بسم الله الرحمن الرحيم

# برنامهریزی نیمهمعین برای طراحی الگوریتمهای تقریبی

جلسه یازدهم: آیا برنامهریزی هممثبت الگوریتم سریع دارد؟



### **Cone Programming**

(P) Maximize  $\langle \mathbf{c}, \mathbf{x} \rangle$ subject to  $\mathbf{b} - A(\mathbf{x}) \in L$  $\mathbf{x} \in K$ .



## SDP

maximize  $C \bullet X$ subject to  $A_i \bullet X = b_i, \quad i = 1, 2, ..., m$  $X \succeq 0.$ 



# LP

 $\begin{array}{ll}
\text{maximize} & c^{\mathsf{T}} x \\
\text{subject to} & Ax = b \\
 & x \ge 0
\end{array}$ 



**7.1.1 Definition.** A matrix 
$$M \in SYM_n$$
 is called copositive if

$$\mathbf{x}^T M \mathbf{x} \geq 0$$
 for all  $\mathbf{x} \geq 0$ .

 $COP_n := \{ M \in SYM_n : \mathbf{x}^T M \mathbf{x} \ge 0 \text{ for all } \mathbf{x} \ge 0 \}$ 

$$POS_n \subseteq PSD_n \subseteq COP_n$$

**7.1.4 Definition.** A matrix  $M \in \text{SYM}_n$  is called completely positive if for some  $\ell$ , there are  $\ell$  nonnegative vectors  $\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_{\ell} \in \mathbb{R}^n_+$ , such that

$$M = \sum_{i=1}^{c} \mathbf{x}_i \mathbf{x}_i^T = AA^T, \tag{7.2}$$

where  $A \in \mathbb{R}^{n \times \ell}$  is the (nonnegative) matrix with columns  $\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_{\ell}$ .

 $POS_n := \{M \in SYM_n : M \text{ is completely positive}\}$ 

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7.1.7 Theorem. 
$$POS_n^* = COP_n$$
.

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$$M = \sum_{i=1}^{t} \mathbf{x}_i \mathbf{x}_i^T = AA^T, \tag{7.2}$$

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برنامهریزی هممثبت برای یک مسئله سخت!

# **Cone Programming**

(P) Maximize  $\langle \mathbf{c}, \mathbf{x} \rangle$ 

subject to  $\mathbf{b} - A(\mathbf{x}) \in L$  $\mathbf{x} \in K$ .

## برنامهریزی هممثبت $C \bullet X$

subject to A(X) = b $X \in COP_n$ 

maximize

**SDP** 

 $C \bullet X$ 

 $X \succeq 0$ .

subject to  $A_i \bullet X = b_i, \quad i = 1, 2, \dots, m$ 

 $C \bullet X$ maximize

برنامهریزی کاملا مثبت

maximize

subject to A(X) = b $X \in POS_n$ 

 $c^{\mathsf{T}}x$ naximize subject to Ax = b $x \ge 0$ 

LP



بیشترین نرخ ارسال با گراف G:

$$\sigma(G) = \sup \left\{ \frac{1}{k} \log \alpha(G^k) : k \in \mathbb{N} \right\},$$

$$\Theta(G) = 2^{\sigma(G)} = \lim_{k \to \infty} \sqrt[k]{\alpha(G^k)}$$

$$\vartheta(G) := \min_{\mathcal{U}} \min_{\|\mathbf{c}\|=1} \max_{i=1}^{n} \frac{1}{(\mathbf{c}^T \mathbf{u}_i)^2}$$

 $\Theta(G) \leq \vartheta(G)$ 

قضىه:

قضیه: برنامهریزی زیر  $\vartheta(G)$  را محاسبه می کند

 $\begin{array}{ll} \textit{Minimize} & t \\ \textit{subject to} & y_{ij} = -1 & \textit{if } \{i,j\} \in \overline{E} \\ & y_{ii} = t-1 & \textit{for all } i = 1, \dots, n \\ & Y \succeq 0. \end{array}$ 

#### **7.2.1 Theorem.** The copositive program

minimize 
$$t$$
  
subject to  $y_{ij} = -1$ , if  $\{i, j\} \in \overline{E}$   
 $y_{ii} = t - 1$ , for all  $i = 1, 2, ..., n$   
 $Y \in \text{COP}_n$ 

minimize tsubject to  $y_{ij} = -1$ , if  $\{i, j\} \in \overline{E}$   $y_{ii} = t - 1$ , for all i = 1, 2, ..., n $Y \in \text{COP}_n$ 

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has value  $\alpha(G)$ , the size of a maximum independent set in G.

 $Y \in COP_n$ 

7.2.1 Theorem. The copositive program minimize subject to  $y_{ij} = -1$ , if  $\{i, j\} \in \overline{E}$  $y_{ii} = t - 1$ , for all i = 1, 2, ..., n $Y \in COP_n$ has value  $\alpha(G)$ , the size of a maximum independent set in G.



$$\begin{array}{ll} \text{minimize} & t \\ \text{subject to} & y_{ij} = -1, \text{ if } \{i, j\} \in \overline{E} \\ & y_{ii} = t - 1, \text{ for all } i = 1, 2, \dots, n \\ & Y \in \text{COP}_n \end{array}$$

$$\alpha(G)$$
 روش: یک جواب شدنی برای دوگان با مقدار  $\alpha$ 

 $\alpha(G) \leq$ 

$$egin{aligned} extbf{.2.1 Ineorem.} & extbf{ Ine copositive program} \ & extbf{minimize} & t \end{aligned}$$

minimize tsubject to  $y_{ij} = -1$ , if  $\{i, j\} \in \overline{E}$   $y_{ii} = t - 1$ , for all i = 1, 2, ..., n $Y \in \text{COP}_n$ 

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(P) Maximize 
$$\langle \mathbf{c}, \mathbf{x} \rangle$$
  
subject to  $\mathbf{b} - A(\mathbf{x}) \in L$   
 $\mathbf{x} \in K$ .

(D) Minimize 
$$\langle \mathbf{b}, \mathbf{y} \rangle$$
  
subject to  $A^T(\mathbf{y}) - \mathbf{c} \in K^*$   
 $\mathbf{y} \in L^*$ .

 $\alpha(G) \leq$ 

$$\alpha(G)$$
 روش: یک جواب شدنی برای دوگان با مقدار

$$\begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix}$$
 (P) Maximize  $\langle \mathbf{c}, \mathbf{x} \rangle$  subject to  $\mathbf{b} - A(\mathbf{x}) \in L$   $\mathbf{x} \in K$ .

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 روش: یک جواب شدنی برای دوگان با مقدار  $\bullet$ 

$$\min \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix}$$
 (P) Maximize  $\langle \mathbf{c}, \mathbf{x} \rangle$  subject to  $\mathbf{b} - A$ 

subject to 
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 $\mathbf{x} \in K$ .

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$$\min \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix}$$

$$\begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix} \bullet \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,i):1 & 0 \\ 0 & 0 & -1 \end{pmatrix} = -1$$

$$(P) \quad \text{Maximi subject}$$

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 روش: یک جواب شدنی برای دوگان با مقدار

$$\min \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix}$$

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$$\alpha(G) \text{ min } \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix}$$

$$\begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix} \bullet \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,i):1 & 0 \\ 0 & 0 & -1 \end{pmatrix} = -1$$

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$$Y \in \text{COP}_n$$

$$(P) \text{ Maximize } \langle \mathbf{c}, \mathbf{x} \rangle$$

$$\text{subject to } \mathbf{b} - A(\mathbf{x})$$

$$\mathbf{x} \in K.$$

$$(D) \text{ Minimize } \langle \mathbf{b}, \mathbf{y} \rangle$$

$$\text{subject to } A^T(\mathbf{y}) - \mathbf{y} \in L^*.$$

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$$Y \in COP_n$$

$$\sum_{i} x_{ii} \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,j):1 & 0 \\ 0 & 0 & -1 \end{pmatrix} = -1$$

$$ij \in \bar{E}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & (i,i) : 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

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$$\begin{pmatrix} Y & 0 \\ 0 & (i,i):1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$\begin{pmatrix} Y & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\sum_{i} \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,i):1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

 $Y \in COP_n$ 

$$\begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix} \bullet \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,i):1 & 0 \\ 0 & 0 & -1 \end{pmatrix} = -1 \qquad \sum_{i} x_{ii} \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,i):1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$
$$\begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix} \bullet \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,j):1 & 0 \\ 0 & 0 & 0 \end{pmatrix} = -1 \qquad \sum_{ij \in \bar{E}} x_{ij} \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,j):1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$
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$$Y \in COP_n$$

$$\begin{pmatrix} 0 \\ : 1 & 0 \\ & -1 \end{pmatrix}$$

$$\begin{vmatrix}
x_{ii} & 0 & 0 & 0 \\
0 & (i,i) : 1 & 0 \\
0 & 0 & -1
\end{vmatrix}$$

$$\begin{bmatrix}
x_{ij} & 0 & 0 & 0 \\
0 & (i,j) : 1 & 0 \\
0 & 0 & 0
\end{bmatrix} - \begin{pmatrix} 0 & 0 \\
0 & -1
\end{pmatrix} \in \mathbb{R}$$

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$$Y \in COP_n$$

$$\sum_{i} x_{ii} \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,i) : 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \begin{pmatrix} 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

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$$\begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix} \bullet \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,j):1 & 0 \\ 0 & 0 & 0 \end{pmatrix} = -1$$

$$Y \in COP_n$$

$$\sum_{ij \in \bar{E}} x_{ij} \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,j):1 & 0 \\ 0 & 0 & 0 \end{pmatrix} -1$$

$$\sum_{ij \in \bar{E}} x_{ij} \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,j):1 & 0 \\ 0 & 0 & 0 \end{pmatrix} -1$$

$$\sum_{ij\in\bar{E}} -x_{ij} + \sum_{i} -x_{ii}$$

$$= \begin{pmatrix} 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix} \bullet \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,i):1 & 0 \\ 0 & 0 & -1 \end{pmatrix} = -1$$

$$\begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix} \bullet \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,j):1 & 0 \\ 0 & 0 & 0 \end{pmatrix} = -1$$

$$Y \in COP_{n}$$

$$ij \in \bar{E}$$

$$\sum_{ij \in \bar{E}} x_{ij} \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,j):1 & 0 \\ 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ 0 & (i,j):1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & (i,j):1 & 0 \\ 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} \in POS_{n}$$

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$$\max \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} \bullet \begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix} \qquad \min \sum_{ij \in \bar{E}} -x_{ij} + \sum_{i} -$$

$$\min \sum_{ij \in \bar{E}} -x_{ij} + \sum_{i} -x_{ii}$$

$$\begin{pmatrix} Y & 0 \\ 0 & t \end{pmatrix} \bullet \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,i):1 & 0 \\ 0 & 0 & -1 \end{pmatrix} = -1 \qquad \sum_{i} x_{ii} \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,i):1 & 0 \\ 0 & 0 & -1 \end{pmatrix} + \begin{pmatrix} Y & 0 \\ 0 & (i,j):1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \bullet \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,j):1 & 0 \\ 0 & 0 & 0 \end{pmatrix} = -1 \qquad \sum_{ij \in \bar{E}} x_{ij} \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,j):1 & 0 \\ 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ 0 & (i,j):1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \in POS_n$$

$$Y \in COP_n$$

(P) Maximize 
$$\langle \mathbf{c}, \mathbf{x} \rangle$$
  
subject to  $\mathbf{b} - A(\mathbf{x}) \in L$   
 $\mathbf{x} \in K$ .

(D) Minimize 
$$\langle \mathbf{b}, \mathbf{y} \rangle$$
  
subject to  $A^T(\mathbf{y}) - \mathbf{c} \in K^*$   
 $\mathbf{y} \in L^*$ .

$$\min \sum_{ij \in \bar{E}} -x_{ij} + \sum_{i} -x_{ii}$$

$$\sum_{i} x_{ii} \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,i) : 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} +$$

$$\sum_{i} x_{ii} \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,i) : 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} + \sum_{ij \in \bar{E}} x_{ij} \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,j) : 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} \in POS_n$$

(P) Maximize 
$$\langle \mathbf{c}, \mathbf{x} \rangle$$
  
subject to  $\mathbf{b} - A(\mathbf{x}) \in L$   
 $\mathbf{x} \in K$ .

(D) Minimize 
$$\langle \mathbf{b}, \mathbf{y} \rangle$$
  
subject to  $A^T(\mathbf{y}) - \mathbf{c} \in K^*$   
 $\mathbf{y} \in L^*$ .

$$\min \sum_{ij \in \bar{E}} -x_{ij} + \sum_{i} -x_{ii}$$

$$\sum_{i} x_{ii} \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,i) : 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} +$$

$$\sum_{ij \in \bar{E}} x_{ij} \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,j) : 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} \in POS_{n}$$

(P) Maximize 
$$\langle \mathbf{c}, \mathbf{x} \rangle$$
  
subject to  $\mathbf{b} - A(\mathbf{x}) \in L$   
 $\mathbf{x} \in K$ .

(D) Minimize 
$$\langle \mathbf{b}, \mathbf{y} \rangle$$
  
subject to  $A^T(\mathbf{y}) - \mathbf{c} \in K^*$   
 $\mathbf{y} \in L^*$ .

$$\sum_{i} -x_{ii} + 1 \ge 0$$

$$\min \sum_{ij \in \bar{E}} -x_{ij} + \sum_{i} -x_{ii}$$

$$\sum_{i} x_{ii} \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i, i) : 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} + \sum_{ij \in \bar{E}} x_{ij} \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i, j) : 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} \in POS_{n}$$

(P) Maximize 
$$\langle \mathbf{c}, \mathbf{x} \rangle$$
  
subject to  $\mathbf{b} - A(\mathbf{x}) \in L$   
 $\mathbf{x} \in K$ .

(D) Minimize 
$$\langle \mathbf{b}, \mathbf{y} \rangle$$
  
subject to  $A^T(\mathbf{y}) - \mathbf{c} \in K^*$   
 $\mathbf{y} \in L^*$ .

$$\sum_{i} -x_{ii} + 1 \ge 0$$

$$\sum_{i} x_{i} < 1$$

$$\sum_{i} x_{ii} \le 1$$

$$\min \sum_{ij \in \bar{E}} -x_{ij} + \sum_{i} -x_{ii}$$

$$\sum_{i} x_{ii} \begin{pmatrix} 0 & 0 & 0 \\ 0 & (i,i) : 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} +$$

$$\sum_{i} x_{ii} \begin{bmatrix} 0 & (i, i) \cdot 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} + \sum_{ij \in \bar{E}} x_{ij} \begin{bmatrix} 0 & 0 & 0 \\ 0 & (i, j) : 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix} \in POS_n$$

(P) Maximize 
$$\langle \mathbf{c}, \mathbf{x} \rangle$$
  
subject to  $\mathbf{b} - A(\mathbf{x}) \in L$   
 $\mathbf{x} \in K$ .

(D) Minimize 
$$\langle \mathbf{b}, \mathbf{y} \rangle$$
  
subject to  $A^T(\mathbf{y}) - \mathbf{c} \in K^*$   
 $\mathbf{y} \in L^*$ .

$$\mathbf{x} \in \mathbf{K}$$
.
$$\ddot{\mathbf{x}} = \mathbf{K}$$

$$\sum_{i} -x_{ii} + 1 \ge 0$$

$$\sum_{i} x_{ii} \le 1$$

$$\min \sum_{ij \in \bar{E}} -x_{ij} + \sum_{i} -x_{ii}$$

$$\sum_{i} x_{ii} \begin{pmatrix} 0 & 0 \\ 0 & (i,i) : 1 \end{pmatrix} +$$

 $\sum_{ij\in\bar{E}} x_{ij} \begin{pmatrix} 0 & 0 \\ 0 & (i,j):1 \end{pmatrix} \quad - \quad (0)$ 

$$\min \sum_{ij \in \bar{E}} -x_{ij} + \sum_{i} -x_{ii}$$

(P) Maximize 
$$\langle \mathbf{c}, \mathbf{x} \rangle$$
  
subject to  $\mathbf{b} - A(\mathbf{x}) \in L$   
 $\mathbf{x} \in K$ .

(D) Minimize 
$$\langle \mathbf{b}, \mathbf{y} \rangle$$
  
subject to  $A^T(\mathbf{y}) - \mathbf{c} \in K^*$   
 $\mathbf{y} \in L^*$ .

$$\min \sum_{ij \in \bar{E}} -x_{ij} + \sum_{i} -x_{ii}$$

$$\max \sum_{ij \in \bar{E}} x_{ij} + \sum_{i} x_{ii}$$

 $\sum_{i:\in\bar{E}} x_{ij} \begin{pmatrix} 0 & 0 \\ 0 & (i,j):1 \end{pmatrix} \quad - \quad (0)$ 

 $\sum x_{ii} \le 1$ 

$$\min \sum_{ij \in \bar{E}} -x_{ij} + \sum_{i} -x_{ii}$$

$$\max \sum_{ij \in \bar{E}} x_{ij} + \sum_{i} x_{ii}$$

$$\sum_{i} x_{ii} \begin{pmatrix} 0 & 0 \\ 0 & (i, i) : 1 \end{pmatrix} +$$

(P) Maximize 
$$\langle \mathbf{c}, \mathbf{x} \rangle$$
  
subject to  $\mathbf{b} - A(\mathbf{x}) \in L$   
 $\mathbf{x} \in K$ .

(D) Minimize 
$$\langle \mathbf{b}, \mathbf{y} \rangle$$
  
subject to  $A^T(\mathbf{y}) - \mathbf{c} \in K^*$   
 $\mathbf{y} \in L^*$ .

 $\in POS_n$ 

$$\mathbf{x} \in K.$$
 
$$\max \quad J_n \bullet X$$
 
$$X \in \mathrm{POS}_n$$

$$X \in POS_n$$

$$Tr(X) \le 1$$

$$\max \sum_{ij \in \bar{E}} x_{ij} + \sum_{i} x_{ii}$$
$$\sum_{i} x_{ii} \begin{pmatrix} 0 & 0 \\ 0 & (i,i) : 1 \end{pmatrix} +$$

 $\sum x_{ii} \le 1$ 

$$\sum_{i} x_{ii} \begin{pmatrix} 0 & 0 \\ 0 & (i, i) : 1 \end{pmatrix} +$$

$$\operatorname{Tr}(X) \leq 1$$

$$\sum_{ij \in \bar{E}} x_{ij} \begin{pmatrix} 0 & 0 \\ 0 & (i, j) : 1 \end{pmatrix} - (0)$$

(P) Maximize 
$$\langle \mathbf{c}, \mathbf{x} \rangle$$
  
subject to  $\mathbf{b} - A(\mathbf{x}) \in L$   
 $\mathbf{x} \in K$ .

(D) Minimize 
$$\langle \mathbf{b}, \mathbf{y} \rangle$$
  
subject to  $A^T(\mathbf{y}) - \mathbf{c} \in K^*$   
 $\mathbf{y} \in L^*$ .

$$\mathbf{x} \in K$$
.

 $\mathbf{max} \quad J_n \bullet X$ 
 $X \in POS_n$ 

$$X \in POS_n$$

$$Tr(X) \le 1$$

$$\max \sum_{ij \in \bar{E}} x_{ij} + \sum_{i} x_{ii}$$
$$\sum_{i} x_{ii} \begin{pmatrix} 0 & 0 \\ 0 & (i,i) : 1 \end{pmatrix} +$$

$$\sum_{i} x_{ii} \begin{pmatrix} 0 & 0 \\ 0 & (i,i) : 1 \end{pmatrix} + \sum_{ij \in \bar{E}} x_{ij} \begin{pmatrix} 0 & 0 \\ 0 & (i,j) : 1 \end{pmatrix} - (0)$$

$$\sum_{i} x_{ii} \le 1$$

 $x_{ij} = 0$   $ij \in E$ 

(P) Maximize 
$$\langle \mathbf{c}, \mathbf{x} \rangle$$
  
subject to  $\mathbf{b} - A(\mathbf{x}) \in L$   
 $\mathbf{x} \in K$ .

$$L \qquad \begin{array}{c} \text{(D)} \quad \text{Minimize} \quad \langle \mathbf{b}, \mathbf{y} \rangle \\ \text{subject to} \quad A^T(\mathbf{y}) - \mathbf{c} \in K^* \\ \mathbf{y} \in L^*. \end{array}$$

 $\in POS_n$ 

$$\max J_n \bullet X$$

$$X \in POS_n$$

$$Tr(X) < 1$$

$$\operatorname{Tr}(X) \leq 1$$

$$\max \sum_{ij \in \bar{E}} x_{ij} + \sum_{i} x_{ii} + \sum_{ij \in E} x_{ij}$$

$$\sum_{i} x_{ii} \begin{pmatrix} 0 & 0 \\ 0 & (i, i) : 1 \end{pmatrix} +$$

$$\sum_{i} x_{ii} \begin{pmatrix} 0 & 0 \\ 0 & (i, i) : 1 \end{pmatrix}$$

$$ij \in E \qquad i \qquad ij \in E$$

$$\sum_{i} x_{ii} \begin{pmatrix} 0 & 0 \\ 0 & (i,i) : 1 \end{pmatrix} \qquad +$$

$$\sum_{i: \in \bar{E}} x_{ij} \begin{pmatrix} 0 & 0 \\ 0 & (i,j) : 1 \end{pmatrix} \qquad - \qquad (0)$$

$$\sum_{ij\in\bar{E}} x_{ij} \begin{pmatrix} 0 & 0 \\ 0 & (i,j) : 1 \end{pmatrix}$$

$$\sum_{i} x_{ii} \le 1$$

$$x_{ii} = 0 \qquad ij$$

$$\overline{x_{ij}} = 0 \qquad ij \in E$$

(P) Maximize 
$$\langle \mathbf{c}, \mathbf{x} \rangle$$
  
subject to  $\mathbf{b} - A(\mathbf{x}) \in L$   
 $\mathbf{x} \in K$ .

$$L \qquad \begin{array}{c} \text{(D)} \quad \text{Minimize} \quad \langle \mathbf{b}, \mathbf{y} \rangle \\ \text{subject to} \quad A^T(\mathbf{y}) - \mathbf{c} \in K^* \\ \mathbf{y} \in L^*. \end{array}$$

$$\max \quad J_n \bullet X$$

$$X \in POS_n$$

$$Tr(X) \le 1$$

$$\max \sum_{ij \in \bar{E}} x_{ij} + \sum_{i} x_{ii} + \sum_{ij \in E} x_{ij}$$

$$\sum_{i} x_{ii} \begin{pmatrix} 0 & 0 \\ 0 & (i,i) : 1 \end{pmatrix} + \sum_{ij \in E} x_{ij} \begin{pmatrix} 0 & 0 \\ 0 & (i,j) : 1 \end{pmatrix}$$

$$\sum_{ij \in \bar{E}} x_{ij} \begin{pmatrix} 0 & 0 \\ 0 & (i,j) : 1 \end{pmatrix} - (0) \in POS$$

$$\sum_{i} x_{ii} \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$
$$\sum_{ij \in \bar{E}} x_{ij} \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$
$$\sum_{i} x_{ii} \le 1$$

$$\sum_{i} x_{ii} \le 1$$

$$x_{ij} = 0 \qquad ij \in E$$

 $\in POS_n$ 

$$X \in POS_n$$

$$Tr(X) \le 1$$

(P) Maximize 
$$\langle \mathbf{c}, \mathbf{x} \rangle$$
  
subject to  $\mathbf{b} - A(\mathbf{x}) \in L$   
 $\mathbf{x} \in K$ .

(D) Minimize 
$$\langle \mathbf{b}, \mathbf{y} \rangle$$
  
subject to  $A^T(\mathbf{y}) - \mathbf{c} \in K^*$   
 $\mathbf{y} \in L^*$ .

$$\mathbf{x} \in K$$
.

 $\mathbf{max} \quad J_n \bullet X$ 
 $X \in POS_n$ 
 $Tr(X) < 1$ 

$$\max \sum_{ij \in \bar{E}} x_{ij} + \sum_{i} x_{ii} + \sum_{ij \in E} x_{ij}$$

$$\sum_{i} x_{ii} \begin{pmatrix} 0 & 0 \\ 0 & (i, i) : 1 \end{pmatrix} + \sum_{ij \in E} x_{ij} \begin{pmatrix} 0 & 0 \\ 0 & (i, j) : 1 \end{pmatrix}$$

 $x_{ij} = 0$   $ij \in E$ 

$$\max \quad J_n \bullet X$$

$$X \in POS_n$$

$$Tr(X) \le 1$$

$$x_{ij} = 0 \quad ij \in E$$

$$\sum_{ij\in\bar{E}} x_{ij} \begin{pmatrix} 0 & 0 \\ 0 & (i,j):1 \end{pmatrix} - (0) \in POS_n$$

$$\sum_{i} x_{ii} \le 1$$

$$X \in POS_n$$

$$Tr(X) \le 1$$

(P) Maximize 
$$\langle \mathbf{c}, \mathbf{x} \rangle$$
  
subject to  $\mathbf{b} - A(\mathbf{x}) \in L$   
 $\mathbf{x} \in K$ .

(D) Minimize 
$$\langle \mathbf{b}, \mathbf{y} \rangle$$
  
subject to  $A^T(\mathbf{y}) - \mathbf{c} \in K^*$   
 $\mathbf{y} \in L^*$ .

 $J_n \bullet X$ 

$$Tr(X) \le 1$$
  $Tr(X) = 1$ 

max

(P) Maximize 
$$\langle \mathbf{c}, \mathbf{x} \rangle$$
  
subject to  $\mathbf{b} - A(\mathbf{x}) \in L$   
 $\mathbf{x} \in K$ .

(D) Minimize 
$$\langle \mathbf{b}, \mathbf{y} \rangle$$
  
subject to  $A^T(\mathbf{y}) - \mathbf{c} \in K^*$   
 $\mathbf{y} \in L^*$ .

 $J_n \bullet X$ 

 $X \in POS_n$ 

$$\operatorname{Tr}(X) \le 1$$
  $\operatorname{Tr}(X) = 1$   $x_{ij} = 0$   $ij \in E$ 

max

 $\alpha(G) \leq$ 

minimize 
$$t$$
  
subject to  $y_{ij} = -1$ , if  $\{i, j\} \in \overline{E}$   
 $y_{ii} = t - 1$ , for all  $i = 1, 2, ..., n$   
 $Y \in \text{COP}_n$ 

has value  $\alpha(G)$ , the size of a maximum independent set in G.

$$\alpha(G)$$
 روش: یک جواب شدنی برای دوگان با مقدار

I: یکی از بزرگترین مجموعههای مستقل

$$\max \quad J_n \bullet X$$

$$X \in POS_n$$

$$Tr(X) = 1$$

$$x_{ij} = 0 \quad ij \in E$$

minimize 
$$t$$
  
subject to  $y_{ij} = -1$ , if  $\{i, j\} \in \overline{E}$   
 $y_{ii} = t - 1$ , for all  $i = 1, 2, ..., n$   
 $Y \in \text{COP}_n$ 

$$\tilde{X}_i = 1_{[i \in I]}$$

$$\tilde{X} = \frac{1}{\alpha(G)} \tilde{\mathbf{x}} \tilde{\mathbf{x}}^T$$

 $\alpha(G) \leq$ 

$$\alpha(G)$$
 روش: یک جواب شدنی برای دوگان با مقدار  $\alpha$ 

I: یکی از بزرگترین مجموعههای مستقل

$$\max \quad J_n \bullet X$$

$$X \in POS_n$$

$$Tr(X) = 1$$

$$x_{ij} = 0 \quad ij \in E$$

$$\tilde{X}_i = 1_{[i \in I]}$$

$$\tilde{X} = \frac{1}{\alpha(G)} \tilde{\mathbf{x}} \tilde{\mathbf{x}}^T$$

 $\alpha(G) \leq$ 

$$\alpha(G)$$
 روش: یک جواب شدنی برای دوگان با مقدار

$$\max \quad J_n \bullet X$$

$$X \in POS_n$$

$$Tr(X) = 1$$

$$x_{ij} = 0 \quad ij \in E$$

 $\alpha(G) \leq$ 

minimize 
$$t$$
  
subject to  $y_{ij} = -1$ , if  $\{i, j\} \in \overline{E}$   
 $y_{ii} = t - 1$ , for all  $i = 1, 2, ..., n$   
 $Y \in \text{COP}_n$ 

has value  $\alpha(G)$ , the size of a maximum independent set in G.

$$\widetilde{x}_i = 1_{[i \in I]}$$
 مقدار  $\alpha(G)$  مستقل  $\widetilde{x}_i = 1_{[i \in I]}$  مستقل و ناز بزرگترین مجموعههای مستقل و ناز بزرگ

$$\tilde{X} = \frac{1}{\alpha(G)} \tilde{\mathbf{x}} \tilde{\mathbf{x}}^T \quad \tilde{x}_{ij} = \frac{1}{\alpha(G)} \tilde{x}_i \tilde{x}_j$$
 max

$$\operatorname{Tr}(X) = 1$$
  
 $x_{ij} = 0$   $ij \in E$ 

 $J_n \bullet X$ 

 $X \in POS_n$ 

 $\alpha(G) \leq$ 

minimize 
$$t$$
  
subject to  $y_{ij} = -1$ , if  $\{i, j\} \in \overline{E}$   
 $y_{ii} = t - 1$ , for all  $i = 1, 2, ..., n$   
 $Y \in \text{COP}_n$ 

$$\alpha(G)$$
 روش: یک جواب شدنی برای دوگان با مقدار  $\widetilde{x}_i=1_{[i\in I]}$  و تابعی از بزرگترین مجموعههای مستقل داد. تابعی از بزرگترین مجموعههای مستقل

$$ilde{X} = rac{1}{lpha(G)} ilde{\mathbf{x}} ilde{\mathbf{x}}^T ilde{x}_{ij} = rac{1}{lpha(G)} ilde{x}_i ilde{x}_j$$
 max  $J_n ullet J_n$ 

$$\max \quad J_n \bullet X$$

$$X \in POS_n$$

$$Tr(X) = 1$$

$$x_{ij} = 0 \quad ij \in E$$

$$\begin{array}{ll} \mbox{minimize} & t \\ \mbox{subject to} & y_{ij} = -1, \mbox{ if } \{i,j\} \in \overline{E} \\ & y_{ii} = t-1, \mbox{ for all } i = 1,2,\ldots,n \\ & Y \in \mathrm{COP}_n \end{array}$$

$$\widetilde{x}_i = 1_{[i \in I]}$$
 مقدار  $\alpha(G)$  مستقل  $\widetilde{x}_i = 1_{[i \in I]}$  مستقل و نارگترین مجموعههای مستقل و نارگ

$$\tilde{X} = \frac{1}{\alpha(G)} \tilde{\mathbf{x}} \tilde{\mathbf{x}}^T \quad \tilde{x}_{ij} = \frac{1}{\alpha(G)} \tilde{x}_i \tilde{x}_j \quad \operatorname{Tr}(\tilde{x} \tilde{x}^T) = \sum \bar{x}_i^2 \qquad \max \qquad J_n \bullet X$$

$$X \in \operatorname{POS}_n$$

$$\operatorname{Tr}(X) = 1$$

$$x_{ij} = 0 \qquad ij \in E$$

$$\begin{array}{ll} \text{minimize} & t \\ \text{subject to} & y_{ij} = -1, \text{ if } \{i, j\} \in \overline{E} \\ & y_{ii} = t - 1, \text{ for all } i = 1, 2, \dots, n \\ & Y \in \text{COP}_n \end{array}$$

$$\widetilde{x}_i = 1_{[i \in I]}$$
 مولان با مقدار  $\alpha(G)$  با مقدار  $\widetilde{x}_i = 1_{[i \in I]}$  وروش: یک جواب شدنی برای دوگان با مقدار  $\widetilde{x}_i = 1_{[i \in I]}$  و مستقل  $\widetilde{x}_i = 1_{[i \in I]}$ 

$$\tilde{X} = \frac{1}{\alpha(G)} \tilde{\mathbf{x}} \tilde{\mathbf{x}}^T \quad \tilde{x}_{ij} = \frac{1}{\alpha(G)} \tilde{x}_i \tilde{x}_j \quad \operatorname{Tr}(\tilde{x} \tilde{x}^T) = \sum_i \bar{x}_i^2 \qquad \max_{X \in POS_n} J_n \bullet X$$

$$Tr(X) = 1$$

$$x_{ij} = 0 \quad ij \in E$$

 $\alpha(G) \leq$ 

minimize 
$$t$$
  
subject to  $y_{ij} = -1$ , if  $\{i, j\} \in \overline{E}$   
 $y_{ii} = t - 1$ , for all  $i = 1, 2, ..., n$   
 $Y \in \text{COP}_n$ 

$$\widetilde{x}_i = 1_{[i \in I]}$$
 مقدار  $\alpha(G)$  با مقدار برای دوگان با مقدار  $\widetilde{x}_i = 1_{[i \in I]}$  و بررگترین مجموعههای مستقل

$$ilde{X}$$
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minimize 
$$t$$
  
subject to  $y_{ij} = -1$ , if  $\{i, j\} \in \overline{E}$   
 $y_{ii} = t - 1$ , for all  $i = 1, 2, ..., n$   
 $Y \in \text{COP}_n$ 

$$\widetilde{x}_i = 1_{[i \in I]}$$
 مقدار  $\alpha(G)$  با مقدار برای دوگان با مقدار  $\widetilde{x}_i = 1_{[i \in I]}$  و بررگترین مجموعههای مستقل  $\widetilde{x}_i = 1_{[i \in I]}$ 

$$\tilde{X} = \frac{1}{\alpha(G)} \tilde{\mathbf{x}} \tilde{\mathbf{x}}^T \quad \tilde{x}_{ij} = \frac{1}{\alpha(G)} \tilde{x}_i \tilde{x}_j \quad \operatorname{Tr}(\tilde{x} \tilde{x}^T) = \sum \bar{x}_i^2 \qquad \max \qquad J_n \bullet X \\ X \in \operatorname{POS}_n \\ \operatorname{Tr}(X) = 1 \\ x_{ij} = 0 \qquad ij \in E$$

.2.1 Theorem. The copositive program 
$$t$$

minimize 
$$t$$
  
subject to  $y_{ij} = -1$ , if  $\{i, j\} \in \overline{E}$   
 $y_{ii} = t - 1$ , for all  $i = 1, 2, ..., n$   
 $Y \in \text{COP}_n$ 

$$\tilde{x}_i = 1_{[i \in I]}$$
 مقدار  $\alpha(G)$  با مقدار برای دوگان با مقدار  $\tilde{x}_i = 1_{[i \in I]}$  و برای مجموعههای مستقل  $\tilde{x}_i = 1_{[i \in I]}$ 

$$\tilde{X} = \frac{1}{\alpha(G)} \tilde{\mathbf{x}} \tilde{\mathbf{x}}^{T} \quad \tilde{x}_{ij} = \frac{1}{\alpha(G)} \tilde{x}_{i} \tilde{x}_{j} \quad \operatorname{Tr}(\tilde{x} \tilde{x}^{T}) = \sum_{i,j} \tilde{x}_{i}^{2} \qquad \max \qquad J_{n} \bullet X$$

$$X \in \operatorname{POS}_{n}$$

$$\operatorname{Tr}(X) = 1$$

$$X_{ij} = 0 \quad ij \in E$$

minimize 
$$t$$
 subject to  $y_{ij} = -1$ , if  $\{i, j\} \in \overline{E}$   $y_{ii} = t - 1$ , for all  $i = 1, 2, ..., n$ 

 $Y \in COP_n$ 

$$\widetilde{x}_i = 1_{[i \in I]}$$
 مقدار  $\alpha(G)$  با مقدار وش: یک جواب شدنی برای دوگان با مقدار  $\widetilde{x}_i = 1_{[i \in I]}$  و تکمی از بزرگترین مجموعه های مستقل و تکمی از بزرگترین مجموعه های مستقل

$$\tilde{X} = \frac{1}{\alpha(G)} \tilde{\mathbf{x}} \tilde{\mathbf{x}}^T \quad \tilde{x}_{ij} = \frac{1}{\alpha(G)} \tilde{x}_i \tilde{x}_j \quad \operatorname{Tr}(\tilde{x} \tilde{x}^T) = \sum_i \bar{x}_i^2 \qquad \max_{X \in POS_n} X \in POS_n$$

$$\operatorname{Tr}(X) = 1$$

$$J_n \bullet \tilde{X} = \sum_{i,j} \tilde{x}_{ij} = \frac{1}{\alpha(G)} \sum_{i,j} i = \frac{1}{\alpha(G)} \sum_{i,j \in I} 1$$

$$x_{ij} = 0 \qquad ij \in E$$

minimize 
$$t$$
  
subject to  $y_{ij} = -1$ , if  $\{i, j\} \in \overline{E}$   
 $y_{ii} = t - 1$ , for all  $i = 1, 2, ..., n$   
 $Y \in \text{COP}_n$ 

$$\tilde{x}_i = 1_{[i \in I]}$$
 مقدار  $\alpha(G)$  با مقدار وش: یک جواب شدنی برای دوگان با مقدار  $\tilde{x}_i = 1_{[i \in I]}$  و تابیخی از بزرگترین مجموعه های مستقل و تابیخی از بزرگترین مجموعه های مستقل و تابیخی از بزرگترین مجموعه های مستقل و تابیخی از بزرگترین محموعه های مستقل و تابیخی از بزرگترین محموعه های مستقل و تابیخی از برگذرین محموعه و تابیخی از برگذرین محموعه های مستقل و تابیخی از برگذرین محموعه های مستقل و تابیخی از برگذرین و تابیخی و تابیخ

ا: یکی از بزرگترین مجموعههای مستقل: 
$$\tilde{X} = \frac{1}{\alpha(G)} \tilde{\mathbf{x}} \tilde{\mathbf{x}}^T$$
  $\tilde{x}_{ij} = \frac{1}{\alpha(G)} \tilde{x}_i \tilde{x}_j$   $\operatorname{Tr}(\tilde{x} \tilde{x}^{\mathsf{T}}) = \sum \bar{x}_i^2$   $\max$   $J_n \bullet X$ 

 $\alpha(G) \qquad X \in POS_n$   $X \in POS_n$  Tr(X) = 1  $J_n \bullet \tilde{X} = \sum_{i,j} \tilde{x}_{ij} = \frac{1}{\alpha(G)} \sum_{i,j} i = \frac{1}{\alpha(G)} \sum_{i,j \in I} 1 = \frac{\alpha(G)^2}{\alpha(G)}$   $x_{ij} = 0 \qquad ij \in E$ 

minimize 
$$t$$
  $subject\ to\ y_{ij}=-1,\ if\ \{i,j\}\in\overline{E}$   $y_{ii}=t-1,\ ext{for all}\ i=1,2,\ldots,n$ 

 $Y \in COP_n$ 

$$\alpha(G)$$
 روش: یک جواب شدنی برای دوگان با مقدار  $\widetilde{x}_i=1_{[i\in I]}$  همتقل از بزرگترین مجموعههای مستقل د.  $\widetilde{x}_i=1_{[i\in I]}$ 

 $J_n \bullet X$ 

 $X \in POS_n$ 

$$\tilde{X} = \frac{1}{\alpha(G)} \tilde{\mathbf{x}} \tilde{\mathbf{x}}^T \quad \tilde{x}_{ij} = \frac{1}{\alpha(G)} \tilde{x}_i \tilde{x}_j \quad \operatorname{Tr}(\tilde{x} \tilde{x}^T) = \sum_i \bar{x}_i^2$$
 max

$$J_n \bullet \tilde{X} = \sum_{i,j} \tilde{x}_{ij} = \frac{1}{\alpha(G)} \sum_{i,j} i = \frac{1}{\alpha(G)} \sum_{i,j \in I} 1 = \frac{\alpha(G)^2}{\alpha(G)}$$

$$Tr(X) = 1$$

$$x_{ij} = 0 \qquad ij \in E$$

 $\alpha(G) \leq$ 

.2.1 Theorem. The copositive program 
$$t$$

$$\begin{array}{ll} \mbox{minimize} & t \\ \mbox{subject to} & y_{ij} = -1, \mbox{ if } \{i,j\} \in \overline{E} \\ & y_{ii} = t-1, \mbox{ for all } i = 1,2,\ldots,n \\ & Y \in \mathrm{COP}_n \end{array}$$

minimize tsubject to  $y_{ij} = -1$ , if  $\{i, j\} \in \overline{E}$   $y_{ii} = t - 1$ , for all i = 1, 2, ..., n $Y \in \text{COP}_n$ 

minimize subject to  $y_{ij} = -1$ , if  $\{i, j\} \in \overline{E}$  $y_{ii} = t - 1$ , for all i = 1, 2, ..., n $Y \in COP_n$ 

has value  $\alpha(G)$ , the siz صفر روى درايه هاى ident set in G.

بدون يال

$$Y = tI_n + Z - J_n$$

minimize tsubject to  $y_{ij} = -1$ , if  $\{i, j\} \in \overline{E}$   $y_{ii} = t - 1$ , for all i = 1, 2, ..., n $Y \in \text{COP}_n$ 

has value  $\alpha(G)$ , the siz مفر روى درايههاى ident set in G.

بدون بال

$$Y = tI_n + Z - J_n$$

$$z = \max_{i,j}(Z_{i,j})$$

$$Y' = tI_n + zA_G - J_n$$

 $\alpha(G) \leq$ 

$$\begin{array}{ll} \mbox{minimize} & t \\ \mbox{subject to} & y_{ij} = -1, \mbox{ if } \{i,j\} \in \overline{E} \\ & y_{ii} = t-1, \mbox{ for all } i = 1,2,\ldots,n \\ & Y \in \mathrm{COP}_n \end{array}$$

has value  $\alpha(G)$ , the size of a maximum independent set in G.

7.2.5 Lemma. The copositive program

Minimize tsubject to  $tI_n + zA_G - J_n \in COP_n$  $t, z \in \mathbb{R}$ 

جوابش برابر با جواب برنامهریزی بالاست.

**7.2.6 Theorem.** For every graph G,

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}.$$

**7.2.6 Theorem.** For every graph G,

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}. \left\{ := m(G) \right\}$$

**7.2.6 Theorem.** For every graph G,

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}.$$
 :=  $m(G)$ 

f(x) :=

**7.2.6 Theorem.** For every graph G,

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f(x) :=

$$m(G) \le \frac{1}{\alpha(G)}$$
 (الف

... (

**7.2.6 Theorem.** For every graph G,

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f(x) :=

$$m(G) \le \frac{1}{\alpha(G)}$$
 (الف

.. •

$$m(G) \ge \frac{1}{\alpha(G)}$$
 (ب

... (

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f(x) :=

$$m(G) \le \frac{1}{\alpha(G)}$$
 (الف

بردار مشخصه مجموعه مستقل $ilde{x}$ 

**7.2.6 Theorem.** For every graph G,

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}. \left\{ := m(G) \right\}$$

f(x) :=

$$m(G) \le \frac{1}{\alpha(G)}$$
 (الف

بردار مشخصه مجموعه مستقل  $\tilde{x}$ 

$$f(\tilde{x}) = \sum_{i,j} (A_G + I)_{i,j} \tilde{x}_i \tilde{x}_j$$

**7.2.6 Theorem.** For every graph G,

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}.$$
 :=  $m(G)$ 

f(x) :=

$$m(G) \le \frac{1}{\alpha(G)}$$
 (الف

بردار مشخصه مجموعه مستقل  $\tilde{x}$ 

$$f(\tilde{x}) = \sum_{i,j} (A_G + I)_{i,j} \tilde{x}_i \tilde{x}_j = |I|$$

**7.2.6 Theorem.** For every graph G,

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}. \left\{ := m(G) \right\}$$

f(x) :=

$$m(G) \le \frac{1}{\alpha(G)}$$
 (الف

بردار مشخصه مجموعه مستقل  $\tilde{x}$ 

$$f(\tilde{x}) = \sum_{i,j} (A_G + I)_{i,j} \tilde{x}_i \tilde{x}_j = |I|$$

$$\frac{1}{\alpha(G)} \tilde{x}$$

**7.2.6 Theorem.** For every graph G,

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}. \left\{ := m(G) \right\}$$

f(x) :=

$$m(G) \leq \frac{1}{\alpha(G)}$$
 (الف $\widetilde{x}$ : بردار مشخصه مجموعه مستقل

 $f(\tilde{x}) = \sum_{i,j} (A_G + I)_{i,j} \tilde{x}_i \tilde{x}_j = |I|$ 

$$\frac{1}{\alpha(G)}\tilde{x}$$
 •

 $f(\frac{1}{\alpha(G)}\tilde{x}) = \frac{1}{\alpha(G)}$ نرم ۱ = ۱، کنج مثبت،  $\alpha(G)$ 

**7.2.6 Theorem.** For every graph G,

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}.$$
 :=  $m(G)$ 

$$f(x) :=$$

$$m(G) \ge \frac{1}{\alpha(G)}$$
 (:

**7.2.6 Theorem.** For every graph G,

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}.$$
 :=  $m(G)$ 

f(x) :=

$$m(G) \ge \frac{1}{\alpha(G)}$$
 (ب

- جواب بهینه  $x^*$
- با بیشترین صفر

**7.2.6 Theorem.** For every graph G,

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}.$$
 :=  $m(G)$ 

f(x) :=

$$m(G) \ge \frac{1}{\alpha(G)}$$
 (ب

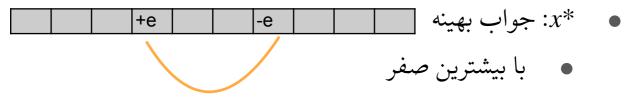
- جواب بهینه  $x^*$
- با بیشترین صفر
- یال i و j که دو سرشان مثبت است

**7.2.6 Theorem.** For every graph G,

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}.$$
 :=  $m(G)$ 

$$f(x) :=$$

$$m(G) \ge \frac{1}{\alpha(G)}$$
 ( $\dot{\varphi}$ 



• یال i و j که دو سرشان مثبت است

**7.2.6 Theorem.** For every graph G,

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}.$$
 :=  $m(G)$ 

f(x) :=

$$m(G) \ge \frac{1}{\alpha(G)}$$
 (:

مال i و j که دو سرشان مثبت است

$$z_k = \left\{ egin{array}{ll} x_k^* + arepsilon & ext{if } k = i \ x_k^* - arepsilon & ext{if } k = j \ x_k^* & ext{otherwise,} \end{array} 
ight.$$

**7.2.6 Theorem.** For every graph G,

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}.$$
 :=  $m(G)$ 

f(x) :=

$$m(G) \ge \frac{1}{\alpha(G)} \ (\because$$

ا یال i و j که دو سرشان مثبت است

$$f(z) = f(x^*) + l(\epsilon) z_k = \begin{cases} x_k^* + \varepsilon & \text{if } k = i \\ x_k^* - \varepsilon & \text{if } k = j \\ x_k^* & \text{otherwise,} \end{cases}$$

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}.$$
 :=  $m(G)$ 

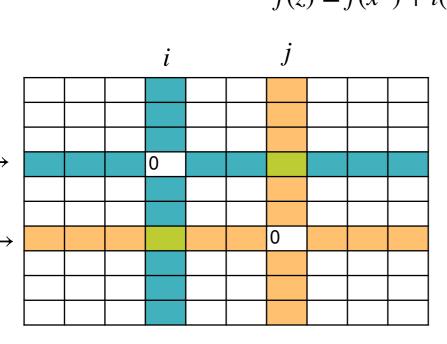
$$f(z) := \begin{cases} f(x) := \\ f(z) = f(x^*) + l(\epsilon) \end{cases} \quad z_k = \begin{cases} x_k^* + \epsilon & \text{if } k = i \\ x_k^* - \epsilon & \text{if } k = j \\ x_k^* & \text{otherwise,} \end{cases}$$

7.2.6 Theorem. For every graph 
$$G$$
, 
$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}. \quad := m(G)$$

$$f(x) :=$$

$$f(z) = f(x^*) + l(\epsilon) \qquad z_k = \begin{cases} x_k^* + \epsilon & \text{if } k = i \\ x_k^* - \epsilon & \text{if } k = j \\ x_k^* & \text{otherwise,} \end{cases}$$

$$i \qquad j$$



7.2.6 Theorem. For every graph 
$$G$$
, 
$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}. \quad := m(G)$$

$$f(x) :=$$

$$f(z) = f(x^*) + l(\epsilon) \qquad z_k = \begin{cases} x_k^* + \epsilon & \text{if } k = i \\ x_k^* - \epsilon & \text{if } k = j \\ x_k^* & \text{otherwise,} \end{cases}$$

$$i \qquad j$$

$$f(z) = \epsilon \mathbf{B} - \epsilon \mathbf{O} + 2(x_i + \epsilon)(x_j - \epsilon) + \epsilon \mathbf{O} + 2(x_i + \epsilon)(x_j - \epsilon)(x_j - \epsilon) + \epsilon \mathbf{O} + 2(x_i + \epsilon)(x_j - \epsilon)(x_j$$

**7.2.6 Theorem.** For every graph 
$$G$$
,

orem. For every graph 
$$G$$
,

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}.$$
 :=  $m(G)$ 

$$f(z) = f(x^*) + i$$

$$i$$

$$j$$

$$0$$

$$0$$

$$f(x) := \begin{cases} f(x) := \\ f(z) = f(x^*) + l(\epsilon) \end{cases} \quad z_k = \begin{cases} x_k^* + \epsilon & \text{if } k = i \\ x_k^* - \epsilon & \text{if } k = j \\ x_k^* & \text{otherwise,} \end{cases}$$

 $f(z) = \epsilon \mathbf{B} - \epsilon \mathbf{O} + 2(x_i + \epsilon)(x_i - \epsilon) +$ 

 $(x_i + \epsilon)^2 + (x_i - \epsilon) + \dots$ 

**7.2.6 Theorem.** For every graph 
$$G$$
,

em. For every graph 
$$G$$
,
$$\frac{1}{1} = \min\{\mathbf{x}^T (A_G + I_G)\}$$

**m.** For every graph 
$$G$$
, 
$$\frac{1}{(G)^2} = \min\{\mathbf{x}^T (A_G + I_n)\}$$

$$\{\mathbf{x}^T(A_G+I_n)\mathbf{x}: \mathbf{x} \geq \mathbf{0}, \sum_{i=1}^n x_i = 1\}.$$

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}.$$
 :=  $m(G)$ 

$$f(x) :=$$

$$f(x) :=$$

$$f(x) :=$$

$$f(x) :=$$

$$(x) :=$$

$$*$$
 +  $c$  if  $k - i$ 

$$k + \varepsilon$$
 if  $k = i$ 

$$k + \varepsilon$$
 if  $k = 0$ 

$$\begin{array}{ll} \text{if } k = i \\ \text{if } k = j \end{array}$$

$$f(z) = f(x^*) + l(\varepsilon) \qquad z_k = \begin{cases} x_k^* + \varepsilon & \text{if } k = i \\ x_k^* - \varepsilon & \text{if } k = j \\ x_k^* & \text{otherwise,} \end{cases}$$

$$f(z) = \epsilon \mathbf{B} - \epsilon \mathbf{O} + 2(x_i + \epsilon)(x_j - \epsilon) +$$
$$(x_i + \epsilon)^2 + (x_j - \epsilon) + \dots$$

نسبت به 
$$l(\epsilon)$$
 خطی است  $\epsilon$ 

**7.2.6 Theorem.** For every graph G,  $\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1} x_i = 1\}.$  := m(G)f(x) := $f(z) = f(x^*) + l(\epsilon) \qquad z_k = \begin{cases} x_k^* + \varepsilon & \text{if } k = i \\ x_k^* - \varepsilon & \text{if } k = j \\ x_k^* & \text{otherwise,} \end{cases}$  $f(z) = \epsilon \mathbf{B} - \epsilon \mathbf{O} + 2(x_i + \epsilon)(x_i - \epsilon) +$  $(x_i + \epsilon)^2 + (x_i - \epsilon) + \dots$ نسبت به f(z) نسبت به  $l(\epsilon)$  خطی است  $\epsilon$  خطی است  $\epsilon$ 0

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}.$$
 :=  $m(G)$ 

$$f(z) := \begin{cases} f(x) := \\ f(z) = f(x^*) + l(\epsilon) \end{cases} \quad z_k = \begin{cases} x_k^* + \epsilon & \text{if } k = i \\ x_k^* - \epsilon & \text{if } k = j \\ x_k^* & \text{otherwise,} \end{cases}$$

نسبت به 
$$f(z)$$
 نسبت به  $l(\epsilon)$  خطی است  $\epsilon$  خطی است  $\epsilon$ 

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}.$$
 :=  $m(G)$ 

$$f(z) := \begin{cases} f(x) := \\ f(z) = f(x^*) + l(\epsilon) \end{cases} \quad z_k = \begin{cases} x_k^* + \epsilon & \text{if } k = i \\ x_k^* - \epsilon & \text{if } k = j \\ x_k^* & \text{otherwise,} \end{cases}$$

ا متحد 
$$l(\epsilon)$$
 متحد  $f(z)$  متحد  $e$  است به  $e$  است  $e$  متحد است  $e$  خطی است  $e$  خطی است  $e$  خطی است  $e$  متحد است  $e$  متحد است به خطی است به خطی است به خطی است به است به خطی است به خطی است است به خطی است به به خطی است به

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}. \left\{ := m(G) \right\}$$

$$f(x) :=$$

$$f(z) = f(x^*) + l(\varepsilon) \qquad z_k = \begin{cases} x_k^* + \varepsilon & \text{if } k = i \\ x_k^* - \varepsilon & \text{if } k = j \\ x_k^* & \text{otherwise,} \end{cases}$$

میتوان یکی از درایههای \*x را صفر کرد

 $l(\epsilon)$  نسبت به f(z) نسبت به e نسبت به e نسبت به خطی است e خطی است e خطی است e

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}.$$
 :=  $m(G)$ 

f(x) :=

$$f(z) = f(x^*) + l(\epsilon) \qquad z_k = \begin{cases} x_k^* + \varepsilon & \text{if } k = i \\ x_k^* - \varepsilon & \text{if } k = j \\ x_k^* & \text{otherwise,} \end{cases}$$

 $l(\epsilon)$  نسبت به f(z) نسبت به f(z) متحد f(z) متحد f(z) نسبت به خطی است f(z) متحد f(z) عضلی است f(z) متحد f(z)

**7.2.6 Theorem.** For every graph G,

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}.$$
 :=  $m(G)$ 

f(x) :=

$$m(G) \ge \frac{1}{\alpha(G)} ( \dot{\varphi} )$$

• با بیشترین *صف*ر

ا یال i و j که دو سرشان مثبت است

$$f(z) = f(x^*) + l(\epsilon) z_k = \begin{cases} x_k^* + \varepsilon & \text{if } k = i \\ x_k^* - \varepsilon & \text{if } k = j \\ x_k^* & \text{otherwise,} \end{cases}$$

**7.2.6 Theorem.** For every graph 
$$G$$
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 :=  $m(G)$ 

f(x) :=

$$m(G) \ge \frac{1}{\alpha(G)}$$
 (ب

$$x^*$$
 • با بیشترین صفر با بیشترین صفر یال  $i$  و  $j$  که دو سرشان مثبت است  $x_k^*$  و  $x_k^*$   $x_k^*$   $x_k^*$  و  $x_k^*$ 

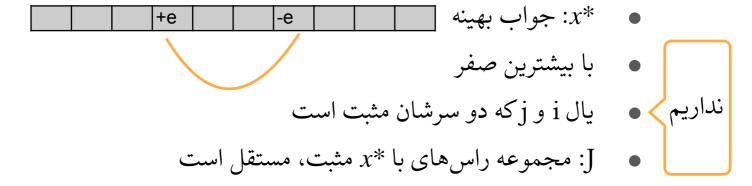
$$f(z) = f(x^*) + l(\epsilon) z_k = \begin{cases} x_k^* + \varepsilon & \text{if } k = i \\ x_k^* - \varepsilon & \text{if } k = j \\ x_k^* & \text{otherwise,} \end{cases}$$

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 :=  $m(G)$ 

$$f(x) :=$$

$$m(G) \ge \frac{1}{\alpha(G)}$$
 (ب



$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}.$$
 :=  $m(G)$ 

$$f(x) :=$$
 
$$m(G) \ge \frac{1}{\alpha(G)} (\because \bullet)$$

است، مستقل است  $x^*$  مثبت، مستقل است :J

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}. \left\{ := m(G) \right\}$$

$$f(x) := m(G) \ge \frac{1}{\alpha(G)} (\mathbf{x} - \mathbf{0})$$

است، مستقل است  $x^*$  است الستان مستقل است

$$f(x) = 2 \sum_{i,j \in E(G)} x_i x_j + \sum_{i \in J} x_i^2$$

**7.2.6 Theorem.** For every graph 
$$G$$
,

 $f(x) = 2 \quad \sum \quad x_i x_j + \sum x_i^2$ 

 $i,j \in E(G)$ 

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}. \left\{ := m(G) \right\}$$

f(x) :=

است، مستقل است  $x^*$  مثبت، مستقل است :J

 $m(G) \ge \frac{1}{\alpha(G)}$  (ب

$$\frac{1}{\mathbf{x}^T(A_G + I_n)\mathbf{x}} = \min\{\mathbf{x}^T(A_G + I_n)\mathbf{x}: \mathbf{x}^T(A_G + I_n)\mathbf{x}\}$$

em. For every graph 
$$G$$
,

**7.2.6 Theorem.** For every graph 
$$G$$
,

 $f(x) = 2 \sum_{i=1}^{n} x_i x_i + \sum_{i=1}^{n} x_i^2$ 

 $i,j \in E(G)$ 

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}.$$
 :=  $m(G)$ 

است، مستقل است  $x^*$  مثبت، مستقل است :J

 $m(G) \ge \frac{1}{\alpha(G)}$  (ب

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x}$$

$$\frac{1}{\mathbf{x}^T} = \min\{\mathbf{x}^T (A_G + I_n)\}$$

**7.2.6 Theorem.** For every graph 
$$G$$
,

**rem.** For every graph 
$$G$$
,

$$\operatorname{raph} G$$
,

f(x) :=

است، مستقل است  $x^*$  مثبت، مستقل است :J

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}.$$
 :=  $m(G)$ 

 $m(G) \ge \frac{1}{\alpha(G)}$  (ب

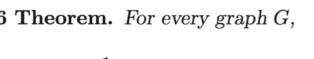
**2.6 Theorem.** For every graph 
$$G$$
,

**neorem.** For every graph 
$$G$$
,

 $||u||^2||v||^2 \ge \langle u, v \rangle^2$ 

 $f(x) = 2 \sum_{i=1}^{n} x_i x_i + \sum_{i=1}^{n} x_i^2$ 

 $i,j \in E(G)$ 



**.6 Theorem.** For every graph 
$$G$$
,

**rem.** For every graph 
$$G$$
,

**7.2.6 Theorem.** For every graph 
$$G$$
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1.2.6 Theorem. For every graph 
$$G$$
,

$$\frac{1}{\mathbf{x}^T} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}$$

f(x) :=

است، مستقل است  $x^*$  مثبت، مستقل است

 $m(G) \ge \frac{1}{\alpha(G)}$  (ب

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$$T = T \cdot T \cdot T$$

 $||u||^2||v||^2 \ge \langle u, v \rangle^2$ 

 $(\sum_{i \in I} x_i^2)(\sum_{i \in I} (\frac{1}{\sqrt{|J|}})^2)$ 

 $f(x) = 2 \quad \sum \quad x_i x_j + \sum x_i^2$ 

 $i,j \in E(G)$ 

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}.$$
 :=  $m(G)$ 

**1.2.6 Theorem.** For every graph 
$$G$$
,

em. For every graph 
$$G$$
,

**7.2.6 Theorem.** For every graph 
$$G$$
,

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T(A_G + I_n)\mathbf{x}: \mathbf{x} \geq \mathbf{0}, \sum_{i=1}^n x_i = 1\}.$$

$$\frac{1}{A_G} = \min\{\mathbf{x}^T (A_G + I_n)\}$$

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1} x_i = 1\}.$$
 :=  $m(G)$ 

$$m(G) \ge \frac{1}{\alpha(G)}$$
 (ب

است، مستقل است 
$$x^*$$
 مثبت، مستقل است

$$||u||^2||v||^2 \ge \langle u, v \rangle^2$$

$$(\sum_{i \in J} x_i^2) (\sum_{i \in J} (\frac{1}{\sqrt{|J|}})^2) \ge (\sum_{i \in J} x_i \frac{1}{\sqrt{|J|}})^2$$

$$f(x) = 2 \sum_{i,j \in E(G)} x_i x_j + \sum_{i \in J} x_i^2$$

**7.2.6 Theorem.** For every graph 
$$G$$
,

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}. \left\{ := m(G) \right\}$$

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)$$

$$f(x) := \int_{i=1}^{\infty} f(x) dx$$

$$m(G) \ge \frac{1}{\alpha(G)}$$
 ( $\dot{\varphi}$ 

است، مستقل است 
$$x^*$$
 مجموعه راسهای با  $x^*$ 

$$||u||^2||v||^2 \ge \langle u, v \rangle^2$$

$$\left(\sum_{i \in I} x_i^2\right) \left(\sum_{i \in I} \left(\frac{1}{\sqrt{|J|}}\right)^2\right) \ge \left(\sum_{i \in I} x_i \frac{1}{\sqrt{|J|}}\right)^2 \ge \frac{1}{|J|} \left(\sum_{i \in I} x_i\right)^2$$

$$f(x) = 2 \sum_{i,j \in E(G)} x_i x_j + \sum_{i \in J} x_i^2$$

**7.2.6 Theorem.** For every graph 
$$G$$
,

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}.$$
 :=  $m(G)$ 

 $||u||^2||v||^2 \ge \langle u, v \rangle^2$ 

 $f(x) = 2 \quad \sum \quad x_i x_j + \sum x_i^2$ 

 $i,j \in E(G)$ 

For every graph 
$$G$$
,

For every graph 
$$G$$
,

$$\operatorname{graph} G,$$

f(x) :=

است، مستقل است  $x^*$  مثبت، مستقل است I

 $\left(\sum_{i \in J} x_i^2\right) \left(\sum_{i \in J} \left(\frac{1}{\sqrt{|J|}}\right)^2\right) \ge \left(\sum_{i \in J} x_i \frac{1}{\sqrt{|J|}}\right)^2 \ge \frac{1}{|J|} \left(\sum_{i \in J} x_i\right)^2 = \frac{1}{|J|}$ 

$$G$$
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$$G$$
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$$n$$

 $m(G) \ge \frac{1}{\alpha(G)}$  (ب

For every graph 
$$G$$
,

**7.2.6 Theorem.** For every graph 
$$G$$
,

f(x) :=

است، مستقل است  $x^*$  مثبت، مستقل است

 $(\sum_{i \in J} x_i^2)(\sum_{i \in J} (\frac{1}{\sqrt{|J|}})^2) \ge (\sum_{i \in J} x_i \frac{1}{\sqrt{|J|}})^2 \ge \frac{1}{|J|}(\sum_{i \in J} x_i)^2 = \frac{1}{|J|}$ 

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}. \left\{ := m(G) \right\}$$

 $m(G) \ge \frac{1}{\alpha(G)}$  (ب

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**rem.** For every graph 
$$G$$
,

 $||u||^2||v||^2 \ge \langle u, v \rangle^2$ 

 $f(x) = 2 \sum_{i \in S} x_i x_j + \sum_{i \in S} x_i^2 \ge \frac{1}{|I|}$ 

**2.6 Theorem.** For every graph 
$$G$$
,

**.6 Theorem.** For every graph 
$$G$$
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**2.6 Theorem.** For every graph 
$$G$$
,

**7.2.6 Theorem.** For every graph 
$$G$$
,

$$+I_n)\mathbf{x}: \mathbf{x} \geq \mathbf{0}, \sum_{i=1}^n x_i = 1\}.$$

$$\frac{1}{1} = \min\{\mathbf{r}^T(A_{n+1}, I_n)\}$$

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} :$$

$$\frac{1}{\mathbf{x}^T(G)} = \min\{\mathbf{x}^T(A_G + I_n)\}$$

$$1 \qquad \qquad T(A + I)$$

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}. \left\{ := m(G) \right\}$$

 $||u||^2||v||^2 \ge \langle u, v \rangle^2$ 

 $f(x) = 2 \sum_{i \in S} x_i x_j + \sum_{i \in S} x_i^2 \ge \frac{1}{|I|} \ge \frac{1}{\alpha(G)}$ 

$$(A_G + I_n)\mathbf{x}: \mathbf{x} \geq \mathbf{0}, \sum_{i=1}^n x_i = 1\}.$$

For every graph 
$$G$$
,

است، مستقل است  $x^*$  مثبت، مستقل است

 $\left(\sum_{i \in J} x_i^2\right) \left(\sum_{i \in J} \left(\frac{1}{\sqrt{|J|}}\right)^2\right) \ge \left(\sum_{i \in J} x_i \frac{1}{\sqrt{|J|}}\right)^2 \ge \frac{1}{|J|} \left(\sum_{i \in J} x_i\right)^2 = \frac{1}{|J|}$ 

$$= m(G)$$

 $m(G) \ge \frac{1}{\alpha(G)}$  (ب

**7.2.6 Theorem.** For every graph G,

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}.$$
 :=  $m(G)$ 

f(x) :=

$$m(G) \le \frac{1}{\alpha(G)}$$
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$$m(G) \ge \frac{1}{\alpha(G)}$$
 (ب

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Minimize 
$$t$$
  
subject to  $tI_n + zA_G - J_n \in COP_n$   
 $t, z \in \mathbb{R}$ 

$$t,z\in\mathbb{R}$$
 . جوابش برابر با جواب برنامه ریزی بالاست.

**7.2.6 Theorem.** For every graph 
$$G$$
,

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}.$$

Minimize 
$$t$$
  
subject to  $tI_n + zA_G - J_n \in COP_n$   
 $t, z \in \mathbb{R}$ 

$$t,z\in\mathbb{R}$$
  $t,z\in\mathbb{R}$ 

**7.2.6 Theorem.** For every graph 
$$G$$
,

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}.$$

$$\mathbf{x}^T(\alpha(G)(A_G + I_n))\mathbf{x} \ge 1$$

برای هر x روی سادک:

$$Minimize \quad t$$

Minimize 
$$t$$
  
subject to  $tI_n + zA_G - J_n \in COP_n$   
 $t, z \in \mathbb{R}$ 

$$t,z\in\mathbb{R}$$
 جوابش برابر با جواب برنامهریزی بالاست.

**7.2.6 Theorem.** For every graph 
$$G$$
,

$$\frac{1}{\alpha(G)} = \min\{\mathbf{x}^T (A_G + I_n)\mathbf{x} : \mathbf{x} \ge \mathbf{0}, \sum_{i=1}^n x_i = 1\}.$$

برای هر x روی سادک:

$$\mathbf{x}^T(\alpha(G)(A_G+I_n))\mathbf{x} \geq 1 = \mathbf{x}^T J_n \mathbf{x},$$

Minimize 
$$t$$
  
subject to  $tI_n + zA_G - J_n \in COP_n$   
 $t, z \in \mathbb{R}$ 

$$t, z \in \mathbb{R}$$

$$= -\epsilon \log t + \epsilon \log t$$

$$= -\epsilon \log t + \epsilon \log t$$

$$\mathbf{x}^T(\alpha(G)(A_G+I_n))\mathbf{x} \ge 1 = \mathbf{x}^T J_n \mathbf{x},$$

$$Minimize \quad t$$

subject to  $tI_n + zA_G - J_n \in COP_n$  $t, z \in \mathbb{R}$ 

$$t,z\in\mathbb{R}$$

$$T(-(\alpha)T + (\alpha)A - T) > 0$$

 $\mathbf{x}^T(\alpha(G)(A_G+I_n))\mathbf{x} \geq 1 = \mathbf{x}^T J_n \mathbf{x},$ 

$$\mathbf{x}^T(lpha(G)I_n+lpha(G)A_G-J_n)\mathbf{x}\geq \mathbf{0}, \;$$
برای هر  $\mathbf{x}$  روی سادک:

Minimize tsubject to  $tI_n + zA_G - J_n \in COP_n$ 

subject to 
$$tI_n + zA_G - J_n \in \mathrm{COI}_n$$
  $t,z \in \mathbb{R}$ 

برای هر x روی سادک:

$$\mathbf{x}^T(\alpha(G)(A_G+I_n))\mathbf{x} \geq 1 = \mathbf{x}^T J_n \mathbf{x},$$

$$\mathbf{x}^T(lpha(G)I_n+lpha(G)A_G-J_n)\mathbf{x}\geq \mathbf{0},$$
 برای هر  $\mathbf{x}$  روی سادک:  $\mathbf{x}$  روی سادک:  $\mathbf{x}$ 

$$\mathbf{x}^T(lpha(G)I_n+lpha(G)A_G-J_n)\mathbf{x}\geq \mathbf{0},$$
 برای هر  $\mathbf{x}$  نامنفی:  $\mathbf{x}$ 

Minimize t

Minimize 
$$t$$
  
subject to  $tI_n + zA_G - J_n \in COP_n$   
 $t, z \in \mathbb{R}$ 

 $\leq \alpha(G)$ 

برای هر x روی سادک:

$$\mathbf{x}^T(\alpha(G)(A_G+I_n))\mathbf{x} \geq 1 = \mathbf{x}^T J_n \mathbf{x},$$

$$\mathbf{x}^T(lpha(G)I_n+lpha(G)A_G-J_n)\mathbf{x}\geq \mathbf{0},$$
 برای هر  $\mathbf{x}$  روی سادک:  $\mathbf{x}$ 

$$\mathbf{x}^T(lpha(G)I_n+lpha(G)A_G-J_n)\mathbf{x}\geq \mathbf{0},$$
 نامنفی:  $\mathbf{x}$  نامنفی:

$$ilde{Y}:=lpha(G)I_n+lpha(G)A_G-J_n$$
 مست

Minimize tsubject to  $tI_n + zA_G - J_n \in COP_n$  $t, z \in \mathbb{R}$ 

 $t,z\in\mathbb{R}$   $t,z\in\mathbb{R}$  جوابش برابر با جواب برنامه ریزی بالاست.

 $\leq \alpha(G)$ 

$$Y:=lpha(G)I_n+lpha(G)A_G-J_n$$
 عممتنت

Minimize 
$$t$$
  
subject to  $tI_n + zA_G - J_n \in COP_n$   
 $t, z \in \mathbb{R}$ 

**7.2.5 Lemma.** The copositive program

جوابش برابر با جواب برنامهریزی بالاست.

 $\alpha(G) \leq$ 

minimize tsubject to  $y_{ij} = -1$ , if  $\{i, j\} \in \overline{E}$   $y_{ii} = t - 1$ , for all i = 1, 2, ..., n $Y \in \text{COP}_n$ 

 $\leq \alpha(G)$ 

has value  $\alpha(G)$ , the size of a maximum independent set in G.

$$ilde{Y}:=lpha(G)I_n+lpha(G)A_G-J_n$$
 ممشت

7.2.5 Lemma. The copositive program

Minimize tsubject to  $tI_n + zA_G - J_n \in COP_n$  $t, z \in \mathbb{R}$ 

جوابش برابر با جواب برنامهریزی بالاست.

 $\alpha(G) \leq$ 

minimize t  $subject\ to\ y_{ij}=-1,\ if\ \{i,j\}\in\overline{E}$   $y_{ii}=t-1,\ for\ all\ i=1,2,\ldots,n$ 



has value  $\alpha(G)$ , the size of a maximum independent set in G.

 $Y \in COP_n$ 

$$ilde{Y}:=lpha(G)I_n+lpha(G)A_G-J_n$$
 ممثنت

7.2.5 Lemma. The copositive program

Minimize tsubject to  $tI_n + zA_G - J_n \in COP_n$  $t, z \in \mathbb{R}$ 

جوابش برابر با جواب برنامهریزی بالاست.

#### 7.2.1 Theorem. The copositive program

minimize tsubject to  $y_{ij} = -1$ , if  $\{i, j\} \in \overline{E}$   $y_{ii} = t - 1$ , for all i = 1, 2, ..., n $Y \in \text{COP}_n$ 

has value  $\alpha(G)$ , the size of a maximum independent set in G.

#### 7.2.1 Theorem. The copositive program

minimize tsubject to  $y_{ij} = -1$ , if  $\{i, j\} \in \overline{E}$   $y_{ii} = t - 1$ , for all i = 1, 2, ..., n $Y \in \text{COP}_n$ 

has value  $\alpha(G)$ , the size of a maximum independent set in G.

برنامهریزی کاملا مثبت سخت است!

#### 7.2.1 Theorem. The copositive program

 $\begin{array}{ll} \mbox{minimize} & t \\ \mbox{subject to} & y_{ij} = -1, \mbox{ if } \{i,j\} \in \overline{E} \\ & y_{ii} = t-1, \mbox{ for all } i = 1,2,\ldots,n \\ & Y \in \mbox{COP}_n \end{array}$ 

has value  $\alpha(G)$ , the size of a maximum independent set in G.

## برنامهریزی کاملا مثبت سخت است!

$$\max \quad J_n \bullet X$$

$$X \in POS_n$$

$$Tr(X) = 1$$

$$x_{ij} = 0 \quad ij \in E$$

## تقريبناپذيري مسئله مجموعه مستقل

- $(NP \not\subseteq ZPP)$  تحت فرضهای خوبی
- هیچ الگوریتم تقریبی برای مسئله بزرگتری مجموعه مستقل
  - با ضریب تقریب  $n^{1-\epsilon}$  برای هیچ  $\epsilon>0$  وجود ندارد.

## **Cone Programming**

(P) Maximize  $\langle \mathbf{c}, \mathbf{x} \rangle$ 

subject to  $\mathbf{b} - A(\mathbf{x}) \in L$  $\mathbf{x} \in K$ .

## برنامهریزی هممثبت $C \bullet X$

subject to A(X) = b $X \in COP_n$ 

maximize

**SDP** 

 $C \bullet X$ 

subject to  $A_i \bullet X = b_i, \quad i = 1, 2, \dots, m$  $X \succeq 0$ .

## LP

 $x \ge 0$ 

 $C \bullet X$ maximize

maximize

subject to A(X) = b

برنامهریزی کاملا مثبت

 $c^{\mathsf{T}}x$ naximize  $X \in POS_n$ subject to Ax = b



# پایان