بسم الله الرحمن الرحیم

برنامه‌ریزی نیمه‌معین برای طراحی الگوریتم‌های تقریبی

جلسه یازدهم: آیا برنامه‌ریزی هم‌مثبت الگوریتم سریع دارد؟

Project: People

Cone Programming

$$\begin{array}{lll} \text{(P)} & \text{Maximize} & \langle \mathbf{c}, \mathbf{x} \rangle \\ & \text{subject to} & \mathbf{b} - A(\mathbf{x}) \in L \\ & & \mathbf{x} \in K. \end{array}$$

?

الگوریتم سریع

SDP

$$\begin{array}{ll} \text{maximize} & C \bullet X \\ \text{subject to} & A_i \bullet X = b_i, \quad i = 1, 2, \ldots, m \\ & X \succeq 0. \end{array}$$

الگوریتم سریع

LP

$$\begin{array}{ll} \text{maximize} & c^\top x \\ \text{subject to} & Ax = b \\ & x \geq 0 \end{array}$$

الگوریتم سریع

7.1.1 Definition. *A matrix $M \in \mathrm{SYM}_n$ is called copositive if*

$$\mathbf{x}^T M \mathbf{x} \geq 0 \text{ for all } \mathbf{x} \geq 0.$$

$$\mathrm{COP}_n := \{M \in \mathrm{SYM}_n : \mathbf{x}^T M \mathbf{x} \geq 0 \text{ for all } \mathbf{x} \geq 0\}$$

$$\mathrm{POS}_n \subseteq \mathrm{PSD}_n \subseteq \mathrm{COP}_n$$

7.1.4 Definition. *A matrix $M \in \mathrm{SYM}_n$ is called completely positive if for some ℓ, there are ℓ nonnegative vectors $\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_\ell \in \mathbb{R}^n_+$, such that*

$$M = \sum_{i=1}^{\ell} \mathbf{x}_i \mathbf{x}_i^T = AA^T, \tag{7.2}$$

where $A \in \mathbb{R}^{n \times \ell}$ is the (nonnegative) matrix with columns $\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_\ell$.

$$\mathrm{POS}_n := \{M \in \mathrm{SYM}_n : M \text{ is completely positive}\}$$

7.1.1 Definition. *A matrix $M \in \mathrm{SYM}_n$ is called copositive if*

$$\mathbf{x}^T M \mathbf{x} \geq 0 \text{ for all } \mathbf{x} \geq 0.$$

$$\mathrm{COP}_n := \{M \in \mathrm{SYM}_n : \mathbf{x}^T M \mathbf{x} \geq 0 \text{ for all } \mathbf{x} \geq 0\}$$

7.1.7 Theorem. $\mathrm{POS}_n^* = \mathrm{COP}_n$.

7.1.4 Definition. *A matrix $M \in \mathrm{SYM}_n$ is called completely positive if for some ℓ, there are ℓ nonnegative vectors $\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_\ell \in \mathbb{R}_+^n$, such that*

$$M = \sum_{i=1}^{\ell} \mathbf{x}_i \mathbf{x}_i^T = AA^T, \tag{7.2}$$

where $A \in \mathbb{R}^{n \times \ell}$ is the (nonnegative) matrix with columns $\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_\ell$.

$$\mathrm{POS}_n := \{M \in \mathrm{SYM}_n : M \text{ is completely positive}\}$$

برنامه‌ریزی هم‌مثبت برای یک مسئله سخت!

Cone Programming

$$\text{(P)} \quad \begin{array}{ll} \text{Maximize} & \langle \mathbf{c}, \mathbf{x} \rangle \\ \text{subject to} & \mathbf{b} - A(\mathbf{x}) \in L \\ & \mathbf{x} \in K. \end{array}$$

?

الگوریتم سریع

برنامه‌ریزی هم‌مثبت

$$\begin{array}{ll} \text{maximize} & C \bullet X \\ \text{subject to} & A(X) = b \\ & X \in \mathrm{COP}_n \end{array}$$

SDP

$$\begin{array}{ll} \text{maximize} & C \bullet X \\ \text{subject to} & A_i \bullet X = b_i, \quad i = 1, 2, \ldots, m \\ & X \succeq 0. \end{array}$$

✓ الگوریتم سریع

برنامه‌ریزی کاملا مثبت

$$\begin{array}{ll} \text{maximize} & C \bullet X \\ \text{subject to} & A(X) = b \\ & X \in \mathrm{POS}_n \end{array}$$

LP

$$\begin{array}{ll} \text{maximize} & c^\top x \\ \text{subject to} & Ax = b \\ & x \ge 0 \end{array}$$

✓ الگوریتم سریع

بیشترین نرخ ارسال با گراف G:

$$\sigma(G) = \sup\left\{\frac{1}{k}\log\alpha(G^k) : k \in \mathbb{N}\right\},$$

$$\Theta(G) = 2^{\sigma(G)} = \lim_{k\to\infty} \sqrt[k]{\alpha(G^k)}$$

$$\vartheta(G) := \min_{\mathcal{U}} \min_{\|\mathbf{c}\|=1} \max_{i=1}^{n} \frac{1}{(\mathbf{c}^T\mathbf{u}_i)^2}$$

قضیه:

$$\Theta(G) \leq \vartheta(G)$$

قضیه: برنامه‌ریزی زیر $\vartheta(G)$ را محاسبه می‌کند

$$\begin{array}{lll} \textit{Minimize} & t & \\ \textit{subject to} & y_{ij} = -1 & \textit{if } \{i,j\} \in \overline{E} \\ & y_{ii} = t-1 & \textit{for all } i = 1,\ldots,n \\ & Y \succeq 0. & \end{array}$$

7.2.1 Theorem. *The copositive program*

$$\begin{array}{ll} \textit{minimize} & t \\ \textit{subject to} & y_{ij} = -1, \ \textit{if } \{i,j\} \in \overline{E} \\ & y_{ii} = t-1, \ \textit{for all } i = 1,2,\ldots,n \\ & Y \in \mathrm{COP}_n \end{array}$$

has value $\alpha(G)$, the size of a maximum independent set in G.

7.2.1 Theorem. *The copositive program*

$$\begin{array}{ll} \text{minimize} & t \\ \text{subject to} & y_{ij} = -1, \ \text{if } \{i,j\} \in \overline{E} \\ & y_{ii} = t-1, \ \text{for all } i = 1,2,\ldots,n \\ & Y \in \mathrm{COP}_n \end{array}$$

has value $\alpha(G)$, the size of a maximum independent set in G.

$\alpha(G) \leq$

7.2.1 Theorem. *The copositive program*

$$\begin{array}{ll} \textit{minimize} & t \\ \textit{subject to} & y_{ij} = -1, \ \textit{if } \{i,j\} \in \overline{E} \\ & y_{ii} = t - 1, \ \textit{for all } i = 1, 2, \ldots, n \\ & Y \in \mathrm{COP}_n \end{array}$$

has value $\alpha(G)$, *the size of a maximum independent set in* G.

$\alpha(G) \leq$

7.2.1 Theorem. *The copositive program*

$$\begin{array}{ll} \textit{minimize} & t \\ \textit{subject to} & y_{ij} = -1, \ \textit{if } \{i,j\} \in \overline{E} \\ & y_{ii} = t - 1, \ \textit{for all } i = 1, 2, \ldots, n \\ & Y \in \mathrm{COP}_n \end{array}$$

has value $\alpha(G)$, the size of a maximum independent set in G.

$\leq \alpha(G)$

$\alpha(G) \leq$

7.2.1 Theorem. *The copositive program*

$$
\begin{array}{ll}
\textit{minimize} & t \\
\textit{subject to} & y_{ij} = -1, \ \textit{if } \{i,j\} \in \overline{E} \\
 & y_{ii} = t - 1, \ \textit{for all } i = 1, 2, \ldots, n \\
 & Y \in \mathrm{COP}_n
\end{array}
$$

has value $\alpha(G)$, the size of a maximum independent set in G.

$\alpha(G) \leq$

7.2.1 Theorem. *The copositive program*

$$\begin{array}{ll} \text{minimize} & t \\ \text{subject to} & y_{ij} = -1, \text{ if } \{i,j\} \in \overline{E} \\ & y_{ii} = t - 1, \text{ for all } i = 1, 2, \ldots, n \\ & Y \in \mathrm{COP}_n \end{array}$$

has value $\alpha(G)$, the size of a maximum independent set in G.

- روش: یک جواب شدنی برای دوگان با مقدار $\alpha(G)$

$\alpha(G) \leq$

7.2.1 Theorem. *The copositive program*

$$
\begin{array}{ll}
\text{minimize} & t \\
\text{subject to} & y_{ij} = -1, \text{ if } \{i,j\} \in \overline{E} \\
& y_{ii} = t - 1, \text{ for all } i = 1, 2, \ldots, n \\
& Y \in \mathrm{COP}_n
\end{array}
$$

has value $\alpha(G)$, the size of a maximum independent set in G.

- روش: یک جواب شدنی برای دوگان با مقدار $\alpha(G)$

$$
\begin{array}{lll}
\text{(P)} & \text{Maximize} & \langle \mathbf{c}, \mathbf{x} \rangle \\
& \text{subject to} & \mathbf{b} - A(\mathbf{x}) \in L \\
& & \mathbf{x} \in K.
\end{array}
$$

$$
\begin{array}{lll}
\text{(D)} & \text{Minimize} & \langle \mathbf{b}, \mathbf{y} \rangle \\
& \text{subject to} & A^T(\mathbf{y}) - \mathbf{c} \in K^* \\
& & \mathbf{y} \in L^*.
\end{array}
$$

$\alpha(G) \leq$

7.2.1 Theorem. *The copositive program*

$$\begin{array}{ll} \textit{minimize} & t \\ \textit{subject to} & y_{ij} = -1, \ \textit{if} \ \{i,j\} \in \overline{E} \\ & y_{ii} = t-1, \ \textit{for all} \ i = 1,2,\ldots,n \\ & Y \in \mathrm{COP}_n \end{array}$$

has value $\alpha(G)$, the size of a maximum independent set in G.

- روش: یک جواب شدنی برای دوگان با مقدار $\alpha(G)$

$$\begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix}$$

$$\begin{array}{lll} \text{(P)} & \text{Maximize} & \langle \mathbf{c}, \mathbf{x} \rangle \\ & \text{subject to} & \mathbf{b} - A(\mathbf{x}) \in L \\ & & \mathbf{x} \in K. \end{array}$$

$$\begin{array}{lll} \text{(D)} & \text{Minimize} & \langle \mathbf{b}, \mathbf{y} \rangle \\ & \text{subject to} & A^T(\mathbf{y}) - \mathbf{c} \in K^* \\ & & \mathbf{y} \in L^*. \end{array}$$

$\alpha(G) \leq$

7.2.1 Theorem. *The copositive program*

$$\begin{array}{ll} \text{minimize} & t \\ \text{subject to} & y_{ij} = -1, \text{ if } \{i,j\} \in \overline{E} \\ & y_{ii} = t-1, \text{ for all } i = 1,2,\ldots,n \\ & Y \in \mathrm{COP}_n \end{array}$$

has value $\alpha(G)$, the size of a maximum independent set in G.

- روش: یک جواب شدنی برای دوگان با مقدار $\alpha(G)$

$$\min \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix}$$

$$\begin{array}{lll} \text{(P)} & \text{Maximize} & \langle \mathbf{c}, \mathbf{x} \rangle \\ & \text{subject to} & \mathbf{b} - A(\mathbf{x}) \in L \\ & & \mathbf{x} \in K. \end{array}$$

$$\begin{array}{lll} \text{(D)} & \text{Minimize} & \langle \mathbf{b}, \mathbf{y} \rangle \\ & \text{subject to} & A^T(\mathbf{y}) - \mathbf{c} \in K^* \\ & & \mathbf{y} \in L^*. \end{array}$$

$\alpha(G) \leq$

7.2.1 Theorem. *The copositive program*

$$\begin{array}{ll} \textit{minimize} & t \\ \textit{subject to} & y_{ij} = -1, \ \textit{if } \{i,j\} \in \overline{E} \\ & y_{ii} = t - 1, \ \textit{for all } i = 1, 2, \ldots, n \\ & Y \in \mathrm{COP}_n \end{array}$$

has value $\alpha(G)$, the size of a maximum independent set in G.

- روش: یک جواب شدنی برای دوگان با مقدار $\alpha(G)$

$$\min \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix}$$

$$\begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix} \bullet \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,i):1 & 0 \\ 0 & 0 & -1 \end{pmatrix} = -1$$

$$\begin{array}{lll} \text{(P)} & \text{Maximize} & \langle \mathbf{c}, \mathbf{x} \rangle \\ & \text{subject to} & \mathbf{b} - A(\mathbf{x}) \in L \\ & & \mathbf{x} \in K. \end{array}$$

$$\begin{array}{lll} \text{(D)} & \text{Minimize} & \langle \mathbf{b}, \mathbf{y} \rangle \\ & \text{subject to} & A^T(\mathbf{y}) - \mathbf{c} \in K^* \\ & & \mathbf{y} \in L^*. \end{array}$$

$\alpha(G) \leq$

7.2.1 Theorem. *The copositive program*

$$\begin{array}{ll} \text{minimize} & t \\ \text{subject to} & y_{ij} = -1, \text{ if } \{i,j\} \in \overline{E} \\ & y_{ii} = t-1, \text{ for all } i = 1,2,\ldots,n \\ & Y \in \mathrm{COP}_n \end{array}$$

has value $\alpha(G)$, the size of a maximum independent set in G.

- روش: یک جواب شدنی برای دوگان با مقدار $\alpha(G)$

$$\min \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix}$$

$$\begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix} \bullet \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,i):1 & 0 \\ 0 & 0 & -1 \end{pmatrix} = -1$$

$ij \in \bar{E}$

$$\begin{array}{lll} \text{(P)} & \text{Maximize} & \langle \mathbf{c}, \mathbf{x} \rangle \\ & \text{subject to} & \mathbf{b} - A(\mathbf{x}) \in L \\ & & \mathbf{x} \in K. \end{array}$$

$$\begin{array}{lll} \text{(D)} & \text{Minimize} & \langle \mathbf{b}, \mathbf{y} \rangle \\ & \text{subject to} & A^T(\mathbf{y}) - \mathbf{c} \in K^* \\ & & \mathbf{y} \in L^*. \end{array}$$

$\alpha(G) \leq$

7.2.1 Theorem. *The copositive program*

$$\begin{array}{ll} \text{minimize} & t \\ \text{subject to} & y_{ij} = -1, \text{ if } \{i,j\} \in \overline{E} \\ & y_{ii} = t-1, \text{ for all } i = 1,2,\ldots,n \\ & Y \in \mathrm{COP}_n \end{array}$$

has value $\alpha(G)$, the size of a maximum independent set in G.

- روش: یک جواب شدنی برای دوگان با مقدار $\alpha(G)$

$$\min \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix}$$

$$\begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix} \bullet \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,i):1 & 0 \\ 0 & 0 & -1 \end{pmatrix} = -1$$

$$\begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix} \bullet \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,j):1 & 0 \\ 0 & 0 & 0 \end{pmatrix} = -1$$

$$ij \in \bar{E}$$

$$\begin{array}{lll} \text{(P)} & \text{Maximize} & \langle \mathbf{c}, \mathbf{x} \rangle \\ & \text{subject to} & \mathbf{b} - A(\mathbf{x}) \in L \\ & & \mathbf{x} \in K. \end{array}$$

$$\begin{array}{lll} \text{(D)} & \text{Minimize} & \langle \mathbf{b}, \mathbf{y} \rangle \\ & \text{subject to} & A^T(\mathbf{y}) - \mathbf{c} \in K^* \\ & & \mathbf{y} \in L^*. \end{array}$$

$\alpha(G) \leq$

7.2.1 Theorem. *The copositive program*

$$\begin{array}{ll} \textit{minimize} & t \\ \textit{subject to} & y_{ij} = -1, \ \textit{if } \{i,j\} \in \overline{E} \\ & y_{ii} = t-1, \ \textit{for all } i = 1,2,\ldots,n \\ & Y \in \mathrm{COP}_n \end{array}$$

has value $\alpha(G)$, the size of a maximum independent set in G.

- روش: یک جواب شدنی برای دوگان با مقدار $\alpha(G)$

$$\min \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix}$$

$$\begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix} \bullet \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,i):1 & 0 \\ 0 & 0 & -1 \end{pmatrix} = -1$$

$$\begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix} \bullet \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,j):1 & 0 \\ 0 & 0 & 0 \end{pmatrix} = -1$$

$$ij \in \bar{E}$$

$$Y \in \mathrm{COP}_n$$

$$\begin{array}{lll} \text{(P)} & \text{Maximize} & \langle \mathbf{c}, \mathbf{x} \rangle \\ & \text{subject to} & \mathbf{b} - A(\mathbf{x}) \in L \\ & & \mathbf{x} \in K. \end{array}$$

$$\begin{array}{lll} \text{(D)} & \text{Minimize} & \langle \mathbf{b}, \mathbf{y} \rangle \\ & \text{subject to} & A^T(\mathbf{y}) - \mathbf{c} \in K^* \\ & & \mathbf{y} \in L^*. \end{array}$$

$$
\begin{array}{llll}
(\mathrm{P}) & \text{Maximize} & \langle \mathbf{c}, \mathbf{x} \rangle \\
 & \text{subject to} & \mathbf{b} - A(\mathbf{x}) \in L \\
 & & \mathbf{x} \in K.
\end{array}
$$

$$
\begin{array}{llll}
(\mathrm{D}) & \text{Minimize} & \langle \mathbf{b}, \mathbf{y} \rangle \\
 & \text{subject to} & A^T(\mathbf{y}) - \mathbf{c} \in K^* \\
 & & \mathbf{y} \in L^*.
\end{array}
$$

$$
\max \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} \bullet \begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix}
$$

$$
\begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix} \bullet \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,i):1 & 0 \\ 0 & 0 & -1 \end{pmatrix} = -1
$$

$$
\begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix} \bullet \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,j):1 & 0 \\ 0 & 0 & 0 \end{pmatrix} = -1 \quad ij \in \bar{E}
$$

$$
Y \in \mathrm{COP}_n
$$

$$\begin{array}{llll} \text{(P)} & \text{Maximize} & \langle \mathbf{c}, \mathbf{x} \rangle \\ & \text{subject to} & \mathbf{b} - A(\mathbf{x}) \in L \\ & & \mathbf{x} \in K. \end{array}$$

$$\begin{array}{llll} \text{(D)} & \text{Minimize} & \langle \mathbf{b}, \mathbf{y} \rangle \\ & \text{subject to} & A^T(\mathbf{y}) - \mathbf{c} \in K^* \\ & & \mathbf{y} \in L^*. \end{array}$$

$$\max \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} \bullet \begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix}$$

$$\begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix} \bullet \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,i):1 & 0 \\ 0 & 0 & -1 \end{pmatrix} = -1$$

$$\begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix} \bullet \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,j):1 & 0 \\ 0 & 0 & 0 \end{pmatrix} = -1 \quad ij \in \bar{E}$$

$$Y \in \mathrm{COP}_n$$

$$\in \mathrm{POS}_n$$

$$\begin{array}{lll} \text{(P)} & \text{Maximize} & \langle \mathbf{c}, \mathbf{x} \rangle \\ & \text{subject to} & \mathbf{b} - A(\mathbf{x}) \in L \\ & & \mathbf{x} \in K. \end{array}$$

$$\begin{array}{lll} \text{(D)} & \text{Minimize} & \langle \mathbf{b}, \mathbf{y} \rangle \\ & \text{subject to} & A^T(\mathbf{y}) - \mathbf{c} \in K^* \\ & & \mathbf{y} \in L^*. \end{array}$$

$$\max \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} \bullet \begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix}$$

$$\begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix} \bullet \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,i):1 & 0 \\ 0 & 0 & -1 \end{pmatrix} = -1$$

$$\begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix} \bullet \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,j):1 & 0 \\ 0 & 0 & 0 \end{pmatrix} = -1$$

$$ij \in \bar{E}$$

$$Y \in \mathrm{COP}_n$$

$$\sum_i x_{ii} \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,i):1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$\in \mathrm{POS}_n$$

$$\begin{array}{lll} \text{(P)} & \text{Maximize} & \langle \mathbf{c}, \mathbf{x} \rangle \\ & \text{subject to} & \mathbf{b} - A(\mathbf{x}) \in L \\ & & \mathbf{x} \in K. \end{array}$$

$$\begin{array}{lll} \text{(D)} & \text{Minimize} & \langle \mathbf{b}, \mathbf{y} \rangle \\ & \text{subject to} & A^T(\mathbf{y}) - \mathbf{c} \in K^* \\ & & \mathbf{y} \in L^*. \end{array}$$

$$\max \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} \bullet \begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix}$$

$$\begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix} \bullet \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,i):1 & 0 \\ 0 & 0 & -1 \end{pmatrix} = -1$$

$$\begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix} \bullet \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,j):1 & 0 \\ 0 & 0 & 0 \end{pmatrix} = -1 \quad ij \in \bar{E}$$

$$Y \in \mathrm{COP}_n$$

$$\sum_i x_{ii} \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,i):1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$\sum_{ij \in \bar{E}} x_{ij} \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,j):1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \in \mathrm{POS}_n$$

$$
\begin{array}{llll}
\text{(P)} & \text{Maximize} & \langle \mathbf{c}, \mathbf{x} \rangle \\
& \text{subject to} & \mathbf{b} - A(\mathbf{x}) \in L \\
& & \mathbf{x} \in K.
\end{array}
$$

$$
\begin{array}{llll}
\text{(D)} & \text{Minimize} & \langle \mathbf{b}, \mathbf{y} \rangle \\
& \text{subject to} & A^T(\mathbf{y}) - \mathbf{c} \in K^* \\
& & \mathbf{y} \in L^*.
\end{array}
$$

$$
\max \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} \bullet \begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix}
$$

$$
\begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix} \bullet \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,i):1 & 0 \\ 0 & 0 & -1 \end{pmatrix} = -1
$$

$$
\begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix} \bullet \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,j):1 & 0 \\ 0 & 0 & 0 \end{pmatrix} = -1 \quad ij \in \bar{E}
$$

$$
Y \in \mathrm{COP}_n
$$

$$
\sum_i x_{ii} \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,i):1 & 0 \\ 0 & 0 & -1 \end{pmatrix}
$$

$$
\sum_{ij \in \bar{E}} x_{ij} \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,j):1 & 0 \\ 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} \in \mathrm{POS}_n
$$

$$
\begin{array}{llll}
\text{(P)} & \text{Maximize} & \langle \mathbf{c}, \mathbf{x} \rangle \\
& \text{subject to} & \mathbf{b} - A(\mathbf{x}) \in L \\
& & \mathbf{x} \in K.
\end{array}
$$

$$
\begin{array}{llll}
\text{(D)} & \text{Minimize} & \langle \mathbf{b}, \mathbf{y} \rangle \\
& \text{subject to} & A^T(\mathbf{y}) - \mathbf{c} \in K^* \\
& & \mathbf{y} \in L^*.
\end{array}
$$

$$
\max \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} \bullet \begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix}
$$

$$
\begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix} \bullet \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,i):1 & 0 \\ 0 & 0 & -1 \end{pmatrix} = -1
$$

$$
\begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix} \bullet \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,j):1 & 0 \\ 0 & 0 & 0 \end{pmatrix} = -1 \quad ij \in \bar{E}
$$

$$
Y \in \mathrm{COP}_n
$$

$$
\sum_i x_{ii} \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,i):1 & 0 \\ 0 & 0 & -1 \end{pmatrix} + \sum_{ij \in \bar{E}} x_{ij} \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,j):1 & 0 \\ 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} \in \mathrm{POS}_n
$$

$$
\begin{array}{lll}
\text{(P)} & \text{Maximize} & \langle \mathbf{c}, \mathbf{x} \rangle \\
& \text{subject to} & \mathbf{b} - A(\mathbf{x}) \in L \\
& & \mathbf{x} \in K.
\end{array}
$$

$$
\begin{array}{lll}
\text{(D)} & \text{Minimize} & \langle \mathbf{b}, \mathbf{y} \rangle \\
& \text{subject to} & A^T(\mathbf{y}) - \mathbf{c} \in K^* \\
& & \mathbf{y} \in L^*.
\end{array}
$$

$$
\max \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} \bullet \begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix}
$$

$$
\begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix} \bullet \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,i):1 & 0 \\ 0 & 0 & -1 \end{pmatrix} = -1
$$

$$
\begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix} \bullet \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,j):1 & 0 \\ 0 & 0 & 0 \end{pmatrix} = -1 \quad ij \in \bar{E}
$$

$$
Y \in \mathrm{COP}_n
$$

$$
\sum_{ij \in \bar{E}} -x_{ij} + \sum_i -x_{ii}
$$

$$
\sum_i x_{ii} \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,i):1 & 0 \\ 0 & 0 & -1 \end{pmatrix} +
$$

$$
\sum_{ij \in \bar{E}} x_{ij} \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,j):1 & 0 \\ 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} \in \mathrm{POS}_n
$$

$$\begin{array}{llc} \text{(P)} & \text{Maximize} & \langle \mathbf{c}, \mathbf{x} \rangle \\ & \text{subject to} & \mathbf{b} - A(\mathbf{x}) \in L \\ & & \mathbf{x} \in K. \end{array}$$

$$\begin{array}{llc} \text{(D)} & \text{Minimize} & \langle \mathbf{b}, \mathbf{y} \rangle \\ & \text{subject to} & A^T(\mathbf{y}) - \mathbf{c} \in K^* \\ & & \mathbf{y} \in L^*. \end{array}$$

$$\max \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} \bullet \begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix}$$

$$\begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix} \bullet \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,i):1 & 0 \\ 0 & 0 & -1 \end{pmatrix} = -1$$

$$\begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix} \bullet \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,j):1 & 0 \\ 0 & 0 & 0 \end{pmatrix} = -1 \quad ij \in \bar{E}$$

$$Y \in \mathrm{COP}_n$$

$$\min \sum_{ij \in \bar{E}} -x_{ij} + \sum_i -x_{ii}$$

$$\sum_i x_{ii} \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,i):1 & 0 \\ 0 & 0 & -1 \end{pmatrix} + \sum_{ij \in \bar{E}} x_{ij} \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,j):1 & 0 \\ 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} \in \mathrm{POS}_n$$

$$\begin{array}{lll} \text{(P)} & \text{Maximize} & \langle \mathbf{c}, \mathbf{x} \rangle \\ & \text{subject to} & \mathbf{b} - A(\mathbf{x}) \in L \\ & & \mathbf{x} \in K. \end{array}$$

$$\begin{array}{lll} \text{(D)} & \text{Minimize} & \langle \mathbf{b}, \mathbf{y} \rangle \\ & \text{subject to} & A^T(\mathbf{y}) - \mathbf{c} \in K^* \\ & & \mathbf{y} \in L^*. \end{array}$$

$$\min \sum_{ij \in \bar{E}} - x_{ij} + \sum_i - x_{ii}$$

$$\sum_i x_{ii} \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,i):1 & 0 \\ 0 & 0 & -1 \end{pmatrix} + \sum_{ij \in \bar{E}} x_{ij} \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,j):1 & 0 \\ 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} \in \mathrm{POS}_n$$

$$\begin{array}{lll}(\mathrm{P}) & \text{Maximize} & \langle \mathbf{c}, \mathbf{x} \rangle \\ & \text{subject to} & \mathbf{b} - A(\mathbf{x}) \in L \\ & & \mathbf{x} \in K.\end{array}$$

$$\begin{array}{lll}(\mathrm{D}) & \text{Minimize} & \langle \mathbf{b}, \mathbf{y} \rangle \\ & \text{subject to} & A^T(\mathbf{y}) - \mathbf{c} \in K^* \\ & & \mathbf{y} \in L^*.\end{array}$$

قید آخر:

$$\min \sum_{ij \in \bar{E}} - x_{ij} + \sum_{i} - x_{ii}$$

$$\sum_i x_{ii} \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,i) : 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} +$$

$$\sum_{ij \in \bar{E}} x_{ij} \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,j) : 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} \in \mathrm{POS}_n$$

$$
\begin{array}{lll}
(\mathrm{P}) & \text{Maximize} & \langle \mathbf{c}, \mathbf{x} \rangle \\
& \text{subject to} & \mathbf{b} - A(\mathbf{x}) \in L \\
& & \mathbf{x} \in K.
\end{array}
$$

$$
\begin{array}{lll}
(\mathrm{D}) & \text{Minimize} & \langle \mathbf{b}, \mathbf{y} \rangle \\
& \text{subject to} & A^T(\mathbf{y}) - \mathbf{c} \in K^* \\
& & \mathbf{y} \in L^*.
\end{array}
$$

قید آخر:

$$\sum_i -x_{ii} + 1 \geq 0$$

$$\min \sum_{ij \in \bar{E}} -x_{ij} + \sum_i -x_{ii}$$

$$\sum_i x_{ii} \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,i):1 & 0 \\ 0 & 0 & -1 \end{pmatrix} +$$

$$\sum_{ij \in \bar{E}} x_{ij} \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,j):1 & 0 \\ 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} \in \mathrm{POS}_n$$

$$
\begin{array}{llll}
(\mathrm{P}) & \text{Maximize} & \langle \mathbf{c}, \mathbf{x} \rangle \\
& \text{subject to} & \mathbf{b} - A(\mathbf{x}) \in L \\
& & \mathbf{x} \in K.
\end{array}
$$

$$
\begin{array}{llll}
(\mathrm{D}) & \text{Minimize} & \langle \mathbf{b}, \mathbf{y} \rangle \\
& \text{subject to} & A^T(\mathbf{y}) - \mathbf{c} \in K^* \\
& & \mathbf{y} \in L^*.
\end{array}
$$

قید آخر:

$$\sum_i -x_{ii} + 1 \geq 0$$

$$\sum_i x_{ii} \leq 1$$

$$\min \sum_{ij \in \bar{E}} -x_{ij} + \sum_i -x_{ii}$$

$$\sum_i x_{ii} \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,i):1 & 0 \\ 0 & 0 & -1 \end{pmatrix} +$$

$$\sum_{ij \in \bar{E}} x_{ij} \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,j):1 & 0 \\ 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} \in \mathrm{POS}_n$$

$$\begin{array}{lll} \text{(P)} & \text{Maximize} & \langle \mathbf{c}, \mathbf{x} \rangle \\ & \text{subject to} & \mathbf{b} - A(\mathbf{x}) \in L \\ & & \mathbf{x} \in K. \end{array}$$

$$\begin{array}{lll} \text{(D)} & \text{Minimize} & \langle \mathbf{b}, \mathbf{y} \rangle \\ & \text{subject to} & A^T(\mathbf{y}) - \mathbf{c} \in K^* \\ & & \mathbf{y} \in L^*. \end{array}$$

قید آخر:

$$\sum_i - x_{ii} + 1 \geq 0$$

$$\sum_i x_{ii} \leq 1$$

$$\min \sum_{ij \in \bar{E}} - x_{ij} + \sum_i - x_{ii}$$

$$\sum_i x_{ii} \begin{pmatrix} 0 & 0 \\ 0 & (i,i):1 \end{pmatrix} \quad +$$

$$\sum_{ij \in \bar{E}} x_{ij} \begin{pmatrix} 0 & 0 \\ 0 & (i,j):1 \end{pmatrix} \quad - \quad (0) \qquad \in \mathrm{POS}_n$$

$$\sum_i x_{ii} \leq 1$$

$$
\begin{array}{lll}
\text{(P)} & \text{Maximize} & \langle \mathbf{c}, \mathbf{x} \rangle \\
& \text{subject to} & \mathbf{b} - A(\mathbf{x}) \in L \\
& & \mathbf{x} \in K.
\end{array}
$$

$$
\begin{array}{lll}
\text{(D)} & \text{Minimize} & \langle \mathbf{b}, \mathbf{y} \rangle \\
& \text{subject to} & A^T(\mathbf{y}) - \mathbf{c} \in K^* \\
& & \mathbf{y} \in L^*.
\end{array}
$$

$$
\min \sum_{ij \in \bar{E}} -x_{ij} + \sum_i -x_{ii}
$$

$$
\max \sum_{ij \in \bar{E}} x_{ij} + \sum_i x_{ii}
$$

$$
\sum_i x_{ii} \begin{pmatrix} 0 & 0 \\ 0 & (i,i):1 \end{pmatrix} \quad +
$$

$$
\sum_{ij \in \bar{E}} x_{ij} \begin{pmatrix} 0 & 0 \\ 0 & (i,j):1 \end{pmatrix} \quad - \quad (0) \qquad \in \mathrm{POS}_n
$$

$$
\sum_i x_{ii} \leq 1
$$

$$
\begin{array}{lll}
\text{(P)} & \text{Maximize} & \langle \mathbf{c}, \mathbf{x} \rangle \\
& \text{subject to} & \mathbf{b} - A(\mathbf{x}) \in L \\
& & \mathbf{x} \in K.
\end{array}
$$

$$
\begin{array}{lll}
\text{(D)} & \text{Minimize} & \langle \mathbf{b}, \mathbf{y} \rangle \\
& \text{subject to} & A^T(\mathbf{y}) - \mathbf{c} \in K^* \\
& & \mathbf{y} \in L^*.
\end{array}
$$

$$
\max \quad J_n \bullet X
$$

$$
X \in \mathrm{POS}_n
$$

$$
\mathrm{Tr}(X) \leq 1
$$

$$
\max \sum_{ij \in \bar{E}} x_{ij} + \sum_i x_{ii}
$$

$$
\sum_i x_{ii} \begin{pmatrix} 0 & 0 \\ 0 & (i,i):1 \end{pmatrix} \quad +
$$

$$
\sum_{ij \in \bar{E}} x_{ij} \begin{pmatrix} 0 & 0 \\ 0 & (i,j):1 \end{pmatrix} \quad - \quad (0) \qquad \in \mathrm{POS}_n
$$

$$
\sum_i x_{ii} \leq 1
$$

$$
\begin{array}{lll}
\text{(P)} & \text{Maximize} & \langle \mathbf{c}, \mathbf{x} \rangle \\
& \text{subject to} & \mathbf{b} - A(\mathbf{x}) \in L \\
& & \mathbf{x} \in K.
\end{array}
$$

$$
\begin{array}{lll}
\text{(D)} & \text{Minimize} & \langle \mathbf{b}, \mathbf{y} \rangle \\
& \text{subject to} & A^T(\mathbf{y}) - \mathbf{c} \in K^* \\
& & \mathbf{y} \in L^*.
\end{array}
$$

$$
\max \quad J_n \bullet X
$$

$$
X \in \mathrm{POS}_n
$$

$$
\mathrm{Tr}(X) \leq 1
$$

$$
\max \sum_{ij \in \bar{E}} x_{ij} + \sum_i x_{ii}
$$

$$
\sum_i x_{ii} \begin{pmatrix} 0 & 0 \\ 0 & (i,i):1 \end{pmatrix} \quad +
$$

$$
\sum_{ij \in \bar{E}} x_{ij} \begin{pmatrix} 0 & 0 \\ 0 & (i,j):1 \end{pmatrix} \quad - \quad (0) \qquad \in \mathrm{POS}_n
$$

$$
\sum_i x_{ii} \leq 1
$$

$$
x_{ij} = 0 \qquad ij \in E
$$

$$
\begin{array}{lll}
\text{(P)} & \text{Maximize} & \langle \mathbf{c}, \mathbf{x} \rangle \\
& \text{subject to} & \mathbf{b} - A(\mathbf{x}) \in L \\
& & \mathbf{x} \in K.
\end{array}
$$

$$
\begin{array}{lll}
\text{(D)} & \text{Minimize} & \langle \mathbf{b}, \mathbf{y} \rangle \\
& \text{subject to} & A^T(\mathbf{y}) - \mathbf{c} \in K^* \\
& & \mathbf{y} \in L^*.
\end{array}
$$

$$\max \quad J_n \bullet X$$

$$X \in \mathrm{POS}_n$$

$$\mathrm{Tr}(X) \leq 1$$

$$\max \sum_{ij \in \bar{E}} x_{ij} + \sum_i x_{ii} + \sum_{ij \in E} x_{ij}$$

$$\sum_i x_{ii} \begin{pmatrix} 0 & 0 \\ 0 & (i,i):1 \end{pmatrix} \quad +$$

$$\sum_{ij \in \bar{E}} x_{ij} \begin{pmatrix} 0 & 0 \\ 0 & (i,j):1 \end{pmatrix} \quad - \quad (0) \qquad \in \mathrm{POS}_n$$

$$\sum_i x_{ii} \leq 1$$

$$x_{ij} = 0 \qquad ij \in E$$

$$\begin{array}{lll} \text{(P)} & \text{Maximize} & \langle \mathbf{c}, \mathbf{x} \rangle \\ & \text{subject to} & \mathbf{b} - A(\mathbf{x}) \in L \\ & & \mathbf{x} \in K. \end{array}$$

$$\begin{array}{lll} \text{(D)} & \text{Minimize} & \langle \mathbf{b}, \mathbf{y} \rangle \\ & \text{subject to} & A^T(\mathbf{y}) - \mathbf{c} \in K^* \\ & & \mathbf{y} \in L^*. \end{array}$$

$$\max \quad J_n \bullet X$$

$$X \in \mathrm{POS}_n$$

$$\mathrm{Tr}(X) \leq 1$$

$$\max \sum_{ij \in \bar{E}} x_{ij} + \sum_{i} x_{ii} + \sum_{ij \in E} x_{ij}$$

$$\sum_i x_{ii} \begin{pmatrix} 0 & 0 \\ 0 & (i,i) : 1 \end{pmatrix} + \sum_{ij \in E} x_{ij} \begin{pmatrix} 0 & 0 \\ 0 & (i,j) : 1 \end{pmatrix}$$

$$\sum_{ij \in \bar{E}} x_{ij} \begin{pmatrix} 0 & 0 \\ 0 & (i,j) : 1 \end{pmatrix} - (0) \quad \in \mathrm{POS}_n$$

$$\sum_i x_{ii} \leq 1$$

$$x_{ij} = 0 \qquad ij \in E$$

$$
\begin{array}{llll}
\text{(P)} & \text{Maximize} & \langle \mathbf{c}, \mathbf{x} \rangle \\
& \text{subject to} & \mathbf{b} - A(\mathbf{x}) \in L \\
& & \mathbf{x} \in K.
\end{array}
$$

$$
\begin{array}{llll}
\text{(D)} & \text{Minimize} & \langle \mathbf{b}, \mathbf{y} \rangle \\
& \text{subject to} & A^T(\mathbf{y}) - \mathbf{c} \in K^* \\
& & \mathbf{y} \in L^*.
\end{array}
$$

$$\max \quad J_n \bullet X$$

$$X \in \mathrm{POS}_n$$

$$\mathrm{Tr}(X) \leq 1$$

$$x_{ij} = 0 \qquad ij \in E$$

$$\max \sum_{ij \in \bar{E}} x_{ij} + \sum_i x_{ii} + \sum_{ij \in E} x_{ij}$$

$$\sum_i x_{ii} \begin{pmatrix} 0 & 0 \\ 0 & (i,i):1 \end{pmatrix} + \sum_{ij \in E} x_{ij} \begin{pmatrix} 0 & 0 \\ 0 & (i,j):1 \end{pmatrix}$$

$$\sum_{ij \in \bar{E}} x_{ij} \begin{pmatrix} 0 & 0 \\ 0 & (i,j):1 \end{pmatrix} - (0) \qquad \in \mathrm{POS}_n$$

$$\sum_i x_{ii} \leq 1$$

$$x_{ij} = 0 \qquad ij \in E$$

$$\text{(P)} \quad \begin{array}{ll} \text{Maximize} & \langle \mathbf{c}, \mathbf{x} \rangle \\ \text{subject to} & \mathbf{b} - A(\mathbf{x}) \in L \\ & \mathbf{x} \in K. \end{array}$$

$$\text{(D)} \quad \begin{array}{ll} \text{Minimize} & \langle \mathbf{b}, \mathbf{y} \rangle \\ \text{subject to} & A^T(\mathbf{y}) - \mathbf{c} \in K^* \\ & \mathbf{y} \in L^*. \end{array}$$

$$\max \quad J_n \bullet X$$

$$X \in \mathrm{POS}_n$$

$$\mathrm{Tr}(X) = 1$$

$$\boxed{\mathrm{Tr}(X) \leq 1}$$

$$
\begin{array}{lll}
\text{(P)} & \text{Maximize} & \langle \mathbf{c}, \mathbf{x} \rangle \\
& \text{subject to} & \mathbf{b} - A(\mathbf{x}) \in L \\
& & \mathbf{x} \in K.
\end{array}
$$

$$
\begin{array}{lll}
\text{(D)} & \text{Minimize} & \langle \mathbf{b}, \mathbf{y} \rangle \\
& \text{subject to} & A^T(\mathbf{y}) - \mathbf{c} \in K^* \\
& & \mathbf{y} \in L^*.
\end{array}
$$

$$
\begin{array}{lll}
\max & J_n \bullet X & \\
& X \in \mathrm{POS}_n & \\
& \mathrm{Tr}(X) = 1 & \\
& x_{ij} = 0 & ij \in E
\end{array}
$$

$\mathrm{Tr}(X) \leq 1$

$\alpha(G) \leq$

7.2.1 Theorem. *The copositive program*

$$\begin{array}{ll} \text{minimize} & t \\ \text{subject to} & y_{ij} = -1, \ \text{if } \{i,j\} \in \overline{E} \\ & y_{ii} = t-1, \ \text{for all } i = 1,2,\ldots,n \\ & Y \in \mathrm{COP}_n \end{array}$$

has value $\alpha(G)$, the size of a maximum independent set in G.

- روش: یک جواب شدنی برای دوگان با مقدار $\alpha(G)$
- I: یکی از بزرگ‌ترین مجموعه‌های مستقل

$$\begin{array}{lll} \max & J_n \bullet X & \\ & X \in \mathrm{POS}_n & \\ & \mathrm{Tr}(X) = 1 & \\ & x_{ij} = 0 & ij \in E \end{array}$$

$\alpha(G) \leq$

7.2.1 Theorem. *The copositive program*

$$\begin{array}{ll} \text{minimize} & t \\ \text{subject to} & y_{ij} = -1, \ \text{if } \{i,j\} \in \overline{E} \\ & y_{ii} = t-1, \ \text{for all } i = 1,2,\ldots,n \\ & Y \in \mathrm{COP}_n \end{array}$$

has value $\alpha(G)$, the size of a maximum independent set in G.

- روش: یک جواب شدنی برای دوگان با مقدار $\alpha(G)$
- I: یکی از بزرگ‌ترین مجموعه‌های مستقل

$$\tilde{x}_i = 1_{[i \in I]}$$

$$\tilde{X} = \frac{1}{\alpha(G)} \tilde{\mathbf{x}} \tilde{\mathbf{x}}^T$$

$$\begin{array}{lll} \max & J_n \bullet X & \\ & X \in \mathrm{POS}_n & \\ & \mathrm{Tr}(X) = 1 & \\ & x_{ij} = 0 & ij \in E \end{array}$$

$\alpha(G) \leq$

7.2.1 Theorem. *The copositive program*

$$\begin{array}{ll} \textit{minimize} & t \\ \textit{subject to} & y_{ij} = -1, \ \textit{if } \{i,j\} \in \overline{E} \\ & y_{ii} = t-1, \ \textit{for all } i = 1,2,\ldots,n \\ & Y \in \mathrm{COP}_n \end{array}$$

has value $\alpha(G)$, the size of a maximum independent set in G.

- روش: یک جواب شدنی برای دوگان با مقدار $\alpha(G)$
- I: یکی از بزرگ‌ترین مجموعه‌های مستقل

$\tilde{x}_i = 1_{[i \in I]}$

$$\tilde{X} = \frac{1}{\alpha(G)} \tilde{\mathbf{x}} \tilde{\mathbf{x}}^T$$

$$\begin{array}{lll} \max & J_n \bullet X & \\ & X \in \mathrm{POS}_n & \\ & \mathrm{Tr}(X) = 1 & \\ & x_{ij} = 0 & ij \in E \end{array}$$

$\alpha(G) \leq$

7.2.1 Theorem. *The copositive program*

$$
\begin{array}{ll}
\textit{minimize} & t \\
\textit{subject to} & y_{ij} = -1, \ \textit{if } \{i,j\} \in \overline{E} \\
 & y_{ii} = t - 1, \ \textit{for all } i = 1, 2, \ldots, n \\
 & Y \in \mathrm{COP}_n
\end{array}
$$

has value $\alpha(G)$, the size of a maximum independent set in G.

- روش: یک جواب شدنی برای دوگان با مقدار $\alpha(G)$
- I: یکی از بزرگ‌ترین مجموعه‌های مستقل

$\tilde{x}_i = 1_{[i \in I]}$

$$\tilde{X} = \frac{1}{\alpha(G)} \tilde{\mathbf{x}} \tilde{\mathbf{x}}^T \quad \tilde{x}_{ij} = \frac{1}{\alpha(G)} \tilde{x}_i \tilde{x}_j$$

$$
\begin{array}{lll}
\max & J_n \bullet X & \\
 & X \in \mathrm{POS}_n & \\
 & \mathrm{Tr}(X) = 1 & \\
 & x_{ij} = 0 & ij \in E
\end{array}
$$

$\alpha(G) \leq$

7.2.1 Theorem. *The copositive program*

$$\begin{array}{ll} \text{minimize} & t \\ \text{subject to} & y_{ij} = -1, \text{ if } \{i,j\} \in \overline{E} \\ & y_{ii} = t-1, \text{ for all } i = 1,2,\ldots,n \\ & Y \in \mathrm{COP}_n \end{array}$$

has value $\alpha(G)$, the size of a maximum independent set in G.

- روش: یک جواب شدنی برای دوگان با مقدار $\alpha(G)$
- I: یکی از بزرگ‌ترین مجموعه‌های مستقل

$\tilde{x}_i = 1_{[i \in I]}$

$\tilde{X} = \frac{1}{\alpha(G)} \tilde{\mathbf{x}}\tilde{\mathbf{x}}^T \quad \tilde{x}_{ij} = \frac{1}{\alpha(G)} \tilde{x}_i \tilde{x}_j$

$$\begin{array}{lll} \max & J_n \bullet X & \\ & X \in \mathrm{POS}_n & \\ & \mathrm{Tr}(X) = 1 & \\ & x_{ij} = 0 & ij \in E \end{array}$$

$\alpha(G) \leq$

7.2.1 Theorem. *The copositive program*

$$\begin{array}{ll} \textit{minimize} & t \\ \textit{subject to} & y_{ij} = -1, \ \textit{if } \{i,j\} \in \overline{E} \\ & y_{ii} = t-1, \ \textit{for all } i = 1,2,\ldots,n \\ & Y \in \mathrm{COP}_n \end{array}$$

has value $\alpha(G)$, the size of a maximum independent set in G.

- روش: یک جواب شدنی برای دوگان با مقدار $\alpha(G)$
- I: یکی از بزرگ‌ترین مجموعه‌های مستقل

$\tilde{x}_i = 1_{[i \in I]}$

$\tilde{X} = \frac{1}{\alpha(G)} \tilde{\mathbf{x}}\tilde{\mathbf{x}}^T$ $\tilde{x}_{ij} = \frac{1}{\alpha(G)} \tilde{x}_i \tilde{x}_j$ $\mathrm{Tr}(\tilde{x}\tilde{x}^\top) = \sum \tilde{x}_i^2$

$$\begin{array}{lll} \max & J_n \bullet X & \\ & X \in \mathrm{POS}_n & \\ & \mathrm{Tr}(X) = 1 & \\ & x_{ij} = 0 & ij \in E \end{array}$$

$\alpha(G) \leq$

7.2.1 Theorem. *The copositive program*

$$\begin{array}{ll} \text{minimize} & t \\ \text{subject to} & y_{ij} = -1, \text{ if } \{i,j\} \in \overline{E} \\ & y_{ii} = t-1, \text{ for all } i = 1,2,\ldots,n \\ & Y \in \mathrm{COP}_n \end{array}$$

has value $\alpha(G)$, the size of a maximum independent set in G.

- روش: یک جواب شدنی برای دوگان با مقدار $\alpha(G)$
- I: یکی از بزرگ‌ترین مجموعه‌های مستقل

$\tilde{x}_i = 1_{[i \in I]}$

$$\tilde{X} = \frac{1}{\alpha(G)}\tilde{\mathbf{x}}\tilde{\mathbf{x}}^T \quad \tilde{x}_{ij} = \frac{1}{\alpha(G)}\tilde{x}_i\tilde{x}_j \quad \mathrm{Tr}(\tilde{x}\tilde{x}^\top) = \sum \tilde{x}_i^2$$

$$\begin{array}{lll} \max & J_n \bullet X & \\ & X \in \mathrm{POS}_n & \\ & \mathrm{Tr}(X) = 1 & \\ & x_{ij} = 0 & ij \in E \end{array}$$

$\alpha(G) \leq$

7.2.1 Theorem. *The copositive program*

$$\begin{array}{ll} \textit{minimize} & t \\ \textit{subject to} & y_{ij} = -1, \ \textit{if } \{i,j\} \in \overline{E} \\ & y_{ii} = t-1, \ \textit{for all } i = 1,2,\ldots,n \\ & Y \in \mathrm{COP}_n \end{array}$$

has value $\alpha(G)$, the size of a maximum independent set in G.

- روش: یک جواب شدنی برای دوگان با مقدار $\alpha(G)$
- I: یکی از بزرگ‌ترین مجموعه‌های مستقل

$\tilde{x}_i = 1_{[i \in I]}$

$$\tilde{X} = \frac{1}{\alpha(G)} \tilde{\mathbf{x}}\tilde{\mathbf{x}}^T \quad \tilde{x}_{ij} = \frac{1}{\alpha(G)} \tilde{x}_i \tilde{x}_j \quad \mathrm{Tr}(\tilde{x}\tilde{x}^{\top}) = \sum \tilde{x}_i^2$$

$$\begin{array}{lll} \max & J_n \bullet X & \\ & X \in \mathrm{POS}_n & \\ & \mathrm{Tr}(X) = 1 & \\ & x_{ij} = 0 & ij \in E \end{array}$$

$J_n \bullet \tilde{X}$

$\alpha(G) \leq$

7.2.1 Theorem. *The copositive program*

$$
\begin{array}{ll}
\text{minimize} & t \\
\text{subject to} & y_{ij} = -1, \text{ if } \{i,j\} \in \overline{E} \\
& y_{ii} = t-1, \text{ for all } i = 1,2,\ldots,n \\
& Y \in \mathrm{COP}_n
\end{array}
$$

has value $\alpha(G)$, the size of a maximum independent set in G.

- روش: یک جواب شدنی برای دوگان با مقدار $\alpha(G)$
- I: یکی از بزرگ‌ترین مجموعه‌های مستقل

$\tilde{x}_i = 1_{[i \in I]}$

$$\tilde{X} = \frac{1}{\alpha(G)} \tilde{\mathbf{x}}\tilde{\mathbf{x}}^T \quad \tilde{x}_{ij} = \frac{1}{\alpha(G)} \tilde{x}_i \tilde{x}_j \quad \mathrm{Tr}(\tilde{x}\tilde{x}^{\mathsf{T}}) = \sum \tilde{x}_i^2$$

$$J_n \bullet \tilde{X} = \sum_{i,j} \tilde{x}_{ij}$$

$$
\begin{array}{ll}
\max & J_n \bullet X \\
& X \in \mathrm{POS}_n \\
& \mathrm{Tr}(X) = 1 \\
& x_{ij} = 0 \qquad ij \in E
\end{array}
$$

$\alpha(G) \leq$

7.2.1 Theorem. *The copositive program*

$$\begin{array}{ll} \text{minimize} & t \\ \text{subject to} & y_{ij} = -1, \text{ if } \{i,j\} \in \overline{E} \\ & y_{ii} = t-1, \text{ for all } i = 1,2,\ldots,n \\ & Y \in \mathrm{COP}_n \end{array}$$

has value $\alpha(G)$, the size of a maximum independent set in G.

- روش: یک جواب شدنی برای دوگان با مقدار $\alpha(G)$
- I: یکی از بزرگ‌ترین مجموعه‌های مستقل

$\tilde{x}_i = 1_{[i \in I]}$

$$\tilde{X} = \frac{1}{\alpha(G)} \tilde{\mathbf{x}}\tilde{\mathbf{x}}^T \quad \tilde{x}_{ij} = \frac{1}{\alpha(G)} \tilde{x}_i \tilde{x}_j \quad \mathrm{Tr}(\tilde{x}\tilde{x}^\top) = \sum \tilde{x}_i^2$$

$$J_n \bullet \tilde{X} = \sum_{i,j} \tilde{x}_{ij} = \frac{1}{\alpha(G)} \sum_{i,j} \tilde{x}_i \tilde{x}_j$$

$$\begin{array}{lll} \max & J_n \bullet X & \\ & X \in \mathrm{POS}_n & \\ & \mathrm{Tr}(X) = 1 & \\ & x_{ij} = 0 & ij \in E \end{array}$$

$\alpha(G) \leq$

7.2.1 Theorem. *The copositive program*

$$
\begin{array}{ll}
\textit{minimize} & t \\
\textit{subject to} & y_{ij} = -1, \ \textit{if } \{i,j\} \in \overline{E} \\
 & y_{ii} = t - 1, \ \textit{for all } i = 1, 2, \ldots, n \\
 & Y \in \mathrm{COP}_n
\end{array}
$$

has value $\alpha(G)$, the size of a maximum independent set in G.

- روش: یک جواب شدنی برای دوگان با مقدار $\alpha(G)$
- I: یکی از بزرگ‌ترین مجموعه‌های مستقل

$$\tilde{x}_i = 1_{[i \in I]}$$

$$\tilde{X} = \frac{1}{\alpha(G)} \tilde{\mathbf{x}} \tilde{\mathbf{x}}^T \quad \tilde{x}_{ij} = \frac{1}{\alpha(G)} \tilde{x}_i \tilde{x}_j \quad \mathrm{Tr}(\tilde{x}\tilde{x}^{\mathsf{T}}) = \sum \tilde{x}_i^2$$

$$J_n \bullet \tilde{X} = \sum_{i,j} \tilde{x}_{ij} = \frac{1}{\alpha(G)} \sum_{i,j} = \frac{1}{\alpha(G)} \sum_{i,j \in I} 1$$

$$
\begin{array}{lll}
\max & J_n \bullet X & \\
 & X \in \mathrm{POS}_n & \\
 & \mathrm{Tr}(X) = 1 & \\
 & x_{ij} = 0 & ij \in E
\end{array}
$$

$\alpha(G) \leq$

7.2.1 Theorem. *The copositive program*

$$
\begin{array}{ll}
\textit{minimize} & t \\
\textit{subject to} & y_{ij} = -1, \ \textit{if } \{i,j\} \in \overline{E} \\
 & y_{ii} = t - 1, \ \textit{for all } i = 1, 2, \ldots, n \\
 & Y \in \mathrm{COP}_n
\end{array}
$$

has value $\alpha(G)$, the size of a maximum independent set in G.

- روش: یک جواب شدنی برای دوگان با مقدار $\alpha(G)$
- I: یکی از بزرگ‌ترین مجموعه‌های مستقل

$\tilde{x}_i = 1_{[i \in I]}$

$$\tilde{X} = \frac{1}{\alpha(G)} \tilde{\mathbf{x}}\tilde{\mathbf{x}}^T \quad \tilde{x}_{ij} = \frac{1}{\alpha(G)} \tilde{x}_i \tilde{x}_j \quad \mathrm{Tr}(\tilde{x}\tilde{x}^\top) = \sum \tilde{x}_i^2$$

$$J_n \bullet \tilde{X} = \sum_{i,j} \tilde{x}_{ij} = \frac{1}{\alpha(G)} \sum_{i,j} : = \frac{1}{\alpha(G)} \sum_{i,j \in I} 1 = \frac{\alpha(G)^2}{\alpha(G)}$$

$$
\begin{array}{lll}
\max & J_n \bullet X & \\
 & X \in \mathrm{POS}_n & \\
 & \mathrm{Tr}(X) = 1 & \\
 & x_{ij} = 0 & ij \in E
\end{array}
$$

$\alpha(G) \leq$

7.2.1 Theorem. *The copositive program*

$$\begin{array}{ll} \text{minimize} & t \\ \text{subject to} & y_{ij} = -1, \ \text{if } \{i,j\} \in \overline{E} \\ & y_{ii} = t-1, \ \text{for all } i = 1,2,\ldots,n \\ & Y \in \mathrm{COP}_n \end{array}$$

has value $\alpha(G)$, the size of a maximum independent set in G.

- روش: یک جواب شدنی برای دوگان با مقدار $\alpha(G)$
- I: یکی از بزرگ‌ترین مجموعه‌های مستقل

$$\tilde{x}_i = 1_{[i \in I]}$$

$$\tilde{X} = \frac{1}{\alpha(G)} \tilde{\mathbf{x}}\tilde{\mathbf{x}}^T \quad \tilde{x}_{ij} = \frac{1}{\alpha(G)} \tilde{x}_i \tilde{x}_j \quad \mathrm{Tr}(\tilde{x}\tilde{x}^\top) = \sum \tilde{x}_i^2$$

$$J_n \bullet \tilde{X} = \sum_{i,j} \tilde{x}_{ij} = \frac{1}{\alpha(G)} \sum_{i,j} \vdots = \frac{1}{\alpha(G)} \sum_{i,j \in I} 1 = \frac{\alpha(G)^2}{\alpha(G)}$$

$$\begin{array}{lll} \max & J_n \bullet X & \\ & X \in \mathrm{POS}_n & \\ & \mathrm{Tr}(X) = 1 & \\ & x_{ij} = 0 & ij \in E \end{array}$$

$\alpha(G) \leq$

7.2.1 Theorem. *The copositive program*

$$\begin{array}{ll} \text{minimize} & t \\ \text{subject to} & y_{ij} = -1, \ \text{if } \{i,j\} \in \overline{E} \\ & y_{ii} = t-1, \ \text{for all } i = 1,2,\ldots,n \\ & Y \in \mathrm{COP}_n \end{array}$$

has value $\alpha(G)$, the size of a maximum independent set in G.

$\leq \alpha(G)$

7.2.1 Theorem. *The copositive program*

$$\begin{array}{ll} \textit{minimize} & t \\ \textit{subject to} & y_{ij} = -1, \ \textit{if } \{i,j\} \in \overline{E} \\ & y_{ii} = t-1, \ \textit{for all } i = 1,2,\ldots,n \\ & Y \in \mathrm{COP}_n \end{array}$$

has value $\alpha(G)$, the size of a maximum independent set in G.

7.2.1 Theorem. *The copositive program*

$$
\begin{array}{ll}
\textit{minimize} & t \\
\textit{subject to} & y_{ij} = -1, \ \textit{if } \{i,j\} \in \overline{E} \\
& y_{ii} = t - 1, \ \textit{for all } i = 1, 2, \ldots, n \\
& Y \in \mathrm{COP}_n
\end{array}
$$

has value $\alpha(G)$, the siz … *ndent set in G.*

صفر روی درایه‌های بدون یال

$$Y = tI_n + Z - J_n$$

7.2.1 Theorem. *The copositive program*

$$\begin{array}{ll} \textit{minimize} & t \\ \textit{subject to} & y_{ij} = -1, \ \textit{if } \{i,j\} \in \overline{E} \\ & y_{ii} = t - 1, \ \textit{for all } i = 1, 2, \ldots, n \\ & Y \in \mathrm{COP}_n \end{array}$$

has value $\alpha(G)$, *the siz* … *dent set in* G.

صفر روی درایه‌های بدون یال

$$Y = tI_n + Z - J_n$$

$$z = \max_{i,j}(Z_{i,j})$$

$$Y' = tI_n + zA_G - J_n$$

$\alpha(G) \leq$

7.2.1 Theorem. *The copositive program*

$$\begin{array}{ll} \textit{minimize} & t \\ \textit{subject to} & y_{ij} = -1, \ \textit{if } \{i,j\} \in \overline{E} \\ & y_{ii} = t-1, \ \textit{for all } i = 1, 2, \ldots, n \\ & Y \in \mathrm{COP}_n \end{array}$$

has value $\alpha(G)$, the size of a maximum independent set in G.

$\leq \alpha(G)$

7.2.5 Lemma. *The copositive program*

$$\begin{array}{ll} \textit{Minimize} & t \\ \textit{subject to} & tI_n + zA_G - J_n \in \mathrm{COP}_n \\ & t, z \in \mathbb{R} \end{array}$$

جوابش برابر با جواب برنامه‌ریزی بالاست.

قضیه Motzkin–Straus:

7.2.6 Theorem. *For every graph G,*

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T(A_G + I_n)\mathbf{x} : \mathbf{x} \geq \mathbf{0}, \sum_{i=1}^{n} x_i = 1\}.$$

قضیه Motzkin–Straus:

7.2.6 Theorem. *For every graph G,*

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T(A_G + I_n)\mathbf{x} : \mathbf{x} \geq \mathbf{0}, \sum_{i=1}^{n} x_i = 1\}.$$

$:= m(G)$

:Motzkin–Straus قضیه

7.2.6 Theorem. *For every graph G,*

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T(A_G + I_n)\mathbf{x} : \mathbf{x} \geq \mathbf{0}, \sum_{i=1}^{n} x_i = 1\}.$$

$:= m(G)$

$f(x) :=$

قضیه Motzkin–Straus:

7.2.6 Theorem. *For every graph G,*

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \geq \mathbf{0}, \sum_{i=1}^{n} x_i = 1\}.$$

$:= m(G)$

$f(x) :=$

- الف) $m(G) \leq \dfrac{1}{\alpha(G)}$
- ...

قضیه Motzkin–Straus:

7.2.6 Theorem. *For every graph G,*

$$\frac{1}{\alpha(G)} = \min\{\underbrace{\mathbf{x}^T(A_G + I_n)\mathbf{x}}_{f(x) :=} :\ \mathbf{x} \geq \mathbf{0}, \sum_{i=1}^{n} x_i = 1\}. \quad := m(G)$$

- الف) $m(G) \leq \dfrac{1}{\alpha(G)}$
- ...
- ب) $m(G) \geq \dfrac{1}{\alpha(G)}$
- ...

قضیه Motzkin–Straus:

7.2.6 Theorem. *For every graph G,*

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T(A_G + I_n)\mathbf{x} : \mathbf{x} \geq \mathbf{0}, \sum_{i=1}^{n} x_i = 1\}.$$

$:= m(G)$

$f(x) :=$

- الف) $m(G) \leq \dfrac{1}{\alpha(G)}$

قضیه Motzkin–Straus:

7.2.6 Theorem. *For every graph G,*

$$\frac{1}{\alpha(G)} = \min\{\underbrace{\mathbf{x}^T(A_G + I_n)\mathbf{x}}_{f(x) :=} : \mathbf{x} \geq \mathbf{0}, \sum_{i=1}^{n} x_i = 1\}. \quad := m(G)$$

- الف) $m(G) \leq \dfrac{1}{\alpha(G)}$
- $\tilde{x}$: بردار مشخصه مجموعه مستقل

قضیه Motzkin–Straus:

7.2.6 Theorem. *For every graph G,*

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T(A_G + I_n)\mathbf{x} : \mathbf{x} \geq \mathbf{0}, \sum_{i=1}^{n} x_i = 1\}.$$

$:= m(G)$

$f(x) :=$

- الف) $m(G) \leq \dfrac{1}{\alpha(G)}$
- $\tilde{x}$: بردار مشخصه مجموعه مستقل

$$f(\tilde{x}) = \sum_{i,j} (A_G + I)_{i,j}\tilde{x}_i\tilde{x}_j$$

قضیه Motzkin–Straus:

7.2.6 Theorem. *For every graph G,*

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \geq \mathbf{0}, \sum_{i=1}^{n} x_i = 1\}.$$

$:= m(G)$

$f(x) :=$

- الف) $m(G) \leq \dfrac{1}{\alpha(G)}$
- $\tilde{x}$: بردار مشخصه مجموعه مستقل

$$f(\tilde{x}) = \sum_{i,j} (A_G + I)_{i,j} \tilde{x}_i \tilde{x}_j \;= |I|$$

قضیه Motzkin–Straus:

7.2.6 Theorem. *For every graph G,*

$$\frac{1}{\alpha(G)} = \min\{\underbrace{\mathbf{x}^T(A_G+I_n)\mathbf{x}}_{f(x):=} : \mathbf{x} \geq \mathbf{0}, \sum_{i=1}^{n} x_i = 1\}. \quad := m(G)$$

- الف) $m(G) \leq \dfrac{1}{\alpha(G)}$
- $\tilde{x}$: بردار مشخصه مجموعه مستقل

$$f(\tilde{x}) = \sum_{i,j} (A_G + I)_{i,j}\tilde{x}_i\tilde{x}_j = |I|$$

- $\dfrac{1}{\alpha(G)}\tilde{x}$

قضیه Motzkin–Straus:

7.2.6 Theorem. *For every graph G,*

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T(A_G + I_n)\mathbf{x} : \mathbf{x} \geq \mathbf{0}, \sum_{i=1}^{n} x_i = 1\}.$$

$:= m(G)$

$f(x) :=$

- الف) $m(G) \leq \frac{1}{\alpha(G)}$
- $\tilde{x}$: بردار مشخصه مجموعه مستقل

$$f(\tilde{x}) = \sum_{i,j} (A_G + I)_{i,j}\tilde{x}_i\tilde{x}_j = |I|$$

- $\frac{1}{\alpha(G)}\tilde{x}$
- نرم ۱ = ۱، کنج مثبت، $f(\frac{1}{\alpha(G)}\tilde{x}) = \frac{1}{\alpha(G)}$

قضیه Motzkin–Straus:

7.2.6 Theorem. *For every graph G,*

$$\frac{1}{\alpha(G)} = \min\{\underbrace{\mathbf{x}^T(A_G + I_n)\mathbf{x}}_{f(x) :=} : \mathbf{x} \geq \mathbf{0}, \sum_{i=1}^{n} x_i = 1\}. \; := m(G)$$

- ب) $m(G) \geq \dfrac{1}{\alpha(G)}$

قضیه Motzkin–Straus:

7.2.6 Theorem. *For every graph G,*

$$\frac{1}{\alpha(G)} = \min\{\underbrace{\mathbf{x}^T(A_G + I_n)\mathbf{x}}_{f(x) :=} : \mathbf{x} \geq \mathbf{0}, \sum_{i=1}^{n} x_i = 1\}. \quad := m(G)$$

- ب) $m(G) \geq \dfrac{1}{\alpha(G)}$
- x^*: جواب بهینه
- با بیشترین صفر

قضیه Motzkin–Straus:

7.2.6 Theorem. *For every graph G,*

$$\frac{1}{\alpha(G)} = \min\{\underbrace{\mathbf{x}^T(A_G + I_n)\mathbf{x}}_{f(x) :=} :\ \mathbf{x} \geq \mathbf{0}, \sum_{i=1}^{n} x_i = 1\}. \quad := m(G)$$

- ب) $m(G) \geq \dfrac{1}{\alpha(G)}$
  - x^*: جواب بهینه
    - با بیشترین صفر
  - یال i و j که دو سرشان مثبت است

قضیه Motzkin–Straus:

7.2.6 Theorem. *For every graph G,*

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T(A_G + I_n)\mathbf{x} : \mathbf{x} \geq \mathbf{0}, \sum_{i=1}^{n} x_i = 1\}.$$

$:= m(G)$

$f(x) :=$

- ب) $m(G) \geq \dfrac{1}{\alpha(G)}$
- x^*: جواب بهینه
- با بیشترین صفر
- یال i و j که دو سرشان مثبت است

			+e			-e			

قضیه Motzkin–Straus:

7.2.6 Theorem. *For every graph G,*

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T(A_G + I_n)\mathbf{x} : \mathbf{x} \geq \mathbf{0}, \sum_{i=1}^{n} x_i = 1\}.$$

$:= m(G)$

$f(x) :=$

- ب) $m(G) \geq \dfrac{1}{\alpha(G)}$

- x^*: جواب بهینه

			+e			-e			

- با بیشترین صفر
- یال i و j که دو سرشان مثبت است

$$z_k = \begin{cases} x_k^* + \varepsilon & \text{if } k = i \\ x_k^* - \varepsilon & \text{if } k = j \\ x_k^* & \text{otherwise,} \end{cases}$$

قضیه Motzkin–Straus:

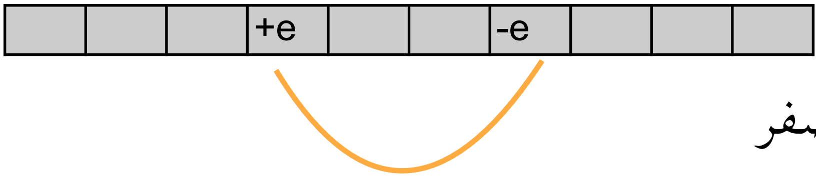
7.2.6 Theorem. *For every graph G,*

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T(A_G + I_n)\mathbf{x} : \mathbf{x} \geq \mathbf{0}, \sum_{i=1}^{n} x_i = 1\}.$$

$:= m(G)$

$f(x) :=$

- ب) $m(G) \geq \dfrac{1}{\alpha(G)}$
- x^*: جواب بهینه
- با بیشترین صفر
- یال i و j که دو سرشان مثبت است



$$f(z) = f(x^*) + l(\epsilon) \qquad z_k = \begin{cases} x_k^* + \varepsilon & \text{if } k = i \\ x_k^* - \varepsilon & \text{if } k = j \\ x_k^* & \text{otherwise,} \end{cases}$$

7.2.6 Theorem. *For every graph G,*

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T(A_G + I_n)\mathbf{x} : \mathbf{x} \geq \mathbf{0}, \sum_{i=1}^{n} x_i = 1\}.$$

$:= m(G)$

$f(x) :=$

$$f(z) = f(x^*) + l(\epsilon) \qquad z_k = \begin{cases} x_k^* + \varepsilon & \text{if } k = i \\ x_k^* - \varepsilon & \text{if } k = j \\ x_k^* & \text{otherwise,} \end{cases}$$

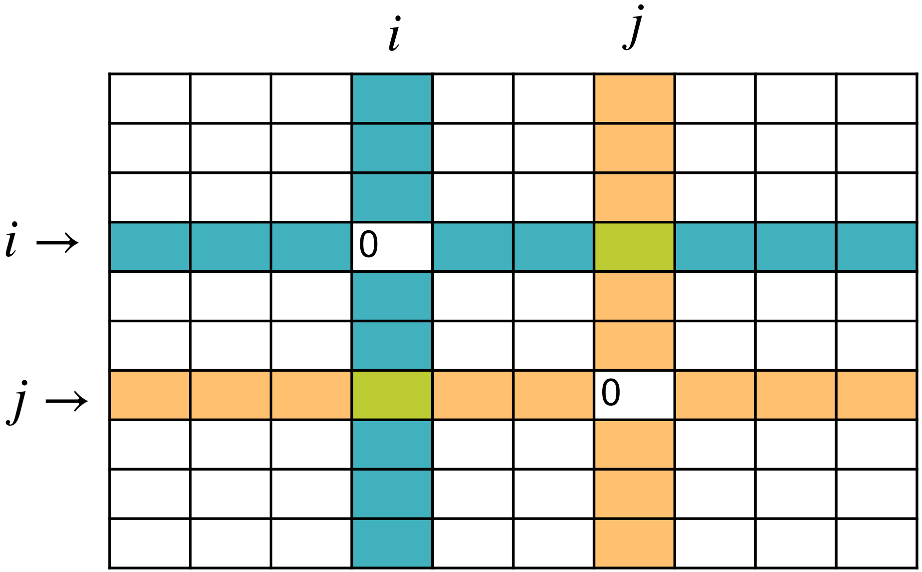
7.2.6 Theorem. *For every graph G,*

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T(A_G + I_n)\mathbf{x} : \mathbf{x} \geq \mathbf{0}, \sum_{i=1}^{n} x_i = 1\}.$$

$:= m(G)$

$f(x) :=$

$$f(z) = f(x^*) + l(\epsilon) \qquad z_k = \begin{cases} x_k^* + \varepsilon & \text{if } k = i \\ x_k^* - \varepsilon & \text{if } k = j \\ x_k^* & \text{otherwise,} \end{cases}$$



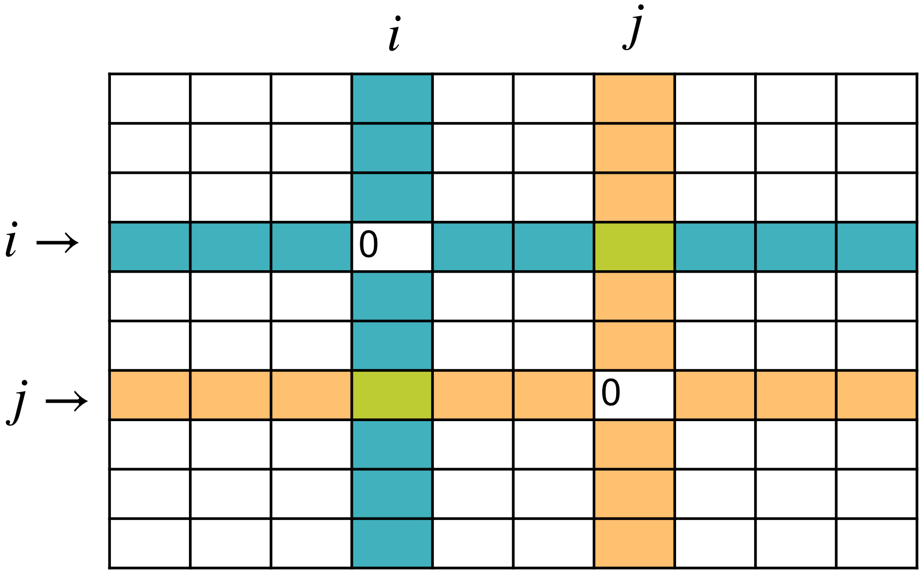
7.2.6 Theorem. *For every graph G,*

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T(A_G + I_n)\mathbf{x} : \mathbf{x} \geq \mathbf{0}, \sum_{i=1}^{n} x_i = 1\}.$$

$:= m(G)$

$f(x) :=$

$$f(z) = f(x^*) + l(\epsilon) \qquad z_k = \begin{cases} x_k^* + \varepsilon & \text{if } k = i \\ x_k^* - \varepsilon & \text{if } k = j \\ x_k^* & \text{otherwise,} \end{cases}$$

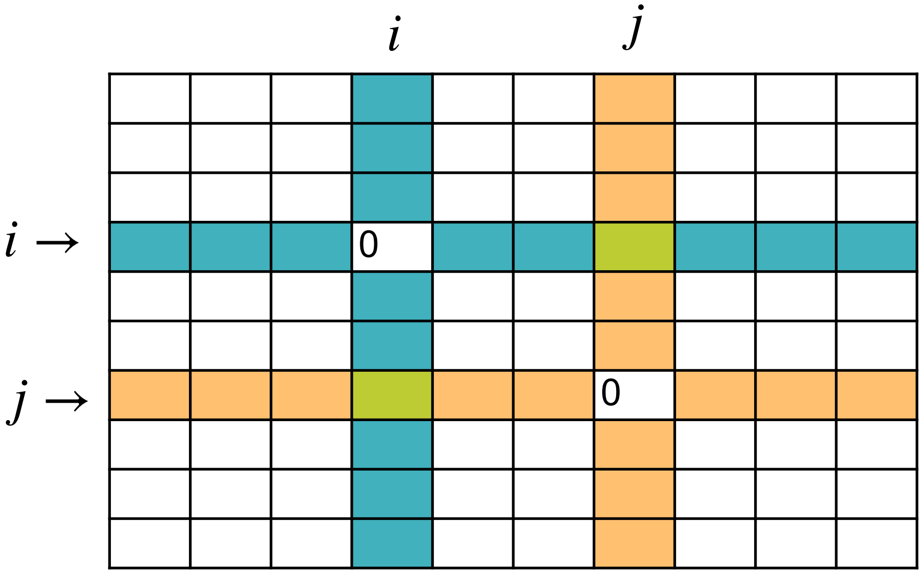


$$f(z) = \epsilon \mathrm{B} - \epsilon \mathrm{O} + 2(x_i + \epsilon)(x_j - \epsilon) +$$

7.2.6 Theorem. *For every graph G,*

$$\frac{1}{\alpha(G)} = \min\{\underbrace{\mathbf{x}^T(A_G + I_n)\mathbf{x}}_{f(x) :=} : \mathbf{x} \geq \mathbf{0}, \sum_{i=1}^{n} x_i = 1\}. \quad := m(G)$$

$$f(z) = f(x^*) + l(\epsilon) \qquad z_k = \begin{cases} x_k^* + \varepsilon & \text{if } k = i \\ x_k^* - \varepsilon & \text{if } k = j \\ x_k^* & \text{otherwise,} \end{cases}$$



$$f(z) = \epsilon \mathrm{B} - \epsilon \mathrm{O} + 2(x_i + \epsilon)(x_j - \epsilon) +$$

$$(x_i + \epsilon)^2 + (x_j - \epsilon) + \dots$$

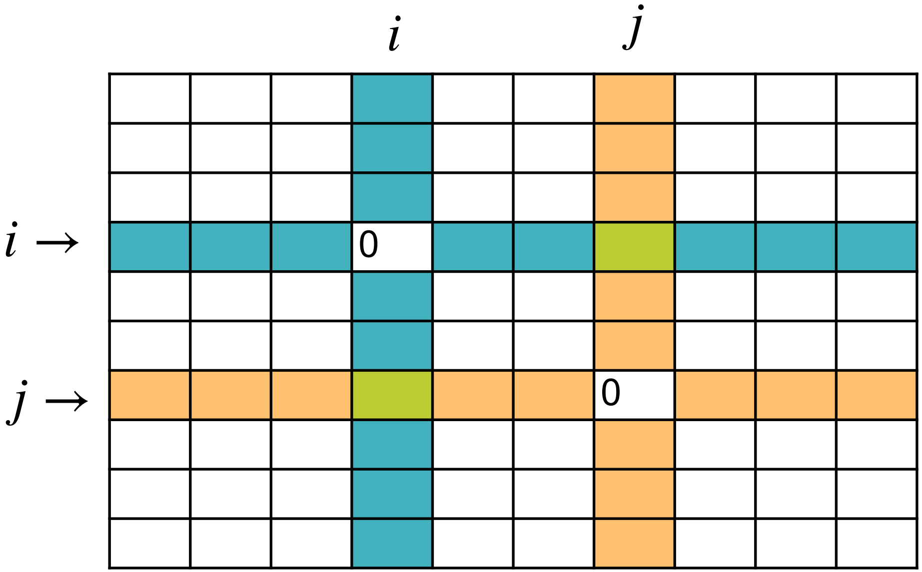
7.2.6 Theorem. *For every graph G,*

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T(A_G + I_n)\mathbf{x} : \mathbf{x} \geq \mathbf{0}, \sum_{i=1}^{n} x_i = 1\}.$$

$:= m(G)$

$f(x) :=$

$$f(z) = f(x^*) + l(\epsilon) \qquad z_k = \begin{cases} x_k^* + \varepsilon & \text{if } k = i \\ x_k^* - \varepsilon & \text{if } k = j \\ x_k^* & \text{otherwise,} \end{cases}$$



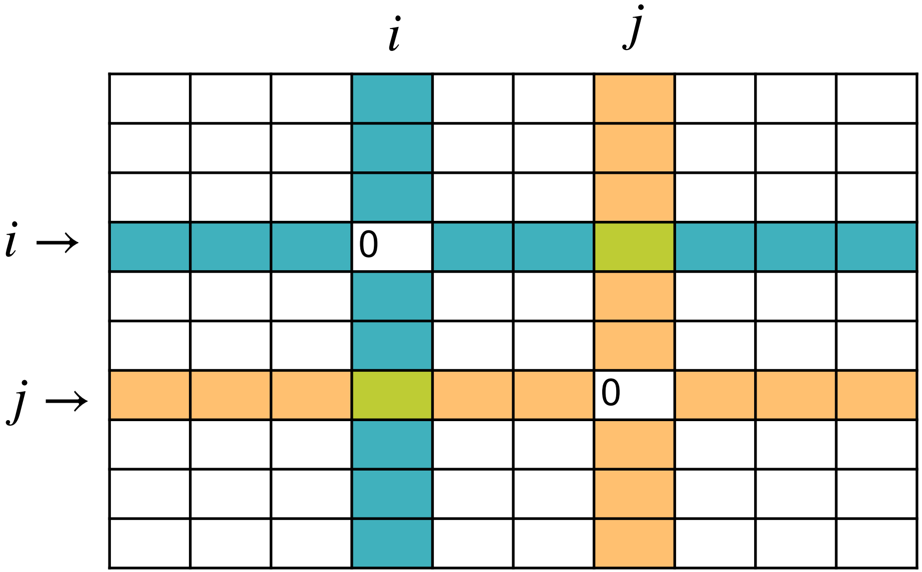
$$f(z) = \epsilon\mathrm{B} - \epsilon\mathrm{O} + 2(x_i + \epsilon)(x_j - \epsilon) +$$
$$(x_i + \epsilon)^2 + (x_j - \epsilon) + \ldots$$

$l(\epsilon)$ نسبت به ϵ خطی است

7.2.6 Theorem. *For every graph G,*

$$\frac{1}{\alpha(G)} = \min\{\underbrace{\mathbf{x}^T(A_G + I_n)\mathbf{x}}_{f(x) :=} : \mathbf{x} \geq \mathbf{0}, \sum_{i=1}^{n} x_i = 1\}. := m(G)$$

$$f(z) = f(x^*) + l(\epsilon) \qquad z_k = \begin{cases} x_k^* + \varepsilon & \text{if } k = i \\ x_k^* - \varepsilon & \text{if } k = j \\ x_k^* & \text{otherwise,} \end{cases}$$



$$f(z) = \epsilon \mathrm{B} - \epsilon \mathrm{O} + 2(x_i + \epsilon)(x_j - \epsilon) +$$

$$(x_i + \epsilon)^2 + (x_j - \epsilon) + \ldots$$

$f(z)$ نسبت به ϵ خطی است

$l(\epsilon)$ نسبت به ϵ خطی است

7.2.6 Theorem. *For every graph G,*

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T(A_G + I_n)\mathbf{x} : \mathbf{x} \geq \mathbf{0}, \sum_{i=1}^{n} x_i = 1\}.$$

$:= m(G)$

$f(x) :=$

$$f(z) = f(x^*) + l(\epsilon) \qquad z_k = \begin{cases} x_k^* + \varepsilon & \text{if } k = i \\ x_k^* - \varepsilon & \text{if } k = j \\ x_k^* & \text{otherwise,} \end{cases}$$

$f(z)$ نسبت به ϵ خطی است

$l(\epsilon)$ نسبت به ϵ خطی است

7.2.6 Theorem. *For every graph G,*

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T(A_G + I_n)\mathbf{x} : \mathbf{x} \geq \mathbf{0}, \sum_{i=1}^{n} x_i = 1\}.$$

$:= m(G)$

$f(x) :=$

$$f(z) = f(x^*) + l(\epsilon) \qquad z_k = \begin{cases} x_k^* + \varepsilon & \text{if } k = i \\ x_k^* - \varepsilon & \text{if } k = j \\ x_k^* & \text{otherwise,} \end{cases}$$

$l(\epsilon)$ نسبت به ϵ خطی است

$f(z)$ نسبت به ϵ خطی است

$l(\epsilon)$ متحد صفر است

7.2.6 Theorem. *For every graph G,*

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T(A_G + I_n)\mathbf{x} : \mathbf{x} \geq \mathbf{0}, \sum_{i=1}^{n} x_i = 1\}.$$

$:= m(G)$

$f(x) :=$

$$f(z) = f(x^*) + l(\epsilon) \qquad z_k = \begin{cases} x_k^* + \varepsilon & \text{if } k = i \\ x_k^* - \varepsilon & \text{if } k = j \\ x_k^* & \text{otherwise,} \end{cases}$$

$l(\epsilon)$ نسبت به ϵ خطی است

$f(z)$ نسبت به ϵ خطی است

$l(\epsilon)$ متحد صفر است

می‌توان یکی از درایه‌های x^* را صفر کرد

7.2.6 Theorem. *For every graph G,*

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T(A_G + I_n)\mathbf{x} : \mathbf{x} \geq \mathbf{0}, \sum_{i=1}^{n} x_i = 1\}.$$

$:= m(G)$

$f(x) :=$

$$f(z) = f(x^*) + l(\epsilon) \qquad z_k = \begin{cases} x_k^* + \varepsilon & \text{if } k = i \\ x_k^* - \varepsilon & \text{if } k = j \\ x_k^* & \text{otherwise,} \end{cases}$$

$l(\epsilon)$ نسبت به ϵ خطی است

$f(z)$ نسبت به ϵ خطی است

$l(\epsilon)$ متحد صفر است

می‌توان یکی از درایه‌های x^* را صفر کرد

تناقض !

قضیه Motzkin–Straus:

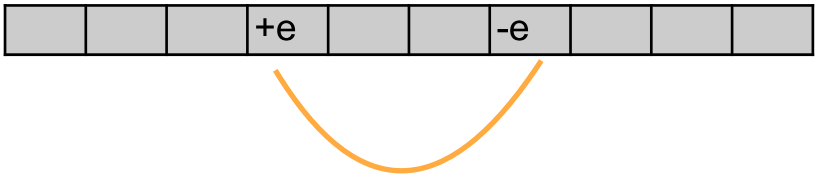
7.2.6 Theorem. *For every graph G,*

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T(A_G + I_n)\mathbf{x} : \mathbf{x} \geq \mathbf{0}, \sum_{i=1}^{n} x_i = 1\}.$$

$:= m(G)$

$f(x) :=$

- ب) $m(G) \geq \dfrac{1}{\alpha(G)}$
- x^*: جواب بهینه
- با بیشترین صفر
- یال i و j که دو سرشان مثبت است



$$f(z) = f(x^*) + l(\epsilon) \qquad z_k = \begin{cases} x_k^* + \varepsilon & \text{if } k = i \\ x_k^* - \varepsilon & \text{if } k = j \\ x_k^* & \text{otherwise,} \end{cases}$$

قضیه Motzkin–Straus:

7.2.6 Theorem. *For every graph G,*

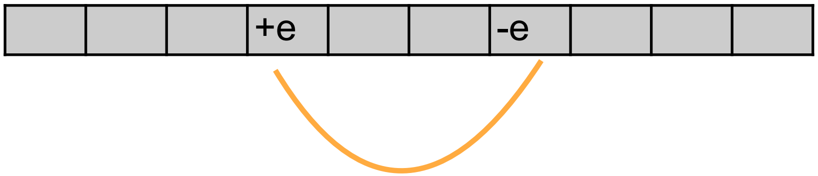
$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T(A_G + I_n)\mathbf{x} : \mathbf{x} \geq \mathbf{0}, \sum_{i=1}^{n} x_i = 1\}.$$

$:= m(G)$

$f(x) :=$

- ب) $m(G) \geq \dfrac{1}{\alpha(G)}$
- x^*: جواب بهینه
- با بیشترین صفر
- یال i و j که دو سرشان مثبت است

نداریم



$$f(z) = f(x^*) + l(\epsilon) \qquad z_k = \begin{cases} x_k^* + \varepsilon & \text{if } k = i \\ x_k^* - \varepsilon & \text{if } k = j \\ x_k^* & \text{otherwise,} \end{cases}$$

قضیه Motzkin–Straus:

7.2.6 Theorem. *For every graph G,*

$$\frac{1}{\alpha(G)} = \min\{\underbrace{\mathbf{x}^T(A_G + I_n)\mathbf{x}}_{f(x) :=} : \mathbf{x} \geq \mathbf{0}, \sum_{i=1}^{n} x_i = 1\}. \quad := m(G)$$

- ب) $m(G) \geq \dfrac{1}{\alpha(G)}$
- x^*: جواب بهینه

			+e			-e			

- با بیشترین صفر
- یال i و j که دو سرشان مثبت است
- J: مجموعه راس‌های با x^* مثبت، مستقل است

نداریم

7.2.6 Theorem. *For every graph G,*

$$\frac{1}{\alpha(G)} = \min\{\underbrace{\mathbf{x}^T(A_G + I_n)\mathbf{x}}_{f(x) :=} : \mathbf{x} \geq \mathbf{0}, \sum_{i=1}^{n} x_i = 1\}. := m(G)$$

- ب) $m(G) \geq \dfrac{1}{\alpha(G)}$
- J: مجموعه راس‌های با x^* مثبت، مستقل است

7.2.6 Theorem. *For every graph G,*

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T(A_G + I_n)\mathbf{x} : \mathbf{x} \geq \mathbf{0}, \sum_{i=1}^{n} x_i = 1\}.$$

$:= m(G)$

$f(x) :=$

- ب) $m(G) \geq \dfrac{1}{\alpha(G)}$
- J: مجموعه راس‌های با x^* مثبت، مستقل است

$$f(x) = 2\sum_{i,j \in E(G)} x_i x_j + \sum_{i \in J} x_i^2$$

7.2.6 Theorem. *For every graph G,*

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T(A_G + I_n)\mathbf{x} : \mathbf{x} \geq \mathbf{0}, \sum_{i=1}^{n} x_i = 1\}.$$

$f(x) :=$

$:= m(G)$

- ب) $m(G) \geq \dfrac{1}{\alpha(G)}$
- J: مجموعه راس‌های با x^* مثبت، مستقل است

$$f(x) = 2\underbrace{\sum_{i,j\in E(G)} x_i x_j}_{0} + \sum_{i\in J} x_i^2$$

7.2.6 Theorem. *For every graph G,*

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T(A_G + I_n)\mathbf{x} : \mathbf{x} \geq \mathbf{0}, \sum_{i=1}^{n} x_i = 1\}.$$

$:= m(G)$

$f(x) :=$

- ب) $m(G) \geq \frac{1}{\alpha(G)}$
- J: مجموعه راس‌های با x^* مثبت، مستقل است

$$f(x) = 2\sum_{i,j\in E(G)} x_i x_j + \sum_{i\in J} x_i^2$$

0

7.2.6 Theorem. *For every graph G,*

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T(A_G + I_n)\mathbf{x} : \mathbf{x} \geq \mathbf{0}, \sum_{i=1}^{n} x_i = 1\}.$$

$:= m(G)$

$f(x) :=$

- ب) $m(G) \geq \dfrac{1}{\alpha(G)}$
- J: مجموعه راس‌های با x^* مثبت، مستقل است

$$\|u\|^2\|v\|^2 \geq \langle u, v\rangle^2$$

$$f(x) = 2\sum_{i,j\in E(G)} x_i x_j + \sum_{i\in J} x_i^2$$

0

7.2.6 Theorem. *For every graph G,*

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T(A_G + I_n)\mathbf{x} : \mathbf{x} \geq \mathbf{0}, \sum_{i=1}^{n} x_i = 1\}.$$

$:= m(G)$

$f(x) :=$

- ب) $m(G) \geq \dfrac{1}{\alpha(G)}$

- J: مجموعه راس‌های با x^* مثبت، مستقل است

$$\|u\|^2\|v\|^2 \geq \langle u, v\rangle^2$$

$$(\sum_{i\in J} x_i^2)(\sum_{i\in J}(\frac{1}{\sqrt{|J|}})^2)$$

$$f(x) = 2\underbrace{\sum_{i,j\in E(G)} x_i x_j}_{0} + \sum_{i\in J} x_i^2$$

7.2.6 Theorem. *For every graph G,*

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T(A_G + I_n)\mathbf{x} : \mathbf{x} \geq \mathbf{0}, \sum_{i=1}^{n} x_i = 1\}.$$

$:= m(G)$

$f(x) :=$

- ب) $m(G) \geq \dfrac{1}{\alpha(G)}$

- J: مجموعه راس‌های با x^* مثبت، مستقل است

$$\|u\|^2\|v\|^2 \geq \langle u, v\rangle^2$$

$$(\sum_{i\in J} x_i^2)(\sum_{i\in J} (\frac{1}{\sqrt{|J|}})^2) \geq (\sum_{i\in J} x_i \frac{1}{\sqrt{|J|}})^2$$

$$f(x) = 2\sum_{i,j\in E(G)} x_i x_j + \sum_{i\in J} x_i^2$$

0

7.2.6 Theorem. *For every graph G,*

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T(A_G + I_n)\mathbf{x} : \mathbf{x} \geq \mathbf{0}, \sum_{i=1}^{n} x_i = 1\}.$$

$:= m(G)$

$f(x) :=$

- ب) $m(G) \geq \dfrac{1}{\alpha(G)}$

- J: مجموعه راس‌های با x^* مثبت، مستقل است

$$\|u\|^2\|v\|^2 \geq \langle u, v\rangle^2$$

$$(\sum_{i\in J} x_i^2)(\sum_{i\in J} (\frac{1}{\sqrt{|J|}})^2) \geq (\sum_{i\in J} x_i \frac{1}{\sqrt{|J|}})^2 \geq \frac{1}{|J|}(\sum_{i\in J} x_i)^2$$

$$f(x) = 2\sum_{i,j\in E(G)} x_i x_j + \sum_{i\in J} x_i^2$$

0

7.2.6 Theorem. *For every graph G,*

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T(A_G + I_n)\mathbf{x} : \mathbf{x} \geq \mathbf{0}, \sum_{i=1}^{n} x_i = 1\}.$$

$:= m(G)$

$f(x) :=$

- ب) $m(G) \geq \dfrac{1}{\alpha(G)}$
- J: مجموعه راس‌های با x^* مثبت، مستقل است

$$\|u\|^2\|v\|^2 \geq \langle u, v\rangle^2$$

$$(\sum_{i\in J} x_i^2)(\sum_{i\in J}(\frac{1}{\sqrt{|J|}})^2) \geq (\sum_{i\in J} x_i \frac{1}{\sqrt{|J|}})^2 \geq \frac{1}{|J|}(\sum_{i\in J} x_i)^2 = \frac{1}{|J|}$$

$$f(x) = 2\underbrace{\sum_{i,j\in E(G)} x_i x_j}_{0} + \sum_{i\in J} x_i^2$$

7.2.6 Theorem. *For every graph G,*

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T(A_G + I_n)\mathbf{x} : \mathbf{x} \geq \mathbf{0}, \sum_{i=1}^{n} x_i = 1\}.$$

$f(x) :=$

$:= m(G)$

- ب) $m(G) \geq \dfrac{1}{\alpha(G)}$

- J: مجموعه راس‌های با x^* مثبت، مستقل است

$$\|u\|^2\|v\|^2 \geq \langle u, v\rangle^2$$

$$(\sum_{i\in J} x_i^2)(\sum_{i\in J}(\frac{1}{\sqrt{|J|}})^2) \geq (\sum_{i\in J} x_i \frac{1}{\sqrt{|J|}})^2 \geq \frac{1}{|J|}(\sum_{i\in J} x_i)^2 = \frac{1}{|J|}$$

$$f(x) = 2\underbrace{\sum_{i,j\in E(G)} x_i x_j}_{0} + \sum_{i\in J} x_i^2 \geq \frac{1}{|J|}$$

7.2.6 Theorem. *For every graph G,*

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T(A_G + I_n)\mathbf{x} : \mathbf{x} \geq \mathbf{0}, \sum_{i=1}^{n} x_i = 1\}.$$

$f(x) :=$

$:= m(G)$

- ب) $m(G) \geq \dfrac{1}{\alpha(G)}$

- J: مجموعه راس‌های با x^* مثبت، مستقل است

$$\|u\|^2\|v\|^2 \geq \langle u, v\rangle^2$$

$$(\sum_{i\in J} x_i^2)(\sum_{i\in J}(\frac{1}{\sqrt{|J|}})^2) \geq (\sum_{i\in J} x_i \frac{1}{\sqrt{|J|}})^2 \geq \frac{1}{|J|}(\sum_{i\in J} x_i)^2 = \frac{1}{|J|}$$

$$f(x) = 2\sum_{i,j\in E(G)} x_i x_j + \sum_{i\in J} x_i^2 \geq \frac{1}{|J|} \geq \frac{1}{\alpha(G)}$$

0

قضیه Motzkin–Straus:

7.2.6 Theorem. *For every graph G,*

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T(A_G + I_n)\mathbf{x} : \mathbf{x} \geq \mathbf{0}, \sum_{i=1}^{n} x_i = 1\}.$$

$:= m(G)$

$f(x) :=$

- الف) $m(G) \leq \dfrac{1}{\alpha(G)}$
  - ...
- ب) $m(G) \geq \dfrac{1}{\alpha(G)}$
  - ...

7.2.5 Lemma. *The copositive program*

$$\begin{array}{ll} \textit{Minimize} & t \\ \textit{subject to} & tI_n + zA_G - J_n \in \mathrm{COP}_n \\ & t, z \in \mathbb{R} \end{array}$$

جوابش برابر با جواب برنامه‌ریزی بالاست.

α $\leq \alpha(G)$

7.2.6 Theorem. *For every graph G,*

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T(A_G + I_n)\mathbf{x} : \ \mathbf{x} \geq \mathbf{0}, \sum_{i=1}^{n} x_i = 1\}.$$

7.2.5 Lemma. *The copositive program*

$$\begin{array}{ll} \textit{Minimize} & t \\ \textit{subject to} & tI_n + zA_G - J_n \in \mathrm{COP}_n \\ & t, z \in \mathbb{R} \end{array}$$

جوابش برابر با جواب برنامه‌ریزی بالاست.

α ... $\leq \alpha(G)$

7.2.6 Theorem. *For every graph G,*

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T(A_G + I_n)\mathbf{x} : \mathbf{x} \geq \mathbf{0}, \sum_{i=1}^{n} x_i = 1\}.$$

برای هر x روی سادک:

$$\mathbf{x}^T(\alpha(G)(A_G + I_n))\mathbf{x} \geq 1$$

7.2.5 Lemma. *The copositive program*

$$\begin{array}{ll} \textit{Minimize} & t \\ \textit{subject to} & tI_n + zA_G - J_n \in \mathrm{COP}_n \\ & t, z \in \mathbb{R} \end{array}$$

جوابش برابر با جواب برنامه‌ریزی بالاست.

$\alpha \quad \leq \alpha(G)$

7.2.6 Theorem. *For every graph G,*

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T(A_G + I_n)\mathbf{x} : \mathbf{x} \geq \mathbf{0}, \sum_{i=1}^{n} x_i = 1\}.$$

برای هر x روی سادک:

$$\mathbf{x}^T(\alpha(G)(A_G + I_n))\mathbf{x} \geq 1 = \mathbf{x}^T J_n \mathbf{x},$$

7.2.5 Lemma. *The copositive program*

$$\begin{array}{ll} \textit{Minimize} & t \\ \textit{subject to} & tI_n + zA_G - J_n \in \mathrm{COP}_n \\ & t, z \in \mathbb{R} \end{array}$$

جوابش برابر با جواب برنامه‌ریزی بالاست.

α ... $\leq \alpha(G)$

برای هر x روی سادک:

$$\mathbf{x}^T(\alpha(G)(A_G + I_n))\mathbf{x} \geq 1 = \mathbf{x}^T J_n \mathbf{x},$$

7.2.5 Lemma. *The copositive program*

$$\begin{array}{ll} \textit{Minimize} & t \\ \textit{subject to} & tI_n + zA_G - J_n \in \mathrm{COP}_n \\ & t, z \in \mathbb{R} \end{array}$$

جوابش برابر با جواب برنامه‌ریزی بالاست.

α … $\leq \alpha(G)$

برای هر x روی سادک:

$$\mathbf{x}^T(\alpha(G)(A_G + I_n))\mathbf{x} \geq 1 = \mathbf{x}^T J_n \mathbf{x},$$

$$\mathbf{x}^T(\alpha(G)I_n + \alpha(G)A_G - J_n)\mathbf{x} \geq \mathbf{0},$$ برای هر x روی سادک:

7.2.5 Lemma. *The copositive program*

$$\begin{array}{ll} \textit{Minimize} & t \\ \textit{subject to} & tI_n + zA_G - J_n \in \mathrm{COP}_n \\ & t, z \in \mathbb{R} \end{array}$$

جوابش برابر با جواب برنامه‌ریزی بالاست.

α ... $\leq \alpha(G)$

برای هر x روی سادک:

$$\mathbf{x}^T(\alpha(G)(A_G + I_n))\mathbf{x} \geq 1 \; = \mathbf{x}^T J_n \mathbf{x},$$

برای هر x روی سادک: $\mathbf{x}^T(\alpha(G)I_n + \alpha(G)A_G - J_n)\mathbf{x} \geq \mathbf{0},$

برای هر x نامنفی: $\mathbf{x}^T(\alpha(G)I_n + \alpha(G)A_G - J_n)\mathbf{x} \geq \mathbf{0},$

7.2.5 Lemma. *The copositive program*

$$\begin{array}{ll} \text{Minimize} & t \\ \text{subject to} & tI_n + zA_G - J_n \in \mathrm{COP}_n \\ & t, z \in \mathbb{R} \end{array}$$

جوابش برابر با جواب برنامه‌ریزی بالاست.

α ... $\leq \alpha(G)$

برای هر x روی سادک:

$$\mathbf{x}^T(\alpha(G)(A_G + I_n))\mathbf{x} \geq 1 \; = \mathbf{x}^T J_n \mathbf{x},$$

برای هر x روی سادک: $\mathbf{x}^T(\alpha(G)I_n + \alpha(G)A_G - J_n)\mathbf{x} \geq \mathbf{0},$

برای هر x نامنفی: $\mathbf{x}^T(\alpha(G)I_n + \alpha(G)A_G - J_n)\mathbf{x} \geq \mathbf{0},$

هم‌مثبت: $\tilde{Y} := \alpha(G)I_n + \alpha(G)A_G - J_n$

7.2.5 Lemma. *The copositive program*

$$\begin{array}{ll} \text{Minimize} & t \\ \text{subject to} & tI_n + zA_G - J_n \in \mathrm{COP}_n \\ & t, z \in \mathbb{R} \end{array}$$

جوابش برابر با جواب برنامه‌ریزی بالاست.

α ... $\leq \alpha(G)$

هم‌مثبت:

$$\tilde{Y} := \alpha(G) I_n + \alpha(G) A_G - J_n$$

7.2.5 Lemma. *The copositive program*

$$\begin{array}{ll} \text{Minimize} & t \\ \text{subject to} & tI_n + zA_G - J_n \in \mathrm{COP}_n \\ & t, z \in \mathbb{R} \end{array}$$

جوابش برابر با جواب برنامه‌ریزی بالاست.

$\alpha(G) \leq$

7.2.1 Theorem. *The copositive program*

$$\begin{array}{ll} \text{minimize} & t \\ \text{subject to} & y_{ij} = -1, \ \text{if } \{i,j\} \in \overline{E} \\ & y_{ii} = t-1, \ \text{for all } i = 1,2,\ldots,n \\ & Y \in \mathrm{COP}_n \end{array}$$

has value $\alpha(G)$, the size of a maximum independent set in G.

$\leq \alpha(G)$

هم‌مثبت:

$$\tilde{Y} := \alpha(G)I_n + \alpha(G)A_G - J_n$$

7.2.5 Lemma. *The copositive program*

$$\begin{array}{ll} \text{Minimize} & t \\ \text{subject to} & tI_n + zA_G - J_n \in \mathrm{COP}_n \\ & t, z \in \mathbb{R} \end{array}$$

جوابش برابر با جواب برنامه‌ریزی بالاست.

$\alpha(G) \leq$

7.2.1 Theorem. *The copositive program*

$$\begin{array}{ll} \textit{minimize} & t \\ \textit{subject to} & y_{ij} = -1, \ \textit{if } \{i,j\} \in \overline{E} \\ & y_{ii} = t - 1, \ \textit{for all } i = 1, 2, \ldots, n \\ & Y \in \mathrm{COP}_n \end{array}$$

has value $\alpha(G)$, the size of a maximum independent set in G.

$\leq \alpha(G)$

$$\tilde{Y} := \alpha(G) I_n + \alpha(G) A_G - J_n$$

هم‌مثبت:

7.2.5 Lemma. *The copositive program*

$$\begin{array}{ll} \textit{Minimize} & t \\ \textit{subject to} & tI_n + zA_G - J_n \in \mathrm{COP}_n \\ & t, z \in \mathbb{R} \end{array}$$

جوابش برابر با جواب برنامه‌ریزی بالاست.

برنامه‌ریزی هم‌مثبت سخت است!

برنامه‌ریزی هم‌مثبت سخت است!

7.2.1 Theorem. *The copositive program*

$$\begin{array}{ll} \textit{minimize} & t \\ \textit{subject to} & y_{ij} = -1, \ \textit{if } \{i,j\} \in \overline{E} \\ & y_{ii} = t-1, \ \textit{for all } i = 1,2,\ldots,n \\ & Y \in \mathrm{COP}_n \end{array}$$

has value $\alpha(G)$, the size of a maximum independent set in G.

برنامه‌ریزی هم‌مثبت سخت است!

7.2.1 Theorem. *The copositive program*

$$\begin{array}{ll} \text{minimize} & t \\ \text{subject to} & y_{ij} = -1, \ \text{if } \{i,j\} \in \overline{E} \\ & y_{ii} = t - 1, \ \text{for all } i = 1, 2, \ldots, n \\ & Y \in \mathrm{COP}_n \end{array}$$

has value $\alpha(G)$, *the size of a maximum independent set in* G.

برنامه‌ریزی کاملا مثبت سخت است!

برنامه‌ریزی هم‌مثبت سخت است!

7.2.1 Theorem. *The copositive program*

$$\begin{array}{ll} \text{minimize} & t \\ \text{subject to} & y_{ij} = -1, \text{ if } \{i,j\} \in \overline{E} \\ & y_{ii} = t-1, \text{ for all } i = 1,2,\ldots,n \\ & Y \in \mathrm{COP}_n \end{array}$$

has value $\alpha(G)$, the size of a maximum independent set in G.

برنامه‌ریزی کاملا مثبت سخت است!

$$\begin{array}{lll} \max & J_n \bullet X & \\ & X \in \mathrm{POS}_n & \\ & \mathrm{Tr}(X) = 1 & \\ & x_{ij} = 0 & ij \in E \end{array}$$

تقریب‌ناپذیری مسئله مجموعه مستقل

- تحت فرض‌های خوبی ($NP \nsubseteq ZPP$)
- هیچ الگوریتم تقریبی برای مسئله بزرگ‌تری مجموعه مستقل
- با ضریب تقریب $n^{1-\epsilon}$ برای هیچ $\epsilon > 0$ وجود ندارد.

Cone Programming

$$\begin{array}{lll} \text{(P)} & \text{Maximize} & \langle \mathbf{c}, \mathbf{x} \rangle \\ & \text{subject to} & \mathbf{b} - A(\mathbf{x}) \in L \\ & & \mathbf{x} \in K. \end{array}$$

?

الگوریتم سریع

برنامه‌ریزی هم‌مثبت

$$\begin{array}{ll} \text{maximize} & C \bullet X \\ \text{subject to} & A(X) = b \\ & X \in \mathrm{COP}_n \end{array}$$

SDP

$$\begin{array}{ll} \text{maximize} & C \bullet X \\ \text{subject to} & A_i \bullet X = b_i, \quad i = 1, 2, \ldots, m \\ & X \succeq 0. \end{array}$$

الگوریتم سریع

برنامه‌ریزی کاملا مثبت

$$\begin{array}{ll} \text{maximize} & C \bullet X \\ \text{subject to} & A(X) = b \\ & X \in \mathrm{POS}_n \end{array}$$

LP

$$\begin{array}{ll} \text{maximize} & c^\top x \\ \text{subject to} & Ax = b \\ & x \ge 0 \end{array}$$

الگوریتم سریع

پایان