10. We assume the opposite: any pair in S with $|S| = 2n$ has in S an odd number of common friends. Let A be one of these persons, and let $M = \{F_1, \dots, F_k\}$ be the set of his friends. We prove the following:

Lemma. The number k is even for every A.

Indeed, for every $F_i \in M$, we consider the list of all his friends in M. The sum of all entries in all k lists is even, since it equals twice the number of pairs in M, and the number of persons in each list is odd by the lemma. Thus k is even.

Let $k = 2m$. Now we consider, for every $F_i \in M$, the list of all his friends, except A (not only in M). Every list contains by the lemma (applied to F_i) an odd number of persons. Hence the sum of all entries in all $2m$ lists is even. But then at least one of the $(2m - 1)$ persons (except A) appears in an even number of lists, that is, this person has an even number of common friends with A.

This contradiction proves that at least two persons in S have an even number of common friends.