جلسه چهارم

درس تحقیق در عملیات

مرور: برنامه‌ریزی خطی

تابع هدف

$$\text{Min/max} \quad c^T x$$

$$\text{s.t.} \quad Ax \leq b$$

قیود

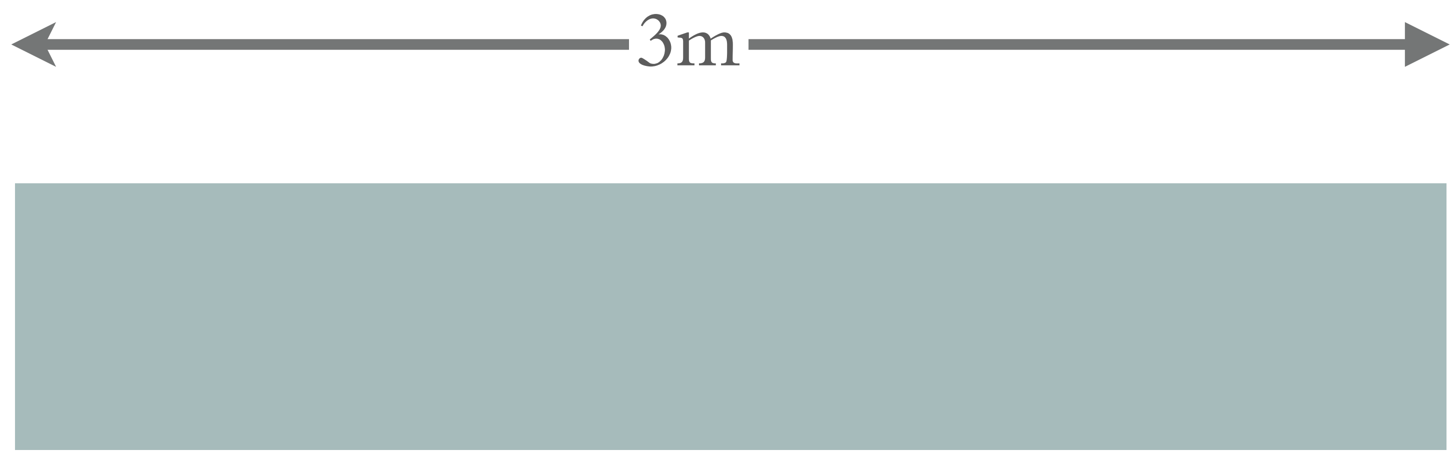
متغیرها

همه چیز خطی!

مثال:
برش رول کاغذ

برش ورقه‌های کاغذ

- 97 rolls of width 135 cm,
- 610 rolls of width 108 cm,
- 395 rolls of width 93 cm,
- 211 rolls of width 42 cm.



P1: 2×135

P2: $135 + 108 + 42$

P3: $135 + 93 + 42$

P4: $135 + 3 \times 42$

P5: $2 \times 108 + 2 \times 42$

P6: $108 + 2 \times 93$

P7: $108 + 93 + 2 \times 42$

P8: $108 + 4 \times 42$

P9: 3×93

P10: $2 \times 93 + 2 \times 42$

P11: $93 + 4 \times 42$

P12: 7×42

$$\sum_{j=1}^{12} x_j$$

$$x_3 + 2x_6 + x_7 + 3x_9 + 2x_{10} + x_{11} \geq 395$$

$$\vdots$$

$$x_1 = 48.5$$

برنامەریزی صحیح

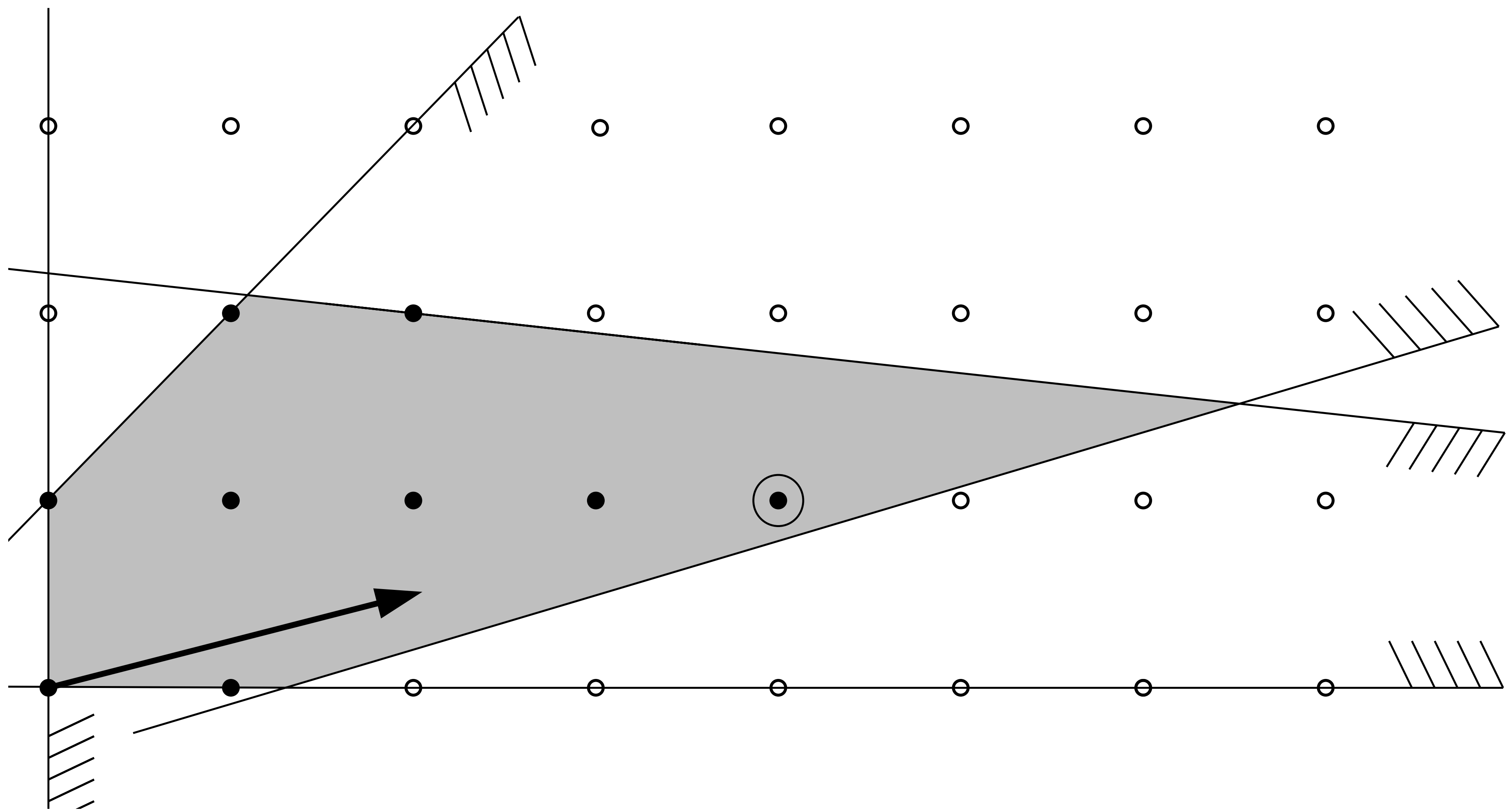
تعریف

خطی

$$\begin{array}{ll} \text{Maximize} & \mathbf{c}^T\mathbf{x} \\ \text{subject to} & A\mathbf{x} \le \mathbf{b} \end{array}$$

صحيح

$$\begin{array}{ll} \text{Maximize} & \mathbf{c}^T\mathbf{x} \\ \text{subject to} & A\mathbf{x} \le \mathbf{b} \\ & \mathbf{x} \in \mathbb{Z}^n. \end{array}$$



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P or **NP**

SAT

$$f = (x_1 \vee x_2 \vee x_4) \wedge (x_3 \vee \bar{x}_4) \ldots.$$

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$$f = (x_1 \vee x_2 \vee x_4) \wedge (x_3 \vee \bar{x}_4)....$$

$$x_1 + x_2 + x_4 \geq 1$$

$$x_3 + (1 - x_4) \geq 1$$

$$\vdots$$

$$f = (x_1 \vee x_2 \vee x_4) \wedge (x_3 \vee \bar{x}_4)....$$

$$x_1 + x_2 + x_4 \geq 1$$

$$x_3 + (1 - x_4) \geq 1$$

$$\vdots$$

$$0 \leq x_i \leq 1$$

$$x_i \in \mathbb{Z}$$

$$f = (x_1 \vee x_2 \vee x_4) \wedge (x_3 \vee \bar{x}_4)....$$

$$x_1 + x_2 + x_4 \geq 1$$

$$x_3 + (1 - x_4) \geq 1$$

$$\vdots$$

$$0 \leq x_i \leq 1$$

$$x_i \in \mathbb{Z}$$

جلسه ۴ کوئیزک ۱

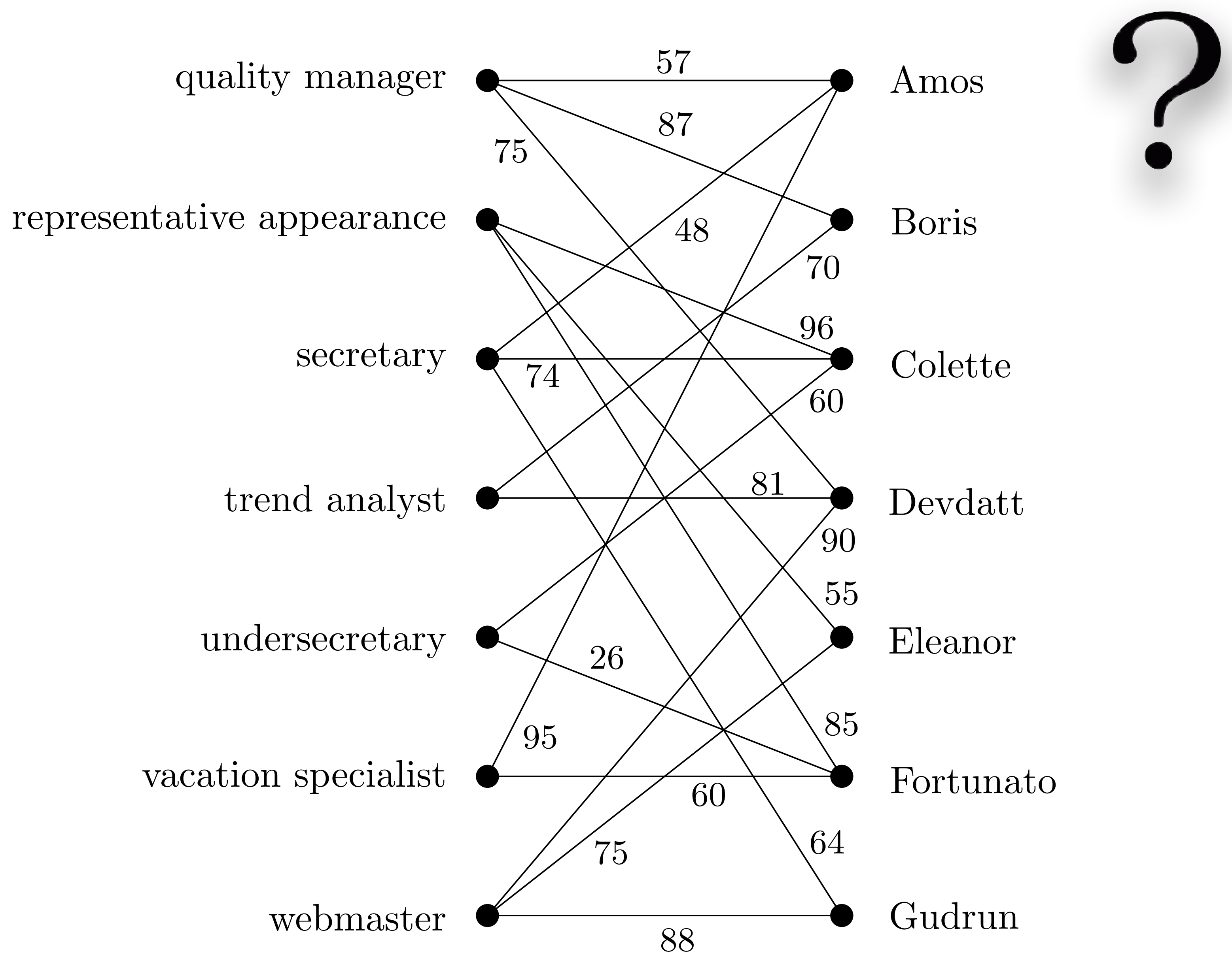
$$(x_1 \vee \neg x_2) \wedge (\neg x_1 \vee x_2 \vee x_3) \wedge \neg x_1$$

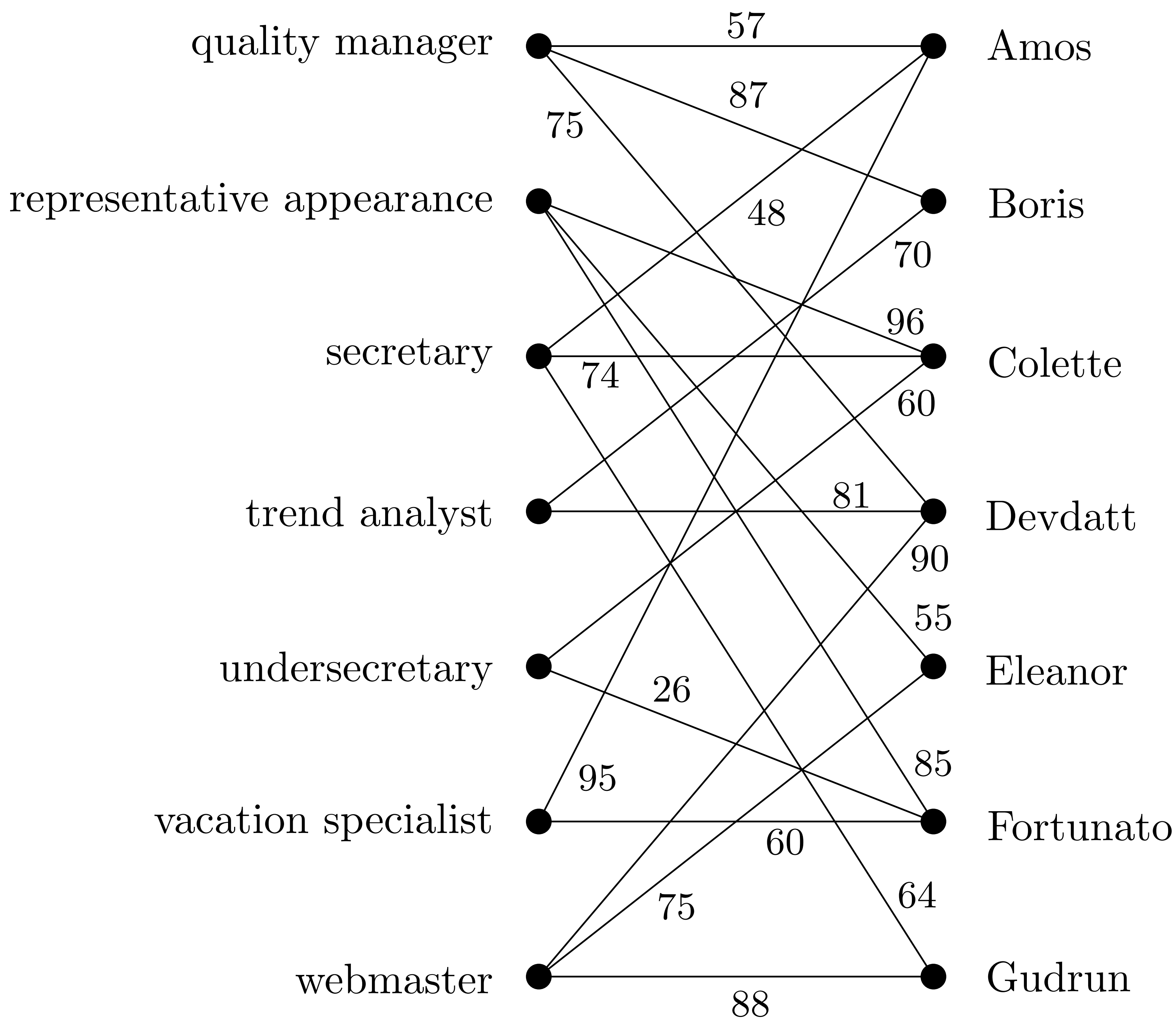
تبدیل

برنامه‌ریزی صحیح

تطابق با بیشترین وزن

یا کی برنامه‌ریزی خطی کافی است

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representative appearance
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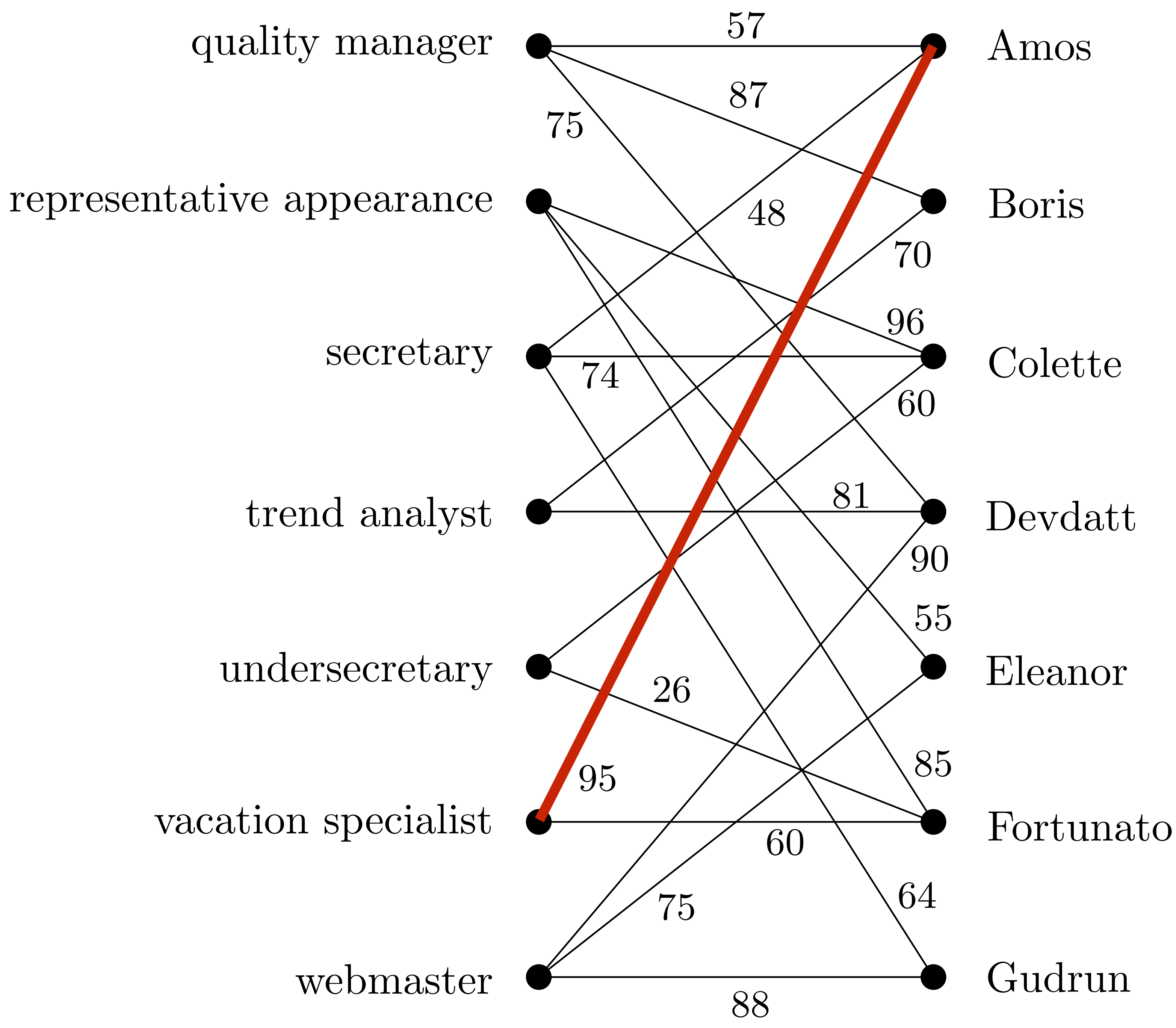
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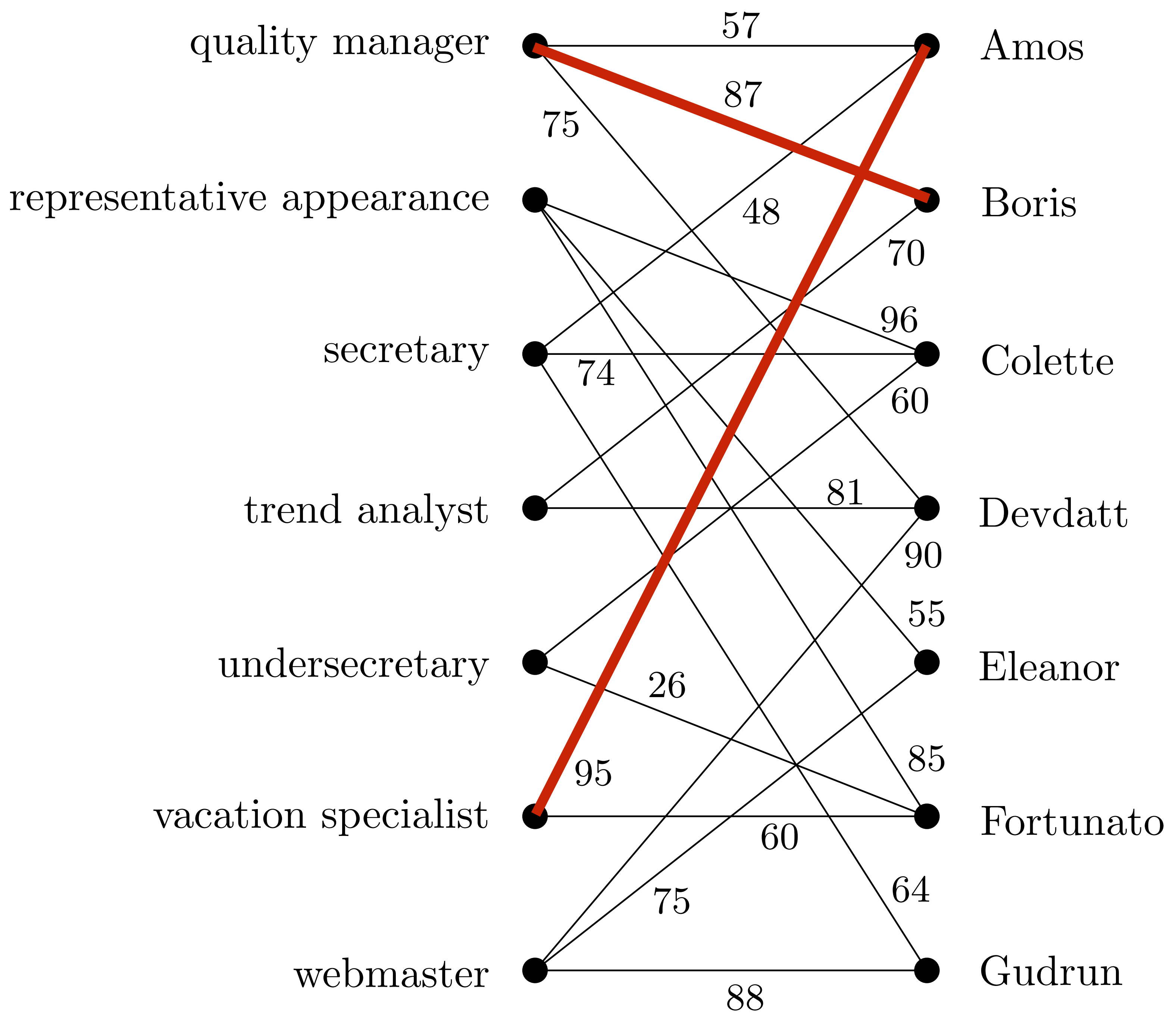

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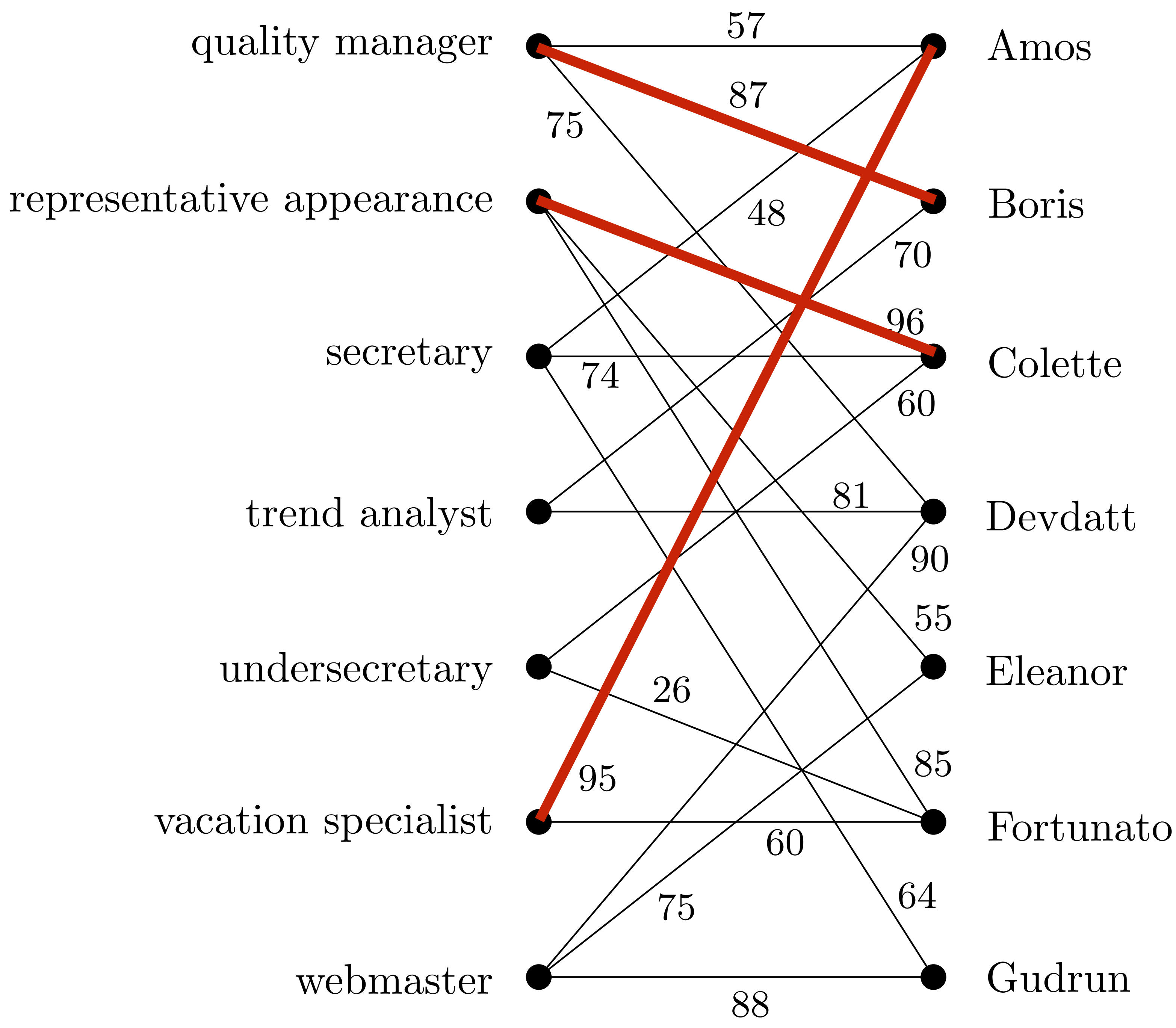
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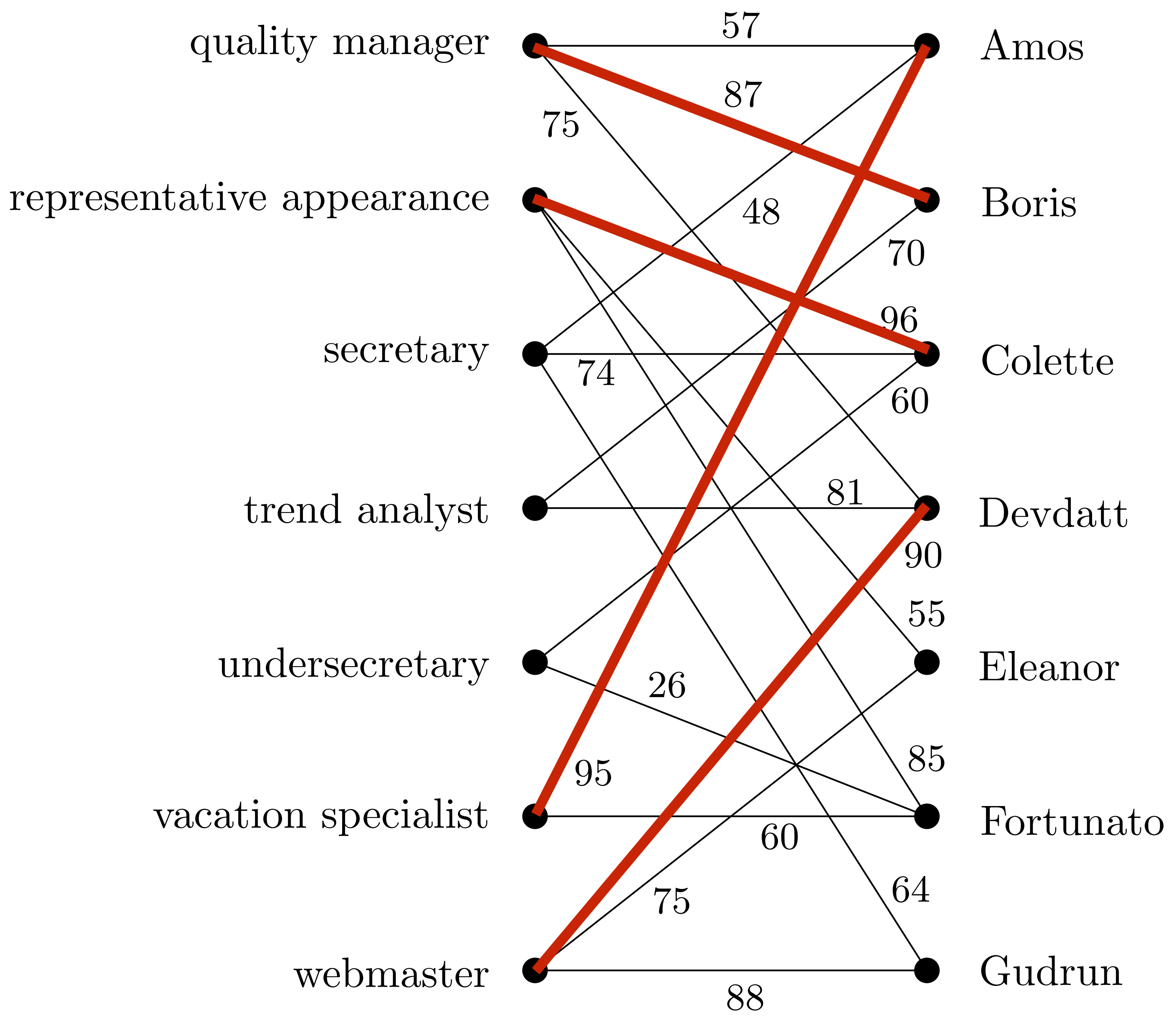
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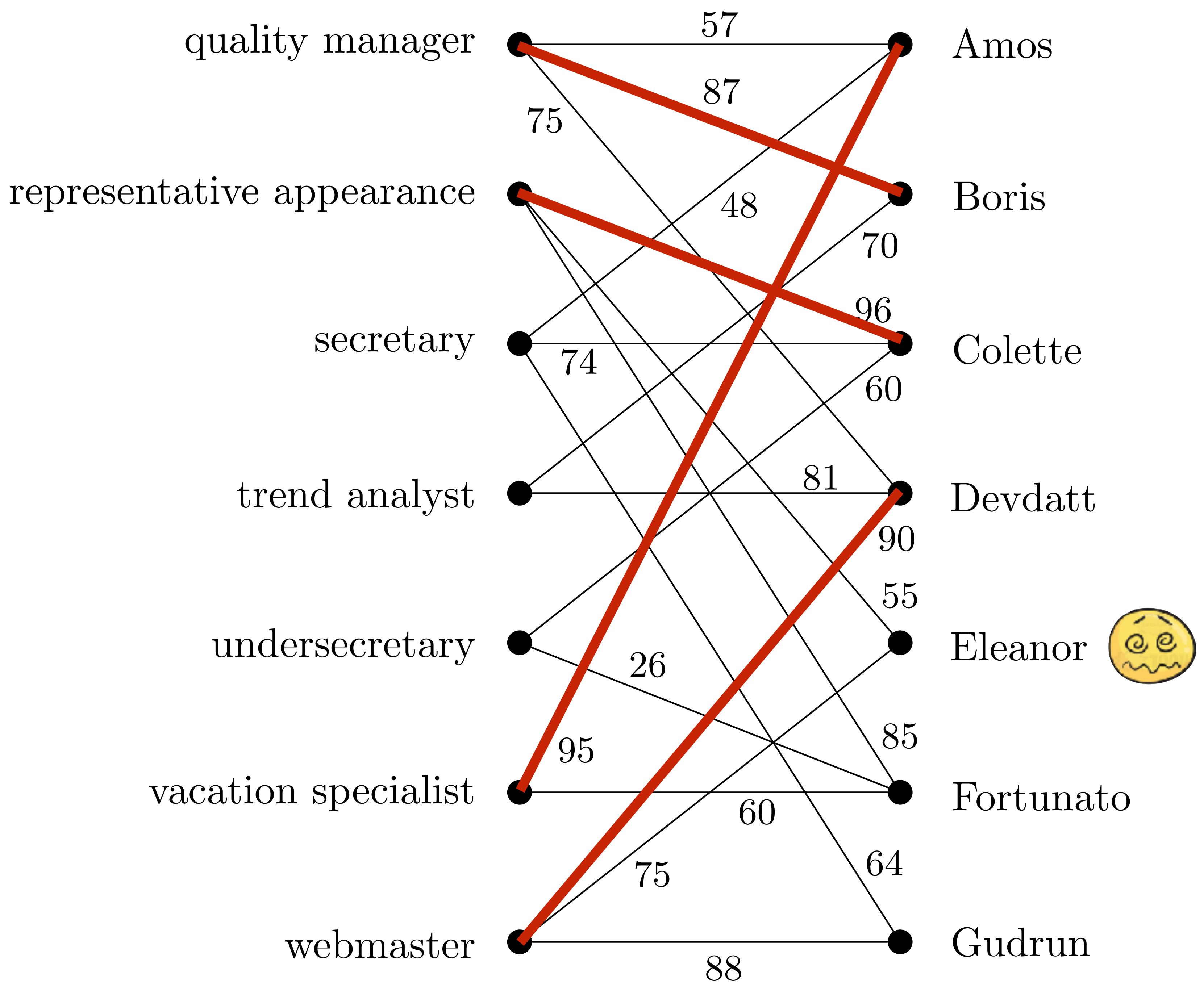
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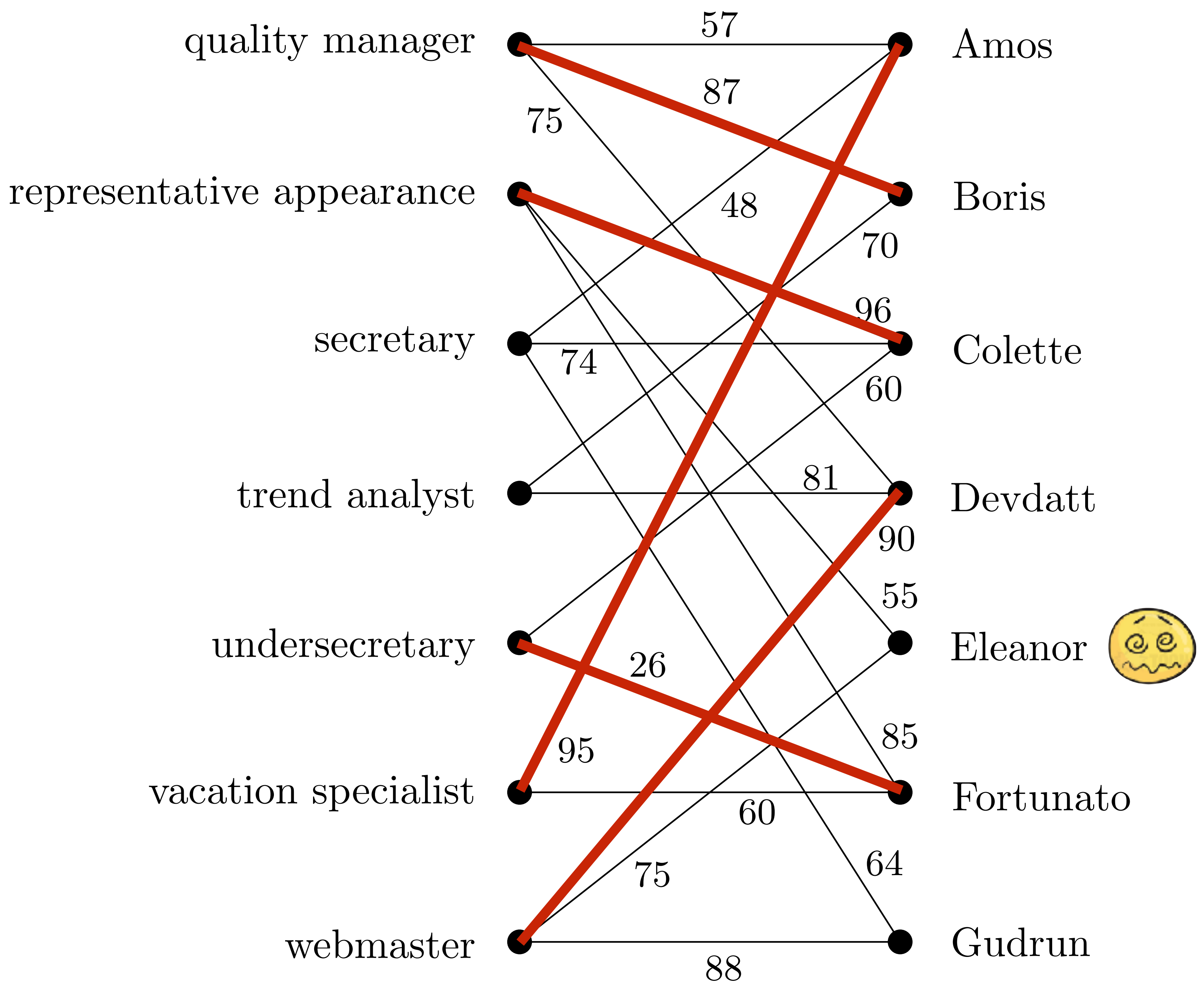
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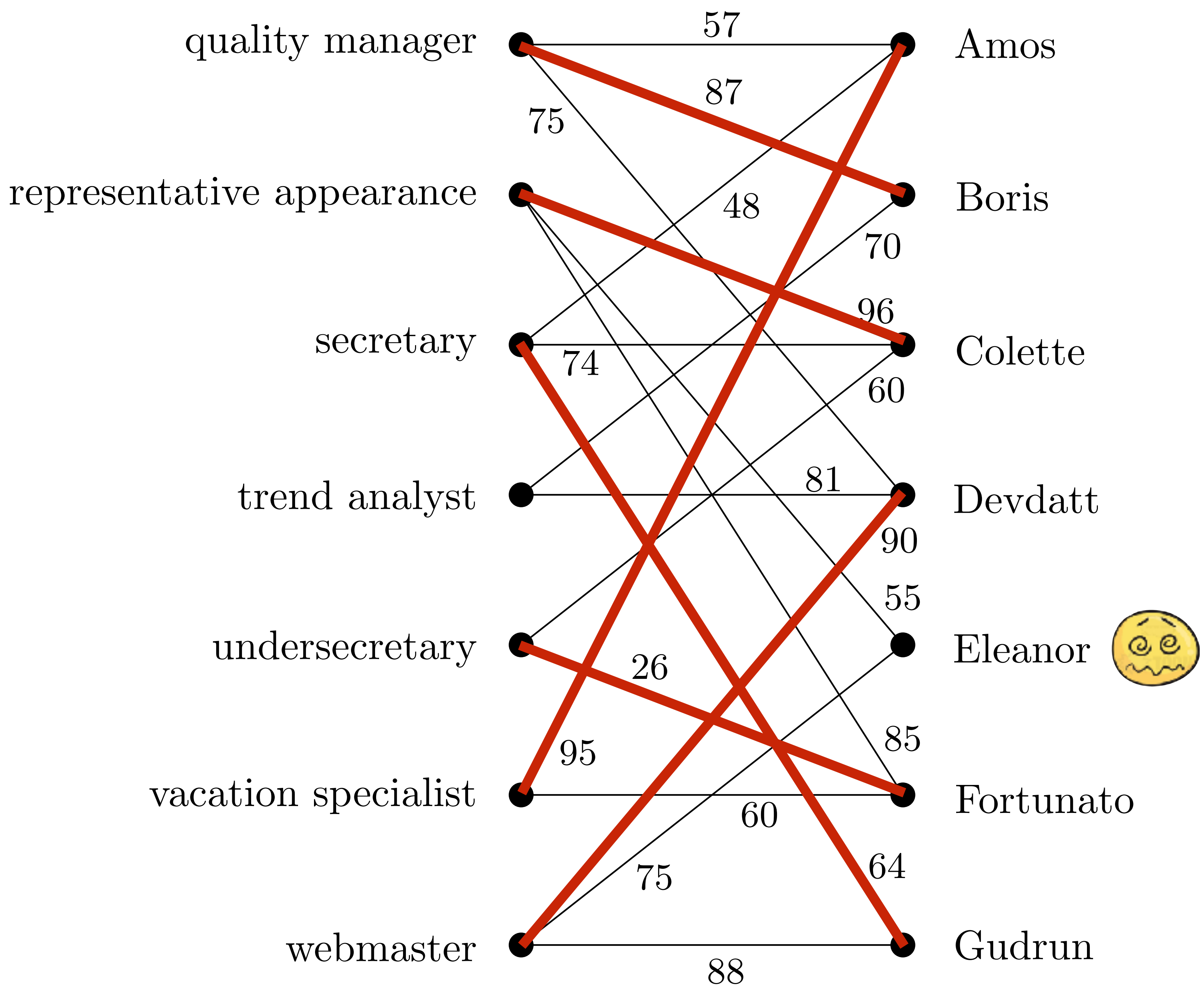
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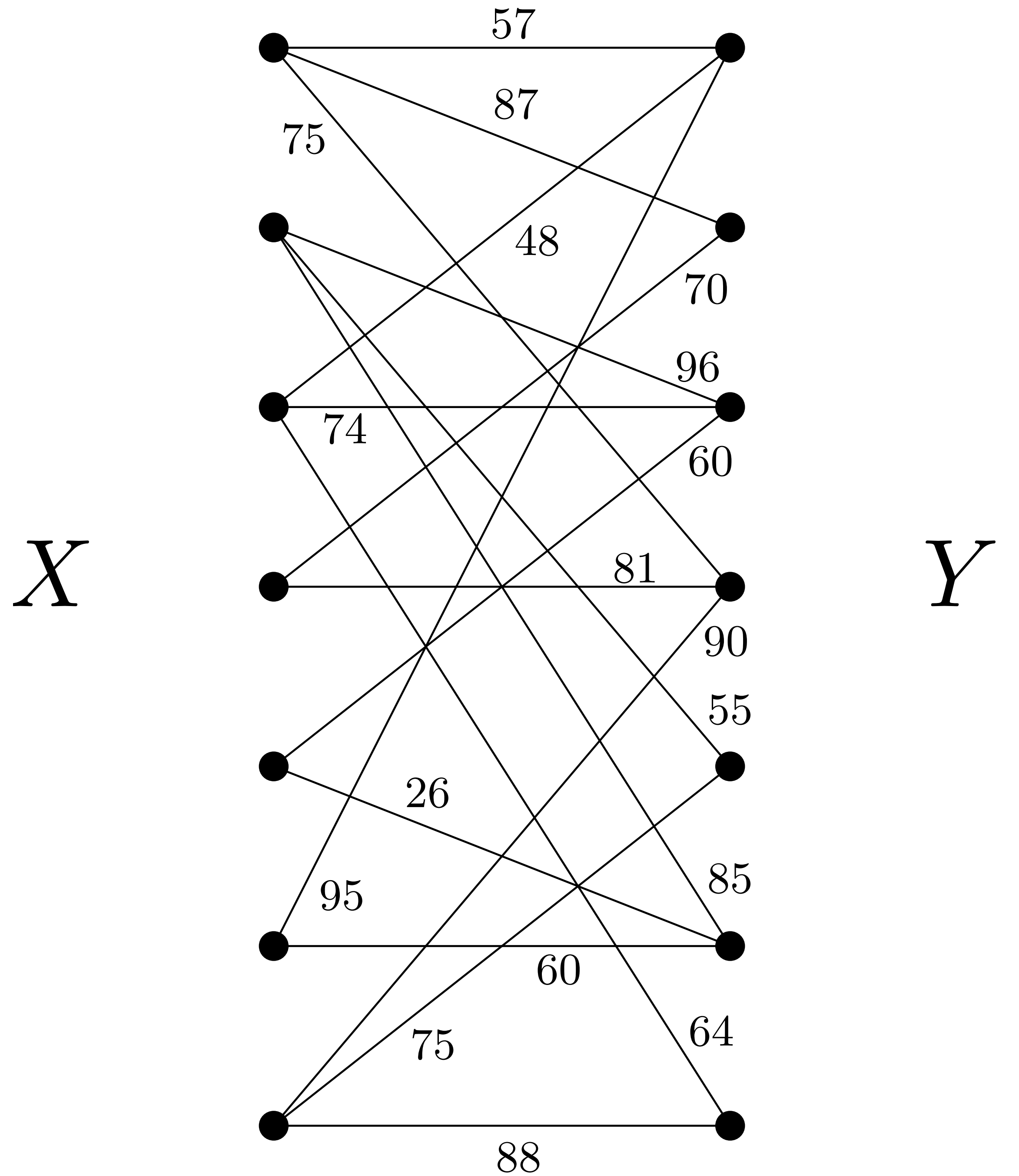

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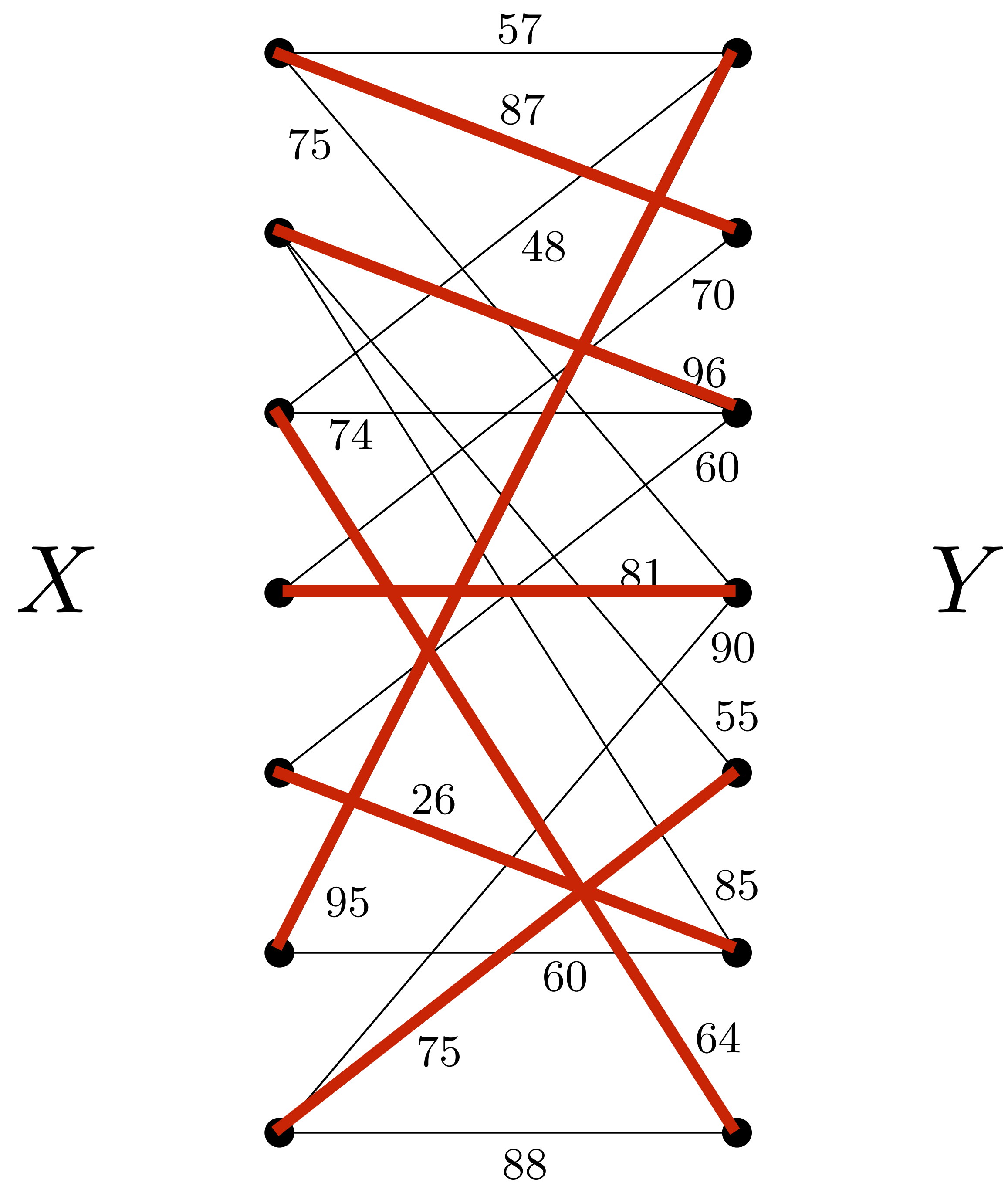
M **perfect matching**



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$$\max \ \textstyle\sum_{e \in M} w_e$$

متغیرها را چه بگذاریم؟

$$\max \ \sum_{e\in M} w_e$$

x_v: راس v از بخش راست به کدام راس از بخش چپ متصل شده باشد؟

$$x_e = 1 \text{ means } e \in M$$

$$x_e = 0 \text{ means } e \notin M$$

$$\begin{array}{ll} \text{maximize} & \sum_{e \in E} w_e x_e \\ \text{subject to} & \sum_{e \in E : v \in e} x_e = 1 \text{ for each vertex } v \in V, \text{ and} \\ & x_e \in \{0, 1\} \text{ for each edge } e \in E. \end{array}$$

$$
\begin{array}{ll}
\text{maximize} & \sum_{e \in E} w_e x_e \\
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 & x_e \in \{0, 1\} \text{ for each edge } e \in E.
\end{array}
$$

(relaxation) آرام‌سازی

$$
\begin{array}{ll}
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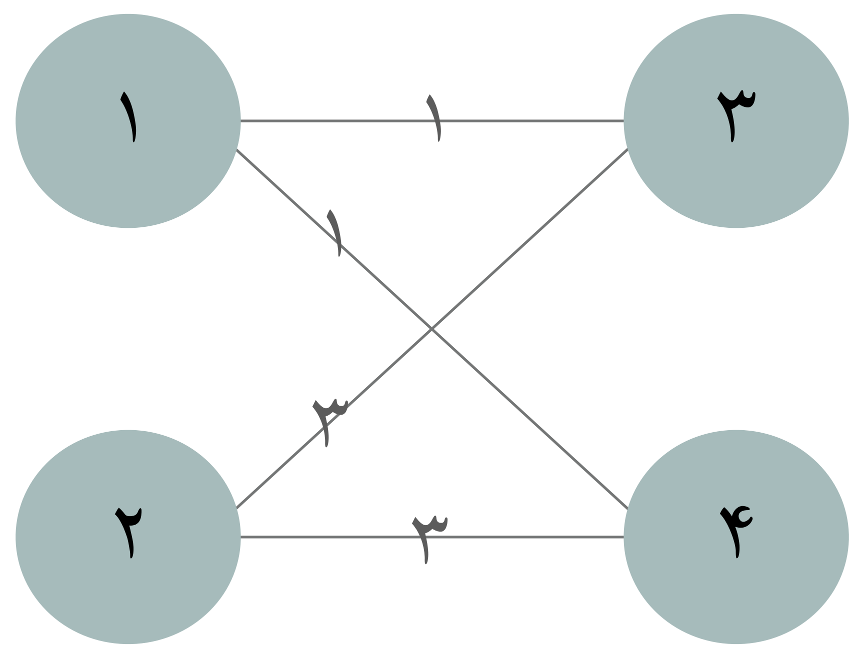
\text{subject to} & \sum_{e \in E : v \in e} x_e = 1 \text{ for each vertex } v \in V, \text{ and} \\
 & 0 \le x_e \le 1 \text{ for each edge } e \in E.
\end{array}
$$

جلسه ۴ کوئیزک ۲

- یک جواب بهینه غیر صحیح برای برنامه‌ریزی خطی تطابق بیشینه گراف دوبخشی زیر:



جلسه ۴ کوئیزک ۲

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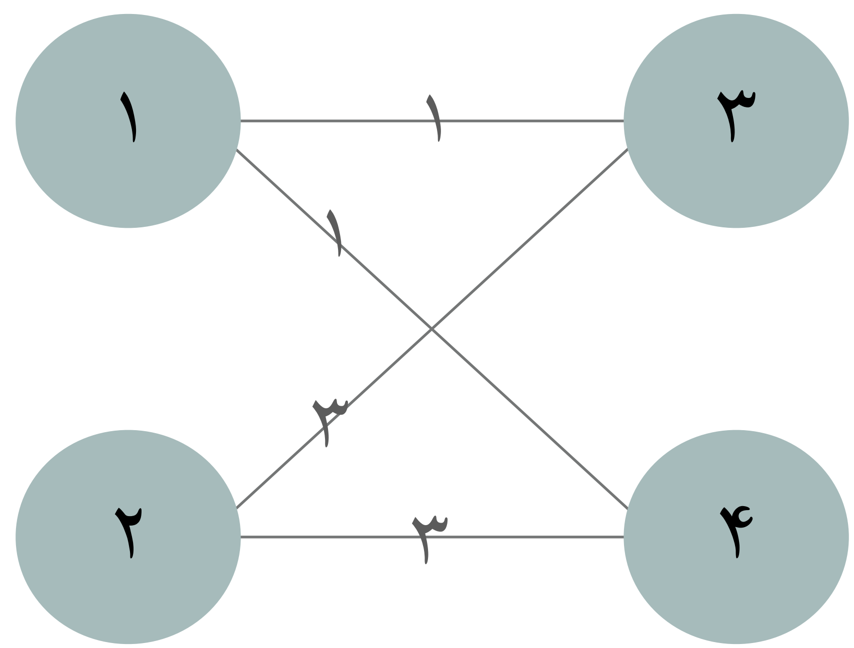

upper bound !

Theorem. *Let $G = (V, E)$ be an arbitrary bipartite graph with real edge weights w_e. If the LP relaxation of the integer program has at least one feasible solution, then it has at least one integral optimal solution.*

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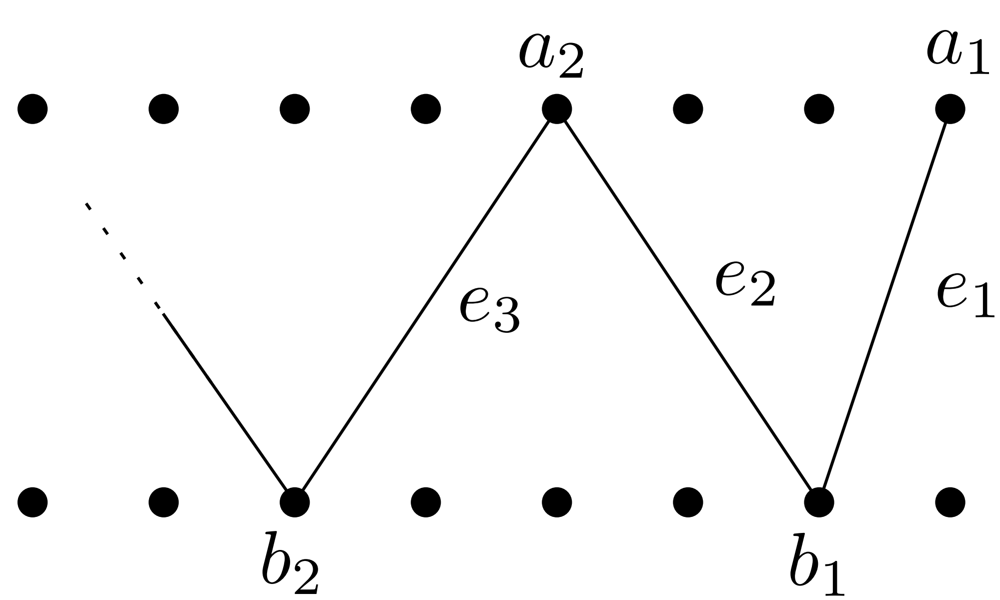
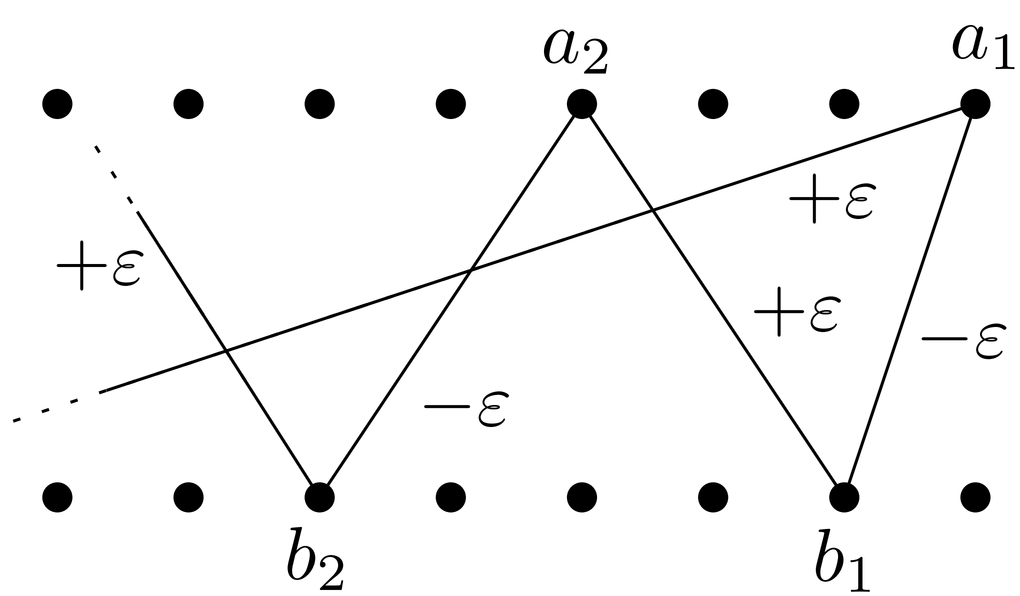
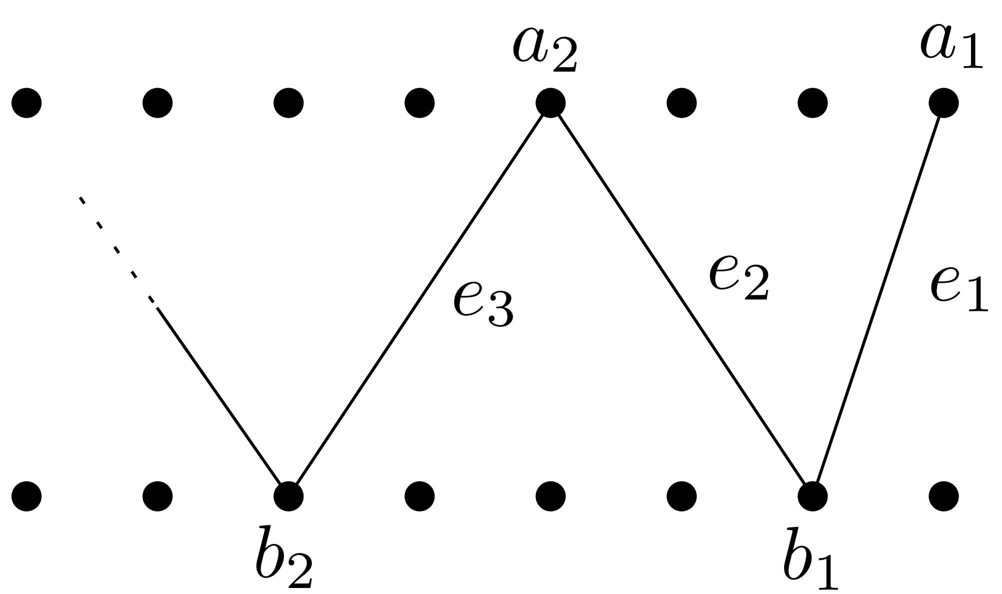


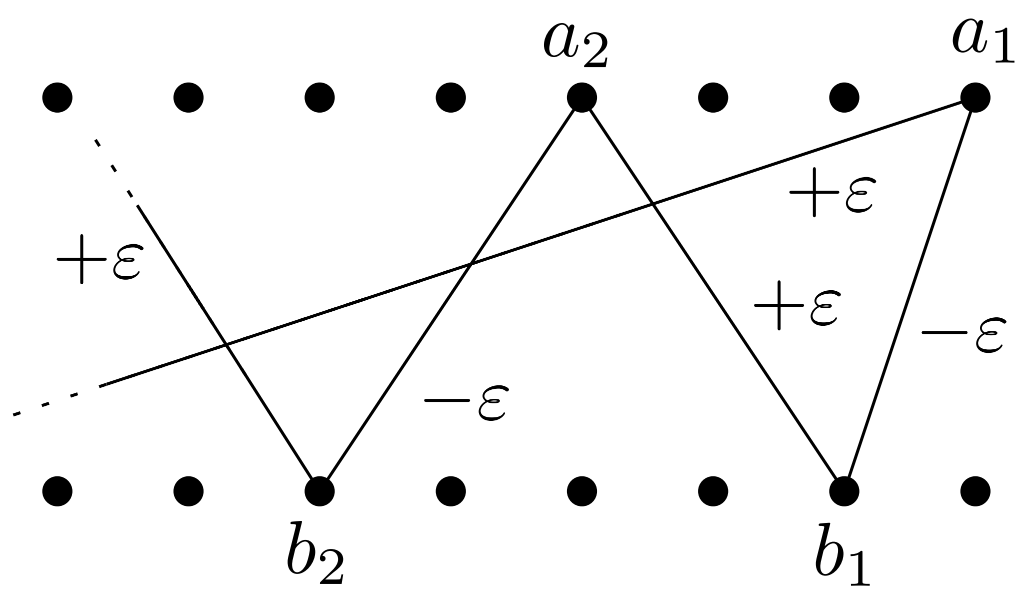
a_2
a_1
e_3
e_2
e_1
b_2
b_1



a_2
a_1
$+\varepsilon$
$+\varepsilon$
$+\varepsilon$
$-\varepsilon$
$-\varepsilon$
b_2
b_1

$$w(\tilde{\mathbf{x}}) = \sum_{e \in E} w_e \tilde{x}_e = w(\mathbf{x}^*) + \varepsilon \sum_{i=1}^{t} (-1)^i w_{e_i} = w(\mathbf{x}^*) + \varepsilon \Delta$$


a_2
a_1
e_3
e_2
e_1
b_2
b_1



a_2
a_1
$+\varepsilon$
$+\varepsilon$
$+\varepsilon$
$-\varepsilon$
$-\varepsilon$
b_2
b_1