بسم الله الرحمن الرحیم

نظریه علوم کامپیوتر

نظریه علوم کامپیوتر - بهار ۱۴۰۰-۱۴۰۱ - جلسه نوزدهم: سیستم اثبات تعاملی (۲)

Theory of computation - 002 - S19 - IP (2)

Legend

Last time:

- Interactive Proof Systems
- The class IP
- Graph isomorphism problem, $\overline{ISO} \in$ IP
- $\#SAT \in$ IP (part 1)

Today: (Sipser §10.4)

- Arithmetization of Boolean formulas
- Finish $\#SAT \in$ IP and conclude that coNP $\subseteq$ IP

Review: Interactive Proofs

Two interacting parties
Verifier (V): Probabilistic polynomial time TM
Prover (P): Unlimited computational power

Both P and V see input w.
They exchange a polynomial number of polynomial-size messages.
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Let $\phi = (x_1 \vee x_2) \wedge (\overline{x_1} \vee \overline{x_2})$

Check all that are true:

a) $\#\phi = 1$ b) $\#\phi = 2$

c) $\#\phi(0) = 1$ d) $\#\phi(0) = 2$

e) $\#\phi(00) = 0$ f) $\#\phi(00) = 1$

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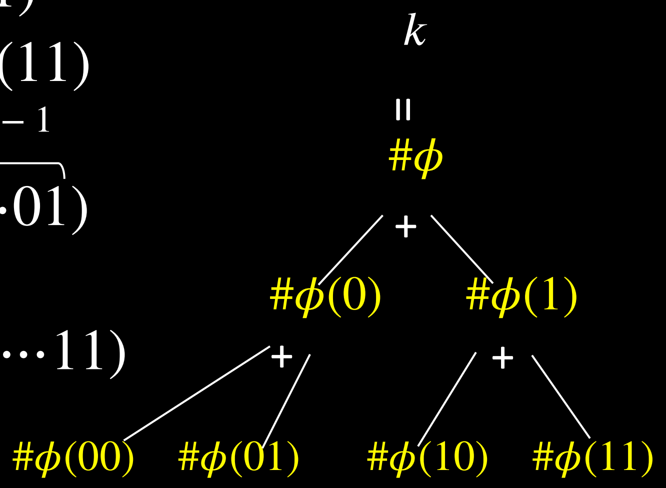

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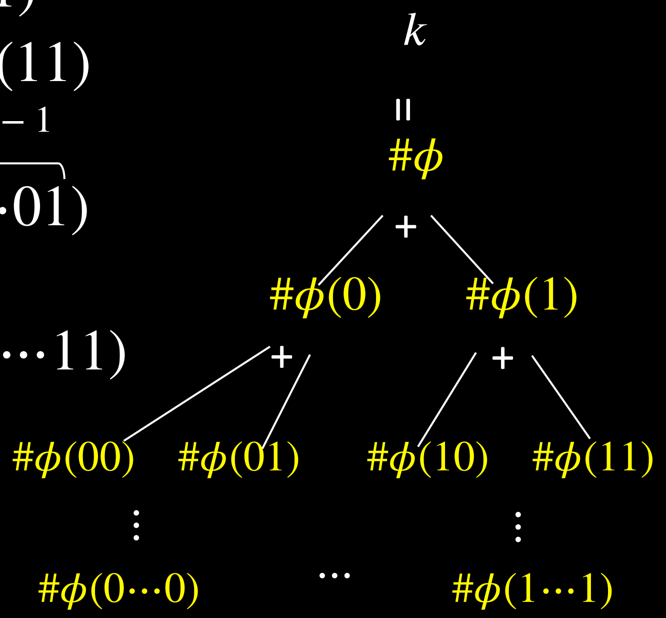

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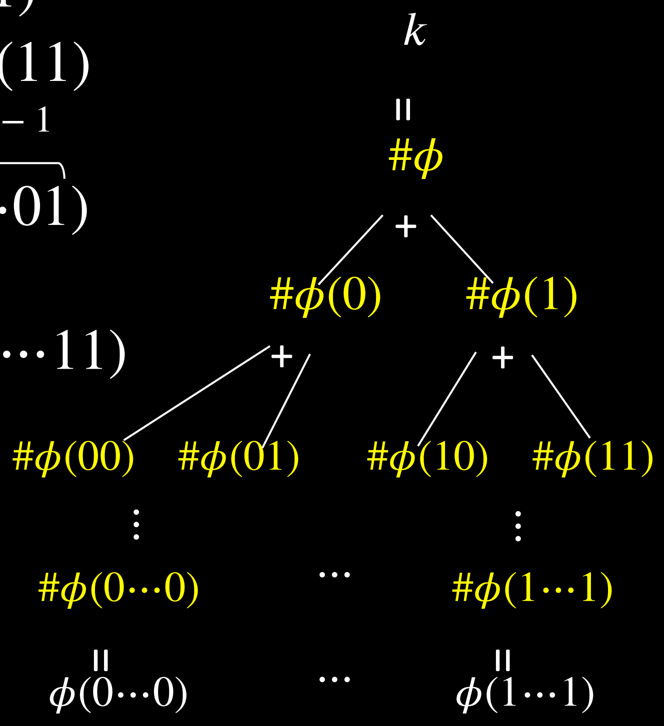

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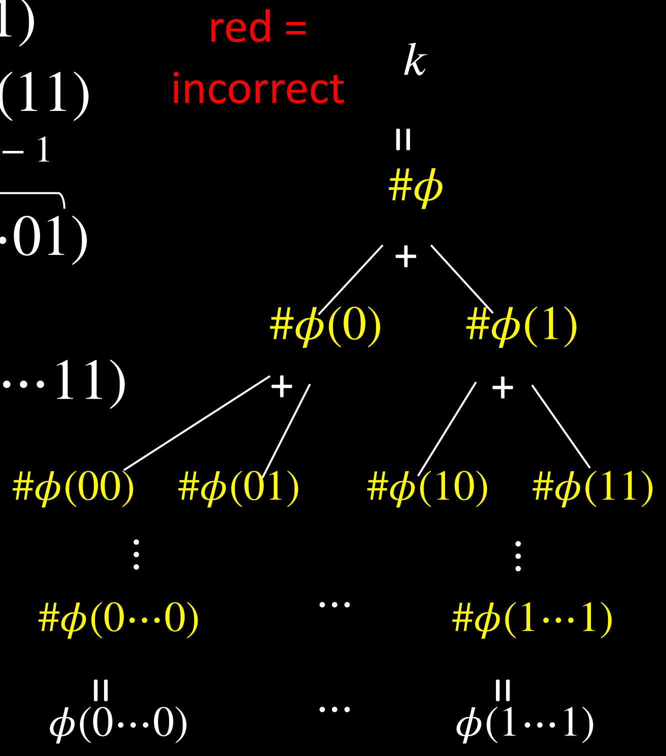

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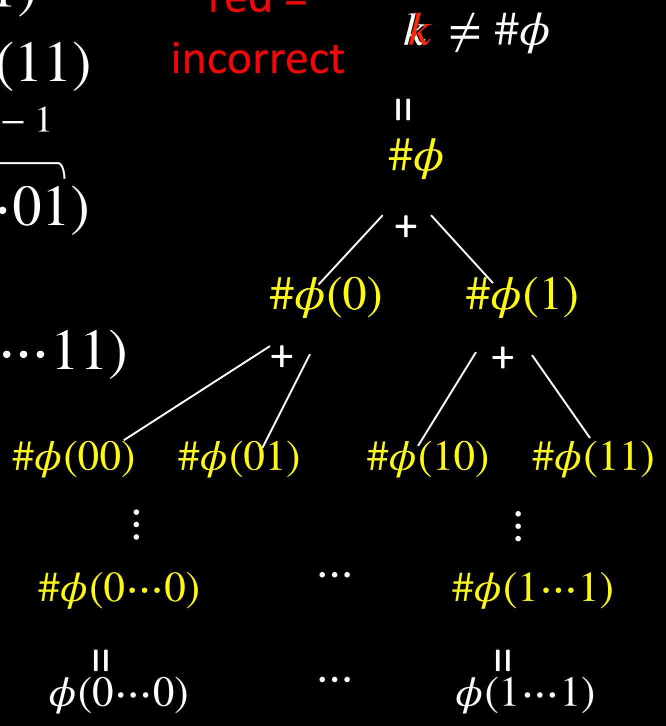

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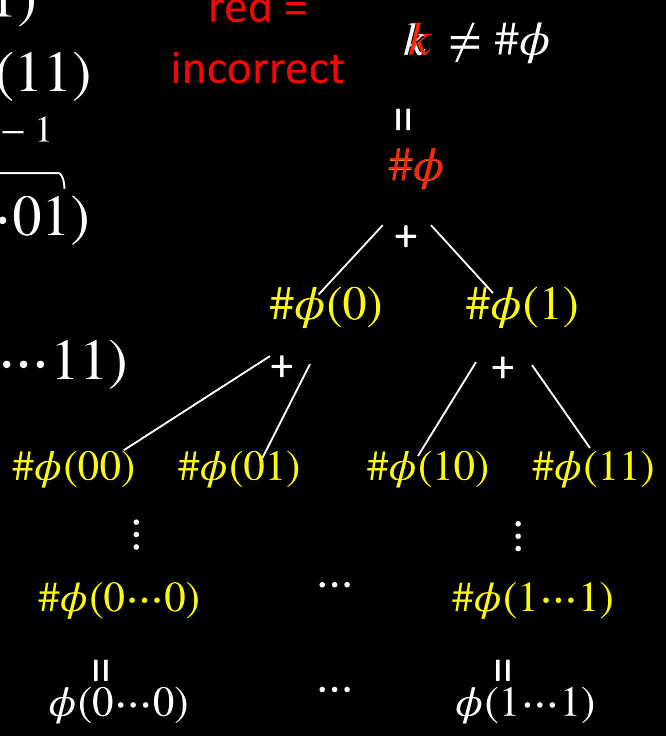

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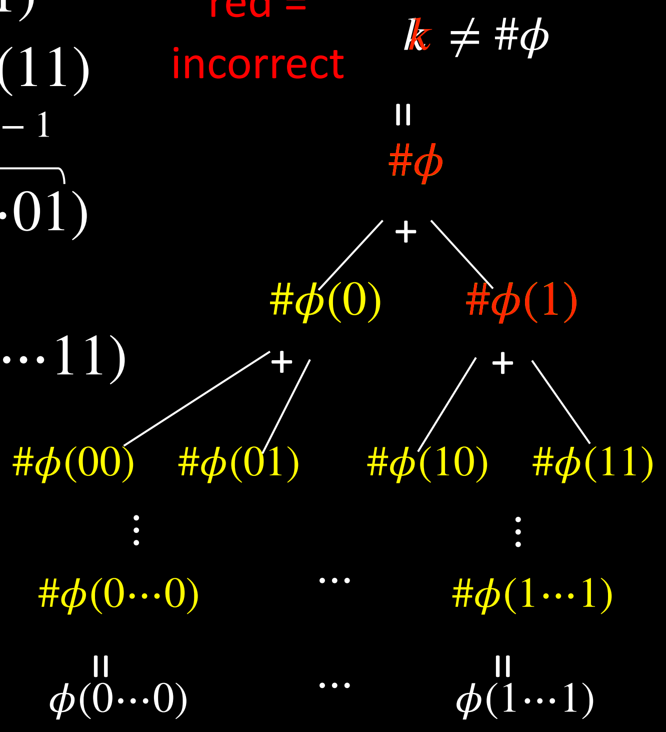

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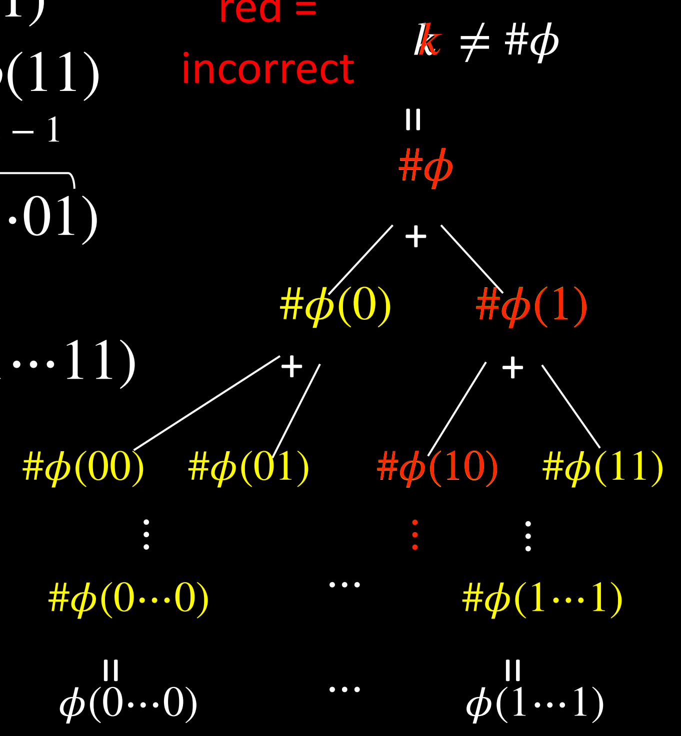

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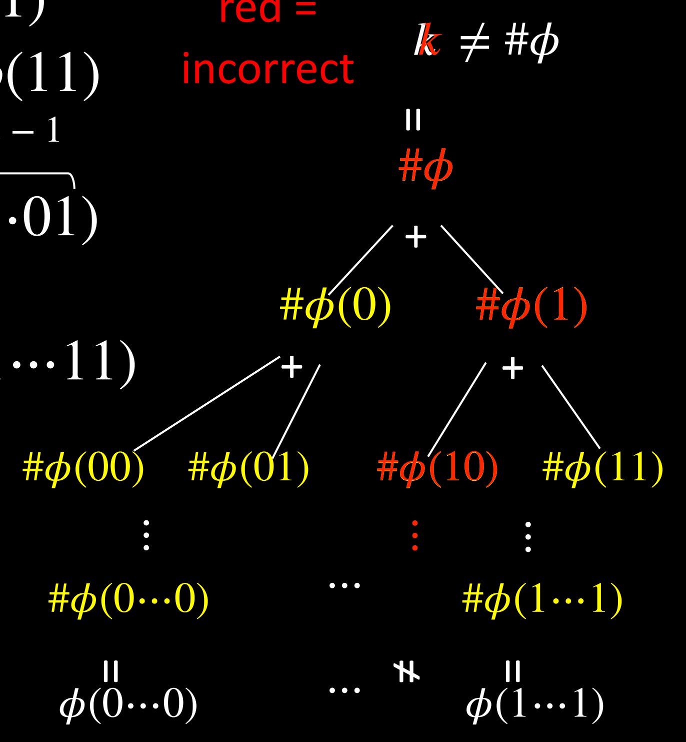

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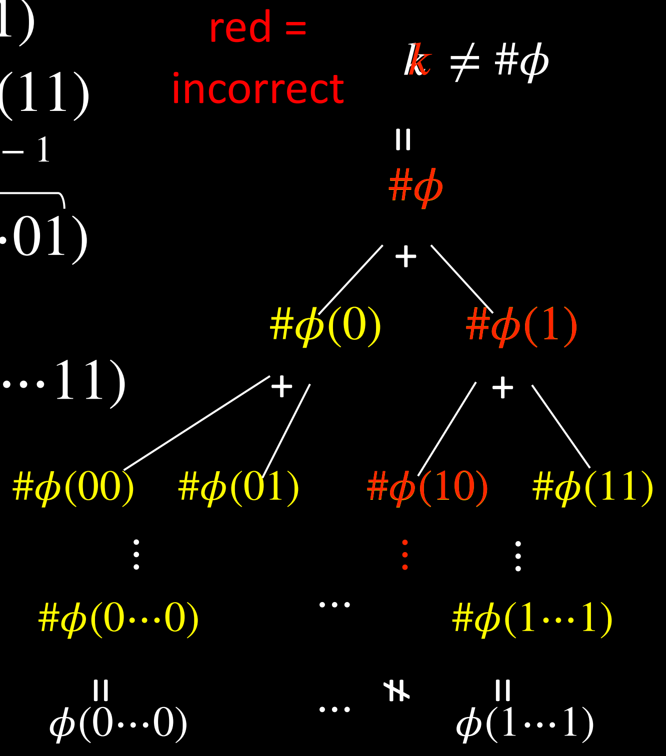

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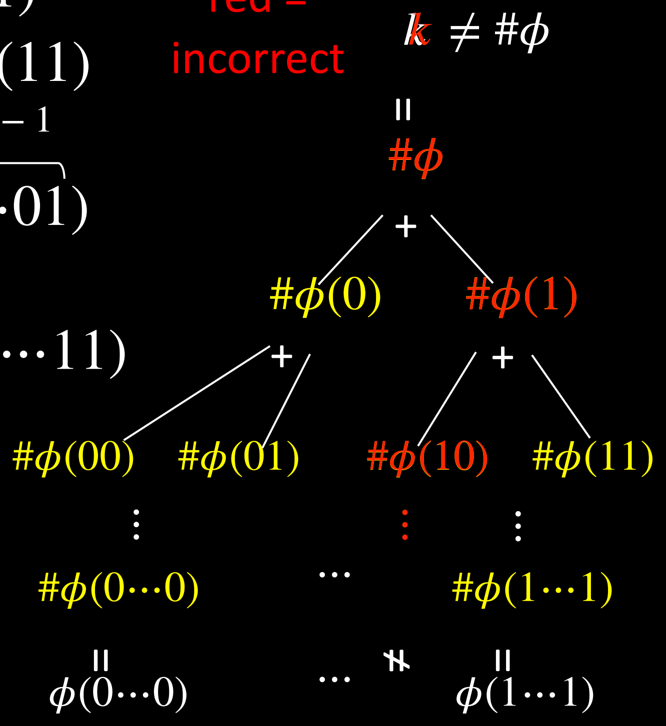

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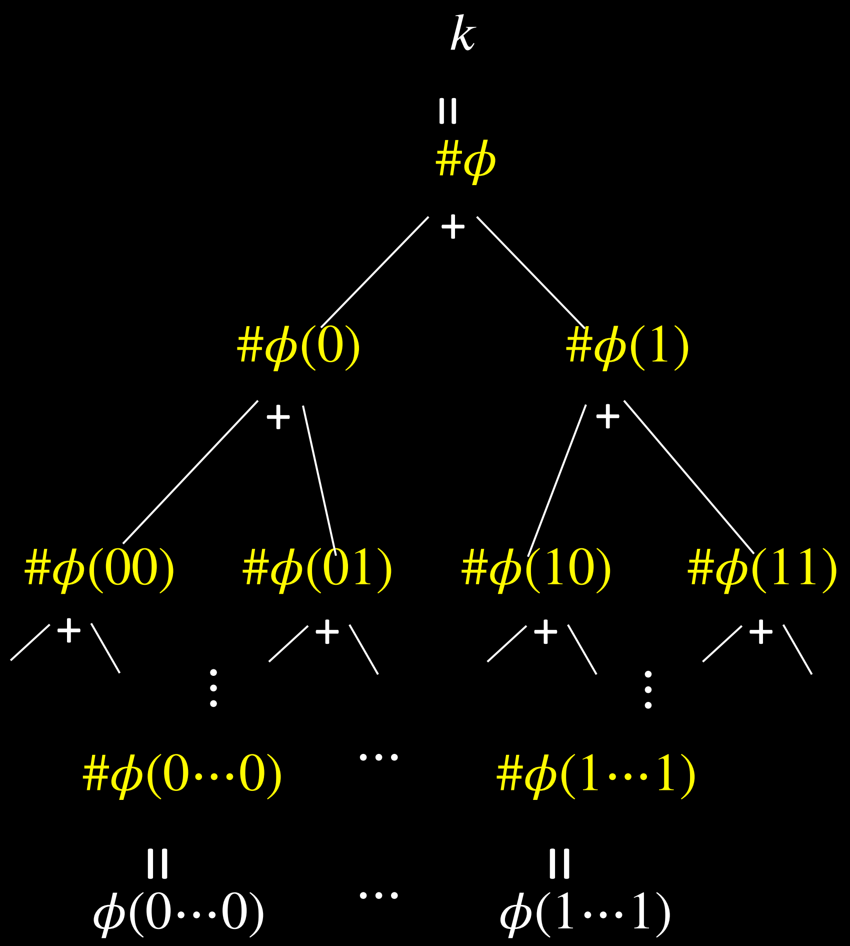

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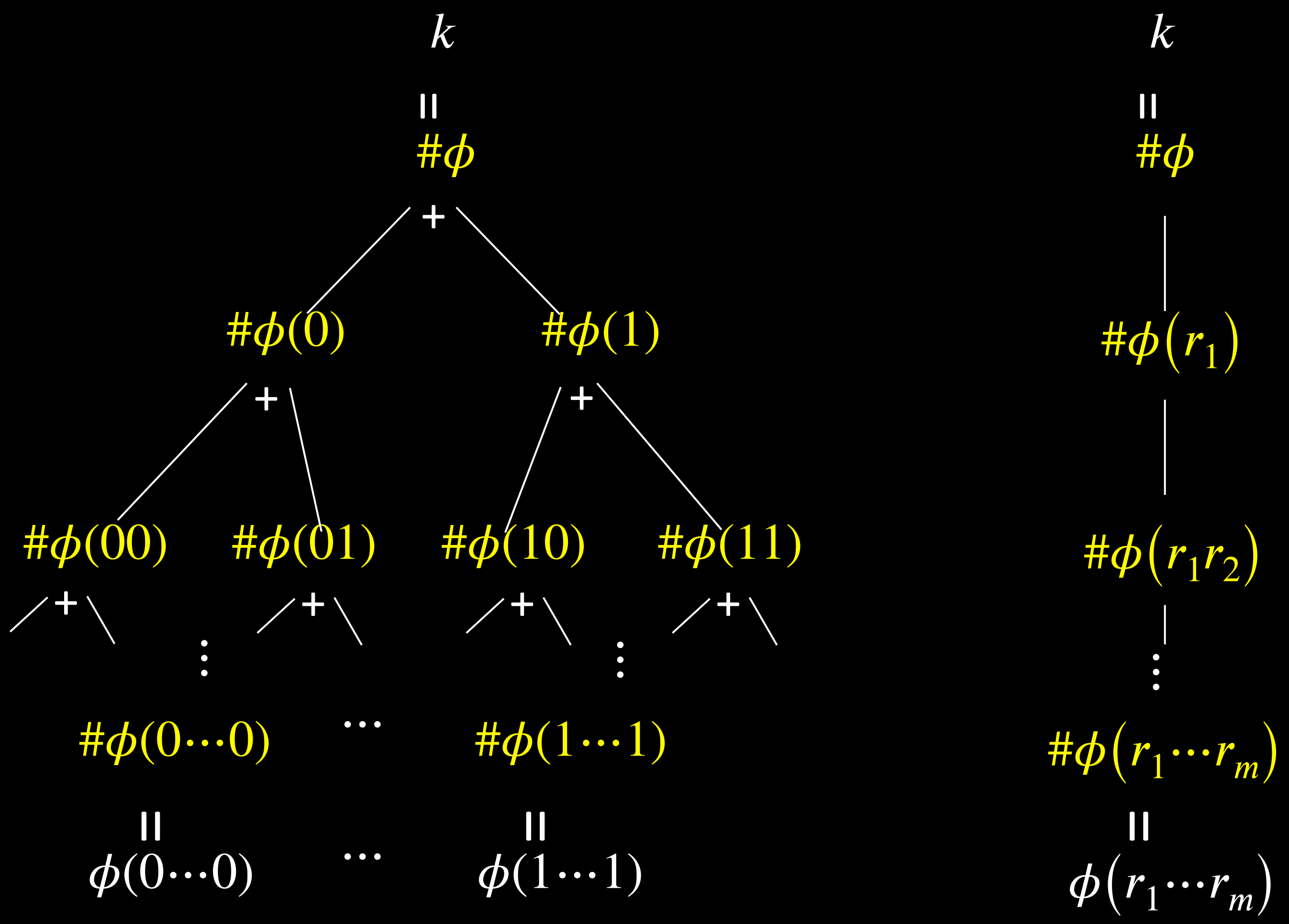

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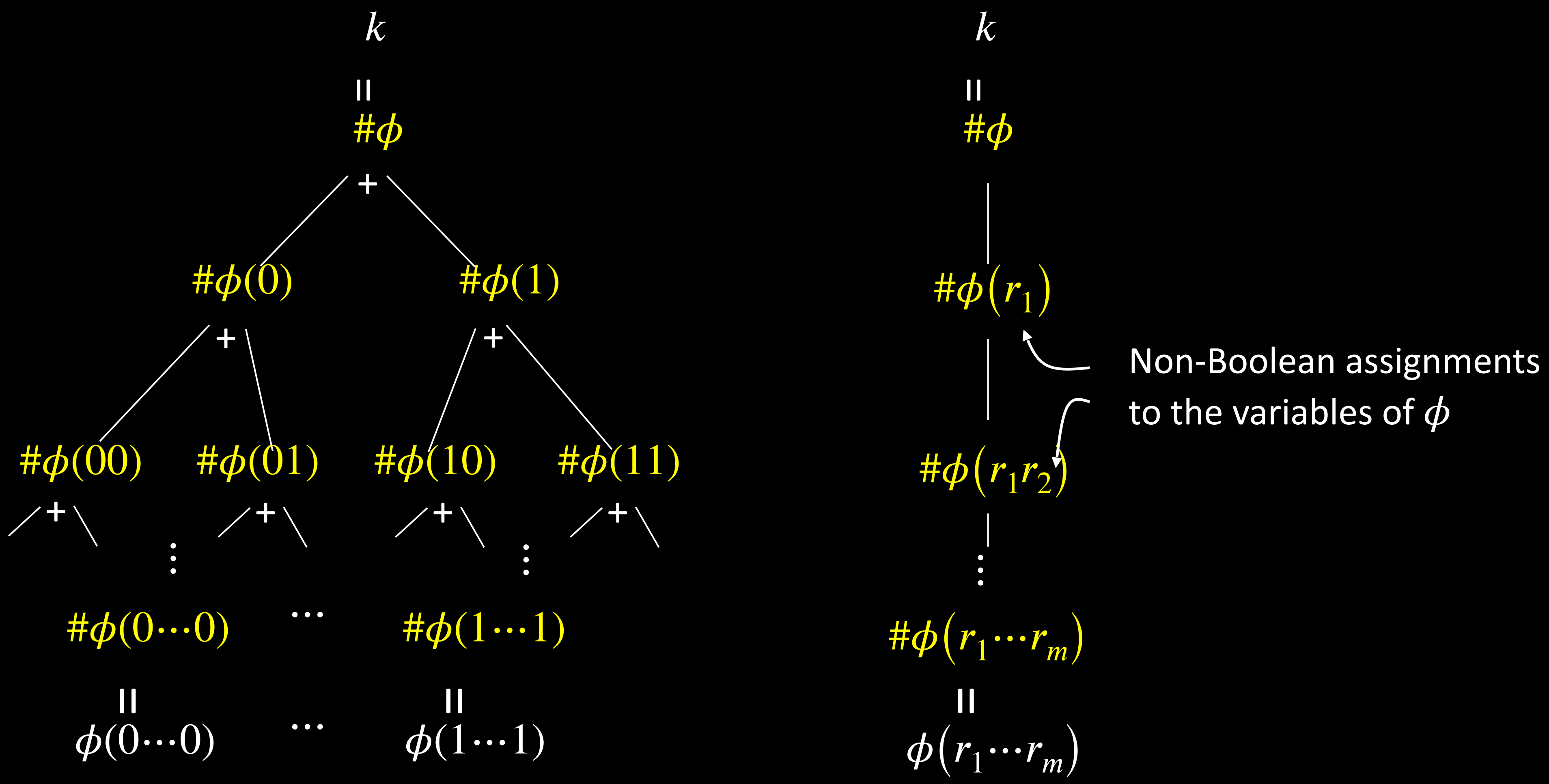

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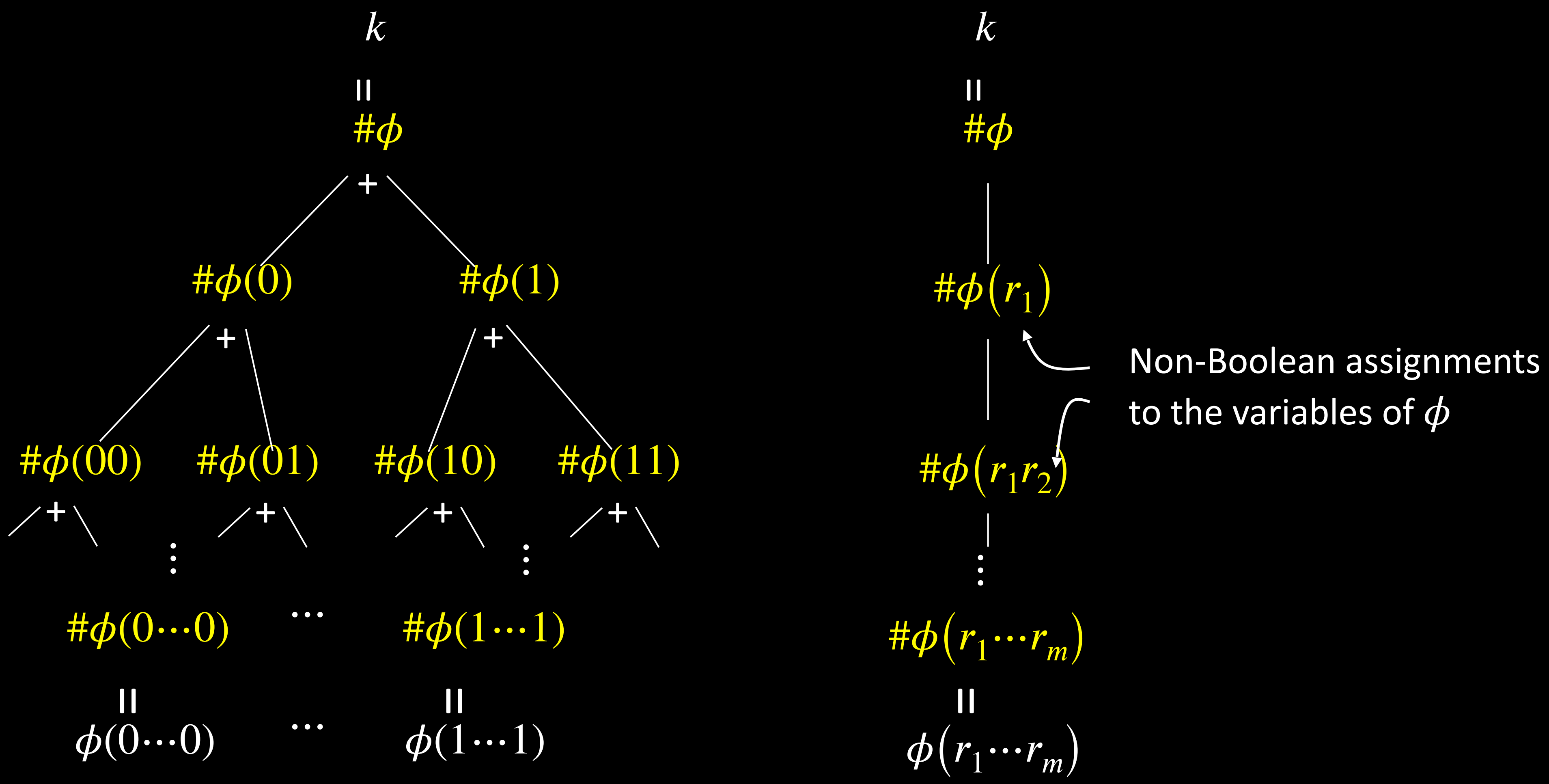

Arithmetizing Boolean formulas

Simulate $\wedge$ and $\vee$ with $+$ and $\times$

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\begin{aligned}
a \wedge b \quad &\rightarrow \quad a \times b = ab \\
\overline{a} \quad &\rightarrow \quad (1 - a) \\
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$$\#\phi(a_1 \ldots a_i 0) + \#\phi(a_1 \ldots a_i 1)$$
2. $\#\phi(a_1 \ldots a_m) = \phi(a_1 \ldots a_m)$

Important: For Boolean $a_1 \ldots a_i$ the values of $\phi(a_1 \ldots a_i)$ and $\#\phi(a_1 \ldots a_i)$ are unchanged from the previous definition.

We have <u>extended</u> these functions to non-Boolean values

Check-in 26.2

Arithmetizing Boolean formulas

Simulate $\wedge$ and $\vee$ with $+$ and $\times$

$$a \wedge b \;\rightarrow\; a \times b = ab$$
$$\bar{a} \;\rightarrow\; (1-a)$$
$$a \vee b \;\rightarrow\; a + b - ab$$
$$\phi \;\rightarrow\; p_\phi \;\; \text{degree}(p_\phi) \le |\phi|$$

Let $\mathbb{F}_q = \{0,1,\ldots,\ q-1\}$ for prime $q > 2^m$ be a finite field ($+, \times$ mod q) and let $a_1, \ldots,\ a_i \in \mathbb{F}_q$

Let $\phi(a_1 \ldots a_i) = p_\phi$ where $x_1 \cdots x_i = a_1 \cdots a_i$ and remaining $x_{i+1},\ \ldots,\ x_m$ stay as unset variables.

Let $$\#\phi(a_1 \ldots\ a_i) = \sum_{a_{i+1},\ldots,\ a_m \in \{0,1\}} \phi(a_1 \ldots\ a_m)$$

identities still true

1. $\#\phi(a_1 \ldots a_i) = \#\phi(a_1 \ldots a_i 0) + \#\phi(a_1 \ldots a_i 1)$
2. $\#\phi(a_1 \ldots a_m) = \phi(a_1 \ldots\ a_m)$

Check-in 26.2

Let $\phi = (x_1 \vee x_2) \wedge (\overline{x_1} \vee \overline{x_2})$. Check all that are true:

a) $p_\phi = (x_1 + x_2 - x_1 x_2)\Big(\ (1-x_1) + (1-x_2) - (1-x_1)(1-x_2)\Big)$

b) $p_\phi = (x_1 + x_2)\Big(\ (1-x_1) + (1-x_2)\Big)$

c) $p_\phi = (x_1 + x_2 - 2x_1 x_2)$

$\#SAT \in$ IP – version 1

$\#SAT \in$ IP – version 1

Theorem: $\#SAT \in$ IP

$\#SAT \in$ IP – version 1

Theorem: $\#SAT \in$ IP

Proof: Protocol for V and (the honest) P on input $\langle \phi, k \rangle$

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$\#SAT \in$ IP – version 1

Theorem: $\#SAT \in$ IP

Proof: Protocol for V and (the honest) P on input $\langle \phi, k \rangle$

0) P sends $\#\phi$; V checks $k = \#\phi$

1) P sends $\#\phi(0)$, $\#\phi(1)$; V checks $\#\phi = \#\phi(0) + \#\phi(1)$

$\#SAT \in$ IP – version 1

Theorem: $\#SAT \in$ IP

Proof: Protocol for V and (the honest) P on input $\langle \phi, k \rangle$

0) P sends $\#\phi$; V checks $k = \#\phi$

1) P sends $\#\phi(z)$ as a polynomial in z [sends coefficients – recall deg $p_\phi \leq |\phi|$]

$\#SAT \in$ IP – version 1

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   V checks $\#\phi = \#\phi(0) + \#\phi(1)$ [by evaluating polynomial for $\#\phi(z)$]

$\#SAT \in$ IP – version 1

Recall $\quad \#\phi(a_1 \ldots a_i) = \sum_{a_{i+1}, \ldots, a_m \in \{0,1\}} \phi(a_1 \ldots a_m)$

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2) P sends $\#\phi\left(r_1 z\right)$ as a polynomial in z

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2) P sends $\#\phi(r_1 z)$ as a polynomial in z

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V sends random $r_2 \in \mathbb{F}_q$ [P needs to show $\#\phi(r_1 r_2)$ is correct]

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V sends random $r_2 \in \mathbb{F}_q$ [P needs to show $\#\phi(r_1 r_2)$ is correct]

⋮

m) P sends $\#\phi(r_1 \cdots r_{m-1} z)$ as a polynomial in z

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⋮

m) P sends $\#\phi\left(r_1 \cdots r_{m-1} z\right)$ as a polynomial in z

   V checks $\#\phi(r_1 \cdots r_{m-1}) = \#\phi\left(r_1 \cdots r_{m-1} 0\right) + \#\phi\left(r_1 \cdots r_{m-1} 1\right)$

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$m+1$) V checks $\#\phi\left(r_1 \cdots r_m\right) = \phi\left(r_1 \cdots r_m\right)$

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$\vdots$

m) P sends $\#\phi\left(r_1 \cdots r_{m-1} z\right)$ as a polynomial in z

V checks $\#\phi(r_1 \cdots r_{m-1}) = \#\phi\left(r_1 \cdots r_{m-1} 0\right) + \#\phi\left(r_1 \cdots r_{m-1} 1\right)$

V sends random $r_m \in \mathbb{F}_q$

$m+1$) V checks $\#\phi\left(r_1 \cdots r_m\right) = \phi\left(r_1 \cdots r_m\right)$

V *accepts* if all checks are correct. Otherwise V *rejects*.

$\#SAT \in$ IP – version 1

Recall $\#\phi(a_1 \ldots a_i) = \sum_{a_{i+1}, \ldots, a_m \in \{0,1\}} \phi(a_1 \ldots a_m)$

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V checks $\#\phi(r_1) = \#\phi(r_1 0) + \#\phi(r_1 1)$ [by evaluating polynomial for $\#\phi(r_1 z)$]

V sends random $r_2 \in \mathbb{F}_q$ [P needs to show $\#\phi(r_1 r_2)$ is correct]

⋮

m) P sends $\#\phi(r_1 \cdots r_{m-1} z)$ as a polynomial in z

V checks $\#\phi(r_1 \cdots r_{m-1}) = \#\phi(r_1 \cdots r_{m-1} 0) + \#\phi(r_1 \cdots r_{m-1} 1)$

V sends random $r_m \in \mathbb{F}_q$

$m+1$) V checks $\#\phi(r_1 \cdots r_m) = \phi(r_1 \cdots r_m)$

V *accepts* if all checks are correct. Otherwise V *rejects*.

$\#SAT \in$ IP – version 2

$\#SAT \in$ IP – version 2

Input $\langle \phi, k \rangle$

$\#SAT \in$ IP – version 2

Input $\langle \phi, k \rangle$

Prover sends **Verifier sends**

$\#SAT \in$ IP – version 2

Input $\langle \phi, k \rangle$

Prover sends **Verifier sends**

$\#\phi$

$\#SAT \in$ IP – version 2

Input $\langle \phi, k \rangle$

Prover sends

$\#\phi$

Verifier sends

Verifier checks

$\#\phi = k$

$\#SAT \in$ IP – version 2

Input $\langle \phi, k \rangle$

Prover sends

$\#\phi$

$\#\phi(z)$
$= 3z^d - 5z^{d-1} + \cdots + 7$

Verifier sends

Verifier checks

$\#\phi = k$

$\#SAT \in$ IP – version 2

Input $\langle \phi, k \rangle$

Prover sends

Verifier sends

Verifier checks

$\#\phi$

$\#\phi = k$

$\#\phi(z)$
$= 3z^d - 5z^{d-1} + \cdots + 7$

$\#\phi(0)$ $\#\phi(1)$

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Verifier checks

$\#\phi = k$

$$\#\phi = \#\phi(0) + \#\phi(1)$$

$\#SAT \in$ IP – version 2

Input $\langle \phi, k \rangle$

Prover sends

$\#\phi$

$\#\phi(z)$
$= 3z^d - 5z^{d-1} + \cdots + 7$

Verifier sends

r_1

Verifier checks

$\#\phi = k$

$$\#\phi = \#\phi(0) + \#\phi(1)$$

$\#SAT \in$ IP – version 2

Input $\langle \phi, k \rangle$

Prover sends

$\#\phi$

$\#\phi(z)$
$= 3z^d - 5z^{d-1} + \cdots + 7$

Verifier sends

r_1

Verifier checks

$\#\phi = k$

$+$

$\#\phi(0)$ $\#\phi(1)$

$\#\phi\left(r_1\right)$

$\#SAT \in$ IP – version 2

Input $\langle \phi, k \rangle$

Prover sends

$\#\phi$

$\#\phi(z)$
$= 3z^d - 5z^{d-1} + \cdots + 7$

$\#\phi(r_1 z) = \cdots$

Verifier sends

r_1

Verifier checks

$\#\phi = k$

+

$\#\phi(0)$ $\#\phi(1)$

$\#\phi(r_1)$

$\#SAT \in$ IP – version 2

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Verifier checks

$\#\phi = k$

$+$

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$\#\phi(r_1)$

$\#\phi(r_1 0)$ $\#\phi(r_1 1)$

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Verifier sends

r_1

Verifier checks

$\#\phi = k$

$+$

$\#\phi(0) \quad \#\phi(1)$

$\#\phi(r_1)$

$+$

$\#\phi(r_1 0) \quad \#\phi(r_1 1)$

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Input $\langle \phi, k \rangle$

Prover sends

$\#\phi$

$\#\phi(z) = 3z^d - 5z^{d-1} + \cdots + 7$

$\#\phi(r_1 z) = \cdots$

Verifier sends

r_1

r_2

Verifier checks

$\#\phi = k$

$+$

$\#\phi(0)$ $\#\phi(1)$

$\#\phi(r_1)$

$+$

$\#\phi(r_1 0)$ $\#\phi(r_1 1)$

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Verifier sends

r_1

r_2

Verifier checks

$\#\phi = k$

$+$

$\#\phi(0) \quad \#\phi(1)$

$\#\phi(r_1)$

$+$

$\#\phi(r_1 0) \quad \#\phi(r_1 1)$

$\#\phi(r_1 r_2)$

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Input $\langle \phi, k \rangle$

Prover sends

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$\#\phi(z) = 3z^d - 5z^{d-1} + \cdots + 7$

$\#\phi(r_1 z) = \cdots$

$\#\phi(r_1 r_2 z) = \cdots$

Verifier sends

r_1

r_2

Verifier checks

$\#\phi = k$

$+$

$\#\phi(0) \quad \#\phi(1)$

$\#\phi(r_1)$

$+$

$\#\phi(r_1 0) \quad \#\phi(r_1 1)$

$\#\phi(r_1 r_2)$

$\#SAT \in$ IP – version 2

Input $\langle \phi, k \rangle$

Prover sends

Verifier sends

Verifier checks

$\#\phi$

$\#\phi = k$

$\#\phi(z) = 3z^d - 5z^{d-1} + \cdots + 7$

$\#\phi = \#\phi(0) + \#\phi(1)$

$\#\phi(r_1)$

r_1

$\#\phi(r_1 z) = \cdots$

$\#\phi(r_1) = \#\phi(r_1 0) + \#\phi(r_1 1)$

$\#\phi(r_1 r_2)$

r_2

$\#\phi(r_1 r_2 z) = \cdots$

$\#\phi(r_1 r_2 0) \qquad \#\phi(r_1 r_2 1)$

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Input $\langle \phi, k \rangle$

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$\#\phi(z) = 3z^d - 5z^{d-1} + \cdots + 7$

$\#\phi(r_1 z) = \cdots$

$\#\phi(r_1 r_2 z) = \cdots$

Verifier sends

r_1

r_2

Verifier checks

$\#\phi = k$

$+$

$\#\phi(0) \quad \#\phi(1)$

$\#\phi(r_1)$

$+$

$\#\phi(r_1 0) \quad \#\phi(r_1 1)$

$\#\phi(r_1 r_2)$

$+$

$\#\phi(r_1 r_2 0) \quad \#\phi(r_1 r_2 1)$

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Input $\langle \phi, k \rangle$

Prover sends

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$\#\phi(z) = 3z^d - 5z^{d-1} + \cdots + 7$

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$\#\phi(r_1 r_2 z) = \cdots$

Verifier sends

r_1

r_2

Verifier checks

$\#\phi = k$

$+$

$\#\phi(0) \quad \#\phi(1)$

$\#\phi(r_1)$

$+$

$\#\phi(r_1 0) \quad \#\phi(r_1 1)$

$\#\phi(r_1 r_2)$

$+$

$\#\phi(r_1 r_2 0) \quad \#\phi(r_1 r_2 1)$

$\vdots$

$\#\phi(r_1 \cdots r_{m-1})$

$\#SAT \in$ IP – version 2

Input $\langle \phi, k \rangle$

Prover sends

Verifier sends

Verifier checks

$\#\phi$

$\#\phi = k$

$\#\phi(z) = 3z^d - 5z^{d-1} + \cdots + 7$

$\#\phi(0) \; + \; \#\phi(1)$

$\#\phi(r_1)$

r_1

$\#\phi(r_1 z) = \cdots$

$\#\phi(r_1 0) \; + \; \#\phi(r_1 1)$

$\#\phi(r_1 r_2)$

r_2

$\#\phi(r_1 r_2 z) = \cdots$

$\#\phi(r_1 r_2 0) \; + \; \#\phi(r_1 r_2 1)$

$\vdots$

$\#\phi(r_1 \cdots r_{m-1})$

$\#\phi(r_1 \cdots r_{m-1} z) = \cdots$

$\#SAT \in$ IP – version 2

Input $\langle \phi, k \rangle$

Prover sends

$\#\phi$

$\#\phi(z) = 3z^d - 5z^{d-1} + \cdots + 7$

$\#\phi(r_1 z) = \cdots$

$\#\phi(r_1 r_2 z) = \cdots$

$\#\phi(r_1 \cdots r_{m-1} z) = \cdots$

Verifier sends

r_1

r_2

Verifier checks

$$\#\phi = k$$

$$\#\phi(0) + \#\phi(1)$$

$$\#\phi(r_1)$$

$$\#\phi(r_1 0) + \#\phi(r_1 1)$$

$$\#\phi(r_1 r_2)$$

$$\#\phi(r_1 r_2 0) + \#\phi(r_1 r_2 1)$$

$$\vdots$$

$$\#\phi(r_1 \cdots r_{m-1})$$

$$\#\phi(r_1 \cdots r_{m-1} 0) \qquad \#\phi(r_1 \cdots r_{m-1} 1)$$

$\#SAT \in$ IP – version 2

Input $\langle \phi, k \rangle$

Prover sends

Verifier sends

Verifier checks

$\#\phi$

$\#\phi(z) = 3z^d - 5z^{d-1} + \cdots + 7$

r_1

$\#\phi(r_1 z) = \cdots$

r_2

$\#\phi(r_1 r_2 z) = \cdots$

$\#\phi(r_1 \cdots r_{m-1} z) = \cdots$

$$\#\phi = k$$

$$\#\phi(0) \; + \; \#\phi(1)$$

$$\#\phi(r_1)$$

$$\#\phi(r_1 0) \; + \; \#\phi(r_1 1)$$

$$\#\phi(r_1 r_2)$$

$$\#\phi(r_1 r_2 0) \; + \; \#\phi(r_1 r_2 1)$$

$$\vdots$$

$$\#\phi(r_1 \cdots r_{m-1})$$

$$\#\phi(r_1 \cdots r_{m-1} 0) \; + \; \#\phi(r_1 \cdots r_{m-1} 1)$$

$\#SAT \in$ IP – version 2

Input $\langle \phi, k \rangle$

Prover sends

$\#\phi$

$\#\phi(z) = 3z^d - 5z^{d-1} + \cdots + 7$

$\#\phi(r_1 z) = \cdots$

$\#\phi(r_1 r_2 z) = \cdots$

$\#\phi(r_1 \cdots r_{m-1} z) = \cdots$

Verifier sends

r_1

r_2

r_m

Verifier checks

$\#\phi = k$

$\#\phi(0) + \#\phi(1)$

$\#\phi(r_1)$

$\#\phi(r_1 0) + \#\phi(r_1 1)$

$\#\phi(r_1 r_2)$

$\#\phi(r_1 r_2 0) + \#\phi(r_1 r_2 1)$

$\vdots$

$\#\phi(r_1 \cdots r_{m-1})$

$\#\phi(r_1 \cdots r_{m-1} 0) + \#\phi(r_1 \cdots r_{m-1} 1)$

$\#SAT \in$ IP – version 2

Input $\langle \phi, k \rangle$

Prover sends

$\#\phi$

$\#\phi(z) = 3z^d - 5z^{d-1} + \cdots + 7$

$\#\phi(r_1 z) = \cdots$

$\#\phi(r_1 r_2 z) = \cdots$

$\#\phi(r_1 \cdots r_{m-1} z) = \cdots$

Verifier sends

r_1

r_2

r_m

Verifier checks

$\#\phi = k$

$\#\phi(0) + \#\phi(1)$

$\#\phi(r_1)$

$\#\phi(r_1 0) + \#\phi(r_1 1)$

$\#\phi(r_1 r_2)$

$\#\phi(r_1 r_2 0) + \#\phi(r_1 r_2 1)$

$\vdots$

$\#\phi(r_1 \cdots r_{m-1})$

$\#\phi(r_1 \cdots r_{m-1} 0) + \#\phi(r_1 \cdots r_{m-1} 1)$

$\#\phi(r_1 \cdots r_m)$

$\#SAT \in$ IP – version 2

Input $\langle \phi, k \rangle$

Prover sends

$\#\phi$

$\#\phi(z) = 3z^d - 5z^{d-1} + \cdots + 7$

$\#\phi(r_1 z) = \cdots$

$\#\phi(r_1 r_2 z) = \cdots$

$\#\phi(r_1 \cdots r_{m-1} z) = \cdots$

Verifier sends

r_1

r_2

r_m

Verifier checks

$\#\phi = k$

$\#\phi(0) + \#\phi(1)$

$\#\phi(r_1)$

$\#\phi(r_1 0) + \#\phi(r_1 1)$

$\#\phi(r_1 r_2)$

$\#\phi(r_1 r_2 0) + \#\phi(r_1 r_2 1)$

$\vdots$

$\#\phi(r_1 \cdots r_{m-1})$

$\#\phi(r_1 \cdots r_{m-1} 0) + \#\phi(r_1 \cdots r_{m-1} 1)$

$\#\phi(r_1 \cdots r_m)$

$\|$

$\phi(r_1 \cdots r_m)$

$\#SAT \in$ IP – version 2

Input $\langle \phi, k \rangle$

Prover sends

$\#\phi$

$\#\phi(z) = 3z^d - 5z^{d-1} + \cdots + 7$

$\#\phi(r_1 z) = \cdots$

$\#\phi(r_1 r_2 z) = \cdots$

$\#\phi(r_1 \cdots r_{m-1} z) = \cdots$

Verifier sends

r_1

r_2

r_m

accept

Verifier checks

$\#\phi = k$

$\#\phi = \#\phi(0) + \#\phi(1)$

$\#\phi(r_1) = \#\phi(r_1 0) + \#\phi(r_1 1)$

$\#\phi(r_1 r_2) = \#\phi(r_1 r_2 0) + \#\phi(r_1 r_2 1)$

$\vdots$

$\#\phi(r_1 \cdots r_{m-1}) = \#\phi(r_1 \cdots r_{m-1} 0) + \#\phi(r_1 \cdots r_{m-1} 1)$

$\#\phi(r_1 \cdots r_m) = \phi(r_1 \cdots r_m)$

$\#SAT \in$ IP – version 2

Input $\langle \phi, k \rangle$

Prover sends

$\#\phi$

$\#\phi(z) = 3z^d - 5z^{d-1} + \cdots + 7$

$\#\phi(r_1 z) = \cdots$

$\#\phi(r_1 r_2 z) = \cdots$

$\#\phi(r_1 \cdots r_{m-1} z) = \cdots$

Verifier sends

r_1

r_2

r_m

accept

Verifier checks

$\#\phi = k$

$+$

$\#\phi(0) \quad \#\phi(1)$

$\#\phi(r_1)$

$+$

$\#\phi(r_1 0) \quad \#\phi(r_1 1)$

$\#\phi(r_1 r_2)$

$+$

$\#\phi(r_1 r_2 0) \quad \#\phi(r_1 r_2 1)$

$\vdots$

$\#\phi(r_1 \cdots r_{m-1})$

$+$

$\#\phi(r_1 \cdots r_{m-1} 0) \quad \#\phi(r_1 \cdots r_{m-1} 1)$

$\#\phi(r_1 \cdots r_m)$

$\|$

$\phi(r_1 \cdots r_m)$

If k is correct, V will accept.

$\#SAT \in$ IP – version 2

Input $\langle \phi, k \rangle$

Prover sends

$\#\phi$

$\#\phi(z) = 3z^d - 5z^{d-1} + \cdots + 7$

$\#\phi(r_1 z) = \cdots$

$\#\phi(r_1 r_2 z) = \cdots$

$\#\phi(r_1 \cdots r_{m-1} z) = \cdots$

Verifier sends

r_1

r_2

r_m

accept

Verifier checks

$\#\phi = k$

$$\#\phi(0) + \#\phi(1)$$

$$\#\phi(r_1) = \#\phi(r_1 0) + \#\phi(r_1 1)$$

$$\#\phi(r_1 r_2) = \#\phi(r_1 r_2 0) + \#\phi(r_1 r_2 1)$$

$\vdots$

$$\#\phi(r_1 \cdots r_{m-1}) = \#\phi(r_1 \cdots r_{m-1} 0) + \#\phi(r_1 \cdots r_{m-1} 1)$$

$$\#\phi(r_1 \cdots r_m) = \phi(r_1 \cdots r_m)$$

If k is correct, V will accept.

If k is wrong, V probably will reject, whatever P does.

$\#SAT \in$ IP – version 2

Input $\langle \phi, k \rangle$

Prover sends	**Verifier sends**	**Verifier checks**
$\#\phi$		$\#\phi = k$
$\#\phi(z) = 3z^d - 5z^{d-1} + \cdots + 7$		$\#\phi(0) + \#\phi(1)$
	r_1	$\#\phi(r_1)$
$\#\phi(r_1 z) = \cdots$		$\#\phi(r_1 0) + \#\phi(r_1 1)$
	r_2	$\#\phi(r_1 r_2)$
$\#\phi(r_1 r_2 z) = \cdots$		$\#\phi(r_1 r_2 0) + \#\phi(r_1 r_2 1)$
		$\vdots$
		$\#\phi(r_1 \cdots r_{m-1})$
$\#\phi(r_1 \cdots r_{m-1} z) = \cdots$		$\#\phi(r_1 \cdots r_{m-1} 0) + \#\phi(r_1 \cdots r_{m-1} 1)$
	r_m	$\#\phi(r_1 \cdots r_m)$
	accept	$\Vert$
		$\phi(r_1 \cdots r_m)$

If k is correct, V will accept.

If k is wrong, V probably will reject, whatever P does.

$\#SAT \in$ IP – version 2

Input $\langle \phi, k \rangle$

Prover sends

$\#\phi$

$\#\phi(z) = 3z^d - 5z^{d-1} + \cdots + 7$

$\#\phi(r_1 z) = \cdots$

$\#\phi(r_1 r_2 z) = \cdots$

$\#\phi(r_1 \cdots r_{m-1} z) = \cdots$

Verifier sends

r_1

r_2

r_m

accept

Verifier checks

$\#\phi = k$

$\#\phi(0) + \#\phi(1)$

$\#\phi(r_1)$

$\#\phi(r_1 0) + \#\phi(r_1 1)$

$\#\phi(r_1 r_2)$

$\#\phi(r_1 r_2 0) + \#\phi(r_1 r_2 1)$

$\vdots$

$\#\phi(r_1 \cdots r_{m-1})$

$\#\phi(r_1 \cdots r_{m-1} 0) + \#\phi(r_1 \cdots r_{m-1} 1)$

$\#\phi(r_1 \cdots r_m)$

$\|$

$\phi(r_1 \cdots r_m)$

If k is correct, V will accept.

If k is wrong, V probably will reject, whatever P does.

$\#SAT \in$ IP – version 2

Input $\langle \phi, k \rangle$

Prover sends

$\#\phi$

$\#\phi(z) = 3z^d - 5z^{d-1} + \cdots + 7$

$\#\phi(r_1 z) = \cdots$

$\#\phi(r_1 r_2 z) = \cdots$

$\#\phi(r_1 \cdots r_{m-1} z) = \cdots$

Verifier sends

r_1

r_2

r_m

accept

Verifier checks

$\#\phi = k$

$+$

$\#\phi(0) \quad \#\phi(1)$

$\#\phi(r_1)$

$+$

$\#\phi(r_1 0) \quad \#\phi(r_1 1)$

$\#\phi(r_1 r_2)$

$+$

$\#\phi(r_1 r_2 0) \quad \#\phi(r_1 r_2 1)$

$\vdots$

$\#\phi(r_1 \cdots r_{m-1})$

$+$

$\#\phi(r_1 \cdots r_{m-1} 0) \quad \#\phi(r_1 \cdots r_{m-1} 1)$

$\#\phi(r_1 \cdots r_m)$

$\parallel$

$\phi(r_1 \cdots r_m)$

If k is correct, V will accept.

If k is wrong, V probably will reject, whatever P does.

$\#SAT \in$ IP – version 2

Input $\langle \phi, k \rangle$

Prover sends

$\#\phi$

$\#\phi(z) = 3z^d - 5z^{d-1} + \cdots + 7$

$\#\phi(r_1 z) = \cdots$

$\#\phi(r_1 r_2 z) = \cdots$

$\#\phi(r_1 \cdots r_{m-1} z) = \cdots$

Verifier sends

r_1

r_2

r_m

accept

Verifier checks

$\#\phi = k$

$$\#\phi(0) + \#\phi(1)$$

$$\#\phi(r_1)$$

$$\#\phi(r_1 0) + \#\phi(r_1 1)$$

$$\#\phi(r_1 r_2)$$

$$\#\phi(r_1 r_2 0) + \#\phi(r_1 r_2 1)$$

$$\vdots$$

$$\#\phi(r_1 \cdots r_{m-1})$$

$$\#\phi(r_1 \cdots r_{m-1} 0) + \#\phi(r_1 \cdots r_{m-1} 1)$$

$$\#\phi(r_1 \cdots r_m) = \phi(r_1 \cdots r_m)$$

If k is correct, V will accept.

If k is wrong, V probably will reject, whatever P does.

$\#SAT \in$ IP – version 2

Input $\langle \phi, k \rangle$

Prover sends

$\#\phi$

$\#\phi(z) = 3z^d - 5z^{d-1} + \cdots + 7$

$\#\phi(r_1 z) = \cdots$

$\#\phi(r_1 r_2 z) = \cdots$

$\#\phi(r_1 \cdots r_{m-1} z) = \cdots$

Verifier sends

r_1

r_2

r_m

accept

Verifier checks

$\#\phi = k$

$\#\phi(0) + \#\phi(1)$

$\#\phi(r_1)$

$\#\phi(r_1 0) + \#\phi(r_1 1)$

$\#\phi(r_1 r_2)$

$\#\phi(r_1 r_2 0) + \#\phi(r_1 r_2 1)$

$\vdots$

$\#\phi(r_1 \cdots r_{m-1})$

$\#\phi(r_1 \cdots r_{m-1} 0) + \#\phi(r_1 \cdots r_{m-1} 1)$

$\#\phi(r_1 \cdots r_m)$

$\|$

$\phi(r_1 \cdots r_m)$

If k is correct, V will accept.

If k is wrong, V probably will reject, whatever P does.

$\#SAT \in$ IP – version 2

Input $\langle \phi, k \rangle$

Prover sends

$\#\phi$

$\#\phi(z) = 3z^d - 5z^{d-1} + \cdots + 7$

$\#\phi(r_1 z) = \cdots$

$\#\phi(r_1 r_2 z) = \cdots$

$\#\phi(r_1 \cdots r_{m-1} z) = \cdots$

Verifier sends

r_1

r_2

r_m

accept

Verifier checks

$\#\phi = k$

$\#\phi(0) + \#\phi(1)$

$\#\phi(r_1)$

$\#\phi(r_1 0) + \#\phi(r_1 1)$

$\#\phi(r_1 r_2)$

$\#\phi(r_1 r_2 0) + \#\phi(r_1 r_2 1)$

$\vdots$

$\#\phi(r_1 \cdots r_{m-1})$

$\#\phi(r_1 \cdots r_{m-1} 0) + \#\phi(r_1 \cdots r_{m-1} 1)$

$\#\phi(r_1 \cdots r_m)$

$\| $

$\phi(r_1 \cdots r_m)$

If k is correct, V will accept.

If k is wrong, V probably will reject, whatever P does.

$\#SAT \in$ IP – version 2

Input $\langle \phi, k \rangle$

Prover sends

$\#\phi$

$\#\phi(z) = 3z^d - 5z^{d-1} + \cdots + 7$

$\#\phi(r_1 z) = \cdots$

$\#\phi(r_1 r_2 z) = \cdots$

$\#\phi(r_1 \cdots r_{m-1} z) = \cdots$

Verifier sends

r_1

r_2

r_m

accept

Verifier checks

$\#\phi = k$

$\#\phi(0) + \#\phi(1)$

$\#\phi(r_1)$

$\#\phi(r_1 0) + \#\phi(r_1 1)$

$\#\phi(r_1 r_2)$

$\#\phi(r_1 r_2 0) + \#\phi(r_1 r_2 1)$

$\vdots$

$\#\phi(r_1 \cdots r_{m-1})$

$\#\phi(r_1 \cdots r_{m-1} 0) + \#\phi(r_1 \cdots r_{m-1} 1)$

$\#\phi(r_1 \cdots r_m)$

$\|$

$\phi(r_1 \cdots r_m)$

If k is correct, V will accept.

If k is wrong, V probably will reject, whatever P does.

$\#SAT \in$ IP – version 2

Input $\langle \phi, k \rangle$

Prover sends

$\#\phi$

$\#\phi(z) = 3z^d - 5z^{d-1} + \cdots + 7$

$\#\phi(r_1 z) = \cdots$

$\#\phi(r_1 r_2 z) = \cdots$

$\#\phi(r_1 \cdots r_{m-1} z) = \cdots$

Verifier sends

r_1

r_2

r_m

accept

Verifier checks

$\#\phi = k$

$\#\phi(0) + \#\phi(1)$

$\#\phi(r_1)$

$\#\phi(r_1 0) + \#\phi(r_1 1)$

$\#\phi(r_1 r_2)$

$\#\phi(r_1 r_2 0) + \#\phi(r_1 r_2 1)$

$\vdots$

$\#\phi(r_1 \cdots r_{m-1})$

$\#\phi(r_1 \cdots r_{m-1} 0) + \#\phi(r_1 \cdots r_{m-1} 1)$

$\#\phi(r_1 \cdots r_m)$

$\|$

$\phi(r_1 \cdots r_m)$

If k is correct, V will accept.

If k is wrong, V probably will reject, whatever P does.

$\#SAT \in$ IP – version 2

Input $\langle \phi, k \rangle$

Prover sends

$\#\phi$

$\#\phi(z) = 3z^d - 5z^{d-1} + \cdots + 7$

$\#\phi(r_1 z) = \cdots$

$\#\phi(r_1 r_2 z) = \cdots$

$\#\phi(r_1 \cdots r_{m-1} z) = \cdots$

Verifier sends

r_1

r_2

r_m

accept

Verifier checks

$\#\phi = k$

$\#\phi(0) + \#\phi(1)$

$\#\phi(r_1)$

$\#\phi(r_1 0) + \#\phi(r_1 1)$

$\#\phi(r_1 r_2)$

$\#\phi(r_1 r_2 0) + \#\phi(r_1 r_2 1)$

$\vdots$

$\#\phi(r_1 \cdots r_{m-1})$

$\#\phi(r_1 \cdots r_{m-1} 0) + \#\phi(r_1 \cdots r_{m-1} 1)$

$\#\phi(r_1 \cdots r_m) = \phi(r_1 \cdots r_m)$

If k is correct, V will accept.

If k is wrong, V probably will reject, whatever P does.

$\#SAT \in$ IP – version 2

Input $\langle \phi, k \rangle$

Prover sends

$\#\phi$

$\#\phi(z) = 3z^d - 5z^{d-1} + \cdots + 7$

$\#\phi(r_1 z) = \cdots$

$\#\phi(r_1 r_2 z) = \cdots$

$\#\phi(r_1 \cdots r_{m-1} z) = \cdots$

Verifier sends

r_1

r_2

r_m

accept

Verifier checks

$\#\phi = k$

$\#\phi(0) + \#\phi(1)$

$\#\phi(r_1)$

$\#\phi(r_1 0) + \#\phi(r_1 1)$

$\#\phi(r_1 r_2)$

$\#\phi(r_1 r_2 0) + \#\phi(r_1 r_2 1)$

$\vdots$

$\#\phi(r_1 \cdots r_{m-1})$

$\#\phi(r_1 \cdots r_{m-1} 0) + \#\phi(r_1 \cdots r_{m-1} 1)$

$\#\phi(r_1 \cdots r_m)$

$\|$

$\phi(r_1 \cdots r_m)$

If k is correct, V will accept.

If k is wrong, V probably will reject, whatever P does.

$\#SAT \in$ IP – version 2

Input $\langle \phi, k \rangle$

Prover sends

$\#\phi$

$\#\phi(z) = 3z^d - 5z^{d-1} + \cdots + 7$

$\#\phi(r_1 z) = \cdots$

$\#\phi(r_1 r_2 z) = \cdots$

$\#\phi(r_1 \cdots r_{m-1} z) = \cdots$

Verifier sends

r_1

r_2

r_m

accept

Verifier checks

$\#\phi = k$

$\#\phi = \#\phi(0) + \#\phi(1)$

$\#\phi(r_1)$

$\#\phi(r_1) = \#\phi(r_1 0) + \#\phi(r_1 1)$

$\#\phi(r_1 r_2)$

$\#\phi(r_1 r_2) = \#\phi(r_1 r_2 0) + \#\phi(r_1 r_2 1)$

$\vdots$

$\#\phi(r_1 \cdots r_{m-1})$

$\#\phi(r_1 \cdots r_{m-1}) = \#\phi(r_1 \cdots r_{m-1} 0) + \#\phi(r_1 \cdots r_{m-1} 1)$

$\#\phi(r_1 \cdots r_m) = \phi(r_1 \cdots r_m)$

If k is correct, V will accept.

If k is wrong, V probably will reject, whatever P does.

$\#SAT \in$ IP – version 2

Input $\langle \phi, k \rangle$

Prover sends

$\#\phi$

$\#\phi(z) = 3z^d - 5z^{d-1} + \cdots + 7$

$\#\phi(r_1 z) = \cdots$

$\#\phi(r_1 r_2 z) = \cdots$

$\#\phi(r_1 \cdots r_{m-1} z) = \cdots$

Verifier sends

r_1

r_2

r_m

accept

Verifier checks

$\#\phi = k$

$\#\phi(0) + \#\phi(1)$

$\#\phi(r_1)$

$\#\phi(r_1 0) + \#\phi(r_1 1)$

$\#\phi(r_1 r_2)$

$\#\phi(r_1 r_2 0) + \#\phi(r_1 r_2 1)$

$\vdots$

$\#\phi(r_1 \cdots r_{m-1})$

$\#\phi(r_1 \cdots r_{m-1} 0) + \#\phi(r_1 \cdots r_{m-1} 1)$

$\#\phi(r_1 \cdots r_m) = \phi(r_1 \cdots r_m)$

If k is correct, V will accept.

If k is wrong, V probably will reject, whatever P does.

$\#SAT \in$ IP – version 2

Input $\langle \phi, k \rangle$

Prover sends

$\#\phi$

$\#\phi(z) = 3z^d - 5z^{d-1} + \cdots + 7$

$\#\phi(r_1 z) = \cdots$

$\#\phi(r_1 r_2 z) = \cdots$

$\#\phi(r_1 \cdots r_{m-1} z) = \cdots$

Verifier sends

r_1

r_2

r_m

accept

Verifier checks

$\#\phi = k$

$+$

$\#\phi(0)$ $\#\phi(1)$

$\#\phi(r_1)$

$+$

$\#\phi(r_1 0)$ $\#\phi(r_1 1)$

$\#\phi(r_1 r_2)$

$+$

$\#\phi(r_1 r_2 0)$ $\#\phi(r_1 r_2 1)$

$\vdots$

$\#\phi(r_1 \cdots r_{m-1})$

$+$

$\#\phi(r_1 \cdots r_{m-1} 0)$ $\#\phi(r_1 \cdots r_{m-1} 1)$

$\#\phi(r_1 \cdots r_m)$

$\|$

$\phi(r_1 \cdots r_m)$

If k is correct, V will accept.

If k is wrong, V probably will reject, whatever P does.

$\#SAT \in$ IP – version 2

Input $\langle \phi, k \rangle$

Prover sends

$\#\phi$

$\#\phi(z) = 3z^d - 5z^{d-1} + \cdots + 7$

$\#\phi(r_1 z) = \cdots$

$\#\phi(r_1 r_2 z) = \cdots$

$\#\phi(r_1 \cdots r_{m-1} z) = \cdots$

Verifier sends

r_1

r_2

r_m

accept

Verifier checks

$\#\phi = k$

$\#\phi(0) + \#\phi(1)$

$\#\phi(r_1)$

$\#\phi(r_1 0) + \#\phi(r_1 1)$

$\#\phi(r_1 r_2)$

$\#\phi(r_1 r_2 0) + \#\phi(r_1 r_2 1)$

$\vdots$

$\#\phi(r_1 \cdots r_{m-1})$

$\#\phi(r_1 \cdots r_{m-1} 0) + \#\phi(r_1 \cdots r_{m-1} 1)$

$\#\phi(r_1 \cdots r_m) = \phi(r_1 \cdots r_m)$

If k is correct, V will accept.

If k is wrong, V probably will reject, whatever P does.

$\#SAT \in$ IP – version 2

Input $\langle \phi, k \rangle$

Prover sends

$\#\phi$

$\#\phi(z) = 3z^d - 5z^{d-1} + \cdots + 7$

$\#\phi(r_1 z) = \cdots$

$\#\phi(r_1 r_2 z) = \cdots$

$\#\phi(r_1 \cdots r_{m-1} z) = \cdots$

Verifier sends

r_1

r_2

r_m

accept

Verifier checks

$\#\phi = k$

$+$

$\#\phi(0)$ $\#\phi(1)$

$\#\phi(r_1)$

$+$

$\#\phi(r_1 0)$ $\#\phi(r_1 1)$

$\#\phi(r_1 r_2)$

$+$

$\#\phi(r_1 r_2 0)$ $\#\phi(r_1 r_2 1)$

$\vdots$

$\#\phi(r_1 \cdots r_{m-1})$

$+$

$\#\phi(r_1 \cdots r_{m-1} 0)$ $\#\phi(r_1 \cdots r_{m-1} 1)$

$\#\phi(r_1 \cdots r_m)$

$\|$

$\phi(r_1 \cdots r_m)$

If k is correct, V will accept.

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$\#SAT \in$ IP – version 2

Input $\langle \phi, k \rangle$

Prover sends	**Verifier sends**	**Verifier checks**
$\#\phi$		$\#\phi = k$
$\#\phi(z) = 3z^d - 5z^{d-1} + \cdots + 7$		$\#\phi(0) + \#\phi(1)$
	r_1	$\#\phi(r_1)$
$\#\phi(r_1 z) = \cdots$		$\#\phi(r_1 0) + \#\phi(r_1 1)$
	r_2	$\#\phi(r_1 r_2)$
$\#\phi(r_1 r_2 z) = \cdots$		$\#\phi(r_1 r_2 0) + \#\phi(r_1 r_2 1)$
		$\vdots$
		$\#\phi(r_1 \cdots r_{m-1})$
$\#\phi(r_1 \cdots r_{m-1} z) = \cdots$		$\#\phi(r_1 \cdots r_{m-1} 0) + \#\phi(r_1 \cdots r_{m-1} 1)$
	r_m	$\#\phi(r_1 \cdots r_m)$
	reject	$\neq$ $\phi(r_1 \cdots r_m)$

If k is correct, V will accept.

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$\#SAT \in$ IP – version 2

Input $\langle \phi, k \rangle$

Prover sends

$\#\phi$

$\#\phi(z) = 3z^d - 5z^{d-1} + \cdots + 7$

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$\#\phi(r_1 r_2 z) = \cdots$

$\#\phi(r_1 \cdots r_{m-1} z) = \cdots$

Verifier sends

r_1

r_2

r_m

reject

Verifier checks

$\#\phi = k$

$+$

$\#\phi(0) \qquad \#\phi(1)$

$\#\phi(r_1)$

$+$

$\#\phi(r_1 0) \qquad \#\phi(r_1 1)$

$\#\phi(r_1 r_2)$

$+$

$\#\phi(r_1 r_2 0) \qquad \#\phi(r_1 r_2 1)$

$\vdots$

$\#\phi(r_1 \cdots r_{m-1})$

$+$

$\#\phi(r_1 \cdots r_{m-1} 0) \qquad \#\phi(r_1 \cdots r_{m-1} 1)$

$\#\phi(r_1 \cdots r_m)$

$\|$

$\phi(r_1 \cdots r_m)$

If k is correct, V will accept.

If k is wrong, V probably will reject, whatever P does.

$\#SAT \in$ IP – correctness

Claim: (1) $\langle \phi, k \rangle \in \#SAT \rightarrow$ Pr [(V $\leftrightarrow$ P) accepts $\langle \phi, k \rangle$] $\geq \frac{2}{3}$

(2) $\langle \phi, k \rangle \notin \#SAT \rightarrow$ for any prover $\widetilde{\text{P}}$ Pr [(V $\leftrightarrow$ $\widetilde{\text{P}}$) accepts $\langle \phi, k \rangle$] $\leq \frac{1}{3}$ }

$\#SAT \in$ IP – correctness

Claim: (1) $\langle \phi, k \rangle \in \#SAT \rightarrow$ Pr [(V $\leftrightarrow$ P) accepts $\langle \phi, k \rangle$] $\geq \frac{2}{3}$

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Proof: (1) All Verifier checks are correct (with the honest P) so V always accepts (Pr = 1).

$\#SAT \in$ IP – correctness

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Proof: (1) All Verifier checks are correct (with the honest P) so V always accepts (Pr = 1).

(2) If V accepts then $\#\phi\left(r_1 \cdots r_m\right)$ is correct even though $\#\phi$ is wrong

$\#SAT \in$ IP – correctness

Claim: (1) $\langle \phi, k \rangle \in \#SAT \rightarrow$ Pr [(V ↔ P) accepts $\langle \phi, k \rangle$] $\geq \frac{2}{3}$

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(2) If V accepts then $\#\phi\left(r_1 \cdots r_m\right)$ is correct even though $\#\phi$ is wrong

So $\#\phi\left(r_1 \cdots r_i\right)$ is correct at some first stage i.

$\#SAT \in$ IP – correctness

Claim: (1) $\langle \phi, k \rangle \in \#SAT \to$ Pr [(V ↔ P) accepts $\langle \phi, k \rangle$] $\geq \frac{2}{3}$

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$\vdots$

$\#\phi(r_1 \cdots r_{i-1})$

$\#SAT \in$ IP – correctness

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$\vdots$

$\#\phi(r_1 \cdots r_{i-1})$

$+$

$\#\phi(r_1 \cdots r_{i-1} 0)$ $\quad$ $\#\phi(r_1 \cdots r_{i-1} 1)$

$\#\phi(r_1 \cdots r_i)$

$\vdots$

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$\vdots$

$\#\phi(r_1 \cdots r_{i-1})$

$+$

$\#\phi(r_1 \cdots r_{i-1} z) \quad \longleftarrow \quad \#\phi(r_1 \cdots r_{i-1} 0) \qquad \#\phi(r_1 \cdots r_{i-1} 1)$

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$\vdots$

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$+$

$\#\phi(r_1 \cdots r_{i-1} z) \qquad \#\phi(r_1 \cdots r_{i-1} z) \longleftarrow \#\phi(r_1 \cdots r_{i-1} 0) \qquad \#\phi(r_1 \cdots r_{i-1} 1)$

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$\vdots$

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$$\vdots$$

$$\#\phi(r_1 \cdots r_{i-1})$$

$$+$$

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$$\vdots$$

Pr [agree at random $r_i \in \mathbb{F}_q$] $\leq \frac{\text{degree}}{q} \leq \frac{|\phi|}{2^m}$

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Claim: (1) $\langle \phi, k \rangle \in \#SAT \rightarrow$ Pr [(V $\leftrightarrow$ P) accepts $\langle \phi, k \rangle$] $\geq \frac{2}{3}$

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Pr [agree at random $r_i \in \mathbb{F}_q$] $\leq \frac{\text{degree}}{q} \leq \frac{|\phi|}{2^m}$

Agreement is necessary for $\#\phi(r_1 \cdots r_i)$ to be correct

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So P is "lucky" at stage i if agreement occurs.

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$$+$$

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Pr [agree at <u>some</u> stage i] $\leq \frac{m|\phi|}{2^m} \leq \frac{1}{m}$ for large m

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Claim: (1) $\langle \phi, k \rangle \in \#SAT \rightarrow$ Pr [(V ↔ P) accepts $\langle \phi, k \rangle$] $\geq \frac{2}{3}$

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$$\vdots$$

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$$\#\phi(r_1 \cdots r_{i-1}z) \qquad \#\phi(r_1 \cdots r_{i-1}z) \longleftarrow \#\phi(r_1 \cdots r_{i-1}0) \qquad \#\phi(r_1 \cdots r_{i-1}1)$$

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Pr [agree at <u>some</u> stage i] $\leq \frac{m|\phi|}{2^m} \leq \frac{1}{m}$ for large m

Therefore, for any prover $\widetilde{\text{P}}$ Pr [(V ↔ $\widetilde{\text{P}}$) accepts $\langle \phi, k \rangle$] $\leq \frac{1}{3}$ }

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Quick review of today

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Finished $\#SAT \in$ IP and coNP $\subseteq$ IP

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Additional subjects:

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