بسم الله الرحمن الرحیم

نظریه علوم کامپیوتر

نظریه علوم کامپیوتر - بهار ۱۴۰۰-۱۴۰۱ - جلسه یازدهم: پیچیدگی محاسبات (۲)

Theory of computation - 002 - S11 - computational complexity (2)

Review & Reduction

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NP = All languages where can verify membership quickly

P = All languages where can test membership quickly

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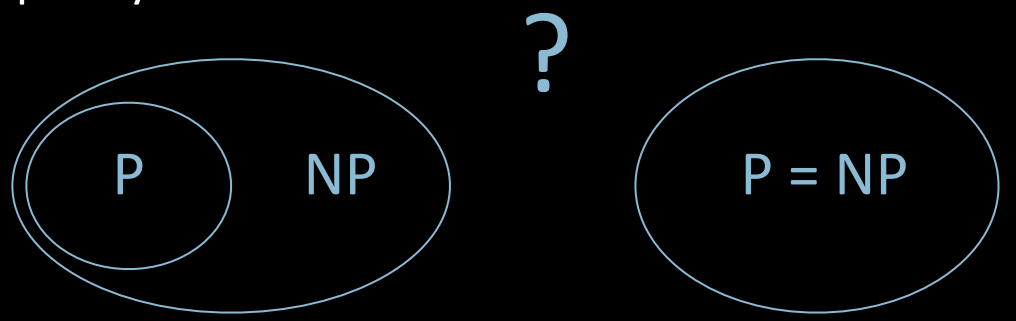
P versus NP question: Does P = NP?



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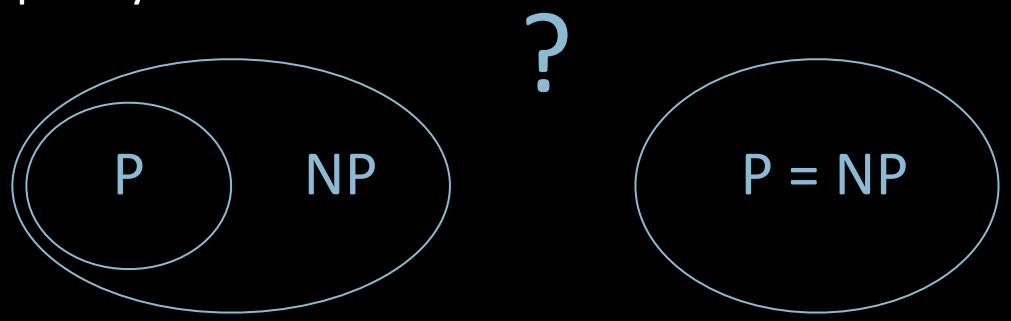

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Is $SAT \in$ NP?

(a) Yes.

(b) No.

(c) I don't know.

(d) No one knows.

Check-in 14.3

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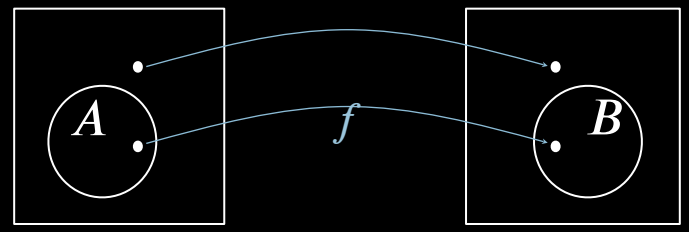

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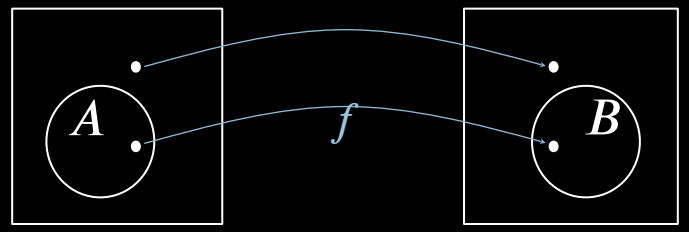

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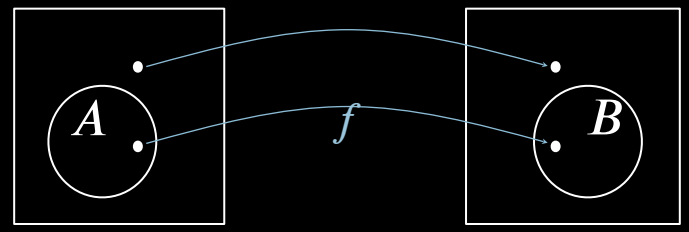

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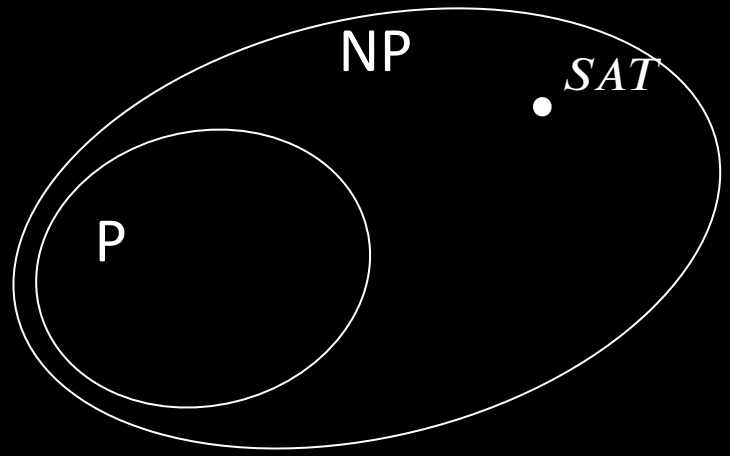

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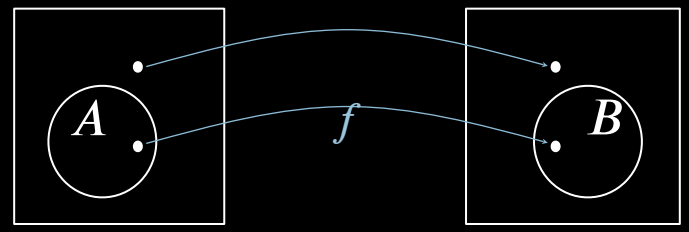

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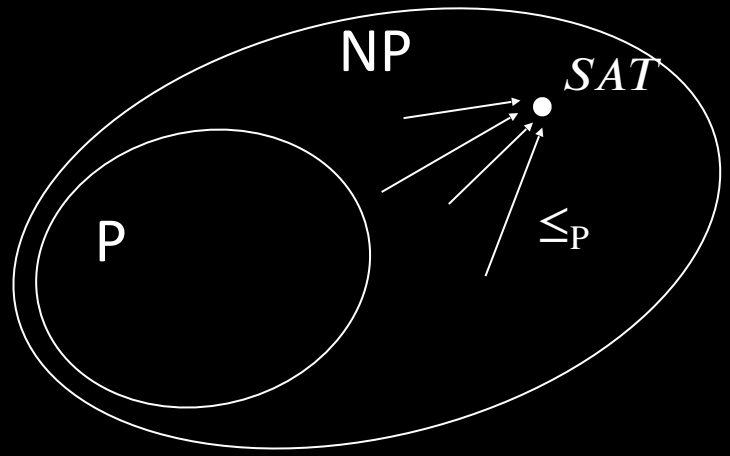

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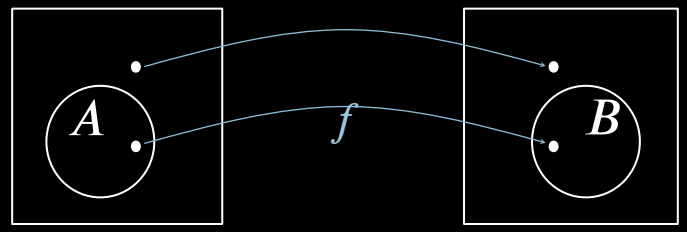

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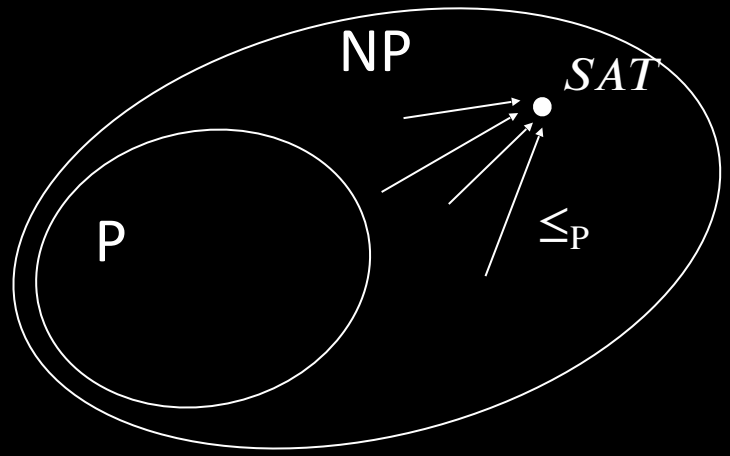

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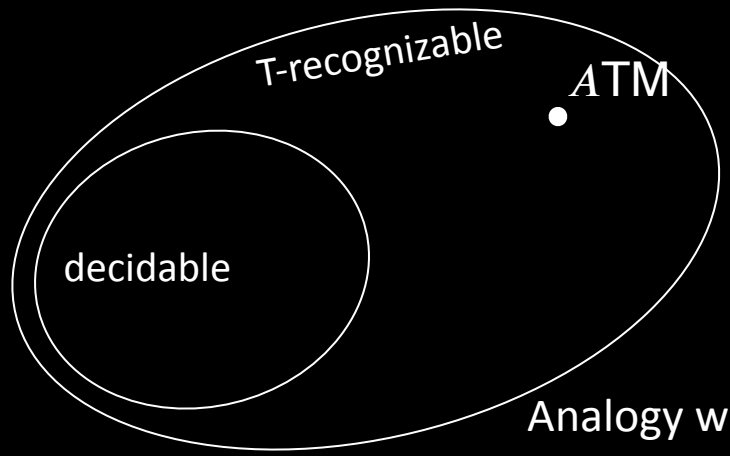

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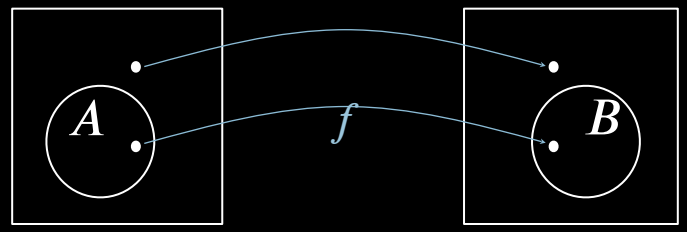

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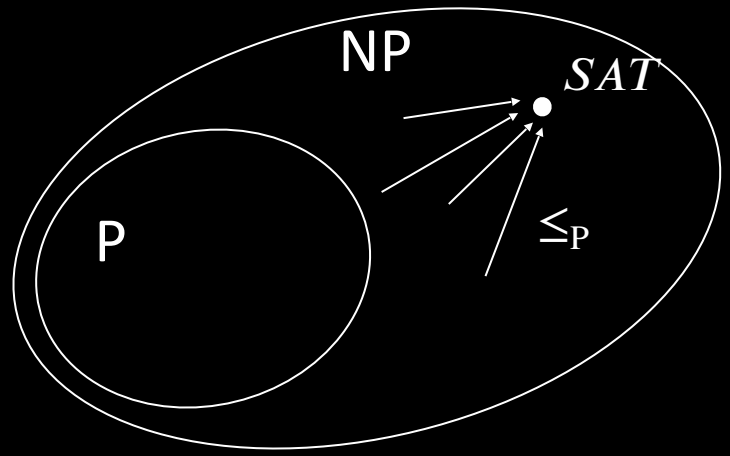

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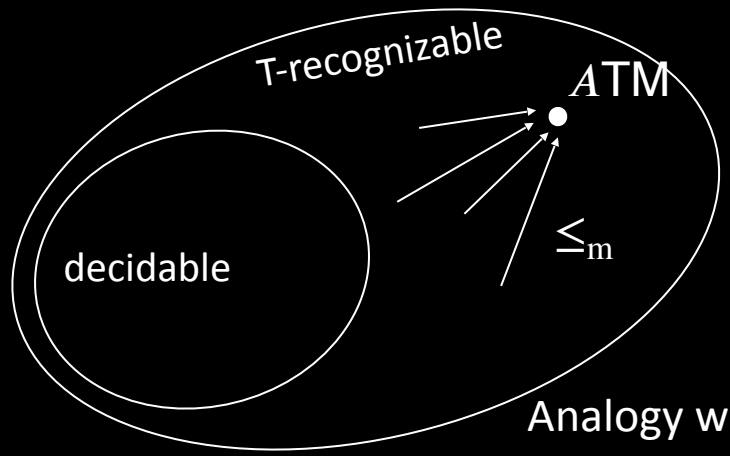

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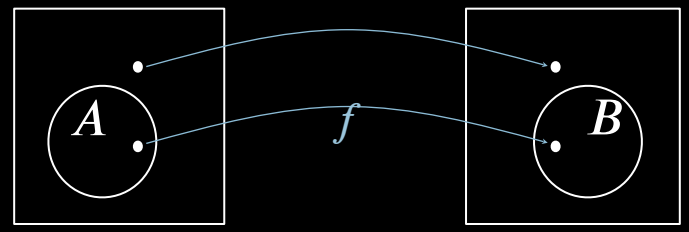

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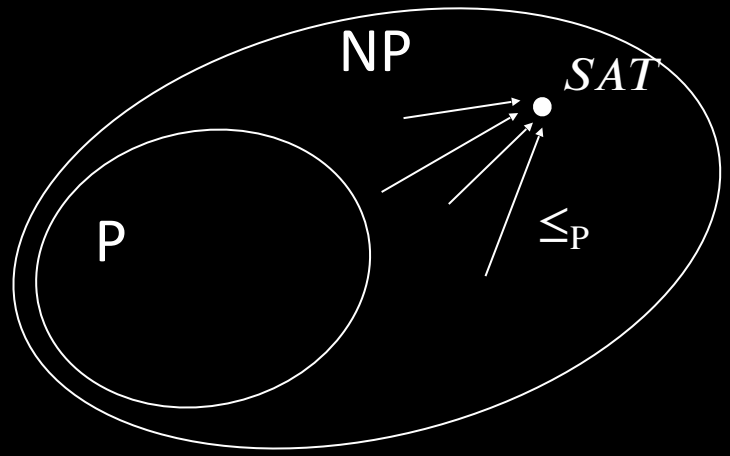

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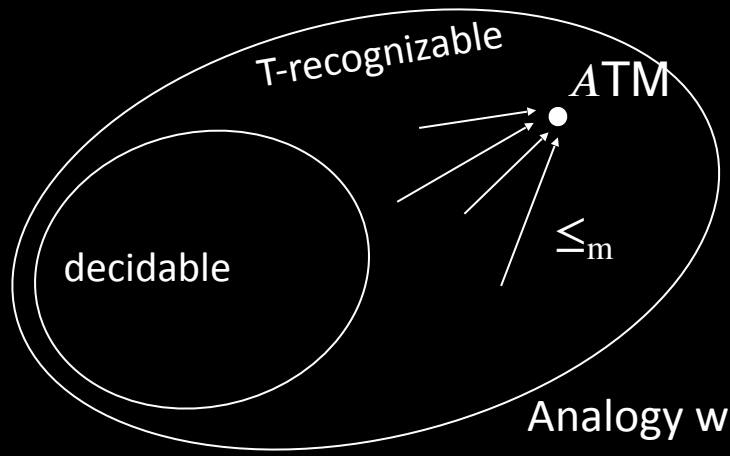

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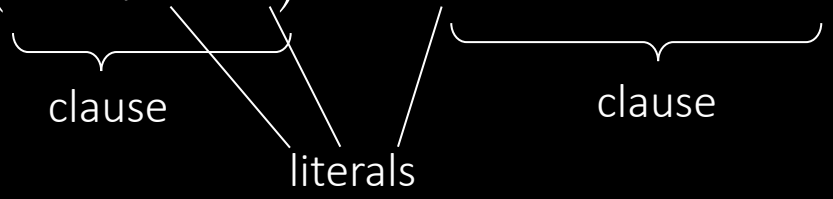

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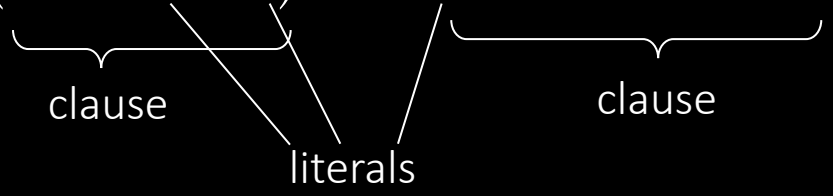

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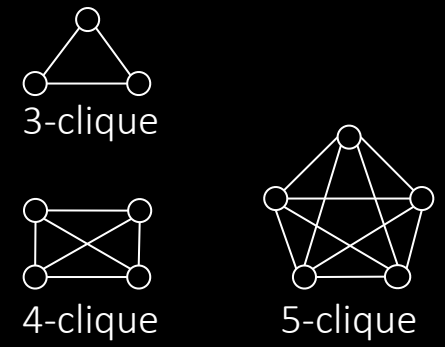

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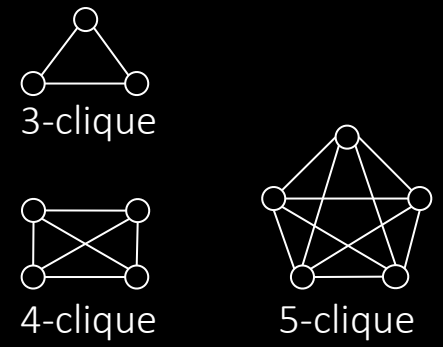

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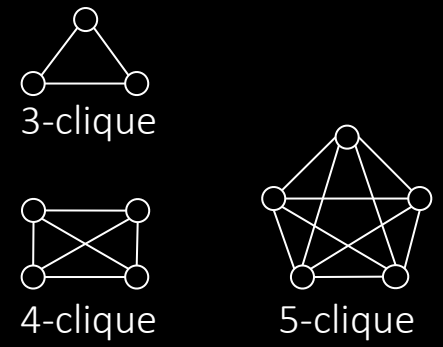

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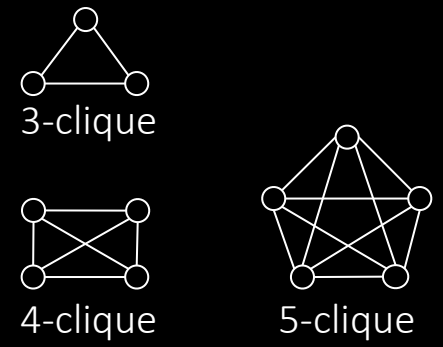

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$$\phi = (a \vee b \vee \bar{c}) \wedge (\bar{a} \vee b \vee d) \wedge (a \vee c \vee \bar{e}) \wedge \cdots \wedge (\bar{x} \vee y \vee \bar{z})$$

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A satisfying assignment to a CNF formula has ≥1 true literal in each clause.

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G

k

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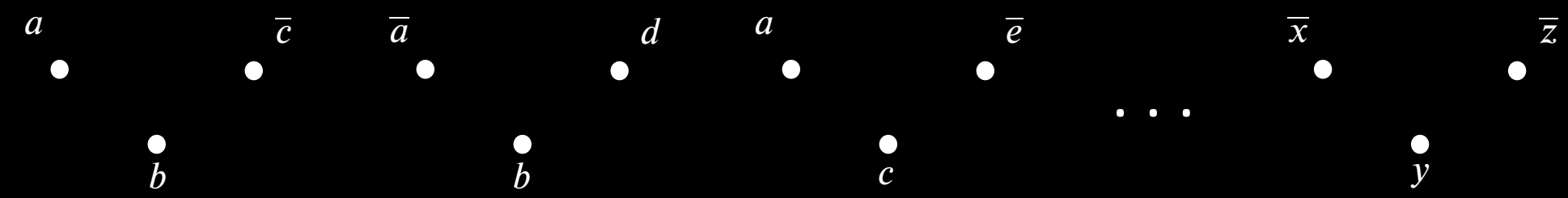
G

k



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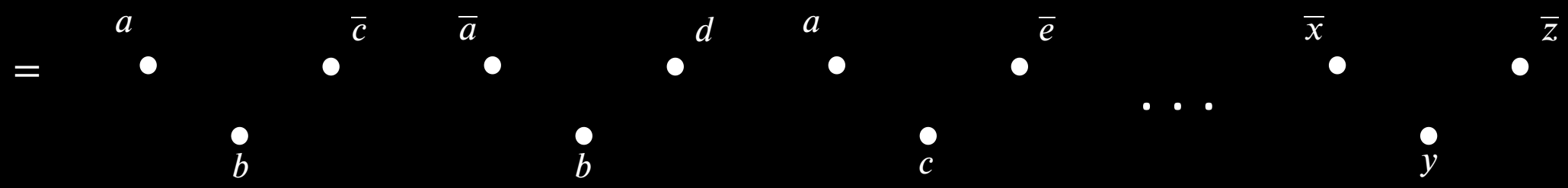

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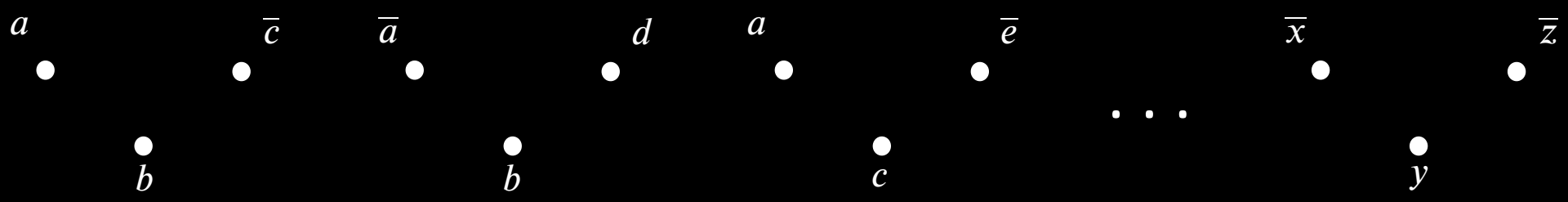

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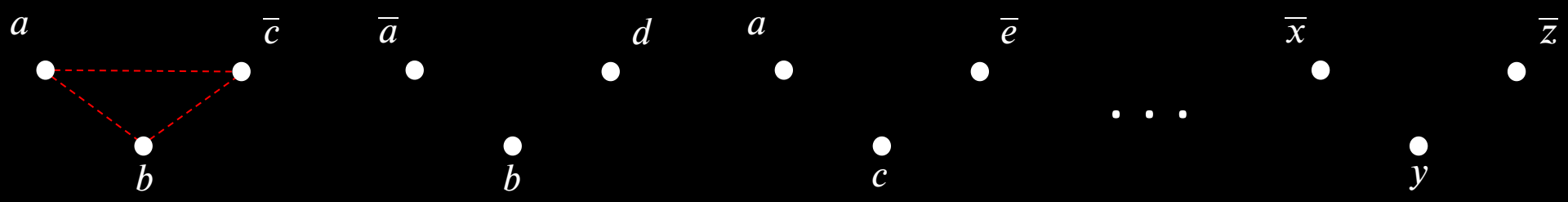

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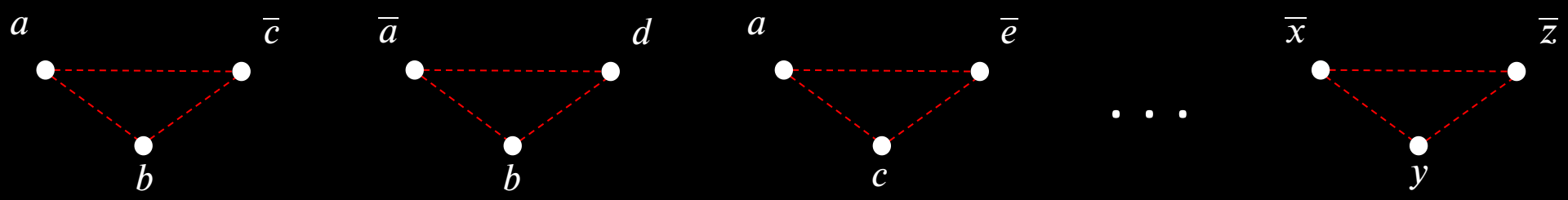

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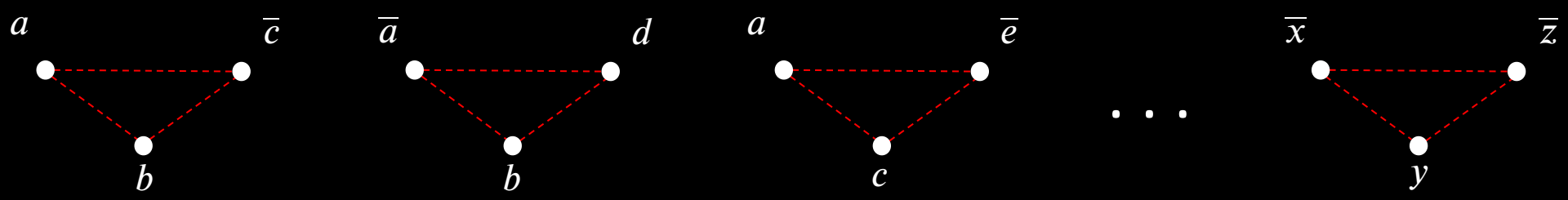

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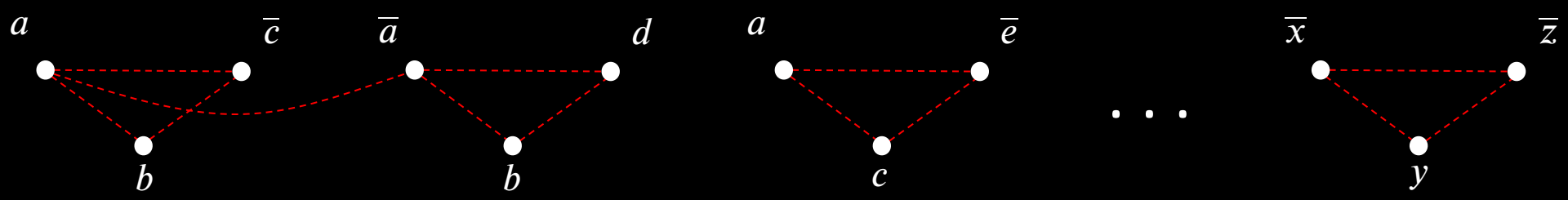

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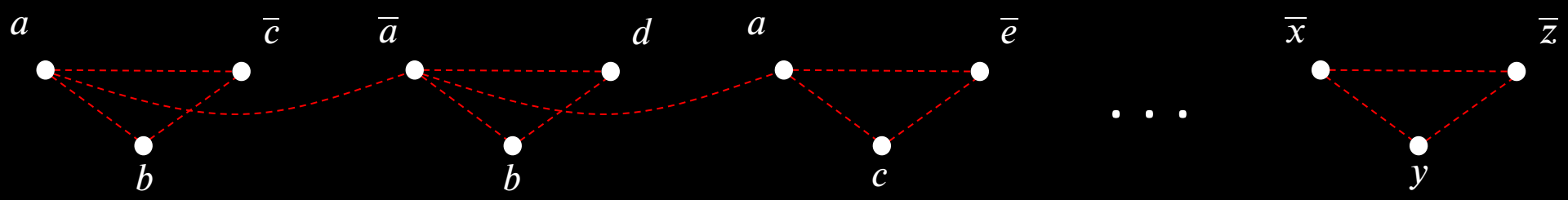

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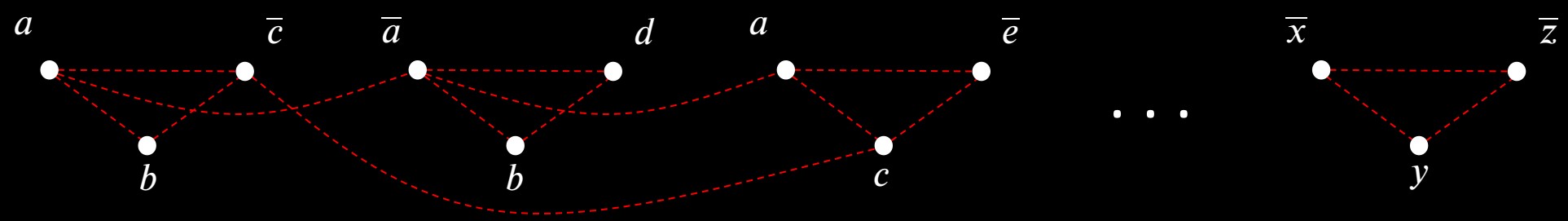

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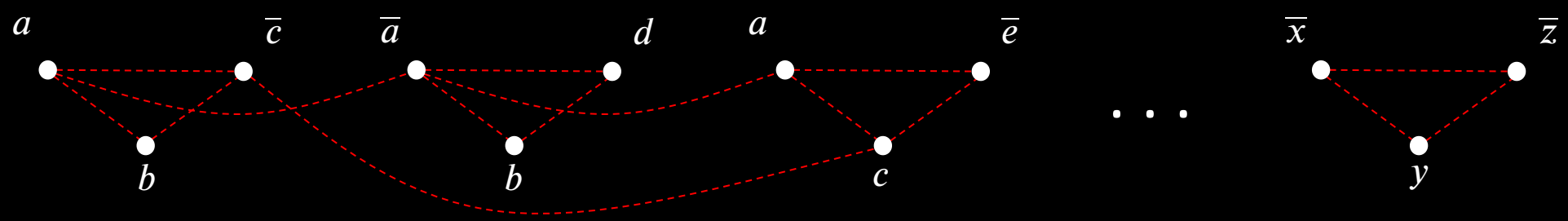

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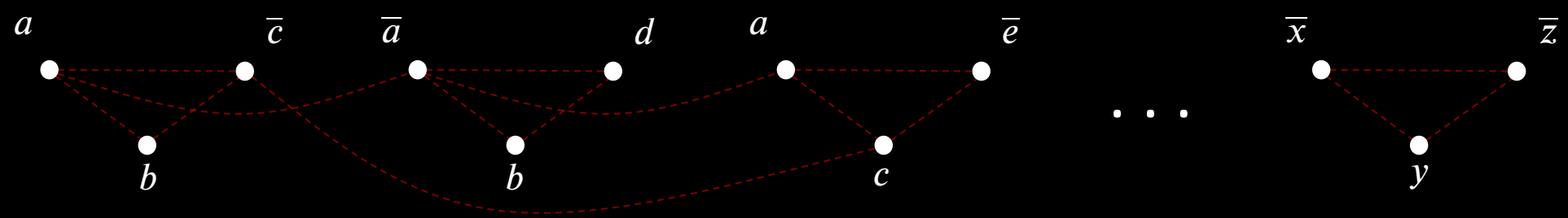

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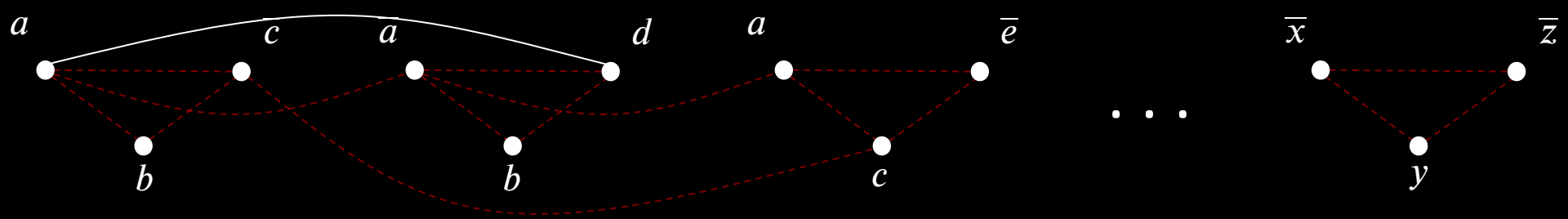

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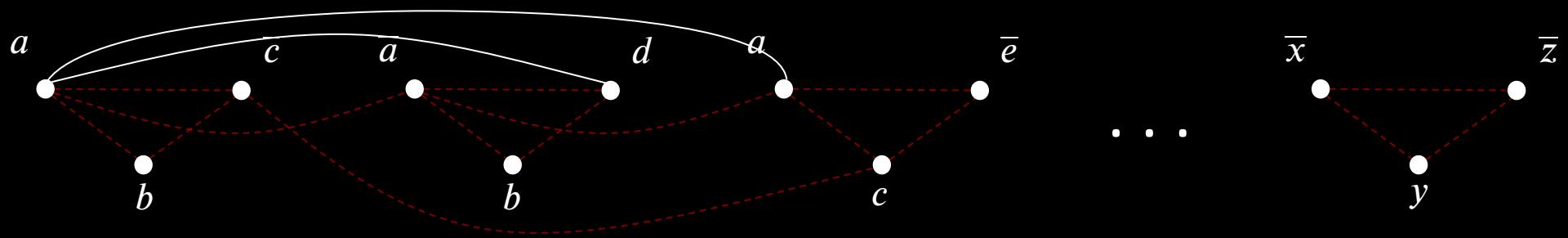

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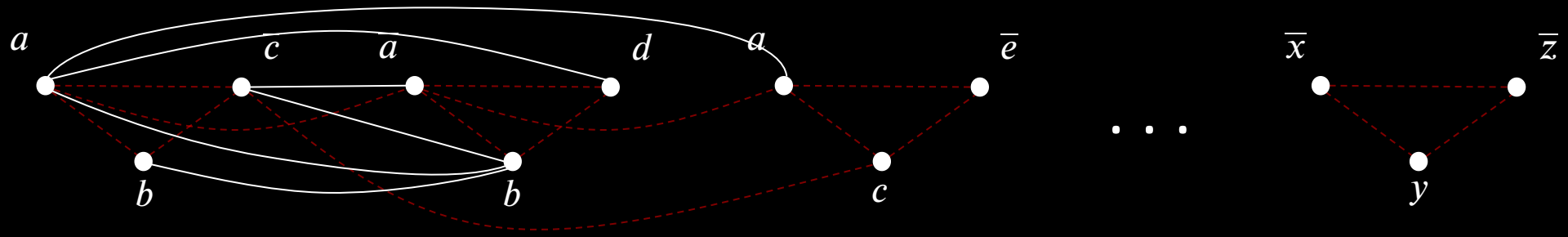

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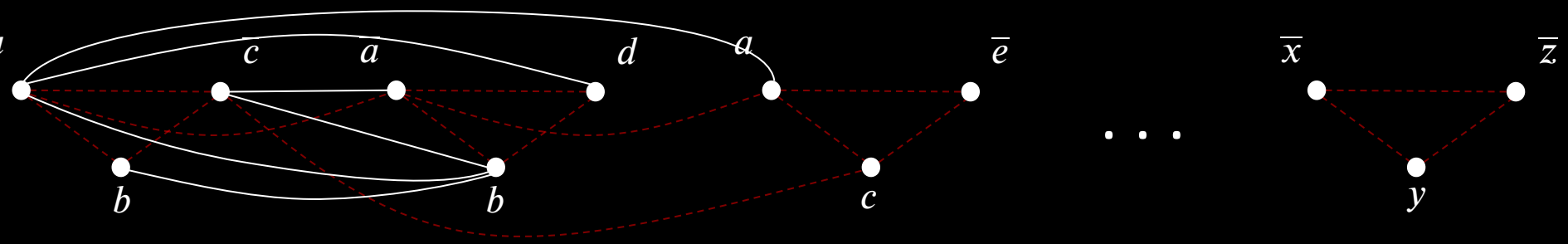
G = 

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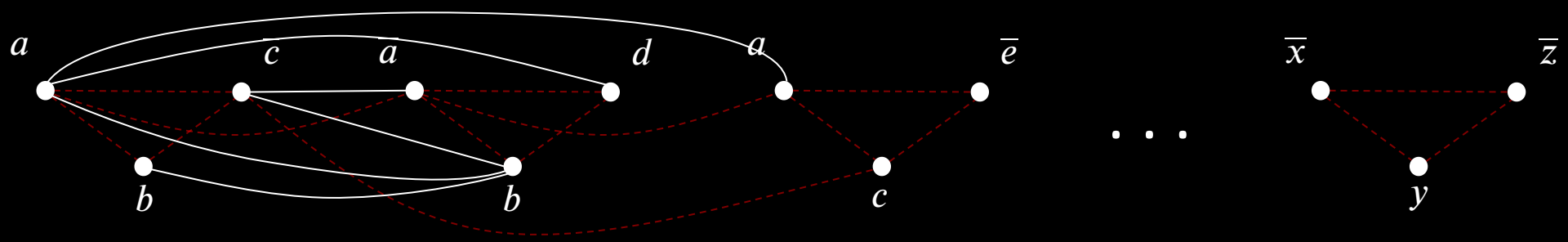
G = 

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$3SAT \leq_P CLIQUE$ conclusion

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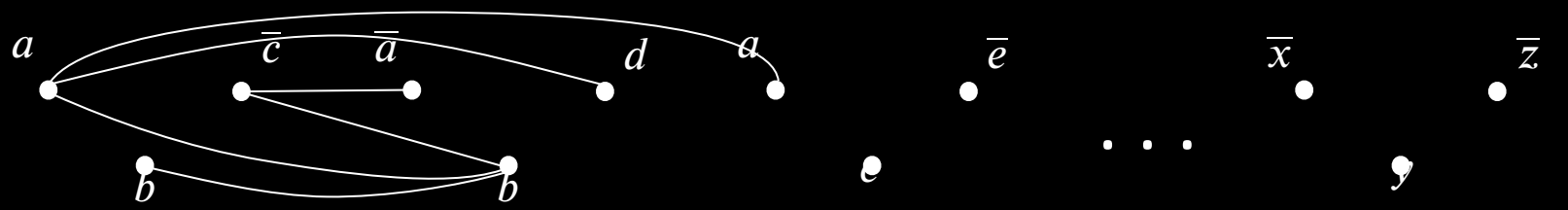
$\boxed{G} =$



$\boxed{k} =$ # clauses

Claim: ϕ is satisfiable iff G has a k-clique

$3SAT \leq_P CLIQUE$ conclusion

$$\phi = (a \vee b \vee \bar{c}) \wedge (\bar{a} \vee b \vee d) \wedge (a \vee c \vee \bar{e}) \wedge \cdots \wedge (\bar{x} \vee y \vee \bar{z})$$

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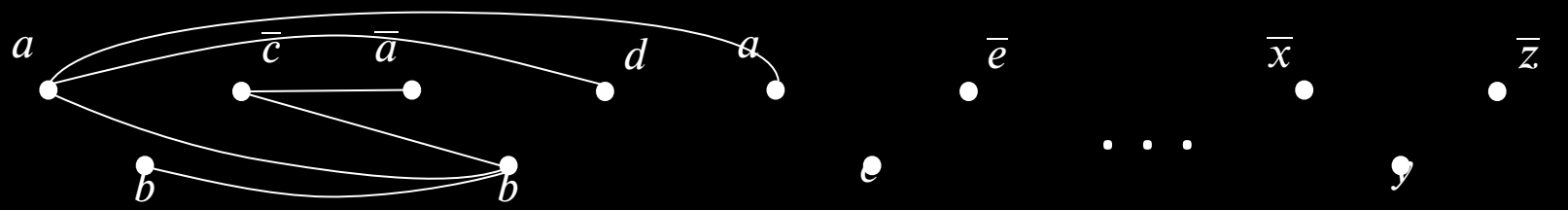

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Claim: ϕ is satisfiable iff G has a k-clique

($\rightarrow$) Take any satisfying assignment to ϕ. Pick 1 true literal in each clause.

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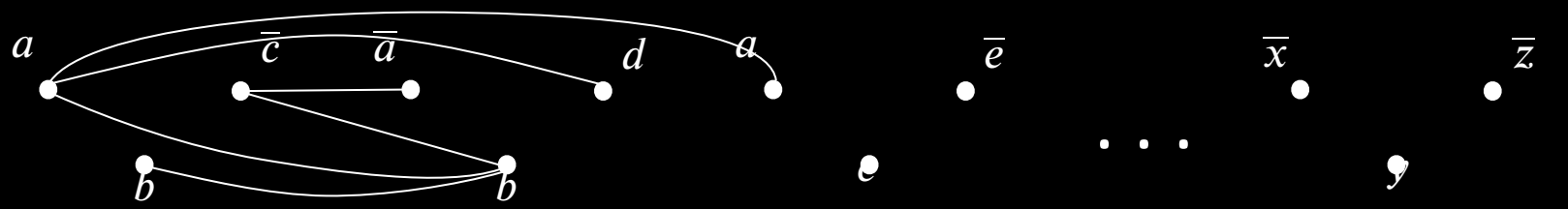

Claim: ϕ is satisfiable iff G has a k-clique

($\rightarrow$) Take any satisfying assignment to ϕ. Pick 1 true literal in each clause.

The corresponding nodes in G are a k-clique because they don't have forbidden edges.

$3SAT \leq_P CLIQUE$ conclusion

$$\phi = (a \vee b \vee \bar{c}) \wedge (\bar{a} \vee b \vee d) \wedge (a \vee c \vee \bar{e}) \wedge \cdots \wedge (\bar{x} \vee y \vee \bar{z})$$

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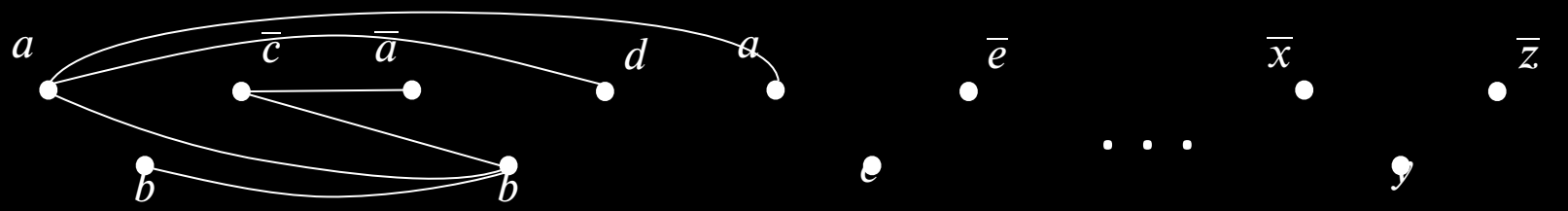

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Claim: ϕ is satisfiable iff G has a k-clique

($\rightarrow$) Take any satisfying assignment to ϕ. Pick 1 true literal in each clause.

The corresponding nodes in G are a k-clique because they don't have forbidden edges.

($\leftarrow$) Take any k-clique in G. It must have 1 node in each clause.

$3SAT \leq_P CLIQUE$ conclusion

$$\phi = (a \vee b \vee \bar{c}) \wedge (\bar{a} \vee b \vee d) \wedge (a \vee c \vee \bar{e}) \wedge \cdots \wedge (\bar{x} \vee y \vee \bar{z})$$

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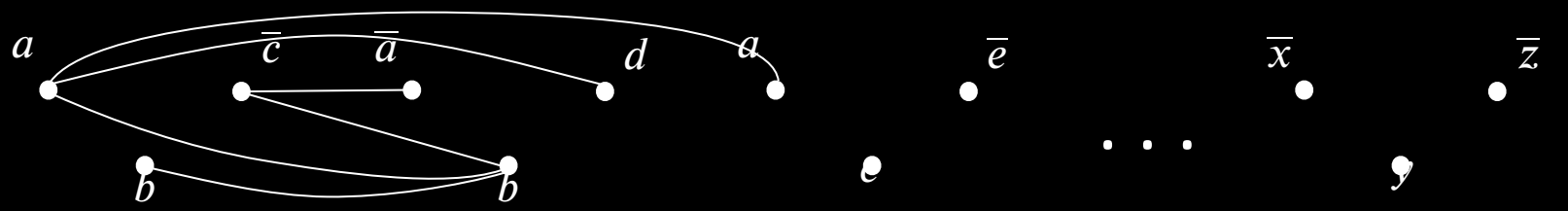

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Set each corresponding literal TRUE. That gives a satisfying assignment to ϕ.

$3SAT \leq_P CLIQUE$ conclusion

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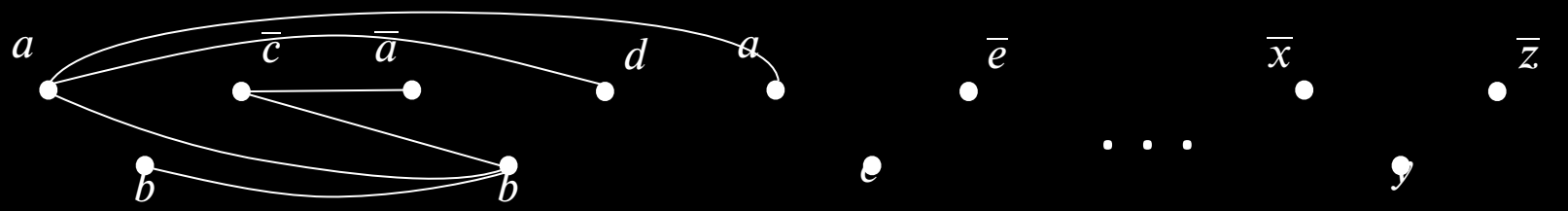

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Set each corresponding literal TRUE. That gives a satisfying assignment to ϕ.

The reduction f is computable in polynomial time.

$3SAT \leq_P CLIQUE$ conclusion

$$\phi = (a \vee b \vee \bar{c}) \wedge (\bar{a} \vee b \vee d) \wedge (a \vee c \vee \bar{e}) \wedge \cdots \wedge (\bar{x} \vee y \vee \bar{z})$$

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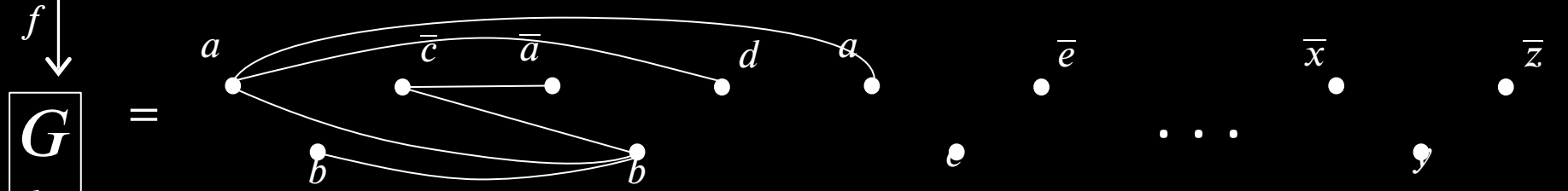

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The reduction f is computable in polynomial time.

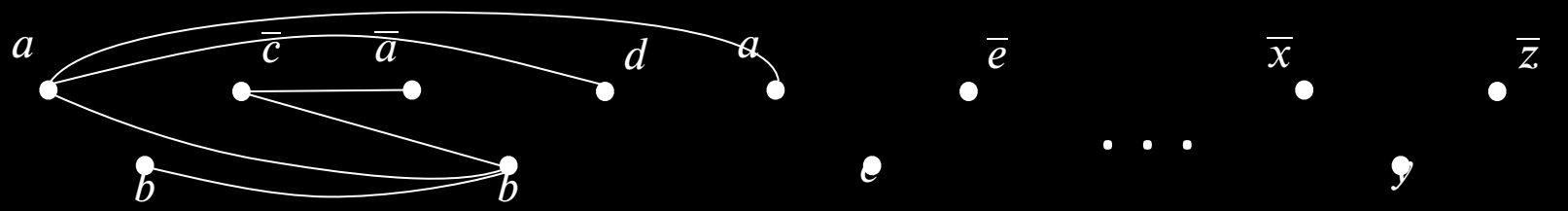
Corollary: $CLIQUE \in \mathrm{P} \rightarrow 3SAT \in \mathrm{P}$

$3SAT \leq_P CLIQUE$ conclusion

$$\phi = (a \vee b \vee \bar{c}) \wedge (\bar{a} \vee b \vee d) \wedge (a \vee c \vee \bar{e}) \wedge \cdots \wedge (\bar{x} \vee y \vee \bar{z})$$

$f \downarrow$

$\boxed{G} =$



$\boxed{k} =$ # clauses

Claim: ϕ is satisfiable iff G has a k-clique

($\rightarrow$) Take any satisfying assignment to ϕ. Pick 1 true literal in each clause.

The corresponding nodes in G are a k-clique because they don't have forbidden edges.

($\leftarrow$) Take any k-clique in G. It must have 1 node in each clause.

Set each corresponding literal TRUE. That gives a satisfying assignment to ϕ.

The reduction f is computable in polynomial time.

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Check-in 15.1

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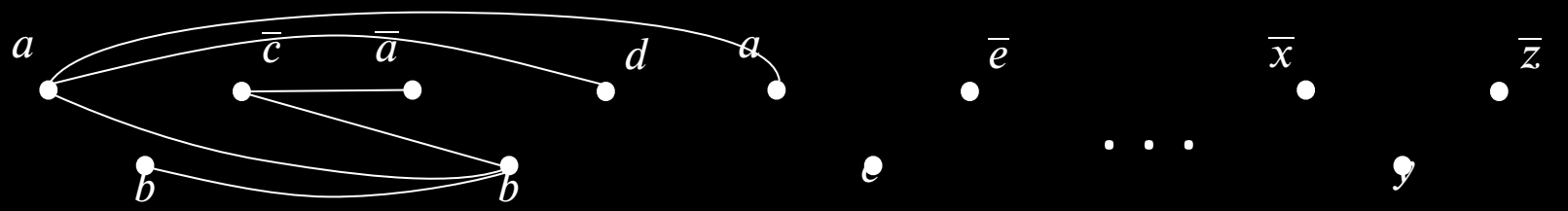

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Does this proof require 3 literals per clause?

(a) Yes, to prove the claim.

(b) Yes, to show it is in poly time.

(c) No, it works for any size clauses.

Check-in 15.1

NP-completeness

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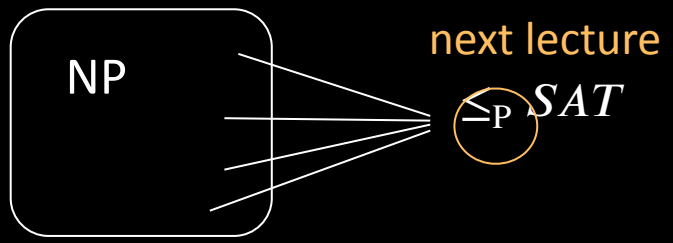

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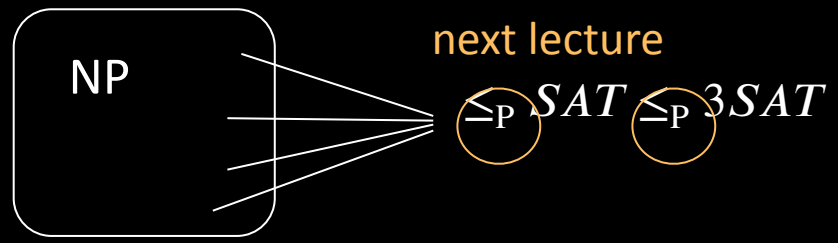

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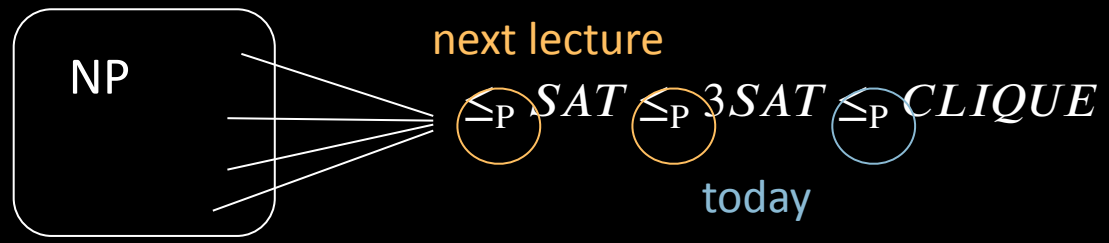

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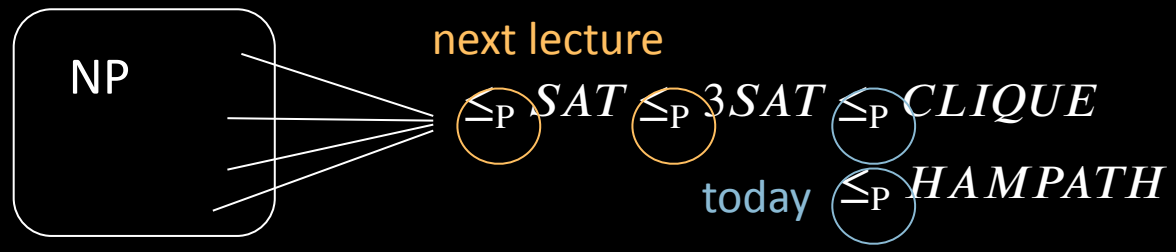

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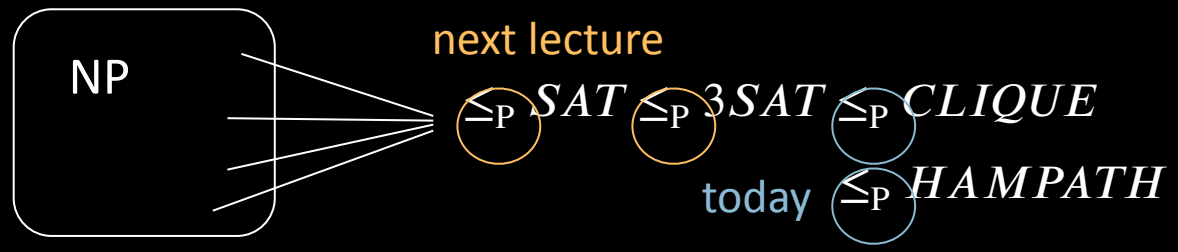

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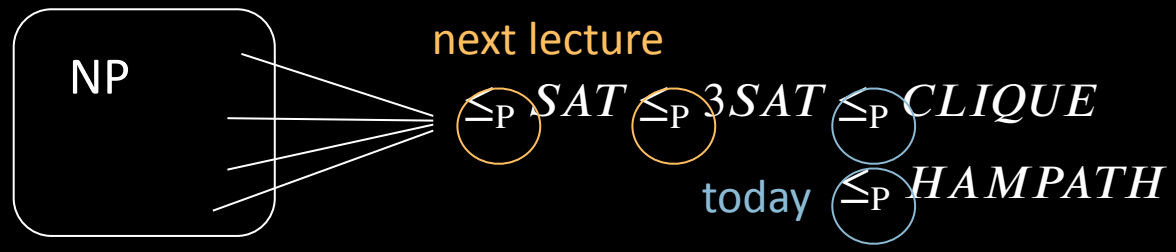

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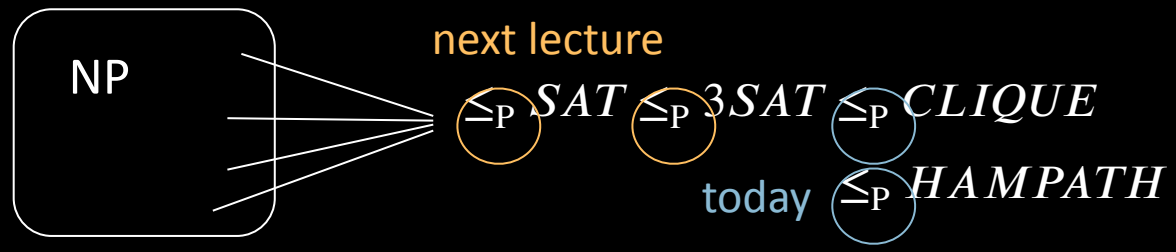

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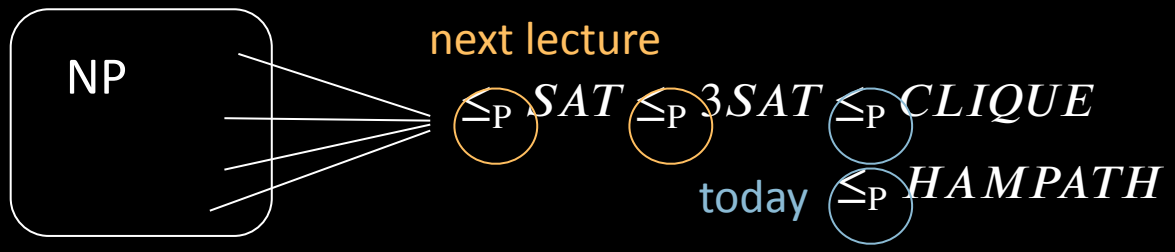

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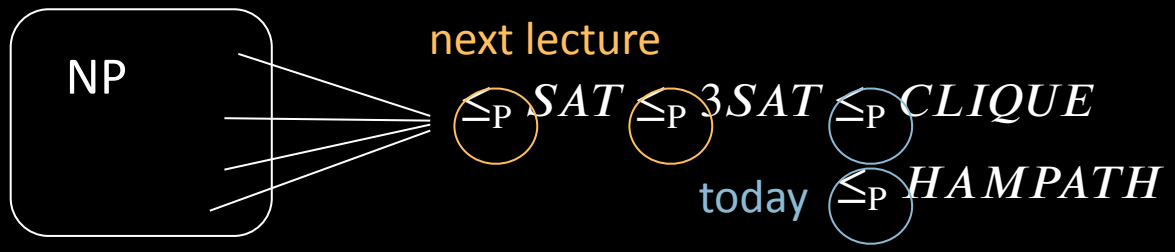

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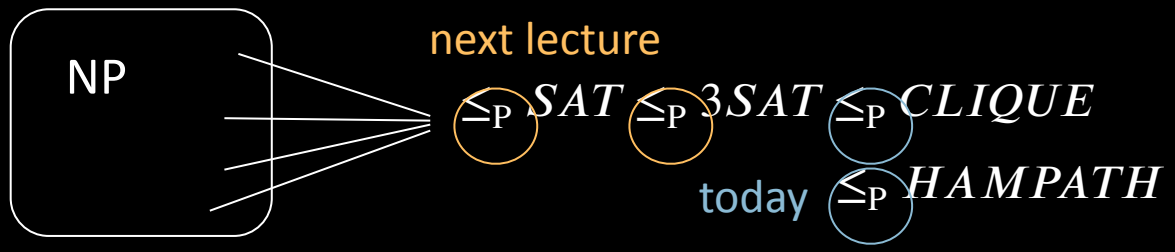

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Check-in 15.2

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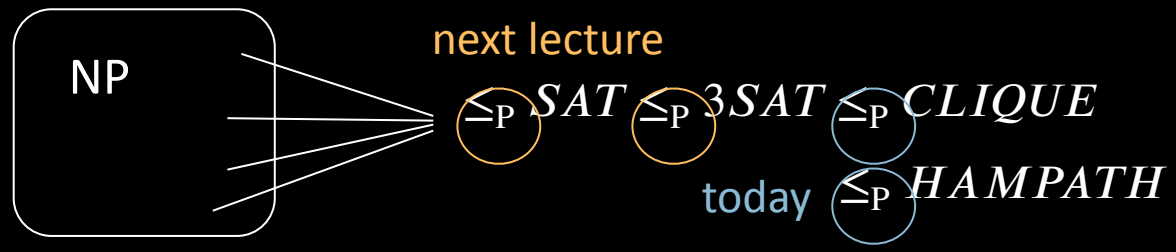

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What language that we've previously seen is most analogous to SAT?

(a) A_{TM}

(b) E_{TM}

(c) $\{0^k 1^k \mid k \geq 0\}$

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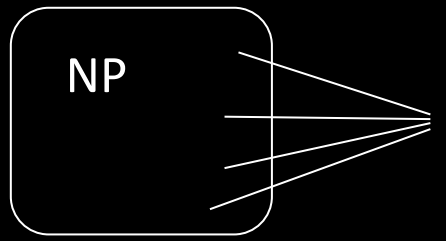

$\leq_P SAT \leq_P 3SAT \leq_P CLIQUE$

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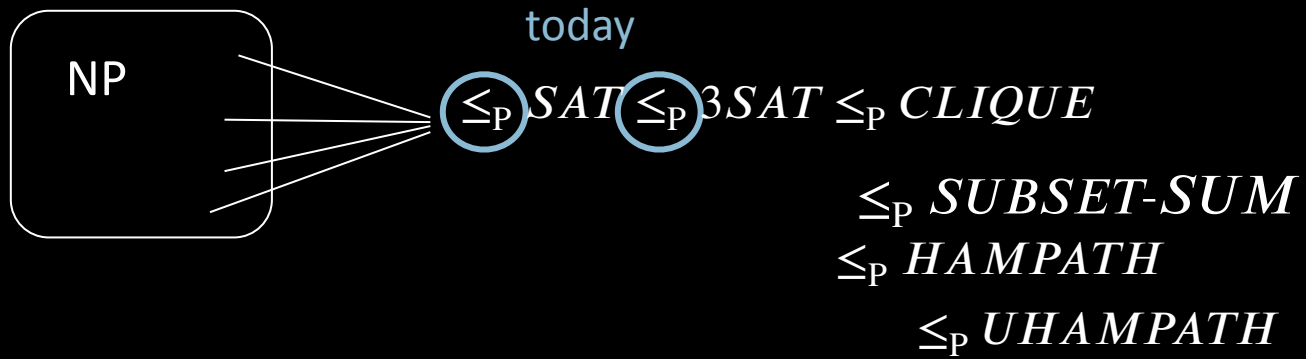

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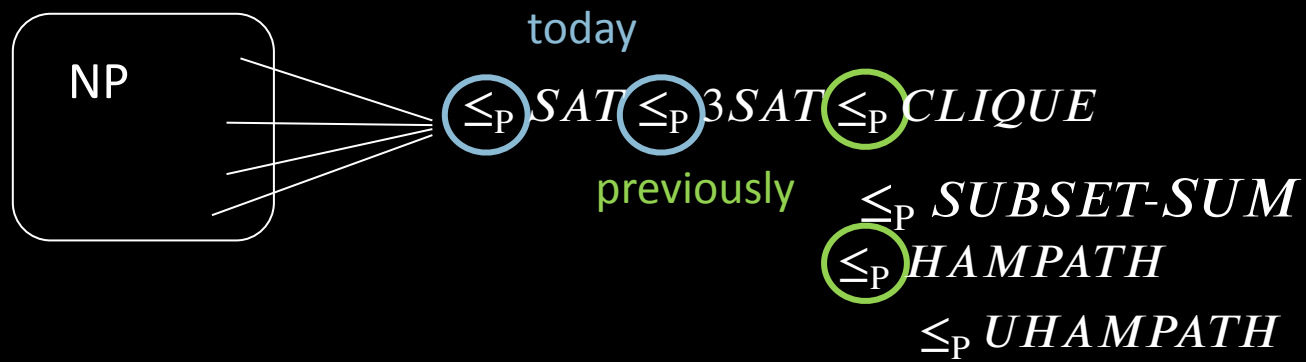

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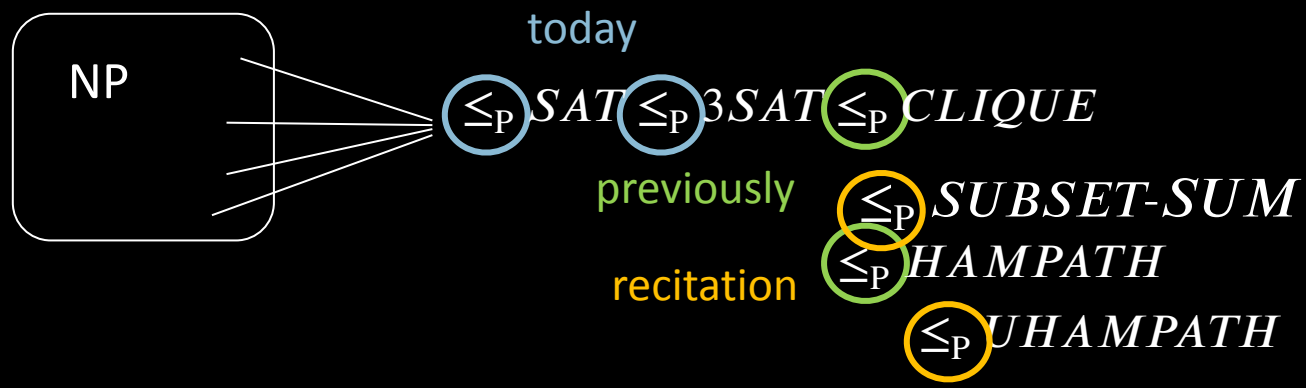

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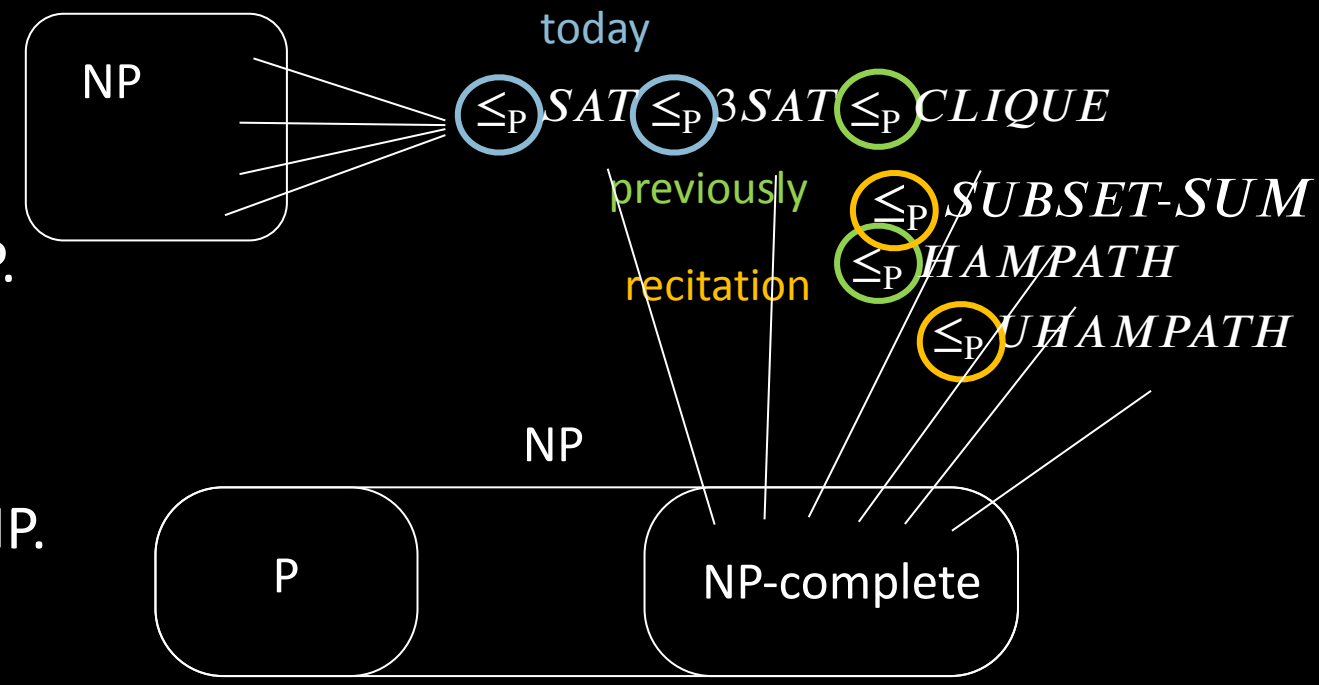

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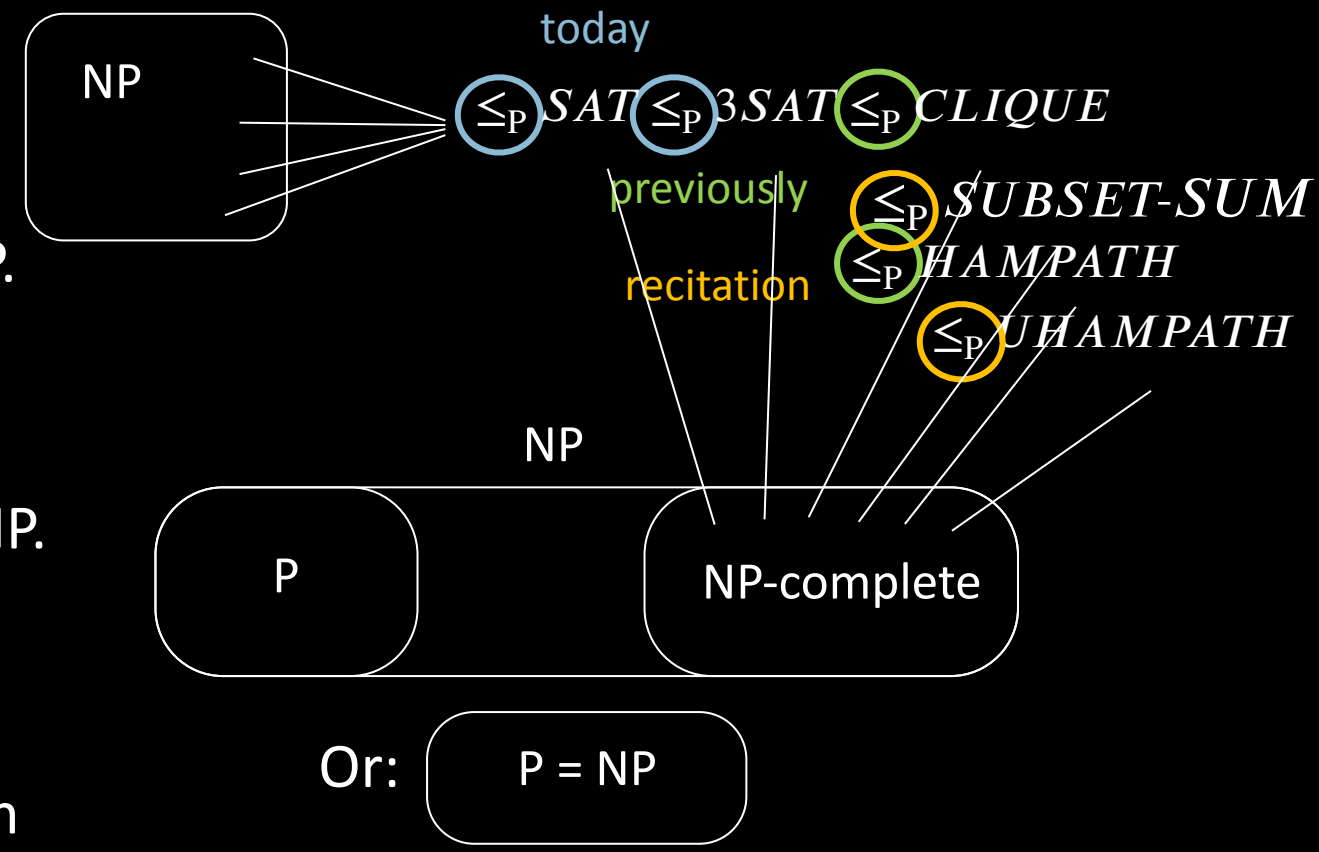

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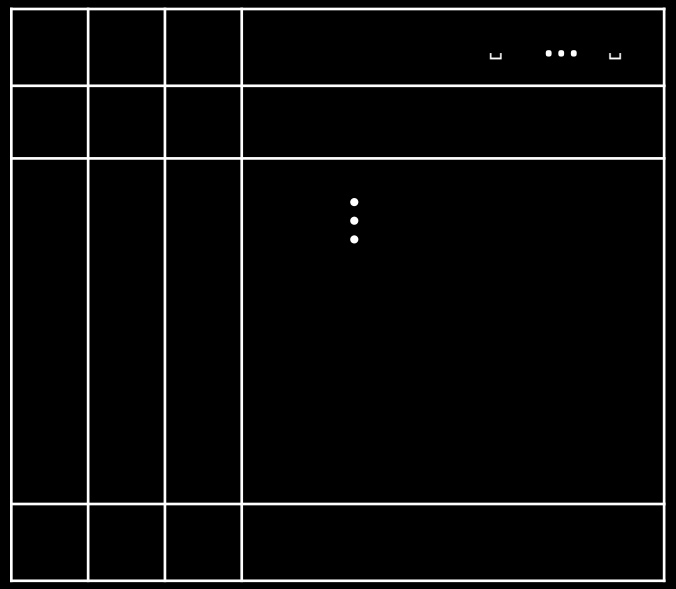

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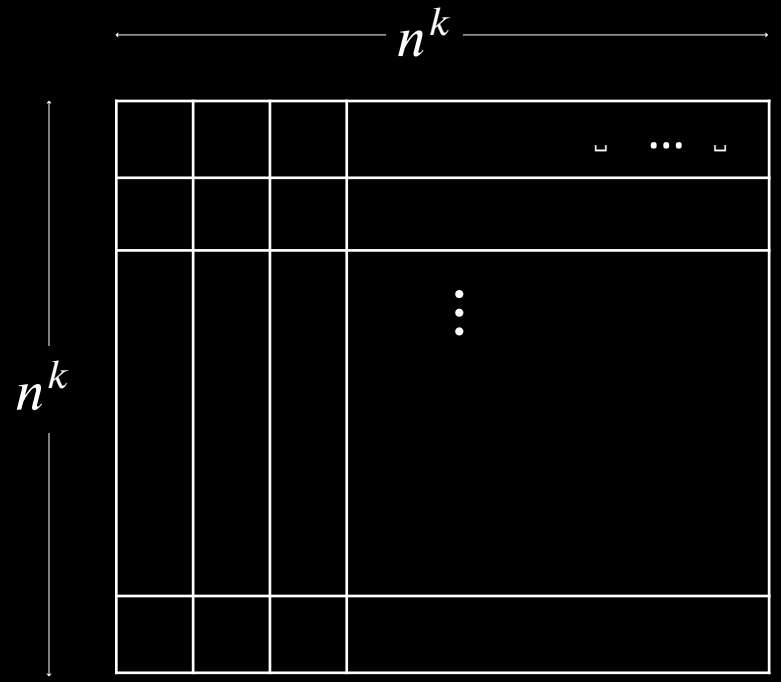

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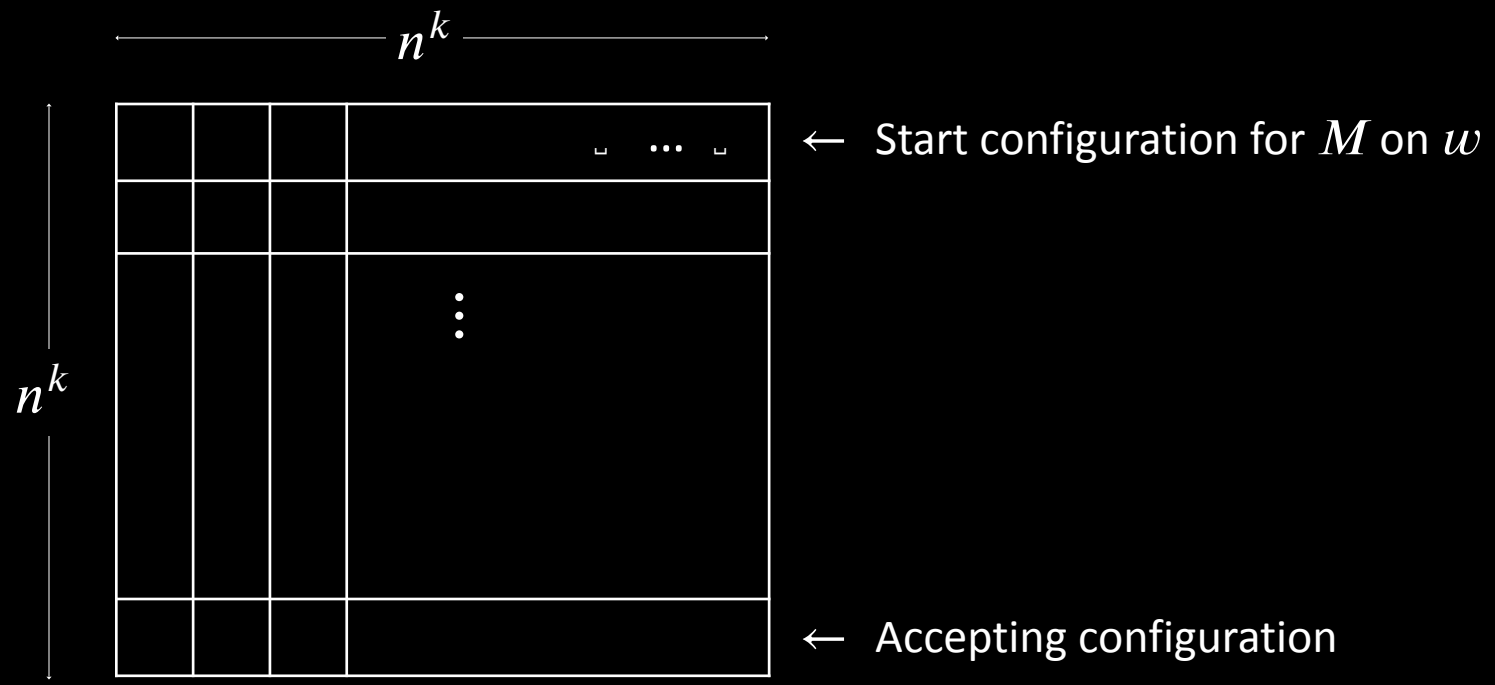

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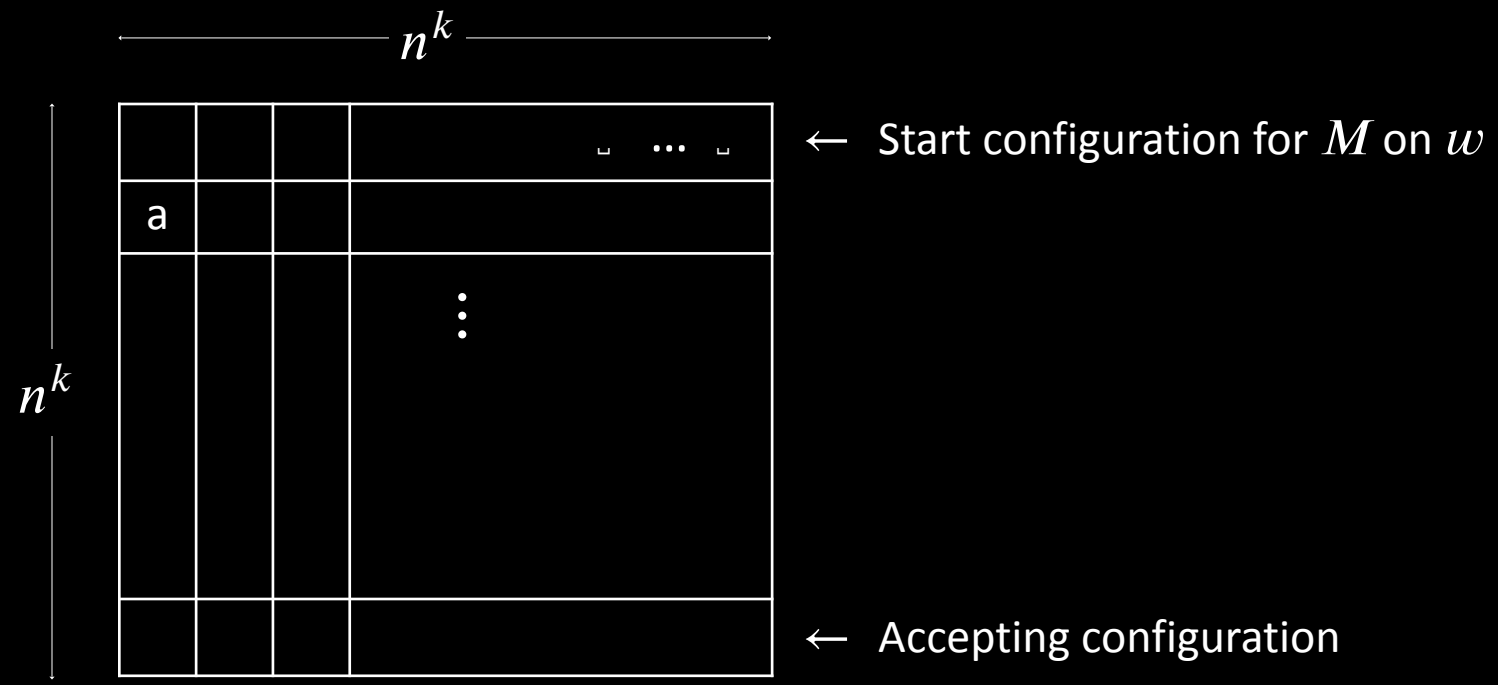

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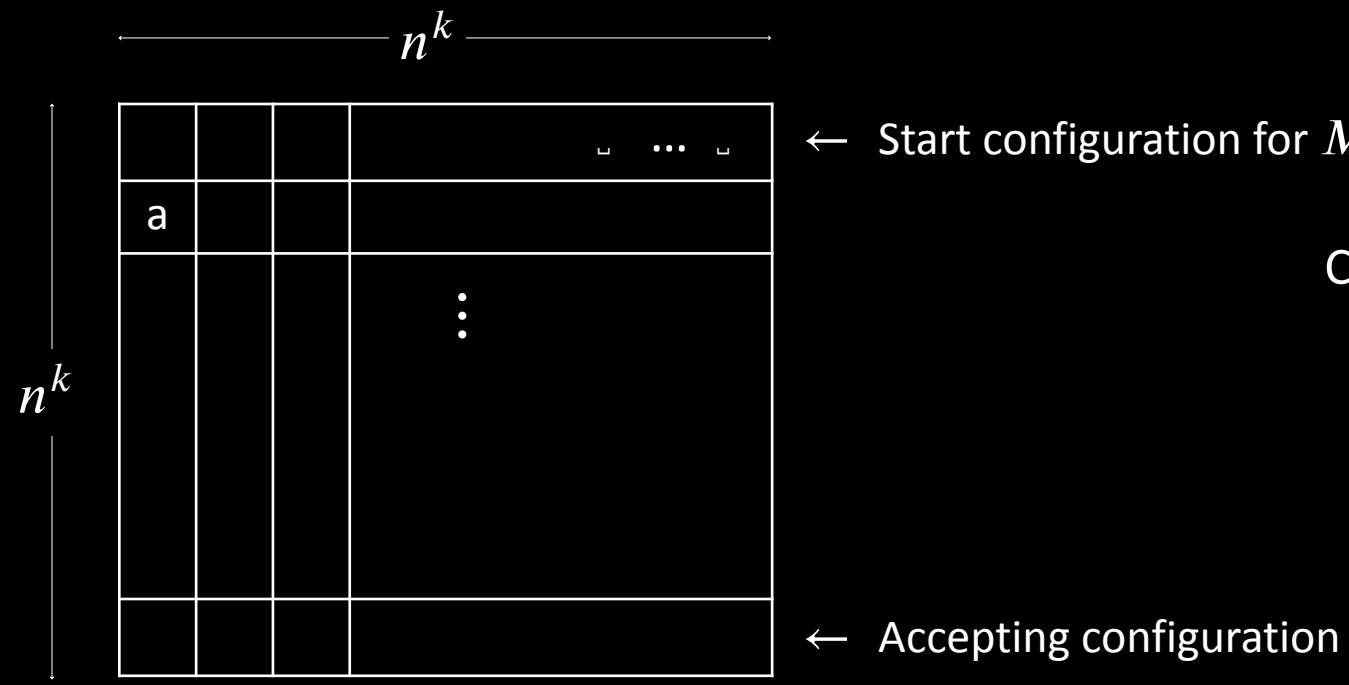

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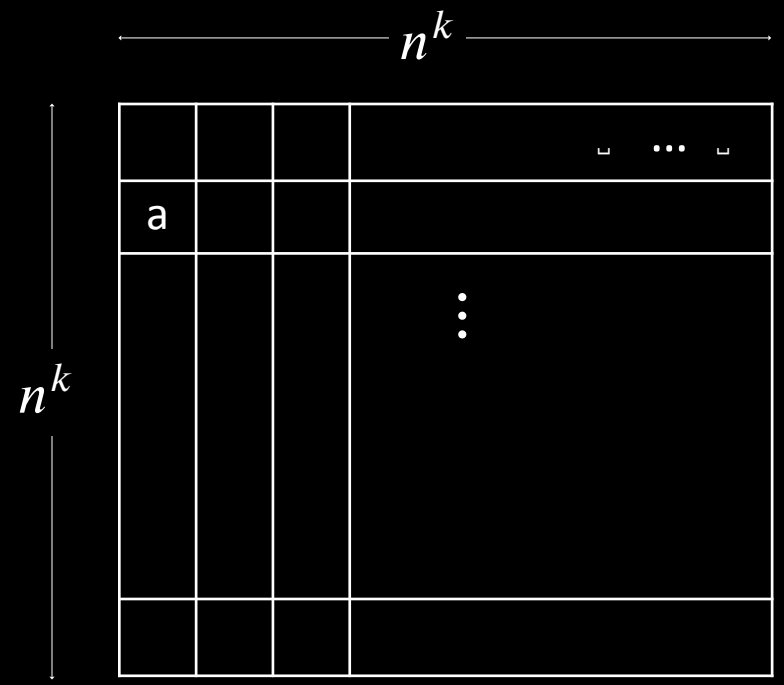

← Start configuration for M on w

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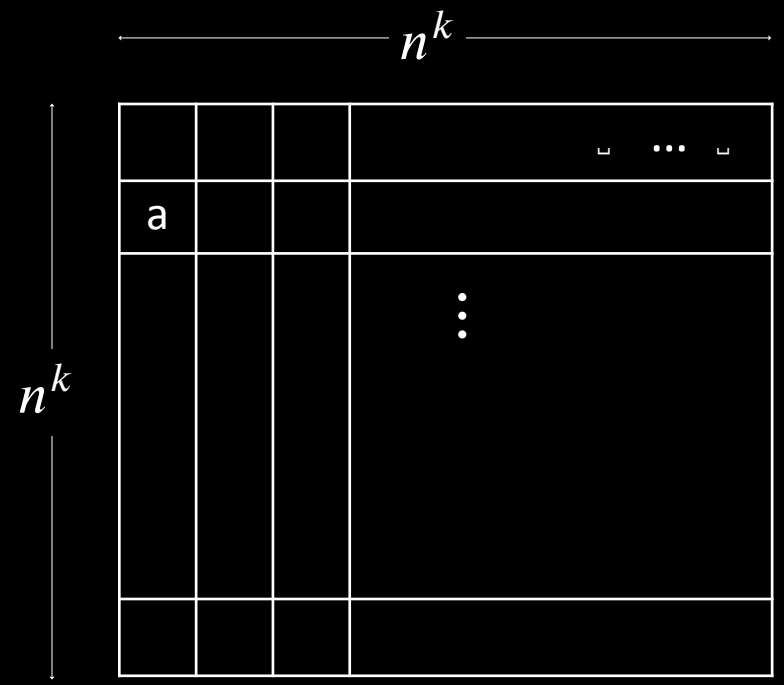

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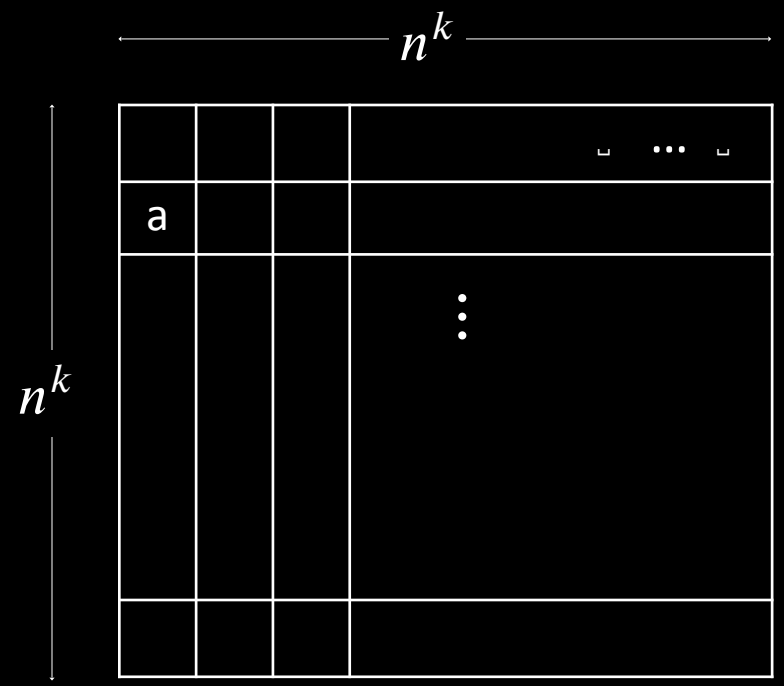

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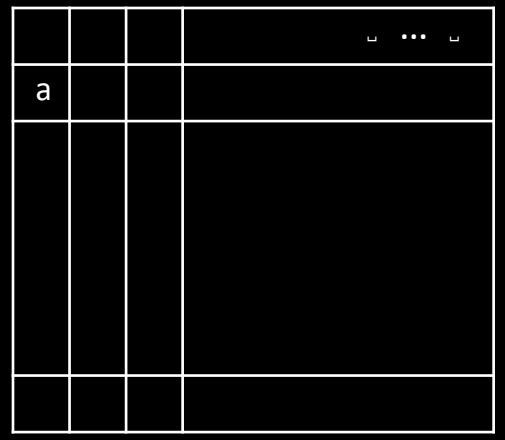

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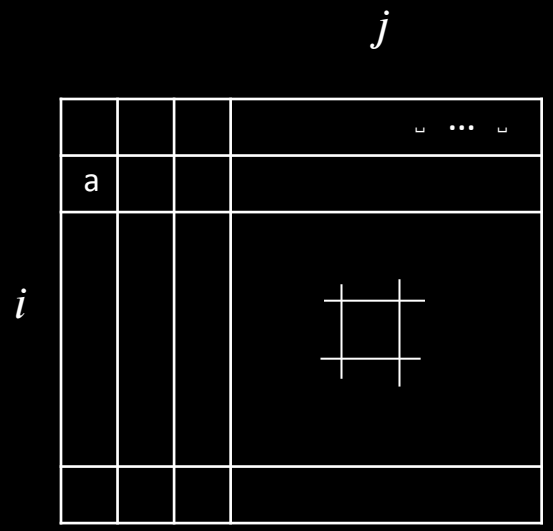

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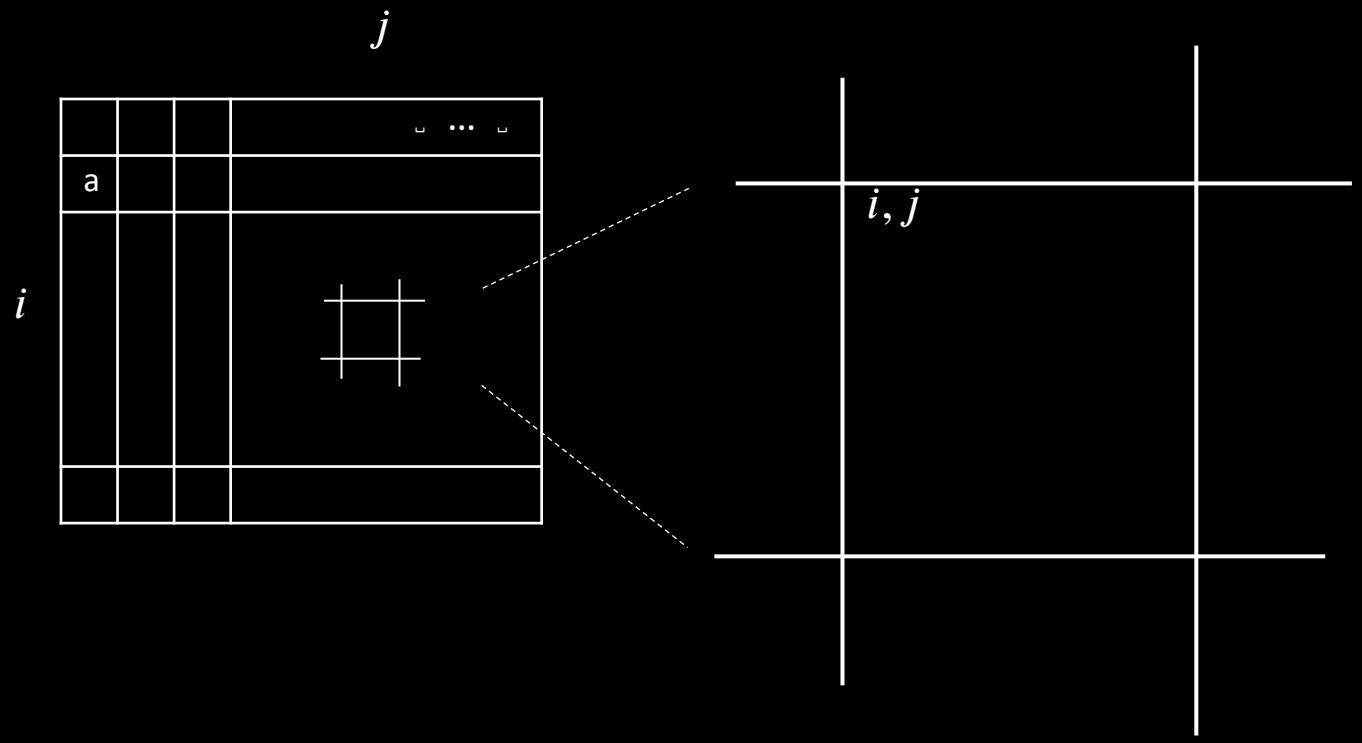

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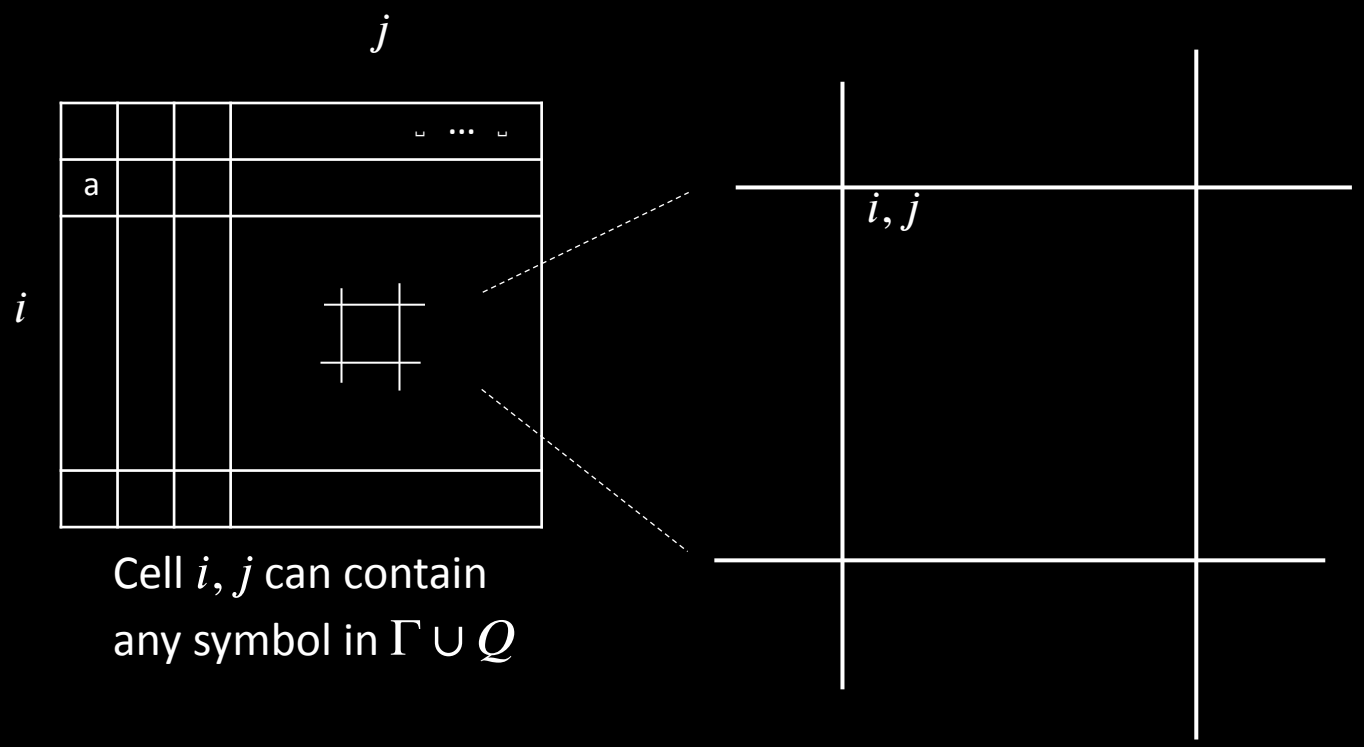

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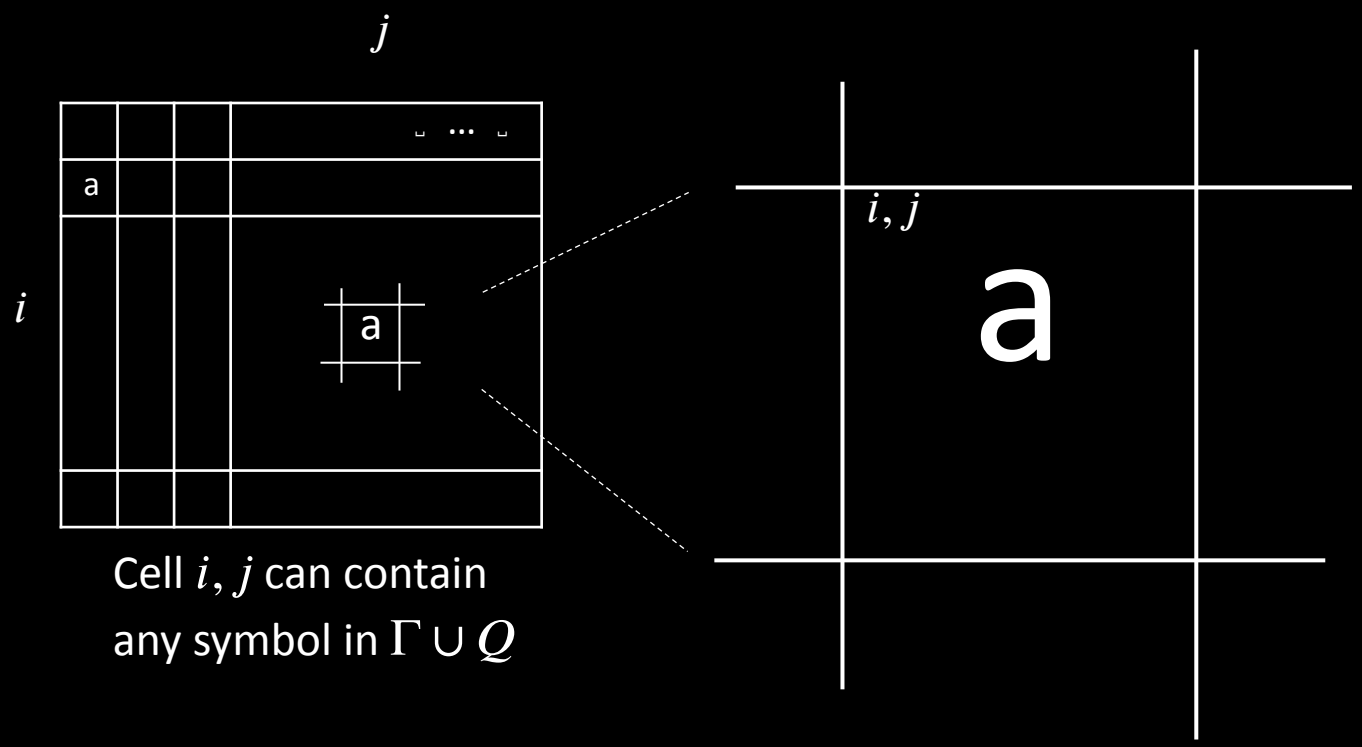

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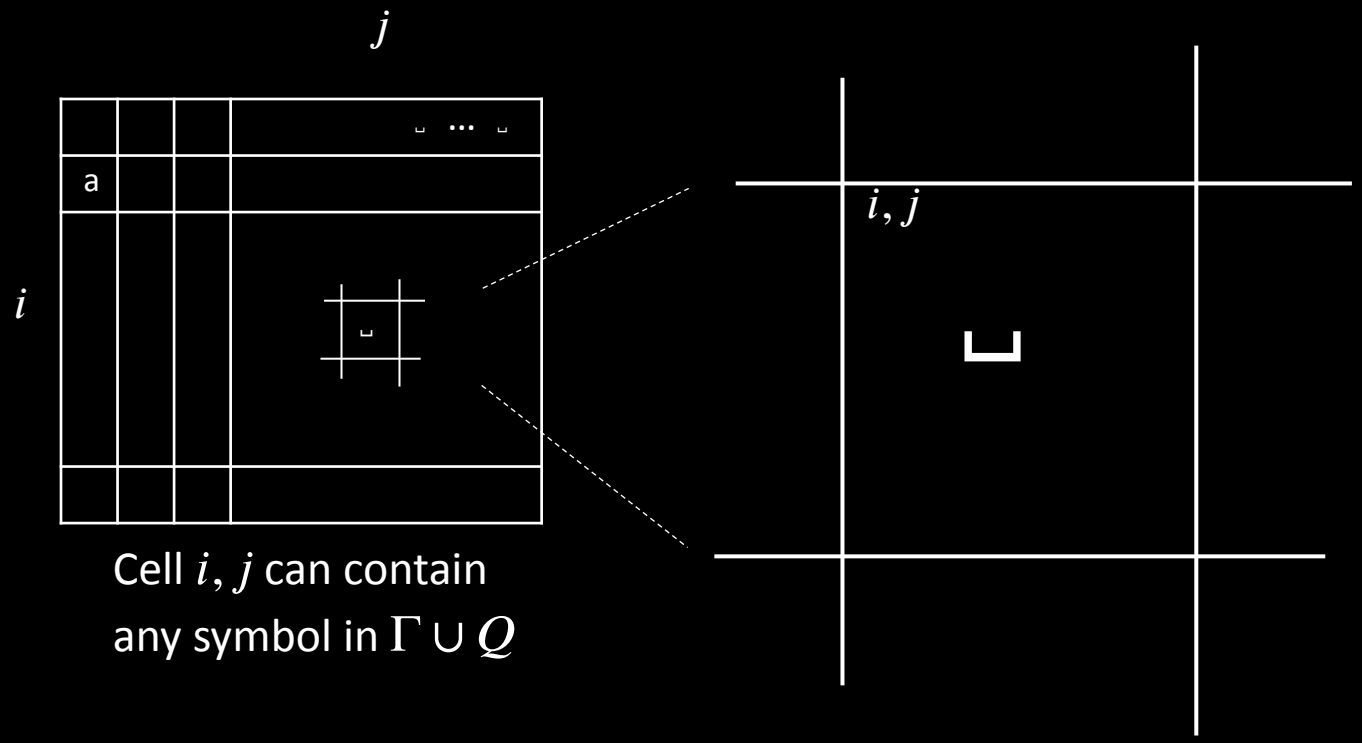

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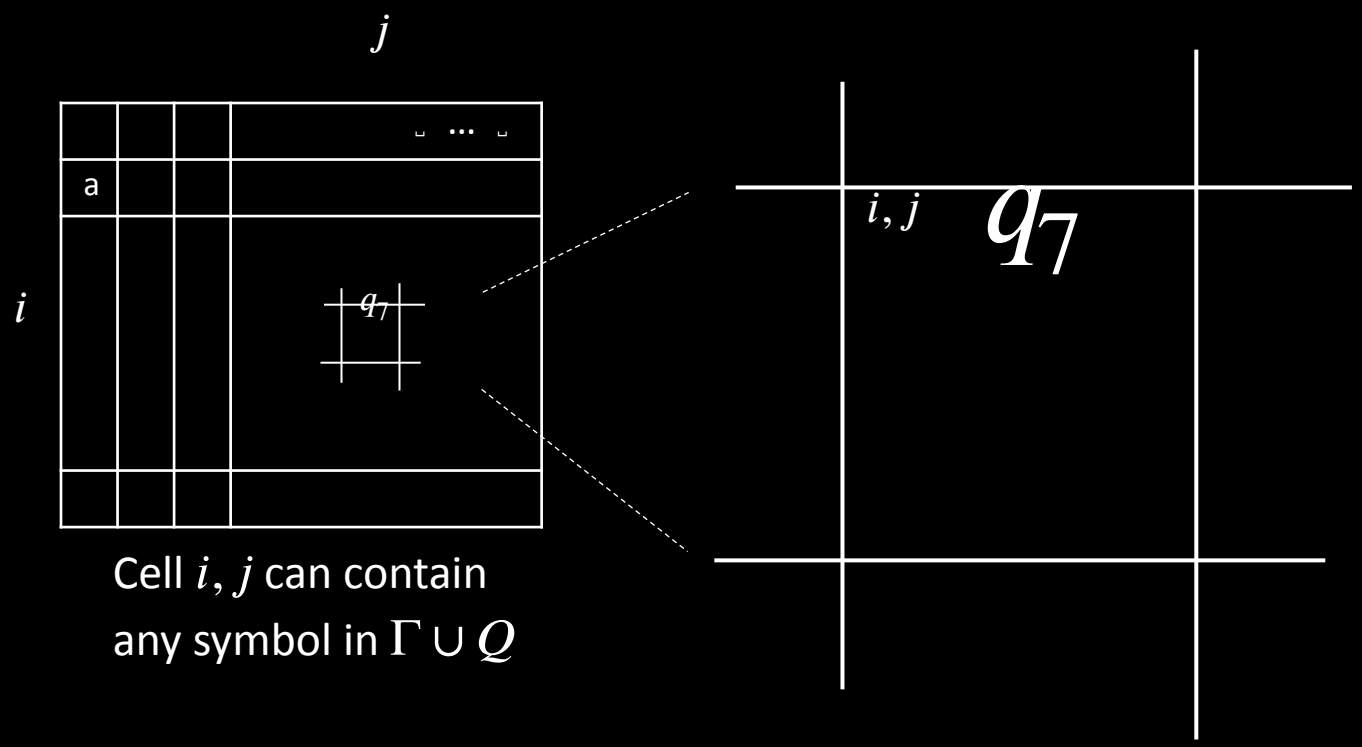

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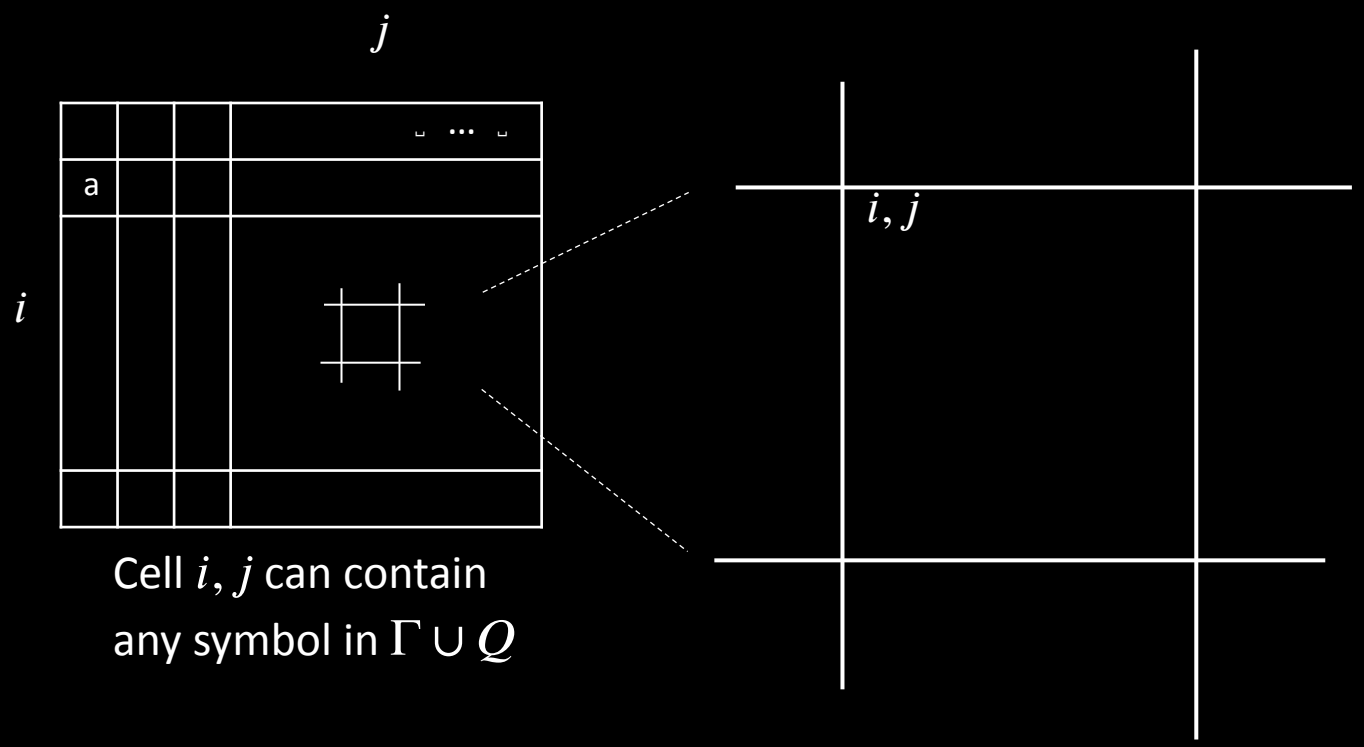

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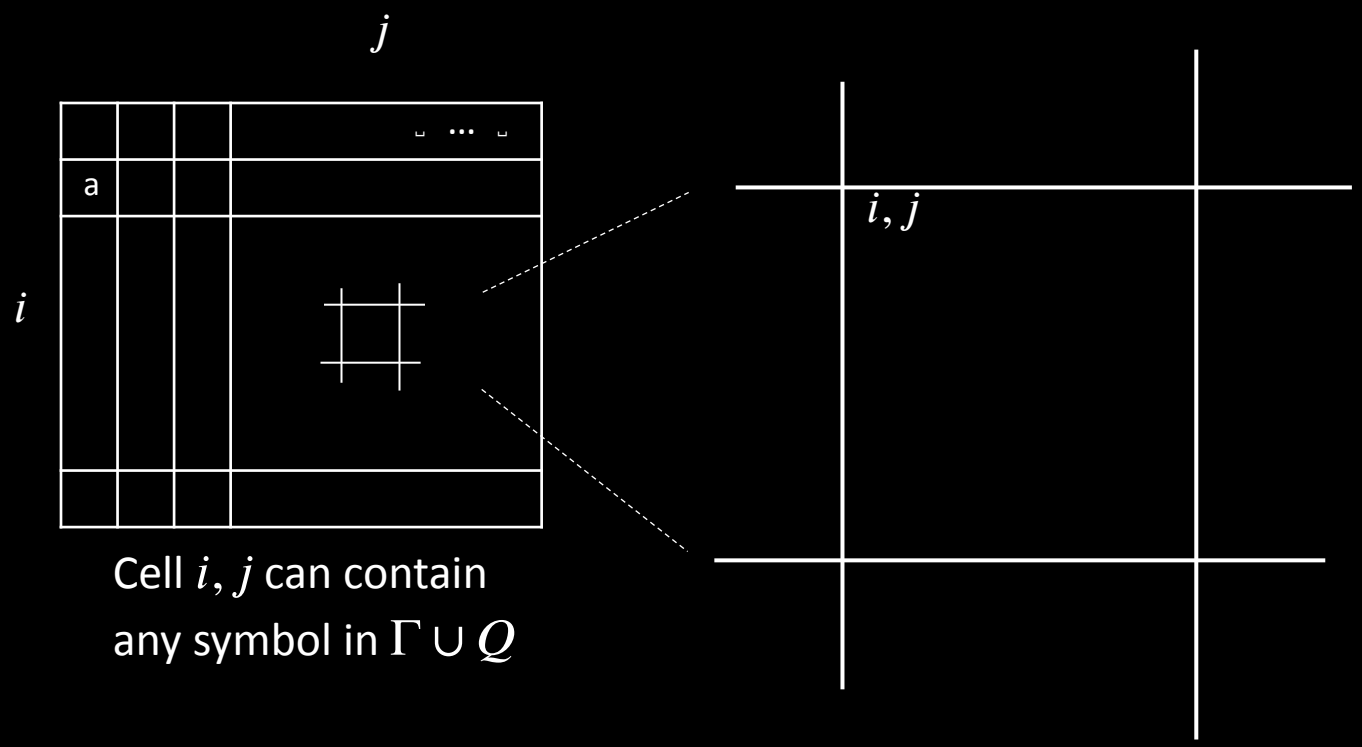

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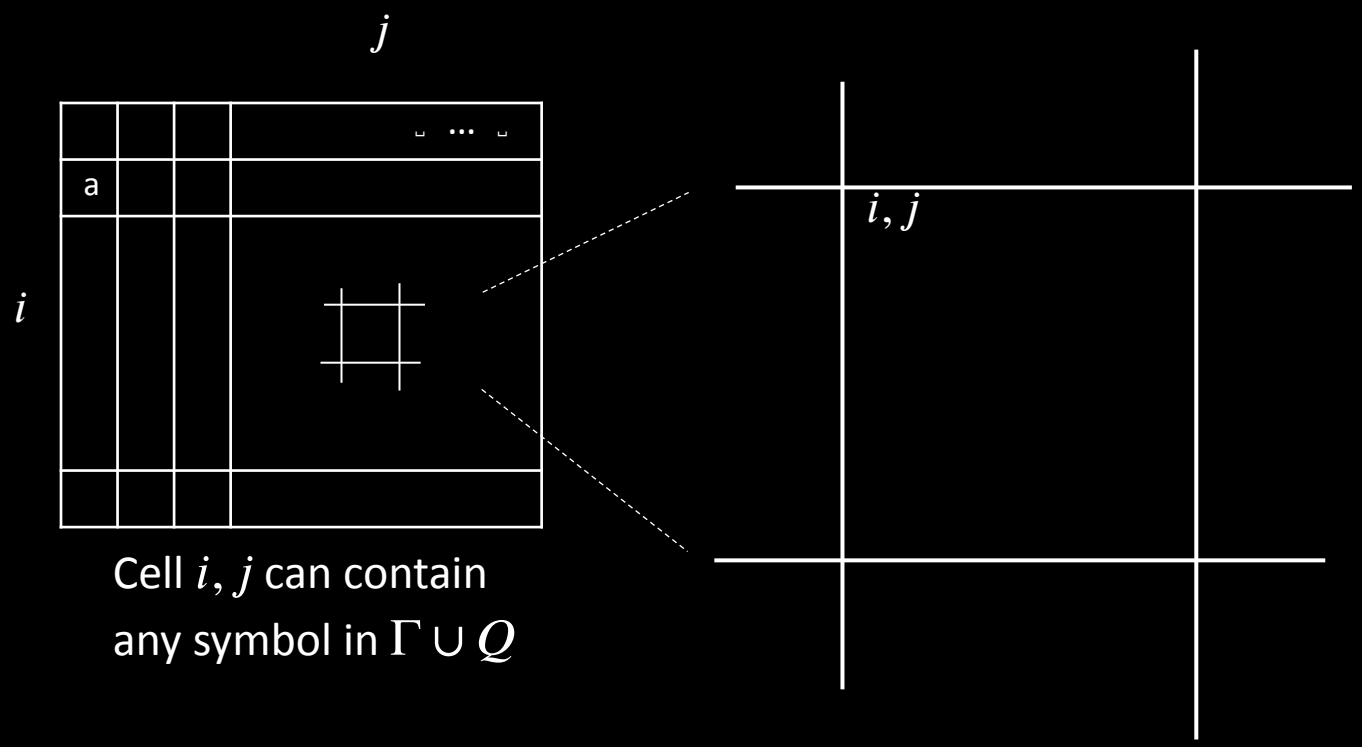

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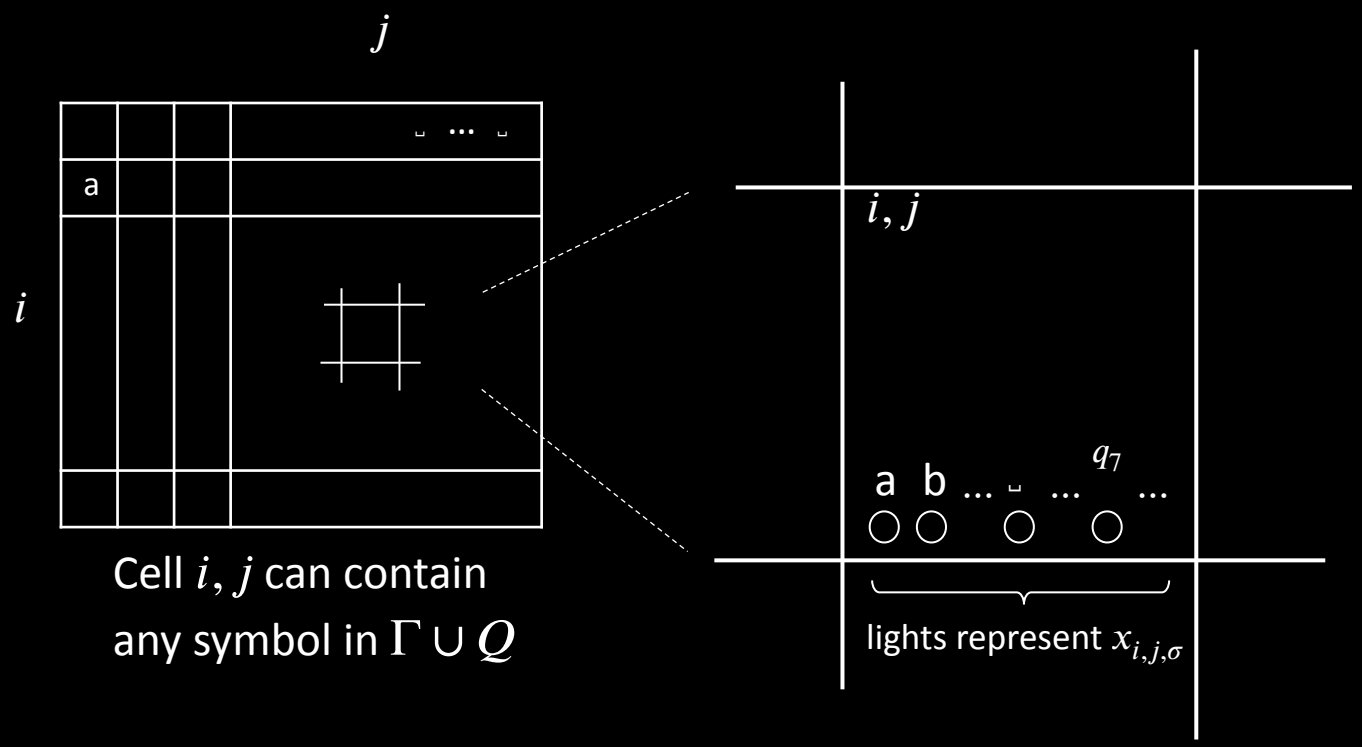

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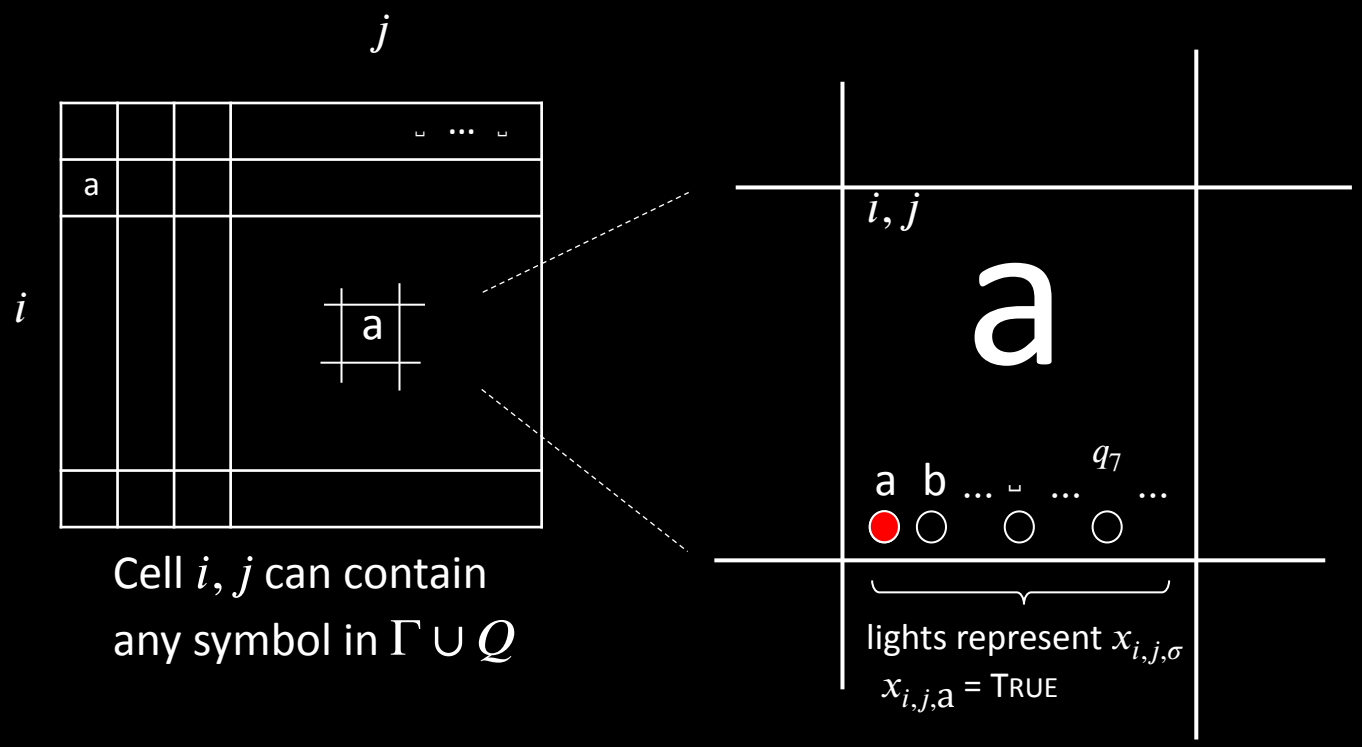

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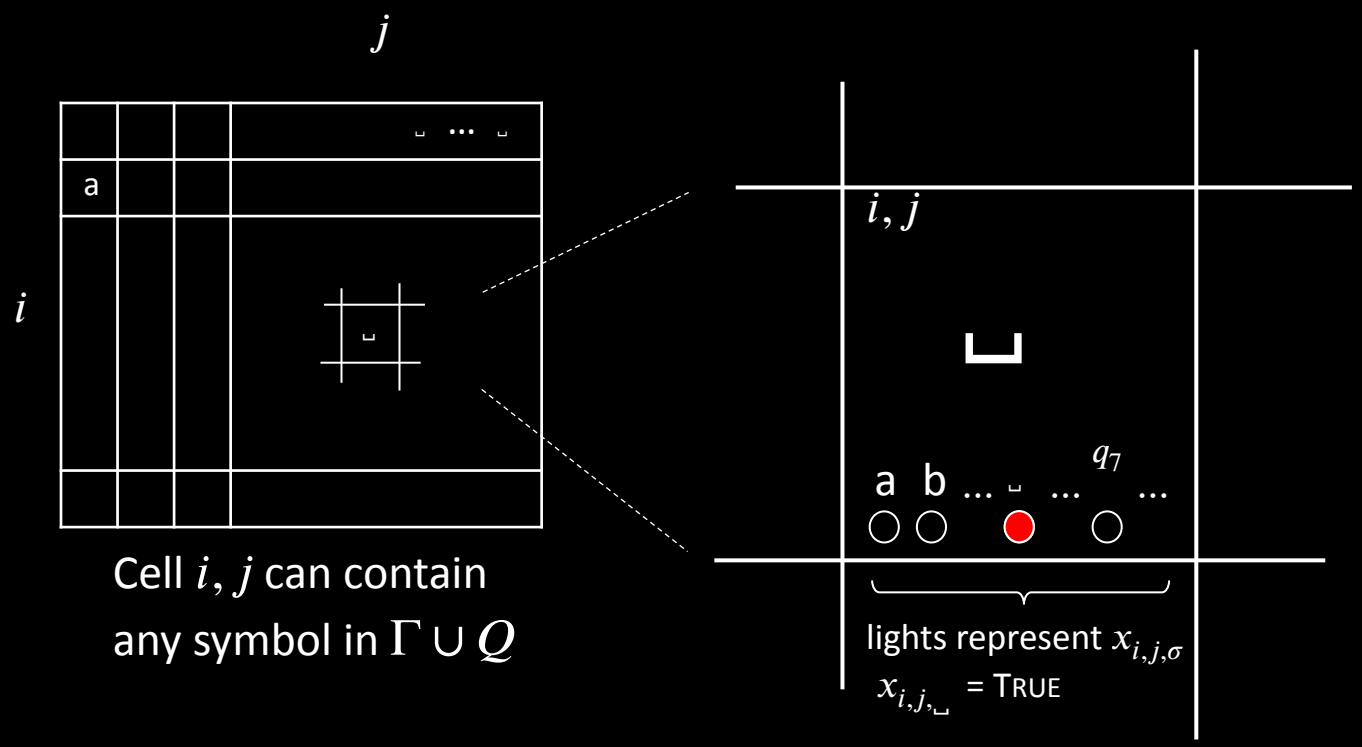

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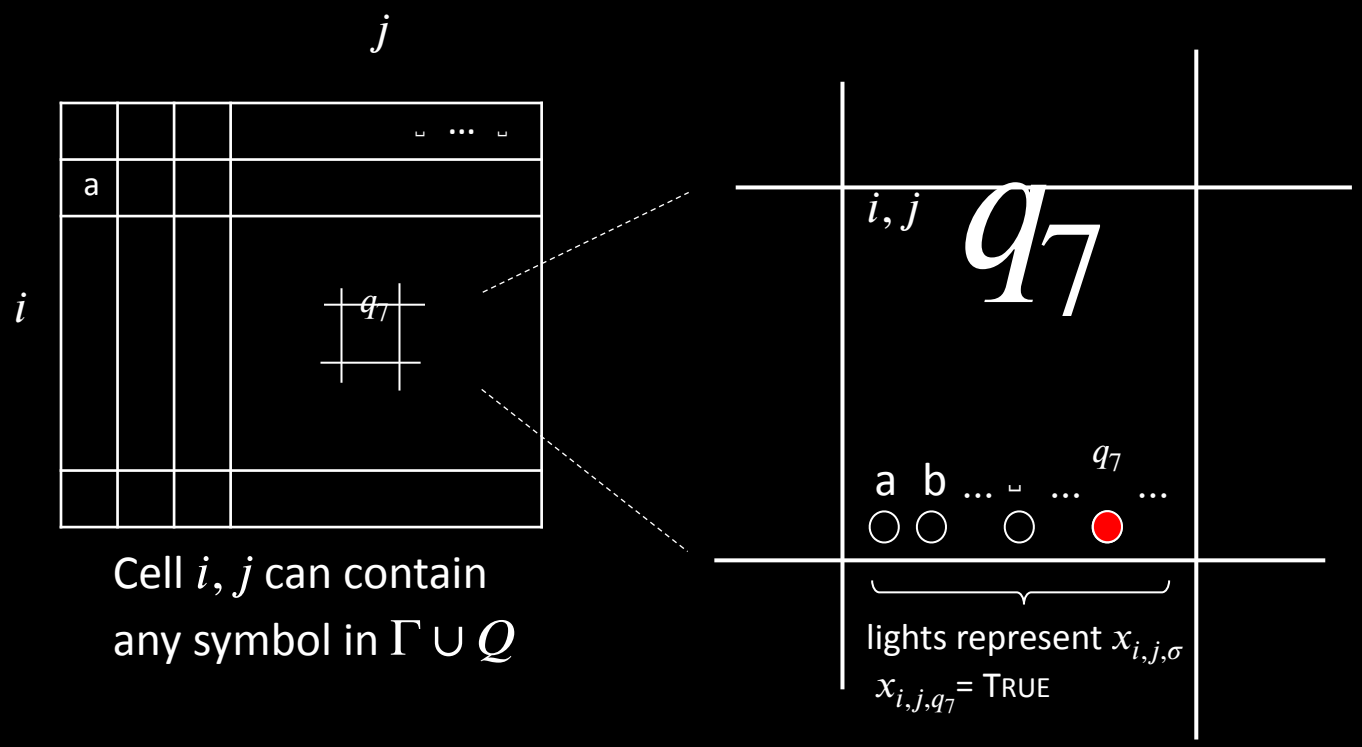

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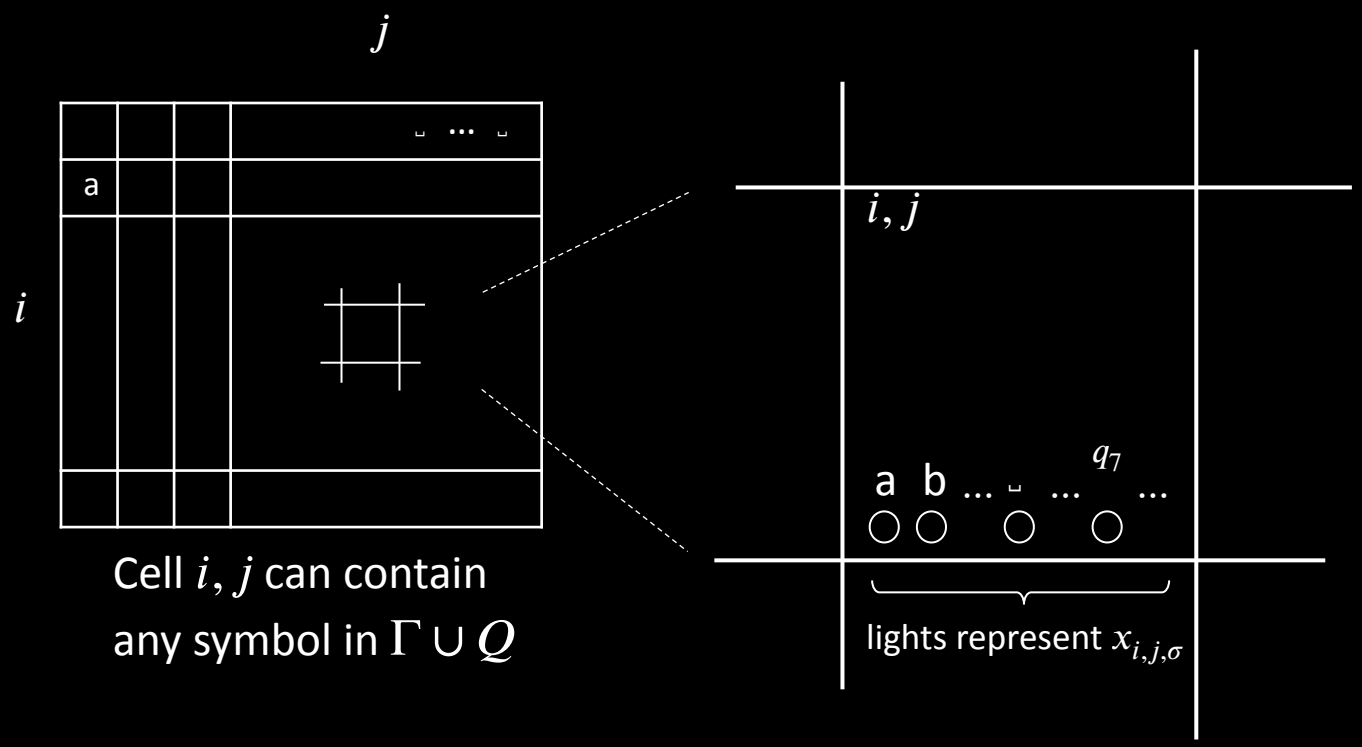

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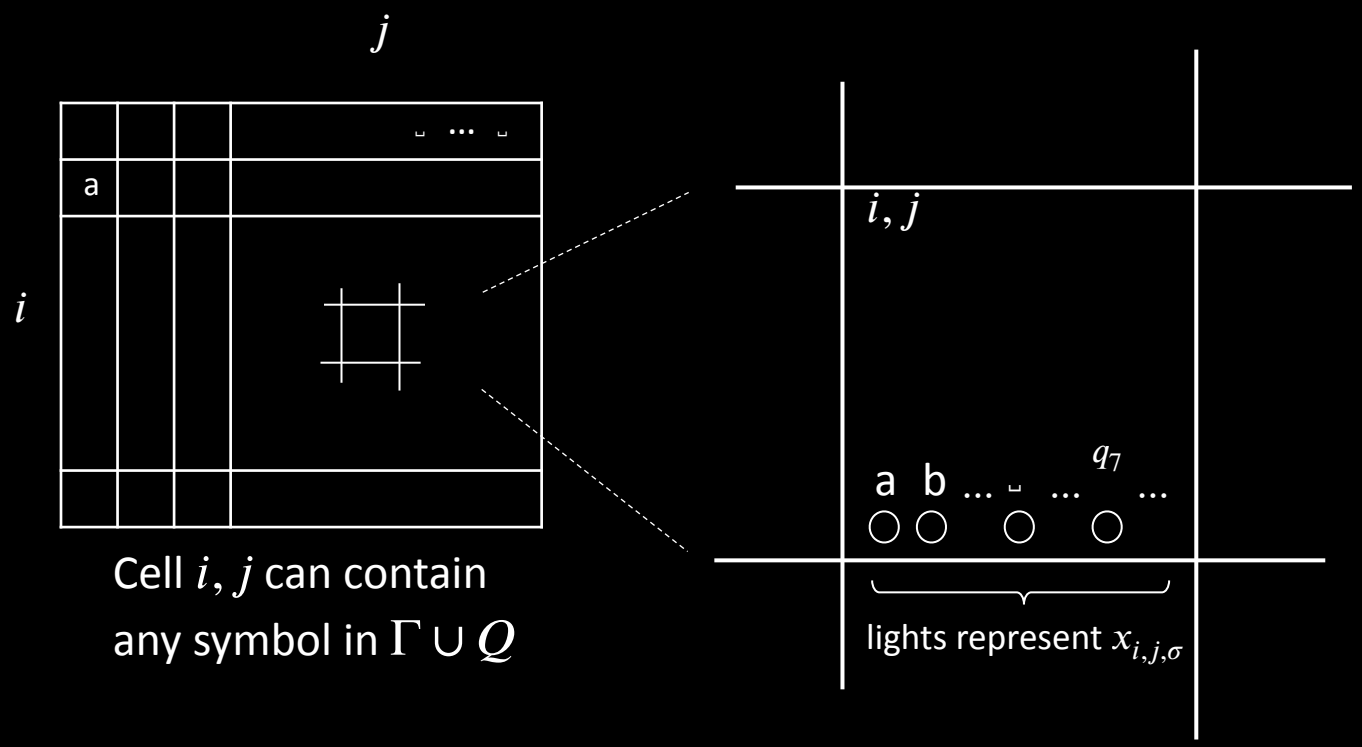

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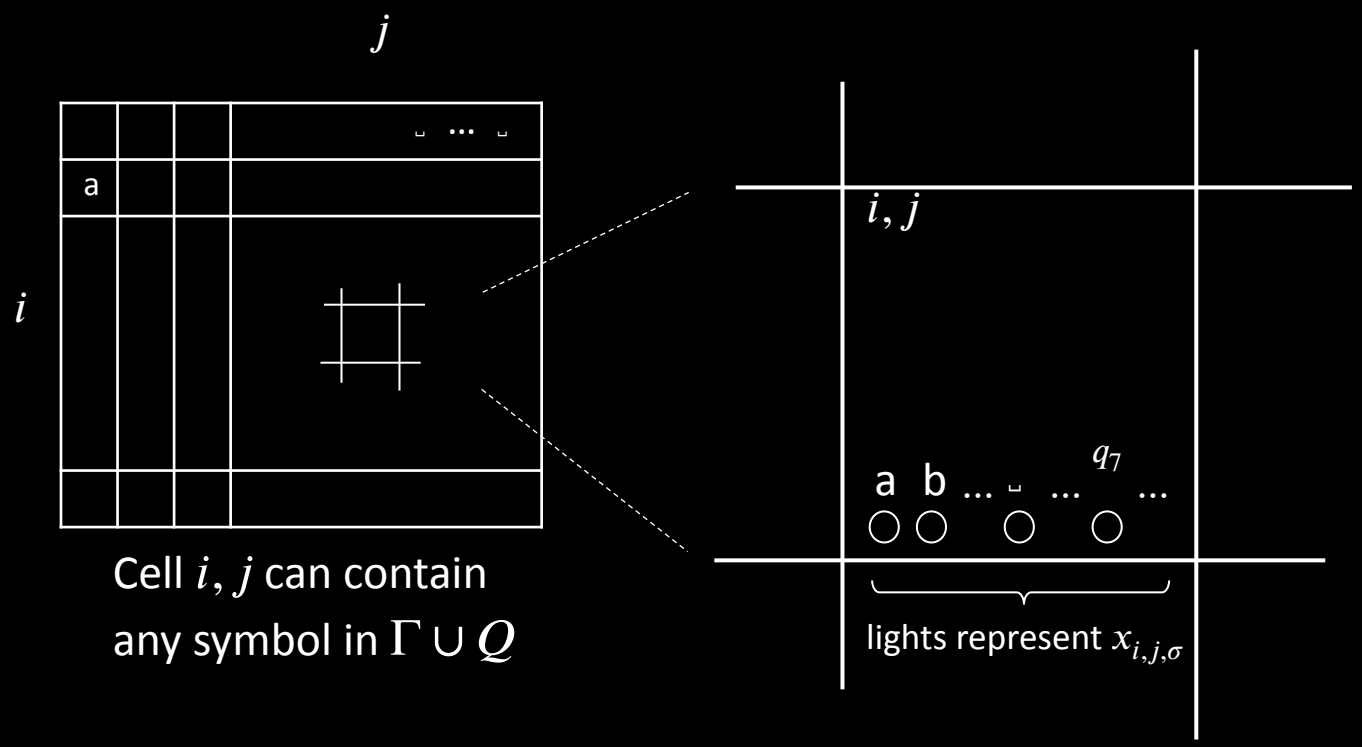

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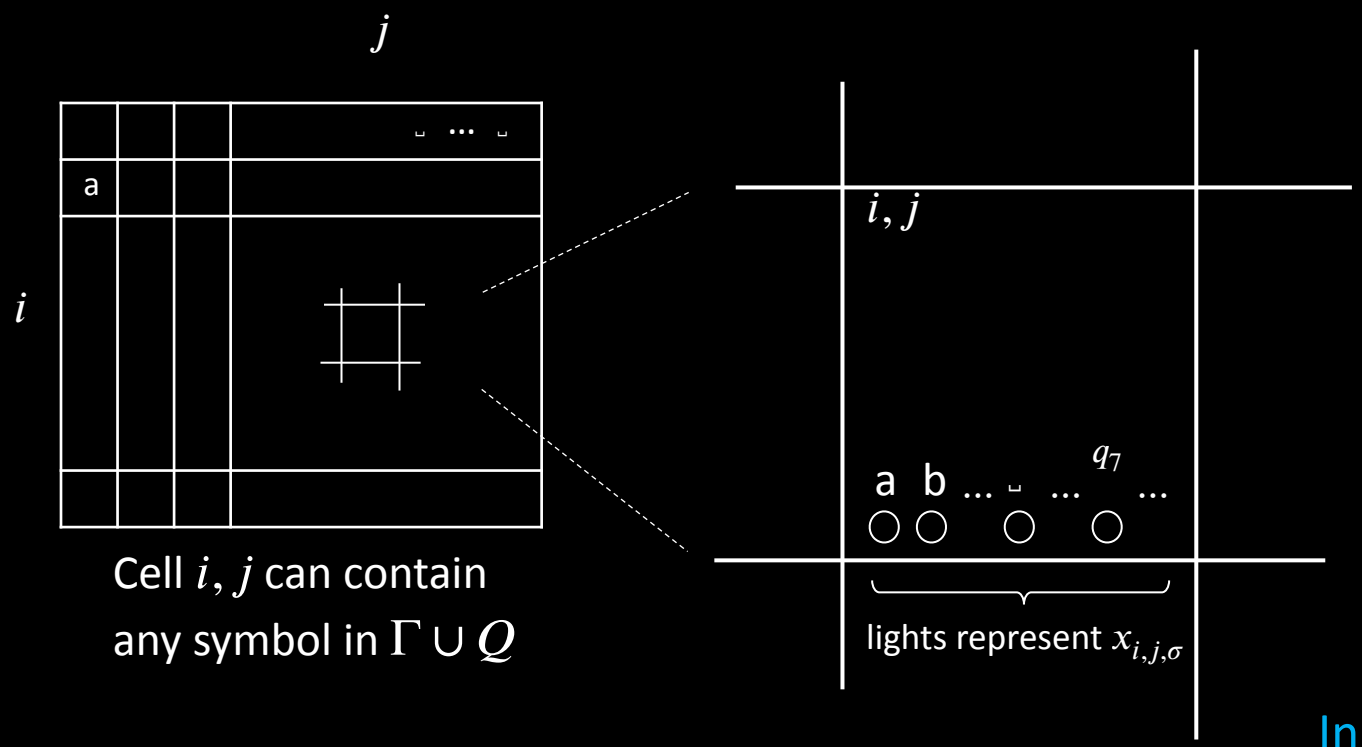

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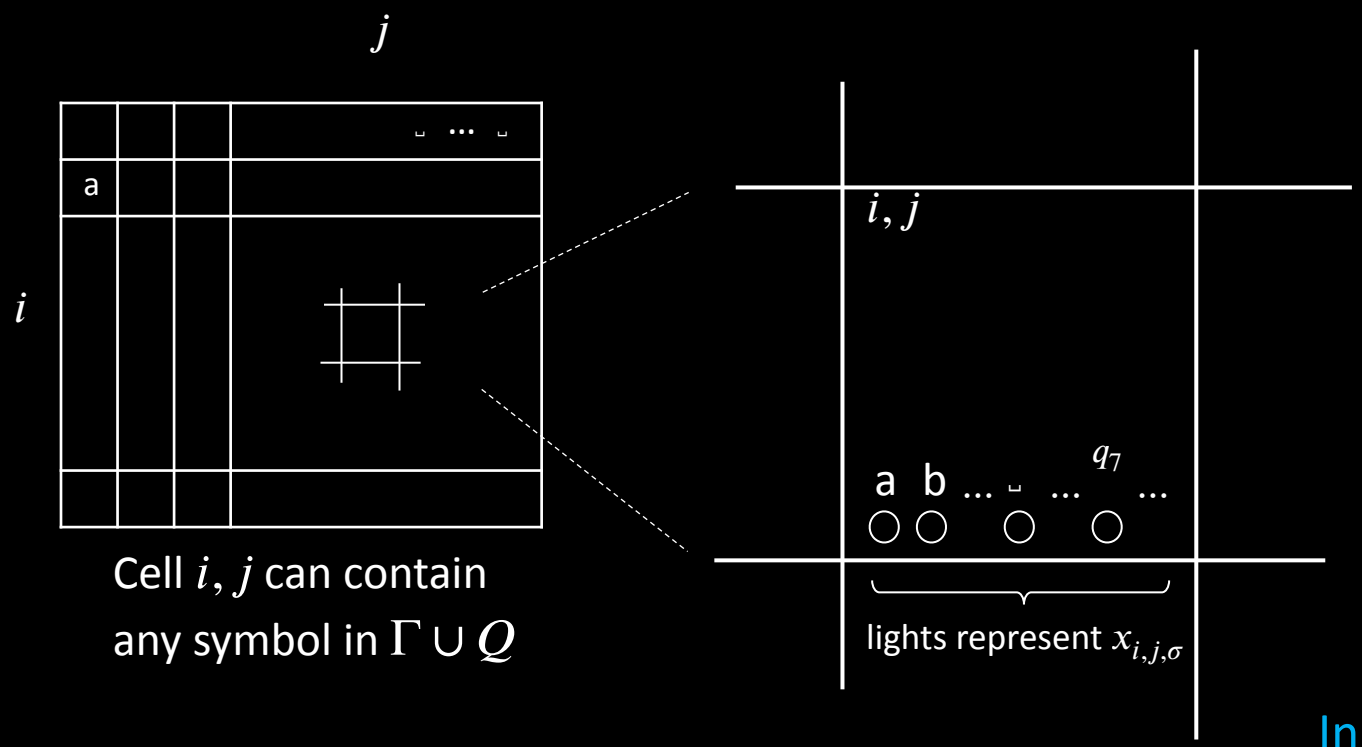

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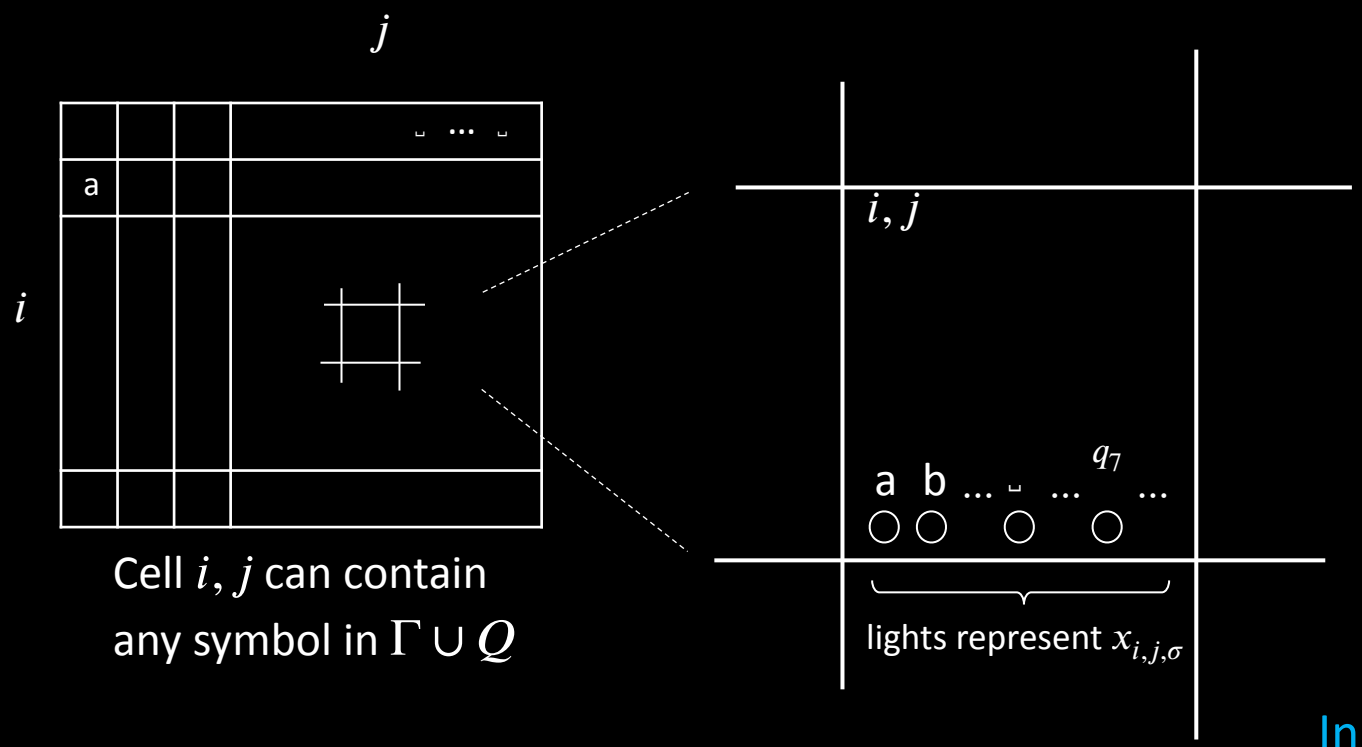

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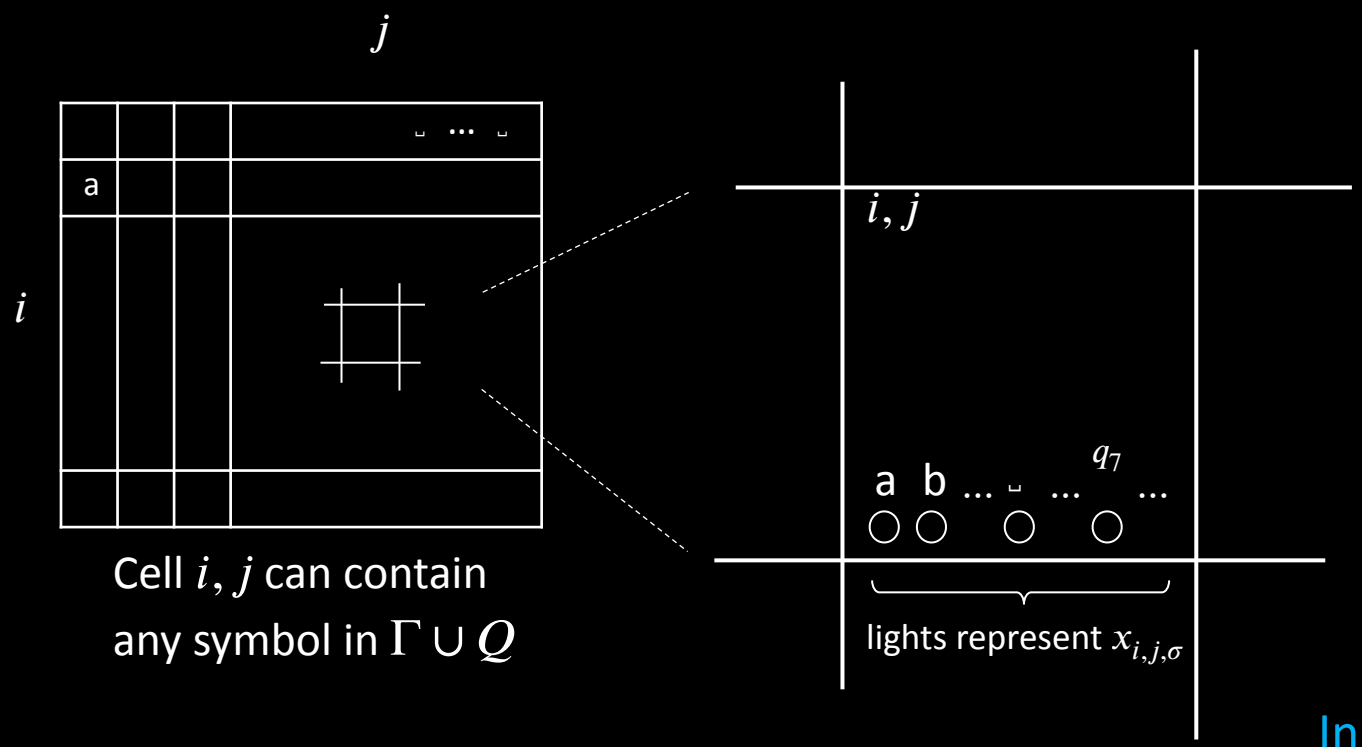

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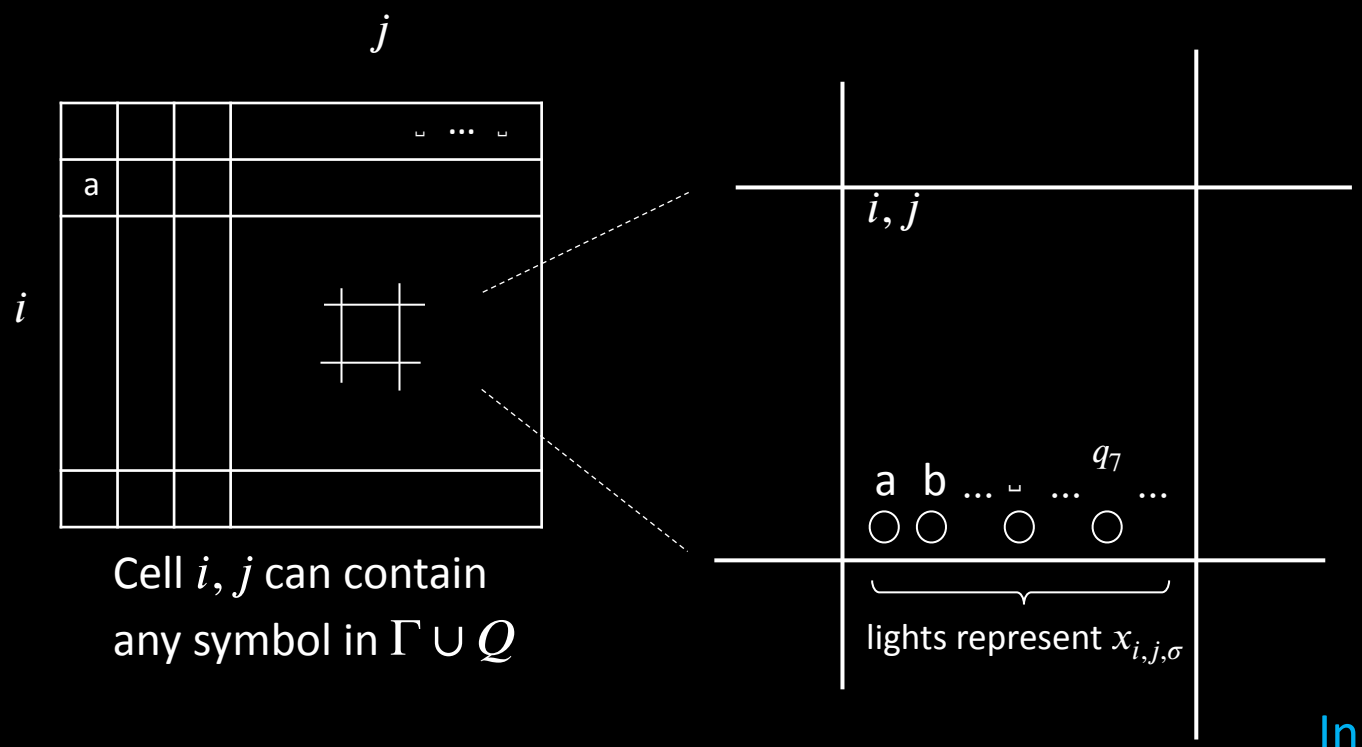

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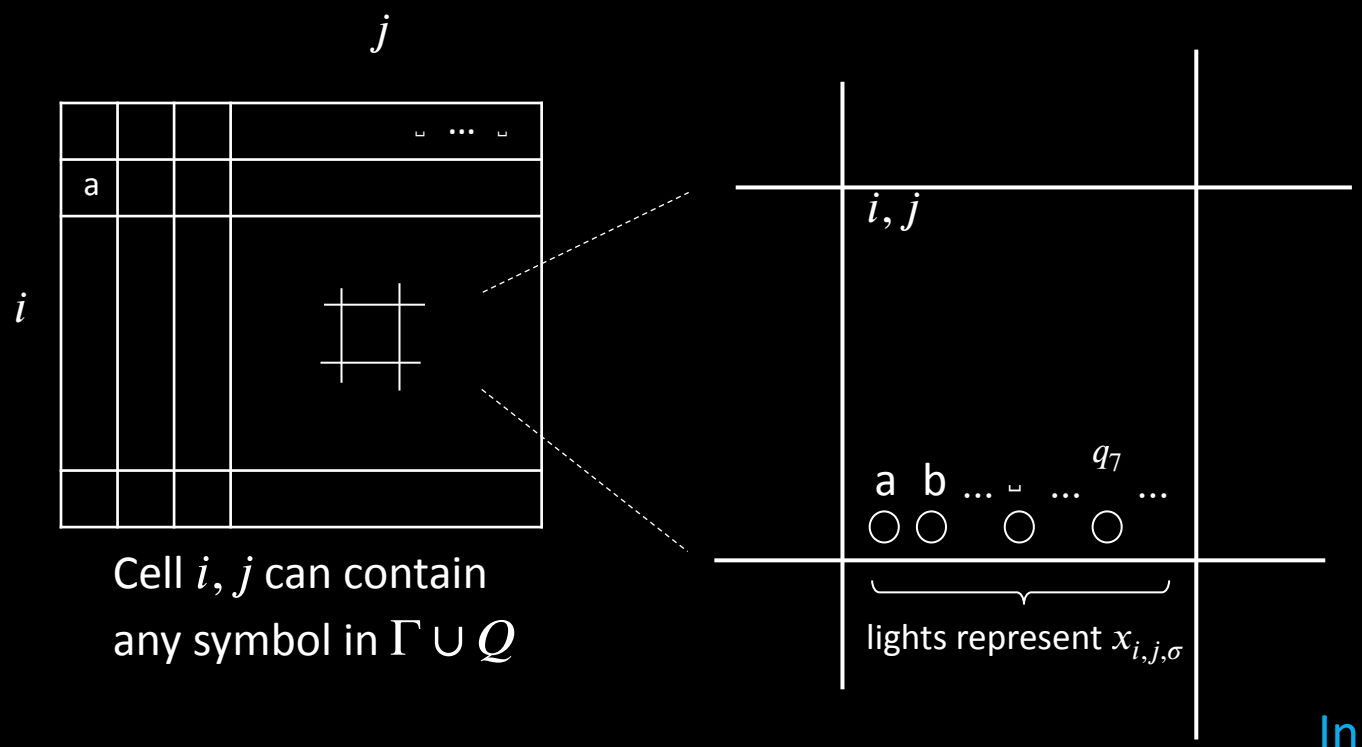

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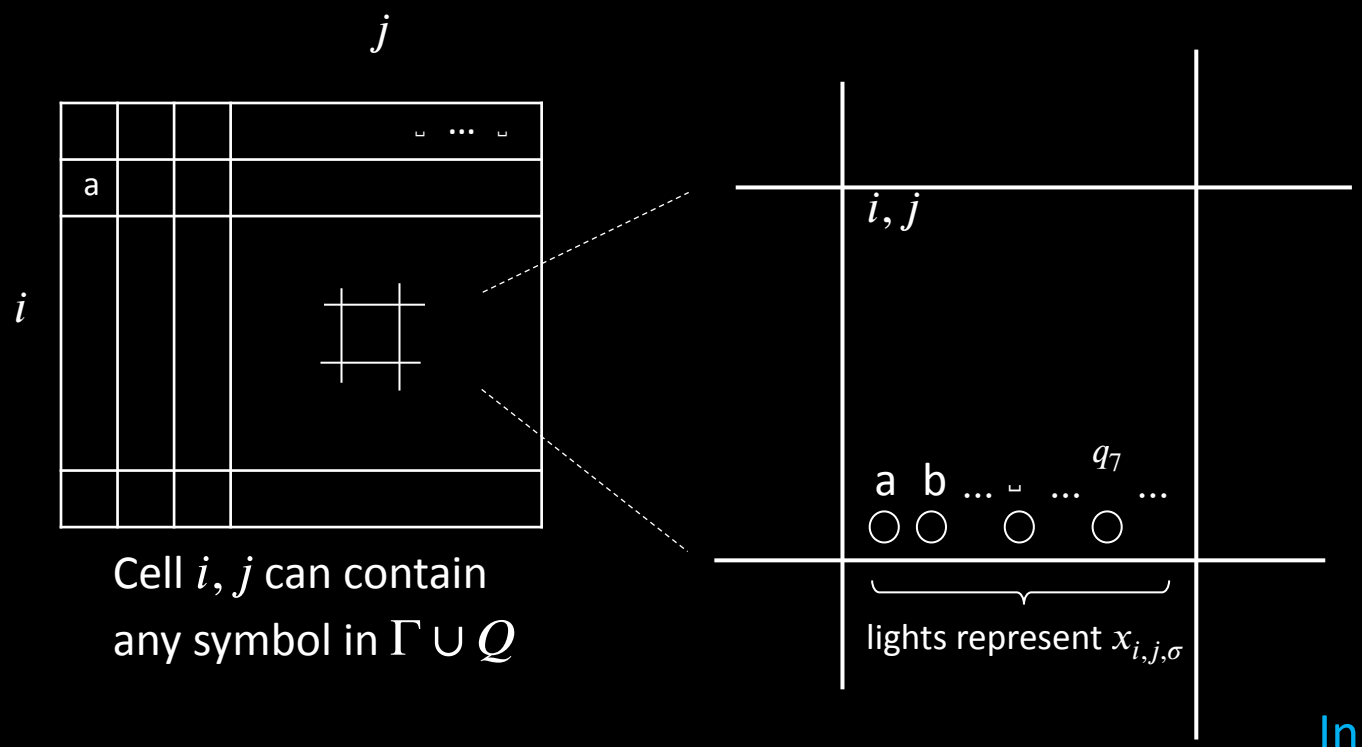

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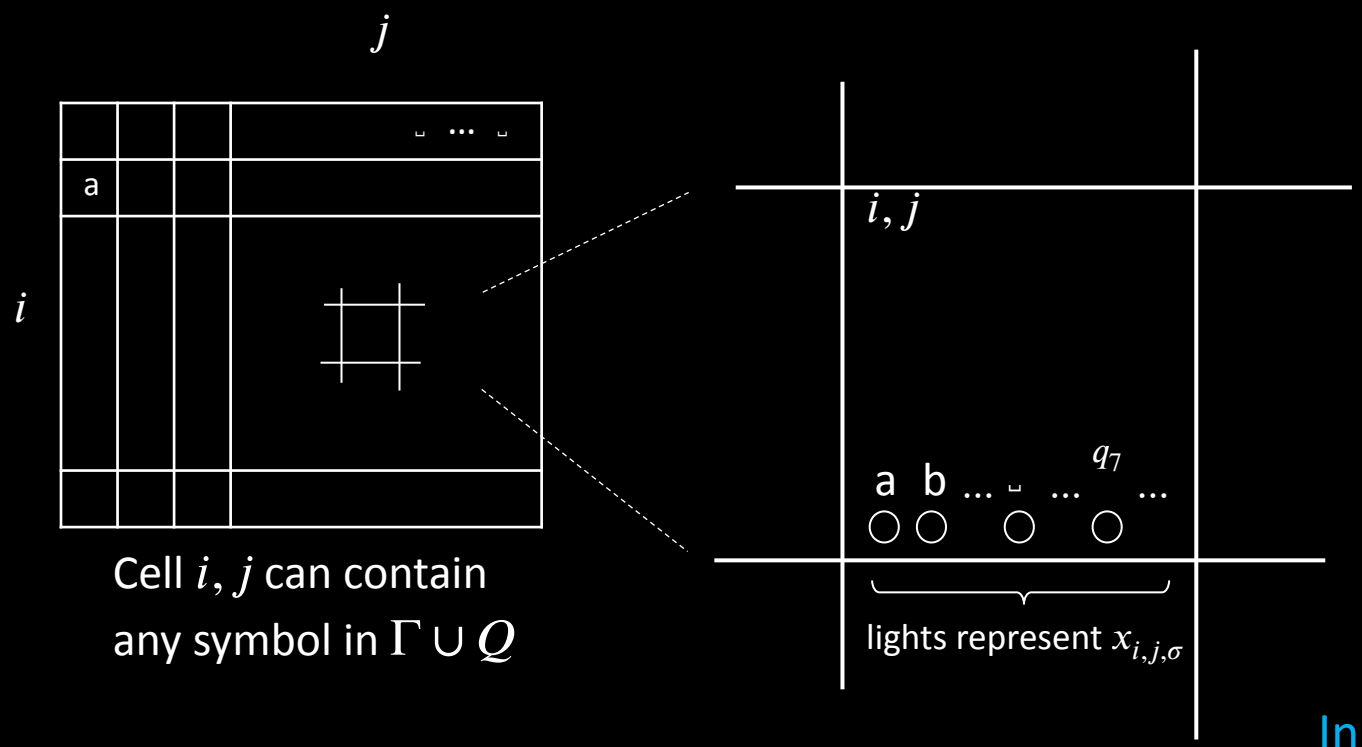

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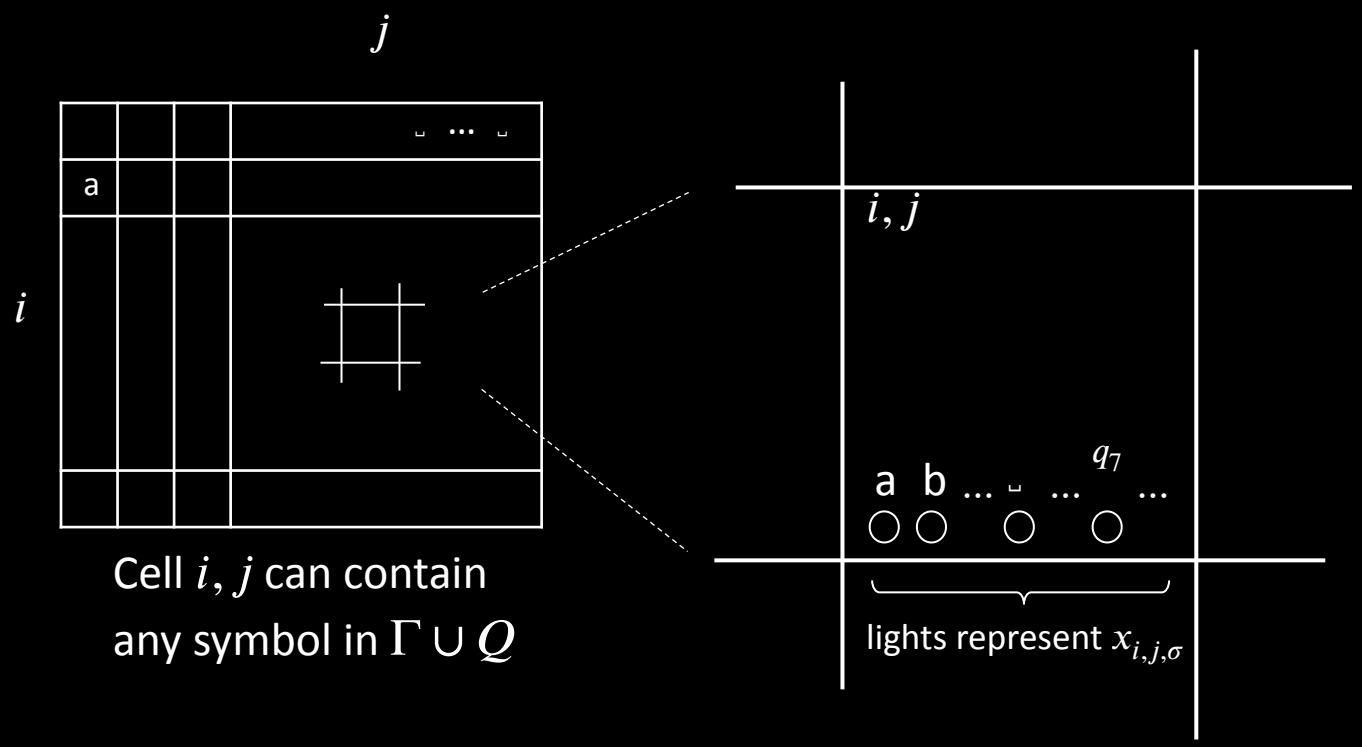
$$x_{i,j,\sigma_1} \vee x_{i,j,\sigma_2} \vee \cdots \vee x_{i,j,\sigma_t}$$

$$\phi\text{cell} = \bigwedge_{1 \le i,j \le n^k} \left(\bigvee_{\sigma \in \Gamma \cup Q} x_{i,j,\sigma} \wedge \bigwedge_{\substack{\sigma,\tau \in \Gamma \cup Q \\ \sigma \neq \tau}} \left(\overline{x_{i,j,\sigma} \wedge x_{i,j,\tau}} \right) \right)$$

Check-in 16.2

Constructing $\phi_{M,w}$: ϕcell



Cell i, j can contain any symbol in $\Gamma \cup Q$

The variables of $\phi_{M,w}$ are $x_{i,j,\sigma}$ for $1 \leq i, j \leq n^k$ and $\sigma \in \Gamma \cup Q$.

$x_{i,j,\sigma} =$ TRUE means cell i, j contains σ.

Check-in 16.2

How many variables does $\phi_{M,w}$ have?

Recall that $n = |w|$.

(a) $O(n)$

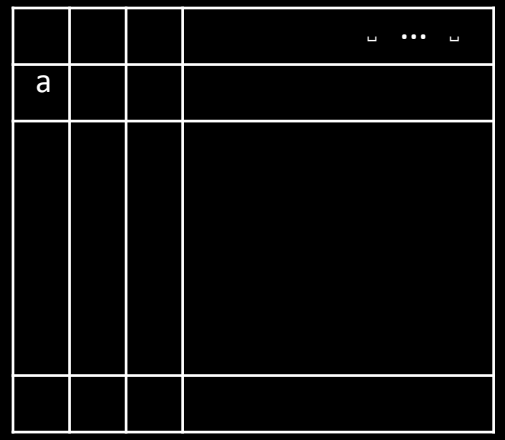
(b) $O(n^2)$

(c) $O(n^k)$

(d) $O(n^{2k})$

Check-in 16.2

Constructing $\phi_{M,w}$: ϕstart and ϕaccept



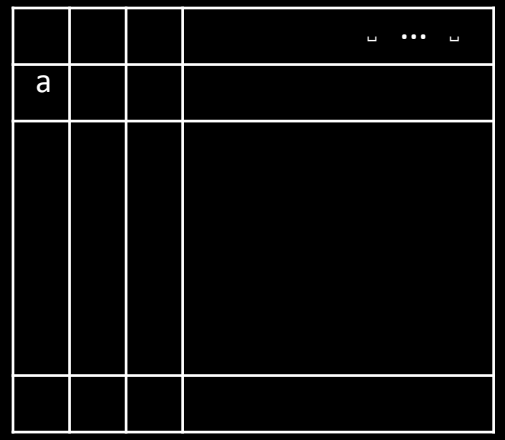
← Start configuration

← Accepting configuration

$\phi_{M,w}$ "says" a tableau for M on w exists.

$\phi_{M,w} = \phi\text{cell} \wedge \phi\text{start} \wedge \phi\text{move} \wedge \phi\text{accept}$

Constructing $\phi_{M,w}$: ϕstart and ϕaccept



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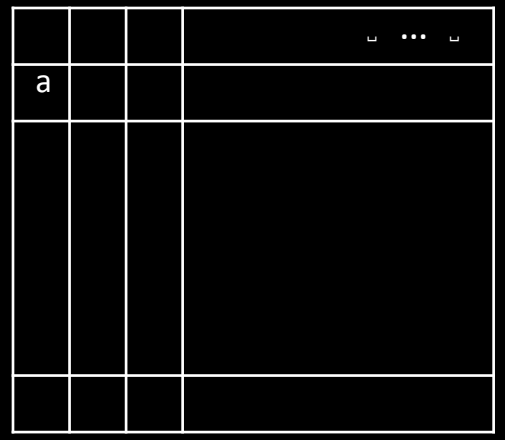
ϕcell done ✓

Constructing $\phi_{M,w}$: ϕstart and ϕaccept



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ϕcell done ✓

ϕstart

ϕaccept =

Constructing $\phi_{M,w}$: ϕstart and ϕaccept

$\phi_{M,w}$ "says" a tableau for M on w exists.

$\phi_{M,w} = \phi\text{cell} \wedge \phi\text{start} \wedge \phi\text{move} \wedge \phi\text{accept}$

ϕcell done ✓

$\phi\text{start} = x_{1,1,q_0}$

$\phi\text{accept} =$

Constructing $\phi_{M,w}$: ϕstart and ϕaccept

$\phi_{M,w}$ "says" a tableau for M on w exists.

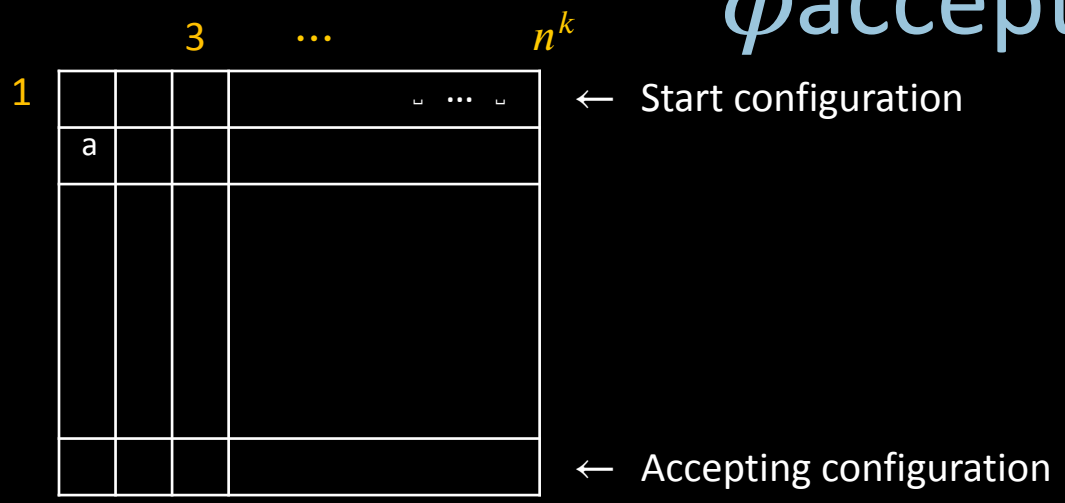
$\phi_{M,w} = \phi\text{cell} \wedge \phi\text{start} \wedge \phi\text{move} \wedge \phi\text{accept}$

ϕcell done ✓

$\phi\text{start} = x_{1,1,q_0} \wedge x_{1,2,w_1}$

$\phi\text{accept} =$

Constructing $\phi_{M,w}$: ϕstart and ϕaccept



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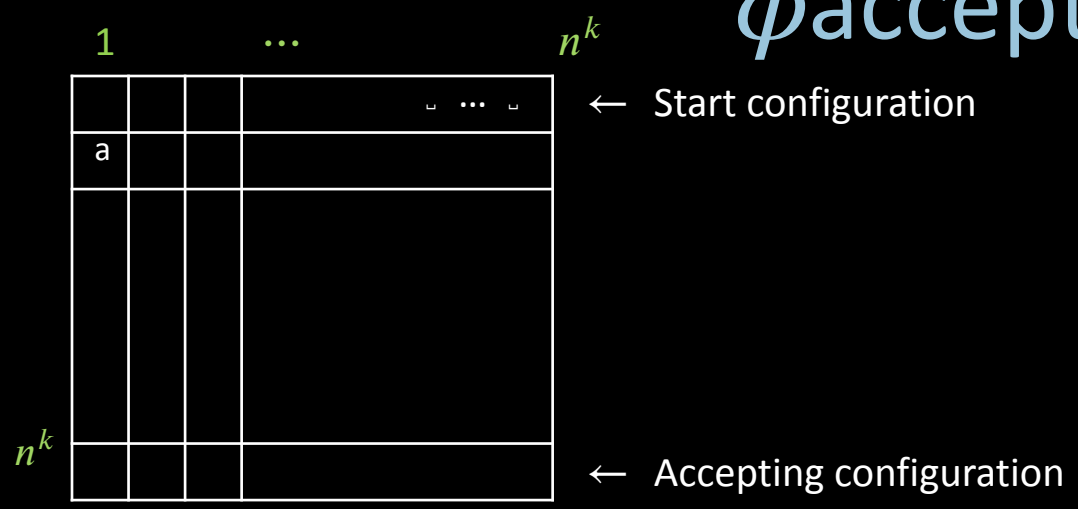
ϕcell done ✓

$\phi\text{start} = x_{1,1,q_0} \wedge x_{1,2,w_1} \wedge x_{1,3,w_2} \wedge \cdots \wedge x_{1,n^k,\sqcup}$

$\phi\text{accept} =$

Constructing $\phi_{M,w}$: ϕstart and ϕaccept



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$\phi_{M,w}$ "says" a tableau for M on w exists.

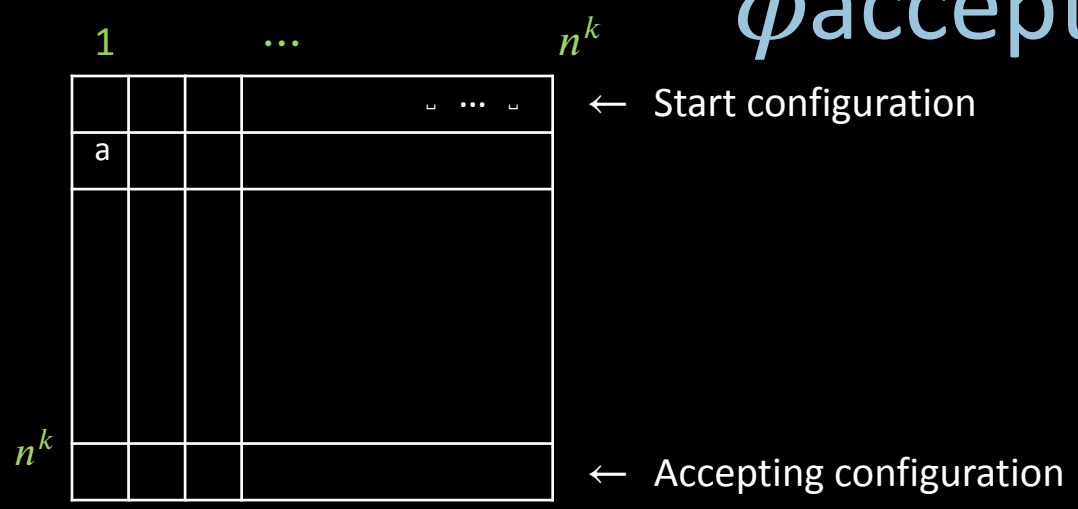
$\phi_{M,w} = \phi\text{cell} \wedge \phi\text{start} \wedge \phi\text{move} \wedge \phi\text{accept}$

ϕcell done ✓

$\phi\text{start} = x_{1,1,q_0} \wedge x_{1,2,w_1} \wedge x_{1,3,w_2} \wedge \cdots \wedge x_{1,n^k,␣}$

$\phi\text{accept} = \bigvee_{1 \le j \le n^k} x_{n^k,j,q\text{accept}}$

Constructing $\phi_{M,w}$: ϕstart and ϕaccept



← Start configuration

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$\phi_{M,w}$ "says" a tableau for M on w exists.

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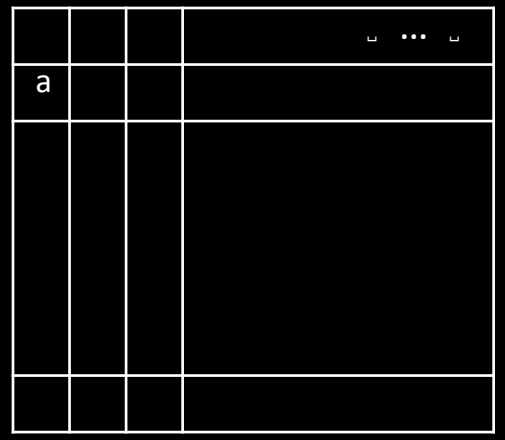
ϕcell done ✓

$\phi\text{start} = x_{1,1,q_0} \wedge x_{1,2,w_1} \wedge x_{1,3,w_2} \wedge \cdots \wedge x_{1,n^k,\sqcup}$

$\phi\text{accept} = \bigvee_{1 \le j \le n^k} x_{n^k,j,q\text{accept}}$

Constructing $\phi_{M,w}$: ϕmove



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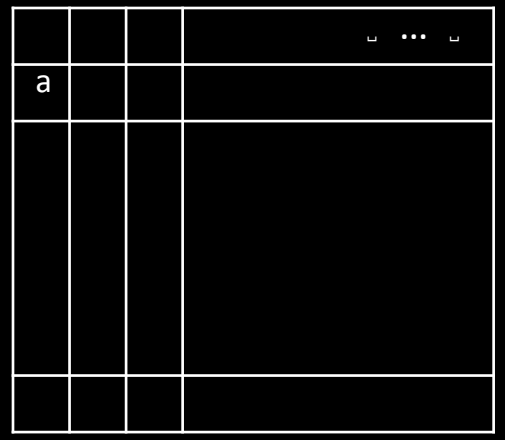

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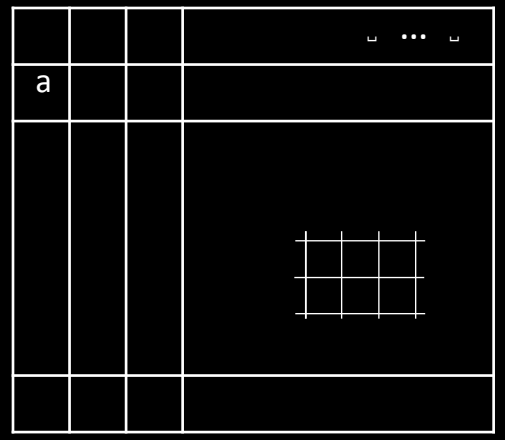
✓ ✓ ✓

Constructing $\phi_{M,w}$: ϕmove



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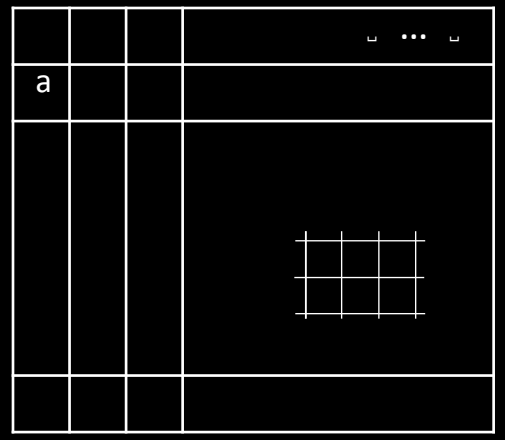
✓ ✓ ✓

Constructing $\phi_{M,w}$: ϕmove



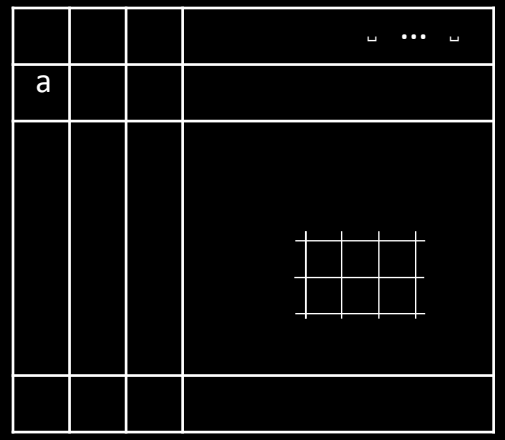
2×3 neighborhood

$\phi_{M,w}$ "says" a tableau for M on w exists.

$\phi_{M,w} = \phi\text{cell} \wedge \phi\text{start} \wedge \phi\text{move} \wedge \phi\text{accept}$

✓ ✓ ✓

Constructing $\phi_{M,w}$: ϕmove



2×3 neighborhood

Legal neighborhoods: consistent with M's transition function

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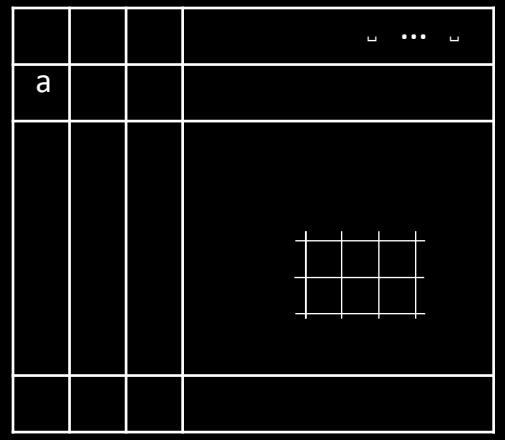
$\phi_{M,w} = \phi\text{cell} \wedge \phi\text{start} \wedge \phi\text{move} \wedge \phi\text{accept}$

✓ ✓ ✓

Constructing $\phi_{M,w}$: ϕmove



2×3 neighborhood

Legal neighborhoods: consistent with M's transition function

potential examples:

a		b
	a	c

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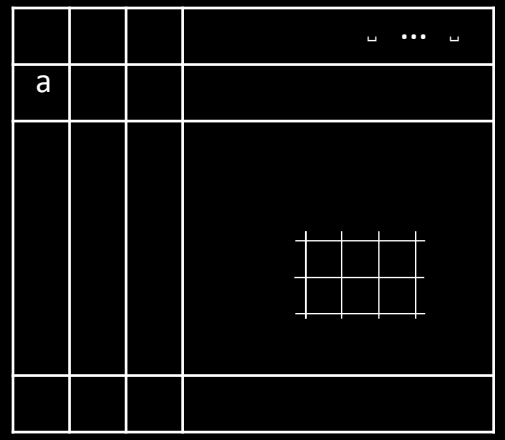
$\phi_{M,w} = \phi\text{cell} \wedge \phi\text{start} \wedge \phi\text{move} \wedge \phi\text{accept}$

✓ ✓ ✓

Constructing $\phi_{M,w}$: ϕmove



2×3 neighborhood

Legal neighborhoods: consistent with M's transition function

potential examples:

a		b
	a	c

a	b	c
a	b	c

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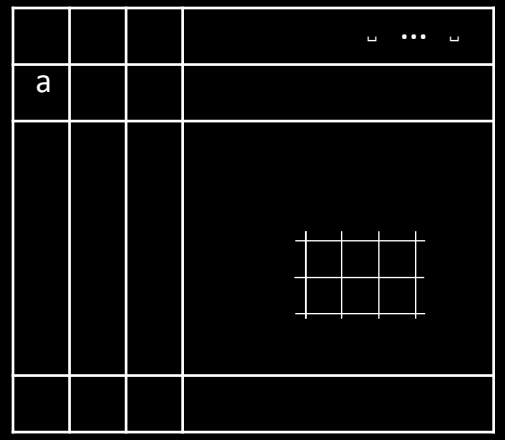
$\phi_{M,w} = \phi\text{cell} \wedge \phi\text{start} \wedge \phi\text{move} \wedge \phi\text{accept}$

✓ ✓ ✓

Constructing $\phi_{M,w}$: ϕmove



2×3 neighborhood

Legal neighborhoods: consistent with M's transition function

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a		b
	a	c

a	b	c
a	b	c

a	b	c
a	b	

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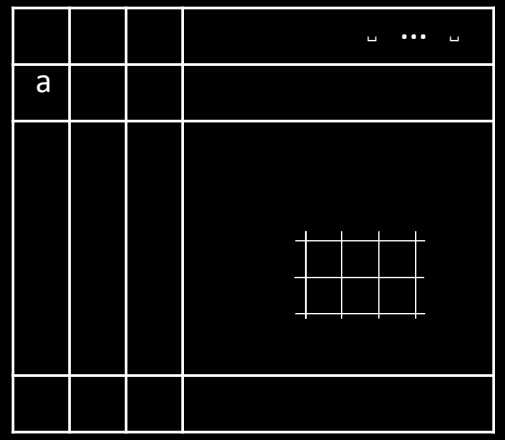
$\phi_{M,w} = \phi\text{cell} \wedge \phi\text{start} \wedge \phi\text{move} \wedge \phi\text{accept}$

✓ ✓ ✓

Constructing $\phi_{M,w}$: ϕmove



2×3 neighborhood

Legal neighborhoods: consistent with M's transition function

potential examples:

a		b
	a	c

a	b	c
a	b	c

a	b	c
a	b	

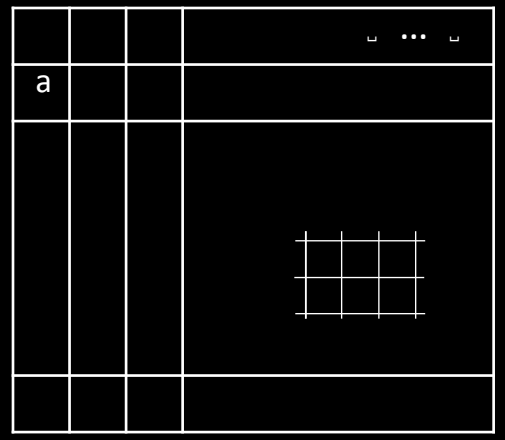
a	b	c
d	b	c

$\phi_{M,w}$ "says" a tableau for M on w exists.

$\phi_{M,w} = \phi\text{cell} \wedge \phi\text{start} \wedge \phi\text{move} \wedge \phi\text{accept}$

✓ ✓ ✓

Constructing $\phi_{M,w}$: ϕmove



2×3 neighborhood

Legal neighborhoods: consistent with M's transition function

potential examples:

a		b
	a	c

a	b	c
a	b	c

a	b	c
a	b	

a	b	c
d	b	c

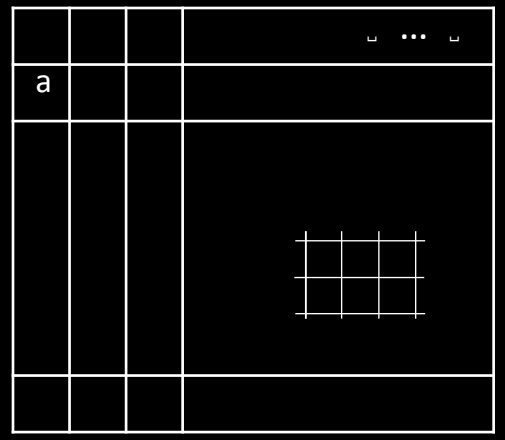
Illegal neighborhoods: not consistent with M's transition function

$\phi_{M,w}$ "says" a tableau for M on w exists.

$\phi_{M,w} = \phi\text{cell} \wedge \phi\text{start} \wedge \phi\text{move} \wedge \phi\text{accept}$

✓ ✓ ✓

Constructing $\phi_{M,w}$: ϕmove



2×3 neighborhood

Legal neighborhoods: consistent with M's transition function

potential examples:

a		b
	a	c

a	b	c
a	b	c

a	b	c
a	b	

a	b	c
d	b	c

Illegal neighborhoods: not consistent with M's transition function

examples:

a	b	c
a	d	c

$\phi_{M,w}$ "says" a tableau for M on w exists.

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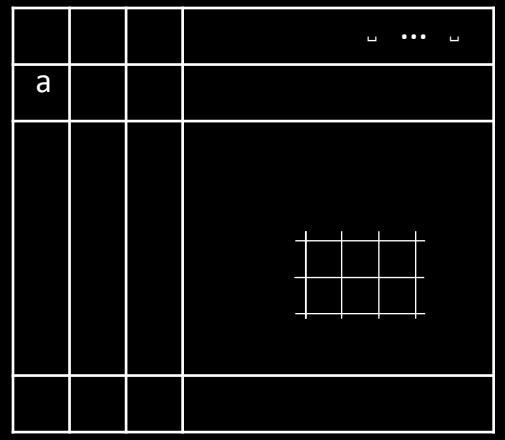
✓ ✓ ✓

Constructing $\phi_{M,w}$: ϕmove



2×3 neighborhood

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potential examples:

a		b
	a	c

a	b	c
a	b	c

a	b	c
a	b	

a	b	c
d	b	c

Illegal neighborhoods: not consistent with M's transition function

examples:

a	b	c
a	d	c

a	b	c
a		c

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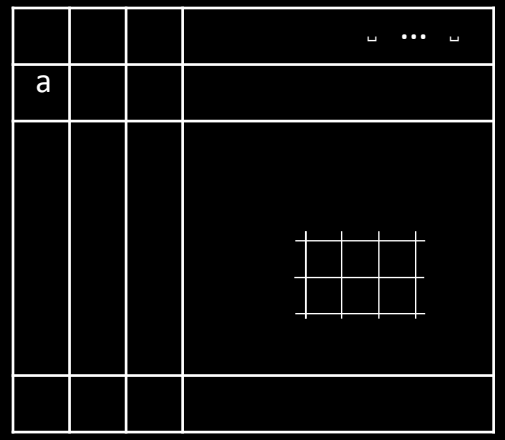
✓ ✓ ✓

Constructing $\phi_{M,w}$: ϕmove



2×3 neighborhood

Legal neighborhoods: consistent with M's transition function

potential examples:

a		b
	a	c

a	b	c
a	b	c

a	b	c
a	b	

a	b	c
d	b	c

Illegal neighborhoods: not consistent with M's transition function

examples:

a	b	c
a	d	c

a	b	c
a		c

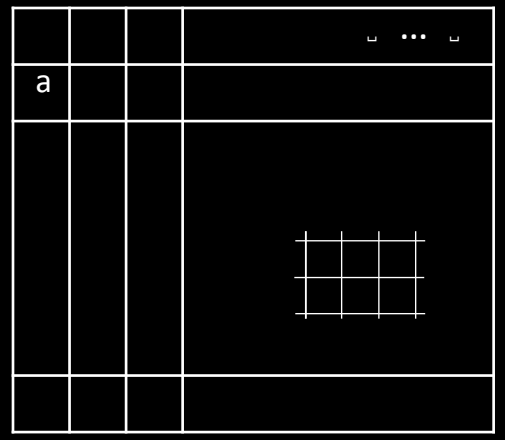
a		c
a	b	c

$\phi_{M,w}$ "says" a tableau for M on w exists.

$\phi_{M,w} = \phi\text{cell} \wedge \phi\text{start} \wedge \phi\text{move} \wedge \phi\text{accept}$

✓ ✓ ✓

Constructing $\phi_{M,w}$: ϕmove



2×3 neighborhood

Legal neighborhoods: consistent with M's transition function

potential examples:

a		b
	a	c

a	b	c
a	b	c

a	b	c
a	b	

a	b	c
d	b	c

Illegal neighborhoods: not consistent with M's transition function

examples:

a	b	c
a	d	c

a	b	c
a		c

a		c
a	b	c

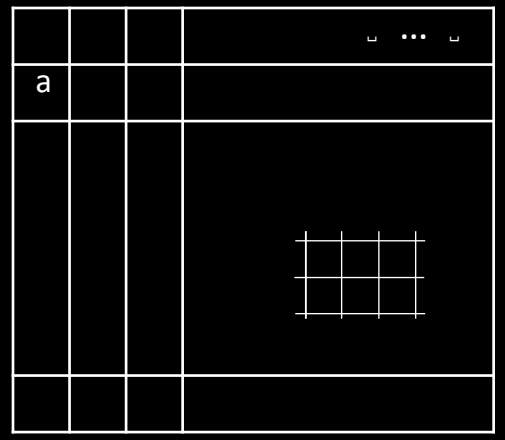
a		c
	d	

$\phi_{M,w}$ "says" a tableau for M on w exists.

$\phi_{M,w} = \phi\text{cell} \wedge \phi\text{start} \wedge \phi\text{move} \wedge \phi\text{accept}$

✓ ✓ ✓

Constructing $\phi_{M,w}$: ϕmove



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✓ ✓ ✓

Legal neighborhoods: consistent with M's transition function

potential examples:

a		b
	a	c

a	b	c
a	b	c

a	b	c
a	b	

a	b	c
d	b	c

Illegal neighborhoods: not consistent with M's transition function

examples:

a	b	c
a	d	c

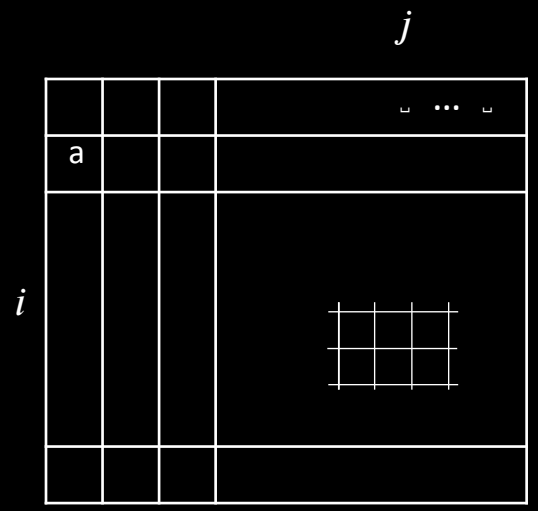
a	b	c
a		c

a		c
a	b	c

a		c
	d	

Claim: If every 2×3 neighborhood is legal then tableau corresponds to a computation history.

Constructing $\phi_{M,w}$: ϕmove



2×3 neighborhood

Legal neighborhoods: consistent with M's transition function

potential examples:

a		b
	a	c

a	b	c
a	b	c

a	b	c
a	b	

a	b	c
d	b	c

Illegal neighborhoods: not consistent with M's transition function

examples:

a	b	c
a	d	c

a	b	c
a		c

a		c
a	b	c

a		c
	d	

$\phi_{M,w}$ "says" a tableau for M on w exists.

Claim: If every 2×3 neighborhood is legal then tableau corresponds to a computation history.

$$\phi_{M,w} = \phi\text{cell} \wedge \phi\text{start} \wedge \phi\text{move} \wedge \phi\text{accept}$$

✓ ✓ ✓

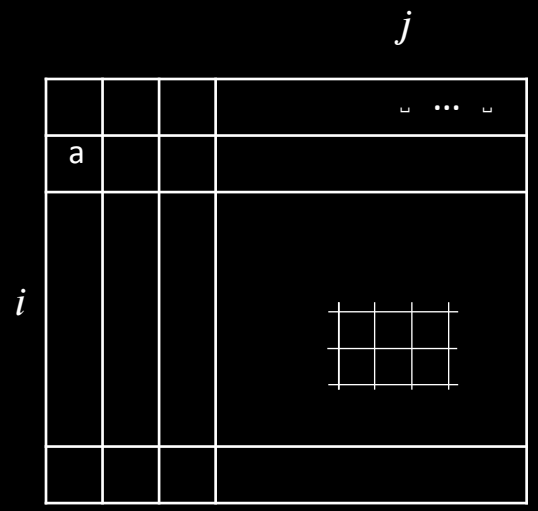
$$\bigvee \left(x_{i,j-1},\mathsf{r} \wedge x_{i,j},\mathsf{s} \wedge x_{i,j+1},\mathsf{t} \wedge x_{i+1,j-1},\mathsf{v} \wedge x_{i+1,j},\mathsf{y} \wedge x_{i+1,j+1},\mathsf{z} \right)$$

Legal

r	s	t
v	y	z

Says that the neighborhood at i, j is legal

Constructing $\phi_{M,w}$: ϕmove



2×3 neighborhood

Legal neighborhoods: consistent with M's transition function

potential examples:

a		b
	a	c

a	b	c
a	b	c

a	b	c
a	b	

a	b	c
d	b	c

Illegal neighborhoods: not consistent with M's transition function

examples:

a	b	c
a	d	c

a	b	c
a		c

a		c
a	b	c

a		c
	d	

$\phi_{M,w}$ "says" a tableau for M on w exists.

Claim: If every 2×3 neighborhood is legal then tableau corresponds to a computation history.

$$\phi_{M,w} = \phi\text{cell} \wedge \phi\text{start} \wedge \phi\text{move} \wedge \phi\text{accept}$$

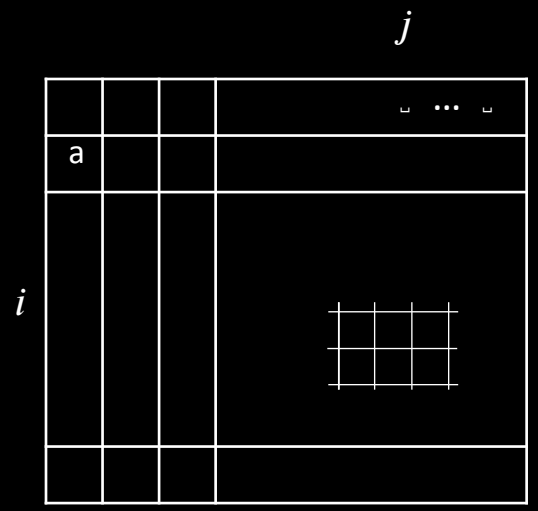
✓ ✓ ✓

$$\bigwedge_{1<i,j<n^k} \left(\bigvee_{\text{Legal}} \left(x_{i,j-1},\mathsf{r} \wedge x_{i,j},\mathsf{s} \wedge x_{i,j+1},\mathsf{t} \wedge x_{i+1,j-1},\mathsf{v} \wedge x_{i+1,j},\mathsf{y} \wedge x_{i+1,j+1},\mathsf{z} \right) \right)$$

r	s	t
v	y	z

Says that the neighborhood at i, j is legal

Constructing $\phi_{M,w}$: ϕmove



2×3 neighborhood

Legal neighborhoods: consistent with M's transition function

potential examples:

a		b
	a	c

a	b	c
a	b	c

a	b	c
a	b	

a	b	c
d	b	c

Illegal neighborhoods: not consistent with M's transition function

examples:

a	b	c
a	d	c

a	b	c
a		c

a		c
a	b	c

a		c
	d	

$\phi_{M,w}$ "says" a tableau for M on w exists.

$\phi_{M,w} = \phi\text{cell} \wedge \phi\text{start} \wedge \phi\text{move} \wedge \phi\text{accept}$

✓ ✓ ✓

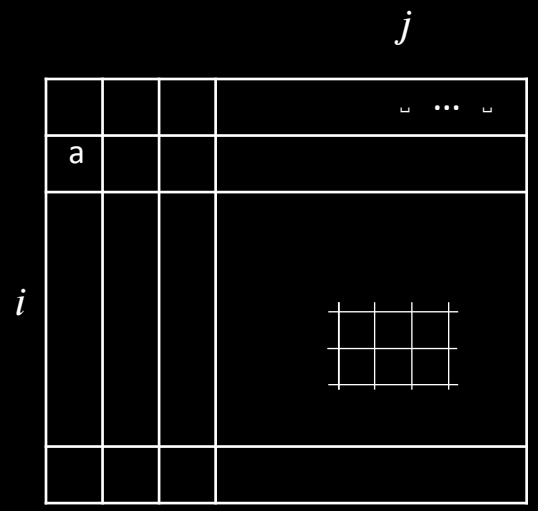
Claim: If every 2×3 neighborhood is legal then tableau corresponds to a computation history.

$$\phi\text{move} = \bigwedge_{1<i,j<n^k} \left(\bigvee_{\text{Legal}} \left(x_{i,j-1},\text{r} \wedge x_{i,j},\text{s} \wedge x_{i,j+1},\text{t} \wedge x_{i+1,j-1},\text{v} \wedge x_{i+1,j},\text{y} \wedge x_{i+1,j+1},\text{z} \right) \right)$$

r	s	t
v	y	z

Says that the neighborhood at i, j is legal

Constructing $\phi_{M,w}$: ϕmove



2×3 neighborhood

Legal neighborhoods: consistent with M's transition function

potential examples:

a		b
	a	c

a	b	c
a	b	c

a	b	c
a	b	

a	b	c
d	b	c

Illegal neighborhoods: not consistent with M's transition function

examples:

a	b	c
a	d	c

a	b	c
a		c

a		c
a	b	c

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	d	

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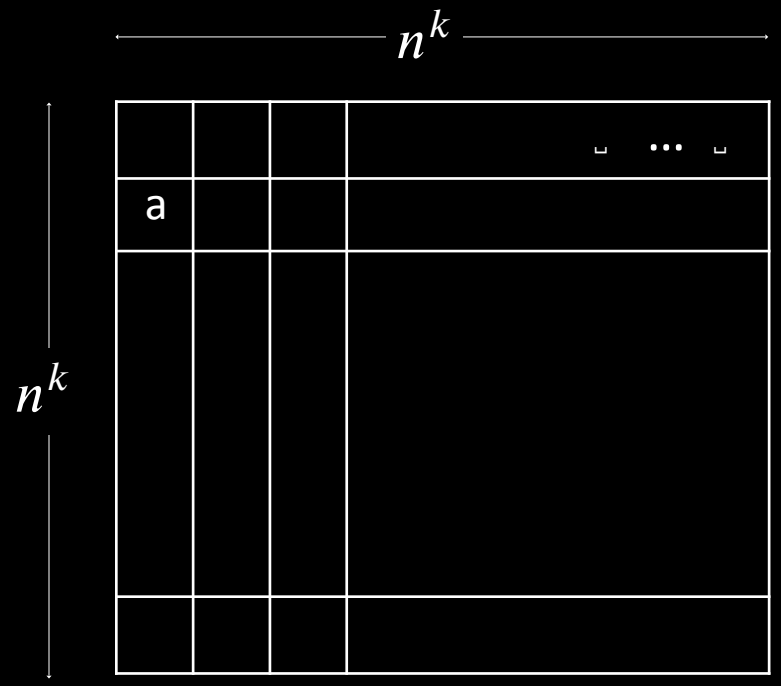

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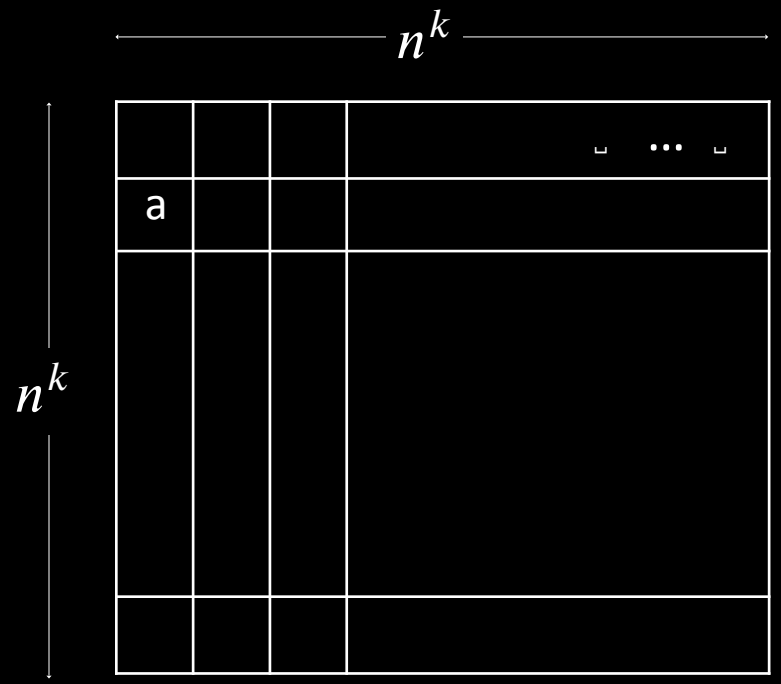

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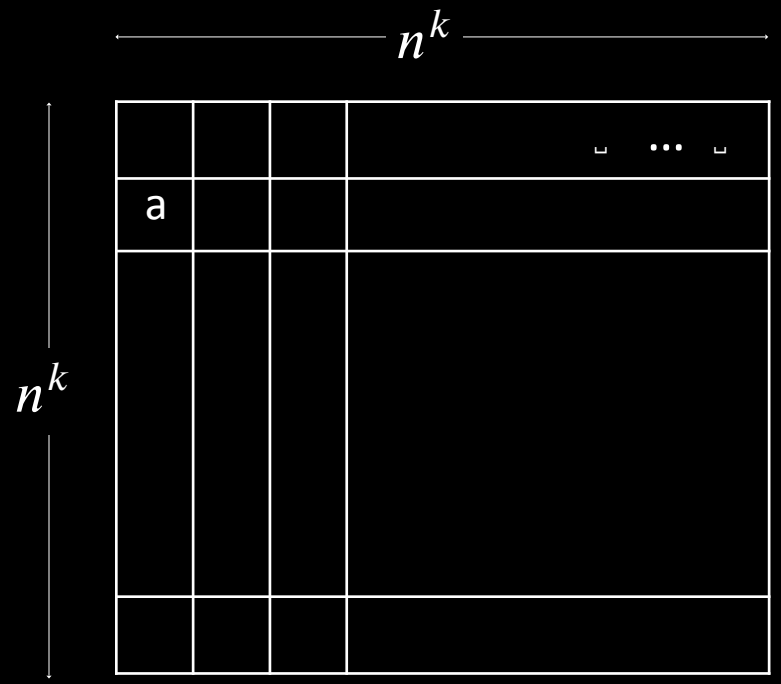

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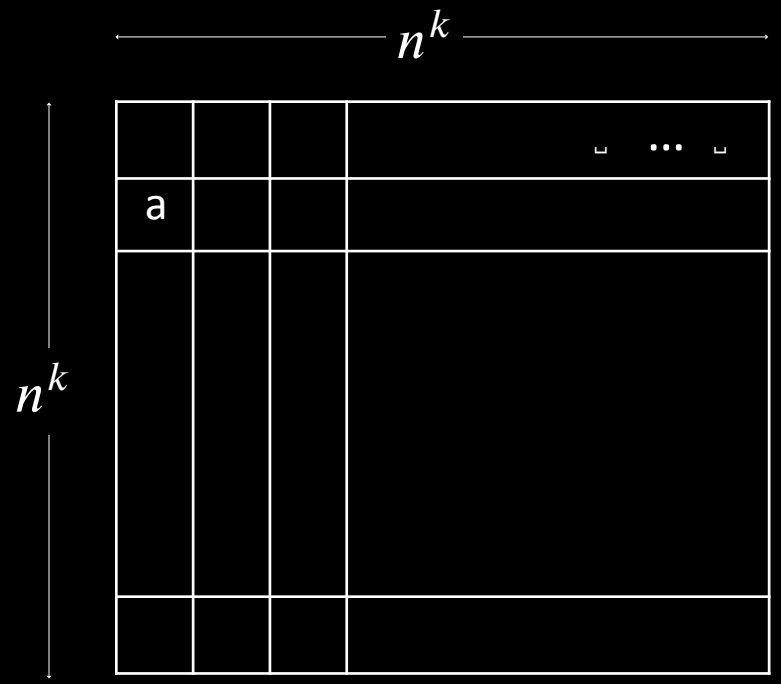

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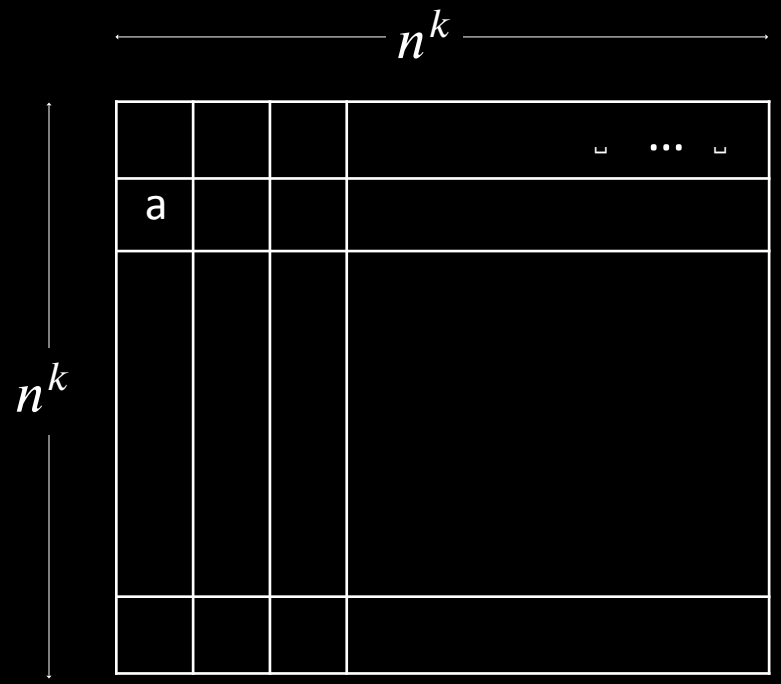

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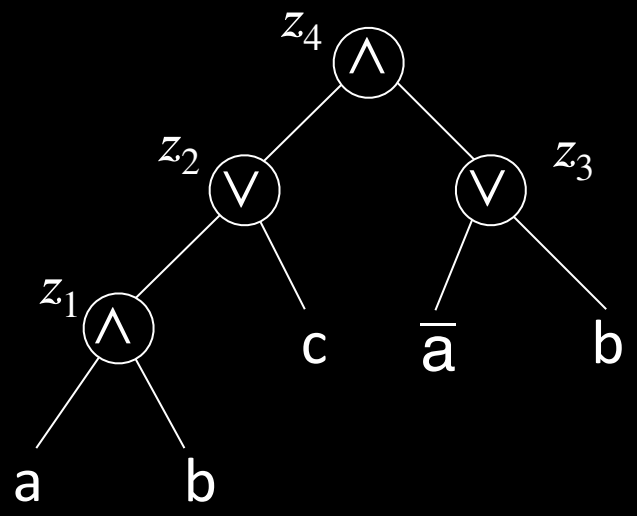

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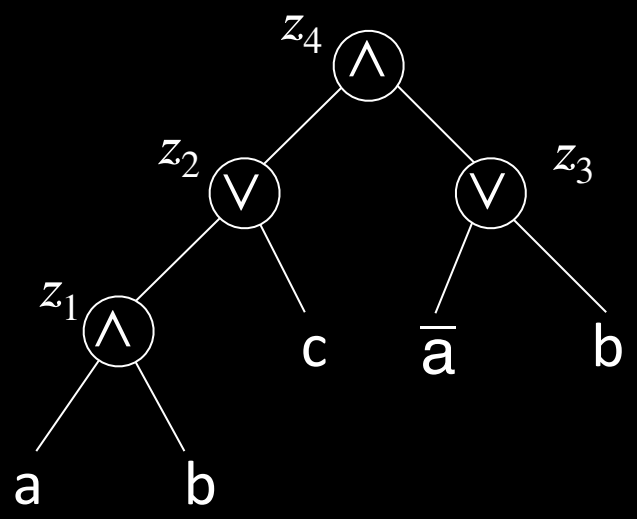

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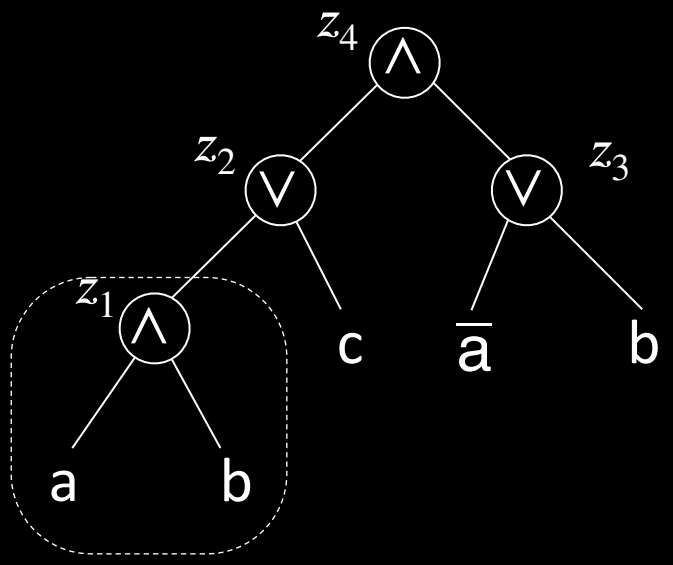

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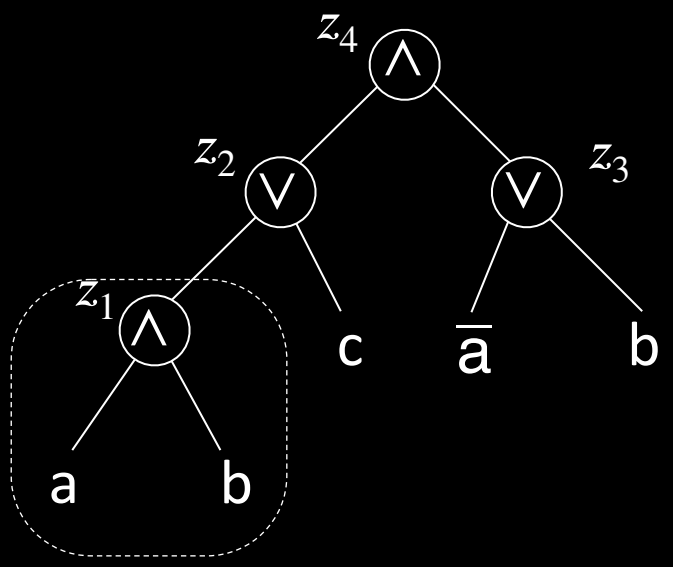

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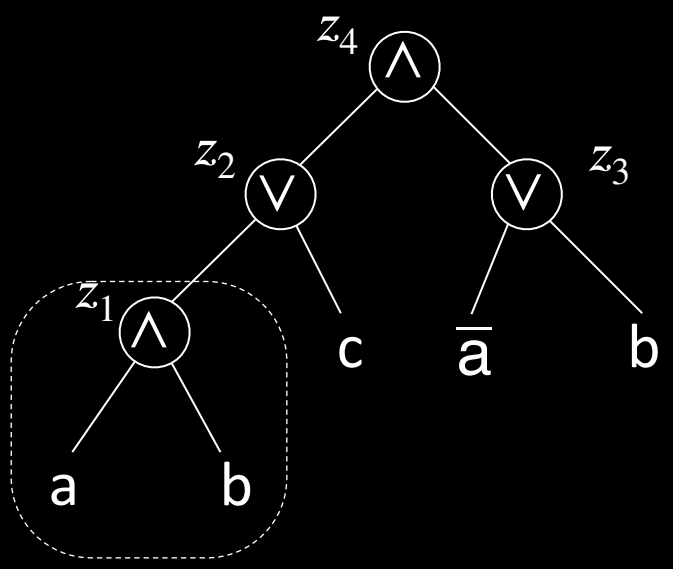

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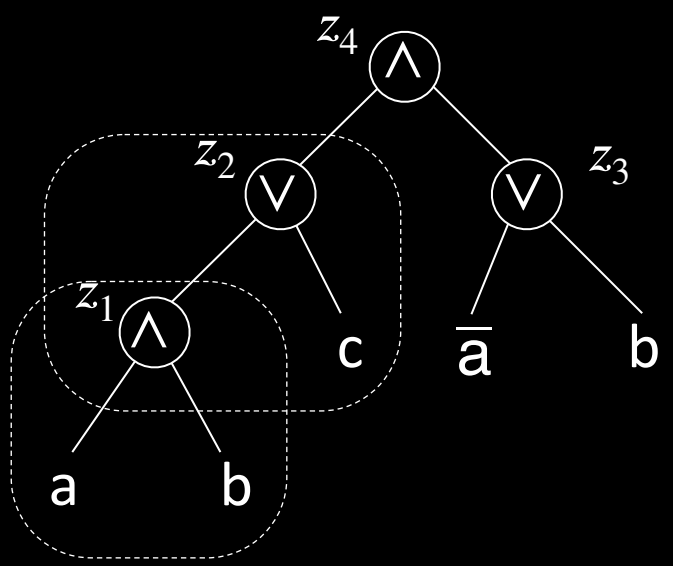

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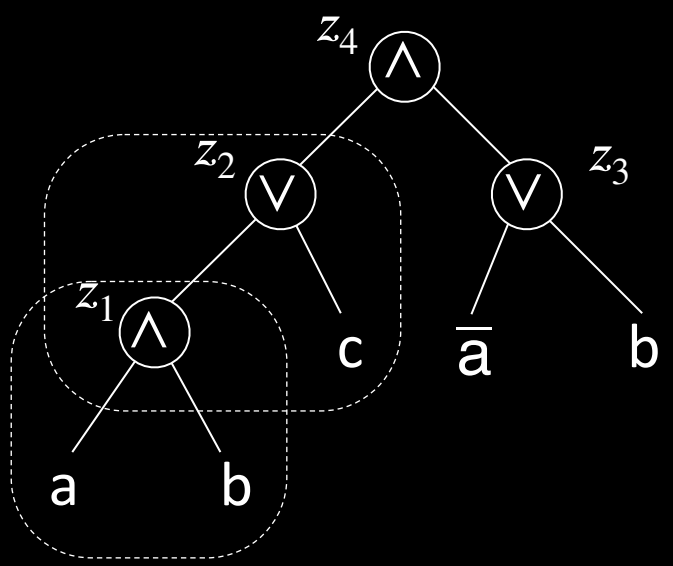

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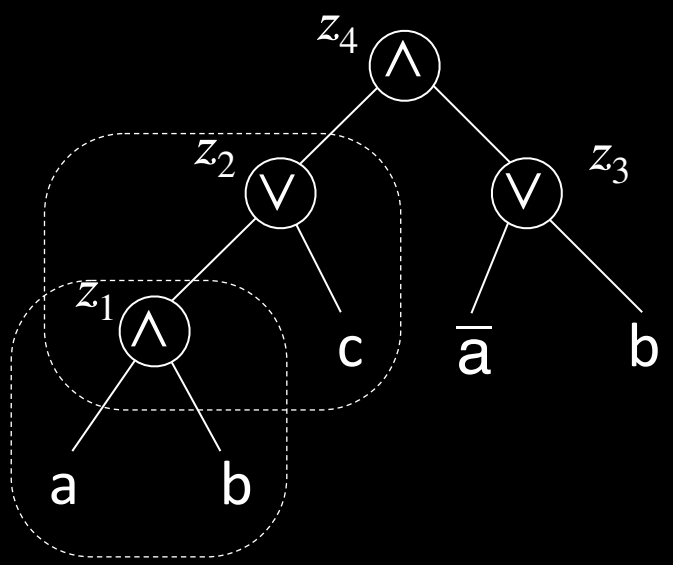

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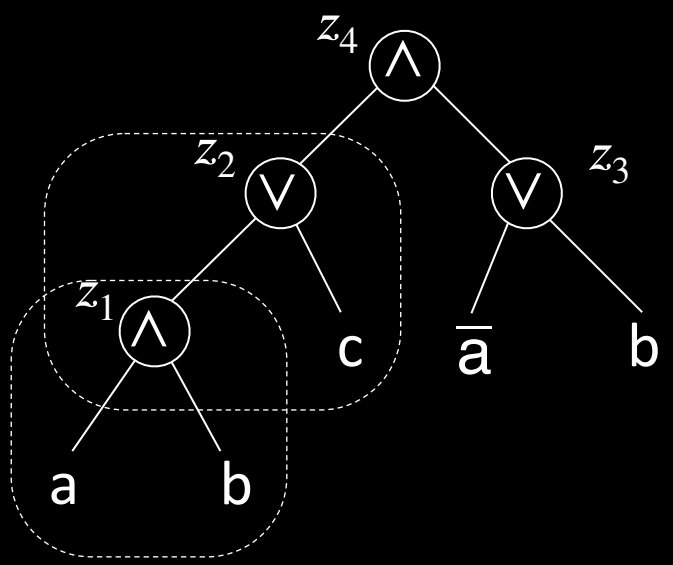

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⋮ repeat for each z_i

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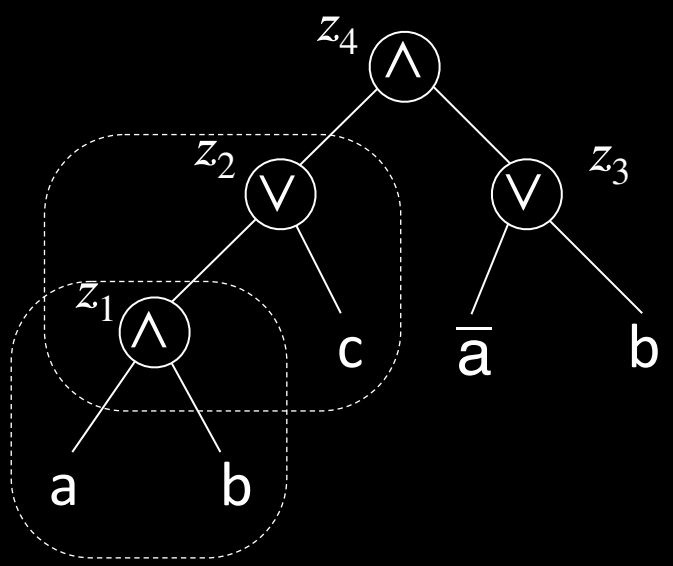

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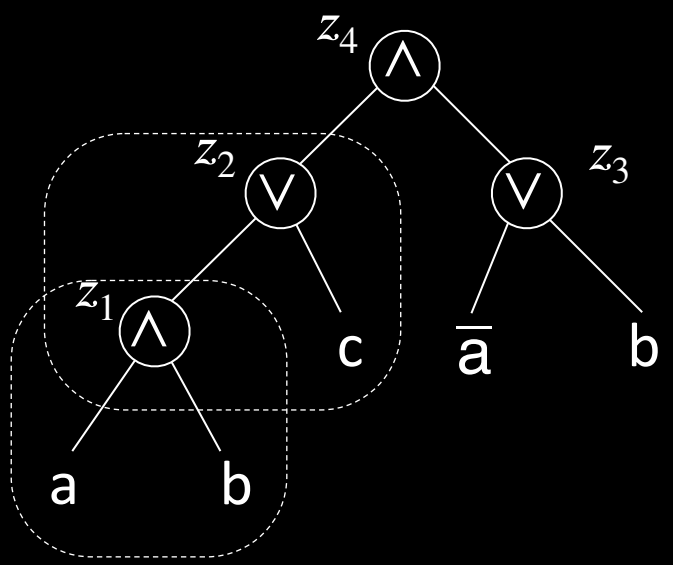

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Observe that $\big((a \wedge b) \rightarrow c\big)$ is logically equivalent to $\big(\bar{a} \vee \bar{b} \vee c\big)$

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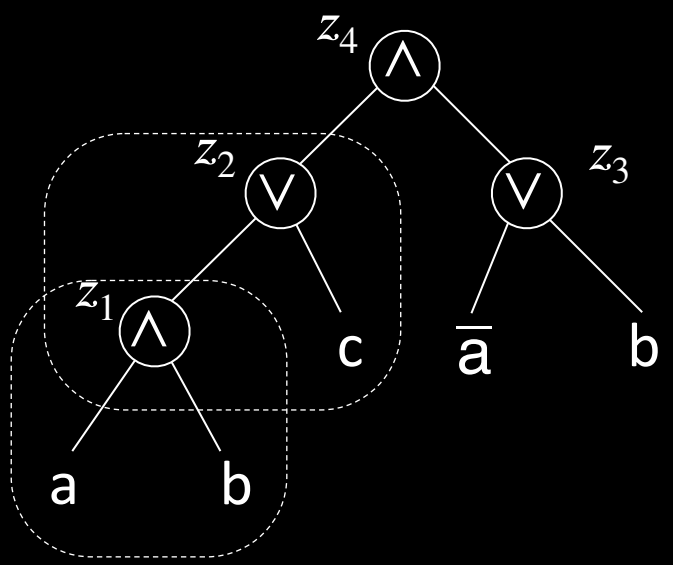
Give reduction f converting formula ϕ to 3CNF formula ϕ', preserving satisfiability.

(Note: ϕ and ϕ' are not logically equivalent)

Example: Say $\phi = \big((a \wedge b) \vee c\big) \wedge (\bar{a} \vee b)$

Tree structure for ϕ:



Logical equivalence: $(A \rightarrow B)$ and $(\bar{A} \vee B)$ $\quad \overline{(A \wedge B)}$ and $(\bar{A} \vee \bar{B})$

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$$\wedge \ (z_4)$$

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$3SAT$ is NP-complete

a	b	a ∨ b = c	
1	1	1	$(a \wedge b) \rightarrow c$
0	1	1	$(\bar{a} \wedge b) \rightarrow c$
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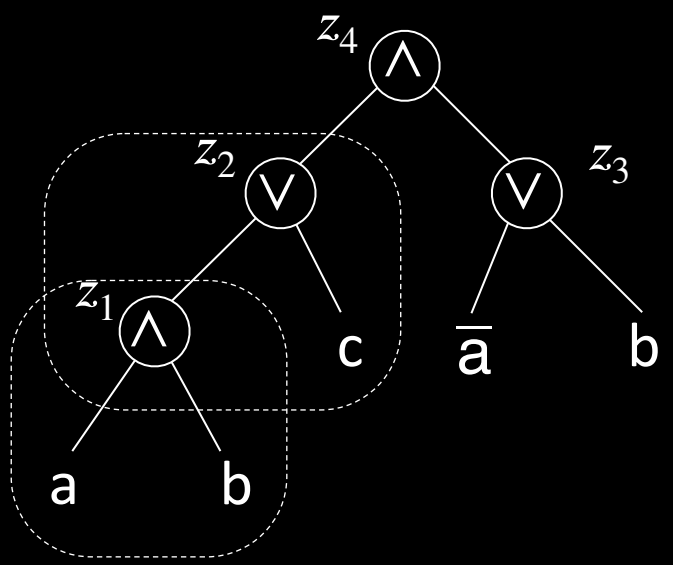

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Check-in 16.3

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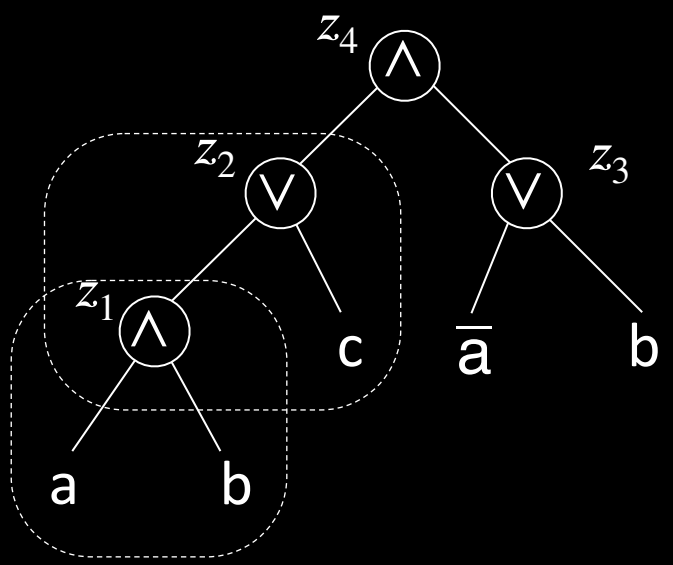

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Check-in 16.3

If ϕ has k operations ($\wedge$ and $\vee$), how many clauses has ϕ'?

(a) $k+1$

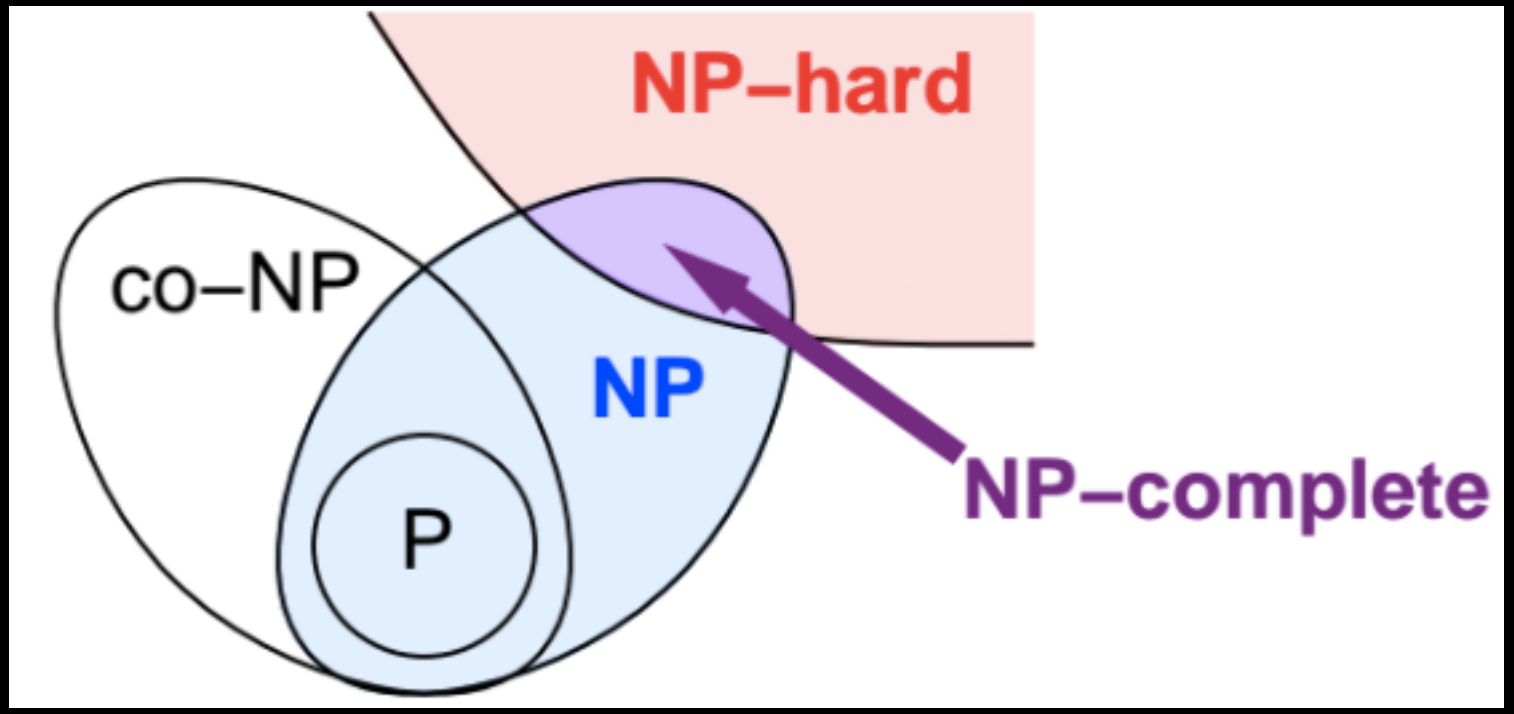
(c) k^2

(b) $4k+1$

(d) $2k^2$

coNP

- Dfn: coNP = { L | $\bar{L} \in$ NP }
- NOT-HAM



NP and certificates

- HAM is NP
- A P-time checkable certificate system:
  - Example HAM:
    - Certificate: permutation
    - Check: if each vertex <-> next vertex
- Theorem: A has P-time checkable certificate system iff A is NP
- Proof:
  - 1) P-time checkable certificate system -> NP
  - 2) NP -> P-time checkable certificate system

P vs NP

- Power of non-determinism
- Checking is easier than finding
  - معما چو حل گشت آسان شود
  - Proofing vs teaching
  - A good music
  - A good idea
  - A good business

What if P = NP

- Mathematicians could be replaced by efficient theorem-discovering programs
- Powerful AIs
- Inventors and engineers would be replaced
- Theoretiacl physics would be easy
- Randomness
- No priavcy, no SSL, no RSA, no PGP, no Bitcoin, …
- Innovation

Quick review of today

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2. NP-Completeness
3. SAT is NP-complete
4. $3SAT$ is NP-complete

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