بسم الله الرحمن الرحيم

برنامهریزی نیمهمعین برای طراحی الگوریتمهای تقریبی

جلسه دهم: آیا برنامهریزی هممثبت الگوریتم سریع دارد؟



Cone Programming

(P) Maximize $\langle \mathbf{c}, \mathbf{x} \rangle$ subject to $\mathbf{b} - A(\mathbf{x}) \in L$ $\mathbf{x} \in K$.



SDP

maximize $C \bullet X$ subject to $A_i \bullet X = b_i, \quad i = 1, 2, ..., m$ $X \succeq 0.$



LP

 $\begin{array}{ll}
\text{maximize} & c^{\mathsf{T}} x \\
\text{subject to} & Ax = b \\
 & x \ge 0
\end{array}$





ماتریس هممثبت و کاملا مثبت

ماتریس هممثبت

7.1.1 Definition. A matrix $M \in SYM_n$ is called copositive if

 $\mathbf{x}^T M \mathbf{x} \geq 0$ for all $\mathbf{x} \geq 0$.

 $COP_n := \{ M \in SYM_n : \mathbf{x}^T M \mathbf{x} \ge 0 \text{ for all } \mathbf{x} \ge 0 \}$

ماتریس هممثبت

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 $PSD_n \subseteq COP_n$

مشاهده:

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 $PSD_n \subsetneq COP_n$

مشاهده:

$$M = \left(\begin{array}{cc} 0 & 1 \\ 1 & 0 \end{array}\right)$$

$$\mathbf{x}^T M \mathbf{x} \geq 0$$
 for all $\mathbf{x} \geq 0$.

$$COP_n := \{ M \in SYM_n : \mathbf{x}^T M \mathbf{x} \ge 0 \text{ for all } \mathbf{x} \ge 0 \}$$

7.1.3 Lemma. The set COP_n is a closed convex cone in SYM_n .

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 COP_n دوگان

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 $x \ge 0$ ماتریس های xx^{T} که

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$$COP_n$$
 دوگان

$$x \ge 0$$
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$$x^{\mathsf{T}} M x = M \bullet x x^{\mathsf{T}}$$

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 $x \ge 0$ ماتریسهای $x \ge 0$ که $x \ge 0$

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- ترکیب محدب این ماتریسها
 - جمع این ماتریسها

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COP_n دوگان

آن را به صورت زیر نوشت

$$M = \sum_{i=1}^{\ell} \mathbf{x}_i \mathbf{x}_i^T$$

$$\mathbf{x}_1,\mathbf{x}_2,\ldots,\mathbf{x}_\ell\in\mathbb{R}^n_+$$
 ک

$$x^{\mathsf{T}} M x = M \bullet x x^{\mathsf{T}}$$

• ترکیب محدب این ماتریسها

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$$M = \sum_{i=1}^{\ell} \mathbf{x}_i \mathbf{x}_i^T = AB$$

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$$\downarrow i$$

$$A = \begin{pmatrix} \vdots \\ x_{i}[j] \end{pmatrix} j$$

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$$\downarrow i \qquad \downarrow k$$

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 $A = (x_1 \quad x_2 \quad \dots \quad x_t)$

 $B = A^{\mathsf{T}}$

$$M = \sum_{i=1}^{\ell} \mathbf{x}_i \mathbf{x}_i^T = AA^T, \tag{7.2}$$

where $A \in \mathbb{R}^{n \times \ell}$ is the (nonnegative) matrix with columns $\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_{\ell}$.

 $POS_n := \{M \in SYM_n : M \text{ is completely positive}\}\$

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برای کاملا مثبت بودن، تعداد ثابتی جمله کافی است.

7.1.5 Lemma. M is completely positive if and only if there are $\binom{n+1}{2}$ nonnegative vectors $\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_{\binom{n+1}{2}} \in \mathbb{R}^n$ such that

$$M = \sum_{i=1}^{\binom{r}{2}} \mathbf{x}_i \mathbf{x}_i^T.$$

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$$\mathbf{x}^T M \mathbf{x} > 0$$
 for all $\mathbf{x} > 0$.

 $COP_n := \{ M \in SYM_n : \mathbf{x}^T M \mathbf{x} \ge 0 \text{ for all } \mathbf{x} \ge 0 \}$

$$M = \sum_{i=1}^{\ell} \mathbf{x}_i \mathbf{x}_i^T = AA^T, \tag{7.2}$$

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کنج محدب بسته است. POS_n

$$\lambda M = \sum_{i=1}^{\ell} (\sqrt{\lambda} \mathbf{x}_i) (\sqrt{\lambda} \mathbf{x}_i)^T$$
 کنج

كنج محدب بسته بودن ماتريسهاى كاملا مثبت

$$M^{(k)} = \sum_{i=1}^{\binom{n+1}{2}} \mathbf{x}_i^{(k)} \mathbf{x}_i^{(k)}^T = A^{(k)} A^{(k)}^T \in POS_n$$

 $\lim_{k\to\infty} M^{(k)} = M \in SYM_n$

 $M \in POS_n$:

 $\mathbf{X}^{(k)}$ نج محدب بسته بودن $\mathbf{a}_i^{(k)}$ کنج محدب بسته بودن

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 $M \in POS_n$:حكم

$$\mathbf{A}^{(k)}$$
نج محدب بسته بودن $\mathbf{a}_i^{(k)}$ کنج محدب بسته بودن

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$$m_{ii} = \lim_{k \to \infty} M_{ii}^{(k)} = \lim_{k \to \infty} \mathbf{a}_i^{(k)T} \mathbf{a}_i^{(k)} = \lim_{k \to \infty} \|\mathbf{a}_i^{(k)}\|^2$$

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$$A^{(k)}$$
نستون i از $A^{(k)}$: \mathbf{A} مثبت $\mathbf{a}_i^{(k)}$

$$\mathbf{a}_i^{(k)}$$
 مثبت $\mathbf{a}_i^{(k)}$

$$M^{(k)} = \sum_{i=1}^{\binom{n+1}{2}} \mathbf{x}_{i}^{(k)} \mathbf{x}_{i}^{(k)}^{T} = A^{(k)} A^{(k)}^{T} \in POS_{n}$$

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بردارهای ai کراندارند

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زبر رشته با حد ai دارند

$\mathbf{a}_i^{(k)}$ ستون \mathbf{i} از $A^{(k)}$ کنج محدب بسته بودن $\mathbf{a}_i^{(k)}$

$$M^{(k)} = \sum_{i=1}^{\binom{n+1}{2}} \mathbf{x}_{i}^{(k)} \mathbf{x}_{i}^{(k)}^{T} = A^{(k)} A^{(k)}^{T} \in POS_{n}$$

$$\lim_{k \to \infty} M^{(k)} = M \in SYM_n$$

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$$M^{(k)} = \sum_{i=1}^{\binom{n+1}{2}} \mathbf{x}_i^{(k)} \mathbf{x}_i^{(k)}^T = A^{(k)} A^{(k)}^T \in POS_n$$

 $\lim_{k\to\infty} M^{(k)} = M \in SYM_n$

$$M = (\lim A)(\lim A)^{\mathsf{T}}$$
 حکم: $M \in \mathrm{POS}_n$ حکم

- بردارهای ai کراندارند
- $m_{ii} = \lim_{k \to \infty} M_{ii}^{(k)} = \lim_{k \to \infty} \mathbf{a}_i^{(k)^T} \mathbf{a}_i^{(k)} = \lim_{k \to \infty} \|\mathbf{a}_i^{(k)}\|^2$
 - زبر رشته با حد ai دارند
 - ماتریس A: با ستونهای ai

$\mathbf{a}_i^{(k)}$ نج محدب بسته بودن $\mathbf{a}_i^{(k)}$ کنج محدب بسته بودن

$$M^{(k)} = \sum_{i=1}^{\binom{n+1}{2}} \mathbf{x}_{i}^{(k)} \mathbf{x}_{i}^{(k)}^{T} = A^{(k)} A^{(k)}^{T} \in POS_{n}$$

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 حکم: $M \in POS_n$:حکم

بردارهای ai کراندارند

$$m_{ii} = \lim_{k \to \infty} M_{ii}^{(k)} = \lim_{k \to \infty} \mathbf{a}_i^{(k)} \mathbf{a}_i^{(k)} = \lim_{k \to \infty} \|\mathbf{a}_i^{(k)}\|^2$$

- زیر رشته با حد ai دارند
- ماتریس A: با ستونهای ai
- حد قطر AA^{\top} = قطر M (حد تابع پیوسته = تابع پیوسته حد)

$$\mathbf{A}^{(k)}$$
 از $\mathbf{A}^{(k)}$ کنج محدب بسته بودن $\mathbf{a}_i^{(k)}$ کنج محدب $\mathbf{A}^{(k)}$ بسته بودن $\mathbf{a}_i^{(k)}$ بسته بودن $\mathbf{A}^{(k)}$

$$M^{(k)} = \sum_{i=1}^{\binom{n+1}{2}} \mathbf{x}_{i}^{(k)} \mathbf{x}_{i}^{(k)}^{T} = A^{(k)} A^{(k)}^{T} \in POS_{n}$$

$$\lim_{k \to \infty} M^{(k)} = M \in SYM_n$$

$$\lim_{k\to\infty} M^{(n)} = M \in S : M_n$$

$$M = (\lim A)(\lim A)^{\top}$$
 حکم: $M \in POS_n$ عکم:

بردارهای ai کراندارند

$$m_{ii} = \lim_{k \to \infty} M_{ii}^{(k)} = \lim_{k \to \infty} \mathbf{a}_i^{(k)} \mathbf{a}_i^{(k)} = \lim_{k \to \infty} \|\mathbf{a}_i^{(k)}\|^2$$
زير رشته با حد ai دارند

- ماتریس A: با ستونهای ai
- حد قطر AA^{\top} = قطر M (حد تابع پیوسته = تابع پیوسته حد)

$$m_{ij} = \lim_{k o \infty} \mathbf{a}_i^{(k)}^T \mathbf{a}_j^{(k)} = \mathbf{a}_i^T \mathbf{a}_j$$
 حد بقیه درایهها

$$M = \sum_{i=1}^{\ell} \mathbf{x}_i \mathbf{x}_i^T = AA^T, \tag{7.2}$$

where $A \in \mathbb{R}^{n \times \ell}$ is the (nonnegative) matrix with columns $\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_{\ell}$.

 $POS_n := \{M \in SYM_n : M \text{ is completely positive}\}\$

کنج محدب بسته است. POS_n



$$\lambda M = \sum_{i=1}^{\ell} (\sqrt{\lambda} \mathbf{x}_i) (\sqrt{\lambda} \mathbf{x}_i)^T$$
 کنج

7.1.7 Theorem. $POS_n^* = COP_n$.

$$M \in \mathrm{POS}_n^*$$
الف $M \in \mathrm{COP}_n$ (الف

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$$M \notin POS_n^*$$
ب $M \notin COP_n$

$$M \in POS_n^*$$
الف $M \in COP_n$ (الف $M \in COP_n$

$$X \in \mathrm{POS}_n$$
معادلا: $M \cdot X \geq 0$ برای هر

$$M \notin POS_n^*$$
ب $M \notin COP_n$ (ب

$$M \in \operatorname{POS}_n^*$$
الف $M \in \operatorname{COP}_n$ (الف $M \in \operatorname{COP}_n$

$$X \in \mathrm{POS}_n$$
معادلا: $0 \leq X \cdot M$ برای هر

 $M \notin POS_n^*$ آنگاه $M \notin COP_n$ (ب

$$M \in \operatorname{POS}_n^*$$
الف $M \in \operatorname{COP}_n$ (الف •

$$X \in \mathrm{POS}_n$$
معادلا: $0 \leq X \bullet M$ برای هر

$$\underbrace{M}_{\in \text{COP}_n} \bullet \underbrace{\sum_{i=1}^{c} \mathbf{x}_i \mathbf{x}_i^T}_{\in \text{POS}_n}$$

 $M \notin POS_n^*$ آنگاه $M \notin COP_n$ (ب

$$M \in POS_n^*$$
الف $M \in COP_n$ (الف •

$$X \in \mathrm{POS}_n$$
معادلا: $0 \le X \le M$ برای هر

$$\underbrace{M}_{\in \text{COP}_n} \bullet \underbrace{\sum_{i=1}^{\ell} \mathbf{x}_i \mathbf{x}_i^T}_{\in \text{POS}_n} = \sum_{i=1}^{\ell} M \bullet \mathbf{x}_i \mathbf{x}_i^T$$

 $M \notin POS_n^*$ ب $M \notin COP_n$ آنگاه

$$M \in \operatorname{POS}_n^*$$
الف $M \in \operatorname{COP}_n$ (الف •

$$X \in \mathrm{POS}_n$$
معادلا: $0 \leq X \bullet M$ برای هر

$$X \in POS_n$$
 معادلا: $0 \ge M \cdot X \ge 0$ برای هر $M \cdot X \ge 0$ معادلا: $\sum_{i=1}^{\ell} \mathbf{x}_i \mathbf{x}_i^T = \sum_{i=1}^{\ell} M \cdot \mathbf{x}_i \mathbf{x}_i^T = \sum_{i=1}^{\ell} \mathbf{x}_i^T M \underbrace{\mathbf{x}_i}_{\ge 0}$

 $M \notin POS_n^*$ آنگاه $M \notin COP_n$ (پ

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$$X \in POS_n$$
 معادلا: $M \cdot X \geq 0$ برای هر $M \cdot X \geq 0$ معادلا: $M \cdot X \geq 0$ معادلا: $M \cdot X \geq 0$ معادلا: $M \cdot X = \sum_{i=1}^{\ell} \mathbf{x}_i \mathbf{x}_i^T = \sum_{i=1}^{\ell} \mathbf{x}_i \mathbf{x}_i^T = \sum_{i=1}^{\ell} \mathbf{x}_i^T M \underbrace{\mathbf{x}_i}_{\geq 0} \geq 0$

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$$M \notin POS_n^*$$
ب) $M \notin COP_n$ آنگاه

 $x^{\mathsf{T}}Mx < 0$ بردار نامنفی x هست که

$$M \in \mathrm{POS}_n^*$$
الف $M \in \mathrm{COP}_n$ (الف •

$$X \in \mathrm{POS}_n$$
معادلا: $0 \leq X \bullet M$ برای هر

$$X \in POS_n$$
 معادلا: $M \cdot X \geq 0$ برای هر $M \cdot X \geq 0$ معادلا: $M \cdot X = \sum_{i=1}^{\ell} \mathbf{x}_i \mathbf{x}_i^T = \sum_{i=1}^{\ell} \mathbf{x}_i^T M \underbrace{\mathbf{x}_i}_{\geq 0} \geq 0$

$$M \notin POS_n^*$$
ب) $M \notin COP_n$ آنگاه

$$x^{\mathsf{T}}Mx < 0$$
 بردار نامنفی x هست که

$$M \bullet xx^{\mathsf{T}} < 0$$
 بردار نامنفی x هست که

1.7 Theorem.
$$100_n - 001_n$$
.

 $POS_n \subseteq PSD_n \subseteq COP_n$

بسم الله الرحمن الرحيم

برنامهریزی نیمهمعین برای طراحی الگوریتمهای تقریبی

جلسه یازدهم: آیا برنامهریزی هممثبت الگوریتم سریع دارد؟



Cone Programming

(P) Maximize $\langle \mathbf{c}, \mathbf{x} \rangle$ subject to $\mathbf{b} - A(\mathbf{x}) \in L$ $\mathbf{x} \in K$.



SDP

maximize $C \bullet X$ subject to $A_i \bullet X = b_i, \quad i = 1, 2, ..., m$ $X \succeq 0.$



LP

maximize $c^{\mathsf{T}}x$ subject to Ax = b $x \ge 0$



7.1.1 Definition. A matrix
$$M \in SYM_n$$
 is called copositive if

$$\mathbf{x}^T M \mathbf{x} \geq 0$$
 for all $\mathbf{x} \geq 0$.

 $COP_n := \{ M \in SYM_n : \mathbf{x}^T M \mathbf{x} \ge 0 \text{ for all } \mathbf{x} \ge 0 \}$

$$POS_n \subseteq PSD_n \subseteq COP_n$$

7.1.4 Definition. A matrix $M \in \text{SYM}_n$ is called completely positive if for some ℓ , there are ℓ nonnegative vectors $\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_{\ell} \in \mathbb{R}^n_+$, such that

$$M = \sum_{i=1}^{c} \mathbf{x}_i \mathbf{x}_i^T = AA^T, \tag{7.2}$$

where $A \in \mathbb{R}^{n \times \ell}$ is the (nonnegative) matrix with columns $\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_{\ell}$.

 $POS_n := \{M \in SYM_n : M \text{ is completely positive}\}$

7.1.1 Definition. A matrix
$$M \in SYM_n$$
 is called copositive if

$$\mathbf{x}^T M \mathbf{x} \geq 0$$
 for all $\mathbf{x} \geq 0$.

$$COP_n := \{ M \in SYM_n : \mathbf{x}^T M \mathbf{x} \ge 0 \text{ for all } \mathbf{x} \ge 0 \}$$

7.1.7 Theorem.
$$POS_n^* = COP_n$$
.

7.1.4 Definition. A matrix
$$M \in \text{SYM}_n$$
 is called *completely positive* if for some ℓ , there are ℓ nonnegative vectors $\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_{\ell} \in \mathbb{R}^n_+$, such that

$$M = \sum_{i=1}^{t} \mathbf{x}_i \mathbf{x}_i^T = AA^T, \tag{7.2}$$

where $A \in \mathbb{R}^{n \times \ell}$ is the (nonnegative) matrix with columns $\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_{\ell}$.

 $POS_n := \{M \in SYM_n : M \text{ is completely positive}\}\$



برنامهریزی هممثبت برای یک مسئله سخت!

Cone Programming

(P) Maximize $\langle \mathbf{c}, \mathbf{x} \rangle$

subject to $\mathbf{b} - A(\mathbf{x}) \in L$ برنامهریزی هممثبت $\mathbf{x} \in K$.

$C \bullet X$

subject to A(X) = b $X \in COP_n$

maximize

SDP

 $C \bullet X$ subject to $A_i \bullet X = b_i, \quad i = 1, 2, \dots, m$ $X \succeq 0$.



 $C \bullet X$ maximize

برنامهریزی کاملا مثبت

maximize

subject to A(X) = b $X \in POS_n$

 $c^{\mathsf{T}}x$ naximize subject to Ax = b

LP



 $x \ge 0$

بیشترین نرخ ارسال با گراف G:

$$\sigma(G) = \sup \left\{ \frac{1}{k} \log \alpha(G^k) : k \in \mathbb{N} \right\},$$

$$\Theta(G) = 2^{\sigma(G)} = \lim_{k \to \infty} \sqrt[k]{\alpha(G^k)}$$

$$\vartheta(G) := \min_{\mathcal{U}} \min_{\|\mathbf{c}\|=1} \max_{i=1}^{n} \frac{1}{(\mathbf{c}^T \mathbf{u}_i)^2}$$

 $\Theta(G) \leq \vartheta(G)$

قضىه:

قضیه: برنامهریزی زیر $\vartheta(G)$ را محاسبه می کند

 $\begin{array}{ll} \textit{Minimize} & t \\ \textit{subject to} & y_{ij} = -1 & \textit{if } \{i,j\} \in \overline{E} \\ & y_{ii} = t-1 & \textit{for all } i = 1, \dots, n \\ & Y \succeq 0. \end{array}$

7.2.1 Theorem. The copositive program

minimize
$$t$$

subject to $y_{ij} = -1$, if $\{i, j\} \in \overline{E}$
 $y_{ii} = t - 1$, for all $i = 1, 2, ..., n$
 $Y \in \text{COP}_n$

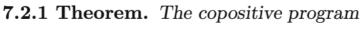
minimize tsubject to $y_{ij} = -1$, if $\{i, j\} \in \overline{E}$ $y_{ii} = t - 1$, for all i = 1, 2, ..., n $Y \in \text{COP}_n$

minimize tsubject to $y_{ij} = -1$, if $\{i, j\} \in \overline{E}$ $y_{ii} = t - 1$, for all i = 1, 2, ..., n $Y \in \text{COP}_n$

minimize
$$t$$

subject to $y_{ij} = -1$, if $\{i, j\} \in \overline{E}$
 $y_{ii} = t - 1$, for all $i = 1, 2, ..., n$
 $Y \in \text{COP}_n$

7.2.1 Theorem. The copositive program minimize subject to $y_{ij} = -1$, if $\{i, j\} \in \overline{E}$ $y_{ii} = t - 1$, for all i = 1, 2, ..., n $Y \in COP_n$ has value $\alpha(G)$, the size of a maximum independent set in G.



minimize
$$t$$
 subject to $y_{ij} = -1$, if $\{i, j\} \in \overline{E}$ $y_{ii} = t - 1$, for all $i = 1, 2, ..., n$

 $Y \in COP_n$

$$\alpha(G)$$
 روش: یک جواب شدنی برای دوگان با مقدار

 $\alpha(G) \leq$

minimize
$$t$$

subject to $y_{ij} = -1$, if $\{i, j\} \in \overline{E}$ $y_{ii} = t - 1$, for all i = 1, 2, ..., n $Y \in \text{COP}_n$

$$\alpha(G)$$
 روش: یک جواب شدنی برای دوگان با مقدار

(P) Maximize
$$\langle \mathbf{c}, \mathbf{x} \rangle$$

subject to $\mathbf{b} - A(\mathbf{x}) \in L$
 $\mathbf{x} \in K$.

(D) Minimize
$$\langle \mathbf{b}, \mathbf{y} \rangle$$

subject to $A^T(\mathbf{y}) - \mathbf{c} \in K^*$
 $\mathbf{y} \in L^*$.

 $\begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix}$

 $\alpha(G) \leq$

$$egin{array}{ll} ext{minimize} & t \ ext{subject to} & y_{ij} = -1, ext{ if } \{i,j\} \in \overline{E} \ & y_{ii} = t-1, ext{ for all } i = 1,2,\ldots,n \end{array}$$

has value $\alpha(G)$, the size of a maximum independent set in G.

 $Y \in COP_n$

$$\alpha(G)$$
 روش: یک جواب شدنی برای دوگان با مقدار

(P) Maximize
$$\langle \mathbf{c}, \mathbf{x} \rangle$$

subject to $\mathbf{b} - A(\mathbf{x}) \in L$
 $\mathbf{x} \in K$.

(D) Minimize
$$\langle \mathbf{b}, \mathbf{y} \rangle$$

subject to $A^T(\mathbf{y}) - \mathbf{c} \in K^*$
 $\mathbf{y} \in L^*$.

 $\alpha(G) \leq$

minimize
$$t$$

subject to $y_{ij} = -1$, if $\{i, j\} \in \overline{E}$
 $y_{ii} = t - 1$, for all $i = 1, 2, ..., n$
 $Y \in \text{COP}_n$

$$\alpha(G)$$
 روش: یک جواب شدنی برای دوگان با مقدار

$$\min \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix}$$
 (P) Maximize $\langle \mathbf{c}, \mathbf{x} \rangle$ subject to $\mathbf{b} - A$

subject to
$$\mathbf{b} - A(\mathbf{x}) \in L$$

 $\mathbf{x} \in K$.

(D) Minimize
$$\langle \mathbf{b}, \mathbf{y} \rangle$$

subject to $A^T(\mathbf{y}) - \mathbf{c} \in K^*$
 $\mathbf{y} \in L^*$.

minimize
$$t$$

subject to $y_{ij} = -1$, if $\{i, j\} \in \overline{E}$
 $y_{ii} = t - 1$, for all $i = 1, 2, ..., n$
 $Y \in \text{COP}_n$

 $\alpha(G)$ روش: یک جواب شدنی برای دوگان با مقدار

$$\min \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix}$$
$$\begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix} \bullet \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,i):1 & 0 \\ 0 & 0 & -1 \end{pmatrix} = -1$$

(P) Maximize
$$\langle \mathbf{c}, \mathbf{x} \rangle$$

subject to $\mathbf{b} - A(\mathbf{x}) \in L$
 $\mathbf{x} \in K$.

(D) Minimize
$$\langle \mathbf{b}, \mathbf{y} \rangle$$

subject to $A^T(\mathbf{y}) - \mathbf{c} \in K^*$
 $\mathbf{y} \in L^*$.

minimize
$$t$$

subject to $y_{ij} = -1$, if $\{i, j\} \in \overline{E}$
 $y_{ii} = t - 1$, for all $i = 1, 2, ..., n$
 $Y \in \text{COP}_n$

$$\alpha(G)$$
 روش: یک جواب شدنی برای دوگان با مقدار •

$$\min \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix}$$
$$\begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix} \bullet \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,i):1 & 0 \\ 0 & 0 & -1 \end{pmatrix} = -1$$

$$ij \in \bar{E}$$

(P) Maximize
$$\langle \mathbf{c}, \mathbf{x} \rangle$$

subject to $\mathbf{b} - A(\mathbf{x}) \in L$
 $\mathbf{x} \in K$.

(D) Minimize
$$\langle \mathbf{b}, \mathbf{y} \rangle$$

subject to $A^T(\mathbf{y}) - \mathbf{c} \in K^*$
 $\mathbf{y} \in L^*$.

$$\alpha(G)$$
 روش: یک جواب شدنی برای دوگان با مقدار

$$\min \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix}$$

$$\begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix} \bullet \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,i):1 & 0 \\ 0 & 0 & -1 \end{pmatrix} = -1$$

$$\begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix} \bullet \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,j):1 & 0 \\ 0 & 0 & 0 \end{pmatrix} = -1$$

$$ij \in \bar{E}$$

(P) Maximize
$$\langle \mathbf{c}, \mathbf{x} \rangle$$

subject to $\mathbf{b} - A(\mathbf{x}) \in L$
 $\mathbf{x} \in K$.

(D) Minimize
$$\langle \mathbf{b}, \mathbf{y} \rangle$$

subject to $A^T(\mathbf{y}) - \mathbf{c} \in K^*$
 $\mathbf{y} \in L^*$.

minimize
$$t$$
 subject to $y_{ij} = -1$, if $\{i, j\} \in \overline{E}$ $y_{ii} = t - 1$, for all $i = 1, 2, ..., n$ $Y \in \text{COP}_n$

 $\alpha(G)$ روش: یک جواب شدنی برای دوگان با مقدار

$$\min \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix}$$

$$\begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix} \bullet \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,i):1 & 0 \\ 0 & 0 & -1 \end{pmatrix} = -1$$

$$\begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix} \bullet \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,j):1 & 0 \\ 0 & 0 & 0 \end{pmatrix} = -1$$

$$V \in COD$$

 $Y \in COP_n$

(P) Maximize
$$\langle \mathbf{c}, \mathbf{x} \rangle$$

subject to $\mathbf{b} - A(\mathbf{x}) \in L$
 $\mathbf{x} \in K$.

(D) Minimize
$$\langle \mathbf{b}, \mathbf{y} \rangle$$

subject to $A^T(\mathbf{y}) - \mathbf{c} \in K^*$
 $\mathbf{y} \in L^*$.

(P) Maximize
$$\langle \mathbf{c}, \mathbf{x} \rangle$$

subject to $\mathbf{b} - A(\mathbf{x}) \in L$
 $\mathbf{x} \in K$.

(D) Minimize
$$\langle \mathbf{b}, \mathbf{y} \rangle$$

subject to $A^T(\mathbf{y}) - \mathbf{c} \in K^*$
 $\mathbf{y} \in L^*$.

$$\max \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} \bullet \begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix}$$
$$\begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix} \bullet \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,i):1 & 0 \\ 0 & 0 & -1 \end{pmatrix} = -1$$
$$\begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix} \bullet \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,j):1 & 0 \\ 0 & 0 & 0 \end{pmatrix} = -1$$
$$Y \in COP_n$$
$$ij \in \bar{E}$$

(P) Maximize
$$\langle \mathbf{c}, \mathbf{x} \rangle$$

subject to $\mathbf{b} - A(\mathbf{x}) \in L$
 $\mathbf{x} \in K$.

(D) Minimize
$$\langle \mathbf{b}, \mathbf{y} \rangle$$

subject to $A^T(\mathbf{y}) - \mathbf{c} \in K^*$
 $\mathbf{y} \in L^*$.

$$\max \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} \bullet \begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix}$$
$$\begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix} \bullet \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,i):1 & 0 \\ 0 & 0 & -1 \end{pmatrix} = -1$$
$$\begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix} \bullet \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,j):1 & 0 \\ 0 & 0 & 0 \end{pmatrix} = -1$$
$$Y \in COP_n$$
$$ij \in \bar{E}$$

 $\in POS_n$

(P) Maximize
$$\langle \mathbf{c}, \mathbf{x} \rangle$$

subject to $\mathbf{b} - A(\mathbf{x}) \in L$
 $\mathbf{x} \in K$.

(D) Minimize
$$\langle \mathbf{b}, \mathbf{y} \rangle$$

subject to $A^T(\mathbf{y}) - \mathbf{c} \in K^*$
 $\mathbf{y} \in L^*$.

$$\max \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} \bullet \begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix}$$

$$\begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix} \bullet \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,i):1 & 0 \\ 0 & 0 & -1 \end{pmatrix} = -1$$

$$\begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix} \bullet \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,j):1 & 0 \\ 0 & 0 & 0 \end{pmatrix} = -1$$

$$Y \in COP_n$$

$$\sum_{i} x_{ii} \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,j):1 & 0 \\ 0 & 0 & -1 \end{pmatrix} = -1$$

$$ij \in \bar{E}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & (i,i) : 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

 $\in POS_n$

(P) Maximize
$$\langle \mathbf{c}, \mathbf{x} \rangle$$

subject to $\mathbf{b} - A(\mathbf{x}) \in L$
 $\mathbf{x} \in K$.

(D) Minimize
$$\langle \mathbf{b}, \mathbf{y} \rangle$$

subject to $A^T(\mathbf{y}) - \mathbf{c} \in K^*$
 $\mathbf{y} \in L^*$.

$$\max \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} \bullet \begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix}$$

$$\begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix} \bullet \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,i):1 & 0 \\ 0 & 0 & -1 \end{pmatrix} = -1$$

$$\begin{pmatrix} Y & 0 \\ 0 & (i,i):1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$\begin{pmatrix} Y & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\sum_{i} \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,i):1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

 $Y \in COP_n$

$$\begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix} \bullet \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,i):1 & 0 \\ 0 & 0 & -1 \end{pmatrix} = -1 \qquad \sum_{i} x_{ii} \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,i):1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$
$$\begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix} \bullet \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,j):1 & 0 \\ 0 & 0 & 0 \end{pmatrix} = -1 \qquad \sum_{ij \in \bar{E}} x_{ij} \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,j):1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$
$$ij \in \bar{E}$$

 $\in POS_n$

(P) Maximize
$$\langle \mathbf{c}, \mathbf{x} \rangle$$

subject to $\mathbf{b} - A(\mathbf{x}) \in L$
 $\mathbf{x} \in K$.

(D) Minimize
$$\langle \mathbf{b}, \mathbf{y} \rangle$$

subject to $A^T(\mathbf{y}) - \mathbf{c} \in K^*$
 $\mathbf{y} \in L^*$.

$$\max \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} \bullet \begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix}$$
$$\begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix} \bullet \begin{pmatrix} 0 & 0 \\ 0 & (i,i) : 1 \end{pmatrix}$$

$$\begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix} \bullet \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,i):1 & 0 \\ 0 & 0 & -1 \end{pmatrix} = -1 \qquad \sum_{i} x_{ii} \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,i):1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$\begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix} \bullet \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,j):1 & 0 \\ 0 & 0 & 0 \end{pmatrix} = -1 \qquad \sum_{ij \in \bar{E}} x_{ij} \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,j):1 & 0 \\ 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ 0 & (i,j):1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \in POS_n$$

$$Y \in COP_n$$

$$\begin{pmatrix} 0 \\ : 1 & 0 \\ & -1 \end{pmatrix}$$

$$\begin{vmatrix}
x_{ii} & 0 & 0 & 0 \\
0 & (i,i) : 1 & 0 \\
0 & 0 & -1
\end{vmatrix}$$

$$\begin{bmatrix}
x_{ij} & 0 & 0 & 0 \\
0 & (i,j) : 1 & 0 \\
0 & 0 & 0
\end{bmatrix} - \begin{pmatrix} 0 & 0 \\
0 & -1
\end{pmatrix} \in \mathbb{R}$$

(P) Maximize
$$\langle \mathbf{c}, \mathbf{x} \rangle$$

subject to $\mathbf{b} - A(\mathbf{x}) \in L$
 $\mathbf{x} \in K$.

(D) Minimize
$$\langle \mathbf{b}, \mathbf{y} \rangle$$

subject to $A^T(\mathbf{y}) - \mathbf{c} \in K^*$
 $\mathbf{y} \in L^*$.

$$\max \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} \bullet \begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix}$$
$$\begin{pmatrix} Y & 0 \\ 0 & (i,i) : 1 \end{pmatrix}$$

$$\begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix} \bullet \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,i):1 & 0 \\ 0 & 0 & -1 \end{pmatrix} = -1$$

$$\begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix} \bullet \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,j):1 & 0 \\ 0 & 0 & 0 \end{pmatrix} = -1$$

$$Y \in COP_{n}$$

$$\sum_{i} x_{ii} \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,i):1 & 0 \\ 0 & 0 & -1 \end{pmatrix} + \sum_{i} x_{ij} \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,j):1 & 0 \\ 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ 0 & (i,j):1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \in POS_{n}$$

$$\sum_{i} x_{ii} \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,i) : 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} + \sum_{i} x_{ij} \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,j) : 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \in PC$$

(P) Maximize
$$\langle \mathbf{c}, \mathbf{x} \rangle$$

subject to $\mathbf{b} - A(\mathbf{x}) \in L$
 $\mathbf{x} \in K$.

(D) Minimize
$$\langle \mathbf{b}, \mathbf{y} \rangle$$

subject to $A^T(\mathbf{y}) - \mathbf{c} \in K^*$
 $\mathbf{y} \in L^*$.

$$\max \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} \bullet \begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix}$$

$$\begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix} \bullet \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,i):1 & 0 \\ 0 & 0 & -1 \end{pmatrix} = -1$$

$$\begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix} \bullet \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,j):1 & 0 \\ 0 & 0 & 0 \end{pmatrix} = -1$$

$$Y \in COP_n$$

$$\sum_{ij \in \bar{E}} x_{ij} \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,j):1 & 0 \\ 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ 0 & (i,j):1 & 0 \\ 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ 0 & (i,j):1 & 0 \\ 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ 0 & (i,j):1 & 0 \\ 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} \in POS_n$$

$$\sum_{ij\in\bar{E}} -x_{ij} + \sum_{i} -x_{ii}$$

$$(0 \quad 0 \quad 0)$$

$$ij \in \bar{E}$$

$$i$$

$$\sum_{i} x_{ii} \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,i) : 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} + \sum_{ij \in \bar{E}} x_{ij} \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,j) : 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix}$$

(P) Maximize
$$\langle \mathbf{c}, \mathbf{x} \rangle$$

subject to $\mathbf{b} - A(\mathbf{x}) \in L$
 $\mathbf{x} \in K$.

(D) Minimize
$$\langle \mathbf{b}, \mathbf{y} \rangle$$

subject to $A^T(\mathbf{y}) - \mathbf{c} \in K^*$
 $\mathbf{y} \in L^*$.

$$\max \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} \bullet \begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix} \qquad \min \sum_{ij \in \bar{E}} -x_{ij} + \sum_{i} -$$

$$\min \sum_{ij \in \bar{E}} -x_{ij} + \sum_{i} -x_{ii}$$

$$\begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix} \bullet \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,i):1 & 0 \\ 0 & 0 & -1 \end{pmatrix} = -1 \qquad \sum_{i} x_{ii} \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,i):1 & 0 \\ 0 & 0 & -1 \end{pmatrix} + \begin{pmatrix} Y & 0 \\ 0 & (i,j):1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \bullet \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,j):1 & 0 \\ 0 & 0 & 0 \end{pmatrix} = -1 \\ Y \in COP$$

$$Y \in COP$$

(P) Maximize
$$\langle \mathbf{c}, \mathbf{x} \rangle$$

subject to $\mathbf{b} - A(\mathbf{x}) \in L$
 $\mathbf{x} \in K$.

(D) Minimize
$$\langle \mathbf{b}, \mathbf{y} \rangle$$

subject to $A^T(\mathbf{y}) - \mathbf{c} \in K^*$
 $\mathbf{y} \in L^*$.

 $\sum_{ij\in\bar{F}} x_{ij} \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,j):1 & 0 \\ 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} \in POS_n$

$$\min \sum_{ij \in \bar{E}} -x_{ij} + \sum_{i} -x_{ii}$$

$$\sum_{i} x_{ii} \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,i) : 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} +$$

(P) Maximize
$$\langle \mathbf{c}, \mathbf{x} \rangle$$

subject to $\mathbf{b} - A(\mathbf{x}) \in L$
 $\mathbf{x} \in K$.

(D) Minimize
$$\langle \mathbf{b}, \mathbf{y} \rangle$$

subject to $A^T(\mathbf{y}) - \mathbf{c} \in K^*$
 $\mathbf{y} \in L^*$.

$$\min \sum_{ij \in \bar{E}} -x_{ij} + \sum_{i} -x_{ii}$$

$$\sum_{i} x_{ii} \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,i) : 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} +$$

$$\sum_{ij \in \bar{E}} x_{ij} \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,j) : 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} \in POS_n$$

(P) Maximize
$$\langle \mathbf{c}, \mathbf{x} \rangle$$

subject to $\mathbf{b} - A(\mathbf{x}) \in L$
 $\mathbf{x} \in K$.

(D) Minimize
$$\langle \mathbf{b}, \mathbf{y} \rangle$$

subject to $A^T(\mathbf{y}) - \mathbf{c} \in K^*$
 $\mathbf{y} \in L^*$.

$$\sum_{i} -x_{ii} + 1 \ge 0$$

$$\min \sum_{ij \in \bar{E}} -x_{ij} + \sum_{i} -x_{ii}$$

$$\sum_{i} x_{ii} \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i, i) : 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} + \sum_{ij \in \bar{E}} x_{ij} \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i, j) : 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} \in POS_{n}$$

(P) Maximize
$$\langle \mathbf{c}, \mathbf{x} \rangle$$

subject to $\mathbf{b} - A(\mathbf{x}) \in L$
 $\mathbf{x} \in K$.

(D) Minimize
$$\langle \mathbf{b}, \mathbf{y} \rangle$$

subject to $A^T(\mathbf{y}) - \mathbf{c} \in K^*$
 $\mathbf{y} \in L^*$.

$$\sum_{i} -x_{ii} + 1 \ge 0$$

$$\sum_{i} x_{ii} \le 1$$

$$\min \sum_{ij \in \bar{E}} -x_{ij} + \sum_{i} -x_{ii}$$

$$\sum \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\sum_{i} x_{ii} \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,i) : 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} + \sum_{ij \in \bar{E}} x_{ij} \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,j) : 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} \in POS_{n}$$

(P) Maximize
$$\langle \mathbf{c}, \mathbf{x} \rangle$$

subject to $\mathbf{b} - A(\mathbf{x}) \in L$
 $\mathbf{x} \in K$.

(D) Minimize
$$\langle \mathbf{b}, \mathbf{y} \rangle$$

subject to $A^T(\mathbf{y}) - \mathbf{c} \in K^*$
 $\mathbf{y} \in L^*$.

$$\mathbf{x} \in \mathbf{K}$$
.
$$\ddot{\mathbf{x}} = \mathbf{K}$$

$$\sum_{i} -x_{ii} + 1 \ge 0$$

$$\sum_{i} x_{ii} \le 1$$

$$\min \sum_{ij \in \bar{E}} -x_{ij} + \sum_{i} -x_{ii}$$

$$\sum_{i} x_{ii} \begin{pmatrix} 0 & 0 \\ 0 & (i,i) : 1 \end{pmatrix} +$$

 $\sum_{ij\in\bar{E}} x_{ij} \begin{pmatrix} 0 & 0 \\ 0 & (i,j):1 \end{pmatrix} \quad - \quad (0)$

 $\in POS_n$

$$\min \sum_{ij \in \bar{E}} -x_{ij} + \sum_{i} -x_{ii}$$

(P) Maximize
$$\langle \mathbf{c}, \mathbf{x} \rangle$$

subject to $\mathbf{b} - A(\mathbf{x}) \in L$
 $\mathbf{x} \in K$.

(D) Minimize
$$\langle \mathbf{b}, \mathbf{y} \rangle$$

subject to $A^T(\mathbf{y}) - \mathbf{c} \in K^*$
 $\mathbf{y} \in L^*$.

$$\min \sum_{ij \in \bar{E}} -x_{ij} + \sum_{i} -x_{ii}$$

$$\max \sum_{ij \in \bar{E}} x_{ij} + \sum_{i} x_{ii}$$

 $\sum_{i:\in\bar{E}} x_{ij} \begin{pmatrix} 0 & 0 \\ 0 & (i,j):1 \end{pmatrix} \quad - \quad (0)$

 $\sum x_{ii} \le 1$

 $\in POS_n$

$$\min \sum_{ij \in \bar{E}} -x_{ij} + \sum_{i} -x_{ii}$$

$$\max \sum_{ij \in \bar{E}} x_{ij} + \sum_{i} x_{ii}$$

$$\sum_{i} x_{ii} \begin{pmatrix} 0 & 0 \\ 0 & (i, i) : 1 \end{pmatrix} +$$

(P) Maximize
$$\langle \mathbf{c}, \mathbf{x} \rangle$$

subject to $\mathbf{b} - A(\mathbf{x}) \in L$
 $\mathbf{x} \in K$.

$$L \qquad \begin{array}{c} \text{(D)} \quad \text{Minimize} \quad \langle \mathbf{b}, \mathbf{y} \rangle \\ \quad \text{subject to} \quad A^T(\mathbf{y}) - \mathbf{c} \in K^* \\ \quad \mathbf{y} \in L^*. \end{array}$$

$$\mathbf{x} \in K.$$

$$\max \quad J_n \bullet X$$

•
$$X$$

$$\equiv POS_n$$

$$X \in POS_n$$

$$Tr(Y) < 1$$

$$\operatorname{Tr}(X) \leq 1$$

$$\max \sum_{ij \in \bar{E}} x_{ij} + \sum_{i} x_{ii}$$
$$\sum_{i} x_{ii} \begin{pmatrix} 0 & 0 \\ 0 & (i,i) : 1 \end{pmatrix} +$$

$$\begin{pmatrix} 0 \\ i \end{pmatrix} : 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ i \end{pmatrix} : 1 \end{pmatrix}$$

$$(0) \in POS_n$$

$$\sum_{ij\in\bar{E}} x_{ij} \begin{pmatrix} 0 & 0 \\ 0 & (i,j):1 \end{pmatrix} \quad - \quad (0)$$

 $\sum x_{ii} \le 1$

(P) Maximize
$$\langle \mathbf{c}, \mathbf{x} \rangle$$

subject to $\mathbf{b} - A(\mathbf{x}) \in L$
 $\mathbf{x} \in K$.

$$L \qquad \begin{array}{c} \text{(D)} \quad \text{Minimize} \quad \langle \mathbf{b}, \mathbf{y} \rangle \\ \quad \text{subject to} \quad A^T(\mathbf{y}) - \mathbf{c} \in K^* \\ \quad \mathbf{y} \in L^*. \end{array}$$

$$\max \quad J_n \bullet X$$

$$X \in POS_n$$

$$X \in POS_n$$

$$Tr(X) \leq 1$$

$$X$$
 POS_n

$$\max \sum_{ij \in \bar{E}} x_{ij} + \sum_{i} x_{ii}$$

$$\sum_{i} x_{ii} \begin{pmatrix} 0 & 0 \\ 0 & (i,i) : 1 \end{pmatrix} \rightarrow$$

$$\sum_{i=\bar{x}}$$

$$\sum_{ij\in\bar{E}} x_{ij} \begin{pmatrix} 0 & 0 \\ 0 & (i,j):1 \end{pmatrix} \quad - \quad (0)$$

 $\sum x_{ii} \le 1$

 $x_{ij} = 0$ $ij \in E$

$$\begin{pmatrix} 0 \\ i \end{pmatrix} : 1$$

$$\sum_{i} x_{ii} \begin{pmatrix} 0 & 0 \\ 0 & (i,i) : 1 \end{pmatrix} +$$



 $\in POS_n$

(P) Maximize
$$\langle \mathbf{c}, \mathbf{x} \rangle$$

subject to $\mathbf{b} - A(\mathbf{x}) \in L$
 $\mathbf{x} \in K$.

$$L \qquad \begin{array}{c} \text{(D)} \quad \text{Minimize} \quad \langle \mathbf{b}, \mathbf{y} \rangle \\ \text{subject to} \quad A^T(\mathbf{y}) - \mathbf{c} \in K^* \\ \mathbf{y} \in L^*. \end{array}$$

 $\in POS_n$

$$\max J_n \bullet X$$

$$X \in POS_n$$

$$Tr(X) < 1$$

$$\operatorname{Tr}(X) \leq 1$$

$$\max \sum_{ij \in \bar{E}} x_{ij} + \sum_{i} x_{ii} + \sum_{ij \in E} x_{ij}$$

$$\sum_{i} x_{ii} \begin{pmatrix} 0 & 0 \\ 0 & (i, i) : 1 \end{pmatrix} +$$

$$\sum_{i} x_{ii} \begin{pmatrix} 0 & 0 \\ 0 & (i, i) : 1 \end{pmatrix}$$

$$ij \in E \qquad i \qquad ij \in E$$

$$\sum_{i} x_{ii} \begin{pmatrix} 0 & 0 \\ 0 & (i,i) : 1 \end{pmatrix} \qquad +$$

$$\sum_{i: \in \bar{E}} x_{ij} \begin{pmatrix} 0 & 0 \\ 0 & (i,j) : 1 \end{pmatrix} \qquad - \qquad (0)$$

$$\sum_{ij\in\bar{E}} x_{ij} \begin{pmatrix} 0 & 0 \\ 0 & (i,j) : 1 \end{pmatrix}$$

$$\sum_{i} x_{ii} \le 1$$

$$x_{ii} = 0 \qquad ij$$

$$\overline{x_{ij}} = 0 \qquad ij \in E$$

(P) Maximize
$$\langle \mathbf{c}, \mathbf{x} \rangle$$

subject to $\mathbf{b} - A(\mathbf{x}) \in L$
 $\mathbf{x} \in K$.

$$L \qquad \begin{array}{c} \text{(D)} \quad \text{Minimize} \quad \langle \mathbf{b}, \mathbf{y} \rangle \\ \text{subject to} \quad A^T(\mathbf{y}) - \mathbf{c} \in K^* \\ \mathbf{y} \in L^*. \end{array}$$

 $\sum_{ij\in\bar{k}} x_{ij} \begin{pmatrix} 0 & 0 \\ 0 & (i,j):1 \end{pmatrix} \quad - \quad (0)$

 $x_{ij} = 0$ $ij \in E$

 $\sum x_{ii} \le 1$

 $\in POS_n$

$$\mathbf{x} \in K$$
.

 $\mathbf{max} \quad J_n \bullet X$
 $X \in POS_n$

$$\max \quad J_n \bullet X \qquad \qquad \max \sum_{ij \in \bar{E}} x_{ij} + \sum_i x_{ii} + \sum_{ij \in E} x_{ij}$$

$$X \in POS_n \qquad \qquad \sum_i x_{ii} \begin{pmatrix} 0 & 0 \\ 0 & (i, i) : 1 \end{pmatrix} + \sum_{ij \in E} x_{ij} \begin{pmatrix} 0 & 0 \\ 0 & (i, j) : 1 \end{pmatrix}$$

$$\operatorname{Tr}(X) < 1$$

$$\max \quad J_n \bullet X$$

$$X \in POS_n$$

$$Tr(X) \le 1$$

(P) Maximize
$$\langle \mathbf{c}, \mathbf{x} \rangle$$

subject to $\mathbf{b} - A(\mathbf{x}) \in L$
 $\mathbf{x} \in K$.

(D) Minimize
$$\langle \mathbf{b}, \mathbf{y} \rangle$$
 subject to $A^T(\mathbf{y}) - \mathbf{c} \in K^*$ $\mathbf{y} \in L^*$.

 $\max \sum x_{ij} + \sum x_{ii} + \sum x_{ij}$

$$\mathbf{x} \in K.$$

$$\max \quad J_n \bullet X$$

$$X \in POS_n$$

$$\mathrm{Tr}(X) \leq 1$$

 $\sum x_{ii} \le 1$

 $x_{ij} = 0$ $ij \in E$

$$\sum_{i} x_{ii} \begin{pmatrix} 0 & 0 \\ 0 & (i,i) : 1 \end{pmatrix} + \sum_{ij \in E} x_{ij} \begin{pmatrix} 0 & 0 \\ 0 & (i,j) : 1 \end{pmatrix}$$

$$\sum_{ij \in \bar{E}} x_{ij} \begin{pmatrix} 0 & 0 \\ 0 & (i,j) : 1 \end{pmatrix} - (0) \in POS_n$$

$$\max \quad J_n \bullet X$$

$$X \in POS_n$$

$$Tr(X) \le 1$$

$$x_{ij} = 0 \quad ij \in E$$

(P) Maximize
$$\langle \mathbf{c}, \mathbf{x} \rangle$$

subject to $\mathbf{b} - A(\mathbf{x}) \in L$
 $\mathbf{x} \in K$.

(D) Minimize
$$\langle \mathbf{b}, \mathbf{y} \rangle$$

subject to $A^T(\mathbf{y}) - \mathbf{c} \in K^*$
 $\mathbf{y} \in L^*$.

 $J_n \bullet X$

 $X \in POS_n$

$$\operatorname{Tr}(X) \le 1$$
 $\operatorname{Tr}(X) = 1$

max

(P) Maximize
$$\langle \mathbf{c}, \mathbf{x} \rangle$$

subject to $\mathbf{b} - A(\mathbf{x}) \in L$
 $\mathbf{x} \in K$.

(D) Minimize
$$\langle \mathbf{b}, \mathbf{y} \rangle$$

subject to $A^T(\mathbf{y}) - \mathbf{c} \in K^*$
 $\mathbf{y} \in L^*$.

 $J_n \bullet X$

 $X \in POS_n$

$$\operatorname{Tr}(X) \le 1$$
 $\operatorname{Tr}(X) = 1$ $x_{ij} = 0$ $ij \in E$

max

 $\alpha(G) \leq$

minimize
$$t$$

subject to $y_{ij} = -1$, if $\{i, j\} \in \overline{E}$
 $y_{ii} = t - 1$, for all $i = 1, 2, ..., n$
 $Y \in \text{COP}_n$

has value $\alpha(G)$, the size of a maximum independent set in G.

$$\alpha(G)$$
 روش: یک جواب شدنی برای دوگان با مقدار

I: یکی از بزرگترین مجموعههای مستقل

$$\max \quad J_n \bullet X$$

$$X \in POS_n$$

$$Tr(X) = 1$$

$$x_{ij} = 0 \quad ij \in E$$

$$\begin{array}{ll} \mbox{minimize} & t \\ \mbox{subject to} & y_{ij} = -1, \mbox{ if } \{i,j\} \in \overline{E} \\ & y_{ii} = t-1, \mbox{ for all } i = 1,2,\ldots,n \\ & Y \in \mathrm{COP}_n \end{array}$$

$$\tilde{x}_i = 1_{[i \in I]}$$

$$\tilde{X} = \frac{1}{\alpha(G)} \tilde{\mathbf{x}} \tilde{\mathbf{x}}^T$$

 $\alpha(G) \leq$

$$\alpha(G)$$
 روش: یک جواب شدنی برای دوگان با مقدار α

$$\max \quad J_n \bullet X$$

$$X \in POS_n$$

$$Tr(X) = 1$$

$$x_{ij} = 0 \quad ij \in E$$

$$\begin{array}{ll} \text{minimize} & t \\ \text{subject to} & y_{ij} = -1, \text{ if } \{i, j\} \in \overline{E} \\ & y_{ii} = t - 1, \text{ for all } i = 1, 2, \dots, n \\ & Y \in \text{COP}_n \end{array}$$

$$\tilde{X}_i = 1_{[i \in I]}$$

$$\tilde{X} = \frac{1}{\alpha(G)} \tilde{\mathbf{x}} \tilde{\mathbf{x}}^T$$

 $\alpha(G) \leq$

$$\alpha(G)$$
 روش: یک جواب شدنی برای دوگان با مقدار \mathbf{G}

$$\max \quad J_n \bullet X$$

$$X \in POS_n$$

$$Tr(X) = 1$$

$$x_{ij} = 0 \quad ij \in E$$

 $\alpha(G) \leq$

minimize
$$t$$

subject to $y_{ij} = -1$, if $\{i, j\} \in \overline{E}$
 $y_{ii} = t - 1$, for all $i = 1, 2, ..., n$
 $Y \in \text{COP}_n$

has value $\alpha(G)$, the size of a maximum independent set in G.

$$\widetilde{x}_i = 1_{[i \in I]}$$
 مقدار $\alpha(G)$ مستقل $\widetilde{x}_i = 1_{[i \in I]}$ مستقل و ناز بزرگترین مجموعههای مستقل و ناز بزرگ

$$\tilde{X} = \frac{1}{\alpha(G)} \tilde{\mathbf{x}} \tilde{\mathbf{x}}^T \quad \tilde{x}_{ij} = \frac{1}{\alpha(G)} \tilde{x}_i \tilde{x}_j$$
 max

$$\operatorname{Tr}(X) = 1$$
 $x_{ij} = 0 \quad ij \in E$

 $X \in POS_n$

 $J_n \bullet X$

 $\alpha(G) \leq$

minimize
$$t$$

subject to $y_{ij} = -1$, if $\{i, j\} \in \overline{E}$
 $y_{ii} = t - 1$, for all $i = 1, 2, ..., n$
 $Y \in \text{COP}_n$

$$\widetilde{x}_i = 1_{[i \in I]}$$
 مقدار $\alpha(G)$ مستقل $\widetilde{x}_i = 1_{[i \in I]}$ مستقل و ناز بزرگترین مجموعههای مستقل و ناز بزرگ

$$\tilde{X} = \frac{1}{\alpha(G)} \tilde{\mathbf{x}} \tilde{\mathbf{x}}^T \quad \tilde{x}_{ij} = \frac{1}{\alpha(G)} \tilde{x}_i \tilde{x}_j$$
 max

$$\max \quad J_n \bullet X$$

$$X \in POS_n$$

$$Tr(X) = 1$$

$$x_{ij} = 0 \quad ij \in E$$

$$\begin{array}{ll} \mbox{minimize} & t \\ \mbox{subject to} & y_{ij} = -1, \mbox{ if } \{i,j\} \in \overline{E} \\ & y_{ii} = t-1, \mbox{ for all } i = 1,2,\ldots,n \\ & Y \in \mathrm{COP}_n \end{array}$$

$$\widetilde{x}_i = 1_{[i \in I]}$$
 مقدار $\alpha(G)$ مستقل وگان با مقدار $\widetilde{x}_i = 1_{[i \in I]}$ و نام برای دوگان با مقدار $\widetilde{x}_i = 1_{[i \in I]}$ و نام برای دوگان با مقدار $\widetilde{x}_i = 1_{[i \in I]}$

$$\tilde{X} = \frac{1}{\alpha(G)} \tilde{\mathbf{x}} \tilde{\mathbf{x}}^T \quad \tilde{x}_{ij} = \frac{1}{\alpha(G)} \tilde{x}_i \tilde{x}_j \quad \operatorname{Tr}(\tilde{x} \tilde{x}^T) = \sum \bar{x}_i^2 \qquad \max \qquad J_n \bullet X$$

$$X \in \operatorname{POS}_n$$

$$\operatorname{Tr}(X) = 1$$

$$x_{ij} = 0 \qquad ij \in E$$

$$\begin{array}{ll} \text{minimize} & t \\ \text{subject to} & y_{ij} = -1, \text{ if } \{i, j\} \in \overline{E} \\ & y_{ii} = t - 1, \text{ for all } i = 1, 2, \dots, n \\ & Y \in \text{COP}_n \end{array}$$

$$\tilde{x}_i = 1_{[i \in I]}$$
 مقدار $\alpha(G)$ مستقل $\tilde{x}_i = 1_{[i \in I]}$ مستقل $\alpha(G)$ مست

$$\tilde{X} = \frac{1}{\alpha(G)} \tilde{\mathbf{x}} \tilde{\mathbf{x}}^T \quad \tilde{x}_{ij} = \frac{1}{\alpha(G)} \tilde{x}_i \tilde{x}_j \quad \operatorname{Tr}(\tilde{x}\tilde{x}^T) = \sum \bar{x}_i^2 \qquad \max_{X \in POS_n} X \in POS_n$$

$$\operatorname{Tr}(X) = 1$$

$$x_{ij} = 0 \quad ij \in E$$

 $\alpha(G) \leq$

minimize
$$t$$

subject to $y_{ij} = -1$, if $\{i, j\} \in \overline{E}$
 $y_{ii} = t - 1$, for all $i = 1, 2, ..., n$
 $Y \in \text{COP}_n$

$$\widetilde{x}_i = 1_{[i \in I]}$$
 مقدار $\alpha(G)$ مقدار برای دوگان با مقدار $\widetilde{x}_i = 1_{[i \in I]}$ و نام برای مجموعه های مستقل المستقل بردگترین مجموعه های مستقل المستقل بردگترین مجموعه های مستقل المستقل بردگترین مجموعه های مستقل المستقل المستقل بردگترین مجموعه های مستقل المستقل المستقل المستقل بردگترین مجموعه های مستقل المستقل ا

$$\tilde{X} = \frac{1}{\alpha(G)} \tilde{\mathbf{x}} \tilde{\mathbf{x}}^T \quad \tilde{x}_{ij} = \frac{1}{\alpha(G)} \tilde{x}_i \tilde{x}_j \quad \operatorname{Tr}(\tilde{x} \tilde{x}^T) = \sum \bar{x}_i^2 \qquad \max \qquad J_n \bullet X$$

$$X \in \operatorname{POS}_n$$

$$\operatorname{Tr}(X) = 1$$

$$x_{ij} = 0 \quad ij \in E$$

 $Y \in COP_n$

$$\tilde{x}_i = 1_{[i \in I]}$$
 مقدار $\alpha(G)$ با مقدار وش: یک جواب شدنی برای دوگان با مقدار $\tilde{x}_i = 1_{[i \in I]}$ هستقل د. $\tilde{x}_i = 1_{[i \in I]}$

$$\tilde{X} = \frac{1}{\alpha(G)} \tilde{\mathbf{x}} \tilde{\mathbf{x}}^{T} \quad \tilde{x}_{ij} = \frac{1}{\alpha(G)} \tilde{x}_{i} \tilde{x}_{j} \quad \operatorname{Tr}(\tilde{x} \tilde{x}^{T}) = \sum \bar{x}_{i}^{2} \qquad \max \qquad J_{n} \bullet X$$

$$X \in \operatorname{POS}_{n}$$

$$\operatorname{Tr}(X) = 1$$

$$J_{n} \bullet \tilde{X} = \sum_{i,j} \tilde{x}_{ij} \qquad x_{ij} = 0 \quad ij \in E$$

minimize
$$t$$

subject to $y_{ij} = -1$, if $\{i, j\} \in \overline{E}$
 $y_{ii} = t - 1$, for all $i = 1, 2, ..., n$
 $Y \in \text{COP}_n$

$$\widetilde{x}_i = 1_{[i \in I]}$$
 مقدار $\alpha(G)$ مستقل $\widetilde{x}_i = 1_{[i \in I]}$ مستقل و ناز بزرگترین مجموعههای مستقل و ناز بزرگ

$$\tilde{X} = \frac{1}{\alpha(G)} \tilde{\mathbf{x}} \tilde{\mathbf{x}}^{T} \quad \tilde{x}_{ij} = \frac{1}{\alpha(G)} \tilde{x}_{i} \tilde{x}_{j} \quad \operatorname{Tr}(\tilde{x} \tilde{x}^{T}) = \sum \bar{x}_{i}^{2} \qquad \max \qquad J_{n} \bullet X$$

$$X \in \operatorname{POS}_{n}$$

$$\operatorname{Tr}(X) = 1$$

$$J_{n} \bullet \tilde{X} = \sum_{i,j} \tilde{x}_{ij} = \frac{1}{\alpha(G)} \sum_{i,j} \tilde{x}_{i} \tilde{x}_{j}$$

$$x_{ij} = 0 \qquad ij \in E$$

minimize
$$t$$

subject to $y_{ij} = -1$, if $\{i, j\} \in \overline{E}$
 $y_{ii} = t - 1$, for all $i = 1, 2, ..., n$
 $Y \in \text{COP}_n$

$$\alpha(G)$$
 روش: یک جواب شدنی برای دوگان با مقدار $\tilde{x}_i = 1_{[i \in I]}$ و نام برای دوگان با مقدار $\tilde{x}_i = 1_{[i \in I]}$ و نام برای مجموعه های مستقل و نام برای دوگان با مقدار و نام برای دوگان با دوگان برای دوگان با داد داد با دوگان بای

$$\tilde{X} = \frac{1}{\alpha(G)} \tilde{\mathbf{x}} \tilde{\mathbf{x}}^T \quad \tilde{x}_{ij} = \frac{1}{\alpha(G)} \tilde{x}_i \tilde{x}_j \quad \operatorname{Tr}(\tilde{x} \tilde{x}^T) = \sum_i \bar{x}_i^2 \qquad \max_{X \in POS_n} J_n \bullet X$$

$$Tr(X) = 1$$

$$J_n \bullet \tilde{X} = \sum_{i,j} \tilde{x}_{ij} = \frac{1}{\alpha(G)} \sum_{i,j} i = \frac{1}{\alpha(G)} \sum_{i,j \in I} 1$$

$$x_{ij} = 0 \quad ij \in E$$

$$egin{aligned} extbf{.2.1 Theorem.} & extbf{ The copositive program} \ & extbf{minimize} & t \end{aligned}$$

minimize
$$t$$

subject to $y_{ij} = -1$, if $\{i, j\} \in \overline{E}$
 $y_{ii} = t - 1$, for all $i = 1, 2, ..., n$
 $Y \in \text{COP}_n$

$$\alpha(G)$$
 روش: یک جواب شدنی برای دوگان با مقدار $\widetilde{x}_i=1_{[i\in I]}$ و ترین مجموعههای مستقل $\widetilde{x}_i=1_{[i\in I]}$

$$X_i = I_{[i \in I]}$$
 یکی از بزرگترین مجموعههای مستقل: I بیرگ $I_i = I_{[i \in I]}$

$$\tilde{X} = \frac{1}{\alpha(G)} \tilde{\mathbf{x}} \tilde{\mathbf{x}}^{T} \quad \tilde{x}_{ij} = \frac{1}{\alpha(G)} \tilde{x}_{i} \tilde{x}_{j} \quad \operatorname{Tr}(\tilde{x} \tilde{x}^{T}) = \sum_{i,j} \tilde{x}_{i}^{2} \qquad \max \qquad J_{n} \bullet X$$

$$X \in \operatorname{POS}_{n}$$

$$\operatorname{Tr}(X) = 1$$

$$X_{ij} = 0 \quad ij \in E$$

minimize
$$t$$
 $subject\ to\ y_{ij}=-1,\ if\ \{i,j\}\in\overline{E}$ $y_{ii}=t-1,\ for\ all\ i=1,2,\ldots,n$

 $Y \in COP_n$

$$\alpha(G)$$
 روش: یک جواب شدنی برای دوگان با مقدار $\widetilde{x}_i=1_{[i\in I]}$ و ترین مجموعههای مستقل $\widetilde{x}_i=1_{[i\in I]}$

$$X_i - I_{[i \in I]}$$
 یکی از بزرگترین مجموعههای مستقل: I $I_{i \in I}$ $I_{i \in I$

$$\tilde{X} = \frac{1}{\alpha(G)} \tilde{\mathbf{x}} \tilde{\mathbf{x}}^{T} \quad \tilde{x}_{ij} = \frac{1}{\alpha(G)} \tilde{x}_{i} \tilde{x}_{j} \quad \operatorname{Tr}(\tilde{x} \tilde{x}^{T}) = \sum_{i,j} \tilde{x}_{i}^{2} \qquad \max_{X \in POS_{n}} J_{n} \bullet \tilde{X}$$

$$Tr(X) = 1$$

$$J_{n} \bullet \tilde{X} = \sum_{i,j} \tilde{x}_{ij} = \frac{1}{\alpha(G)} \sum_{i,j} \tilde{x}_{i} = \frac{1}{\alpha(G)} \sum_{i,j} 1 = \frac{\alpha(G)^{2}}{\alpha(G)} \qquad x_{ij} = 0 \quad ij \in E$$

 $\alpha(G) \leq$

.2.1 Theorem. The copositive program
$$t$$

minimize
$$t$$

subject to $y_{ij} = -1$, if $\{i, j\} \in \overline{E}$
 $y_{ii} = t - 1$, for all $i = 1, 2, ..., n$
 $Y \in \text{COP}_n$

7.2.1 Theorem. The copositive program

minimize tsubject to $y_{ij} = -1$, if $\{i, j\} \in \overline{E}$ $y_{ii} = t - 1$, for all i = 1, 2, ..., n $Y \in \text{COP}_n$

7.2.1 Theorem. The copositive program

minimize subject to $y_{ij} = -1$, if $\{i, j\} \in \overline{E}$ $y_{ii} = t - 1$, for all i = 1, 2, ..., n $Y \in COP_n$

has value $\alpha(G)$, the siz صفر روى درايه هاى ident set in G.

بدون يال

 $Y = tI_n + Z - J_n$

7.2.1 Theorem. The copositive program
$$t$$

minimize tsubject to $y_{ij} = -1$, if $\{i, j\} \in \overline{E}$ $y_{ii} = t - 1$, for all i = 1, 2, ..., n $Y \in \text{COP}_n$

has value $\alpha(G)$, the siz مفر روى درايههاى ident set in G.

بدون يال

$$Y = tI_n + Z - J_n$$

$$z = \max_{i,j}(Z_{i,j})$$

$$Y' = tI_n + zA_G - J_n$$

7.2.1 Theorem. The copositive program

 $\alpha(G) \leq$

$$\begin{array}{ll} \mbox{minimize} & t \\ \mbox{subject to} & y_{ij} = -1, \mbox{ if } \{i,j\} \in \overline{E} \\ \mbox{ } y_{ii} = t-1, \mbox{ for all } i = 1,2,\ldots,n \\ \mbox{ } Y \in \mathrm{COP}_n \end{array}$$

has value $\alpha(G)$, the size of a maximum independent set in G.

7.2.5 Lemma. The copositive program

Minimize
$$t$$

subject to $tI_n + zA_G - J_n \in COP_n$
 $t, z \in \mathbb{R}$

جوابش برابر با جواب برنامهریزی بالاست.

قضیه Motzkin-Straus:

7.2.6 Theorem. For every graph G,

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}.$$

قضیه Motzkin-Straus:

7.2.6 Theorem. For every graph G,

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}. \left\{ := m(G) \right\}$$

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f(x) :=

$$m(G) \le \frac{1}{\alpha(G)}$$
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 := $m(G)$

f(x) :=

$$m(G) \leq \frac{1}{\alpha(G)}$$
 (الف

بردار مشخصه مجموعه مستقل $ilde{x}$

7.2.6 Theorem. For every graph G,

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}. \left\{ := m(G) \right\}$$

f(x) :=

$$m(G) \le \frac{1}{\alpha(G)}$$
 (الف

بردار مشخصه مجموعه مستقل \tilde{x}

$$f(\tilde{x}) = \sum_{i,j} (A_G + I)_{i,j} \tilde{x}_i \tilde{x}_j$$

7.2.6 Theorem. For every graph G,

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}.$$
 := $m(G)$

f(x) :=

$$m(G) \le \frac{1}{\alpha(G)}$$
 (الف

بردار مشخصه مجموعه مستقل \tilde{x}

$$f(\tilde{x}) = \sum_{i,j} (A_G + I)_{i,j} \tilde{x}_i \tilde{x}_j = |I|$$

7.2.6 Theorem. For every graph G,

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}. \left\{ := m(G) \right\}$$

$$f(x) :=$$

$$m(G) \le \frac{1}{\alpha(G)}$$
 (الف

بردار مشخصه مجموعه مستقل $ilde{x}$

$$f(\tilde{x}) = \sum_{i,j} (A_G + I)_{i,j} \tilde{x}_i \tilde{x}_j = |I|$$

$$\frac{1}{\alpha(G)} \tilde{x}$$

7.2.6 Theorem. For every graph G,

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}.$$
 := $m(G)$

f(x) :=

$$m(G) \leq \frac{1}{\alpha(G)}$$
 الف \widetilde{x} : بردار مشخصه مجموعه مستقل

 $f(\tilde{x}) = \sum_{i,j} (A_G + I)_{i,j} \tilde{x}_i \tilde{x}_j = |I|$

$$\frac{1}{\alpha(G)}\tilde{x}$$
 •

 $f(\frac{1}{\alpha(G)}\tilde{x}) = \frac{1}{\alpha(G)}$ نرم ۱ = ۱، کنج مثبت، $\alpha(G)$

7.2.6 Theorem. For every graph G,

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}.$$
 := $m(G)$

$$f(x) :=$$

$$m(G) \ge \frac{1}{\alpha(G)}$$
 (.

7.2.6 Theorem. For every graph G,

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f(x) :=

$$m(G) \ge \frac{1}{\alpha(G)}$$
 (ب

- جواب بهینه x^*
- با بیشترین صفر

7.2.6 Theorem. For every graph G,

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 := $m(G)$

f(x) :=

$$m(G) \ge \frac{1}{\alpha(G)}$$
 (ب

- جواب بهینه: x^*
- با بیشترین صفر
- یال i و j که دو سرشان مثبت است

7.2.6 Theorem. For every graph G,

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}.$$
 := $m(G)$

$$f(x) :=$$

$$m(G) \ge \frac{1}{\alpha(G)} (\smile$$

• یال i و j که دو سرشان مثبت است

7.2.6 Theorem. For every graph G,

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}.$$
 := $m(G)$

$$f(x) :=$$

$$m(G) \ge \frac{1}{\alpha(G)}$$
 (ب

$$z_k = \begin{cases} x_k^* + \varepsilon & \text{if } k = i \\ x_k^* - \varepsilon & \text{if } k = j \\ x_k^* & \text{otherwise,} \end{cases}$$

7.2.6 Theorem. For every graph G,

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}.$$
 := $m(G)$

f(x) :=

$$m(G) \ge \frac{1}{\alpha(G)} \ (\because$$

ا یال i و j که دو سرشان مثبت است

$$f(z) = f(x^*) + l(\epsilon) z_k = \begin{cases} x_k^* + \varepsilon & \text{if } k = i \\ x_k^* - \varepsilon & \text{if } k = j \\ x_k^* & \text{otherwise,} \end{cases}$$

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}.$$
 := $m(G)$

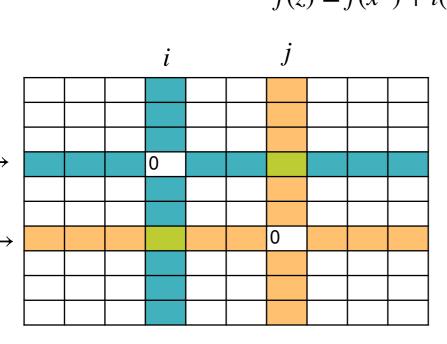
$$f(z) = f(x^*) + l(\epsilon) \qquad z_k = \begin{cases} x_k^* + \varepsilon & \text{if } k = i \\ x_k^* - \varepsilon & \text{if } k = j \\ x_k^* & \text{otherwise,} \end{cases}$$

7.2.6 Theorem. For every graph
$$G$$
,
$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}. \quad := m(G)$$

$$f(x) :=$$

$$f(z) = f(x^*) + l(\epsilon) \qquad z_k = \begin{cases} x_k^* + \epsilon & \text{if } k = i \\ x_k^* - \epsilon & \text{if } k = j \\ x_k^* & \text{otherwise,} \end{cases}$$

$$i \qquad j$$



7.2.6 Theorem. For every graph
$$G$$
,
$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}. \quad := m(G)$$

$$f(x) :=$$

$$f(z) = f(x^*) + l(\epsilon) \qquad z_k = \begin{cases} x_k^* + \epsilon & \text{if } k = i \\ x_k^* - \epsilon & \text{if } k = j \\ x_k^* & \text{otherwise,} \end{cases}$$

$$i \qquad j$$

$$f(z) = \epsilon \mathbf{B} - \epsilon \mathbf{O} + 2(x_i + \epsilon)(x_j - \epsilon) + \epsilon \mathbf{O} + 2(x_i + \epsilon)(x_j - \epsilon)(x_j - \epsilon) + \epsilon \mathbf{O} + 2(x_i + \epsilon)(x_j - \epsilon)(x_j$$

7.2.6 Theorem. For every graph
$$G$$
,

orem. For every graph
$$G$$
,

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}.$$
 := $m(G)$

$$f(z) = f(x^*) + i$$

$$i$$

$$j$$

$$0$$

$$0$$

$$f(x) := \begin{cases} f(x) := \\ f(z) = f(x^*) + l(\epsilon) \end{cases} \quad z_k = \begin{cases} x_k^* + \epsilon & \text{if } k = i \\ x_k^* - \epsilon & \text{if } k = j \\ x_k^* & \text{otherwise,} \end{cases}$$

 $f(z) = \epsilon \mathbf{B} - \epsilon \mathbf{O} + 2(x_i + \epsilon)(x_i - \epsilon) +$

 $(x_i + \epsilon)^2 + (x_i - \epsilon) + \dots$

7.2.6 Theorem. For every graph
$$G$$
,

.2.6 Theorem. For every graph
$$G$$
,
$$\frac{1}{m} = \min\{\mathbf{x}^T (A_G + I_m)\}$$

For every graph
$$G$$
,

0



$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}.$$
 := $m(G)$

 $f(z) = \epsilon \mathbf{B} - \epsilon \mathbf{O} + 2(x_i + \epsilon)(x_i - \epsilon) +$

 $(x_i + \epsilon)^2 + (x_i - \epsilon) + \dots$

نسبت به $l(\epsilon)$ خطی است ϵ

$$f(x) :=$$

$$f(x) :=$$

$$f(z) = f(x^*) + l(\varepsilon) \qquad z_k = \begin{cases} x_k^* + \varepsilon & \text{if } k = i \\ x_k^* - \varepsilon & \text{if } k = j \\ x_k^* & \text{otherwise,} \end{cases}$$

$$(x) :=$$

$$(\mathbf{x}:\mathbf{x})$$

$$I_n)\mathbf{x}:\mathbf{x}$$

$$(n)\mathbf{x}:\mathbf{x}$$

$$(\mathbf{x}:\mathbf{x})$$

$$G$$
,



7.2.6 Theorem. For every graph G, $\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1} x_i = 1\}. \left\{ := m(G) \right\}$ f(x) := $f(z) = f(x^*) + l(\epsilon) \qquad z_k = \begin{cases} x_k^* + \varepsilon & \text{if } k = i \\ x_k^* - \varepsilon & \text{if } k = j \\ x_k^* & \text{otherwise,} \end{cases}$ $f(z) = \epsilon \mathbf{B} - \epsilon \mathbf{O} + 2(x_i + \epsilon)(x_i - \epsilon) +$ $(x_i + \epsilon)^2 + (x_i - \epsilon) + \dots$ نسبت به f(z) نسبت به $l(\epsilon)$ خطی است ϵ خطی است ϵ 0

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}.$$
 := $m(G)$

$$f(z) := \begin{cases} f(x) := \\ f(z) = f(x^*) + l(\epsilon) \end{cases} \quad z_k = \begin{cases} x_k^* + \epsilon & \text{if } k = i \\ x_k^* - \epsilon & \text{if } k = j \\ x_k^* & \text{otherwise,} \end{cases}$$

نسبت به
$$f(z)$$
 نسبت به ϵ خطی است ϵ خطی است ϵ

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}.$$
 := $m(G)$

$$f(z) := \begin{cases} f(x) := \\ f(z) = f(x^*) + l(\epsilon) \end{cases} \quad z_k = \begin{cases} x_k^* + \epsilon & \text{if } k = i \\ x_k^* - \epsilon & \text{if } k = j \\ x_k^* & \text{otherwise,} \end{cases}$$

نسبت به
$$l(\epsilon)$$
 نسبت به $f(z)$ نسبت به خطی است ϵ خطی است ϵ

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}.$$
 := $m(G)$

$$f(x) :=$$

$$f(z) = f(x^*) + l(\epsilon) \qquad z_k = \begin{cases} x_k^* + \varepsilon & \text{if } k = i \\ x_k^* - \varepsilon & \text{if } k = j \\ x_k^* & \text{otherwise,} \end{cases}$$

میتوان یکی از درایههای *x را صفر کرد

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}.$$
 := $m(G)$

$$f(z) = f(x^*) + l(\epsilon) \qquad z_k = \begin{cases} x_k^* + \varepsilon & \text{if } k = i \\ x_k^* - \varepsilon & \text{if } k = j \\ x_k^* & \text{otherwise,} \end{cases}$$

$$l(\epsilon)$$
 نسبت به $f(z)$ نسبت به $f(z)$ متحد $f(z)$ متحد $f(z)$ نسبت به خطی است $f(z)$ متحد $f(z)$ عضلی است $f(z)$ متحد $f(z)$

نسبت به
$$f(z)$$
 نسبت به $\ell(\epsilon)$ نسبت به خطی است ϵ خطی است ϵ

7.2.6 Theorem. For every graph G,

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}.$$
 := $m(G)$

$$f(x) :=$$

$$m(G) \ge \frac{1}{\alpha(G)} \ (\because$$

ا یال i و j که دو سرشان مثبت است

$$f(z) = f(x^*) + l(\epsilon) z_k = \begin{cases} x_k^* + \varepsilon & \text{if } k = i \\ x_k^* - \varepsilon & \text{if } k = j \\ x_k^* & \text{otherwise,} \end{cases}$$

7.2.6 Theorem. For every graph
$$G$$
,

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}.$$
 := $m(G)$

$$f(x) :=$$

$$m(G) \ge \frac{1}{\alpha(G)}$$
 (ب

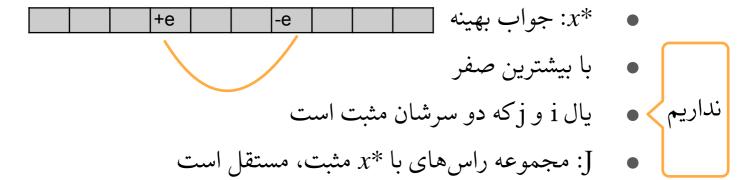
الداريم بهينه بهينه واب بهينه به بيشترين صفر با بيشترين صفر با بيشترين صفر يال او
$$j$$
 و i له دو سرشان مثبت است $f(z) = f(x^*) + l(\epsilon)$ $z_k = \begin{cases} x_k^* + \epsilon & \text{if } k = i \\ x_k^* - \epsilon & \text{if } k = j \\ x_k^* & \text{otherwise,} \end{cases}$

7.2.6 Theorem. For every graph G,

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}.$$
 := $m(G)$

f(x) :=

$$m(G) \ge \frac{1}{\alpha(G)}$$
 (ب



$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}.$$
 := $m(G)$

$$f(x) :=$$

$$m(G) \ge \frac{1}{\alpha(G)} (\because \bullet)$$

است، مستقل است x^* مثبت، مستقل است :J

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}. \left\{ := m(G) \right\}$$

$$f(x) := m(G) \ge \frac{1}{\alpha(G)} (\mathbf{x} - \mathbf{0})$$

است، مستقل است x^* مثبت، مستقل است

$$f(x) = 2 \sum_{i,j \in E(G)} x_i x_j + \sum_{i \in J} x_i^2$$

7.2.6 Theorem. For every graph
$$G$$
,

1 =
$$\min\{\mathbf{x}^T(A_{n+1}, A_{n+1})\}$$

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1} x_i = 1\}.$$
 := $m(G)$

$$f(x) :=$$

$$m(G) \ge \frac{1}{\alpha(G)} \ (\because$$

ات مجموعه راسهای با
$$x^*$$
 مثبت، مستقل است J

$$f(x) = 2\sum_{i,j \in E(G)} x_i x_j + \sum_{i \in J} x_i^2$$

7.2.6 Theorem. For every graph
$$G$$
,

 $f(x) = 2 \sum_{i=1}^{n} x_i x_i + \sum_{i=1}^{n} x_i^2$

 $i,j \in E(G)$

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}.$$
 := $m(G)$

است، مستقل است x^* مثبت، مستقل است :J

 $m(G) \ge \frac{1}{\alpha(G)}$ (ب

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x}$$

$$\frac{1}{\mathbf{x}^T} = \min\{\mathbf{x}^T (A_G + I_n)\}$$

7.2.6 Theorem. For every graph
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rem. For every graph
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است، مستقل است x^* مثبت، مستقل است :J

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}.$$
 := $m(G)$

 $m(G) \ge \frac{1}{\alpha(G)}$ (ب

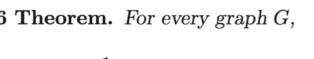
2.6 Theorem. For every graph
$$G$$
,

neorem. For every graph
$$G$$
,

 $||u||^2||v||^2 \ge \langle u, v \rangle^2$

 $f(x) = 2 \sum_{i=1}^{n} x_i x_i + \sum_{i=1}^{n} x_i^2$

 $i,j \in E(G)$



.6 Theorem. For every graph
$$G$$
,

rem. For every graph
$$G$$
,

7.2.6 Theorem. For every graph
$$G$$
,

1.2.6 Theorem. For every graph
$$G$$
,

$$\frac{1}{\mathbf{x}^T} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}$$

است، مستقل است x^* مثبت، مستقل است

 $m(G) \ge \frac{1}{\alpha(G)}$ (ب

1
$$T = T \cdot T \cdot T$$

 $||u||^2||v||^2 \ge \langle u, v \rangle^2$

 $(\sum_{i \in I} x_i^2)(\sum_{i \in I} (\frac{1}{\sqrt{|J|}})^2)$

 $f(x) = 2 \quad \sum \quad x_i x_j + \sum x_i^2$

 $i,j \in E(G)$

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}.$$
 := $m(G)$

1.2.6 Theorem. For every graph
$$G$$
,

em. For every graph
$$G$$
,

7.2.6 Theorem. For every graph
$$G$$
,

$$\frac{1}{1} = \min\{\mathbf{x}^T(A_{n+1}, I_{n})\}$$

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}. \left\{ := m(G) \right\}$$

است، مستقل است x^* مثبت، مستقل است

$$\frac{1}{A(G)} = \min\{\mathbf{x}^T (A_G + I_n)\}$$

 $||u||^2||v||^2 \ge \langle u, v \rangle^2$

 $(\sum_{i \in I} x_i^2)(\sum_{i \in I} (\frac{1}{\sqrt{|J|}})^2) \ge (\sum_{i \in J} x_i \frac{1}{\sqrt{|J|}})^2$

 $f(x) = 2 \quad \sum \quad x_i x_j + \sum x_i^2$

 $i,j \in E(G)$

Theorem. For every graph
$$G$$
,

For every graph
$$G$$
,

$$\frac{1}{n}$$

$$\frac{n}{G}$$

 $m(G) \ge \frac{1}{\alpha(G)}$ (ب

7.2.6 Theorem. For every graph
$$G$$
,

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}. \left\{ := m(G) \right\}$$

$$rac{1}{lpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x}$$

$$m(G) \ge \frac{1}{\alpha(G)}$$
 (ب

است، مستقل است x^* مثبت، مستقل است

$$||u||^2||v||^2 \ge \langle u, v \rangle^2$$

$$(\sum_{i \in J} x_i^2)(\sum_{i \in J} (\frac{1}{\sqrt{|J|}})^2) \ge (\sum_{i \in J} x_i \frac{1}{\sqrt{|J|}})^2 \ge \frac{1}{|J|} (\sum_{i \in J} x_i)^2$$

$$f(x) = 2 \sum_{i,j \in E(G)} x_i x_j + \sum_{i \in J} x_i^2$$

7.2.6 Theorem. For every graph
$$G$$
,

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}.$$
 := $m(G)$

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} :$$

$$f(x) := \frac{1}{\alpha(G)} (\cdot)$$

است، مجموعه راسهای با
$$x^*$$
 مثبت، مستقل است

ا مجموعه راسهای با
$$x^*$$
 متبت، مستقل است x^*

$$||u||^{2}||v||^{2} \ge \langle u, v \rangle^{2}$$

$$(\sum_{i \in J} x_{i}^{2})(\sum_{i \in J} (\frac{1}{\sqrt{|J|}})^{2}) \ge (\sum_{i \in J} x_{i} \frac{1}{\sqrt{|J|}})^{2} \ge \frac{1}{|J|}(\sum_{i \in J} x_{i})^{2} = \frac{1}{|J|}$$

$$f(x) = 2 \sum_{i,j \in E(G)} x_i x_j + \sum_{i \in J} x_i^2$$

7.2.6 Theorem. For every graph
$$G$$
,

$$I > r \cdot r > 0$$
 $\sum_{n=1}^{n} r = 1$

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1} x_i = 1\}.$$
 := $m(G)$

$$f(x) := \int_{i=1}^{i=1}$$

$$m(G) \ge \frac{1}{\alpha(G)} \ (\because$$

است، مجموعه راسهای با
$$x^*$$
 مثبت، مستقل است نا

$$||u||^2||v||^2 \ge \langle u, v \rangle^2$$

$$\begin{array}{c|c}
|u| & ||v|| & \geq \langle u, v \rangle \\
1 & & \\
\end{array}$$

$$(\sum_{i \in J} x_i^2)(\sum_{i \in J} (\frac{1}{\sqrt{|J|}})^2) \ge (\sum_{i \in J} x_i \frac{1}{\sqrt{|J|}})^2 \ge \frac{1}{|J|} (\sum_{i \in J} x_i)^2 = \frac{1}{|J|}$$

$$f(x) = 2 \sum_{i \in J} x_i x_j + \sum_{i \in J} x_i^2 \ge \frac{1}{|J|}$$

7.2.6 Theorem. For every graph
$$G$$
,

$$|L| |\mathbf{v} \cdot \mathbf{v}| \ge 0$$
 $\sum_{m=1}^{n} m = 1$

$$\frac{1}{\mathbf{x}^T} = \min\{\mathbf{x}^T (A_G + I_n)\}$$

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}.$$
 := $m(G)$

$$m(G) \ge \frac{1}{\alpha(G)} \ (\because$$

است، مجموعه راسهای با
$$x^*$$
 مثبت، مستقل است

$$||u||^2||v||^2 \ge \langle u, v \rangle^2$$

$$||u||^2||v||^2 \ge \langle u, v \rangle^2$$

$$1$$

$$(\sum_{i \in J} x_i^2)(\sum_{i \in J} (\frac{1}{\sqrt{|J|}})^2) \ge (\sum_{i \in J} x_i \frac{1}{\sqrt{|J|}})^2 \ge \frac{1}{|J|} (\sum_{i \in J} x_i)^2 = \frac{1}{|J|}$$

$$f(x) = 2 \sum_{i \in J} x_i x_i + \sum_{i \in J} x_i^2 \ge \frac{1}{|J|} \ge \frac{1}{\alpha(G)}$$

$$\overline{i \in J} \quad \sqrt{|J|} \qquad \overline{i \in J} \quad \sqrt{|J|}$$

$$(x) = 2 \quad \sum x_i x_i + \sum x_i^2 > \frac{1}{|I|}$$

قضیه Motzkin-Straus:

7.2.6 Theorem. For every graph G,

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}. \left\{ := m(G) \right\}$$

f(x) :=

$$m(G) \le \frac{1}{\alpha(G)}$$
 (الف

.. •

$$m(G) \ge \frac{1}{\alpha(G)}$$
 (ب

... (

$$\begin{array}{ll} \textit{Minimize} & t \\ \textit{subject to} & tI_n + zA_G - J_n \in \mathsf{COP}_n \\ & t,z \in \mathbb{R} \end{array}$$

$$t,z\in\mathbb{R}$$
 . جوابش برابر با جواب برنامه ریزی بالاست

7.2.6 Theorem. For every graph
$$G$$
,

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}.$$

Minimize
$$t$$

Minimize
$$t$$

subject to $tI_n + zA_G - J_n \in COP_n$
 $t, z \in \mathbb{R}$

$$t,z\in\mathbb{R}$$
 $t,z\in\mathbb{R}$. جوابش برابر با جواب برنامه ریزی بالاست.

7.2.6 Theorem. For every graph
$$G$$
,

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}.$$

برای هر
$${f x}$$
 روی سادک: ${f x}^T(lpha(G)(A_G+I_n)){f x}\geq 1$

Minimize
$$t$$

Minimize
$$t$$

subject to $tI_n + zA_G - J_n \in COP_n$
 $t, z \in \mathbb{R}$

 $t,z\in\mathbb{R}$ $t,z\in\mathbb{R}$

7.2.6 Theorem. For every graph
$$G$$
,

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}.$$

$$\mathbf{x}^T(\alpha(G)(A_G+I_n))\mathbf{x} \geq 1 = \mathbf{x}^T J_n \mathbf{x},$$

Minimize
$$t$$

subject to $tI_n + zA_G - J_n \in COP_n$
 $t, z \in \mathbb{R}$

 $t, z \in \mathbb{R}$

$$t, z \in \mathbb{R}$$

$$= t, z \in \mathbb{R}$$

$$\mathbf{x}^{T}(\alpha(G)(A_G+I_n))\mathbf{x}\geq 1=\mathbf{x}^{T}J_n\mathbf{x},$$

subject to
$$tI_n + zA_G - J_n \in COP_n$$

 $t, z \in \mathbb{R}$

$$t,z\in\mathbb{R}$$

برای هر
$$x$$
 روی سادک:

$$T(-(\alpha) + -(\alpha) + -(\alpha)$$

 $\mathbf{x}^T(\alpha(G)(A_G+I_n))\mathbf{x} \geq 1 = \mathbf{x}^T J_n \mathbf{x},$

$$\mathbf{x}^T(lpha(G)I_n+lpha(G)A_G-J_n)\mathbf{x}\geq \mathbf{0},\;$$
برای هر $_{\mathrm{X}}$ روی سادک:

Minimize tsubject to $tI_n + zA_G - J_n \in COP_n$

subject to
$$tI_n + zA_G - J_n \in \mathrm{COP}_n$$
 $t,z \in \mathbb{R}$

برای هر x روی سادک:

$$\mathbf{x}^{T}(\alpha(G)(A_G+I_n))\mathbf{x} \geq 1 = \mathbf{x}^{T}J_n\mathbf{x},$$

$$\mathbf{x}^T(lpha(G)I_n+lpha(G)A_G-J_n)\mathbf{x}\geq \mathbf{0},$$
 برای هر \mathbf{x} روی سادک: \mathbf{x}

$$\mathbf{x}^T(lpha(G)I_n+lpha(G)A_G-J_n)\mathbf{x}\geq \mathbf{0},$$
 نامنفی: \mathbf{x} نامنفی:

Minimize t

$$n \in COP_n$$

 $\leq \alpha(G)$

subject to $tI_n + zA_G - J_n \in COP_n$ $t, z \in \mathbb{R}$

برای هر x روی سادک:

$$\mathbf{x}^T(\alpha(G)(A_G+I_n))\mathbf{x} \geq 1 = \mathbf{x}^T J_n \mathbf{x},$$

$$\mathbf{x}^T(lpha(G)I_n+lpha(G)A_G-J_n)\mathbf{x}\geq \mathbf{0},\;$$
برای هر \mathbf{x} روی سادک: $\mathbf{x}^T(lpha(G)I_n+lpha(G)A_G-J_n)\mathbf{x}\geq \mathbf{0},\;$ برای هر \mathbf{x} نامنفی: \mathbf{x} نامنفی: \mathbf{x}

$$ilde{Y}:=lpha(G)I_n+lpha(G)A_G-J_n$$
 عمشت:

Minimize tsubject to $tI_n + zA_G - J_n \in COP_n$ $t, z \in \mathbb{R}$

 $t,z\in\mathbb{R}$ $t,z\in\mathbb{R}$ جوابش برابر با جواب برنامه ریزی بالاست.

 $\leq \alpha(G)$

$$Y:=lpha(G)I_n+lpha(G)A_G-J_n$$
 نمشت

Minimize
$$t$$

subject to $tI_n + zA_G - J_n \in COP_n$
 $t, z \in \mathbb{R}$

7.2.5 Lemma. The copositive program

جوابش برابر با جواب برنامهریزی بالاست.

 $\alpha(G) \leq$

minimize tsubject to $y_{ij} = -1$, if $\{i, j\} \in \overline{E}$ $y_{ii} = t - 1$, for all i = 1, 2, ..., n $Y \in \text{COP}_n$

 $\leq \alpha(G)$

$$ilde{Y}:=lpha(G)I_n+lpha(G)A_G-J_n$$
 عممثت:

has value $\alpha(G)$, the size of a maximum independent set in G.

Minimize
$$t$$

subject to $tI_n + zA_G - J_n \in COP_n$
 $t, z \in \mathbb{R}$

جوابش برابر با جواب برنامه ریزی بالاست.

 $\alpha(G) \leq$

minimize t $subject\ to\ y_{ij}=-1,\ if\ \{i,j\}\in\overline{E}$ $y_{ii}=t-1,\ for\ all\ i=1,2,\ldots,n$



 $\geq \alpha$

has value $\alpha(G)$, the size of a maximum independent set in G.

 $Y \in COP_n$

$$ilde{Y}:=lpha(G)I_n+lpha(G)A_G-J_n$$
 ممثنت

7.2.5 Lemma. The copositive program

Minimize tsubject to $tI_n + zA_G - J_n \in COP_n$ $t, z \in \mathbb{R}$

جوابش برابر با جواب برنامهریزی بالاست.

7.2.1 Theorem. The copositive program

minimize tsubject to $y_{ij} = -1$, if $\{i, j\} \in \overline{E}$ $y_{ii} = t - 1$, for all i = 1, 2, ..., n $Y \in \text{COP}_n$

has value $\alpha(G)$, the size of a maximum independent set in G.

7.2.1 Theorem. The copositive program

minimize tsubject to $y_{ij} = -1$, if $\{i, j\} \in \overline{E}$ $y_{ii} = t - 1$, for all i = 1, 2, ..., n $Y \in \text{COP}_n$

has value $\alpha(G)$, the size of a maximum independent set in G.

برنامهریزی کاملا مثبت سخت است!

7.2.1 Theorem. The copositive program

 $\begin{array}{ll} \mbox{minimize} & t \\ \mbox{subject to} & y_{ij} = -1, \mbox{ if } \{i,j\} \in \overline{E} \\ & y_{ii} = t-1, \mbox{ for all } i = 1,2,\ldots,n \\ & Y \in \mbox{COP}_n \end{array}$

has value $\alpha(G)$, the size of a maximum independent set in G.

برنامهریزی کاملا مثبت سخت است!

$$\max \quad J_n \bullet X$$

$$X \in POS_n$$

$$Tr(X) = 1$$

$$x_{ij} = 0 \quad ij \in E$$

تقريبناپذيري مسئله مجموعه مستقل

- $(NP \not\subseteq ZPP)$ تحت فرضهای خوبی
- هیچ الگوریتم تقریبی برای مسئله بزرگتری مجموعه مستقل
 - با ضریب تقریب $n^{1-\epsilon}$ برای هیچ $\epsilon>0$ وجود ندارد.

Cone Programming

(P) Maximize $\langle \mathbf{c}, \mathbf{x} \rangle$

subject to $\mathbf{b} - A(\mathbf{x}) \in L$ برنامهریزی هممثبت $\mathbf{x} \in K$.

$C \bullet X$

subject to A(X) = b $X \in COP_n$

maximize

SDP

 $C \bullet X$ subject to $A_i \bullet X = b_i, \quad i = 1, 2, \dots, m$ $X \succeq 0$.



 $C \bullet X$ maximize

برنامهریزی کاملا مثبت

maximize

subject to A(X) = b $X \in POS_n$

 $c^{\mathsf{T}}x$ naximize subject to Ax = b

LP



 $x \ge 0$

پایان