بسم الله الرحمن الرحیم

نظریه علوم کامپیوتر

نظریه علوم کامپیوتر - بهار ۱۴۰۰-۱۴۰۱ - جلسه دوم: زبان‌های منظم

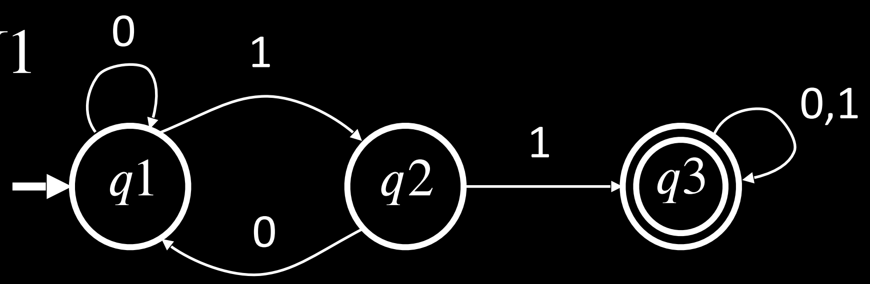
Theory of computation - 002 - S02 - Regular Languanges

نکات صنفی

- در CW عضو شوید

Let's begin: Finite Automata

$M1$



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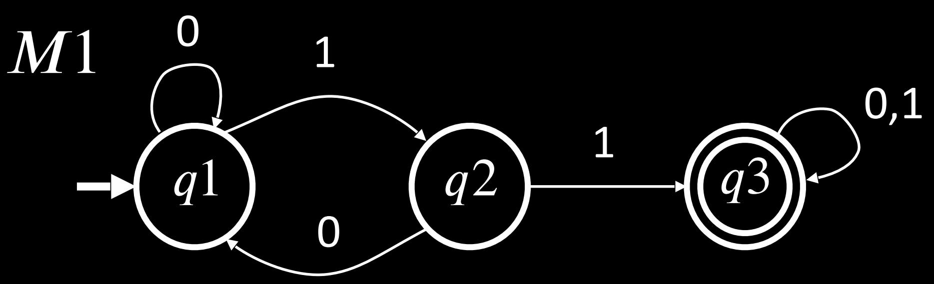

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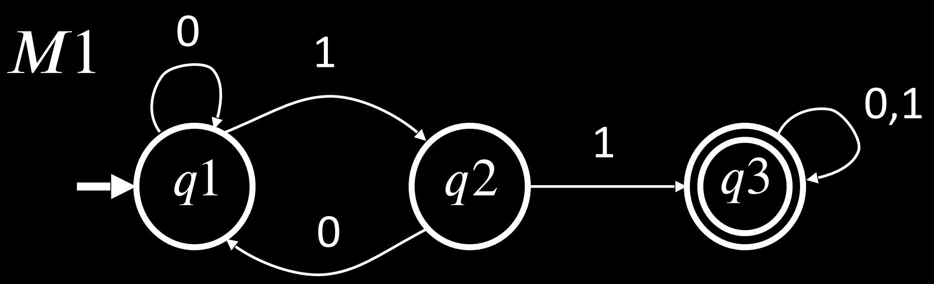

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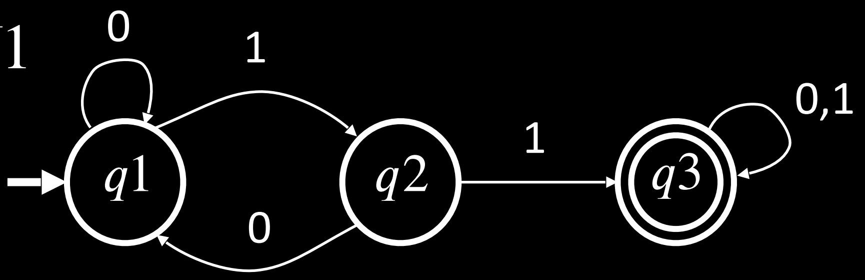

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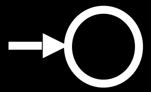

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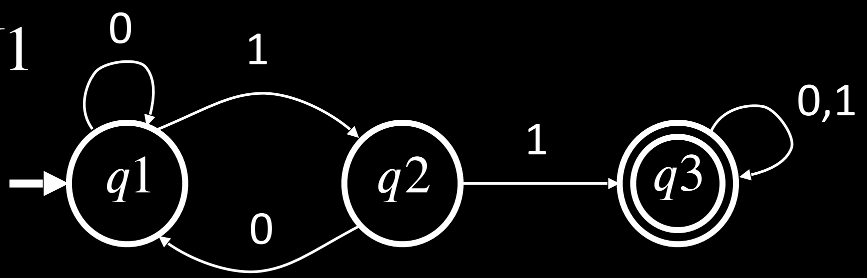

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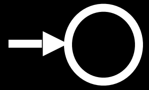
Start state: 

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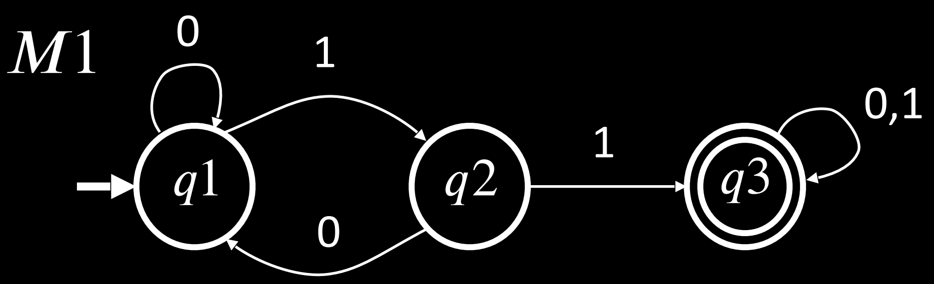

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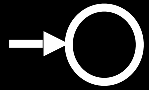
Start state: 

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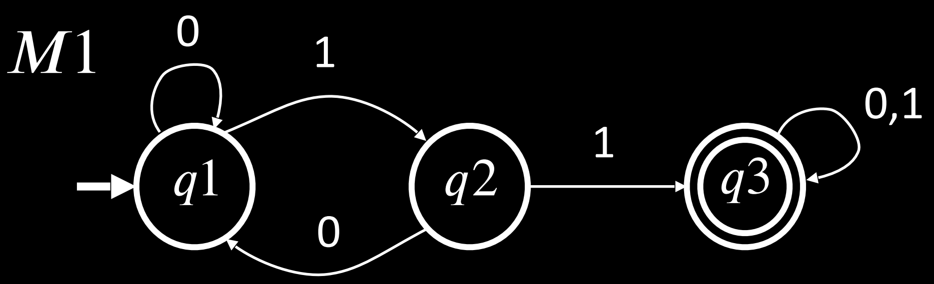

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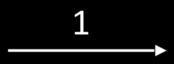
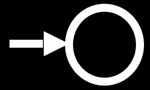
Transitions: 

Start state: 

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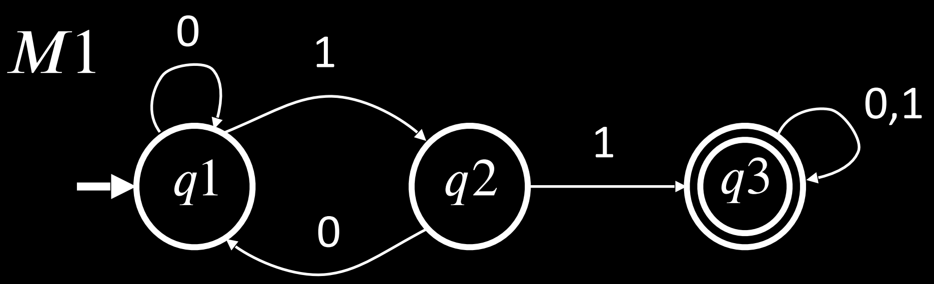

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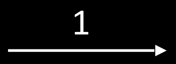
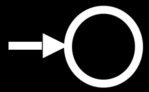
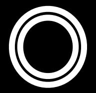
Transitions: 

Start state: 

Accept states: 

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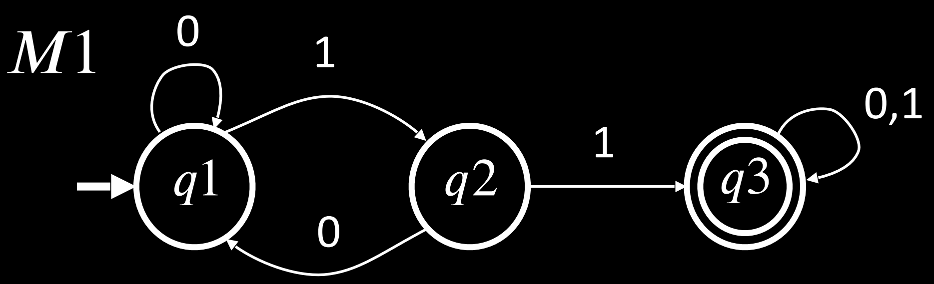

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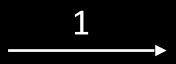
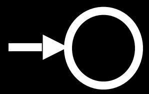
Transitions: 

Start state: 

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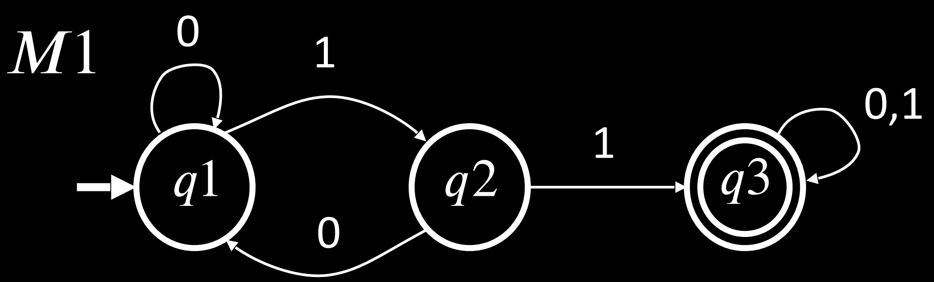

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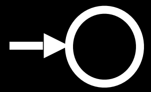
Start state: 

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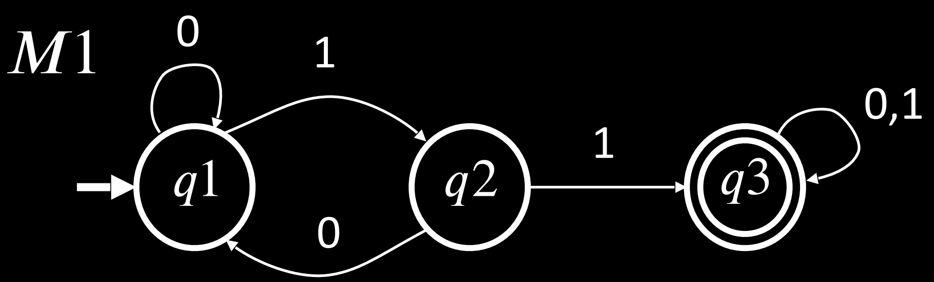

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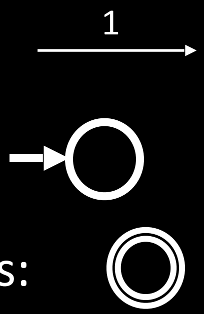

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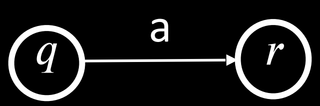
$\delta\,(q,\ \ a)\ =\ \ r$ means 

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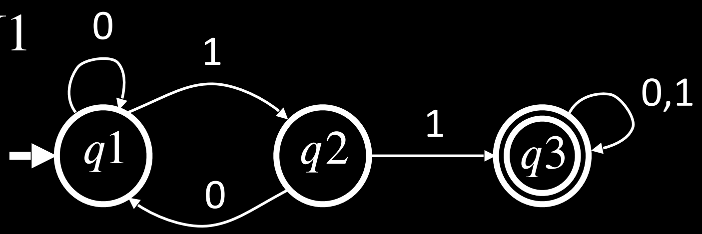

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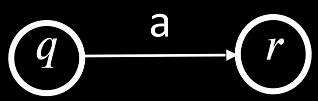
$q0$ start state $\delta\,(q,\ \ a)\ =\ \ r$ means 

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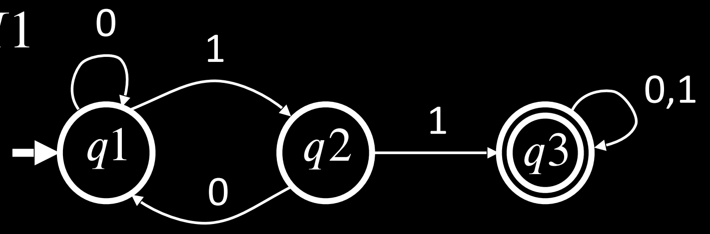

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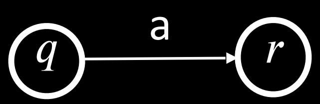
$\delta\,(q,\ a) \ =\ r$ means 

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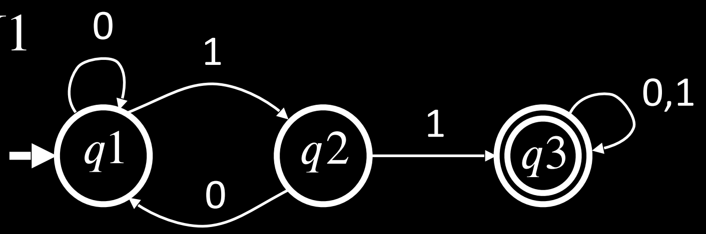

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$\delta =$	0	1
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q2	q1	q3
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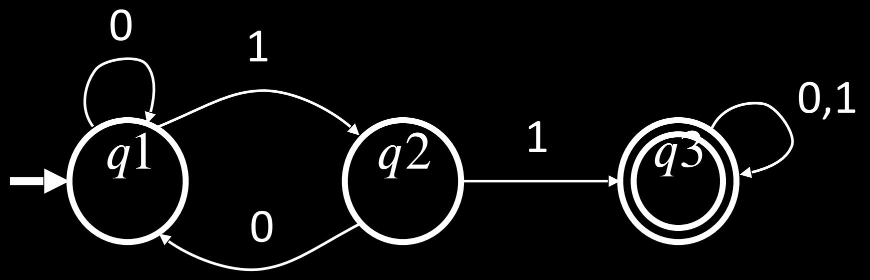

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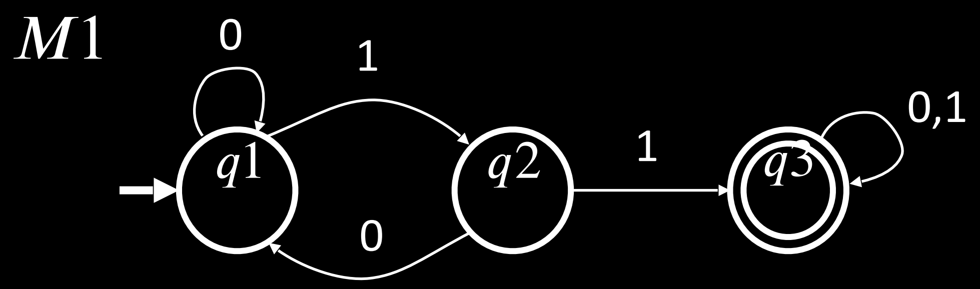

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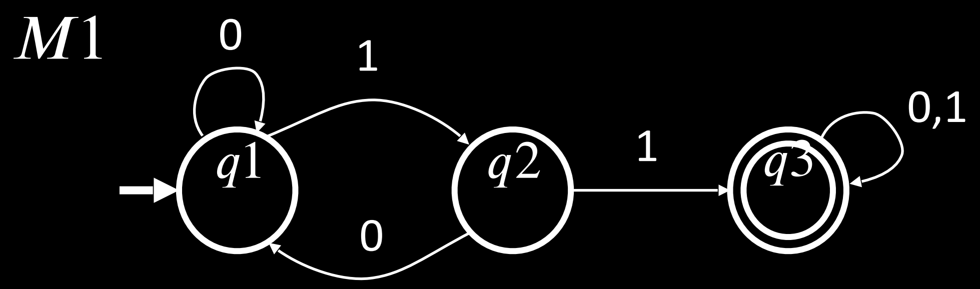

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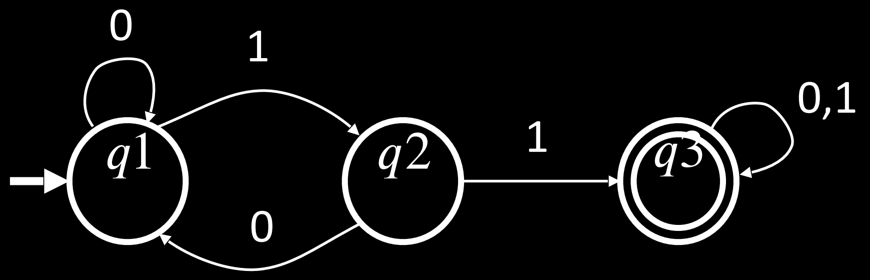

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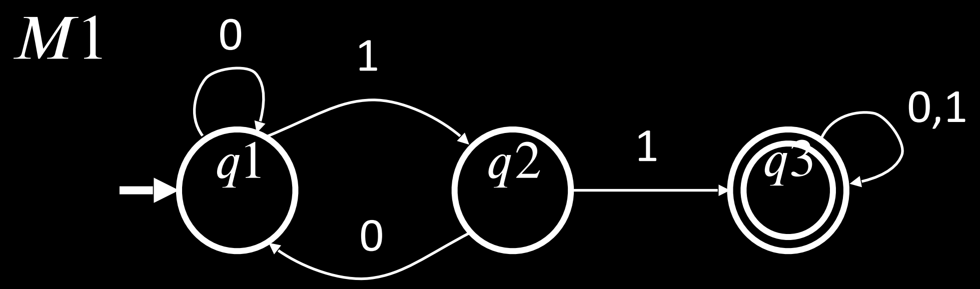

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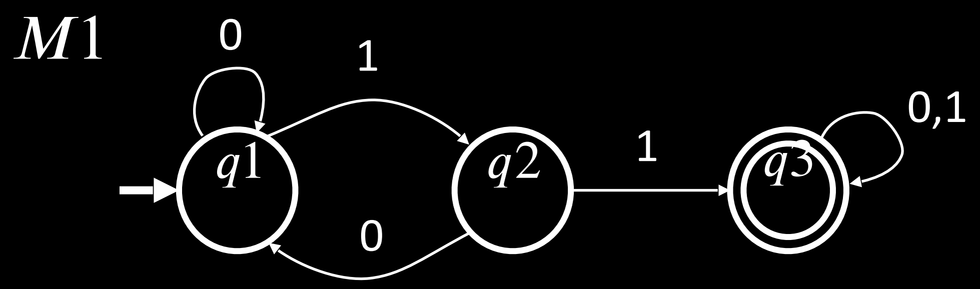

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Goal: Understand the regular languages

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Goal: Show finite automata equivalent to regular expressions

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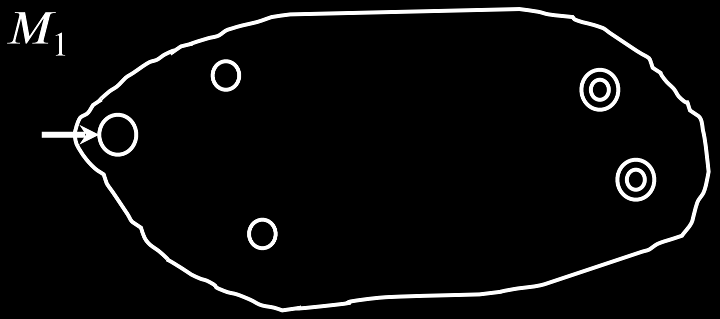

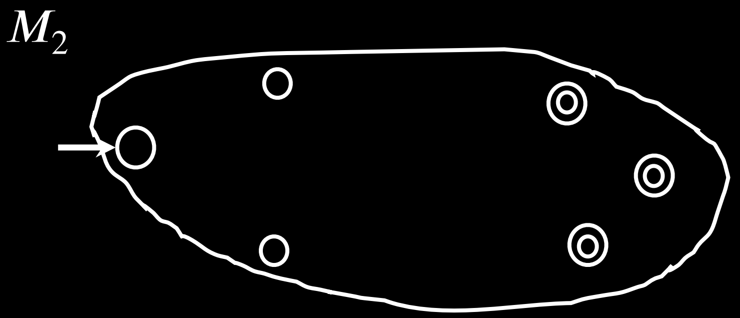

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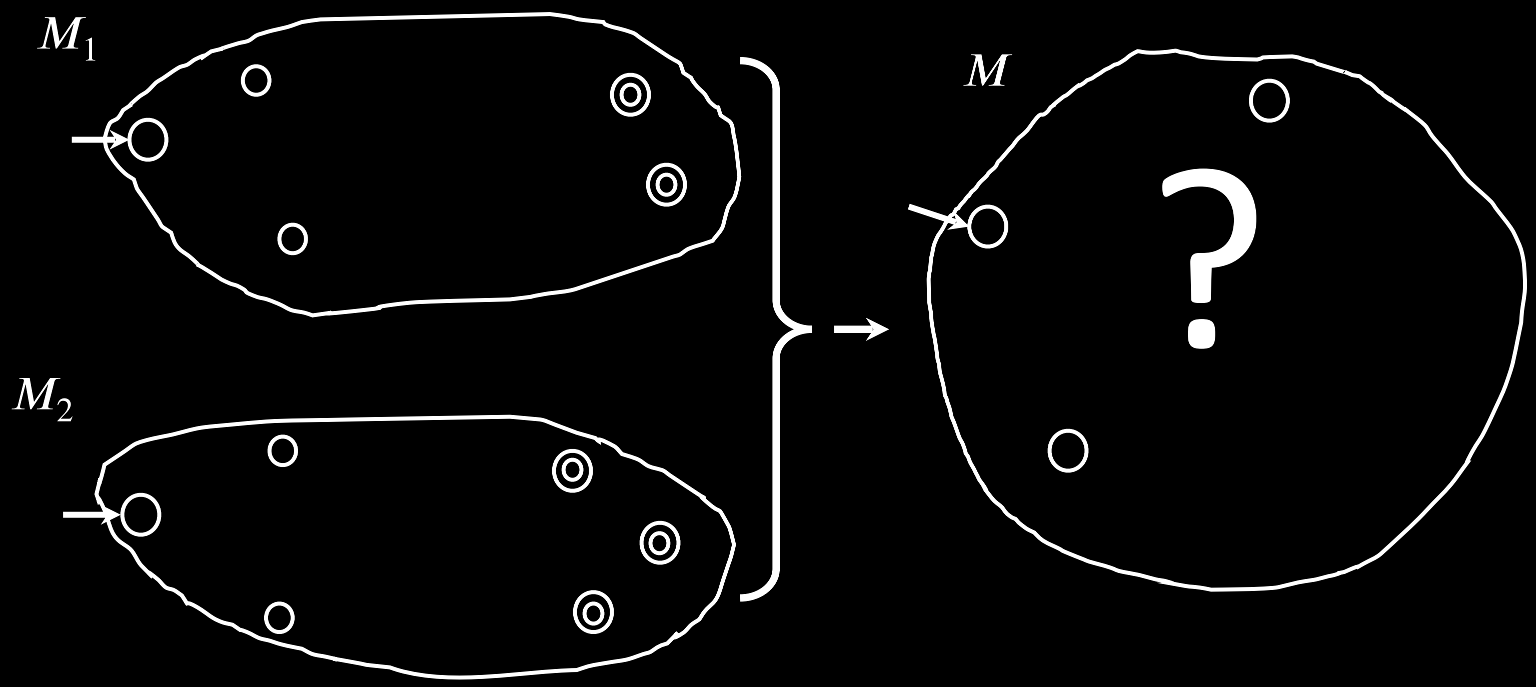

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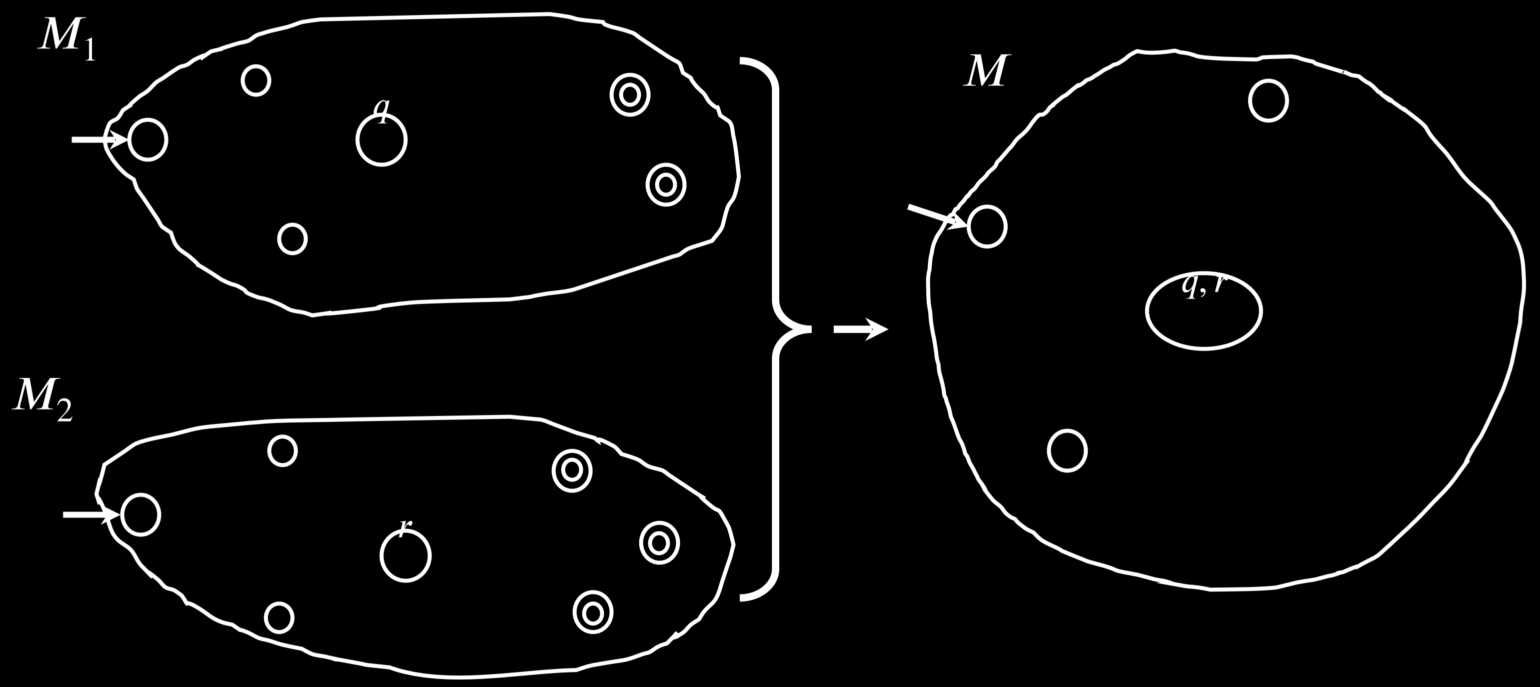

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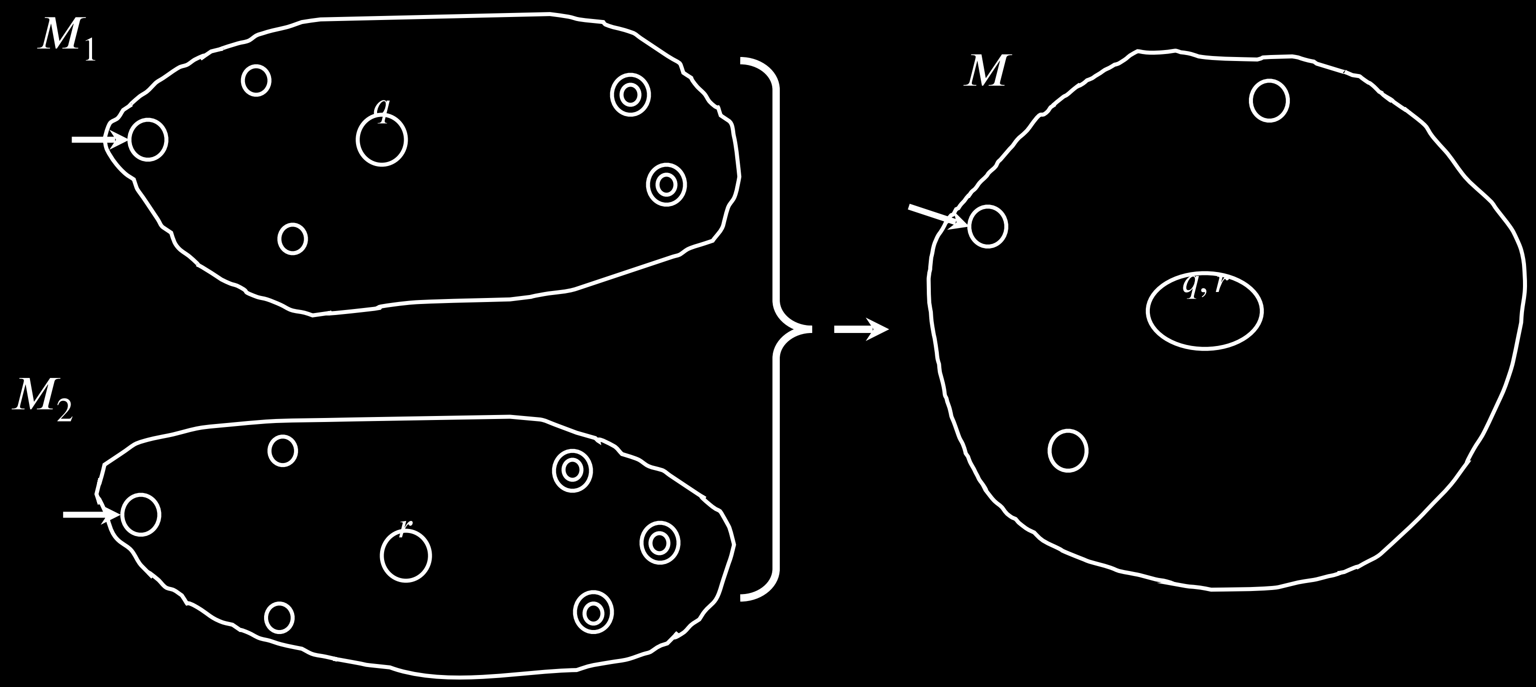

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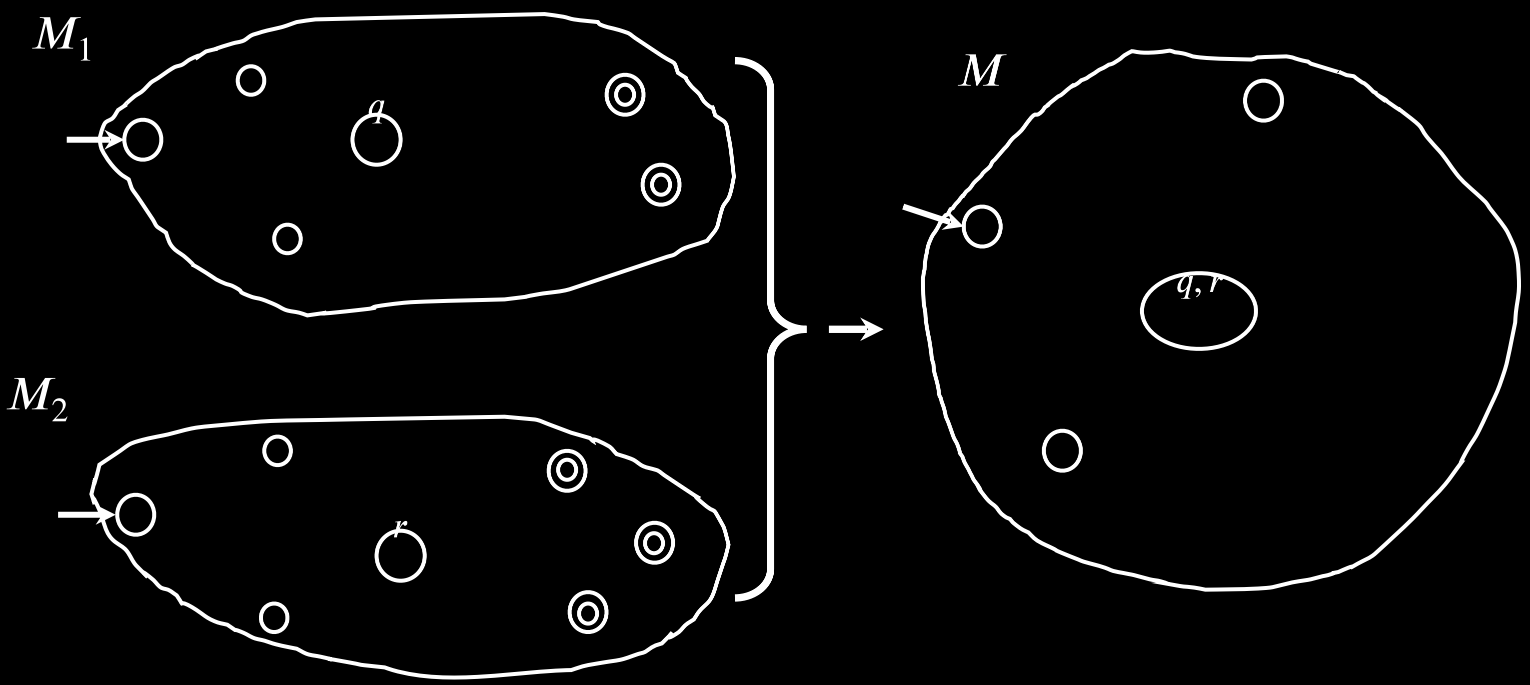

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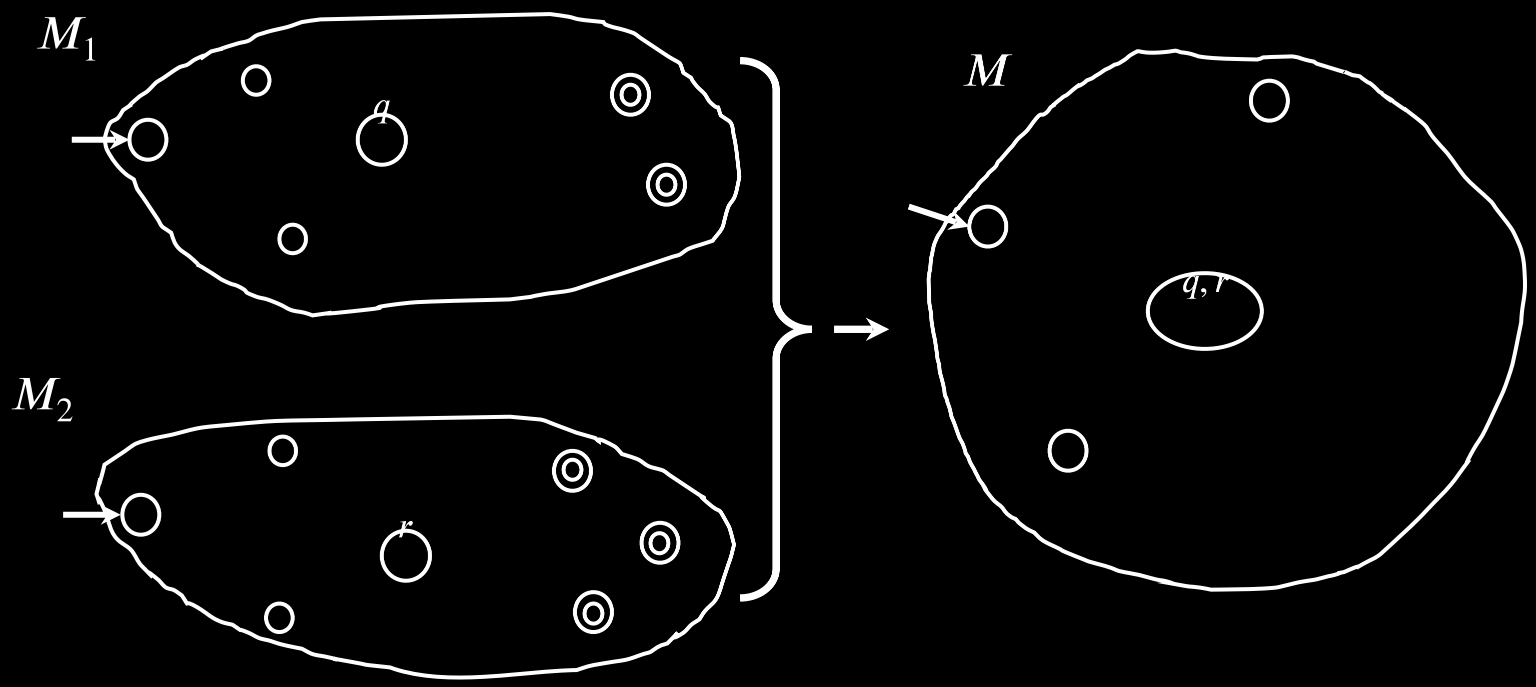

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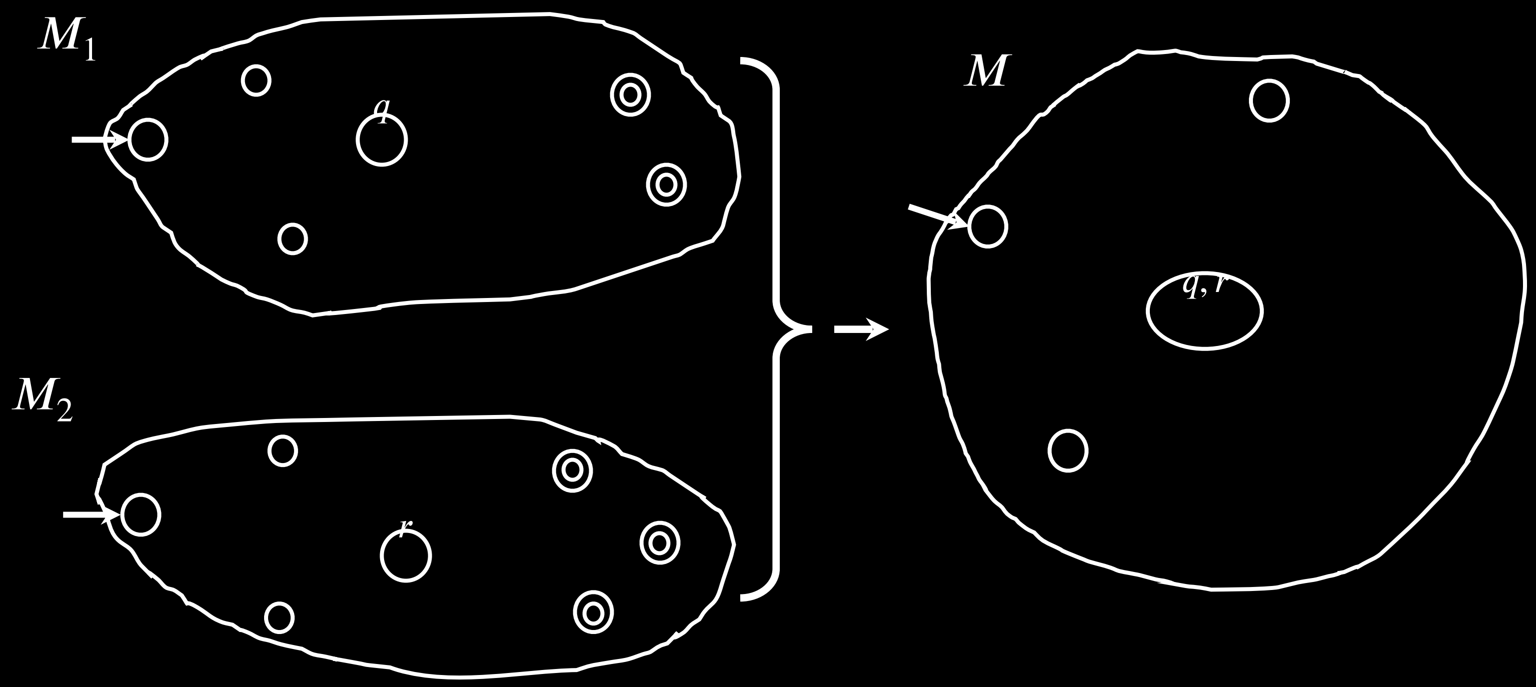

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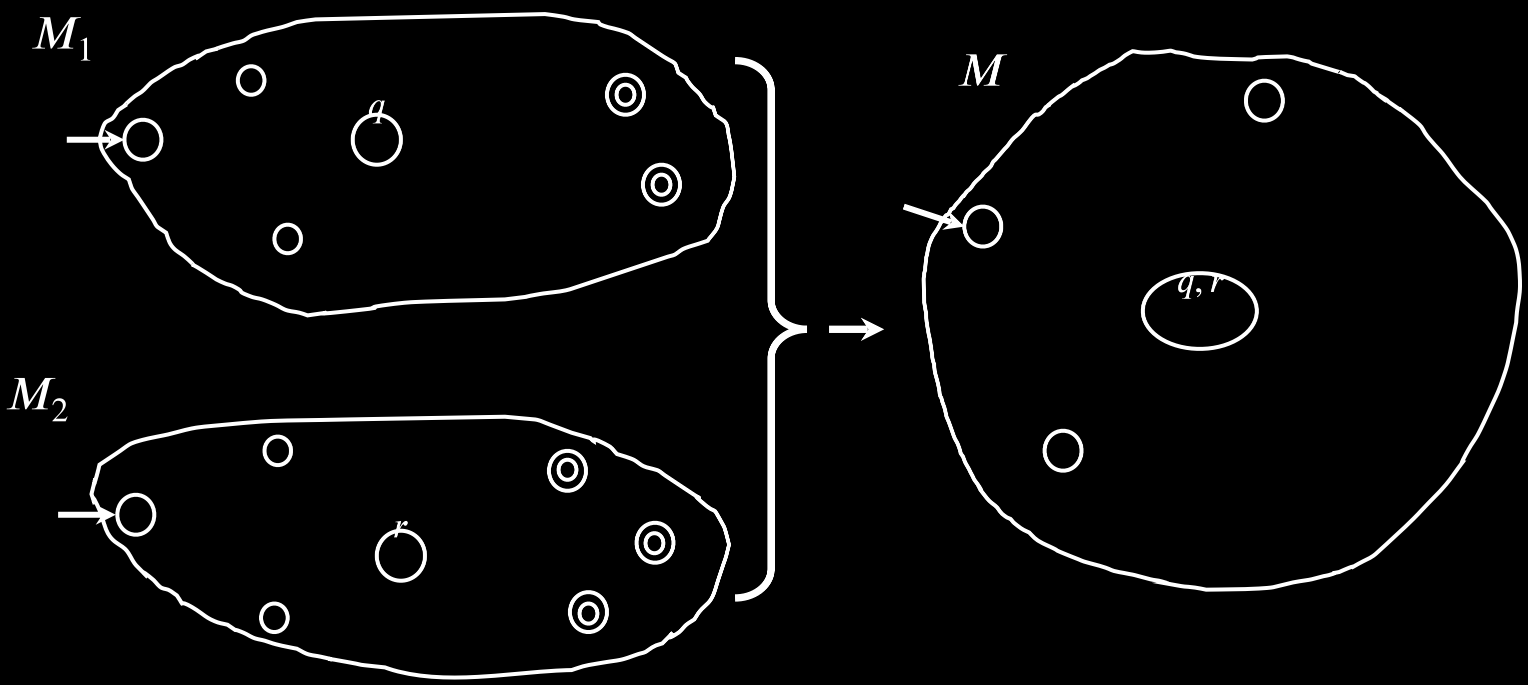

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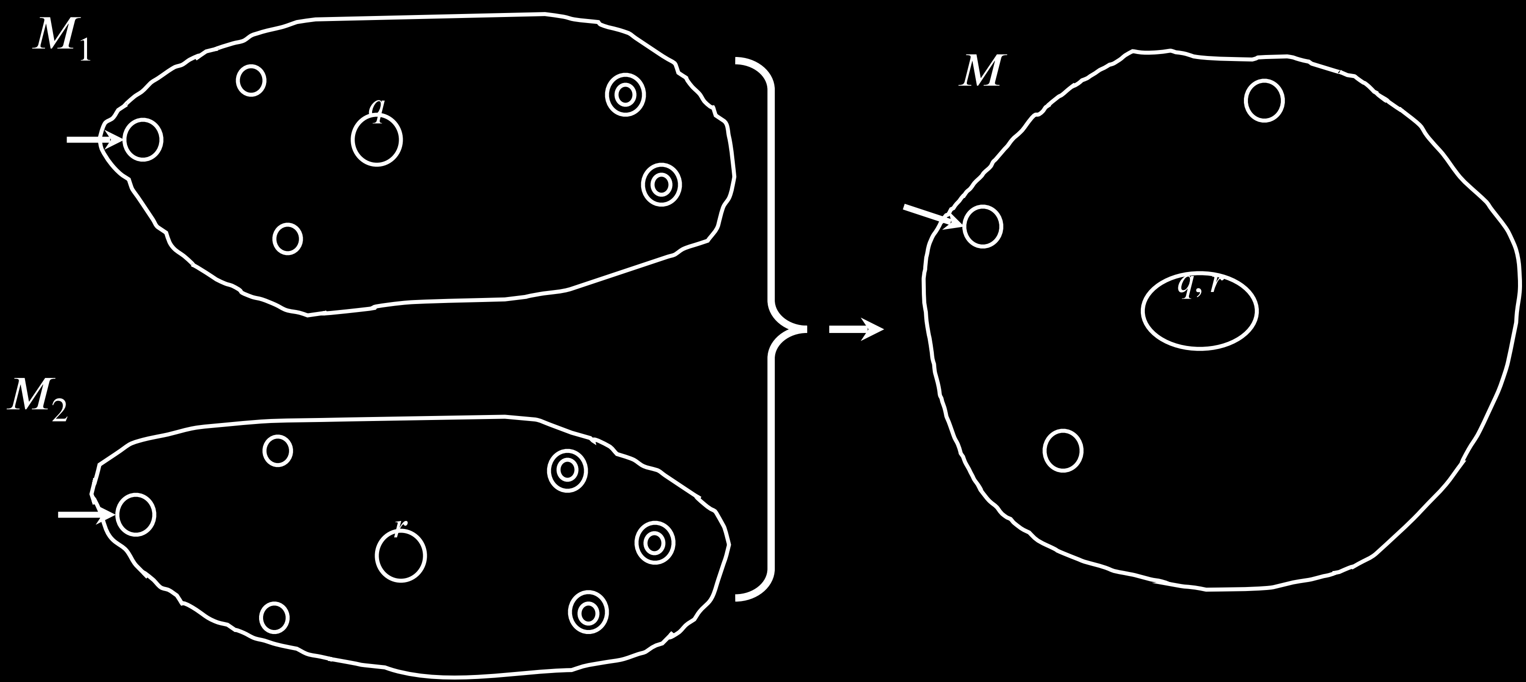

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In the proof, if M_1 and M_2 are finite automata where M_1 has k_1 states and M_2 has k_2 states Then how many states does M have?

(a) $k_1 + k_2$

(b) $(k_1)^2 + (k_2)^2$

(c) $k_1 \times k_2$

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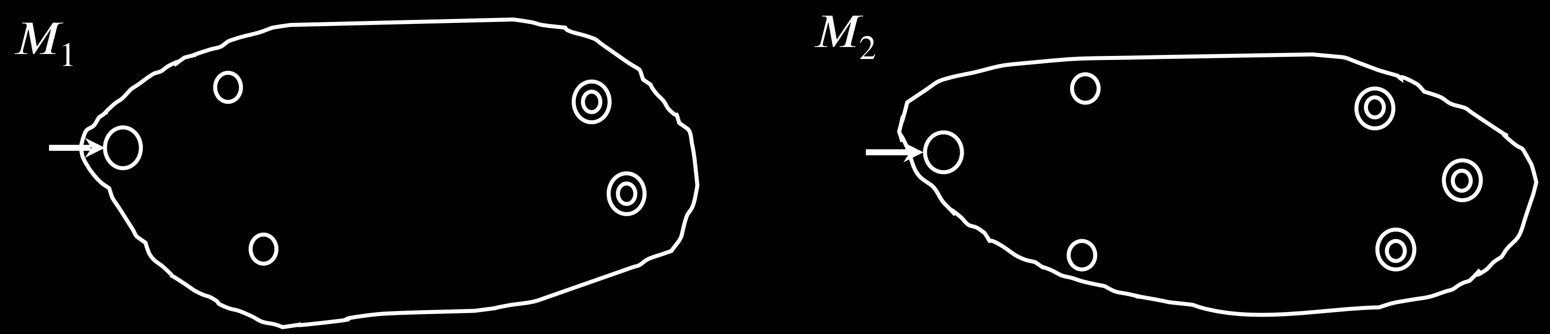

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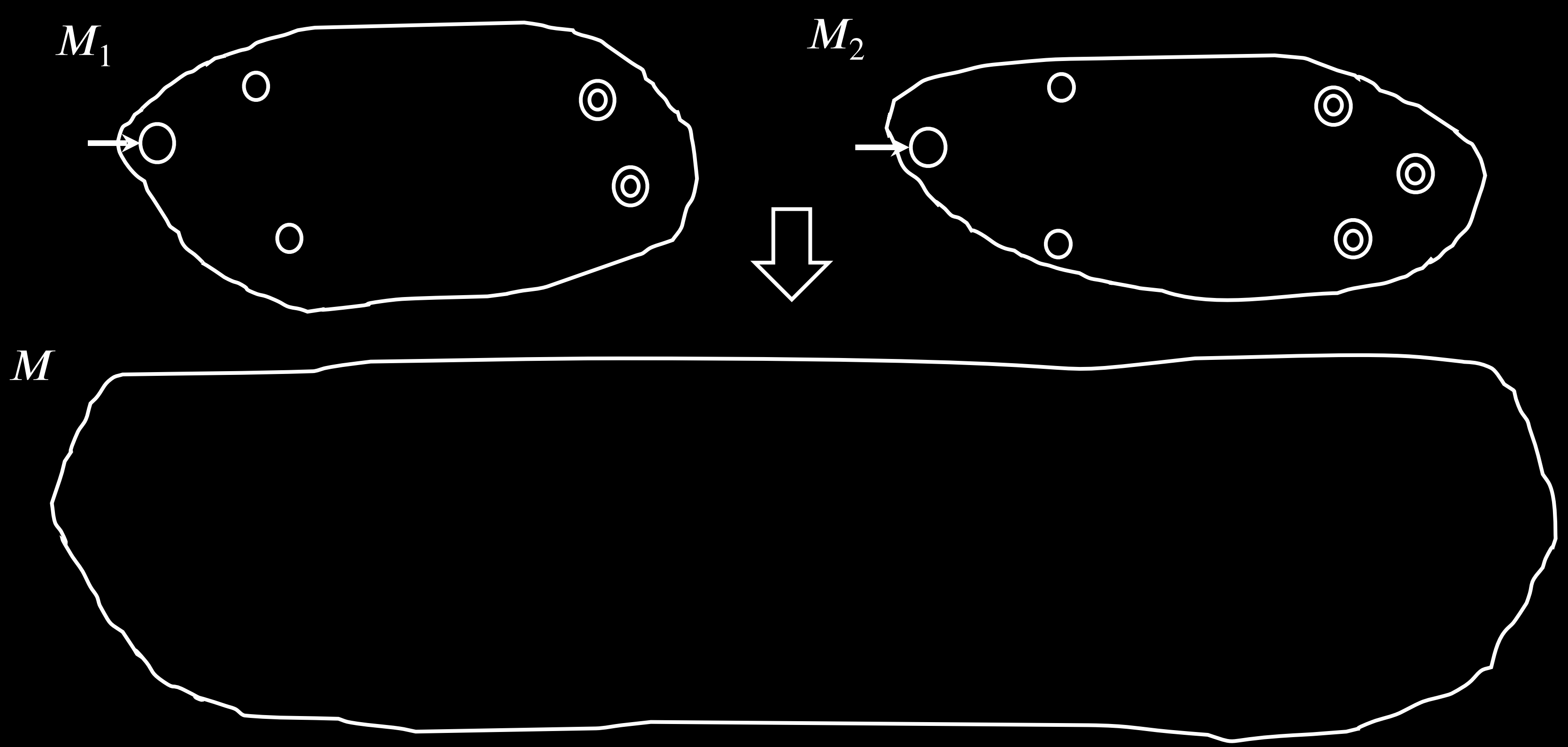

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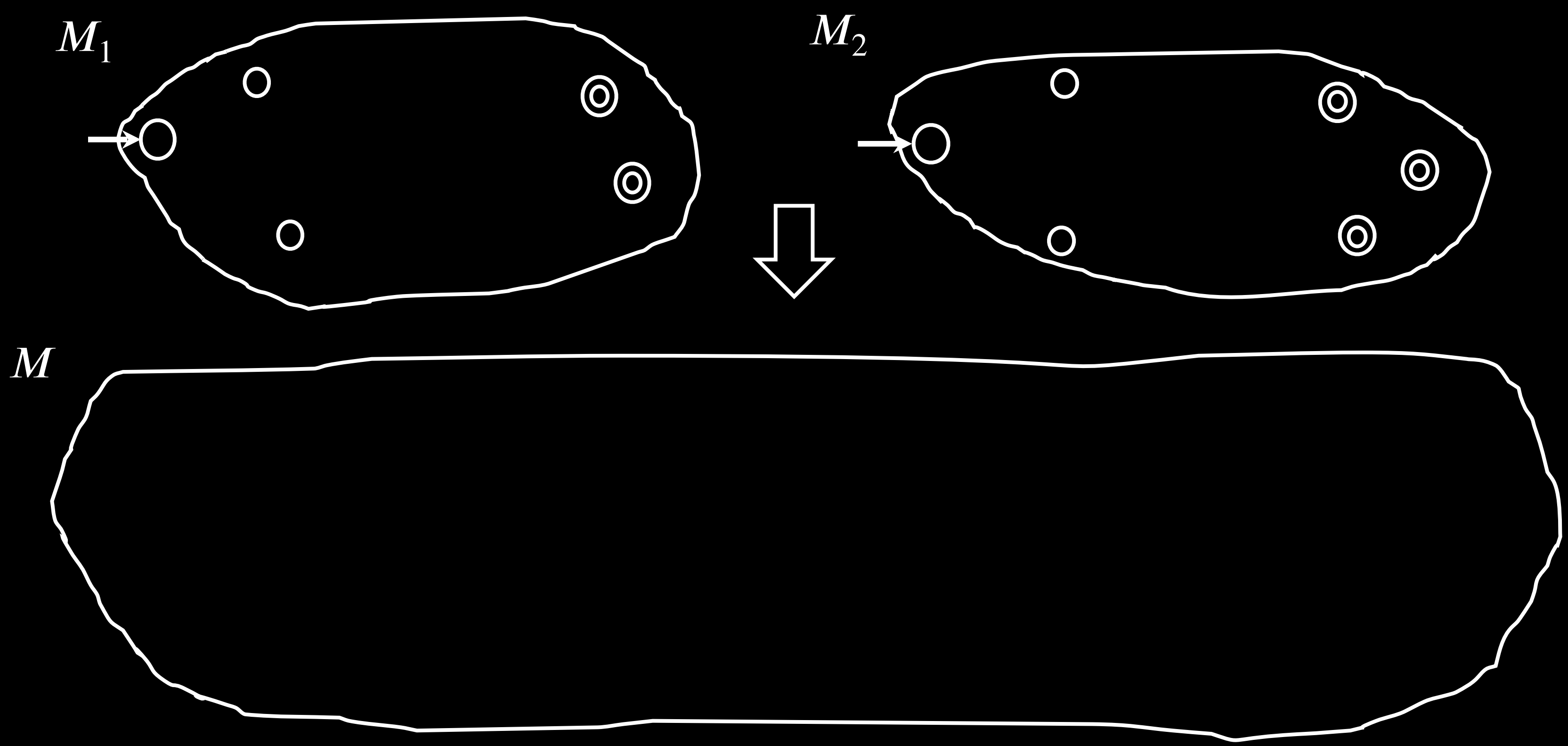

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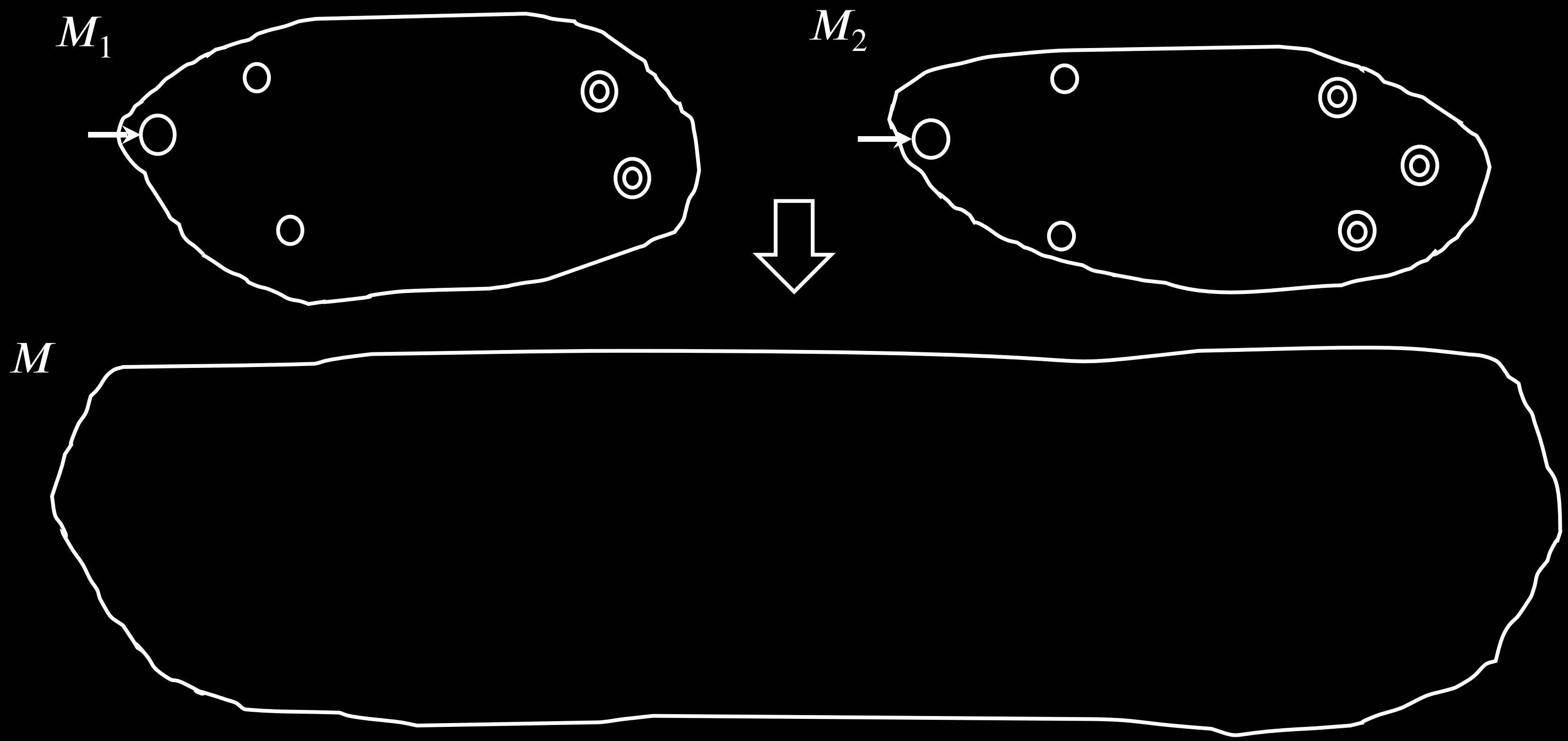

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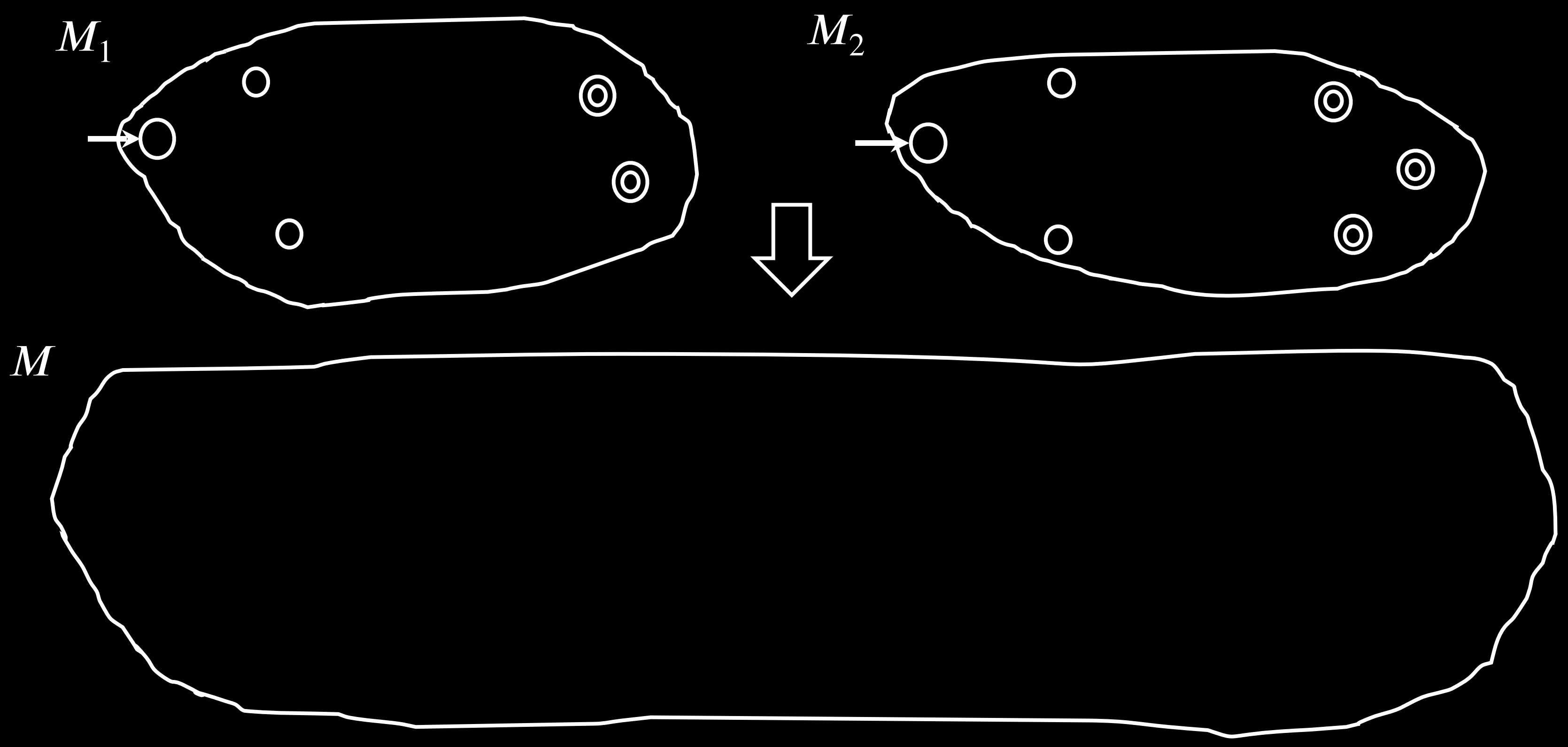

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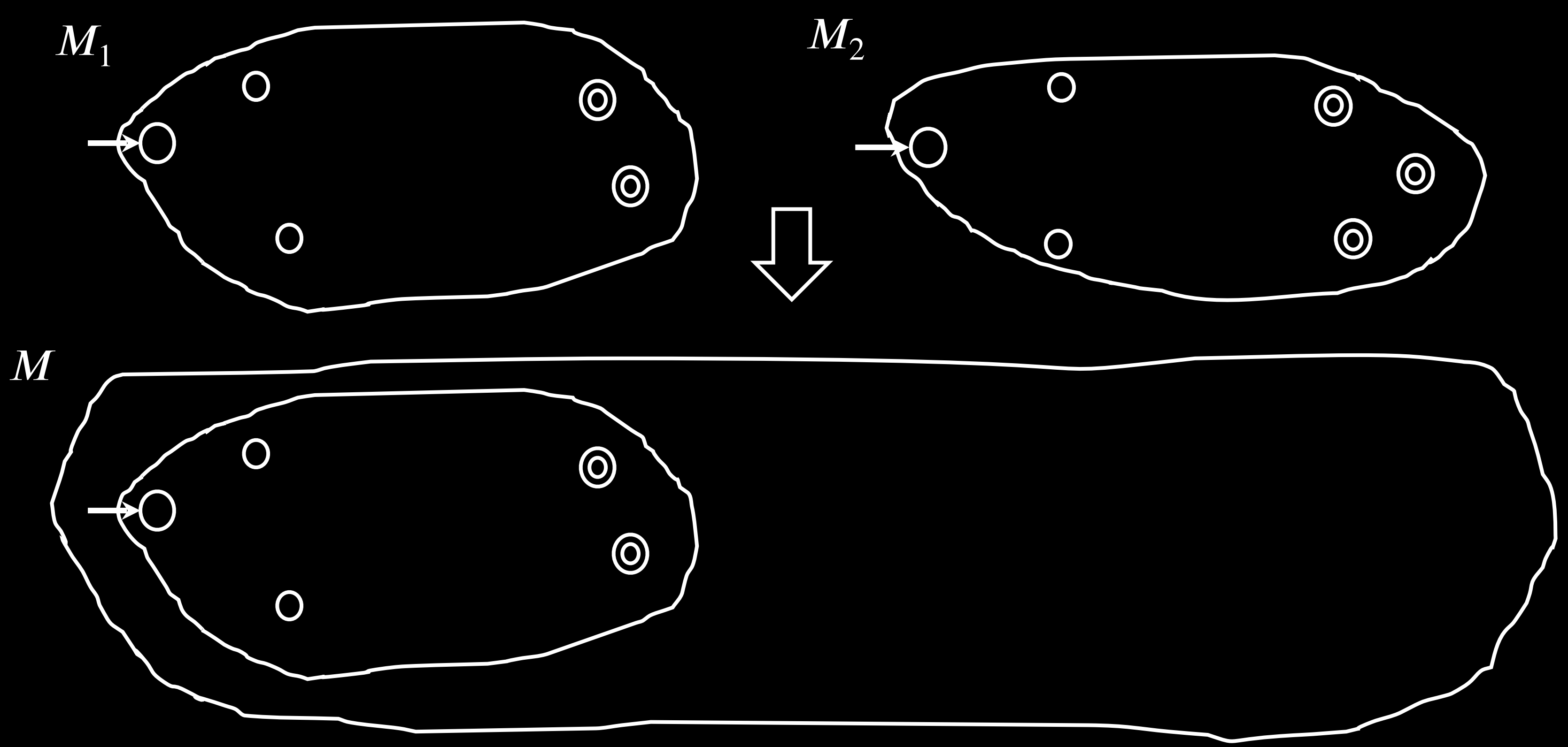

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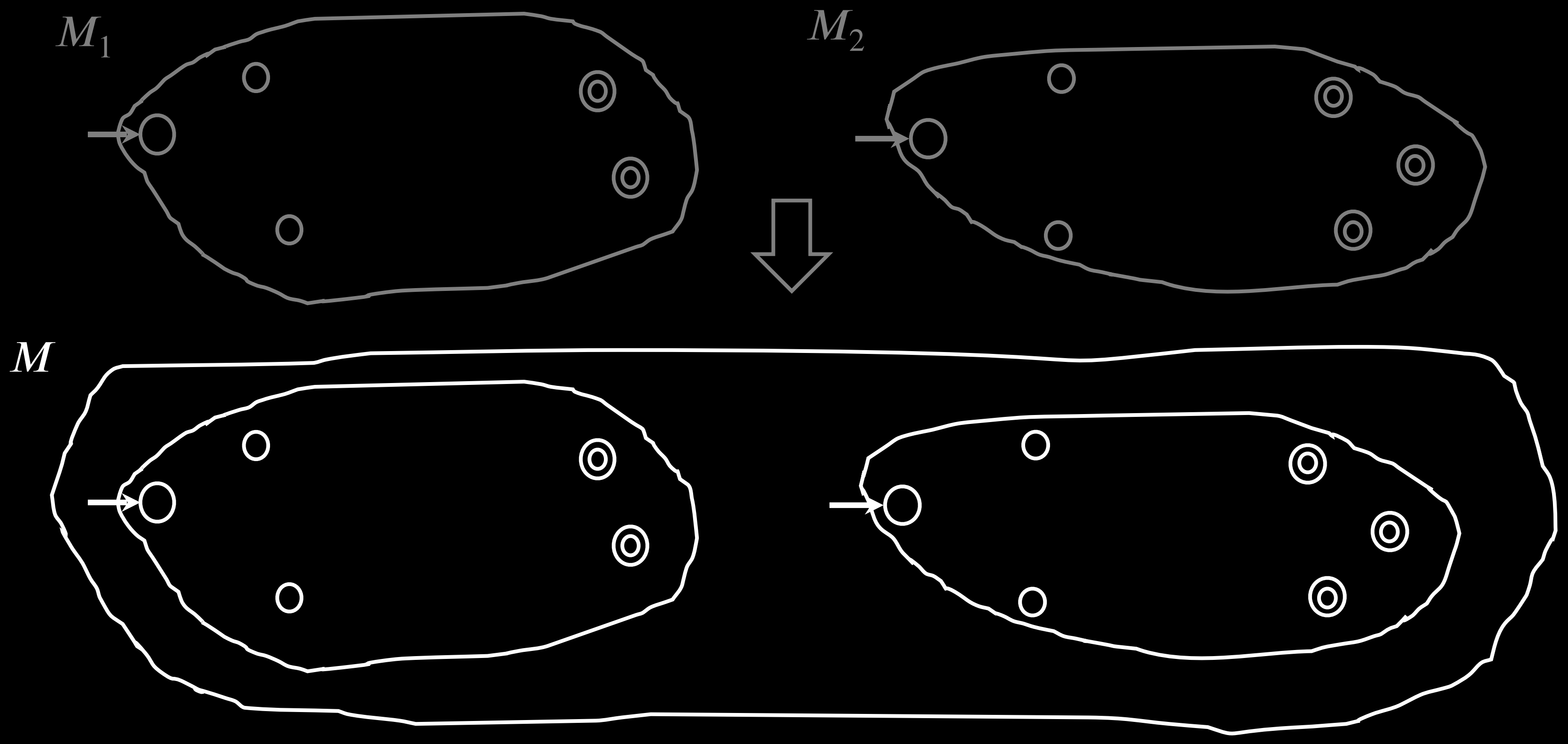

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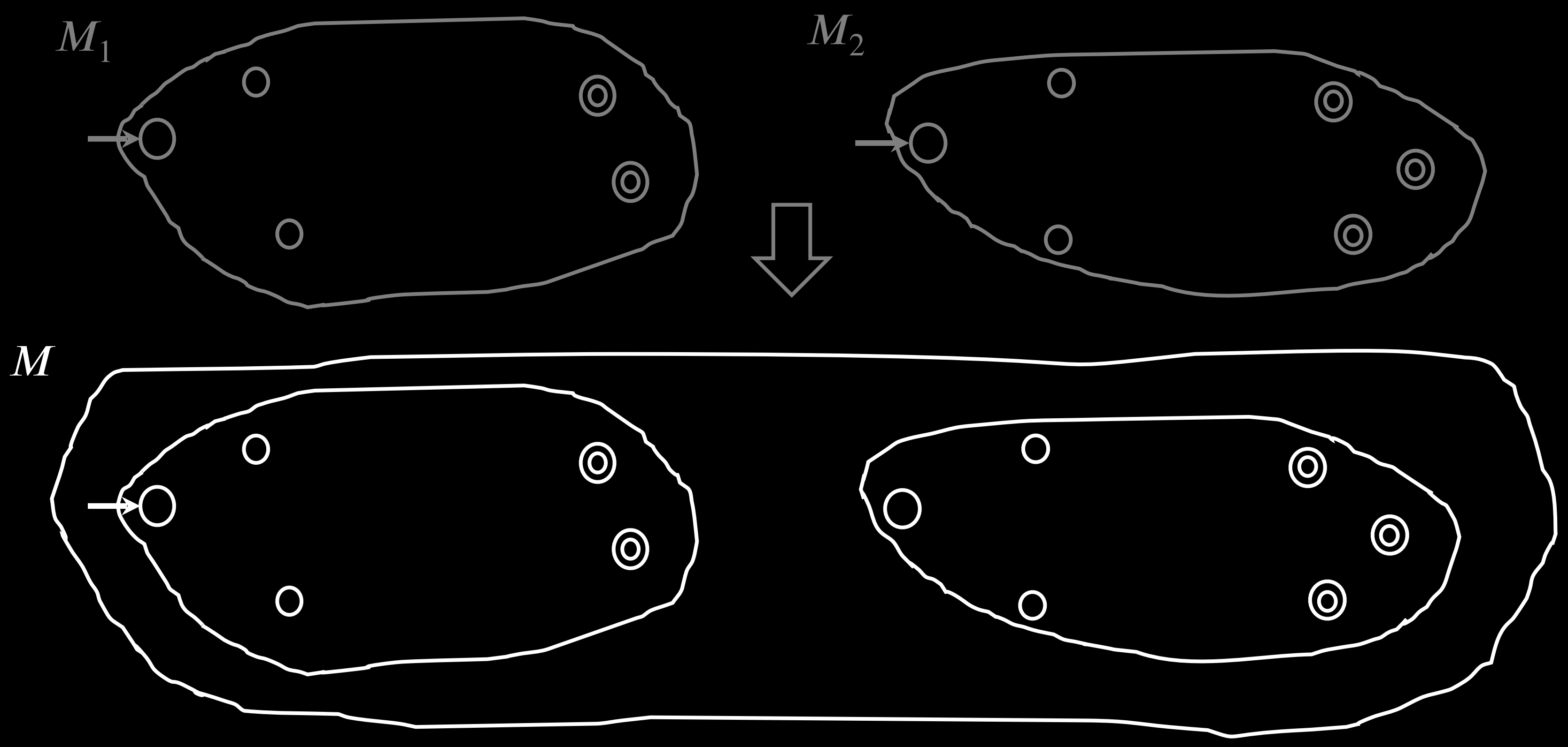

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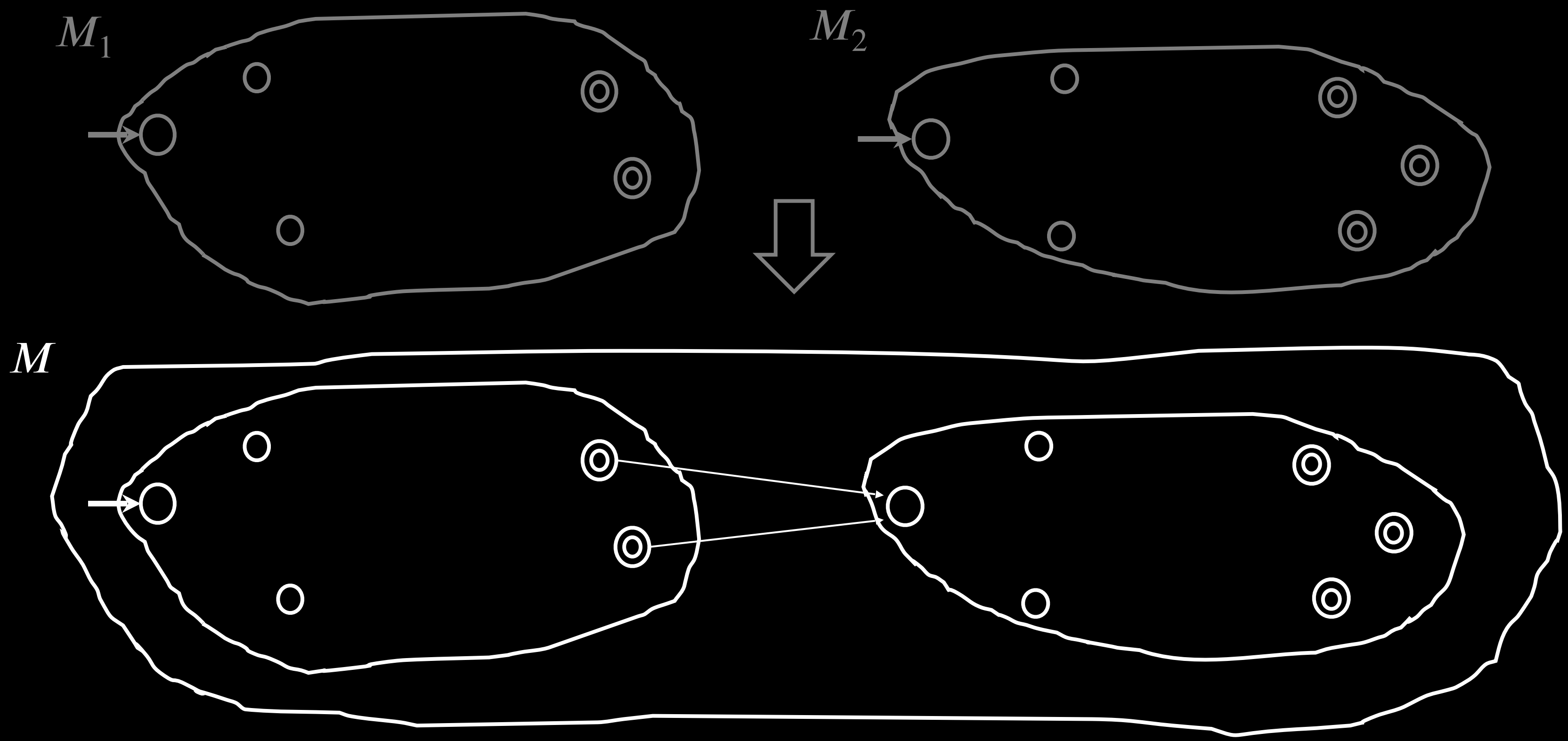

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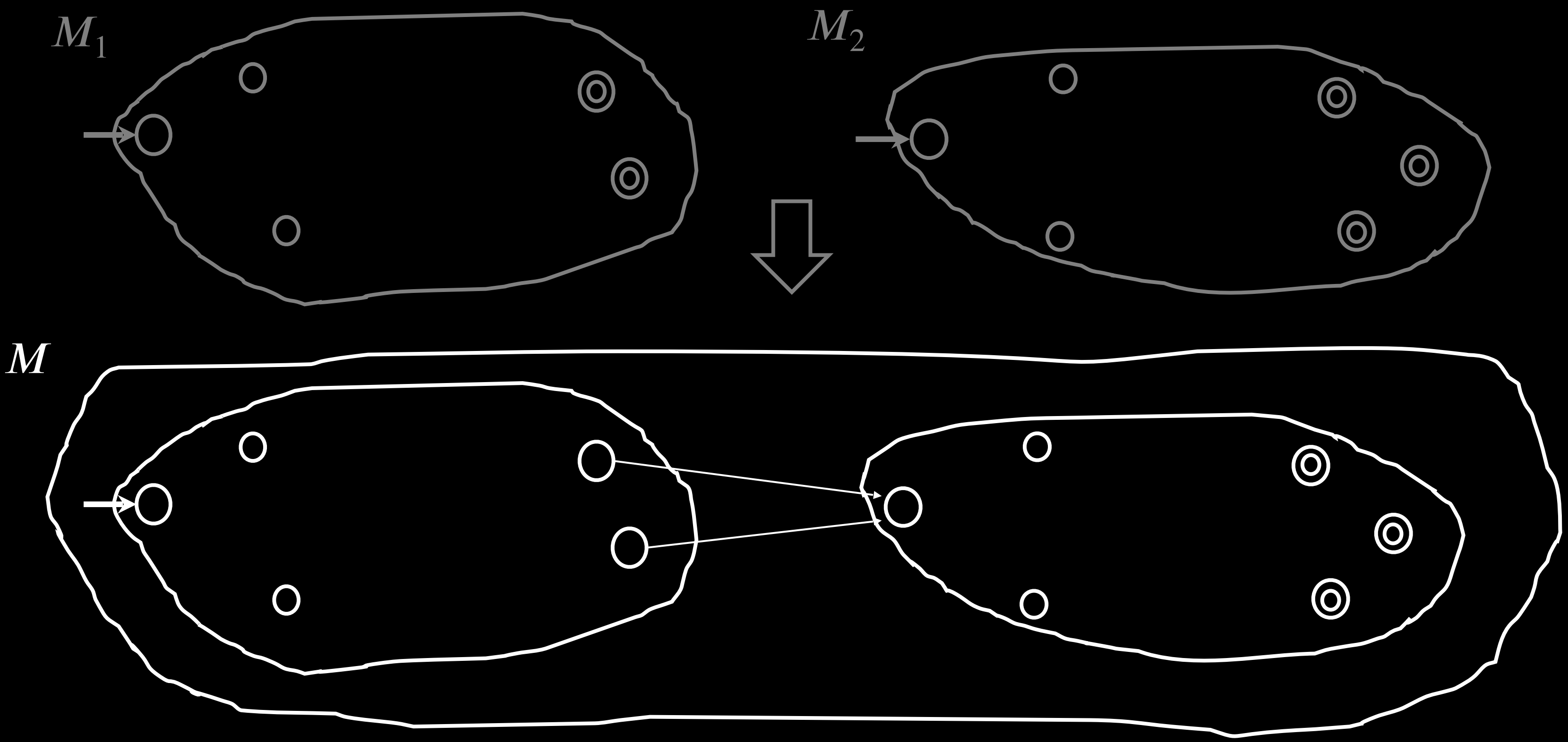

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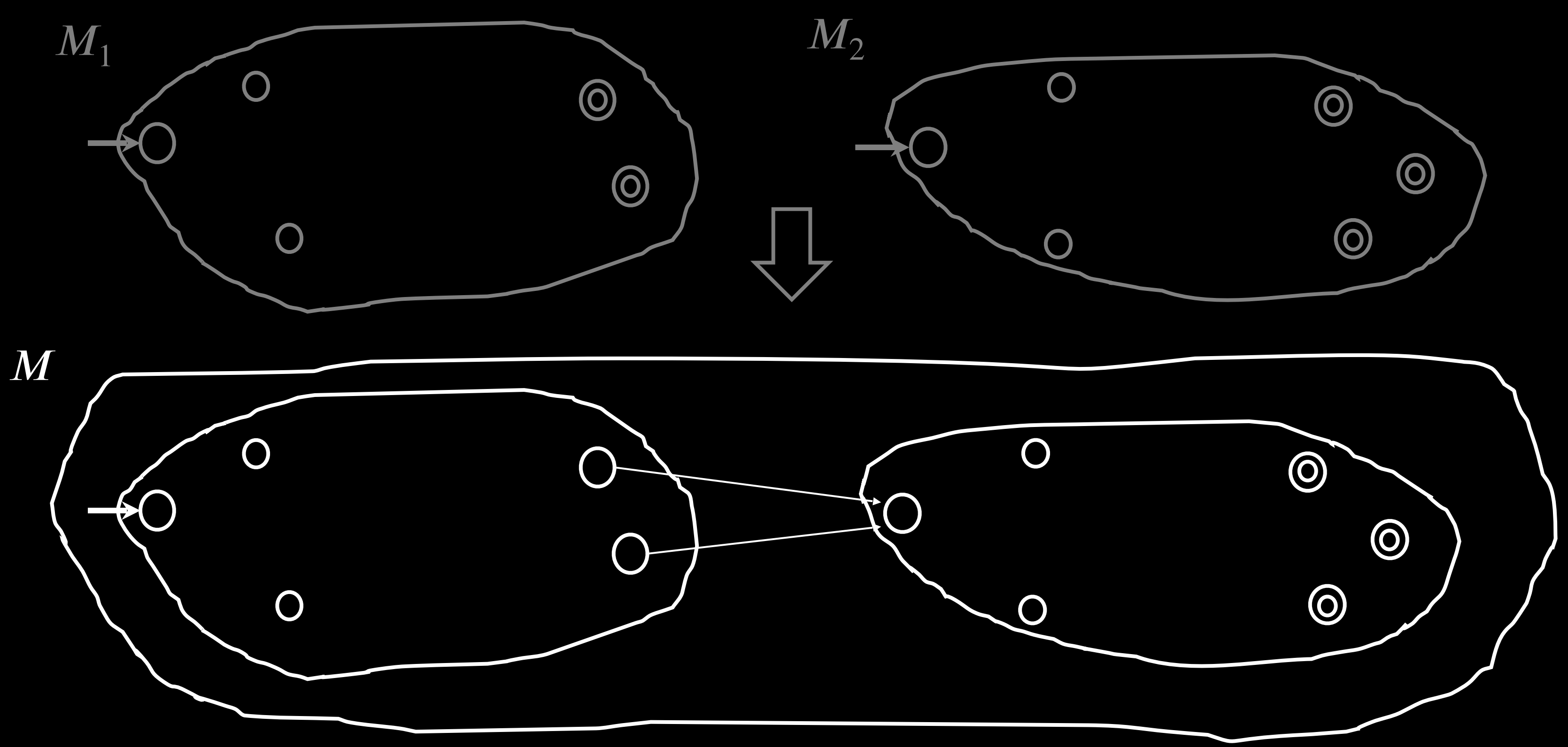

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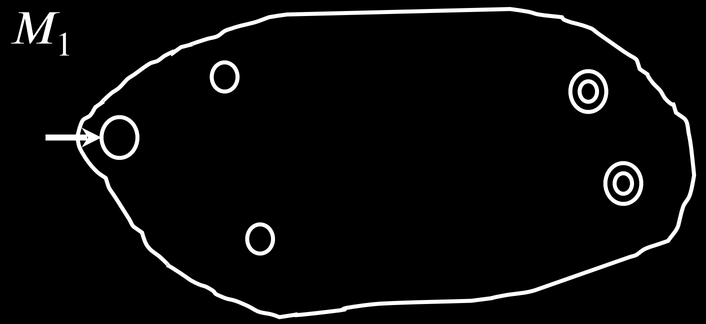

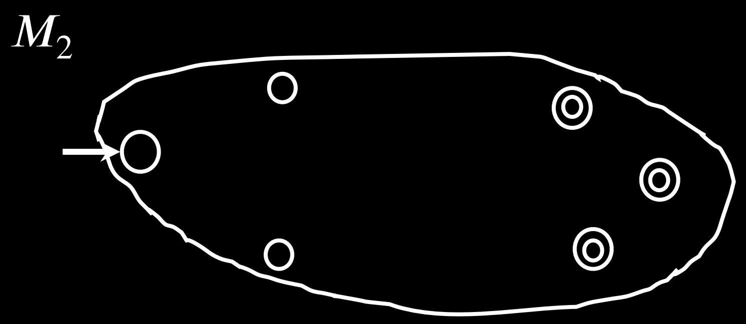

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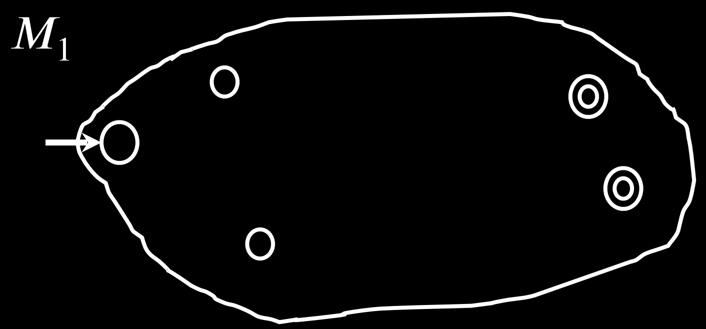

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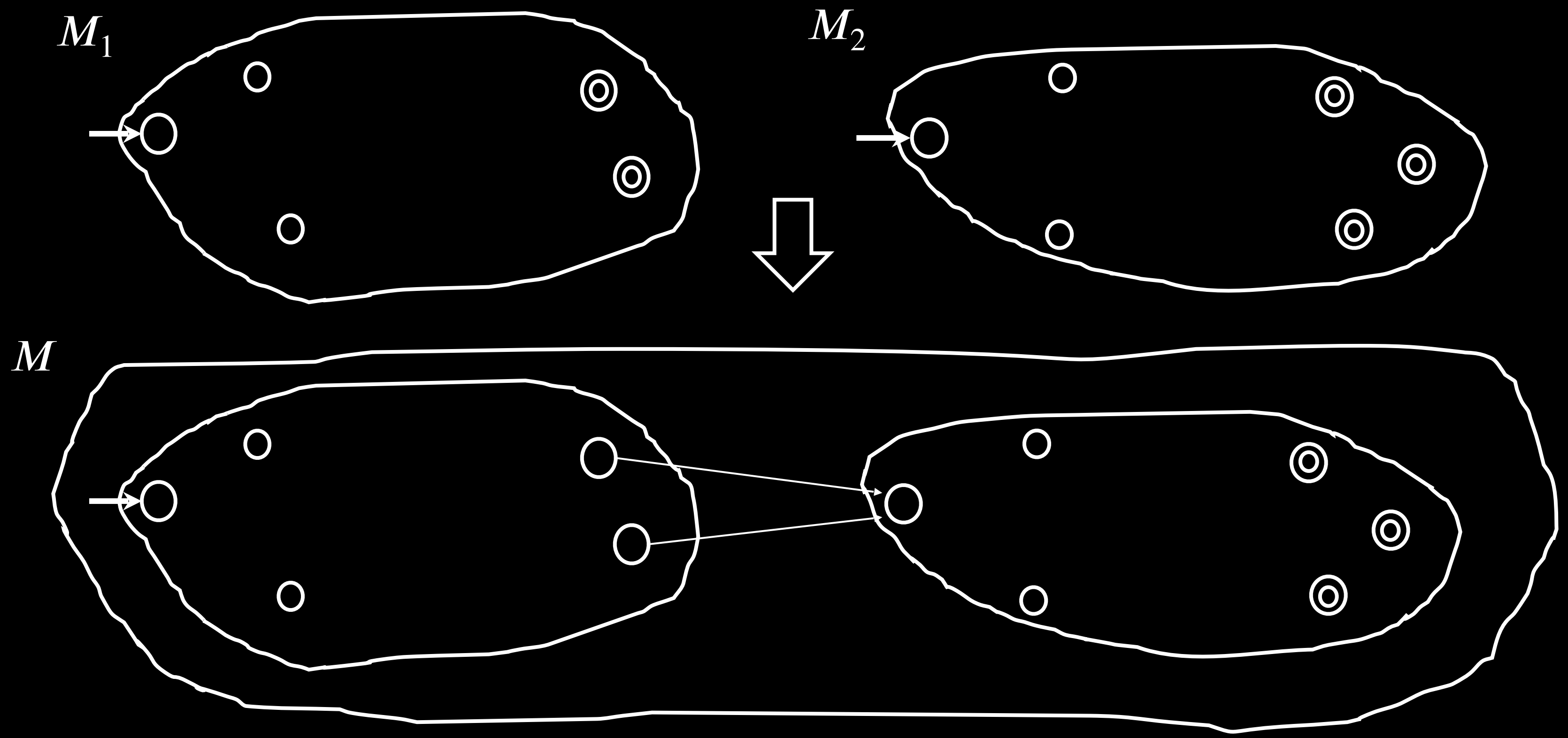

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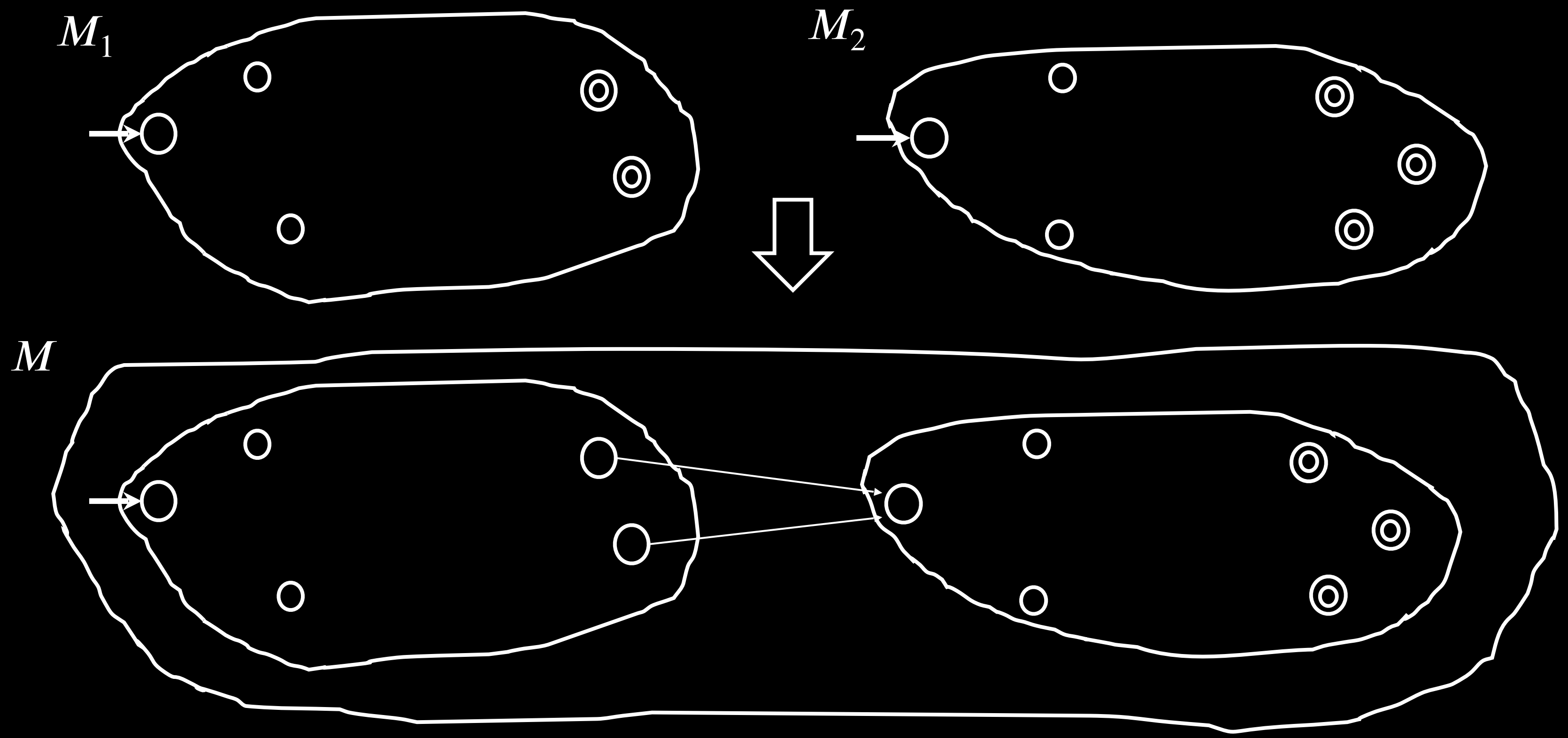

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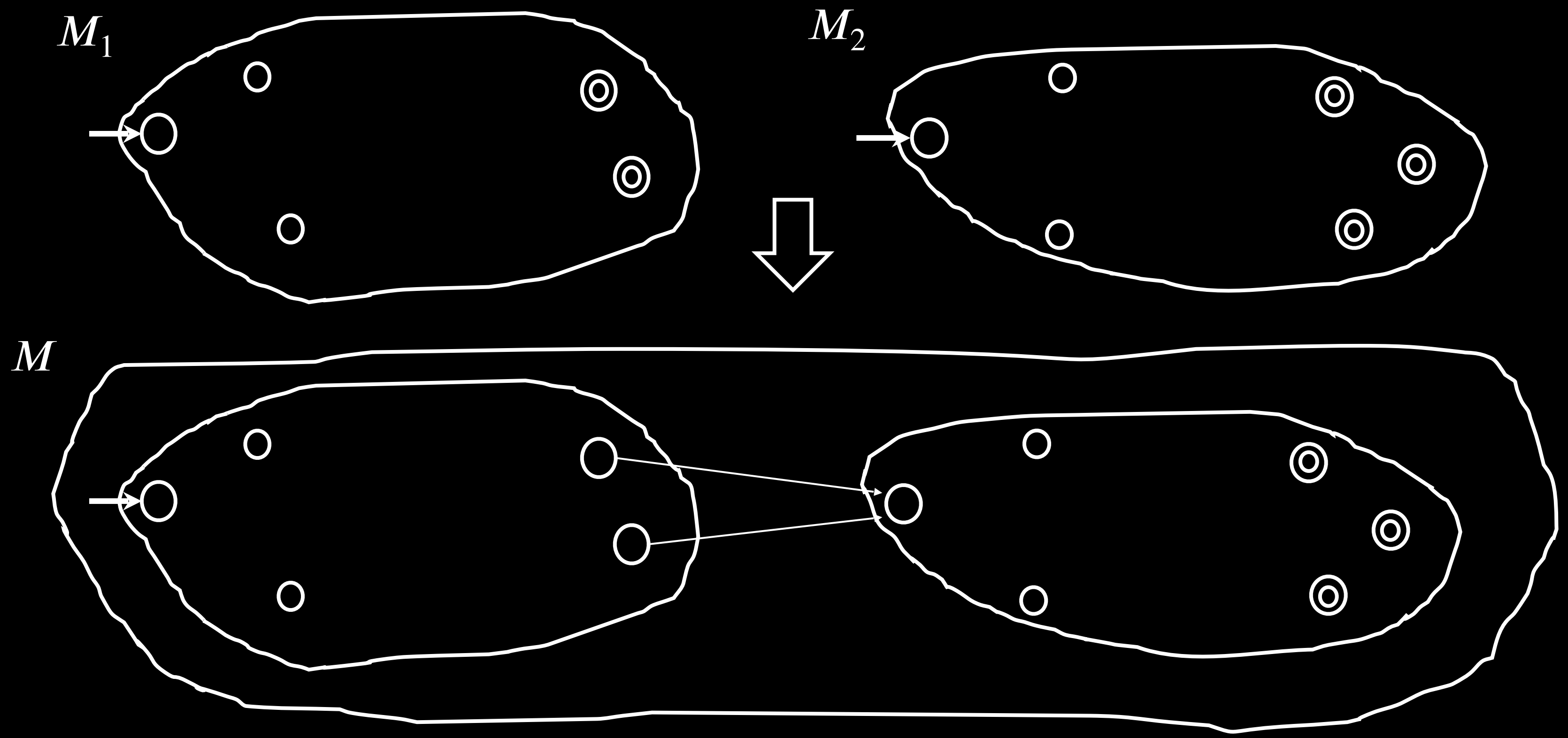

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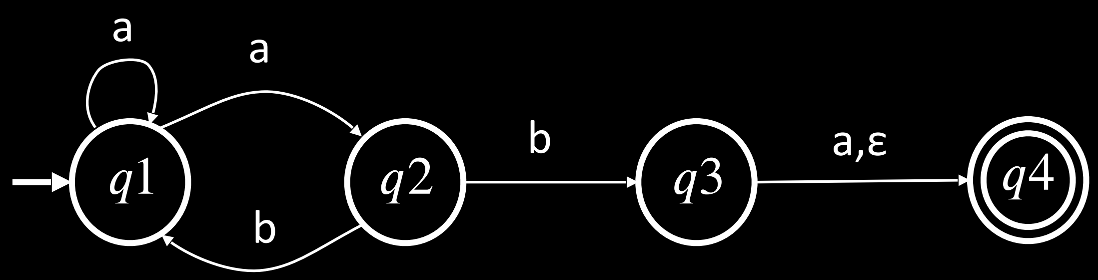
Hold off. Need new concept.

Nondeterministic Finite Automata

Nondeterministic Finite Automata

$N1$



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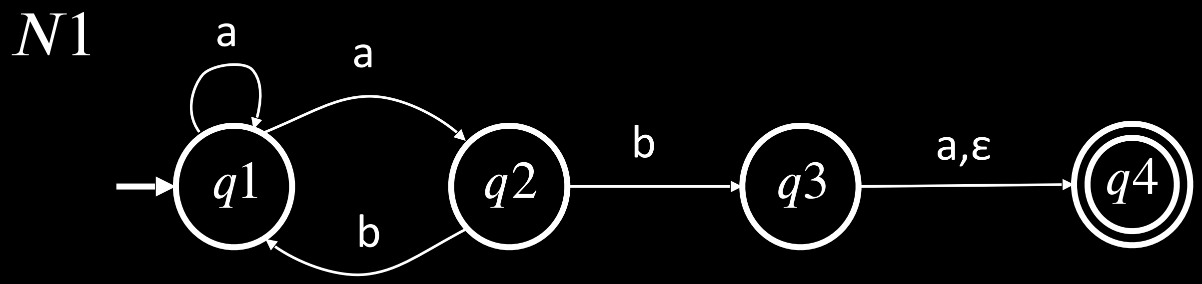

New features of nondeterminism:

- multiple paths possible (0, 1 or many at each step)

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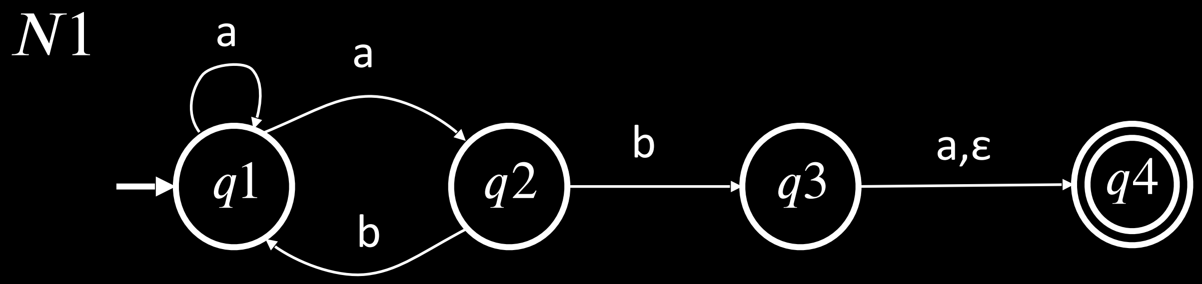

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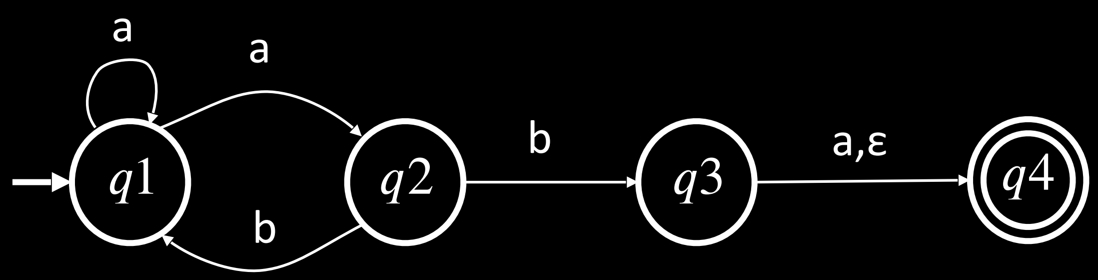

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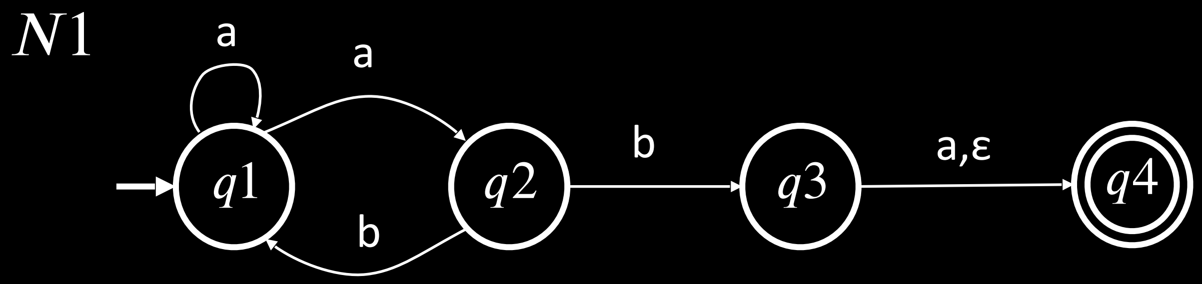

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Example inputs:

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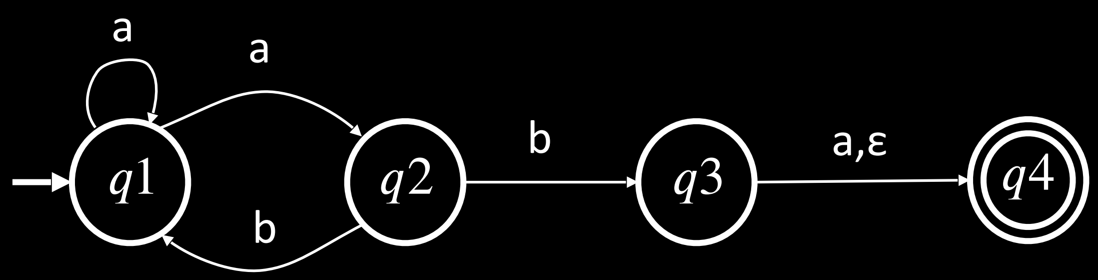

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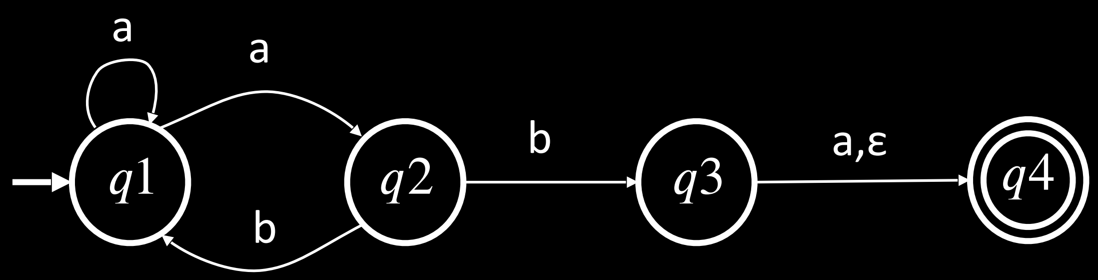

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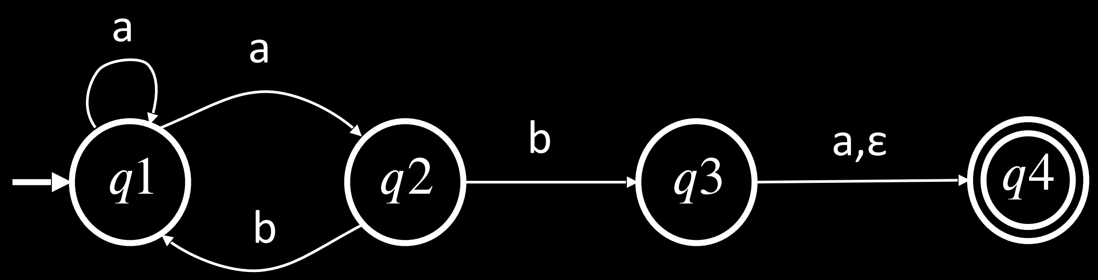

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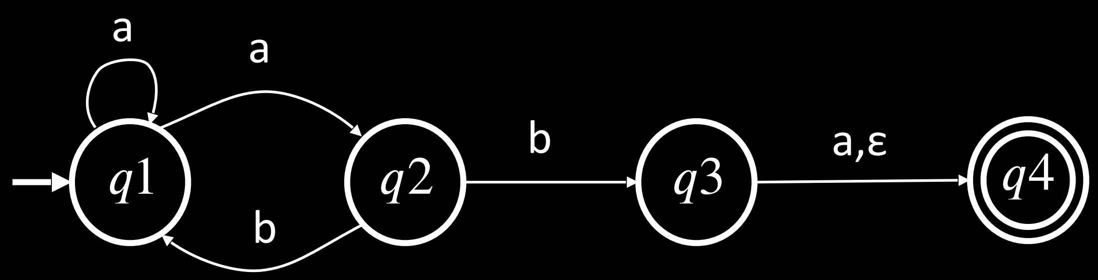

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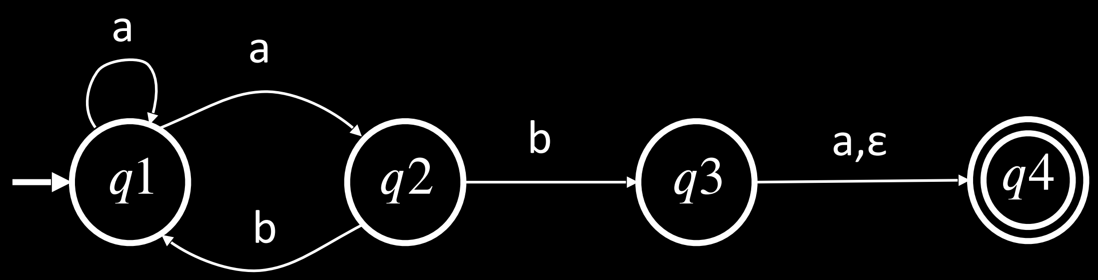

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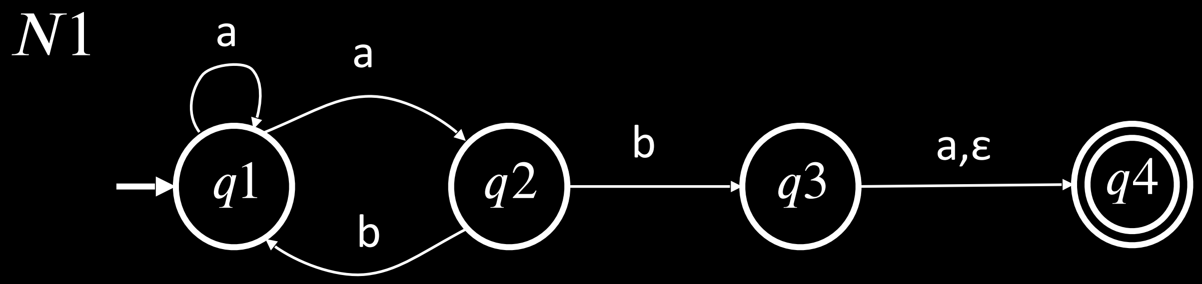

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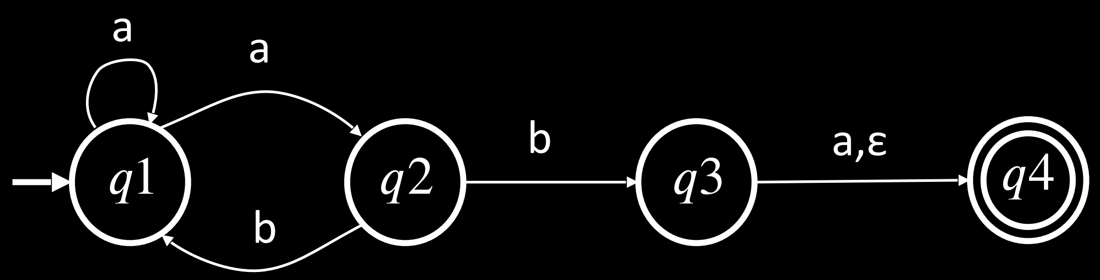

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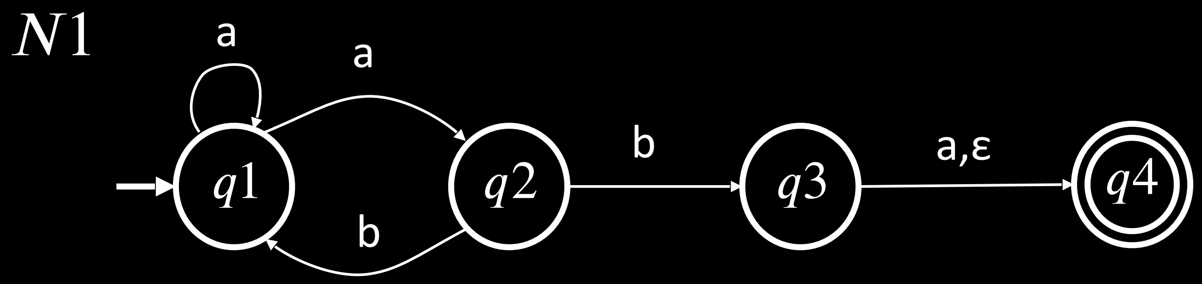

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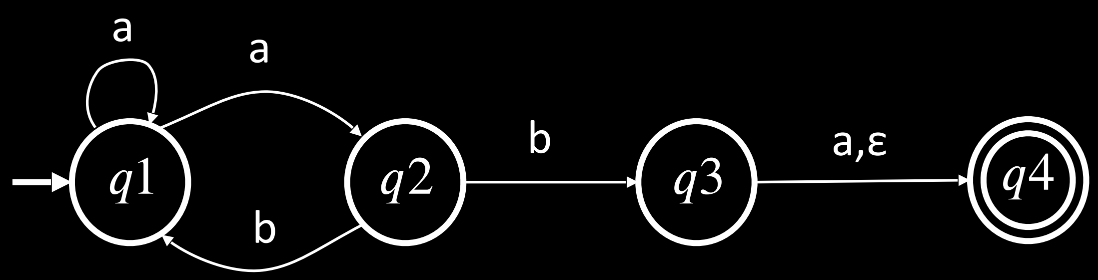

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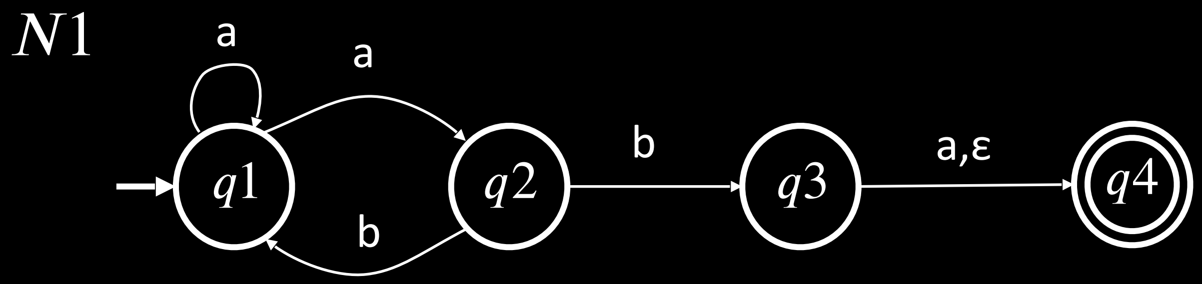
Check-in 2.1

Nondeterministic Finite Automata



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Check-in 2.1

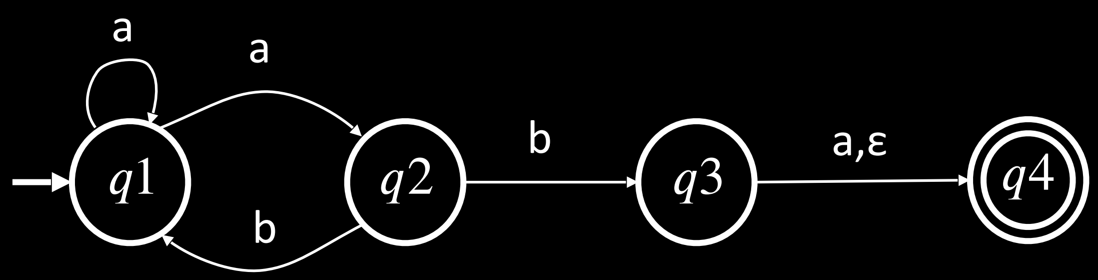
What does N_1 do on input aab ?

(a) Accept
(b) Reject
(c) Both Accept and Reject

Check-in 2.1

NFA – Formal Definition

$N1$



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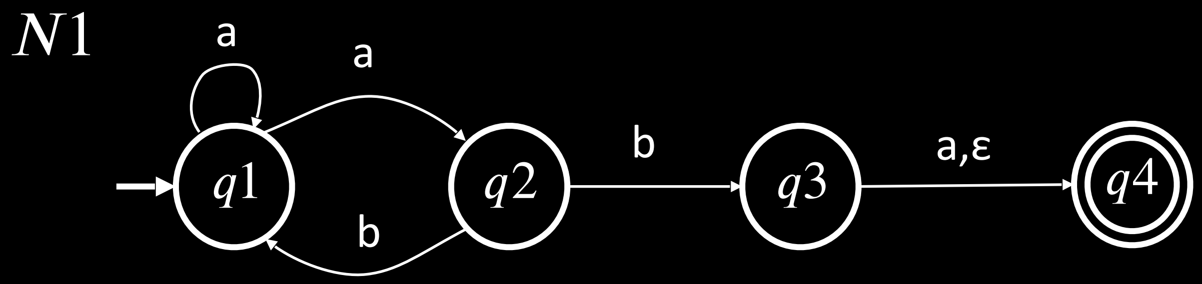
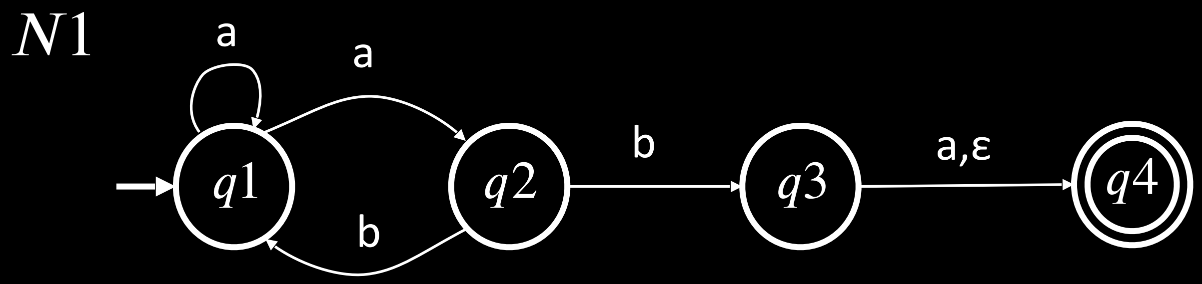


Defn: A <u>nondeterministic finite automaton (NFA)</u>

N is a 5-tuple $(Q,\ \Sigma,\ \delta,\ q0,\ F)$

states alphabet transition function start state accept states

NFA – Formal Definition



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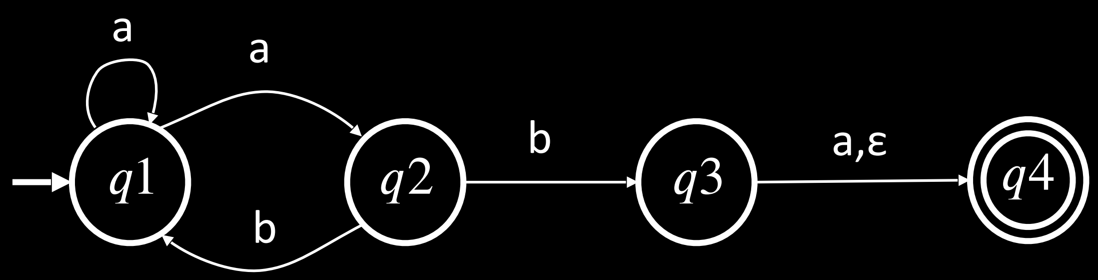

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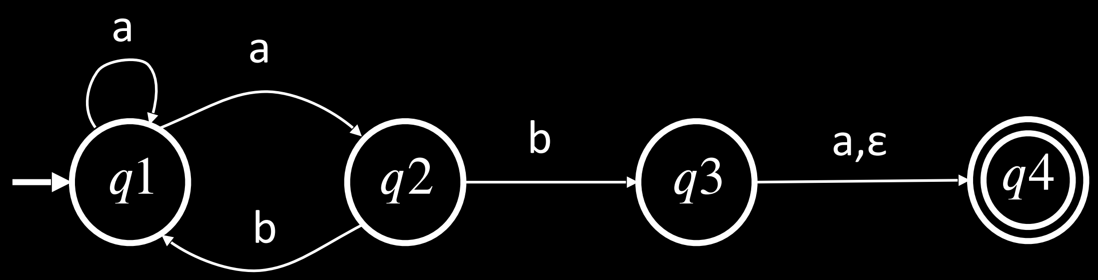

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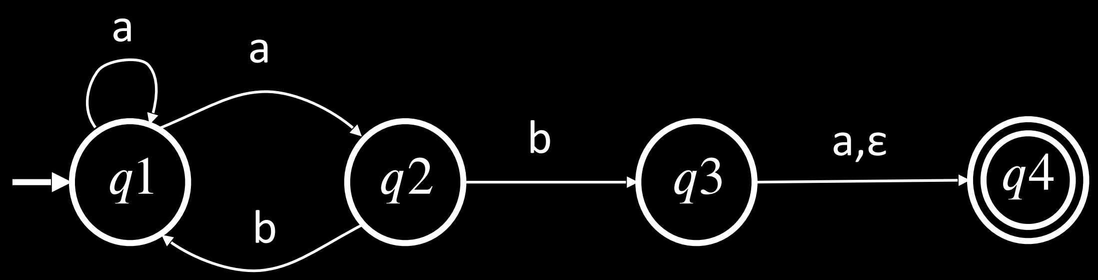

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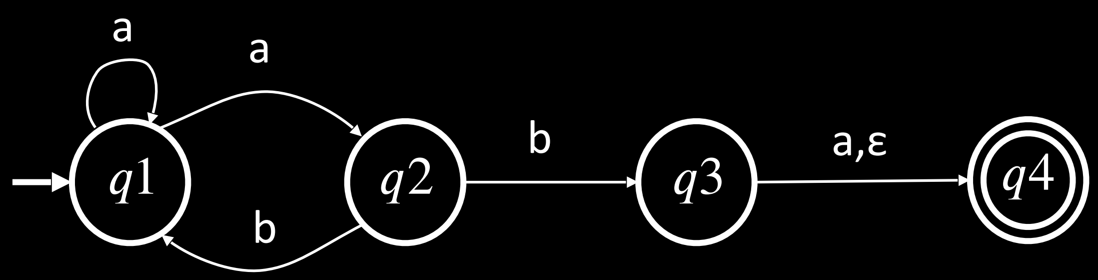

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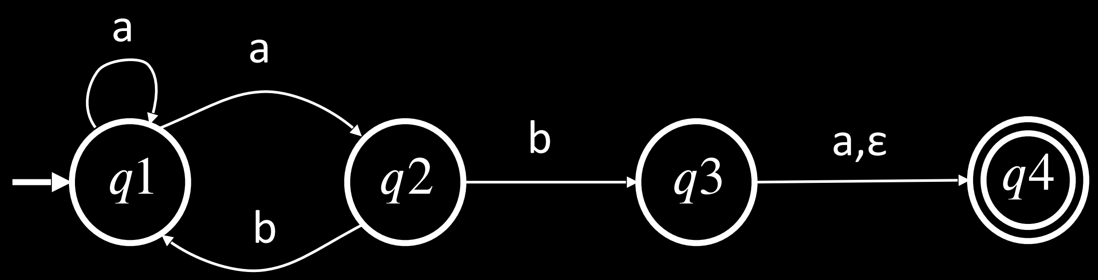

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  $\delta(q_1, \mathrm{b}) = \varnothing$

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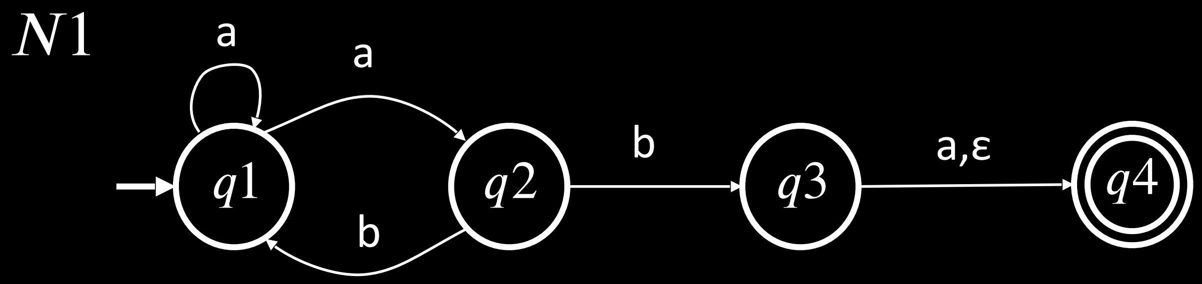

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states, alphabet, transition function, start state, accept states

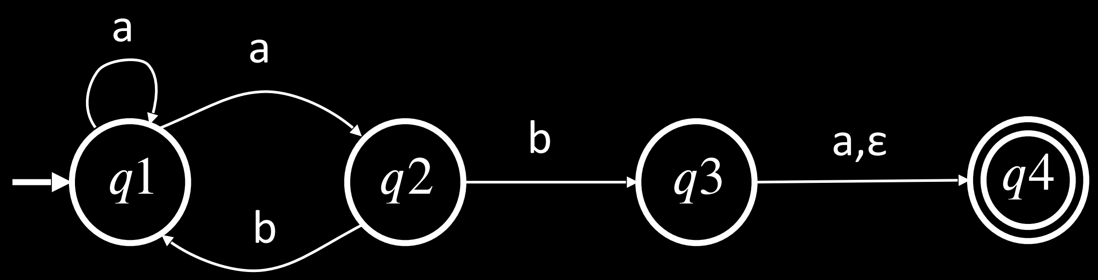
Ways to think about nondeterminism:

- all same as before except δ
- $\delta : Q \times \underbrace{\Sigma_\varepsilon}_{\Sigma \cup \{\varepsilon\}} \to \underbrace{\mathscr{P}(Q)}_{\text{power set}} = \{R \mid R \subseteq Q\}$
- In the N_1 example: $\delta(q_1, \mathrm{a}) = \{q_1, q_2\}$

  $\delta(q_1, \mathrm{b}) = \varnothing$

NFA – Formal Definition

$N1$



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<u>Computational:</u> Fork new parallel thread and accept if any thread leads to an accept state.

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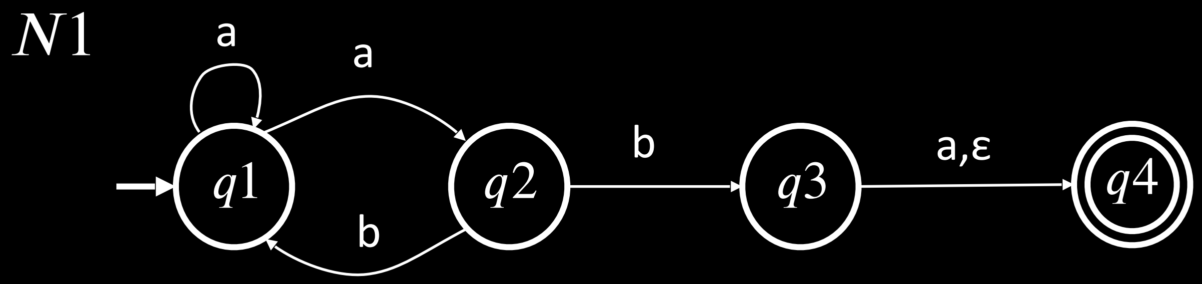

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<u>Mathematical:</u> Tree with branches. Accept if any branch leads to an accept state.

NFA – Formal Definition

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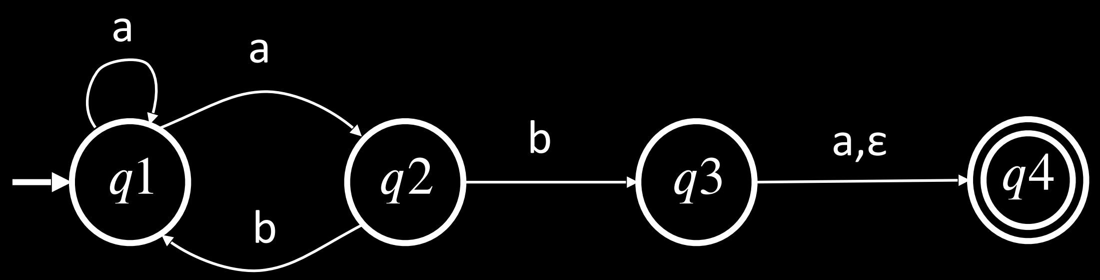

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<u>Computational:</u> Fork new parallel thread and accept if any thread leads to an accept state.

<u>Mathematical:</u> Tree with branches. Accept if any branch leads to an accept state.

<u>Magical:</u> Guess at each nondeterministic step which way to go. Machine always makes the right guess that leads to accepting, if possible.

Converting NFAs to DFAs

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Theorem: If an NFA recognizes A then A is regular

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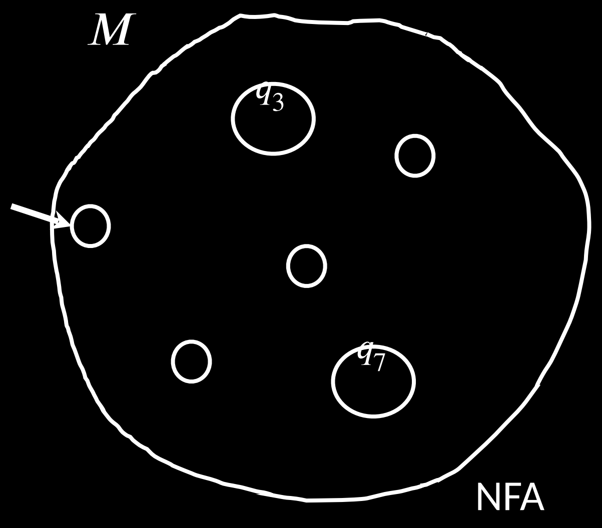

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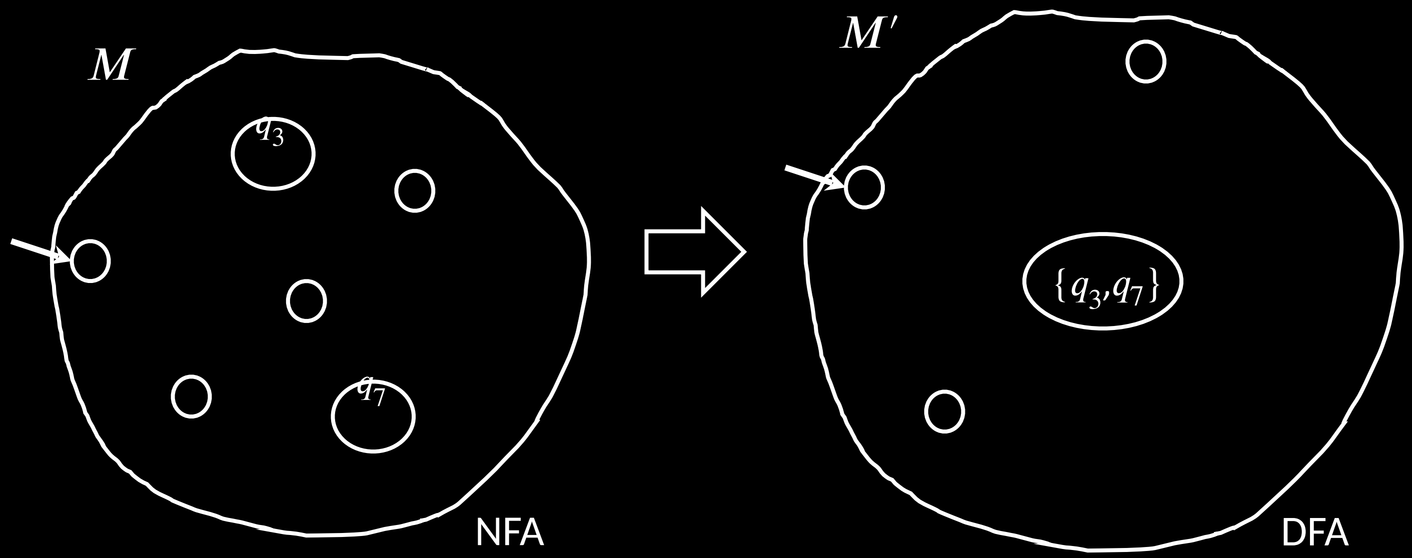

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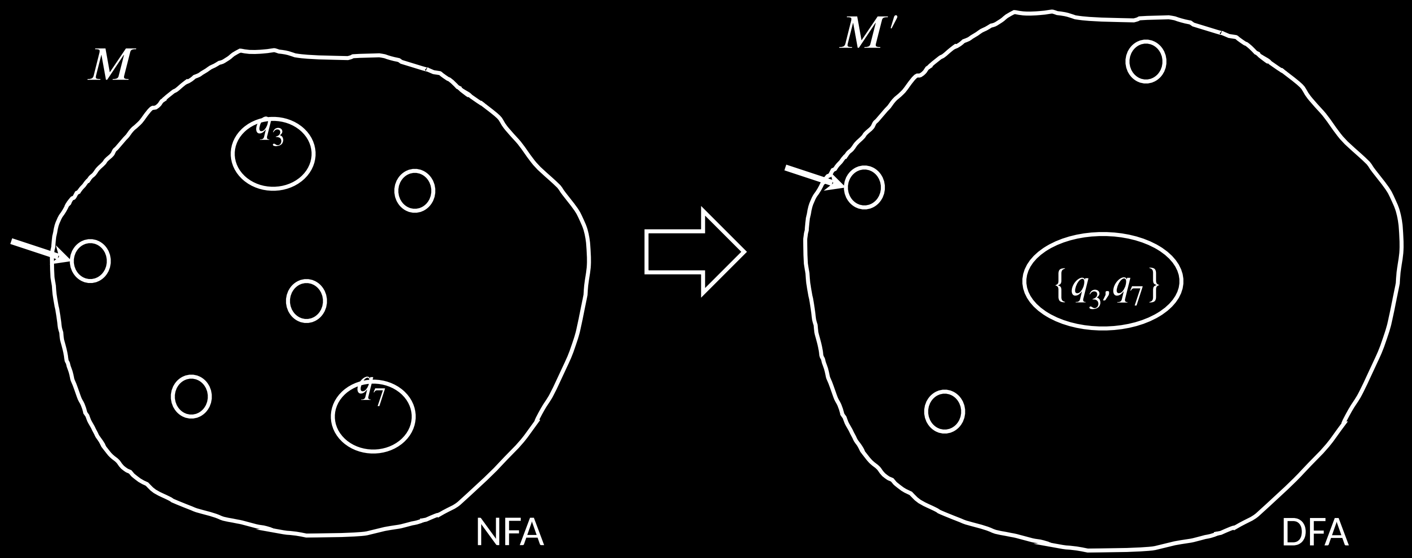

Construction of M':

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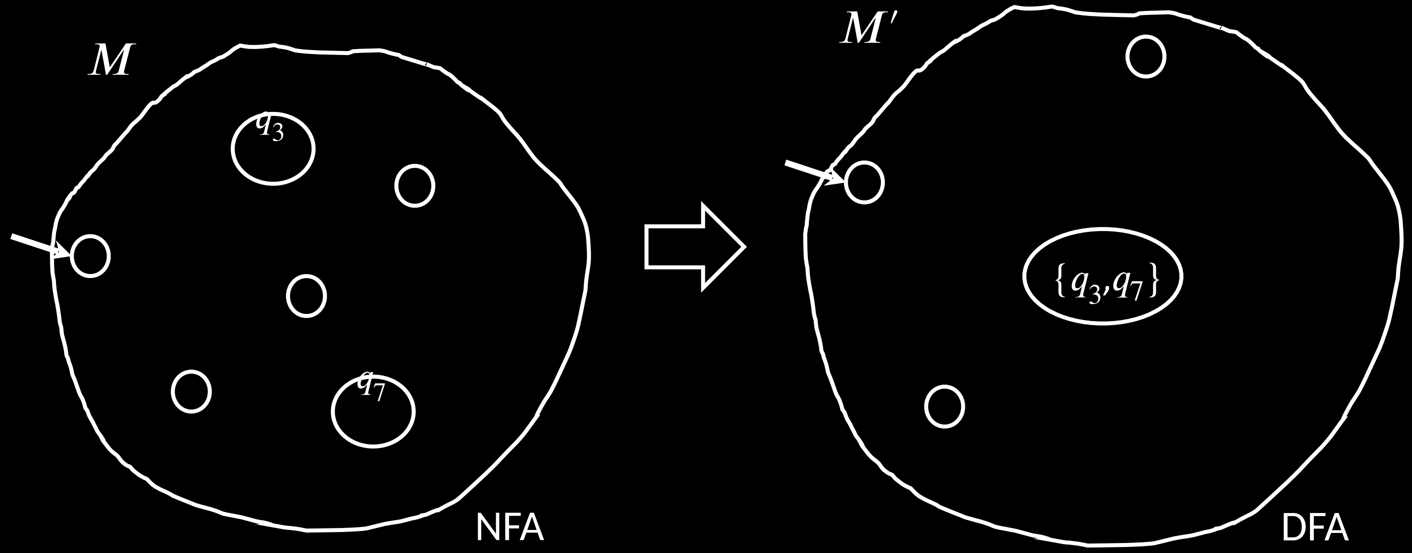

Construction of $\boldsymbol{M'}$:

$Q' = \mathscr{P}(Q)$

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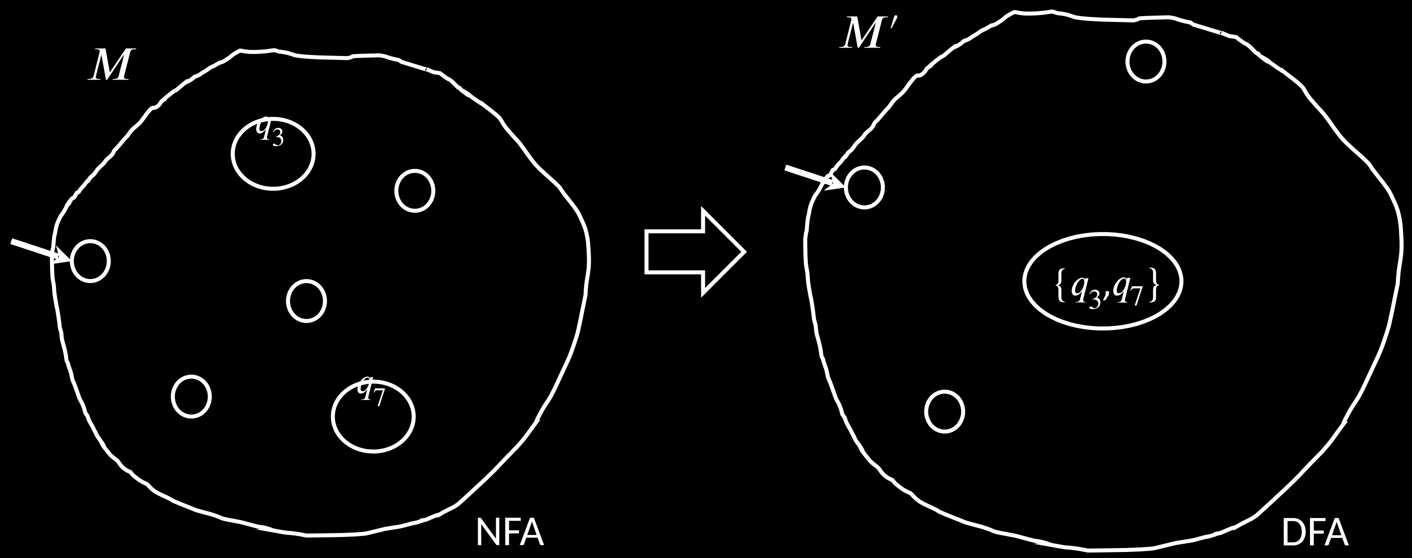

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$\delta'(R, a) =$

$R \in Q'$

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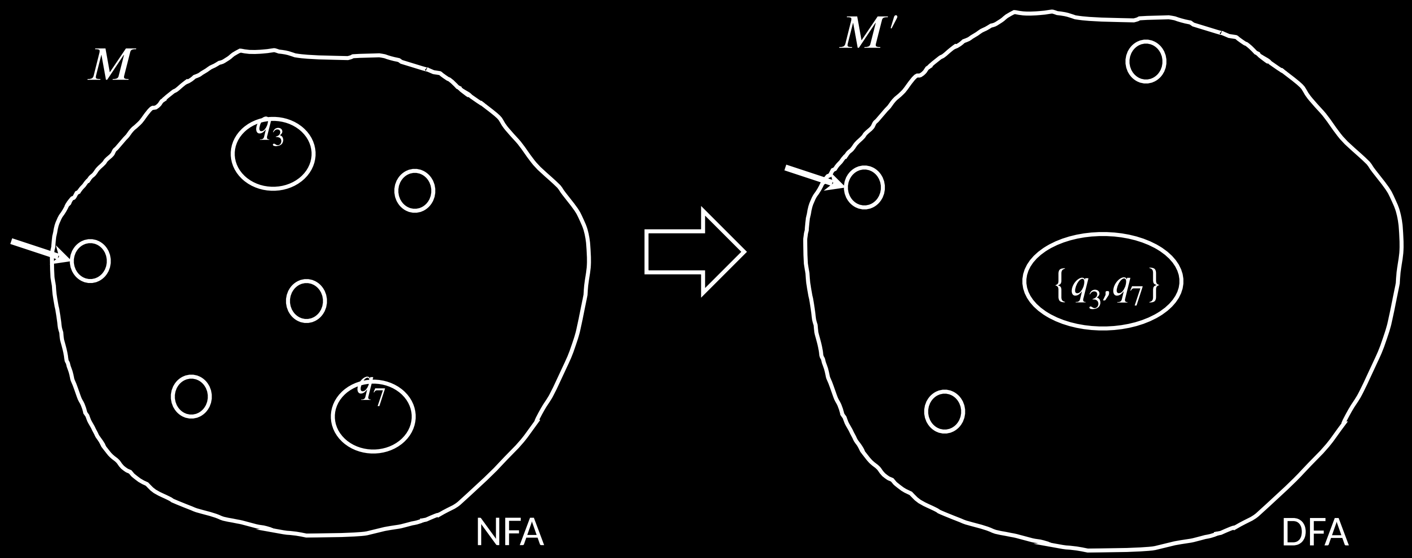

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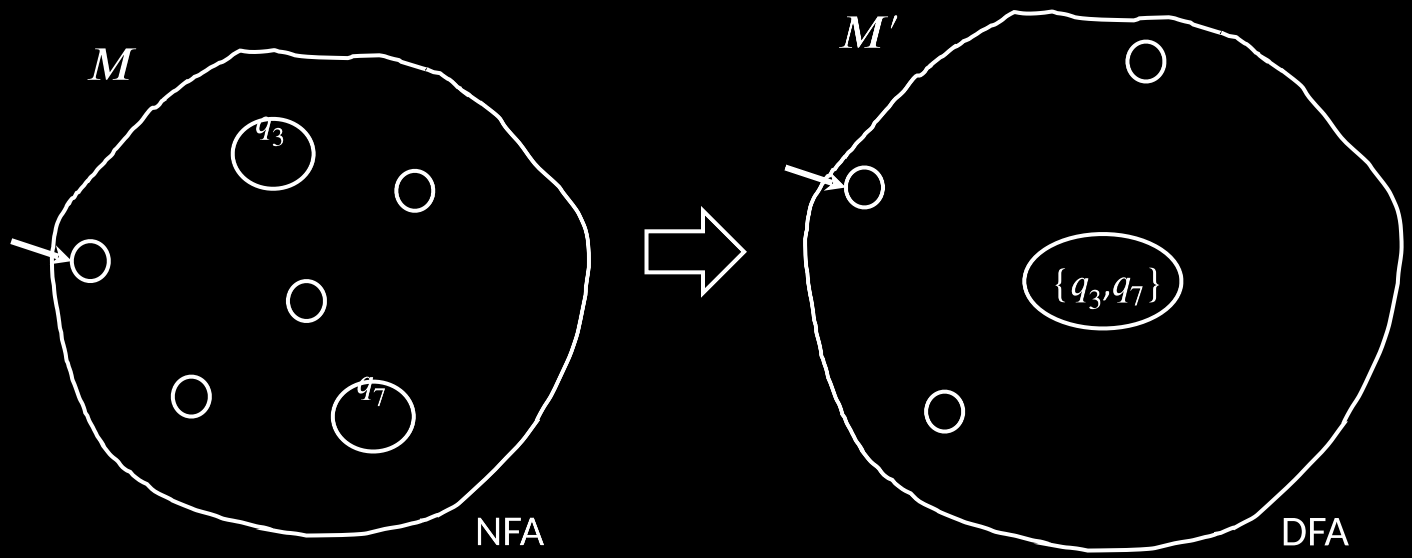

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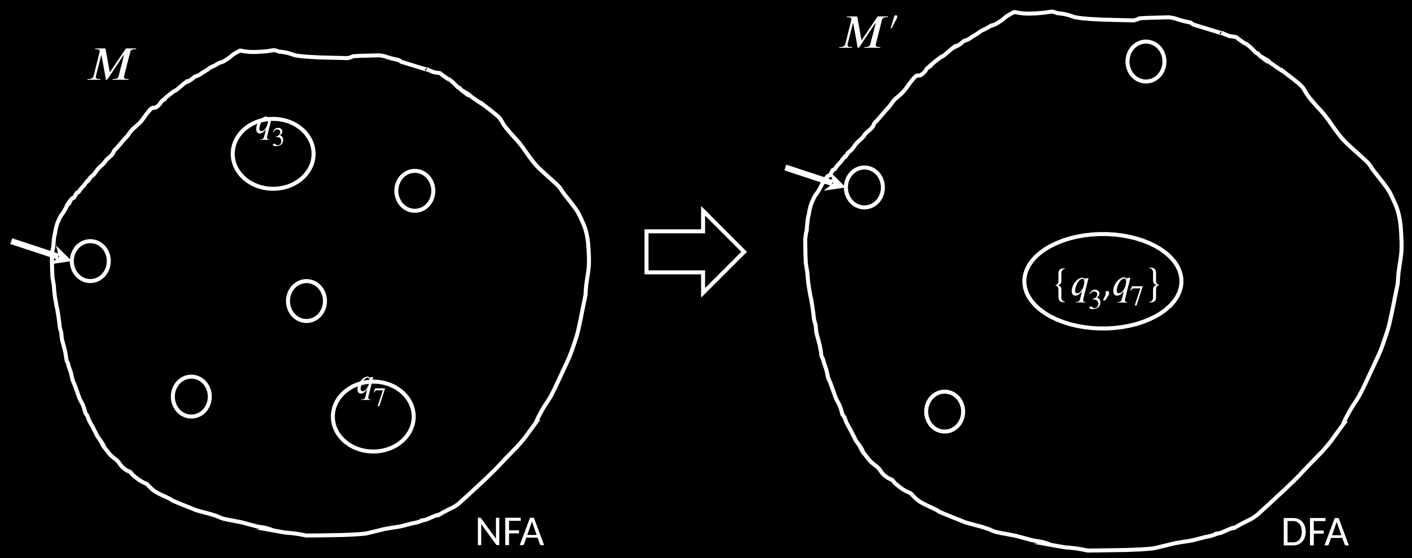

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$F' = \{ R \in Q' \mid R$ intersects $F \}$

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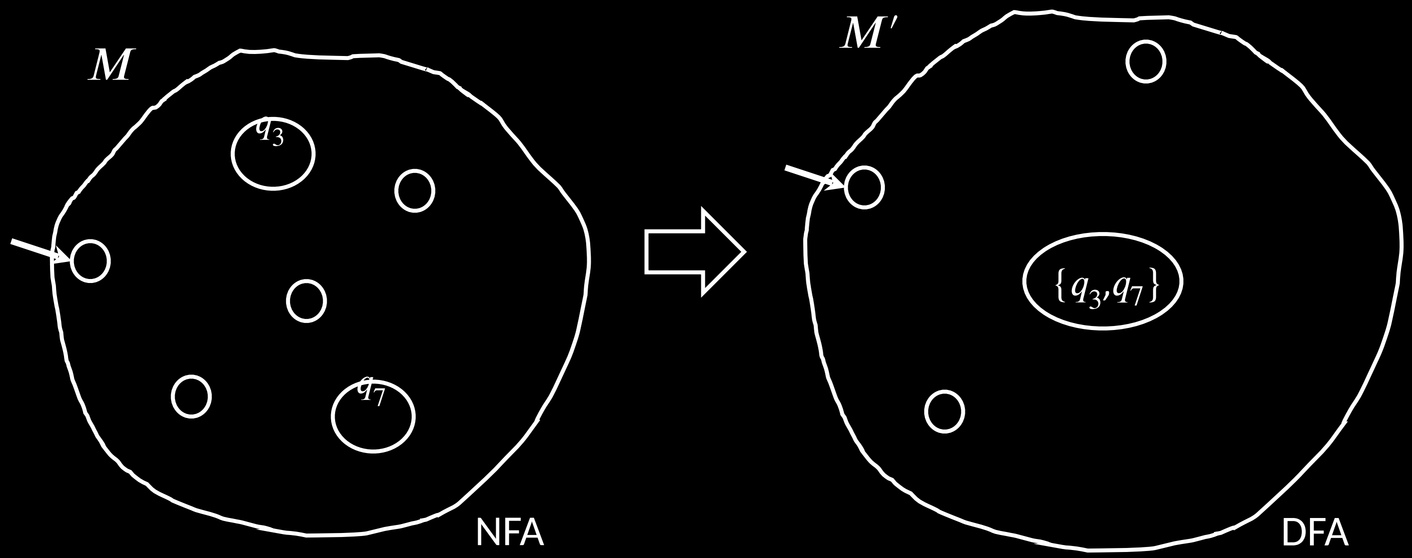

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Check-in 2.2

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IDEA: DFA M' keeps track of the subset of possible states in NFA M.

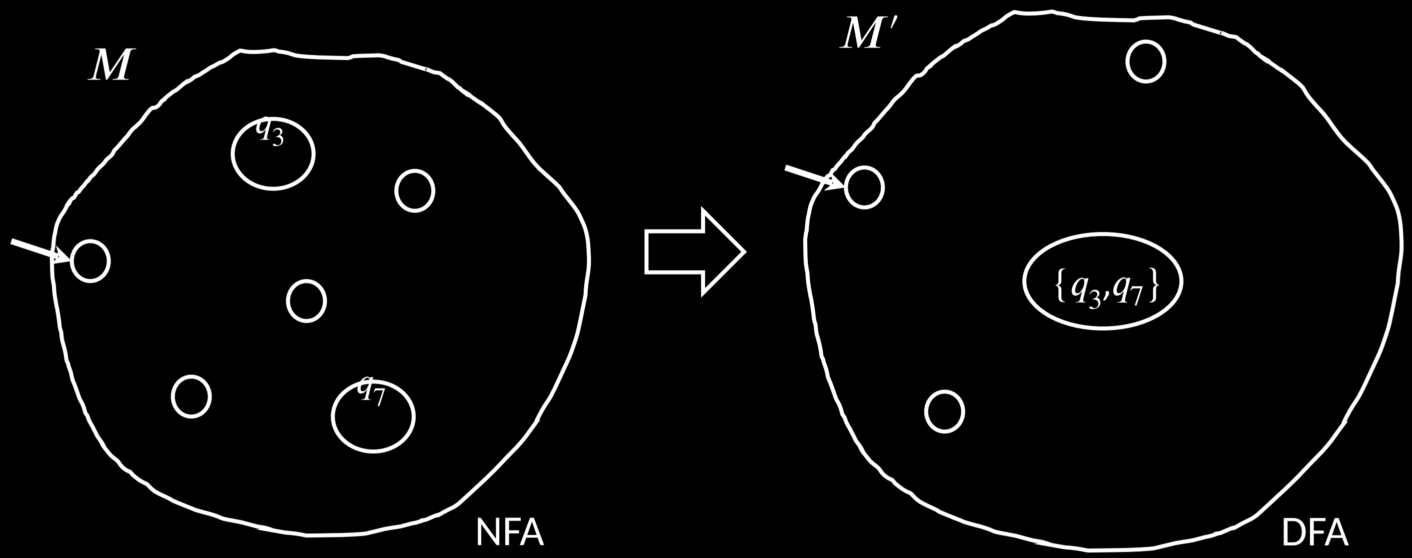
Check-in 2.2

If M has n states, how many states does M' have by this construction?

(a) $2n$

(b) n^2

(c) 2^n



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Check-in 2.2

Return to Closure Properties

Recall Theorem: If A_1, A_2 are regular languages, so is $A_1 \cup A_2$

(The class of regular languages is closed under union)

New Proof (sketch): Given DFAs M_1 and M_2 recognizing A_1 and A_2

Construct NFA M recognizing $A_1 \cup A_2$

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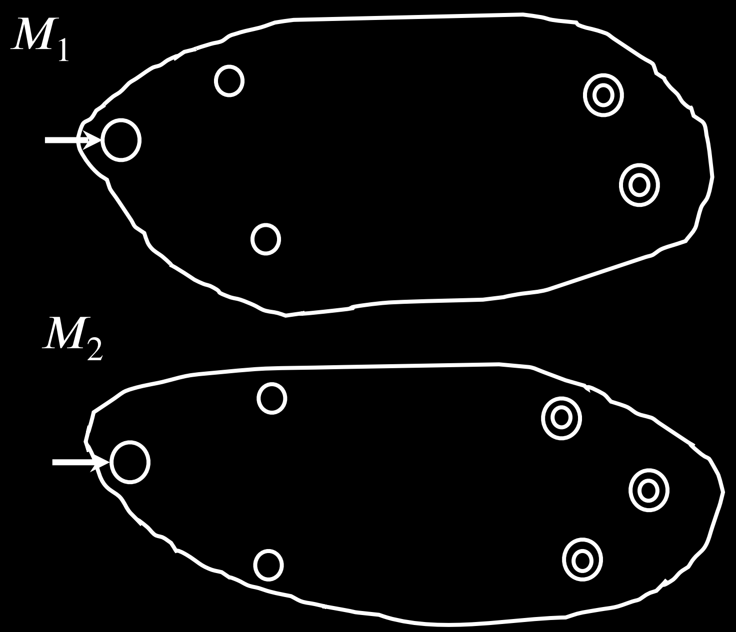

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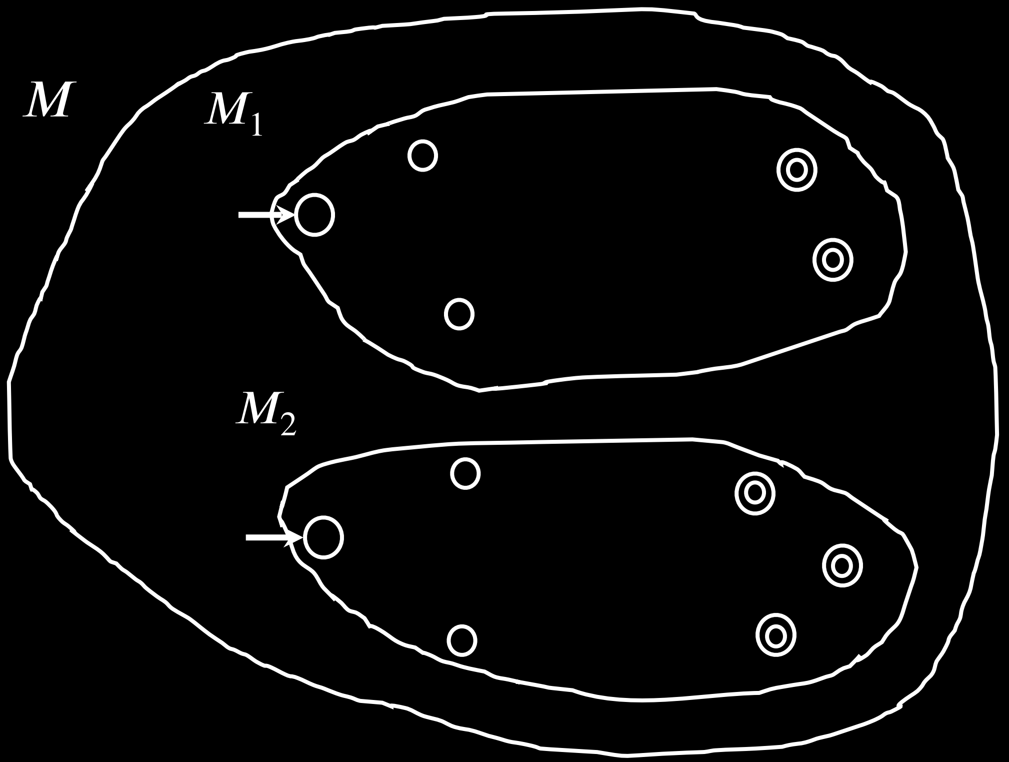

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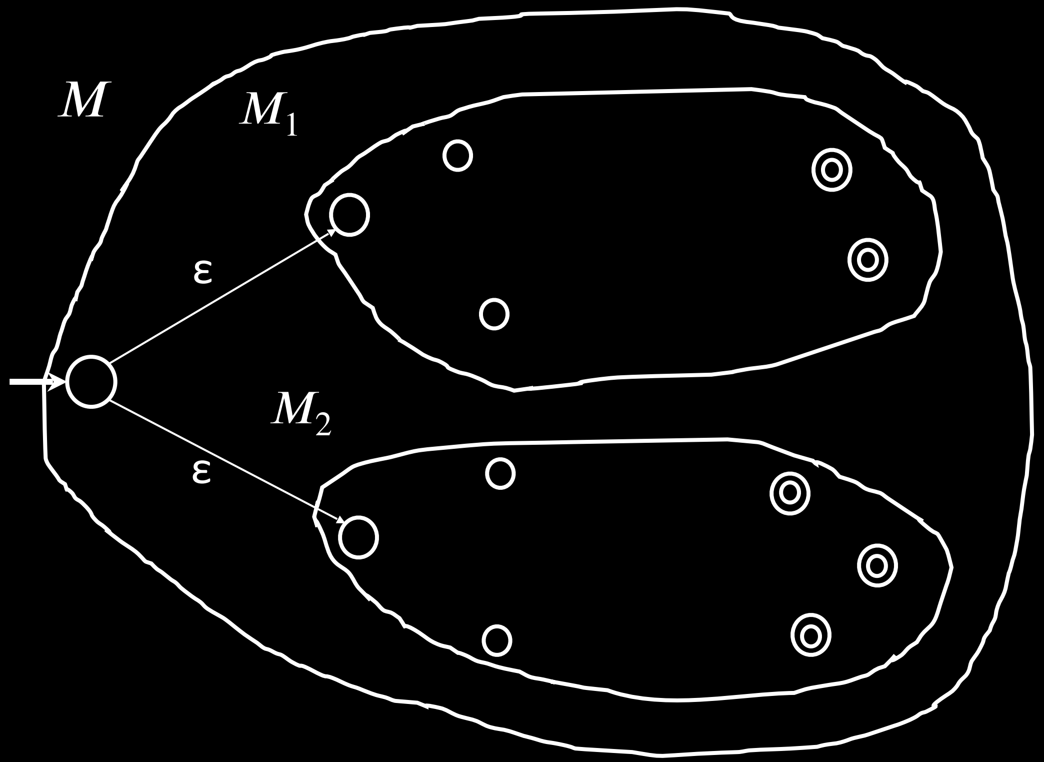

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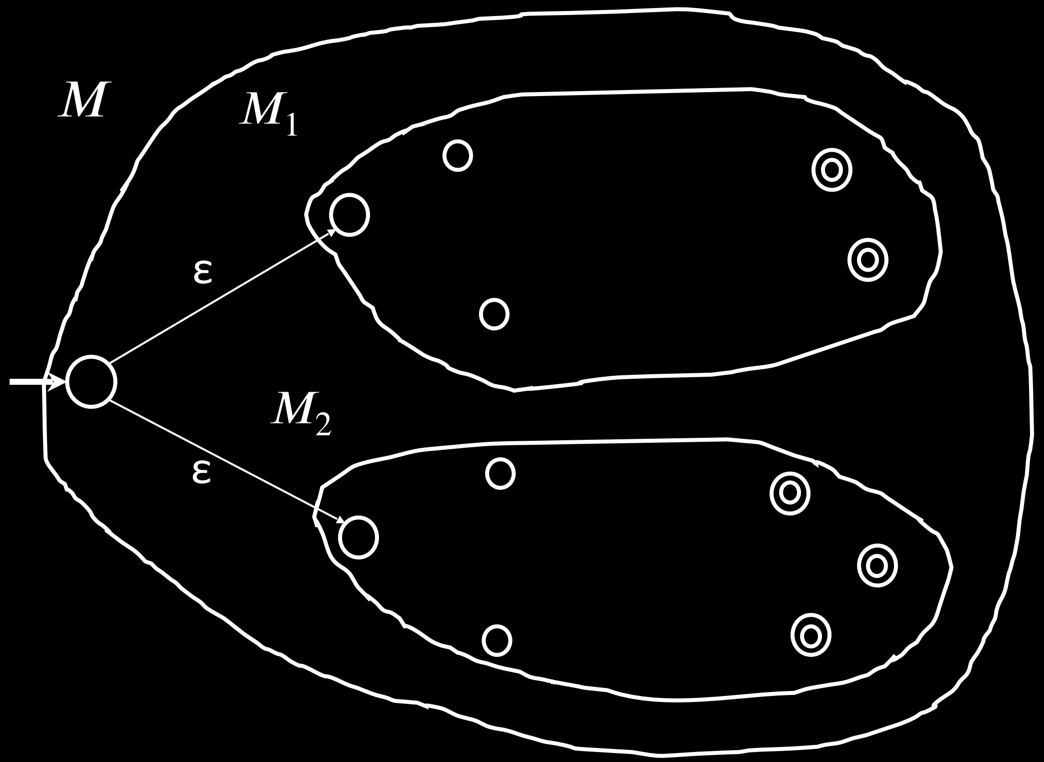

<u>Nondeterminism</u>

parallelism

vs

guessing

Closure under $\circ$ (concatenation)

Theorem: If A_1, A_2 are regular languages, so is A_1A_2

Closure under ∘ (concatenation)

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Proof sketch: Given DFAs M_1 and M_1 recognizing A_1 and A_2

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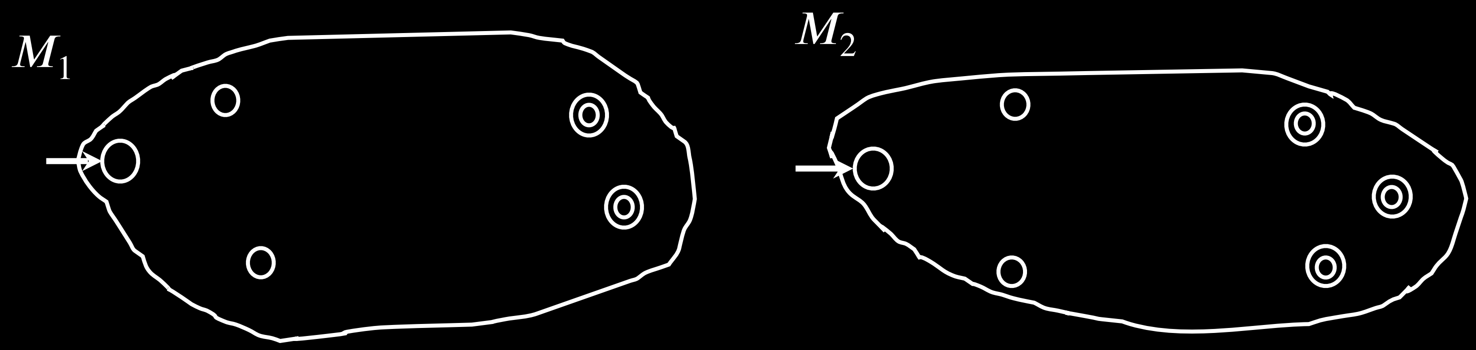

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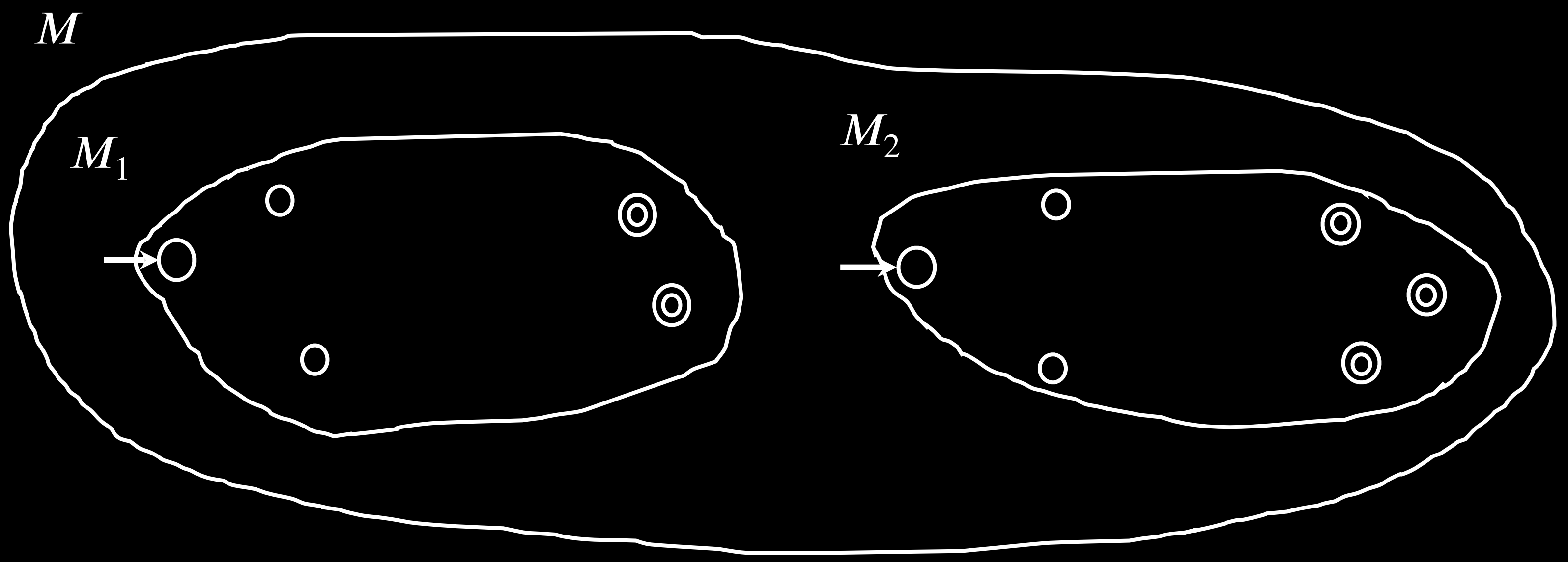

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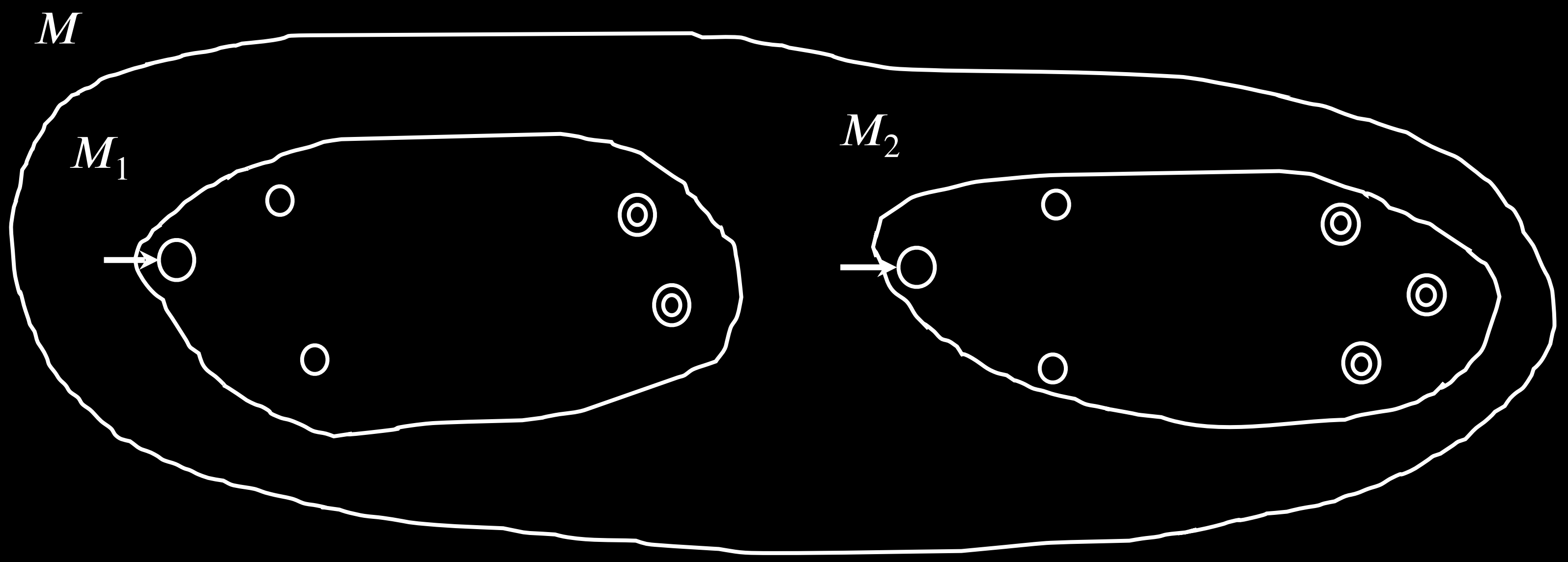

M should accept input w

if $w = xy$ where

M_1 accepts x and M_2 accepts y.

$w = x \mid y$

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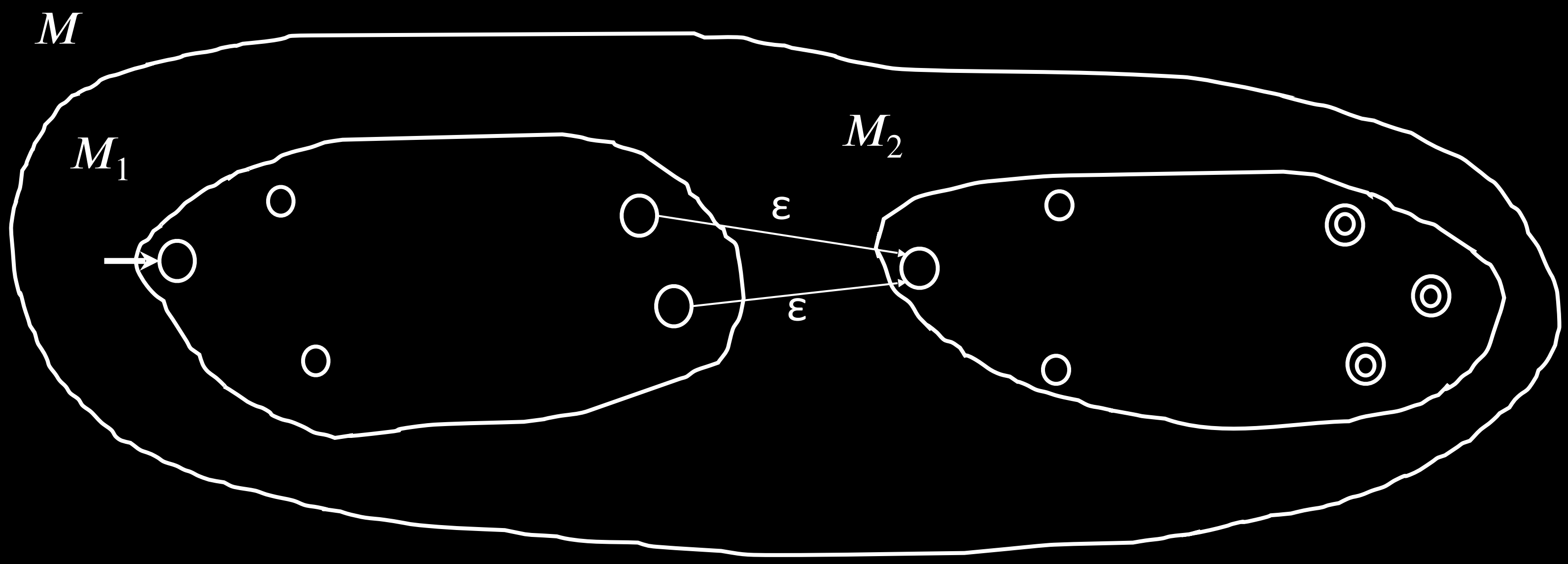

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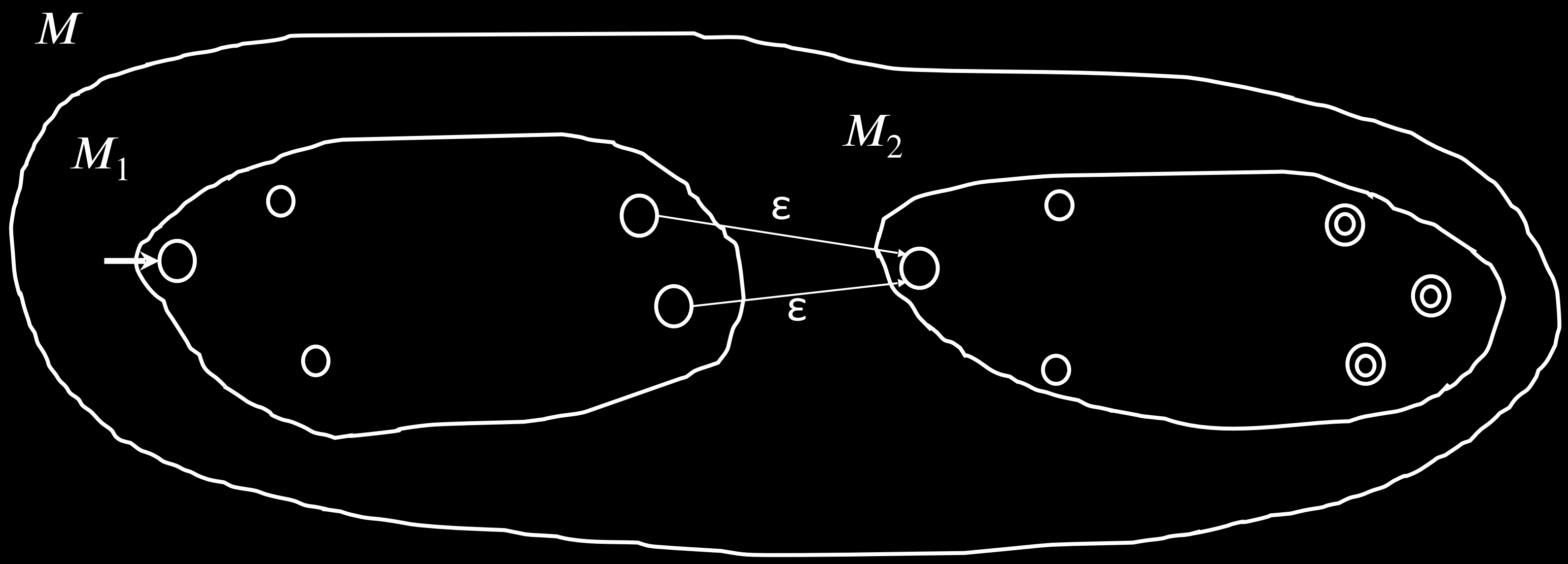

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Nondeterministic M' has the option to jump to M_2 when M_1 accepts.

Closure under $*$ (star)

Theorem: If A is a regular language, so is A^*

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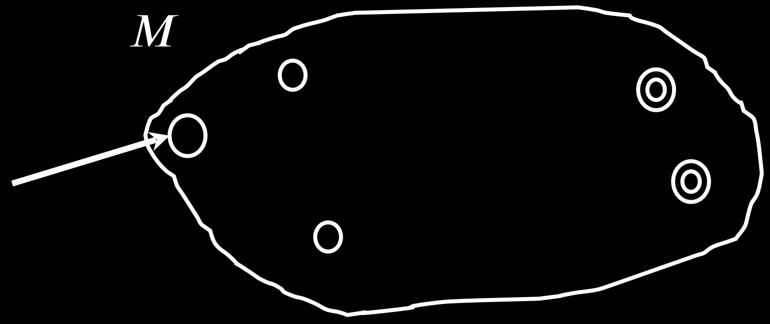

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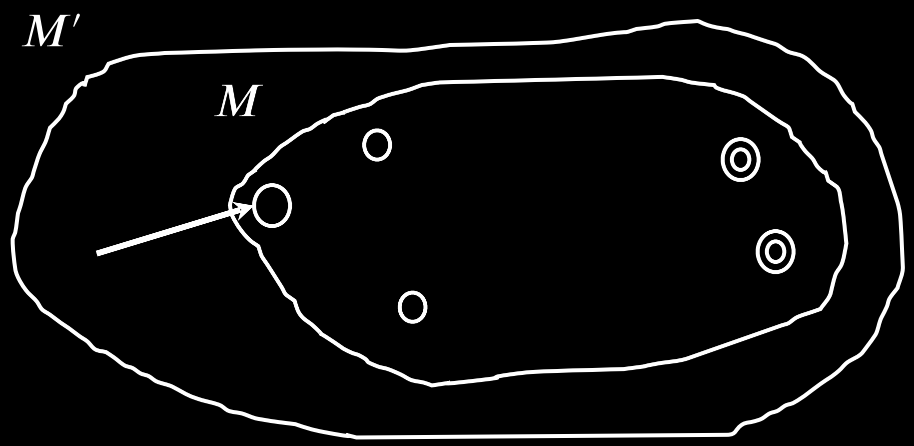

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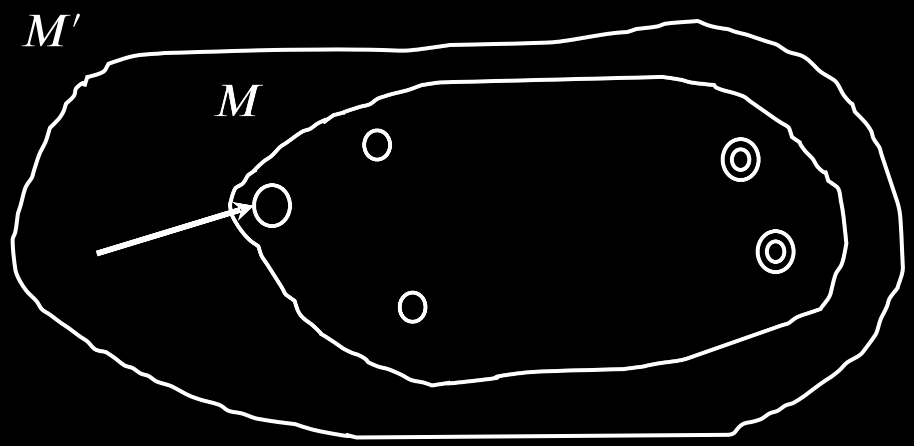

M' should accept input w

if $w = x_1 x_2 \ldots\ x_k$

where $k \geq 0$ and M accepts each x_i

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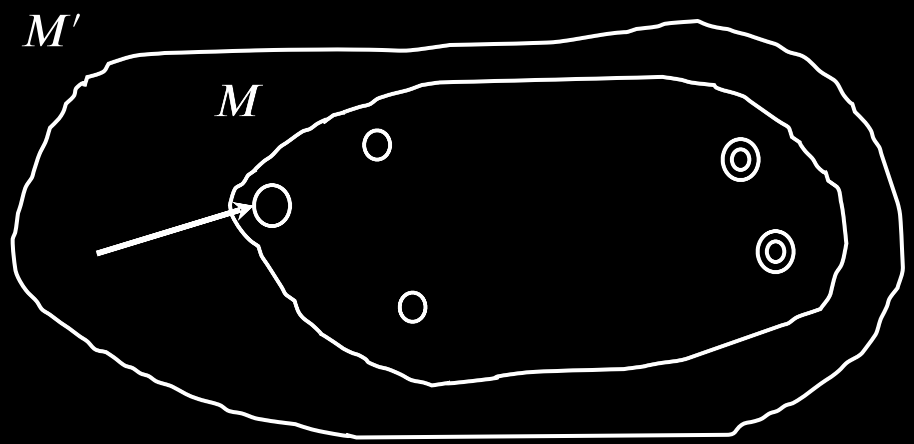

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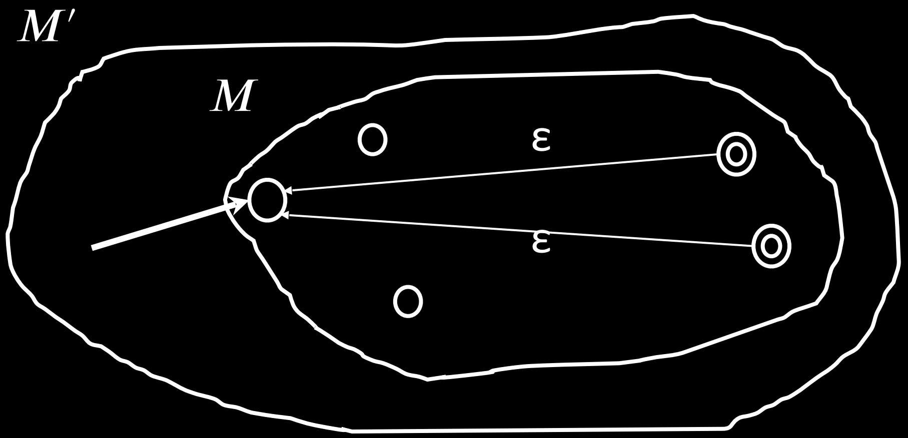

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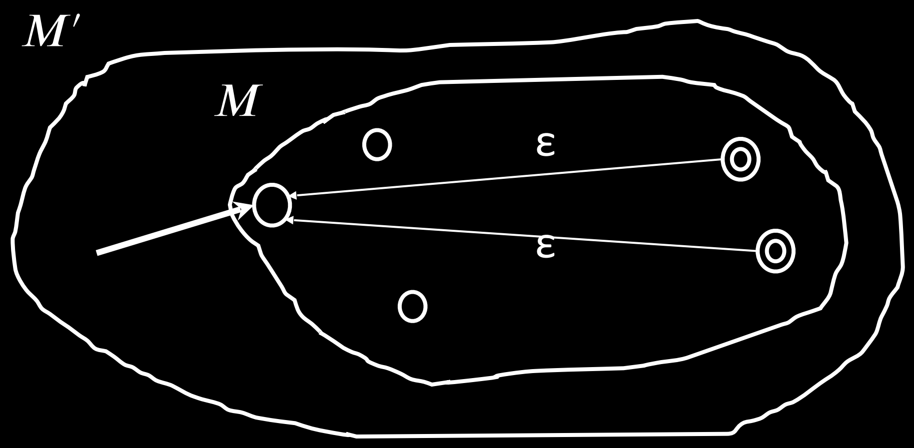

Make sure M' accepts ε

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if $w = x_1 x_2 \ldots x_k$

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$w = x_1 \mid x_2 \mid x_3 \mid x_4$

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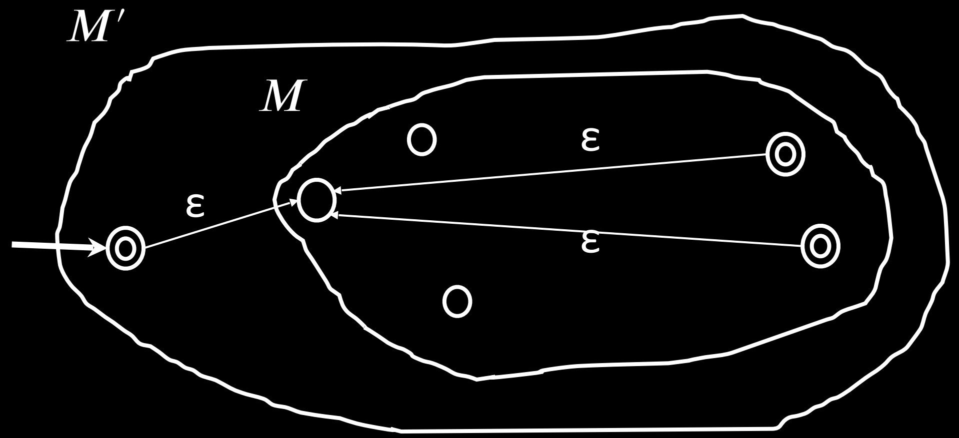

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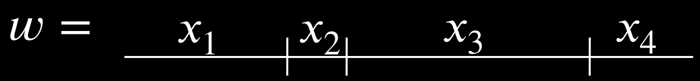

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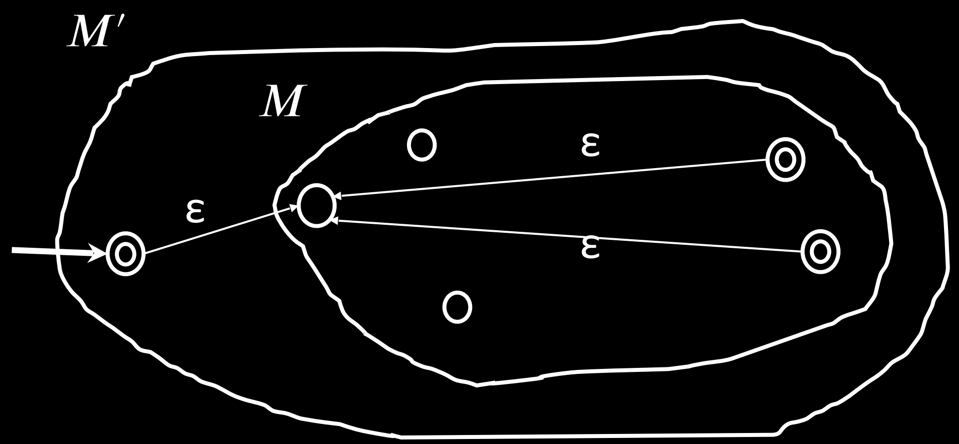

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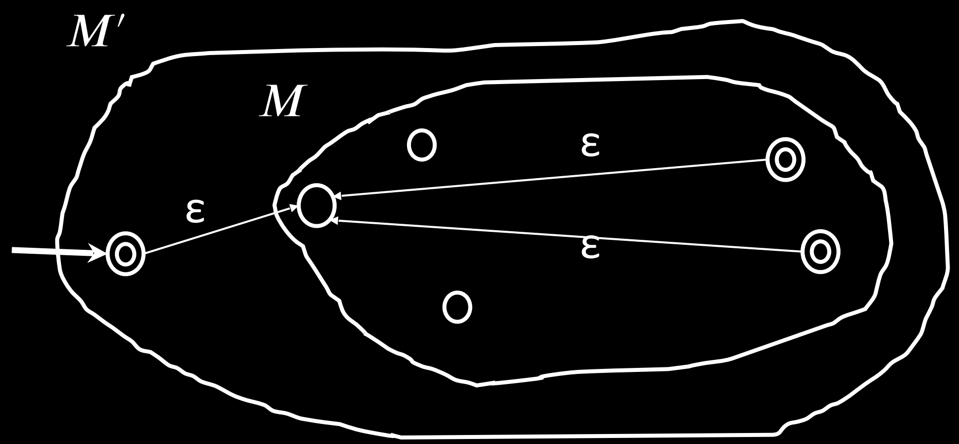
Check-in 2.3

Closure under $*$ (star)

Theorem: If A is a regular language, so is A^*

Proof sketch: Given DFA M recognizing A

Construct NFA M' recognizing A^*



Make sure M' accepts ε

Check-in 2.3

If M has n states, how many states does M' have by this construction?

(a) n

(b) $n+1$

(c) $2n$

Check-in 2.3

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Example:

$$(a \cup ab)^*$$

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Theorem: If R is a regular expr and $A = L(R)$ then A is regular

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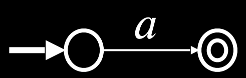

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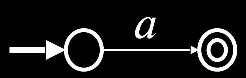

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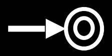

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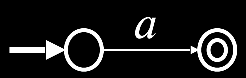

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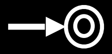

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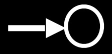

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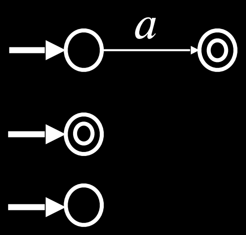

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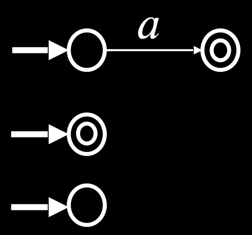

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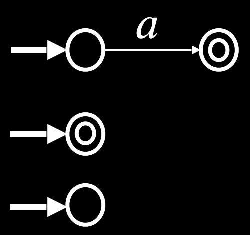

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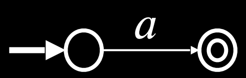

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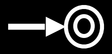

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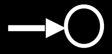

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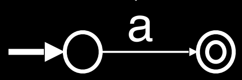

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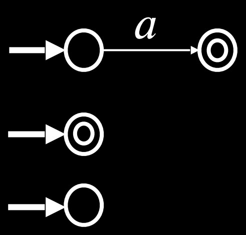

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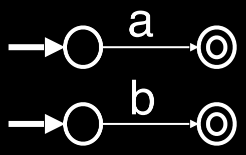

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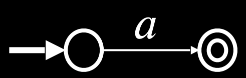
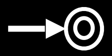
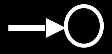
$R = a$ for $a \in \Sigma$ 

$R = \varepsilon$ 

$R = \varnothing$ 

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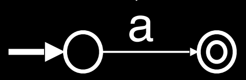
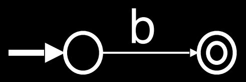
a: 

b: 

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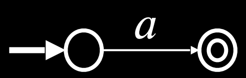

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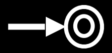

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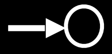

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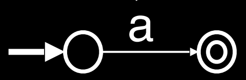

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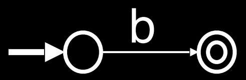

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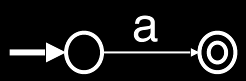

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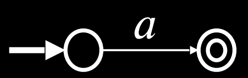

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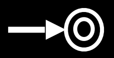

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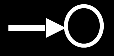

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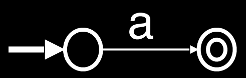

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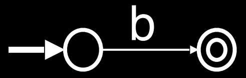

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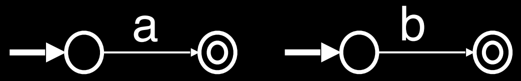

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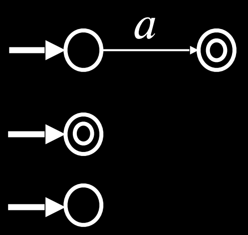

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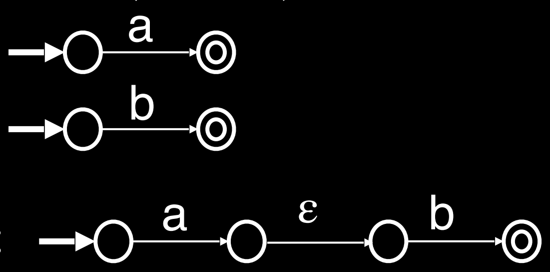

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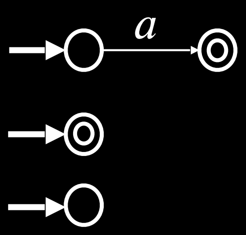

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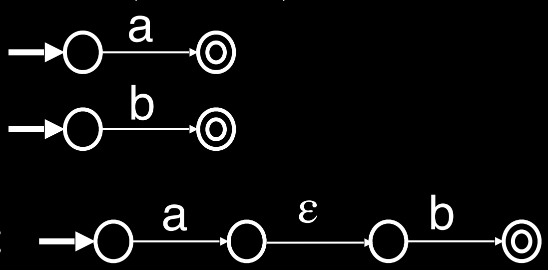

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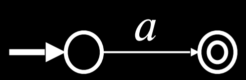

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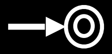

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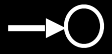

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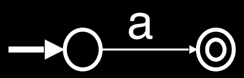

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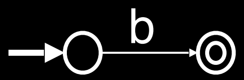

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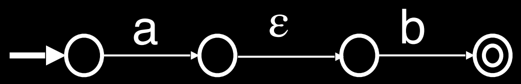

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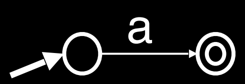

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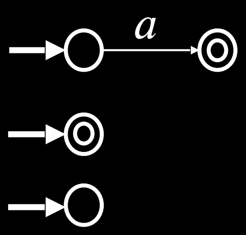

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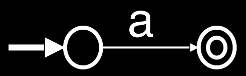

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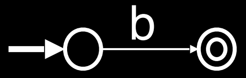

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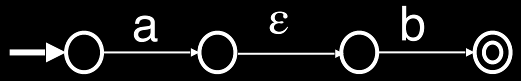

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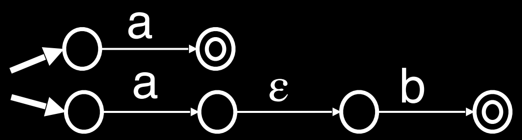

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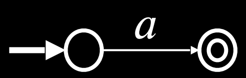

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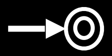

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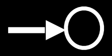

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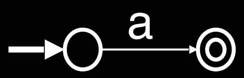

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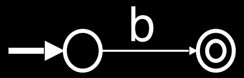

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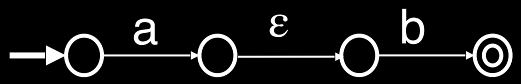

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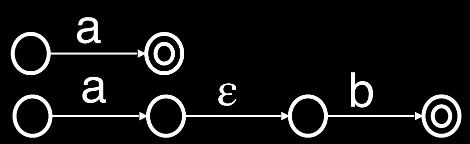

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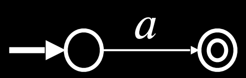

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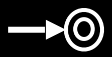

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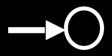

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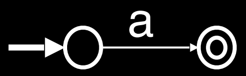

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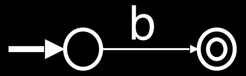

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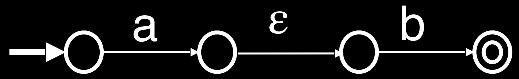

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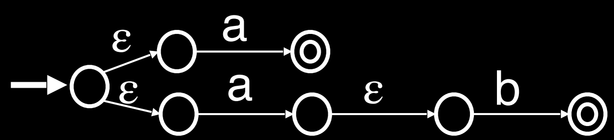

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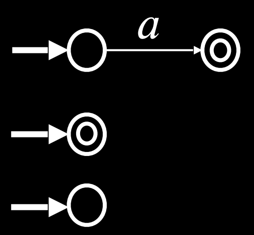

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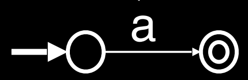

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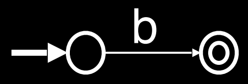

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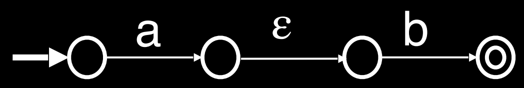

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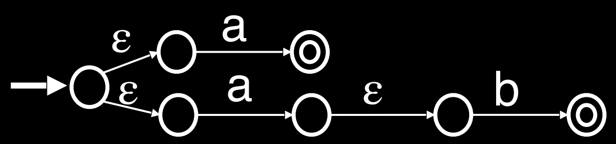

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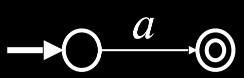

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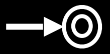

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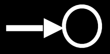

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Convert $(a \cup ab)^*$ to equivalent NFA

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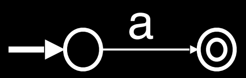

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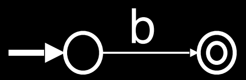

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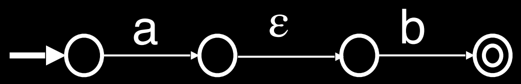

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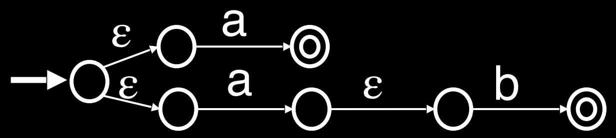

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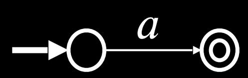

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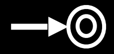

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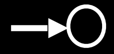

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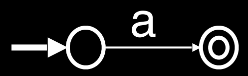

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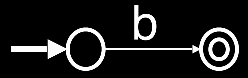

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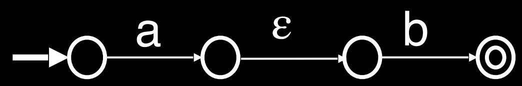

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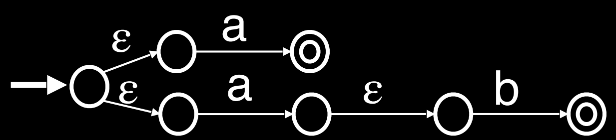

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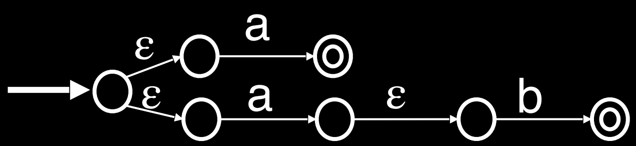

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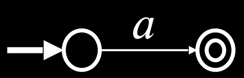

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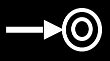

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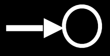

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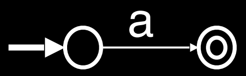

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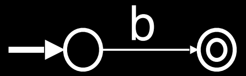

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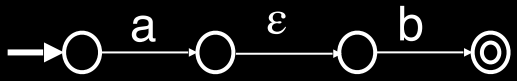

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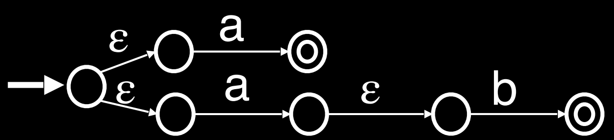

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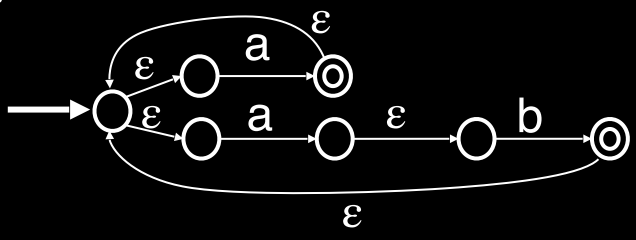

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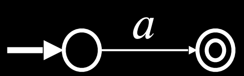

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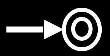

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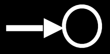

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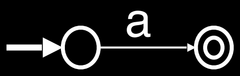

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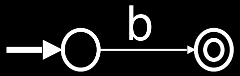

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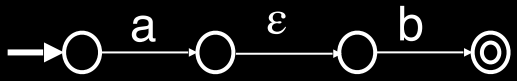

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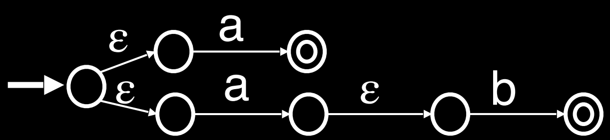

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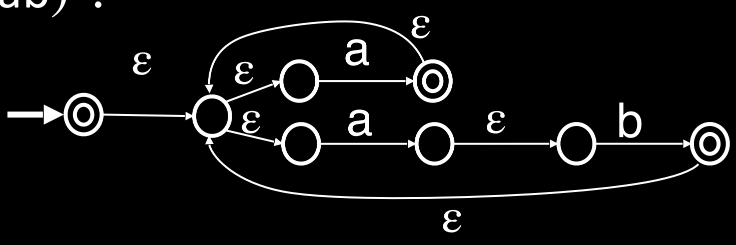

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Regular expression R $\Rightarrow$ M Finite automaton

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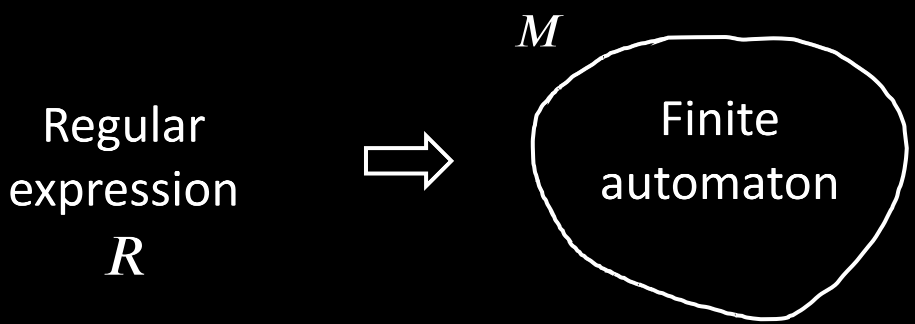

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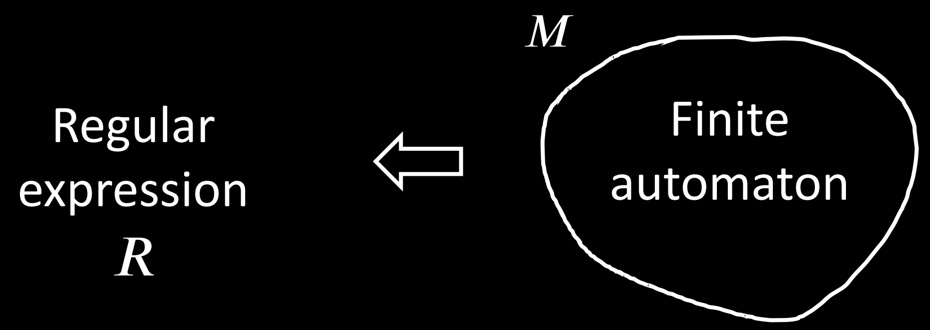

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Today's Theorem: If A is regular then $A = L(R)$ for some regular expr R

Proof: Give conversion DFA $M \rightarrow R$

WAIT! Need new concept first.

Generalized NFA

Defn: A Generalized Nondeterministic Finite Automaton (GNFA) is similar to an NFA, but allows regular expressions as transition labels

Generalized NFA

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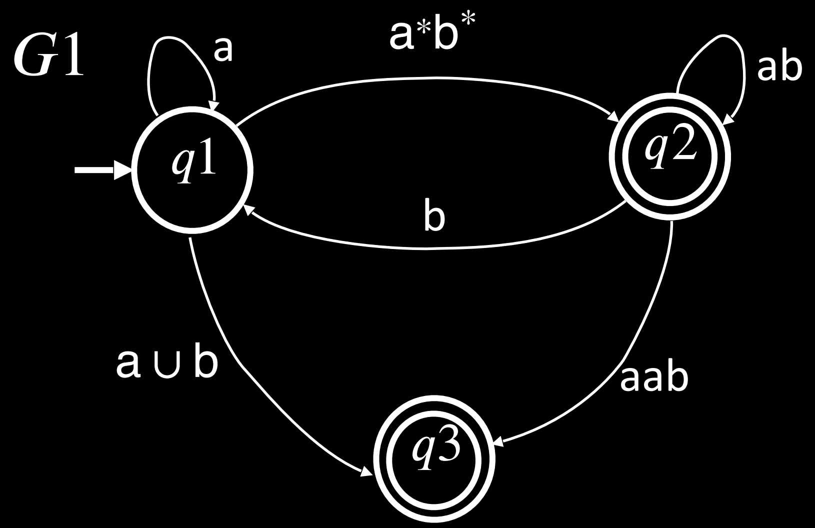

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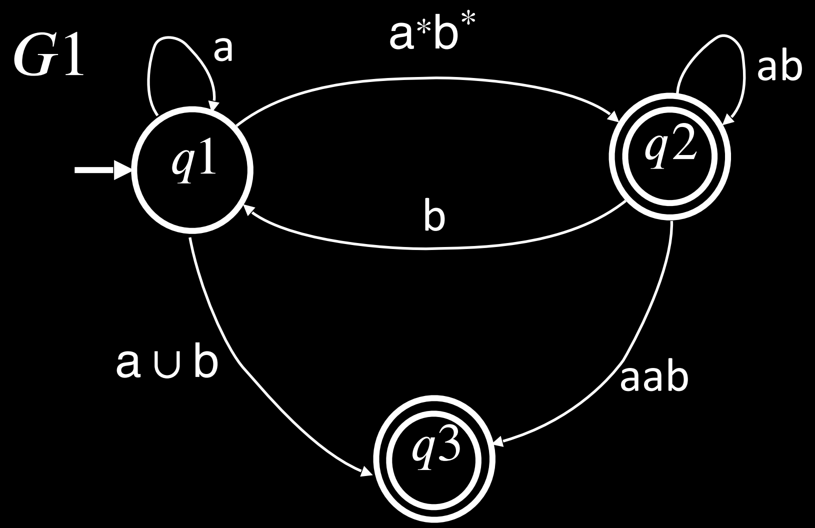

For convenience we will assume:

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We can easily modify a GNFA to have this <u>special form</u>.

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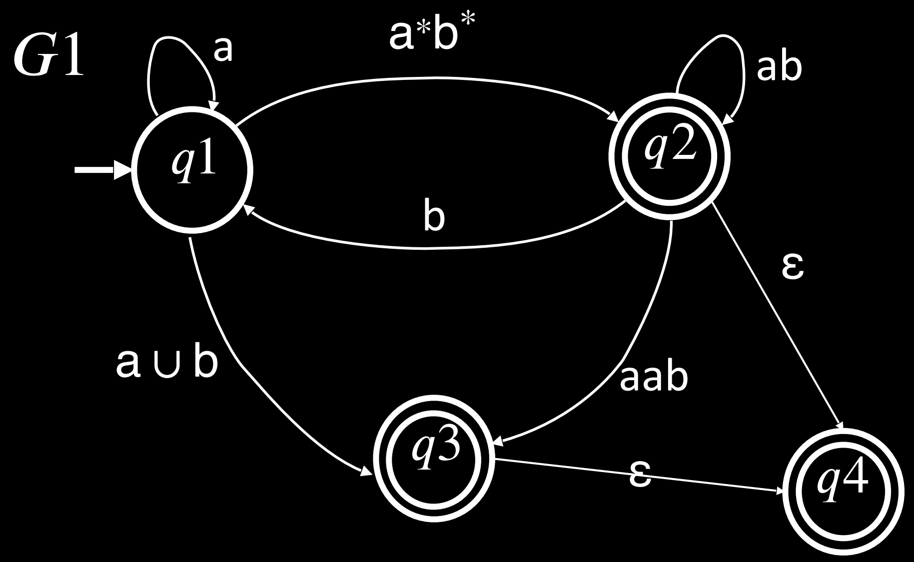

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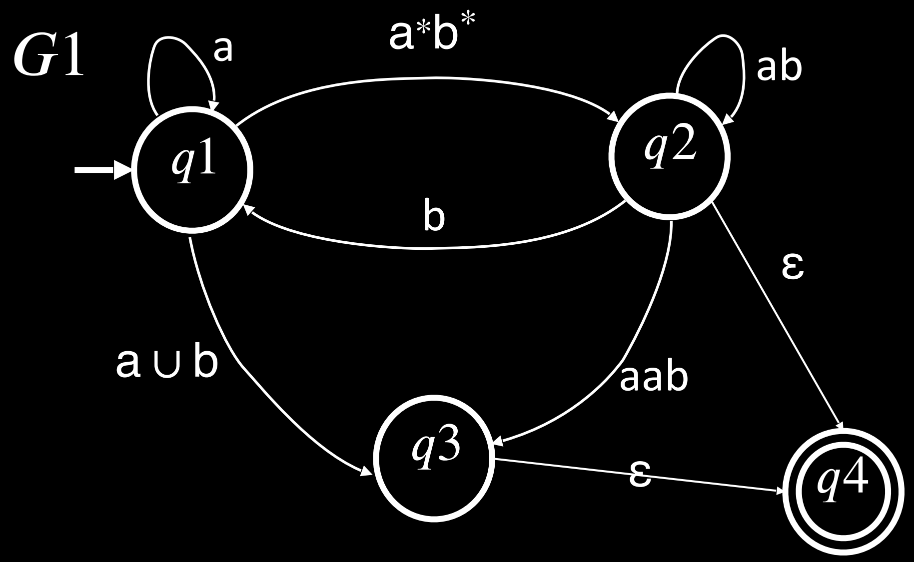

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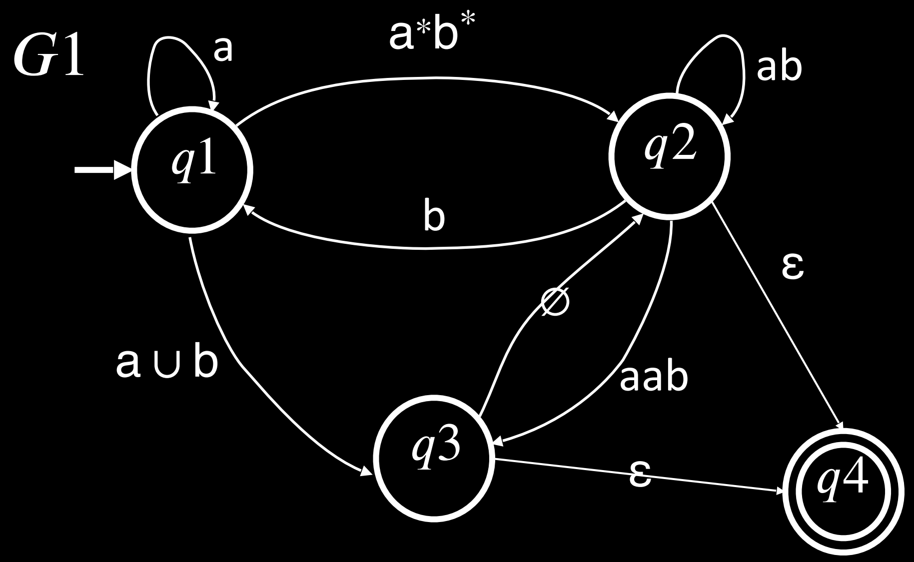

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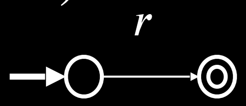
$G =$ 

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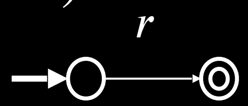
$G =$ 

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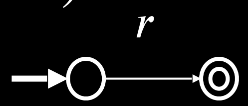
$G =$ 

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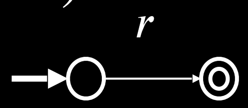
$G =$ 

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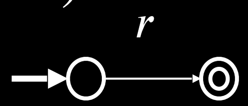
$G =$ 

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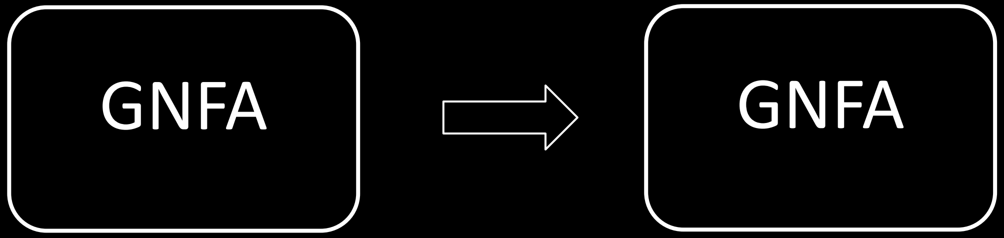

k states $\qquad$ $k - 1$ states

k-state GNFA $\rightarrow$ $(k-1)$-state GNFA

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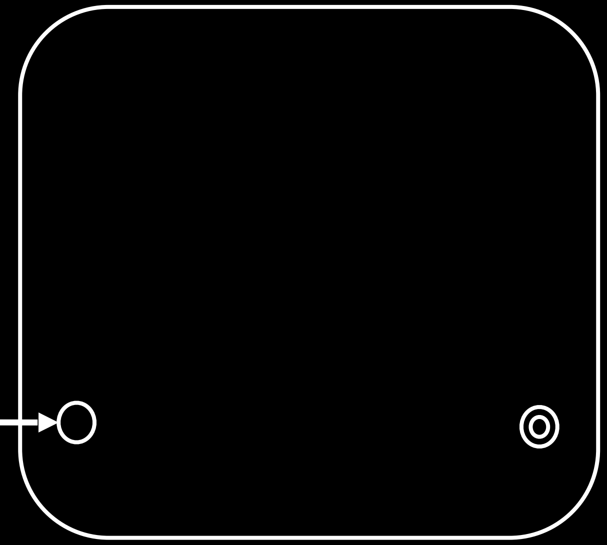

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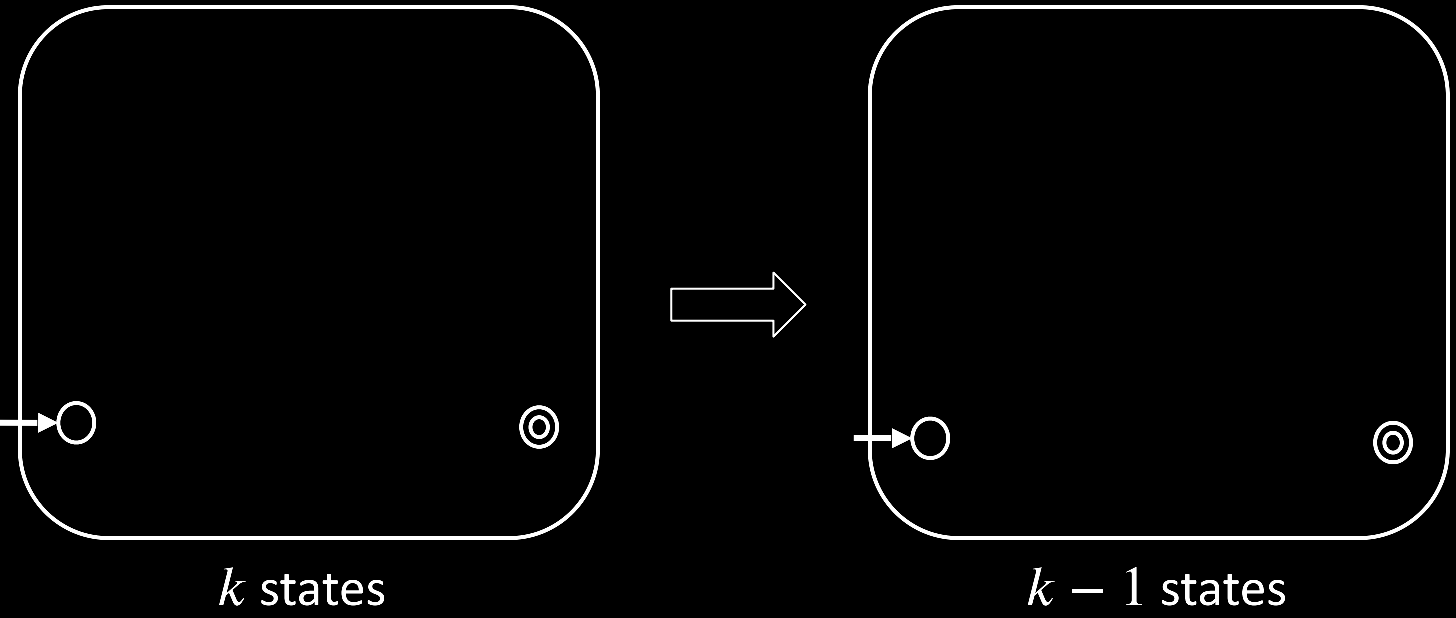

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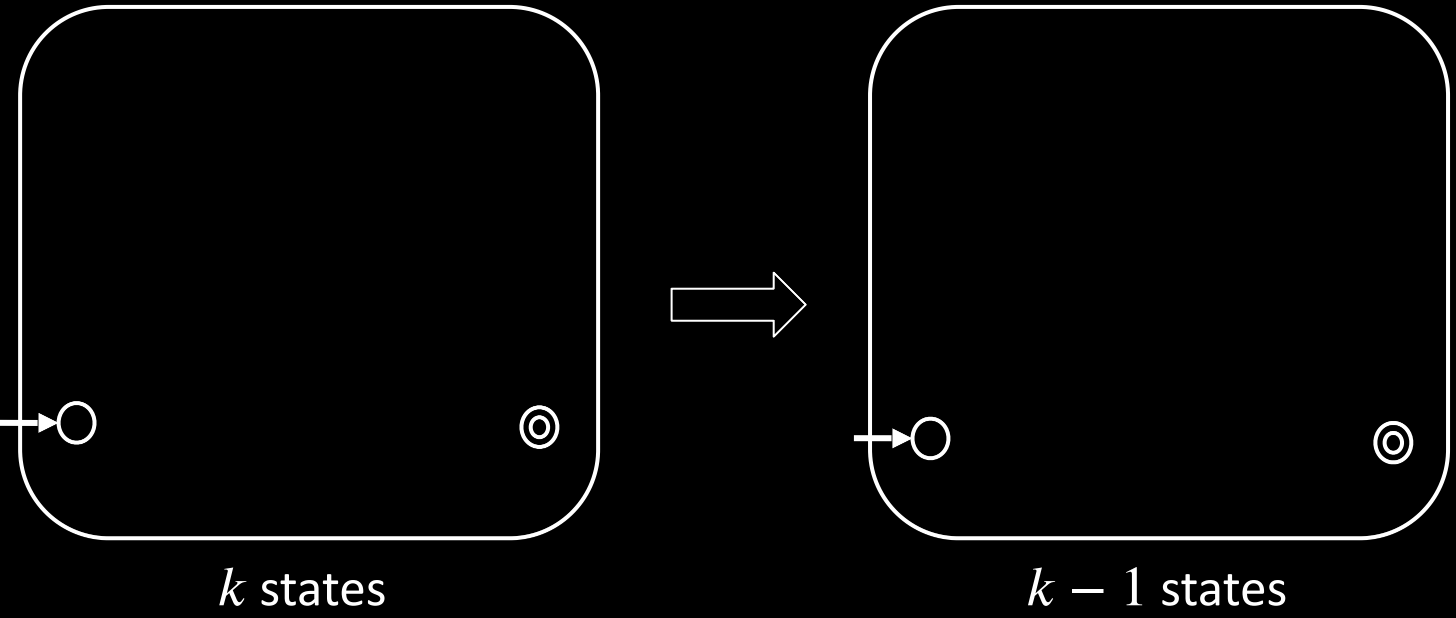

1. Pick any state x except the start and accept states.

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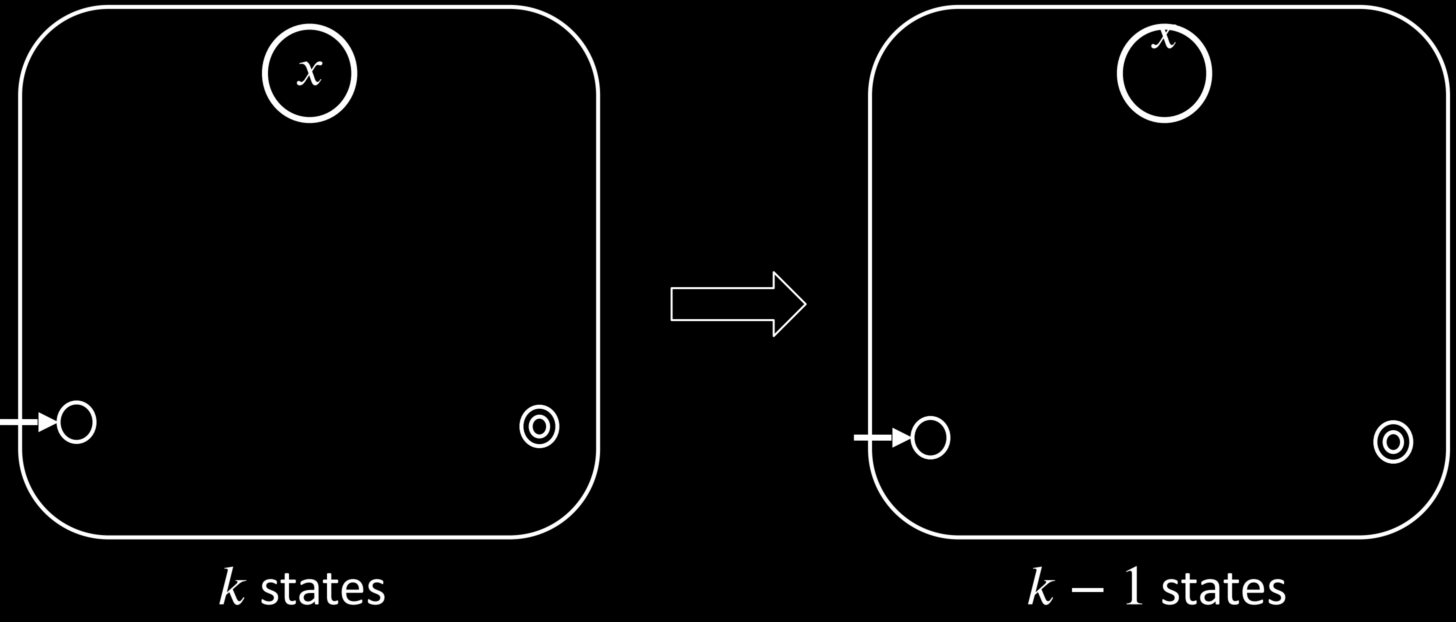

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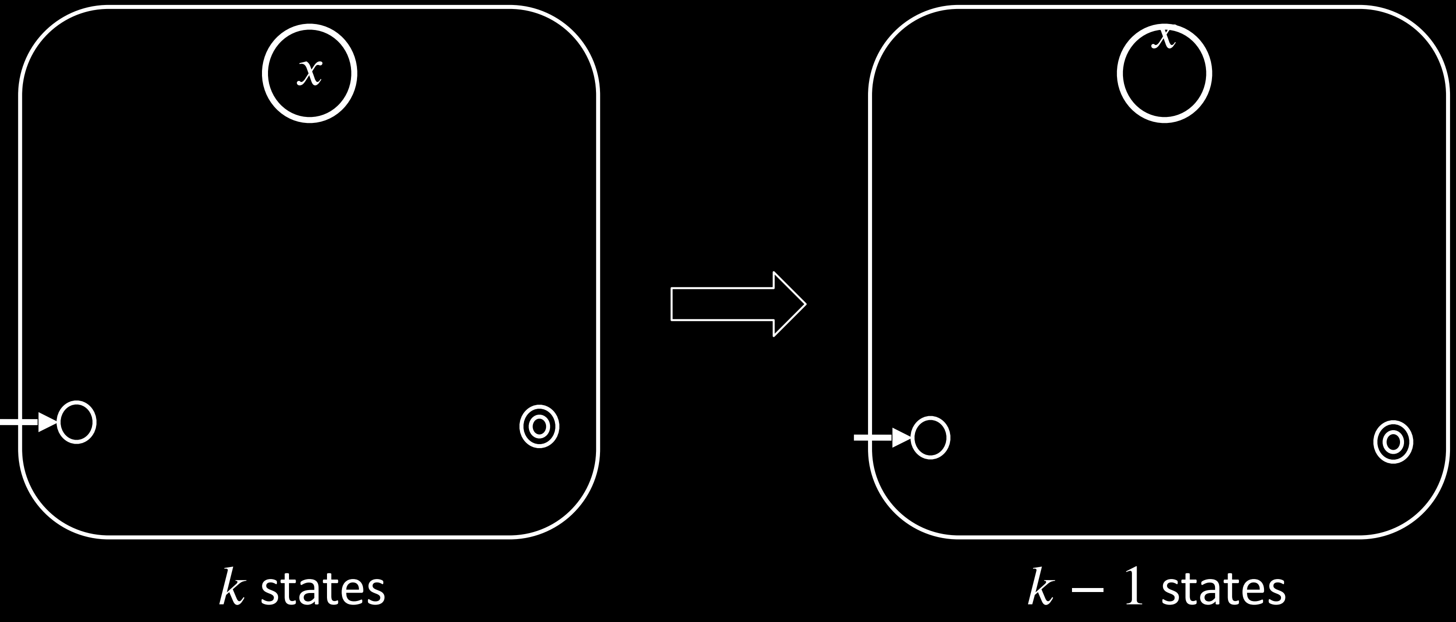

1. Pick any state x except the start and accept states.
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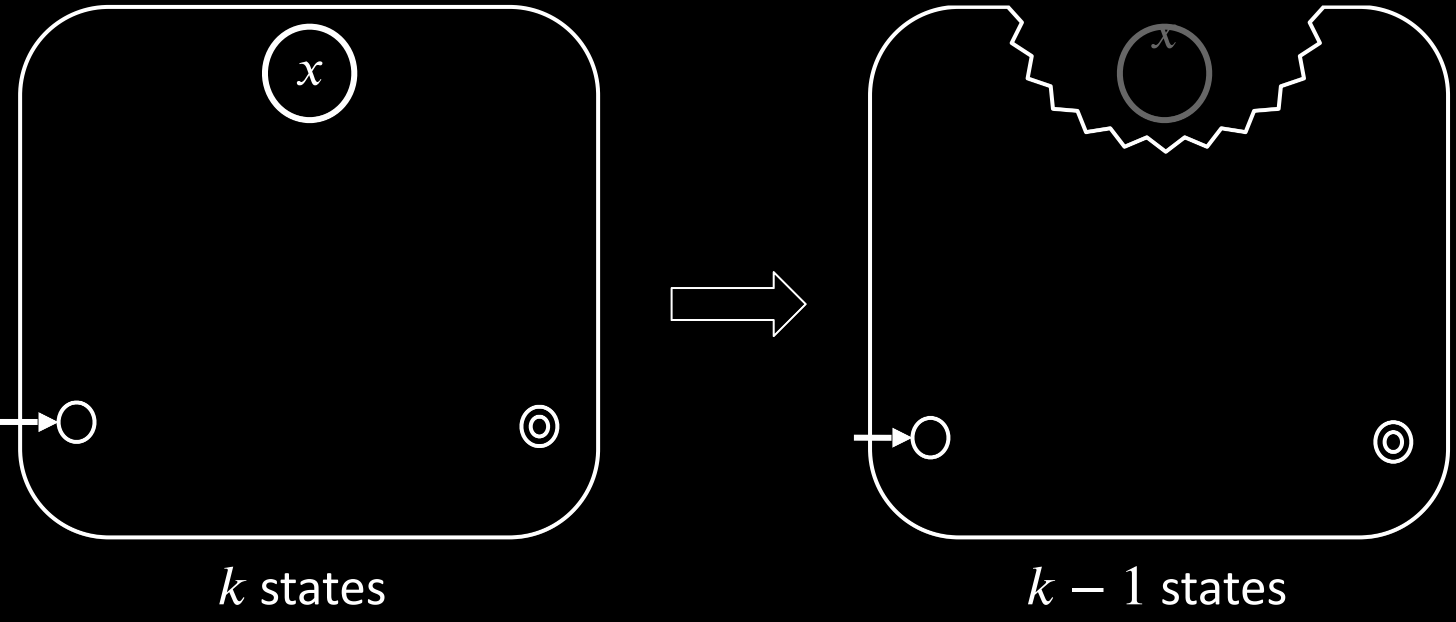

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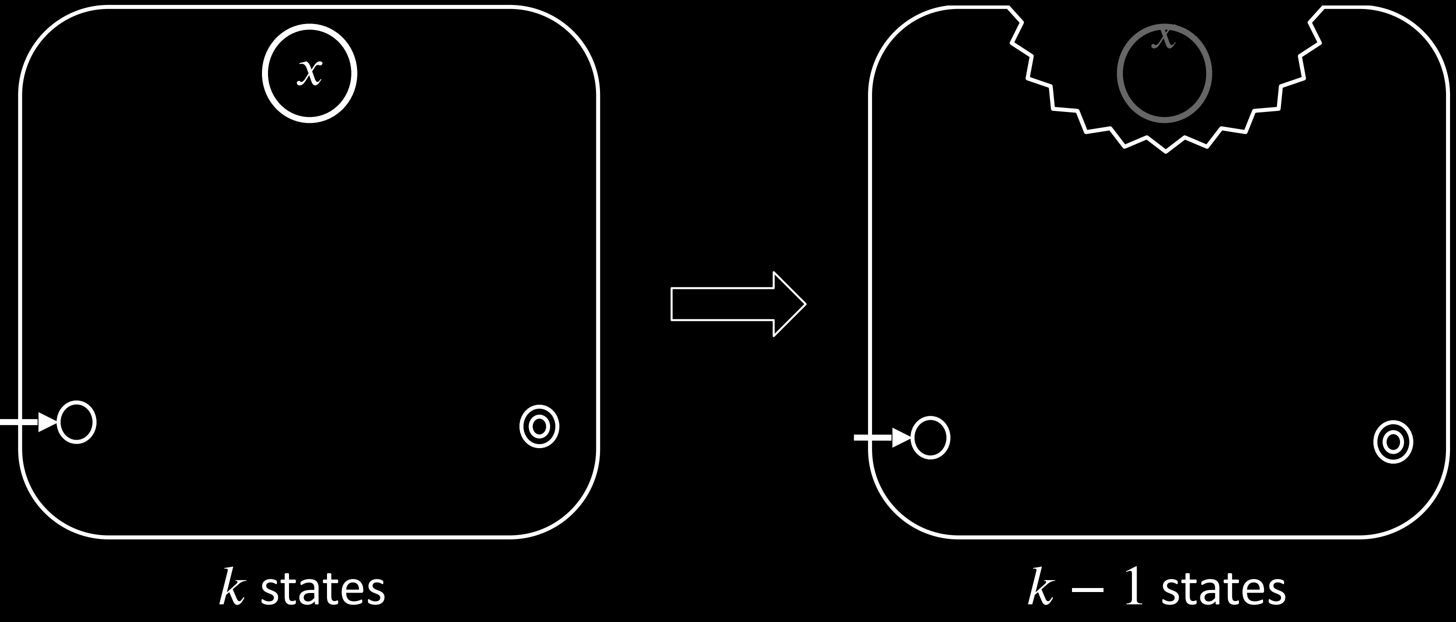

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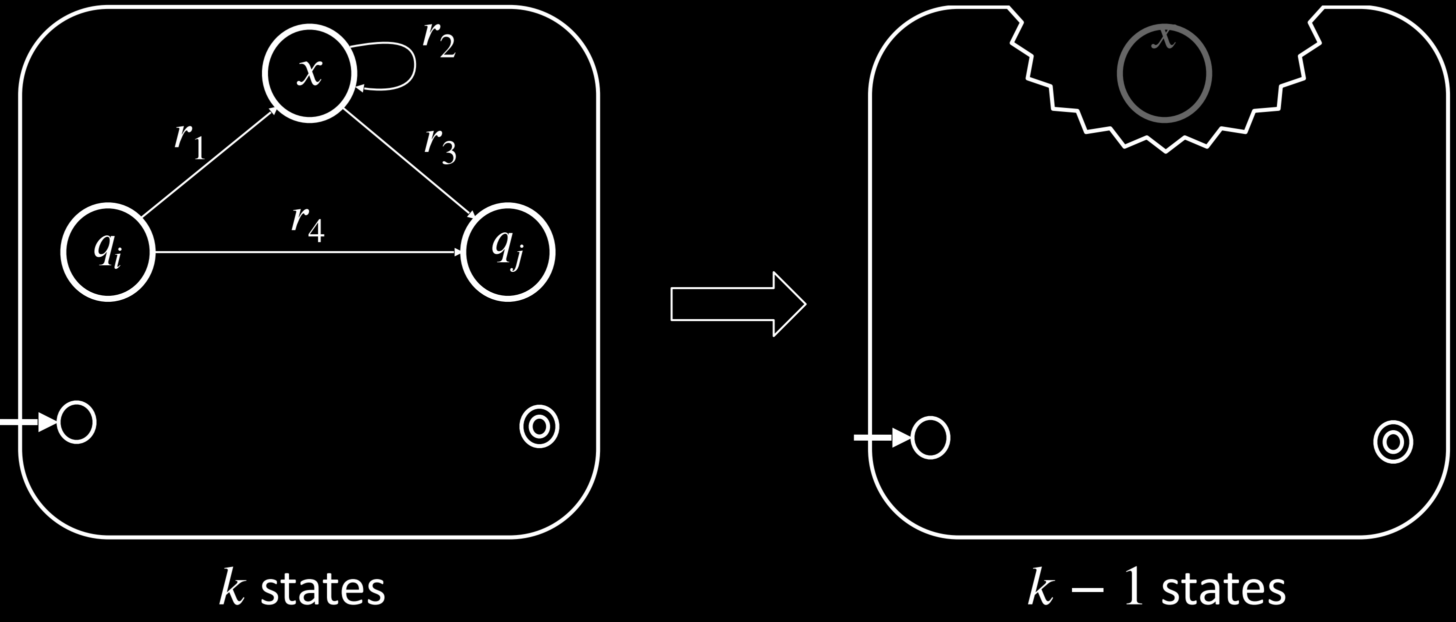

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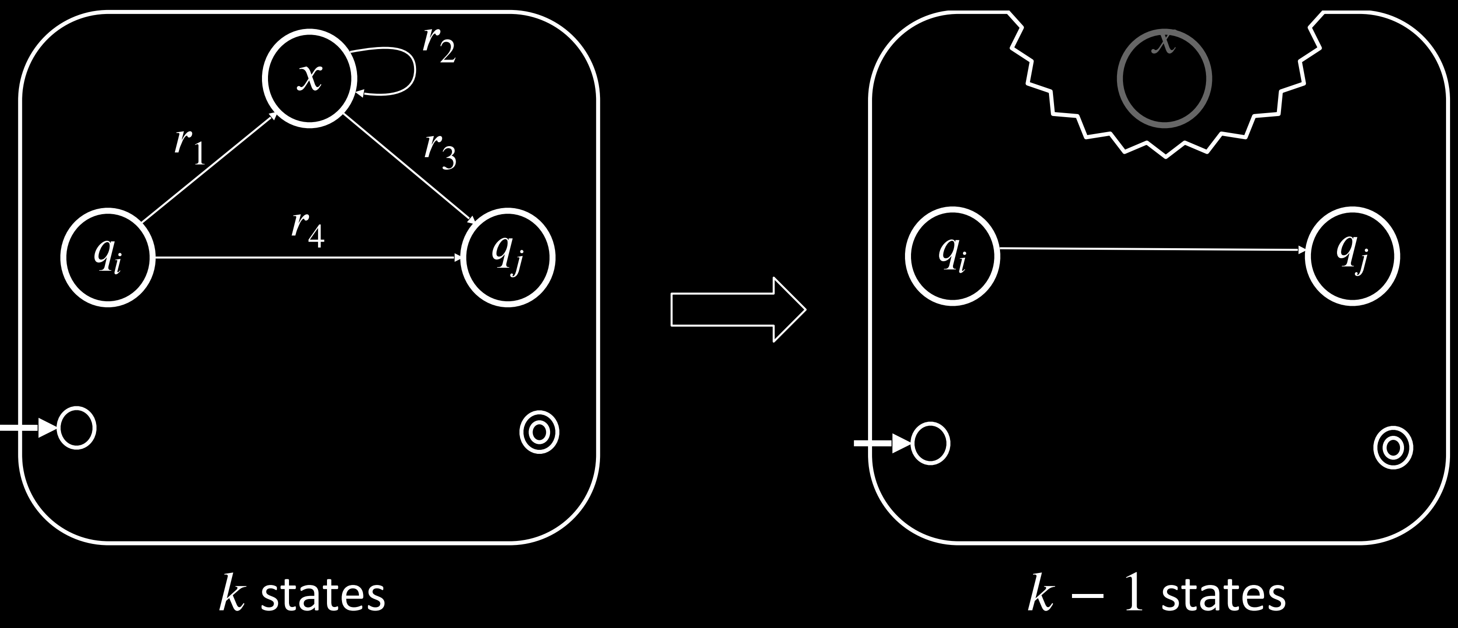

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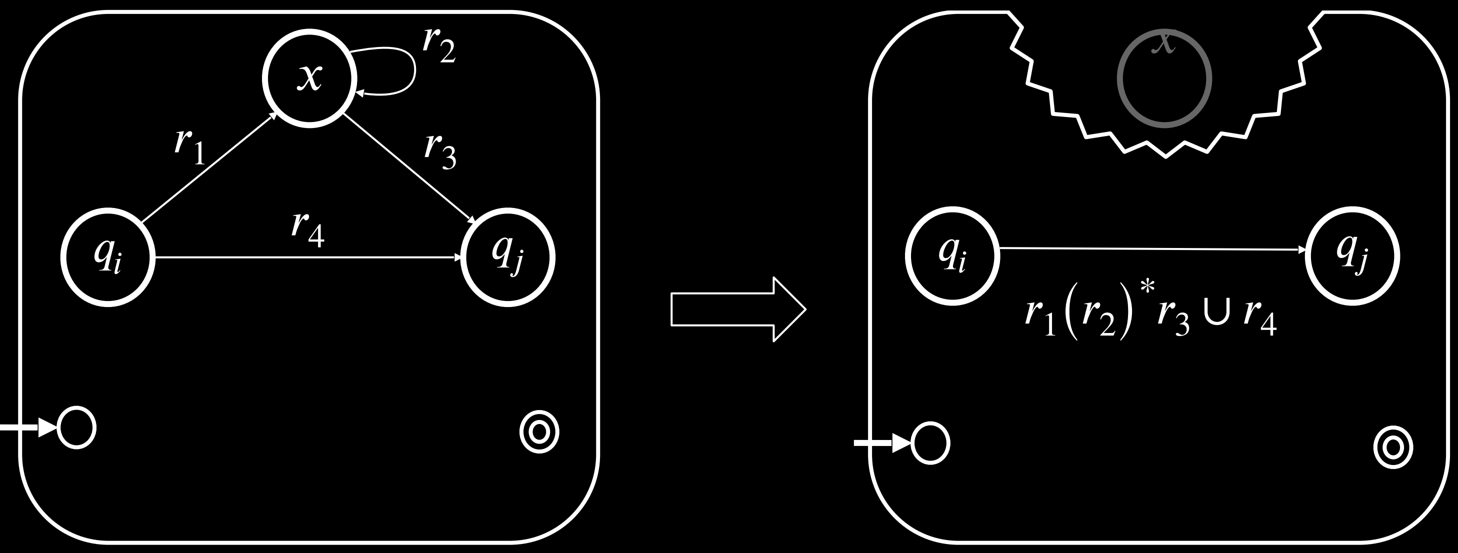

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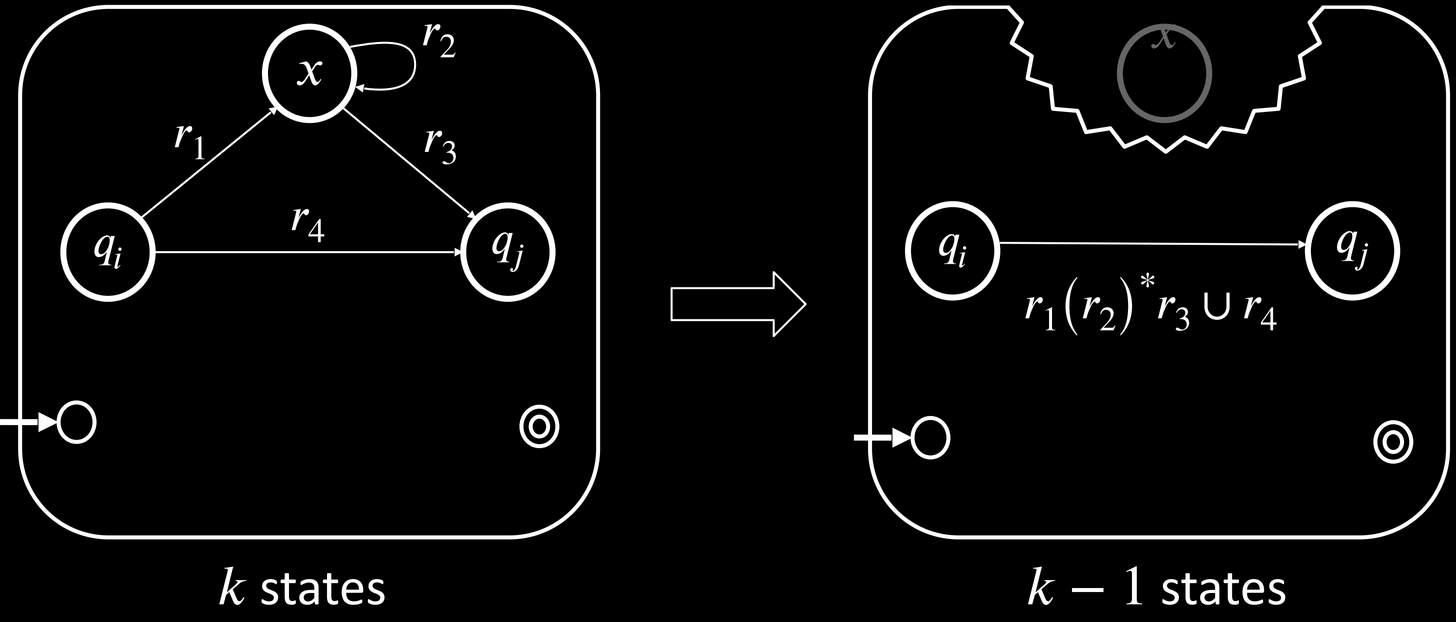

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4. Make the indicated change for each pair of states q_i, q_j.

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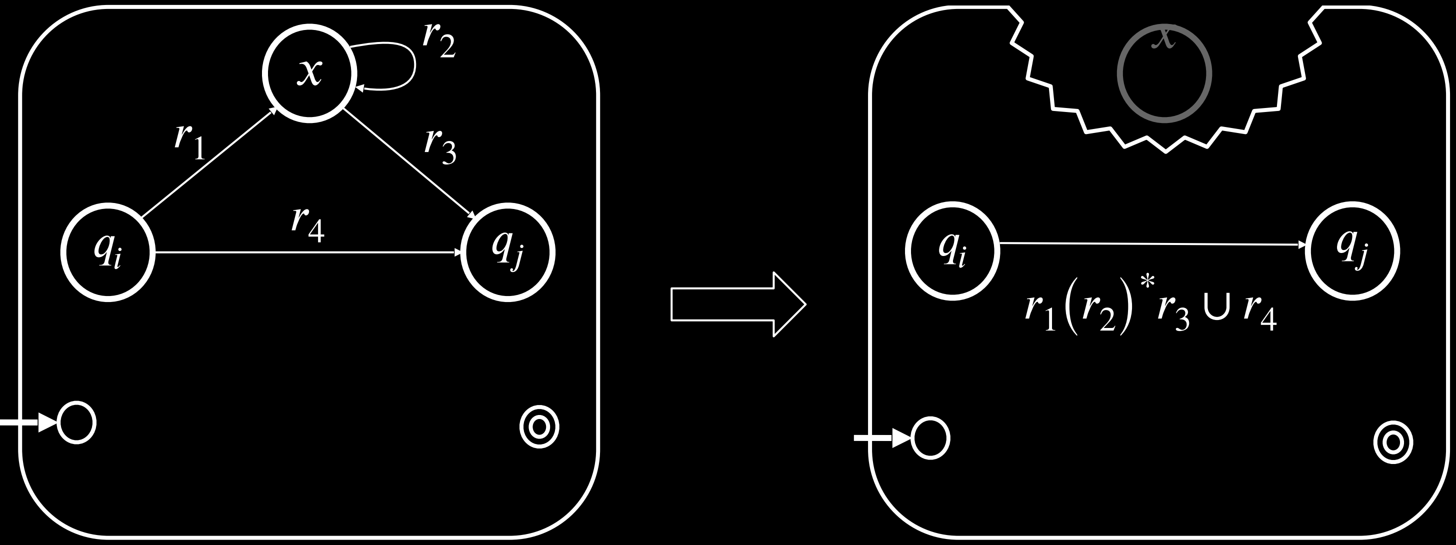

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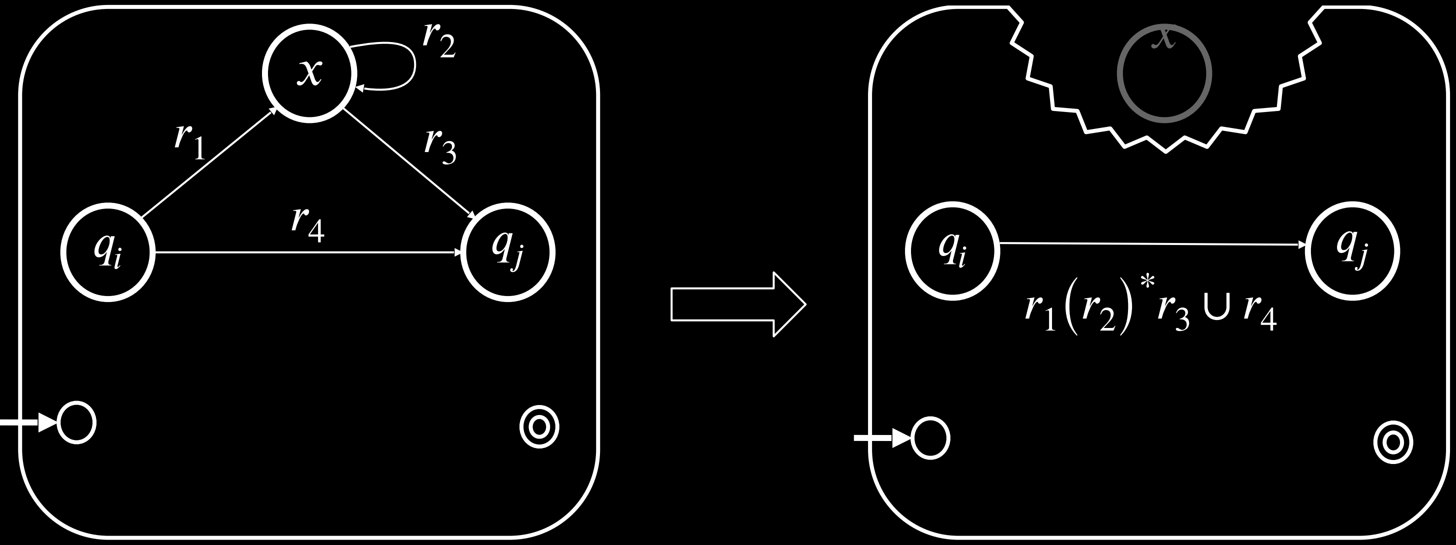

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Check-in 3.1

k-state GNFA $\rightarrow$ $(k-1)$-state GNFA

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We just showed how to convert GNFAs to regular expressions but our goal was to show that how to convert DFAs to regular expressions. How do we finish our goal?

(a) Show how to convert DFAs to GNFAs

(b) Show how to convert GNFAs to DFAs

(c) We are already done. DFAs are a type of GNFAs.

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