بهینه‌سازی ترکیبیاتی (پاییز ۹۵ـ۹۴)

تمرین دوم

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1. Let $S \subseteq \{0,1\}^n$, and let $P = CH(S)$. Show that S is the set of vertices of P.

2. Let G be a digraph, $c : E(G) \to \mathbb{R}_+$, $E_1, E_2 \subseteq E(G)$, and $s, t \in V(G)$. Consider the following linear program:

$$\begin{array}{lll} \min & \sum_{e \in E(G)} c(e) y_e & \\ s.t. & y_e \geq z_w - z_v & (e = (v,w) \in E(G)) \\ & z_t - z_s = 1 & \\ & y_e \geq 0 & (e \in E_1) \\ & y_e \leq 0 & (e \in E_2) \end{array}$$

   Prove that there is an optimum solution (y, z) and $(s \in X \subseteq V(G) \backslash \{t\}$ with $y_e = 1$ for $e \in \delta^+(X)$, $y_e = -1$ for $e \in \delta^-(X) \backslash E_1$, and $y_e = 0$ for all other edges e.

   Hint: Consider the complementary slackness conditions for the edges entering or leaving $\{v \in V(G) : z_v \leq q z_s\}$.

3. Let P be a nonempty polytope. Consider the graph $G(P)$ whose vertices are the vertices of P and whose edges correspond to the 1-dimensional faces of P. Let x be any vertex of P, and c a vector with $c^T x < \max\{c^T z : z \in P\}$. Prove that then there is a neighbour y of x in $G(P)$ with $c^T x < c^T y$.

   Note that, a face F of a polytope P is a set of its points for which there is a vector c such that $F = \{x \in P : c^T x = \max\{c^T y : y \in P\}\}$, for some vector c.

   Note that, You can use this exercise to prove that $G(P)$ is n-connected for any n-dimensional polytope P ($n \geq 1$).

Extra Using a polynomial-time algorithm for linear programming as subroutine, give a polynomial-time algorithm to solve the following problem. Given two systems of linear inequalities $Ax \leq b$, $Cx \leq d$ in n variables, decide whether $\{x \in \mathbb{R}^n : Ax \leq b\} = \{x \in \mathbb{R}^n : Cx \leq d\}$.

موفق باشید