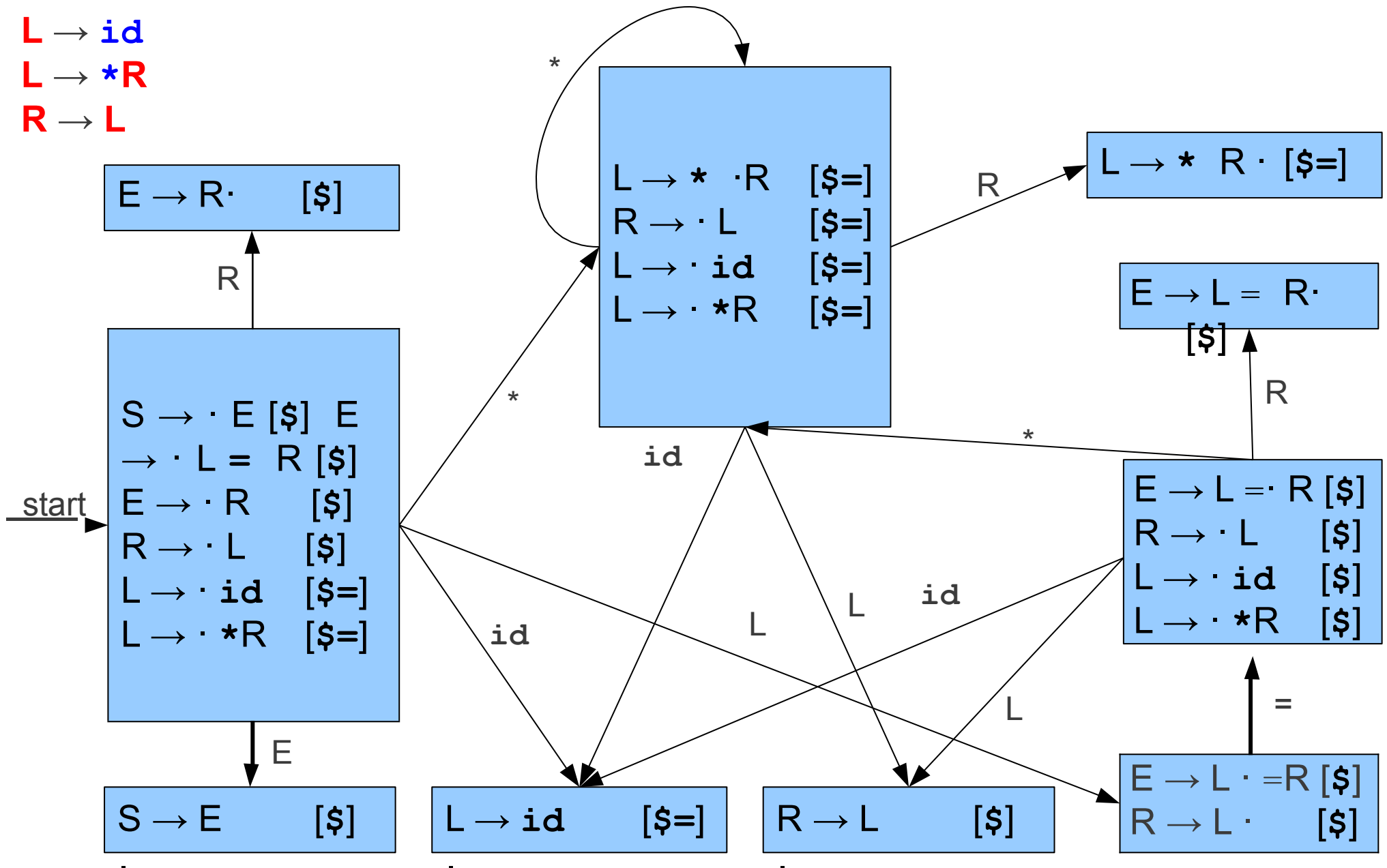
$S \rightarrow E$   
 $E \rightarrow L = R$   
 $E \rightarrow R$   
 $L \rightarrow id$   
 $L \rightarrow *R$   
 $R \rightarrow L$





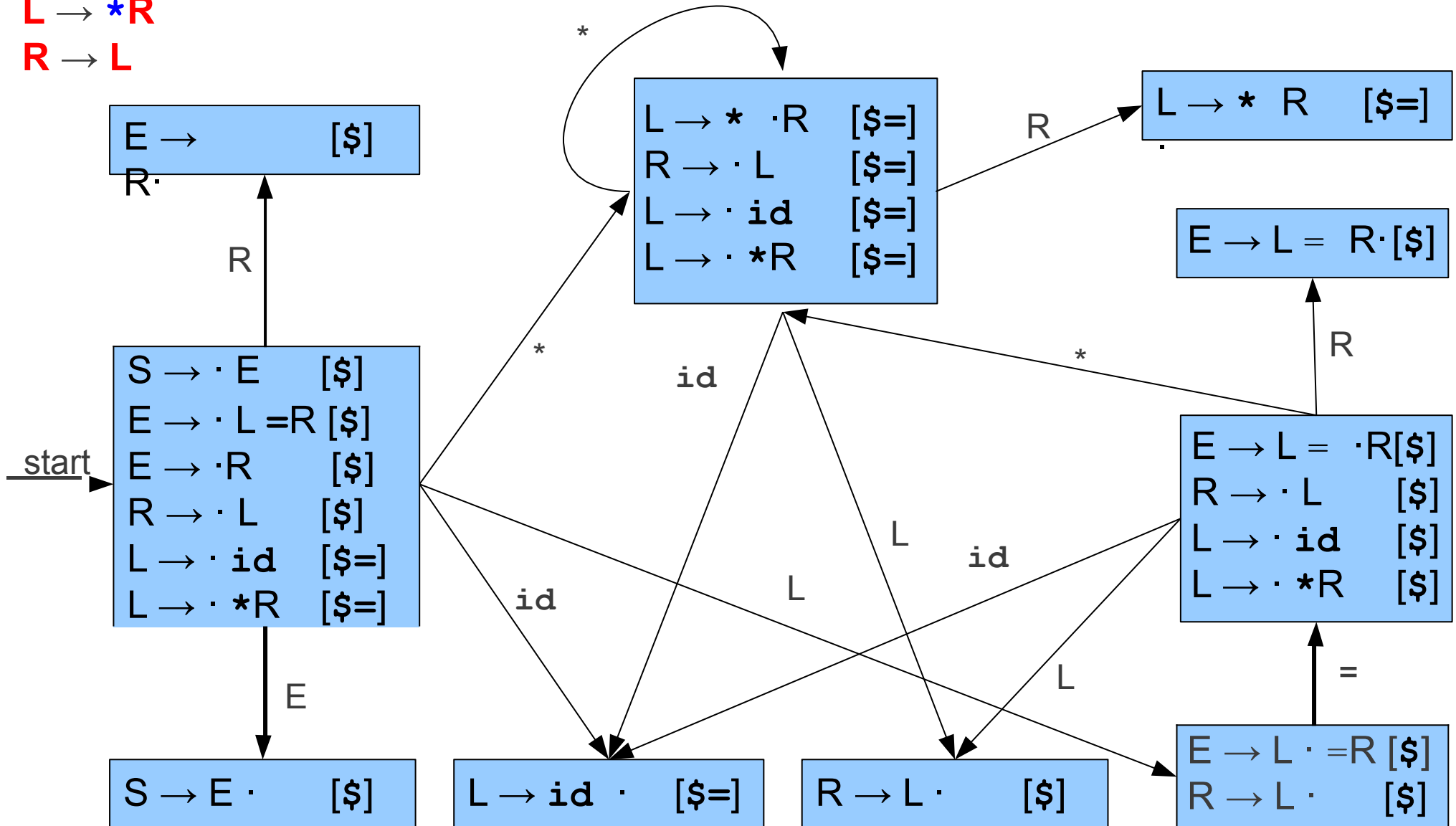
# From LR(1) to LALR(1)

$S \rightarrow E$   
 $E \rightarrow L = R$   
 $E \rightarrow R$   
 $L \rightarrow id$   
 $L \rightarrow *R$   
 $R \rightarrow L$



# From LR(1) to LALR(1)

$S \rightarrow E$   
 $E \rightarrow L = R$   
 $E \rightarrow R$   
 $L \rightarrow id$   
 $L \rightarrow *R$   
 $R \rightarrow L$



# Advantages of LALR(1)

- Maintains context.
  - Lookup sets based on the fine-grained LR(1) automaton.
  - Each state's lookup relevant only for that state.
- Keeps automaton small.
  - Resulting automaton has same size as LR(0) automaton.

# LALR(1) is Powerful

- Every LR(0) grammar is LALR(1).
- Every SLR(1) grammar is LALR(1)
- *Most* (but not all) LR(1) grammars are LALR(1).
- It is powerful enough! i.e. most of PL's structures fit with LALR(1)

# LALR(1) isn't LR(1)

- Merging LR(1) states cannot introduce a shift/reduce conflict.
- **Why?**
- Since the items have the same core, a shift/reduce conflict in a LALR(1) state would have to also exist in one of the LR(1) states it was merged from.
- Merging LR(1) states **can** introduce a reduce/reduce conflict.
- Often these conflicts appear without any good reason; this is one limitation of LALR(1).

# Constructing LALR(1) Automata

- It's not a good idea to build LALR(1) automata from LR(1) automata.
- **Why?**
- LR(1) automata are impractically large.
- Are there more efficient methods for LALR(1) automata construction?
- **Yes**; we'll see two.

# The “Lazy Merging” Technique

- Idea: Merge together LR(1) states as they're generated.
- Maintain a worklist of states to process; begin with the initial LR(1) state.
- When adding a new state, if it has the same core (kernel/base) as an old state, update the old state and put it back in the worklist.

$S \rightarrow E$   
 $E \rightarrow L = R$   
 $E \rightarrow R$   
 $L \rightarrow id$   
 $L \rightarrow *R$   
 $R \rightarrow L$

# LALR(1) Construction



$S \rightarrow E$   
 $E \rightarrow L = R$   
 $E \rightarrow R$   
 $L \rightarrow id$   
 $L \rightarrow *R$   
 $R \rightarrow L$

# LALR(1) Construction



$S \rightarrow E$   
 $E \rightarrow L = R$   
 $E \rightarrow R$   
 $L \rightarrow id$   
 $L \rightarrow *R$   
 $R \rightarrow L$

# LALR(1) Construction



$S \rightarrow E$   
 $E \rightarrow L = R$   
 $E \rightarrow R$   
 $L \rightarrow id$   
 $L \rightarrow *R$   
 $R \rightarrow L$

# LALR(1) Construction

start

$S \rightarrow \cdot E [\$]$   
 $E \rightarrow \cdot L = R [\$]$   
 $E \rightarrow \cdot R [\$]$   
 $R \rightarrow \cdot L [\$]$

$S \rightarrow E$   
 $E \rightarrow L = R$   
 $E \rightarrow R$   
 $L \rightarrow id$   
 $L \rightarrow *R$   
 $R \rightarrow L$

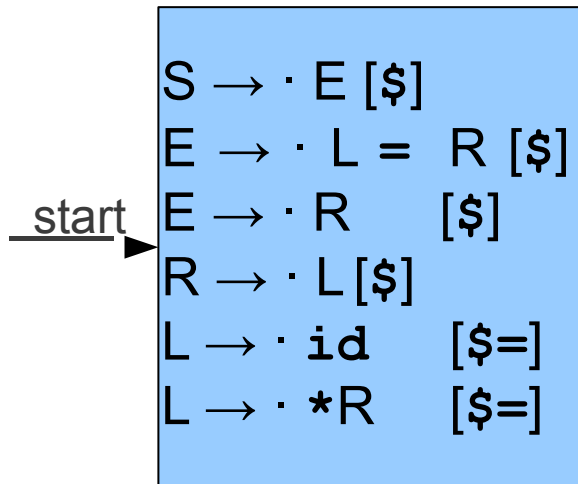
# LALR(1) Construction

start 

$S \rightarrow \cdot E [\$]$   
 $E \rightarrow \cdot L = R [\$]$   
 $E \rightarrow \cdot R [\$]$   
 $R \rightarrow \cdot L [\$]$   
 $L \rightarrow \cdot id [\$]$   
 $L \rightarrow \cdot *R [\$]$

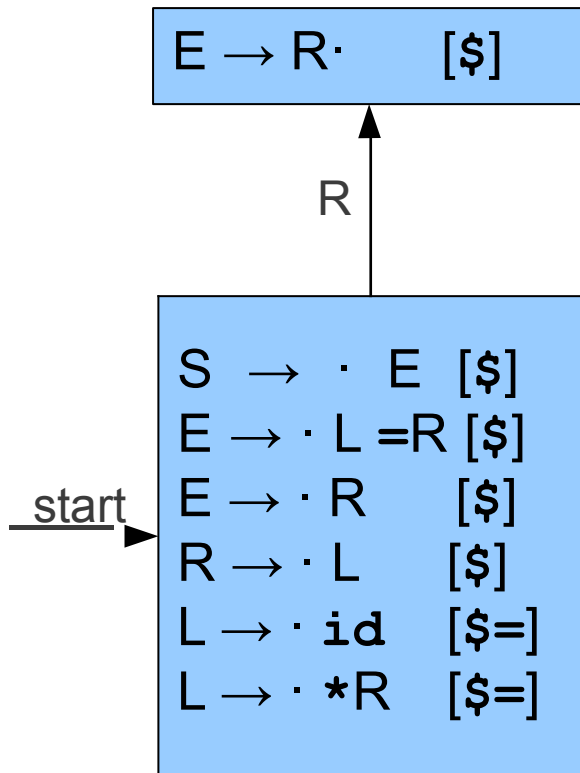
$S \rightarrow E$   
 $E \rightarrow L = R$   
 $E \rightarrow R$   
 $L \rightarrow id$   
 $L \rightarrow *R$   
 $R \rightarrow L$

# LALR(1) Construction



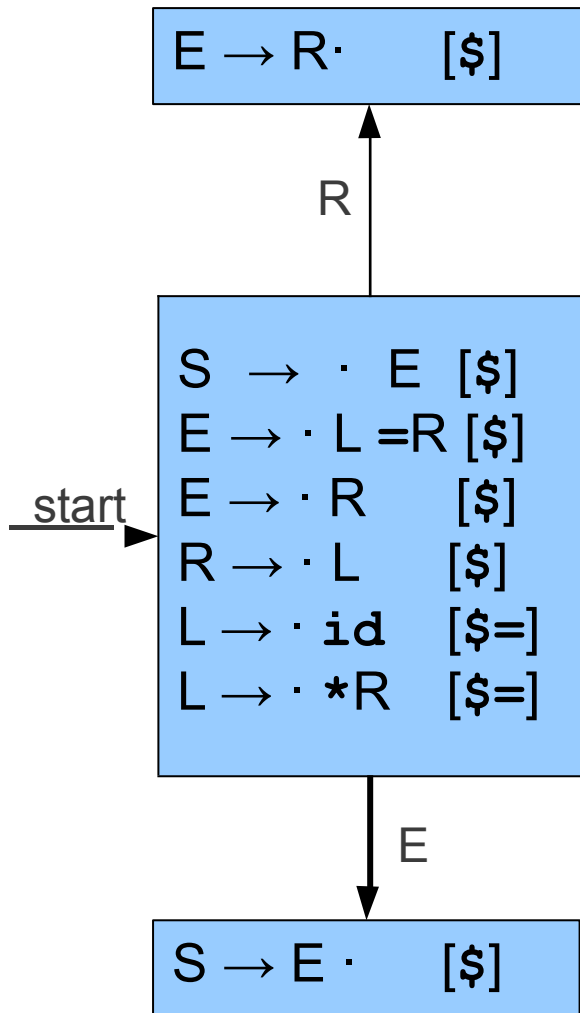
$S \rightarrow E$   
 $E \rightarrow L = R$   
 $E \rightarrow R$   
 $L \rightarrow id$   
 $L \rightarrow *R$   
 $R \rightarrow L$

# LALR(1) Construction



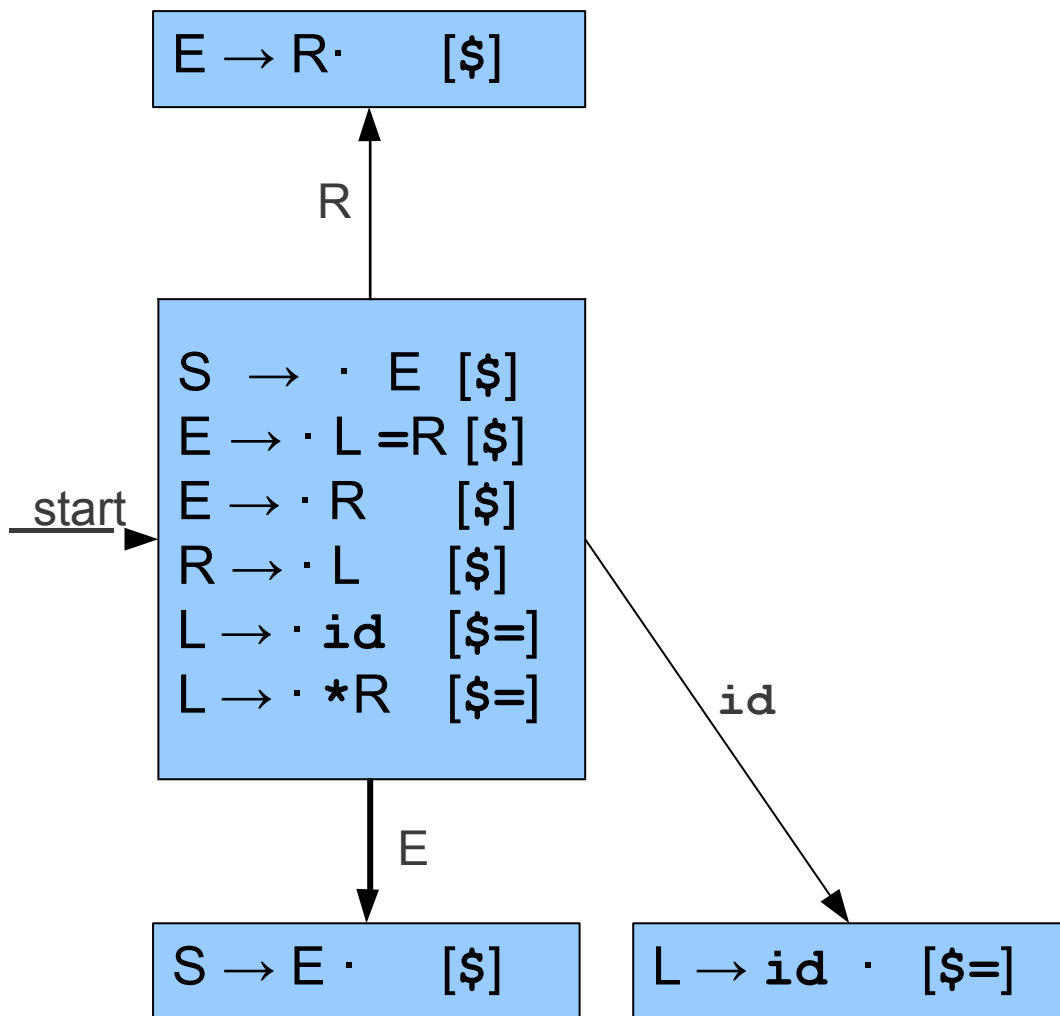
$S \rightarrow E$   
 $E \rightarrow L = R$   
 $E \rightarrow R$   
 $L \rightarrow id$   
 $L \rightarrow *R$   
 $R \rightarrow L$

# LALR(1) Construction



$S \rightarrow E$   
 $E \rightarrow L = R$   
 $E \rightarrow R$   
 $L \rightarrow id$   
 $L \rightarrow *R$   
 $R \rightarrow L$

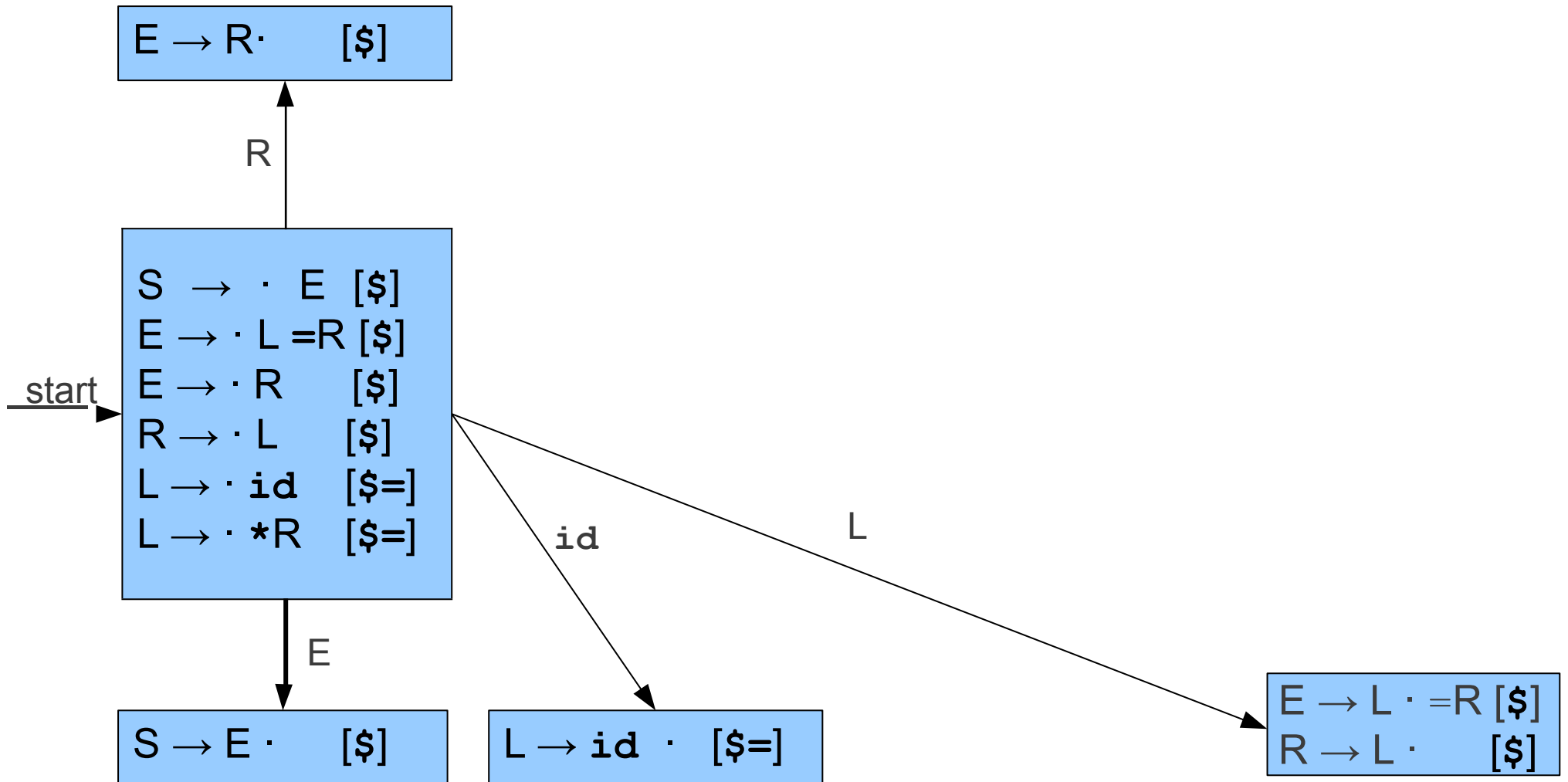
# LALR(1) Construction





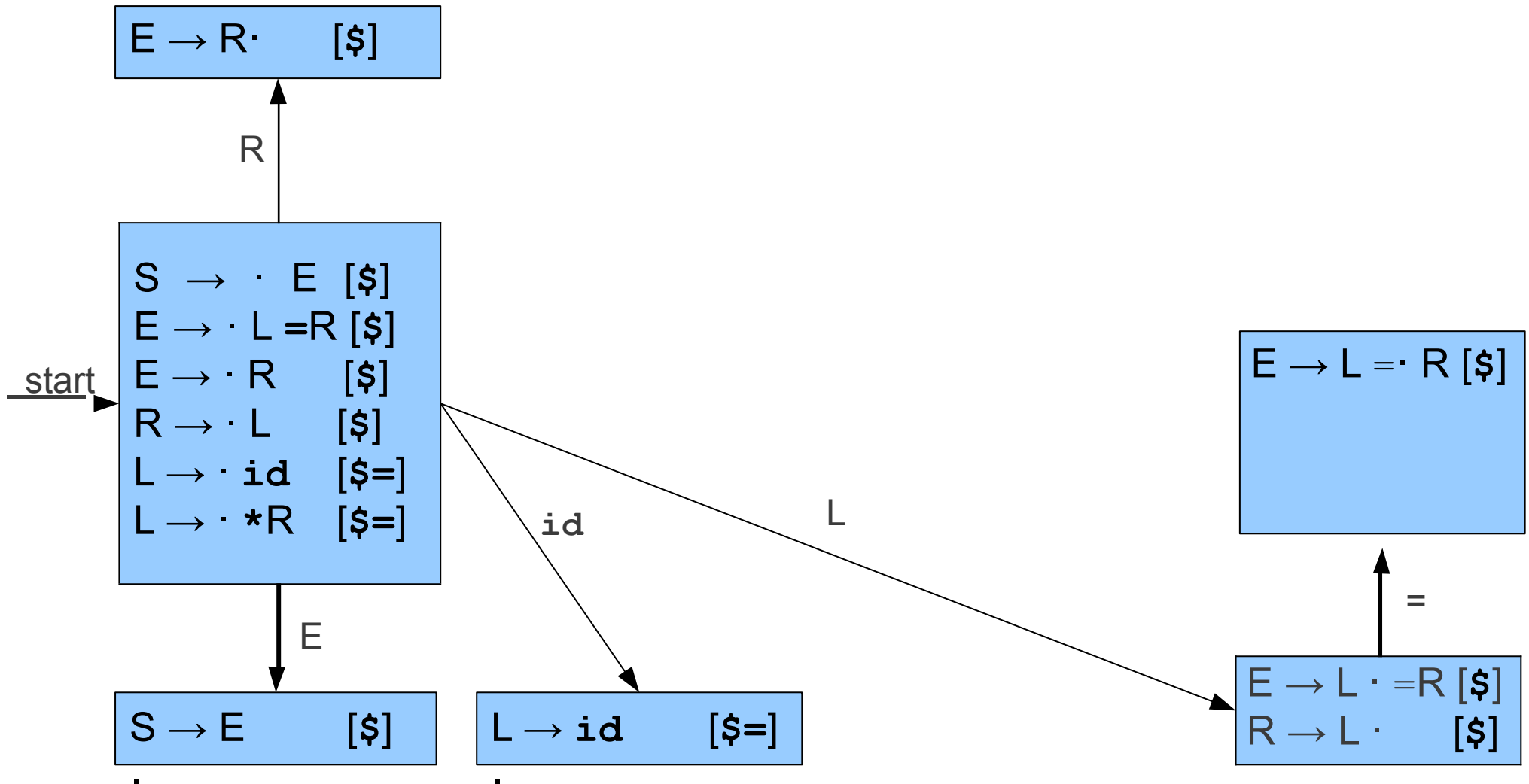
$S \rightarrow E$   
 $E \rightarrow L = R$   
 $E \rightarrow R$   
 $L \rightarrow id$   
 $L \rightarrow *R$   
 $R \rightarrow L$

# LALR(1) Construction



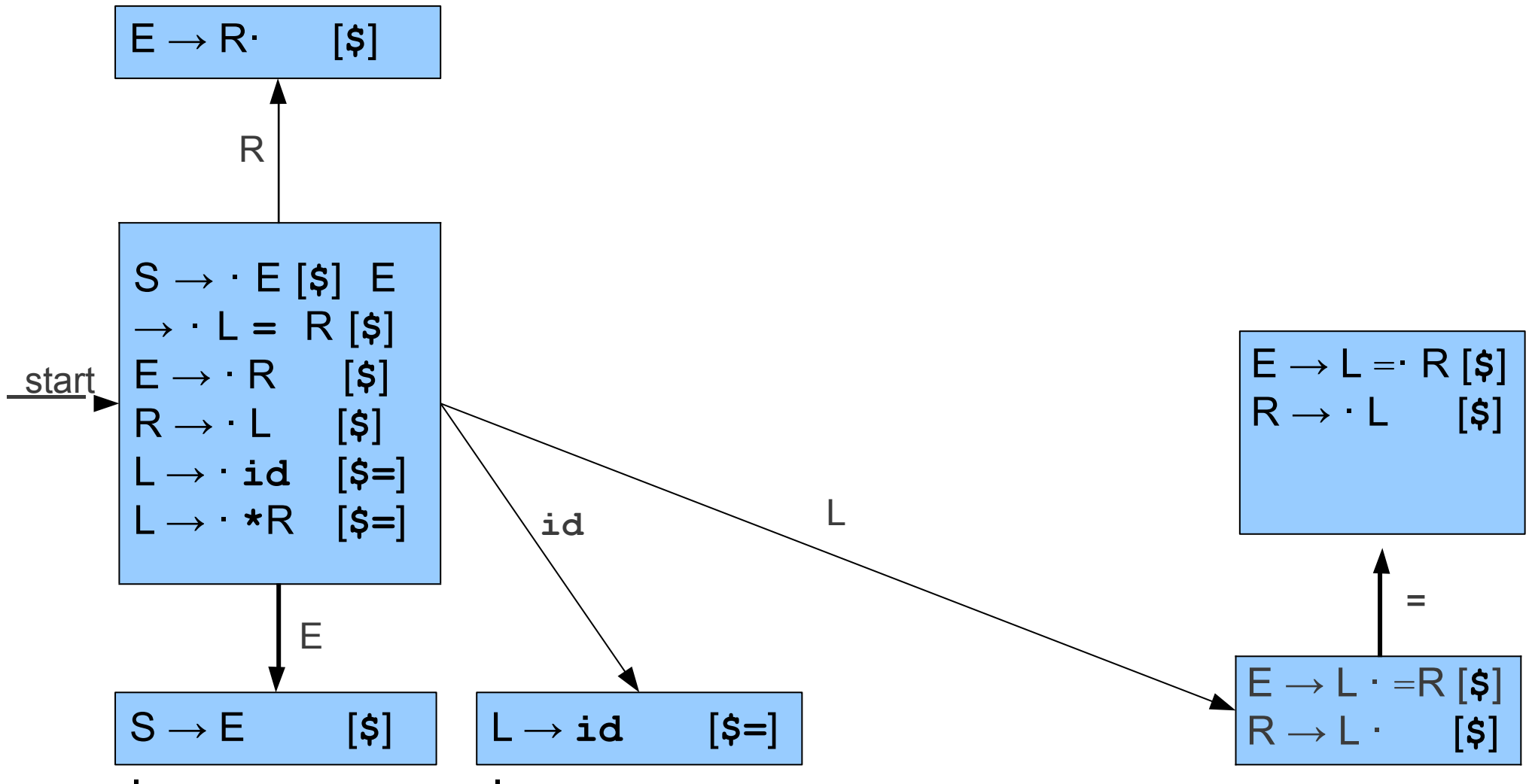
$S \rightarrow E$   
 $E \rightarrow L = R$   
 $E \rightarrow R$   
 $L \rightarrow id$   
 $L \rightarrow *R$   
 $R \rightarrow L$

# LALR(1) Construction



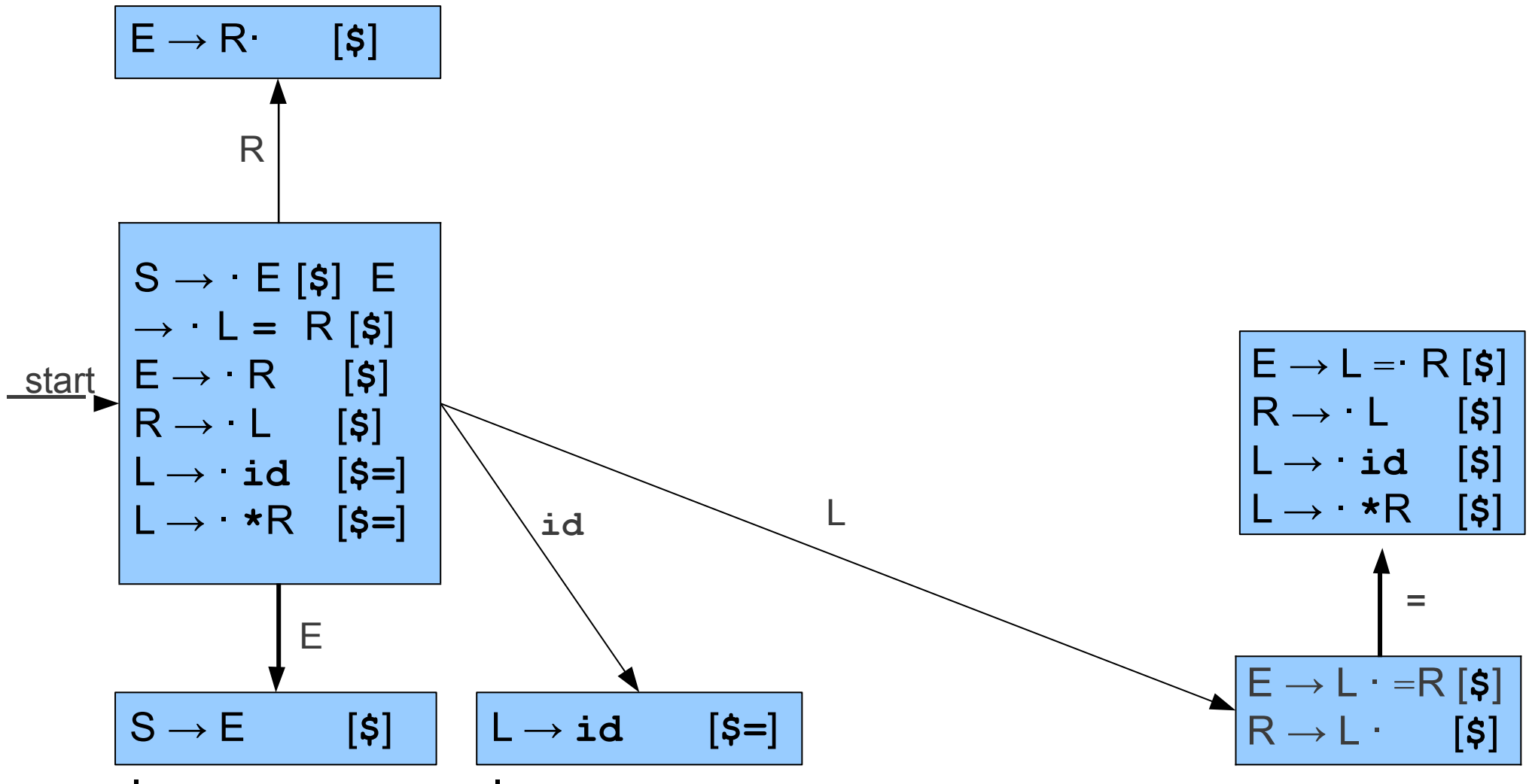
$S \rightarrow E$   
 $E \rightarrow L = R$   
 $E \rightarrow R$   
 $L \rightarrow id$   
 $L \rightarrow *R$   
 $R \rightarrow L$

# LALR(1) Construction



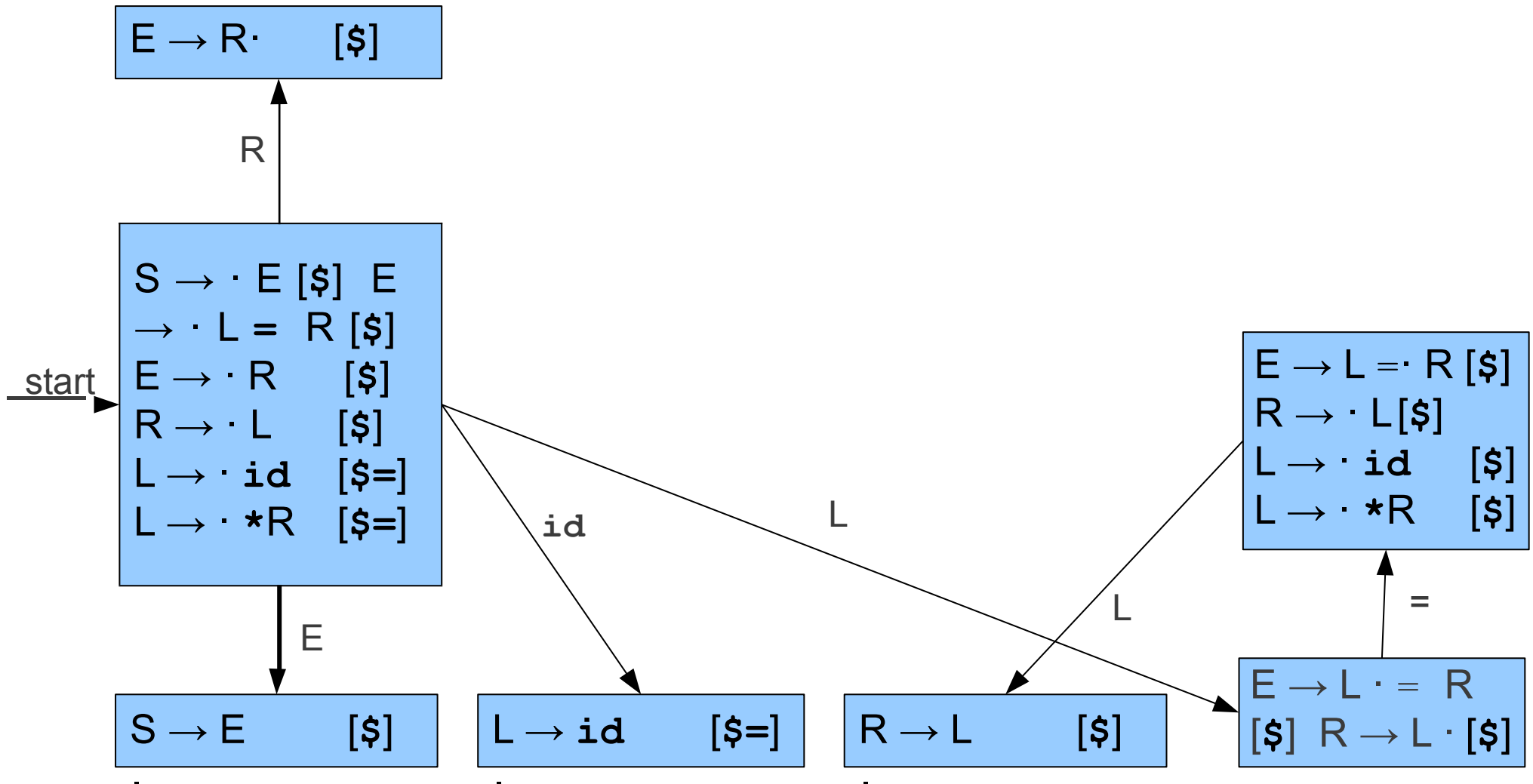
$S \rightarrow E$   
 $E \rightarrow L = R$   
 $E \rightarrow R$   
 $L \rightarrow id$   
 $L \rightarrow *R$   
 $R \rightarrow L$

# LALR(1) Construction



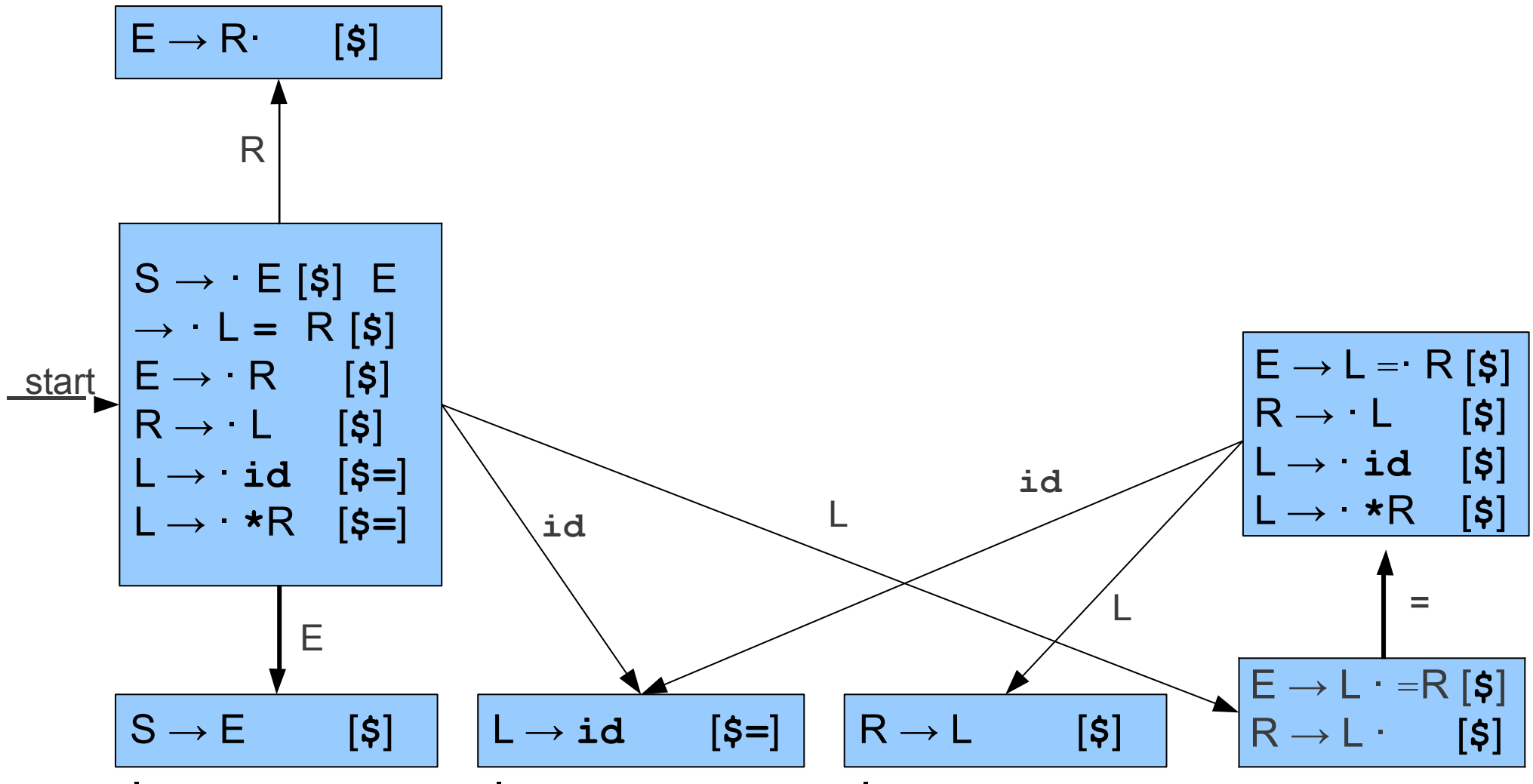
$S \rightarrow E$   
 $E \rightarrow L = R$   
 $E \rightarrow R$   
 $L \rightarrow id$   
 $L \rightarrow *R$   
 $R \rightarrow L$

# LALR(1) Construction



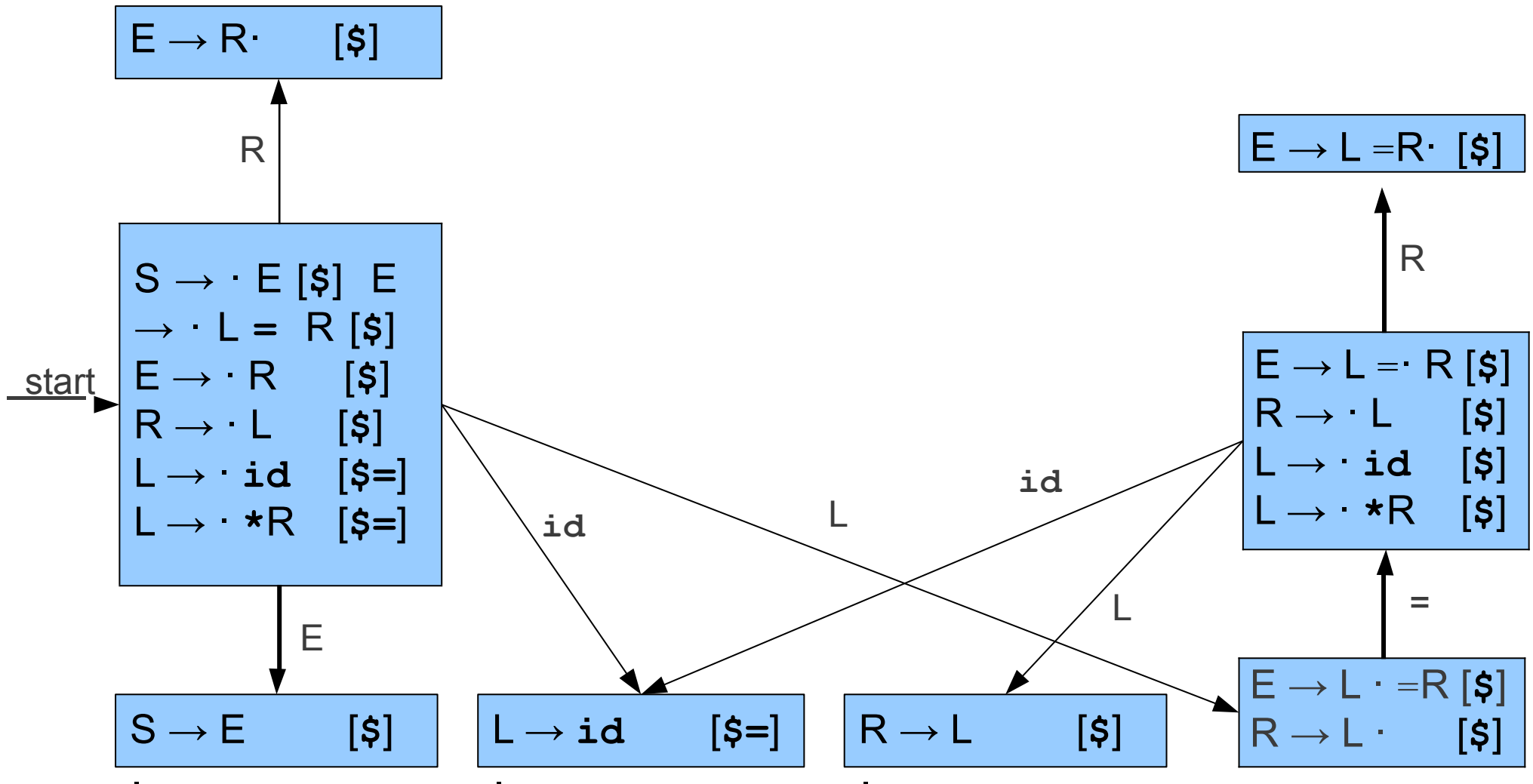
$S \rightarrow E$   
 $E \rightarrow L = R$   
 $E \rightarrow R$   
 $L \rightarrow id$   
 $L \rightarrow *R$   
 $R \rightarrow L$

# LALR(1) Construction



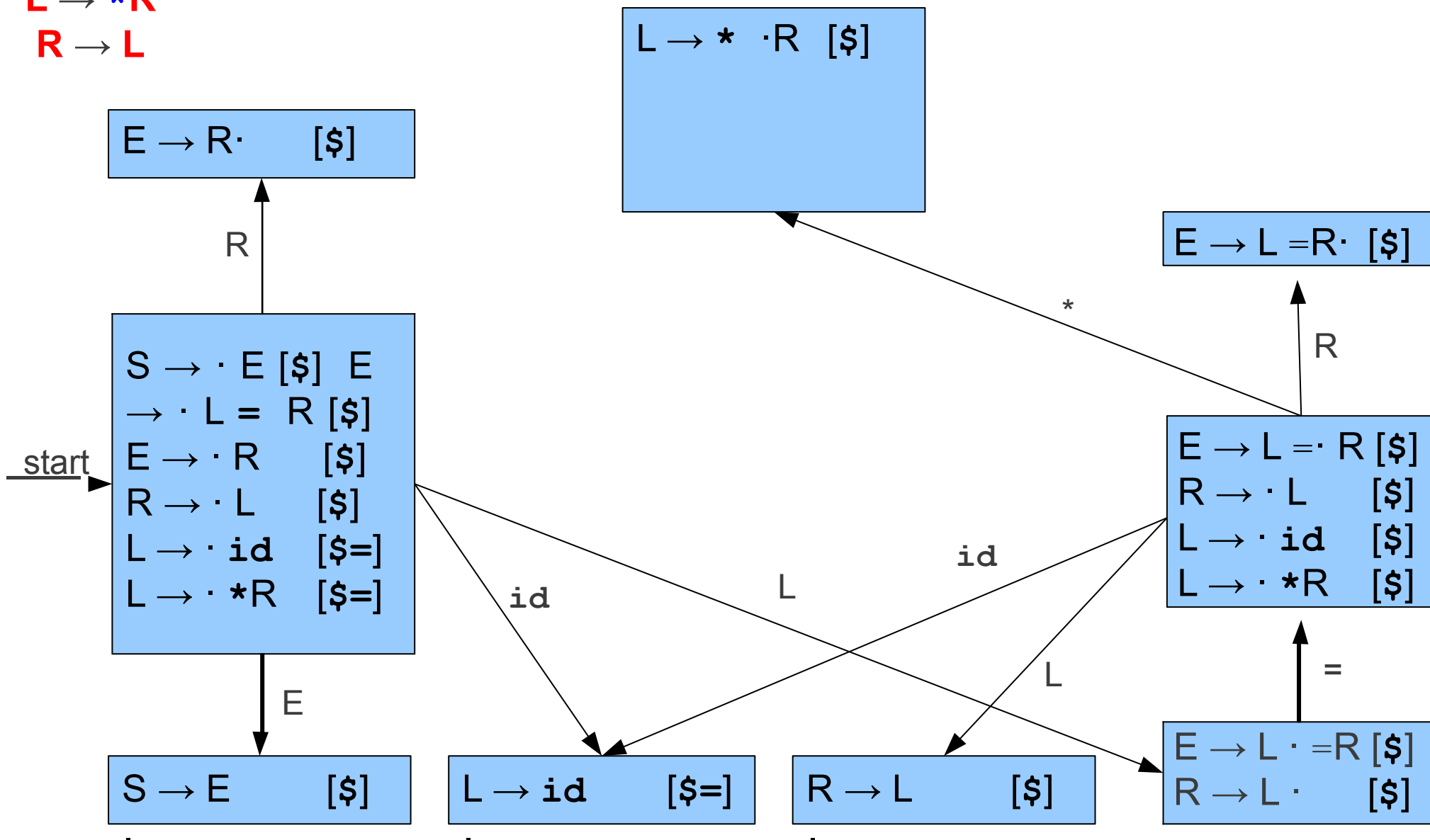
$S \rightarrow E$   
 $E \rightarrow L = R$   
 $E \rightarrow R$   
 $L \rightarrow id$   
 $L \rightarrow *R$   
 $R \rightarrow L$

# LALR(1) Construction



$S \rightarrow E$   
 $E \rightarrow L = R$   
 $E \rightarrow R$   
 $L \rightarrow id$   
 $L \rightarrow *R$   
 $R \rightarrow L$

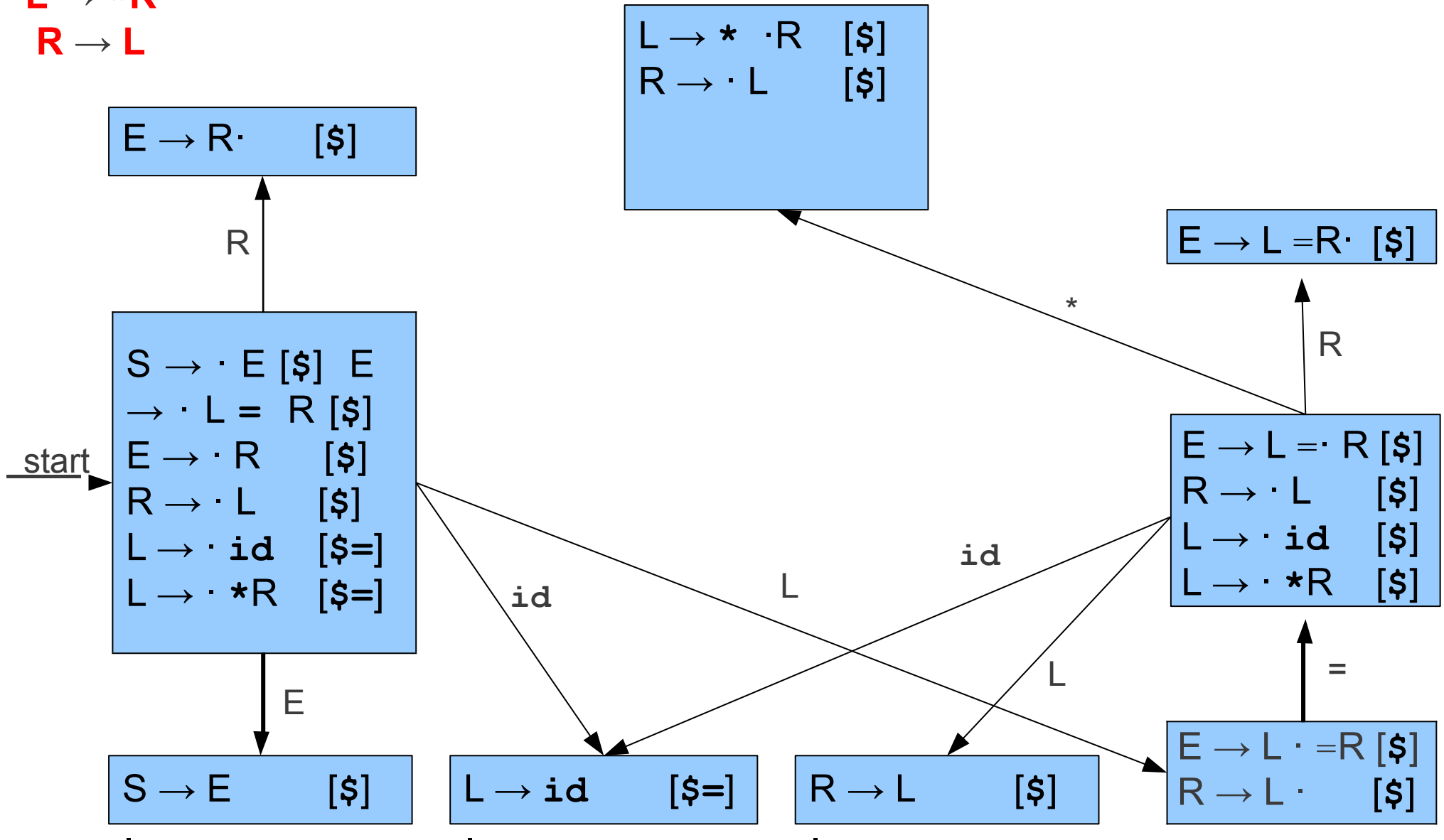
# LALR(1) Construction





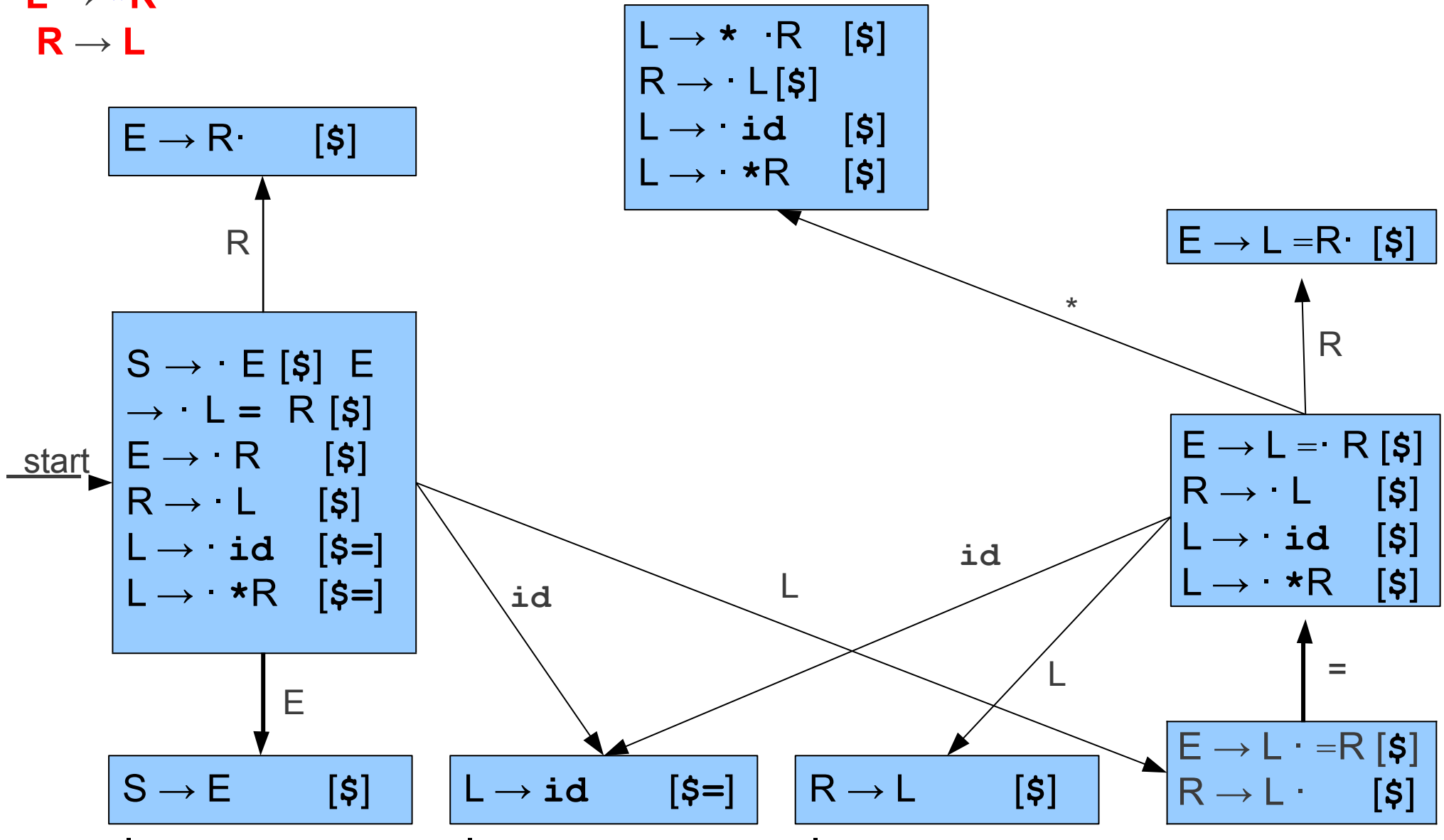
$S \rightarrow E$   
 $E \rightarrow L = R$   
 $E \rightarrow R$   
 $L \rightarrow id$   
 $L \rightarrow *R$   
 $R \rightarrow L$

# LALR(1) Construction



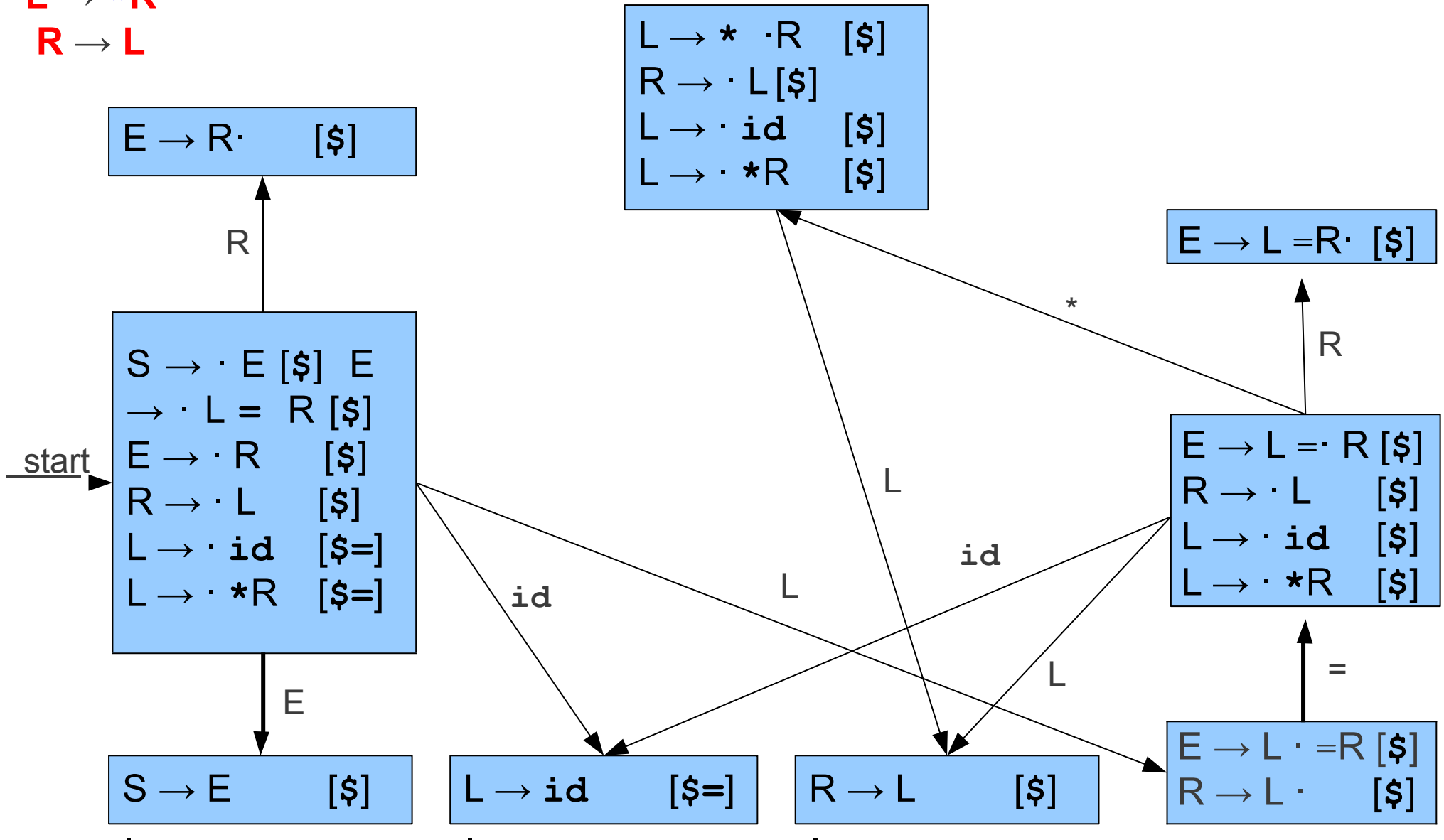
$S \rightarrow E$   
 $E \rightarrow L = R$   
 $E \rightarrow R$   
 $L \rightarrow id$   
 $L \rightarrow *R$   
 $R \rightarrow L$

# LALR(1) Construction



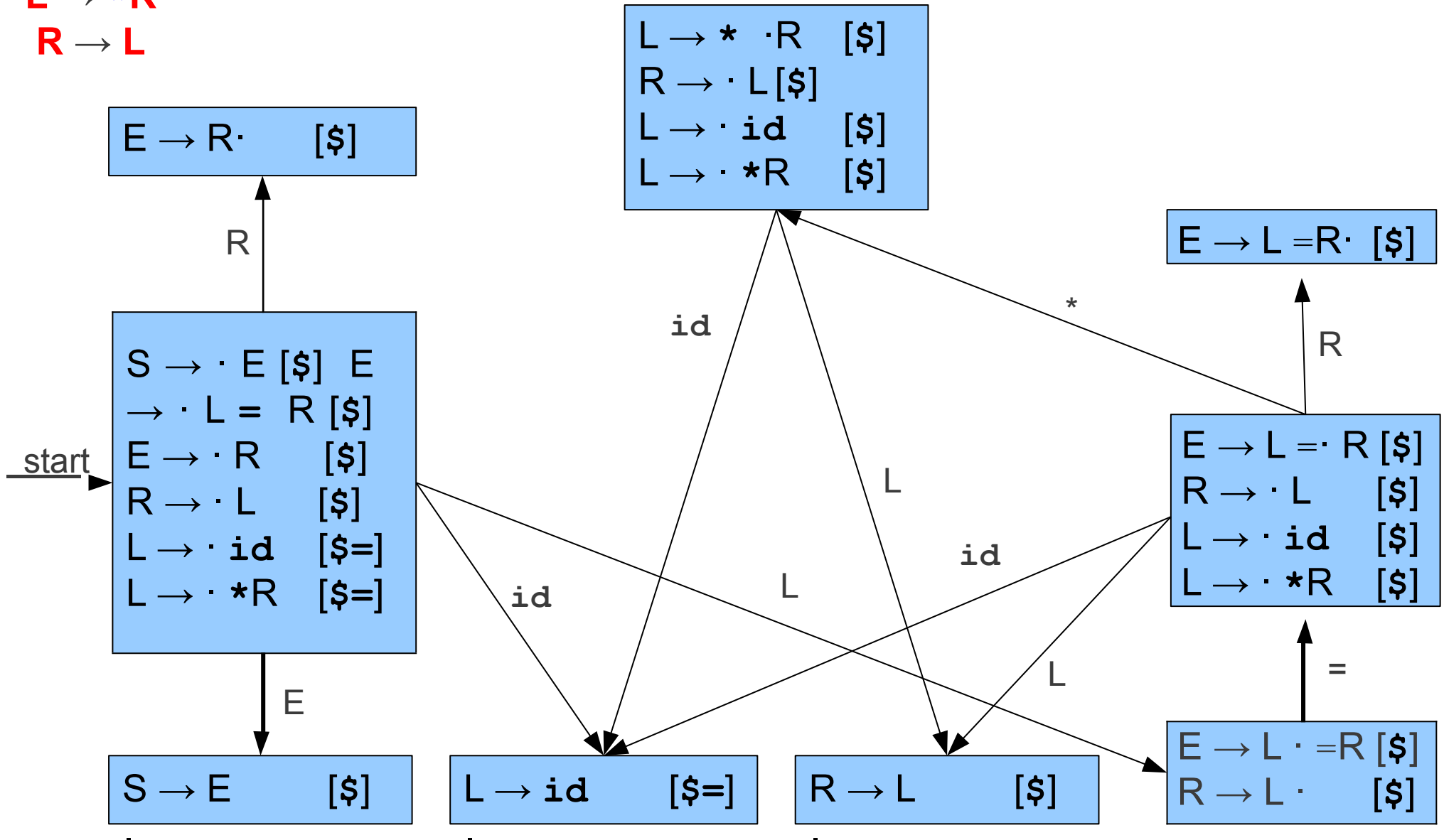
$S \rightarrow E$   
 $E \rightarrow L = R$   
 $E \rightarrow R$   
 $L \rightarrow id$   
 $L \rightarrow *R$   
 $R \rightarrow L$

# LALR(1) Construction



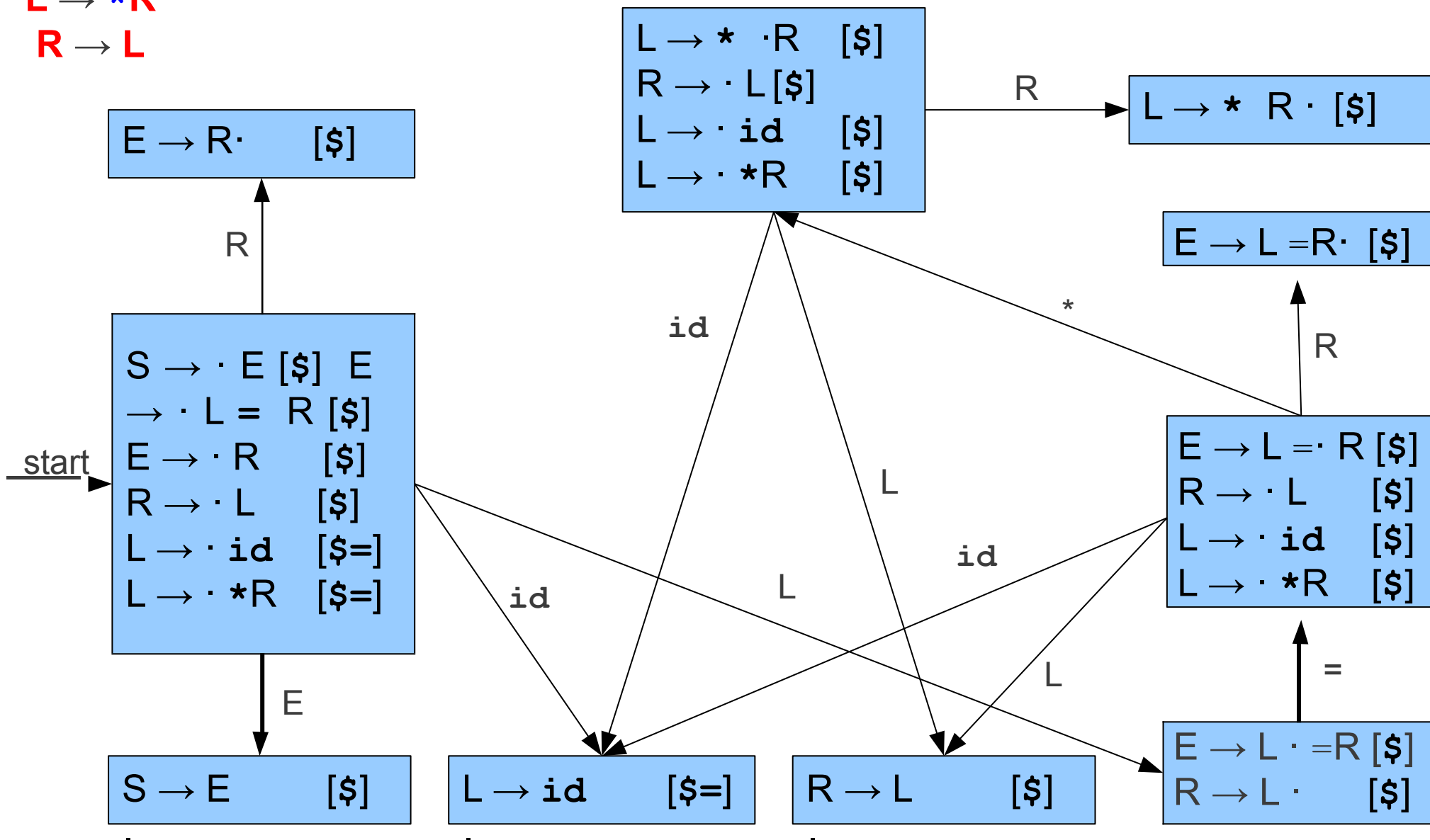
$S \rightarrow E$   
 $E \rightarrow L = R$   
 $E \rightarrow R$   
 $L \rightarrow id$   
 $L \rightarrow *R$   
 $R \rightarrow L$

# LALR(1) Construction



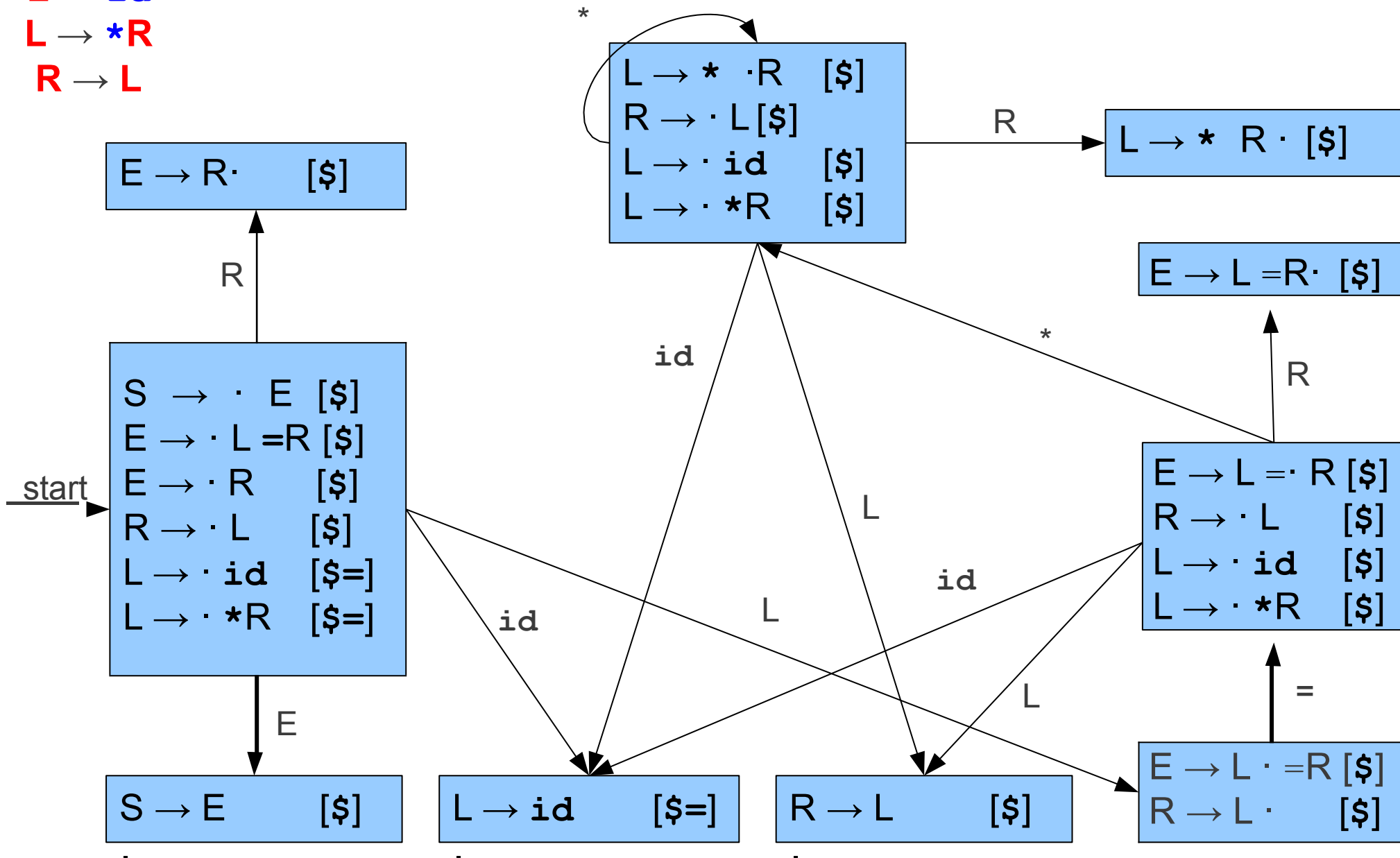
$S \rightarrow E$   
 $E \rightarrow L = R$   
 $E \rightarrow R$   
 $L \rightarrow id$   
 $L \rightarrow *R$   
 $R \rightarrow L$

# LALR(1) Construction



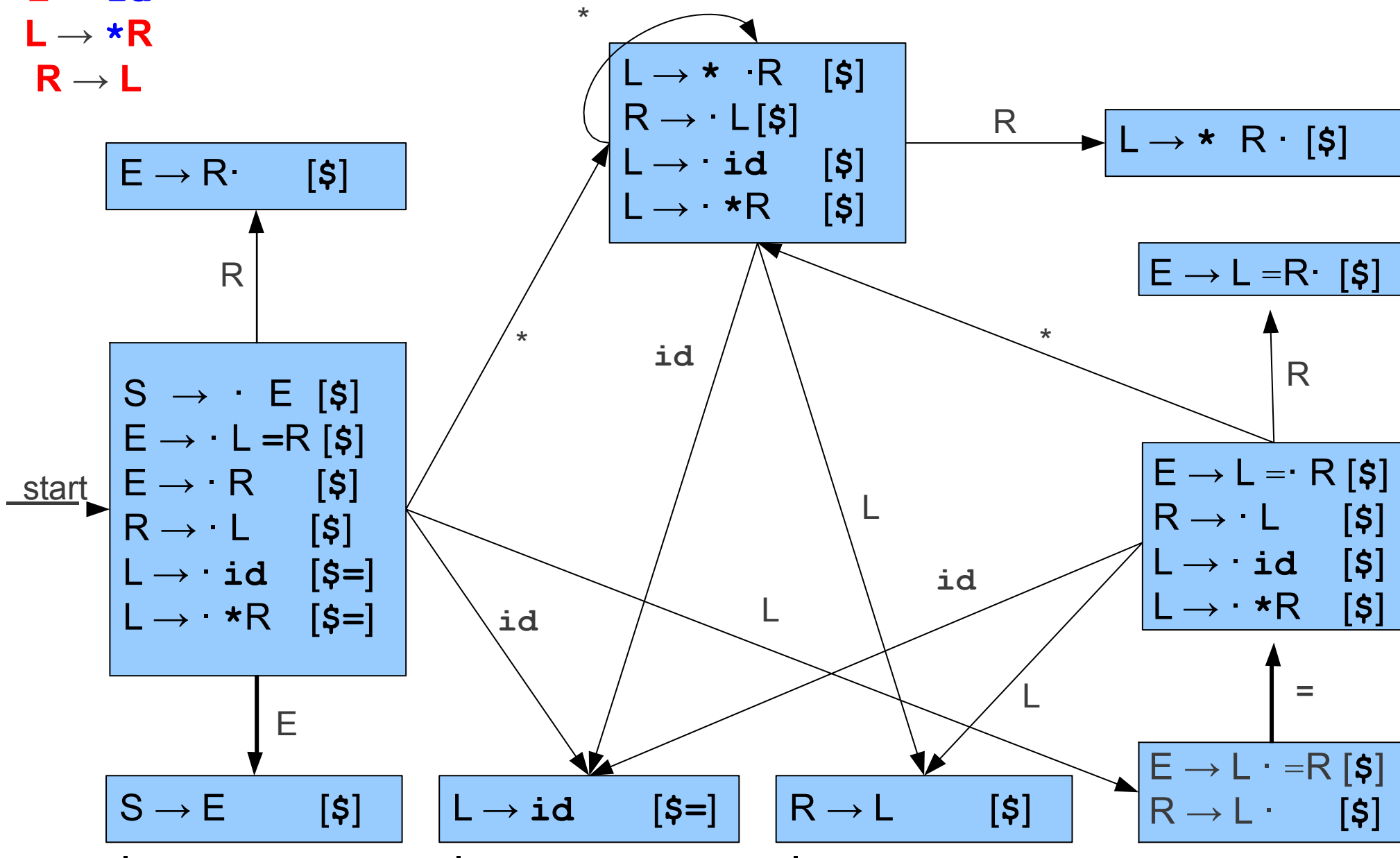
$S \rightarrow E$   
 $E \rightarrow L = R$   
 $E \rightarrow R$   
 $L \rightarrow id$   
 $L \rightarrow *R$   
 $R \rightarrow L$

# LALR(1) Construction



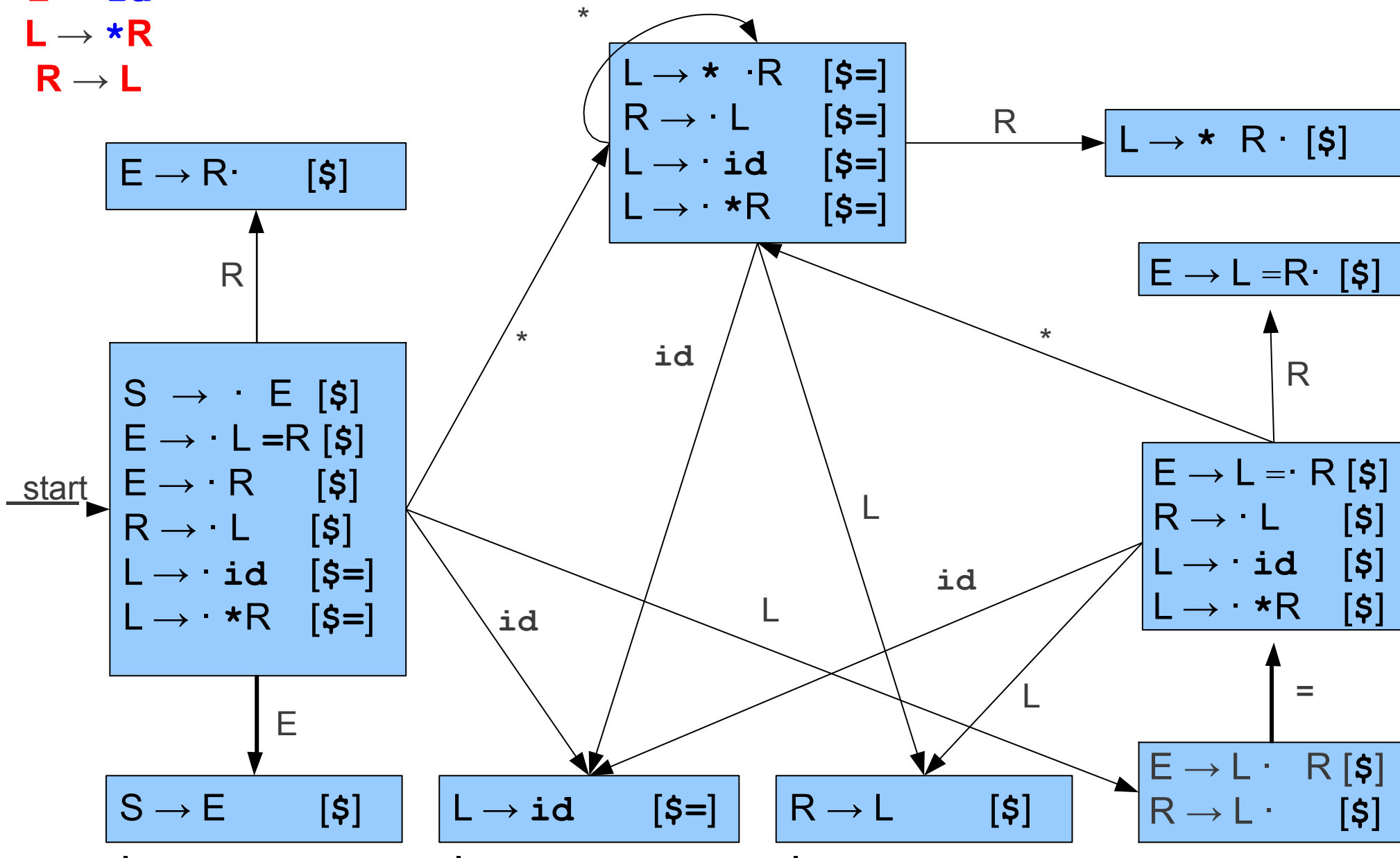
$S \rightarrow E$   
 $E \rightarrow L = R$   
 $E \rightarrow R$   
 $L \rightarrow id$   
 $L \rightarrow *R$   
 $R \rightarrow L$

# LALR(1) Construction



$S \rightarrow E$   
 $E \rightarrow L = R$   
 $E \rightarrow R$   
 $L \rightarrow id$   
 $L \rightarrow *R$   
 $R \rightarrow L$

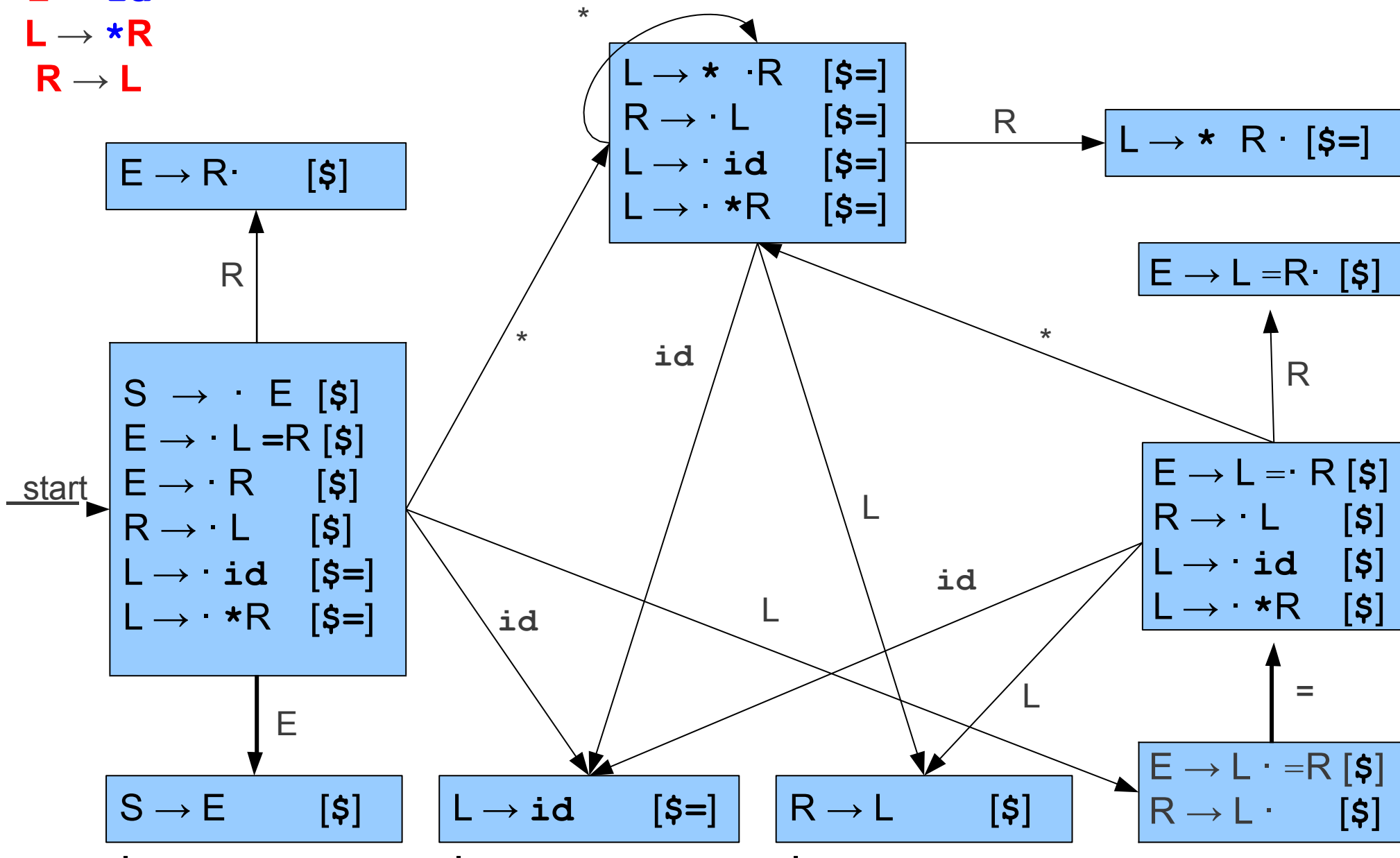
# LALR(1) Construction





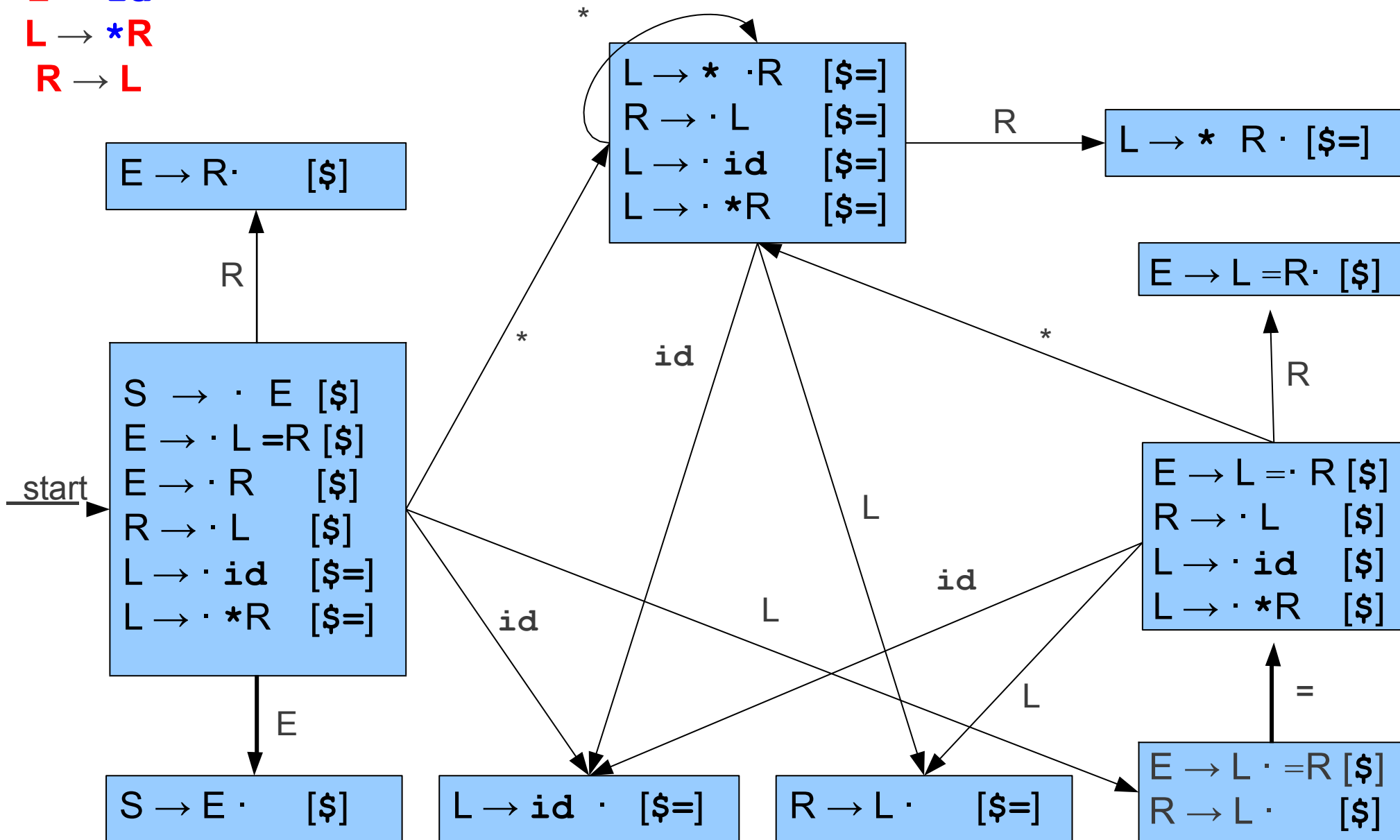
$S \rightarrow E$   
 $E \rightarrow L = R$   
 $E \rightarrow R$   
 $L \rightarrow id$   
 $L \rightarrow *R$   
 $R \rightarrow L$

# LALR(1) Construction



$S \rightarrow E$   
 $E \rightarrow L = R$   
 $E \rightarrow R$   
 $L \rightarrow id$   
 $L \rightarrow *R$   
 $R \rightarrow L$

# LALR(1) Construction



# Analysis of our Algorithm

- Since we merge as we go, size of the partial automaton never exceeds size of overall automaton.
- However, this algorithm could be very slow in practice.
  - We might still have to generate all the LR(1) states, even if they immediately get merged.
- This can be very slow.

# SLR uses FOLLOW sets

- Recall: FOLLOW(**A**) is the set of terminals that can follow **A** in a derivation:

$$\text{FOLLOW}(\mathbf{A}) = \{ \mathbf{t} \mid \mathbf{S} \Rightarrow^* \alpha \mathbf{A} \mathbf{t} \omega \}$$

- SLR is LR(0), with reductions augmented using FOLLOW sets.
- This is too weak for two reasons:
  - It ignores context (what state we're in). It
  - ignores which reduction we're doing.

# LALR uses LA sets

- Given an LR(0) state  $q$  and a production  $A \rightarrow \gamma$ , the **lookahead set**  $LA(q, A \rightarrow \gamma)$  is defined as

$$LA(q, A \rightarrow \gamma) = \{ t \mid S \Rightarrow^* \alpha A t \omega \text{ and } \alpha \gamma \text{ reaches } q \}$$

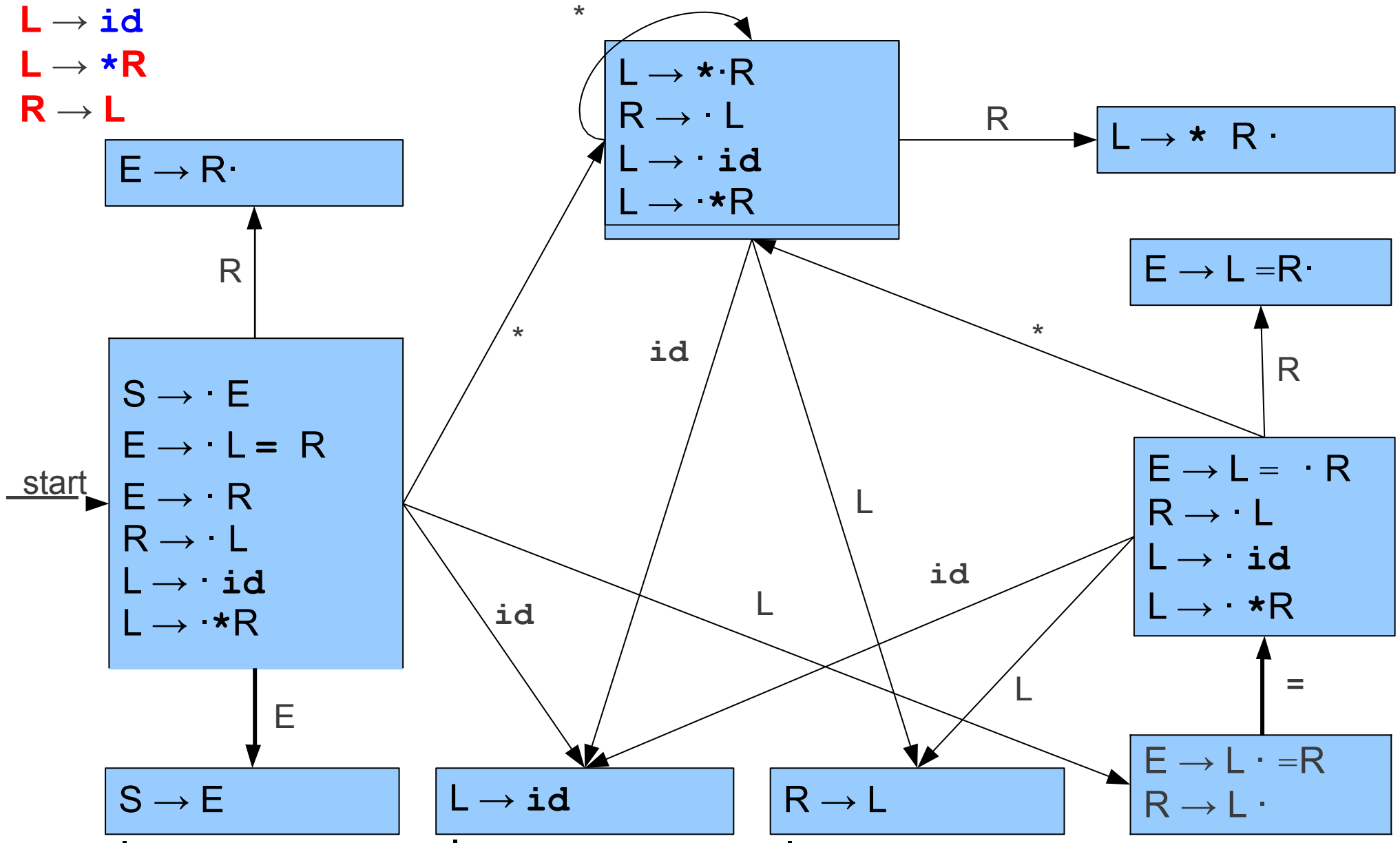
- Here, “ $\alpha \gamma$  reaches  $q$ ” means that the LR(0) automaton, when run on  $\alpha \gamma$ , reaches state  $q$ .
- Intuitively, if we're in some state  $q$  and are going to reduce  $A$  to  $\gamma$ ,  $LA(q, A \rightarrow \gamma)$  is the set of terminals that could actually follow  $A$  at this point, given that we're reducing  $A \rightarrow \gamma$ .
- Much more precise than FOLLOW sets.

# LA and FOLLOW

- The **lookahead set**  $LA(q, A \rightarrow \gamma)$  is defined as
$$LA(q, A \rightarrow \gamma) = \{ t \mid S \Rightarrow^* \alpha A t \omega \text{ and } \alpha \gamma \text{ reaches } q \}$$
- The **follow set**  $FOLLOW(A)$  is defined as
$$FOLLOW(A) = \{ t \mid S \Rightarrow^* \alpha A t \omega \}$$
- Note that  $LA(q, A \rightarrow \gamma) \subseteq FOLLOW(A)$ ; that is, LA sets are “more precise” than FOLLOW sets.
- If we can compute LA from FOLLOW, we can construct a LALR(1) parser efficiently.

# An LR(0) Automaton

$S \rightarrow E$   
 $E \rightarrow L = R$   
 $E \rightarrow R$   
 $L \rightarrow id$   
 $L \rightarrow *R$   
 $R \rightarrow L$



# An LR(0) Automaton

$S \rightarrow E$

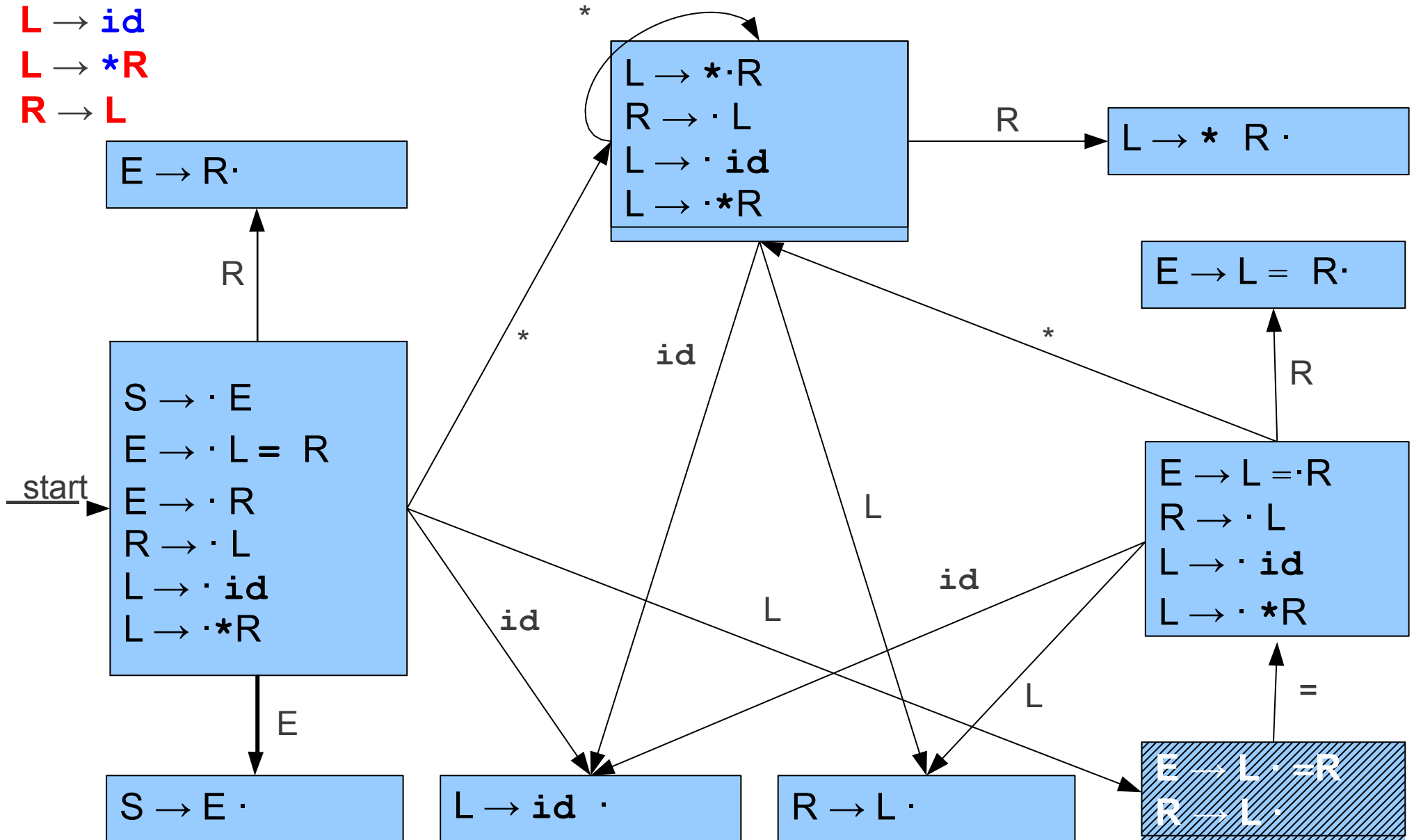
$E \rightarrow L = R$

$E \rightarrow R$

$L \rightarrow id$

$L \rightarrow *R$

$R \rightarrow L$

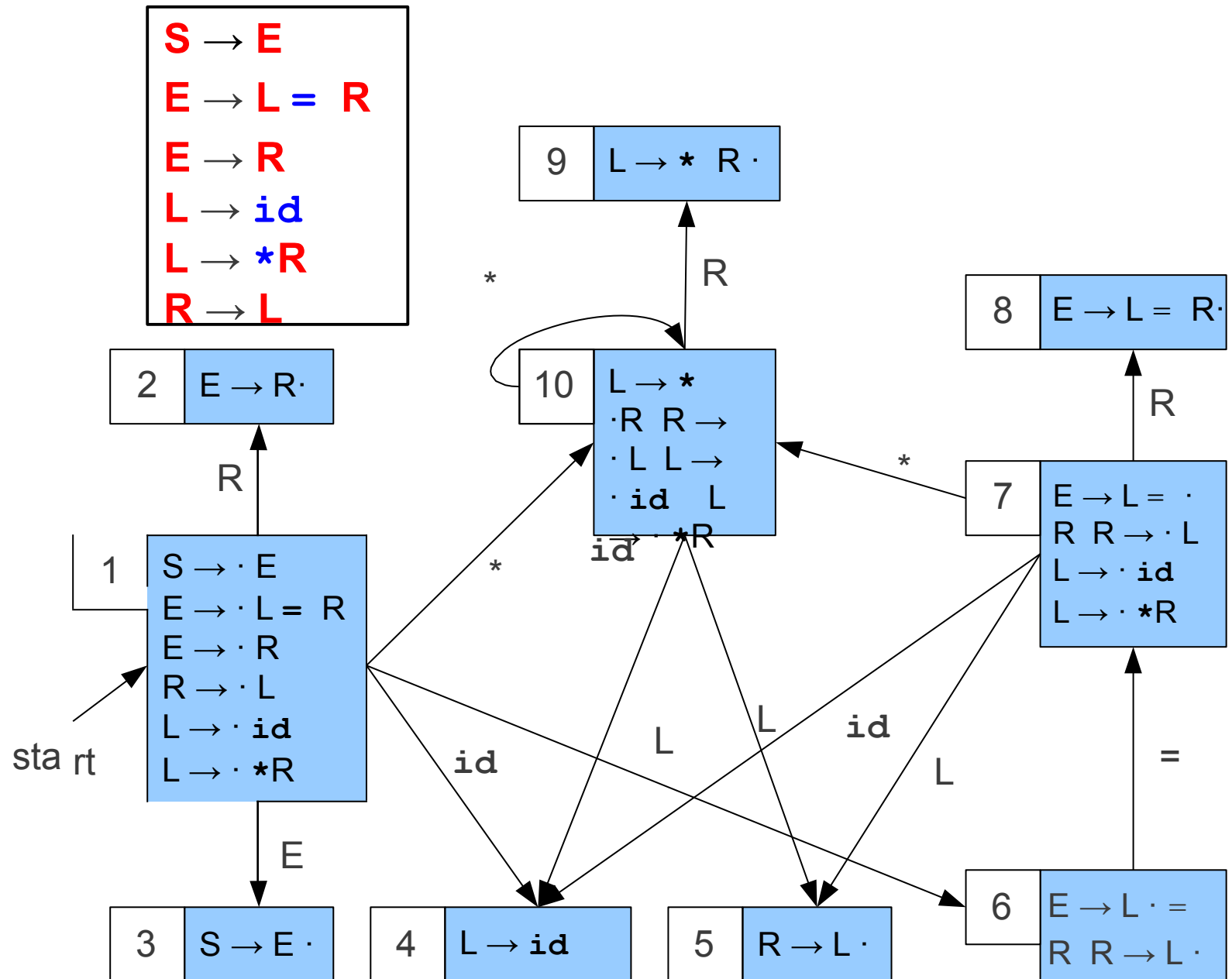




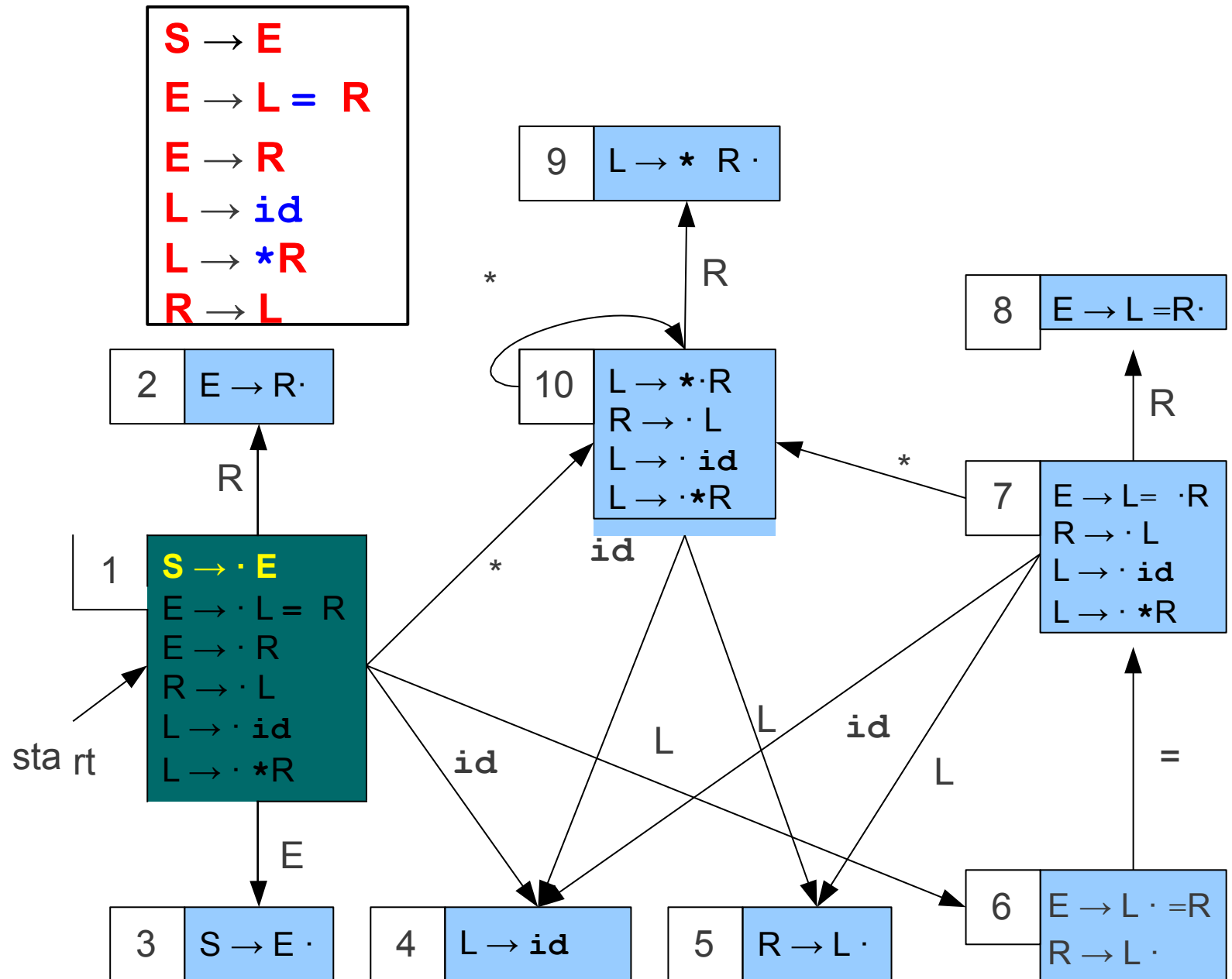
What if we used the LR(0) automaton to  
add context to the grammar?

Prepare for one of the most beautiful  
constructions of the quarter...

# Augmenting the Grammar

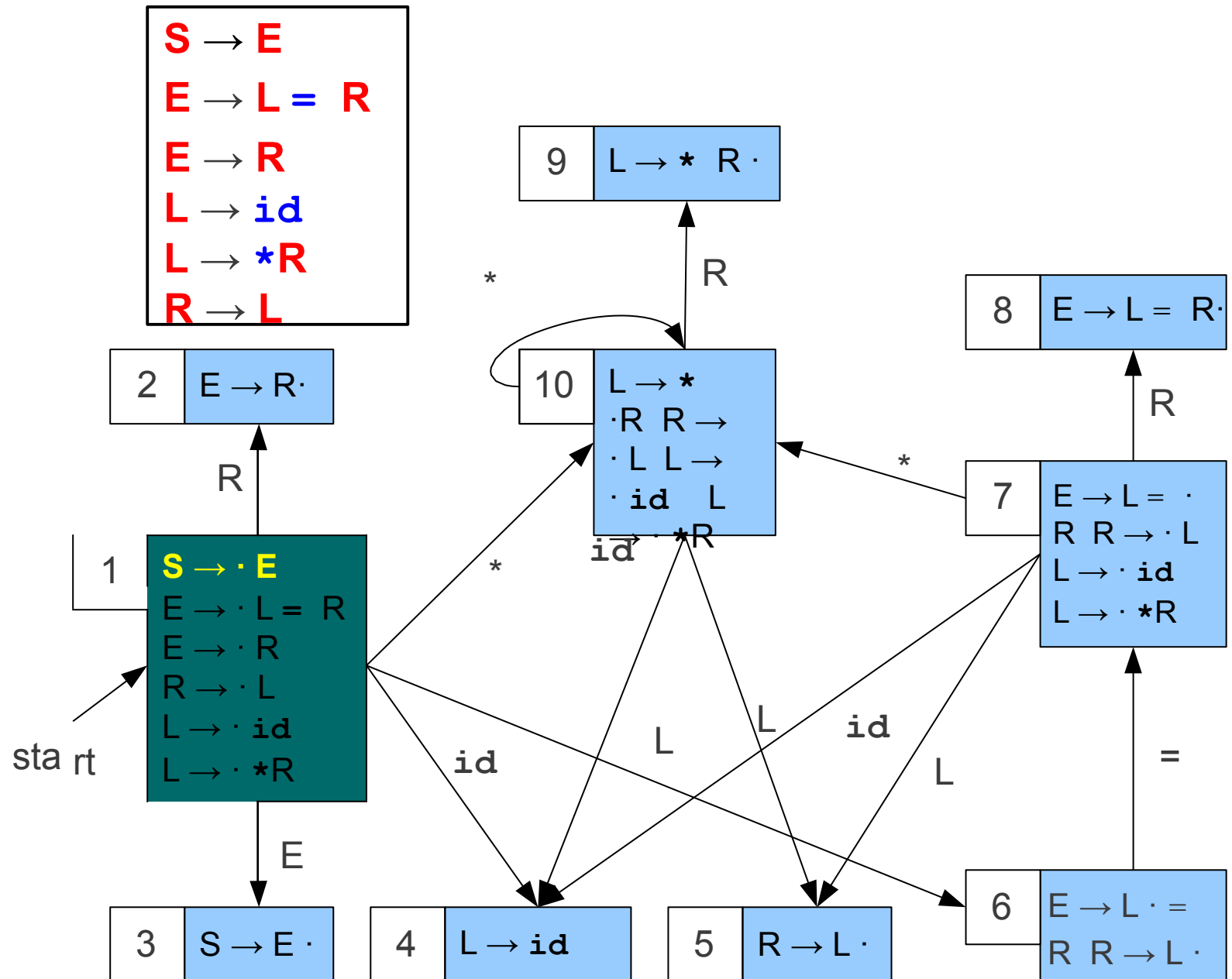


# Augmenting the Grammar



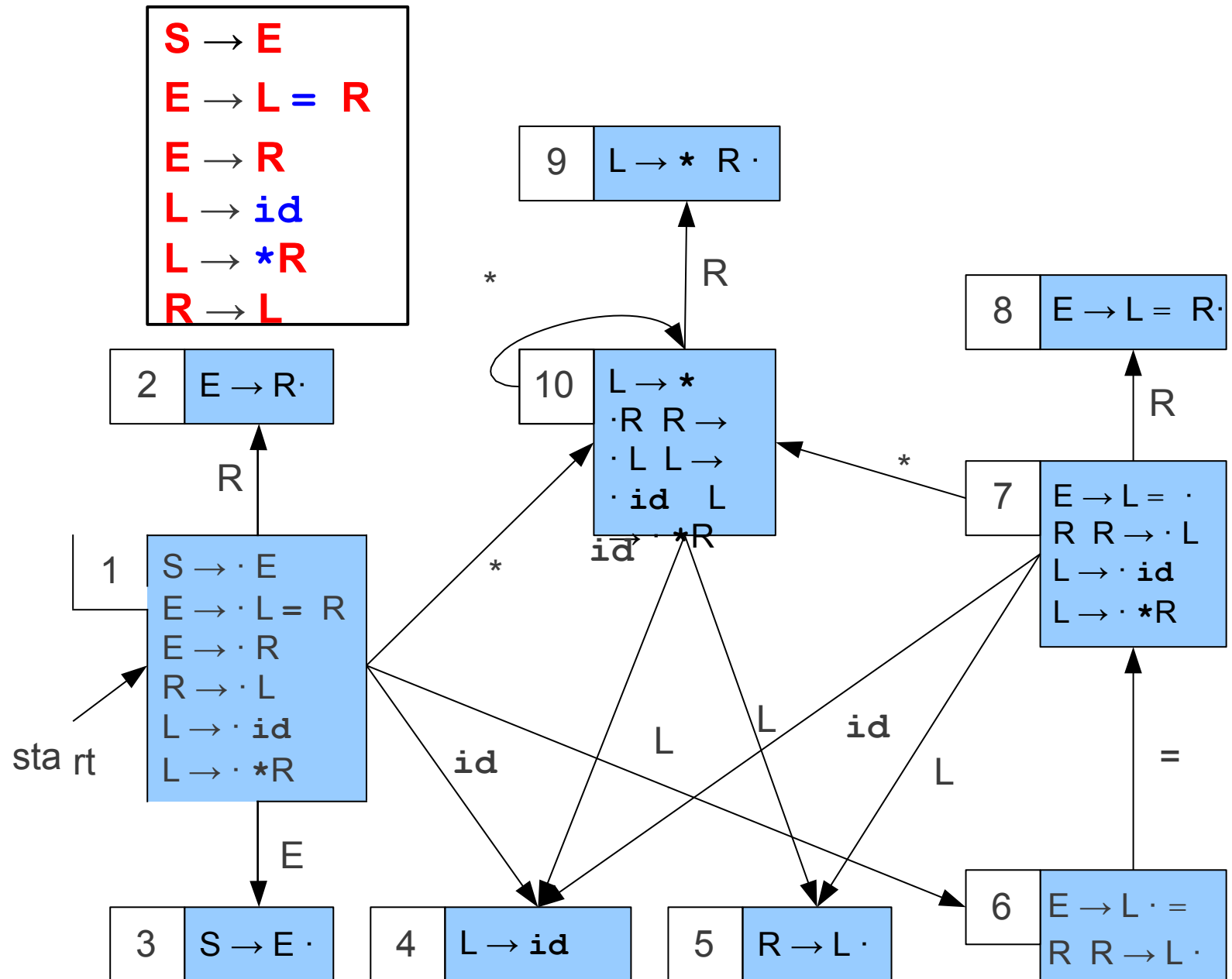
# Augmenting the Grammar

$$S_1 \rightarrow E_{1-3}$$



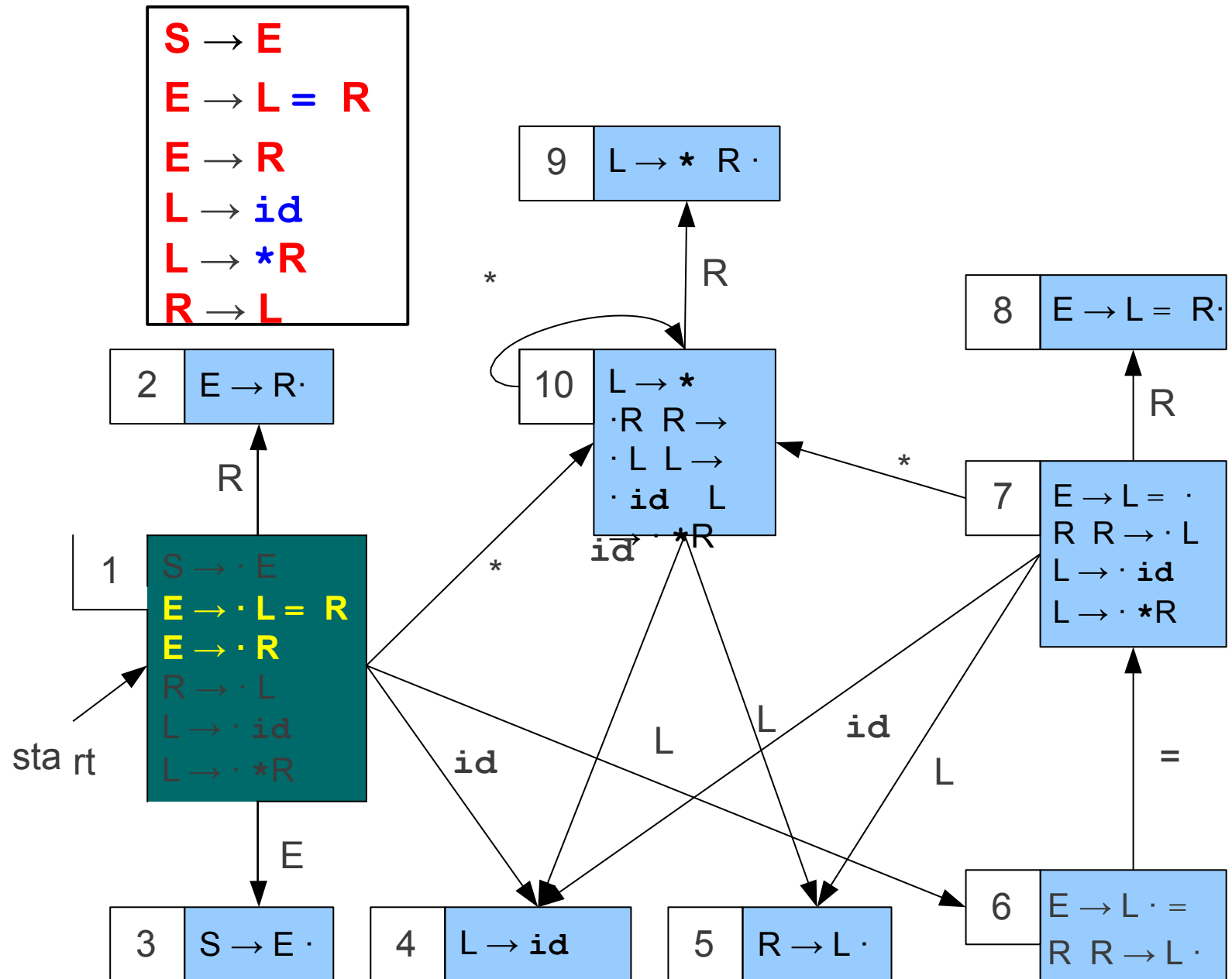
# Augmenting the Grammar

$$S_1 \rightarrow E_{1-3}$$



# Augmenting the Grammar

$$S_1 \rightarrow E_{1-3}$$



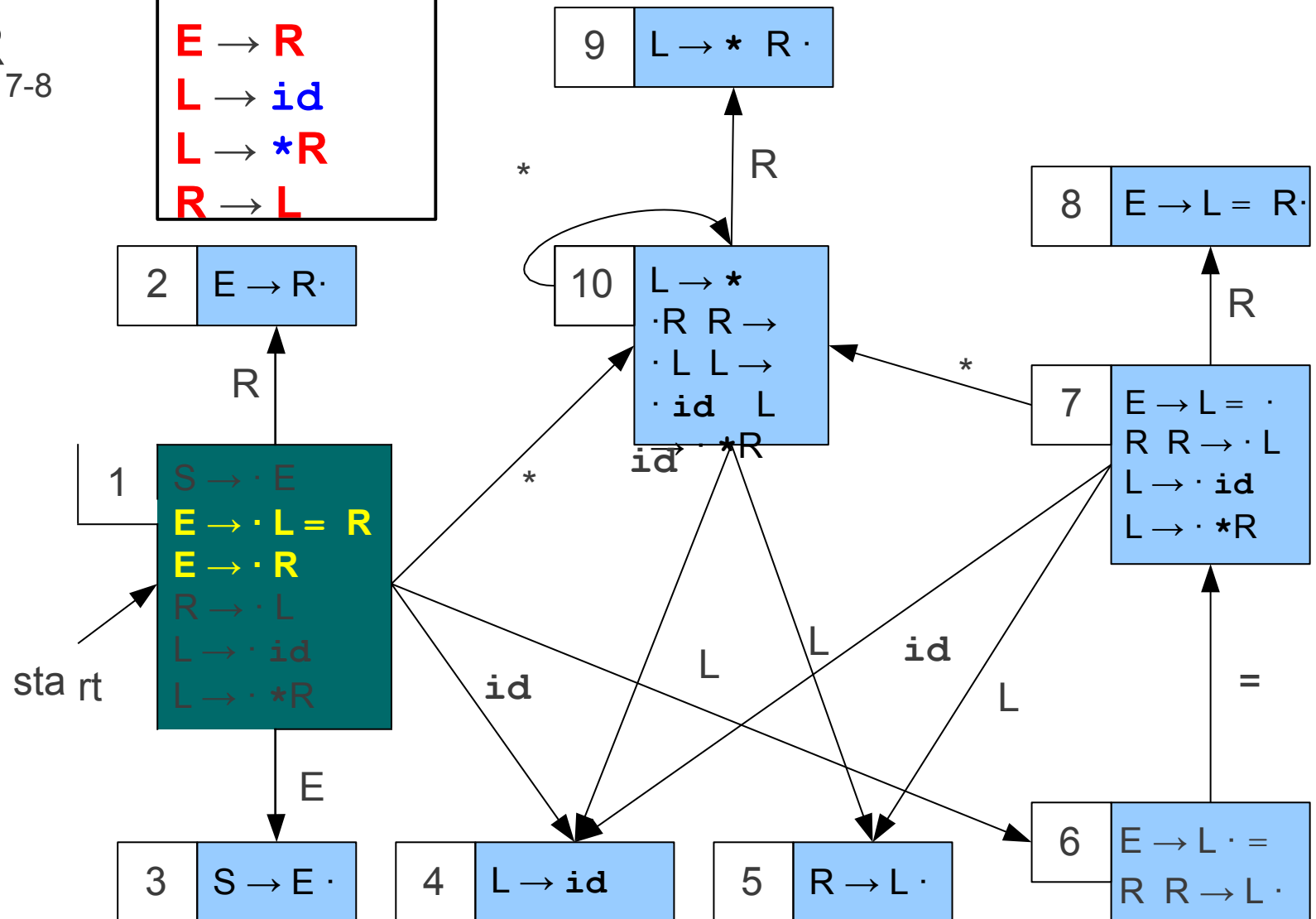
# Augmenting the Grammar

$$S_1 \rightarrow E_{1-3}$$

$$E_{1-3} \rightarrow L_{1-6} = R_{7-8}$$

$$E_{1-3} \rightarrow R_{1-2}$$

$S \rightarrow E$
$E \rightarrow L = R$
$E \rightarrow R$
$L \rightarrow id$
$L \rightarrow *R$
$R \rightarrow L$





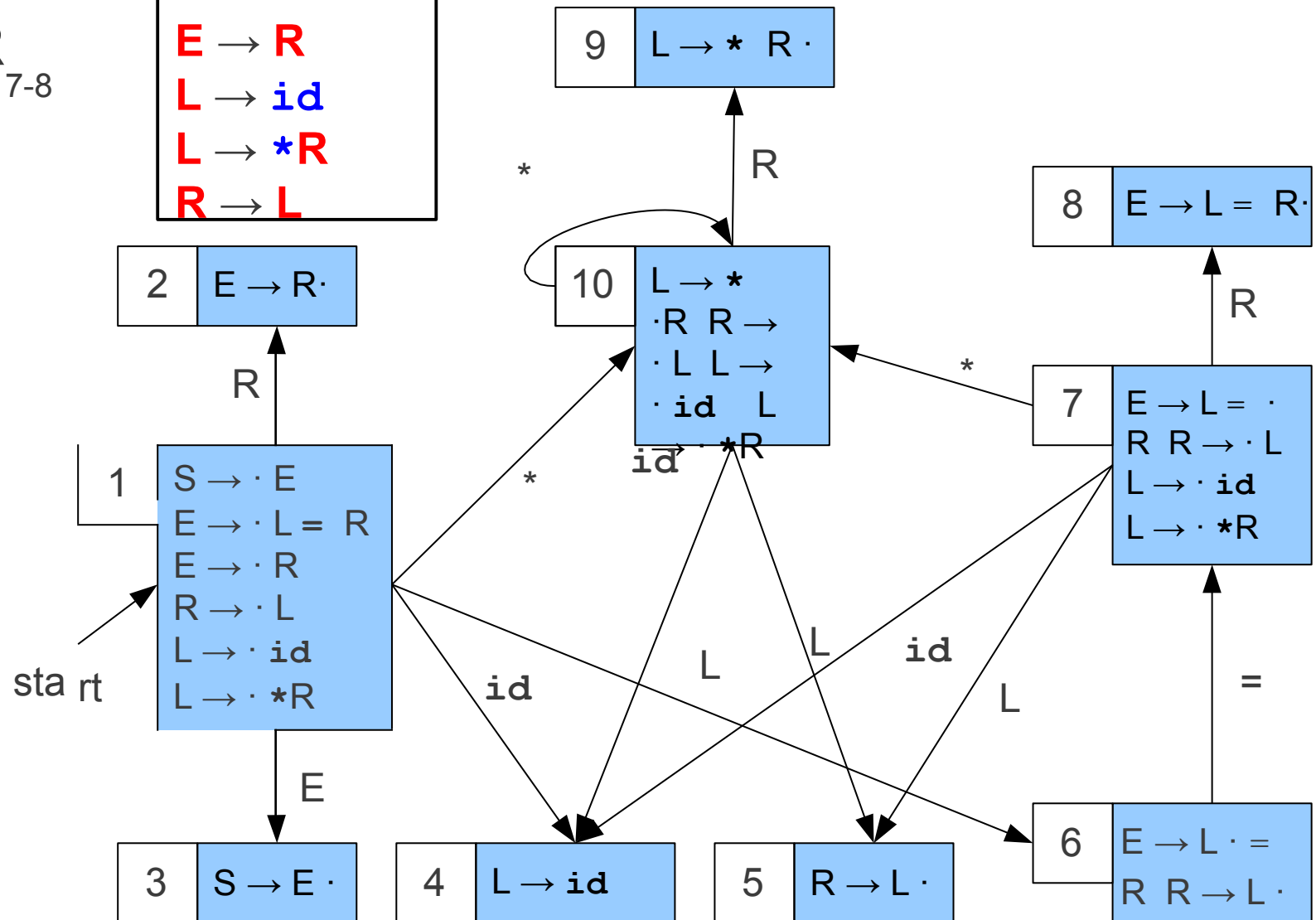
# Augmenting the Grammar

$$S_1 \rightarrow E_{1-3}$$

$$E_{1-3} \rightarrow L_{1-6} = R_{7-8}$$

$$E_{1-3} \rightarrow R_{1-2}$$

$S \rightarrow E$
$E \rightarrow L = R$
$E \rightarrow R$
$L \rightarrow id$
$L \rightarrow *R$
$R \rightarrow L$



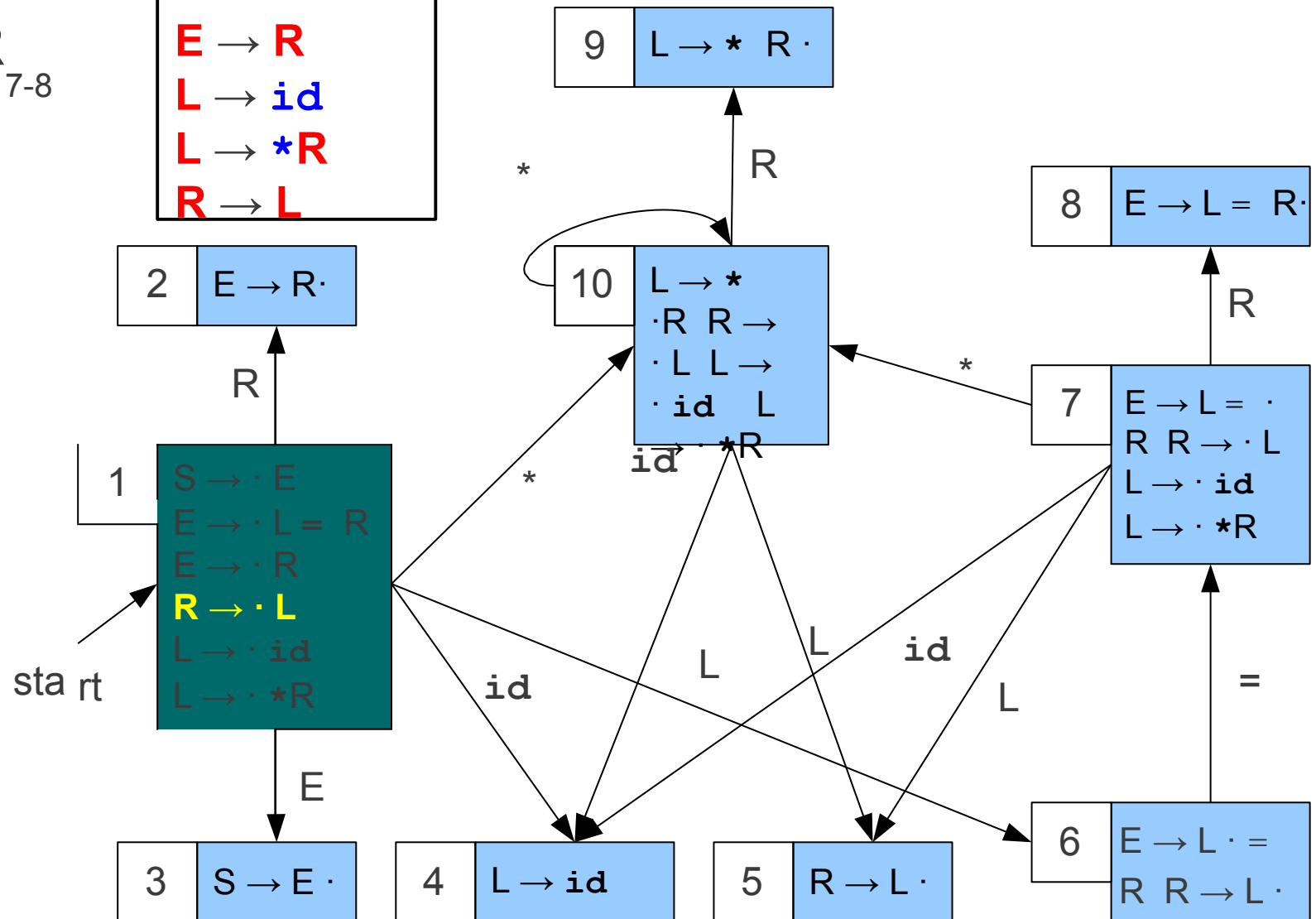
# Augmenting the Grammar

$$S_1 \rightarrow E_{1-3}$$

$$E_{1-3} \rightarrow L_{1-6} = R_{7-8}$$

$$E_{1-3} \rightarrow R_{1-2}$$

<b>S</b>	<b>→</b>	<b>E</b>
<b>E</b>	<b>→</b>	<b>L = R</b>
<b>E</b>	<b>→</b>	<b>R</b>
<b>L</b>	<b>→</b>	<b>id</b>
<b>L</b>	<b>→</b>	<b>*R</b>
<b>R</b>	<b>→</b>	<b>L</b>



# Augmenting the Grammar

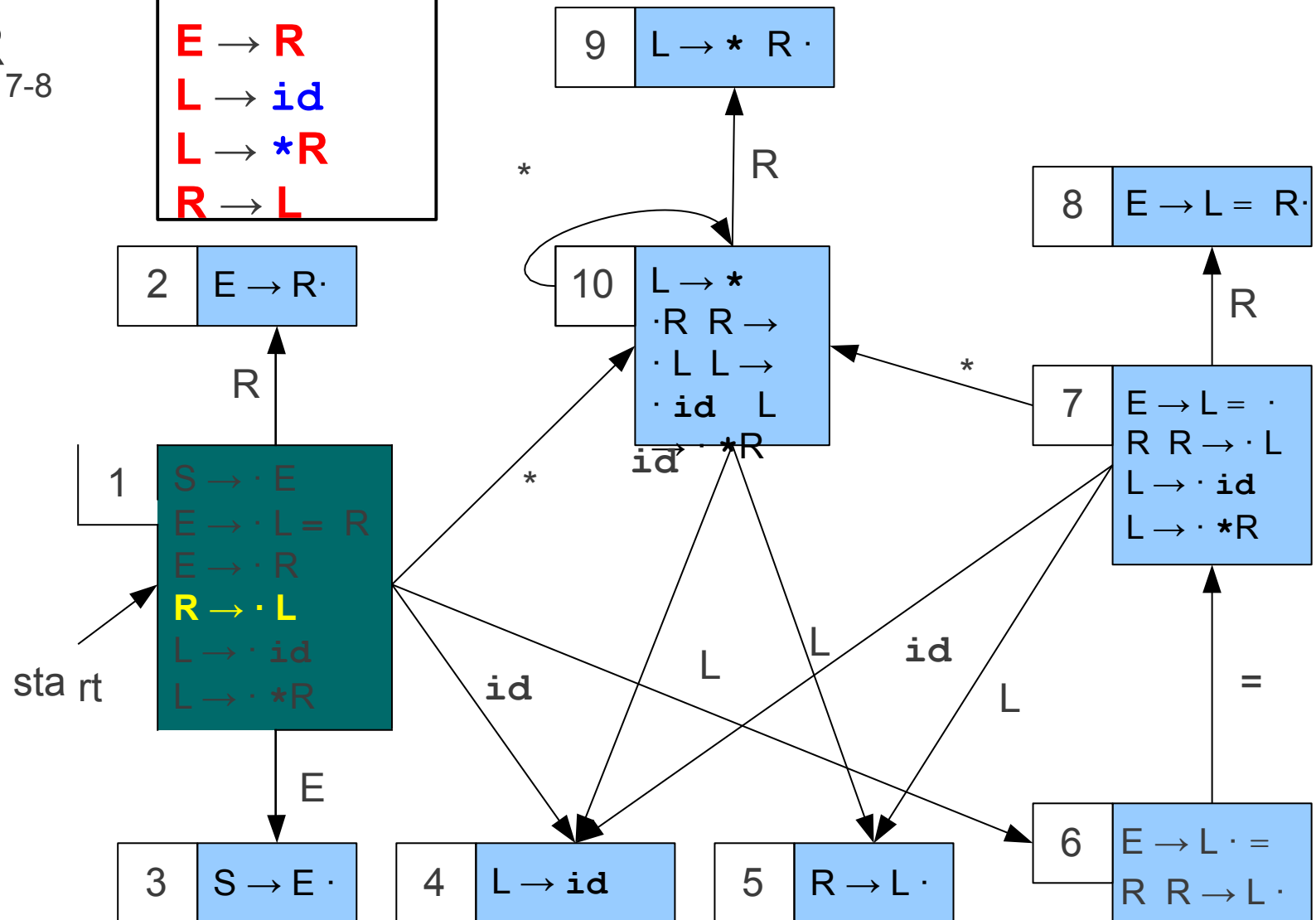
$$S_1 \rightarrow E_{1-3}$$

$$E_{1-3} \rightarrow L_{1-6} = R_{7-8}$$

$$E_{1-3} \rightarrow R_{1-2}$$

$$R_{1-2} \rightarrow L_{1-6}$$

<b>S</b>	<b>→</b>	<b>E</b>
<b>E</b>	<b>→</b>	<b>L = R</b>
<b>E</b>	<b>→</b>	<b>R</b>
<b>L</b>	<b>→</b>	<b>id</b>
<b>L</b>	<b>→</b>	<b>*R</b>
<b>R</b>	<b>→</b>	<b>L</b>



# Augmenting the Grammar

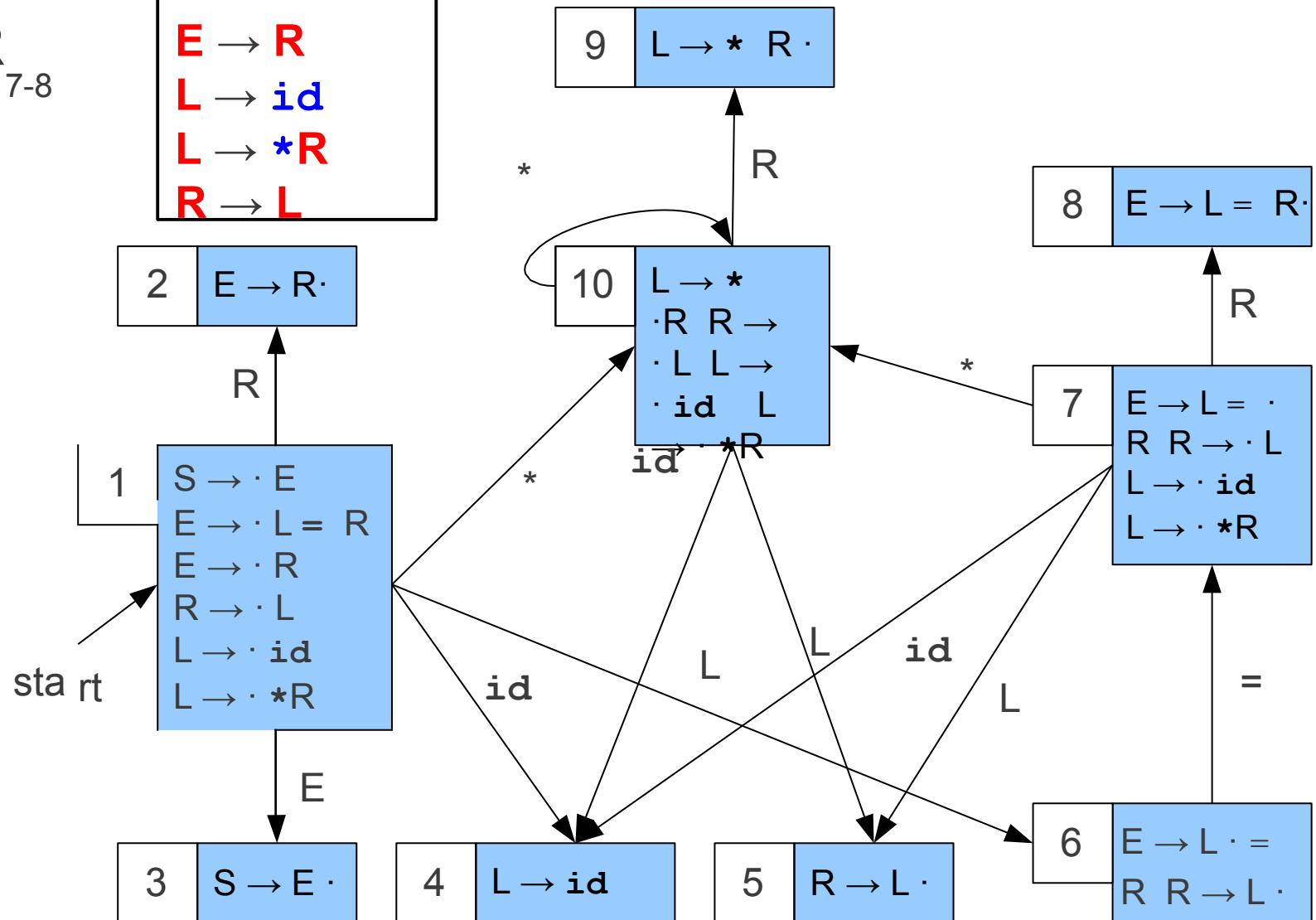
$$S_1 \rightarrow E_{1-3}$$

$$E_{1-3} \rightarrow L_{1-6} = R_{7-8}$$

$$E_{1-3} \rightarrow R_{1-2}$$

$$R_{1-2} \rightarrow L_{1-6}$$

<b>S</b>	<b>→</b>	<b>E</b>
<b>E</b>	<b>→</b>	<b>L = R</b>
<b>E</b>	<b>→</b>	<b>R</b>
<b>L</b>	<b>→</b>	<b>id</b>
<b>L</b>	<b>→</b>	<b>*R</b>
<b>R</b>	<b>→</b>	<b>L</b>



# Augmenting the Grammar

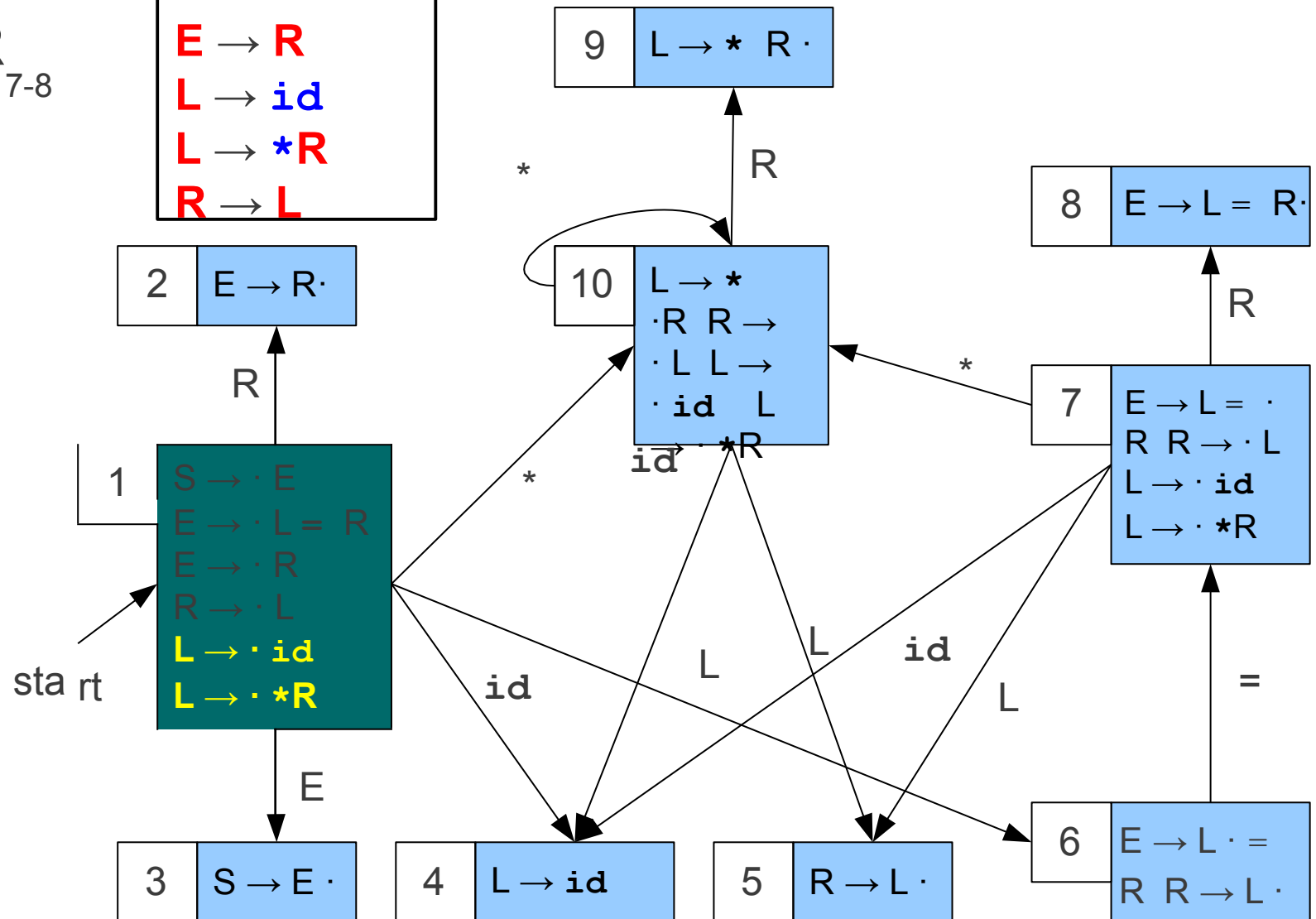
$$S_1 \rightarrow E_{1-3}$$

$$E_{1-3} \rightarrow L_{1-6} = R_{7-8}$$

$$E_{1-3} \rightarrow R_{1-2}$$

$$R_{1-2} \rightarrow L_{1-6}$$

$S \rightarrow E$
$E \rightarrow L = R$
$E \rightarrow R$
$L \rightarrow id$
$L \rightarrow *R$
$R \rightarrow L$



# Augmenting the Grammar

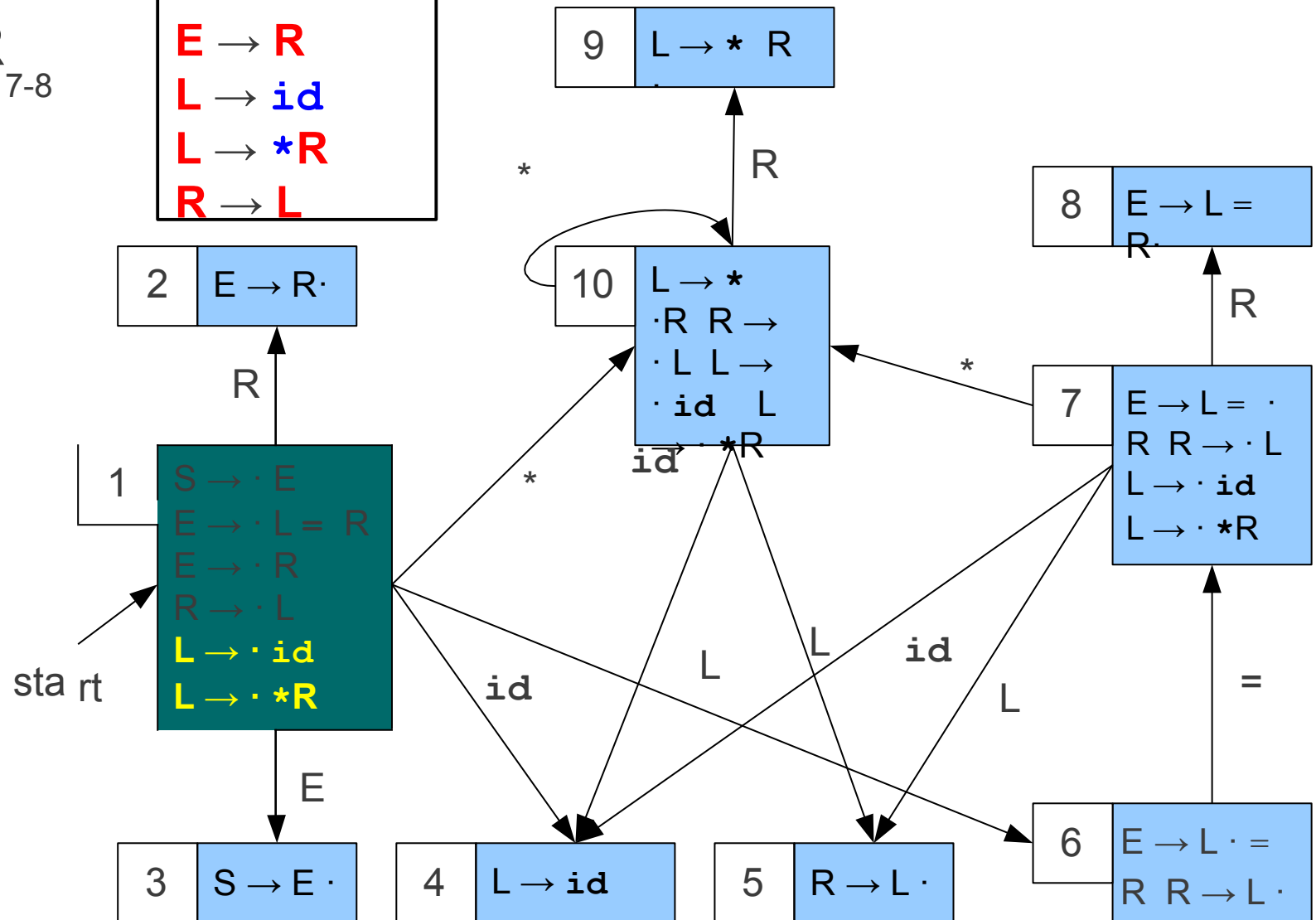
$$S_1 \rightarrow E_{1-3}$$

$$E_{1-3} \rightarrow L_{1-6} = R_{7-8}$$

$$E_{1-3} \rightarrow R_{1-2}$$

$$\begin{array}{l} R_{1-2} \rightarrow L_{1-6} \\ L_{1-6} \rightarrow *R_{10-9} \\ L_{1-6} \rightarrow id \end{array}$$

$S \rightarrow E$
$E \rightarrow L = R$
$E \rightarrow R$
$L \rightarrow id$
$L \rightarrow *R$
$R \rightarrow L$



# Augmenting the Grammar

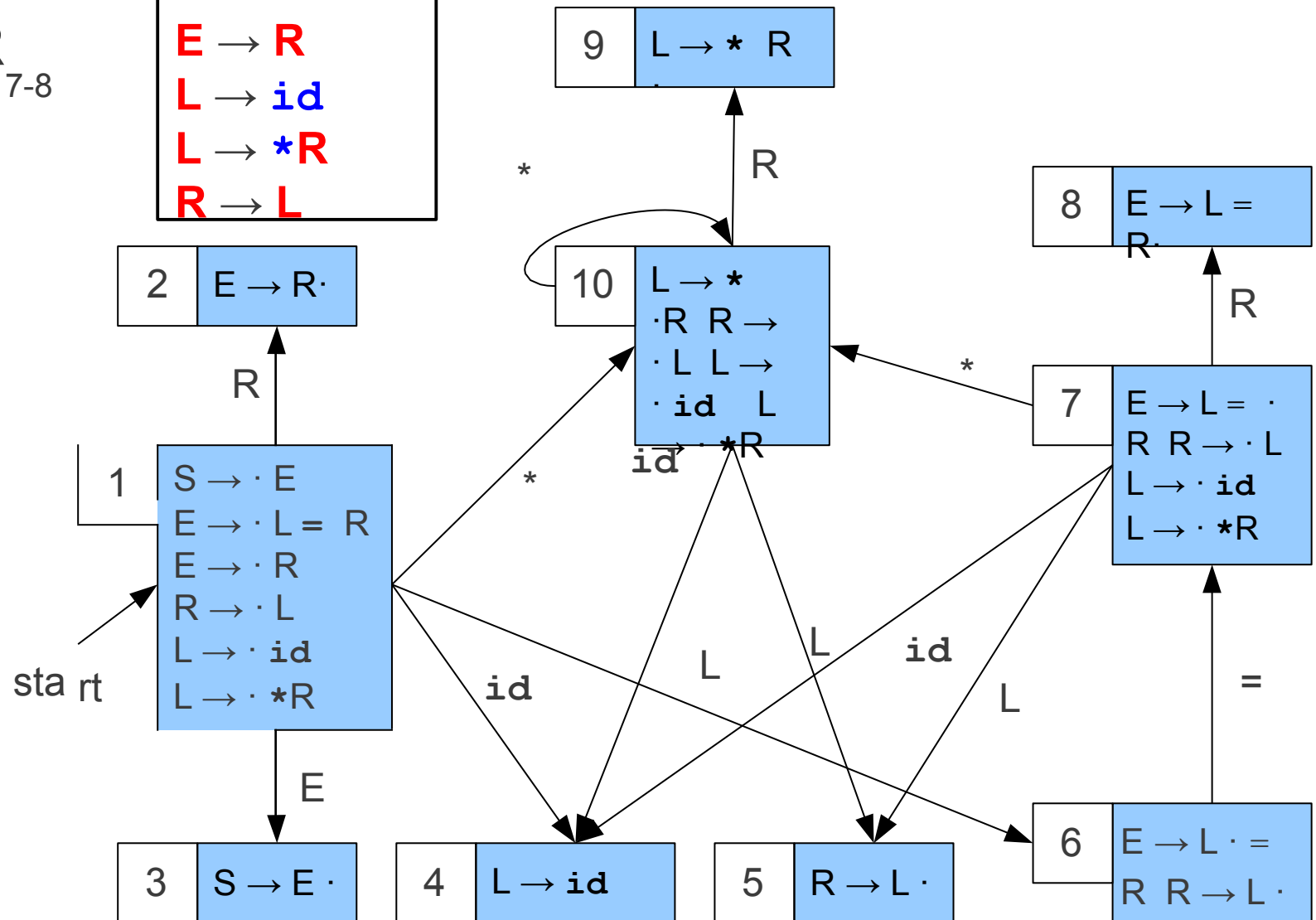
$$S_1 \rightarrow E_{1-3}$$

$$E_{1-3} \rightarrow L_{1-6} = R_{7-8}$$

$$E_{1-3} \rightarrow R_{1-2}$$

$$\begin{array}{l} R_{1-2} \rightarrow L_{1-6} \\ L_{1-6} \rightarrow *R_{10-9} \\ L_{1-6} \rightarrow id \end{array}$$

$S \rightarrow E$
$E \rightarrow L = R$
$E \rightarrow R$
$L \rightarrow id$
$L \rightarrow *R$
$R \rightarrow L$



# Augmenting the Grammar

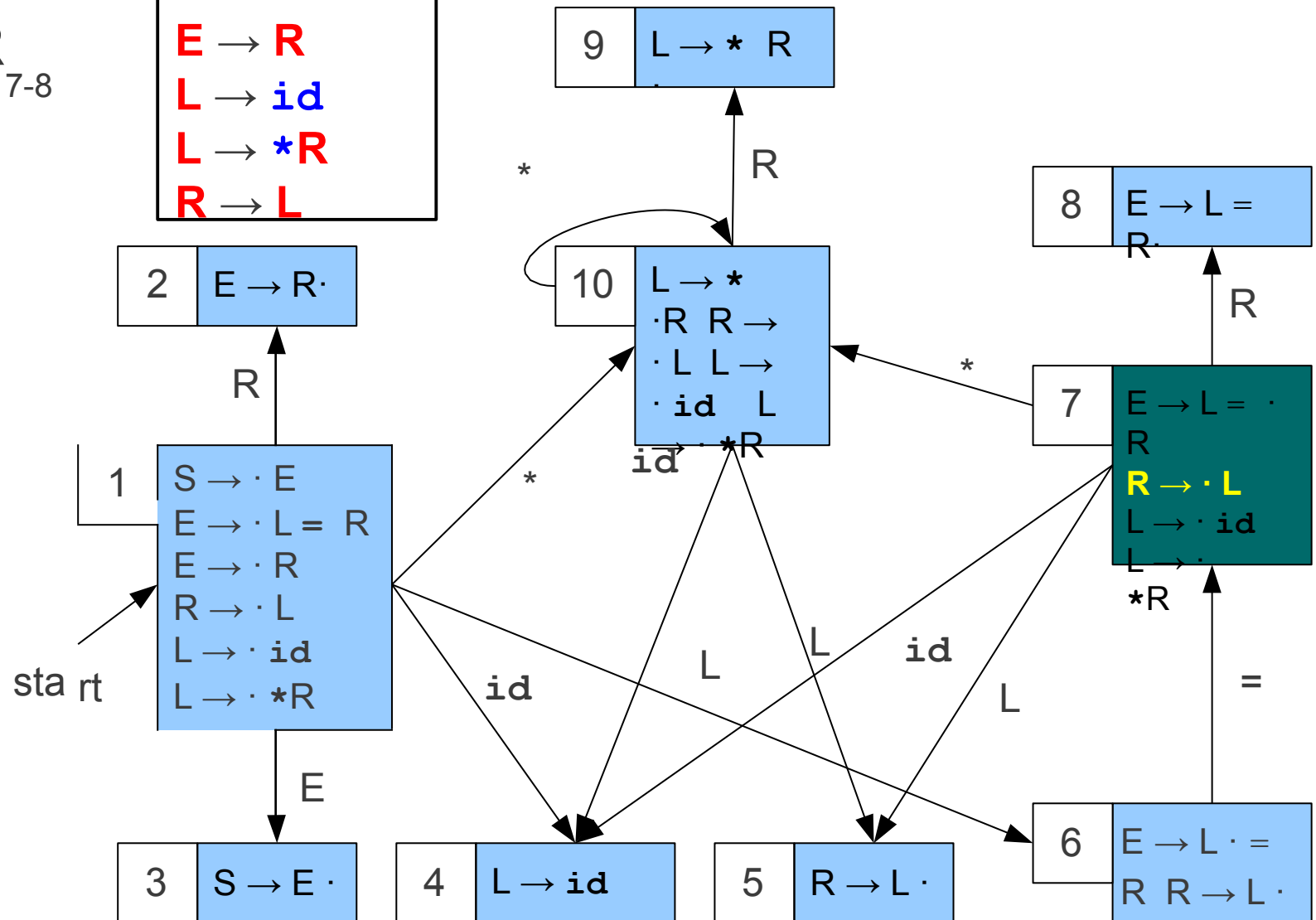
$$S_1 \rightarrow E_{1-3}$$

$$E_{1-3} \rightarrow L_{1-6} = R_{7-8}$$

$$E_{1-3} \rightarrow R_{1-2}$$

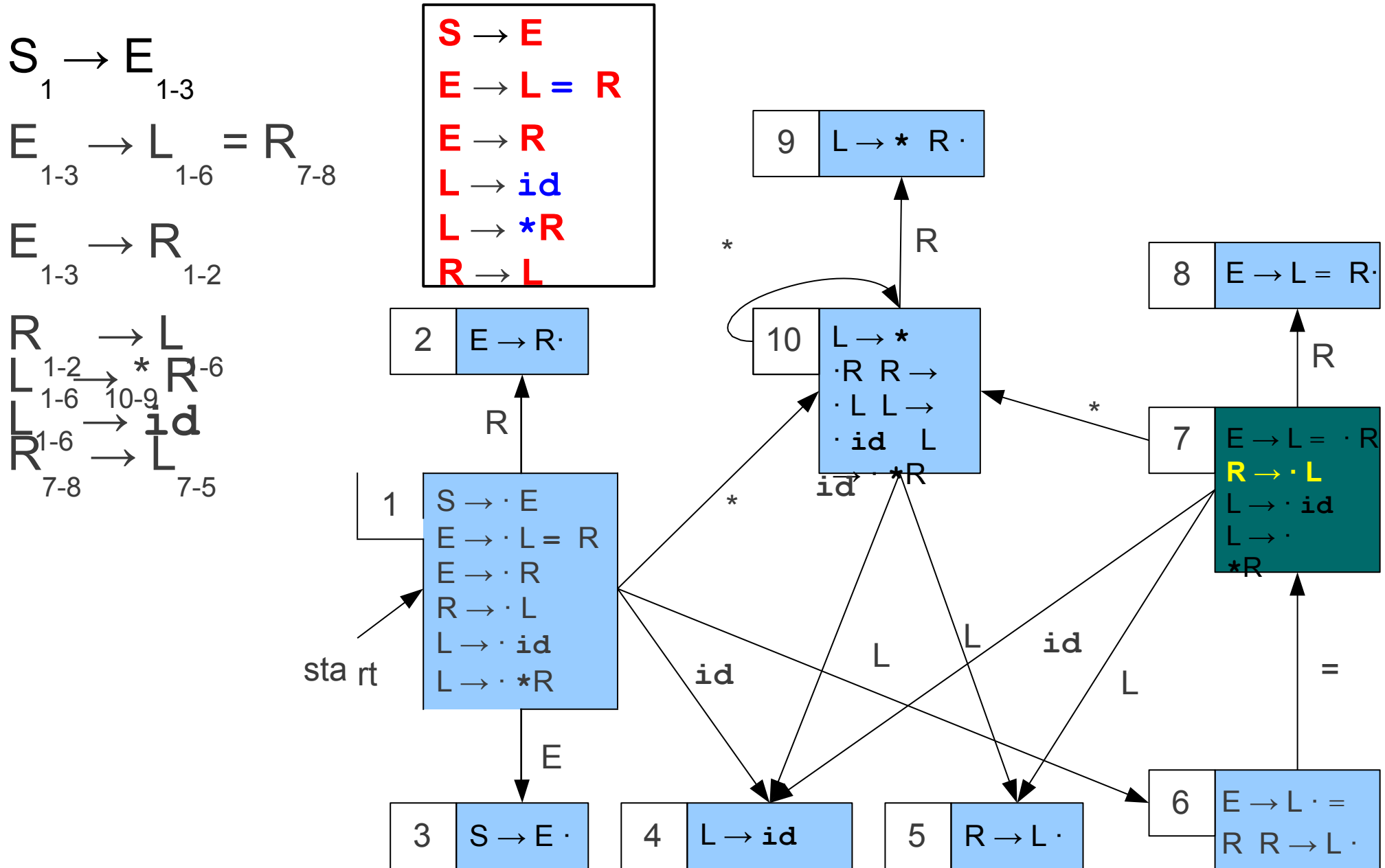
$$\begin{array}{l} R_{1-2} \rightarrow L_{1-6} \\ L_{1-6} \rightarrow *R_{10-9} \\ L_{1-6} \rightarrow id \end{array}$$

$S \rightarrow E$   
 $E \rightarrow L = R$   
 $E \rightarrow R$   
 $L \rightarrow id$   
 $L \rightarrow *R$   
 $R \rightarrow L$





# Augmenting the Grammar



# Augmenting the Grammar

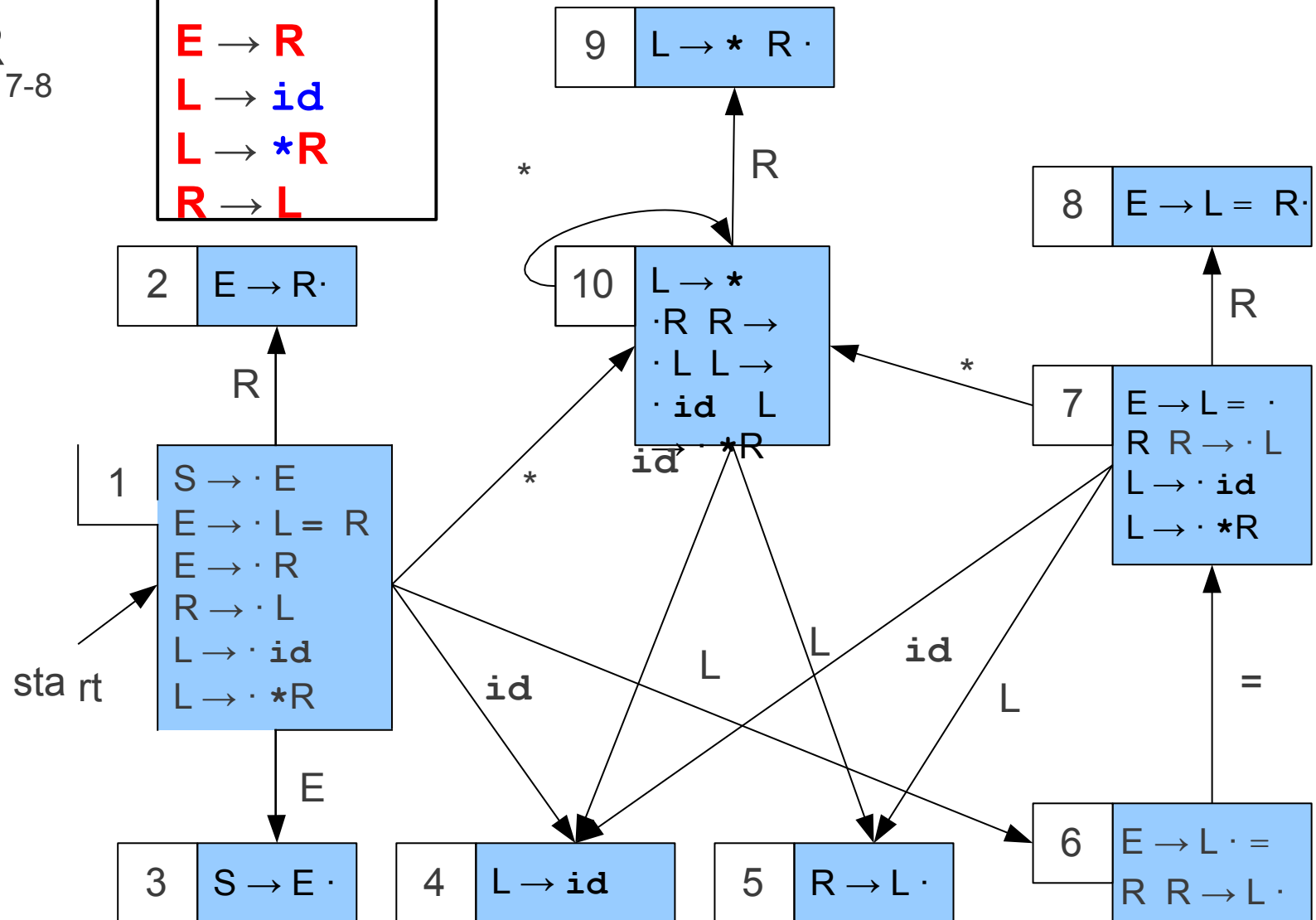
$$S_1 \rightarrow E_{1-3}$$

$$E_{1-3} \rightarrow L_{1-6} = R_{7-8}$$

$$E_{1-3} \rightarrow R_{1-2}$$

$$\begin{aligned} R_{1-2} &\rightarrow L_{1-6} \\ L_{1-6} &\rightarrow *R_{10-9} \\ L_{1-6} &\rightarrow id \\ R_{1-6} &\rightarrow L_{7-5} \end{aligned}$$

$S \rightarrow E$
$E \rightarrow L = R$
$E \rightarrow R$
$L \rightarrow id$
$L \rightarrow *R$
$R \rightarrow L$



# Augmenting the Grammar

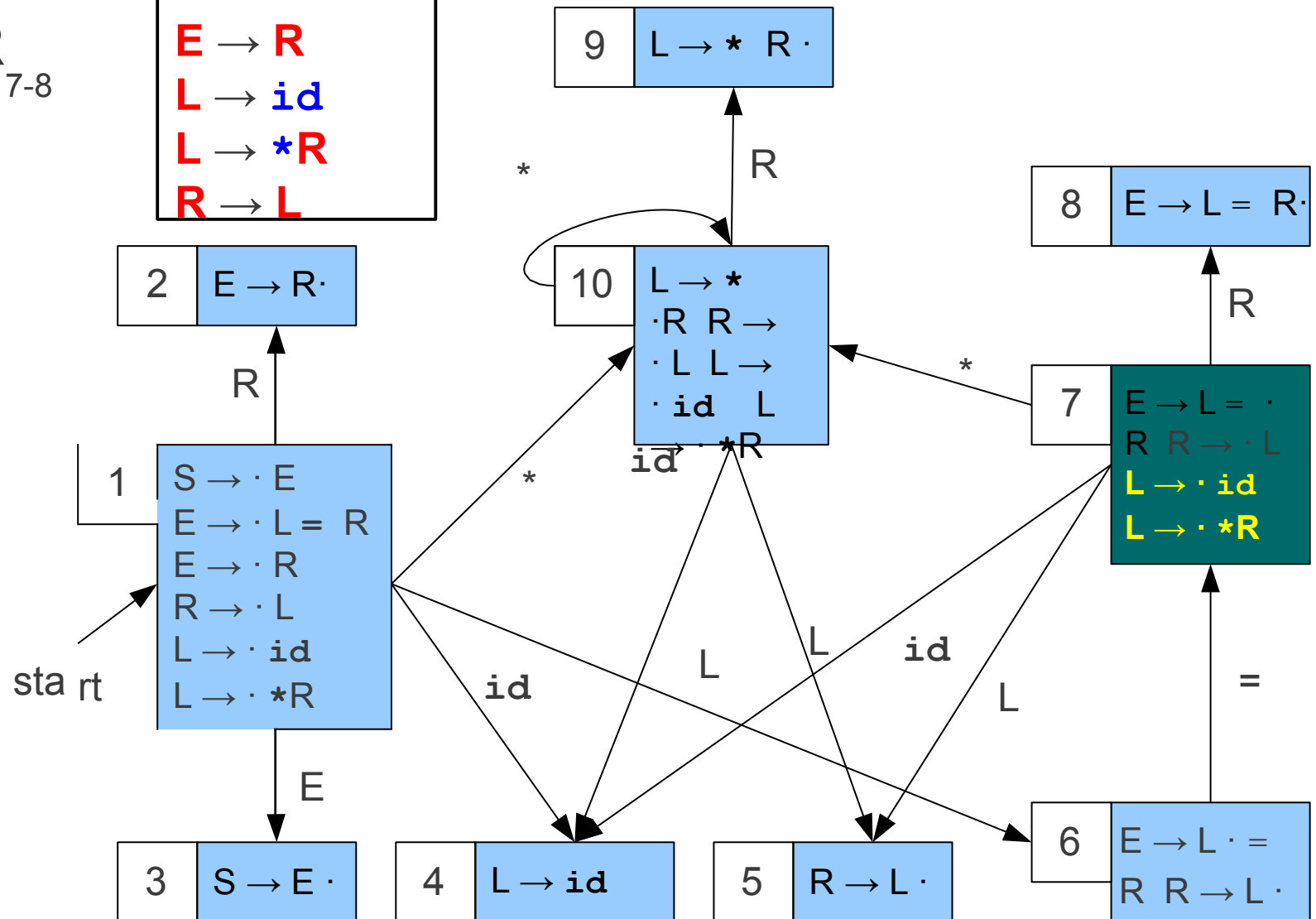
$$S_1 \rightarrow E_{1-3}$$

$$E_{1-3} \rightarrow L_{1-6} = R_{7-8}$$

$$E_{1-3} \rightarrow R_{1-2}$$

$$\begin{aligned} R_{1-2} &\rightarrow L_{1-6} \\ L_{1-6} &\rightarrow *R_{10-9} \\ L_{1-6} &\rightarrow id \\ R_{1-6} &\rightarrow L_{7-5} \end{aligned}$$

$S \rightarrow E$   
 $E \rightarrow L = R$   
 $E \rightarrow R$   
 $L \rightarrow id$   
 $L \rightarrow *R$   
 $R \rightarrow L$



# Augmenting the Grammar

$$S_1 \rightarrow E_{1-3}$$

$$E_{1-3} \rightarrow L_{1-6} = R_{7-8}$$

$$E_{1-3} \rightarrow R_{1-2}$$

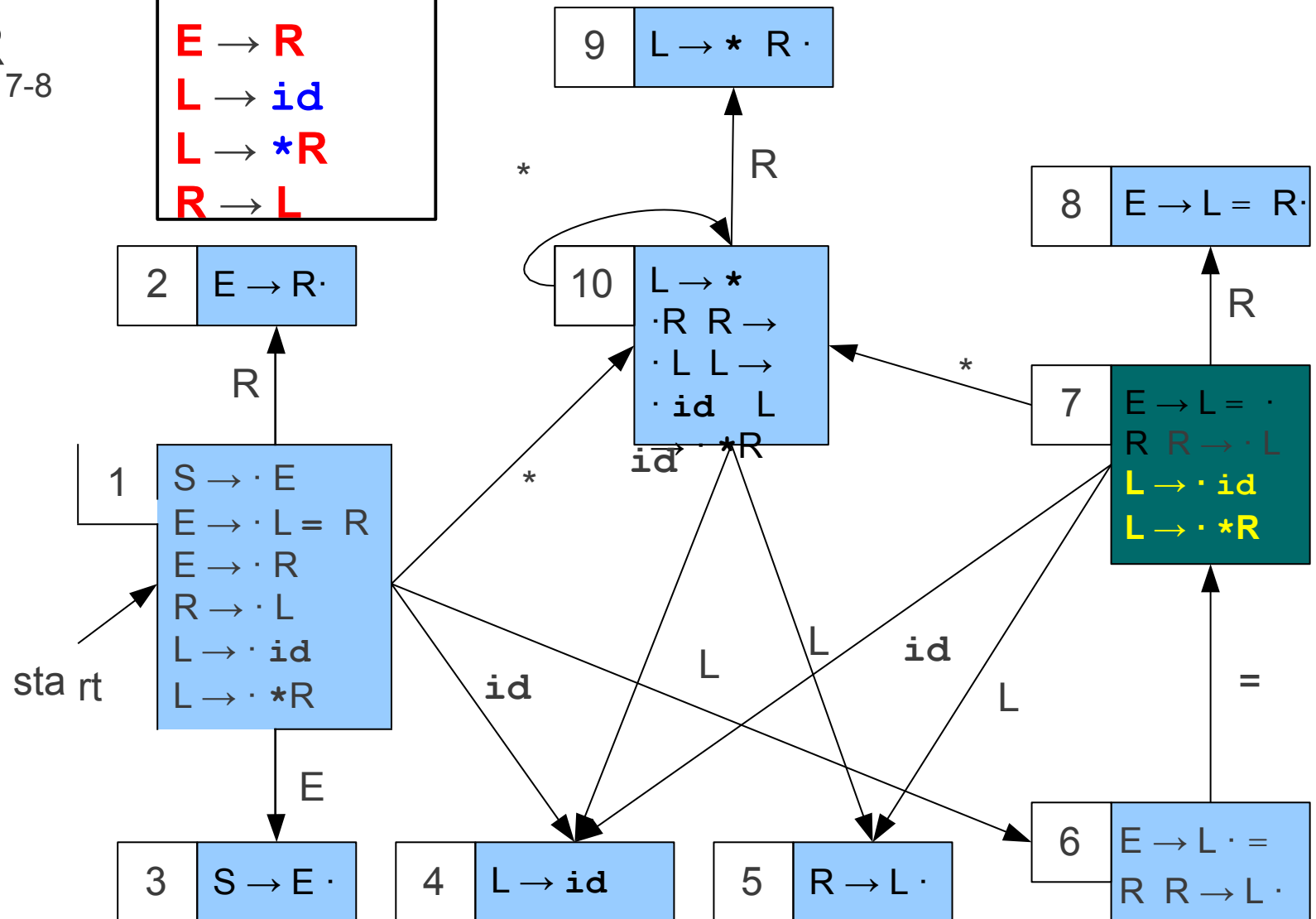
$$R_{1-2} \rightarrow L_{1-6} \rightarrow * R_{10-9}$$

$$L_{1-6} \rightarrow id$$

$$R_{1-6} \rightarrow L_{7-8} \rightarrow id$$

$$L_{7-5} \rightarrow * R_{10-9}$$

$S \rightarrow E$   
 $E \rightarrow L = R$   
 $E \rightarrow R$   
 $L \rightarrow id$   
 $L \rightarrow * R$   
 $R \rightarrow L$



# Augmenting the Grammar

$$S_1 \rightarrow E_{1-3}$$

$$E_{1-3} \rightarrow L_{1-6} = R_{7-8}$$

$$E_{1-3} \rightarrow R_{1-2}$$

$$R_{1-2} \rightarrow L_{1-6} \rightarrow * R_{10-9}$$

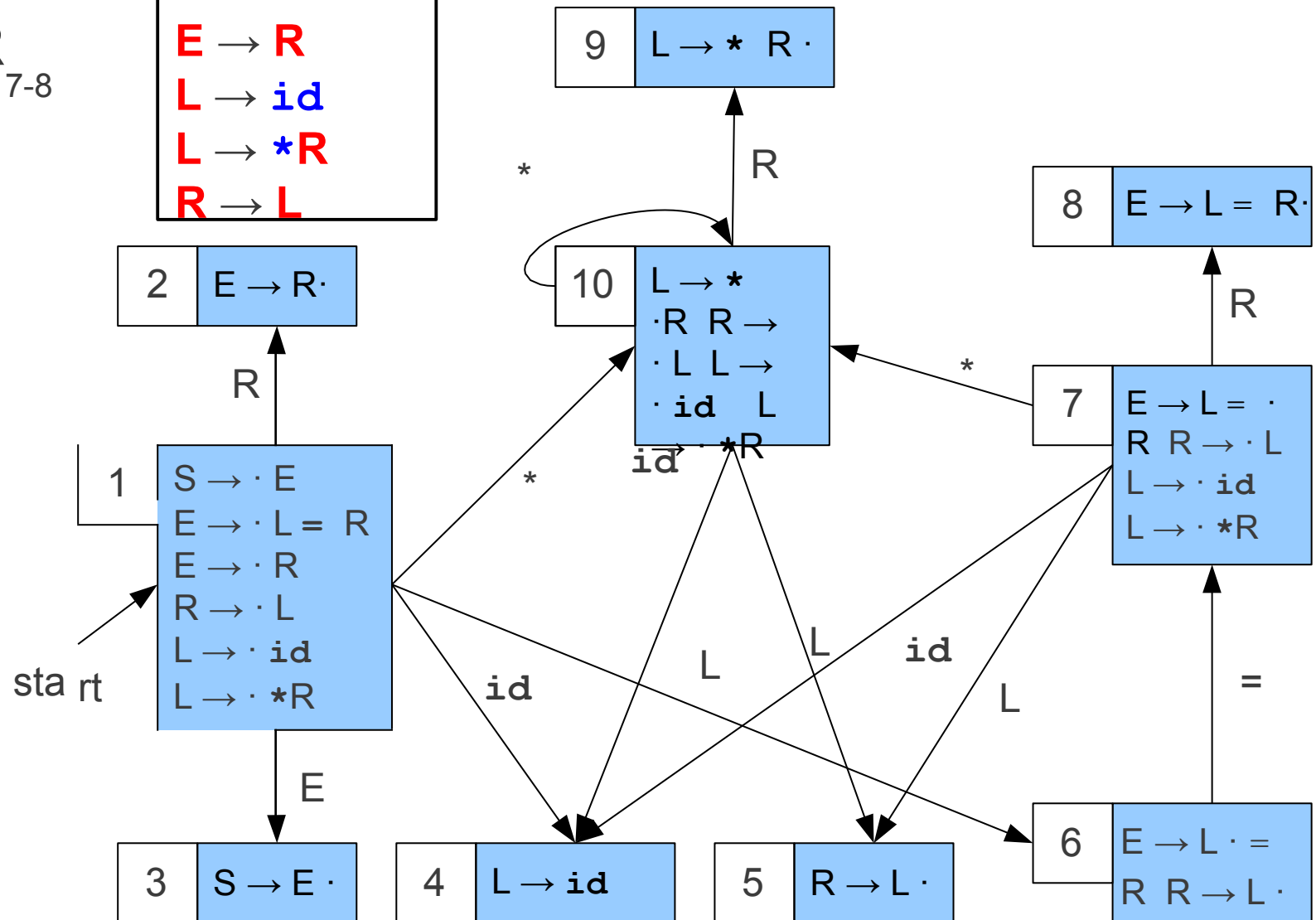
$$L_{1-6} \rightarrow id$$

$$R_{1-6} \rightarrow L_{7-8} \rightarrow id$$

$$L_{7-8} \rightarrow id$$

$$L_{7-5} \rightarrow * R_{10-9}$$

$S \rightarrow E$   
 $E \rightarrow L = R$   
 $E \rightarrow R$   
 $L \rightarrow id$   
 $L \rightarrow * R$   
 $R \rightarrow L$



# Augmenting the Grammar

$$S_1 \rightarrow E_{1-3}$$

$$E_{1-3} \rightarrow L_{1-6} = R_{7-8}$$

$$E_{1-3} \rightarrow R_{1-2}$$

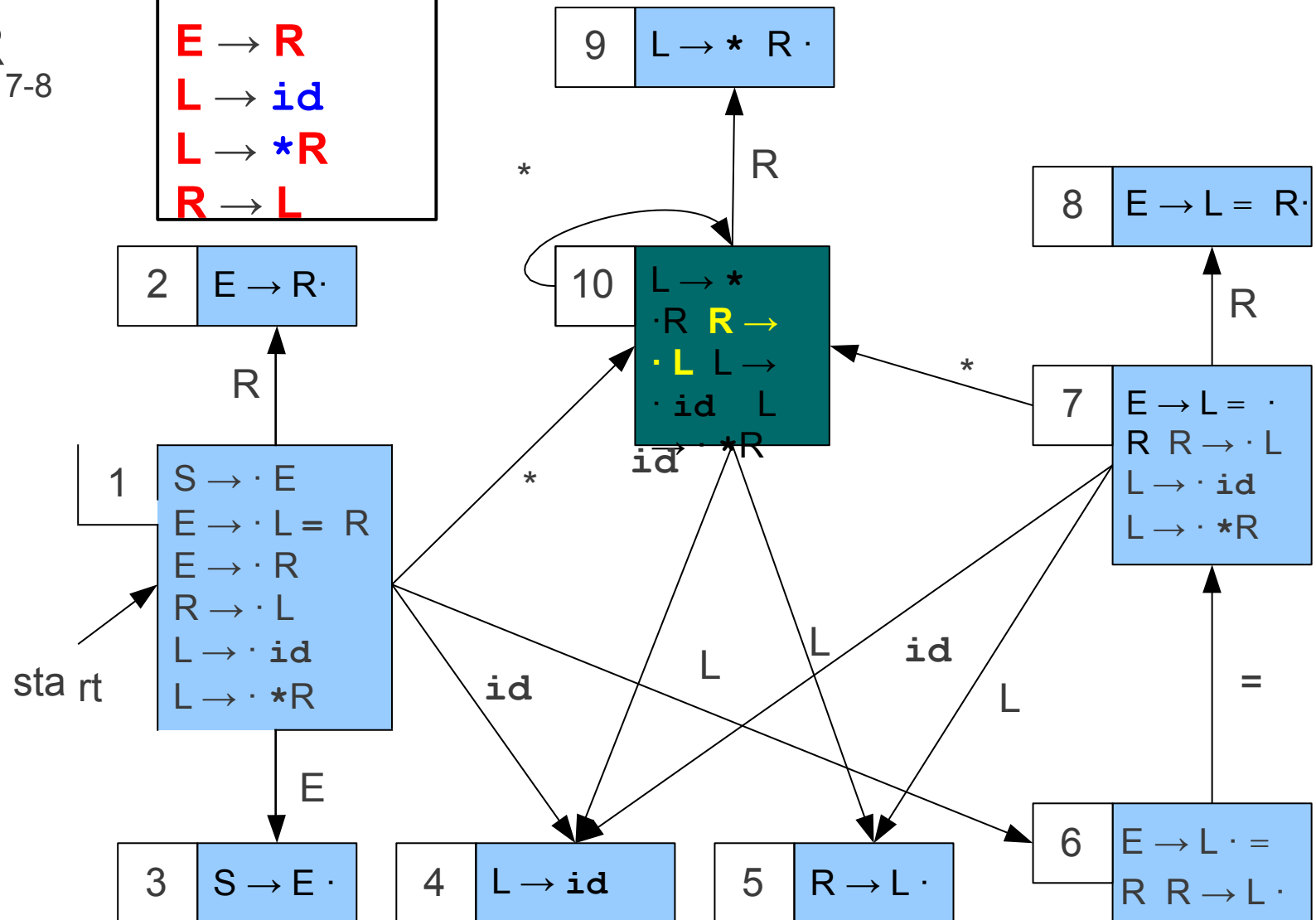
$$R_{1-2} \rightarrow L_{1-6} \rightarrow *R_{10-9}$$

$$L_{1-6} \rightarrow id_{7-5}$$

$$L_{7-5} \rightarrow id_{7-5}$$

$$L_{7-5} \rightarrow *R_{10-9}$$

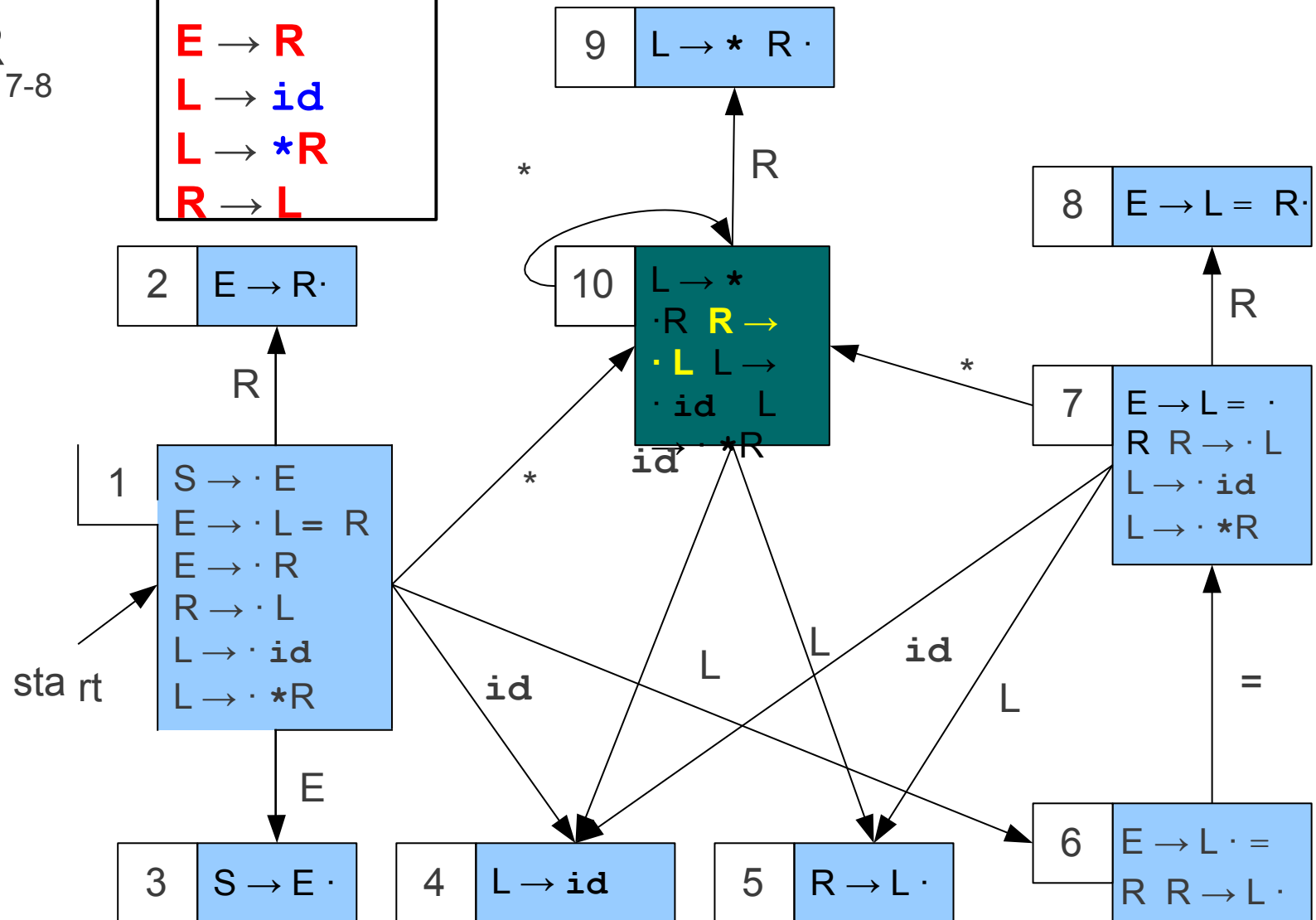
$S \rightarrow E$   
 $E \rightarrow L = R$   
 $E \rightarrow R$   
 $L \rightarrow id$   
 $L \rightarrow *R$   
 $R \rightarrow L$



# Augmenting the Grammar

$S \xrightarrow{1} E_{1-3}$   
 $E \xrightarrow{1-3} L_{1-6} = R_{7-8}$   
 $E \xrightarrow{1-3} R_{1-2}$   
 $R \xrightarrow{1-2} L_{1-6}$   
 $L \xrightarrow{1-6} *R_{10-9}$   
 $L \xrightarrow{1-6} id_{7-5}$   
 $R \xrightarrow{1-6} L_{10-5}$   
 $L \xrightarrow{7-8} id_{7-5}$   
 $L \xrightarrow{7-5} *R_{10-9}$

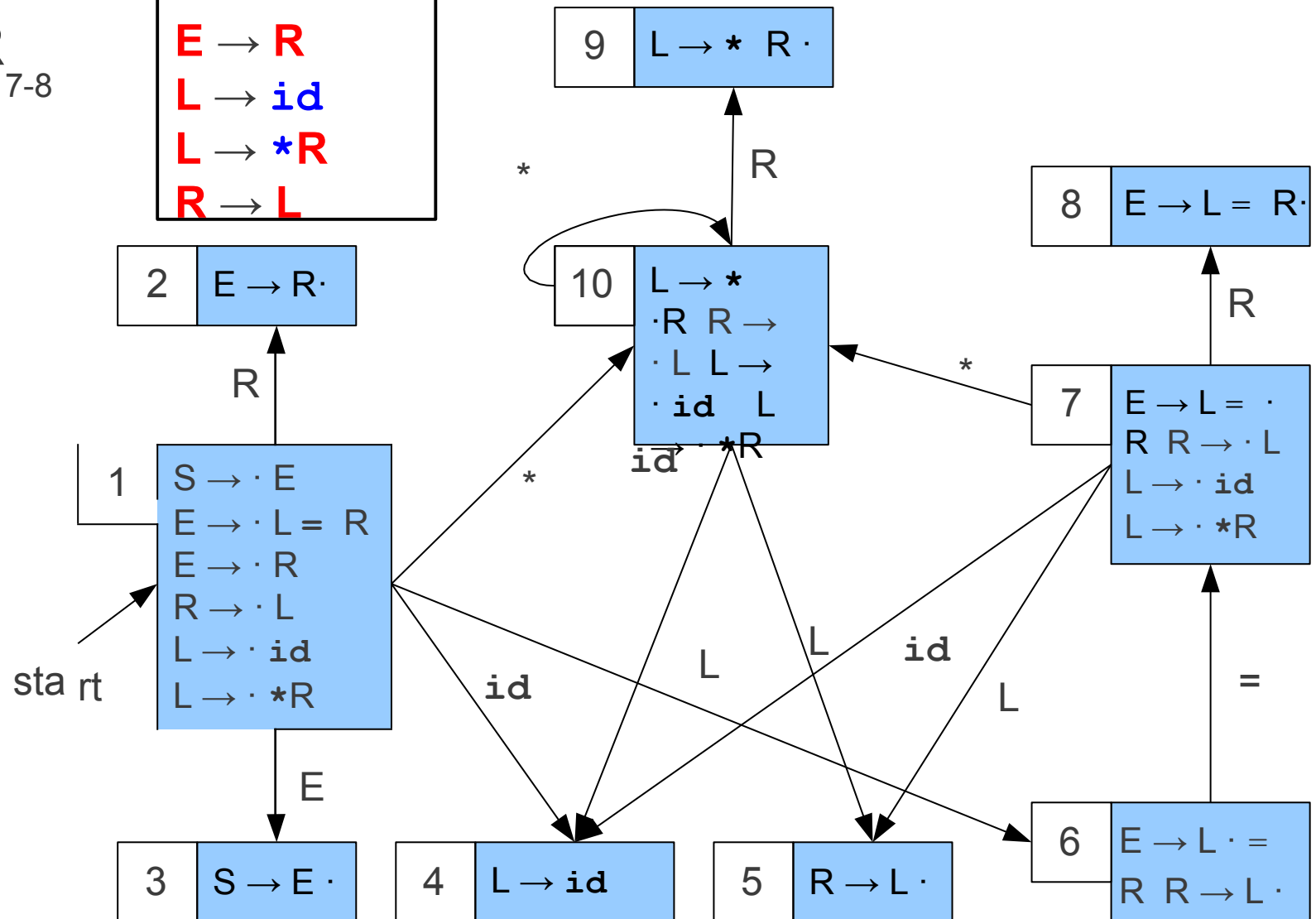
$S \rightarrow E$   
 $E \rightarrow L = R$   
 $E \rightarrow R$   
 $L \rightarrow id$   
 $L \rightarrow *R$   
 $R \rightarrow L$



# Augmenting the Grammar

$S \xrightarrow{1} E_{1-3}$   
 $E \xrightarrow{1-3} L_{1-6} = R_{7-8}$   
 $E \xrightarrow{1-3} R_{1-2}$   
 $R \xrightarrow{1-2} L_{1-6}$   
 $L \xrightarrow{1-6} *R_{10-9}$   
 $L \xrightarrow{1-6} id_{7-5}$   
 $R \xrightarrow{1-6} L_{7-5}$   
 $L \xrightarrow{7-8} id_{7-5}$   
 $L \xrightarrow{7-5} *R_{10-9}$   
 $R \xrightarrow{10-9} L_{10-5}$

$S \rightarrow E$   
 $E \rightarrow L = R$   
 $E \rightarrow R$   
 $L \rightarrow id$   
 $L \rightarrow *R$   
 $R \rightarrow L$





# Augmenting the Grammar

$S \xrightarrow{1} E_{1-3}$

$E \xrightarrow{1-3} L_{1-6} = R_{7-8}$

$E \xrightarrow{1-3} R_{1-2}$

$R \xrightarrow{1-2} L_{1-6}$

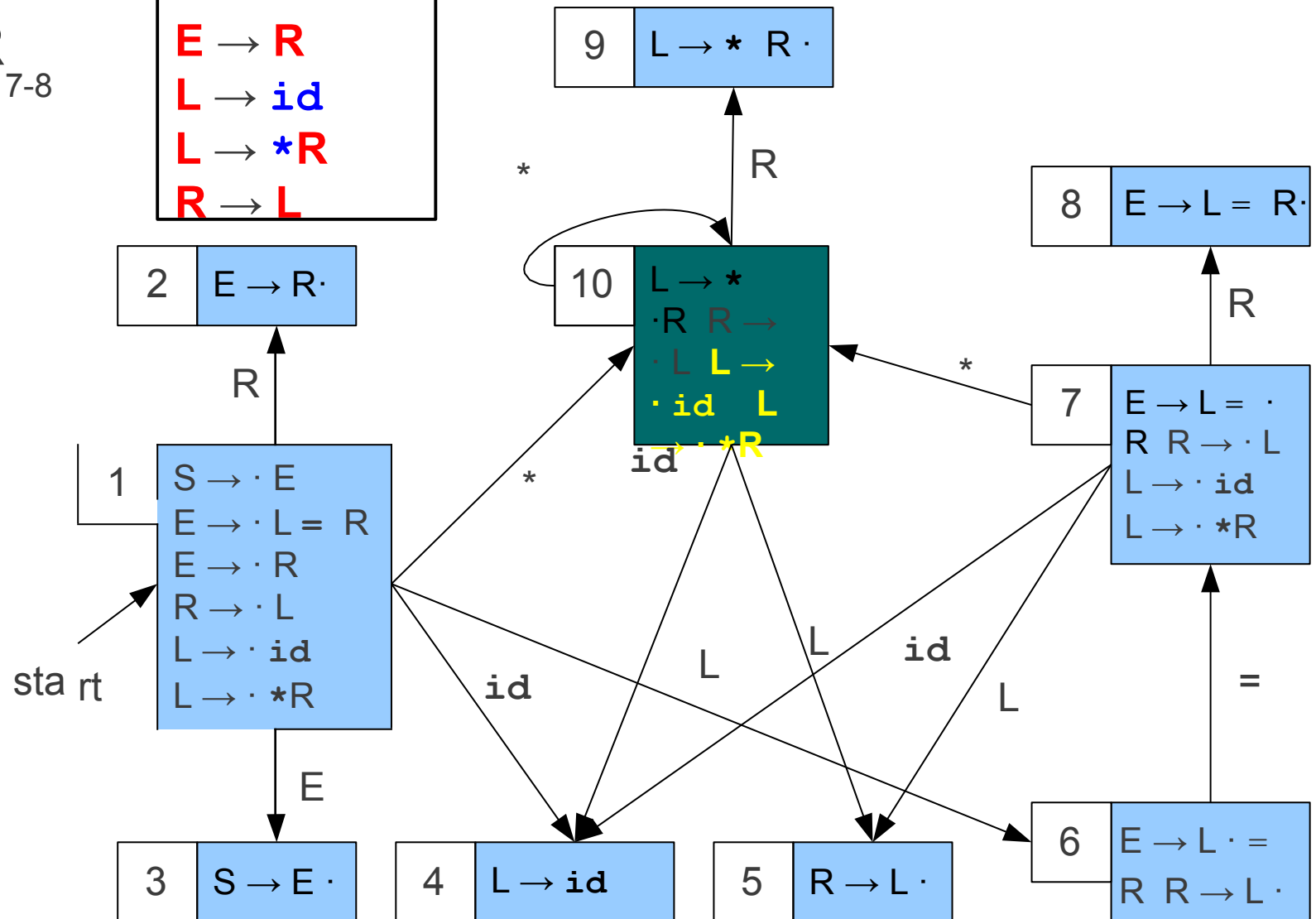
$L \xrightarrow{1-6} id_{7-5}$

$L \xrightarrow{7-8} id_{7-5}$

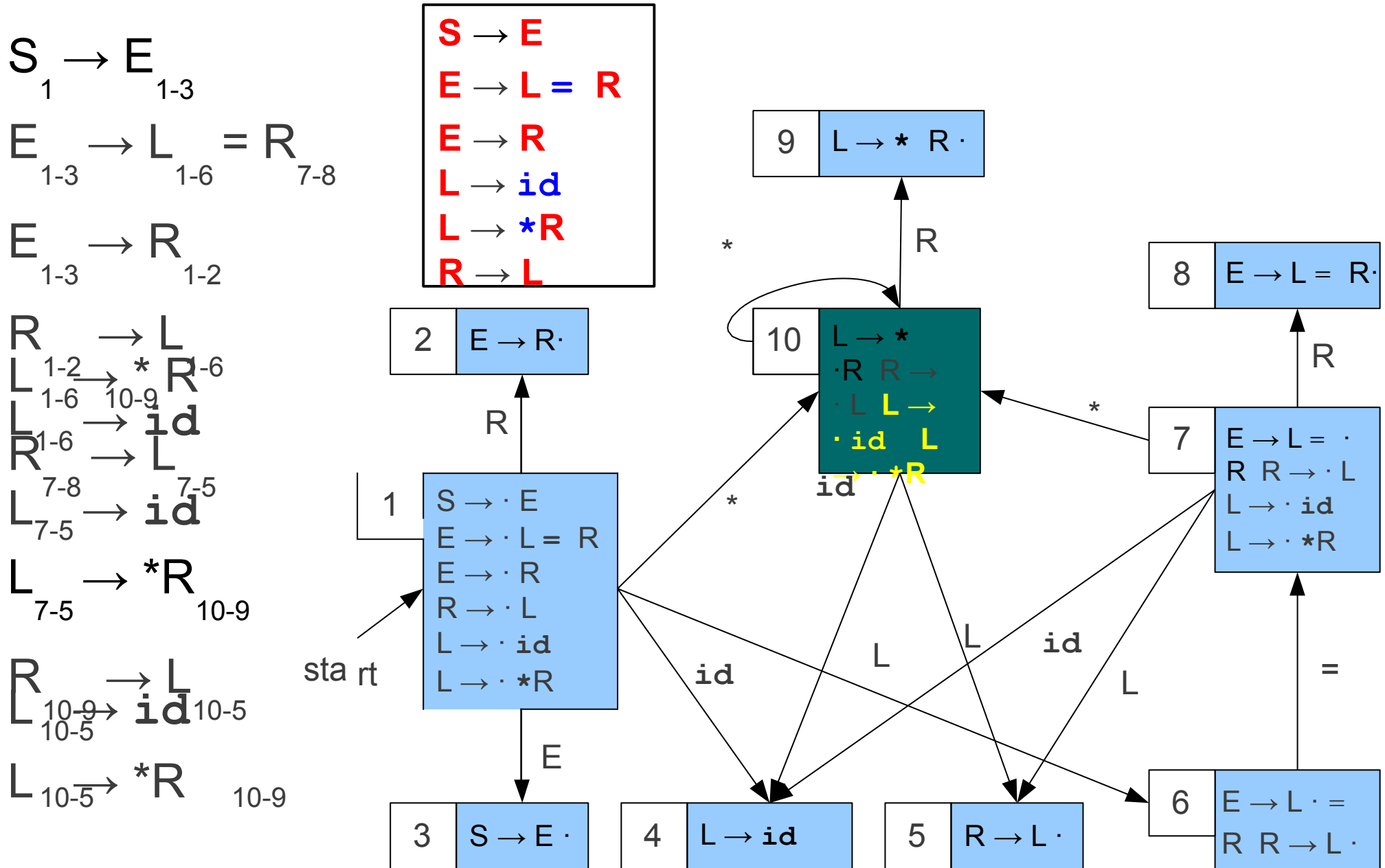
$L \xrightarrow{7-5} *R_{10-9}$

$R \xrightarrow{10-9} L_{10-5}$

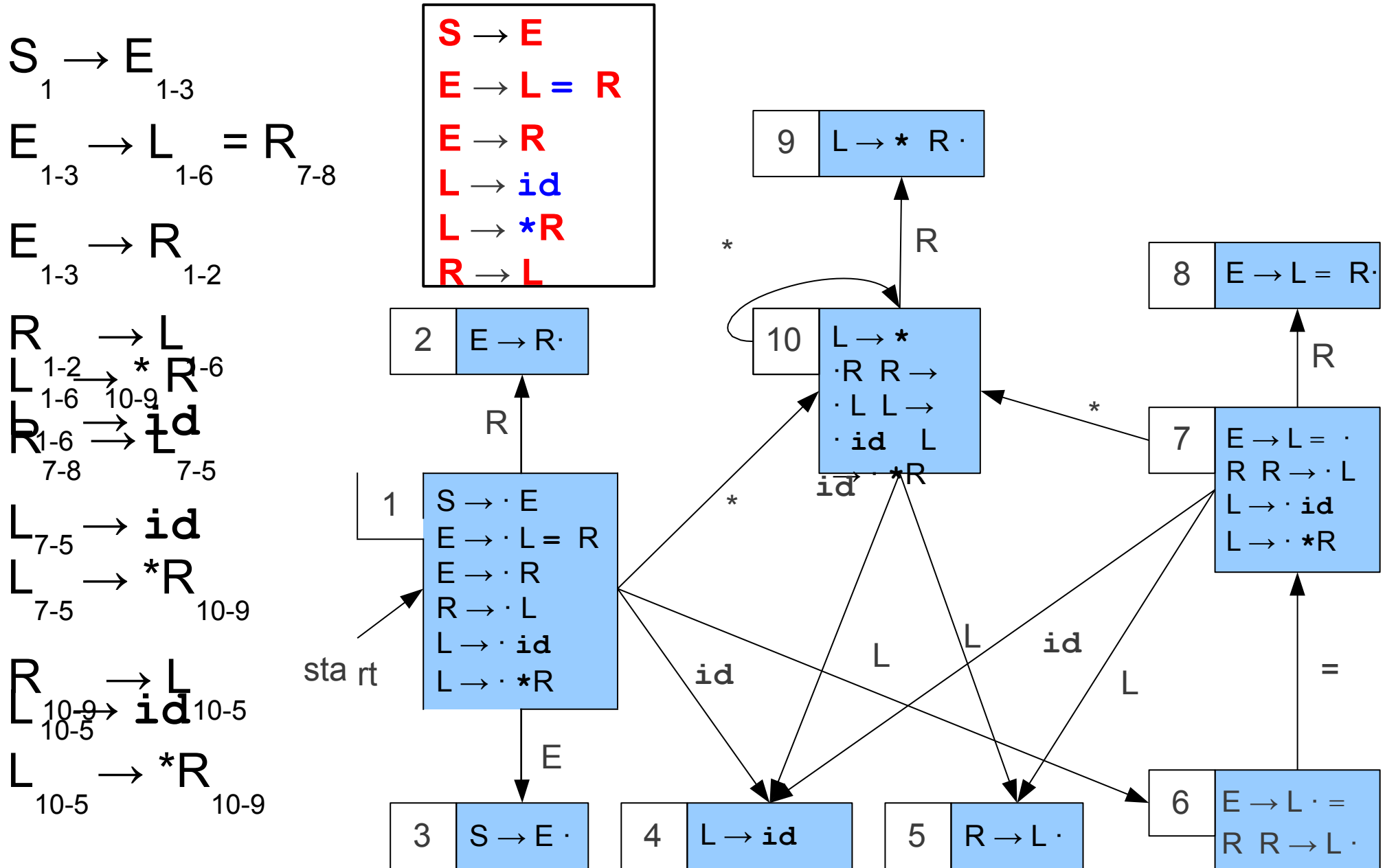
$S \rightarrow E$   
 $E \rightarrow L = R$   
 $E \rightarrow R$   
 $L \rightarrow id$   
 $L \rightarrow *R$   
 $R \rightarrow L$



# Augmenting the Grammar



# Augmenting the Grammar



# Constructing Augmented Grammars

For each item  $A \rightarrow \cdot \omega$  in some state  $q$ :

Trace out the path  $\omega$  takes through the LR(0) automaton starting at  $q$ .

Replace each nonterminal in  $\omega$  with a nonterminal annotated with the state transitioned to and from by the edge labeled with that nonterminal.

Replace  $A$  with a nonterminal annotated with the start and end state of the transition on  $A$  out of  $q$ .

Result is a larger grammar with more precise productions.

# Why is this Grammar Useful?

- At a high-level, separates out the nonterminals based on their context.
- This makes the FOLLOW sets more precise for their nonterminals.
- In fact, the FOLLOW sets are surprisingly precise.

# Augmented FOLLOW Sets

$$S_1 \rightarrow E_{1-3}$$

$$E_{1-3} \rightarrow L_{1-6} = R_{7-8}$$

$$E_{1-3} \rightarrow R_{1-2}$$

$$R_{1-2} \rightarrow L_{1-6}$$

$$L_{1-6} \rightarrow \mathbf{id}$$

$$L_{1-6} \rightarrow * R_{10-9}$$

$$R_{7-8} \rightarrow L_{7-5}$$

$$L_{7-5} \rightarrow \mathbf{id}$$

$$L_{7-5} \rightarrow * R_{10-9}$$

$$R_{10-9} \rightarrow L_{10-5}$$

# Augmented FOLLOW Sets

$$S_1 \rightarrow E_{1-3}$$

$$E_{1-3} \rightarrow L_{1-6} = R_{7-8}$$

$$E_{1-3} \rightarrow R_{1-2}$$

$$R_{1-2} \rightarrow L_{1-6}$$

$$L_{1-6} \rightarrow \mathbf{id}$$

$$L_{1-6} \rightarrow * R_{10-9}$$

$$R_{7-8} \rightarrow L_{7-5}$$

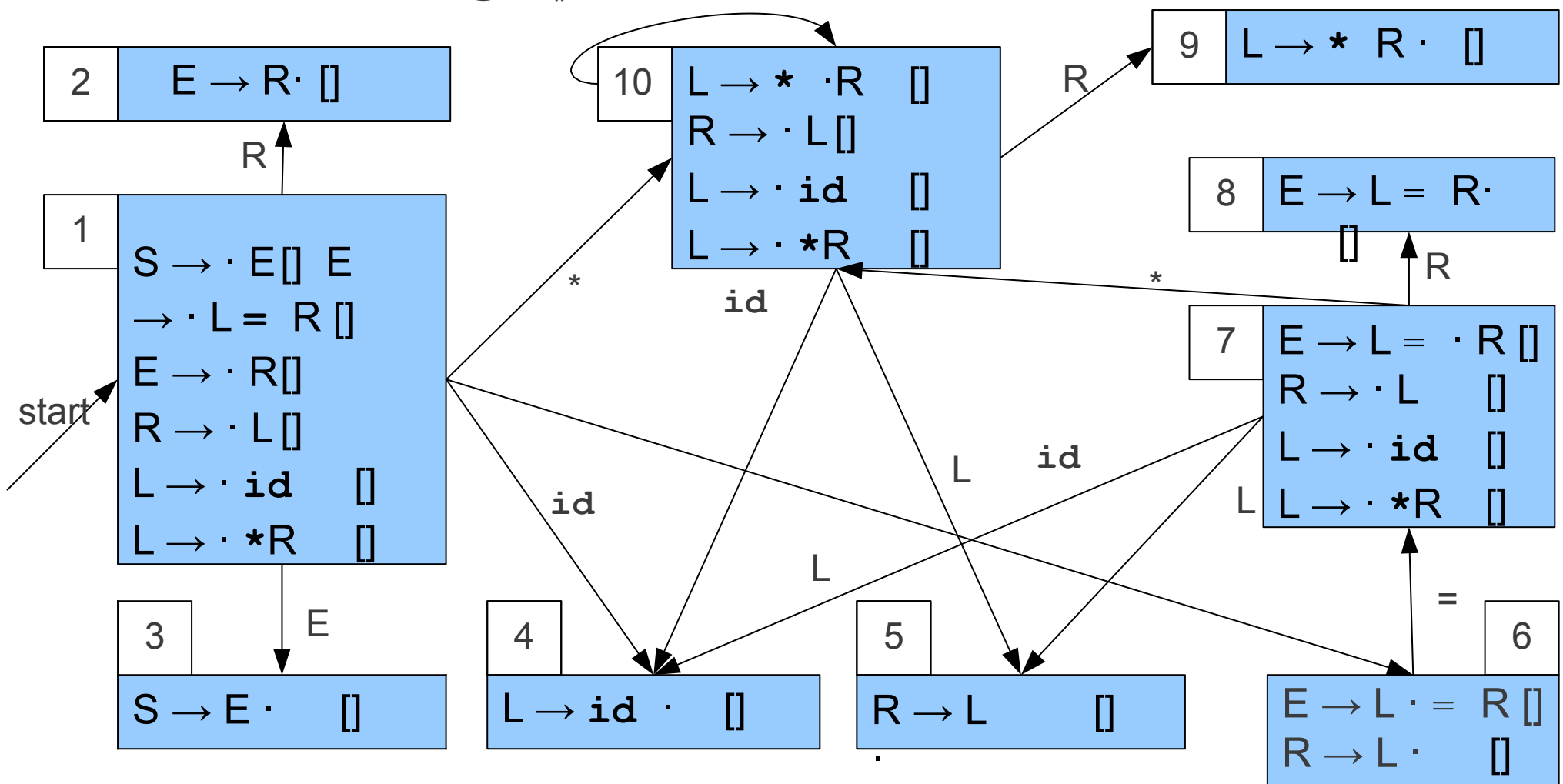
$$L_{7-5} \rightarrow \mathbf{id}$$

$$L_{7-5} \rightarrow * R_{10-9}$$

$$R_{10-9} \rightarrow L_{10-5}$$

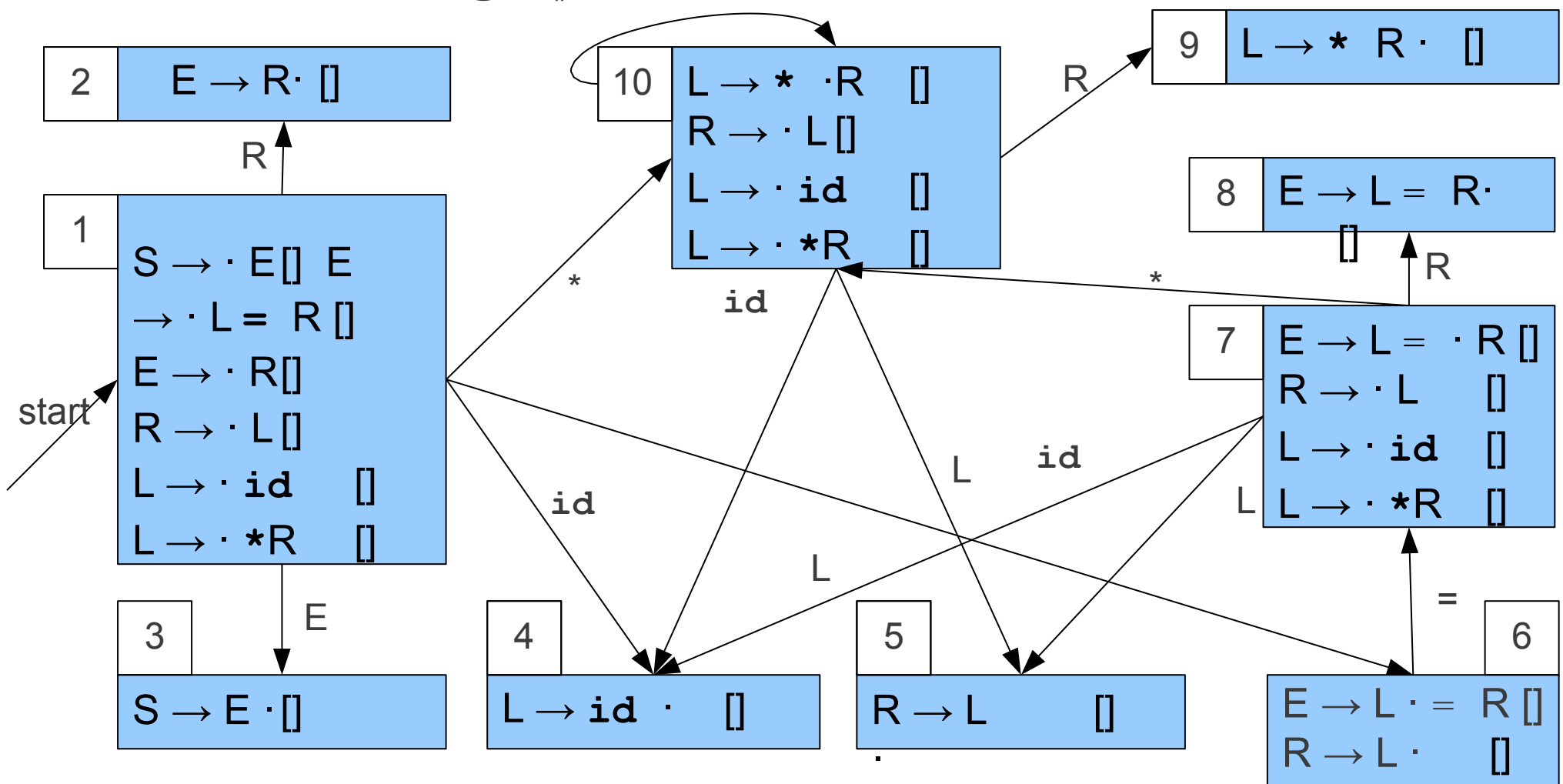
$S_1$	$E_{1-3}$	$L_{1-6}$	$L_{7-5}$	$L_{10-5}$	$R_{1-2}$	$R_{7-8}$	$R_{10-9}$
\$	\$	=	\$	=	\$	\$	=

# Using Our FOLLOW Sets



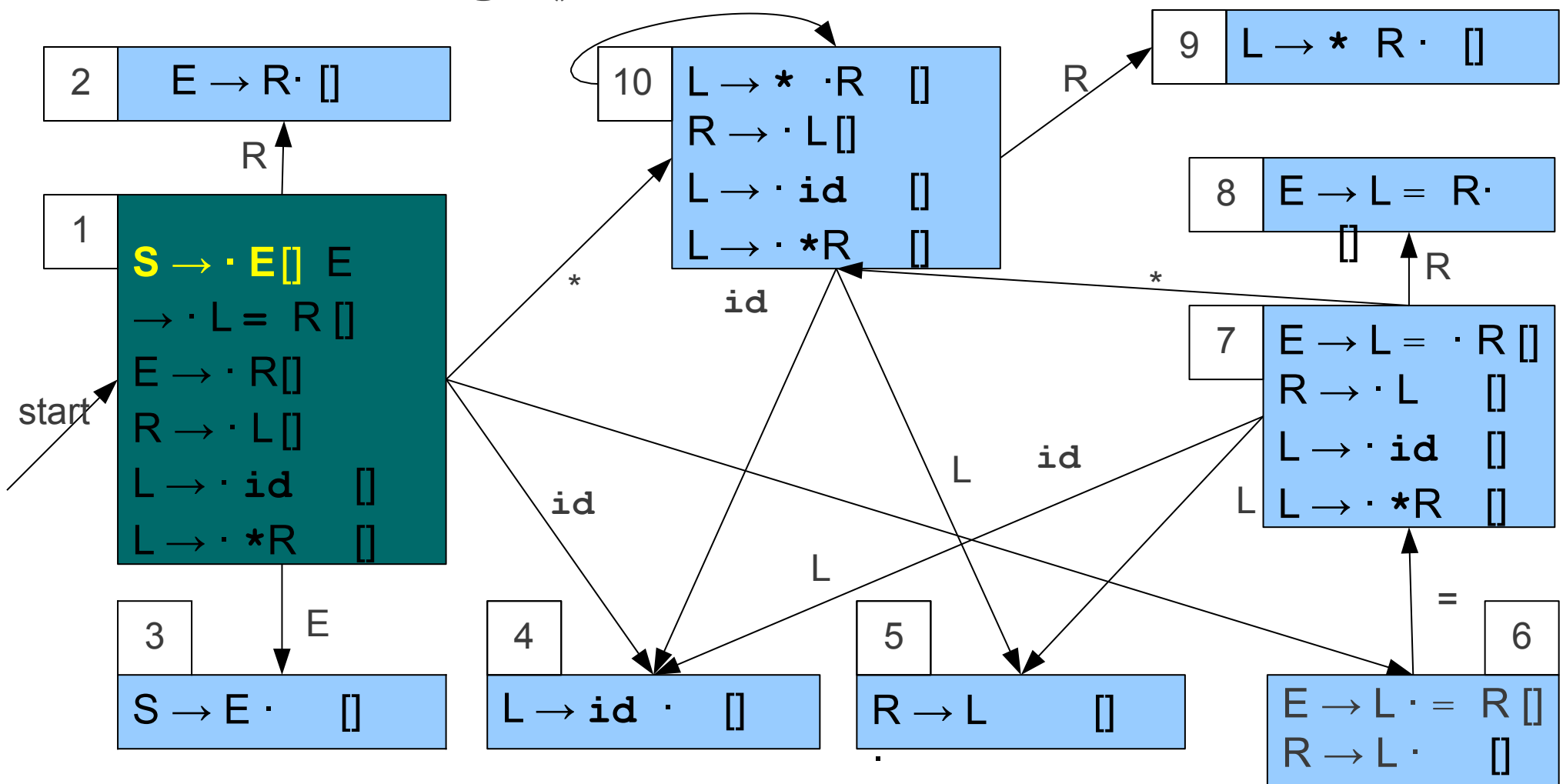


# Using Our FOLLOW Sets



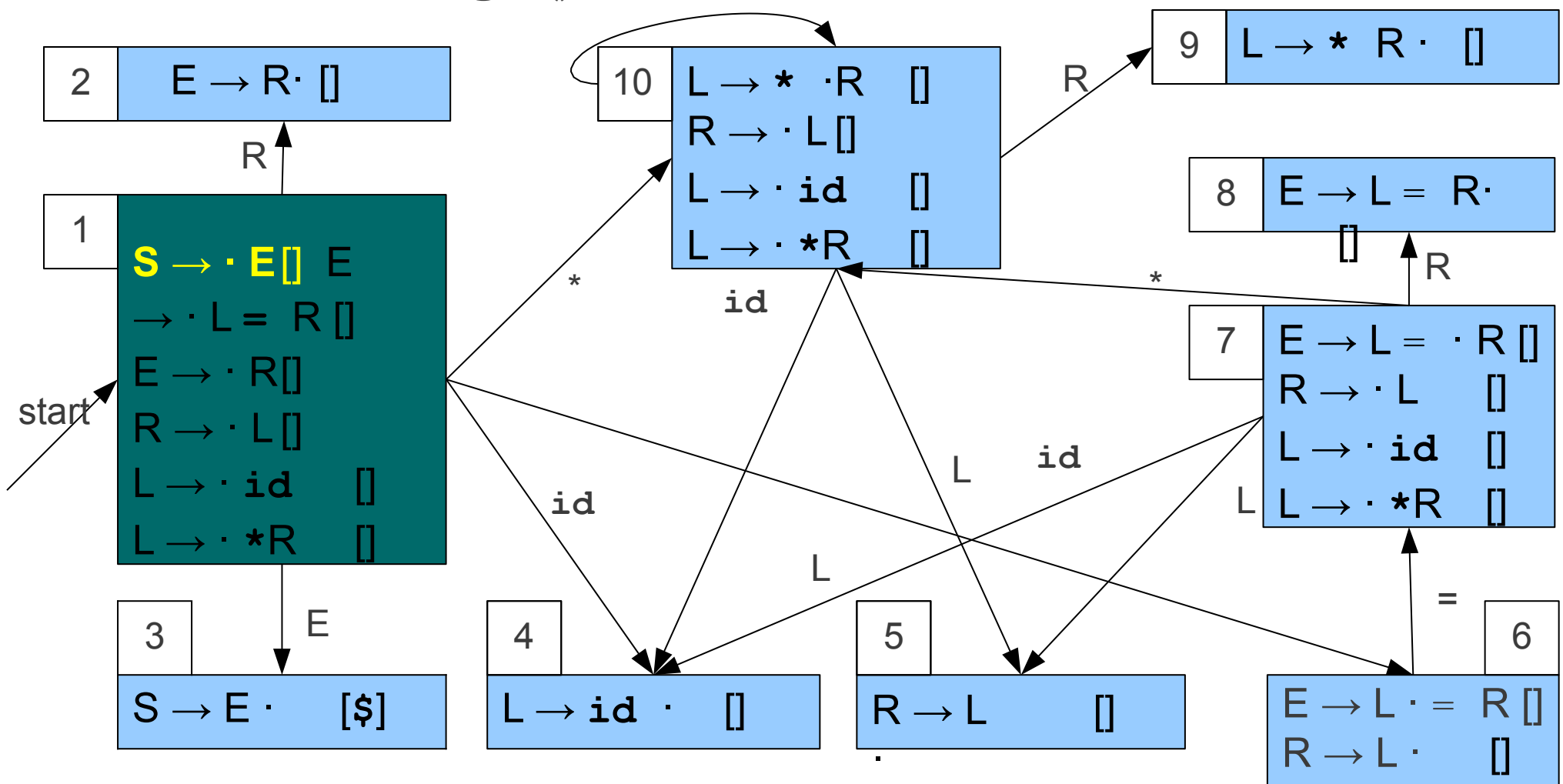
$S_1$	$E_{1-3}$	$L_{1-6}$	$L_{7-5}$	$L_{10-5}$	$R_{1-2}$	$R_{7-8}$	$R_{10-9}$
\$	\$	=	\$	=	\$	\$	=

# Using Our FOLLOW Sets



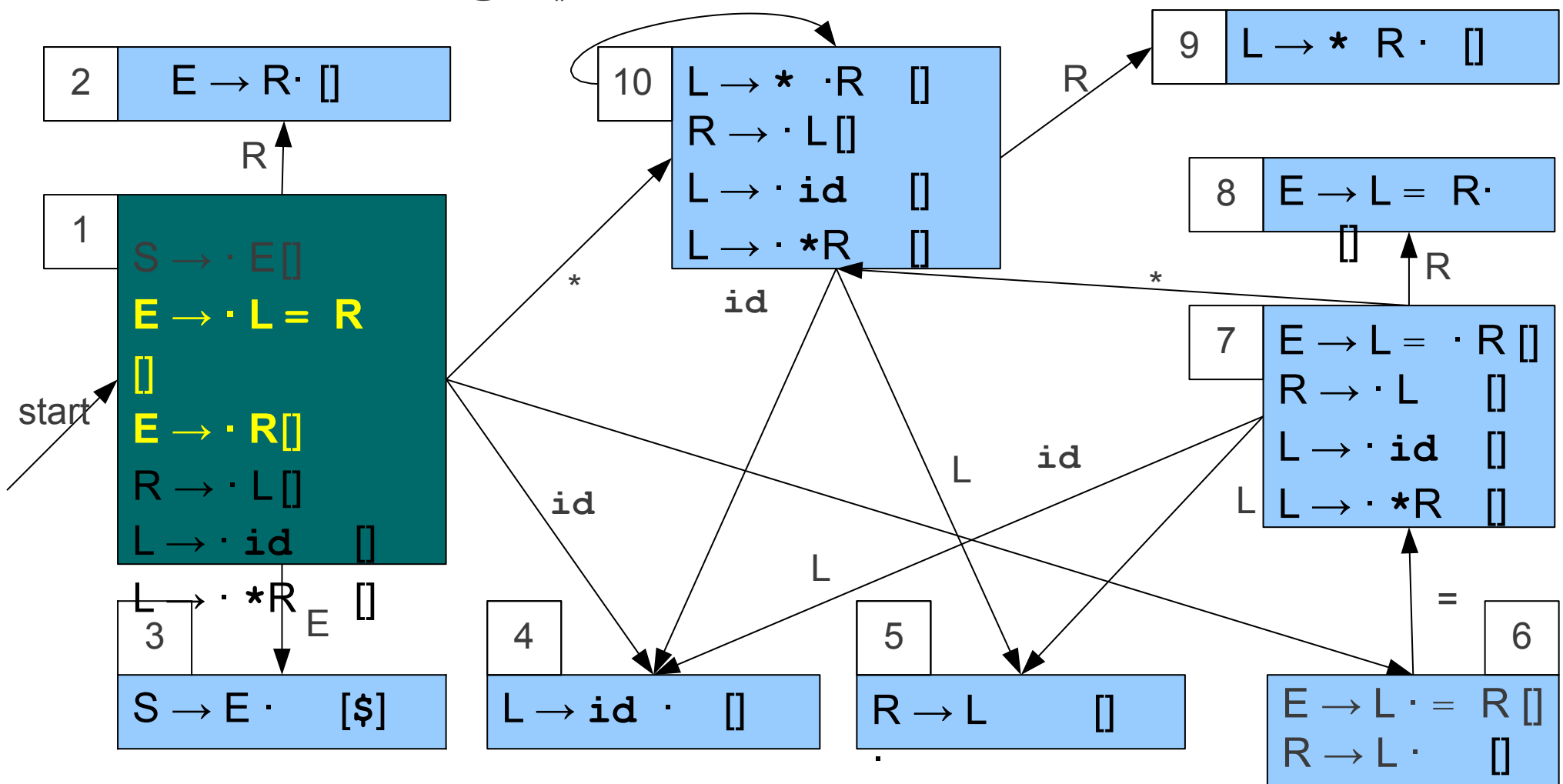
$S_1$	$E_{1-3}$	$L_{1-6}$	$L_{7-5}$	$L_{10-5}$	$R_{1-2}$	$R_{7-8}$	$R_{10-9}$
\$	\$	=	=	=	\$	\$	=

# Using Our FOLLOW Sets



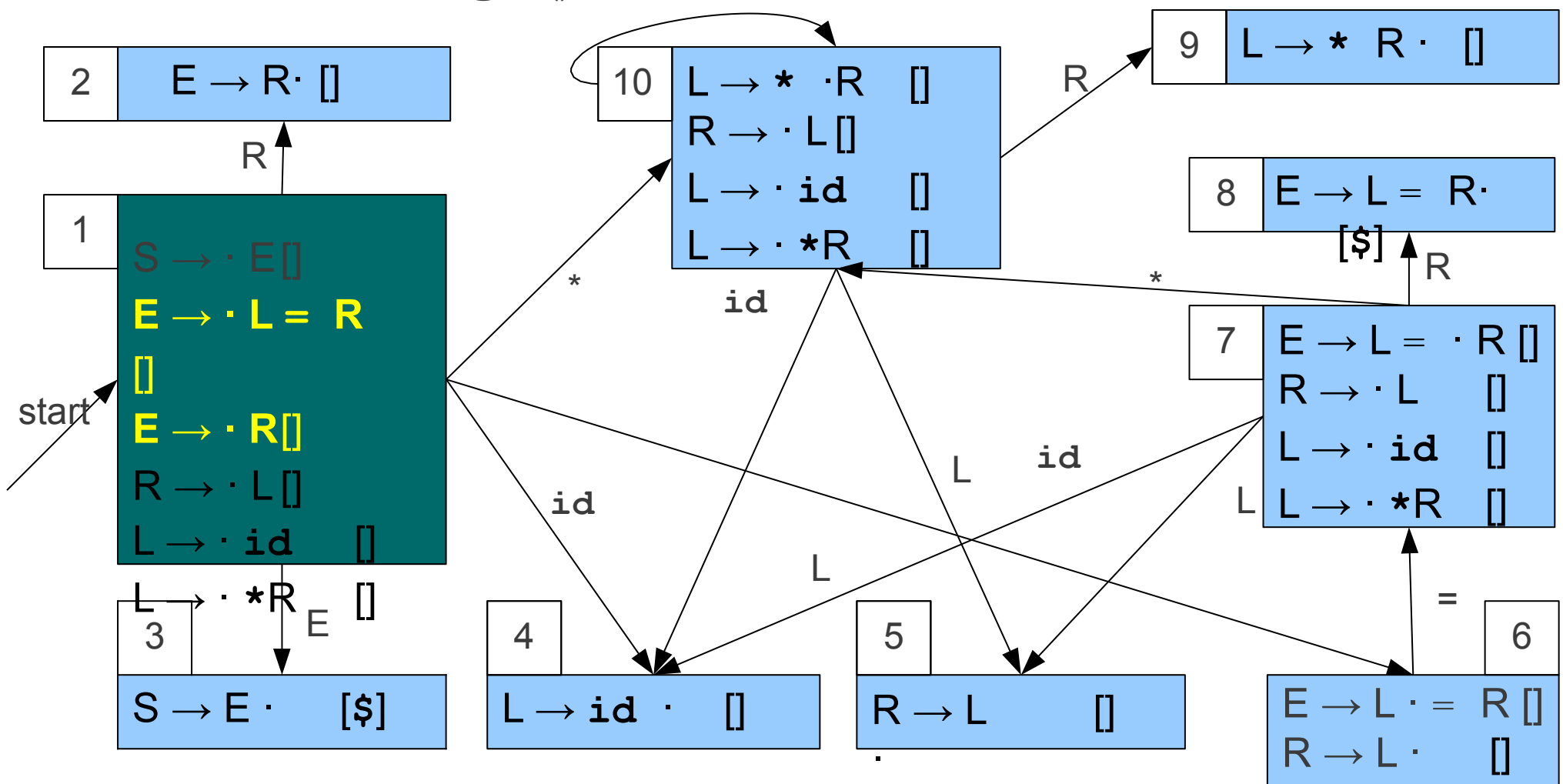
$S_1$	$E_{1-3}$	$L_{1-6}$	$L_{7-5}$	$L_{10-5}$	$R_{1-2}$	$R_{7-8}$	$R_{10-9}$
\$	\$	=	=	=	\$	\$	=
		\$					

# Using Our FOLLOW Sets



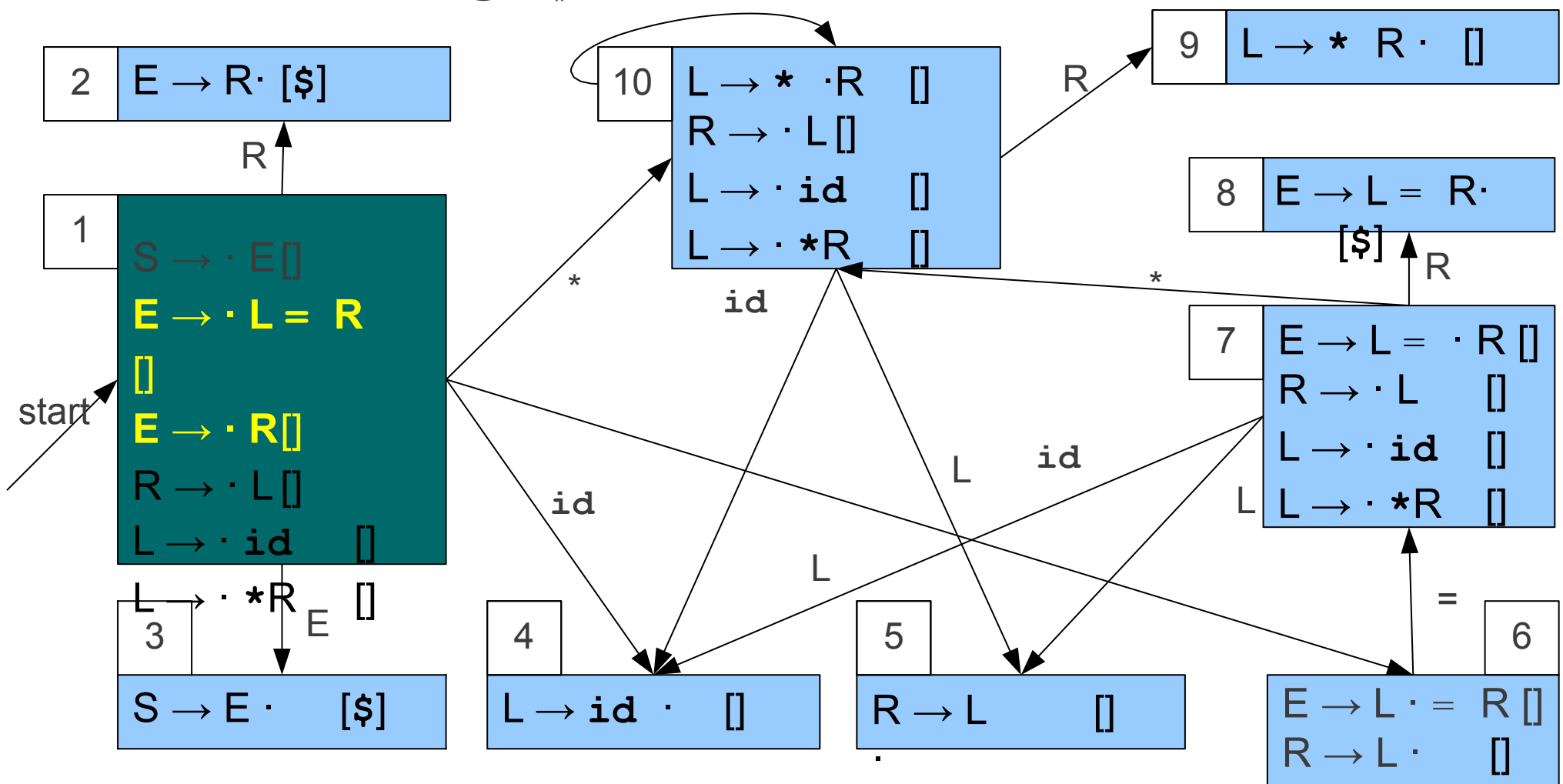
$S_1$	$E_{1-3}$	$L_{1-6}$	$L_{7-5}$	$L_{10-5}$	$R_{1-2}$	$R_{7-8}$	$R_{10-9}$
\$	\$	=	=	=	\$	\$	=
		\$					

# Using Our FOLLOW Sets



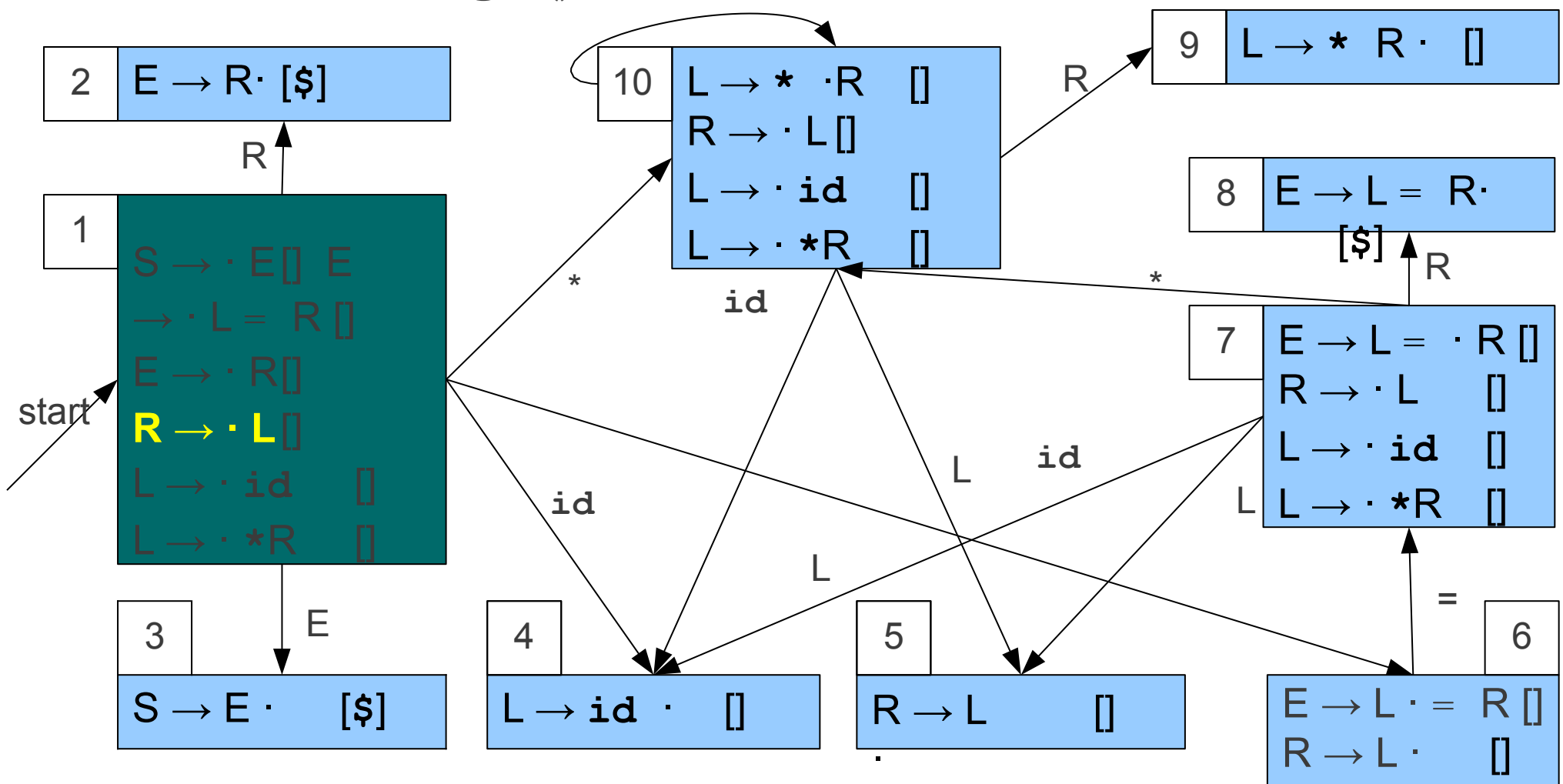
$S_1$	$E_{1-3}$	$L_{1-6}$	$L_{7-5}$	$L_{10-5}$	$R_{1-2}$	$R_{7-8}$	$R_{10-9}$
\$	\$	=	=	=	\$	\$	=
		\$					

# Using Our FOLLOW Sets



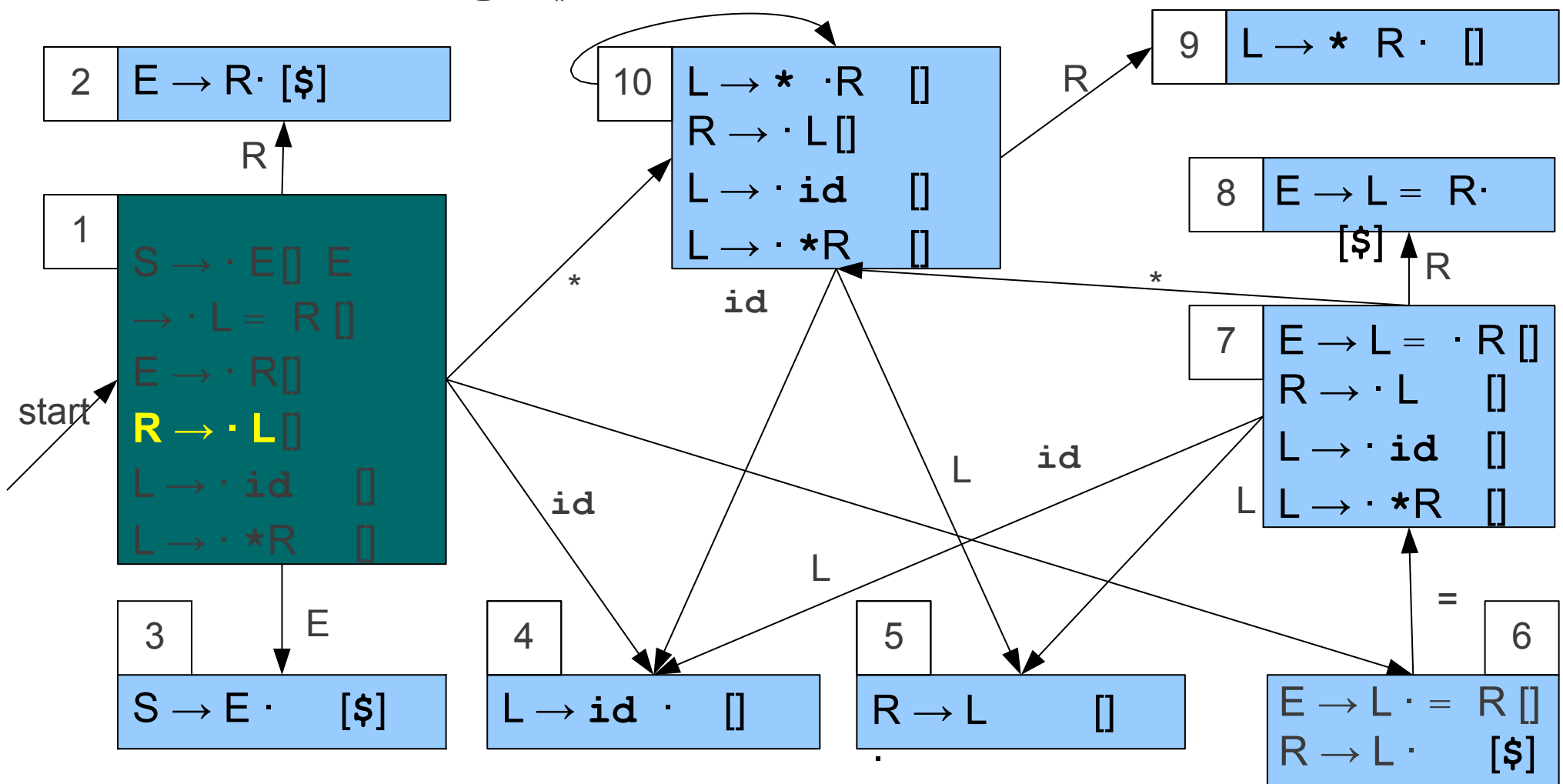
$S_1$	$E_{1-3}$	$L_{1-6}$	$L_{7-5}$	$L_{10-5}$	$R_{1-2}$	$R_{7-8}$	$R_{10-9}$
\$	\$	=	=	=	\$	\$	=
		\$					

# Using Our FOLLOW Sets



$S_1$	$E_{1-3}$	$L_{1-6}$	$L_{7-5}$	$L_{10-5}$	$R_{1-2}$	$R_{7-8}$	$R_{10-9}$
\$	\$	=	=	=	\$	\$	=
		\$					

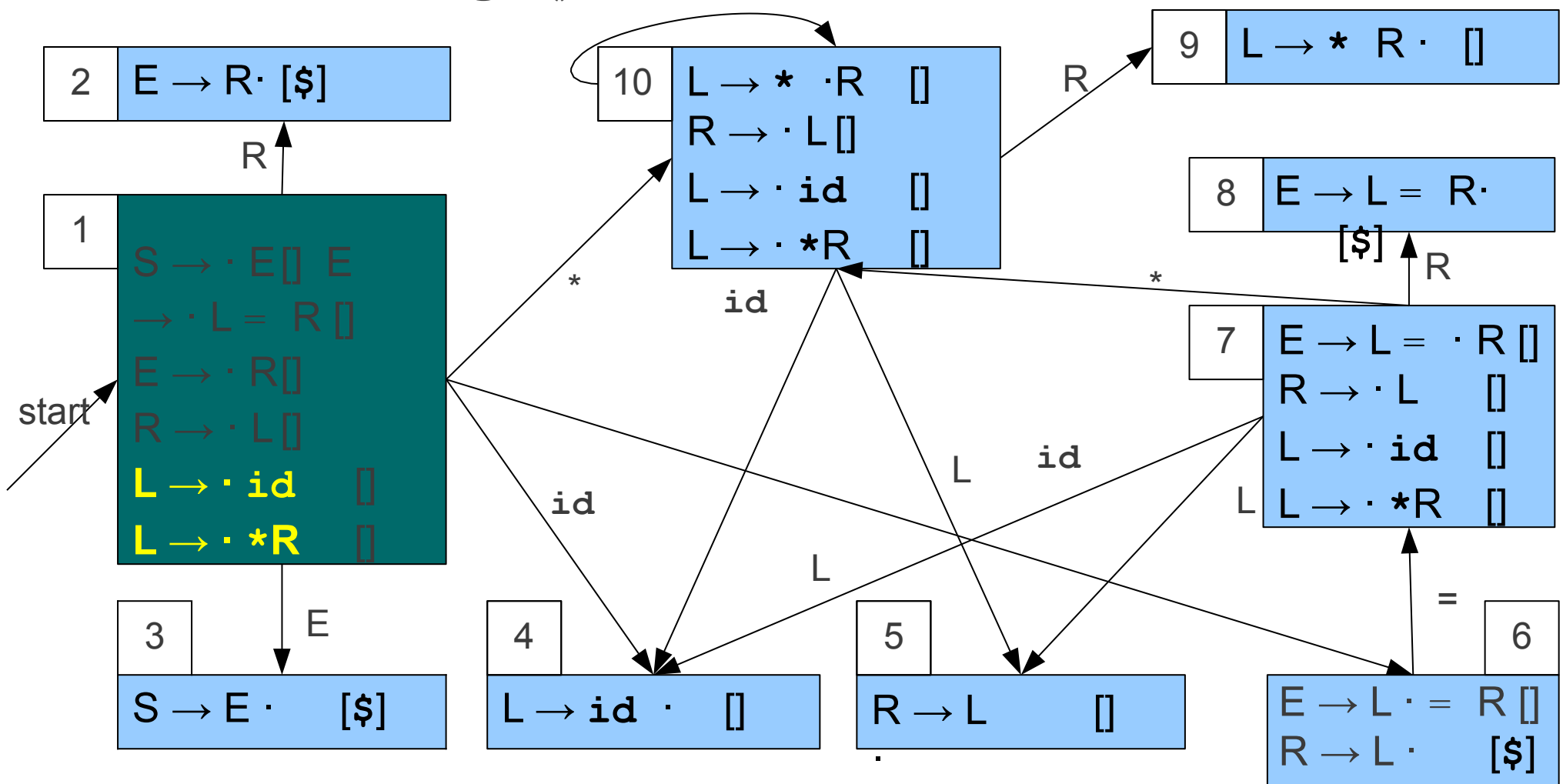
# Using Our FOLLOW Sets



$S_1$	$E_{1-3}$	$L_{1-6}$	$L_{7-5}$	$L_{10-5}$	$R_{1-2}$	$R_{7-8}$	$R_{10-9}$
\$	\$	=	=	=	\$	\$	=
		\$					

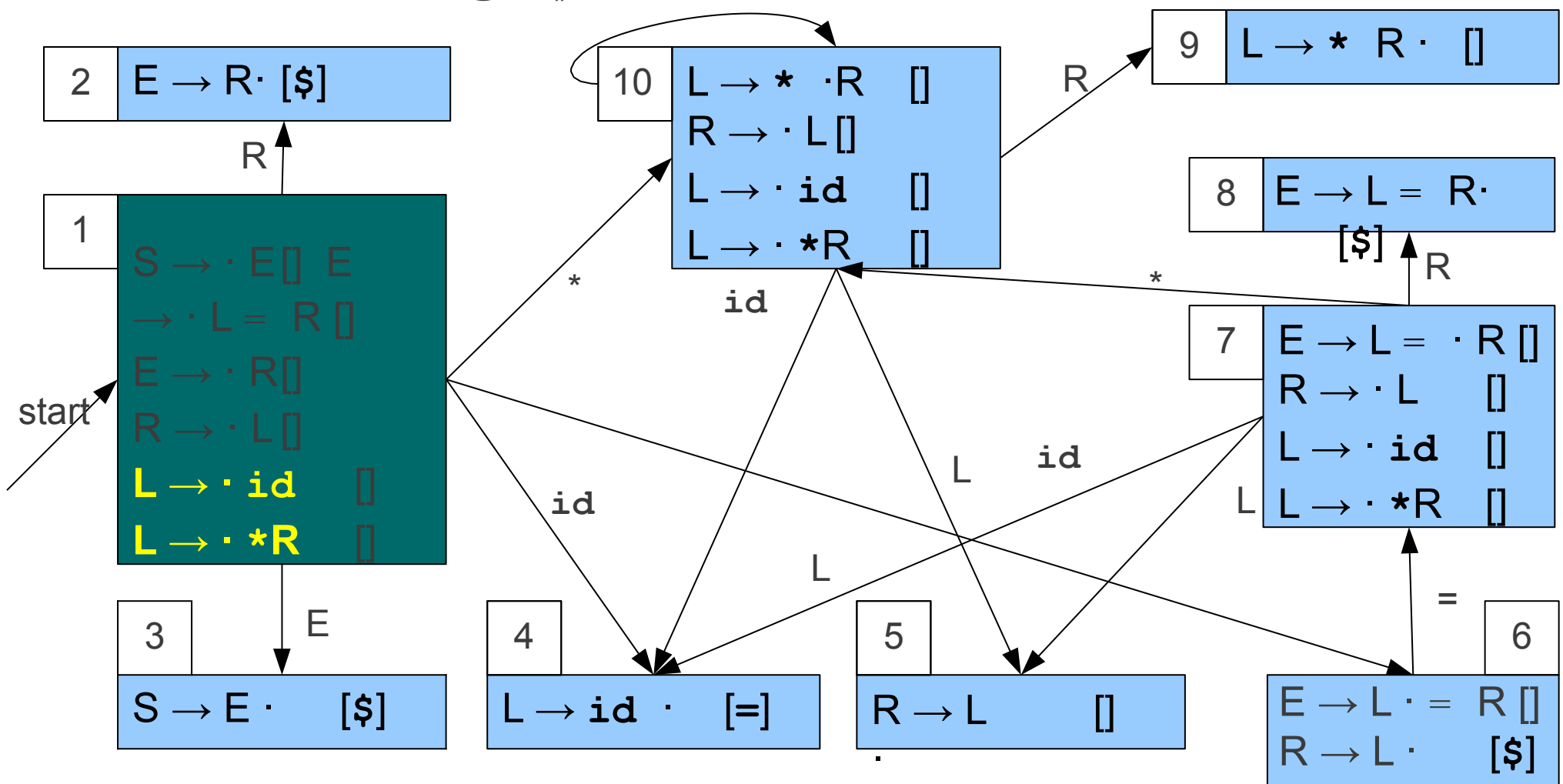


# Using Our FOLLOW Sets



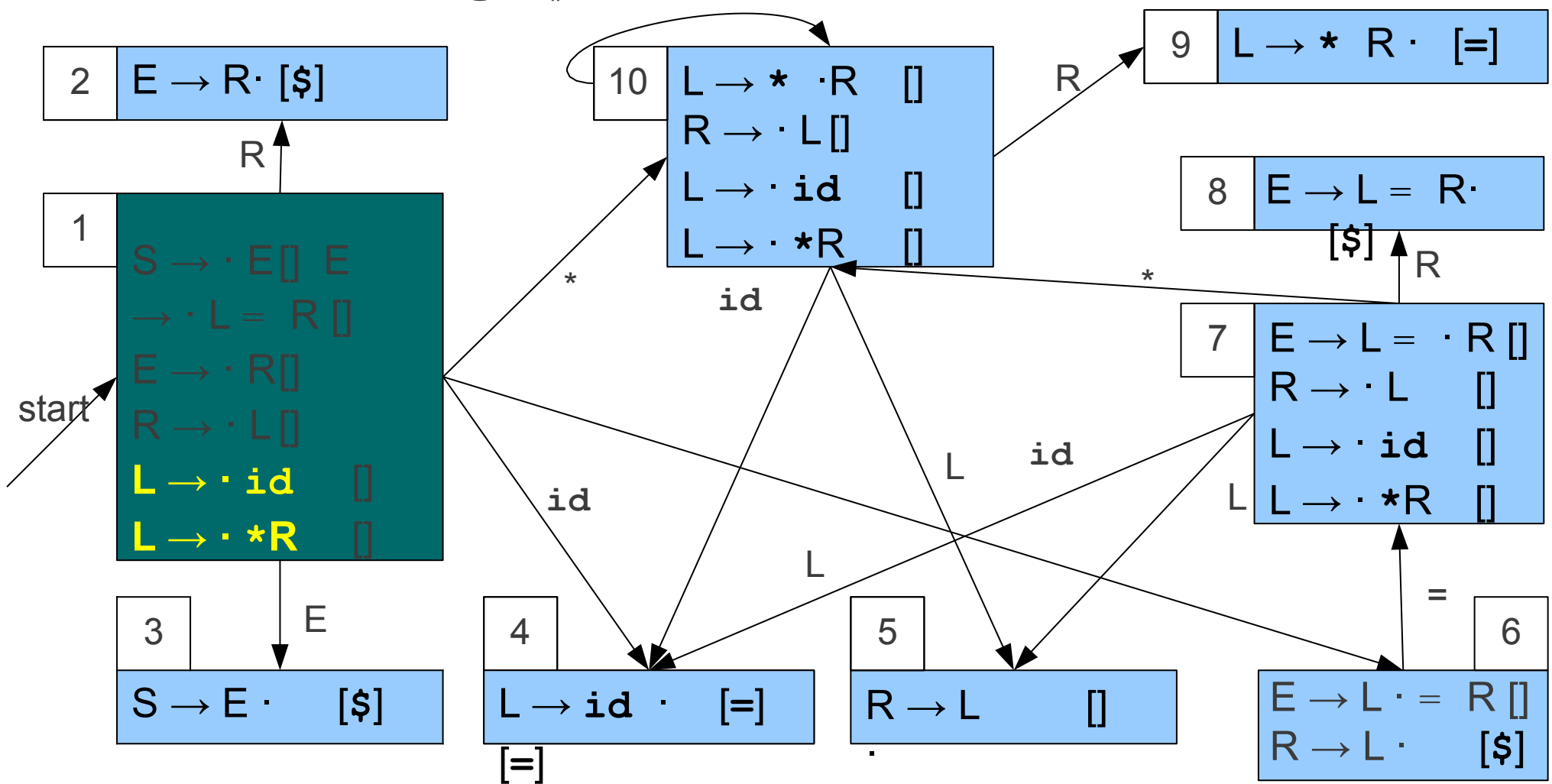
$S_1$	$E_{1-3}$	$L_{1-6}$	$L_{7-5}$	$L_{10-5}$	$R_{1-2}$	$R_{7-8}$	$R_{10-9}$
\$	\$	=	=	=	\$	\$	=
		\$					

# Using Our FOLLOW Sets



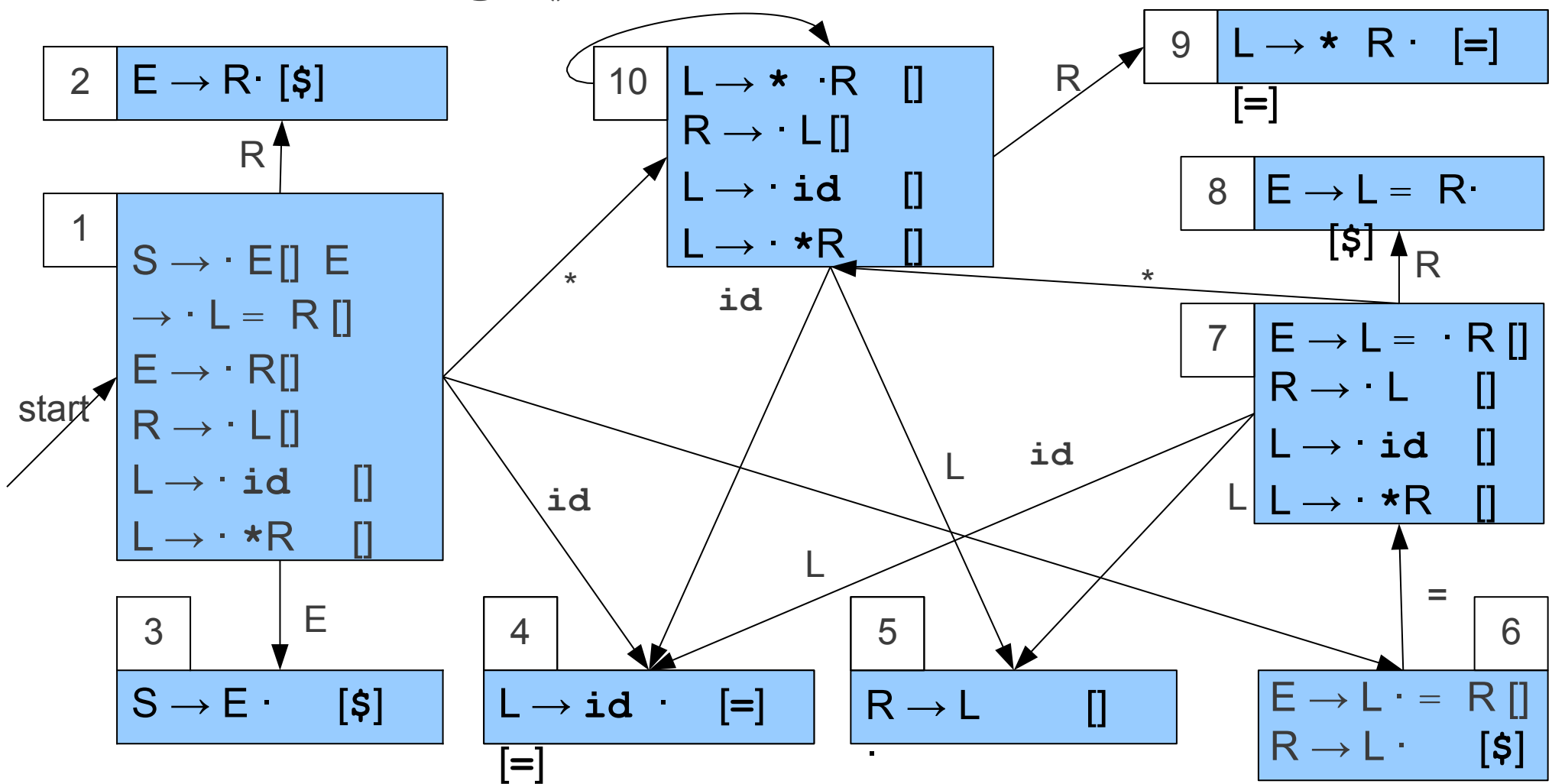
$S_1$	$E_{1-3}$	$L_{1-6}$	$L_{7-5}$	$L_{10-5}$	$R_{1-2}$	$R_{7-8}$	$R_{10-9}$
\$	\$	=	=	=	\$	\$	=
		\$					

# Using Our FOLLOW Sets



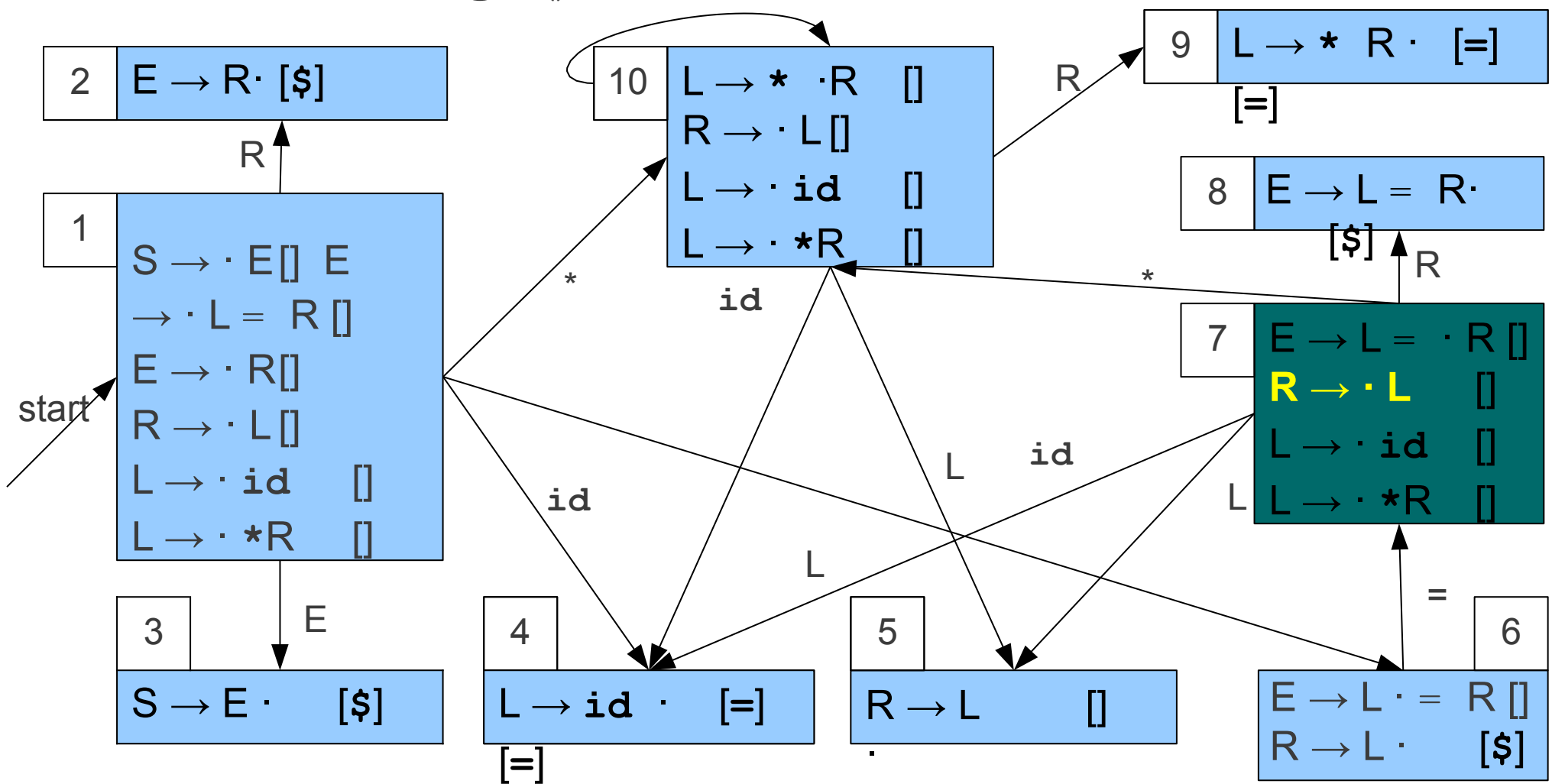
$S_1$	$E_{1-3}$	$L_{1-6}$	$L_{7-5}$	$L_{10-5}$	$R_{1-2}$	$R_{7-8}$	$R_{10-9}$
\$	\$	=	=	=	\$	\$	=
		\$					

# Using Our FOLLOW Sets



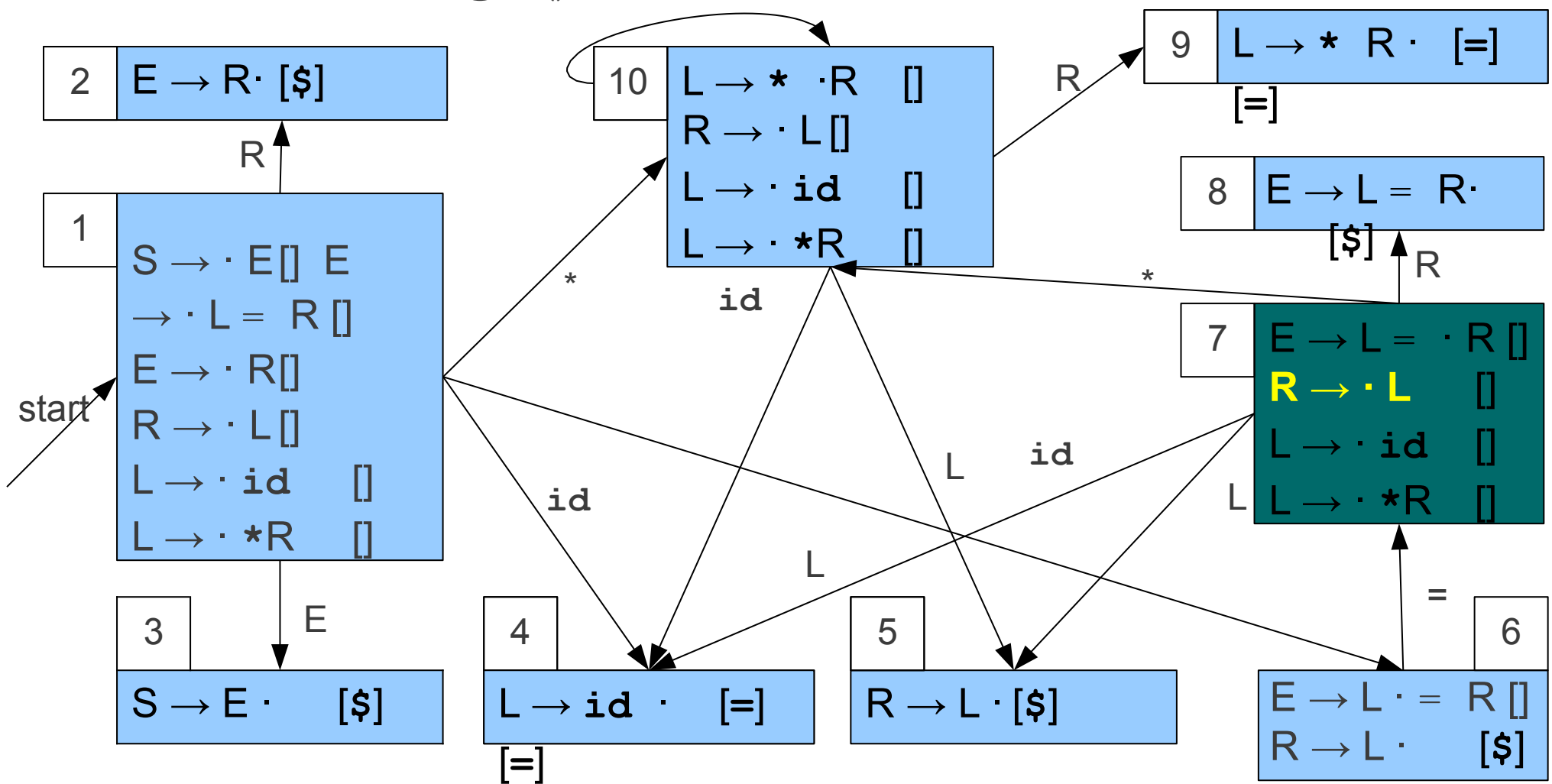
$S_1$	$E_{1-3}$	$L_{1-6}$	$L_{7-5}$	$L_{10-5}$	$R_{1-2}$	$R_{7-8}$	$R_{10-9}$
\$	\$	=	=	=	\$	\$	=
		\$					

# Using Our FOLLOW Sets



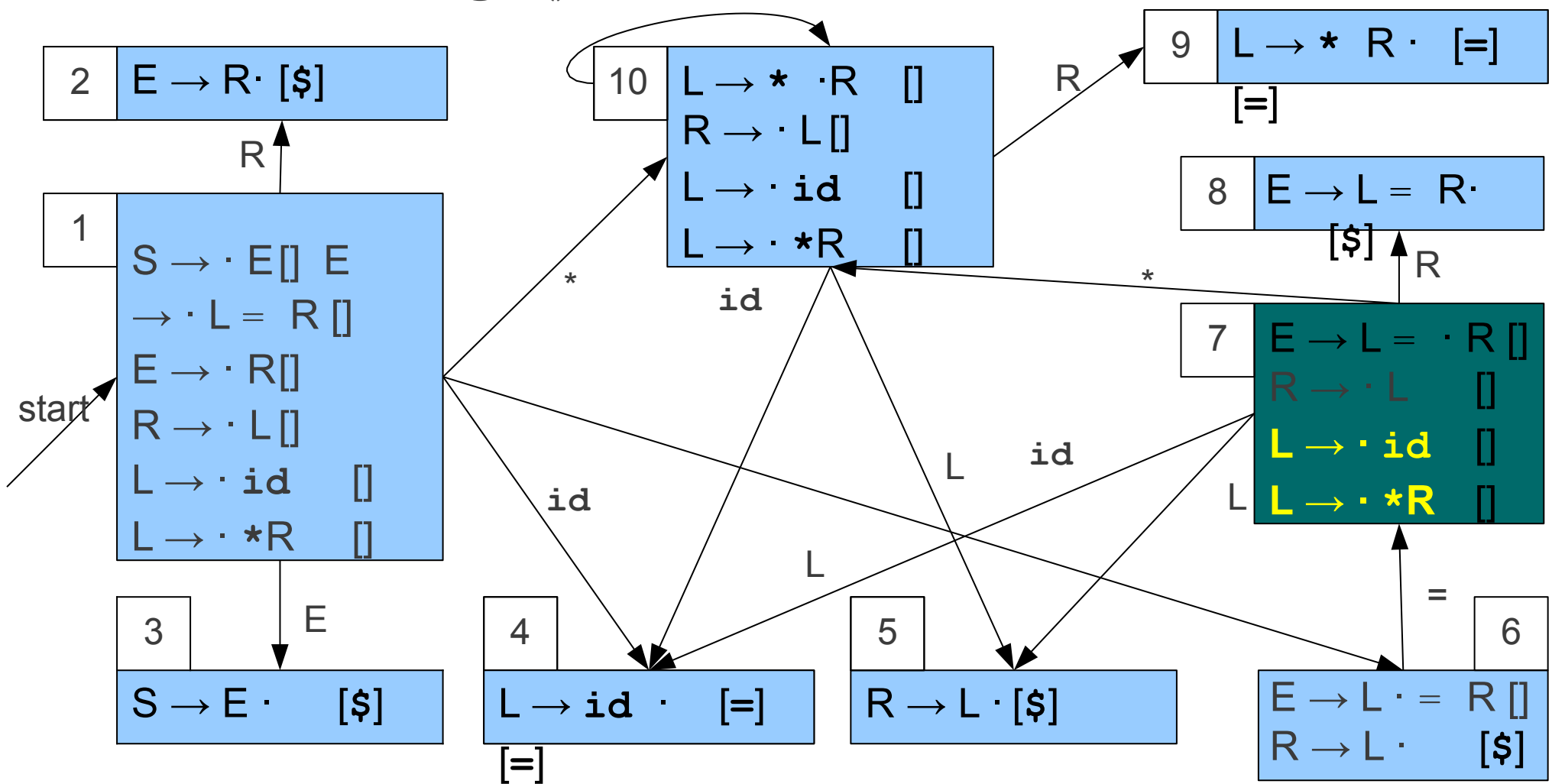
$S_1$	$E_{1-3}$	$L_{1-6}$	$L_{7-5}$	$L_{10-5}$	$R_{1-2}$	$R_{7-8}$	$R_{10-9}$
\$	\$	=	=	=	\$	\$	=
		\$					

# Using Our FOLLOW Sets



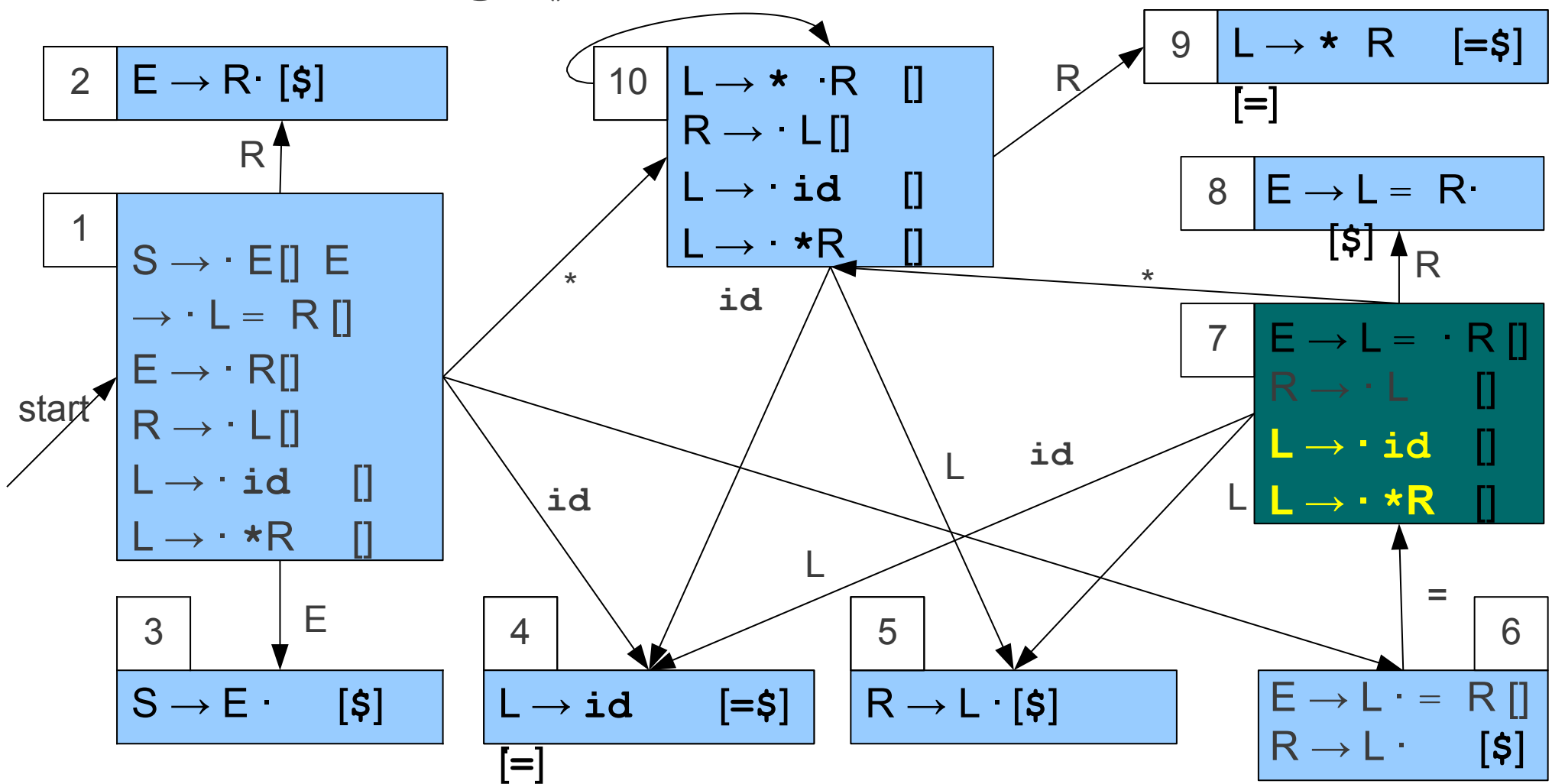
$S_1$	$E_{1-3}$	$L_{1-6}$	$L_{7-5}$	$L_{10-5}$	$R_{1-2}$	$R_{7-8}$	$R_{10-9}$
\$	\$	=	=	=	\$	\$	=
		\$					

# Using Our FOLLOW Sets



$S_1$	$E_{1-3}$	$L_{1-6}$	$L_{7-5}$	$L_{10-5}$	$R_{1-2}$	$R_{7-8}$	$R_{10-9}$
\$	\$	=	=	=	\$	\$	=
		\$					

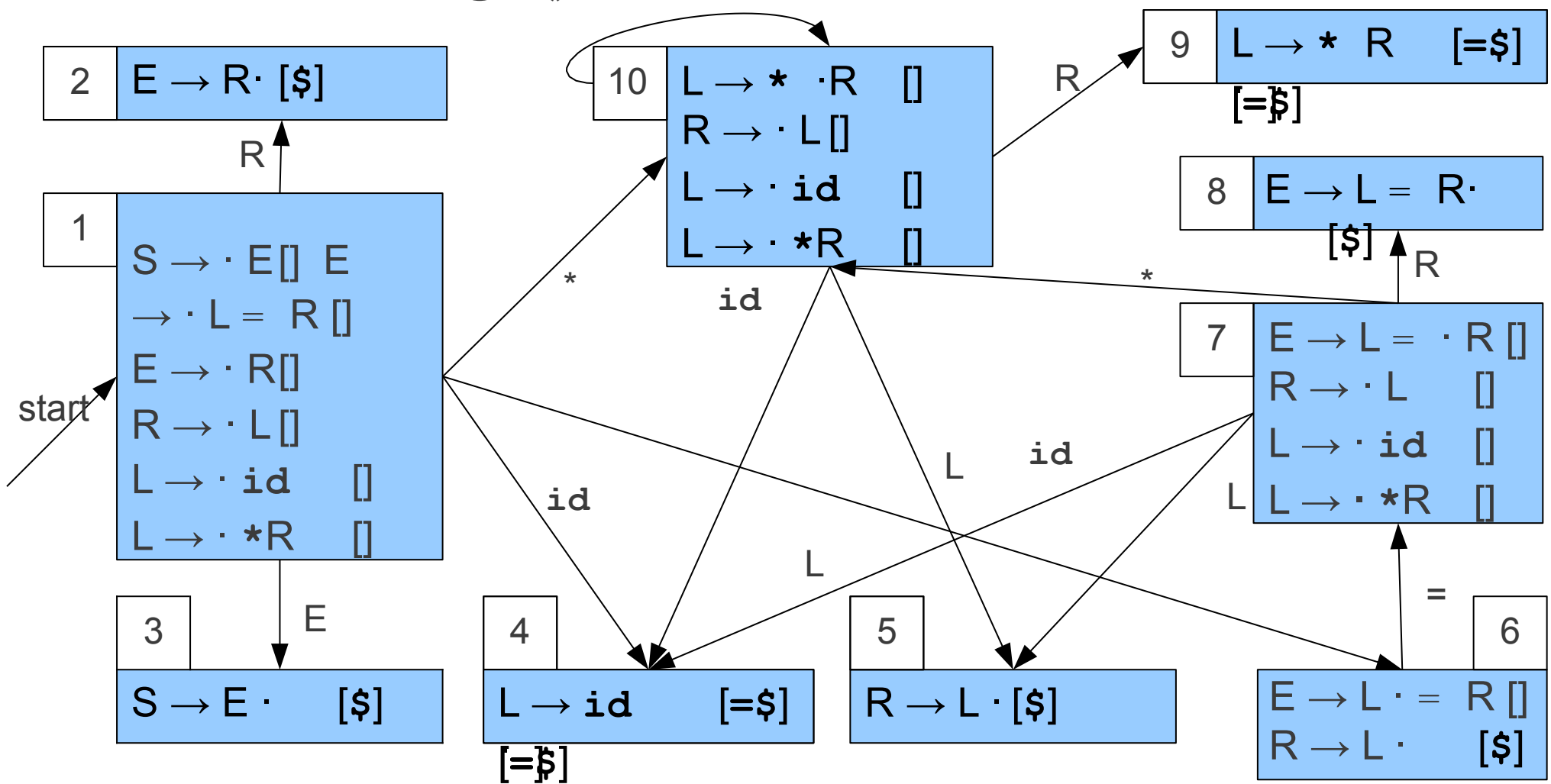
# Using Our FOLLOW Sets



$S_1$	$E_{1-3}$	$L_{1-6}$	$L_{7-5}$	$L_{10-5}$	$R_{1-2}$	$R_{7-8}$	$R_{10-9}$
\$	\$	=	=	=	\$	\$	=
		\$					

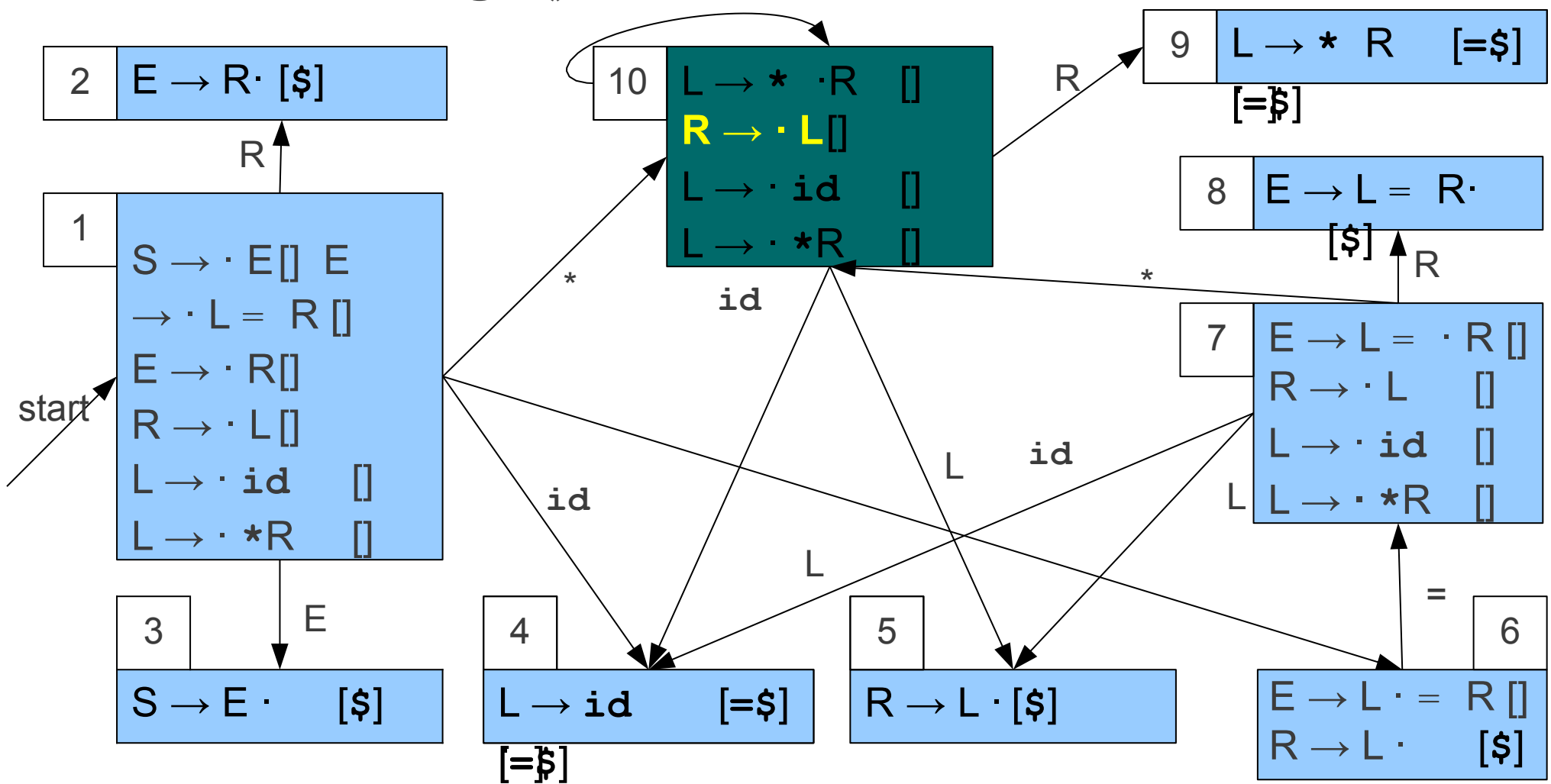


# Using Our FOLLOW Sets



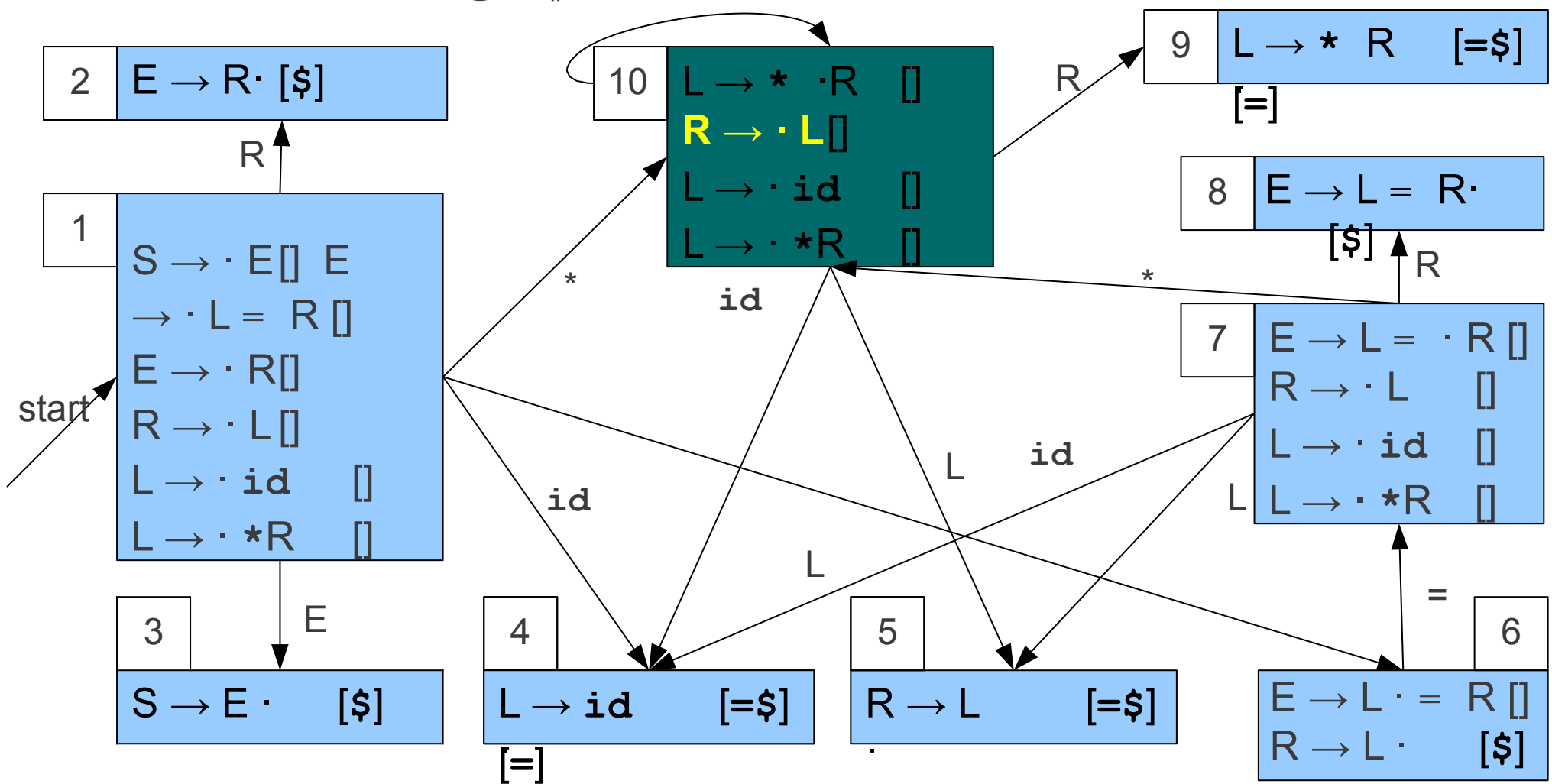
$S_1$	$E_{1-3}$	$L_{1-6}$	$L_{7-5}$	$L_{10-5}$	$R_{1-2}$	$R_{7-8}$	$R_{10-9}$
\$	\$	=	=	=	\$	\$	=
		\$					

# Using Our FOLLOW Sets



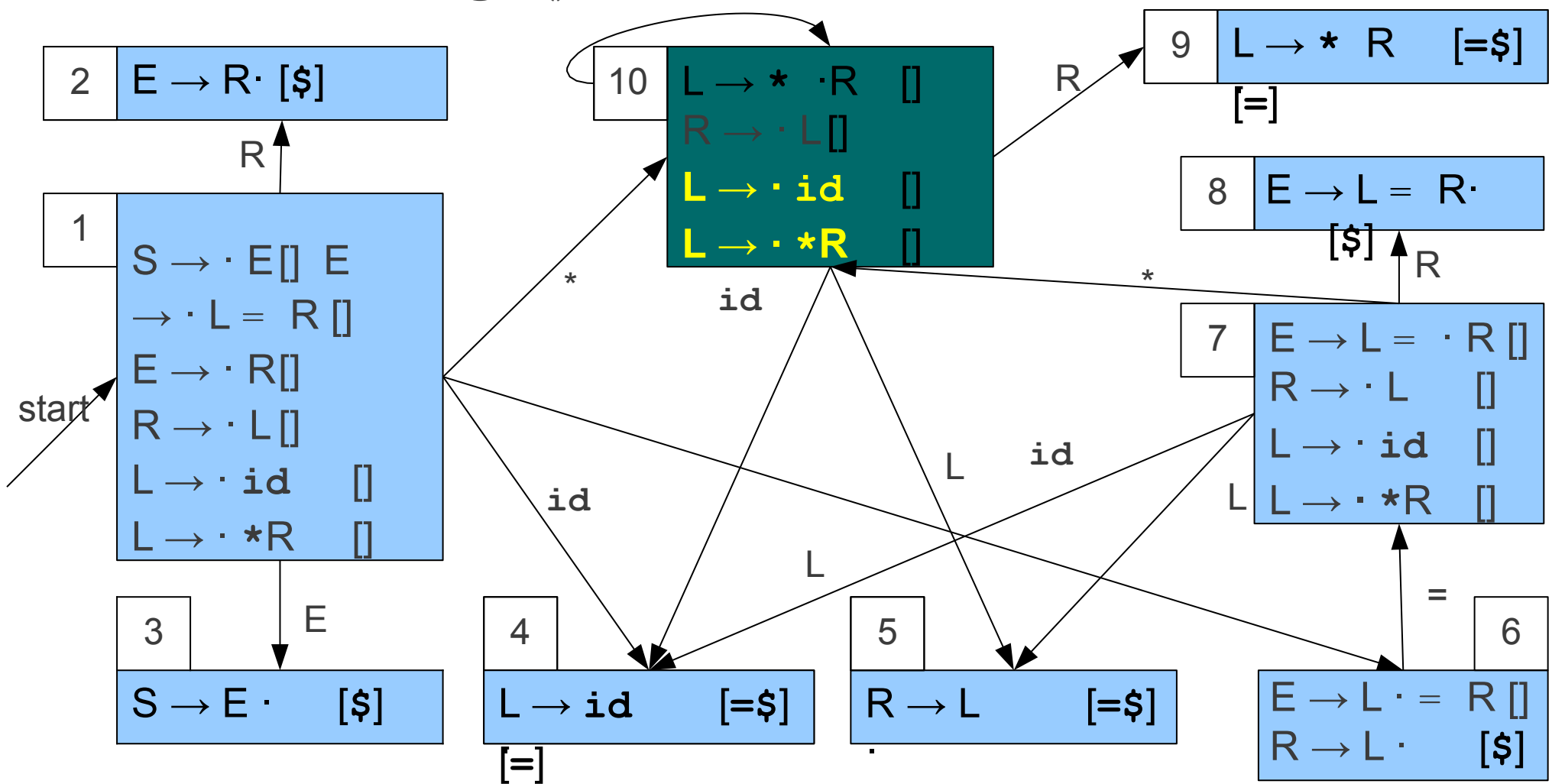
$S_1$	$E_{1-3}$	$L_{1-6}$	$L_{7-5}$	$L_{10-5}$	$R_{1-2}$	$R_{7-8}$	$R_{10-9}$
\$	\$	=	=	=	\$	\$	=
		\$					

# Using Our FOLLOW Sets



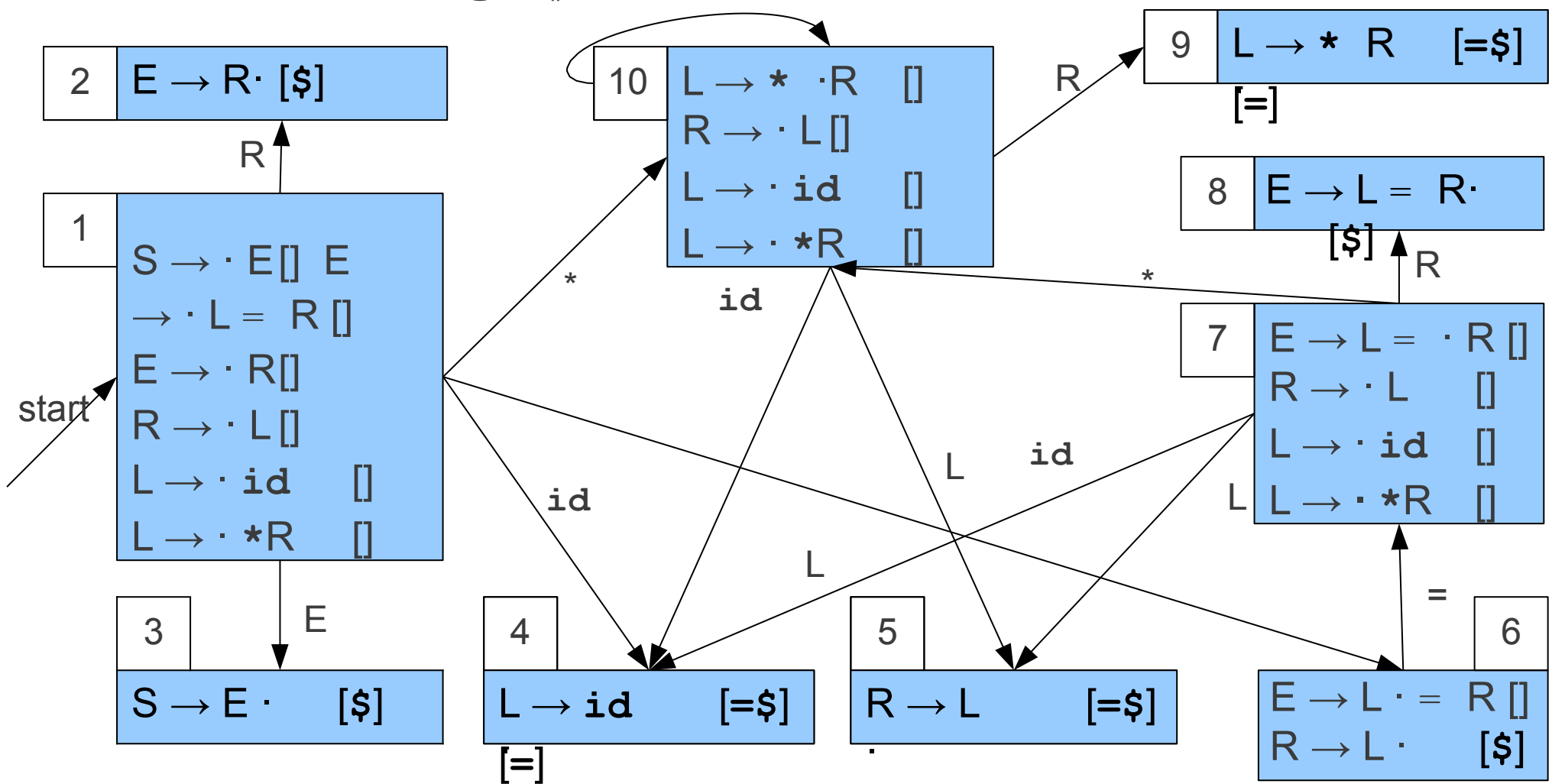
$S_1$	$E_{1-3}$	$L_{1-6}$	$L_{7-5}$	$L_{10-5}$	$R_{1-2}$	$R_{7-8}$	$R_{10-9}$
\$	\$	=	=	=	\$	\$	=
		\$					

# Using Our FOLLOW Sets



$S_1$	$E_{1-3}$	$L_{1-6}$	$L_{7-5}$	$L_{10-5}$	$R_{1-2}$	$R_{7-8}$	$R_{10-9}$
\$	\$	=	=	=	\$	\$	=
		\$					

# Using Our FOLLOW Sets



$S_1$	$E_{1-3}$	$L_{1-6}$	$L_{7-5}$	$L_{10-5}$	$R_{1-2}$	$R_{7-8}$	$R_{10-9}$
\$	\$	=	=	=	\$	\$	=
		\$					

# Propagating Changes

- For each item  $A \rightarrow \cdot \omega$  in a state  $q$ :
  - Let  $A_{q-r}$  be the nonterminal corresponding to  $A$  following the transition out of  $q$  into some state  $r$ .
  - Trace through the automaton along the path labeled by  $\omega$ . This will lead to a state containing an item  $A \rightarrow \omega \cdot$ .
  - Add to the lookahead of  $A \rightarrow \omega \cdot$  the contents of  $\text{FOLLOW}(A_{q-r})$

# LALR(1)-by-SLR(1)

Fast and simple construction of LALR(1)

- lookaheads:
  - Construct the LR(0) automaton for the grammar.
    - Construct the augmented grammar by replacing
      - nonterminals with new nonterminals based on the LR(0) transitions.
    - Compute the FOLLOW sets for these nonterminals.
    - Propagate changes through the LR(0) automaton.
  - **Theorem** (*Bermudez and Logothetis*): This correctly computes LALR(1) lookaheads.

# Summary of LALR(1)

- Along with  $LL(k)$ , one of the most popular parsing algorithms in use today.
- Produced by the **bison** parser generator; rarely generated by hand.
- Can handle most, but not all, LR(1) languages.



# Practical Concerns

# Where Theory Meets Practice

- We've just covered eight powerful parsing algorithms:

- Leftmost DFS
- Syntax Graph
- LL(1)
- RD-Parser
- LR(0)
- SLR(1)
- LALR(1)
- LR(1)

How do we make them  
work in practice?

# Two Practical Concerns

- **Ambiguity**
  - Real grammars are often ambiguous.
  - Also they are more readable.
  - Programmers are *terrible* at eliminating it.
  - How do you build a parser to try to combat it?
- **Error-handling**
  - How do you report errors intelligently?
  - How do you continue parsing after an error?

# Ambiguity and Predictive Parsing

- The parsers we have seen so far only work on unambiguous grammars.
  - Intuitively: if grammar is ambiguous, cannot uniquely guess which production/reduction to use.
- Most grammars for programming languages, unless cleverly written, are ambiguous.
- How can we handle this?

# Parsing Ambiguous Grammars

Consider this simple grammar for arithmetic expressions:

$$S \rightarrow E$$
$$E \rightarrow E + E$$
$$E \rightarrow E * E$$
$$E \rightarrow \text{int}$$
$$E \rightarrow (E)$$

This grammar is ambiguous.

e.g. Two trees for `int + int * int`

What happens if we try parsing it?

# SLR(1) Parsing with Ambiguity

1
$S \rightarrow \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

2
$S \rightarrow E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E$
$\cdot * E$

6
$E \rightarrow E * E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E$
$\cdot * E$

1.  $S \rightarrow E$
2.  $E \rightarrow E + E$
3.  $E \rightarrow E * E$
4.  $E \rightarrow (E)$
5.  $E \rightarrow int$

3
$E \rightarrow E + \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

7
$E \rightarrow (\cdot E)$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

4
$E \rightarrow E * \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

8
$E \rightarrow (E \cdot )$
$E \rightarrow E \cdot +$
$E \rightarrow E$
$\cdot * E$

5
$E \rightarrow E + E$
$\cdot E \rightarrow E \cdot +$
$E \rightarrow E$
$\cdot * E$

9
$E \rightarrow (E) \cdot$
10
$E \rightarrow int \cdot$

$FOLLOW(S) = \{ \$ \}$

$FOLLOW(E) = \{ +, *, ), \$ \}$

	int	+	*	(	)	\$	E
1							
2							
3							
4							
5							
6							
7							
8							
9							
10							

# SLR(1) Parsing with Ambiguity

1
$S \rightarrow \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

2
$S \rightarrow E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E$
$\cdot * E$

6
$E \rightarrow E * E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E$
$\cdot * E$

1.  $S \rightarrow E$
2.  $E \rightarrow E + E$
3.  $E \rightarrow E * E$
4.  $E \rightarrow (E)$
5.  $E \rightarrow int$

3
$E \rightarrow E + \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

7
$E \rightarrow (\cdot E)$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

$E \rightarrow \cdot (E)$

4
$E \rightarrow E * \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

8
$E \rightarrow (E \cdot)$
$E \rightarrow E \cdot +$
$E \rightarrow E$
$\cdot * E$

5
$E \rightarrow E + E$
$\cdot E \rightarrow E \cdot +$
$E \rightarrow E$
$\cdot * E$

9
$E \rightarrow (E) \cdot$
10
$E \rightarrow int \cdot$

$FOLLOW(S) = \{ \$ \}$

$FOLLOW(E) = \{ +, *, ), \$ \}$

	int	+	*	(	)	\$	E
1							
2							
3							
4							
5							
6							
7							
8							
9							
10							

# SLR(1) Parsing with Ambiguity

1
$S \rightarrow \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

2
$S \rightarrow E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$\cdot * E$

6
$E \rightarrow E * E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

1.  $S \rightarrow E$
2.  $E \rightarrow E + E$
3.  $E \rightarrow E * E$
4.  $E \rightarrow (E)$
5.  $E \rightarrow int$

3
$E \rightarrow E + \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

7
$E \rightarrow (\cdot E)$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

4
$E \rightarrow E * \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

8
$E \rightarrow (E \cdot )$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$\cdot * E$

5
$E \rightarrow E + E$
$\cdot E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

9
$E \rightarrow (E) \cdot$
10
$E \rightarrow int \cdot$

$FOLLOW(S) = \{ \$ \}$

$FOLLOW(E) = \{ +, *, ), \$ \}$

	int	+	*	(	)	\$	E
1	s10						
2							
3							
4							
5							
6							
7							
8							
9							
10							



# SLR(1) Parsing with Ambiguity

1
$S \rightarrow \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

2
$S \rightarrow E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$\cdot * E$

6
$E \rightarrow E * E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

1.  $S \rightarrow E$
2.  $E \rightarrow E + E$
3.  $E \rightarrow E * E$
4.  $E \rightarrow (E)$
5.  $E \rightarrow int$

3
$E \rightarrow E + \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

7
$E \rightarrow (\cdot E)$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

4
$E \rightarrow E * \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

8
$E \rightarrow (E \cdot )$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$\cdot * E$

5
$E \rightarrow E + E$
$\cdot E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

9
$E \rightarrow (E) \cdot$
10
$E \rightarrow int \cdot$

$FOLLOW(S) = \{ \$ \}$

$FOLLOW(E) = \{ +, *, ), \$ \}$

	int	+	*	(	)	\$	E
1	s10			s7			
2							
3							
4							
5							
6							
7							
8							
9							
10							

# SLR(1) Parsing with Ambiguity

1
$S \rightarrow \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

2
$S \rightarrow E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E$
$\cdot * E$

6
$E \rightarrow E * E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E$
$\cdot * E$

1.  $S \rightarrow E$
2.  $E \rightarrow E + E$
3.  $E \rightarrow E * E$
4.  $E \rightarrow (E)$
5.  $E \rightarrow int$

3
$E \rightarrow E + \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

7
$E \rightarrow (\cdot E)$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

4
$E \rightarrow E * \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

8
$E \rightarrow (E \cdot )$
$E \rightarrow E \cdot +$
$E \rightarrow E$
$\cdot * E$

5
$E \rightarrow E + E$
$\cdot E \rightarrow E \cdot +$
$E \rightarrow E$
$\cdot * E$

9
$E \rightarrow (E) \cdot$
10
$E \rightarrow int \cdot$

$FOLLOW(S) = \{ \$ \}$

$FOLLOW(E) = \{ +, *, ), \$ \}$

	int	+	*	(	)	\$	E
1	s10			s7			s2
2							
3							
4							
5							
6							
7							
8							
9							
10							

# SLR(1) Parsing with Ambiguity

1
$S \rightarrow \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

2
$S \rightarrow E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E$
$\cdot * E$

6
$E \rightarrow E * E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E$
$\cdot * E$

1.  $S \rightarrow E$
2.  $E \rightarrow E + E$
3.  $E \rightarrow E * E$
4.  $E \rightarrow (E)$
5.  $E \rightarrow int$

3
$E \rightarrow E + \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

7
$E \rightarrow (\cdot E)$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

4
$E \rightarrow E * \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

8
$E \rightarrow (E \cdot )$
$E \rightarrow E \cdot +$
$E \rightarrow E$
$\cdot * E$

5
$E \rightarrow E + E$
$\cdot E \rightarrow E \cdot +$
$E \rightarrow E$
$\cdot * E$

9
$E \rightarrow (E) \cdot$
10
$E \rightarrow int \cdot$

$FOLLOW(S) = \{ \$ \}$

$FOLLOW(E) = \{ +, *, ), \$ \}$

	int	+	*	(	)	\$	E
1	s10			s7			s2
2							
3							
4							
5							
6							
7							
8							
9							
10							

# SLR(1) Parsing with Ambiguity

1
$S \rightarrow \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

2
$S \rightarrow E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E$
$\cdot * E$

6
$E \rightarrow E * E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E$
$\cdot * E$

1.  $S \rightarrow E$
2.  $E \rightarrow E + E$
3.  $E \rightarrow E * E$
4.  $E \rightarrow (E)$
5.  $E \rightarrow int$

3
$E \rightarrow E + \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

7
$E \rightarrow (\cdot E)$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

4
$E \rightarrow E * \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

8
$E \rightarrow (E \cdot )$
$E \rightarrow E \cdot +$
$E \rightarrow E$
$\cdot * E$

5
$E \rightarrow E + E$
$\cdot E \rightarrow E \cdot +$
$E \rightarrow E$
$\cdot * E$

9
$E \rightarrow (E) \cdot$
10
$E \rightarrow int \cdot$

$FOLLOW(S) = \{ \$ \}$

$FOLLOW(E) = \{ +, *, ), \$ \}$

	int	+	*	(	)	\$	E
1	s10			s7			s2
2		s3					
3							
4							
5							
6							
7							
8							
9							
10							

# SLR(1) Parsing with Ambiguity

1
$S \rightarrow \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

2
$S \rightarrow E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E$
$\cdot * E$

6
$E \rightarrow E * E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E$
$\cdot * E$

1.  $S \rightarrow E$
2.  $E \rightarrow E + E$
3.  $E \rightarrow E * E$
4.  $E \rightarrow (E)$
5.  $E \rightarrow int$

3
$E \rightarrow E + \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

7
$E \rightarrow (\cdot E)$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

4
$E \rightarrow E * \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

8
$E \rightarrow (E \cdot )$
$E \rightarrow E \cdot +$
$E \rightarrow E$
$\cdot * E$

5
$E \rightarrow E + E$
$\cdot E \rightarrow E \cdot +$
$E \rightarrow E$
$\cdot * E$

9
$E \rightarrow (E) \cdot$
10
$E \rightarrow int \cdot$

$FOLLOW(S) = \{ \$ \}$

$FOLLOW(E) = \{ +, *, ), \$ \}$

	int	+	*	(	)	\$	E
1	s10			s7			s2
2		s3	s4				
3							
4							
5							
6							
7							
8							
9							
10							

# SLR(1) Parsing with Ambiguity

1
$S \rightarrow \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

2
$S \rightarrow E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E$
$\cdot * E$

6
$E \rightarrow E * E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E$
$\cdot * E$

1.  $S \rightarrow E$
2.  $E \rightarrow E + E$
3.  $E \rightarrow E * E$
4.  $E \rightarrow (E)$
5.  $E \rightarrow int$

3
$E \rightarrow E + \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

7
$E \rightarrow (\cdot E)$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

4
$E \rightarrow E * \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

8
$E \rightarrow (E \cdot )$
$E \rightarrow E \cdot +$
$E \rightarrow E$
$\cdot * E$

5
$E \rightarrow E + E$
$\cdot E \rightarrow E \cdot +$
$E \rightarrow E$
$\cdot * E$

9
$E \rightarrow (E) \cdot$
10
$E \rightarrow int \cdot$

$FOLLOW(S) = \{ \$ \}$

$FOLLOW(E) = \{ +, *, ), \$ \}$

	int	+	*	(	)	\$	E
1	s10			s7			s2
2		s3	s4			acc	
3							
4							
5							
6							
7							
8							
9							
10							

# SLR(1) Parsing with Ambiguity

1
$S \rightarrow \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

2
$S \rightarrow E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E$
$\cdot * E$

6
$E \rightarrow E * E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E$
$\cdot * E$

1.  $S \rightarrow E$
2.  $E \rightarrow E + E$
3.  $E \rightarrow E * E$
4.  $E \rightarrow (E)$
5.  $E \rightarrow int$

3
$E \rightarrow E + \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

7
$E \rightarrow (\cdot E)$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

$E \rightarrow \cdot (E)$

4
$E \rightarrow E * \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

8
$E \rightarrow (E \cdot )$
$E \rightarrow E \cdot +$
$E \rightarrow E$
$\cdot * E$

5
$E \rightarrow E + E$
$\cdot E \rightarrow E \cdot +$
$E \rightarrow E$
$\cdot * E$

9
$E \rightarrow (E) \cdot$
10
$E \rightarrow int \cdot$

$FOLLOW(S) = \{ \$ \}$

$FOLLOW(E) = \{ +, *, ), \$ \}$

	int	+	*	(	)	\$	E
1	s10			s7			s2
2		s3	s4			acc	
3							
4							
5							
6							
7							
8							
9							
10							

# SLR(1) Parsing with Ambiguity

1
$S \rightarrow \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

2
$S \rightarrow E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E$
$\cdot * E$

6
$E \rightarrow E * E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E$
$\cdot * E$

1.  $S \rightarrow E$
2.  $E \rightarrow E + E$
3.  $E \rightarrow E * E$
4.  $E \rightarrow (E)$
5.  $E \rightarrow int$

3
$E \rightarrow E + \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

7
$E \rightarrow (\cdot E)$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

4
$E \rightarrow E * \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

8
$E \rightarrow (E \cdot )$
$E \rightarrow E \cdot +$
$E \rightarrow E$
$\cdot * E$

5
$E \rightarrow E + E$
$\cdot E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

9
$E \rightarrow (E) \cdot$
10
$E \rightarrow int \cdot$

$FOLLOW(S) = \{ \$ \}$

$FOLLOW(E) = \{ +, *, ), \$ \}$

	int	+	*	(	)	\$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10						
4							
5							
6							
7							
8							
9							
10							



# SLR(1) Parsing with Ambiguity

1
$S \rightarrow \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

2
$S \rightarrow E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$\cdot * E$

6
$E \rightarrow E * E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$\cdot * E$

1.  $S \rightarrow E$
2.  $E \rightarrow E + E$
3.  $E \rightarrow E * E$
4.  $E \rightarrow (E)$
5.  $E \rightarrow int$

3
$E \rightarrow E + \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

7
$E \rightarrow (\cdot E)$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

$E \rightarrow \cdot (E)$

4
$E \rightarrow E * \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

8
$E \rightarrow (E \cdot)$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$\cdot * E$

5
$E \rightarrow E + E$
$\cdot E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

9
$E \rightarrow (E) \cdot$
10
$E \rightarrow int \cdot$

$FOLLOW(S) = \{ \$ \}$

$FOLLOW(E) = \{ +, *, ), \$ \}$

	int	+	*	(	)	\$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			
4							
5							
6							
7							
8							
9							
10							

# SLR(1) Parsing with Ambiguity

1
$S \rightarrow \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

2
$S \rightarrow E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E$
$\cdot * E$

6
$E \rightarrow E * E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E$
$\cdot * E$

1.  $S \rightarrow E$
2.  $E \rightarrow E + E$
3.  $E \rightarrow E * E$
4.  $E \rightarrow (E)$
5.  $E \rightarrow int$

3
$E \rightarrow E + \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

7
$E \rightarrow (\cdot E)$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

$E \rightarrow \cdot (E)$

4
$E \rightarrow E * \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

8
$E \rightarrow (E \cdot )$
$E \rightarrow E \cdot +$
$E \rightarrow E$
$\cdot * E$

5
$E \rightarrow E + E$
$\cdot E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

9
$E \rightarrow (E) \cdot$
10
$E \rightarrow int \cdot$

$FOLLOW(S) = \{ \$ \}$

$FOLLOW(E) = \{ +, *, ), \$ \}$

	int	+	*	(	)	\$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4							
5							
6							
7							
8							
9							
10							

# SLR(1) Parsing with Ambiguity

1
$S \rightarrow \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

2
$S \rightarrow E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$\cdot * E$

6
$E \rightarrow E * E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

1.  $S \rightarrow E$
2.  $E \rightarrow E + E$
3.  $E \rightarrow E * E$
4.  $E \rightarrow (E)$
5.  $E \rightarrow int$

3
$E \rightarrow E + \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

7
$E \rightarrow (\cdot E)$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

$E \rightarrow \cdot (E)$

4
$E \rightarrow E * \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

8
$E \rightarrow (E \cdot)$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$\cdot * E$

5
$E \rightarrow E + E$
$\cdot E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

9
$E \rightarrow (E) \cdot$
10
$E \rightarrow int \cdot$

$FOLLOW(S) = \{ \$ \}$

$FOLLOW(E) = \{ +, *, ), \$ \}$

	int	+	*	(	)	\$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4							
5							
6							
7							
8							
9							
10							

# SLR(1) Parsing with Ambiguity

1
$S \rightarrow \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

2
$S \rightarrow E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$\cdot * E$

6
$E \rightarrow E * E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

1.  $S \rightarrow E$
2.  $E \rightarrow E + E$
3.  $E \rightarrow E * E$
4.  $E \rightarrow (E)$
5.  $E \rightarrow int$

3
$E \rightarrow E + \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

7
$E \rightarrow (\cdot E)$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

4
$E \rightarrow E * \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

8
$E \rightarrow (E \cdot )$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$\cdot * E$

5
$E \rightarrow E + E$
$\cdot E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

9
$E \rightarrow (E) \cdot$
10
$E \rightarrow int \cdot$

$FOLLOW(S) = \{ \$ \}$

$FOLLOW(E) = \{ +, *, ), \$ \}$

	int	+	*	(	)	\$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10						
5							
6							
7							
8							
9							
10							

# SLR(1) Parsing with Ambiguity

1
$S \rightarrow \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

2
$S \rightarrow E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$\cdot * E$

6
$E \rightarrow E * E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

1.  $S \rightarrow E$
2.  $E \rightarrow E + E$
3.  $E \rightarrow E * E$
4.  $E \rightarrow (E)$
5.  $E \rightarrow int$

3
$E \rightarrow E + \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

7
$E \rightarrow (\cdot E)$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

$E \rightarrow \cdot (E)$

4
$E \rightarrow E * \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

8
$E \rightarrow (E \cdot)$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$\cdot * E$

5
$E \rightarrow E + E$
$\cdot E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

9
$E \rightarrow (E) \cdot$
10
$E \rightarrow int \cdot$

$FOLLOW(S) = \{ \$ \}$

$FOLLOW(E) = \{ +, *, ), \$ \}$

	int	+	*	(	)	\$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			
5							
6							
7							
8							
9							
10							

# SLR(1) Parsing with Ambiguity

1
$S \rightarrow \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

2
$S \rightarrow E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$\cdot * E$

6
$E \rightarrow E * E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$\cdot * E$

1.  $S \rightarrow E$
2.  $E \rightarrow E + E$
3.  $E \rightarrow E * E$
4.  $E \rightarrow (E)$
5.  $E \rightarrow int$

3
$E \rightarrow E + \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

7
$E \rightarrow (\cdot E)$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

$E \rightarrow \cdot (E)$

4
$E \rightarrow E * \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

8
$E \rightarrow (E \cdot)$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$\cdot * E$

5
$E \rightarrow E + E$
$\cdot E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

9
$E \rightarrow (E) \cdot$
10
$E \rightarrow int \cdot$

$FOLLOW(S) = \{ \$ \}$

$FOLLOW(E) = \{ +, *, ), \$ \}$

	int	+	*	(	)	\$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5							
6							
7							
8							
9							
10							

# SLR(1) Parsing with Ambiguity

1
$S \rightarrow \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

2
$S \rightarrow E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E$
$\cdot * E$

6
$E \rightarrow E * E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E$
$\cdot * E$

1.  $S \rightarrow E$
2.  $E \rightarrow E + E$
3.  $E \rightarrow E * E$
4.  $E \rightarrow (E)$
5.  $E \rightarrow int$

3
$E \rightarrow E + \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

7
$E \rightarrow (\cdot E)$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

4
$E \rightarrow E * \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

8
$E \rightarrow (E \cdot )$
$E \rightarrow E \cdot +$
$E \rightarrow E$
$\cdot * E$

5
$E \rightarrow E + E$
$\cdot E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

9
$E \rightarrow (E) \cdot$
10
$E \rightarrow int \cdot$

$FOLLOW(S) = \{ \$ \}$

$FOLLOW(E) = \{ +, *, ), \$ \}$

	int	+	*	(	)	\$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5							
6							
7							
8							
9							
10							

# SLR(1) Parsing with Ambiguity

1
$S \rightarrow \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

2
$S \rightarrow E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$\cdot * E$

6
$E \rightarrow E * E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$\cdot * E$

1.  $S \rightarrow E$
2.  $E \rightarrow E + E$
3.  $E \rightarrow E * E$
4.  $E \rightarrow (E)$
5.  $E \rightarrow int$

3
$E \rightarrow E + \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

7
$E \rightarrow (\cdot E)$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

4
$E \rightarrow E * \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

8
$E \rightarrow (E \cdot)$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$\cdot * E$

5
$E \rightarrow E + E$
$\cdot E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$\cdot * E$

9
$E \rightarrow (E) \cdot$
10
$E \rightarrow int \cdot$

$FOLLOW(S) = \{ \$ \}$

$FOLLOW(E) = \{ +, *, ), \$ \}$

	int	+	*	(	)	\$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5		s3					
6							
7							
8							
9							
10							



# SLR(1) Parsing with Ambiguity

1
$S \rightarrow \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

2
$S \rightarrow E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$\cdot * E$

6
$E \rightarrow E * E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

1.  $S \rightarrow E$
2.  $E \rightarrow E + E$
3.  $E \rightarrow E * E$
4.  $E \rightarrow (E)$
5.  $E \rightarrow int$

3
$E \rightarrow E + \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

7
$E \rightarrow (\cdot E)$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

4
$E \rightarrow E * \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

8
$E \rightarrow (E \cdot )$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$\cdot * E$

5
$E \rightarrow E + E$
$\cdot E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

9
$E \rightarrow (E) \cdot$
10
$E \rightarrow int \cdot$

$FOLLOW(S) = \{ \$ \}$

$FOLLOW(E) = \{ +, *, ), \$ \}$

	int	+	*	(	)	\$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5		s3	s4				
6							
7							
8							
9							
10							

# SLR(1) Parsing with Ambiguity

1
$S \rightarrow \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

2
$S \rightarrow E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$\cdot * E$

6
$E \rightarrow E * E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$\cdot * E$

1.  $S \rightarrow E$
2.  $E \rightarrow E + E$
3.  $E \rightarrow E * E$
4.  $E \rightarrow (E)$
5.  $E \rightarrow int$

3
$E \rightarrow E + \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

7
$E \rightarrow (\cdot E)$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

4
$E \rightarrow E * \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

8
$E \rightarrow (E \cdot )$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$\cdot * E$

5
$E \rightarrow E + E$
$\cdot E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

9
$E \rightarrow (E) \cdot$
10
$E \rightarrow int \cdot$

$FOLLOW(S) = \{ \$ \}$

$FOLLOW(E) = \{ +, *, ), \$ \}$

	int	+	*	(	)	\$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5		s3 r2	s4				
6							
7							
8							
9							
10							

# SLR(1) Parsing with Ambiguity

1
$S \rightarrow \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

2
$S \rightarrow E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$\cdot * E$

6
$E \rightarrow E * E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$\cdot * E$

1.  $S \rightarrow E$
2.  $E \rightarrow E + E$
3.  $E \rightarrow E * E$
4.  $E \rightarrow (E)$
5.  $E \rightarrow int$

3
$E \rightarrow E + \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

7
$E \rightarrow (\cdot E)$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

4
$E \rightarrow E * \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

8
$E \rightarrow (E \cdot )$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$\cdot * E$

5
$E \rightarrow E + E$
$\cdot E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$\cdot * E$

9
$E \rightarrow (E) \cdot$
10
$E \rightarrow int \cdot$

$FOLLOW(S) = \{ \$ \}$

$FOLLOW(E) = \{ +, *, ), \$ \}$

	int	+	*	(	)	\$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5		s3 r2	s4 r2				
6							
7							
8							
9							
10							

# SLR(1) Parsing with Ambiguity

1
$S \rightarrow \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

2
$S \rightarrow E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$\cdot * E$

6
$E \rightarrow E * E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$\cdot * E$

1.  $S \rightarrow E$
2.  $E \rightarrow E + E$
3.  $E \rightarrow E * E$
4.  $E \rightarrow (E)$
5.  $E \rightarrow int$

3
$E \rightarrow E + \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

7
$E \rightarrow (\cdot E)$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

4
$E \rightarrow E * \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

8
$E \rightarrow (E \cdot)$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$\cdot * E$

5
$E \rightarrow E + E$
$\cdot E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

9
$E \rightarrow (E) \cdot$
10
$E \rightarrow int \cdot$

$FOLLOW(S) = \{ \$ \}$

$FOLLOW(E) = \{ +, *, ), \$ \}$

	int	+	*	(	)	\$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5		s3 r2	s4 r2		r2		
6							
7							
8							
9							
10							

# SLR(1) Parsing with Ambiguity

1
$S \rightarrow \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

2
$S \rightarrow E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$\cdot * E$

6
$E \rightarrow E * E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$\cdot * E$

1.  $S \rightarrow E$
2.  $E \rightarrow E + E$
3.  $E \rightarrow E * E$
4.  $E \rightarrow (E)$
5.  $E \rightarrow int$

3
$E \rightarrow E + \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

7
$E \rightarrow (\cdot E)$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

4
$E \rightarrow E * \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

8
$E \rightarrow (E \cdot)$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$\cdot * E$

5
$E \rightarrow E + E$
$\cdot E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

9
$E \rightarrow (E) \cdot$
10
$E \rightarrow int \cdot$

$FOLLOW(S) = \{ \$ \}$

$FOLLOW(E) = \{ +, *, ), \$ \}$

	int	+	*	(	)	\$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5		s3 r2	s4 r2		r2	r2	
6							
7							
8							
9							
10							

# SLR(1) Parsing with Ambiguity

1
$S \rightarrow \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

2
$S \rightarrow E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E$
$\cdot * E$

6
$E \rightarrow E * E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E$
$\cdot * E$

1.  $S \rightarrow E$
2.  $E \rightarrow E + E$
3.  $E \rightarrow E * E$
4.  $E \rightarrow (E)$
5.  $E \rightarrow int$

3
$E \rightarrow E + \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

7
$E \rightarrow (\cdot E)$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

$E \rightarrow \cdot (E)$

4
$E \rightarrow E * \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

8
$E \rightarrow (E \cdot )$
$E \rightarrow E \cdot +$
$E \rightarrow E$
$\cdot * E$

5
$E \rightarrow E + E$
$\cdot E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

9
$E \rightarrow (E) \cdot$
10
$E \rightarrow int \cdot$

$FOLLOW(S) = \{ \$ \}$

$FOLLOW(E) = \{ +, *, ), \$ \}$

	int	+	*	(	)	\$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5		s3 r2	s4 r2		r2	r2	
6							
7							
8							
9							
10							

# SLR(1) Parsing with Ambiguity

1
$S \rightarrow \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

2
$S \rightarrow E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E$
$\cdot * E$

6
$E \rightarrow E * E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E$
$\cdot * E$

1.  $S \rightarrow E$
2.  $E \rightarrow E + E$
3.  $E \rightarrow E * E$
4.  $E \rightarrow (E)$
5.  $E \rightarrow int$

3
$E \rightarrow E + \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

7
$E \rightarrow (\cdot E)$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

4
$E \rightarrow E * \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

8
$E \rightarrow (E \cdot )$
$E \rightarrow E \cdot +$
$E \rightarrow E$
$\cdot * E$

5
$E \rightarrow E + E$
$\cdot E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

9
$E \rightarrow (E) \cdot$
10
$E \rightarrow int \cdot$

$FOLLOW(S) = \{ \$ \}$

$FOLLOW(E) = \{ +, *, ), \$ \}$

	int	+	*	(	)	\$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5		s3 r2	s4 r2		r2	r2	
6		s3					
7							
8							
9							
10							

# SLR(1) Parsing with Ambiguity

1
$S \rightarrow \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

2
$S \rightarrow E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$\cdot * E$

6
$E \rightarrow E * E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

1.  $S \rightarrow E$
2.  $E \rightarrow E + E$
3.  $E \rightarrow E * E$
4.  $E \rightarrow (E)$
5.  $E \rightarrow int$

3
$E \rightarrow E + \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

7
$E \rightarrow (\cdot E)$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

4
$E \rightarrow E * \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

8
$E \rightarrow (E \cdot )$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$\cdot * E$

5
$E \rightarrow E + E$
$\cdot E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

9
$E \rightarrow (E) \cdot$
10
$E \rightarrow int \cdot$

$FOLLOW(S) = \{ \$ \}$

$FOLLOW(E) = \{ +, *, ), \$ \}$

	int	+	*	(	)	\$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5		s3 r2	s4 r2		r2	r2	
6		s3	s4				
7							
8							
9							
10							



# SLR(1) Parsing with Ambiguity

1
$S \rightarrow \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

2
$S \rightarrow E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E$
$\cdot * E$

6
$E \rightarrow E * E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E$
$\cdot * E$

1.  $S \rightarrow E$
2.  $E \rightarrow E + E$
3.  $E \rightarrow E * E$
4.  $E \rightarrow (E)$
5.  $E \rightarrow int$

3
$E \rightarrow E + \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

7
$E \rightarrow (\cdot E)$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

4
$E \rightarrow E * \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

8
$E \rightarrow (E \cdot)$
$E \rightarrow E \cdot +$
$E \rightarrow E$
$\cdot * E$

5
$E \rightarrow E + E$
$\cdot E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

9
$E \rightarrow (E) \cdot$
10
$E \rightarrow int \cdot$

$FOLLOW(S) = \{ \$ \}$

$FOLLOW(E) = \{ +, *, ), \$ \}$

	int	+	*	(	)	\$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5		s3 r2	s4 r2		r2	r2	
6		s3 r3	s4				
7							
8							
9							
10							

# SLR(1) Parsing with Ambiguity

1
$S \rightarrow \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
int

2
$S \rightarrow E \cdot$
$E \rightarrow E \cdot + E$
$E \rightarrow E \cdot * E$

6
$E \rightarrow E * E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

1.  $S \rightarrow E$
2.  $E \rightarrow E + E$
3.  $E \rightarrow E * E$
4.  $E \rightarrow (E)$
5.  $E \rightarrow \text{int}$

3
$E \rightarrow E + \cdot E$
$E \rightarrow \cdot E + E$
$E \rightarrow \cdot E * E$
$E \rightarrow \cdot \text{int}$
$E \rightarrow \cdot (E)$

4
$E \rightarrow E * \cdot E$
$E \rightarrow \cdot E + E$
$E \rightarrow \cdot E * E$
$E \rightarrow \cdot \text{int}$
$E \rightarrow \cdot (E)$

5
$E \rightarrow E + E$
$\cdot E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

8
$E \rightarrow (E \cdot)$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

9
$E \rightarrow (E) \cdot$
10
$E \rightarrow \text{int} \cdot$

$\text{FOLLOW}(S) = \{ \$ \}$

$\text{FOLLOW}(E) = \{ +, *, ), \$ \}$

	int	+	*	(	)	\$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5		s3 r2	s4 r2		r2	r2	
6		s3 r3	s4 r3				
7							
8							
9							
10							

(E)

# SLR(1) Parsing with Ambiguity

1
$S \rightarrow \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

2
$S \rightarrow E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E$
$\cdot * E$

6
$E \rightarrow E * E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E$
$* E$

1.  $S \rightarrow E$
2.  $E \rightarrow E + E$
3.  $E \rightarrow E * E$
4.  $E \rightarrow (E)$
5.  $E \rightarrow int$

3
$E \rightarrow E + \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

7
$E \rightarrow (\cdot E)$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

$E \rightarrow \cdot (E)$

4
$E \rightarrow E * \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

8
$E \rightarrow (E \cdot )$
$E \rightarrow E \cdot +$
$E \rightarrow E$
$\cdot * E$

5
$E \rightarrow E + E$
$\cdot E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

9
$E \rightarrow (E) \cdot$
10
$E \rightarrow int \cdot$

$FOLLOW(S) = \{ \$ \}$

$FOLLOW(E) = \{ +, *, ), \$ \}$

	int	+	*	(	)	\$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5		s3 r2	s4 r2		r2	r2	
6		s3 r3	s4 r3		r3		
7							
8							
9							
10							

# SLR(1) Parsing with Ambiguity

1
$S \rightarrow \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

2
---

$S \rightarrow E \cdot$   
 $E \rightarrow E \cdot + E$   
 $E \rightarrow E \cdot * E$

6
---

$E \rightarrow E * E \cdot$   
 $E \rightarrow E \cdot +$   
 $E \rightarrow E \cdot$   
 $* E$

1.  $S \rightarrow E$
2.  $E \rightarrow E + E$
3.  $E \rightarrow E * E$
4.  $E \rightarrow (E)$
5.  $E \rightarrow int$

3
---

$E \rightarrow E + \cdot E$   
 $E \rightarrow \cdot E + E$   
 $E \rightarrow \cdot E * E$   
 $E \rightarrow \cdot int$   
 $E \rightarrow \cdot (E)$

7
---

$E \rightarrow (\cdot E)$   
 $E \rightarrow \cdot E +$   
 $E \rightarrow \cdot E$   
 $E \rightarrow \cdot int$   
 $* E$   
 $E \rightarrow \cdot$

(E)

4
---

$E \rightarrow E * \cdot E$   
 $E \rightarrow \cdot E + E$   
 $E \rightarrow \cdot E * E$   
 $E \rightarrow \cdot int$   
 $E \rightarrow \cdot (E)$

8
---

$E \rightarrow (E \cdot)$   
 $E \rightarrow E \cdot +$   
 $E \rightarrow E \cdot$   
 $* E$

5
---

$E \rightarrow E + E$   
 $\cdot E \rightarrow E \cdot +$   
 $E \rightarrow E \cdot$   
 $* E$

9
---

$E \rightarrow (E) \cdot$

10
----

$E \rightarrow int \cdot$

$FOLLOW(S) = \{ \$ \}$

$FOLLOW(E) = \{ +, *, ), \$ \}$

	int	+	*	(	)	\$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5		s3 r2	s4 r2		r2	r2	
6		s3 r3	s4 r3		r3	r3	
7							
8							
9							
10							

# SLR(1) Parsing with Ambiguity

1
$S \rightarrow \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

$E \rightarrow \cdot (E)$

2
---

$S \rightarrow E \cdot$   
 $E \rightarrow E \cdot + E$   
 $E \rightarrow E \cdot * E$

6
---

$E \rightarrow E * E \cdot$   
 $E \rightarrow E \cdot +$   
 $E \rightarrow E \cdot$   
 $* E$

1.  $S \rightarrow E$
2.  $E \rightarrow E + E$
3.  $E \rightarrow E * E$
4.  $E \rightarrow (E)$
5.  $E \rightarrow int$

3
---

$E \rightarrow E + \cdot E$   
 $E \rightarrow \cdot E + E$   
 $E \rightarrow \cdot E * E$   
 $E \rightarrow \cdot int$   
 $E \rightarrow \cdot (E)$

7
---

$E \rightarrow (\cdot E)$   
 $E \rightarrow \cdot E +$   
 $E \rightarrow \cdot E$   
 $E \rightarrow \cdot int$   
 $* E$   
 $E \rightarrow \cdot$

(E)

4
---

$E \rightarrow E * \cdot E$   
 $E \rightarrow \cdot E + E$   
 $E \rightarrow \cdot E * E$   
 $E \rightarrow \cdot int$   
 $E \rightarrow \cdot (E)$

8
---

$E \rightarrow (E \cdot)$   
 $E \rightarrow E \cdot +$   
 $E \rightarrow E \cdot$   
 $* E$

5
---

$E \rightarrow E + E$   
 $\cdot E \rightarrow E \cdot +$   
 $E \rightarrow E \cdot$   
 $* E$

9
---

$E \rightarrow (E) \cdot$

10
----

$E \rightarrow int \cdot$

$FOLLOW(S) = \{ \$ \}$

$FOLLOW(E) = \{ +, *, ), \$ \}$

	int	+	*	(	)	\$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5		s3 r2	s4 r2		r2	r2	
6		s3 r3	s4 r3		r3	r3	
7							
8							
9							
10							

# SLR(1) Parsing with Ambiguity

1
$S \rightarrow \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

2
$S \rightarrow E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E$
$\cdot * E$

6
$E \rightarrow E * E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E$
$\cdot * E$

1.  $S \rightarrow E$
2.  $E \rightarrow E + E$
3.  $E \rightarrow E * E$
4.  $E \rightarrow (E)$
5.  $E \rightarrow int$

3
$E \rightarrow E + \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

7
$E \rightarrow (\cdot E)$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

$E \rightarrow \cdot (E)$

4
$E \rightarrow E * \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

8
$E \rightarrow (E \cdot)$
$E \rightarrow E \cdot +$
$E \rightarrow E$
$\cdot * E$

5
$E \rightarrow E + E$
$\cdot E \rightarrow E \cdot +$
$E \rightarrow E$
$\cdot * E$

9
$E \rightarrow (E) \cdot$
10
$E \rightarrow int \cdot$

$FOLLOW(S) = \{ \$ \}$

$FOLLOW(E) = \{ +, *, ), \$ \}$

	int	+	*	(	)	\$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5		s3 r2	s4 r2		r2	r2	
6		s3 r3	s4 r3		r3	r3	
7	s10						
8							
9							
10							

# SLR(1) Parsing with Ambiguity

1
$S \rightarrow \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

2
---

$S \rightarrow E \cdot$   
 $E \rightarrow E \cdot + E$   
 $E \rightarrow E \cdot * E$

6
---

$E \rightarrow E * E \cdot$   
 $E \rightarrow E \cdot +$   
 $E \rightarrow E \cdot$   
 $* E$

1.  $S \rightarrow E$
2.  $E \rightarrow E + E$
3.  $E \rightarrow E * E$
4.  $E \rightarrow (E)$
5.  $E \rightarrow int$

3
---

$E \rightarrow E + \cdot E$   
 $E \rightarrow \cdot E + E$   
 $E \rightarrow \cdot E * E$   
 $E \rightarrow \cdot int$   
 $E \rightarrow \cdot (E)$

7
---

$E \rightarrow (\cdot E)$   
 $E \rightarrow \cdot E +$   
 $E \rightarrow \cdot E$   
 $E \rightarrow \cdot int$   
 $* E$   
 $E \rightarrow \cdot$

(E)

4
---

$E \rightarrow E * \cdot E$   
 $E \rightarrow \cdot E + E$   
 $E \rightarrow \cdot E * E$   
 $E \rightarrow \cdot int$   
 $E \rightarrow \cdot (E)$

8
---

$E \rightarrow (E \cdot)$   
 $E \rightarrow E \cdot +$   
 $E \rightarrow E \cdot$   
 $* E$

5
---

$E \rightarrow E + E$   
 $\cdot E \rightarrow E \cdot +$   
 $E \rightarrow E \cdot$   
 $* E$

9
---

$E \rightarrow (E) \cdot$

10
----

$E \rightarrow int \cdot$

$FOLLOW(S) = \{ \$ \}$

$FOLLOW(E) = \{ +, *, ), \$ \}$

	int	+	*	(	)	\$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5		s3 r2	s4 r2		r2	r2	
6		s3 r3	s4 r3		r3	r3	
7	s10			s7			
8							
9							
10							

# SLR(1) Parsing with Ambiguity

1
$S \rightarrow \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

$E \rightarrow \cdot (E)$

2
---

$S \rightarrow E \cdot$   
 $E \rightarrow E \cdot + E$   
 $E \rightarrow E \cdot * E$

6
---

$E \rightarrow E * E \cdot$   
 $E \rightarrow E \cdot +$   
 $E \rightarrow E \cdot$   
 $* E$

1.  $S \rightarrow E$
2.  $E \rightarrow E + E$
3.  $E \rightarrow E * E$
4.  $E \rightarrow (E)$
5.  $E \rightarrow int$

3
---

$E \rightarrow E + \cdot E$   
 $E \rightarrow \cdot E + E$   
 $E \rightarrow \cdot E * E$   
 $E \rightarrow \cdot int$   
 $E \rightarrow \cdot (E)$

7
---

$E \rightarrow (\cdot E)$   
 $E \rightarrow \cdot E +$   
 $E \rightarrow \cdot E$   
 $E \rightarrow \cdot int$   
 $* E$   
 $E \rightarrow \cdot$

(E)

4
---

$E \rightarrow E * \cdot E$   
 $E \rightarrow \cdot E + E$   
 $E \rightarrow \cdot E * E$   
 $E \rightarrow \cdot int$   
 $E \rightarrow \cdot (E)$

8
---

$E \rightarrow (E \cdot)$   
 $E \rightarrow E \cdot +$   
 $E \rightarrow E \cdot$   
 $* E$

5
---

$E \rightarrow E + E$   
 $\cdot E \rightarrow E \cdot +$   
 $E \rightarrow E \cdot$   
 $* E$

9
---

$E \rightarrow (E) \cdot$

10
----

$E \rightarrow int \cdot$

$FOLLOW(S) = \{ \$ \}$

$FOLLOW(E) = \{ +, *, ), \$ \}$

	int	+	*	(	)	\$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5		s3 r2	s4 r2		r2	r2	
6		s3 r3	s4 r3		r3	r3	
7	s10			s7			s8
8							
9							
10							



# SLR(1) Parsing with Ambiguity

1
$S \rightarrow \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

2
$S \rightarrow E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E$
$\cdot * E$

6
$E \rightarrow E * E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E$
$\cdot * E$

1.  $S \rightarrow E$
2.  $E \rightarrow E + E$
3.  $E \rightarrow E * E$
4.  $E \rightarrow (E)$
5.  $E \rightarrow int$

3
$E \rightarrow E + \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

7
$E \rightarrow (\cdot E)$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

4
$E \rightarrow E * \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

8
$E \rightarrow (E \cdot )$
$E \rightarrow E \cdot +$
$E \rightarrow E$
$\cdot * E$

5
$E \rightarrow E + E$
$\cdot E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

9
$E \rightarrow (E) \cdot$
10
$E \rightarrow int \cdot$

$FOLLOW(S) = \{ \$ \}$

$FOLLOW(E) = \{ +, *, ), \$ \}$

	int	+	*	(	)	\$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5		s3 r2	s4 r2		r2	r2	
6		s3 r3	s4 r3		r3	r3	
7	s10			s7			s8
8							
9							
10							

# SLR(1) Parsing with Ambiguity

1
$S \rightarrow \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

2
$S \rightarrow E \cdot$
$E \rightarrow E \cdot + E$
$E \rightarrow E \cdot * E$

6
$E \rightarrow E * E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

1.  $S \rightarrow E$
2.  $E \rightarrow E + E$
3.  $E \rightarrow E * E$
4.  $E \rightarrow (E)$
5.  $E \rightarrow int$

3
$E \rightarrow E + \cdot E$
$E \rightarrow \cdot E + E$
$E \rightarrow \cdot E * E$
$E \rightarrow \cdot int$
$E \rightarrow \cdot (E)$

7
$E \rightarrow (\cdot E)$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$E \rightarrow \cdot int$
$* E$
$E \rightarrow \cdot$

(E)

4
$E \rightarrow E * \cdot E$
$E \rightarrow \cdot E + E$
$E \rightarrow \cdot E * E$
$E \rightarrow \cdot int$
$E \rightarrow \cdot (E)$

8
$E \rightarrow (E \cdot)$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

5
$E \rightarrow E + E$
$\cdot E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

9
$E \rightarrow (E) \cdot$
10
$E \rightarrow int \cdot$

$FOLLOW(S) = \{ \$ \}$

$FOLLOW(E) = \{ +, *, ), \$ \}$

	int	+	*	(	)	\$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5		s3 r2	s4 r2		r2	r2	
6		s3 r3	s4 r3		r3	r3	
7	s10			s7			s8
8		s3					
9							
10							

# SLR(1) Parsing with Ambiguity

1
$S \rightarrow \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
int

2
---

$S \rightarrow E \cdot$   
 $E \rightarrow E \cdot + E$   
 $E \rightarrow E \cdot * E$

6
---

$E \rightarrow E * E \cdot$   
 $E \rightarrow E \cdot +$   
 $E \rightarrow E \cdot$   
 $* E$

1.  $S \rightarrow E$
2.  $E \rightarrow E + E$
3.  $E \rightarrow E * E$
4.  $E \rightarrow (E)$
5.  $E \rightarrow \text{int}$

3
---

$E \rightarrow E + \cdot E$   
 $E \rightarrow \cdot E + E$   
 $E \rightarrow \cdot E * E$   
 $E \rightarrow \cdot \text{int}$   
 $E \rightarrow \cdot (E)$

7
---

$E \rightarrow (\cdot E)$   
 $E \rightarrow \cdot E +$   
 $E \rightarrow \cdot E$   
 $E \rightarrow \cdot \text{int}$   
 $* E$   
 $E \rightarrow \cdot$

(E)

4
---

$E \rightarrow E * \cdot E$   
 $E \rightarrow \cdot E + E$   
 $E \rightarrow \cdot E * E$   
 $E \rightarrow \cdot \text{int}$   
 $E \rightarrow \cdot (E)$

8
---

$E \rightarrow (E \cdot)$   
 $E \rightarrow E \cdot +$   
 $E \rightarrow E \cdot$   
 $* E$

5
---

$E \rightarrow E + E$   
 $\cdot E \rightarrow E \cdot +$   
 $E \rightarrow E \cdot$   
 $* E$

9
---

$E \rightarrow (E) \cdot$

10
----

$E \rightarrow \text{int} \cdot$

$\text{FOLLOW}(S) = \{ \$ \}$

$\text{FOLLOW}(E) = \{ +, *, ), \$ \}$

	int	+	*	(	)	\$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5		s3 r2	s4 r2		r2	r2	
6		s3 r3	s4 r3		r3	r3	
7	s10			s7			s8
8		s3	s4				
9							
10							

# SLR(1) Parsing with Ambiguity

1
$S \rightarrow \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

2
$S \rightarrow E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E$
$\cdot * E$

6
$E \rightarrow E * E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E$
$\cdot * E$

1.  $S \rightarrow E$
2.  $E \rightarrow E + E$
3.  $E \rightarrow E * E$
4.  $E \rightarrow (E)$
5.  $E \rightarrow int$

3
$E \rightarrow E + \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

7
$E \rightarrow (\cdot E)$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

$E \rightarrow \cdot (E)$

4
$E \rightarrow E * \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

8
$E \rightarrow (E \cdot)$
$E \rightarrow E \cdot +$
$E \rightarrow E$
$\cdot * E$

5
$E \rightarrow E + E$
$\cdot E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

9
$E \rightarrow (E) \cdot$
10
$E \rightarrow int \cdot$

$FOLLOW(S) = \{ \$ \}$

$FOLLOW(E) = \{ +, *, ), \$ \}$

	int	+	*	(	)	\$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5		s3 r2	s4 r2		r2	r2	
6		s3 r3	s4 r3		r3	r3	
7	s10			s7			s8
8		s3	s4		s9		
9							
10							

# SLR(1) Parsing with Ambiguity

1
$S \rightarrow \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

$E \rightarrow \cdot (E)$

2
---

$S \rightarrow E \cdot$   
 $E \rightarrow E \cdot + E$   
 $E \rightarrow E \cdot * E$

6
---

$E \rightarrow E * E \cdot$   
 $E \rightarrow E \cdot +$   
 $E \rightarrow E \cdot$   
 $* E$

1.  $S \rightarrow E$
2.  $E \rightarrow E + E$
3.  $E \rightarrow E * E$
4.  $E \rightarrow (E)$
5.  $E \rightarrow int$

3
---

$E \rightarrow E + \cdot E$   
 $E \rightarrow \cdot E + E$   
 $E \rightarrow \cdot E * E$   
 $E \rightarrow \cdot int$   
 $E \rightarrow \cdot (E)$

7
---

$E \rightarrow (\cdot E)$   
 $E \rightarrow \cdot E +$   
 $E \rightarrow \cdot E$   
 $E \rightarrow \cdot int$   
 $* E$   
 $E \rightarrow \cdot$

(E)

4
---

$E \rightarrow E * \cdot E$   
 $E \rightarrow \cdot E + E$   
 $E \rightarrow \cdot E * E$   
 $E \rightarrow \cdot int$   
 $E \rightarrow \cdot (E)$

8
---

$E \rightarrow (E \cdot)$   
 $E \rightarrow E \cdot +$   
 $E \rightarrow E \cdot$   
 $* E$

5
---

$E \rightarrow E + E$   
 $\cdot E \rightarrow E \cdot +$   
 $E \rightarrow E \cdot$   
 $* E$

9
---

$E \rightarrow (E) \cdot$

10
----

$E \rightarrow int \cdot$

$FOLLOW(S) = \{ \$ \}$

$FOLLOW(E) = \{ +, *, ), \$ \}$

	int	+	*	(	)	\$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5		s3 r2	s4 r2		r2	r2	
6		s3 r3	s4 r3		r3	r3	
7	s10			s7			s8
8		s3	s4		s9		
9							
10							

# SLR(1) Parsing with Ambiguity

1
$S \rightarrow \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
int

2
---

$S \rightarrow E \cdot$   
 $E \rightarrow E \cdot + E$   
 $E \rightarrow E \cdot * E$

6
$E \rightarrow E * E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

1.  $S \rightarrow E$
2.  $E \rightarrow E + E$
3.  $E \rightarrow E * E$
4.  $E \rightarrow (E)$
5.  $E \rightarrow \text{int}$

3
---

$E \rightarrow E + \cdot E$   
 $E \rightarrow \cdot E + E$   
 $E \rightarrow \cdot E * E$   
 $E \rightarrow \cdot \text{int}$   
 $E \rightarrow \cdot (E)$

7
$E \rightarrow (\cdot E)$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E$

(E)

4
$E \rightarrow E * \cdot E$
$E \rightarrow \cdot E + E$
$E \rightarrow \cdot E * E$
$E \rightarrow \cdot \text{int}$
$E \rightarrow \cdot (E)$

5
$E \rightarrow E + E$
$\cdot E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

9
$E \rightarrow (E) \cdot$
10
$E \rightarrow \text{int} \cdot$

$\text{FOLLOW}(S) = \{ \$ \}$

$\text{FOLLOW}(E) = \{ +, *, ), \$ \}$

	int	+	*	(	)	\$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5		s3 r2	s4 r2		r2	r2	
6		s3 r3	s4 r3		r3	r3	
7	s10			s7			s8
8		s3	s4		s9		
9		r4					
10							

# SLR(1) Parsing with Ambiguity

1
$S \rightarrow \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

2
$S \rightarrow E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E$
$\cdot * E$

6
$E \rightarrow E * E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E$
$\cdot * E$

1.  $S \rightarrow E$
2.  $E \rightarrow E + E$
3.  $E \rightarrow E * E$
4.  $E \rightarrow (E)$
5.  $E \rightarrow int$

3
$E \rightarrow E + \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

7
$E \rightarrow (\cdot E)$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

$E \rightarrow \cdot (E)$

4
$E \rightarrow E * \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

8
$E \rightarrow (E \cdot)$
$E \rightarrow E \cdot +$
$E \rightarrow E$
$\cdot * E$

5
$E \rightarrow E + E$
$\cdot E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

9
$E \rightarrow (E) \cdot$
10
$E \rightarrow int \cdot$

$FOLLOW(S) = \{ \$ \}$

$FOLLOW(E) = \{ +, *, ), \$ \}$

	int	+	*	(	)	\$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5		s3 r2	s4 r2		r2	r2	
6		s3 r3	s4 r3		r3	r3	
7	s10			s7			s8
8		s3	s4		s9		
9		r4	r4				
10							

# SLR(1) Parsing with Ambiguity

1
$S \rightarrow \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

$E \rightarrow \cdot (E)$

2
---

$S \rightarrow E \cdot$   
 $E \rightarrow E \cdot + E$   
 $E \rightarrow E \cdot * E$

6
---

$E \rightarrow E * E \cdot$   
 $E \rightarrow E \cdot +$   
 $E \rightarrow E \cdot$   
 $* E$

1.  $S \rightarrow E$
2.  $E \rightarrow E + E$
3.  $E \rightarrow E * E$
4.  $E \rightarrow (E)$
5.  $E \rightarrow int$

3
---

$E \rightarrow E + \cdot E$   
 $E \rightarrow \cdot E + E$   
 $E \rightarrow \cdot E * E$   
 $E \rightarrow \cdot int$   
 $E \rightarrow \cdot (E)$

7
---

$E \rightarrow (\cdot E)$   
 $E \rightarrow \cdot E +$   
 $E \rightarrow \cdot E$   
 $E \rightarrow \cdot int$   
 $* E$   
 $E \rightarrow \cdot$

(E)

4
---

$E \rightarrow E * \cdot E$   
 $E \rightarrow \cdot E + E$   
 $E \rightarrow \cdot E * E$   
 $E \rightarrow \cdot int$   
 $E \rightarrow \cdot (E)$

8
---

$E \rightarrow (E \cdot)$   
 $E \rightarrow E \cdot +$   
 $E \rightarrow E \cdot$   
 $* E$

5
---

$E \rightarrow E + E$   
 $\cdot E \rightarrow E \cdot +$   
 $E \rightarrow E \cdot$   
 $* E$

9
---

$E \rightarrow (E) \cdot$

10
----

$E \rightarrow int \cdot$

$FOLLOW(S) = \{ \$ \}$

$FOLLOW(E) = \{ +, *, ), \$ \}$

	int	+	*	(	)	\$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5		s3 r2	s4 r2		r2	r2	
6		s3 r3	s4 r3		r3	r3	
7	s10			s7			s8
8		s3	s4		s9		
9		r4	r4		r4		
10							



# SLR(1) Parsing with Ambiguity

1
$S \rightarrow \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

2
$S \rightarrow E \cdot$
$E \rightarrow E \cdot + E$
$E \rightarrow E \cdot * E$

6
$E \rightarrow E * E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

1.  $S \rightarrow E$
2.  $E \rightarrow E + E$
3.  $E \rightarrow E * E$
4.  $E \rightarrow (E)$
5.  $E \rightarrow int$

3
$E \rightarrow E + \cdot E$
$E \rightarrow \cdot E + E$
$E \rightarrow \cdot E * E$
$E \rightarrow \cdot int$
$E \rightarrow \cdot (E)$

4
$E \rightarrow E * \cdot E$
$E \rightarrow \cdot E + E$
$E \rightarrow \cdot E * E$
$E \rightarrow \cdot int$
$E \rightarrow \cdot (E)$

5
$E \rightarrow E + E$
$\cdot E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

8
$E \rightarrow (E \cdot)$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

9
$E \rightarrow (E) \cdot$
10
$E \rightarrow int \cdot$

$FOLLOW(S) = \{ \$ \}$

$FOLLOW(E) = \{ +, *, ), \$ \}$

	int	+	*	(	)	\$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5		s3 r2	s4 r2		r2	r2	
6		s3 r3	s4 r3		r3	r3	
7	s10			s7			s8
8		s3	s4		s9		
9		r4	r4		r4	r4	
10							

(E)

# SLR(1) Parsing with Ambiguity

1
$S \rightarrow \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

2
$S \rightarrow E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$\cdot * E$

6
$E \rightarrow E * E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

1.  $S \rightarrow E$
2.  $E \rightarrow E + E$
3.  $E \rightarrow E * E$
4.  $E \rightarrow (E)$
5.  $E \rightarrow int$

3
$E \rightarrow E + \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

7
$E \rightarrow (\cdot E)$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

$E \rightarrow \cdot (E)$

4
$E \rightarrow E * \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

8
$E \rightarrow (E \cdot)$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$\cdot * E$

5
$E \rightarrow E + E$
$\cdot E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

9
$E \rightarrow (E) \cdot$
10
$E \rightarrow int \cdot$

$FOLLOW(S) = \{ \$ \}$

$FOLLOW(E) = \{ +, *, ), \$ \}$

	int	+	*	(	)	\$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5		s3 r2	s4 r2		r2	r2	
6		s3 r3	s4 r3		r3	r3	
7	s10			s7			s8
8		s3	s4		s9		
9		r4	r4		r4	r4	
10							

# SLR(1) Parsing with Ambiguity

1
$S \rightarrow \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

$E \rightarrow \cdot (E)$

2
---

$S \rightarrow E \cdot$   
 $E \rightarrow E \cdot + E$   
 $E \rightarrow E \cdot * E$

6
---

$E \rightarrow E * E \cdot$   
 $E \rightarrow E \cdot +$   
 $E \rightarrow E \cdot$   
 $* E$

1.  $S \rightarrow E$
2.  $E \rightarrow E + E$
3.  $E \rightarrow E * E$
4.  $E \rightarrow (E)$
5.  $E \rightarrow int$

3
---

$E \rightarrow E + \cdot E$   
 $E \rightarrow \cdot E + E$   
 $E \rightarrow \cdot E * E$   
 $E \rightarrow \cdot int$   
 $E \rightarrow \cdot (E)$

7
---

$E \rightarrow (\cdot E)$   
 $E \rightarrow \cdot E +$   
 $E \rightarrow \cdot E$   
 $E \rightarrow \cdot int$   
 $* E$   
 $E \rightarrow \cdot$

(E)

4
---

$E \rightarrow E * \cdot E$   
 $E \rightarrow \cdot E + E$   
 $E \rightarrow \cdot E * E$   
 $E \rightarrow \cdot int$   
 $E \rightarrow \cdot (E)$

8
---

$E \rightarrow (E \cdot)$   
 $E \rightarrow E \cdot +$   
 $E \rightarrow E \cdot$   
 $* E$

5
---

$E \rightarrow E + E$   
 $\cdot E \rightarrow E \cdot +$   
 $E \rightarrow E \cdot$   
 $* E$

9
---

$E \rightarrow (E) \cdot$

10
$E \rightarrow int \cdot$

$FOLLOW(S) = \{ \$ \}$

$FOLLOW(E) = \{ +, *, ), \$ \}$

	int	+	*	(	)	\$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5		s3 r2	s4 r2		r2	r2	
6		s3 r3	s4 r3		r3	r3	
7	s10			s7			s8
8		s3	s4		s9		
9		r4	r4		r4	r4	
10		r5					

# SLR(1) Parsing with Ambiguity

1
$S \rightarrow \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

2
$S \rightarrow E \cdot$
$E \rightarrow E \cdot + E$
$E \rightarrow E \cdot * E$

6
$E \rightarrow E * E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

1.  $S \rightarrow E$
2.  $E \rightarrow E + E$
3.  $E \rightarrow E * E$
4.  $E \rightarrow (E)$
5.  $E \rightarrow int$

3
$E \rightarrow E + \cdot E$
$E \rightarrow \cdot E + E$
$E \rightarrow \cdot E * E$
$E \rightarrow \cdot int$
$E \rightarrow \cdot (E)$

7
$E \rightarrow (\cdot E)$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$E \rightarrow \cdot int$
$* E$
$E \rightarrow \cdot$

(E)

4
$E \rightarrow E * \cdot E$
$E \rightarrow \cdot E + E$
$E \rightarrow \cdot E * E$
$E \rightarrow \cdot int$
$E \rightarrow \cdot (E)$

8
$E \rightarrow (E \cdot)$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

5
$E \rightarrow E + E$
$\cdot E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

9
$E \rightarrow (E) \cdot$
10
$E \rightarrow int \cdot$

$FOLLOW(S) = \{ \$ \}$

$FOLLOW(E) = \{ +, *, ), \$ \}$

	int	+	*	(	)	\$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5		s3 r2	s4 r2		r2	r2	
6		s3 r3	s4 r3		r3	r3	
7	s10			s7			s8
8		s3	s4		s9		
9		r4	r4		r4	r4	
10		r5	r5				

# SLR(1) Parsing with Ambiguity

1
$S \rightarrow \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

2
$S \rightarrow E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$\cdot * E$

6
$E \rightarrow E * E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

1.  $S \rightarrow E$
2.  $E \rightarrow E + E$
3.  $E \rightarrow E * E$
4.  $E \rightarrow (E)$
5.  $E \rightarrow int$

3
$E \rightarrow E + \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

7
$E \rightarrow (\cdot E)$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

$E \rightarrow \cdot (E)$

4
$E \rightarrow E * \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

8
$E \rightarrow (E \cdot)$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$\cdot * E$

5
$E \rightarrow E + E$
$\cdot E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

9
$E \rightarrow (E) \cdot$
10
$E \rightarrow int \cdot$

$FOLLOW(S) = \{ \$ \}$

$FOLLOW(E) = \{ +, *, ), \$ \}$

	int	+	*	(	)	\$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5		s3 r2	s4 r2		r2	r2	
6		s3 r3	s4 r3		r3	r3	
7	s10			s7			s8
8		s3	s4		s9		
9		r4	r4		r4	r4	
10		r5	r5		r5		

# SLR(1) Parsing with Ambiguity

1
$S \rightarrow \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

$E \rightarrow \cdot (E)$

2
---

$S \rightarrow E \cdot$   
 $E \rightarrow E \cdot + E$   
 $E \rightarrow E \cdot * E$

6
---

$E \rightarrow E * E \cdot$   
 $E \rightarrow E \cdot +$   
 $E \rightarrow E \cdot$   
 $* E$

1.  $S \rightarrow E$
2.  $E \rightarrow E + E$
3.  $E \rightarrow E * E$
4.  $E \rightarrow (E)$
5.  $E \rightarrow int$

3
---

$E \rightarrow E + \cdot E$   
 $E \rightarrow \cdot E + E$   
 $E \rightarrow \cdot E * E$   
 $E \rightarrow \cdot int$   
 $E \rightarrow \cdot (E)$

7
---

$E \rightarrow (\cdot E)$   
 $E \rightarrow \cdot E +$   
 $E \rightarrow \cdot E$   
 $E \rightarrow \cdot int$   
 $* E$   
 $E \rightarrow \cdot$

(E)

4
---

$E \rightarrow E * \cdot E$   
 $E \rightarrow \cdot E + E$   
 $E \rightarrow \cdot E * E$   
 $E \rightarrow \cdot int$   
 $E \rightarrow \cdot (E)$

8
---

$E \rightarrow (E \cdot)$   
 $E \rightarrow E \cdot +$   
 $E \rightarrow E \cdot$   
 $* E$

5
---

$E \rightarrow E + E$   
 $\cdot E \rightarrow E \cdot +$   
 $E \rightarrow E \cdot$   
 $* E$

9
---

$E \rightarrow (E) \cdot$

10
----

$E \rightarrow int \cdot$

$FOLLOW(S) = \{ \$ \}$

$FOLLOW(E) = \{ +, *, ), \$ \}$

	int	+	*	(	)	\$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5		s3 r2	s4 r2		r2	r2	
6		s3 r3	s4 r3		r3	r3	
7	s10			s7			s8
8		s3	s4		s9		
9		r4	r4		r4	r4	
10		r5	r5		r5	r5	

# SLR(1) Parsing with Ambiguity

1
$S \rightarrow \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

$E \rightarrow \cdot (E)$

2
---

$S \rightarrow E \cdot$   
 $E \rightarrow E \cdot + E$   
 $E \rightarrow E \cdot * E$

6
---

$E \rightarrow E * E \cdot$   
 $E \rightarrow E \cdot +$   
 $E \rightarrow E \cdot$   
 $* E$

1.  $S \rightarrow E$
2.  $E \rightarrow E + E$
3.  $E \rightarrow E * E$
4.  $E \rightarrow (E)$
5.  $E \rightarrow int$

3
---

$E \rightarrow E + \cdot E$   
 $E \rightarrow \cdot E + E$   
 $E \rightarrow \cdot E * E$   
 $E \rightarrow \cdot int$   
 $E \rightarrow \cdot (E)$

7
---

$E \rightarrow (\cdot E)$   
 $E \rightarrow \cdot E +$   
 $E \rightarrow \cdot E$   
 $E \rightarrow \cdot int$   
 $* E$   
 $E \rightarrow \cdot$

(E)

4
---

$E \rightarrow E * \cdot E$   
 $E \rightarrow \cdot E + E$   
 $E \rightarrow \cdot E * E$   
 $E \rightarrow \cdot int$   
 $E \rightarrow \cdot (E)$

8
---

$E \rightarrow (E \cdot)$   
 $E \rightarrow E \cdot +$   
 $E \rightarrow E \cdot$   
 $* E$

5
---

$E \rightarrow E + E$   
 $\cdot E \rightarrow E \cdot +$   
 $E \rightarrow E \cdot$   
 $* E$

9
---

$E \rightarrow (E) \cdot$

10
----

$E \rightarrow int \cdot$

$FOLLOW(S) = \{ \$ \}$

$FOLLOW(E) = \{ +, *, ), \$ \}$

	int	+	*	(	)	\$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5		s3 r2	s4 r2		r2	r2	
6		s3 r3	s4 r3		r3	r3	
7	s10			s7			s8
8		s3	s4		s9		
9		r4	r4		r4	r4	
10		r5	r5		r5	r5	

# SLR(1) Parsing with Ambiguity

1
$S \rightarrow \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

2
$S \rightarrow E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$\cdot * E$

6
$E \rightarrow E * E \cdot$
$E \rightarrow E \cdot + E$
$E \rightarrow E \cdot * E$

1.  $S \rightarrow E$
2.  $E \rightarrow E + E$
3.  $E \rightarrow E * E$
4.  $E \rightarrow (E)$
5.  $E \rightarrow int$

3
$E \rightarrow E + \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

7
$E \rightarrow (E) \cdot$
$E \rightarrow (E) \cdot + E$
$E \rightarrow (E) \cdot * E$
$E \rightarrow (E) \cdot$
$int$

4
$E \rightarrow E * \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$
$E \rightarrow \cdot (E)$

8
$E \rightarrow (E \cdot)$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$\cdot * E$

5
$E \rightarrow E + E$
$\cdot E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

9
$E \rightarrow (E) \cdot$
10
$E \rightarrow int \cdot$

$FOLLOW(S) = \{ \$ \}$

$FOLLOW(E) = \{ +, *, ), \$ \}$

	int	+	*	(	)	\$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5		s3 r2	s4 r2		r2	r2	
6		s3 r3	s4 r3		r3	r3	
7	s10			s7			s8
8		s3	s4		s9		
9		r4	r4		r4	r4	
10		r5	r5		r5	r5	



# Resolving Ambiguity

- Although the grammar is ambiguous, there is clearly one intended parse tree because of **operator precedence**.
- How can we use this precedence information to avoid LR conflicts?

# Precedence Declarations

- Tell the parser generator about the *associativity* and *precedence* of certain rules.
- Productions can be left-associative, right-associative, or nonassociative.
- Productions can have their *priorities* **ranked** against one another.

# SLR(1) Parsing with Ambiguity

1
$S \rightarrow \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
int

2
$S \rightarrow E \cdot$
$E \rightarrow E \cdot + E$
$E \rightarrow E \cdot * E$

6
$E \rightarrow E * E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

1.  $S \rightarrow E$
2.  $E \rightarrow E + E$
3.  $E \rightarrow E * E$
4.  $E \rightarrow (E)$
5.  $E \rightarrow \text{int}$

3
$E \rightarrow E + \cdot E$
$E \rightarrow \cdot E + E$
$E \rightarrow \cdot E * E$
$E \rightarrow \cdot \text{int}$
$E \rightarrow \cdot (E)$

7
$E \rightarrow (\cdot E)$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$E \rightarrow \cdot \text{int}$
$* E$
$E \rightarrow \cdot$

(E)

4
$E \rightarrow E * \cdot E$
$E \rightarrow \cdot E + E$
$E \rightarrow \cdot E * E$
$E \rightarrow \cdot \text{int}$
$E \rightarrow \cdot (E)$

8
$E \rightarrow (E \cdot)$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

5
$E \rightarrow E + E$
$\cdot E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

9
$E \rightarrow (E) \cdot$
10
$E \rightarrow \text{int} \cdot$

	int	+	*	(	)	\$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5		s3 r2	s4 r2		r2	r2	
6		s3 r3	s4 r3		r3	r3	
7	s10			s7			s8
8		s3	s4		s9		
9		r4	r4		r4	r4	
10		r5	r5		r5	r5	

# SLR(1) Parsing with Ambiguity

1
$S \rightarrow \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
int

2
$S \rightarrow E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

6
$E \rightarrow E * E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

1.  $S \rightarrow E$
2.  $E \rightarrow E + E$  (Left-assoc, pri. 0)
3.  $E \rightarrow E * E$  (Left-assoc, pri. 1)
4.  $E \rightarrow (E)$
5.  $E \rightarrow \text{int}$

3
$E \rightarrow E + \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
int

7
$E \rightarrow (\cdot E)$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
int

(E)

4
$E \rightarrow E * \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
int

8
$E \rightarrow (E \cdot)$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

5
$E \rightarrow E + E$
$\cdot E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

9
$E \rightarrow (E) \cdot$
10
$E \rightarrow \text{int} \cdot$

1)	int	+	*	(	)	\$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5		s3 r2	s4 r2		r2	r2	
6		s3 r3	s4 r3		r3	r3	
7	s10			s7			s8
8		s3	s4		s9		
9		r4	r4		r4	r4	
10		r5	r5		r5	r5	

# SLR(1) Parsing with Ambiguity

1
$S \rightarrow \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

2
$S \rightarrow E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$\cdot * E$

6
$E \rightarrow E * E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$\cdot * E$

1.  $S \rightarrow E$
2.  $E \rightarrow E + E$  (Left-assoc, pri. 0)
3.  $E \rightarrow E * E$  (Left-assoc, pri. 1)
4.  $E \rightarrow (E)$
5.  $E \rightarrow int$

3
$E \rightarrow E + \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

7
$E \rightarrow (\cdot E)$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

$E \rightarrow \cdot (E)$

4
$E \rightarrow E * \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

8
$E \rightarrow (E \cdot)$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$\cdot * E$

5
$E \rightarrow E + E$
$\cdot E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

9
$E \rightarrow (E) \cdot$
10
$E \rightarrow int \cdot$

1)	int	+	*	(	)	\$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5		s3 r2	s4 r2		r2	r2	
6		s3 r3	s4 r3		r3	r3	
7	s10			s7			s8
8		s3	s4		s9		
9		r4	r4		r4	r4	
10		r5	r5		r5	r5	

# SLR(1) Parsing with Ambiguity

1
$S \rightarrow \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
int

2
$S \rightarrow E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

6
$E \rightarrow E * E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

1.  $S \rightarrow E$
2.  $E \rightarrow E + E$  (Left-assoc, pri. 0)
3.  $E \rightarrow E * E$  (Left-assoc, pri. 1)
4.  $E \rightarrow (E)$
5.  $E \rightarrow \text{int}$

3
$E \rightarrow E + \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
int

7
$E \rightarrow (\cdot E)$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
int

(E)

4
$E \rightarrow E * \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
int

8
$E \rightarrow (E \cdot)$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

5
$E \rightarrow E + E$
$\cdot E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

9
$E \rightarrow (E) \cdot$
10
$E \rightarrow \text{int} \cdot$

1)	int	+	*	(	)	\$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5		s3 r2	s4		r2	r2	
6		s3 r3	s4 r3		r3	r3	
7	s10			s7			s8
8		s3	s4		s9		
9		r4	r4		r4	r4	
10		r5	r5		r5	r5	

# SLR(1) Parsing with Ambiguity

1
$S \rightarrow \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

2
$S \rightarrow E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

6
$E \rightarrow E * E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

1.  $S \rightarrow E$
2.  $E \rightarrow E + E$  (Left-assoc, pri. 0)
3.  $E \rightarrow E * E$  (Left-assoc, pri. 1)
4.  $E \rightarrow (E)$
5.  $E \rightarrow int$

3
$E \rightarrow E + \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

7
$E \rightarrow (\cdot E)$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

(E)

4
$E \rightarrow E * \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

8
$E \rightarrow (E \cdot)$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

5
$E \rightarrow E + E$
$\cdot E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

9
$E \rightarrow (E) \cdot$
10
$E \rightarrow int \cdot$

1)	int	+	*	(	)	\$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5		s3 r2	s4		r2	r2	
6		s3 r3	s4 r3		r3	r3	
7	s10			s7			s8
8		s3	s4		s9		
9		r4	r4		r4	r4	
10		r5	r5		r5	r5	

# SLR(1) Parsing with Ambiguity

1
$S \rightarrow \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

2
$S \rightarrow E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$\cdot * E$

6
$E \rightarrow E * E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$\cdot * E$

1.  $S \rightarrow E$
2.  $E \rightarrow E + E$  (Left-assoc, pri. 0)
3.  $E \rightarrow E * E$  (Left-assoc, pri. 1)
4.  $E \rightarrow (E)$
5.  $E \rightarrow int$

3
$E \rightarrow E + \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

7
$E \rightarrow (\cdot E)$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

$E \rightarrow \cdot (E)$

4
$E \rightarrow E * \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

8
$E \rightarrow (E \cdot)$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$\cdot * E$

5
$E \rightarrow E + E$
$\cdot E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

9
$E \rightarrow (E) \cdot$
10
$E \rightarrow int \cdot$

1)	int	+	*	(	)	\$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5		s3 r2	s4		r2	r2	
6		s3 r3	s4 r3		r3	r3	
7	s10			s7			s8
8		s3	s4		s9		
9		r4	r4		r4	r4	
10		r5	r5		r5	r5	



# SLR(1) Parsing with Ambiguity

1
$S \rightarrow \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
int

2
$S \rightarrow E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

6
$E \rightarrow E * E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

1.  $S \rightarrow E$
2.  $E \rightarrow E + E$  (Left-assoc, pri. 0)
3.  $E \rightarrow E * E$  (Left-assoc, pri. 1)
4.  $E \rightarrow (E)$
5.  $E \rightarrow \text{int}$

3
$E \rightarrow E + \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
int

7
$E \rightarrow (\cdot E)$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
int

(E)

4
$E \rightarrow E * \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
int

8
$E \rightarrow (E \cdot)$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

5
$E \rightarrow E + E$
$\cdot E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

9
$E \rightarrow (E) \cdot$
10
$E \rightarrow \text{int} \cdot$

1)	int	+	*	(	)	\$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5		s3 r2	s4		r2	r2	
6		r3	s4 r3		r3	r3	
7	s10			s7			s8
8		s3	s4		s9		
9		r4	r4		r4	r4	
10		r5	r5		r5	r5	

# SLR(1) Parsing with Ambiguity

1
$S \rightarrow \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
int

2
$S \rightarrow E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

6
$E \rightarrow E * E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

1.  $S \rightarrow E$
2.  $E \rightarrow E + E$  (Left-assoc, pri. 0)
3.  $E \rightarrow E * E$  (Left-assoc, pri. 1)
4.  $E \rightarrow (E)$
5.  $E \rightarrow \text{int}$

3
$E \rightarrow E + \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
int

7
$E \rightarrow (\cdot E)$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
int

(E)

4
$E \rightarrow E * \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
int

8
$E \rightarrow (E \cdot)$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

5
$E \rightarrow E + E$
$\cdot E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

9
$E \rightarrow (E) \cdot$
10
$E \rightarrow \text{int} \cdot$

1)	int	+	*	(	)	\$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5		s3 r2	s4		r2	r2	
6		r3	s4 r3		r3	r3	
7	s10			s7			s8
8		s3	s4		s9		
9		r4	r4		r4	r4	
10		r5	r5		r5	r5	

# SLR(1) Parsing with Ambiguity

1
$S \rightarrow \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

2
$S \rightarrow E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

6
$E \rightarrow E * E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

1.  $S \rightarrow E$
2.  $E \rightarrow E + E$  (Left-assoc, pri. 0)
3.  $E \rightarrow E * E$  (Left-assoc, pri. 1)
4.  $E \rightarrow (E)$
5.  $E \rightarrow int$

3
$E \rightarrow E + \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

7
$E \rightarrow (\cdot E)$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

$E \rightarrow \cdot (E)$

4
$E \rightarrow E * \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

8
$E \rightarrow (E \cdot)$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

5
$E \rightarrow E + E$
$\cdot E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

9
$E \rightarrow (E) \cdot$
10
$E \rightarrow int \cdot$

1)	int	+	*	(	)	\$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5		s3 r2	s4		r2	r2	
6		r3	s4 r3		r3	r3	
7	s10			s7			s8
8		s3	s4		s9		
9		r4	r4		r4	r4	
10		r5	r5		r5	r5	

# SLR(1) Parsing with Ambiguity

1
$S \rightarrow \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
int

2
$S \rightarrow E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

6
$E \rightarrow E * E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

1.  $S \rightarrow E$
2.  $E \rightarrow E + E$  (Left-assoc, pri. 0)
3.  $E \rightarrow E * E$  (Left-assoc, pri. 1)
4.  $E \rightarrow (E)$
5.  $E \rightarrow \text{int}$

3
$E \rightarrow E + \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
int

7
$E \rightarrow (\cdot E)$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
int

$E \rightarrow \cdot (E)$

4
$E \rightarrow E * \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
int

8
$E \rightarrow (E \cdot)$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

5
$E \rightarrow E + E$
$\cdot E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

9
$E \rightarrow (E) \cdot$
10
$E \rightarrow \text{int} \cdot$

1)	int	+	*	(	)	\$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5		r2	s4		r2	r2	
6		r3	r3		r3	r3	
7	s10			s7			s8
8		s3	s4		s9		
9		r4	r4		r4	r4	
10		r5	r5		r5	r5	

# SLR(1) Parsing with Ambiguity

1
$S \rightarrow \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
int

2
$S \rightarrow E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$\cdot * E$

6
$E \rightarrow E * E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$\cdot * E$

1.  $S \rightarrow E$
2.  $E \rightarrow E + E$  (Left-assoc, pri. 0)
3.  $E \rightarrow E * E$  (Left-assoc, pri. 1)
4.  $E \rightarrow (E)$
5.  $E \rightarrow \text{int}$

3
$E \rightarrow E + \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
int

7
$E \rightarrow (\cdot E)$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
int

$E \rightarrow \cdot (E)$

4
$E \rightarrow E * \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
int

8
$E \rightarrow (E \cdot)$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$\cdot * E$

5
$E \rightarrow E + E$
$\cdot E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

9
$E \rightarrow (E) \cdot$
10
$E \rightarrow \text{int} \cdot$

1)	int	+	*	(	)	\$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5		r2	s4		r2	r2	
6		r3	r3		r3	r3	
7	s10			s7			s8
8		s3	s4		s9		
9		r4	r4		r4	r4	
10		r5	r5		r5	r5	

# SLR(1) Parsing with Ambiguity

1
$S \rightarrow \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

2
$S \rightarrow E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

6
$E \rightarrow E * E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

1.  $S \rightarrow E$
2.  $E \rightarrow E + E$  (Rgt.-assoc, pri. 0)
3.  $E \rightarrow E * E$  (Rgt.-assoc, pri. 1)
4.  $E \rightarrow (E)$
5.  $E \rightarrow int$

3
$E \rightarrow E + \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

7
$E \rightarrow (\cdot E)$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

$E \rightarrow \cdot (E)$

4
$E \rightarrow E * \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

8
$E \rightarrow (E \cdot)$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

5
$E \rightarrow E + E$
$\cdot E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

9
$E \rightarrow (E) \cdot$
10
$E \rightarrow int \cdot$

1)	int	+	*	(	)	\$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5		s3 r2	s4		r2	r2	
6		r3	s4 r3		r3	r3	
7	s10			s7			s8
8		s3	s4		s9		
9		r4	r4		r4	r4	
10		r5	r5		r5	r5	

# SLR(1) Parsing with Ambiguity

1
$S \rightarrow \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
int

2
$S \rightarrow E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

6
$E \rightarrow E * E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

1.  $S \rightarrow E$
2.  $E \rightarrow E + E$  (Rgt.-assoc, pri. 0)
3.  $E \rightarrow E * E$  (Rgt.-assoc, pri. 1)
4.  $E \rightarrow (E)$
5.  $E \rightarrow \text{int}$

3
$E \rightarrow E + \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
int

7
$E \rightarrow (\cdot E)$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
int

(E)

4
$E \rightarrow E * \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
int

8
$E \rightarrow (E \cdot)$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

5
$E \rightarrow E + E$
$\cdot E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

9
$E \rightarrow (E) \cdot$
10
$E \rightarrow \text{int} \cdot$

1)	int	+	*	(	)	\$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5		s3 r2	s4		r2	r2	
6		r3	s4 r3		r3	r3	
7	s10			s7			s8
8		s3	s4		s9		
9		r4	r4		r4	r4	
10		r5	r5		r5	r5	

# SLR(1) Parsing with Ambiguity

1
$S \rightarrow \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

2
$S \rightarrow E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

6
$E \rightarrow E * E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

1.  $S \rightarrow E$
2.  $E \rightarrow E + E$  (Rgt.-assoc, pri. 0)
3.  $E \rightarrow E * E$  (Rgt.-assoc, pri. 1)
4.  $E \rightarrow (E)$
5.  $E \rightarrow int$

3
$E \rightarrow E + \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

7
$E \rightarrow (\cdot E)$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

$E \rightarrow \cdot (E)$

4
$E \rightarrow E * \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

8
$E \rightarrow (E \cdot)$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

5
$E \rightarrow E + E$
$\cdot E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

9
$E \rightarrow (E) \cdot$
10
$E \rightarrow int \cdot$

1)	int	+	*	(	)	\$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5		s3	s4		r2	r2	
6		r3	s4		r3	r3	
7	s10			s7			s8
8		s3	s4		s9		
9		r4	r4		r4	r4	
10		r5	r5		r5	r5	



# SLR(1) Parsing with Ambiguity

1
$S \rightarrow \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
int

2
$S \rightarrow E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$\cdot * E$

6
$E \rightarrow E * E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$\cdot * E$

1.  $S \rightarrow E$
2.  $E \rightarrow E + E$  (Rgt.-assoc, pri. 0)
3.  $E \rightarrow E * E$  (Rgt.-assoc, pri. 1)
4.  $E \rightarrow (E)$
5.  $E \rightarrow \text{int}$

3
$E \rightarrow E + \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
int

7
$E \rightarrow (\cdot E)$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
int

$E \rightarrow \cdot (E)$

4
$E \rightarrow E * \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
int

8
$E \rightarrow (E \cdot)$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$\cdot * E$

5
$E \rightarrow E + E$
$\cdot E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

9
$E \rightarrow (E) \cdot$
10
$E \rightarrow \text{int} \cdot$

1)	int	+	*	(	)	\$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5		s3	s4		r2	r2	
6		r3	s4		r3	r3	
7	s10			s7			s8
8		s3	s4		s9		
9		r4	r4		r4	r4	
10		r5	r5		r5	r5	

# Resolving Conflicts with Precedence

- When choosing whether to reduce a rule containing **t** or shift the terminal **r**:
  - If **t** has higher priority, **reduce**. If **r**
    - has higher priority, **shift**.
  - If **t** and **r** have the same priority:
    - If **t** is left-associative, **reduce**.
    - If **t** is right-associative, **shift**.
    - If **t** is non-associative, **error**.

# SLR(1) Parsing with Ambiguity

1
$S \rightarrow \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

2
$S \rightarrow E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$\cdot * E$

6
$E \rightarrow E * E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$\cdot * E$

1.  $S \rightarrow E$
2.  $E \rightarrow E + E$  (Left-assoc, pri. 0)
3.  $E \rightarrow E * E$  (Left-assoc, pri. 1)
4.  $E \rightarrow (E)$
5.  $E \rightarrow int$

3
$E \rightarrow E + \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

7
$E \rightarrow (\cdot E)$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

$E \rightarrow \cdot (E)$

4
$E \rightarrow E * \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

8
$E \rightarrow (E \cdot)$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$\cdot * E$

5
$E \rightarrow E + E$
$\cdot E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

9
$E \rightarrow (E) \cdot$
10
$E \rightarrow int \cdot$

1)	int	+	*	(	)	\$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5		r2	s4		r2	r2	
6		r3	r3		r3	r3	
7	s10			s7			s8
8		s3	s4		s9		
9		r4	r4		r4	r4	
10		r5	r5		r5	r5	

# Error Handling

- What should the parser do when it encounters an error?
- Could just say “syntax error,” but we'd like more detailed messages.
- How do we resume parsing after an error?

# SLR(1) Parsing with Ambiguity

1
$S \rightarrow \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

2
$S \rightarrow E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$\cdot * E$

6
$E \rightarrow E * E \cdot$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$\cdot * E$

1.  $S \rightarrow E$
2.  $E \rightarrow E + E$  (Left-assoc, pri. 0)
3.  $E \rightarrow E * E$  (Left-assoc, pri. 1)
4.  $E \rightarrow (E)$
5.  $E \rightarrow int$

3
$E \rightarrow E + \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

7
$E \rightarrow (\cdot E)$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

$E \rightarrow \cdot (E)$

4
$E \rightarrow E * \cdot E$
$E \rightarrow \cdot E +$
$E \rightarrow \cdot E$
$* E \rightarrow \cdot$
$int$

8
$E \rightarrow (E \cdot)$
$E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$\cdot * E$

5
$E \rightarrow E + E$
$\cdot E \rightarrow E \cdot +$
$E \rightarrow E \cdot$
$* E$

9
$E \rightarrow (E) \cdot$
10
$E \rightarrow int \cdot$

1)	int	+	*	(	)	\$	E
1	s10			s7			s2
2		s3	s4			acc	
3	s10			s7			s5
4	s10			s7			s6
5		r2	s4		r2	r2	
6		r3	r3		r3	r3	
7	s10			s7			s8
8		s3	s4		s9		
9		r4	r4		r4	r4	
10		r5	r5		r5	r5	

# Next Time

- **Error Recovery**
  - Report all the errors!