بسم الله الرحمن الرحیم

نظریه علوم کامپیوتر

نظریه علوم کامپیوتر - بهار ۱۴۰۰-۱۴۰۱ - جلسه چهاردهم: پیچیدگی حافظه (۳)

Theory of computation - 002 - S14 - space complexity (3)

Games and Complexity

Games and Complexity

Geography game

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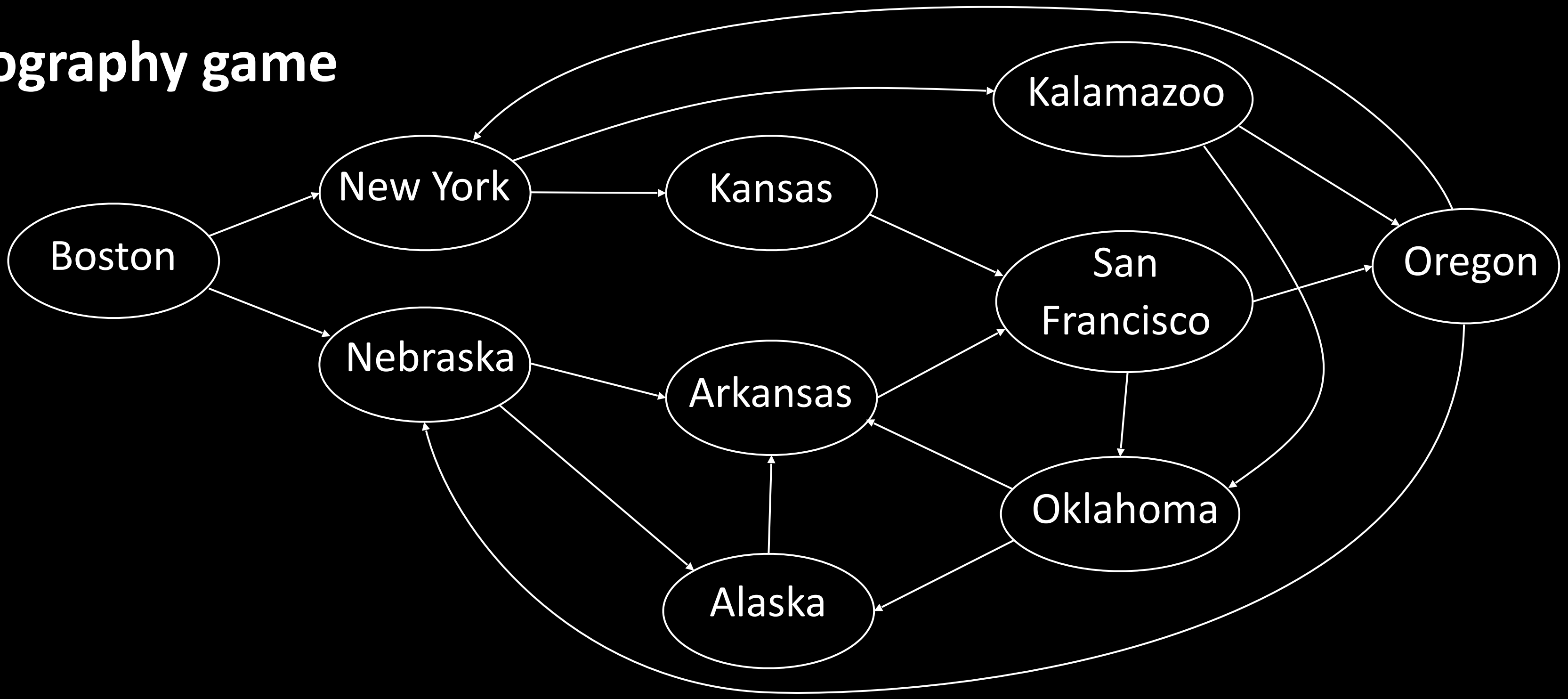
Geography game



Games and Complexity

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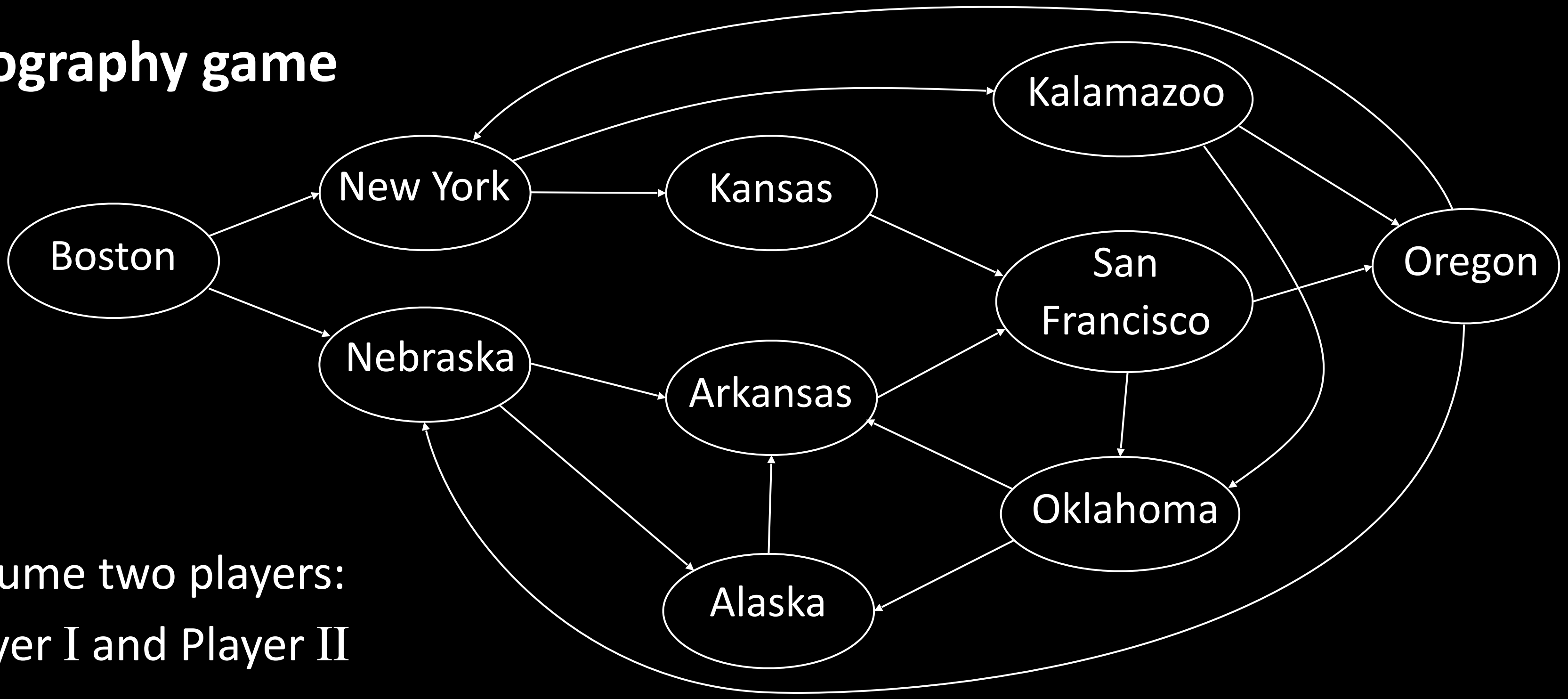

Assume two players:

Player I and Player II

Players take turns picking places that start with the letter which ended the previous place. No repeats allowed. The first player stuck (= cannot move) loses.

Games and Complexity

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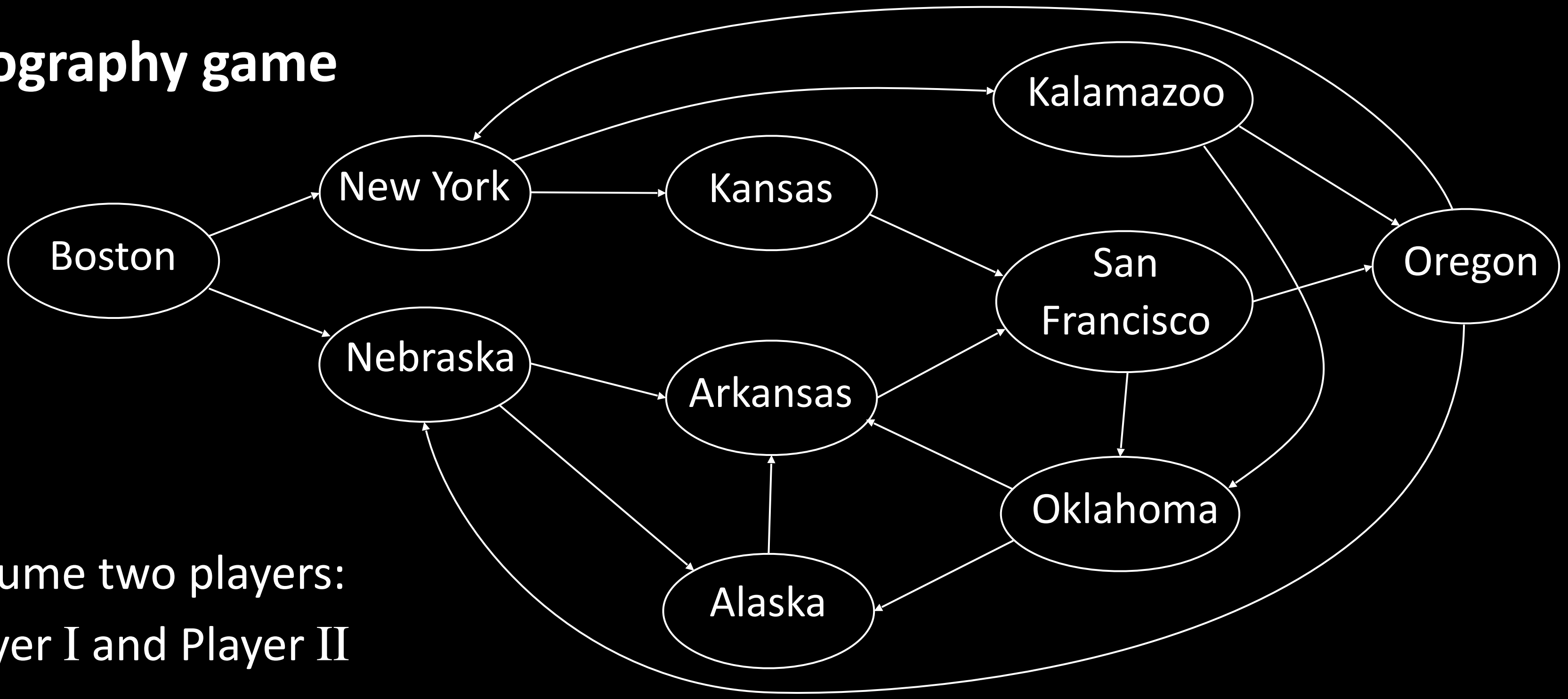

Generalized Geography Game

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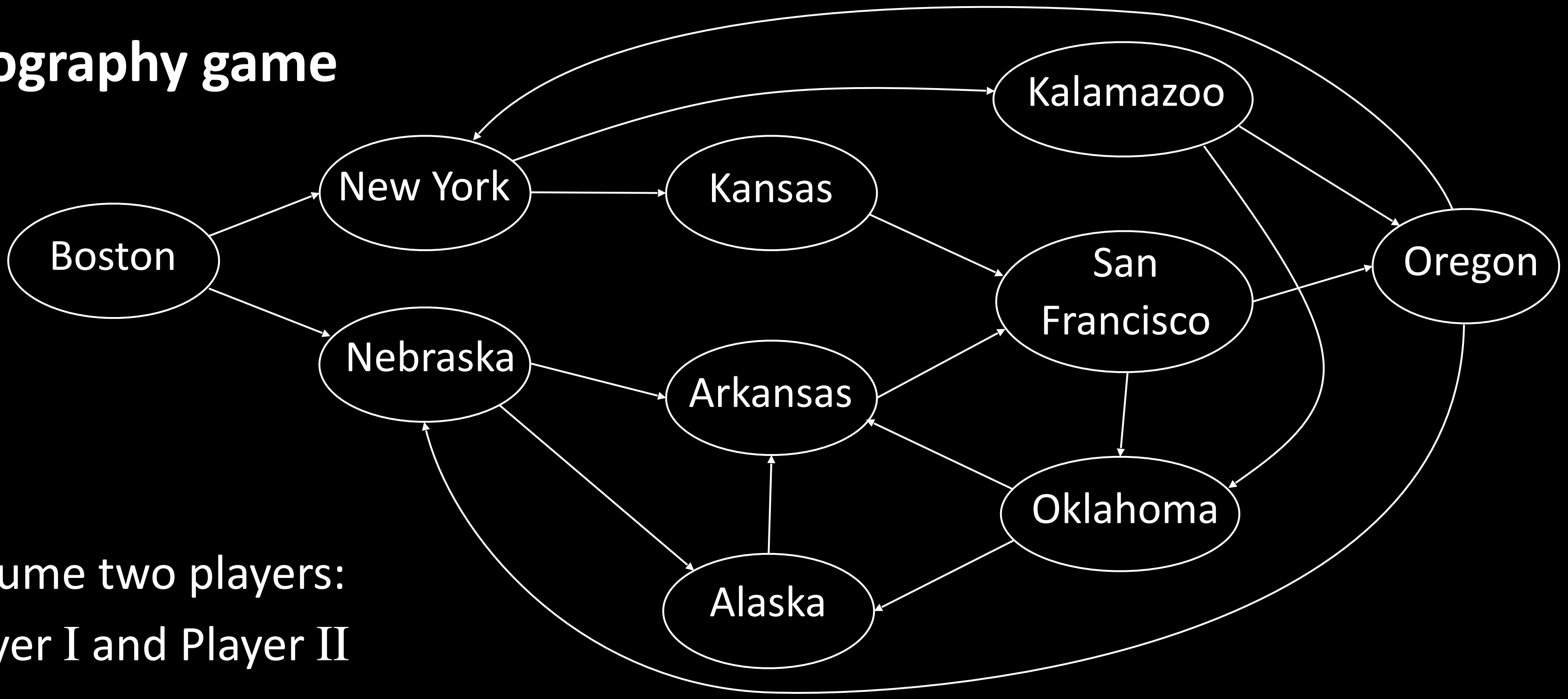

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Played on any directed graph.

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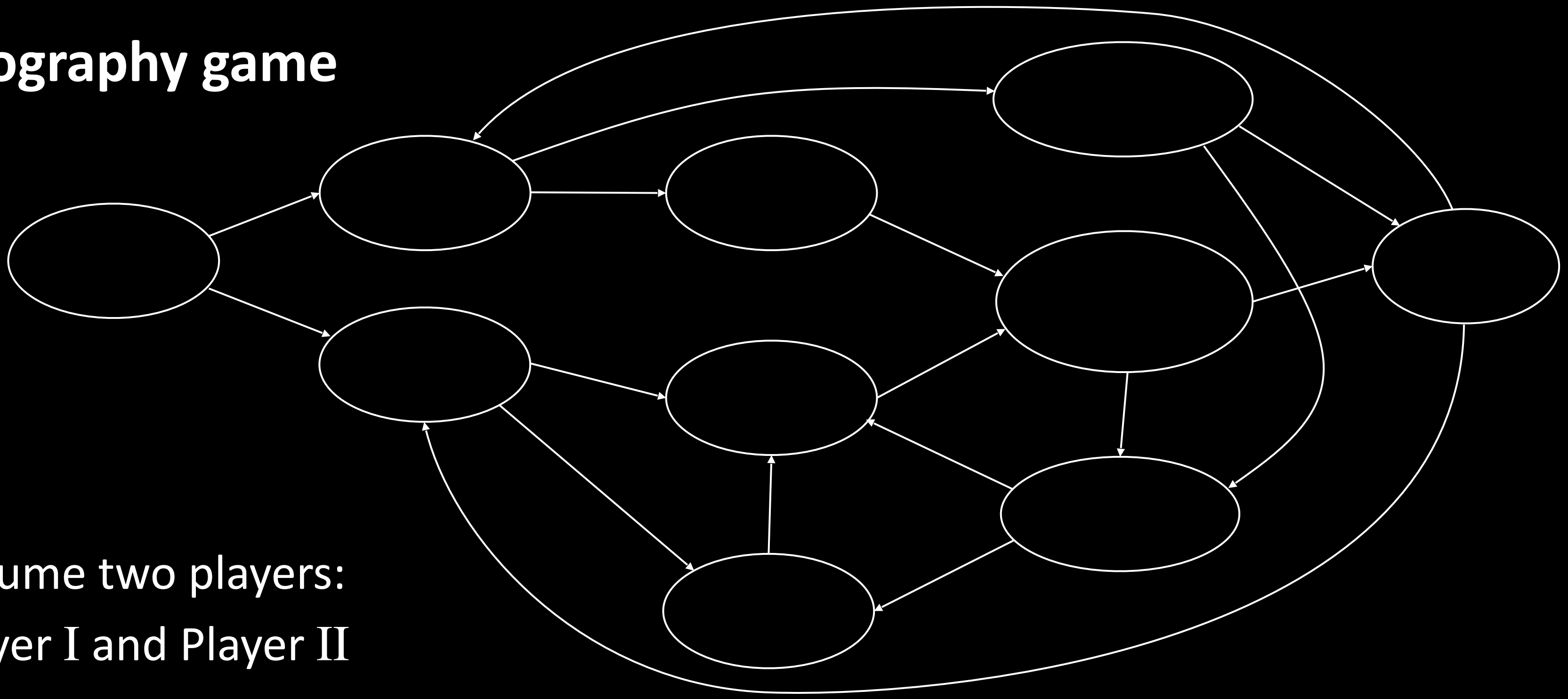

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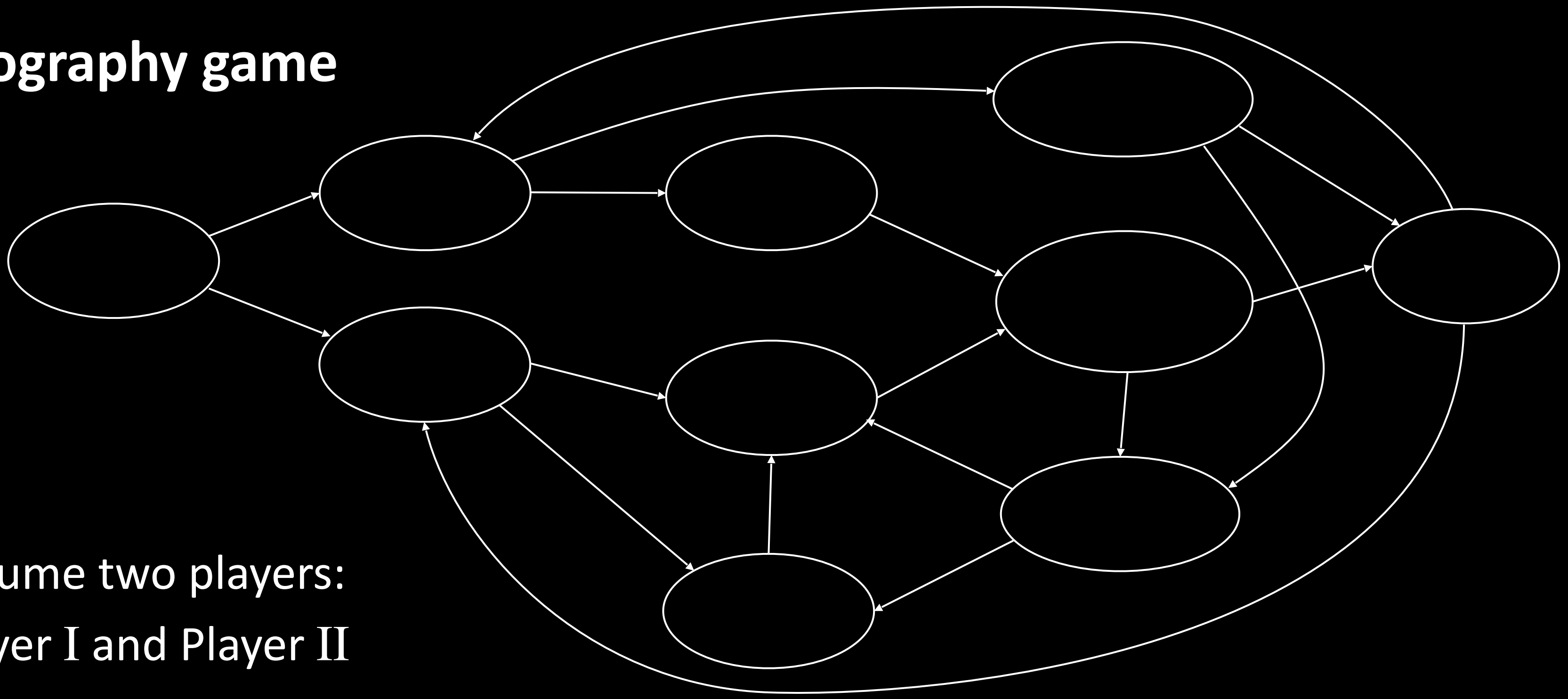

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Games and Complexity

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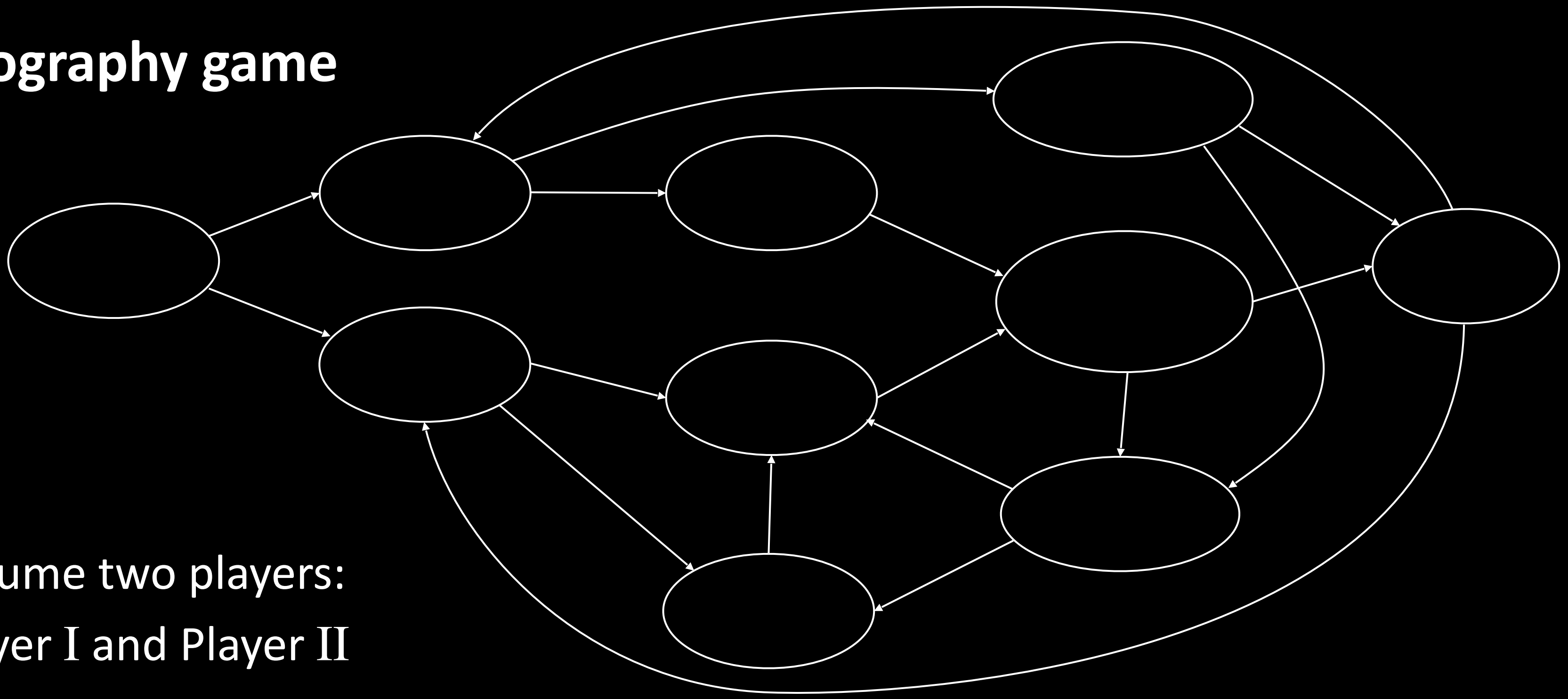

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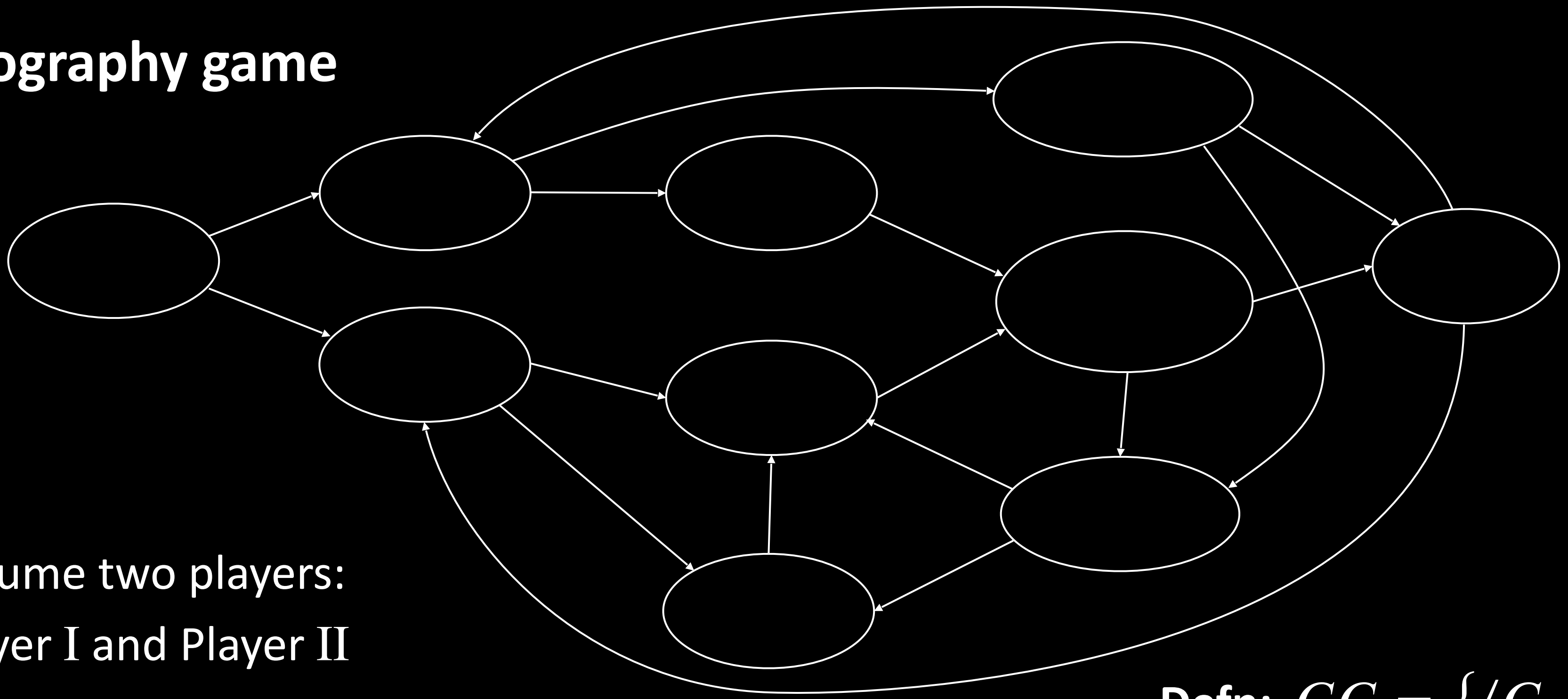

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Defn: $GG = \{\langle G, a\rangle \mid$ Player I has a forced win in Generalized Geography on graph G starting at node $a\}$.

Games and Complexity

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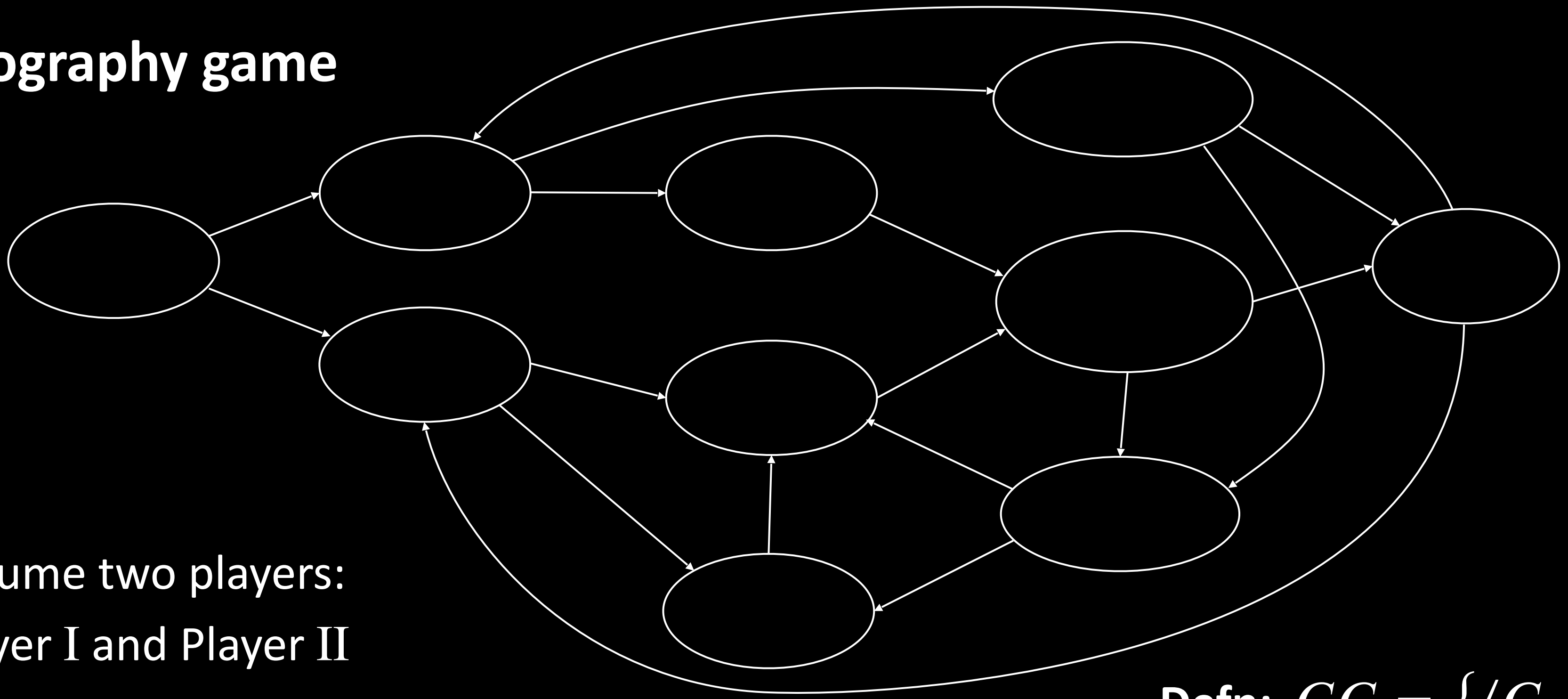

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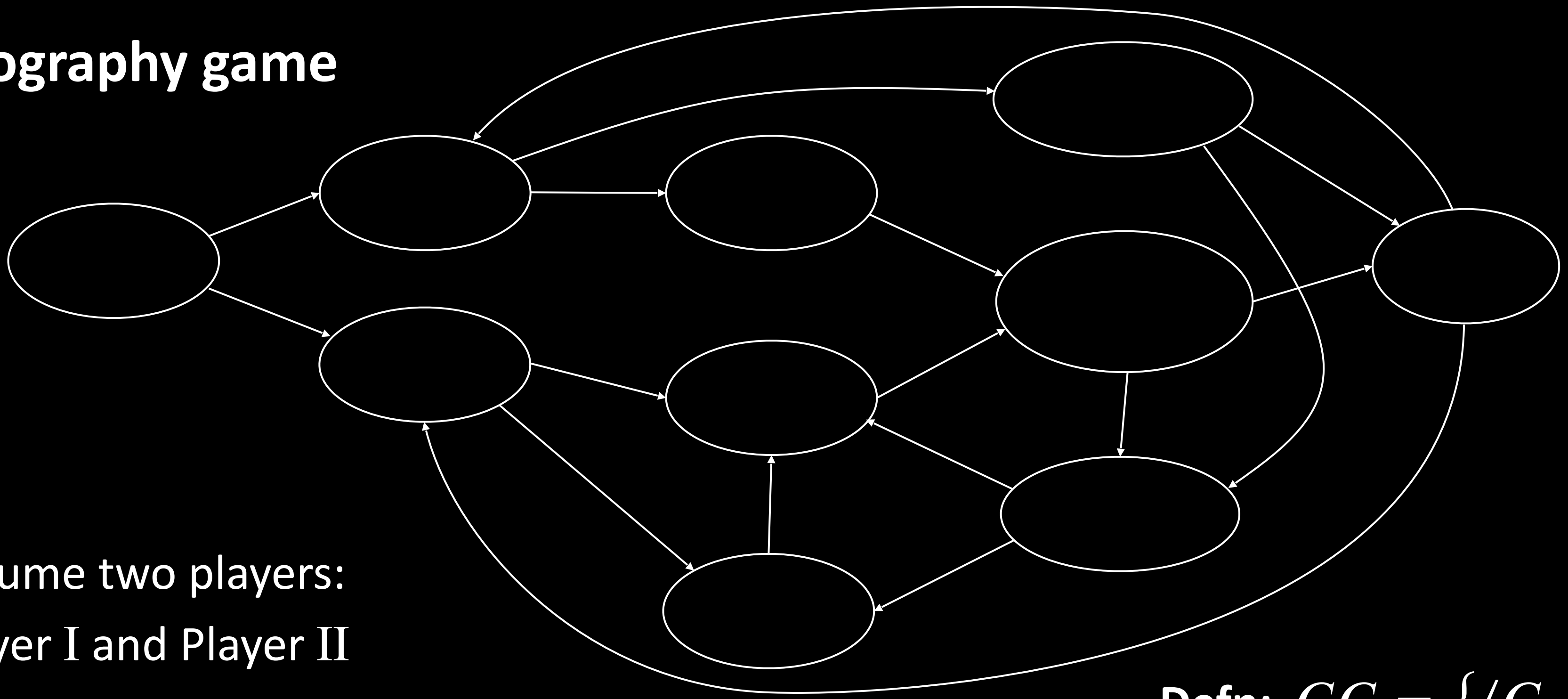

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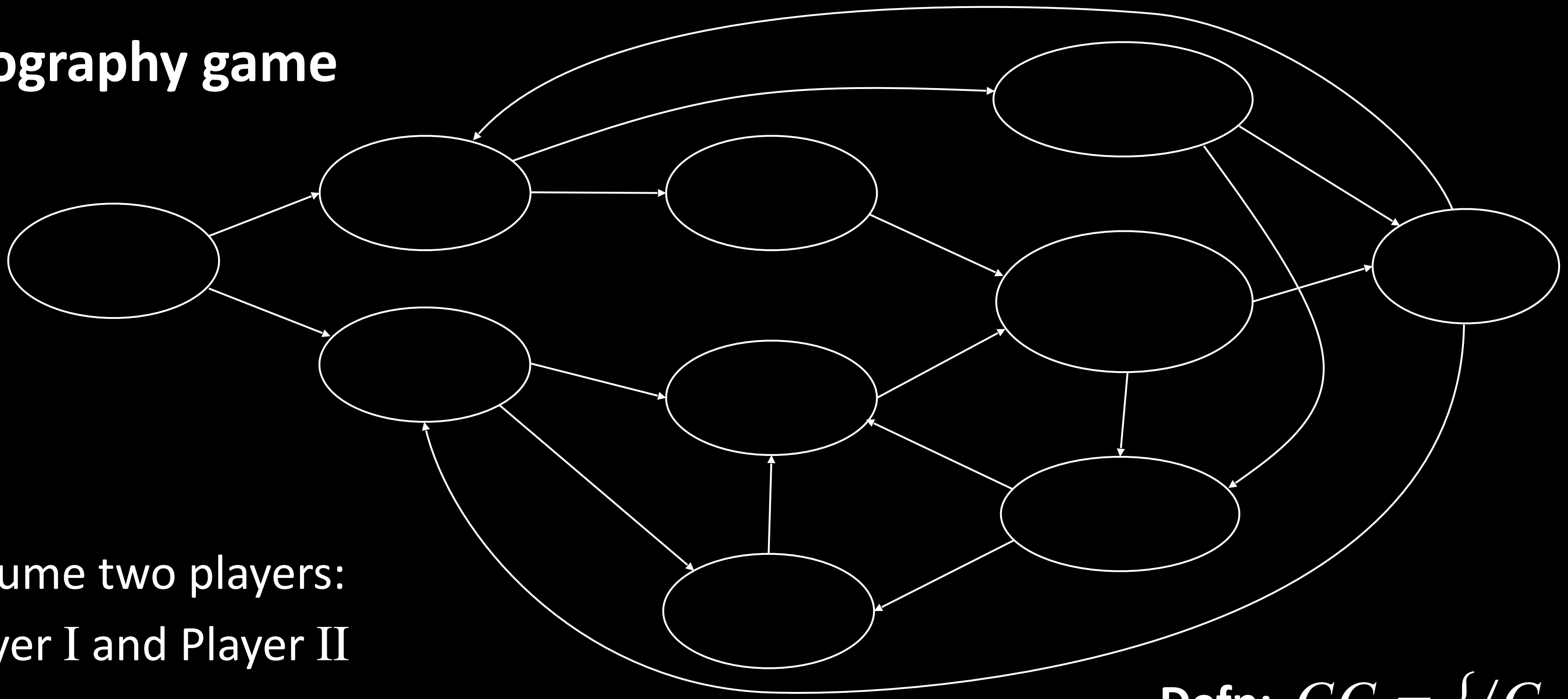

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Check-in 19.1

Games and Complexity

Geography game

Check-in 19.1

Let G be the graph below.

Which player has a winning strategy in the Generalized Geography game starting at node a?

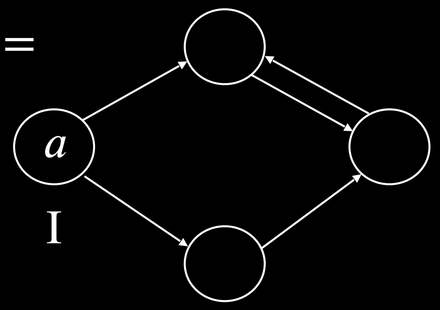
(a) Player I

(b) Player II

(c) Neither player

(d) Both players

$G =$



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Games and Quantifiers

The Formula Game

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Given QBF $\phi = \exists x_1 \; \forall x_2 \; \exists x_3 \cdots (\exists / \forall) x_k \overbrace{\left[(\cdots) \land \cdots \land (\cdots) \right]}^{\psi}$

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There are two Players "∃" and "∀".

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After all variables have been assigned values, we determine the winner:

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Claim: Player ∃ has a forced win in the formula game on ϕ iff ϕ is TRUE.

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Next: show $TQBF \leq_P GG$.

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Check-in 19.2

Games and Quantifiers

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Check-in 19.2

Which player has a winning strategy in the formula game on

$$\phi \ = \ \exists x \ \forall y \left[(x \vee y) \wedge (\overline{x} \vee \overline{y}) \right]$$

(a) ∃-player

(b) ∀-player

(c) Neither player

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Proof: 1) $GG \in$ PSPACE (recursive algorithm, exercise)

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Give reduction f from $TQBF$ to GG. $f\big(\langle \phi \rangle\big) = \langle G, a \rangle$

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Construct G to mimic the formula game on ϕ.

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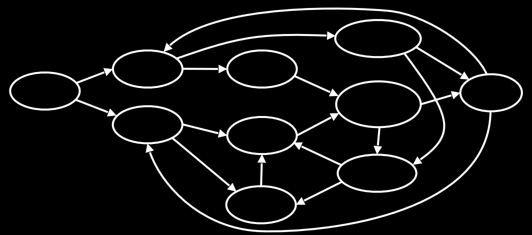
$\big\downarrow f$

$G =$ 

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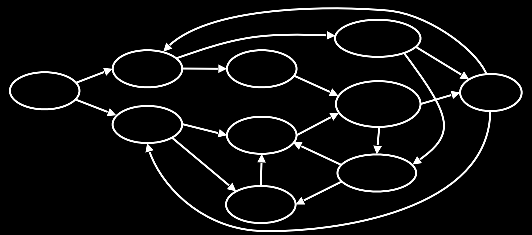
$G =$ 

Constructing the GG graph G

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Illustrate construction by example

Say $\phi = \exists x_1 \; \forall x_2 \; \exists x_3 \cdots \; \forall x_k \, [\, (\, x_1 \vee \overline{x_2} \vee x_3 \,) \wedge (\overline{x_1} \vee \overline{x_2} \vee x_4) \wedge \cdots \wedge (\, \cdots) \,]$

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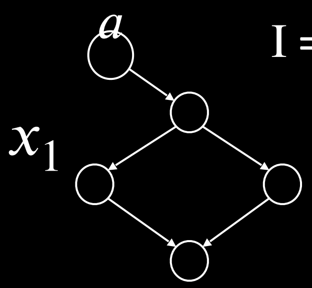

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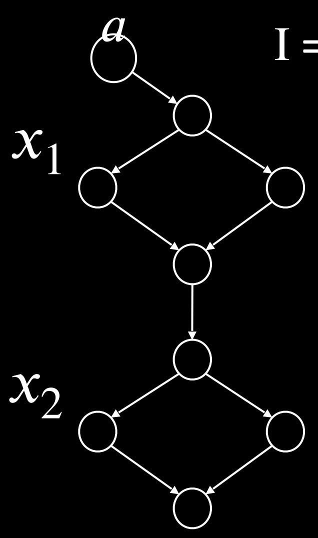

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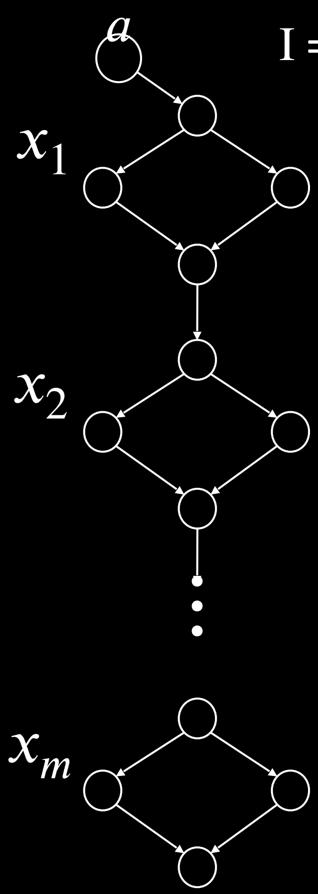

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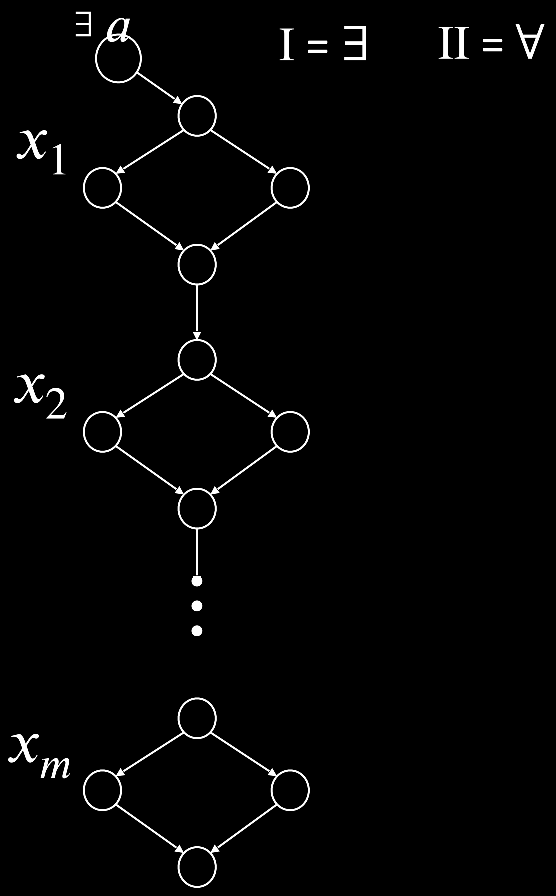

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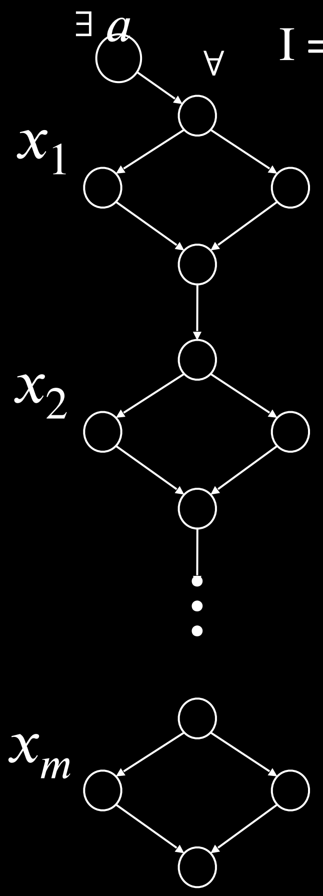

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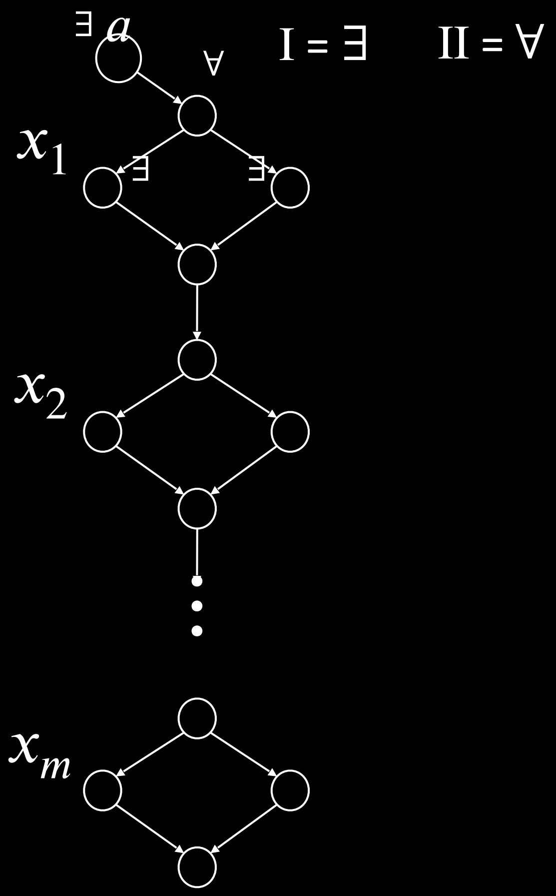
Constructing the GG graph G

Illustrate construction by example

Say $\phi = \exists x_1\ \forall x_2\ \exists x_3 \cdots\ \forall x_k\, [\, (\underbrace{x_1 \vee \overline{x_2} \vee x_3}_{c_1}) \wedge (\underbrace{\overline{x_1} \vee \overline{x_2} \vee x_4}_{c_2}) \wedge \cdots \wedge (\underbrace{\cdots}_{c_k})\,]$

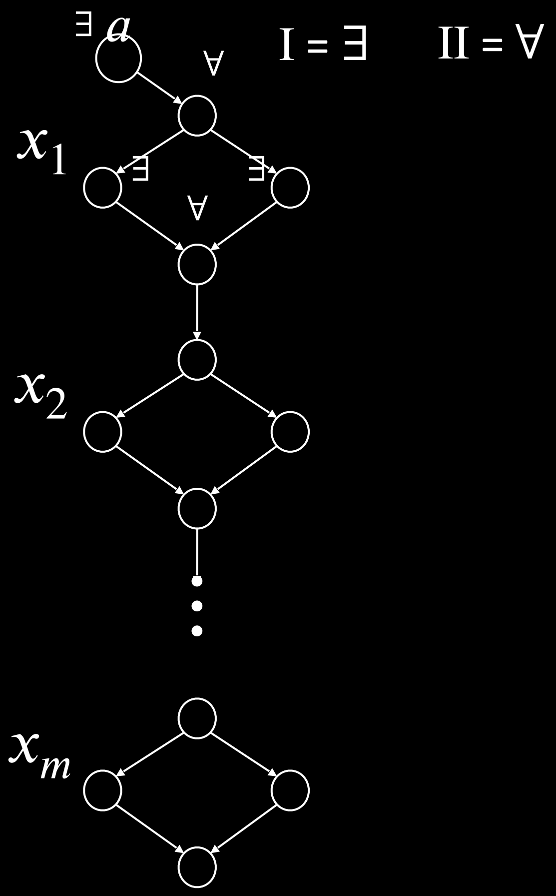
$G =$ 

Constructing the GG graph G

Illustrate construction by example

Say $\phi = \exists x_1\ \forall x_2\ \exists x_3 \cdots\ \forall x_k\,[\,(\underbrace{x_1 \vee \overline{x_2} \vee x_3}_{c_1}) \wedge (\underbrace{\overline{x_1} \vee \overline{x_2} \vee x_4}_{c_2}) \wedge \cdots \wedge (\underbrace{\cdots}_{c_k})\,]$

$G =$

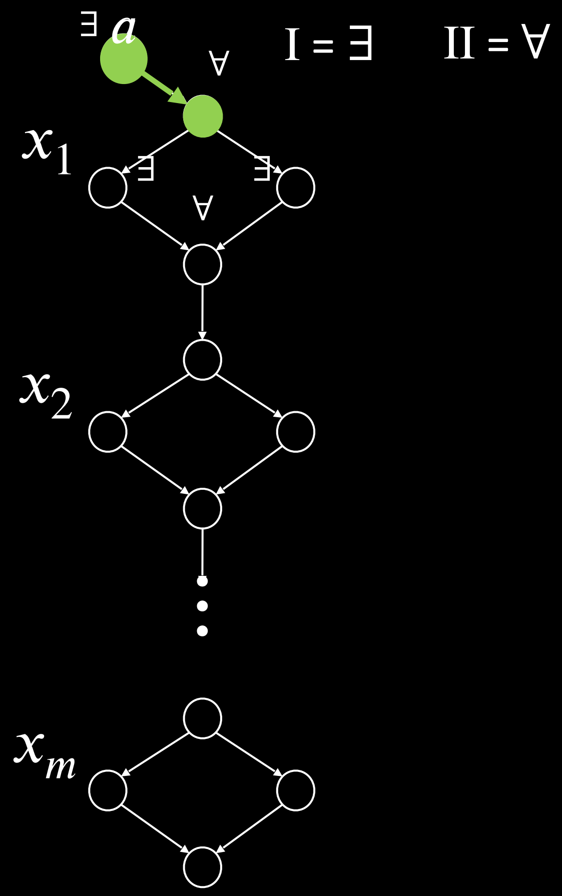


Constructing the GG graph G

Illustrate construction by example

Say $\phi = \exists x_1\ \forall x_2\ \exists x_3 \cdots\ \forall x_k\,[\,\underbrace{(\,x_1 \vee \overline{x_2} \vee x_3\,)}_{c_1} \wedge \underbrace{(\overline{x_1} \vee \overline{x_2} \vee x_4)}_{c_2} \wedge \cdots \wedge \underbrace{(\,\cdots)}_{c_k}\,]$

$G =$

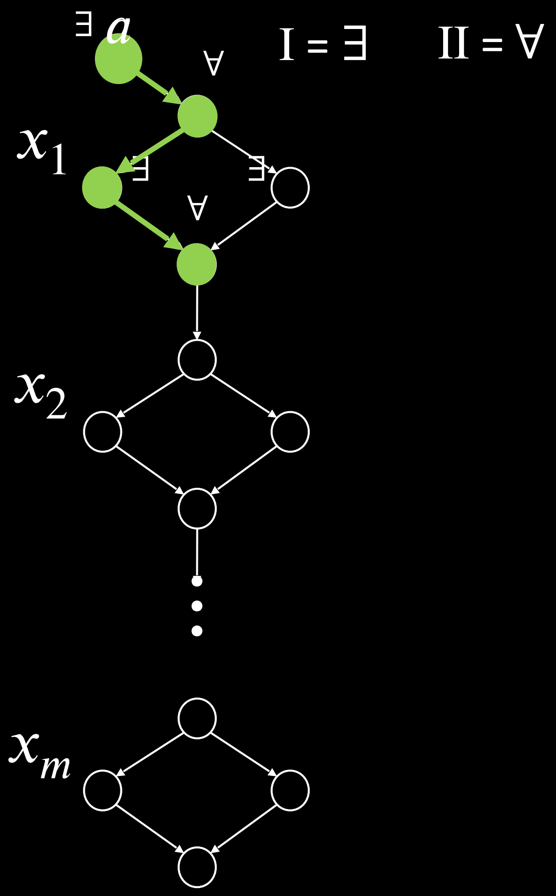


Constructing the GG graph G

Illustrate construction by example

Say $\phi = \exists x_1\ \forall x_2\ \exists x_3 \cdots\ \forall x_k\ [\ \underbrace{(\ x_1 \vee \overline{x_2} \vee x_3\ )}_{c_1} \wedge \underbrace{(\overline{x_1} \vee \overline{x_2} \vee x_4)}_{c_2} \wedge \cdots \wedge \underbrace{(\ \cdots)}_{c_k}\ ]$

$G =$

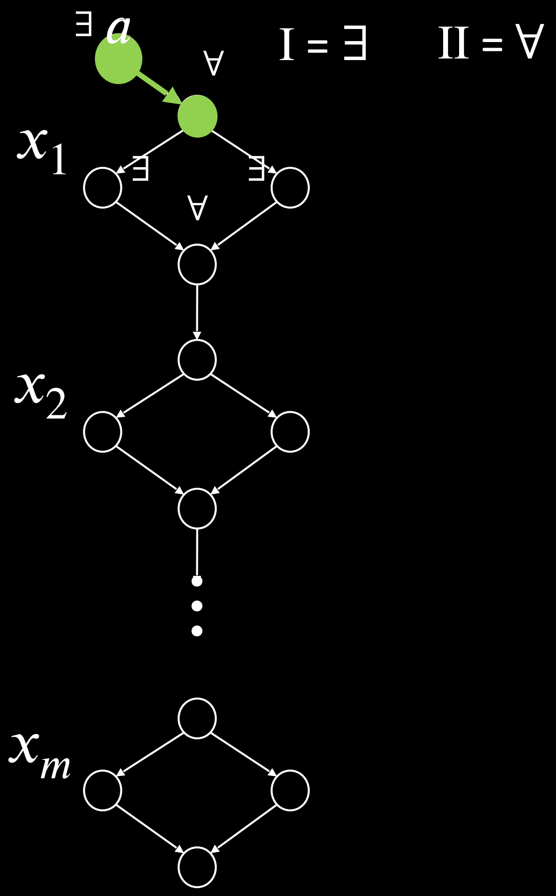


Constructing the GG graph G

Illustrate construction by example

Say $\phi = \exists x_1 \, \forall x_2 \, \exists x_3 \cdots \, \forall x_k \, [\, \underbrace{(\, x_1 \vee \overline{x_2} \vee x_3 \,)}_{c_1} \wedge \underbrace{(\overline{x_1} \vee \overline{x_2} \vee x_4)}_{c_2} \wedge \cdots \wedge \underbrace{(\, \cdots)}_{c_k} \,]$

$G =$

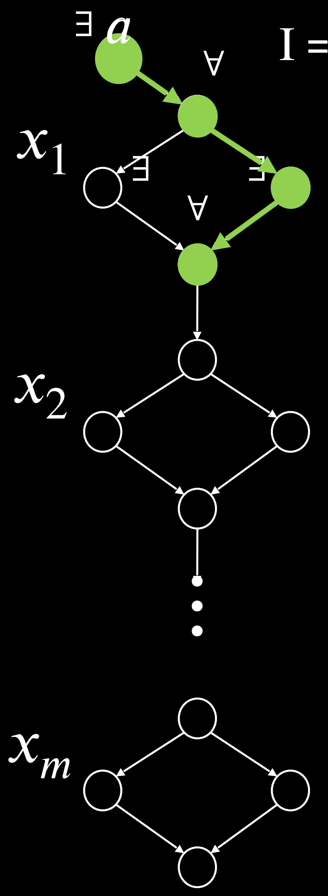


Constructing the GG graph G

Illustrate construction by example

Say $\phi = \exists x_1\ \forall x_2\ \exists x_3 \cdots\ \forall x_k\ [\ \underbrace{(\ x_1 \vee \overline{x_2} \vee x_3\ )}_{c_1} \wedge \underbrace{(\overline{x_1} \vee \overline{x_2} \vee x_4)}_{c_2} \wedge \cdots \wedge \underbrace{(\ \cdots)}_{c_k}\ ]$

$G =$



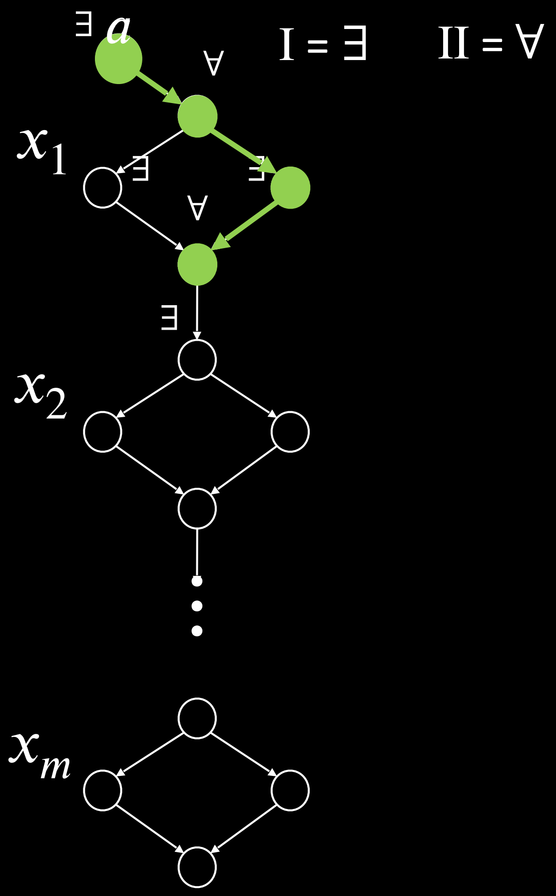
I = ∃ II = ∀

Constructing the GG graph G

Illustrate construction by example

Say $\phi = \exists x_1\ \forall x_2\ \exists x_3 \cdots\ \forall x_k\,[\,(\underbrace{x_1 \vee \overline{x_2} \vee x_3}_{c_1}) \wedge (\underbrace{\overline{x_1} \vee \overline{x_2} \vee x_4}_{c_2}) \wedge \cdots \wedge (\underbrace{\cdots}_{c_k})\,]$

$G =$

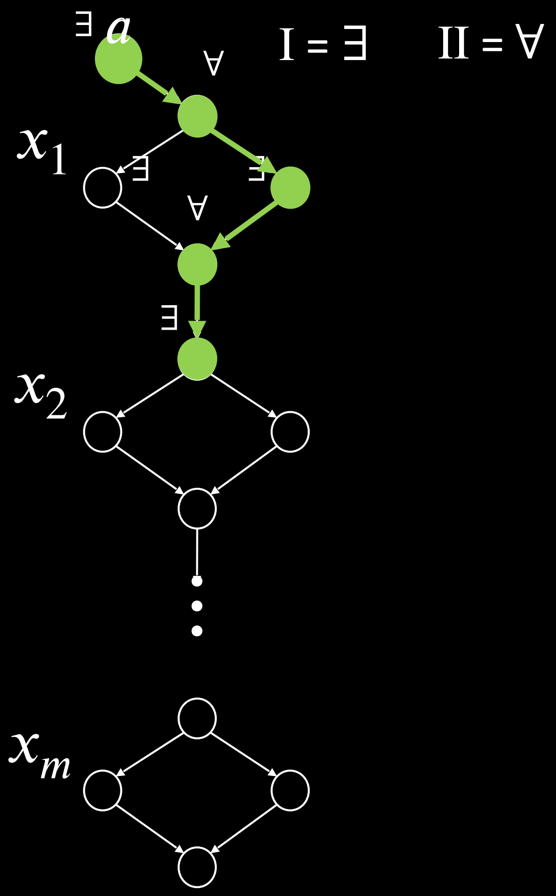


Constructing the GG graph G

Illustrate construction by example

Say $\phi = \exists x_1 \; \forall x_2 \; \exists x_3 \cdots \; \forall x_k \, [\, (\underbrace{x_1 \vee \overline{x_2} \vee x_3}_{c_1}) \wedge (\underbrace{\overline{x_1} \vee \overline{x_2} \vee x_4}_{c_2}) \wedge \cdots \wedge (\underbrace{\cdots}_{c_k}) \,]$

$G =$

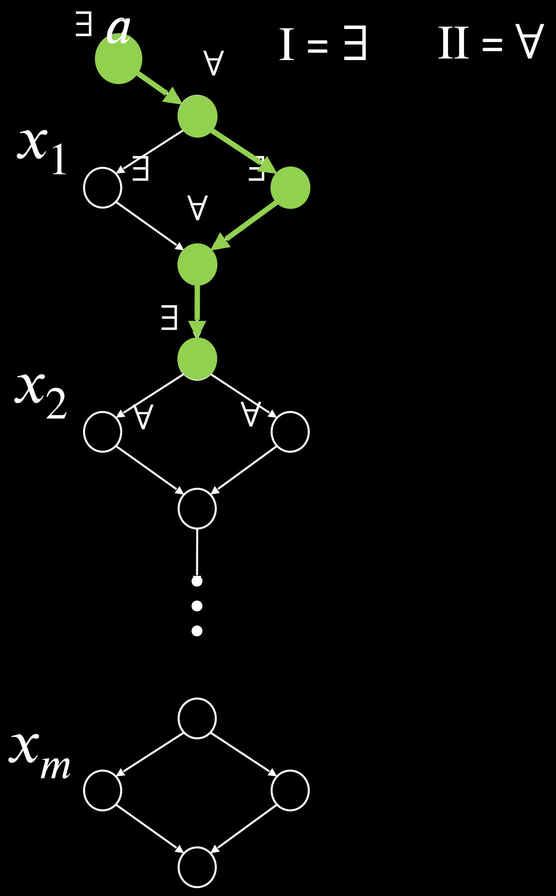


Constructing the GG graph G

Illustrate construction by example

Say $\phi = \exists x_1\ \forall x_2\ \exists x_3 \cdots\ \forall x_k\, [\, \underbrace{(\, x_1 \vee \overline{x_2} \vee x_3\,)}_{c_1} \wedge \underbrace{(\overline{x_1} \vee \overline{x_2} \vee x_4)}_{c_2} \wedge \cdots \wedge \underbrace{(\, \cdots)}_{c_k}\,]$

$G =$

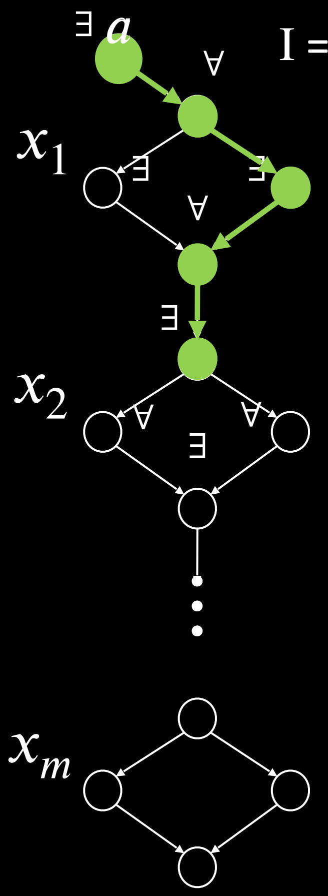


Constructing the GG graph G

Illustrate construction by example

Say $\phi = \exists x_1\ \forall x_2\ \exists x_3 \cdots\ \forall x_k\,[\,(\underbrace{x_1 \vee \overline{x_2} \vee x_3}_{c_1}) \wedge (\underbrace{\overline{x_1} \vee \overline{x_2} \vee x_4}_{c_2}) \wedge \cdots \wedge (\underbrace{\cdots}_{c_k})\,]$

$G =$



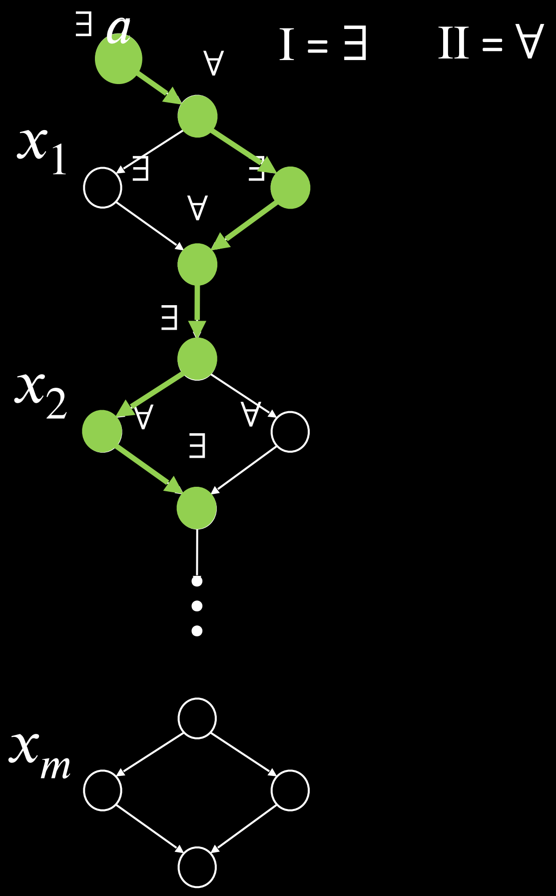
I = ∃ II = ∀

Constructing the GG graph G

Illustrate construction by example

Say $\phi = \exists x_1\ \forall x_2\ \exists x_3 \cdots\ \forall x_k\ [\ \underbrace{(\ x_1 \vee \overline{x_2} \vee x_3\ )}_{c_1} \wedge \underbrace{(\overline{x_1} \vee \overline{x_2} \vee x_4)}_{c_2} \wedge \cdots \wedge \underbrace{(\ \cdots)}_{c_k}\ ]$

$G =$

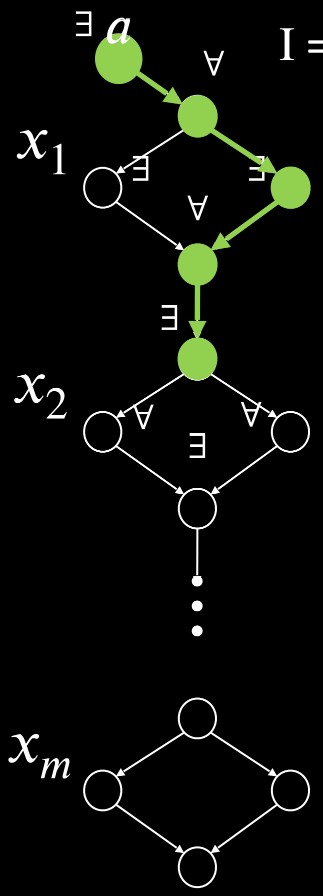


Constructing the GG graph G

Illustrate construction by example

Say $\phi = \exists x_1 \ \forall x_2 \ \exists x_3 \cdots \ \forall x_k \, [\, \underbrace{(x_1 \vee \overline{x_2} \vee x_3)}_{c_1} \wedge \underbrace{(\overline{x_1} \vee \overline{x_2} \vee x_4)}_{c_2} \wedge \cdots \wedge \underbrace{(\cdots)}_{c_k} \,]$

$G =$



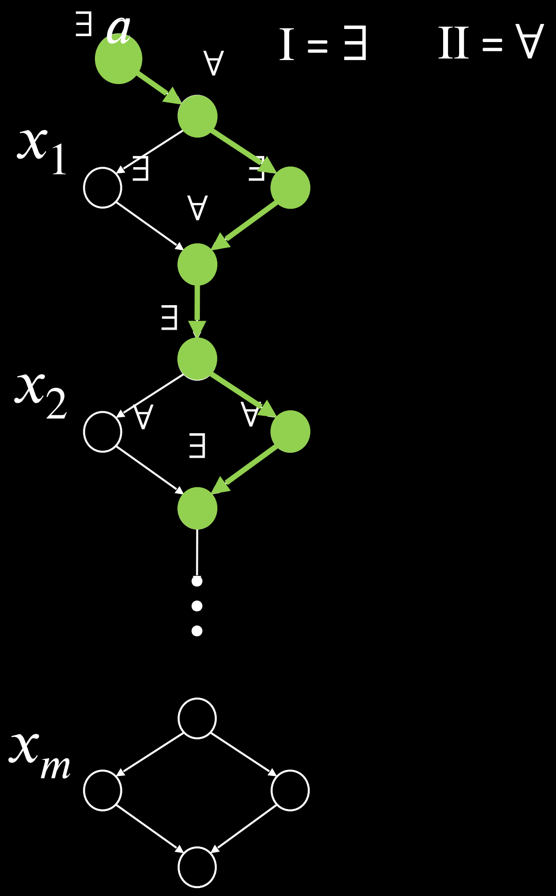
I = ∃ II = ∀

Constructing the GG graph G

Illustrate construction by example

Say $\phi = \exists x_1 \ \forall x_2 \ \exists x_3 \cdots \ \forall x_k \left[(\underbrace{x_1 \vee \overline{x_2} \vee x_3}_{c_1}) \wedge (\underbrace{\overline{x_1} \vee \overline{x_2} \vee x_4}_{c_2}) \wedge \cdots \wedge (\underbrace{\cdots}_{c_k}) \right]$

$G =$

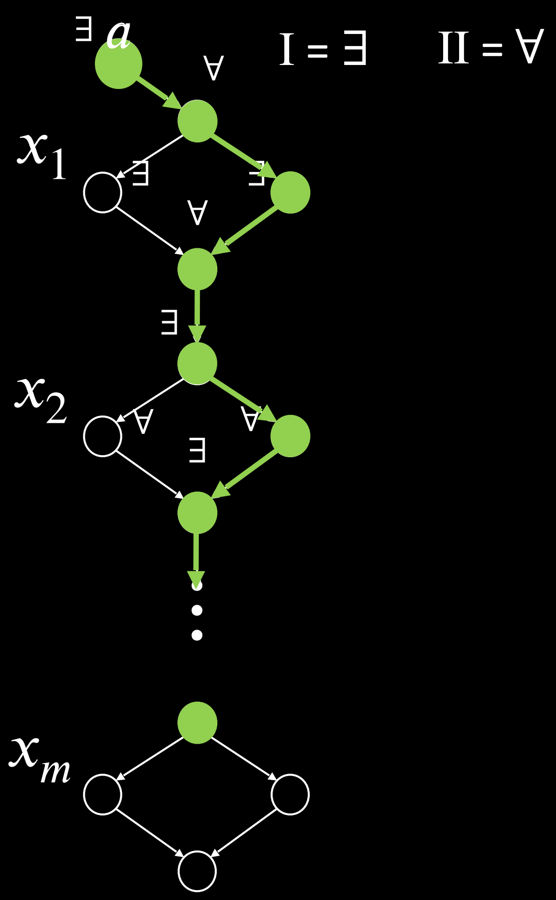


Constructing the GG graph G

Illustrate construction by example

Say $\phi = \exists x_1\ \forall x_2\ \exists x_3 \cdots\ \forall x_k\, [\, \underbrace{(\, x_1 \vee \overline{x_2} \vee x_3\,)}_{c_1} \wedge \underbrace{(\overline{x_1} \vee \overline{x_2} \vee x_4)}_{c_2} \wedge \cdots \wedge \underbrace{(\, \cdots)}_{c_k}\,]$

$G =$

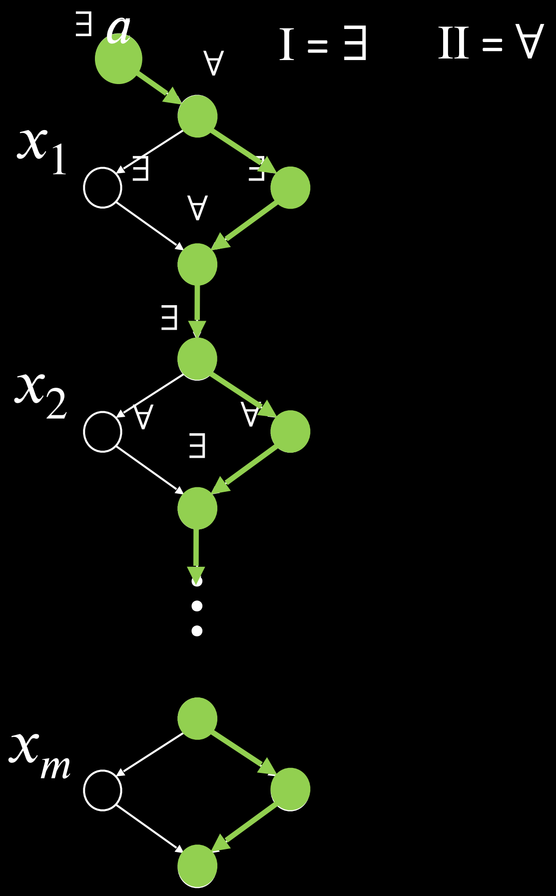


Constructing the GG graph G

Illustrate construction by example

Say $\phi = \exists x_1 \; \forall x_2 \; \exists x_3 \cdots \; \forall x_k \, [\, (\underbrace{x_1 \vee \overline{x_2} \vee x_3}_{c_1}) \wedge (\underbrace{\overline{x_1} \vee \overline{x_2} \vee x_4}_{c_2}) \wedge \cdots \wedge (\underbrace{\cdots}_{c_k}) \,]$

$G =$

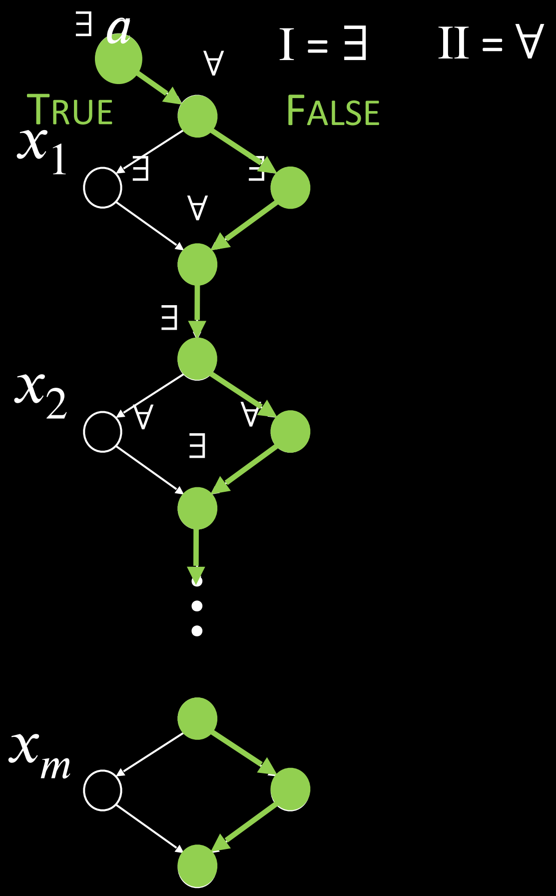


Constructing the GG graph G

Illustrate construction by example

Say $\phi = \exists x_1\ \forall x_2\ \exists x_3 \cdots\ \forall x_k\, [\, \underbrace{(\, x_1 \vee \overline{x_2} \vee x_3\,)}_{c_1} \wedge \underbrace{(\overline{x_1} \vee \overline{x_2} \vee x_4)}_{c_2} \wedge \cdots \wedge \underbrace{(\, \cdots)}_{c_k}\,]$

$G =$

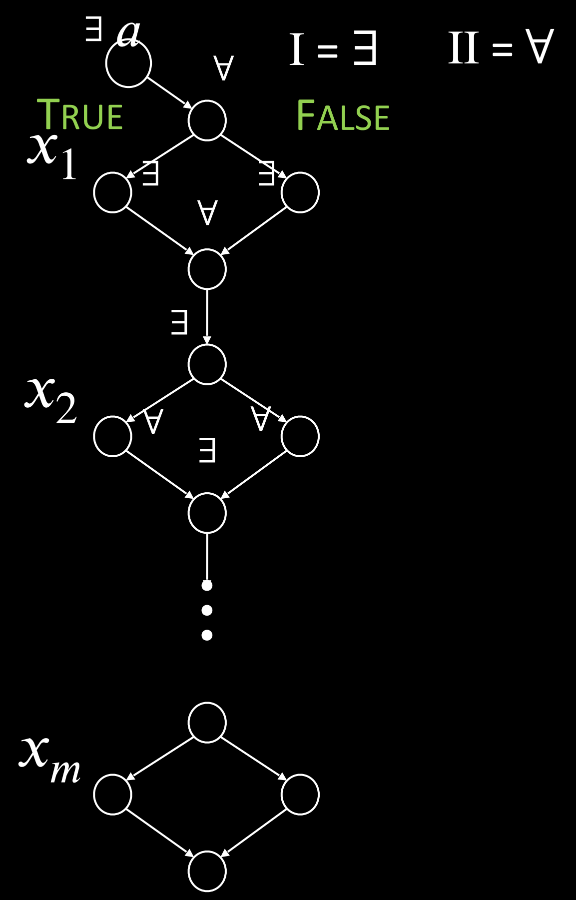


Constructing the GG graph G

Illustrate construction by example

Say $\phi = \exists x_1 \; \forall x_2 \; \exists x_3 \cdots \; \forall x_k \, [\, \underbrace{(\, x_1 \vee \overline{x_2} \vee x_3 \,)}_{c_1} \wedge \underbrace{(\overline{x_1} \vee \overline{x_2} \vee x_4)}_{c_2} \wedge \cdots \wedge \underbrace{(\, \cdots)}_{c_k} \,]$

$G =$

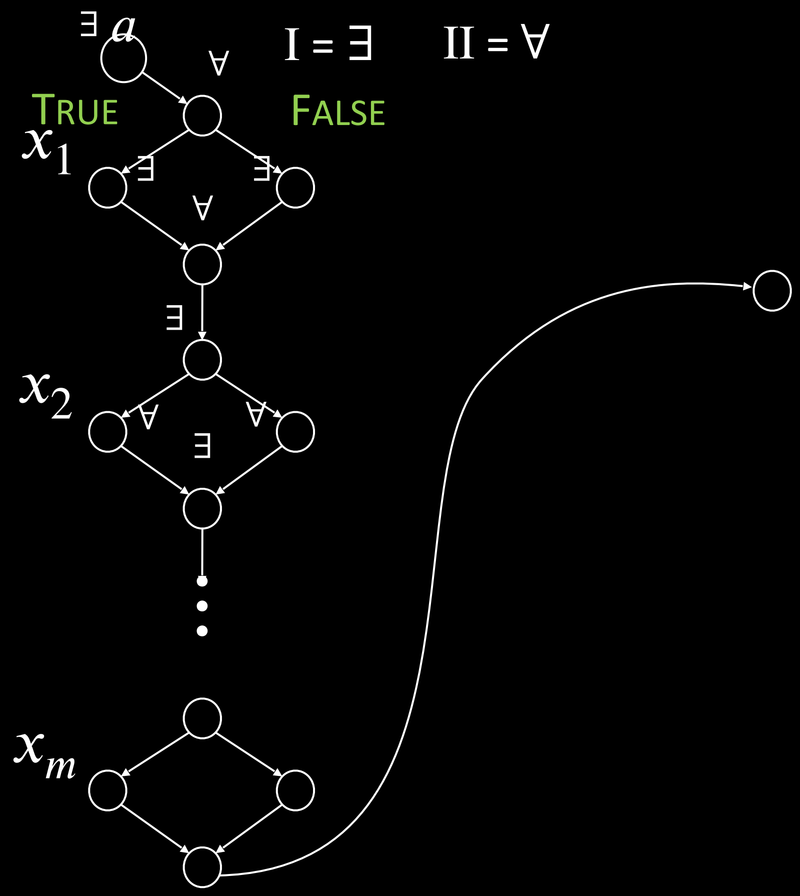


Constructing the GG graph G

Illustrate construction by example

Say $\phi = \exists x_1\ \forall x_2\ \exists x_3 \cdots\ \forall x_k\ [\ \underbrace{(\ x_1 \vee \overline{x_2} \vee x_3\ )}_{c_1} \wedge \underbrace{(\overline{x_1} \vee \overline{x_2} \vee x_4)}_{c_2} \wedge \cdots \wedge \underbrace{(\ \cdots)}_{c_k}\ ]$

$G =$

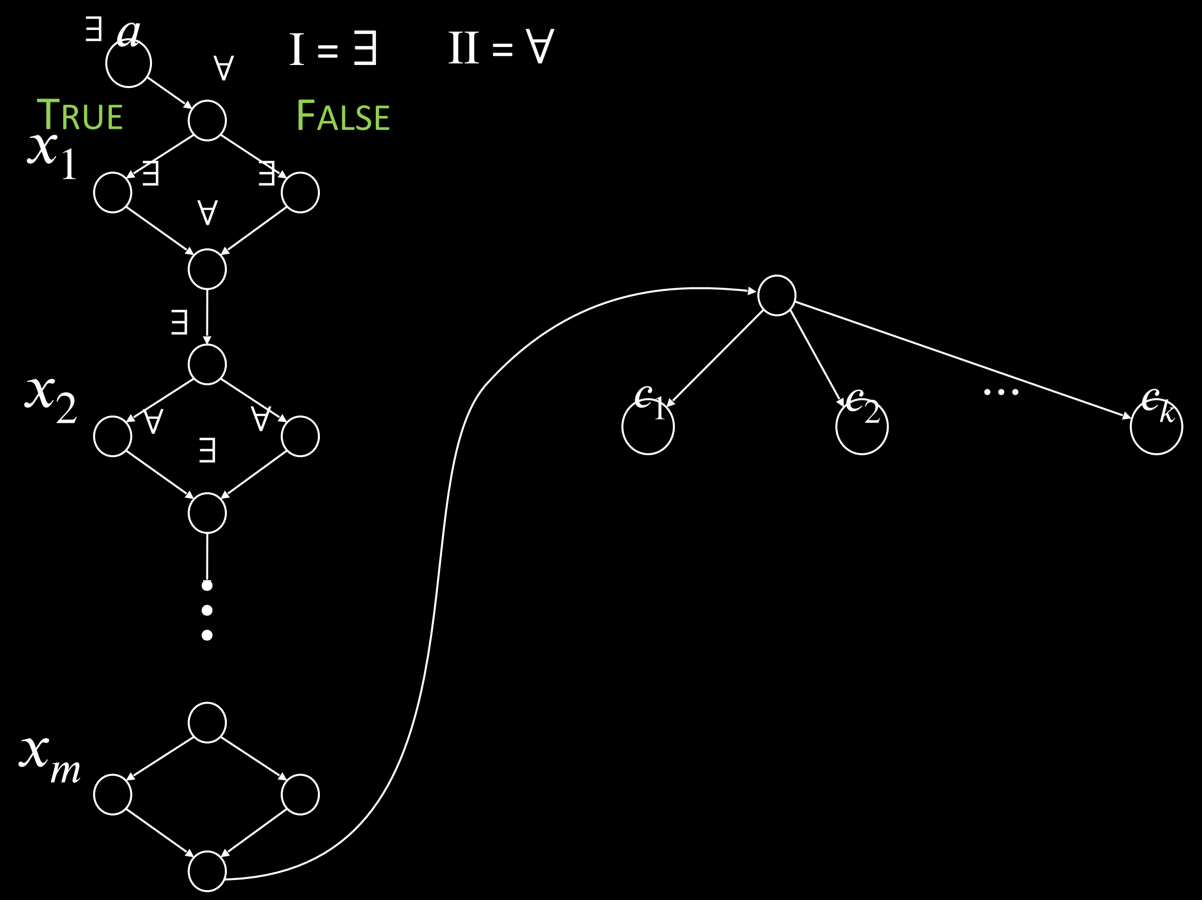


Constructing the GG graph G

Illustrate construction by example

Say $\phi = \exists x_1\ \forall x_2\ \exists x_3 \cdots\ \forall x_k\, [\, \underbrace{(\, x_1 \vee \overline{x_2} \vee x_3\,)}_{c_1} \wedge \underbrace{(\overline{x_1} \vee \overline{x_2} \vee x_4)}_{c_2} \wedge \cdots \wedge \underbrace{(\ \cdots)}_{c_k}\,]$

$G =$

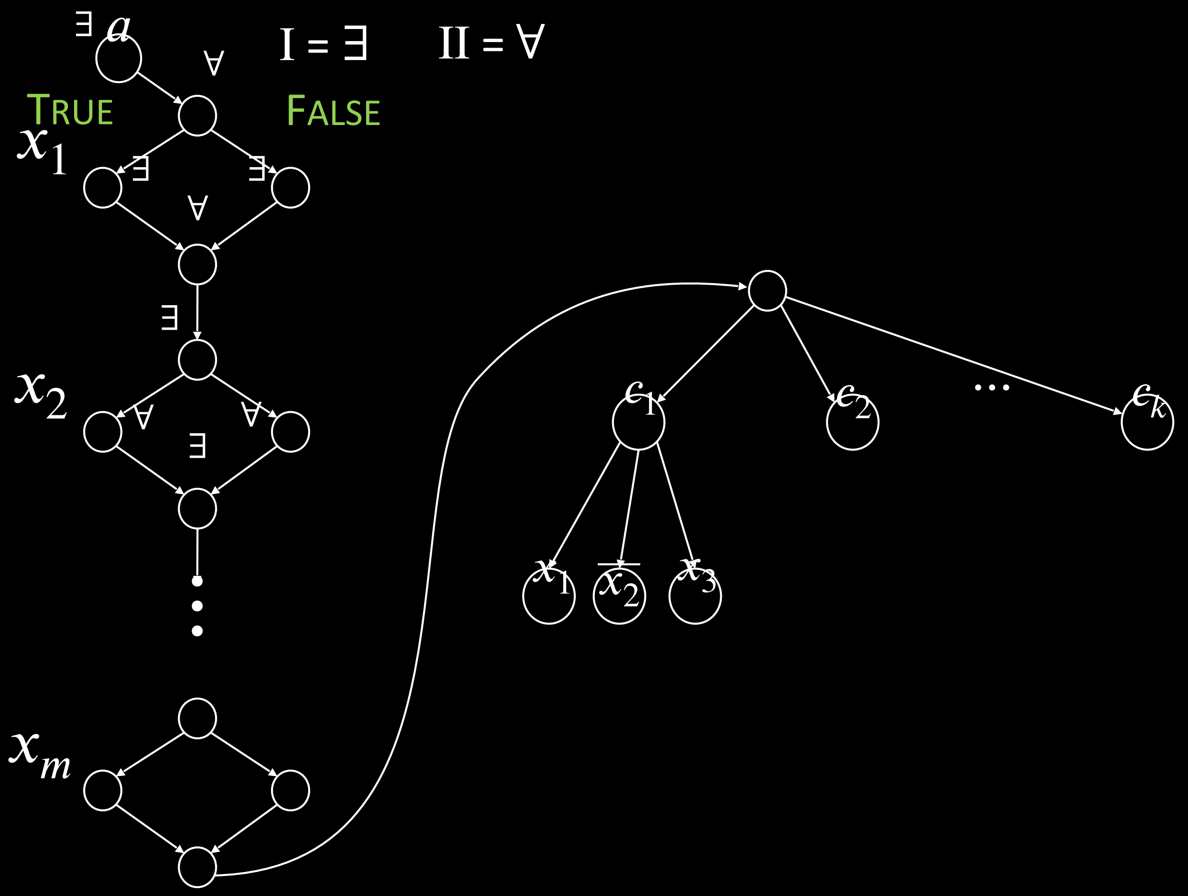


Constructing the GG graph G

Illustrate construction by example

Say $\phi = \exists x_1\ \forall x_2\ \exists x_3 \cdots\ \forall x_k\ [\ \underbrace{(\ x_1 \vee \overline{x_2} \vee x_3\ )}_{c_1} \wedge \underbrace{(\overline{x_1} \vee \overline{x_2} \vee x_4)}_{c_2} \wedge \cdots \wedge \underbrace{(\ \cdots)}_{c_k}\ ]$

$G =$

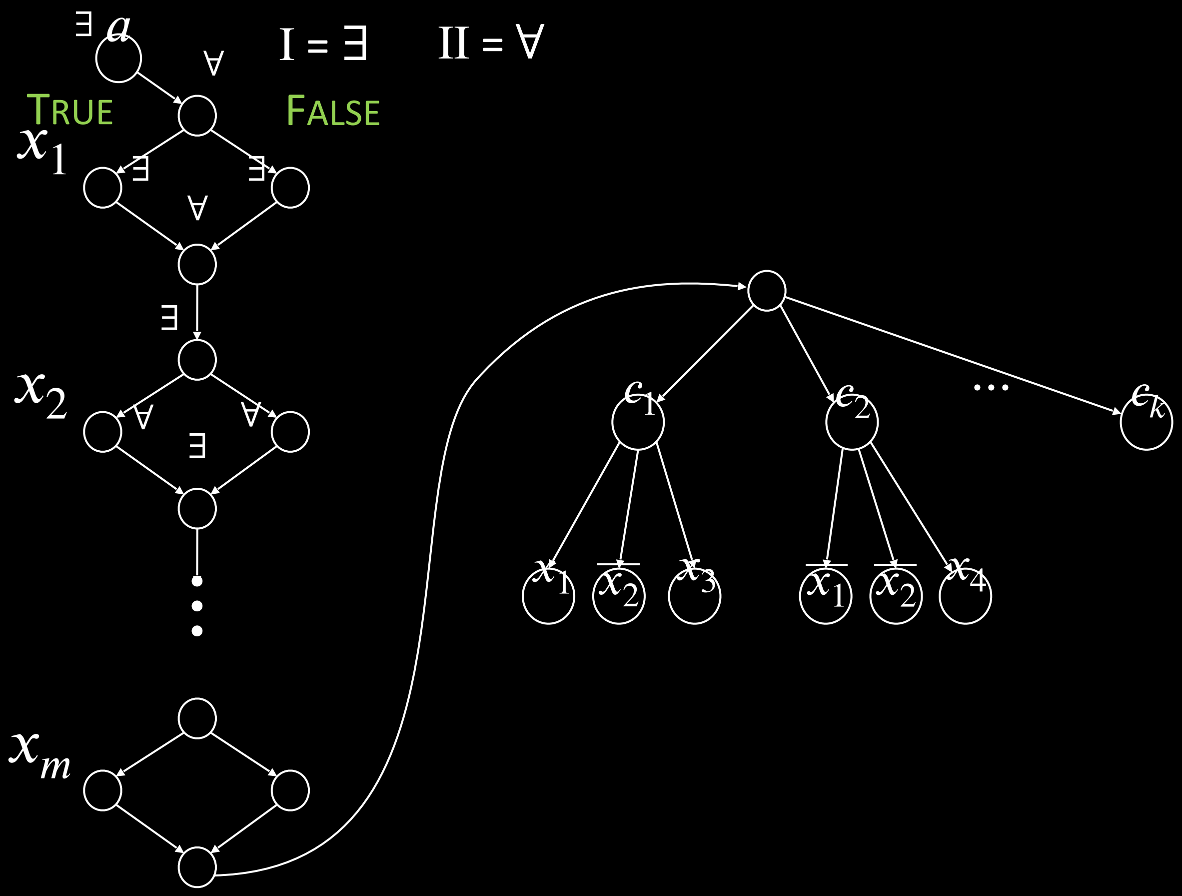


Constructing the GG graph G

Illustrate construction by example

Say $\phi = \exists x_1 \; \forall x_2 \; \exists x_3 \cdots \; \forall x_k \, [\, (\underbrace{x_1 \vee \overline{x_2} \vee x_3}_{c_1}) \wedge (\underbrace{\overline{x_1} \vee \overline{x_2} \vee x_4}_{c_2}) \wedge \cdots \wedge (\underbrace{\cdots}_{c_k})\,]$

$G =$

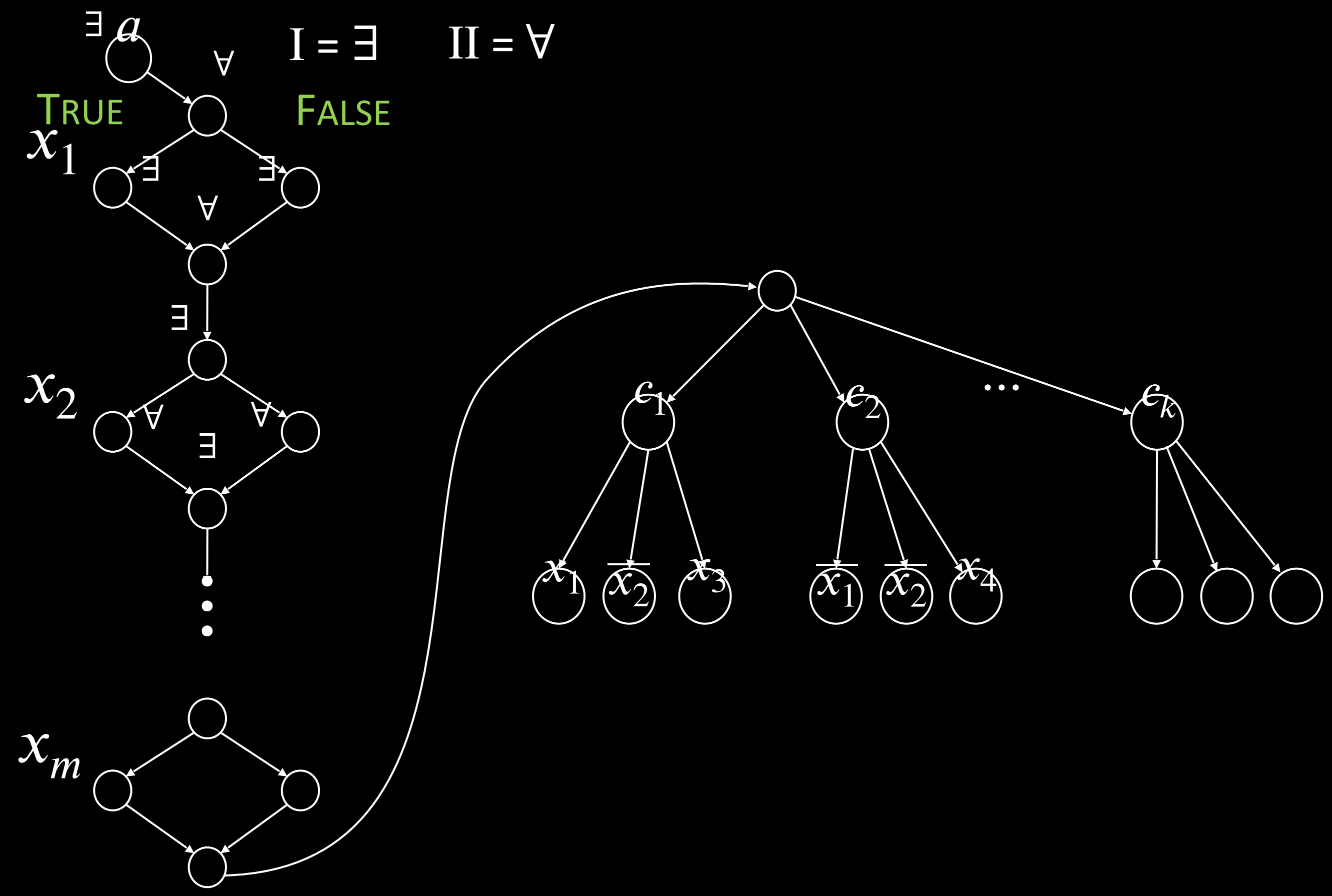


Constructing the GG graph G

Illustrate construction by example

Say $\phi = \exists x_1\ \forall x_2\ \exists x_3 \cdots\ \forall x_k\,[\,\underbrace{(x_1 \vee \overline{x_2} \vee x_3)}_{c_1} \wedge \underbrace{(\overline{x_1} \vee \overline{x_2} \vee x_4)}_{c_2} \wedge \cdots \wedge \underbrace{(\cdots)}_{c_k}\,]$

$G =$

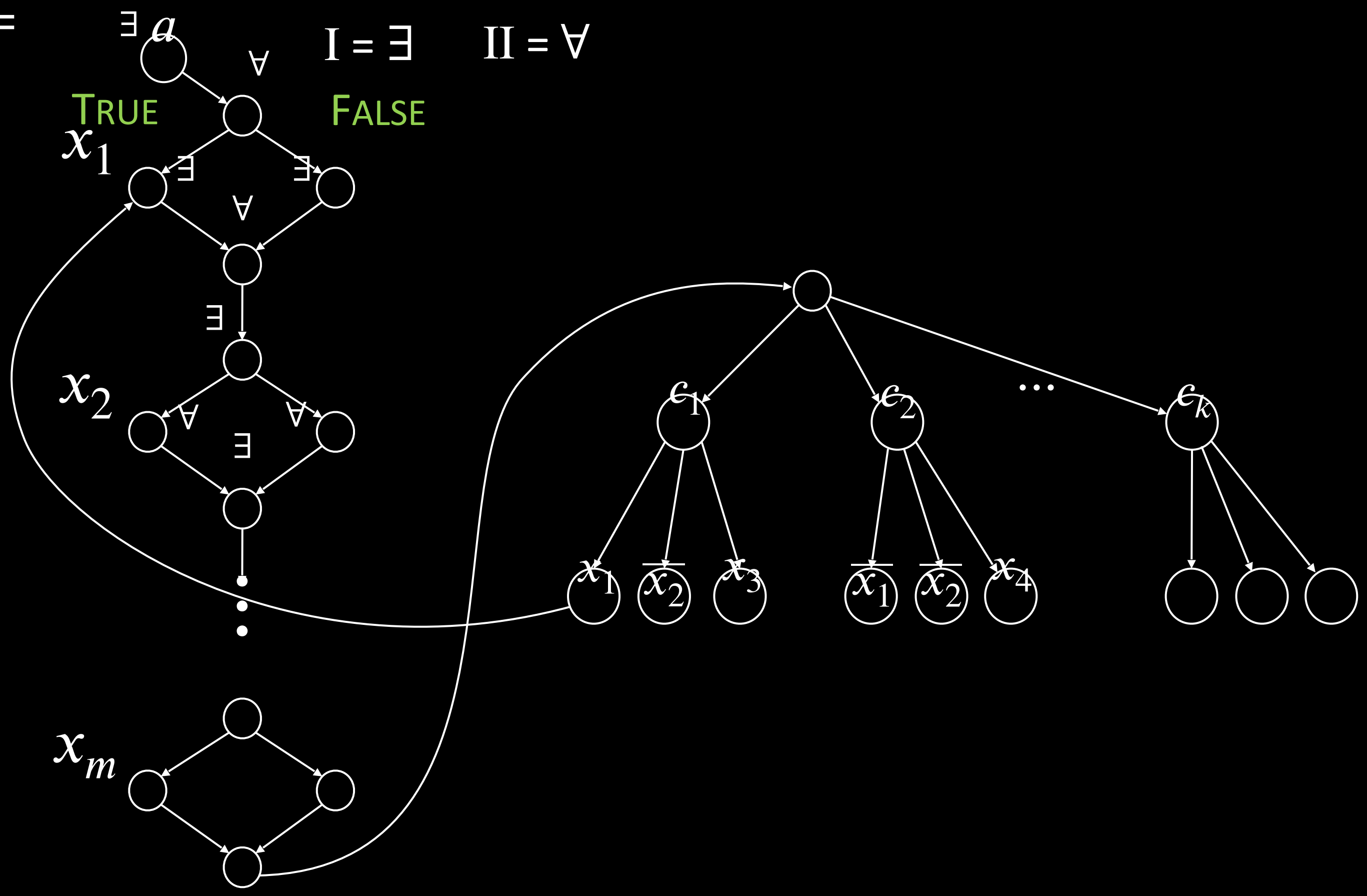


Constructing the GG graph G

Illustrate construction by example

Say $\phi = \exists x_1 \; \forall x_2 \; \exists x_3 \cdots \; \forall x_k \, [\, \underbrace{(\, x_1 \vee \overline{x_2} \vee x_3 \,)}_{c_1} \wedge \underbrace{(\overline{x_1} \vee \overline{x_2} \vee x_4)}_{c_2} \wedge \cdots \wedge \underbrace{(\, \cdots)}_{c_k} \,]$

$G =$

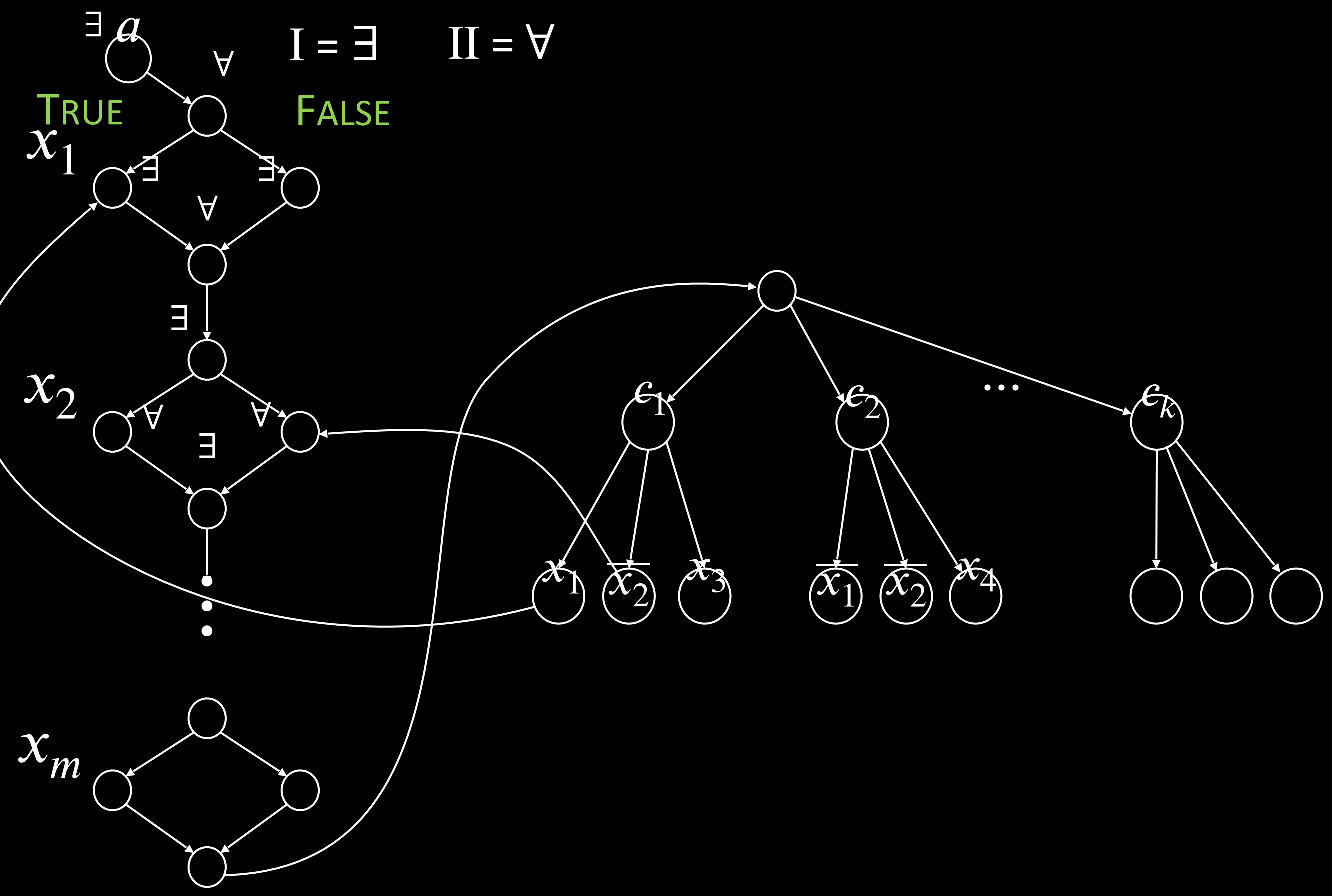


Constructing the GG graph G

Illustrate construction by example

Say $\phi = \exists x_1\ \forall x_2\ \exists x_3 \cdots\ \forall x_k\,[\,\underbrace{(\,x_1 \vee \overline{x_2} \vee x_3\,)}_{c_1} \wedge \underbrace{(\overline{x_1} \vee \overline{x_2} \vee x_4)}_{c_2} \wedge \cdots \wedge \underbrace{(\,\cdots)}_{c_k}\,]$

$G =$

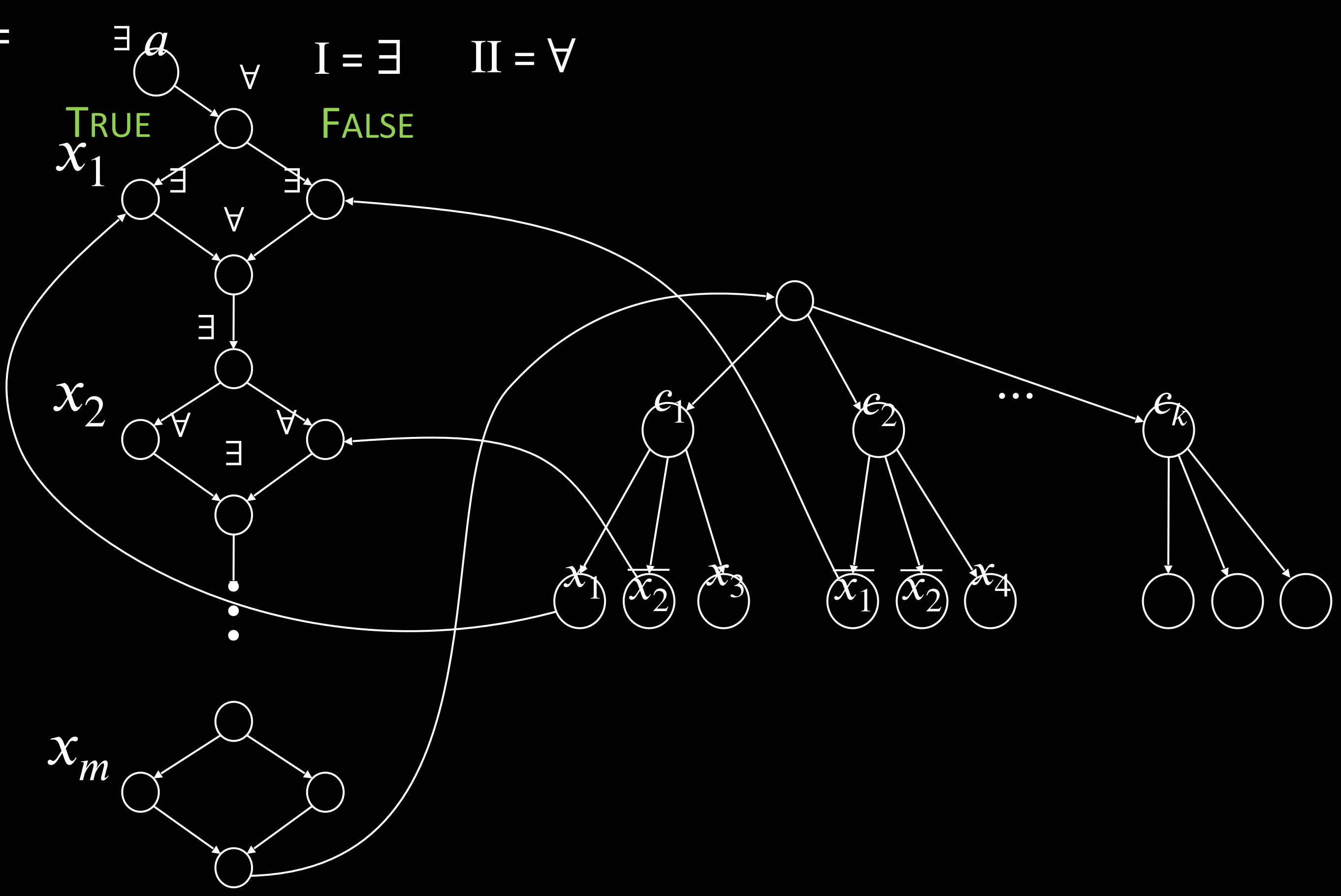


Constructing the GG graph G

Illustrate construction by example

Say $\phi = \exists x_1 \; \forall x_2 \; \exists x_3 \cdots \; \forall x_k \, [\, \underbrace{(x_1 \vee \overline{x_2} \vee x_3)}_{c_1} \wedge \underbrace{(\overline{x_1} \vee \overline{x_2} \vee x_4)}_{c_2} \wedge \cdots \wedge \underbrace{(\cdots)}_{c_k} \,]$

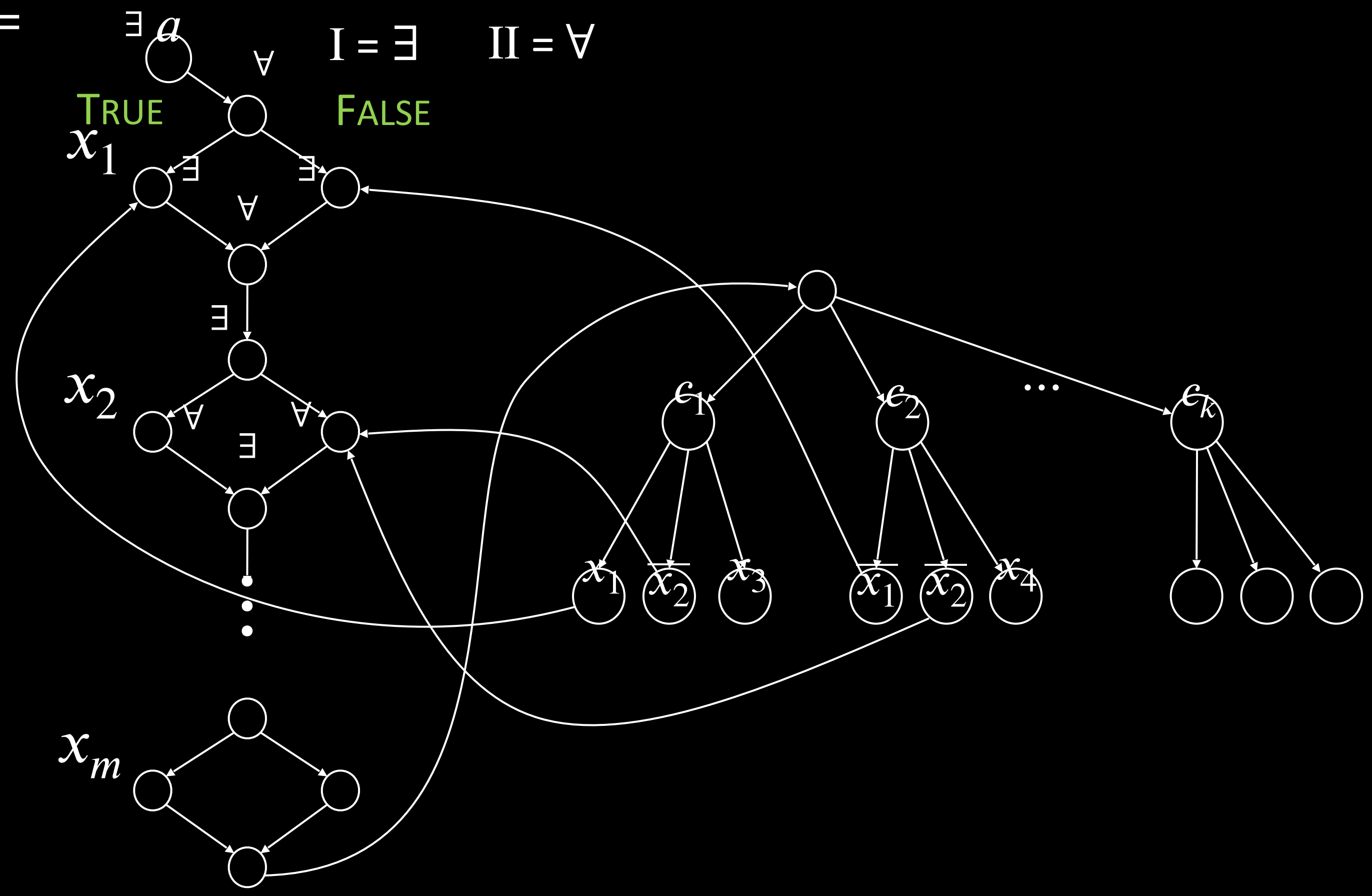
$G =$



Constructing the GG graph G

Illustrate construction by example

Say $\phi = \exists x_1\ \forall x_2\ \exists x_3 \cdots\ \forall x_k\ [\ \underbrace{(\ x_1 \vee \overline{x_2} \vee x_3\ )}_{c_1} \wedge \underbrace{(\overline{x_1} \vee \overline{x_2} \vee x_4)}_{c_2} \wedge \cdots \wedge \underbrace{(\ \cdots)}_{c_k}\ ]$

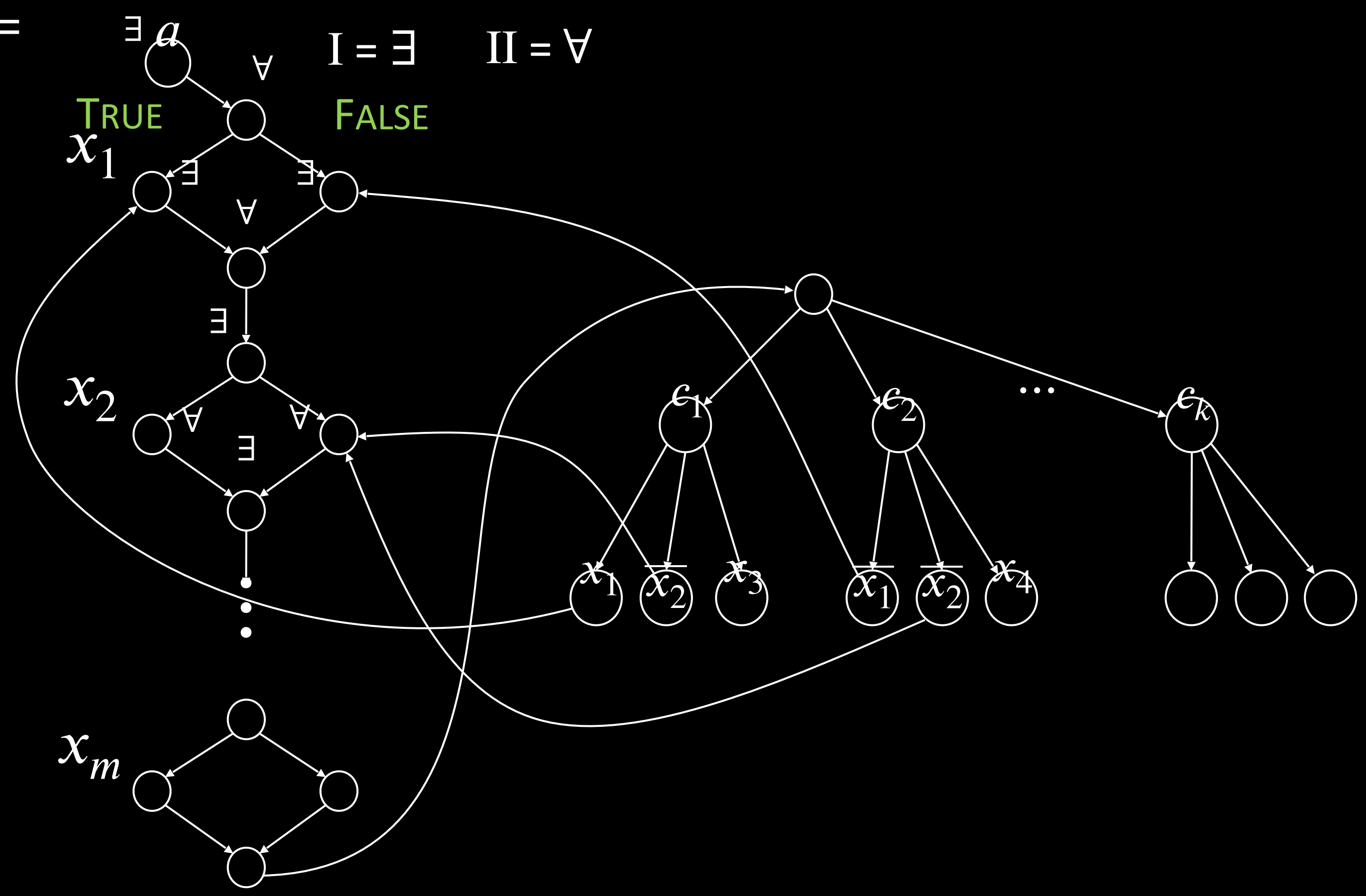
$G =$ 

Constructing the GG graph G

Illustrate construction by example

Say $\phi = \exists x_1\ \forall x_2\ \exists x_3 \cdots\ \forall x_k\ [\ \underbrace{(\ x_1 \vee \overline{x_2} \vee x_3\ )}_{c_1} \wedge \underbrace{(\overline{x_1} \vee \overline{x_2} \vee x_4)}_{c_2} \wedge \cdots \wedge \underbrace{(\ \cdots)}_{c_k}\ ]$

$G =$



Endgame

∃ should win if assignment satisfied all clauses

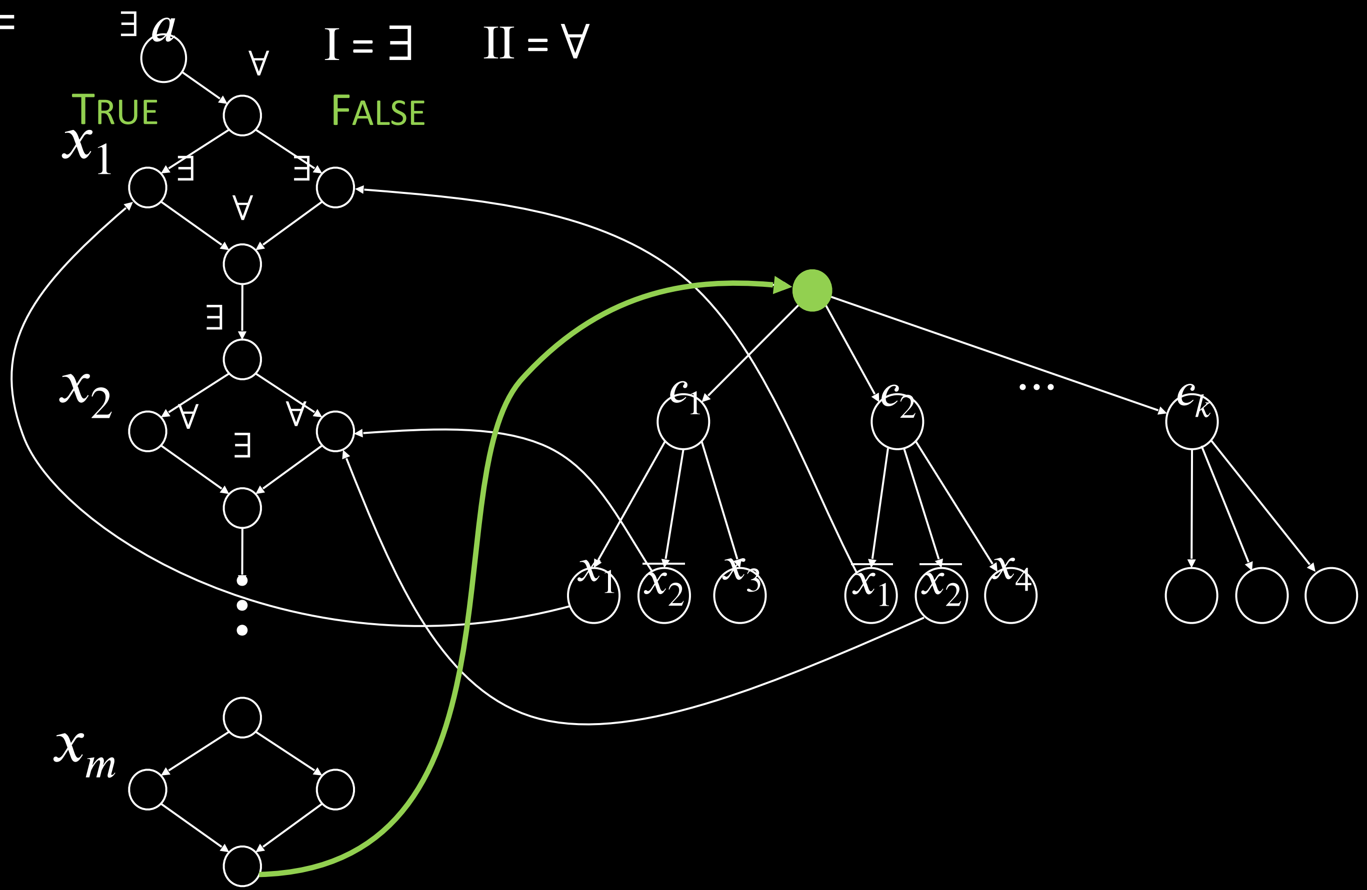
∀ should win if some unsatisfied clause

Constructing the GG graph G

Illustrate construction by example

Say $\phi = \exists x_1 \; \forall x_2 \; \exists x_3 \cdots \; \forall x_k \, [\, (\underbrace{x_1 \vee \overline{x_2} \vee x_3}_{c_1}) \wedge (\underbrace{\overline{x_1} \vee \overline{x_2} \vee x_4}_{c_2}) \wedge \cdots \wedge (\underbrace{\cdots}_{c_k})\,]$

$G =$



Endgame

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Constructing the GG graph G

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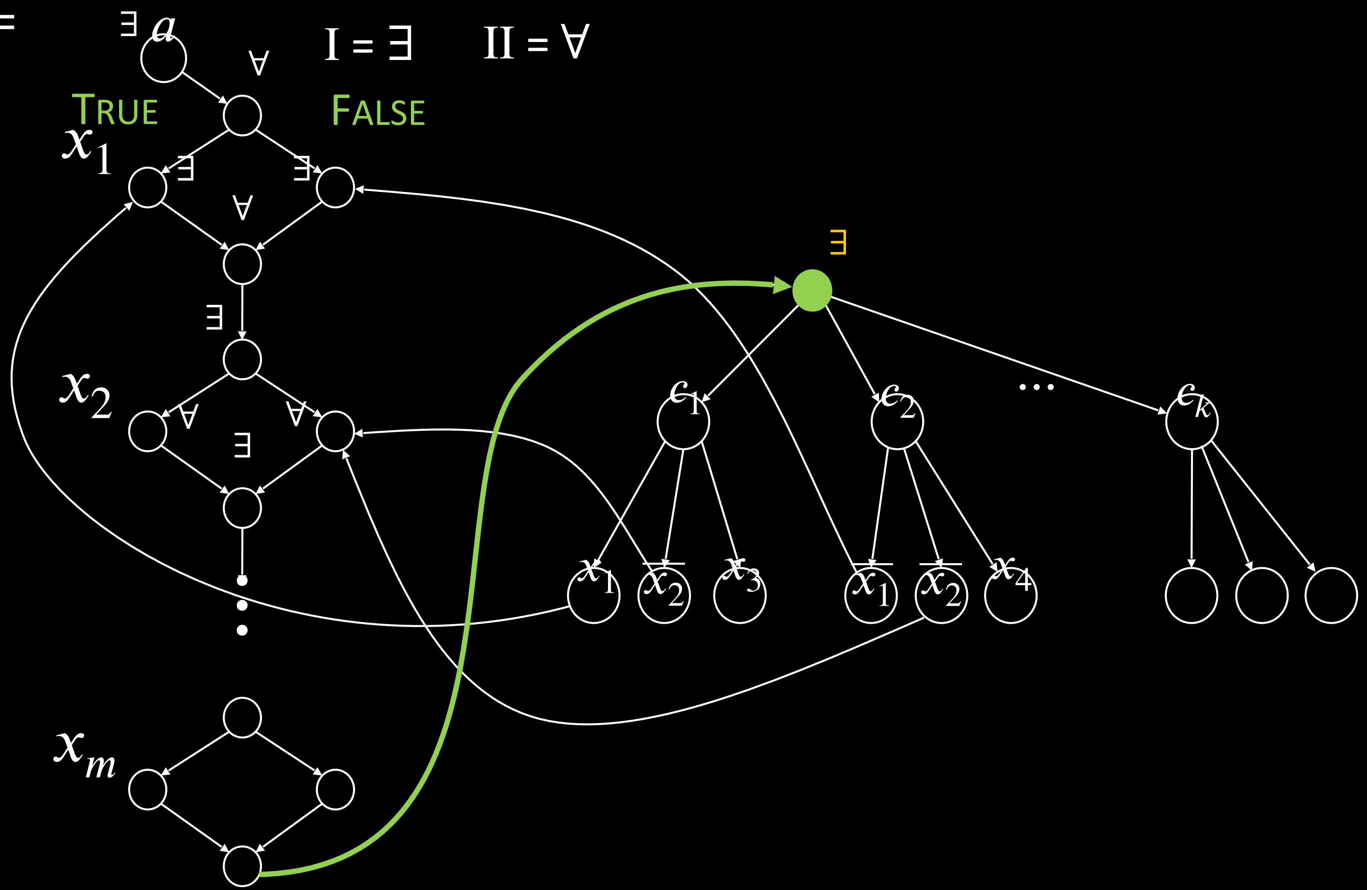
Say $\phi = \exists x_1\ \forall x_2\ \exists x_3 \cdots\ \forall x_k\ [\ (\underbrace{x_1 \vee \overline{x_2} \vee x_3}_{c_1}) \wedge (\underbrace{\overline{x_1} \vee \overline{x_2} \vee x_4}_{c_2}) \wedge \cdots \wedge (\underbrace{\cdots}_{c_k})\ ]$

$G =$



Endgame

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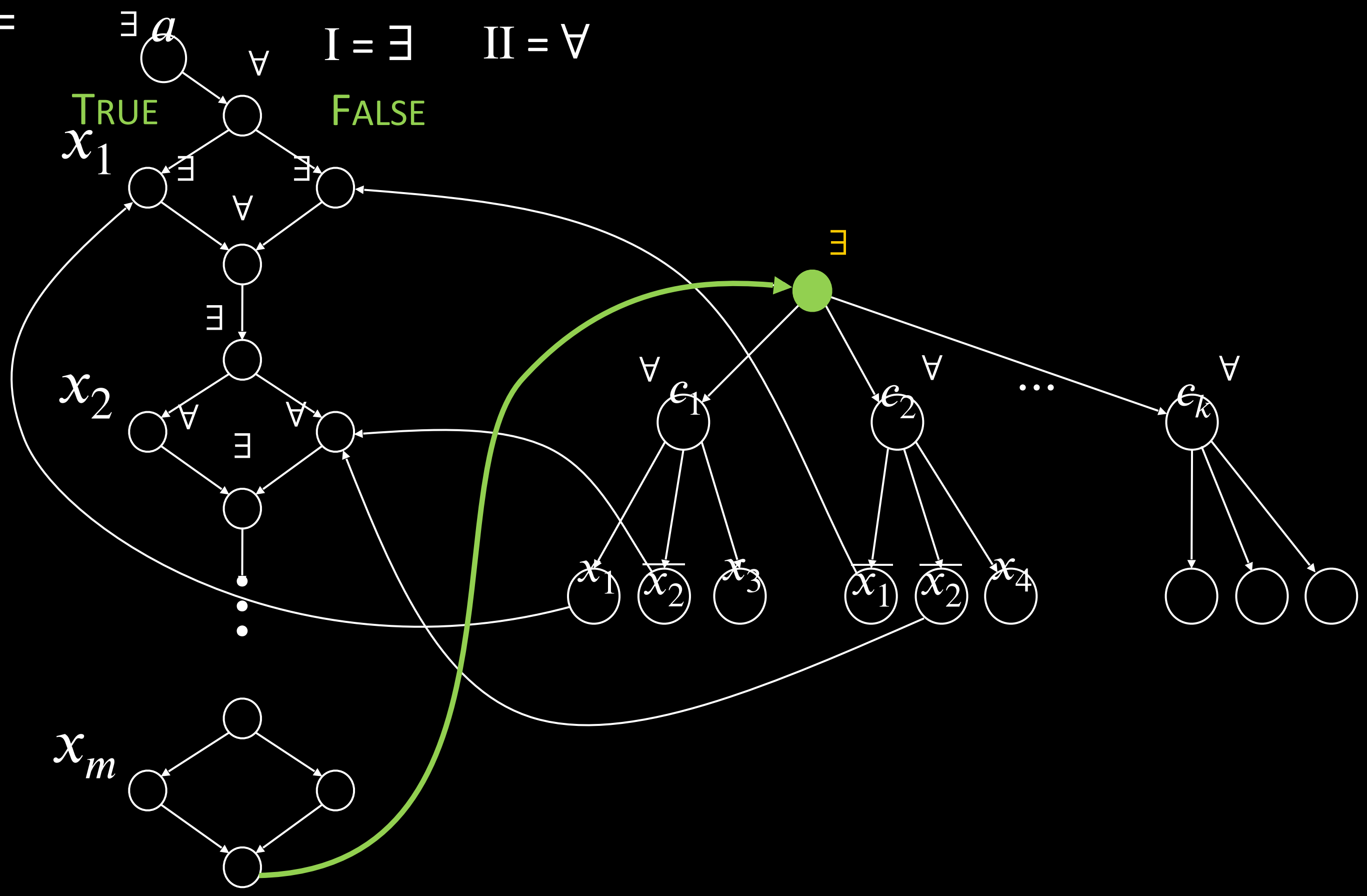
Constructing the GG graph G

Illustrate construction by example

Say $\phi = \exists x_1\ \forall x_2\ \exists x_3 \cdots\ \forall x_k\, [\, \underbrace{(\, x_1 \vee \overline{x_2} \vee x_3\,)}_{c_1} \wedge \underbrace{(\overline{x_1} \vee \overline{x_2} \vee x_4)}_{c_2} \wedge \cdots \wedge \underbrace{(\,\cdots)}_{c_k}\,]$

$G =$



Endgame

∃ should win if assignment satisfied all clauses

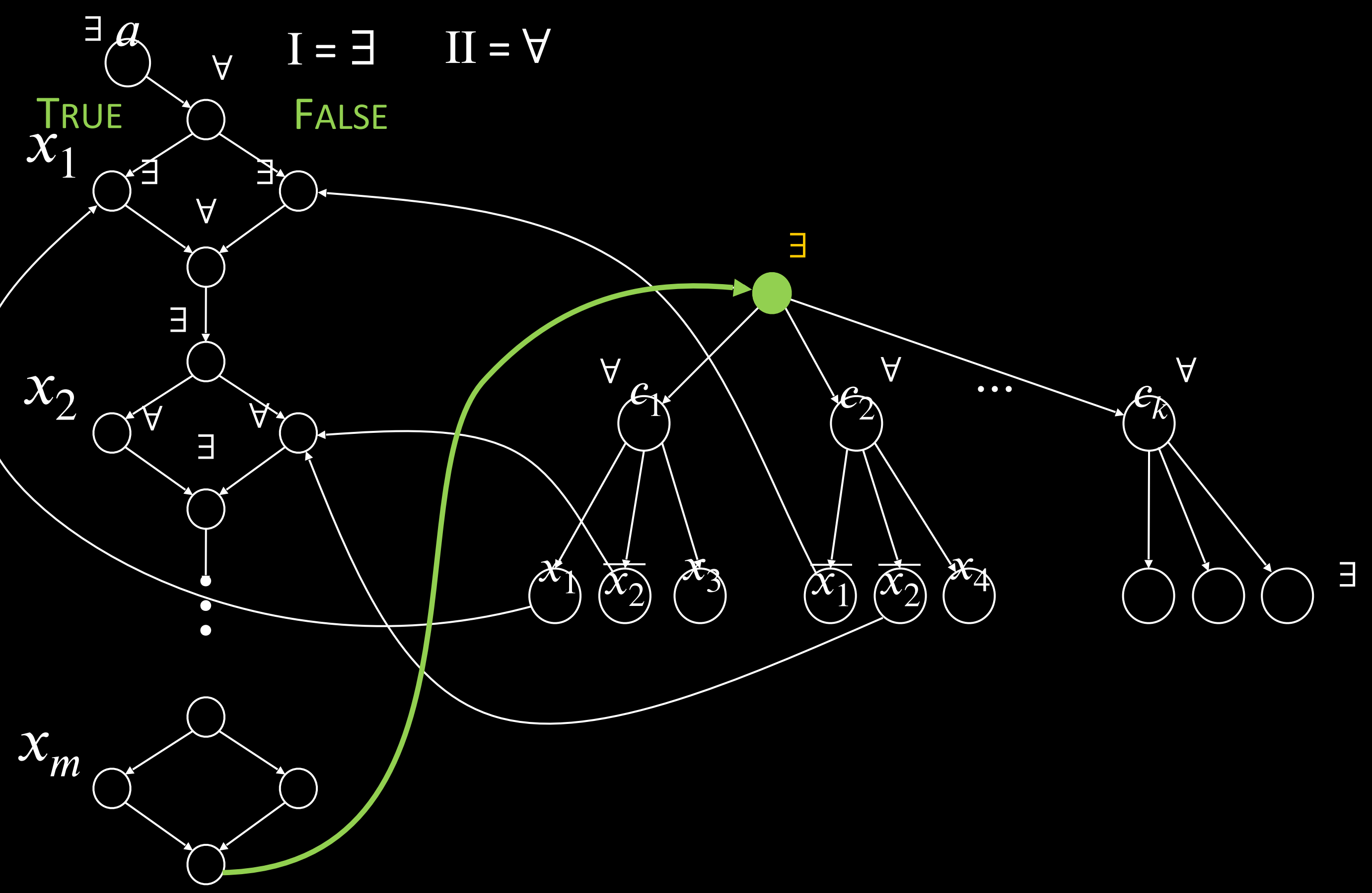
∀ should win if some unsatisfied clause

Constructing the GG graph G

Illustrate construction by example

Say $\phi = \exists x_1 \; \forall x_2 \; \exists x_3 \cdots \; \forall x_k \, [\, (\underbrace{x_1 \vee \overline{x_2} \vee x_3}_{c_1}) \wedge (\underbrace{\overline{x_1} \vee \overline{x_2} \vee x_4}_{c_2}) \wedge \cdots \wedge (\underbrace{\cdots}_{c_k})\,]$

$G =$



Endgame

∃ should win if assignment satisfied all clauses

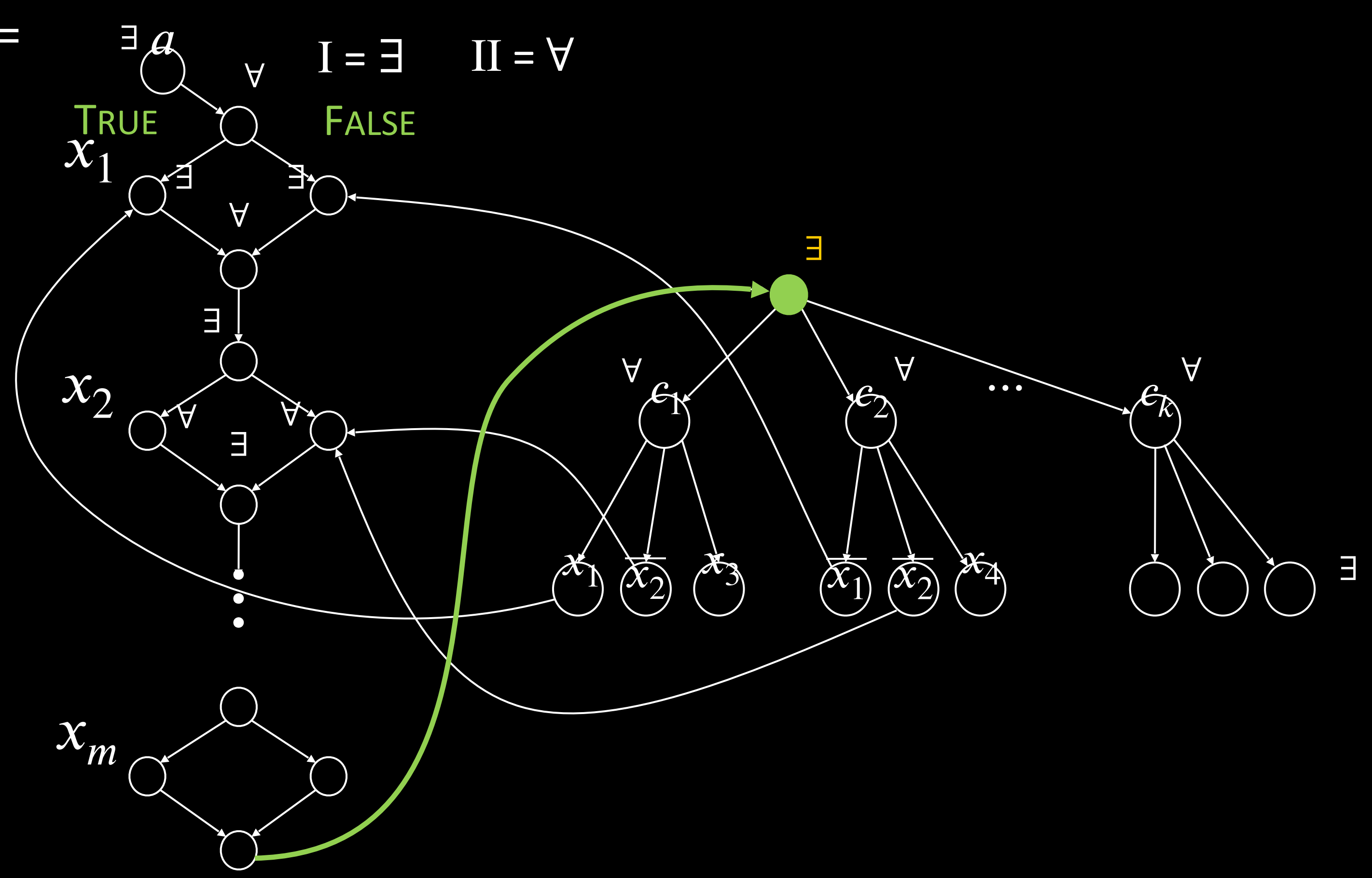
∀ should win if some unsatisfied clause

Constructing the GG graph G

Illustrate construction by example

Say $\phi = \exists x_1\ \forall x_2\ \exists x_3 \cdots\ \forall x_k\, [\, \underbrace{(\, x_1 \vee \overline{x_2} \vee x_3\,)}_{c_1} \wedge \underbrace{(\overline{x_1} \vee \overline{x_2} \vee x_4)}_{c_2} \wedge \cdots \wedge \underbrace{(\, \cdots)}_{c_k}\,]$

$G =$



Endgame

$\exists$ should win if assignment satisfied all clauses

$\forall$ should win if some unsatisfied clause

Implementation

$\forall$ picks clause node claimed unsatisfied

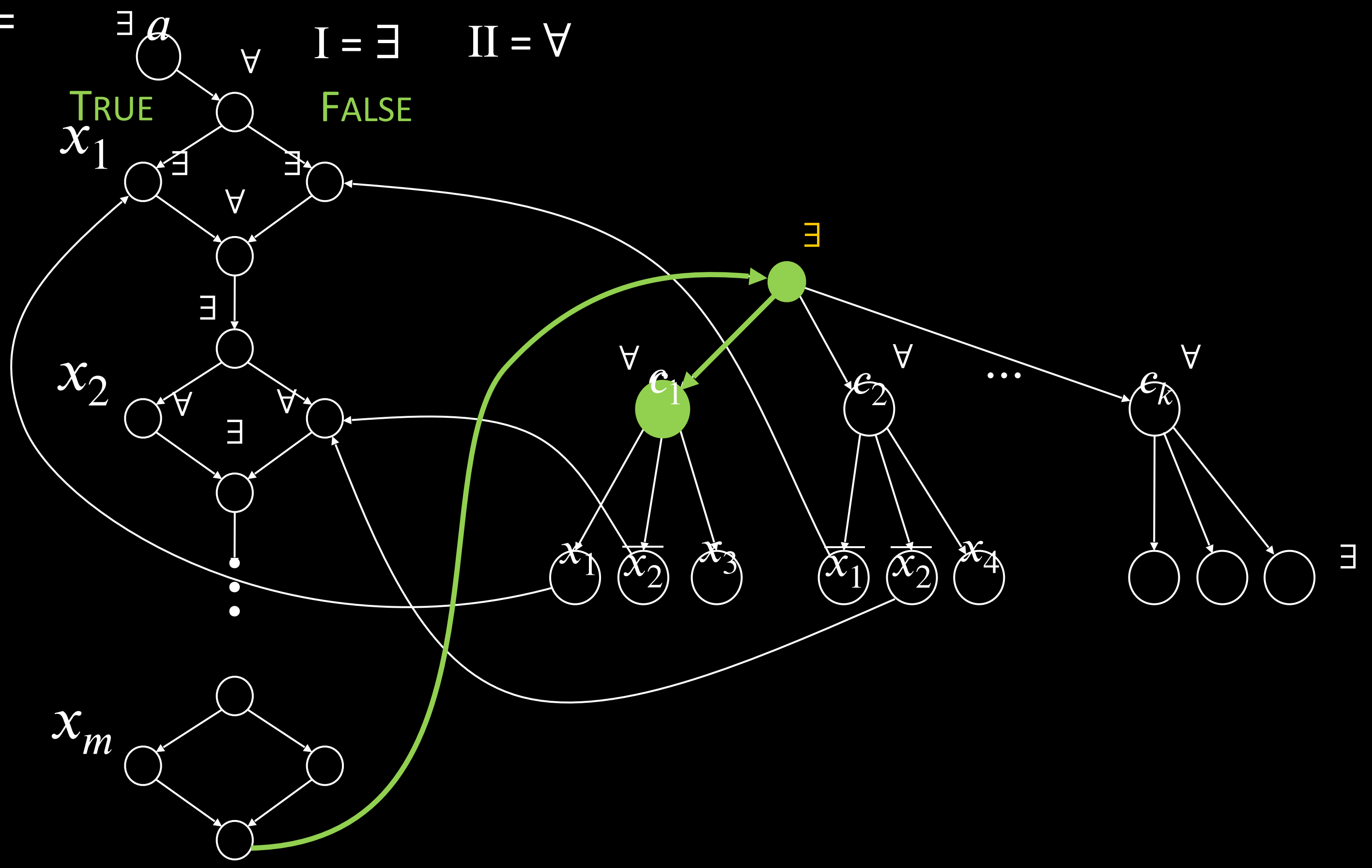
$\exists$ picks literal node claimed to satisfy the clause

liar will be stuck

Constructing the GG graph G

Illustrate construction by example

Say $\phi = \exists x_1\ \forall x_2\ \exists x_3 \cdots\ \forall x_k\ [\ \underbrace{(\ x_1 \vee \overline{x_2} \vee x_3\ )}_{c_1} \wedge \underbrace{(\overline{x_1} \vee \overline{x_2} \vee x_4)}_{c_2} \wedge \cdots \wedge \underbrace{(\ \cdots)}_{c_k}\ ]$

$G =$ 

Endgame

∃ should win if assignment satisfied all clauses

∀ should win if some unsatisfied clause

Implementation

∀ picks clause node claimed unsatisfied

∃ picks literal node claimed to satisfy the clause

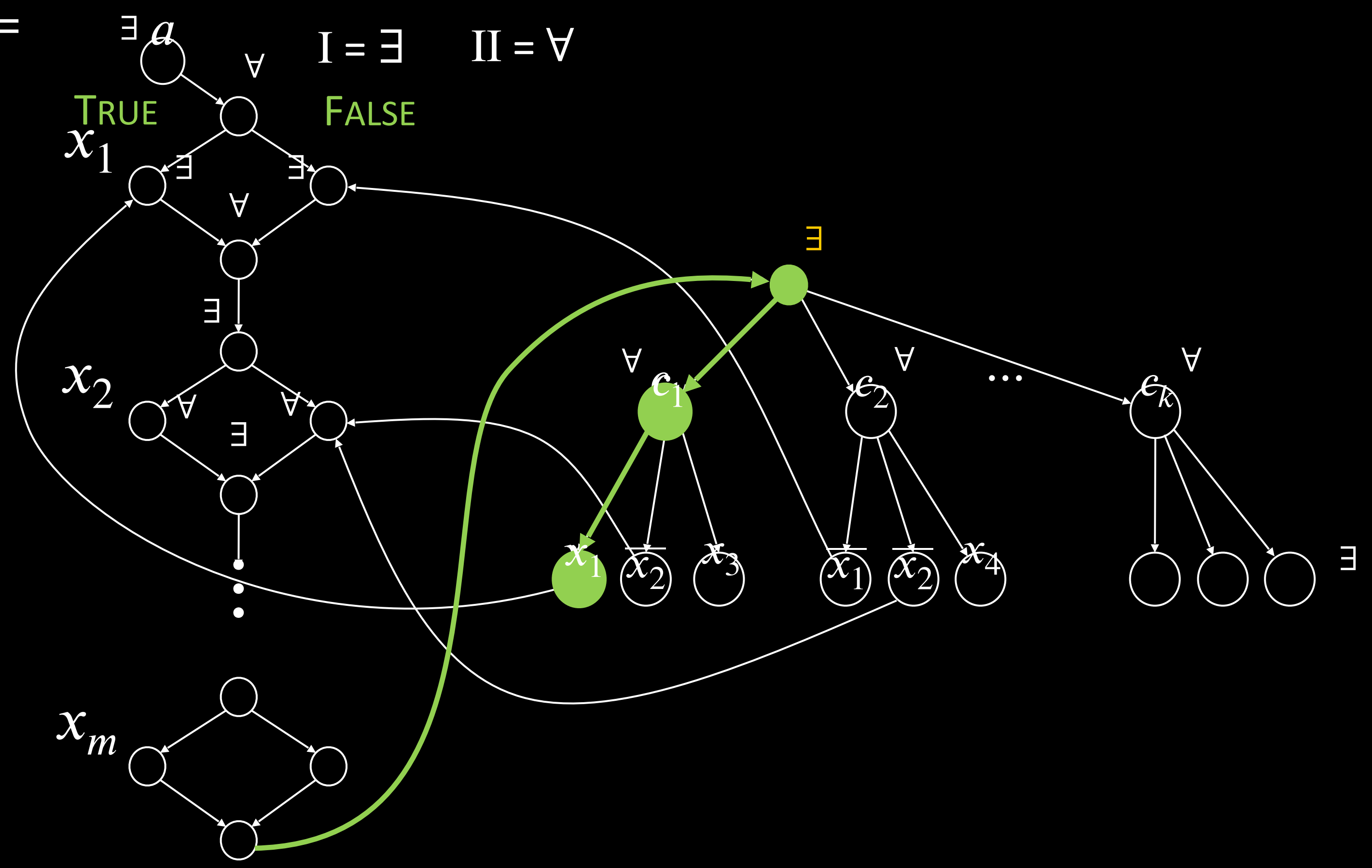
liar will be stuck

Constructing the GG graph G

Illustrate construction by example

Say $\phi = \exists x_1\ \forall x_2\ \exists x_3 \cdots\ \forall x_k\ [\ \underbrace{(\ x_1 \vee \overline{x_2} \vee x_3\ )}_{c_1} \wedge \underbrace{(\overline{x_1} \vee \overline{x_2} \vee x_4)}_{c_2} \wedge \cdots \wedge \underbrace{(\ \cdots)}_{c_k}\ ]$

$G =$



Endgame

∃ should win if assignment satisfied all clauses

∀ should win if some unsatisfied clause

Implementation

∀ picks clause node claimed unsatisfied

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liar will be stuck

Constructing the GG graph G

Illustrate construction by example

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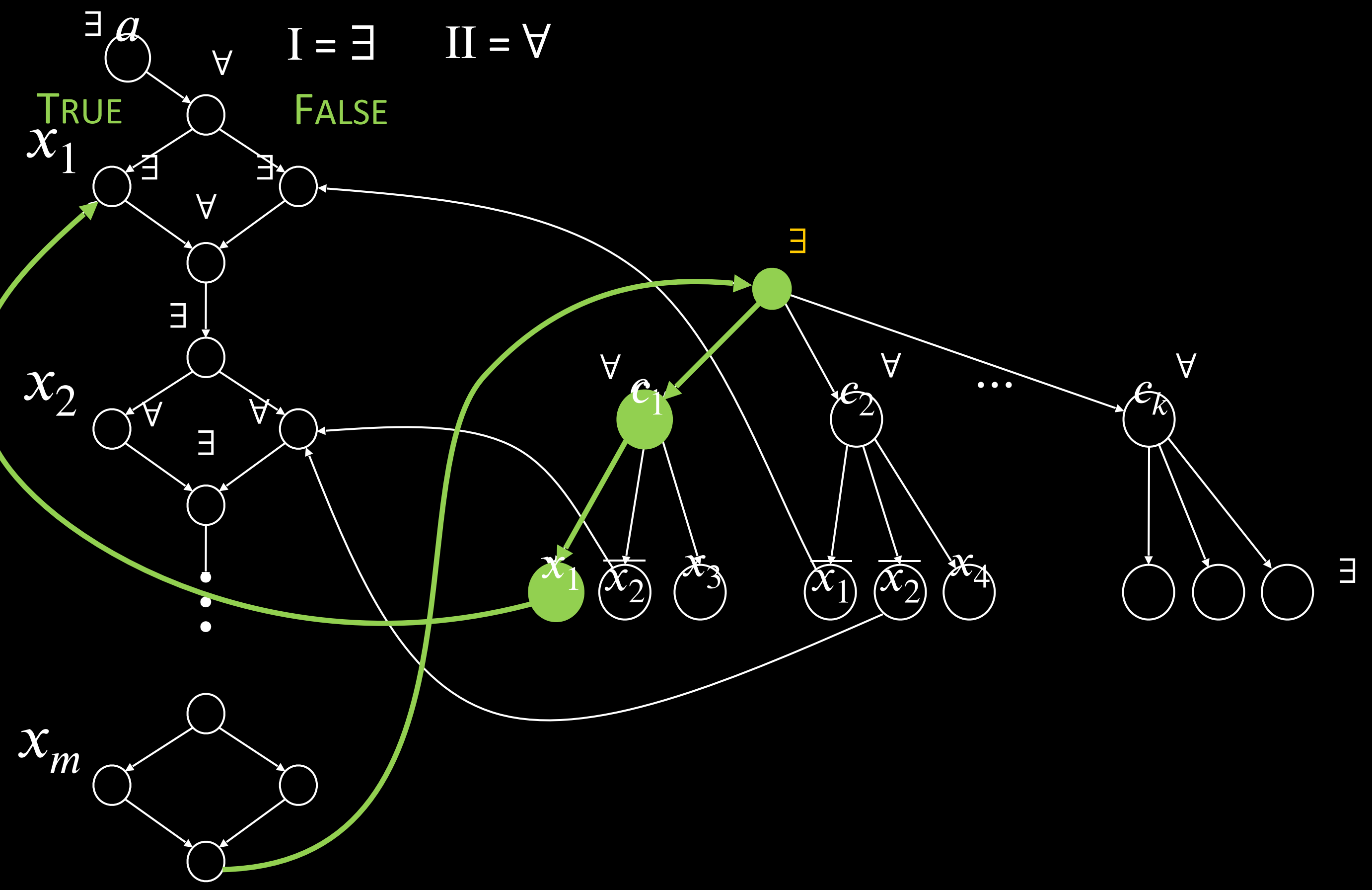

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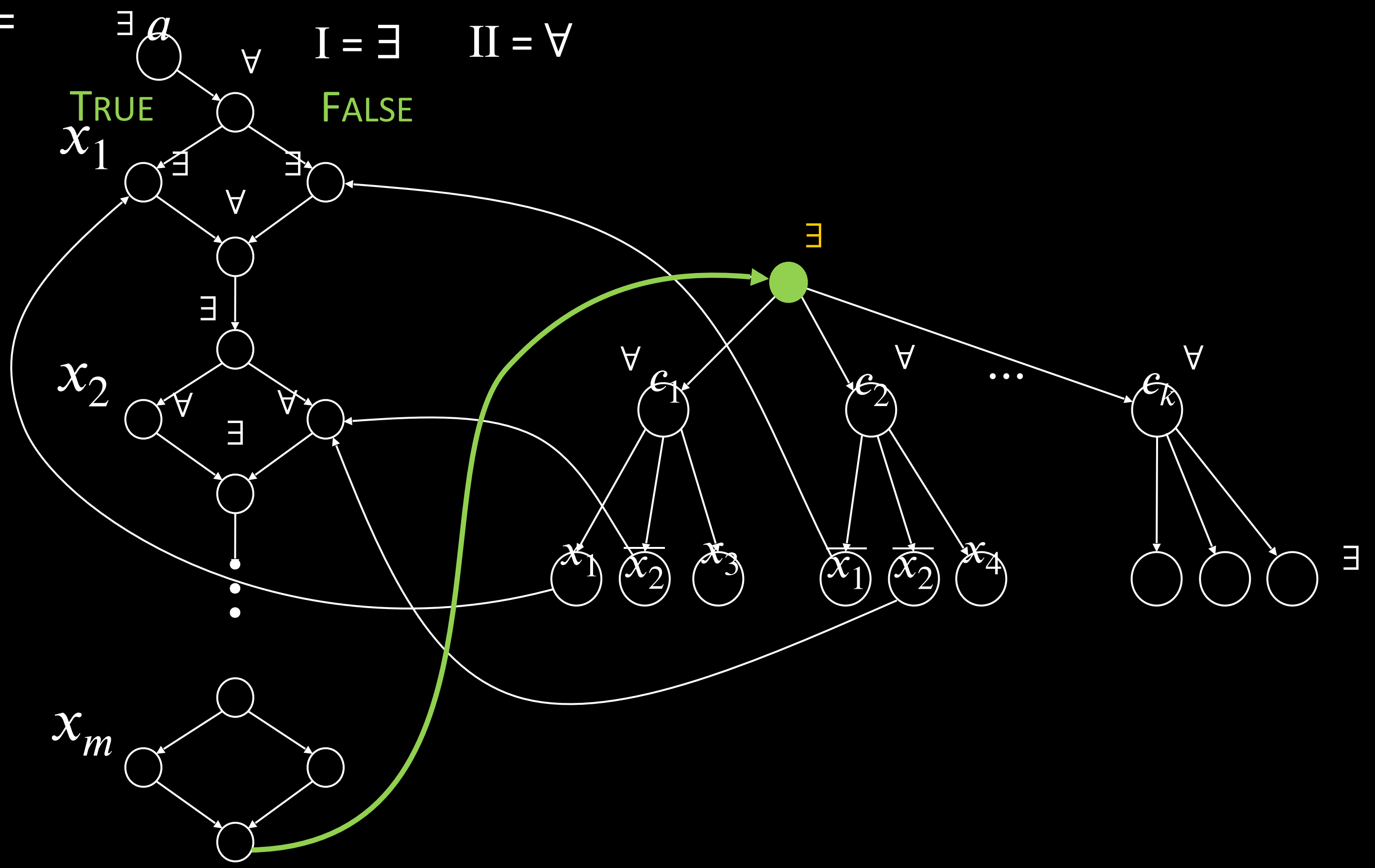

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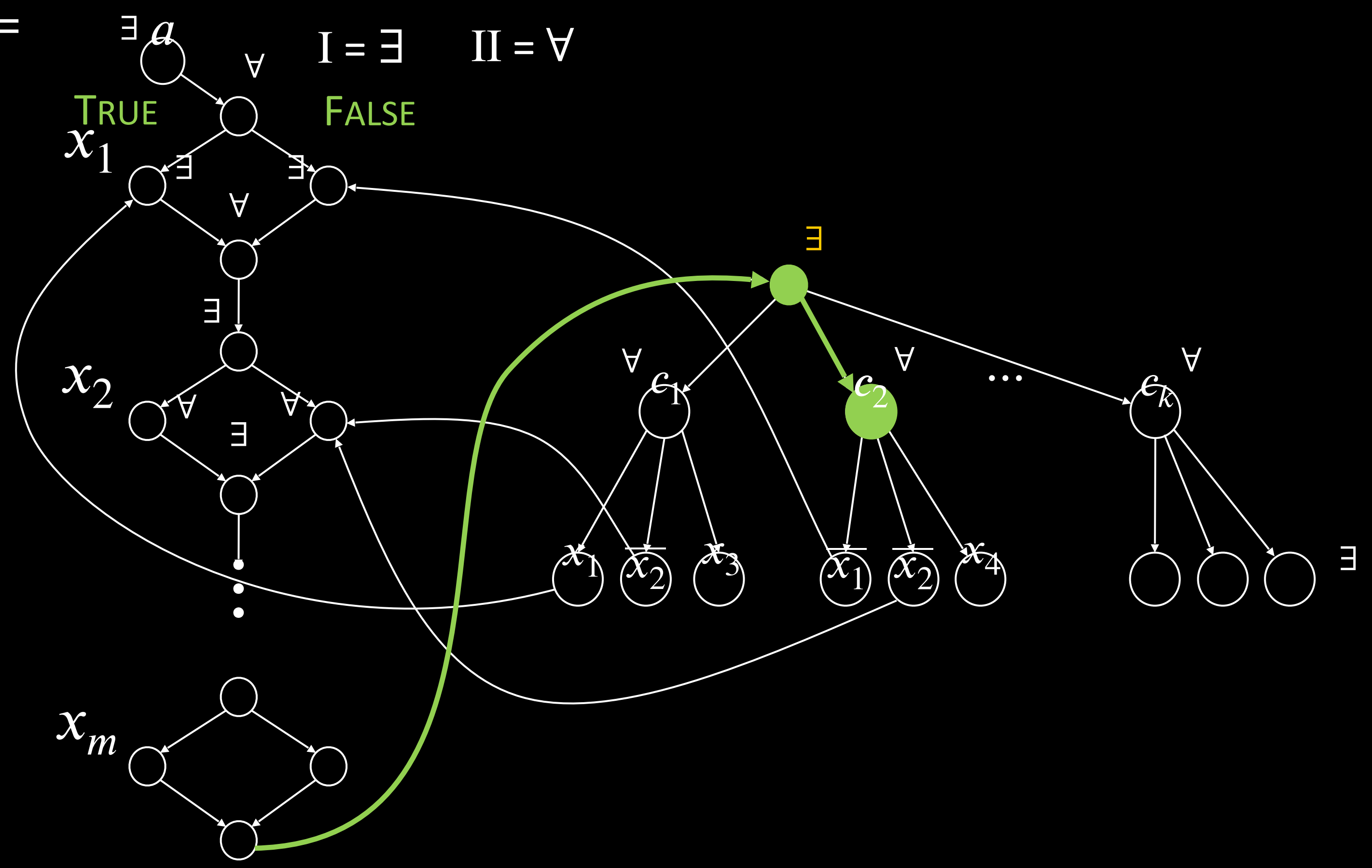

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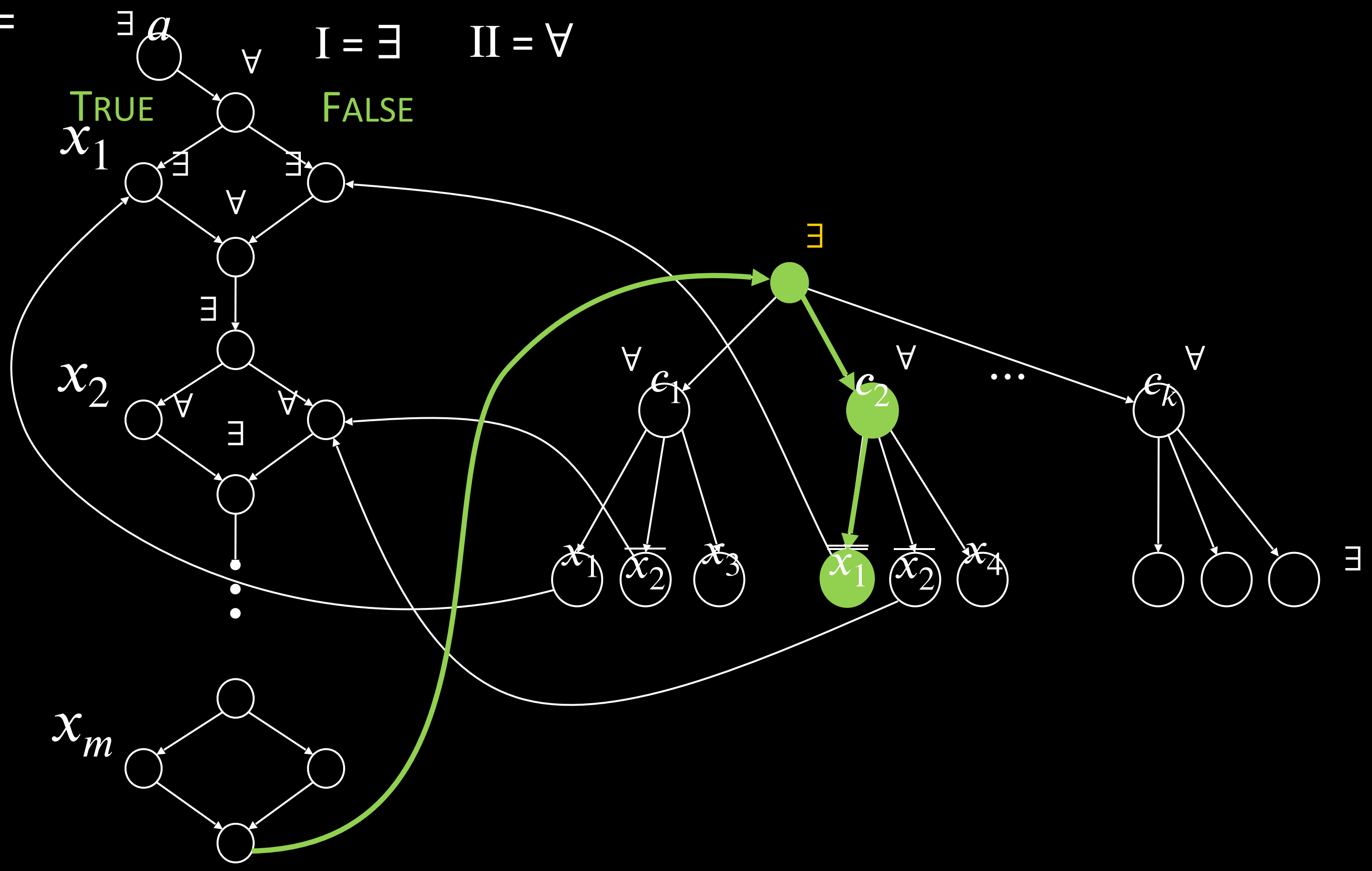

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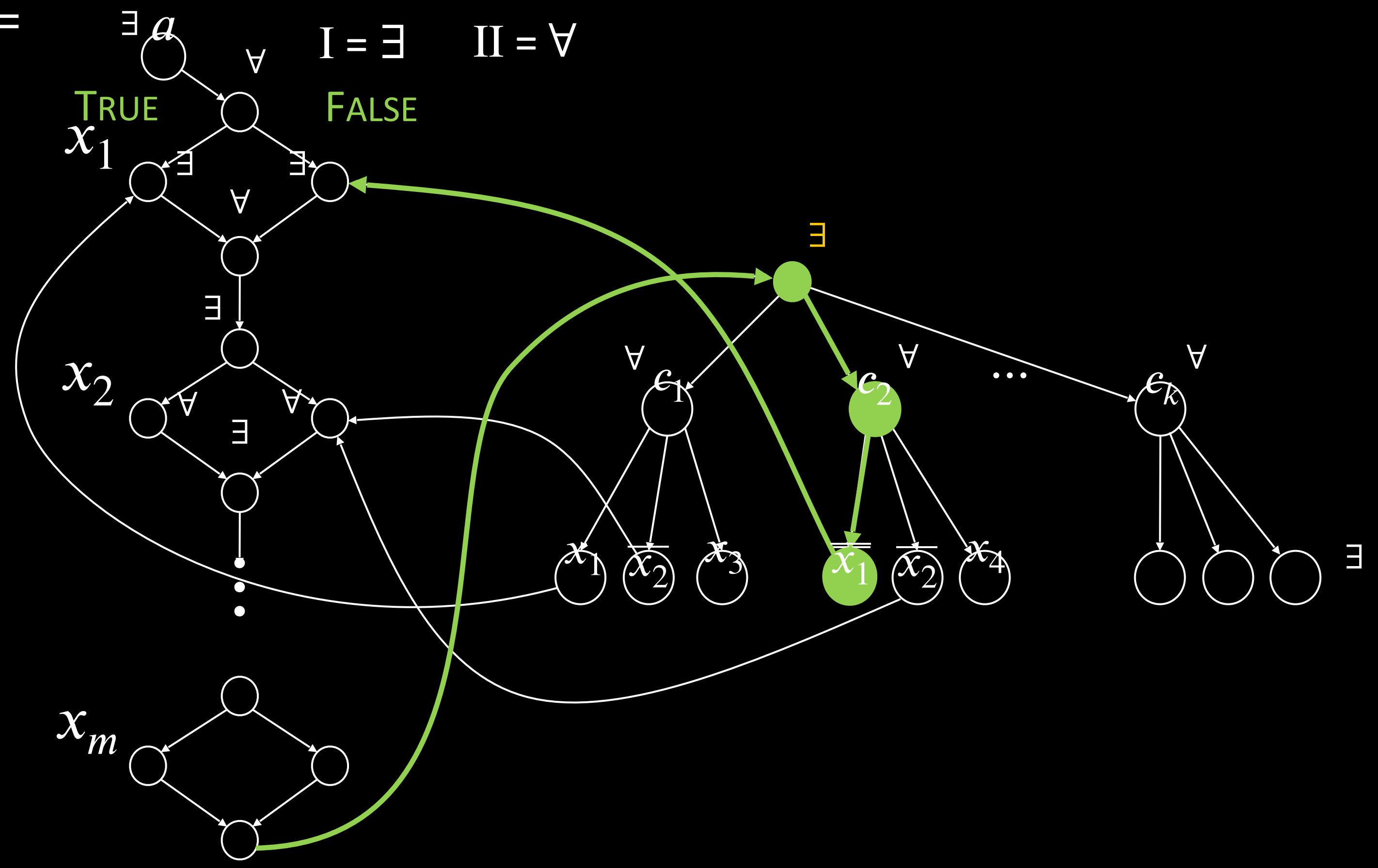

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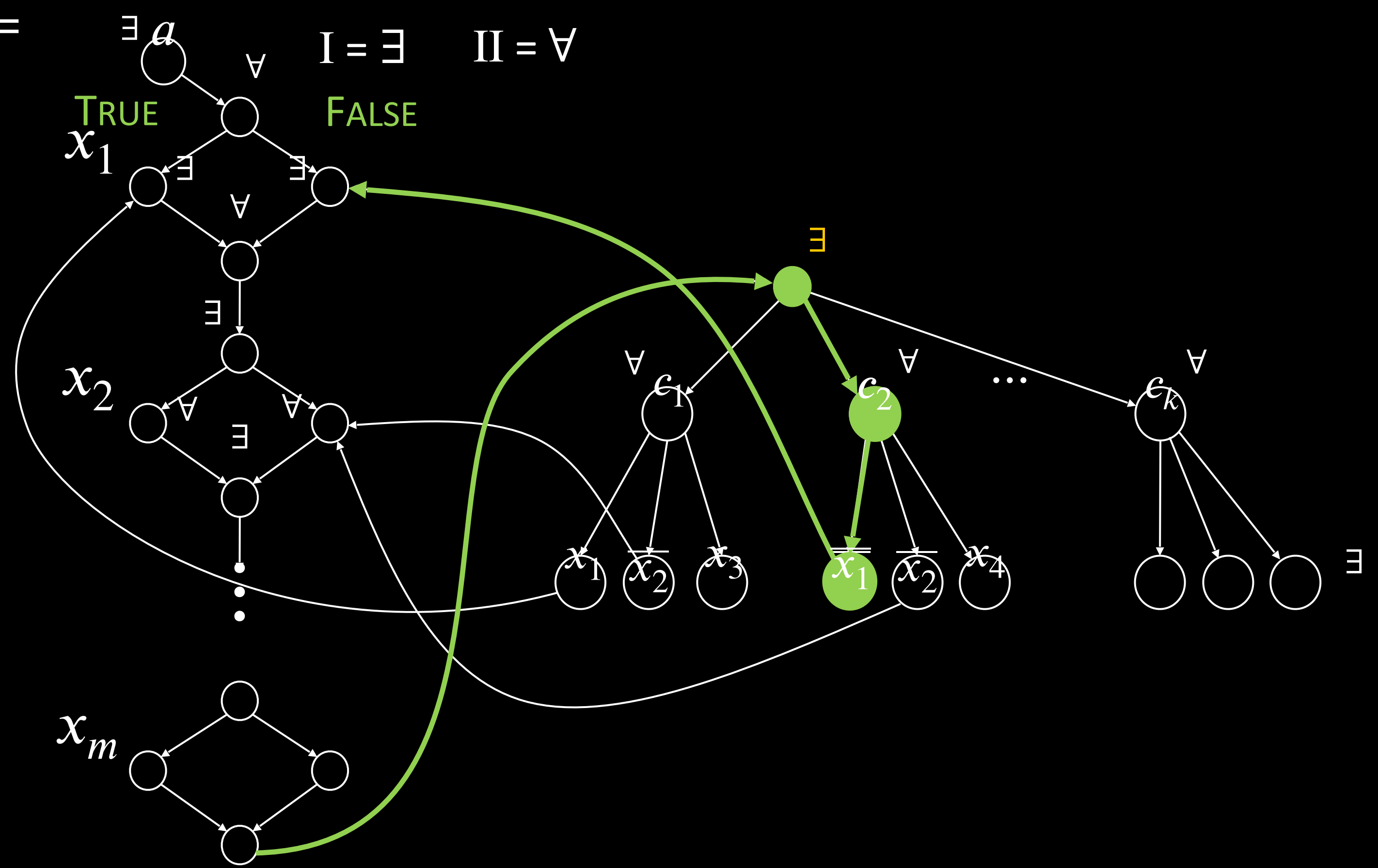

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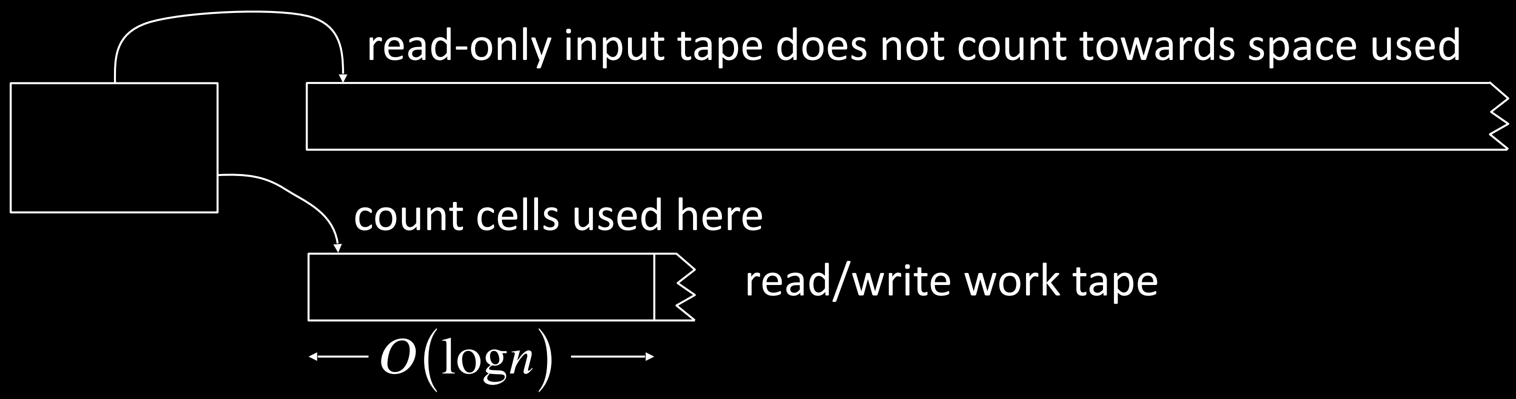

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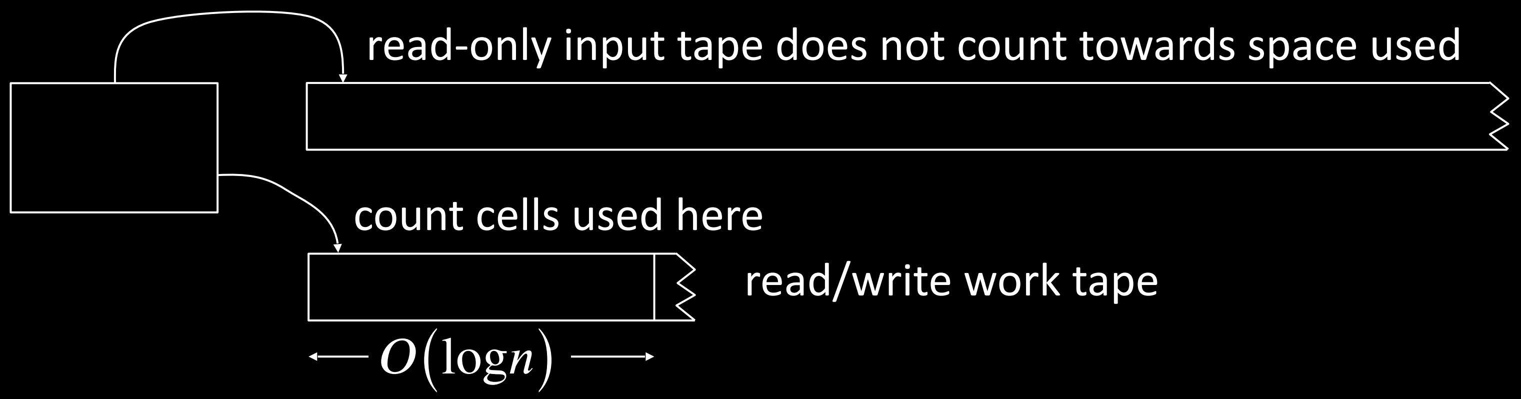
Log space can represent a constant number of pointers into the input.



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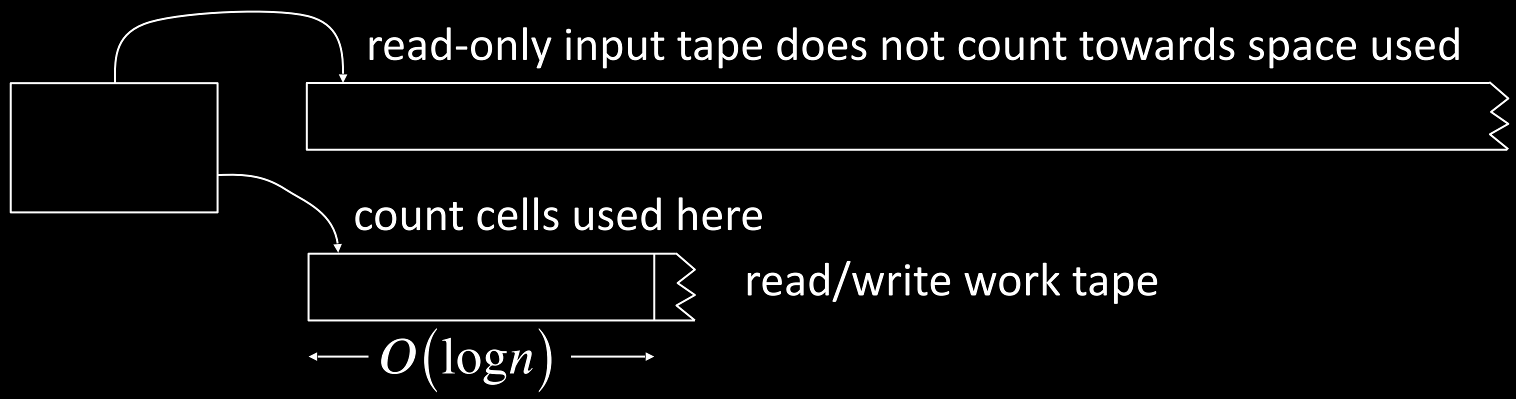

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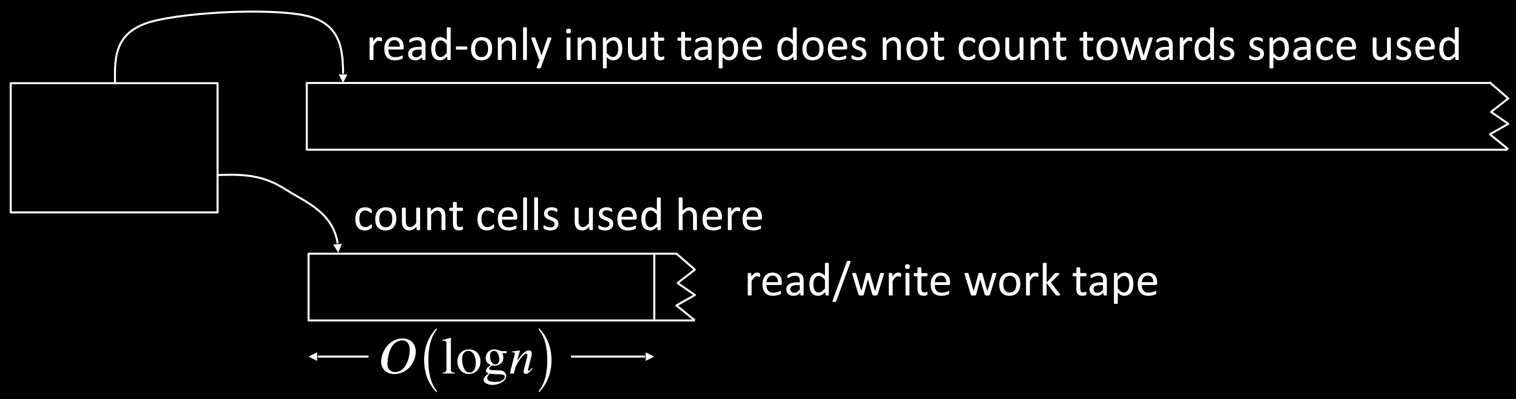

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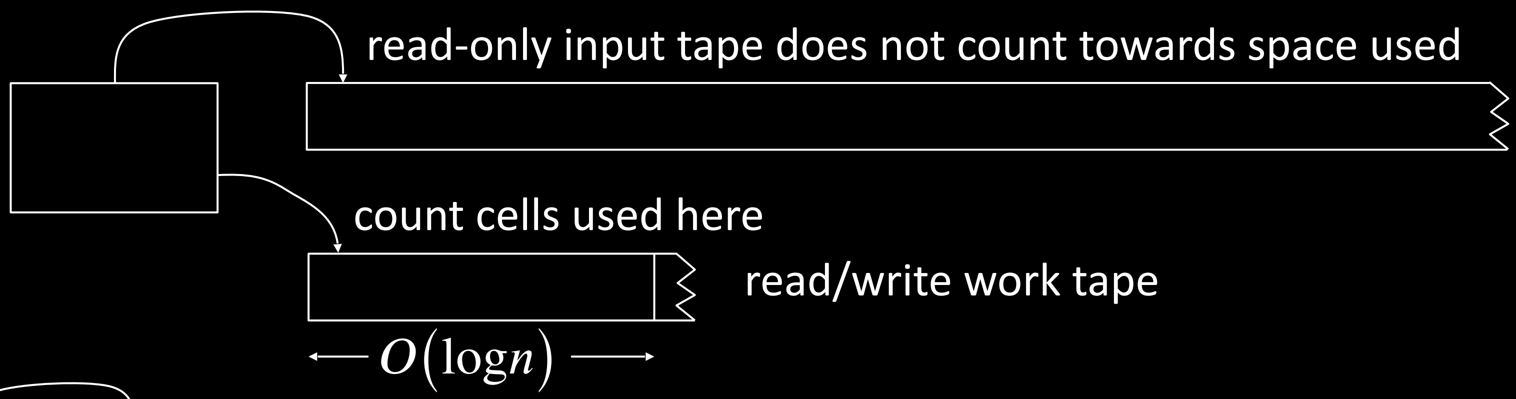

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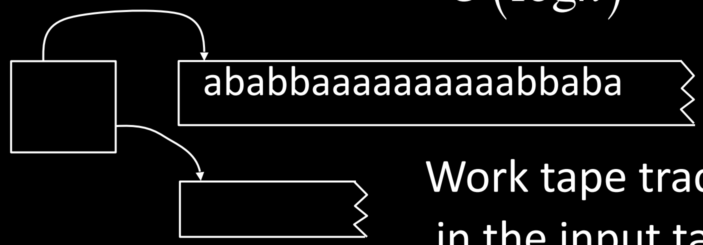

Work tape tracks corresponding locations in the input tape.

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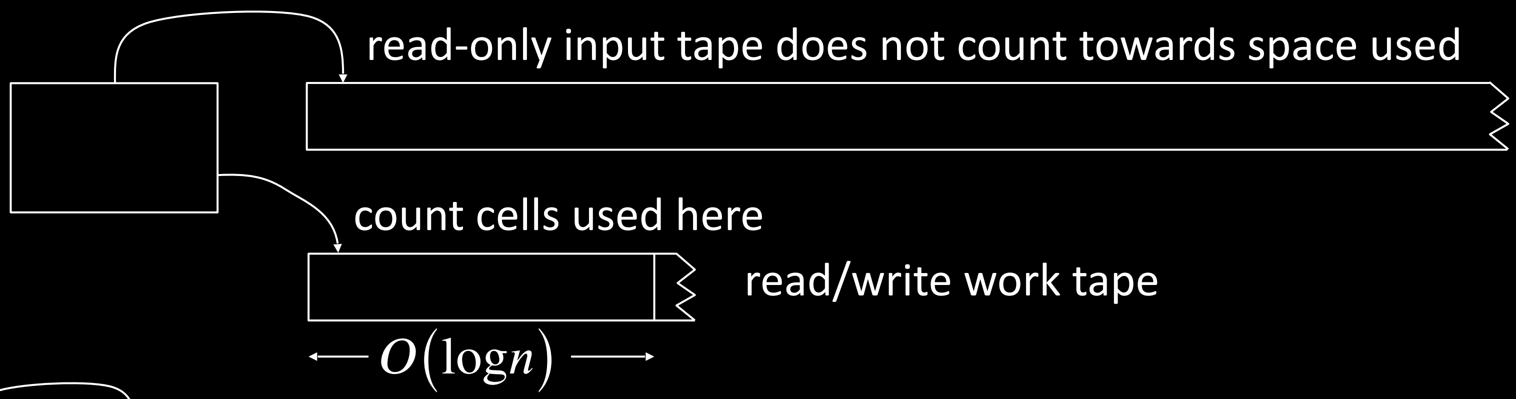

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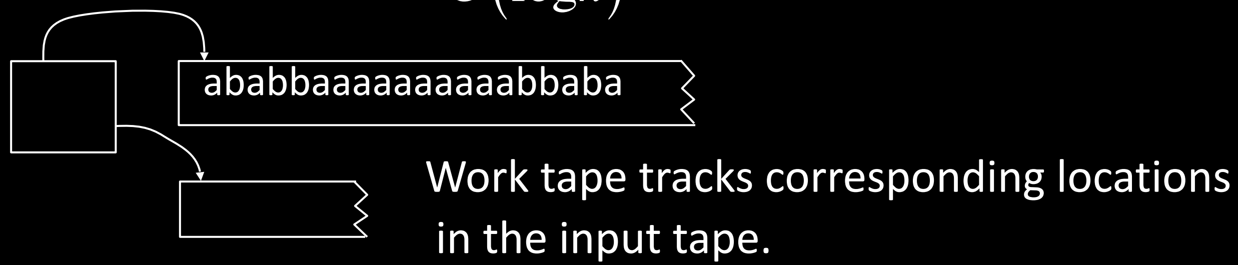

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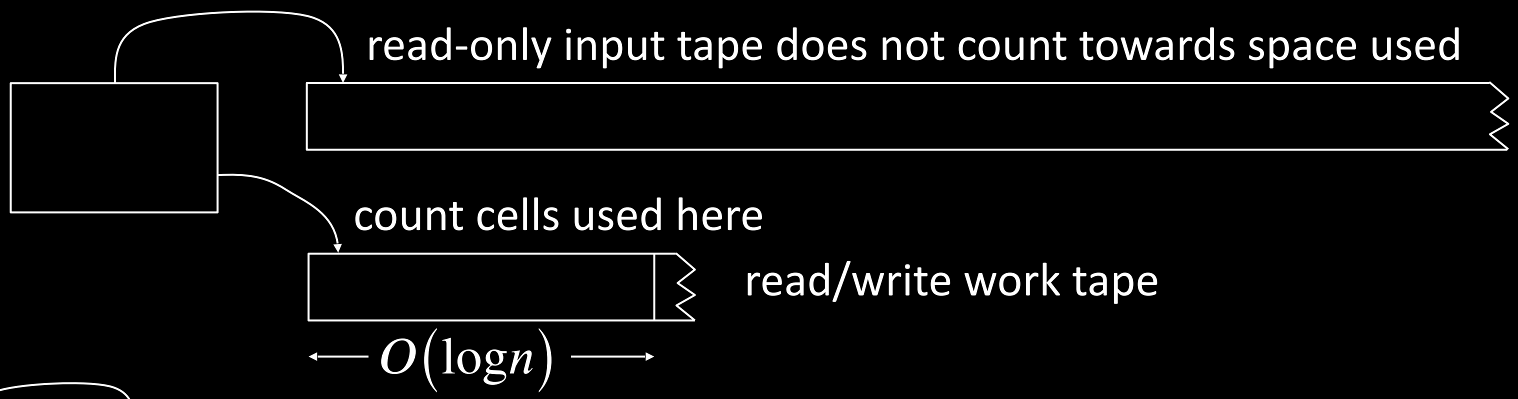

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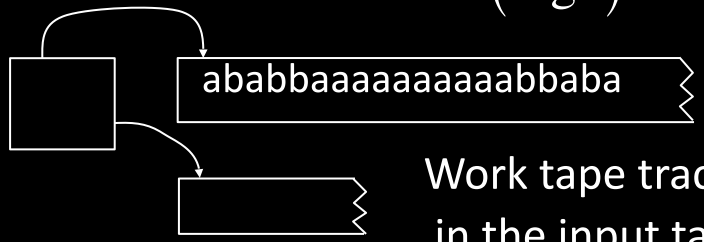

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Nondeterministically select the nodes of a path connecting s to t.

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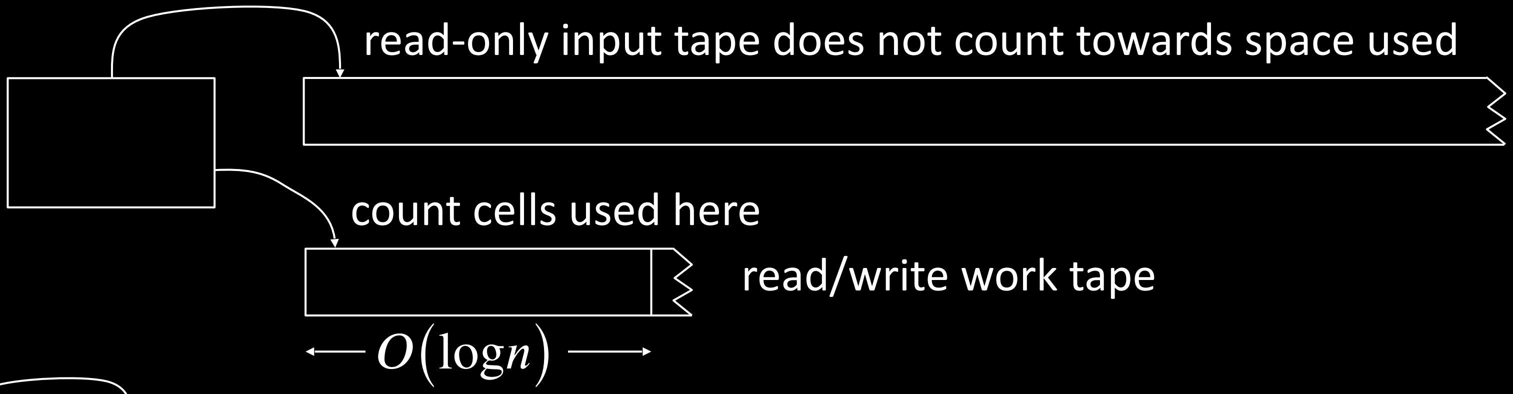

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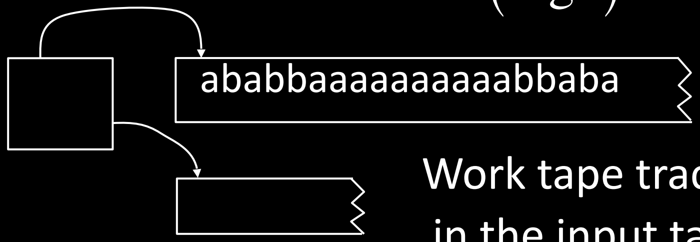

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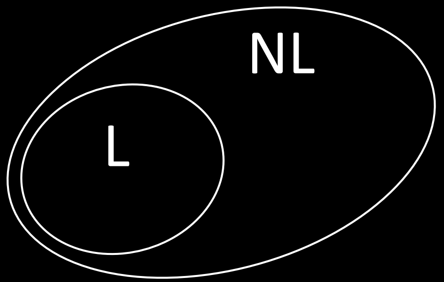

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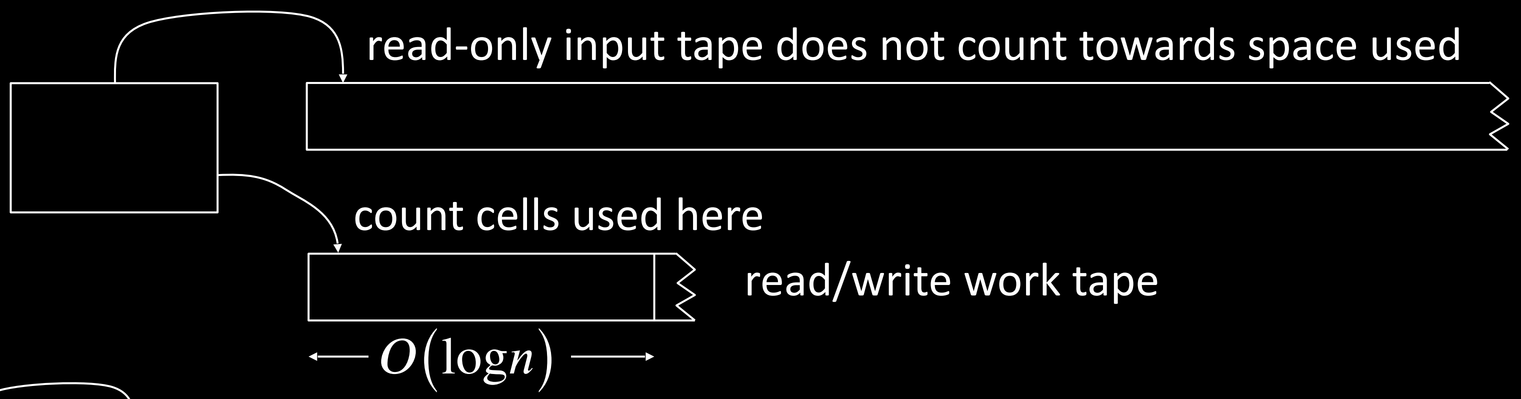

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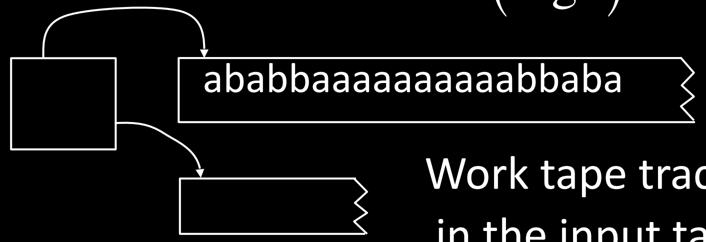

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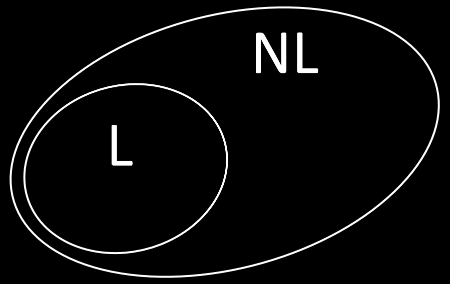

L = NL? Unsolved

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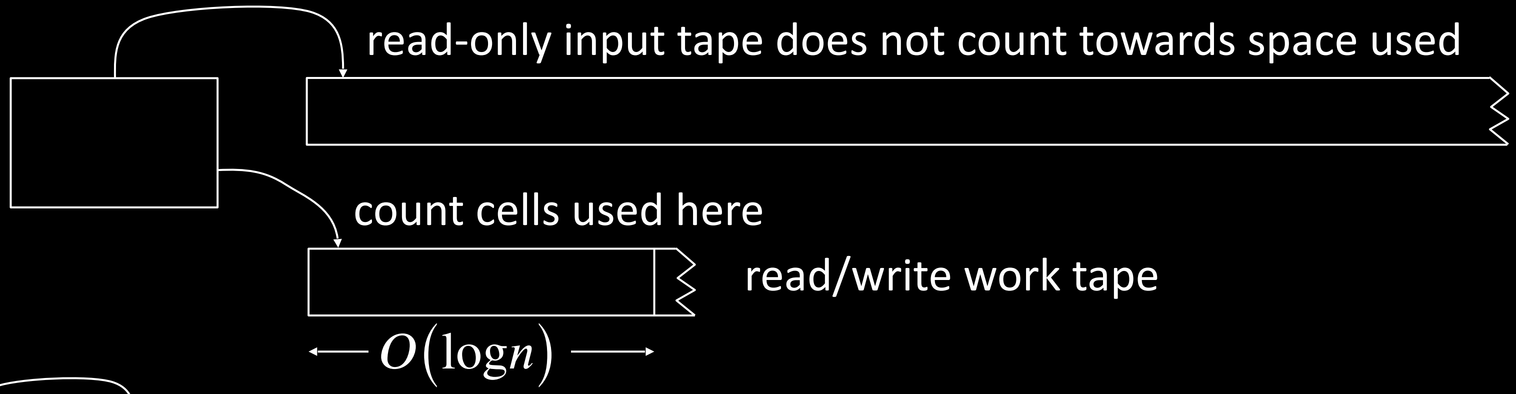

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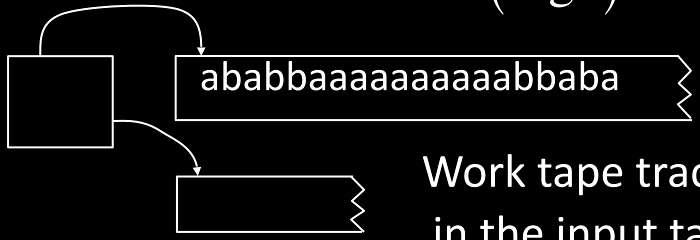

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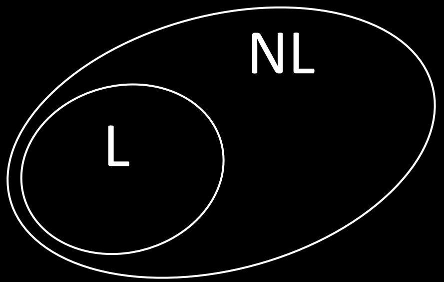

L = NL? Unsolved

Log space properties

Theorem: $L \subseteq P$

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Proof: Say M decides A in space $O(\log n)$.

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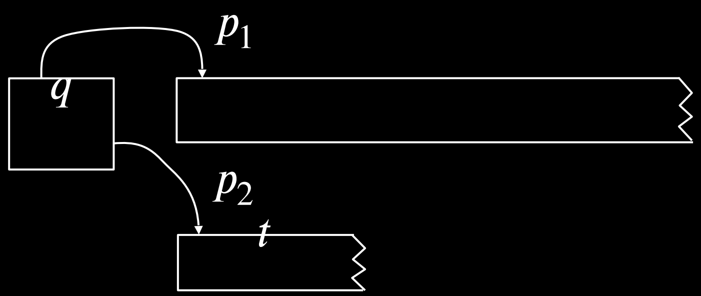

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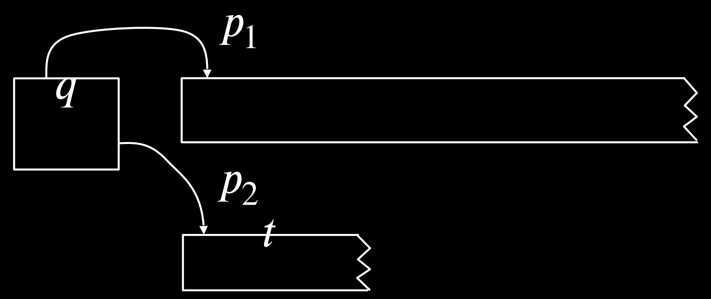

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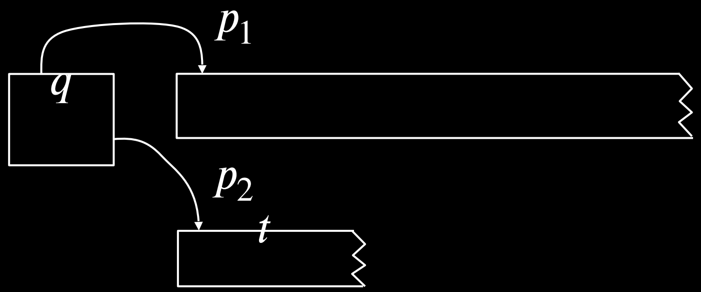
Therefore M runs in polynomial time.



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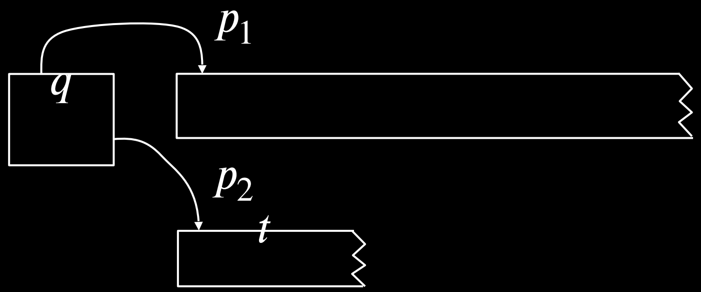

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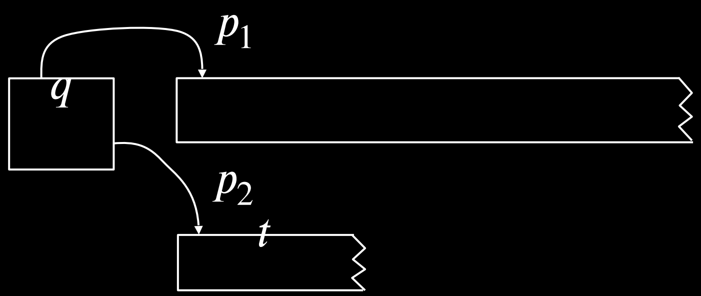

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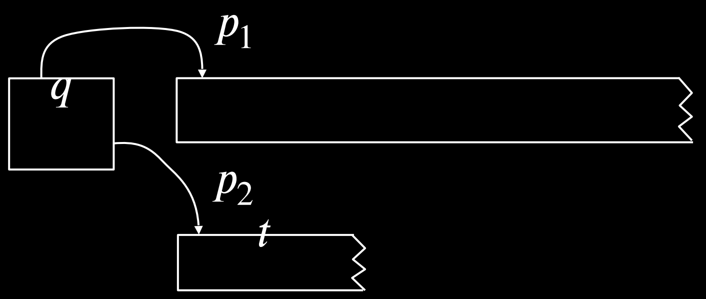

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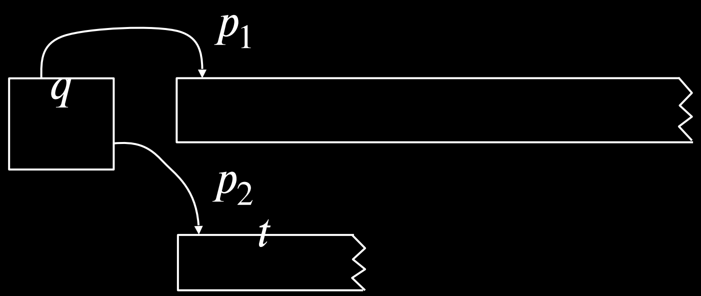

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Defn: The configuration graph $G_{M,w}$ for M on w has

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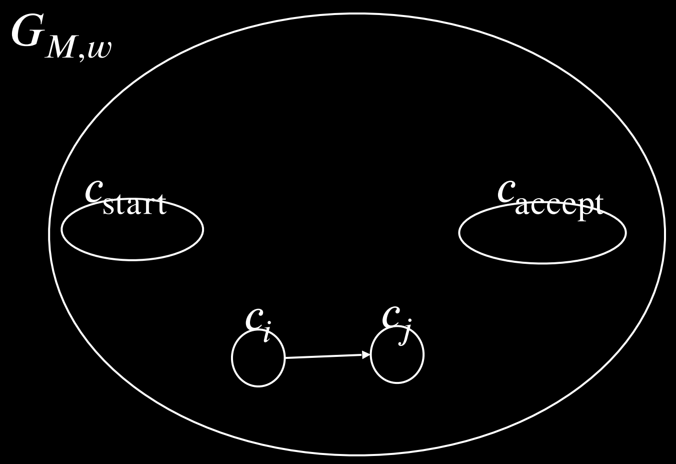

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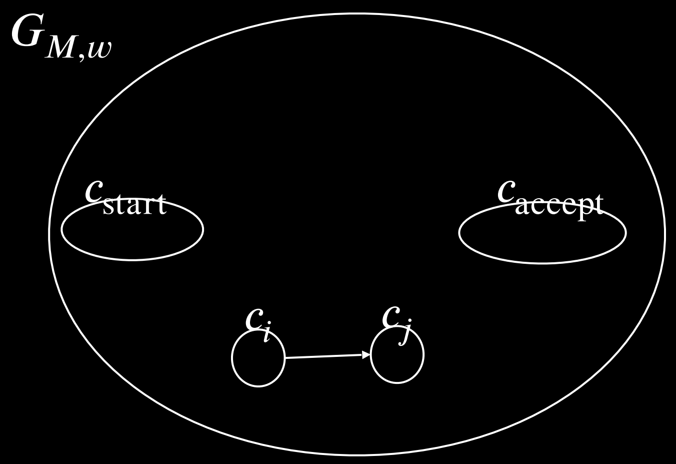
Claim: M accepts w iff the configuration graph $G_{M,w}$ has a path from c_{start} to c_{accept}



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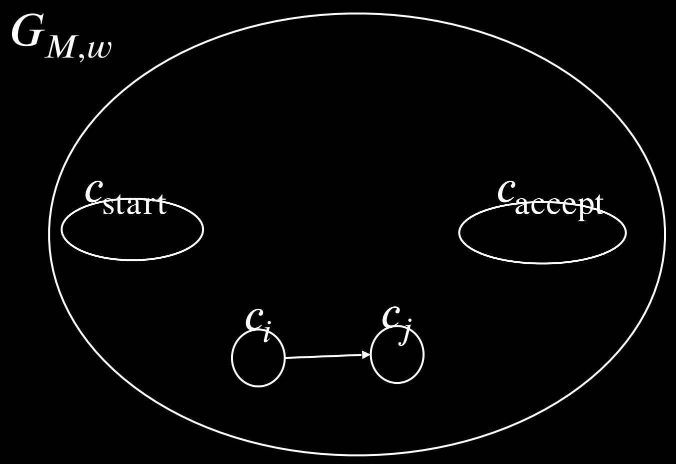
Polynomial time algorithm T for A:



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Claim: M accepts w iff the configuration graph $G_{M,w}$ has a path from c_{start} to c_{accept}

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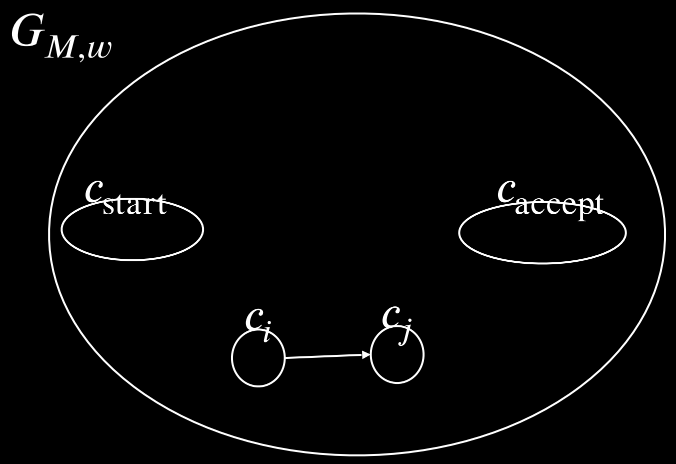

NL properties

Theorem: NL ⊆ P

Proof: Say NTM M decides A in space $O(\log n)$.

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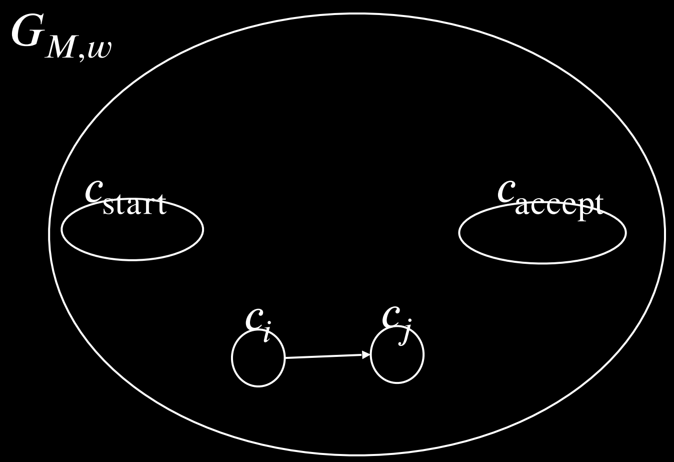

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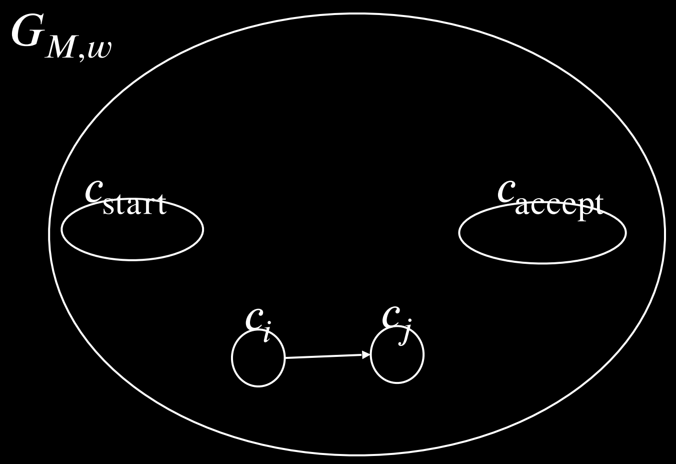

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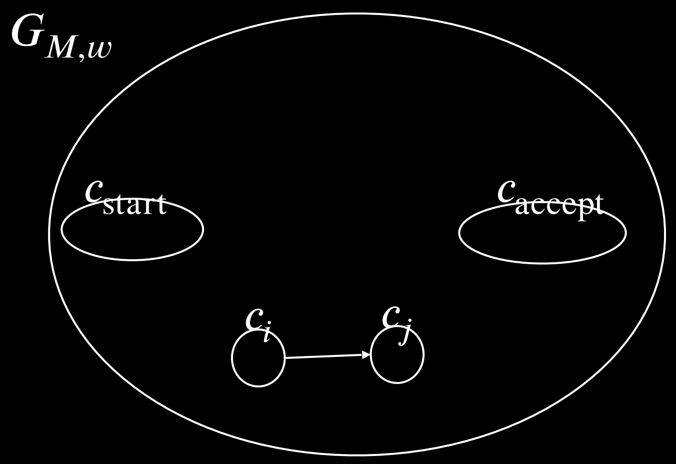

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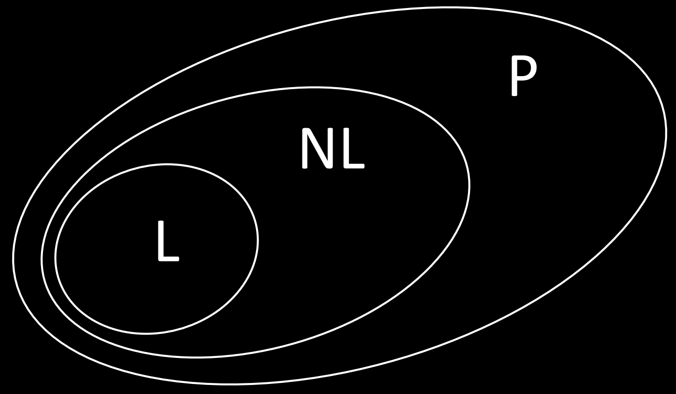

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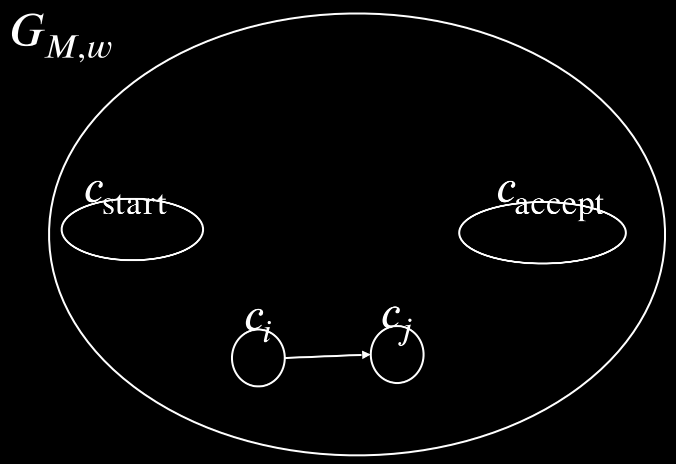
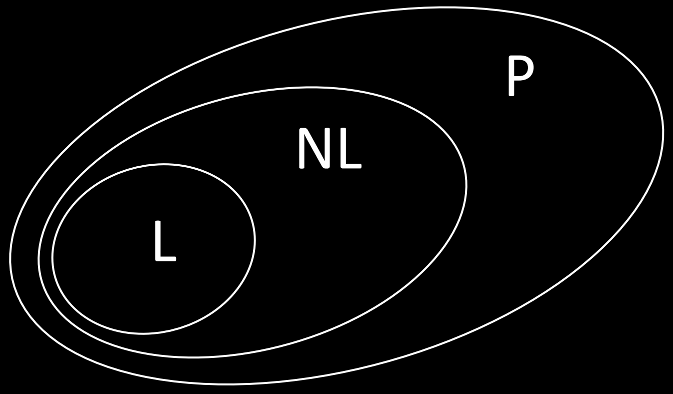


L = P? Unsolved



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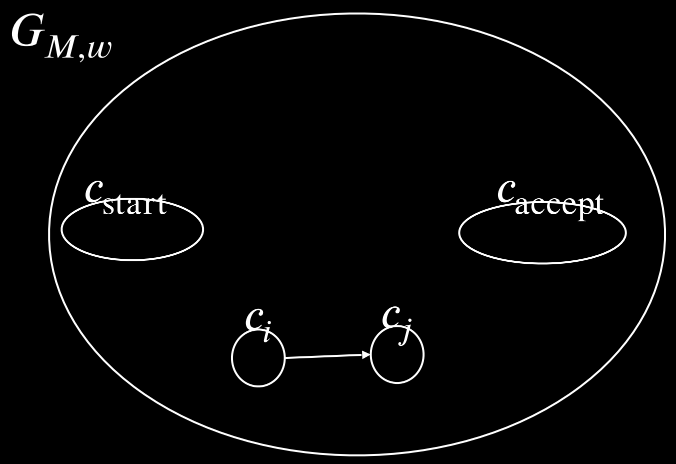

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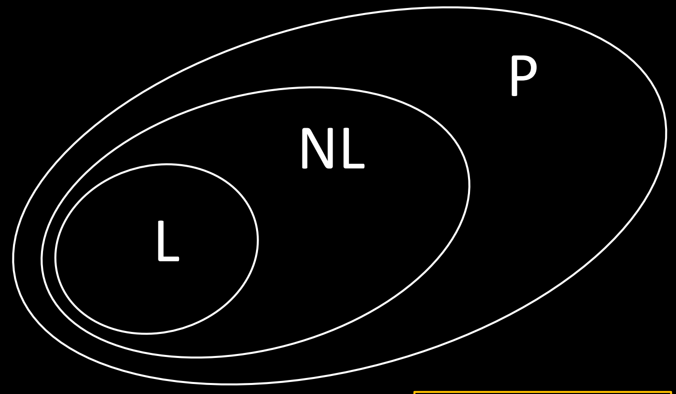

Check-in 19.3

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Check-in 19.3

We showed that $PATH \in$ NL.
What is the best we know about the deterministic space complexity of $PATH$?

(a) $PATH \in$ PSPACE

(b) $PATH \in$ SPACE(n)

(c) $PATH \in$ SPACE$\left(\log^2 n\right)$

(d) $PATH \in$ SPACE$\left(\log n\right)$

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2. Generalized Geography is PSPACE-complete
3. Log space: L and NL
4. Configuration graph
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