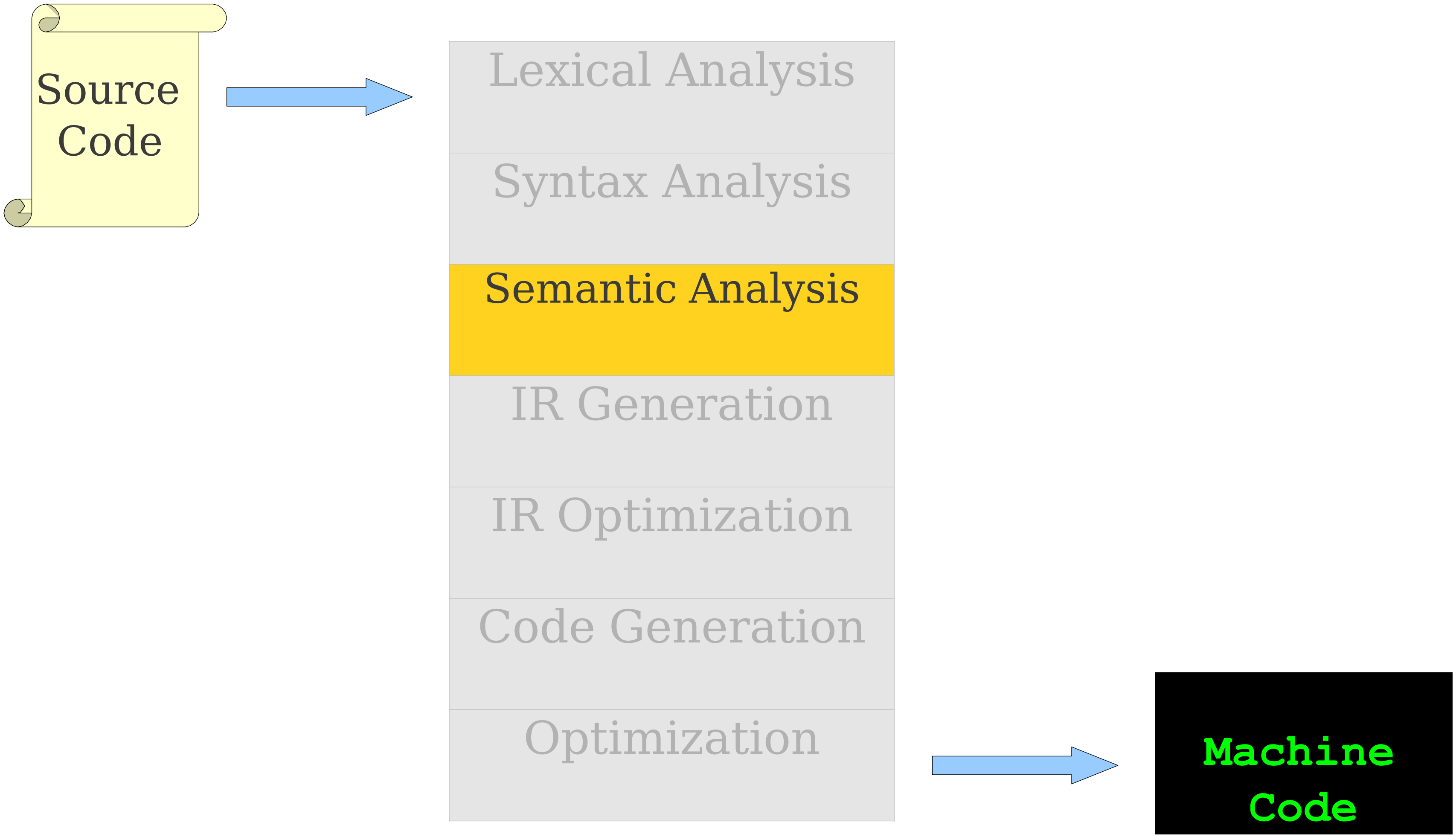
بسم الله الرحمن الرحيم

# Semantic Analysis, Type checking (2)

# Where We Are


Source Code
Lexical Analysis
Syntax Analysis
Semantic Analysis
IR Generation
IR Optimization
Code Generation
Optimization
Machine Code

# Review from Last Time

- Static type checking in Decaf consists of two separate processes:
  - Inferring the type of each expression from the types of its components.
  - Confirming that the types of expressions in certain contexts matches what is expected.
- Logically two steps, but you will probably combine into one pass.

# Overview for Today

- Type-checking **statements**.
- Practical type-checking considerations.
- Type-checking practical language constructs:
  - Function overloading.
  - Specializing overrides.

# Using our Type Proofs

- We can now prove the types of various expressions.
- How do we check...
  - ... that **`if`** statements have well-formed conditional expressions?
  - ... that **`return`** statements actually return the right type of value?
- Use another proof system!

# Proofs of Structural Soundness

- Idea: extend our proof system to statements to confirm that they are well-formed.
- We say that

$$S \vdash \mathrm{WF}(\mathit{stmt})$$

  if the statement *stmt* is **well-formed** in scope S.
- The type system is satisfied if for every function *f* with body B in scope S, we can show $S \vdash \mathrm{WF}(B)$.

# A Simple Well-Formedness Rule

$$\frac{S \vdash \mathit{expr} : T}{S \vdash \mathrm{WF}(\mathit{expr};)}$$

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If we can assign a valid type to an expression in scope S…

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# A Simple Well-Formedness Rule

$$\frac{S \vdash expr : T}{S \vdash \mathrm{WF}(expr;\ )}$$

If we can assign a valid type to an expression in scope S…

… then it is a valid statement in scope S.

# A More Complex Rule

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$$\frac{S \vdash \mathrm{WF}(\mathit{stmt}_1) \qquad S \vdash \mathrm{WF}(\mathit{stmt}_2)}{S \vdash \mathrm{WF}(\mathit{stmt}_1\ \mathit{stmt}_2)}$$

# Rules for `break`

# Rules for **break**

$$\frac{S \text{ is in a } \texttt{for} \text{ or } \texttt{while} \text{ loop.}}{S \vdash \mathrm{WF}(\texttt{break;})}$$

# A Rule for Loops

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$$\frac{\begin{array}{c} S \vdash \textit{expr} : \texttt{bool} \\ \text{S' is the scope inside the loop.} \\ S' \vdash \text{WF}(\textit{stmt}) \end{array}}{S \vdash \text{WF}(\texttt{while (}\textit{expr}\texttt{)}\ \textit{stmt})}$$

# Rules for Block Statements

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$$\frac{\text{S' is the scope formed by adding } \textit{decls} \text{ to S} \quad \text{S'} \vdash \text{WF}(\textit{stmt})}{\text{S} \vdash \text{WF}(\texttt{\{}\ \textit{decls stmt}\ \texttt{\}})}$$

# Rules for `return`

# Rules for `return`

$$\frac{\begin{array}{c}\text{S is in a function returning T} \\ S \vdash \mathit{expr} : T' \\ T' \leq T\end{array}}{S \vdash \text{WF}(\texttt{return}\ \mathit{expr}\texttt{;})}$$

$$\frac{\text{S is in a function returning } \texttt{void}}{S \vdash \text{WF}(\texttt{return;})}$$

# Checking Well-Formedness

- Recursively walk the AST.
- For each statement:
  - Typecheck any subexpressions it contains.
    - Report errors if **no** type can be assigned.
    - Report errors if the **wrong** type is assigned.
  - Typecheck child statements.
  - Check the overall correctness.

# Practical Concerns

# Something is Very Wrong Here

```
int x, y, z;
if (((x == y)  > 5 && x + y < z) || x == z) {
     /* … */
}
```

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Facts

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Facts

$$\frac{x \text{ is an identifier.} \quad x \text{ is a variable in scope S with type T.}}{S \vdash x : T}$$

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```
int x, y, z;
if (((x == y)  > 5 && x + y < z) || x == z) {
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**Facts**

S ⊢ **x : int**

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int x, y, z;
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**Facts**

S ⊢ **x : int**

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| Facts |
|---|
| S ⊢ **x : int** |
| S ⊢ **y : int** |
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int x, y, z;
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| Facts |
|---|
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$$\frac{S \vdash e_1 : T_1 \quad S \vdash e_2 : T_2 \quad T_1 \leq T_2 \text{ or } T_2 \leq T_1}{S \vdash e_1 == e_2 : \textbf{bool}}$$

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$$\frac{i \text{ is an integer constant}}{S \vdash i : \texttt{int}}$$

| Facts |
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| $S \vdash \texttt{5} : \texttt{int}$ |

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int x, y, z;
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$$\frac{S \vdash e_1 : \mathbf{int} \qquad S \vdash e_2 : \mathbf{int}}{S \vdash e_1 + e_2 : \mathbf{int}}$$

| Facts |
|---|
| $S \vdash$ **x** : **int** |
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>

| Facts |
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$$\frac{S \vdash e_1 : \texttt{int} \quad S \vdash e_2 : \texttt{int}}{S \vdash e_1 > e_2 : \texttt{bool}}$$

```
> Error: Cannot compare int and bool
```

| Facts |
|---|
| $S \vdash$ **x** : **int** |
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```

| Facts |
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| $S \vdash \texttt{x} : \texttt{int}$ |
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```
int x, y, z;
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     /* … */
}
```

$$\frac{S \vdash e_1 : \mathbf{bool} \qquad S \vdash e_2 : \mathbf{bool}}{S \vdash e_1 \;\&\&\; e_2 : \mathbf{bool}}$$

```
> Error: Cannot compare int and bool
```

| Facts |
|---|
| $S \vdash$ **x** : **int** |
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# Something is Very Wrong Here

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    /* … */
}
```

$$\frac{S \vdash e_1 : \mathtt{bool} \qquad S \vdash e_2 : \mathtt{bool}}{S \vdash e_1 \;\&\&\; e_2 : \mathtt{bool}}$$

```
> Error: Cannot compare int and bool
  Error: Cannot compare ??? and bool
```

| Facts |
|---|
| $S \vdash \mathtt{x} : \mathtt{int}$ |
| $S \vdash \mathtt{y} : \mathtt{int}$ |
| $S \vdash \mathtt{z} : \mathtt{int}$ |
| $S \vdash \mathtt{x == y} : \mathtt{bool}$ |
| $S \vdash \mathtt{5} : \mathtt{int}$ |
| $S \vdash \mathtt{x + y} : \mathtt{int}$ |
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# Something is Very Wrong Here

```
int x, y, z;
if (((x == y)  > 5 && x + y < z) || x == z) {
     /* … */
}
```

$$\frac{S \vdash e_1 : \textbf{bool} \qquad S \vdash e_2 : \textbf{bool}}{S \vdash e_1 \;\&\&\; e_2 : \textbf{bool}}$$

```
> Error: Cannot compare int and bool
  Error: Cannot compare ??? and bool
```

| Facts |
|---|
| $S \vdash$ **x** : **int** |
| $S \vdash$ **y** : **int** |
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| $S \vdash$ **x == y** : **bool** |
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int x, y, z;
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    /* … */
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```

$$\frac{S \vdash e_1 : \mathbf{bool} \quad S \vdash e_2 : \mathbf{bool}}{S \vdash e_1 \mid\mid e_2 : \mathbf{bool}}$$

```
> Error: Cannot compare int and bool
  Error: Cannot compare ??? and bool
```

| Facts |
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| $S \vdash \mathtt{x} : \mathtt{int}$ |
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| $S \vdash \mathtt{x + y} : \mathtt{int}$ |
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int x, y, z;
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```
> Error:  Cannot compare int and  bool
  Error:  Cannot compare ??? and  bool
  Error:  Cannot compare ??? and  bool
```

| Facts |
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| $S \vdash$ **x** : **int** |
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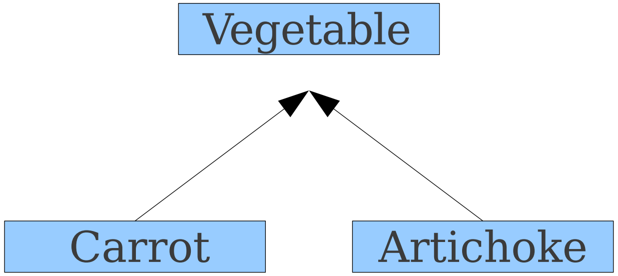
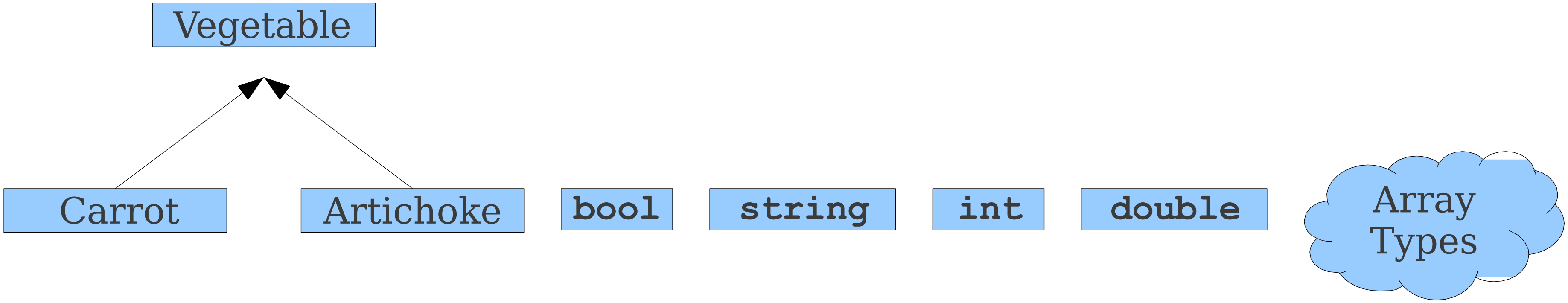
# Cascading Errors

- A **static type error** occurs when we cannot prove that an expression has a given type.
- Type errors can easily cascade:
  - Can't prove a type for $\mathbf{e_1}$, so can't prove a type for $\mathbf{e_1 + e_2}$, so can't prove a type for $\mathbf{(e_1 + e_2) + e_3}$, etc.
- How do we resolve this?

# The Shape of Types



# The Shape of Types



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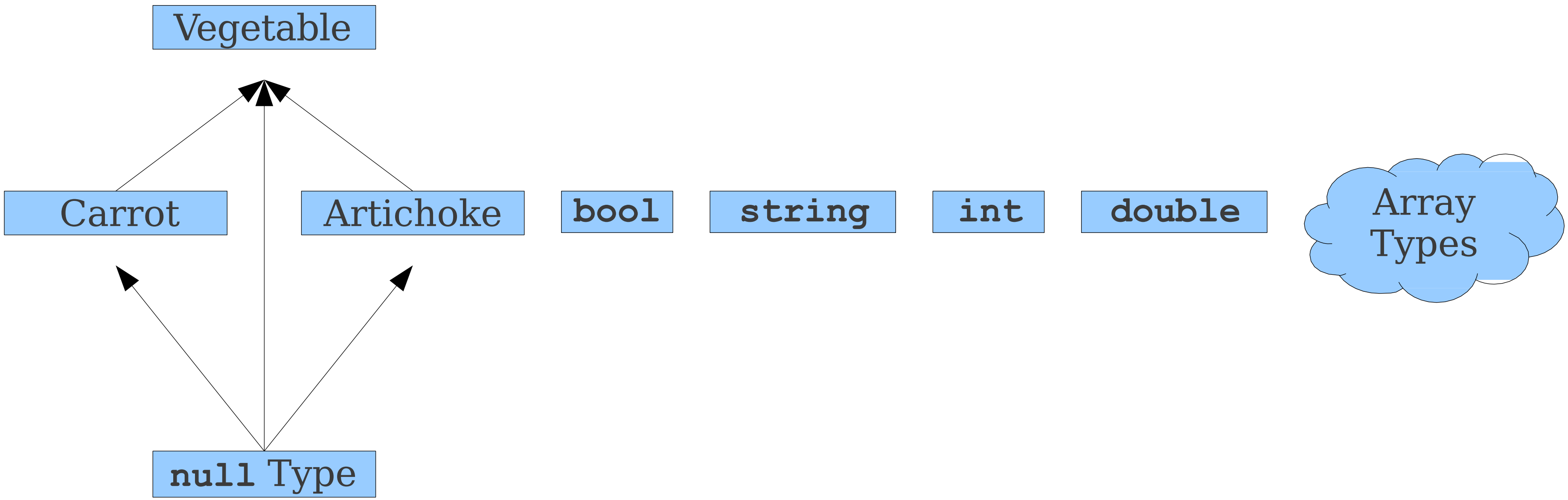
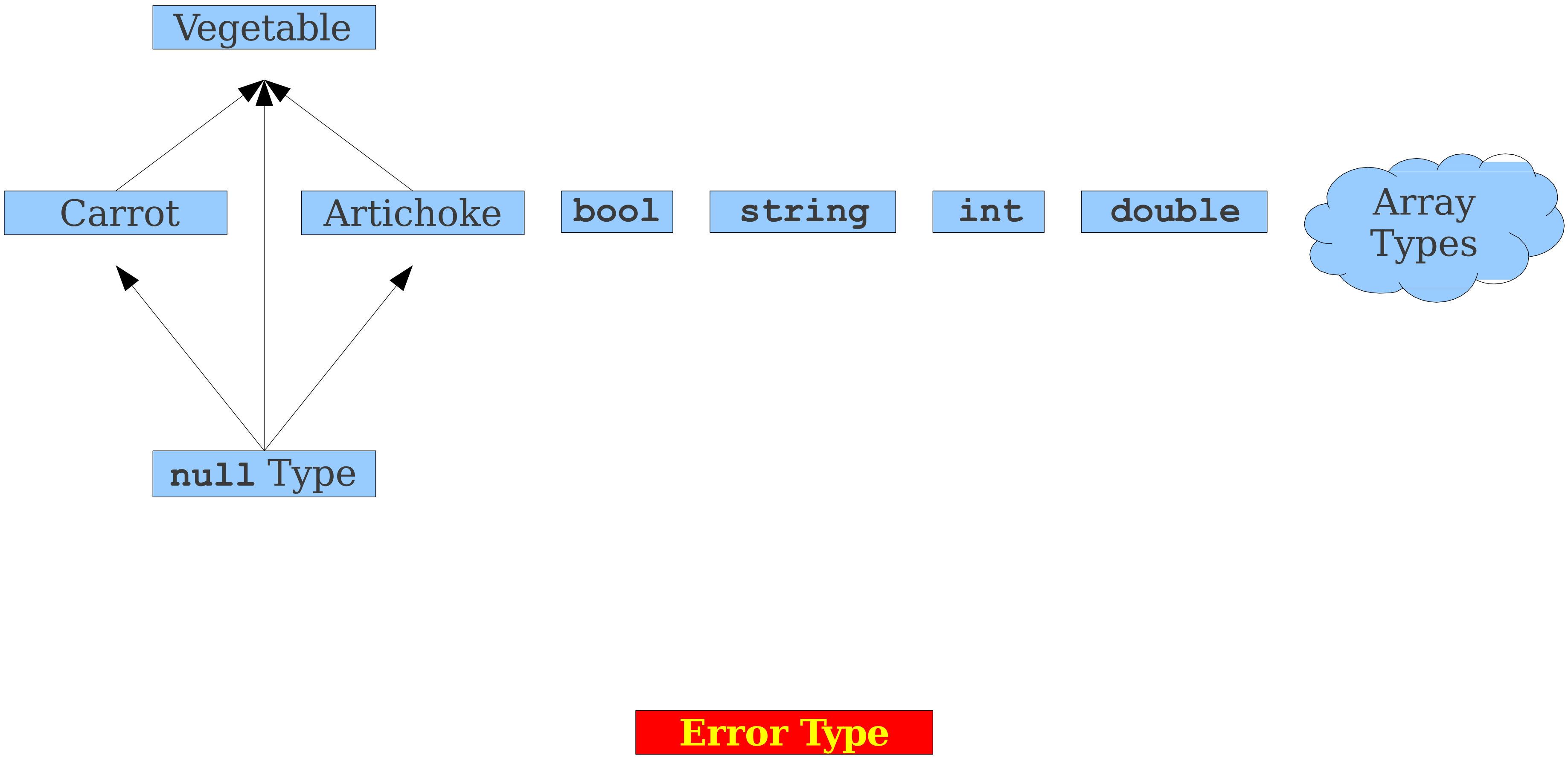
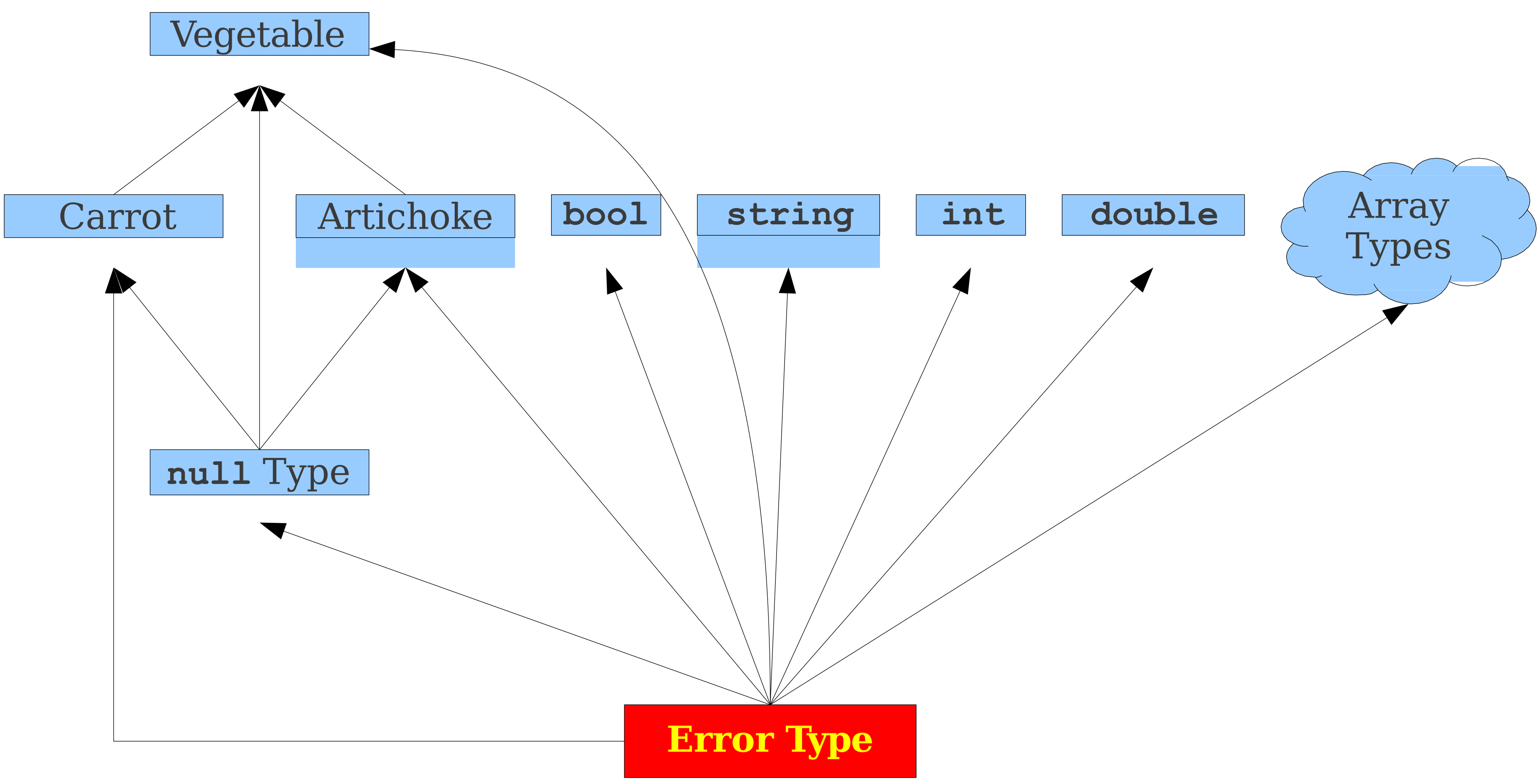


# The Shape of Types



# The Shape of Types



# The Error Type

- Introduce a new type representing an error into the type system.
- The **error type** is less than all other types and is denoted **$\perp$**.
  - It is sometimes called the **bottom type**.
- By definition, $\perp \leq A$ for any type A.
- On discovery of a type error, pretend that we can prove the expression has type $\perp$.
- Update our inference rules to support $\perp$.

# Updated Rules for Addition

$$\frac{S \vdash e_1 : \textbf{double} \qquad S \vdash e_2 : \textbf{double}}{S \vdash e_1 + e_2 : \textbf{double}}$$

# Updated Rules for Addition

$$\frac{S \vdash e_1 : T_1 \qquad S \vdash e_2 : T_2}{S \vdash e_1 + e_2 : \texttt{double}}$$

# Updated Rules for Addition

$$\frac{\begin{array}{c} S \vdash e_1 : T_1 \\ S \vdash e_2 : T_2 \\ T_1 \leq \texttt{double} \\ T_2 \leq \texttt{double} \end{array}}{S \vdash e_1 + e_2 : \texttt{double}}$$

# Updated Rules for Addition

$$\frac{\begin{array}{c} S \vdash e_1 : T_1 \\ S \vdash e_2 : T_2 \\ T_1 \leq \textbf{double} \\ T_2 \leq \textbf{double} \end{array}}{S \vdash e_1 + e_2 : \textbf{double}}$$

What does this mean?

# Updated Rules for Addition

$$\frac{S \vdash e_1 : T_1 \quad S \vdash e_2 : T_2 \quad T_1 \leq \texttt{double} \quad T_2 \leq \texttt{double}}{S \vdash e_1 + e_2 : \texttt{double}}$$

$$\frac{S \vdash e_1 : T_1 \quad S \vdash e_2 : T_2 \quad T_1 \leq \texttt{int} \quad T_2 \leq \texttt{int}}{S \vdash e_1 + e_2 : \texttt{int}}$$

# Updated Rules for Addition

$$\frac{\begin{array}{c} S \vdash e_1 : T_1 \\ S \vdash e_2 : T_2 \\ \boxed{\begin{array}{l} T_1 \leq \texttt{double} \\ T_2 \leq \texttt{double} \end{array}} \end{array}}{S \vdash e_1 + e_2 : \texttt{double}}$$

$$\frac{\begin{array}{c} S \vdash e_1 : T_1 \\ S \vdash e_2 : T_2 \\ \boxed{\begin{array}{l} T_1 \leq \texttt{int} \\ T_2 \leq \texttt{int} \end{array}} \end{array}}{S \vdash e_1 + e_2 : \texttt{int}}$$

Prevents errors from propagating.

# Updated Rules for Addition

$$\frac{S \vdash e_1 : T_1 \qquad S \vdash e_2 : T_2 \qquad T_1 \le \texttt{double} \qquad T_2 \le \texttt{double}}{S \vdash e_1 + e_2 : \texttt{double}}$$

$$\frac{S \vdash e_1 : T_1 \qquad S \vdash e_2 : T_2 \qquad T_1 \le \texttt{int} \qquad T_2 \le \texttt{int}}{S \vdash e_1 + e_2 : \texttt{int}}$$

# Error-Recovery in Practice

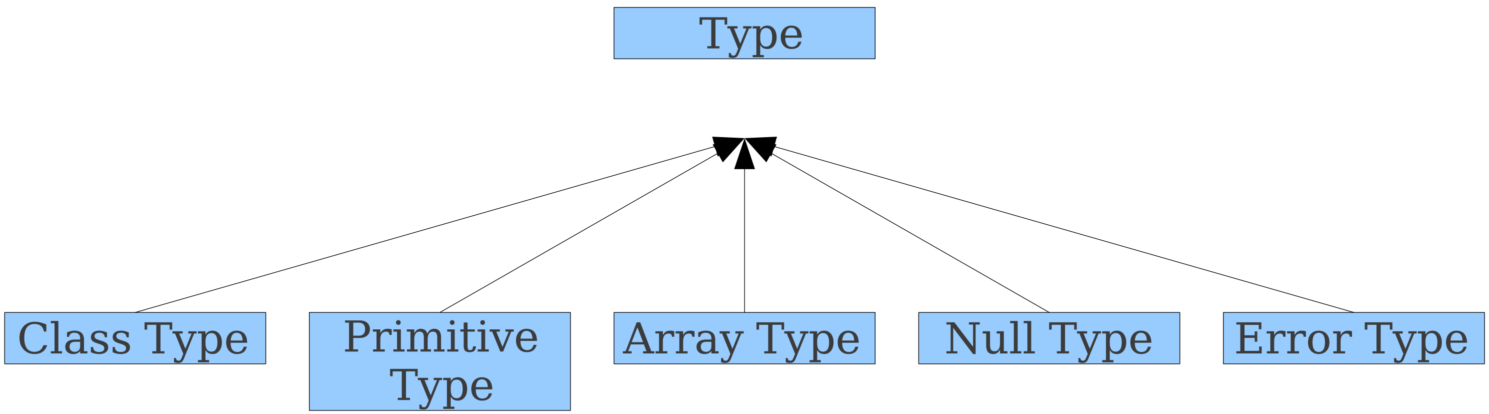
- In your semantic analyzer, you will need to do some sort of error recovery.
- We provide an error type **`Type::errorType`**.
- But what about other cases?
  - Calling a nonexistent function.
  - Declaring a variable of a bad type.
  - Treating a non-array as an array.

# Implementing Convertibility

- How do we implement the ≤ operator we've described so far?
- Lots of cases:

| From \ To | Class Type | Primitive Type | Array Type | Null Type | Error Type |
|---|---|---|---|---|---|
| Class Type | If same or inherits from | No | No | No | No |
| Primitive Type | No | If same type | No | No | No |
| Array Type | No | No | If underlying types match | No | No |
| Null Type | **Yes** | No | No | **Yes** | No |
| Error Type | **Yes** | **Yes** | **Yes** | **Yes** | **Yes** |

# A Hierarchy for Types



# Methods You Might Want...

- `virtual bool Type::IsIdenticalTo(Type* other);`
  - Returns whether two types represent the same actual type.
- `virtual bool Type::IsConvertibleTo(Type* other);`
  - Returns whether one type is convertible to some other type.

# Function Overloading

# Function Overloading

- Two functions are said to be **overloads** of one another if they have the same name but a different set of arguments.
- At compile-time, determine which function is meant by inspecting the types of the arguments.
- Report an error if no one function is the best function.

# Overloading Example

```
void Function();
void Function(int x);
void Function(double x);
void Function(Base b);
void Function(Derived d);
```

```
Function();
Function(137);
Function(42.0);
Function(new Base);
Function(new Derived);
```

# Overloading Example

```
void Function();
void Function(int x);
void Function(double x);
void Function(Base b);
void Function(Derived d);
```

```
Function();
Function(137);
Function(42.0);
Function(new Base);
Function(new Derived);
```

# Implementing Overloading

```
void Function();
void Function(int x);
void Function(double x);
void Function(Base b);
void Function(Derived d);
```

# Implementing Overloading

```
void Function();
void Function(int x);
void Function(double x);
void Function(Base b);
void Function(Derived d);
```

```
Function(137);
```

# Implementing Overloading

```
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void Function(double x);
void Function(Base b);
void Function(Derived d);
```
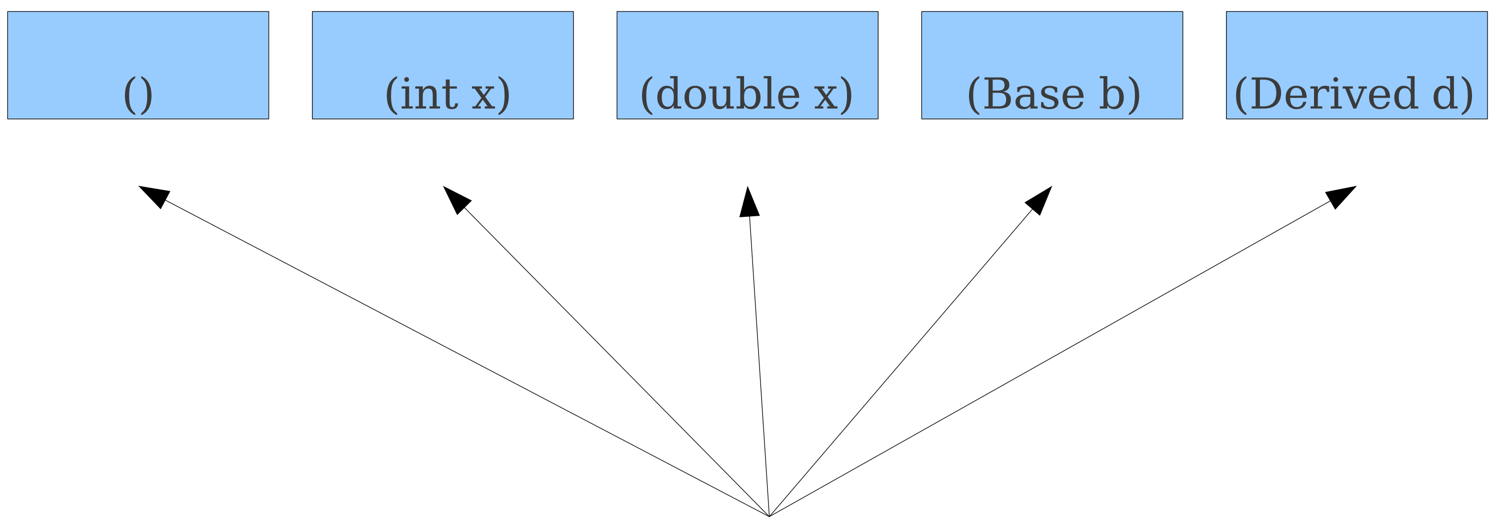
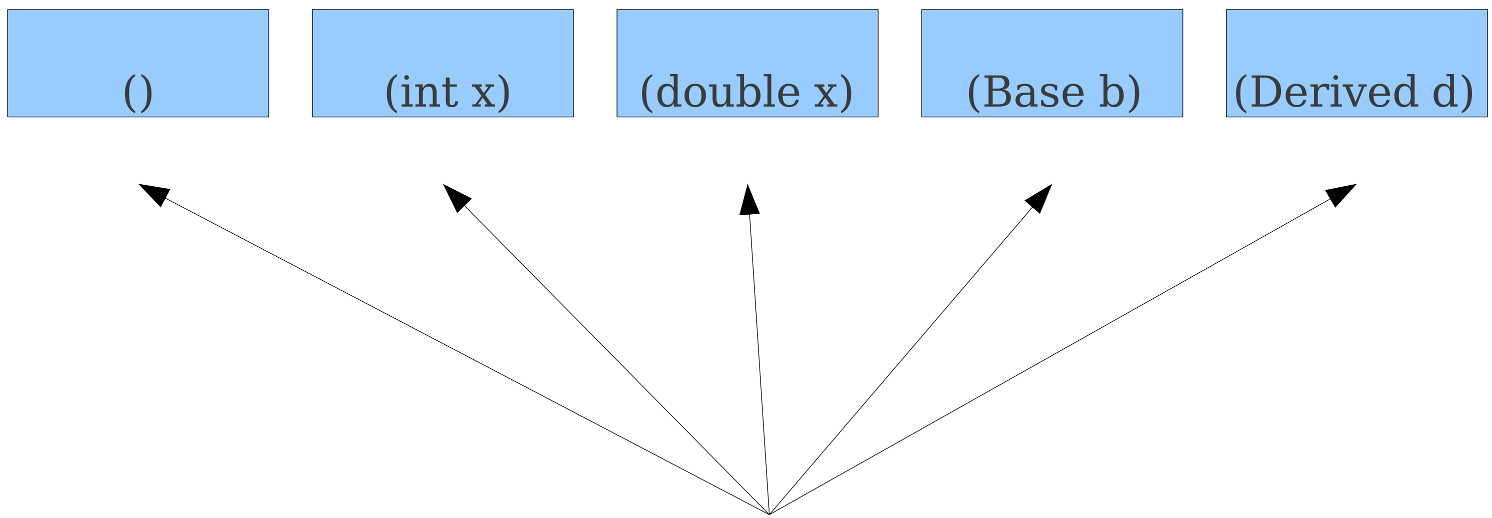




**`Function(137);`**

# Implementing Overloading

```
void Function();
void Function(int x);
void Function(double x);
void Function(Base b);
void Function(Derived d);
```




```
Function(137);
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# Implementing Overloading

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void Function();
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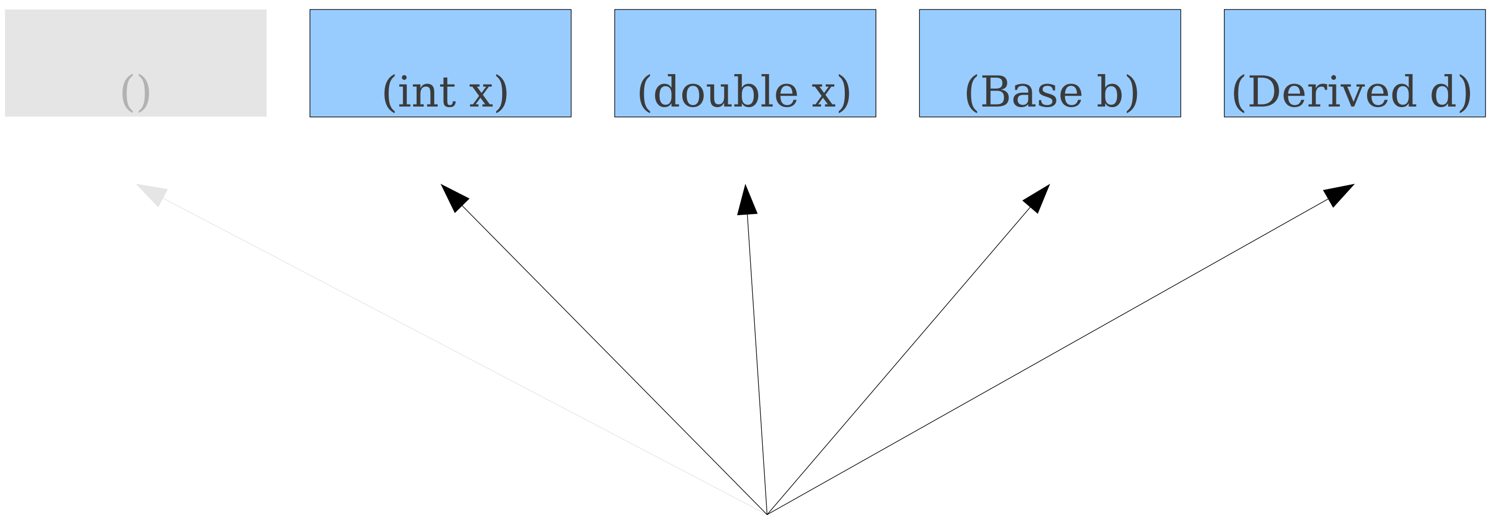


```
Function(137);
```

# Implementing Overloading

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void Function();
void Function(int x);
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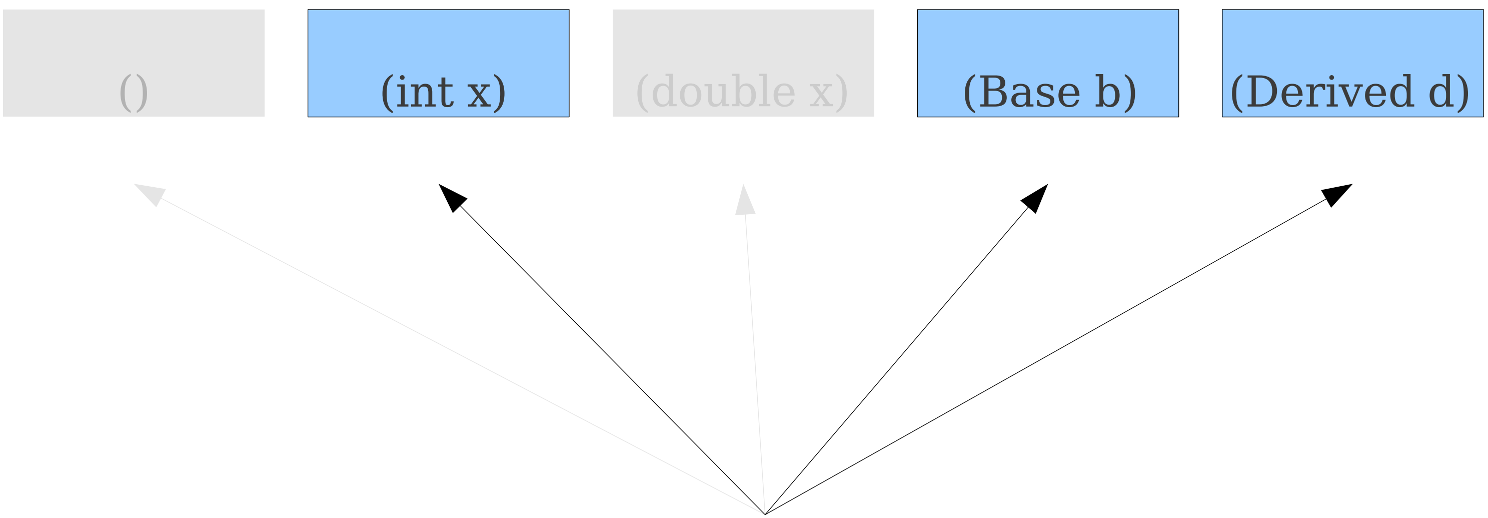


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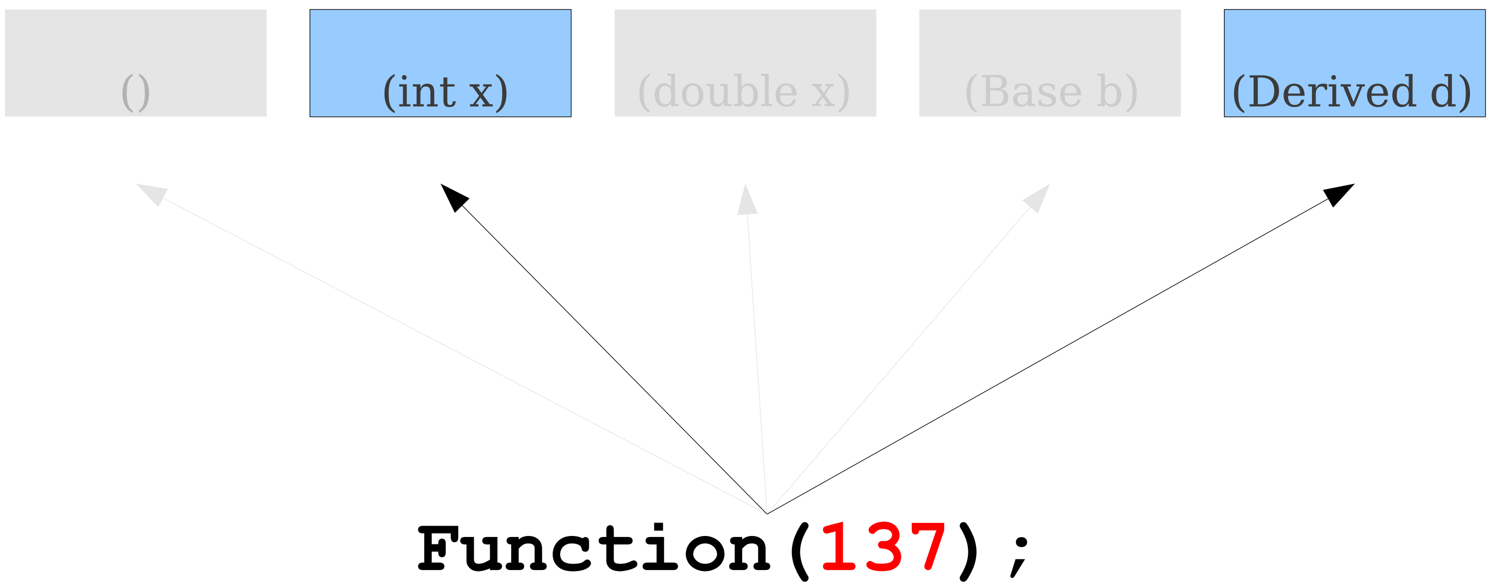

# Implementing Overloading

```
void Function();
void Function(int x);
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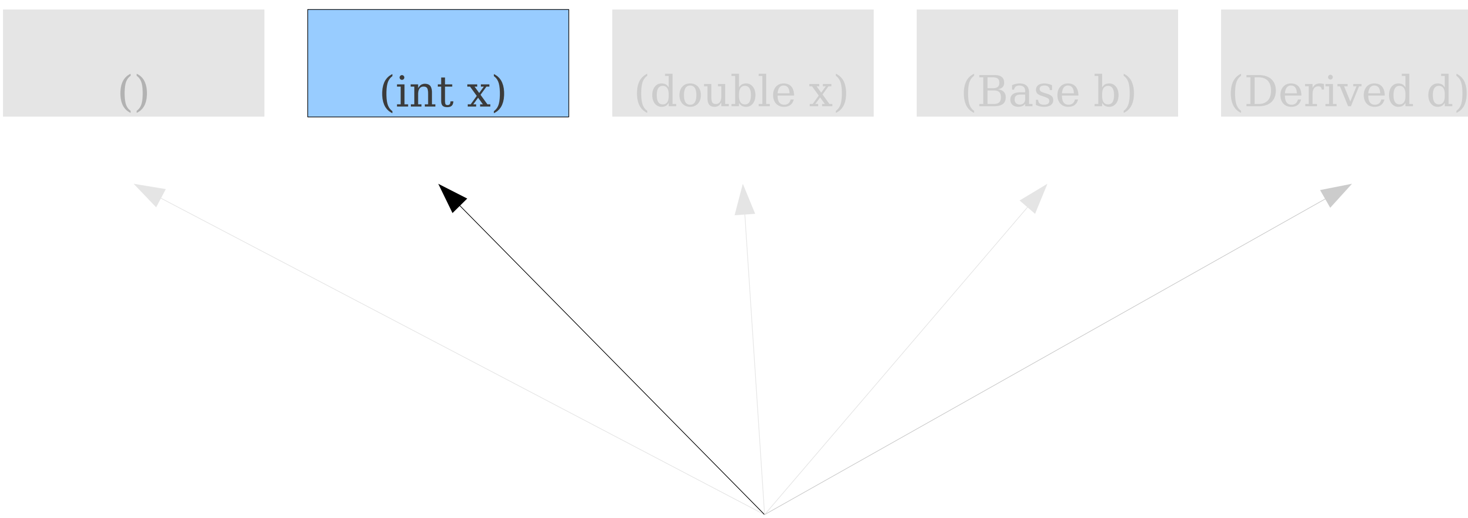
void Function(Base b);
void Function(Derived d);
```




```
Function(137);
```

# Implementing Overloading

```
void Function();
void Function(int x);
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void Function(Base b);
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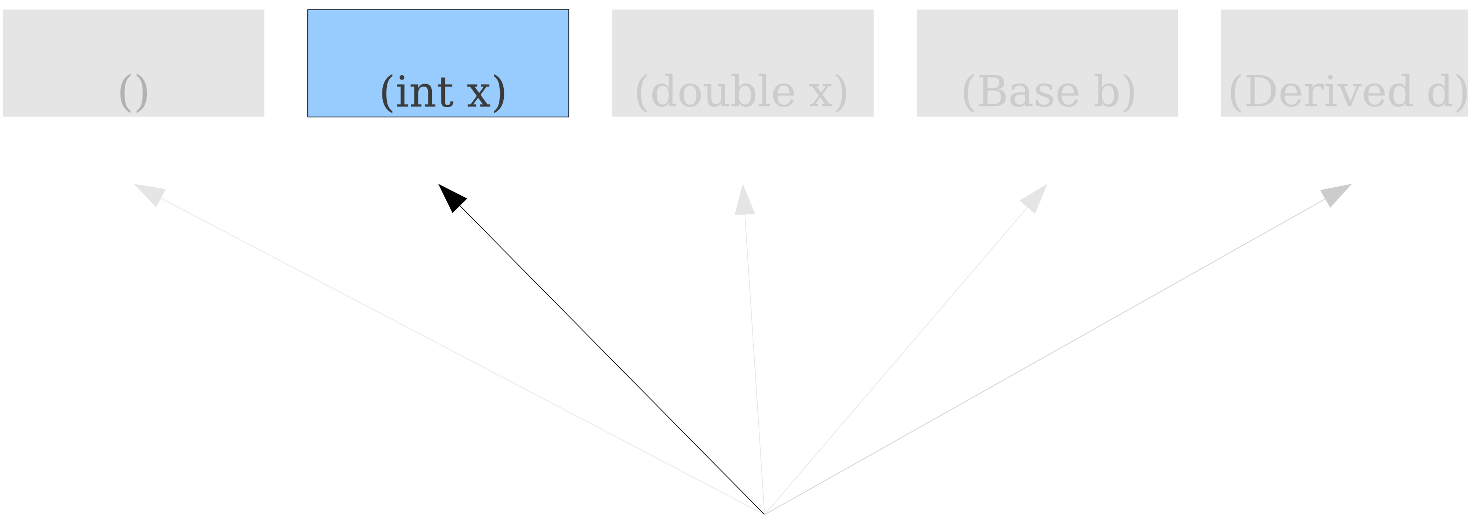
```




```
Function(137);
```

# Simple Overloading

- We begin with a set of overloaded functions.
- After filtering out functions that cannot match, we have a **candidate set** (C++ terminology) or set of **potentially applicable methods** (Java-speak).
- If no functions are left, report an error.
- If exactly one function left, choose it.
- (We'll deal with two or more in a second)

# Overloading with Inheritance

```
void Function();  void
Function(int x);
void Function(double x);
void Function(Base b);  void
Function(Derived d);
```

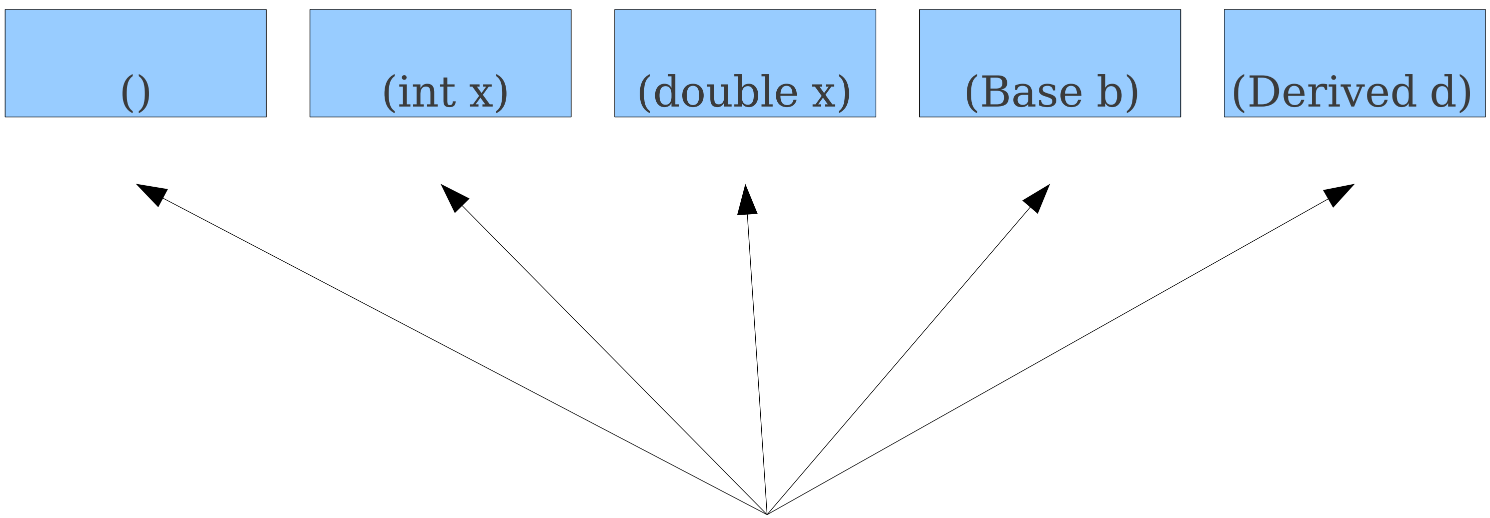
# Overloading with Inheritance

```
void Function();  void
Function(int x);
void Function(double x);
void Function(Base b);  void
Function(Derived d);
```

```
Function(new Derived);
```

# Overloading with Inheritance

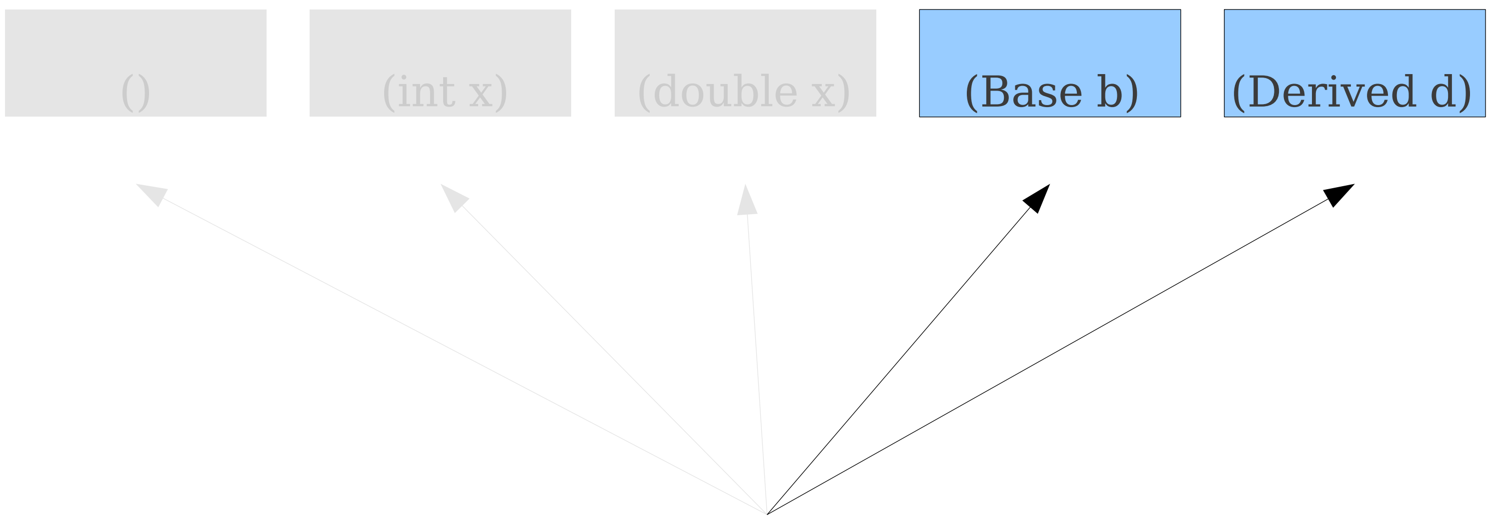
```
void Function();  void
Function(int x);
void Function(double x);
void Function(Base b);  void
Function(Derived d);
```




**`Function(new Derived);`**

# Overloading with Inheritance

```
void Function();
void Function(int x);
void Function(double x);
void Function(Base b);
void Function(Derived d);
```

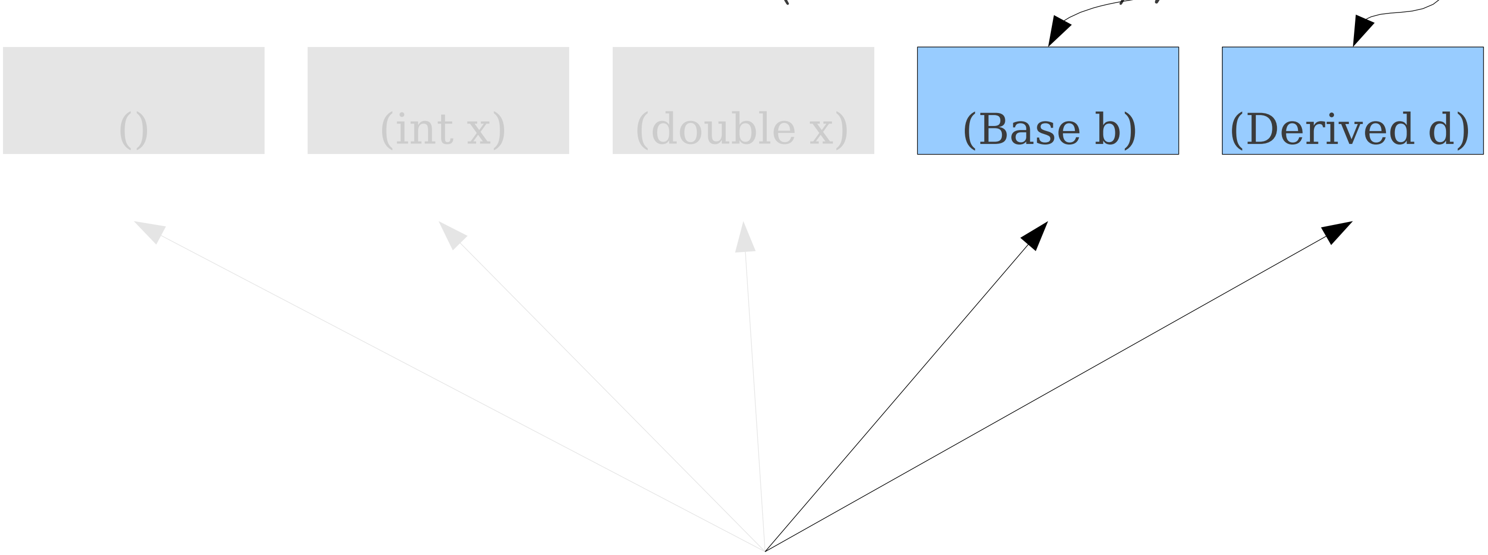



```
Function(new Derived);
```

# Overloading with Inheritance

```
void Function();
void Function(int x);
void Function(double x);
void Function(Base b);
void Function(Derived d);
```

How do we compare these?



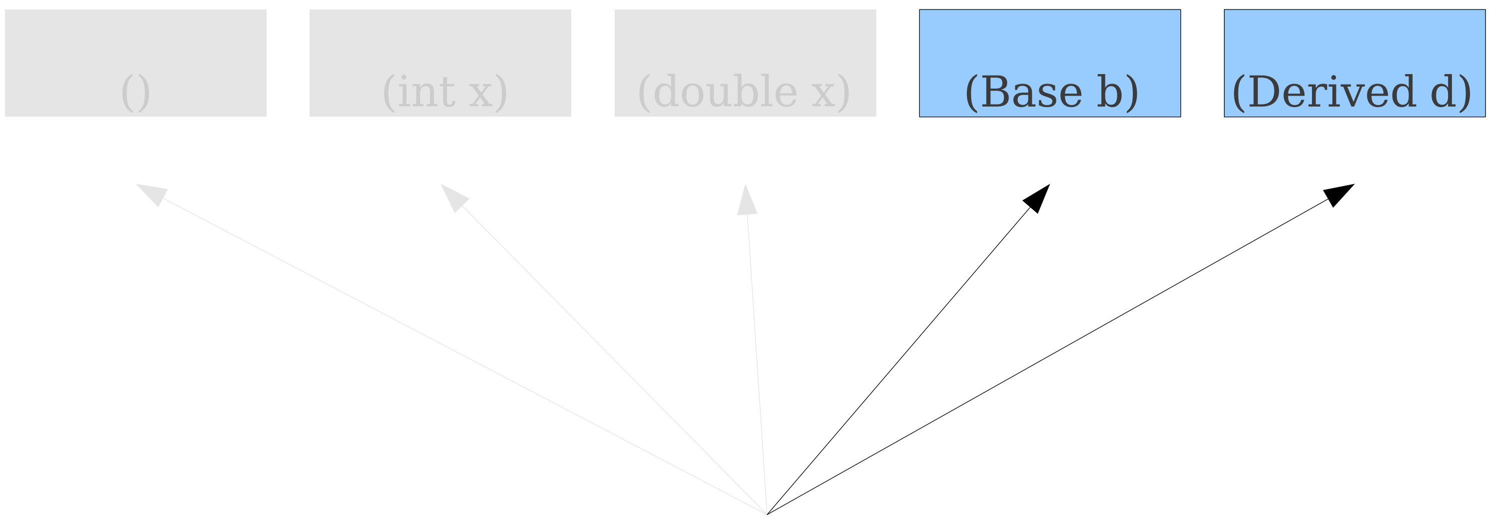

```
Function(new Derived);
```

# Finding the Best Match

- Choose one function over another if it's strictly more specific.
- Given two candidate functions A and B with argument types $A_1, A_2, \ldots, A_n$ and $B_1, B_2, \ldots, B_n$, we say that A <: B if $A_i \leq B_i$ for all i, $1 \leq i \leq n$.
  - This relation is also a **partial order**.
- A candidate function A is the **best match** if for any candidate function B, A <: B.
  - It's at least as good any other match.
- If there is a best match, we choose that function. Otherwise, the call is ambiguous.

# Overloading with Inheritance

```
void Function();  void
Function(int x);
void Function(double x);
void Function(Base b);  void
Function(Derived d);
```




**`Function(new Derived);`**

# Overloading with Inheritance

```
void Function();  void
Function(int x);
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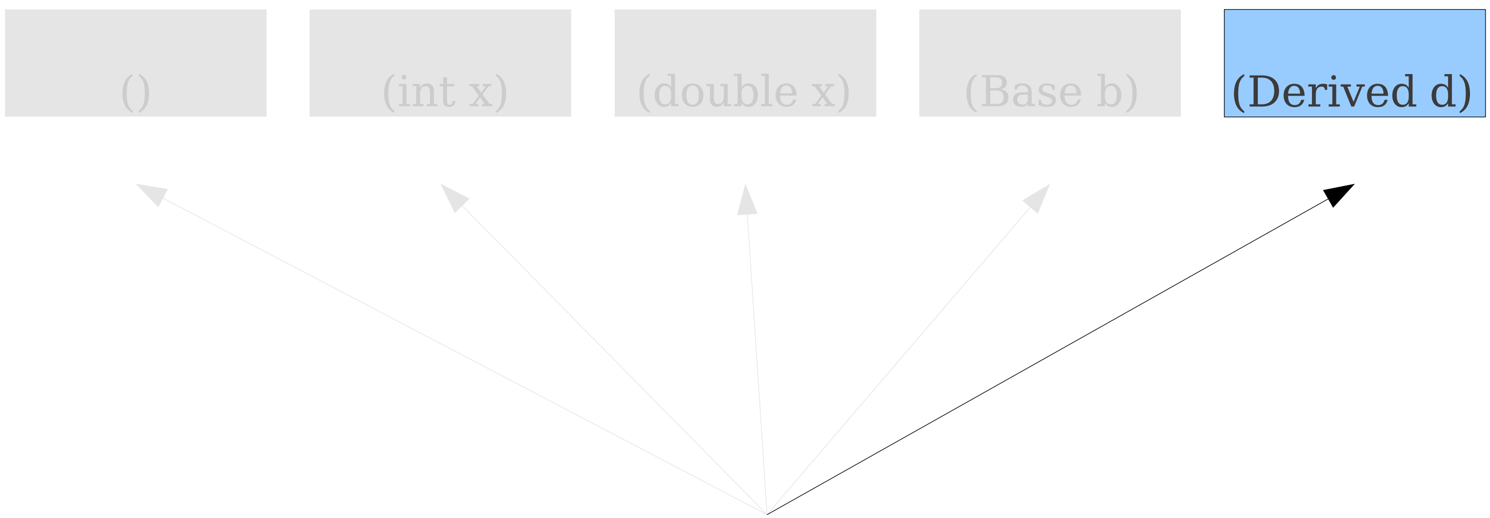


```
Function(new Derived);
```

# Overloading with Inheritance

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void Function();
void Function(int x);
void Function(double x);
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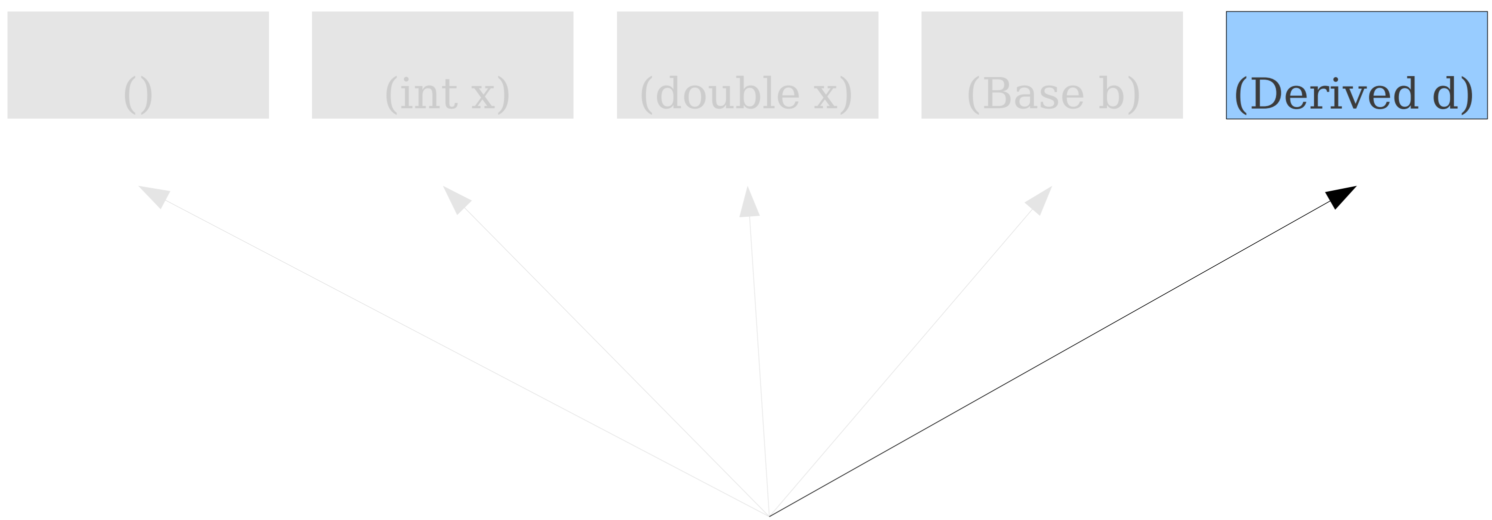
```




```
Function(new Derived);
```

# Ambiguous Calls

```
void Function(Base b1, Base b2);
void Function(Derived d1, Base b2);
void Function(Base b1, Derived d2);
```

# Ambiguous Calls

```
void Function(Base b1, Base b2);
void Function(Derived d1, Base b2);
void Function(Base b1, Derived d2);
```

```
Function(new Derived, new Derived);
```

# Ambiguous Calls

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void Function(Base b1, Base b2);
void Function(Derived d1, Base b2);
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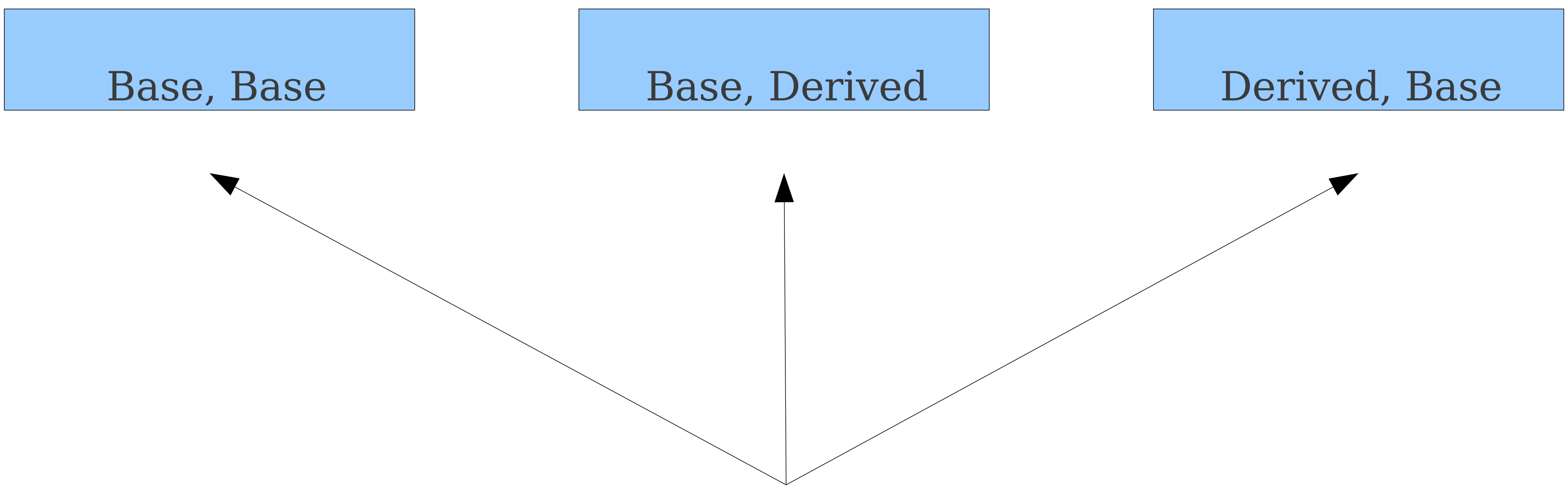


```
Function(new Derived, new Derived);
```

# Ambiguous Calls

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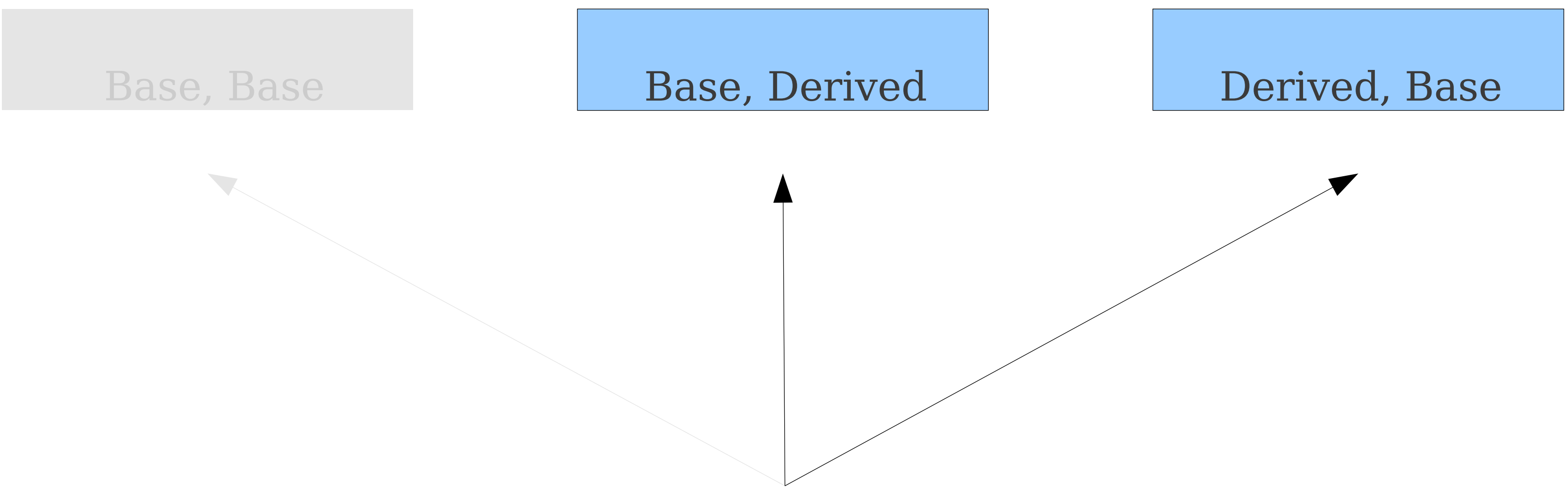


```
Function(new Derived, new Derived);
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# Ambiguous Calls

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void Function(Base b1, Base b2);
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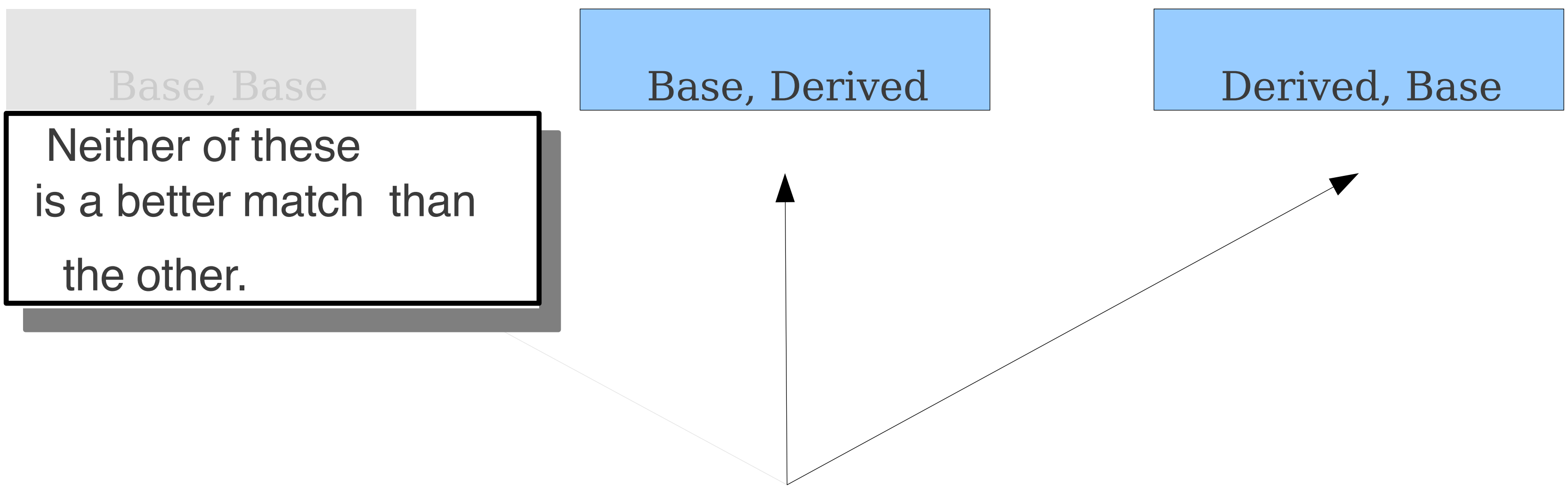


```
Function(new Derived, new Derived);
```

# Variadics

- Often much more complex than this.
- Example: **variadic functions**.
    - Functions that can take multiple arguments.
- Supported by C, C++, and Java.

# Overloading with Variadic Functions

```
void Function(Base b1, Base b2);
void Function(Derived d1, Base b2);
void Function(Derived d1, ...);
```

# Overloading with Variadic Functions

```
void Function(Base b1, Base b2);
void Function(Derived d1, Base b2);
void Function(Derived d1, ...);
```

```
Function(new Derived, new Derived);
```

# Overloading with Variadic Functions

```
void Function(Base b1, Base b2);
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void Function(Derived d1, ...);
```
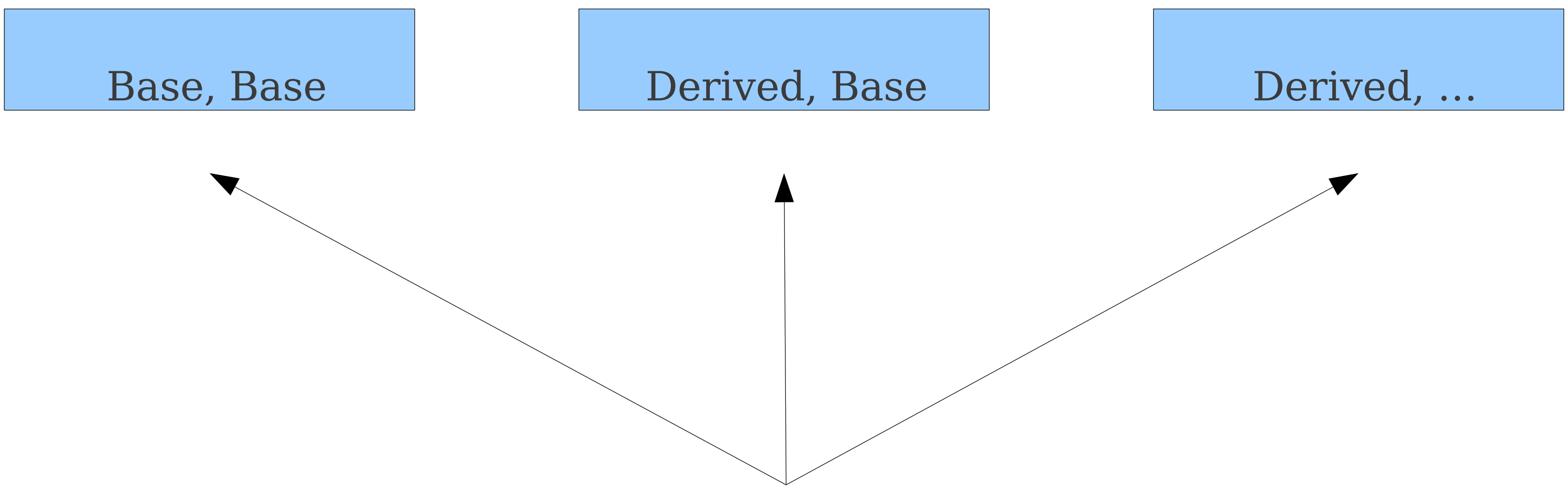
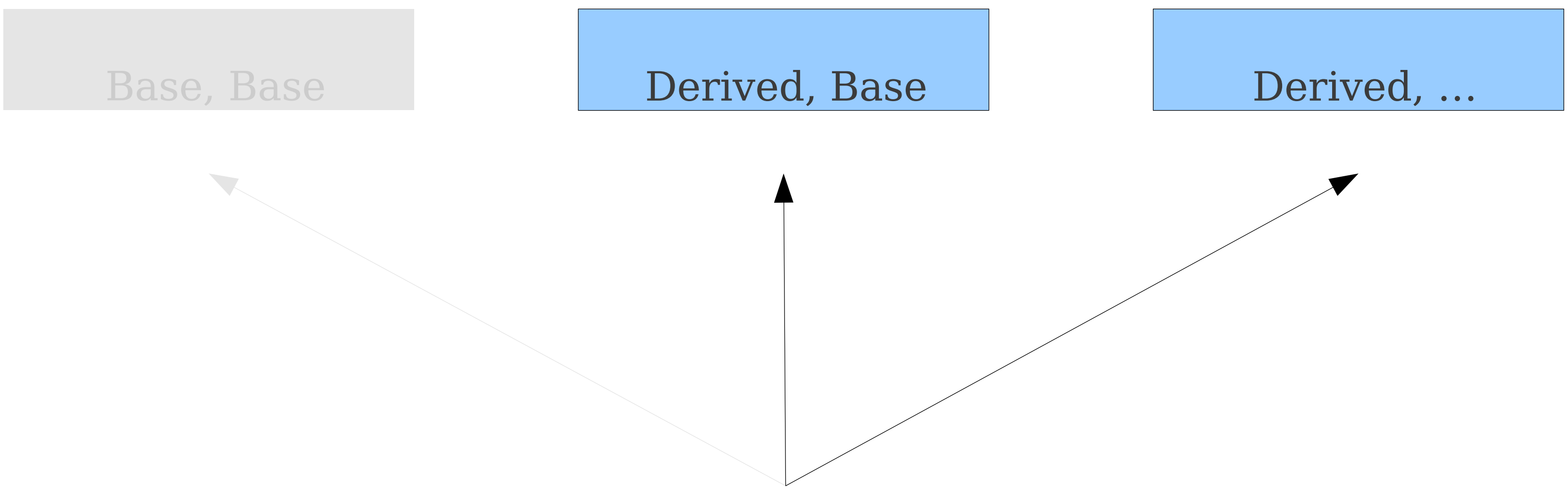




```
Function(new Derived, new Derived);
```

# Overloading  with Variadic Functions

```
void Function(Base b1, Base b2);
void Function(Derived d1, Base b2);
void Function(Derived d1, ...);
```




```
Function(new Derived, new Derived);
```

# Overloading with Variadic Functions

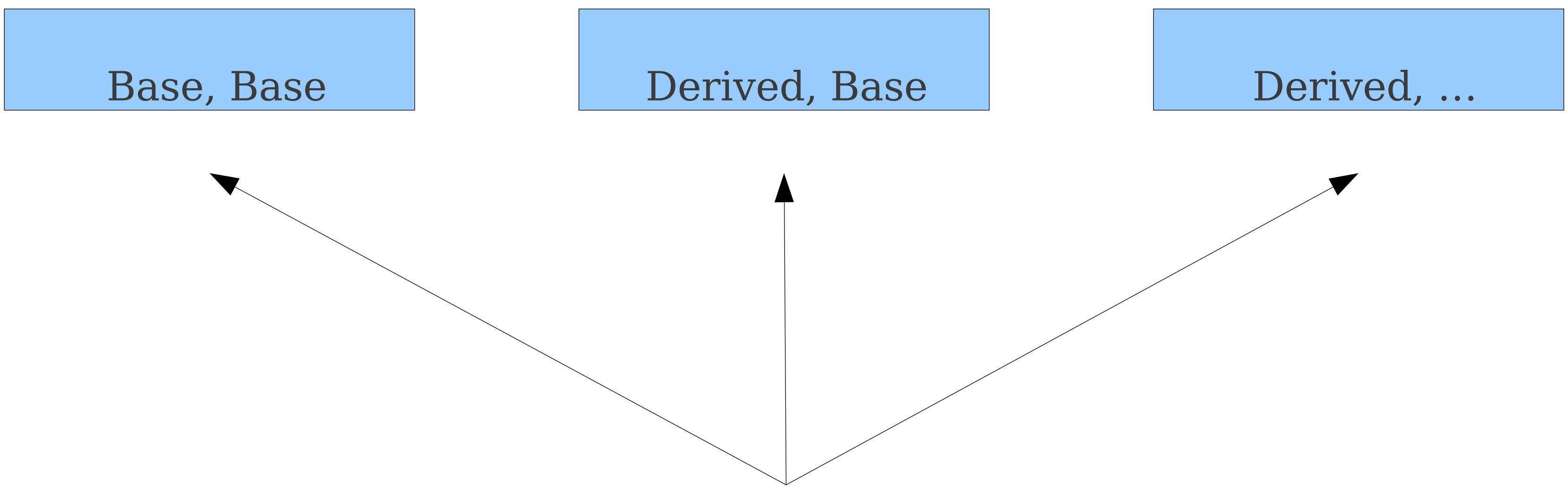
- Option one: **Consider the call ambiguous**.
  - There are indeed multiple valid function calls, and that's that!
- Option two: **Prefer the non-variadic function**.
  - A function specifically designed to handle a set of arguments is probably a better match than one designed to handle arbitrarily many parameters.
  - Used in both C++ and (with minor modifications) Java.

# Hierarchical Function Overloads

- Idea: Have a hierarchy of candidate functions.
- Conceptually similar to a scope chain:
  - Start with the lowest hierarchy level and look for an overload.
  - If a match is found, choose it.
  - If multiple functions match, report an ambiguous call.
  - If no match is found, go to the next level in the chain.
- Similar techniques used in other places:
  - Template / generic functions.
  - Implicit conversions

# Overloading with Variadic Functions

```
void Function(Base b1, Base b2);
void Function(Derived d1, Base b2);
void Function(Derived d1, ...);
```




```
Function(new Derived, new Derived);
```

# Overloading with Variadic Functions

```
void Function(Base b1, Base b2);
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void Function(Derived d1, ...);
```
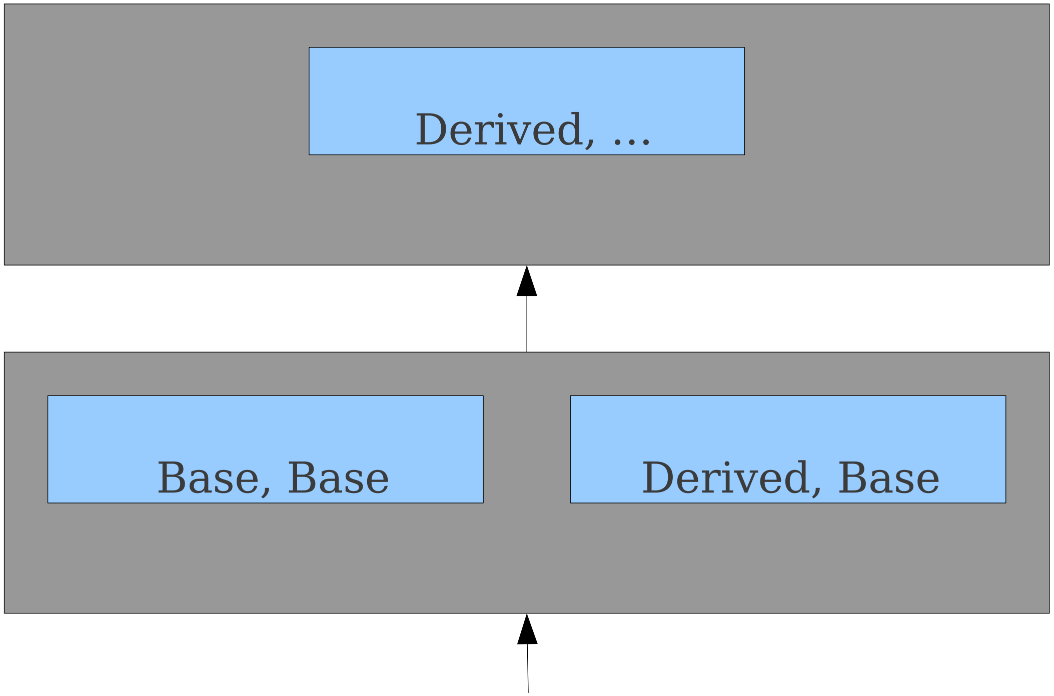
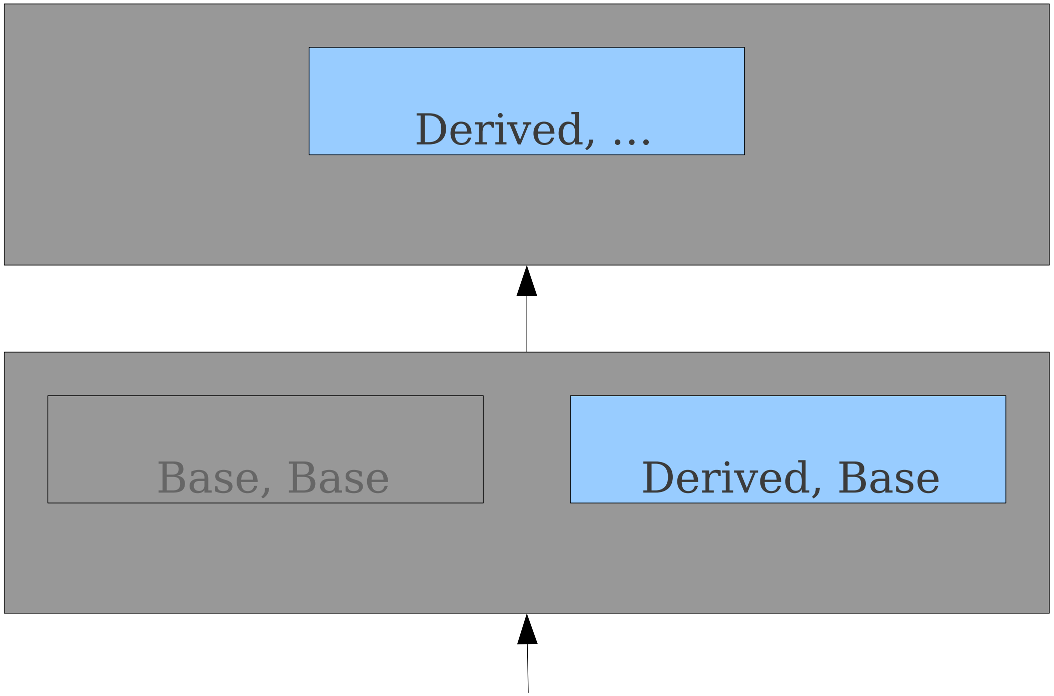




```
Function(new Derived, new Derived);
```

# Overloading with Variadic Functions

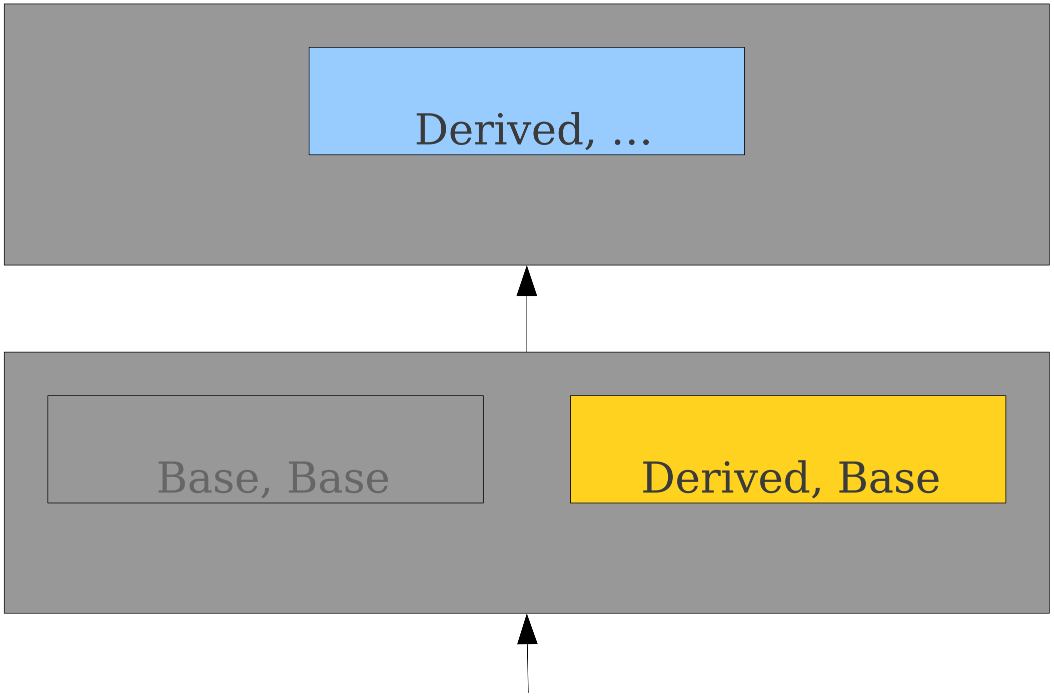
```
void Function(Base b1, Base b2);
void Function(Derived d1, Base b2);
void Function(Derived d1, ...);
```




```
Function(new Derived, new Derived);
```

# Overloading with Variadic Functions

```
void Function(Base b1, Base b2);
void Function(Derived d1, Base b2);
void Function(Derived d1, ...);
```




```
Function(new Derived, new Derived);
```

# Overloading with Variadic Functions

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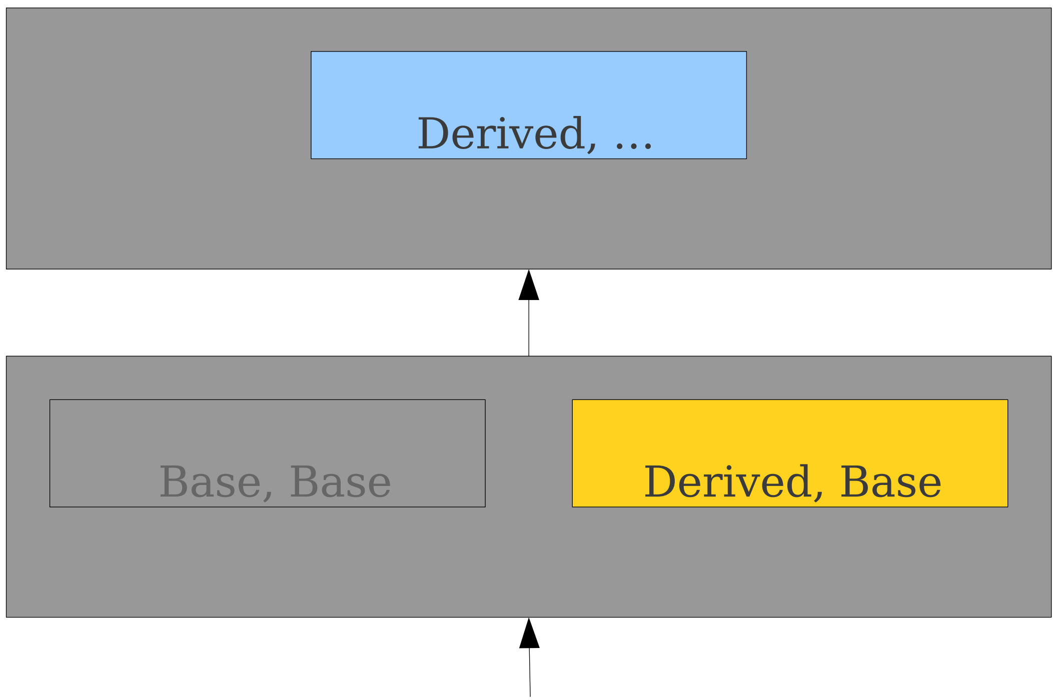


```
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# Overloading with Variadic Functions

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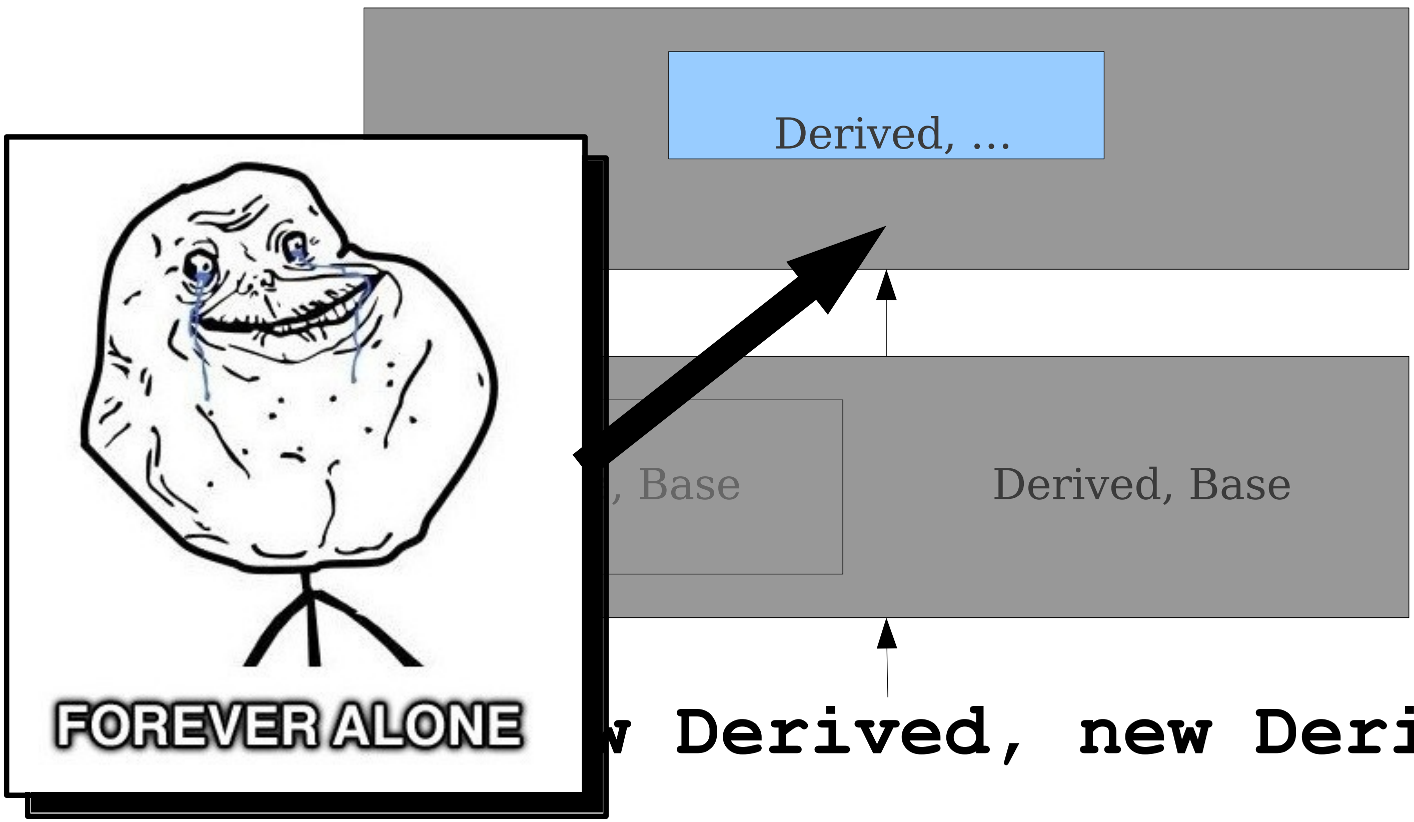
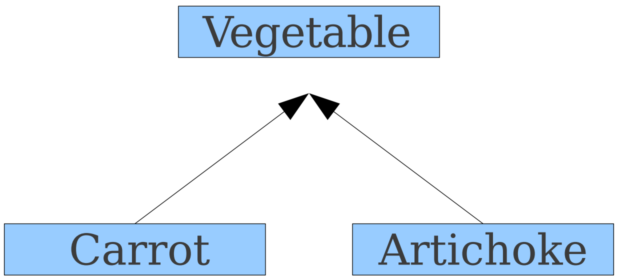



```
w Derived, new Derived);
```

# Covariance and Contravariance

# What are the?

- Consider following:
  - Object to object
  - Array to Array
  - IComparable to IComparable



# A Rule for Member Functions

$$\frac{}{S \vdash e_0.f(e_1, \ldots, e_n) \quad : ?}$$

# A Rule for Member Functions

*f* is an identifier.

$$\frac{}{S \vdash e_0.f(e_1, \ldots, e_n) \quad : ?}$$

# A Rule for Member Functions

$$\frac{\begin{array}{c} f \text{ is an identifier.} \\ S \vdash e_0 : M \end{array}}{S \vdash e_0.f(e_1, \ldots, e_n) \quad : ?}$$

# A Rule for Member Functions

*f* is an identifier.

$S \vdash e_0 : M$

*f* is a member function in class M.

$$\frac{}{S \vdash e_0.f(e_1, \ldots, e_n) \quad : ?}$$

# A Rule for Member Functions

*f* is an identifier.

$S \vdash e_0 : M$

*f* is a member function in class M.

*f* has type $(T_1, \ldots, T_n) \to U$

$$\frac{\quad}{S \vdash e_0.f(e_1, \ldots, e_n) \quad : ?}$$

# A Rule for Member Functions

$$\frac{\begin{array}{c} f \text{ is an identifier.} \\ S \vdash e_0 : M \\ f \text{ is a member function in class } M. \\ f \text{ has type } (T_1, \dots, T_n) \to U \\ S \vdash e_i : R_i \quad \text{for } 1 \le i \le n \\ R_i \le T_i \quad \text{for } 1 \le i \le n \end{array}}{S \vdash e_0.f(e_1, \dots, e_n) \quad : ?}$$

# A Rule for Member Functions

$f$ is an identifier.

$S \vdash e_0 : M$

$f$ is a member function in class M.

$f$ has type $(T_1, \ldots, T_n) \to U$

$S \vdash e_i : R_i$ for $1 \leq i \leq n$

$R_i \leq T_i$ for $1 \leq i \leq n$

$$\frac{}{S \vdash e_0.f(e_1, \ldots, e_n) \quad : U}$$

# Legality and Safety

```
class Id {
     Id me() {
          return this;
     }
     void beSelfish() {
          /* … */
     }
}

class Ego extends Id
     {  void bePractical()
     {
          /* … */
     }
}

int main() {
     (new Ego).me().bePractical();
}
```

# Legality and Safety

```
class Id {
     Id me() {
          return this;
     }
     void beSelfish() {
          /* … */
     }
}

class Ego extends Id
     {  void bePractical()
     {
          /* … */
     }
}

int main() {
     (new Ego).me().bePractical();
}
```

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```

**Ego**

# Legality and Safety

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          /* … */
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     }
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int main() {
     (new Ego).me().bePractical();
}
```

**Ego**

# Legality and Safety

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int main() {
      (new Ego).me().bePractical();
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```

$\underbrace{\texttt{(new Ego).me()}}_{\texttt{Ego}}$

$$
\frac{
\begin{array}{c}
f \text{ is an identifier.} \\
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$\underbrace{\underbrace{\texttt{(new Ego)}}_{\texttt{Ego}}\texttt{.me()}}_{\texttt{Id}}$

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class Ego extends Id
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      {
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       }
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int main() {
      (new Ego).me().bePractical();
}
```

Ego    Id

*f* is an identifier.

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$$\frac{}{S \vdash e_0.f(e_1, \ldots, e_n) : U}$$

bePractical is not in Id!

# Limitations of Static Type Systems

- Static type systems are often **incomplete**.
  - There are valid programs that are rejected.
- Tension between the **static** and **dynamic** types of objects.
  - Static type is the type declared in the program source.
  - Dynamic type is the actual type of the object at runtime.

# Soundness and Completeness

- Static type systems sometimes reject valid programs because they cannot prove the absence of a type error.
- A type system like this is called **incomplete**.
- Instead, try to prove for every expression that

$$\text{DynamicType}(E) \leq \text{StaticType}(E)$$

- A type system like this is called **sound**.

# An Impossibility Result

- Unfortunately, for most programming languages, it is provably impossible to have a sound and complete static type checker.
- Intuition: Could build a program that makes a type error iff a certain Turing machine accepts a given string.
- Type-checking equivalent to  solving the halting problem!

# Building a Good Static Checker

- It is difficult to  build a good static type checker.
  - Easy to have unsound rules.
  - Impossible to accept all valid programs.
- Goal: make the language as complete as possible with sound type-checking rules.

# Relaxing our Restrictions

```
class Base {
     Base clone()
          {  return new
          Base;
     }
}

class Derived extends Base {
     Base clone() {
          return new Derived;
     }
}
```

# Relaxing our Restrictions

```
class Base {
     Base clone()
          {   return new
          Base;
     }
}

class Derived extends Base {
     Base clone() {
          return new Derived;
     }
}
```

# Relaxing our Restrictions

```
class Base {
      Base clone()
            {  return new
            Base;
      }
}

class Derived extends Base {
      Derived clone()
            {  return new
            Derived;
      }
}
```

# Relaxing our Restrictions

```
class Base {
    Base clone()
        { return new
        Base;
    }
}

class Derived extends Base {
    Derived clone() {
        return new
        Derived;
    }
}
```

Is this safe?

# The Intuition

```
Base b = new Base;
Derived d = new Derived;
```

# The Intuition

```
Base b = new Base;
Derived d = new Derived;

Base b2 = b.clone();
```

# The Intuition

```
Base b = new Base;
Derived d = new Derived;

Base  b2  =     b.clone();
Base  b3  =     d.clone();
```

# The Intuition

```
Base b = new Base;
Derived d = new Derived;

Base  b2  =     b.clone();

Base  b3  =     d.clone();
Derived d2 = b.clone();
```

# The Intuition

```
Base b = new Base;
Derived d = new Derived;

Base  b2  =     b.clone();
Base  b3  =     d.clone();
Derived d2 = b.clone();
```

# The Intuition

```
Base b = new Base;
Derived d = new Derived;

Base   b2   =      b.clone();
Base   b3   =      d.clone();
Derived d2 = b.clone();
Derived d3 = d.clone();
```

# The Intuition

```
Base b = new Base;
Derived d = new Derived;

Base   b2   =      b.clone();
Base   b3   =      d.clone();
Derived d2 = b.clone();
Derived d3 = d.clone();

Base reallyD = new Derived;
```

# The Intuition

```
Base b = new Base;
Derived d = new Derived;

Base  b2  =     b.clone();
Base  b3  =     d.clone();
Derived d2 = b.clone();
Derived d3 = d.clone();

Base reallyD = new Derived;
Base b4 = reallyD.clone();
```

# The Intuition

```
Base b = new Base;
Derived d = new Derived;

Base   b2   =     b.clone();
Base   b3   =     d.clone();
Derived d2 = b.clone();
Derived d3 = d.clone();

Base reallyD = new Derived;
Base b4 = reallyD.clone();
Derived d4 = reallyD.clone();
```

# The Intuition

```
Base b = new Base;
Derived d = new Derived;

Base   b2   =      b.clone();
Base   b3   =      d.clone();
Derived d2 = b.clone();
Derived d3 = d.clone();

Base reallyD = new Derived;
Base b4 = reallyD.clone();
Derived d4 = reallyD.clone();
```

# Is this Safe?

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This refers to the **static** type of the function.

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f has **dynamic** type

$(T_1, T_2, \ldots, T_n) \to V$

and we know that

$V \le U$

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f has **dynamic** type

$(T_1, T_2, \ldots, T_n) \to V$

and we know that

$V \leq U$

So the rule is sound!

# Covariant Return Types

- Two functions A and B are **covariant** in their return types if the return type of A is convertible to the return type of B.
- Many programming language support covariant return types.
  - C++ and Java, for example.
- Not supported in Decaf.

# Relaxing our Restrictions (Again)

```
class Base {
    bool equalTo(Base B) {
        /* … */
    }
}

class Derived extends Base {
    bool equalTo(Base B) {
        /* … */
    }
}
```

# Relaxing our Restrictions (Again)

```
class Base {
    bool equalTo(Base B) {
        /* … */
    }
}

class Derived extends Base {
    bool equalTo(Base B) {
        /* … */
    }
}
```

# Relaxing our Restrictions (Again)

```
class Base {
    bool equalTo(Base B) {
        /* … */
    }
}

class Derived extends Base
    {  bool equalTo(Derived
    B) {
        /* … */
    }
}
```

# Relaxing our Restrictions (Again)

```
class Base {
    bool equalTo(Base B) {
        /* … */
    }
}

class Derived extends Base
    {   bool equalTo(Derived
    D) {
        /* … */
    }
}
```

# Relaxing our Restrictions (Again)

```
class Base {
     bool equalTo(Base B) {
          /* … */
     }
}

class Derived extends Base
     {  bool equalTo(Derived
     D) {
          /* … */
     }
}
```

Is this safe?

# Is this Safe?

*f* is an identifier.

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*f* is a member function in class M.

*f* has type $(T_1, \ldots, T_n) \rightarrow U$

$S \vdash e_i : R_i$ for $1 \leq i \leq n$

$R_i \leq T_i$ for $1 \leq i \leq n$

---

$S \vdash e_0.f(e_1, \ldots, e_n) \quad : U$

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This refers to the **static** type of the function.

f has **dynamic** type

$(V_1, V_2, \ldots, V_n) \to U$

and we know that

$V_i \le T_i$ for $1 \le i \le n$

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$R_i \leq T_i \quad \text{for } 1 \leq i \leq n$

$$\frac{}{S \vdash e_0.f(e_1, \ldots, e_n) : U}$$

This refers to the **static** type of the function.

$R_i \leq T_i \quad \text{for } 1 \leq i \leq n$
$V_i \leq T_i \quad \text{for } 1 \leq i \leq n$

f has **dynamic** type

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f has **dynamic** type

$(V_1, V_2, \ldots, V_n) \rightarrow U$

and we know that

$V_i \leq T_i$ for $1 \leq i \leq n$

$R_i \leq T_i \quad \text{for } 1 \leq i \leq n$
$V_i \leq T_i \quad \text{for } 1 \leq i \leq n$

This doesn't mean that
$R_i \leq V_i \quad \text{for } 1 \leq i \leq n$

# A Concrete Example

# A Concrete Example

```
class Fine {
     void nothingFancy(Fine f) {
          /* … do nothing … */
     }
}
```

# A Concrete Example

```
class Fine {
    void nothingFancy(Fine f) {
        /* … do nothing … */
    }
}

class Borken extends Fine
    {  int missingFn() {
        return 137;
    }
    void nothingFancy(Borken b)
        {  Print(b.missingFn());
    }
}
```

# A Concrete Example

```
class Fine {
     void nothingFancy(Fine f) {
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     {  int missingFn() {
          return 137;
     }
     void nothingFancy(Borken b)
          {  Print(b.missingFn());
     }
}

int main() {
     Fine f = new Borken;
     f.nothingFancy(new Fine);
}
```

# A Concrete Example

```
class Fine {
    void nothingFancy(Fine f) {
        /* … do nothing … */
    }
}

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      Fine f = new Borken;
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(That calls this one)

# A Concrete Example

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(That calls this one)

# Covariant Arguments are Unsafe

- Allowing subclasses to restrict their parameter types is **fundamentally unsafe**.
- Calls through base class can send objects of the wrong type down to base classes.
- This is why Java's **`Object.equals`** takes another **`Object`**.
- Some languages got this wrong.
  - Eiffel allows functions to be covariant in their arguments; can cause runtime errors.

# Contravariant Arguments

```
class Super {}
class Base extends Super {
    bool equalTo(Base B) {
        /* … */
    }
}

class Derived extends Base {
    bool equalTo(Base B) {
        /* … */
    }
}
```

# Contravariant Arguments

```
class    Super  {}
class   Base    extends    Super    {

    bool equalTo(Base B) {
        /* … */
    }
}

class Derived extends Base {
    bool equalTo(Base B) {
        /* … */
    }
}
```

# Contravariant Arguments

```
class   Super   {}
class   Base    extends    Super    {
     bool equalTo(Base B) {
          /* … */
     }
}

class Derived extends Base {
     bool equalTo(Super B) {
          /* … */
     }
}
```

# Contravariant Arguments

```
class   Super   {}
class   Base    extends    Super    {
    bool equalTo(Base B) {
        /* … */
    }
}

class Derived extends Base {
    bool equalTo(Super B) {
        /* … */
    }
}
```

# Contravariant Arguments

```
class   Super   {}
class   Base    extends    Super    {
     bool equalTo(Base B) {
          /* … */
     }
}

class Derived extends Base {
     bool equalTo(Super B) {
          /* … */
     }
}
```

Is this safe?

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This refers to the **static** type of the function.

f has **dynamic** type

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and we know that

$T_i \le V_i$ for $1 \le i \le n$

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$T_i \le V_i$ for $1 \le i \le n$

$R_i \le T_i$ for $1 \le i \le n$

$T_i \le V_i$ for $1 \le i \le n$

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and we know that

$T_i \leq V_i$ for $1 \leq i \leq n$

$R_i \leq T_i$ for $1 \leq i \leq n$

$T_i \leq V_i$ for $1 \leq i \leq n$

so

**$R_i \leq V_i$ for $1 \leq i \leq n$**

# Contravariant Arguments are Safe

- Intuition: When called through base class, will accept anything the base class already would.
- Most languages do not support contravariant arguments.
- Why?
    - Increases the complexity of the compiler and the language specification.
    - Increases the complexity of checking method overrides.

# Contravariant Overrides

```
class Super {}
class Duper extends Super {}
class Base extends Super {
     bool equalTo(Base B) {
          /* … */
     }
}

class Derived extends Base
     {  bool equalTo(Super B) {
          /* … */
     }
     bool equalTo(Duper B) {
          /* … */
     }
}
```

# Contravariant Overrides

```
class Super {}
class Duper extends Super {}
class Base extends Super {
     bool equalTo(Base B) {
          /* … */
     }
}

class Derived extends Base {
    bool equalTo(Super B) {
         /* … */
    }
    bool equalTo(Duper B) {
         /* … */
    }
}
```

Two overrides?

Or an over**load** and

an over**ride**?

# So What?

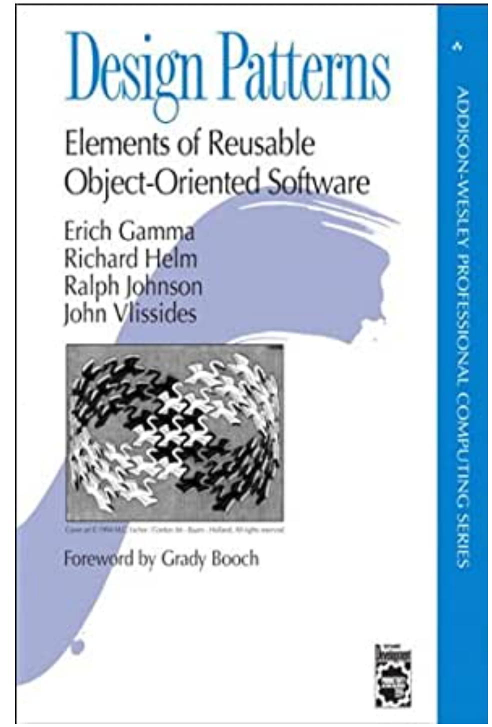
- Need to be **very careful** when introducing language features into a statically-typed language.
- Easy to design language features; hard to design language features that are type-safe.
- Type proof system can sometimes help detect these errors in the abstract.

# Summary

- We can extend our type proofs to handle well-formedness proofs.
- The **error type** is convertible to all other types and helps prevent cascading errors.
- Overloading is resolved at compile-time and determines which of many functions to call.
- Overloading ranks functions against one another to determine the best match.

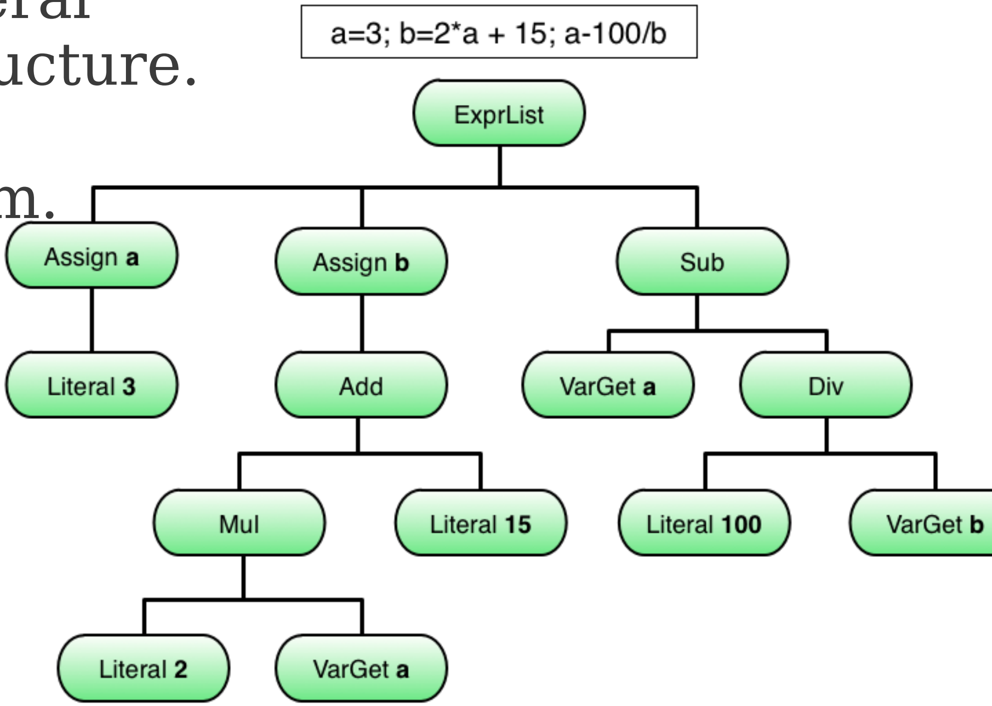
# How to Implement these analysis?

- We have to traverse the AST and perform various analysis.
- There is design pattern for that.
- We Use this book for this class.



# What is our Problem?

- There is an AST for a piece of code.
- We need to perform several analysis on the same structure.
- We may add/remove them.



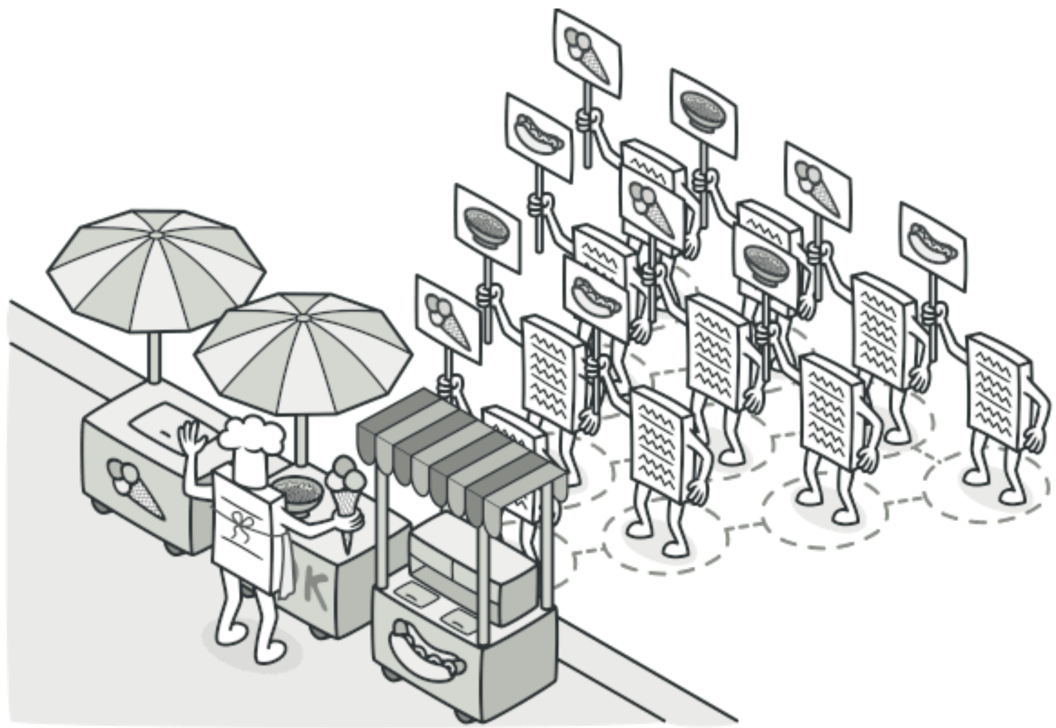
# Visitor Pattern-Intent

- Represent an **operation** to be performed on the **elements** of an *object structure*.

- Visitor lets you define a new operation without changing the classes of the elements on which it operates.

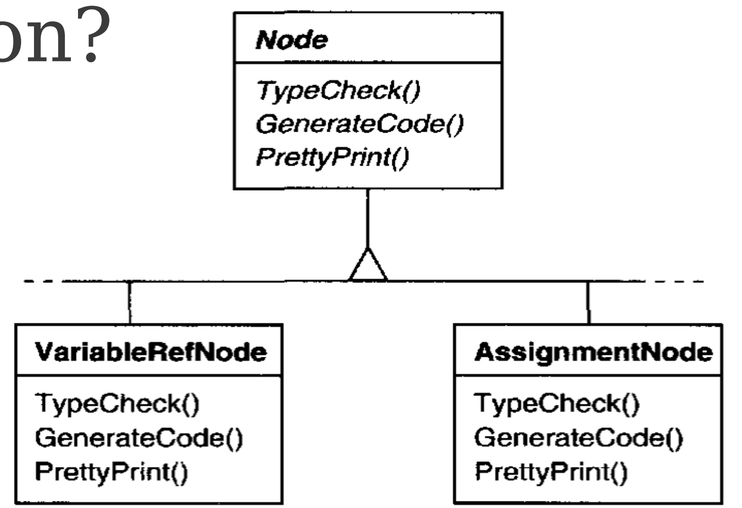
# Visitor Pattern-Intent

- Simply: **Visitor** is a behavioral design pattern that lets you separate algorithms from the objects on which they operate.

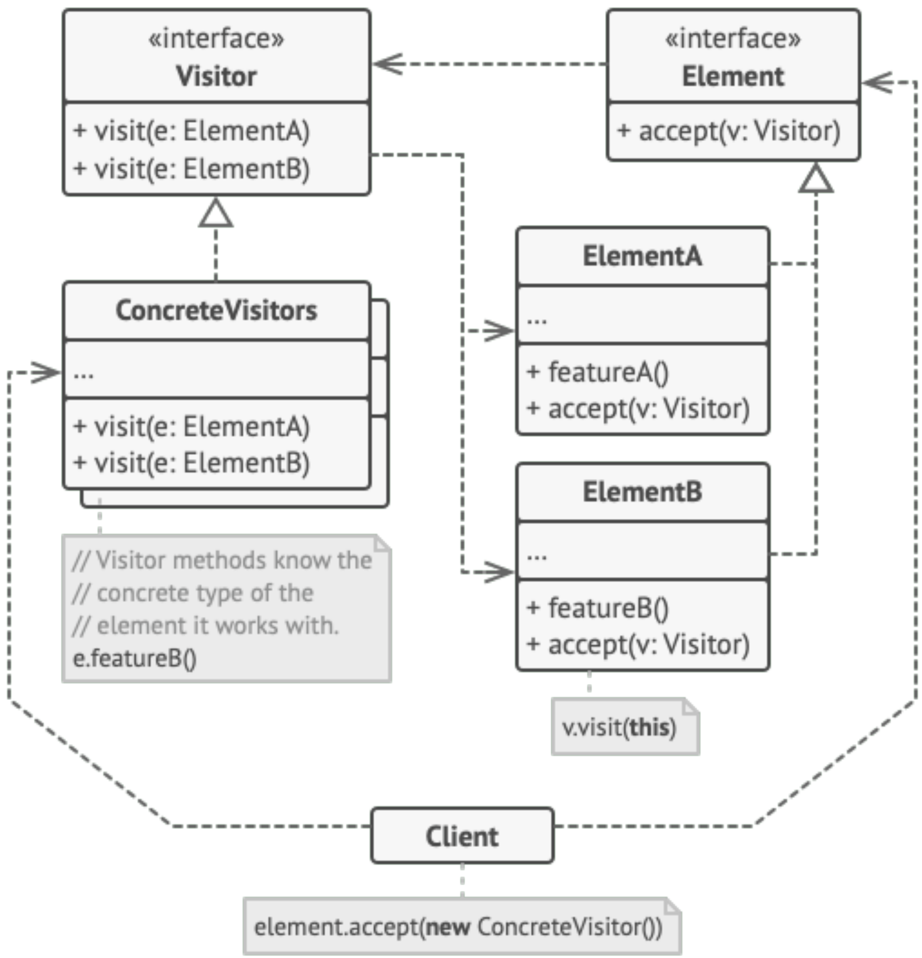


# Visitor Pattern-Motivation

- We need to perform action on AST for: Scope analysis, type checking, code generation, and etc.
- We might use AST also for: program restructuring, code instrumentation, and metric computation.
- What is your initial Solution?

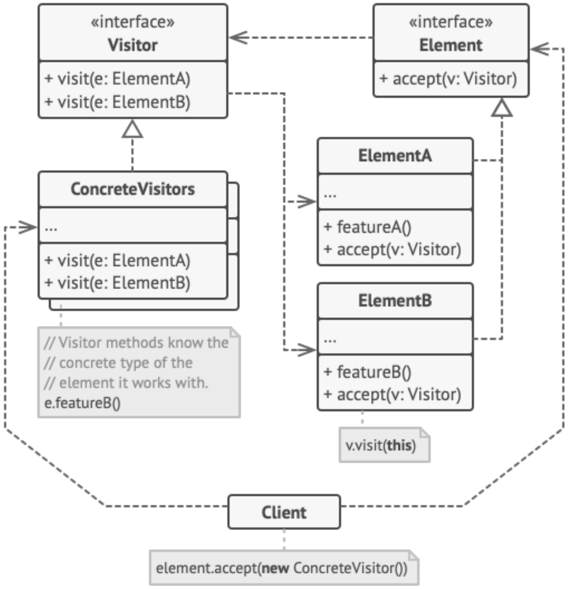


# Visitor Pattern: Structure



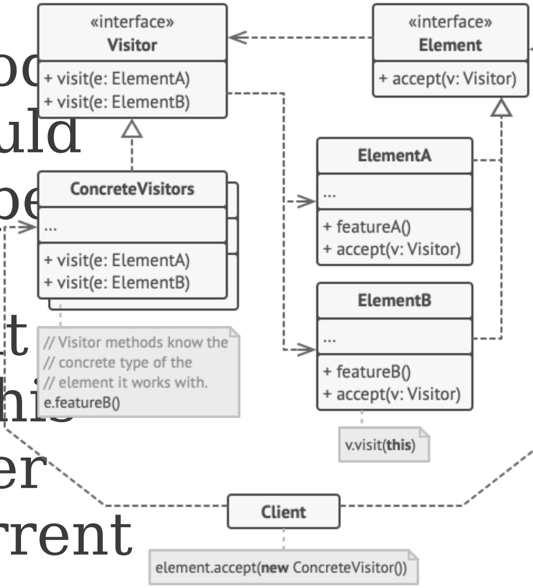
# Visitor Pattern: Elements

- The Visitor **interface** declares a set of visiting methods that can take concrete elements of an object structure as arguments. These methods may have the same names if the program is written in a language that supports overloading, but the type of their parameters must be different.
- Each **Concrete Visitor** implements several versions of the same behaviors, tailored for different concrete element classes.

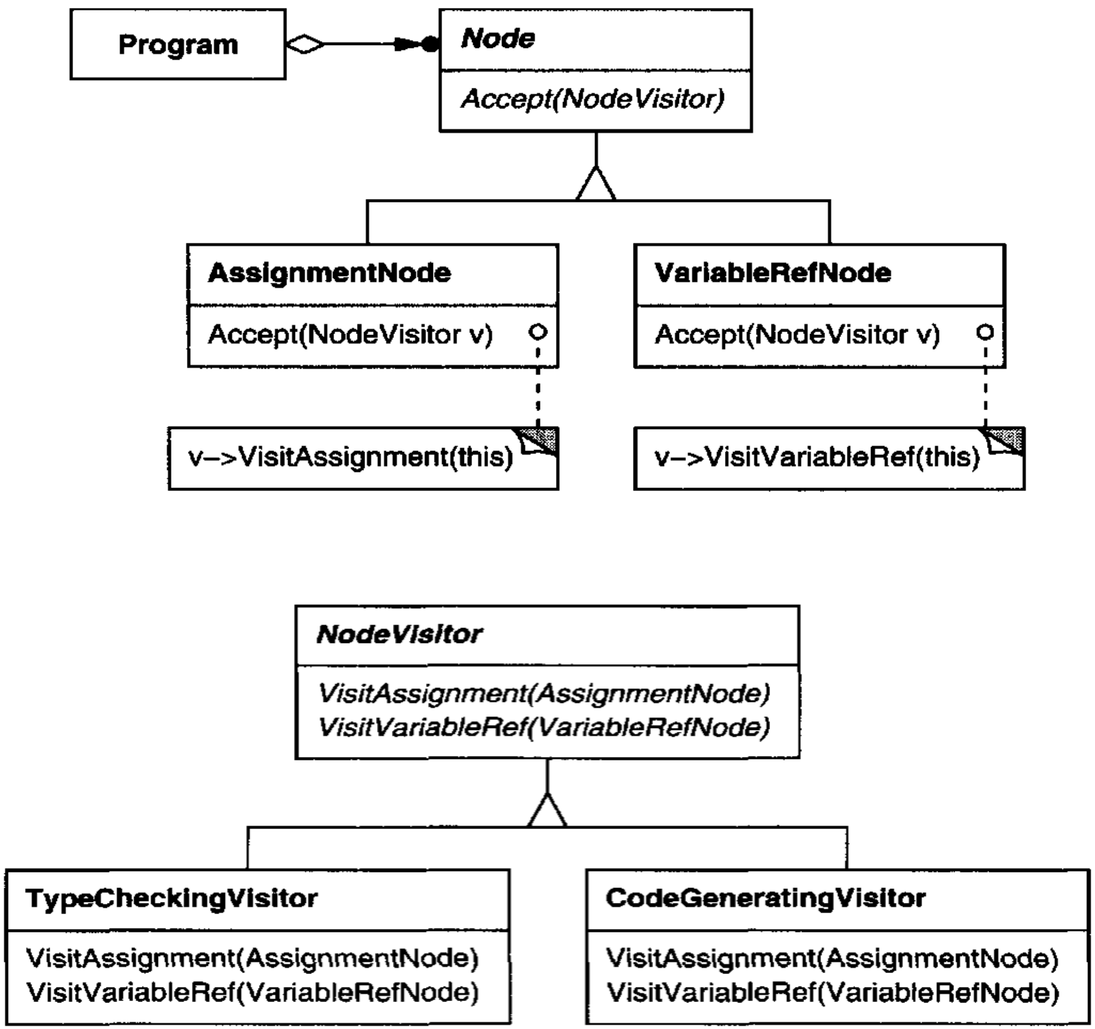


# Visitor Pattern: Elements

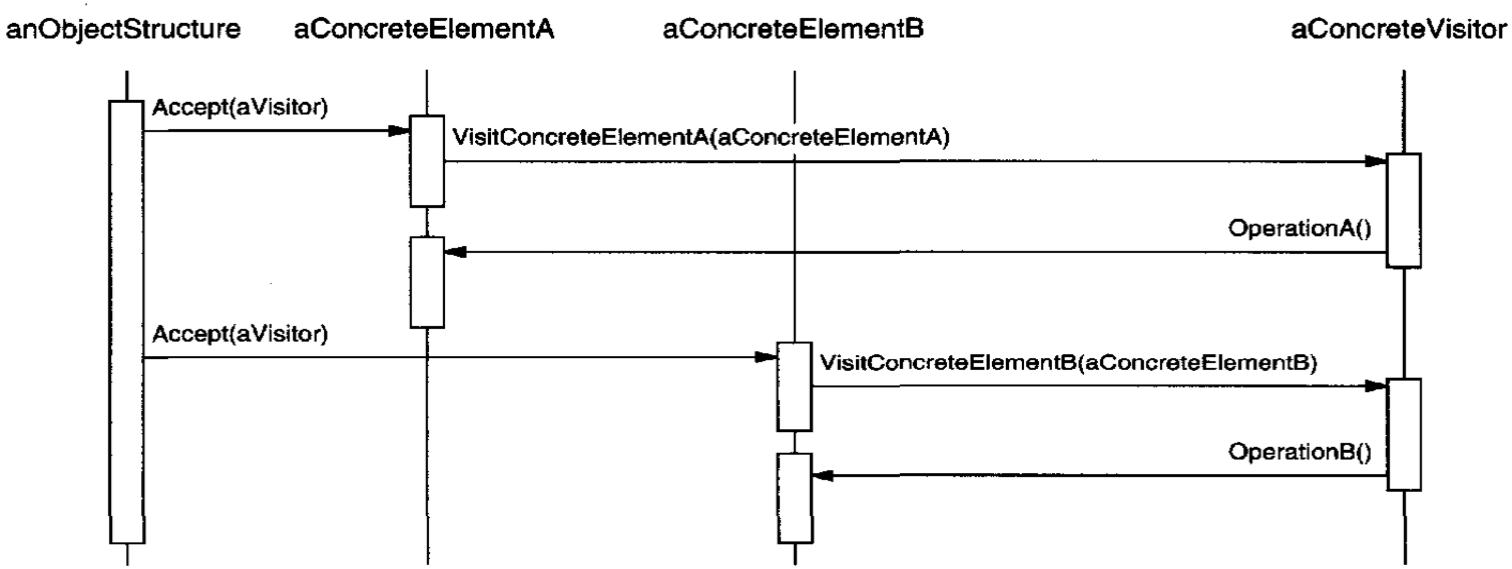
- The ***Element interface*** declares a method for “accepting” visitors. This method should have one parameter declared with the type the visitor interface.
- Each **Concrete Element** must implement the acceptance method. The purpose of this method is to redirect the call to the proper visitor’s method corresponding to the current element class. Be aware that even if a base element class implements this method, all subclasses must still override this method in their own classes and call the appropriate method on the visitor object.



# Visitor Pattern: Structure



# Visitor Pattern: Structure



# Visitor Pattern: Implementation

```
class Visitor{
public:
    void visit(Main& m) = 0;
    void visit(Statement& stmt) = 0;
    void visit(BinaryExpression& expr) = 0;
};

class TypeChecker: public Visitor{
public:
    void visit(Main& m) {}
    void visit(Statement& stmt) {}
    void visit(BinaryExpression& expr) {
        expr.getLeft().accept(this);
        expr.getRight().accept(this);
    }
};
```

# Visitor Pattern: Implementation

```
class Node {
public:
    void accept(Visitor& v) = 0;
};

class Expression: public Node {};

class BinaryExpression: public Expression {
public:
    void accept(Visitor &v){
        v.visit(this);
    }
};
```

# Next Time

- **Run-time environment!**