

یادگیری برخط

جلسه بیست و دوم:
بندیت ترکیبیاتی (۲)

کاهش آینه‌ای

$$a_1 = \operatorname{argmin}_{a \in \mathcal{A}} F(a)$$

$$a_{t+1} = \operatorname{argmin}_{a \in \mathcal{A}} (\eta \langle a, y_t \rangle + D_F(a, a_t))$$

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قضیه:

$$R_n(a) \leq \frac{F(a) - F(a_1)}{\eta} + \sum_{t=1}^n \langle a_t - a_{t+1}, y_t \rangle - \frac{1}{\eta} \sum_{t=1}^n D(a_{t+1}, a_t).$$

$$R_n(a) \leq \frac{1}{\eta} \left(F(a) - F(a_1) + \sum_{t=1}^n D(a_t, \tilde{a}_{t+1}) \right)$$

الگوریتم کاهش آینده‌ای/پیروی از پیش‌روی منظم شده برای بندیت

- 1: **Input** Legendre potential F , action set \mathcal{A} and learning rate $\eta > 0$
- 2: Choose $\bar{A}_1 = \operatorname{argmin}_{a \in \mathcal{A} \cap \operatorname{dom}(F)} F(a)$
- 3: **for** $t = 1, \dots, n$ **do**
- 4: Choose measure P_t on \mathcal{A} with mean \bar{A}_t
- 5: Sample action A_t from P_t and observe $\langle A_t, y_t \rangle$
- 6: Compute estimate \hat{Y}_t of the loss vector y_t
- 7: Update:
$$\bar{A}_{t+1} = \operatorname{argmin}_{a \in \mathcal{A} \cap \operatorname{dom}(F)} \eta \langle a, \hat{Y}_t \rangle + D_F(a, \bar{A}_t) \quad (\text{Mirror descent})$$
$$\bar{A}_{t+1} = \operatorname{argmin}_{a \in \mathcal{A} \cap \operatorname{dom}(F)} \eta \sum_{s=1}^t \langle a, \hat{Y}_s \rangle + F(a) \quad (\text{follow-the-regularised-leader})$$
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THEOREM 28.10 (Regret of Mirror-Descent and FTRL with bandit feedback). *Suppose that Algorithm 16 is run with Legendre potential F , convex action set $\mathcal{A} \subset \mathbb{R}^d$ and learning rate $\eta > 0$ such that the loss estimators are unbiased: $\mathbb{E}[\hat{Y}_t \mid \bar{A}_t] = y_t$ for all $t \in [n]$. Then the regret for either variant of Algorithm 16, provided that they are well defined, is bounded by*

$$R_n(a) \leq \mathbb{E} \left[\frac{F(a) - F(\bar{A}_1)}{\eta} + \sum_{t=1}^n \langle \bar{A}_t - \bar{A}_{t+1}, \hat{Y}_t \rangle - \frac{1}{\eta} \sum_{t=1}^n D(\bar{A}_{t+1}, \bar{A}_t) \right].$$

بندیت معمولی

بندیت ترکیبیاتی

$$\mathcal{A} \subseteq \{a \in \{0, 1\}^d : \|a\|_1 \leq m\}$$

مجموعه اعمال

بندیت معمولی

$$y_t \in \{y : \sup_{a \in \mathcal{A}} |\langle a, y \rangle| \leq 1\}$$

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$$(A_{t1}y_{t1}, \dots, A_{td}y_{td})$$

نیمه - بندیت

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$$R_n = \max_{a \in \mathcal{A}} \mathbb{E} \left[\sum_{t=1}^n \langle A_t - a, y_t \rangle \right]$$

مجموعه اعمال

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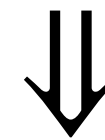
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بازخورد

پشیمانی

صورت ۲: نیمه-بندیت + کاهش آینه‌ای

بازخورد: $(A_{t1}y_{t1}, \dots, A_{td}y_{td})$

صورت ۲: نیمه-بندیت + کاهش آینده‌ای

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$$\hat{Y}_{ti} = \frac{A_{ti}y_{ti}}{\bar{A}_{ti}}$$

$$\bar{A}_{ti} = \mathbb{E}[A_{ti} | \mathcal{F}_{t-1}]$$

صورت ۲: نیمه-بندیت + کاهش آینده‌ای

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$$\Leftarrow \hat{Y}_{ti} = \frac{A_{ti}y_{ti}}{\bar{A}_{ti}}$$

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صورت ۲: نیمه-بندیت + کاهش آینده‌ای

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$$\mathbb{E}[\hat{Y}_{ti}] = \sum_{a \in A} P[a] \frac{a_i y_{ti}}{\bar{A}_{ti}} \iff \hat{Y}_{ti} = \frac{A_{ti} y_{ti}}{\bar{A}_{ti}}$$

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$$\mathbb{E}[\hat{Y}_t | \mathcal{F}_{t-1}] = y_t$$

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Input \mathcal{A}, η, F

$\bar{A}_1 = \operatorname{argmin}_{a \in \operatorname{co}(\mathcal{A})} F(a)$

for $t = 1, \dots, n$ **do**

 Choose distribution P_t on \mathcal{A} such that $\sum_{a \in \mathcal{A}} P_t(a)a = \bar{A}_t$

 Sample $A_t \sim P_t$ and observe $A_{t1}y_{t1}, \dots, A_{td}y_{td}$

 Compute $\hat{Y}_{ti} = A_{ti}y_{ti}/\bar{A}_{ti}$ for all $i \in [d]$

 Update $\bar{A}_{t+1} = \operatorname{argmin}_{a \in \operatorname{co}(\mathcal{A})} \eta \langle a, \hat{Y}_t \rangle + D_F(a, \bar{A}_t)$

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الگوریتم کاهش آینه‌ای

$$F(a) = \sum_{i=1}^d (a_i \log(a_i) - a_i)$$

$$R_n \leq \frac{\text{diam}_F(\text{co}(\mathcal{A}))}{\eta} + \mathbb{E} \left[\sum_{t=1}^n \langle \bar{A}_t - \bar{A}_{t+1}, \hat{Y}_t \rangle - \frac{1}{\eta} D_F(\bar{A}_{t+1}, \bar{A}_t) \right]$$

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$$\text{diam}_F(\text{co}(\mathcal{A})) \leq \sup_{a \in \text{co}(\mathcal{A})} \sum_{i=1}^d \left(a_i + a_i \log \left(\frac{1}{a_i} \right) \right)$$

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$$\begin{aligned} \text{diam}_F(\text{co}(\mathcal{A})) &\leq \sup_{a \in \text{co}(\mathcal{A})} \sum_{i=1}^d \left(a_i + a_i \log \left(\frac{1}{a_i} \right) \right) \\ &\leq m(1 + \log(d/m)). \end{aligned}$$

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$$\bar{A}_{ti} = \mathbb{E}[A_{ti} | \mathcal{F}_{t-1}]$$

$$\hat{Y}'_{ti} = \hat{Y}_{ti} \mathbb{I} \{ \bar{A}_{t+1,i} \leq \bar{A}_{ti} \}$$

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$$= \frac{\eta}{2} \sum_i \hat{Y}'_{t,i} Z_i \hat{Y}'_{t,i}$$

$$\begin{aligned} \nabla F(x) &= \log(x) - 1 \\ \nabla^2 F(x) &= \text{diam}(1/x) \end{aligned}$$

الگوریتم کاهش آینه‌ای

$$F(a) = \sum_{i=1}^d (a_i \log(a_i) - a_i)$$

$$R_n \leq \frac{\text{diam}_F(\text{co}(\mathcal{A}))}{\eta} + \mathbb{E} \left[\sum_{t=1}^n \langle \bar{A}_t - \bar{A}_{t+1}, \hat{Y}_t \rangle - \frac{1}{\eta} D_F(\bar{A}_{t+1}, \bar{A}_t) \right]$$

$$\leq m(1 + \log(d/m))/\eta$$

$$\bar{A}_{ti} = \mathbb{E}[A_{ti} | \mathcal{F}_{t-1}]$$

$$\hat{Y}'_{ti} = \hat{Y}_{ti} \mathbb{I} \{ \bar{A}_{t+1,i} \leq \bar{A}_{ti} \}$$

$$\leq \langle \bar{A}_t - \bar{A}_{t+1}, \hat{Y}'_t \rangle - \frac{1}{\eta} D_F(\bar{A}_{t+1}, \bar{A}_t)$$

$$\leq \frac{\eta}{2} \|\hat{Y}'_t\|_{\nabla^2 F(Z_t)^{-1}}^2 = \frac{\eta}{2} \hat{Y}'_t{}^\top \nabla^2 F(Z)^{-1} \hat{Y}'_t$$

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$$Z_t = \bar{A}_t: \text{بیشینه}$$

$$\begin{aligned} \nabla F(x) &= \log(x) - 1 \\ \nabla^2 F(x) &= \text{diam}(1/x) \end{aligned}$$

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$$\begin{aligned} \nabla F(x) &= \log(x) - 1 \\ \nabla^2 F(x) &= \text{diag}(1/x) \end{aligned}$$

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الگوریتم کاهش آینه‌ای

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$$\leq \frac{\eta}{2} \sum_{i=1}^d \frac{A_{ti}}{\bar{A}_{ti}}$$

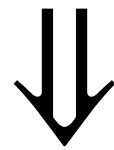
$$\frac{\eta}{2} \mathbb{E} \left[\sum_{t=1}^n \sum_{i=1}^d \frac{A_{ti}}{\bar{A}_{ti}} \right] = \frac{\eta n d}{2}$$

$$\leq \frac{m(1 + \log(d/m))}{\eta} + \frac{\eta n d}{2} = \sqrt{2nmd(1 + \log(d/m))}$$

قضيه:

$$F(a) = \sum_{i=1}^d (a_i \log(a_i) - a_i) \quad a \in [0, \infty)^d$$
$$F(a) = \infty \quad \textit{otherwise.}$$

$$\eta = \sqrt{2m(1 + \log(d/m))/(nd)},$$



$$R_n \leq \sqrt{2nmd(1 + \log(d/m))}$$

از لحاظ محاسبات

Input \mathcal{A}, η, F

$\bar{A}_1 = \operatorname{argmin}_{a \in \operatorname{co}(\mathcal{A})} F(a)$

for $t = 1, \dots, n$ **do**

Choose distribution P_t on \mathcal{A} such that $\sum_{a \in \mathcal{A}} P_t(a)a = \bar{A}_t$

Sample $A_t \sim P_t$ and observe $A_{t1}y_{t1}, \dots, A_{td}y_{td}$

Compute $\hat{Y}_{ti} = A_{ti}y_{ti}/\bar{A}_{ti}$ for all $i \in [d]$

Update $\bar{A}_{t+1} = \operatorname{argmin}_{a \in \operatorname{co}(\mathcal{A})} \eta \langle a, \hat{Y}_t \rangle + D_F(a, \bar{A}_t)$

end for

برای مسئله کوتاه‌ترین مسیر نیمه-بندیتی

Input \mathcal{A}, η, F

$\bar{A}_1 = \operatorname{argmin}_{a \in \operatorname{co}(\mathcal{A})} F(a)$

for $t = 1, \dots, n$ **do**

Choose distribution P_t on \mathcal{A} such that $\sum_{a \in \mathcal{A}} P_t(a)a = \bar{A}_t$

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