بسم الله الرحمن الرحیم

برنامه‌ریزی نیمه‌معین برای طراحی الگوریتم‌های تقریبی

جلسه پنجم: ظرفیت شنون و تتای لواژ (۲)

Project: People

مرور ظرفیت شنون

- اندازه بزرگ‌ترین واژه‌نامه خوب برای ارسال == $\alpha(G^k)$

3.1.2 Lemma. *For all* $k, \ell \in \mathbb{N}$,

$$\alpha(G^{k+\ell}) \geq \alpha(G^k)\alpha(G^\ell).$$

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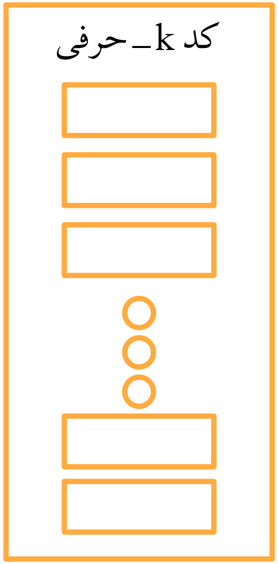

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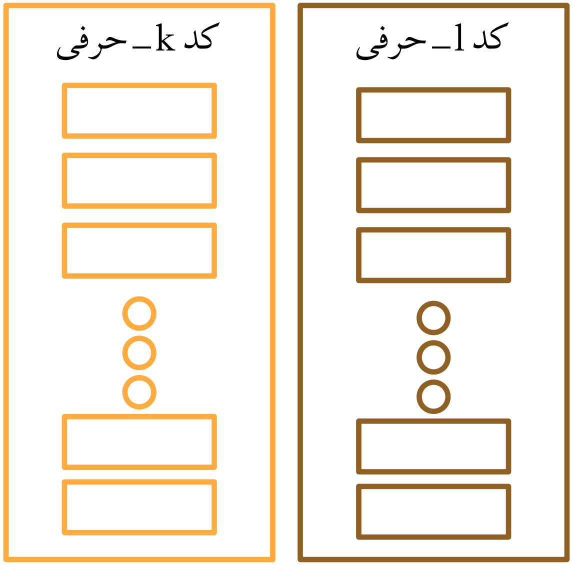

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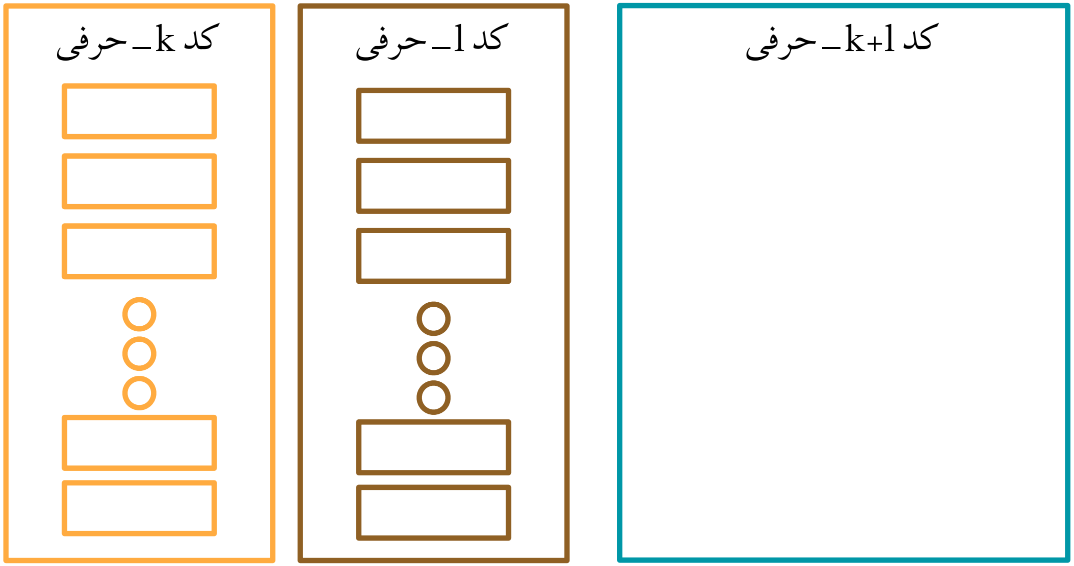

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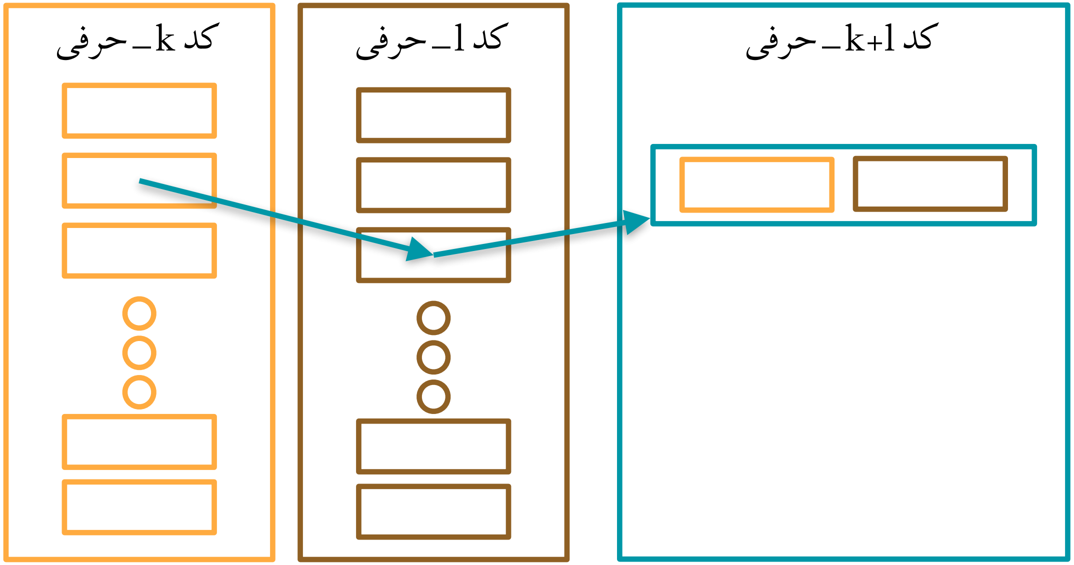

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$$\sigma(C_5)\geq\frac{1}{2}\log 5,\qquad \alpha(C_5^2)\geq 5$$

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ابرجمعی $$(x_k)_{k\in\mathbb{N}} = (\log\alpha(G^k))_{k\in\mathbb{N}}$$

همگرا به سوپریمم $$\left(\frac{x_k}{k}\right)_{k\in\mathbb{N}}$$

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تابع ϑ

نمایش متعامد یکه برای گراف

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$\vartheta(G) = n$ $\quad$ $\mathbf{c} = \sum_{i=1}^{n} \mathbf{u}_i / \sqrt{n}$ $\quad$ $\mathbf{u}_i = \mathbf{e}_i$

کران بالا برای ظرفیت شنون گراف

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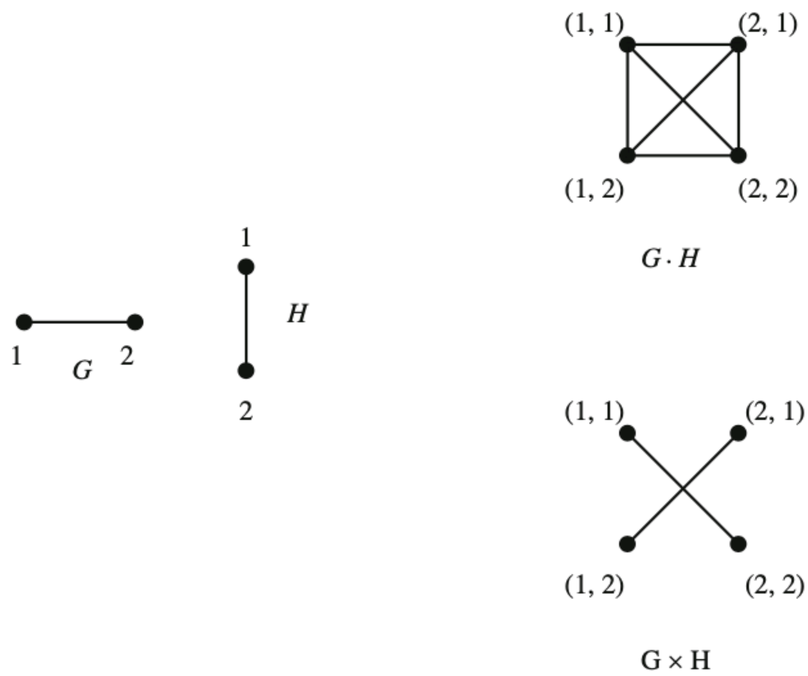
$$\Theta(G) = 2^{\sigma(G)} = \lim_{k\to\infty} \sqrt[k]{\alpha(G^k)}$$

3.4.1 Definition. *Let $G = (V, E)$ and $H = (W, F)$ be graphs. The strong product of G and H is the graph $G \cdot H$ with vertex set $V \times W$, and an edge between (v, w) and (v', w') if v is similar to v' in G and w is similar to w' in H.*



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$$(\mathbf{x} \otimes \mathbf{y})^T(\mathbf{x}' \otimes \mathbf{y}') = \sum_{i=1}^{m}\sum_{j=1}^{n} x_iy_jx_i'y_j'$$

3.4.2 Lemma. *For all graphs G and H,*

$$\vartheta(G \cdot H) \leq \vartheta(G)\vartheta(H).$$

$$G = (V, E)$$

$$\mathcal{U} = (\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_m)$$ و $\mathbf{c}$

$$H = (W, F)$$

$$\mathcal{V} = (\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n)$$ و $\mathbf{d}$

$$G \cdot H$$

$\mathcal{U} \otimes \mathcal{V}$ و $\mathbf{c} \otimes \mathbf{d}$

$$\mathbf{x} \otimes \mathbf{y} = (x_1 y_1, \ldots, x_1 y_n, x_2 y_1, \ldots, x_2 y_n, \ldots, x_m y_1, \ldots, x_m y_n)$$

$$(\mathbf{x} \otimes \mathbf{y})^T (\mathbf{x}' \otimes \mathbf{y}') = \sum_{i=1}^{m} \sum_{j=1}^{n} x_i y_j x'_i y'_j = \sum_{i=1}^{m} x_i x'_i \sum_{j=1}^{n} y_j y'_j$$

3.4.2 Lemma. *For all graphs G and H,*

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$$G \cdot H$$

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$$\mathbf{x} \otimes \mathbf{y} = (x_1y_1, \ldots, x_1y_n, x_2y_1, \ldots, x_2y_n, \ldots, x_my_1, \ldots, x_my_n)$$

$$(\mathbf{x} \otimes \mathbf{y})^T(\mathbf{x}' \otimes \mathbf{y}') = \sum_{i=1}^{m}\sum_{j=1}^{n} x_iy_jx'_iy'_j = \sum_{i=1}^{m} x_ix'_i \sum_{j=1}^{n} y_jy'_j = (\mathbf{x}^T\mathbf{x}')(\mathbf{y}^T\mathbf{y}').$$

3.4.2 Lemma. *For all graphs* G *and* H,

$$\vartheta(G \cdot H) \leq \vartheta(G)\vartheta(H).$$

3.4.2 Lemma. *For all graphs G and H,*

$$\vartheta(G \cdot H) \leq \vartheta(G)\vartheta(H).$$

الف) نمایش متعامد یکه از گراف G . H است.

3.4.2 Lemma. *For all graphs G and H,*

$$\vartheta(G \cdot H) \leq \vartheta(G)\vartheta(H).$$

الف) نمایش متعامد یکه از گراف H . G است.

ب) در نامساوی صدق می‌کند $\vartheta(G)\vartheta(H) \leq \vartheta(\mathcal{U} \otimes \mathcal{V})$

3.4.2 Lemma. *For all graphs G and H,*

$$\vartheta(G \cdot H) \leq \vartheta(G)\vartheta(H).$$

الف) نمایش متعامد یکه از گراف G . H است.

$$(i, j) \qquad\qquad (i', j')$$

ب) در نامساوی صدق می‌کند $\vartheta(\mathcal{U} \otimes \mathcal{V}) \leq \vartheta(G)\vartheta(H)$

3.4.2 Lemma. *For all graphs G and H,*

$$\vartheta(G \cdot H) \leq \vartheta(G)\vartheta(H).$$

الف) نمایش متعامد یکه از گراف H . G است.

$$(i, j) \qquad\qquad (i', j')$$

$$(\mathbf{u}_i \otimes \mathbf{v}_j)^T(\mathbf{u}_{i'} \otimes \mathbf{v}_{j'}) = (\mathbf{u}_i^T \mathbf{u}_{i'})(\mathbf{u}_j^T \mathbf{u}_{j'})$$

ب) در نامساوی صدق می‌کند $\vartheta(G)\vartheta(H) \quad \leq \quad \vartheta(\mathcal{U} \otimes \mathcal{V})$

3.4.2 Lemma. *For all graphs G and H,*

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الف) نمایش متعامد یکه از گراف H . G است.

اگر در یکی از گراف‌ها مشابه نباشند

$$(i, j) \qquad (i', j')$$

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$$(i, j) \qquad\qquad (i', j')$$

اگر در یکی از گراف‌ها مشابه نباشند

$$(\mathbf{u}_i \otimes \mathbf{v}_j)^T(\mathbf{u}_{i'} \otimes \mathbf{v}_{j'}) = (\mathbf{u}_i^T \mathbf{u}_{i'})(\mathbf{u}_j^T \mathbf{u}_{j'}) = 0$$

ب) در نامساوی صدق می‌کند $\vartheta(\mathcal{U} \otimes \mathcal{V}) \leq \vartheta(G)\vartheta(H)$

3.4.2 Lemma. *For all graphs G and H,*

$$\vartheta(G \cdot H) \leq \vartheta(G)\vartheta(H).$$

الف) نمایش متعامد یکه از گراف H . G است.

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ب) در نامساوی صدق می‌کند $\vartheta(\mathcal{U} \otimes \mathcal{V}) \leq \vartheta(G)\vartheta(H)$

$$\vartheta(\mathcal{U}) := \min_{\|\mathbf{c}\|=1} \max_{i=1}^{n} \frac{1}{(\mathbf{c}^T \mathbf{u}_i)^2}$$

$$\vartheta(\mathcal{U} \otimes \mathcal{V}) \leq$$

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$$\vartheta(\mathcal{U} \otimes \mathcal{V}) \leq \max_{i \in V, j \in W} \frac{1}{((\mathbf{c} \otimes \mathbf{d})^T (\mathbf{u}_i \otimes \mathbf{v}_j))^2}$$

3.4.2 Lemma. *For all graphs G and H,*

$$\vartheta(G \cdot H) \leq \vartheta(G)\vartheta(H).$$

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ب) در نامساوی صدق می‌کند $\vartheta(\mathcal{U} \otimes \mathcal{V}) \leq \vartheta(G)\vartheta(H)$

$$\vartheta(\mathcal{U}) := \min_{\|\mathbf{c}\|=1} \max_{i=1}^{n} \frac{1}{(\mathbf{c}^T \mathbf{u}_i)^2}$$

$$\begin{aligned}\vartheta(\mathcal{U} \otimes \mathcal{V}) &\leq \max_{i \in V, j \in W} \frac{1}{((\mathbf{c} \otimes \mathbf{d})^T (\mathbf{u}_i \otimes \mathbf{v}_j))^2} \\ &= \max_{i \in V, j \in W} \frac{1}{(\mathbf{c}^T \mathbf{u}_i)^2 (\mathbf{d}^T \mathbf{v}_j)^2}\end{aligned}$$

3.4.2 Lemma. *For all graphs G and H,*

$$\vartheta(G \cdot H) \leq \vartheta(G)\vartheta(H).$$

الف) نمایش متعامد یکه از گراف G . H است.

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$$\vartheta(\mathcal{U}) := \min_{\|\mathbf{c}\|=1} \max_{i=1}^{n} \frac{1}{(\mathbf{c}^T \mathbf{u}_i)^2}$$

$$\vartheta(\mathcal{U} \otimes \mathcal{V}) \leq \max_{i \in V, j \in W} \frac{1}{((\mathbf{c} \otimes \mathbf{d})^T (\mathbf{u}_i \otimes \mathbf{v}_j))^2}$$

$$= \max_{i \in V, j \in W} \frac{1}{(\mathbf{c}^T \mathbf{u}_i)^2 (\mathbf{d}^T \mathbf{v}_j)^2} = \vartheta(G)\vartheta(H)$$

3.4.4 Lemma. *For every graph G, $\alpha(G) \leq \vartheta(G)$.*

3.4.4 Lemma. *For every graph G, $\alpha(G) \leq \vartheta(G)$.*

$$\mathbf{c}^T\mathbf{c} \geq \sum_{i \in I} (\mathbf{c}^T\mathbf{u}_i)^2$$

3.4.4 Lemma. *For every graph G, $\alpha(G) \leq \vartheta(G)$.*

$$\mathbf{c}^T\mathbf{c} \geq \sum_{i\in I}(\mathbf{c}^T\mathbf{u}_i)^2 \geq |I| \min_{i\in I}(\mathbf{c}^T\mathbf{u}_i)^2$$

3.4.4 Lemma. *For every graph G, $\alpha(G) \leq \vartheta(G)$.*

$$\mathbf{c}^T\mathbf{c} \geq \sum_{i\in I}(\mathbf{c}^T\mathbf{u}_i)^2 \geq |I|\min_{i\in I}(\mathbf{c}^T\mathbf{u}_i)^2 = \alpha(G)\min_{i\in I}(\mathbf{c}^T\mathbf{u}_i)^2$$

3.4.4 Lemma. *For every graph G, $\alpha(G) \leq \vartheta(G)$.*

$$1 = \mathbf{c}^T\mathbf{c} \geq \sum_{i\in I}(\mathbf{c}^T\mathbf{u}_i)^2 \geq |I| \min_{i\in I}(\mathbf{c}^T\mathbf{u}_i)^2 = \alpha(G) \min_{i\in I}(\mathbf{c}^T\mathbf{u}_i)^2$$

3.4.4 Lemma. *For every graph G, $\alpha(G) \leq \vartheta(G)$.*

$$1 = \mathbf{c}^T\mathbf{c} \geq \sum_{i \in I} (\mathbf{c}^T\mathbf{u}_i)^2 \geq |I| \min_{i \in I} (\mathbf{c}^T\mathbf{u}_i)^2 = \alpha(G) \min_{i \in I} (\mathbf{c}^T\mathbf{u}_i)^2$$

$$\alpha(G) \leq \frac{1}{\min_{i \in I} (\mathbf{c}^T\mathbf{u}_i)^2}$$

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$$1 = \mathbf{c}^T\mathbf{c} \geq \sum_{i \in I} (\mathbf{c}^T\mathbf{u}_i)^2 \geq |I| \min_{i \in I} (\mathbf{c}^T\mathbf{u}_i)^2 = \alpha(G) \min_{i \in I} (\mathbf{c}^T\mathbf{u}_i)^2$$

$$\alpha(G) \leq \frac{1}{\min_{i \in I} (\mathbf{c}^T\mathbf{u}_i)^2} = \max_{i \in I} \frac{1}{(\mathbf{c}^T\mathbf{u}_i)^2}$$

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$$1 = \mathbf{c}^T\mathbf{c} \geq \sum_{i \in I}(\mathbf{c}^T\mathbf{u}_i)^2 \geq |I| \min_{i \in I}(\mathbf{c}^T\mathbf{u}_i)^2 = \alpha(G) \min_{i \in I}(\mathbf{c}^T\mathbf{u}_i)^2$$

$$\alpha(G) \leq \frac{1}{\min_{i \in I}(\mathbf{c}^T\mathbf{u}_i)^2} = \max_{i \in I} \frac{1}{(\mathbf{c}^T\mathbf{u}_i)^2} \leq \max_{i \in V} \frac{1}{(\mathbf{c}^T\mathbf{u}_i)^2}$$

3.4.4 Lemma. *For every graph G, $\alpha(G) \leq \vartheta(G)$.*

$$1 = \mathbf{c}^T\mathbf{c} \geq \sum_{i\in I}(\mathbf{c}^T\mathbf{u}_i)^2 \geq |I| \min_{i\in I}(\mathbf{c}^T\mathbf{u}_i)^2 = \alpha(G) \min_{i\in I}(\mathbf{c}^T\mathbf{u}_i)^2$$

$$\alpha(G) \leq \frac{1}{\min_{i\in I}(\mathbf{c}^T\mathbf{u}_i)^2} = \max_{i\in I} \frac{1}{(\mathbf{c}^T\mathbf{u}_i)^2} \leq \max_{i\in V} \frac{1}{(\mathbf{c}^T\mathbf{u}_i)^2} = \vartheta(G)$$

3.4.5 Theorem (Lovász' bound). *For every graph G, $\Theta(G) \leq \vartheta(G)$.*

3.4.2 Lemma. *For all graphs G and H,*

$$\vartheta(G \cdot H) \leq \vartheta(G)\vartheta(H).$$

$$\lim_{k\to\infty} \sqrt[k]{\alpha(G^k)} \leq \lim_{k\to\infty} \sqrt[k]{\vartheta(G^k)} \leq \lim_{k\to\infty} \sqrt[k]{\vartheta(G)^k} \leq \vartheta(G)$$

3.4.4 Lemma. *For every graph G, $\alpha(G) \leq \vartheta(G)$.*

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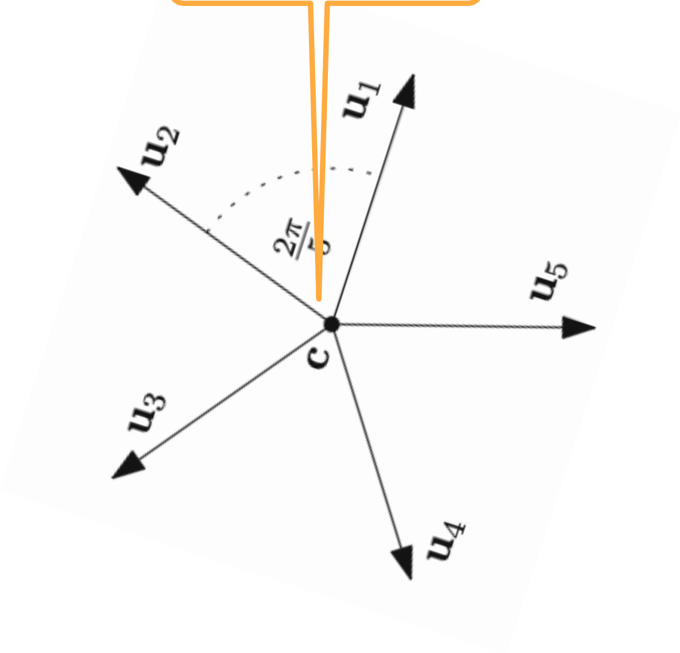
3.4.4 Lemma. *For every graph G, $\alpha(G) \leq \vartheta(G)$.*

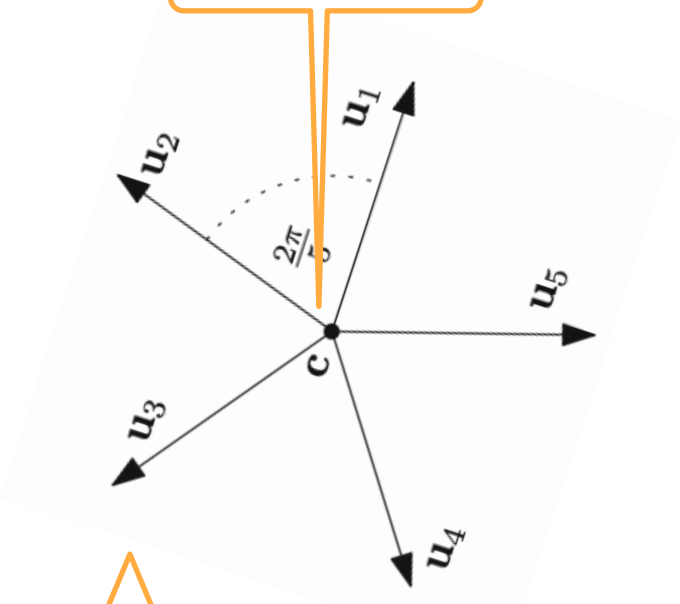
دور ۵ راسی

$$\Theta(C_5) = \sqrt{5}.$$

$$\mathbf{c} = (0, 0, 1)$$

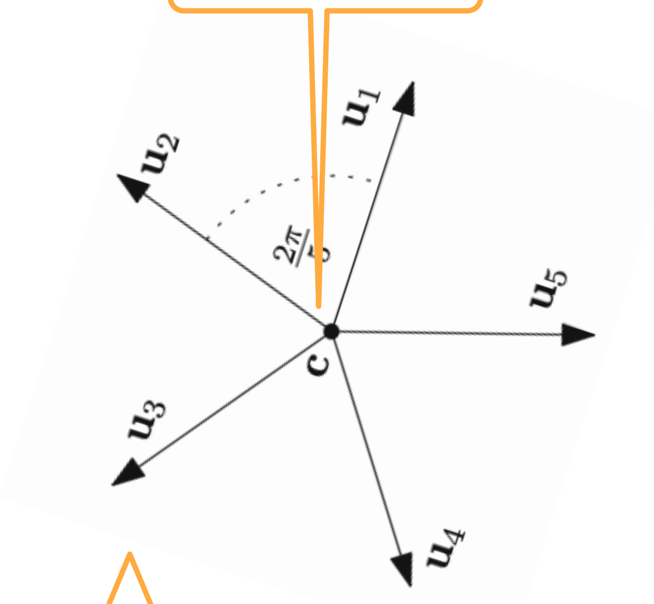

$\mathbf{u}_1$
$\mathbf{u}_2$
$\mathbf{u}_3$
$\mathbf{u}_4$
$\mathbf{u}_5$
$\mathbf{c}$
$\frac{2\pi}{5}$

$$\mathbf{c} = (0, 0, 1)$$



$$\mathbf{u}_i = \frac{(\cos\frac{2\pi i}{5}, \sin\frac{2\pi i}{5}, z)}{\|(\cos\frac{2\pi i}{5}, \sin\frac{2\pi i}{5}, z)\|}$$

$$\mathbf{c} = (0, 0, 1)$$



$$\mathbf{u}_i = \frac{(\cos\frac{2\pi i}{5}, \sin\frac{2\pi i}{5}, z)}{\|(\cos\frac{2\pi i}{5}, \sin\frac{2\pi i}{5}, z)\|}$$

نمایش متعامد یکه از گراف است!

$$\mathbf{u}_i = \frac{(\cos\frac{2\pi i}{5}, \sin\frac{2\pi i}{5}, z)}{\|(\cos\frac{2\pi i}{5}, \sin\frac{2\pi i}{5}, z)\|}$$

$$0 = \mathbf{u}_5^T \mathbf{u}_2$$

نمایش متعامد یکه از گراف است!

$$\mathbf{u}_i = \frac{(\cos\frac{2\pi i}{5}, \sin\frac{2\pi i}{5}, z)}{\|(\cos\frac{2\pi i}{5}, \sin\frac{2\pi i}{5}, z)\|}$$

$$0 = \mathbf{u}_5^T \mathbf{u}_2 \quad \Leftrightarrow \quad (1, 0, z)\begin{pmatrix} \cos\frac{4\pi}{5} \\ \sin\frac{4\pi}{5} \\ z \end{pmatrix}$$

نمایش متعامد یکه از گراف است!

$$\mathbf{u}_i = \frac{(\cos\frac{2\pi i}{5}, \sin\frac{2\pi i}{5}, z)}{\|(\cos\frac{2\pi i}{5}, \sin\frac{2\pi i}{5}, z)\|}$$

$$0 = \mathbf{u}_5^T \mathbf{u}_2 \quad \Leftrightarrow \quad (1, 0, z) \begin{pmatrix} \cos\frac{4\pi}{5} \\ \sin\frac{4\pi}{5} \\ z \end{pmatrix} = \cos\frac{4\pi}{5} + z^2 = 0$$

نمایش متعامد یکه از گراف است!

$$\mathbf{u}_i = \frac{(\cos\frac{2\pi i}{5}, \sin\frac{2\pi i}{5}, z)}{\|(\cos\frac{2\pi i}{5}, \sin\frac{2\pi i}{5}, z)\|}$$

$$0 = \mathbf{u}_5^T \mathbf{u}_2 \quad \Leftrightarrow \quad (1, 0, z)\begin{pmatrix} \cos\frac{4\pi}{5} \\ \sin\frac{4\pi}{5} \\ z \end{pmatrix} = \cos\frac{4\pi}{5} + z^2 = 0$$

$$z = \sqrt{-\cos\frac{4\pi}{5}}$$

نمایش متعامد یکه از گراف است!

$$\mathbf{u}_i = \frac{(\cos\frac{2\pi i}{5}, \sin\frac{2\pi i}{5}, z)}{\|(\cos\frac{2\pi i}{5}, \sin\frac{2\pi i}{5}, z)\|}$$

$$0 = \mathbf{u}_5^T \mathbf{u}_2 \quad \Leftrightarrow \quad (1, 0, z)\begin{pmatrix} \cos\frac{4\pi}{5} \\ \sin\frac{4\pi}{5} \\ z \end{pmatrix} = \cos\frac{4\pi}{5} + z^2 = 0$$

$$z = \sqrt{-\cos\frac{4\pi}{5}} \qquad \mathbf{u}_5 = \frac{\left(1, 0, \sqrt{-\cos\frac{4\pi}{5}}\right)}{\sqrt{1 - \cos\frac{4\pi}{5}}}$$

$\vartheta(C_5)$

$$\vartheta(C_5) \leq \vartheta(\mathcal{U})$$

$$\vartheta(C_5) \leq \vartheta(\mathcal{U}) \leq \max_{i=1}^{5} \frac{1}{(\mathbf{c}^T \mathbf{u}_i)^2}$$

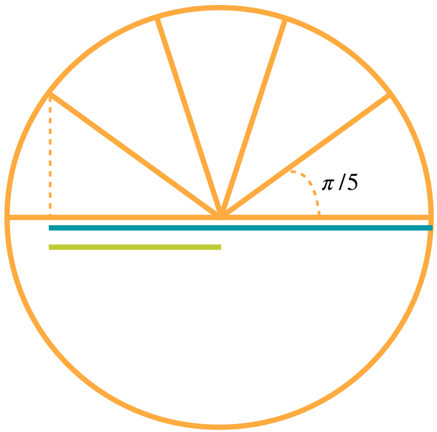
$$\vartheta(C_5) \leq \vartheta(\mathcal{U}) \leq \max_{i=1}^{5} \frac{1}{(\mathbf{c}^T \mathbf{u}_i)^2} = \frac{1}{(\mathbf{c}^T \mathbf{u}_5)^2}$$

$$\vartheta(C_5) \le \vartheta(\mathcal{U}) \le \max_{i=1}^{5} \frac{1}{(\mathbf{c}^T\mathbf{u}_i)^2} = \frac{1}{(\mathbf{c}^T\mathbf{u}_5)^2}$$

$$= \frac{1 - \cos\frac{4\pi}{5}}{-\cos\frac{4\pi}{5}};$$

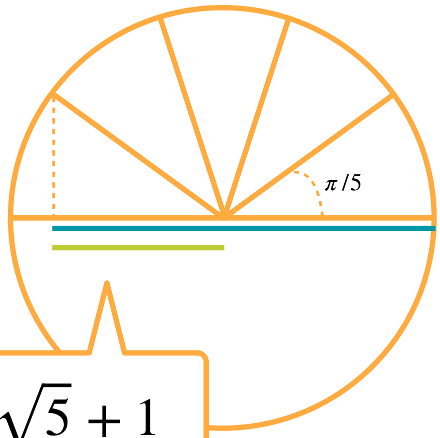
$$\vartheta(C_5) \leq \vartheta(\mathcal{U}) \leq \max_{i=1}^{5} \frac{1}{(\mathbf{c}^T \mathbf{u}_i)^2} = \frac{1}{(\mathbf{c}^T \mathbf{u}_5)^2}$$

$$= \frac{1 - \cos\frac{4\pi}{5}}{- \cos\frac{4\pi}{5}};$$



$$\vartheta(C_5) \le \vartheta(\mathcal{U}) \le \max_{i=1}^{5} \frac{1}{(\mathbf{c}^T\mathbf{u}_i)^2} = \frac{1}{(\mathbf{c}^T\mathbf{u}_5)^2}$$

$$= \frac{1-\cos\frac{4\pi}{5}}{-\cos\frac{4\pi}{5}};$$

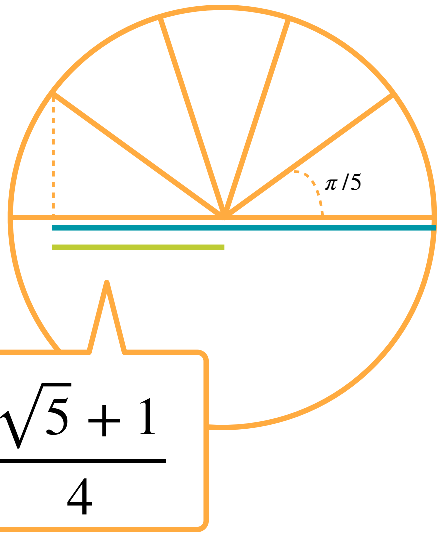


$$\frac{\sqrt{5}+1}{4}$$

$$\vartheta(C_5) \leq \vartheta(\mathcal{U}) \leq \max_{i=1}^{5} \frac{1}{(\mathbf{c}^T \mathbf{u}_i)^2} = \frac{1}{(\mathbf{c}^T \mathbf{u}_5)^2}$$

$$= \frac{1 - \cos\frac{4\pi}{5}}{-\cos\frac{4\pi}{5}}$$

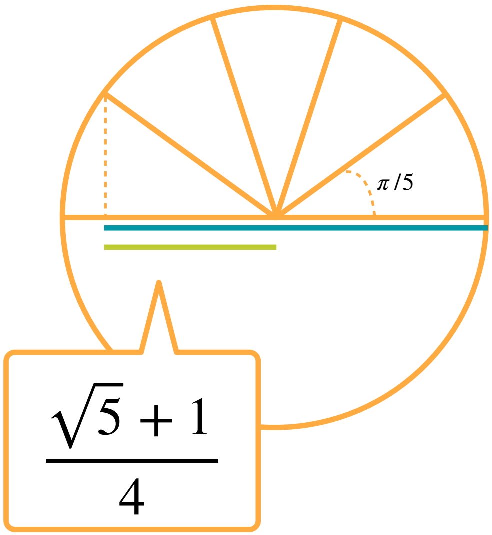
$$= \frac{1 + \frac{\sqrt{5}+1}{4}}{\frac{\sqrt{5}+1}{4}}$$



$$\vartheta(C_5) \le \vartheta(\mathcal{U}) \le \max_{i=1}^{5} \frac{1}{(\mathbf{c}^T \mathbf{u}_i)^2} = \frac{1}{(\mathbf{c}^T \mathbf{u}_5)^2}$$

$$= \frac{1 - \cos\frac{4\pi}{5}}{-\cos\frac{4\pi}{5}}$$

$$= \frac{1 + \frac{\sqrt{5}+1}{4}}{\frac{\sqrt{5}+1}{4}} = \frac{4 + \sqrt{5} + 1}{\sqrt{5} + 1}$$

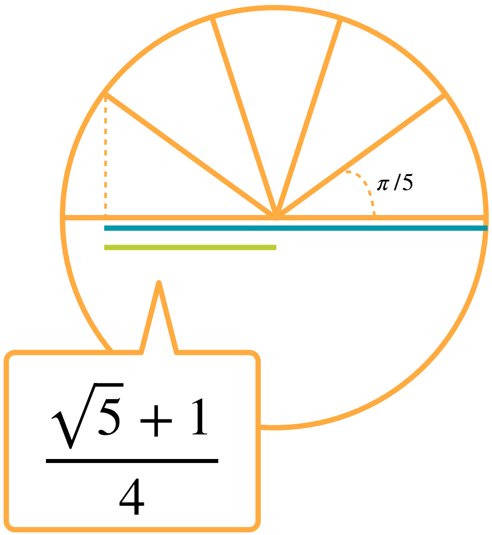


$$\vartheta(C_5) \leq \vartheta(\mathcal{U}) \leq \max_{i=1}^{5} \frac{1}{(\mathbf{c}^T \mathbf{u}_i)^2} = \frac{1}{(\mathbf{c}^T \mathbf{u}_5)^2}$$

$$= \frac{1 - \cos\frac{4\pi}{5}}{-\cos\frac{4\pi}{5}}$$

$$= \frac{1 + \frac{\sqrt{5}+1}{4}}{\frac{\sqrt{5}+1}{4}} = \frac{4 + \sqrt{5} + 1}{\sqrt{5} + 1}$$

$$= \frac{5 + \sqrt{5}}{\sqrt{5} + 1}$$

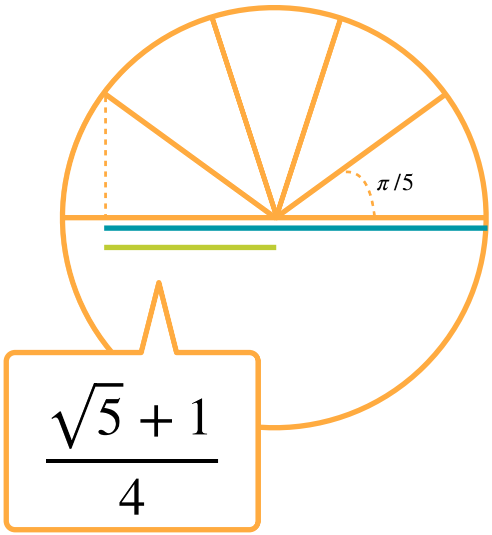


$$\vartheta(C_5) \leq \vartheta(\mathcal{U}) \leq \max_{i=1}^{5} \frac{1}{(\mathbf{c}^T \mathbf{u}_i)^2} = \frac{1}{(\mathbf{c}^T \mathbf{u}_5)^2}$$

$$= \frac{1 - \cos\frac{4\pi}{5}}{-\cos\frac{4\pi}{5}};$$

$$= \frac{1 + \frac{\sqrt{5}+1}{4}}{\frac{\sqrt{5}+1}{4}} = \frac{4 + \sqrt{5} + 1}{\sqrt{5} + 1}$$

$$= \frac{5 + \sqrt{5}}{\sqrt{5} + 1} = \frac{\sqrt{5}(\sqrt{5} + 1)}{\sqrt{5} + 1}$$

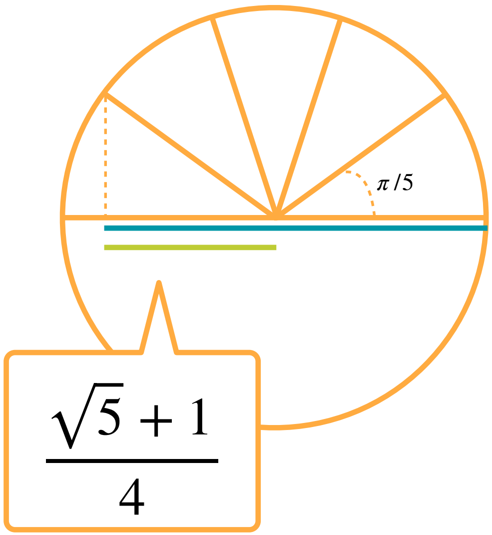


$$\vartheta(C_5) \leq \vartheta(\mathcal{U}) \leq \max_{i=1}^{5} \frac{1}{(\mathbf{c}^T \mathbf{u}_i)^2} = \frac{1}{(\mathbf{c}^T \mathbf{u}_5)^2}$$

$$= \frac{1 - \cos\frac{4\pi}{5}}{-\cos\frac{4\pi}{5}}$$

$$= \frac{1 + \frac{\sqrt{5}+1}{4}}{\frac{\sqrt{5}+1}{4}} = \frac{4 + \sqrt{5} + 1}{\sqrt{5}+1}$$

$$= \frac{5 + \sqrt{5}}{\sqrt{5}+1} = \frac{\sqrt{5}(\sqrt{5}+1)}{\sqrt{5}+1} = \sqrt{5}$$



قضیه:

$$\Theta(G) \leq \vartheta(G)$$

واقعا چند است؟

$$\Theta(C_2) \geq \sqrt{5}$$

$$\sigma(C_5) \geq \frac{1}{2}\log 5,$$

3.5.1 Lemma. $\vartheta(C_5) \leq \sqrt{5}.$

$$\Theta(C_5) = \sqrt{5}.$$

قضیه:

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3.5.1 Lemma. $\vartheta(C_5) \leq \sqrt{5}.$

$$\Theta(C_5) = \sqrt{5}.$$

برنامه‌ریزی نیمه‌معین برای محاسبه ϑ

چگونه $\vartheta(G)$ را محاسبه کنیم؟

- محاسبه $\vartheta(G)$ مفید است:

قضیه:

$$\Theta(G) \leq \vartheta(G)$$

چگونه $\vartheta(G)$ را محاسبه کنیم؟

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- چگونه محاسبه کنیم؟

$$\vartheta(G) := \min_{\mathcal{U}} \min_{\|\mathbf{c}\|=1} \max_{i=1}^{n} \frac{1}{(\mathbf{c}^T \mathbf{u}_i)^2}$$

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$$= \max_{\mathcal{U}} \frac{1}{\sqrt{\vartheta(\mathcal{U})}}$$

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$$\frac{1}{\sqrt{\vartheta(G)}} = \max_{\mathcal{U}} \frac{1}{\sqrt{\vartheta(\mathcal{U})}}$$

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$$\begin{array}{ll} \text{maximize} & t \\ \text{subject to} & \mathbf{u}_i^T \mathbf{u}_j = 0 \text{ for all } \{i,j\} \in \overline{E} \\ & \mathbf{c}^T \mathbf{u}_i \geq t, \;\; i \in V \\ & \|\mathbf{u}_i\| = 1, \;\;\; i \in V \\ & \|\mathbf{c}\| = 1. \end{array}$$

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SDP?

گراف تام

3.7.5 Definition. *A graph G is called perfect if $\omega(G') = \chi(G')$ for every induced subgraph G' of G.*

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الگوریتم محاسبه عدد رنگی و بزرگ‌ترین خوشه

پایان