بسم الله الرحمن الرحیم

نظریه علوم کامپیوتر

نظریه علوم کامپیوتر - بهار ۱۴۰۰-۱۴۰۱ - جلسه هجدهم: سیستم اثبات تعاملی

Theory of computation - 002 - S18 - IP

18.404/6.840 Lecture 25

Last time:

- BPP
- Schwartz-Zippel Theorem
- EQROBP $\in$ BPP

Today: (Sipser §10.4)

- Interactive Proof Systems
- The class IP
- Graph isomorphism problem
- coNP $\subseteq$ IP (part 1)

Interactive Proofs – Introduction

Illustration: Graph isomorphism testing

Defn: Undirected graphs G and H are isomorphic if they are identical except for a permutation (rearrangement) of the nodes.

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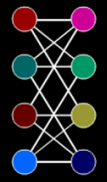

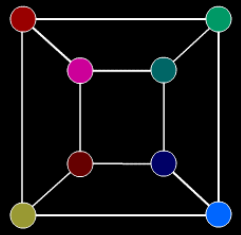

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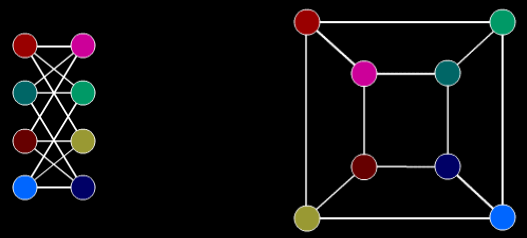

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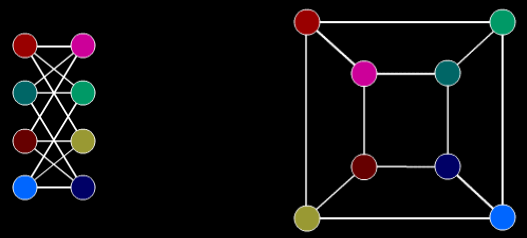

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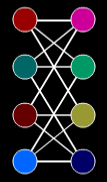

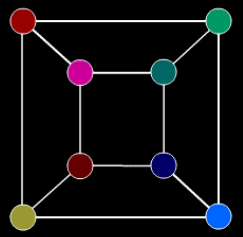

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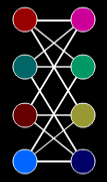

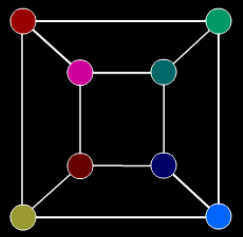

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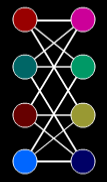

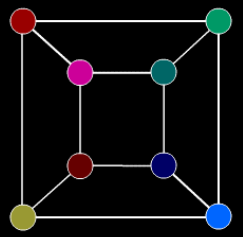

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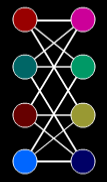

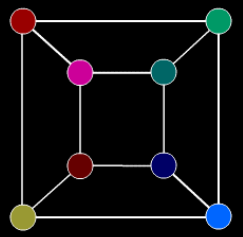

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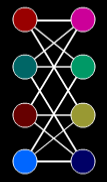

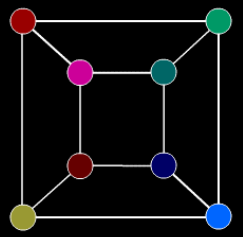

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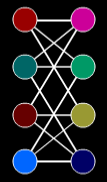

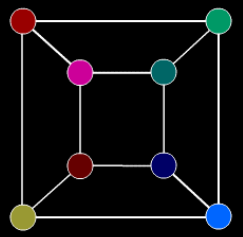

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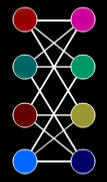

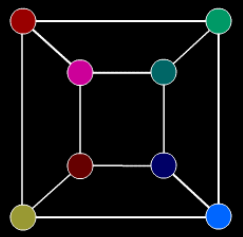

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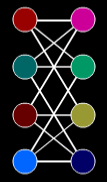

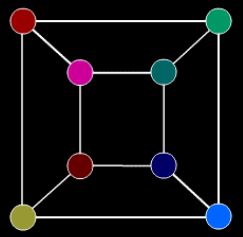

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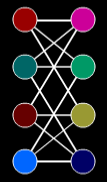

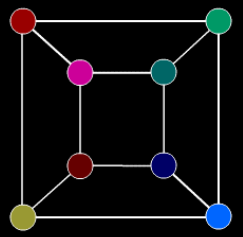

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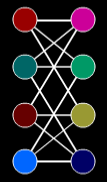

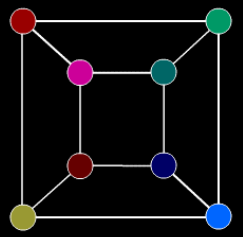

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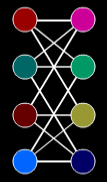

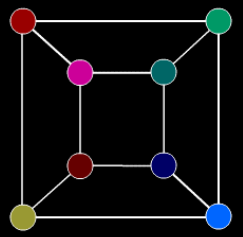

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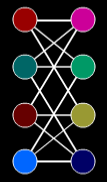

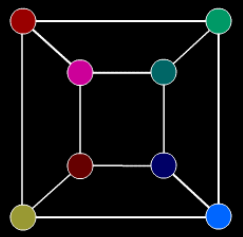

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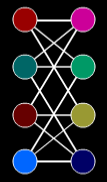

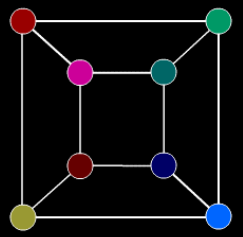

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Probabilistic
polynomial time TM

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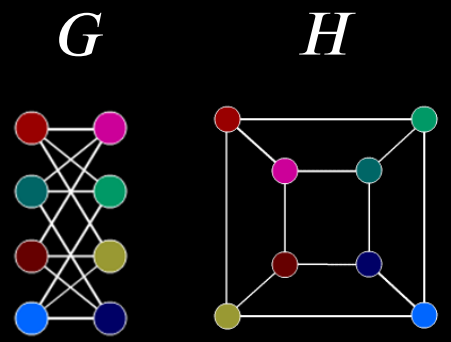

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Graduate Students

Unlimited
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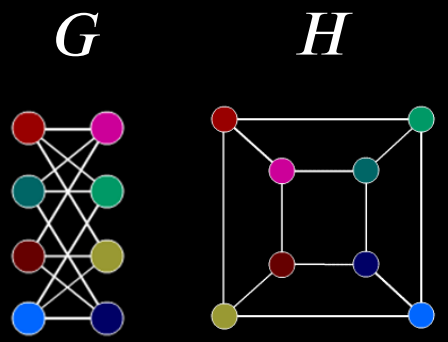

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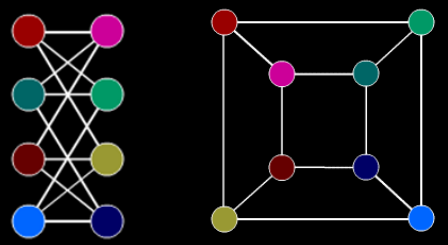
G H



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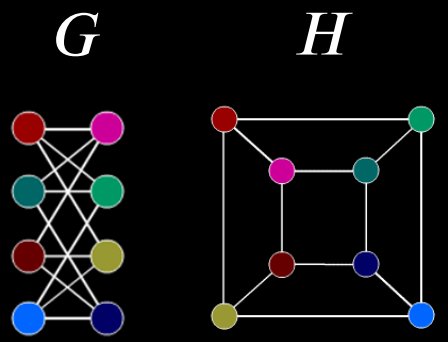

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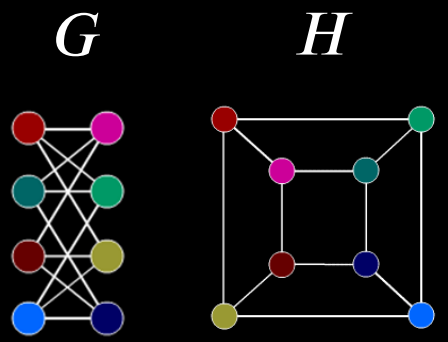

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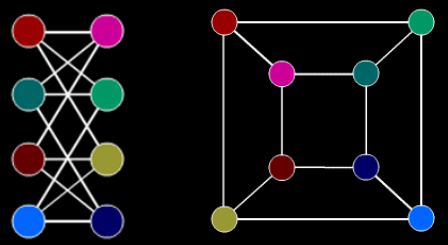

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But what if they claim that G and H are not isomorphic?
- The Professor randomly and secretly picks G or H and permutes it, then sends the result to the Students.

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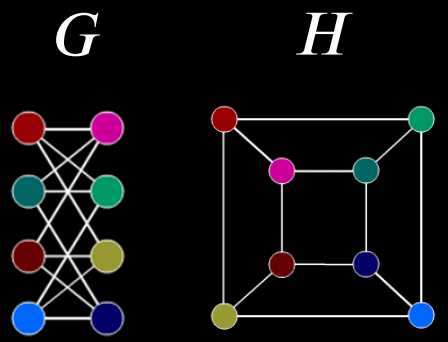

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Interactive Proofs – informal model

Probabilistic
polynomial time TM

Professor = Verifier (V)

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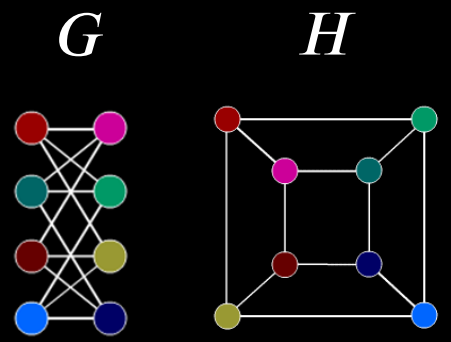
Graduate Students



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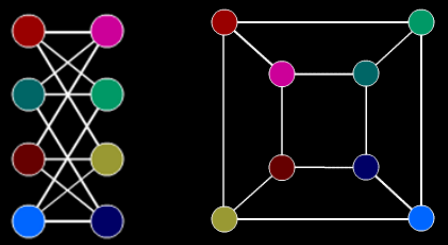

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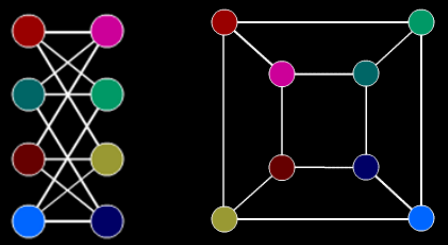

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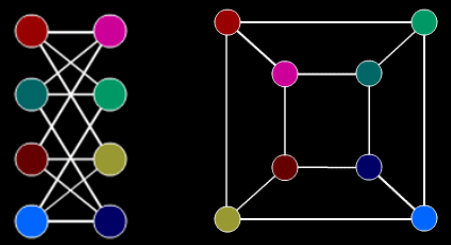

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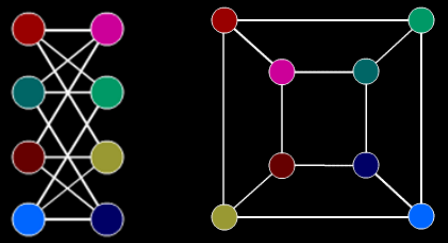

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An amplification lemma can improve the error probability from $\frac{1}{3}$ to $\frac{1}{2^{\text{poly}(n)}}$

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Theorem: $\overline{ISO} \in$ IP

Proof: Protocol for V and (the honest) P on input $\langle G, H \rangle$

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Check-in 25.1

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Suppose we change the model to allow the Prover access to the Verifier's random choices. Now consider the same protocol as described above. What language does it describe?

(a) $\{\langle G, H \rangle | \ G \neq H\}$

(b) $\{\langle G, H \rangle |$ G and H are not isomorphic $\}$

(c) $\{\langle G, H \rangle |$ G and H are any two graphs $\}$

(d) $\varnothing$

Facts about IP – Checkin 25.2

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Which of the following is true?
Check all that apply

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We will prove only a weaker statement: coNP $\subseteq$ IP

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Let $\phi(a_1 \ldots a_i)$ be ϕ with $x_1 = a_1$, … , $x_i = a_i$ for $a_1, \ldots, a_i \in \{0,1\}$.

Call $a_1, \ldots, a_i$ presets. The remaining $x_{i+1}, \ldots, x_m$ stay as unset variables.

Let $\#\phi$ = the number of satisfying assignments of ϕ.

Let $\#\phi(0)$ = the number of satisfying assignments of $\phi(0)$.

$\#SAT \in$ IP – notation

$\#SAT = \{\langle \phi, k \rangle \mid$ Boolean formula ϕ has exactly k satisfying assignments$\}$

Theorem: $\#SAT \in$ IP

Proof: First some notation. Assume ϕ has m variables $x_1, \ldots, x_m$.

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Equivalently:

$$\#\phi(a_1 \ldots a_i) = \sum_{\substack{a_{i+1}, \ldots, a_m \\ \in \{0,1\}}} \phi(a_1 \ldots a_m)$$

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Equivalently:

$$\#\phi(a_1 \ldots a_i) = \sum_{a_{i+1}, \ldots, a_m \in \{0,1\}} \phi(a_1 \ldots a_m)$$

Two useful facts

1. $\#\phi(a_1 \ldots a_i) =$

$$\#\phi(a_1 \ldots a_i 0) + \#\phi(a_1 \ldots a_i 1)$$

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Check-in 25.3

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Equivalently: $\#\phi(a_1 \ldots a_i) = \sum_{a_{i+1}, \ldots, a_m \in \{0,1\}} \phi(a_1 \ldots a_m)$

Check-in 25.3

If $\#\phi = 9$ and $\#\phi(0) = 6$ then what do we know?

a) $\#\phi(1) = 3$ c) $\#\phi(00) \leq 5$

b) $\#\phi(1) = 15$ d) none of these

1. $\#\phi(a_1 \ldots a_i) = \#\phi(a_1 \ldots a_i 0) + \#\phi(a_1 \ldots a_i 1)$

2. $\#\phi(a_1 \ldots a_m) = \phi(a_1 \ldots a_m)$

Check-in 25.3

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Let $\#\phi$ = the number of satisfying assignments of ϕ.

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Check-in 25.3

$\#SAT \in$ IP – 1st attempt

Theorem: $\#SAT \in$ IP

Proof: Protocol for V and (the honest) P on input $\langle \phi, k \rangle$

$\#SAT \in$ IP – 1st attempt

Theorem: $\#SAT \in$ IP

Proof: Protocol for V and (the honest) P on input $\langle \phi, k \rangle$

0) P sends $\#\phi$; V checks $k = \#\phi$

$\#SAT \in$ IP – 1st attempt

Theorem: $\#SAT \in$ IP

Proof: Protocol for V and (the honest) P on input $\langle \phi, k \rangle$

0) P sends $\#\phi$; V checks $k = \#\phi$

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$\#SAT \in$ IP – 1st attempt

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$\#SAT \in$ IP – 1st attempt

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$\vdots$

m) P sends $\#\phi(\overbrace{0\cdots0}^{m}), \ldots, \#\phi(1\cdots1)$; V checks $\#\phi(0\cdots0) = \#\phi(\overbrace{0\cdots00}^{m-1}) + \#\phi(\overbrace{0\cdots01}^{m-1})$

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V checks $\#\phi(1\cdots1) = \#\phi(1\cdots10) + \#\phi(1\cdots11)$

$\#SAT \in$ IP – 1st attempt

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$\vdots$

V checks $\#\phi(1\cdots1) = \#\phi(1\cdots10) + \#\phi(1\cdots11)$

$m+1$) V checks $\#\phi(\overbrace{0\cdots0}^{m}) = \phi(0\cdots0)$

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$\#SAT \in$ IP – 1st attempt

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$\#\phi(1\cdots1) = \phi(1\cdots1)$

V *accepts* if all checks are correct. Otherwise V *rejects*.

$\#SAT \in$ IP – 1st attempt

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k

=

$\#\phi$

$\vdots$

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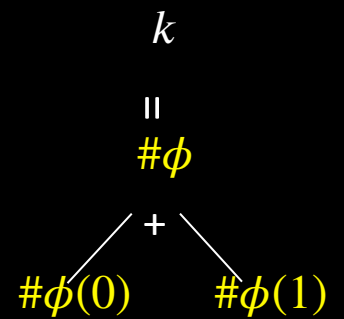

$\#SAT \in$ IP – 1st attempt

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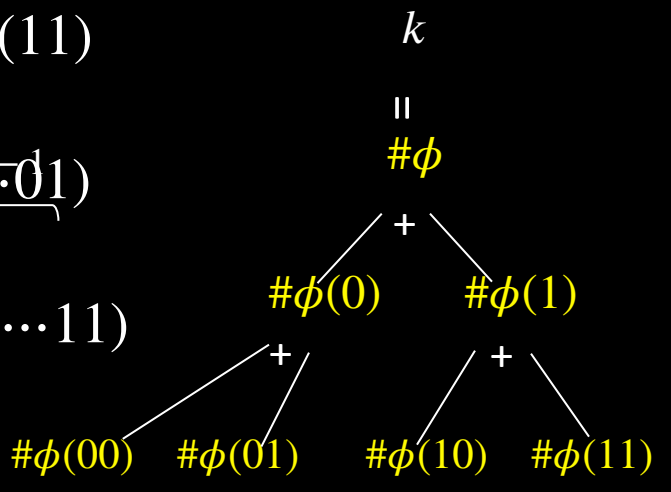

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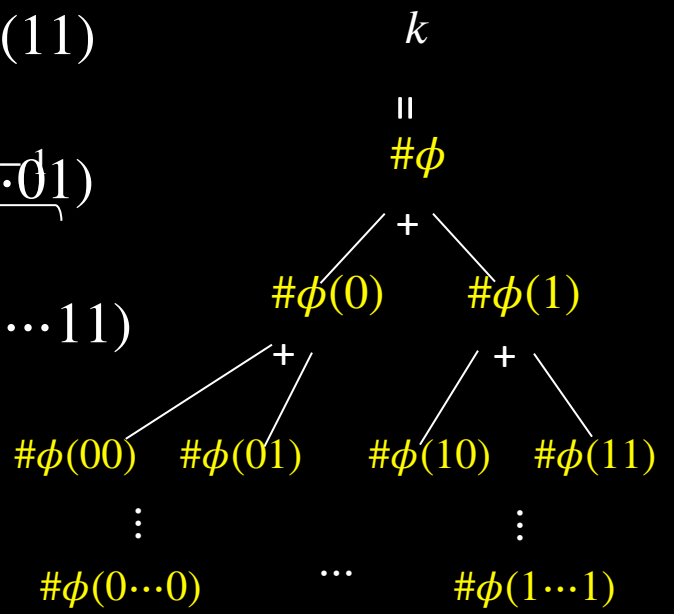

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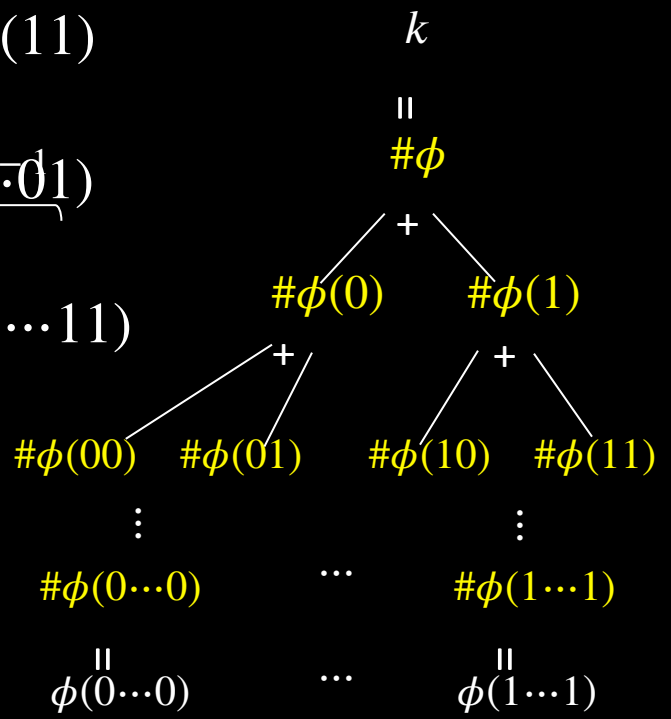

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$\vdots$

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$m+1$) V checks $\#\phi(\overbrace{0\cdots0}^{m}) = \phi(0\cdots0)$

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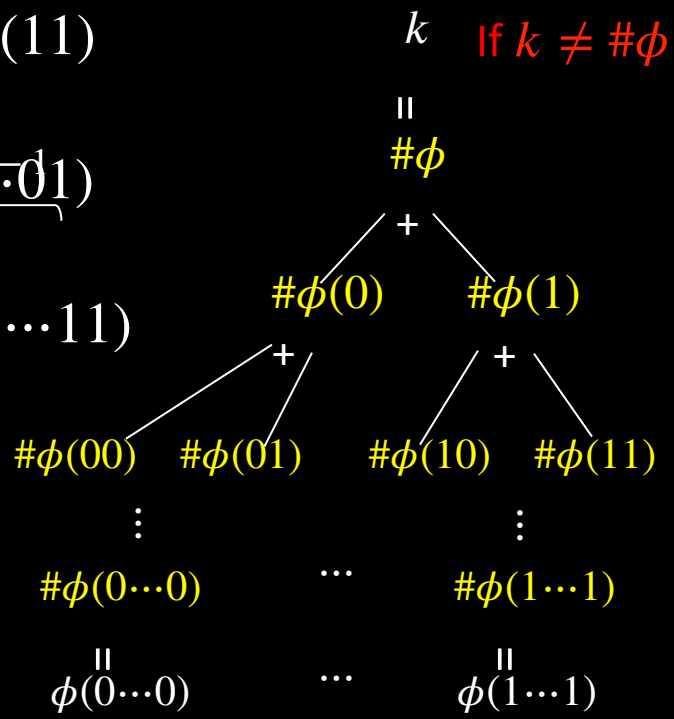

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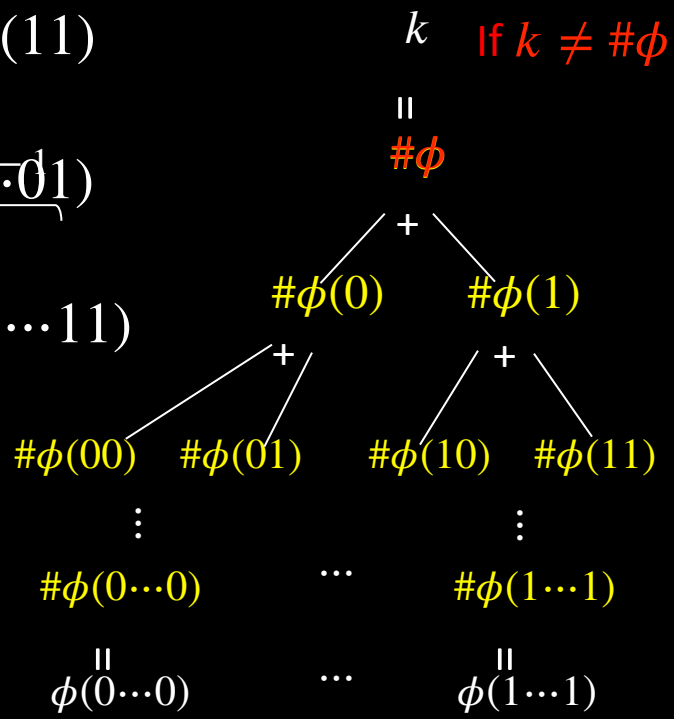
V *accepts* if all checks are correct. Otherwise V *rejects*.



$\#SAT \in$ IP – 1st attempt

Theorem: $\#SAT \in$ IP

Proof: Protocol for V and (the honest) P on input $\langle \phi, k \rangle$

0) P sends $\#\phi$; V checks $k = \#\phi$

1) P sends $\#\phi(0), \#\phi(1)$; V checks $\#\phi = \#\phi(0) + \#\phi(1)$

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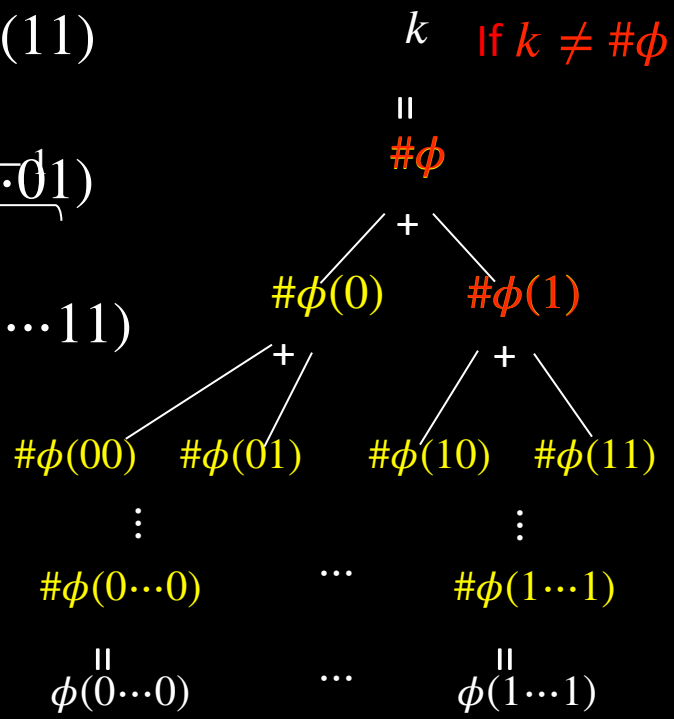

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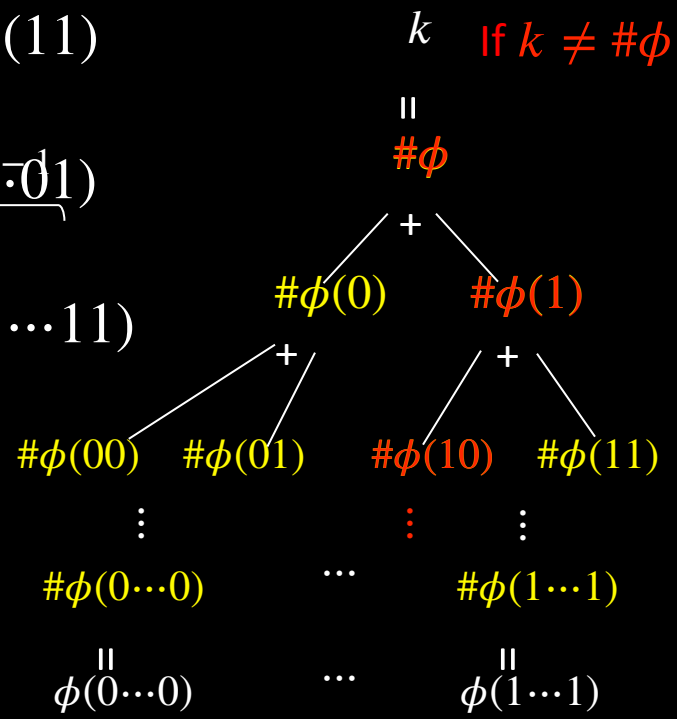

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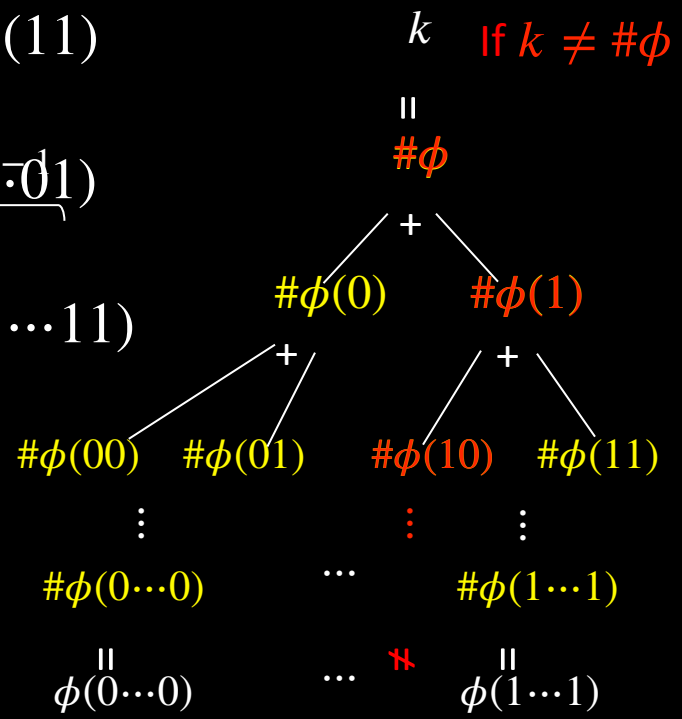

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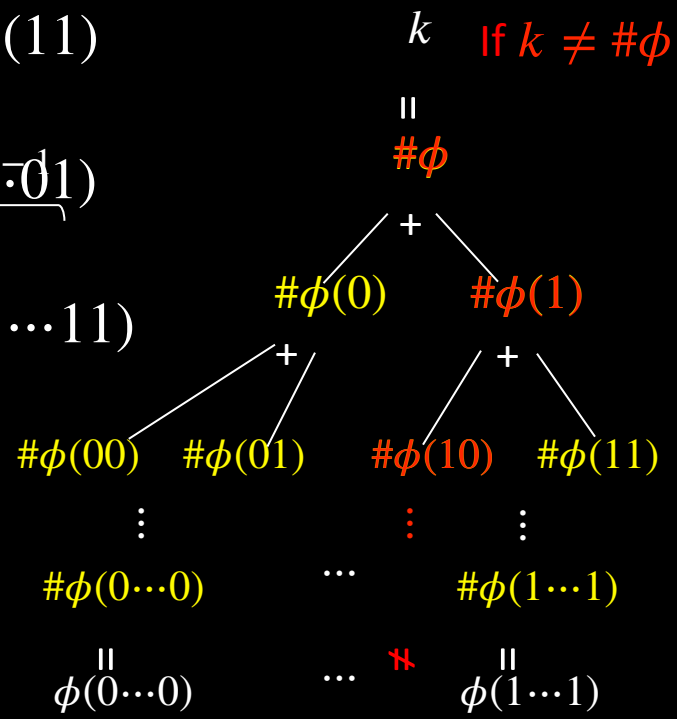

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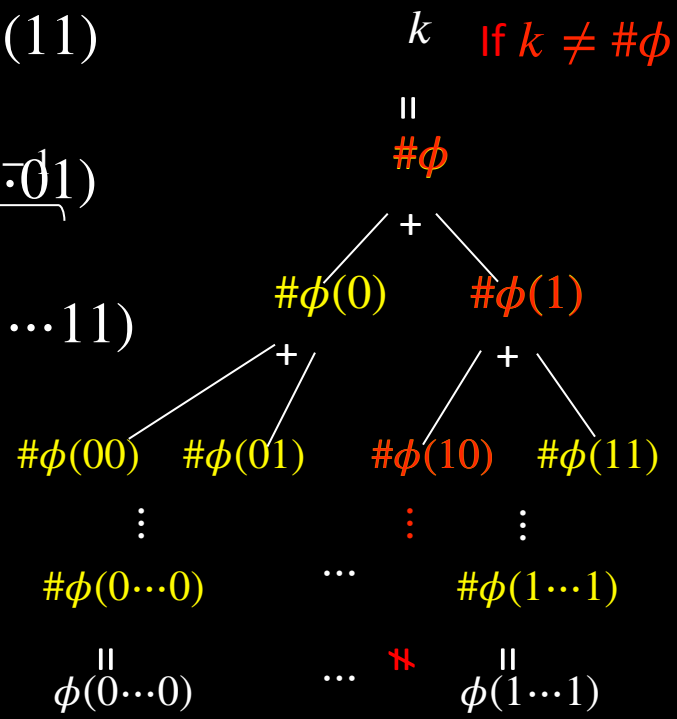

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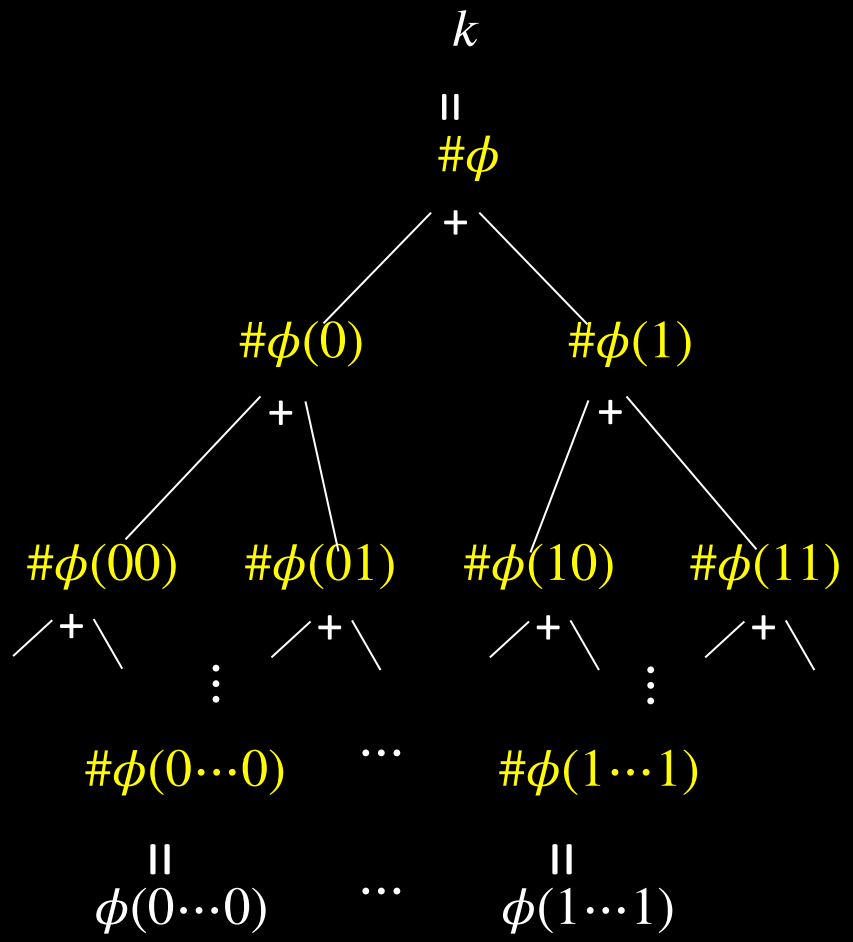
Idea for fixing $\#SAT \in$ IP protocol



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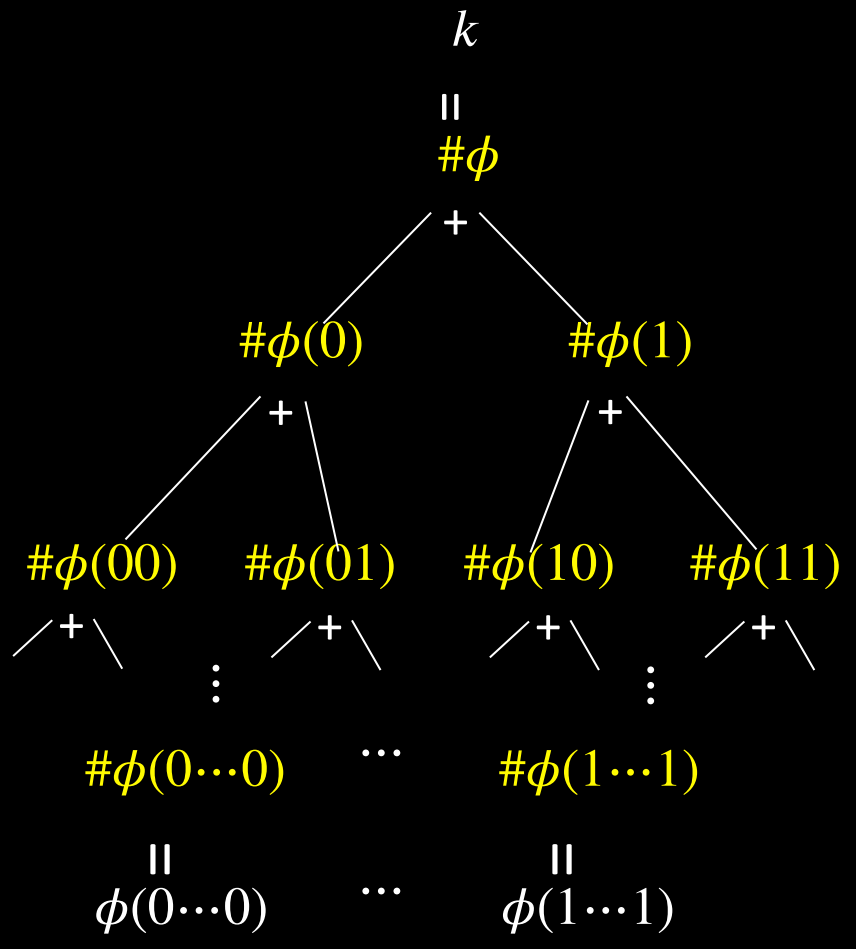

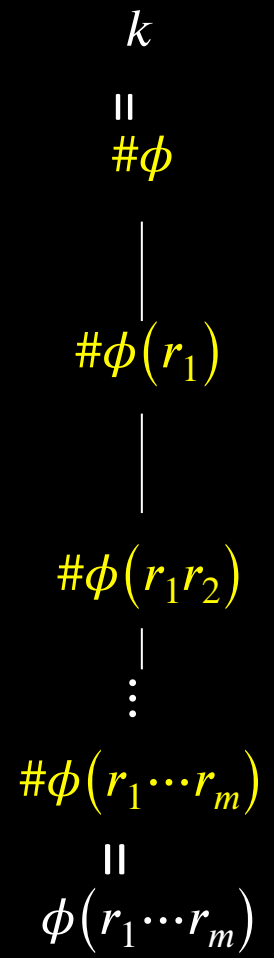

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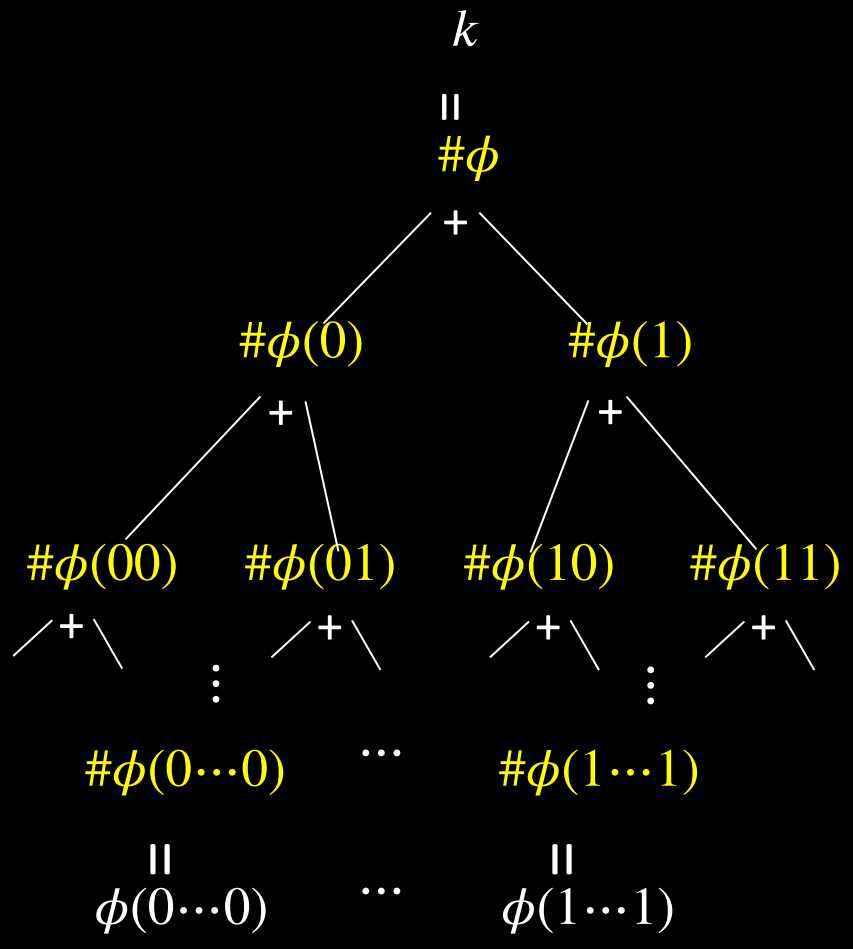

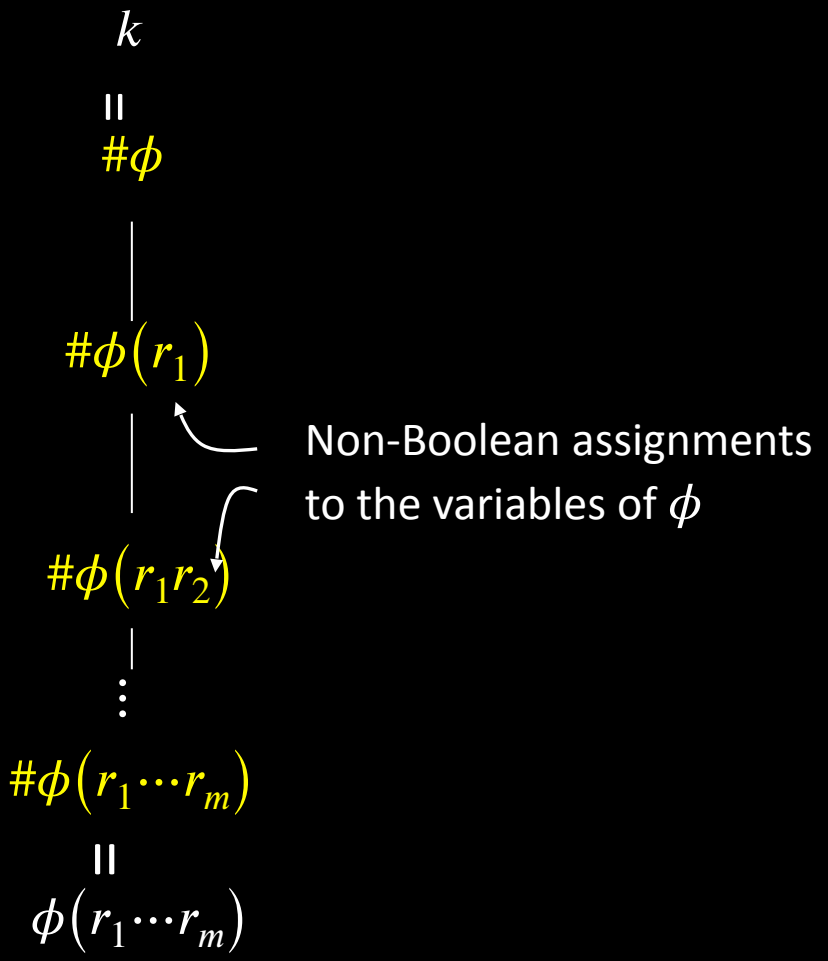

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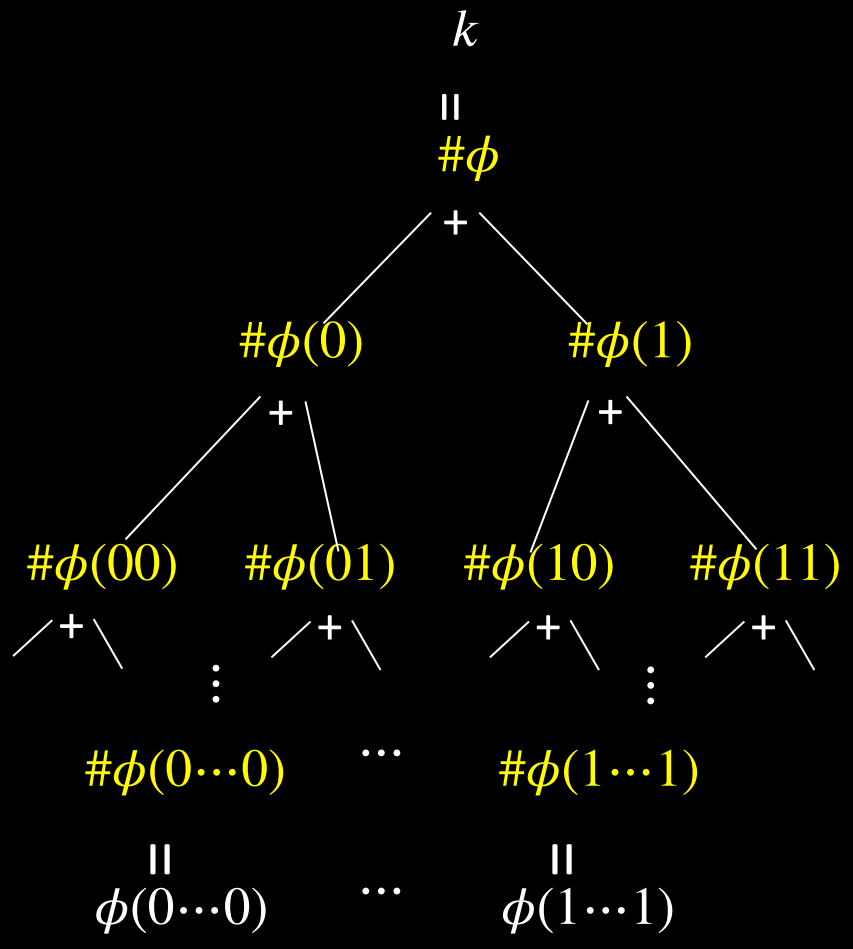

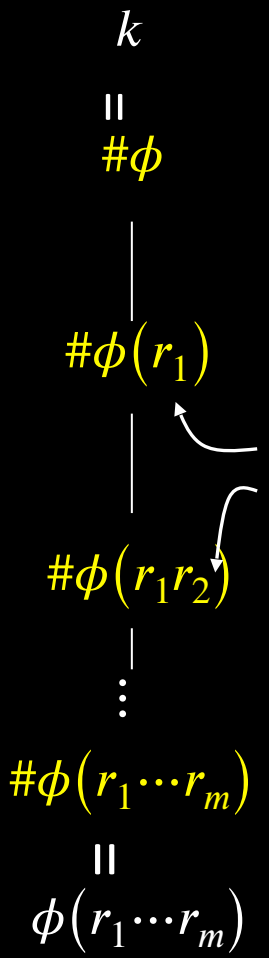

Non-Boolean assignments to the variables of ϕ

To be continued…

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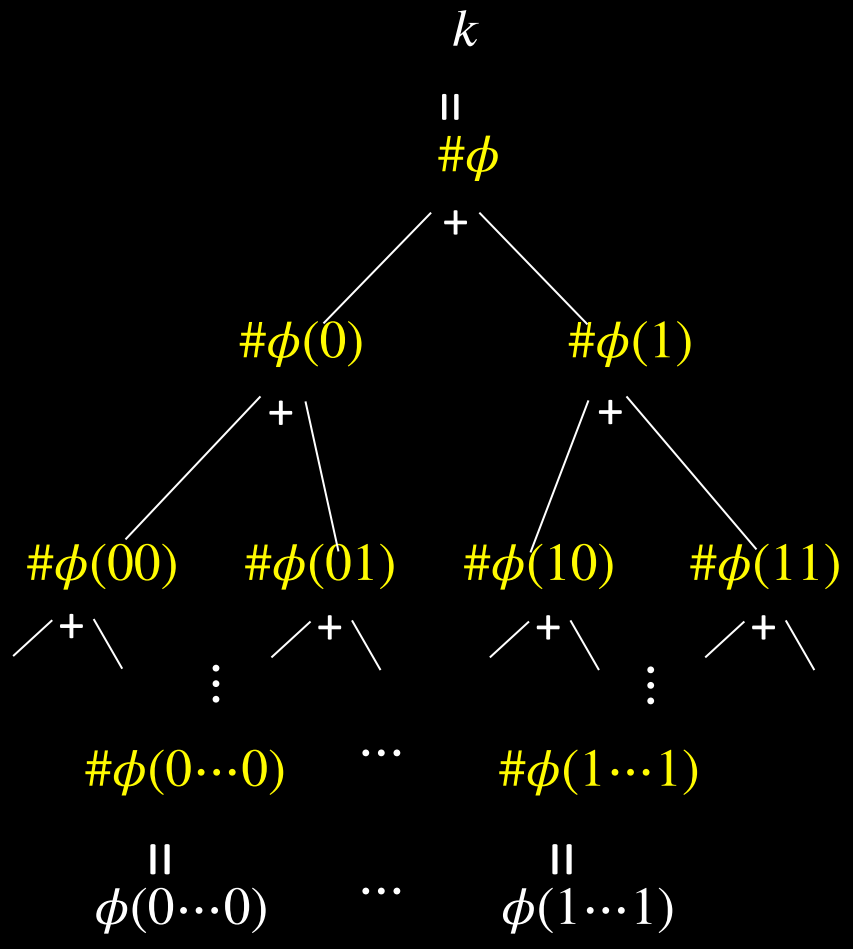

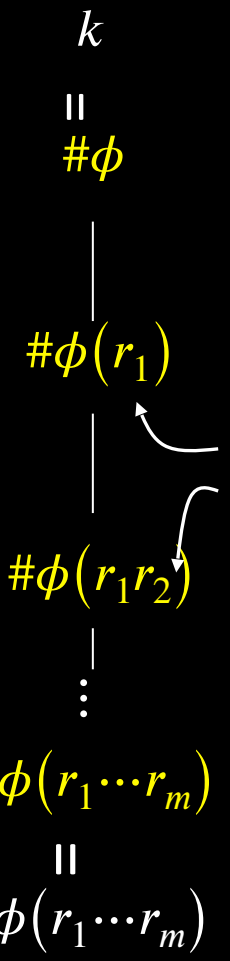

Non-Boolean assignments to the variables of ϕ

To be continued…

Quick review of today

1. Introduced the interactive proof system model
2. Defined the class IP
3. Showed $\overline{ISO} \in$ IP
4. Started showing $\#SAT \in$ IP to prove that coNP $\subseteq$ IP

18.404/6.840 Lecture 26

Last time:

- Interactive Proof Systems
- The class IP
- Graph isomorphism problem, $\overline{ISO} \in$ IP
- $\#SAT \in$ IP (part 1)

Today: (Sipser §10.4)

- Arithmetization of Boolean formulas
- Finish $\#SAT \in$ IP and conclude that coNP $\subseteq$ IP

Review: Interactive Proofs

Two interacting parties
Verifier (V): Probabilistic polynomial time TM
Prover (P): Unlimited computational power

Both P and V see input w.
They exchange a polynomial number of polynomial-size messages.
Then V *accepts* or *rejects*.

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Call $a_1, \ldots, a_i$ presets. The remaining $x_{i+1}, \ldots, x_m$ stay as unset variables.

Two identities

Let $\#\phi$ = the number of satisfying assignments of ϕ.

Let $\#\phi(0)$ = the number of satisfying assignments of $\phi(0)$.

Let $\#\phi(a_1 \ldots a_i)$ = the number of satisfying assignments of $\phi(a_1 \ldots a_i)$

coNP ⊆ IP

Surprising Theorem: IP = PSPACE

IP ⊆ PSPACE: standard simulation, similar to NP ⊆ PSPACE

PSPACE ⊆ IP: show $TQBF \in$ IP, we won't prove

coNP ⊆ IP: weaker but similar, show $\#SAT \in$ IP ($\#SAT$ is coNP-hard)

$\#SAT = \{\langle \phi, k \rangle \mid$ Boolean formula ϕ has exactly k satisfying assignments$\}$

Theorem: $\#SAT \in$ IP

Proof: First some notation. Assume ϕ has m variables $x_1, \ldots, x_m$.

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Check-in 26.1

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Check-in 26.1

Let $\phi = (x_1 \vee x_2) \wedge (\overline{x_1} \vee \overline{x_2})$

Check all that are true:

a) $\#\phi = 1$ b) $\#\phi = 2$

c) $\#\phi(0) = 1$ d) $\#\phi(0) = 2$

e) $\#\phi(00) = 0$ f) $\#\phi(00) = 1$

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$\#SAT \in$ IP – 1st attempt

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Proof: Protocol for V and (the honest) P on input $\langle \phi, k \rangle$

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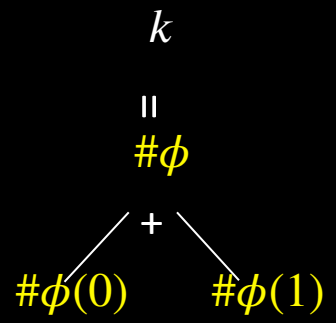

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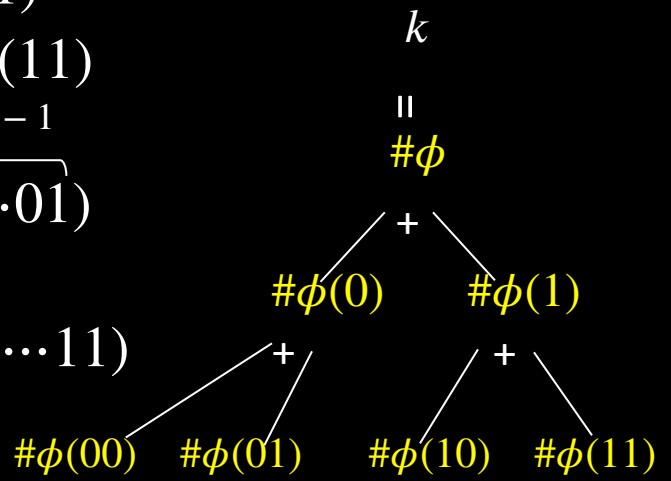

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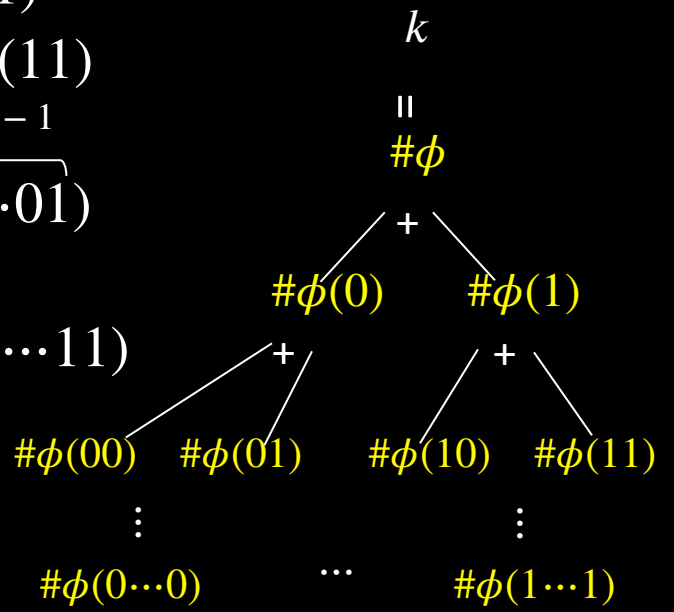

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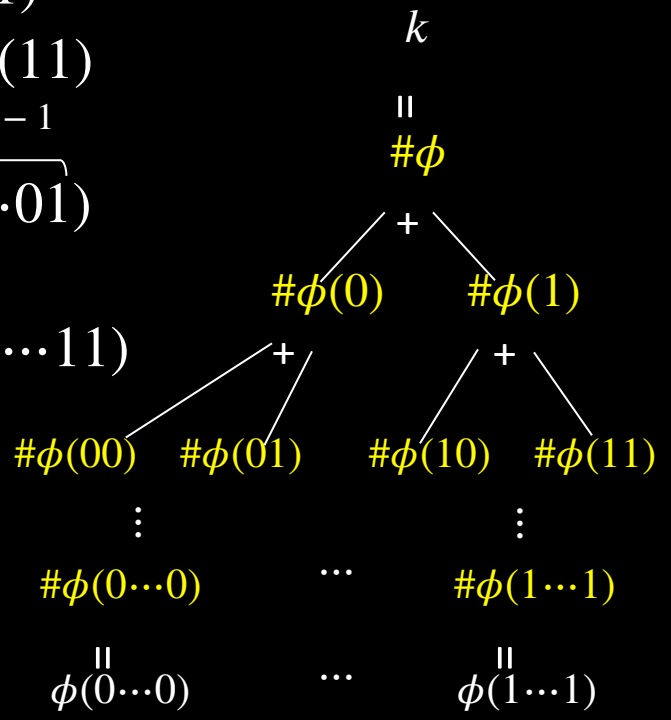

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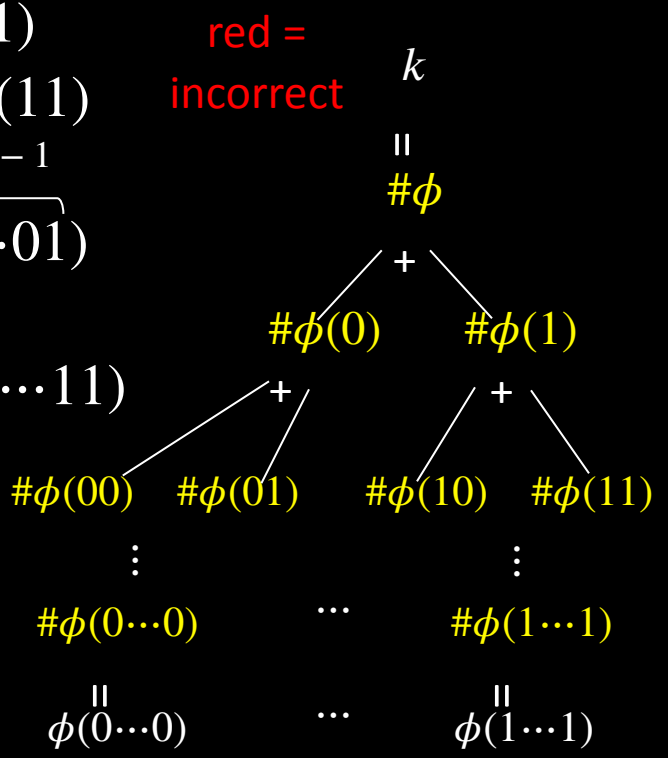

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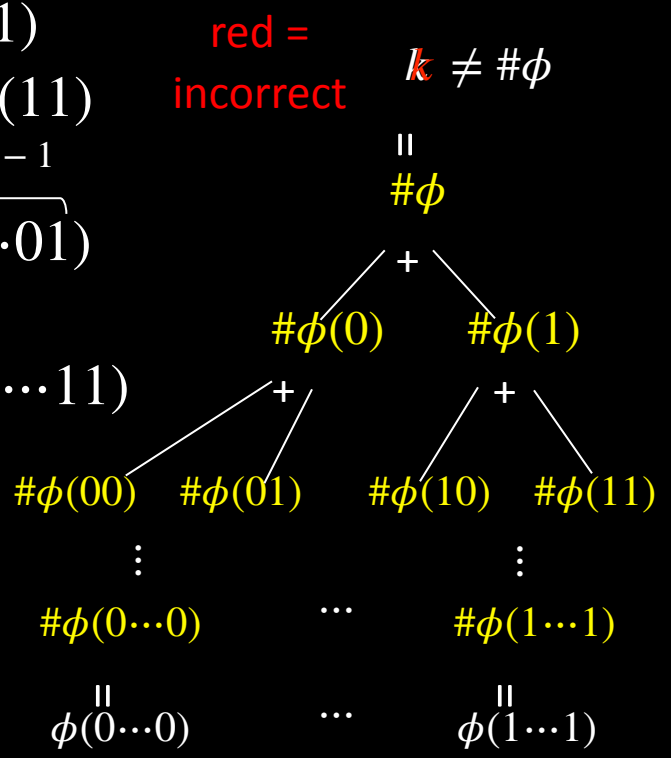

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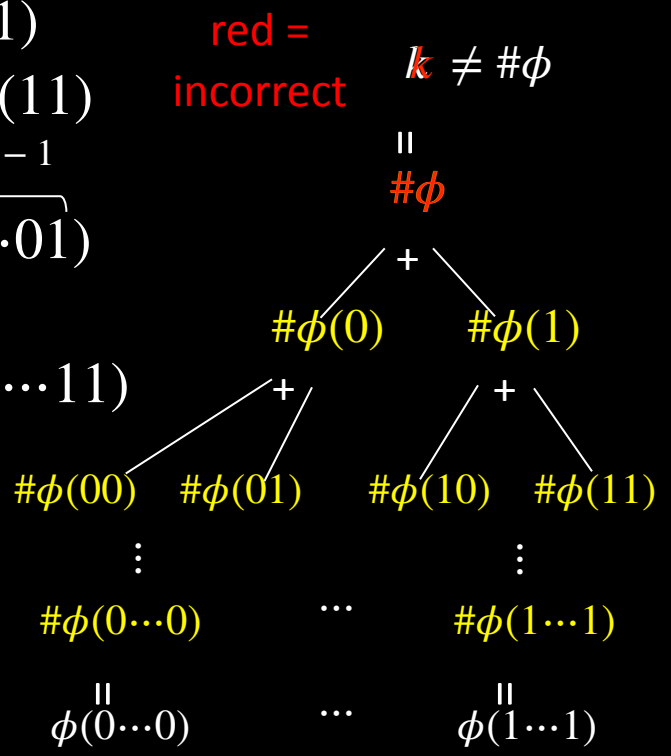

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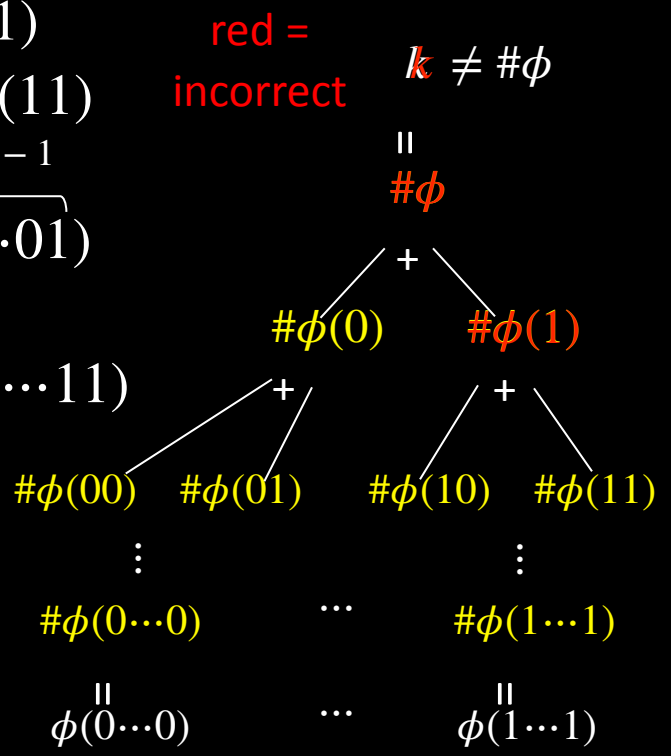

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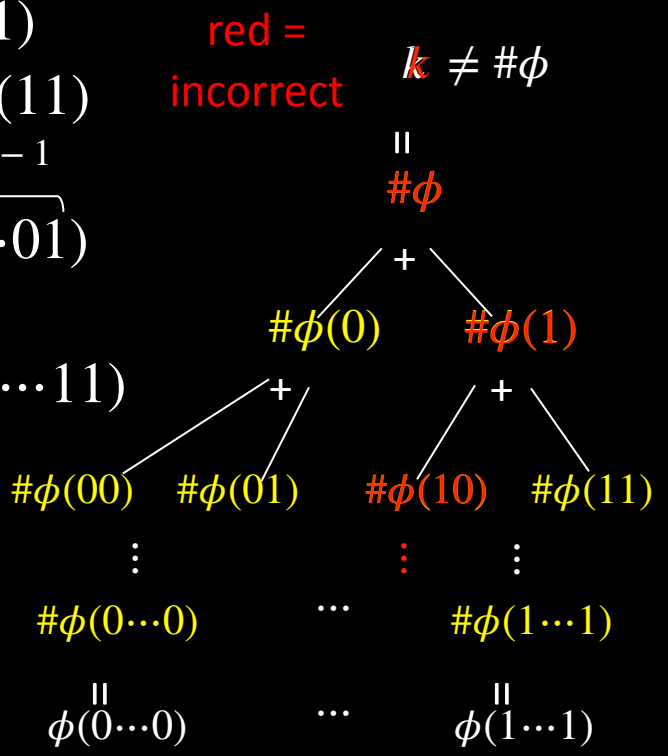
V *accepts* if all checks are correct. Otherwise V *rejects*.



$\#SAT \in$ IP – 1st attempt

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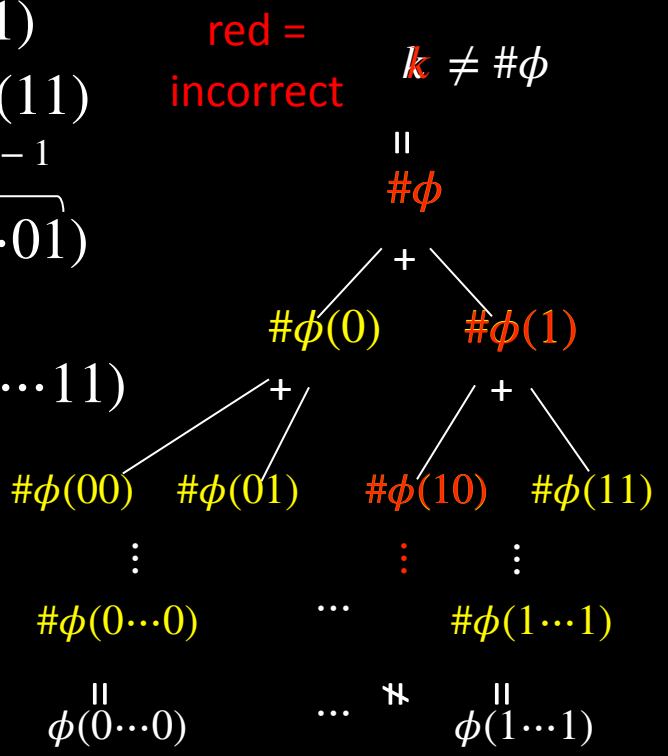

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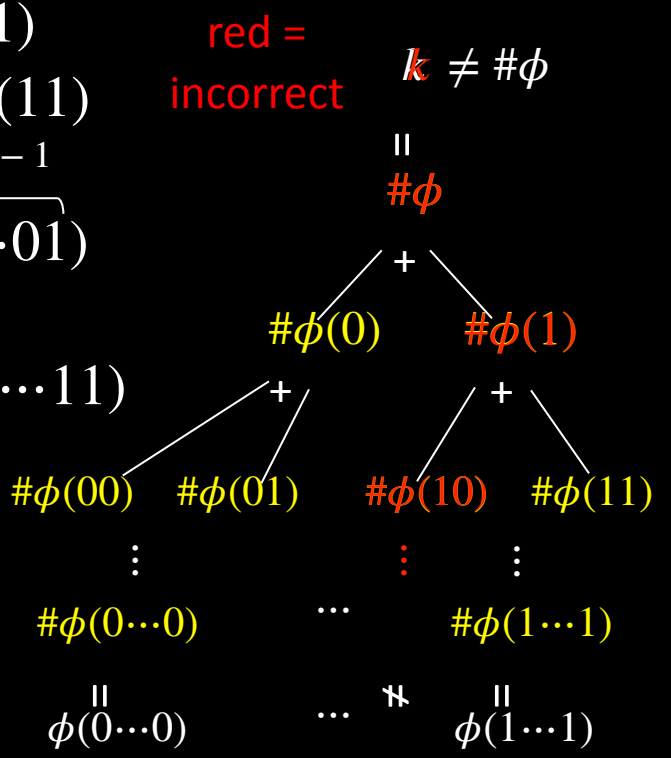

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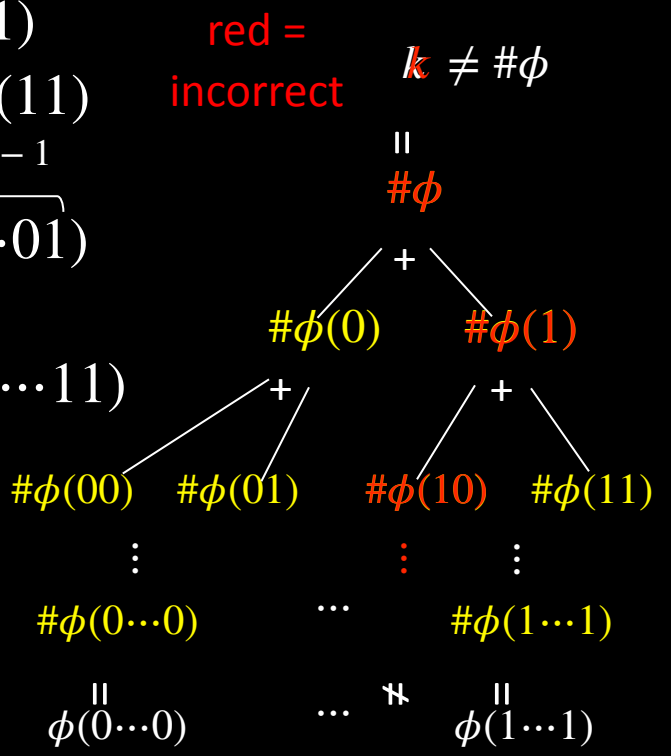

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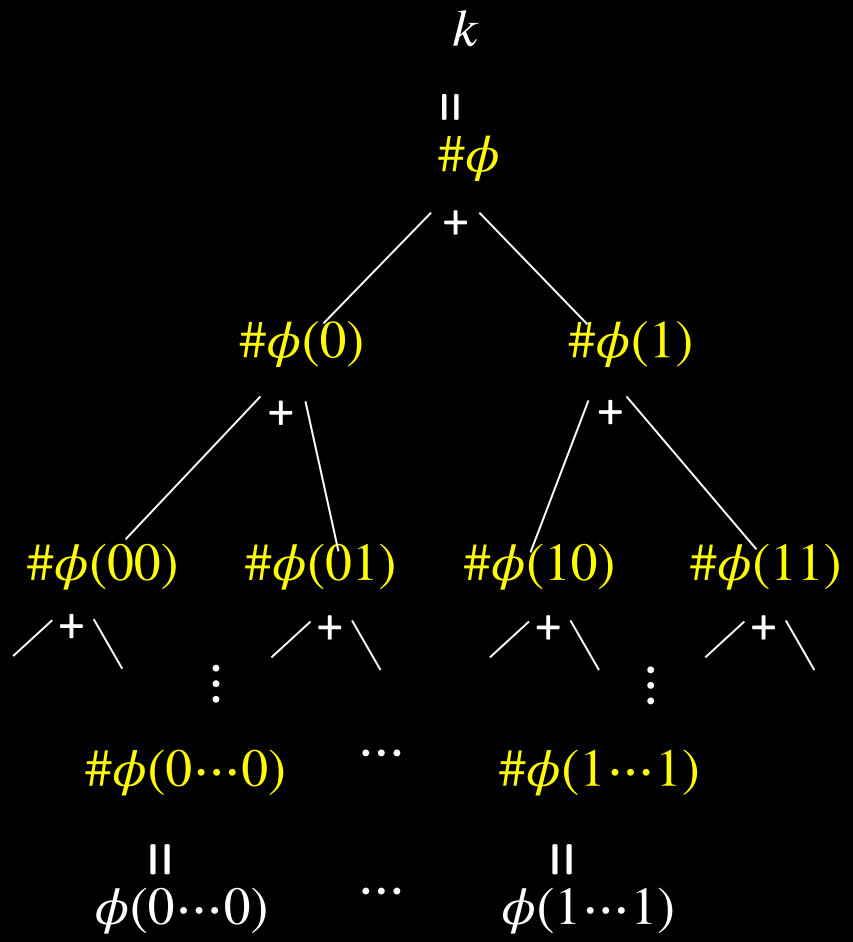
Idea for fixing $\#SAT \in$ IP protocol



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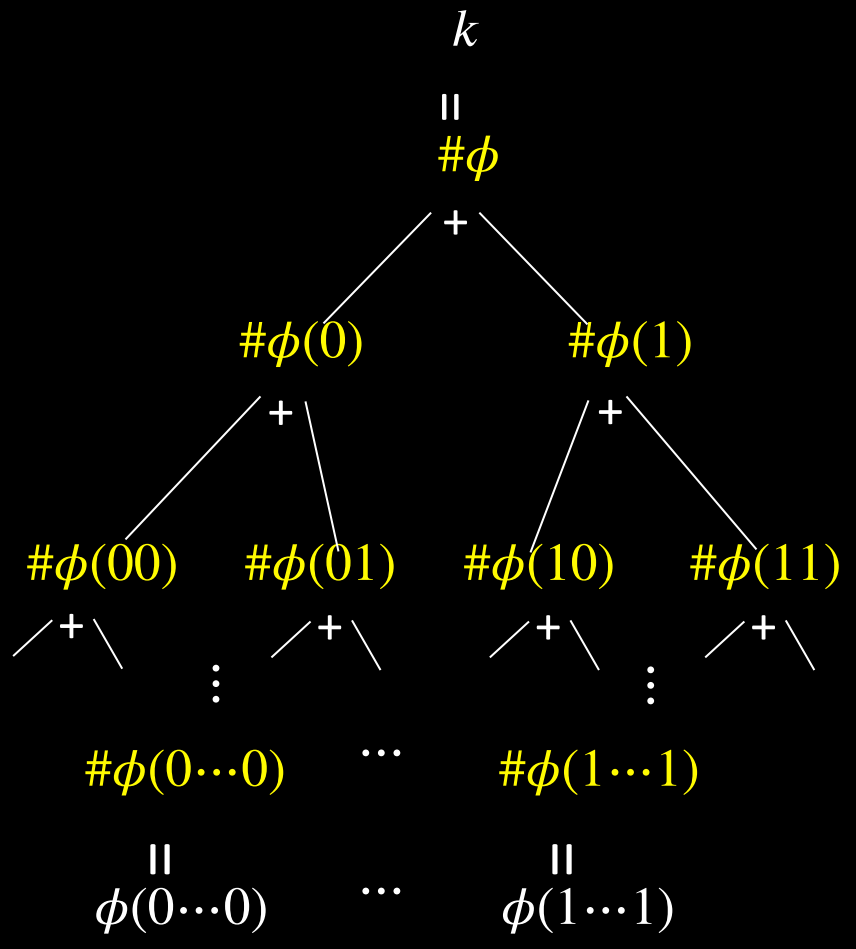

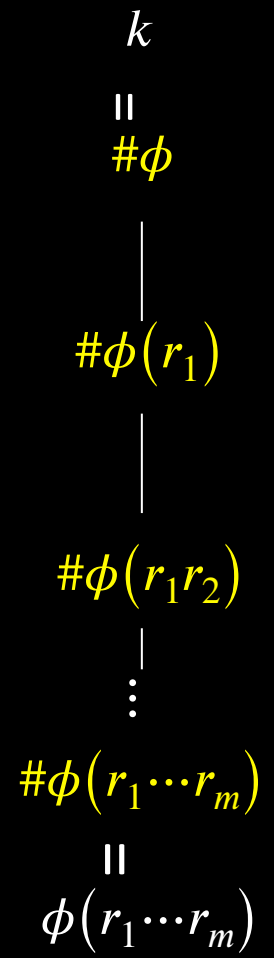

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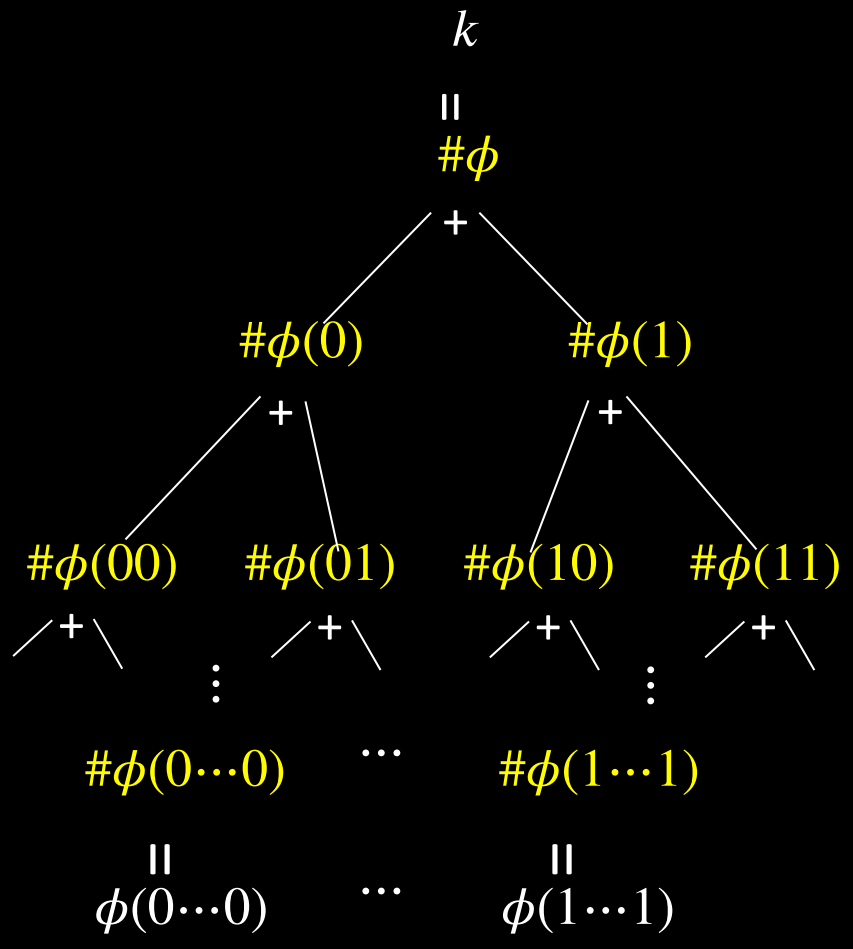

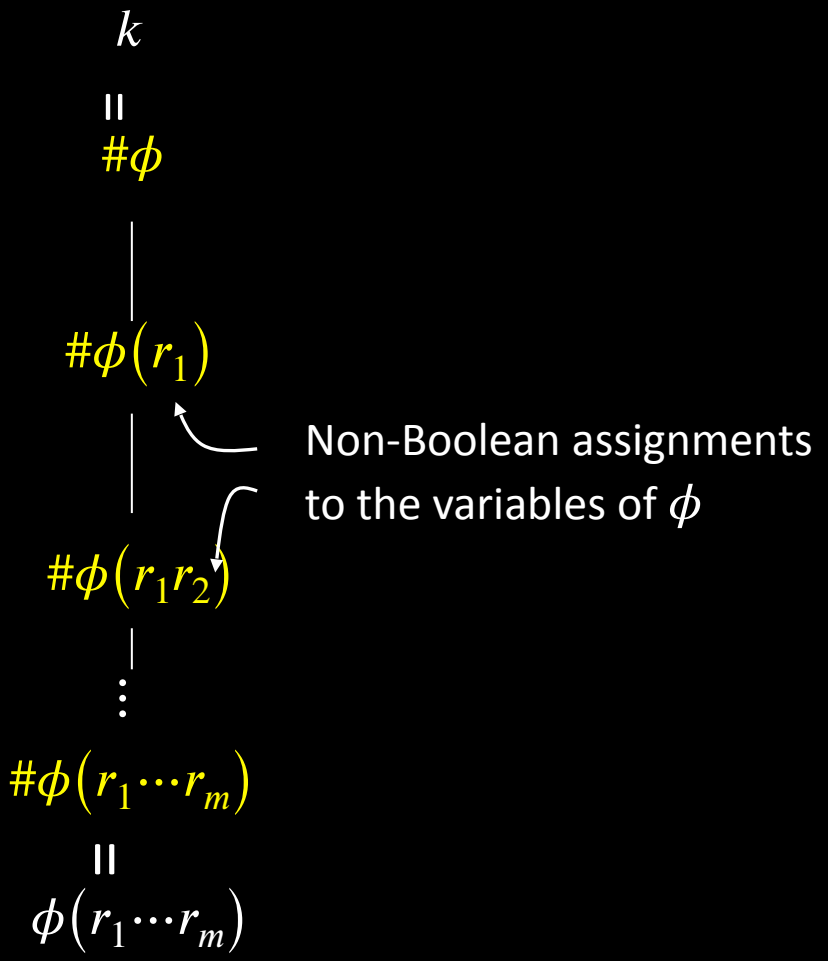

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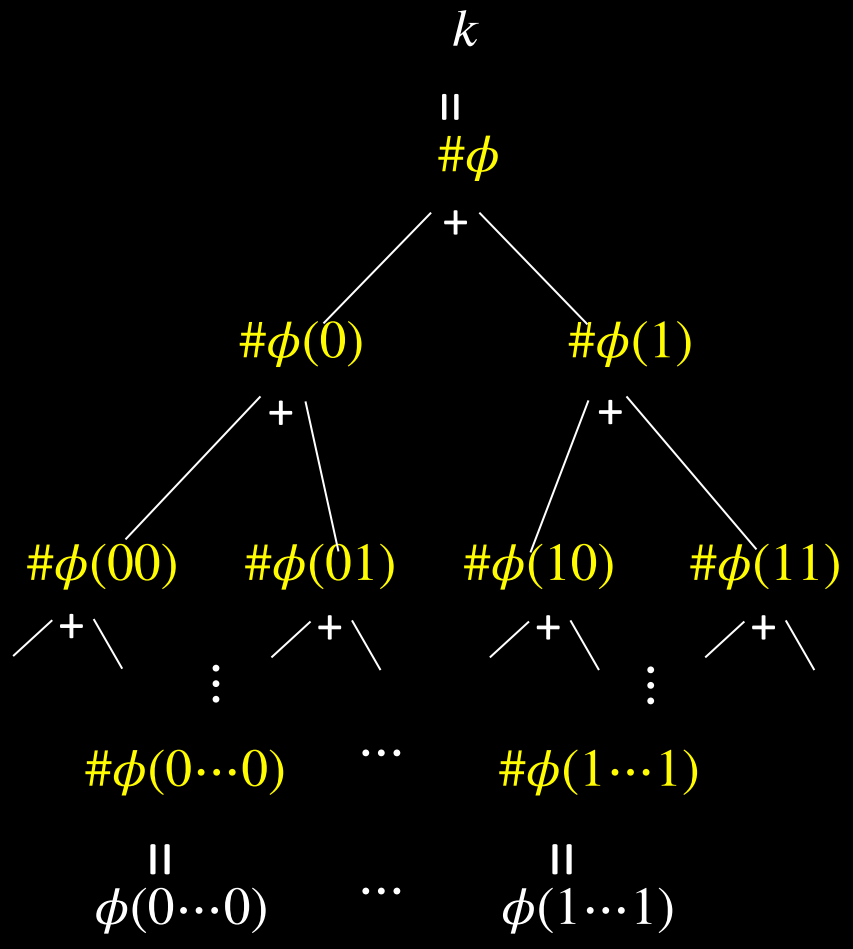

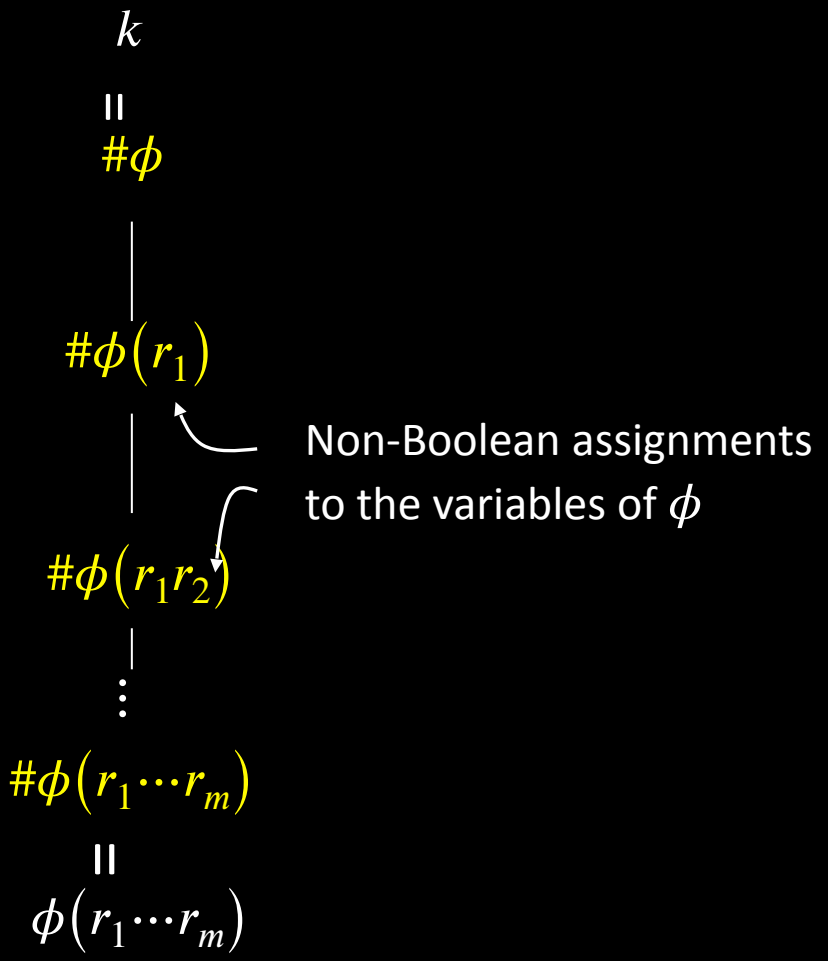

Arithmetizing Boolean formulas

Simulate $\wedge$ and $\vee$ with $+$ and $\times$

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\begin{aligned}
a \wedge b &\rightarrow a \times b = ab \\
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Important: For Boolean $a_1 \ldots\ a_i$ the values of $\phi(a_1 \ldots a_i)$ and $\#\phi(a_1 \ldots\ a_i)$ are unchanged from the previous definition.

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Check-in 26.2

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Check-in 26.2

Let $\phi = (x_1 \vee x_2) \wedge (\overline{x_1} \vee \overline{x_2})$. Check all that are true:

a) $p_\phi = (x_1 + x_2 - x_1 x_2)\Big((1-x_1) + (1-x_2) - (1-x_1)(1-x_2) \Big)$

b) $p_\phi = (x_1 + x_2)\Big((1-x_1) + (1-x_2) \Big)$

c) $p_\phi = (x_1 + x_2 - 2x_1 x_2)$

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V sends random $r_2 \in \mathbb{F}_q$ [P needs to show $\#\phi(r_1 r_2)$ is correct]

$\#SAT \in$ IP – version 1

Recall $\#\phi(a_1 \dots a_i) = \sum_{a_{i+1}, \dots, a_m \in \{0,1\}} \phi(a_1 \dots a_m)$

Theorem: $\#SAT \in$ IP

Proof: Protocol for V and (the honest) P on input $\langle \phi, k \rangle$

0) P sends $\#\phi$; V checks $k = \#\phi$

1) P sends $\#\phi(z)$ as a polynomial in z [sends coefficients – recall deg $p_\phi \leq |\phi|$]

V checks $\#\phi = \#\phi(0) + \#\phi(1)$ [by evaluating polynomial for $\#\phi(z)$]

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⋮

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⋮

m) P sends $\#\phi(r_1 \cdots r_{m-1} z)$ as a polynomial in z

$\#SAT \in$ IP – version 1

Recall $\#\phi(a_1 \dots a_i) = \sum_{a_{i+1}, \dots, a_m \in \{0,1\}} \phi(a_1 \dots a_m)$

Theorem: $\#SAT \in$ IP

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$\vdots$

m) P sends $\#\phi(r_1 \cdots r_{m-1} z)$ as a polynomial in z

V checks $\#\phi(r_1 \cdots r_{m-1}) = \#\phi(r_1 \cdots r_{m-1} 0) + \#\phi(r_1 \cdots r_{m-1} 1)$

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⋮

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V sends random $r_m \in \mathbb{F}_q$

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V sends random $r_2 \in \mathbb{F}_q$ [P needs to show $\#\phi(r_1 r_2)$ is correct]

$\vdots$

m) P sends $\#\phi(r_1 \cdots r_{m-1} z)$ as a polynomial in z

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V sends random $r_m \in \mathbb{F}_q$

$m+1$) V checks $\#\phi(r_1 \cdots r_m) = \phi(r_1 \cdots r_m)$

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$\vdots$

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V checks $\#\phi(r_1 \cdots r_{m-1}) = \#\phi(r_1 \cdots r_{m-1} 0) + \#\phi(r_1 \cdots r_{m-1} 1)$

V sends random $r_m \in \mathbb{F}_q$

$m+1$) V checks $\#\phi(r_1 \cdots r_m) = \phi(r_1 \cdots r_m)$

V *accepts* if all checks are correct. Otherwise V *rejects*.

$\#SAT \in$ IP – version 1

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$m+1$) V checks $\#\phi(r_1 \cdots r_m) = \phi(r_1 \cdots r_m)$

V *accepts* if all checks are correct. Otherwise V *rejects*.

$\#SAT \in$ IP – version 2

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Input $\langle \phi, k \rangle$

$\#SAT \in$ IP – version 2

Input $\langle \phi, k \rangle$

Prover sends **Verifier sends**

$\#SAT \in$ IP – version 2

Input $\langle \phi, k \rangle$

Prover sends **Verifier sends**

$\#\phi$

$\#SAT \in$ IP – version 2

Input $\langle \phi, k \rangle$

Prover sends

Verifier sends

Verifier checks

$\#\phi$

$\#\phi = k$

$\#SAT \in$ IP – version 2

Input $\langle \phi, k \rangle$

Prover sends

Verifier sends

Verifier checks

$\#\phi$

$\#\phi(z)$

$= 3z^d - 5z^{d-1} + \cdots + 7$

$\#\phi = k$

$\#SAT \in$ IP – version 2

Input $\langle \phi, k \rangle$

Prover sends

$\#\phi$

$\#\phi(z)$

$= 3z^d - 5z^{d-1} + \cdots + 7$

Verifier sends

Verifier checks

$\#\phi = k$

$\#\phi(0)$ $\#\phi(1)$

$\#SAT \in$ IP – version 2

Input $\langle \phi, k \rangle$

Prover sends

$\#\phi$

$\#\phi(z)$

$= 3z^d - 5z^{d-1} + \cdots + 7$

Verifier sends

Verifier checks

$\#\phi = k$

$+$

$\#\phi(0)$ $\#\phi(1)$

$\#SAT \in$ IP – version 2

Input $\langle \phi, k \rangle$

Prover sends

$\#\phi$

$\#\phi(z)$

$= 3z^d - 5z^{d-1} + \cdots + 7$

Verifier sends

r_1

Verifier checks

$\#\phi = k$

$+$

$\#\phi(0)$ $\#\phi(1)$

$\#SAT \in$ IP – version 2

Input $\langle \phi, k \rangle$

Prover sends

$\#\phi$

$\#\phi(z)$

$= 3z^d - 5z^{d-1} + \cdots + 7$

Verifier sends

r_1

Verifier checks

$\#\phi = k$

$+$

$\#\phi(0)$ $\#\phi(1)$

$\#\phi(r_1)$

$\#SAT \in$ IP – version 2

Input $\langle \phi, k \rangle$

Prover sends

$\#\phi$

$\#\phi(z) = 3z^d - 5z^{d-1} + \cdots + 7$

$\#\phi(r_1 z) = \cdots$

Verifier sends

r_1

Verifier checks

$\#\phi = k$

$+$

$\#\phi(0)$ $\#\phi(1)$

$\#\phi(r_1)$

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r_1

Verifier checks

$\#\phi = k$

$+$

$\#\phi(0)$ $\#\phi(1)$

$\#\phi(r_1)$

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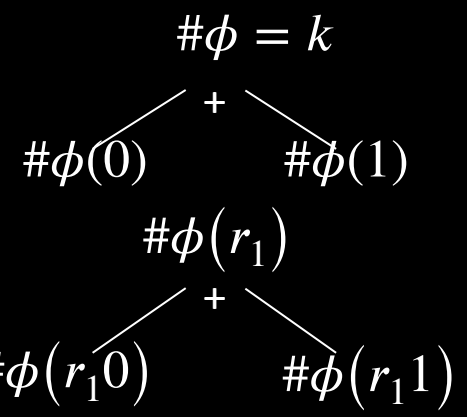
$\#\phi(r_1 z) = \cdots$

Verifier sends

r_1

Verifier checks



$\#SAT \in$ IP – version 2

Input $\langle \phi, k \rangle$

Prover sends

$\#\phi$

$\#\phi(z) = 3z^d - 5z^{d-1} + \cdots + 7$

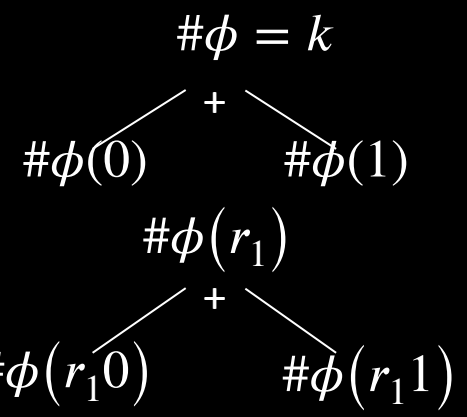
$\#\phi(r_1 z) = \cdots$

Verifier sends

r_1

r_2

Verifier checks



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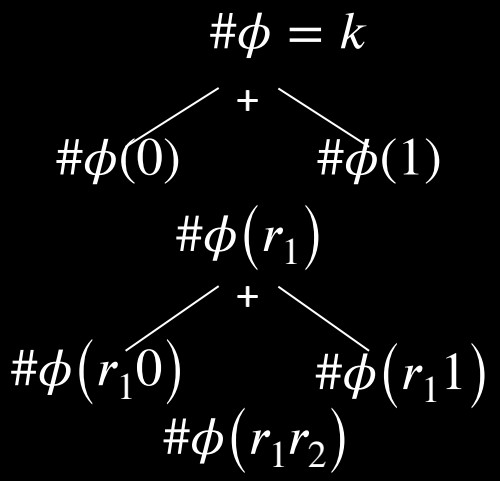
Verifier sends

r_1

r_2

Verifier checks



$\#SAT \in$ IP – version 2

Input $\langle \phi, k \rangle$

Prover sends

$\#\phi$

$\#\phi(z) = 3z^d - 5z^{d-1} + \cdots + 7$

$\#\phi(r_1 z) = \cdots$

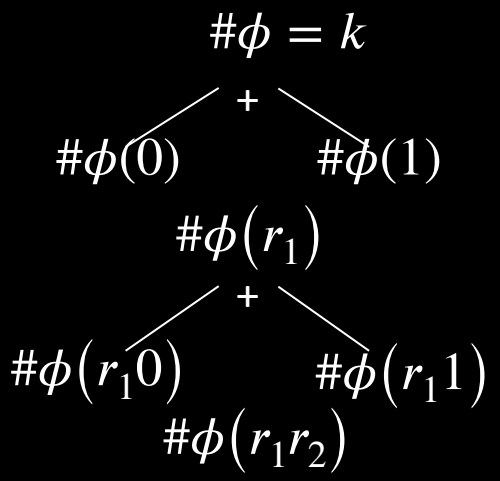
$\#\phi(r_1 r_2 z) = \cdots$

Verifier sends

r_1

r_2

Verifier checks



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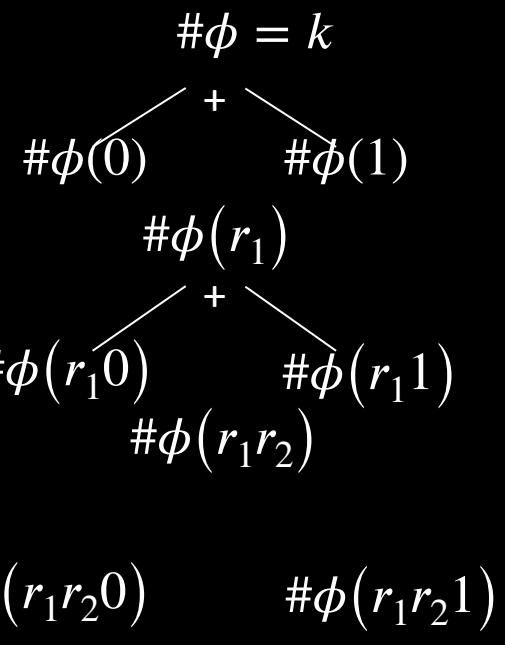
$\#\phi(r_1 r_2 z) = \cdots$

Verifier sends

r_1

r_2

Verifier checks



$\#\phi(r_1 r_2 0)$ $\#\phi(r_1 r_2 1)$

$\#SAT \in$ IP – version 2

Input $\langle \phi, k \rangle$

Prover sends

$\#\phi$

$\#\phi(z) = 3z^d - 5z^{d-1} + \cdots + 7$

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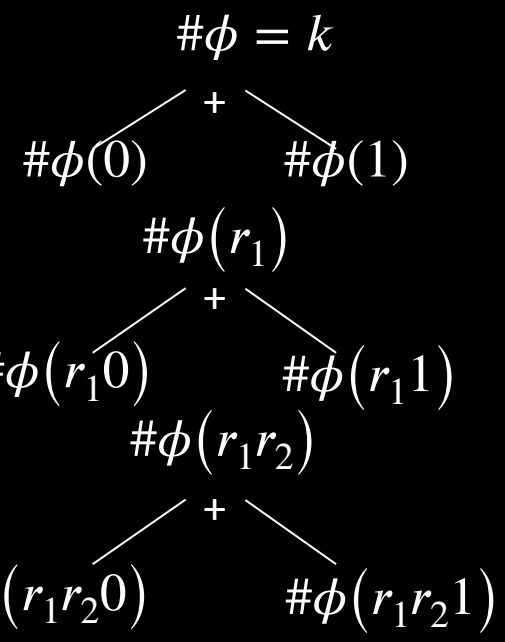
$\#\phi(r_1 r_2 z) = \cdots$

Verifier sends

r_1

r_2

Verifier checks



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Input $\langle \phi, k \rangle$

Prover sends

$\#\phi$

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$\#\phi(r_1 z) = \cdots$

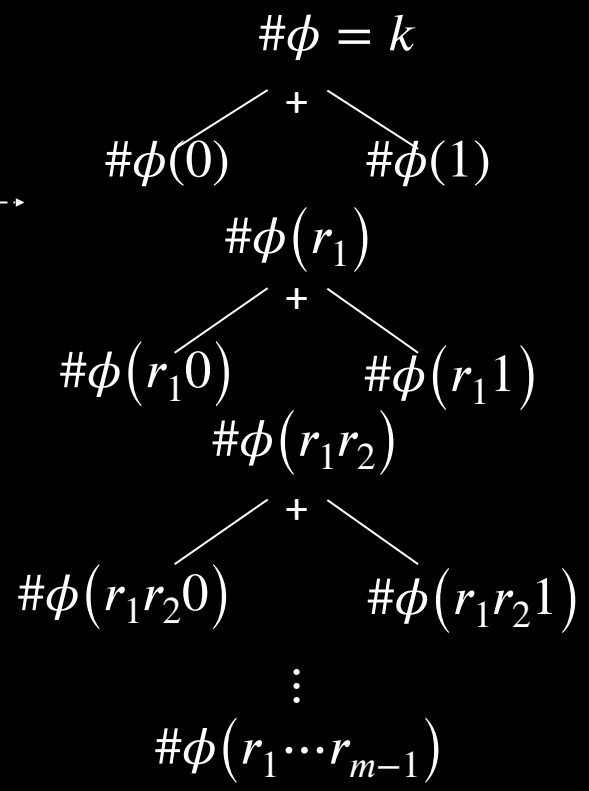
$\#\phi(r_1 r_2 z) = \cdots$

Verifier sends

r_1

r_2

Verifier checks



$\#SAT \in$ IP – version 2

Input $\langle \phi, k \rangle$

Prover sends

$\#\phi$

$\#\phi(z) = 3z^d - 5z^{d-1} + \cdots + 7$

$\#\phi(r_1 z) = \cdots$

$\#\phi(r_1 r_2 z) = \cdots$

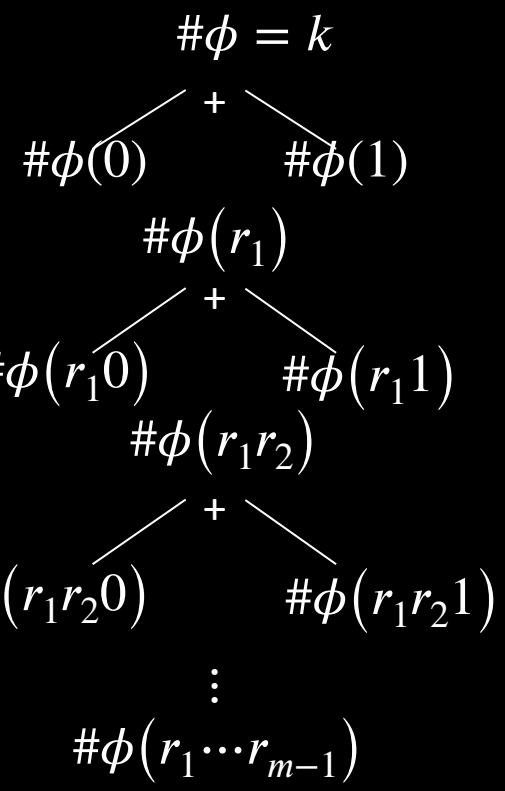
$\#\phi(r_1 \cdots r_{m-1} z) = \cdots$

Verifier sends

r_1

r_2

Verifier checks



$\#SAT \in$ IP – version 2

Input $\langle \phi, k \rangle$

Prover sends

Verifier sends

Verifier checks

$\#\phi$

$\#\phi(z) = 3z^d - 5z^{d-1} + \cdots + 7$

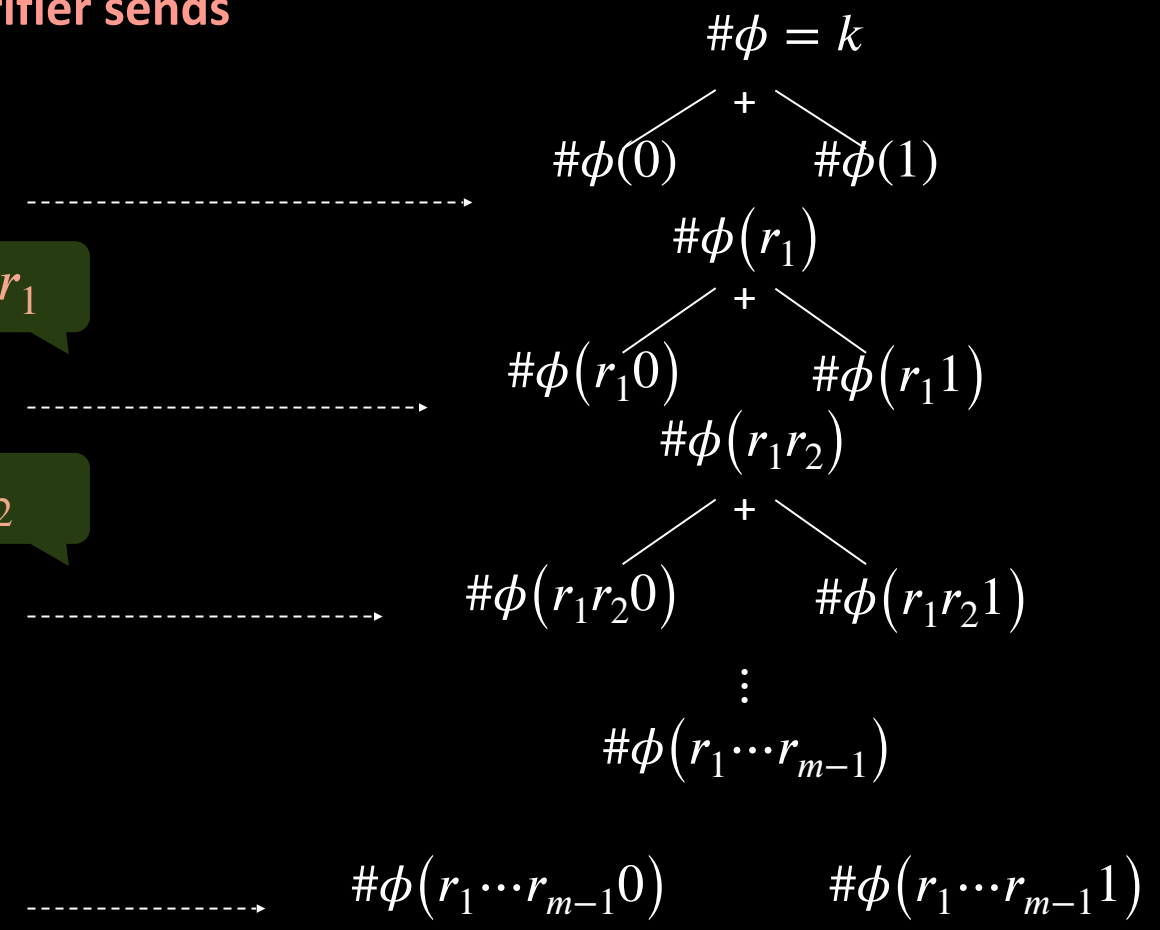
r_1

$\#\phi(r_1 z) = \cdots$

r_2

$\#\phi(r_1 r_2 z) = \cdots$

$\#\phi(r_1 \cdots r_{m-1} z) = \cdots$



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Input $\langle \phi, k \rangle$

Prover sends

$\#\phi$

$\#\phi(z) = 3z^d - 5z^{d-1} + \cdots + 7$

$\#\phi(r_1 z) = \cdots$

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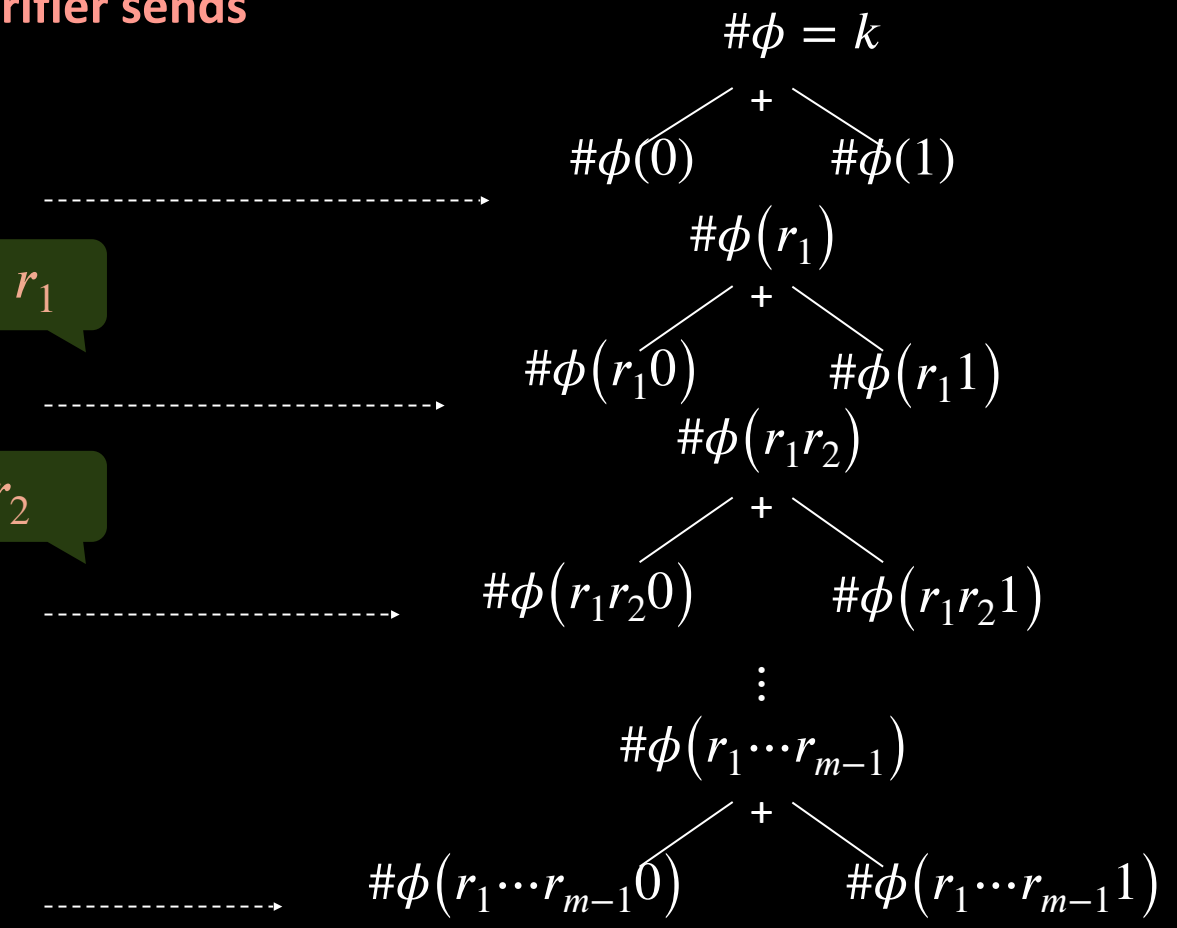
Verifier sends

r_1

r_2

Verifier checks



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$\#\phi(r_1 r_2 z) = \cdots$

$\#\phi(r_1 \cdots r_{m-1} z) = \cdots$

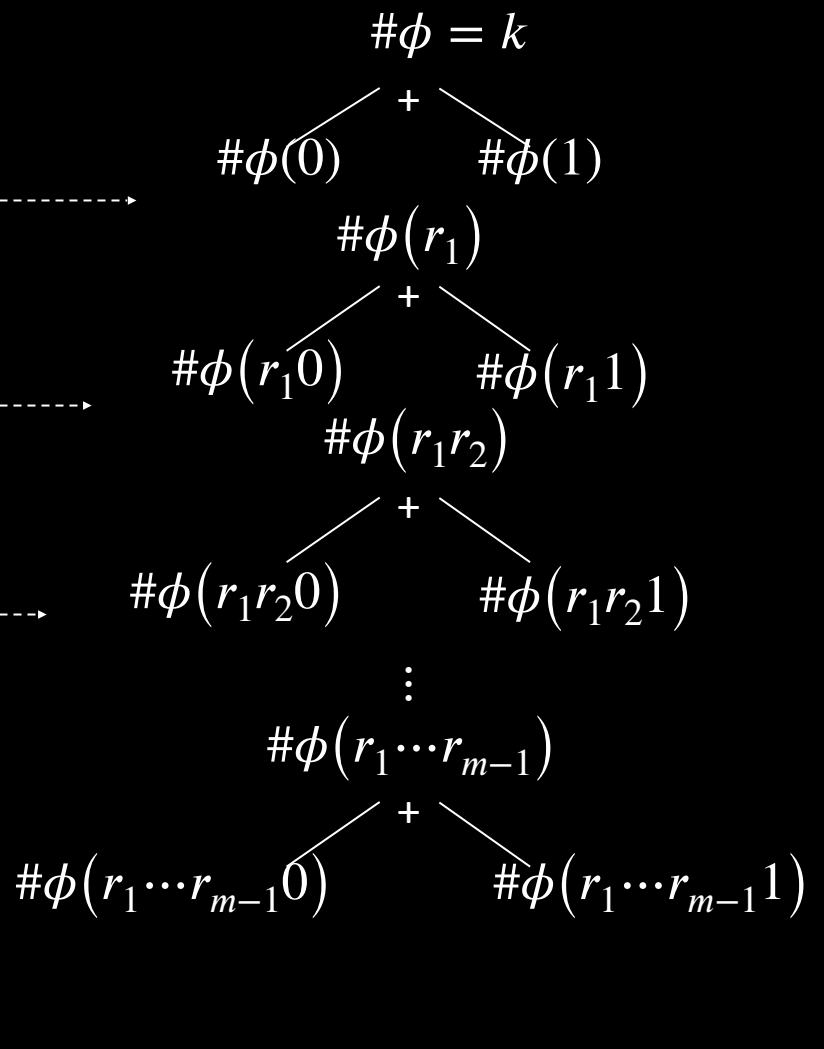
Verifier sends

r_1

r_2

r_m

Verifier checks



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$\#\phi(r_1 z) = \cdots$

$\#\phi(r_1 r_2 z) = \cdots$

$\#\phi(r_1 \cdots r_{m-1} z) = \cdots$

Verifier sends

r_1

r_2

r_m

Verifier checks

$\#\phi = k$

$+$

$\#\phi(0)$ $\#\phi(1)$

$\#\phi(r_1)$

$+$

$\#\phi(r_1 0)$ $\#\phi(r_1 1)$

$\#\phi(r_1 r_2)$

$+$

$\#\phi(r_1 r_2 0)$ $\#\phi(r_1 r_2 1)$

$\vdots$

$\#\phi(r_1 \cdots r_{m-1})$

$+$

$\#\phi(r_1 \cdots r_{m-1} 0)$ $\#\phi(r_1 \cdots r_{m-1} 1)$

$\#\phi(r_1 \cdots r_m)$

$\#SAT \in$ IP – version 2

Input $\langle \phi, k \rangle$

Prover sends

$\#\phi$

$\#\phi(z) = 3z^d - 5z^{d-1} + \cdots + 7$

$\#\phi(r_1 z) = \cdots$

$\#\phi(r_1 r_2 z) = \cdots$

$\#\phi(r_1 \cdots r_{m-1} z) = \cdots$

Verifier sends

r_1

r_2

r_m

Verifier checks

$\#\phi = k$

$+$

$\#\phi(0)$ $\#\phi(1)$

$\#\phi(r_1)$

$+$

$\#\phi(r_1 0)$ $\#\phi(r_1 1)$

$\#\phi(r_1 r_2)$

$+$

$\#\phi(r_1 r_2 0)$ $\#\phi(r_1 r_2 1)$

$\vdots$

$\#\phi(r_1 \cdots r_{m-1})$

$+$

$\#\phi(r_1 \cdots r_{m-1} 0)$ $\#\phi(r_1 \cdots r_{m-1} 1)$

$\#\phi(r_1 \cdots r_m)$

$\phi(r_1 \cdots r_m)$

$\#SAT \in$ IP – version 2

Input $\langle \phi, k \rangle$

Prover sends

$\#\phi$

$\#\phi(z) = 3z^d - 5z^{d-1} + \cdots + 7$

$\#\phi(r_1 z) = \cdots$

$\#\phi(r_1 r_2 z) = \cdots$

$\#\phi(r_1 \cdots r_{m-1} z) = \cdots$

Verifier sends

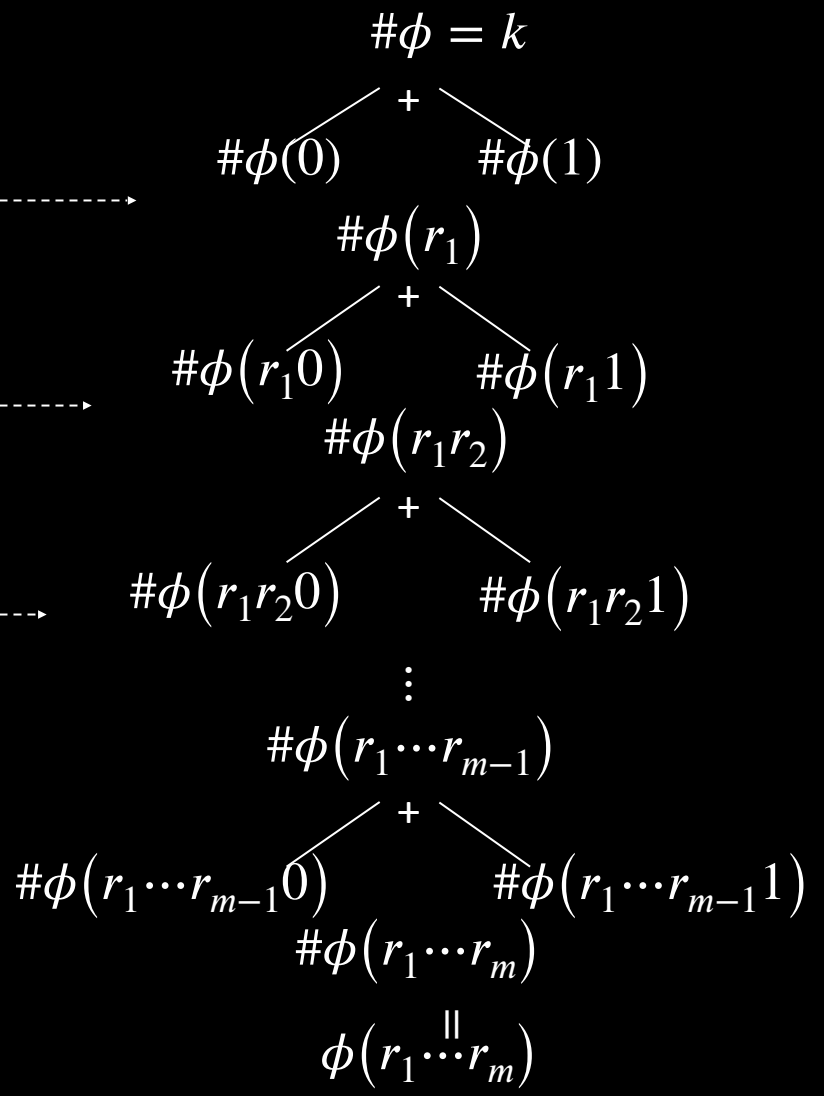
r_1

r_2

r_m

accept

Verifier checks



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Verifier sends

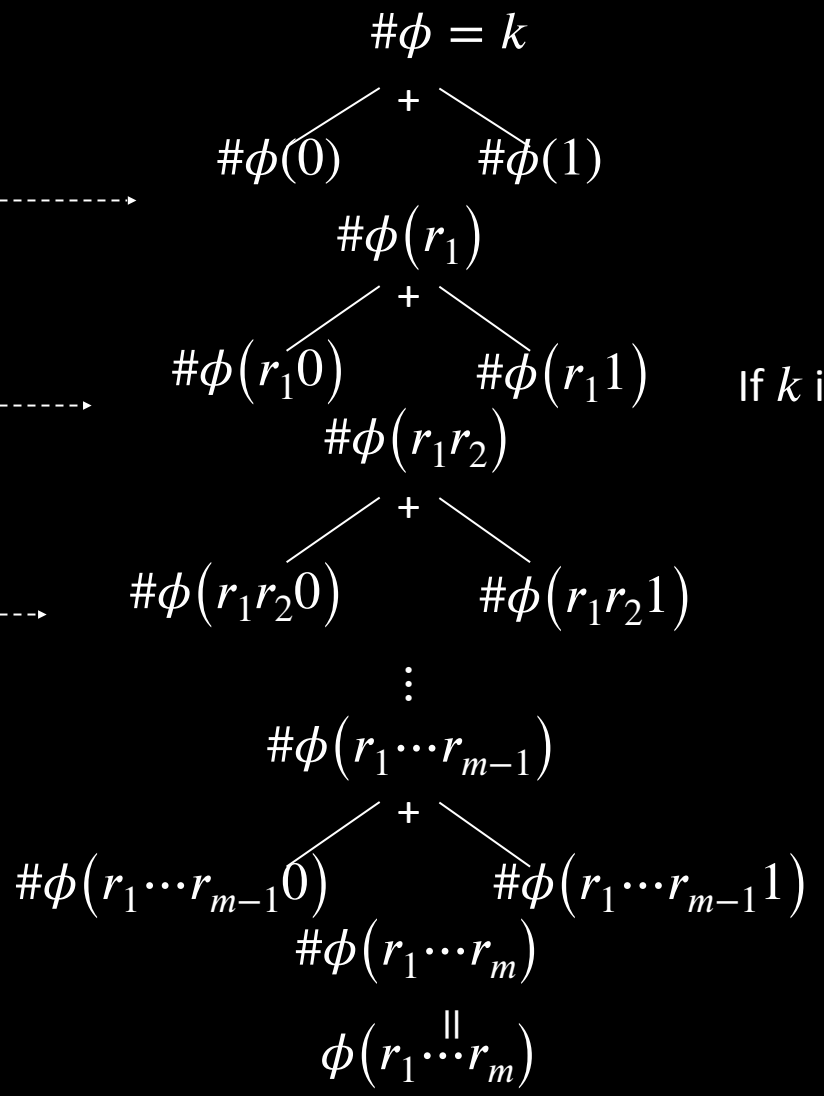
r_1

r_2

r_m

accept

Verifier checks



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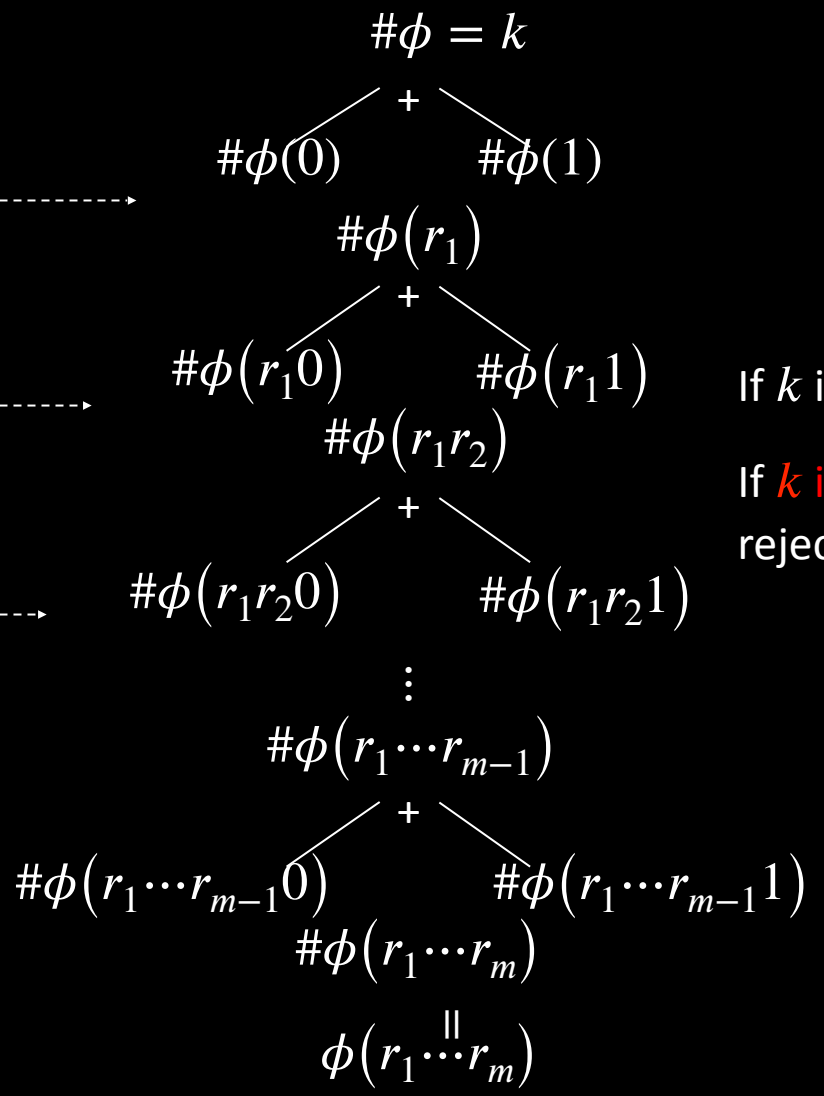

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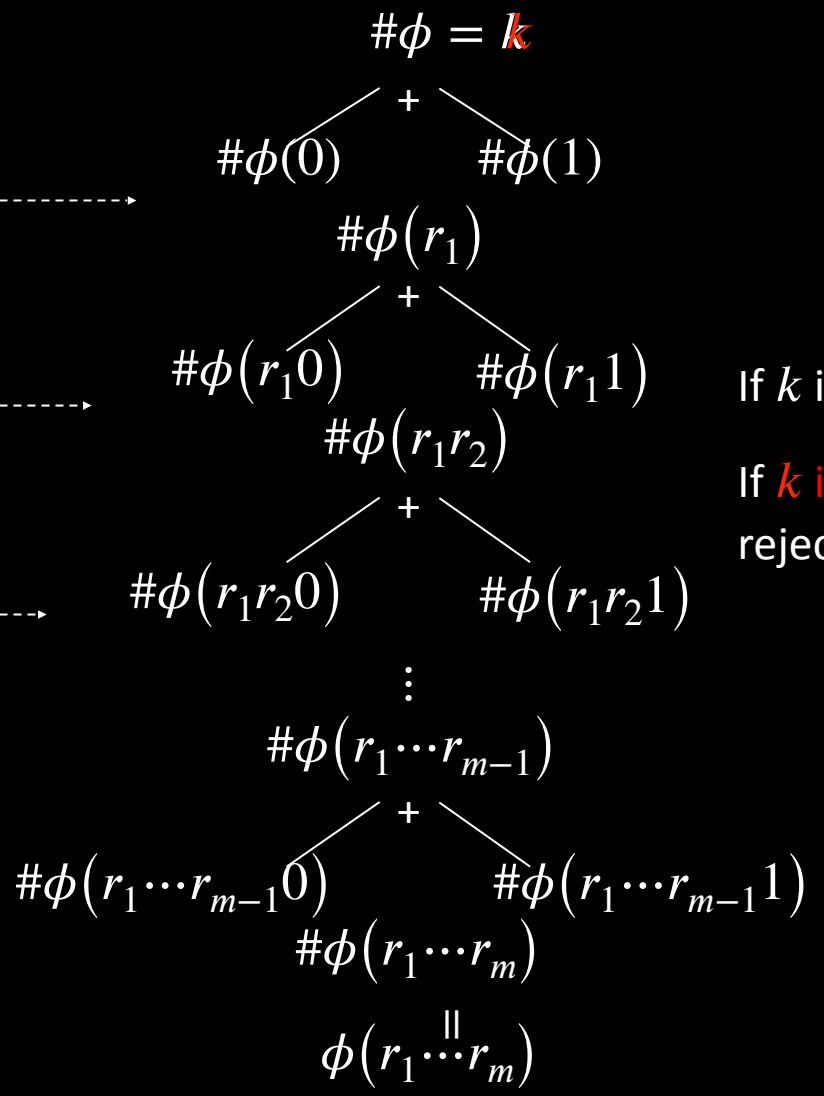

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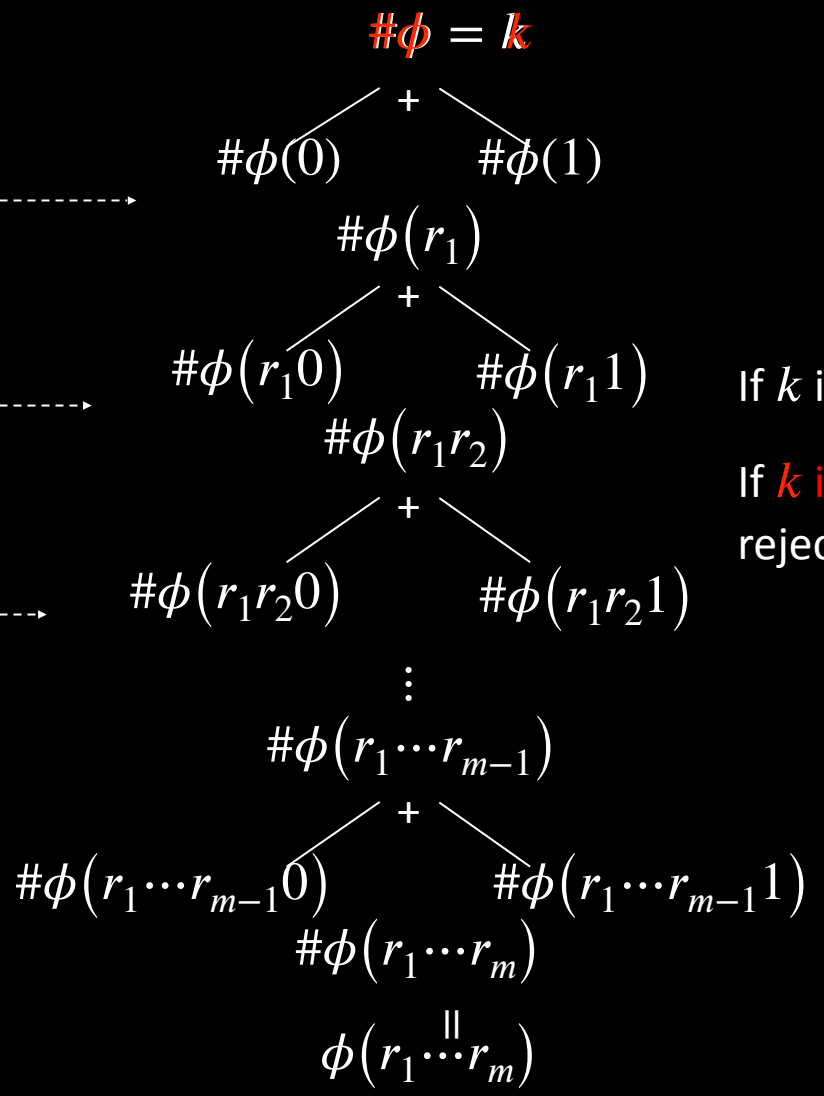

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Verifier checks

$\#\phi = k$

$\#\phi(0) + \#\phi(1)$

$\#\phi(r_1) = \#\phi(r_1 0) + \#\phi(r_1 1)$

$\#\phi(r_1 r_2) = \#\phi(r_1 r_2 0) + \#\phi(r_1 r_2 1)$

$\vdots$

$\#\phi(r_1 \cdots r_{m-1}) = \#\phi(r_1 \cdots r_{m-1} 0) + \#\phi(r_1 \cdots r_{m-1} 1)$

$\#\phi(r_1 \cdots r_m) \stackrel{?}{=} \phi(r_1 \cdots r_m)$

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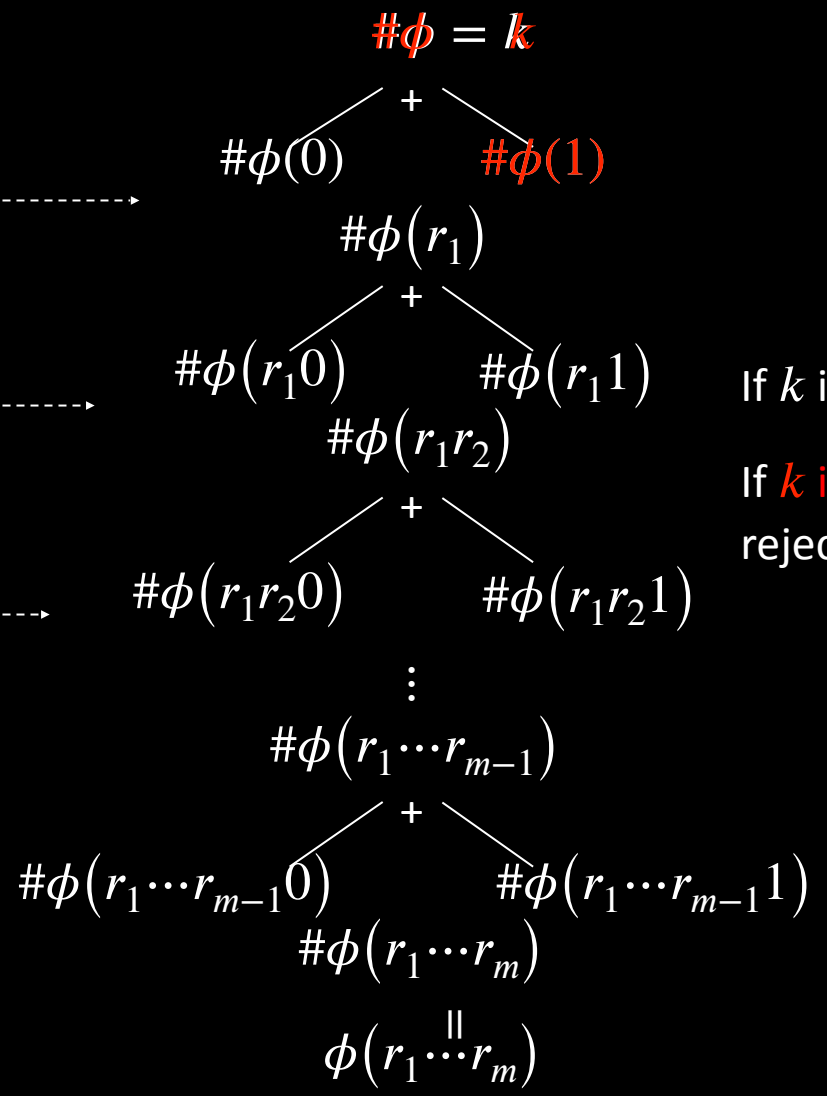

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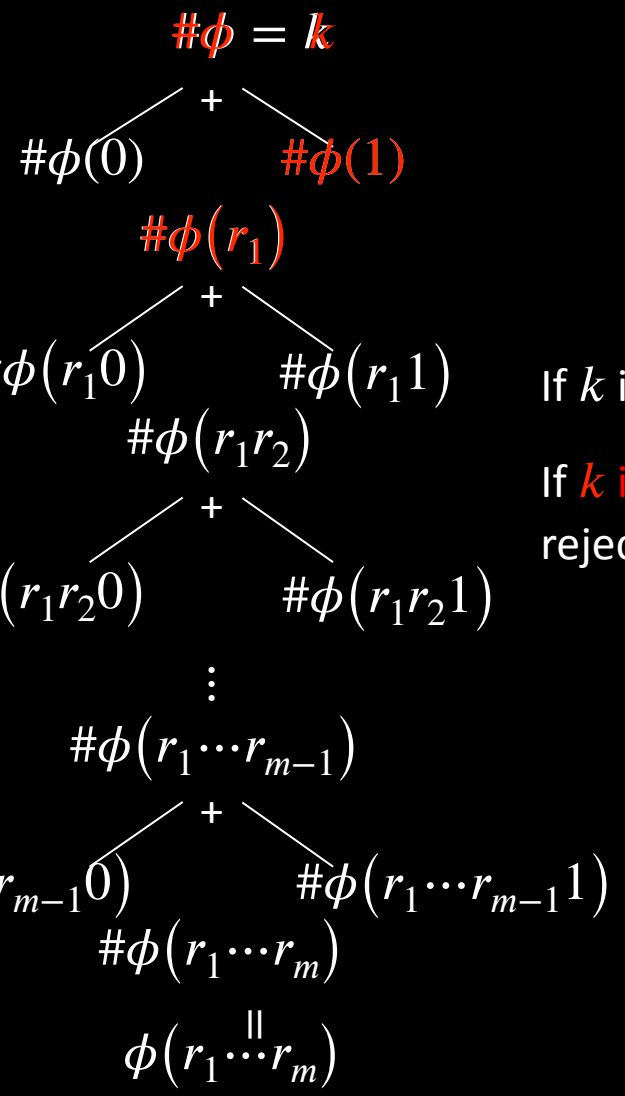

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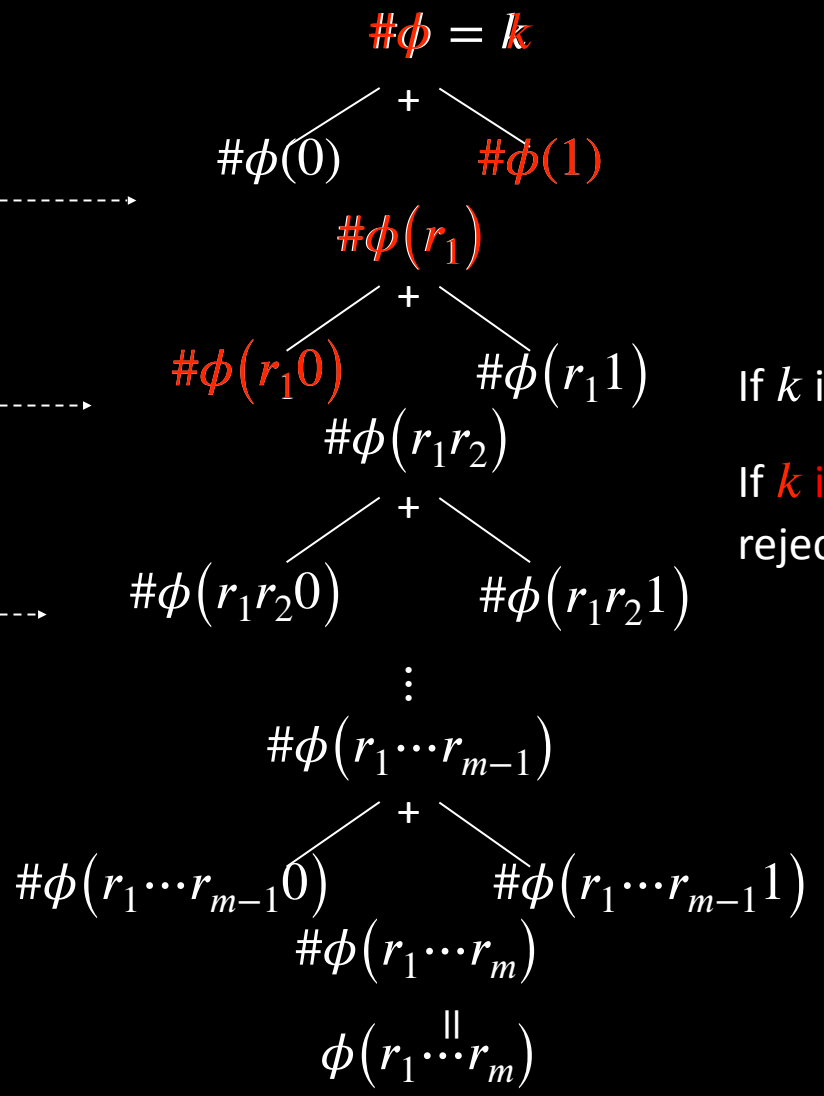

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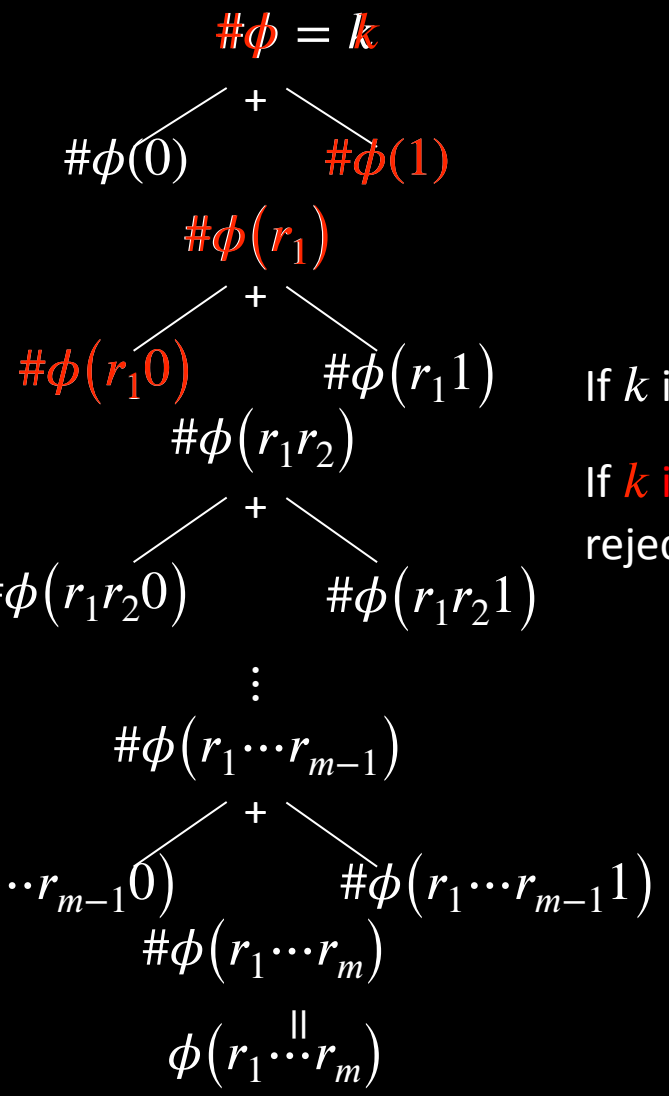

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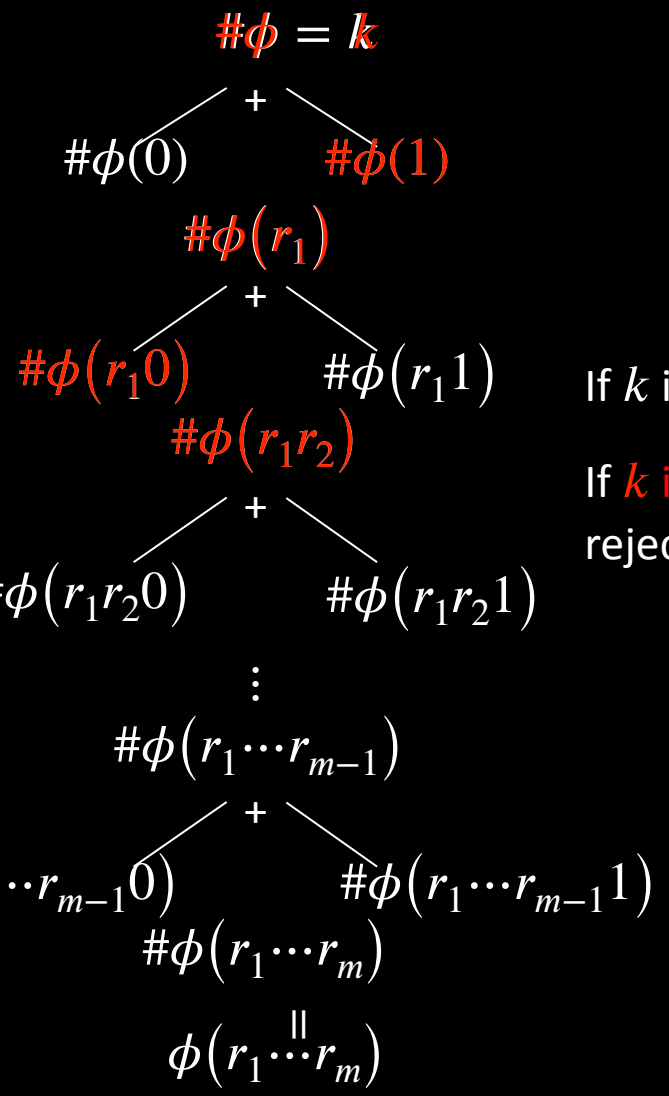

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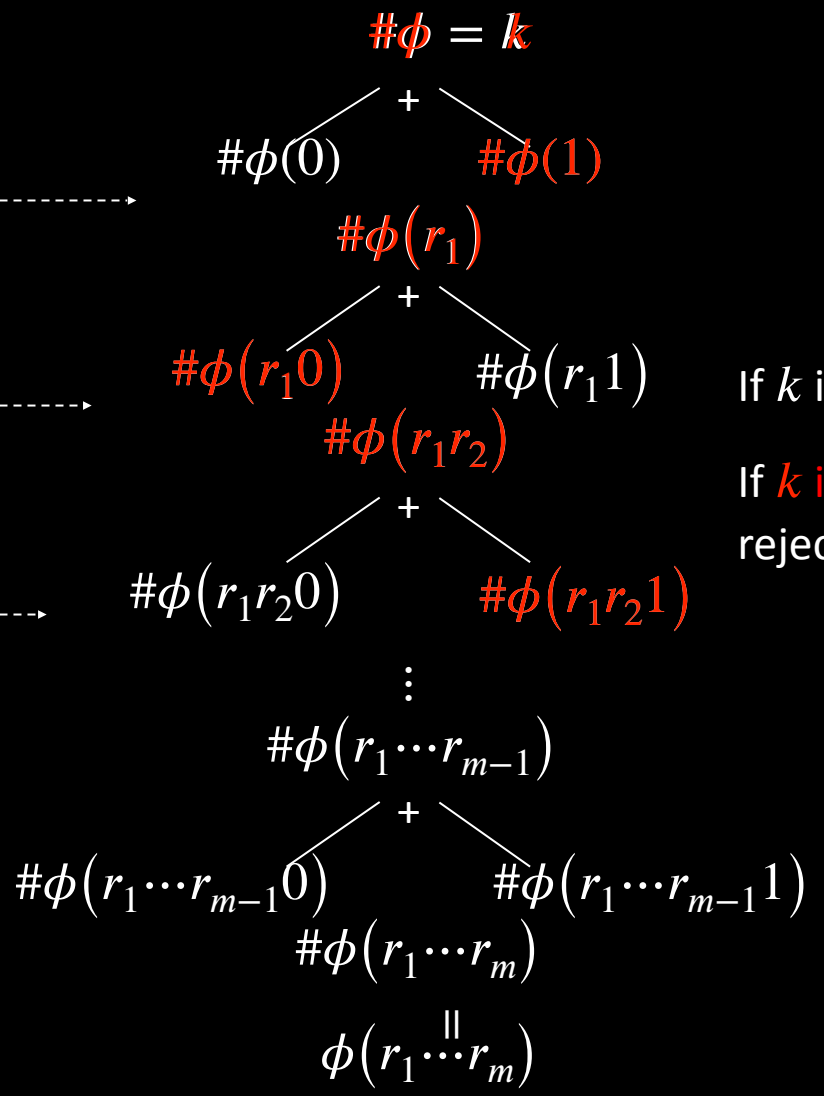

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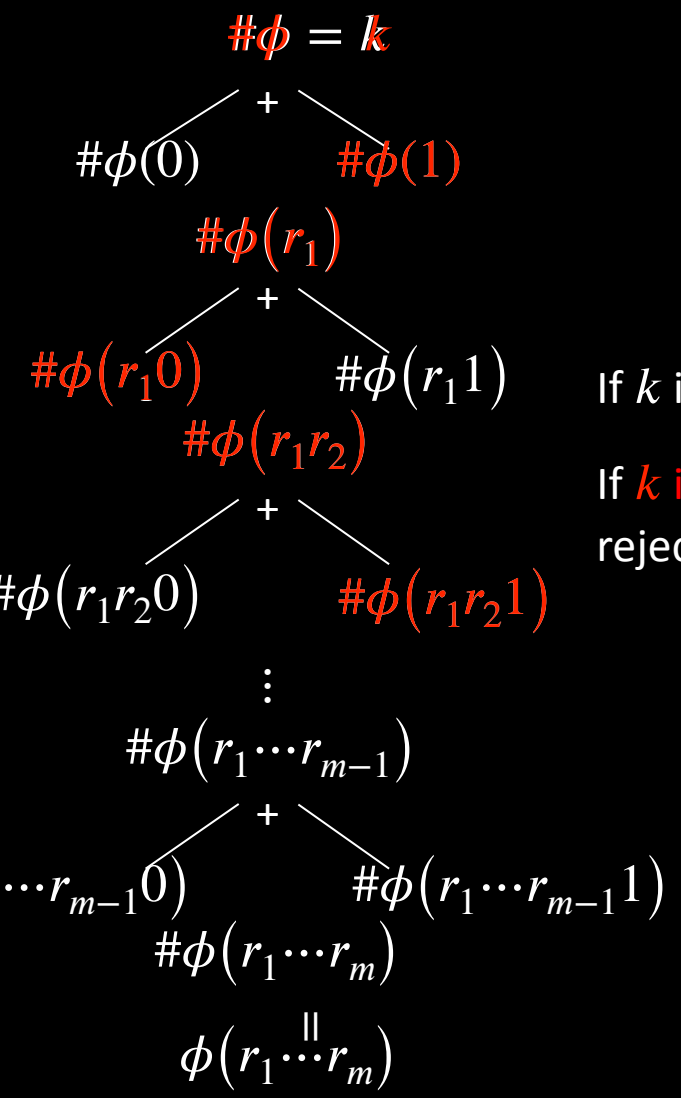

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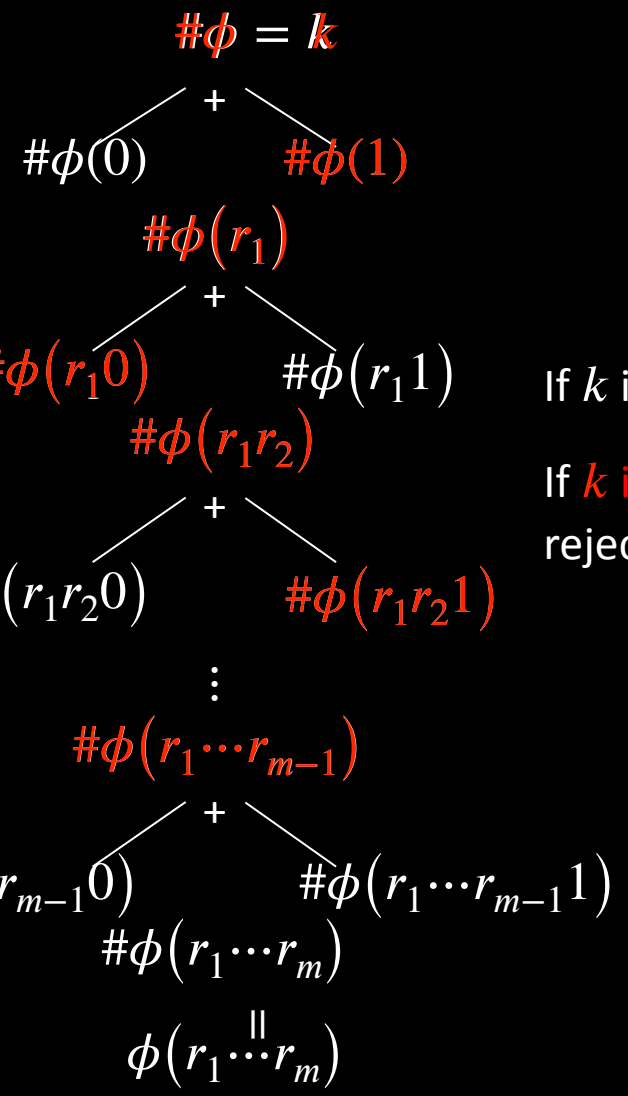

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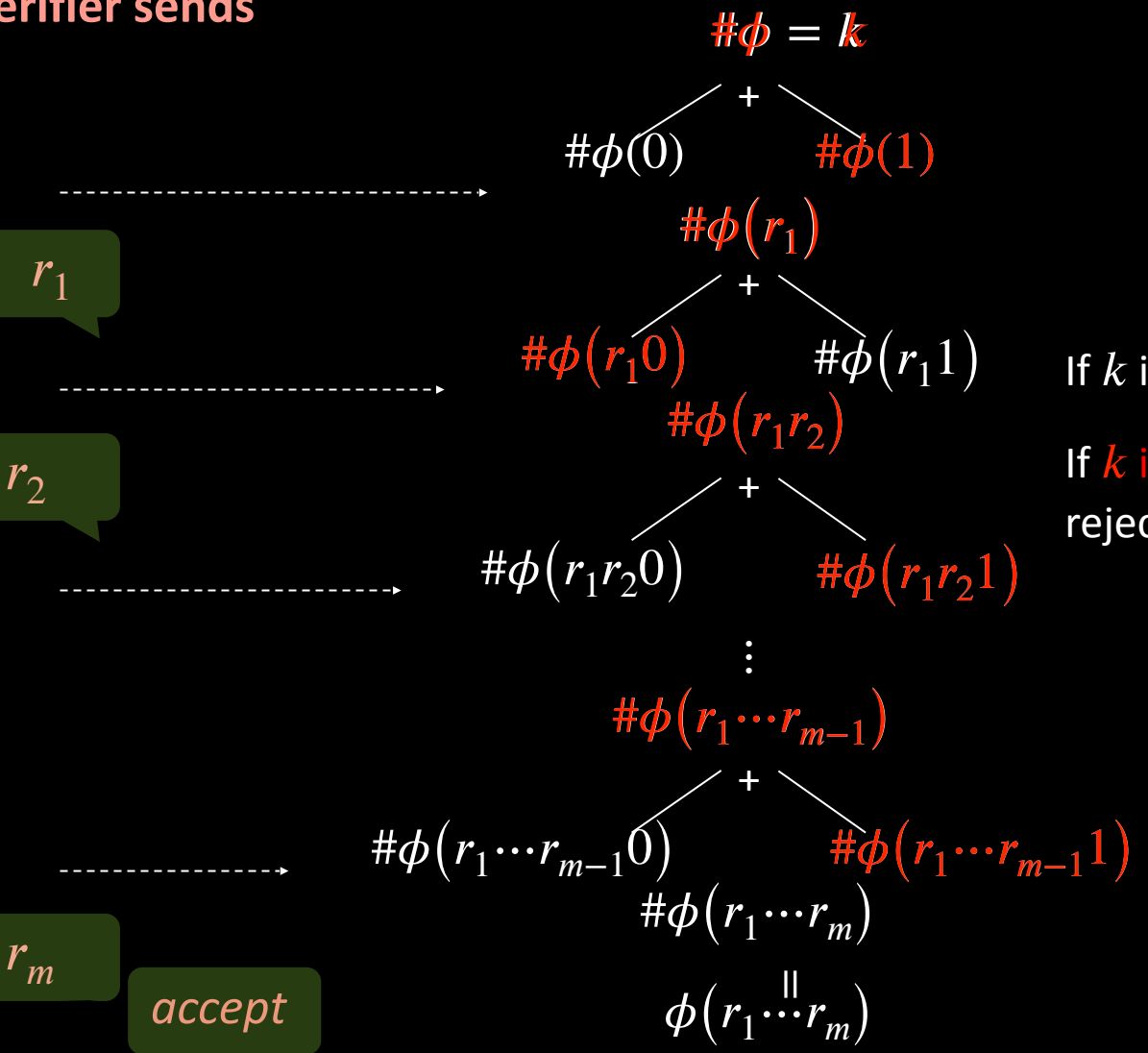

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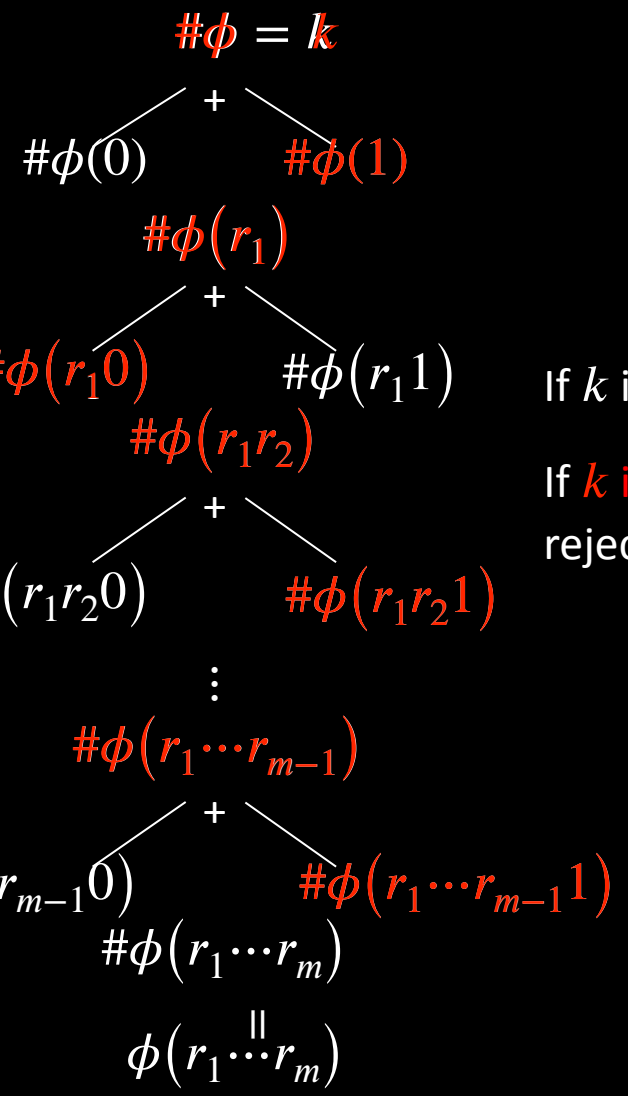

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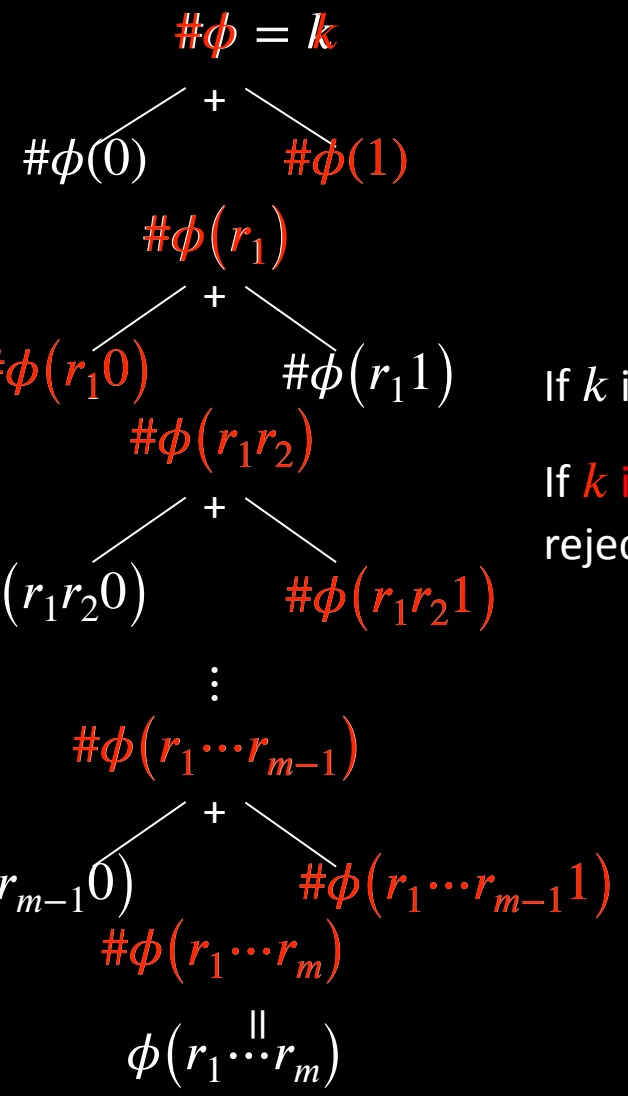

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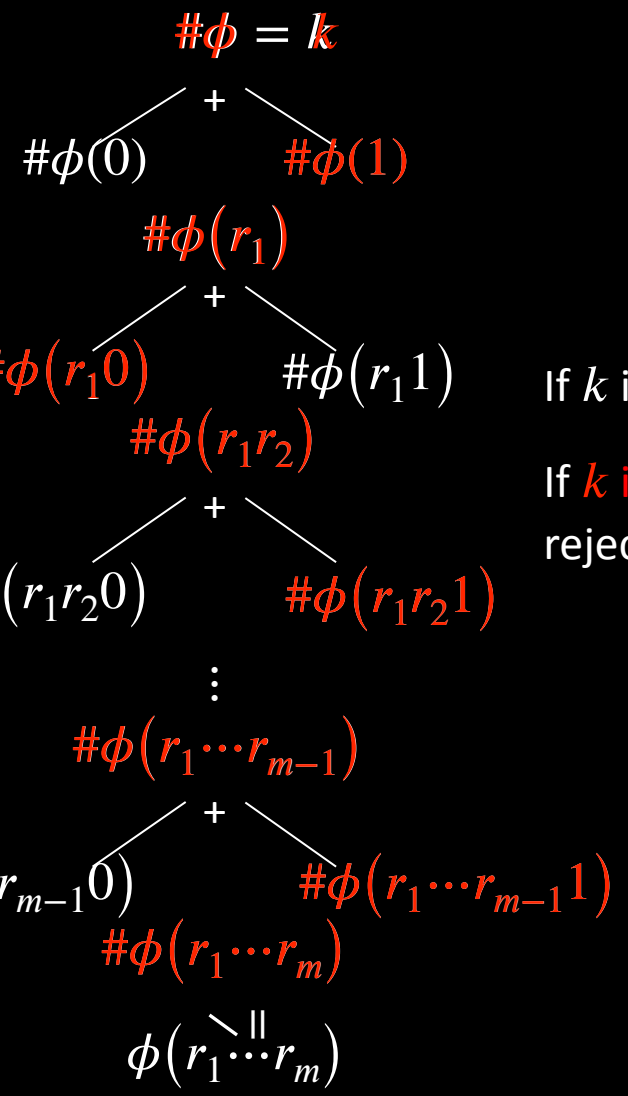

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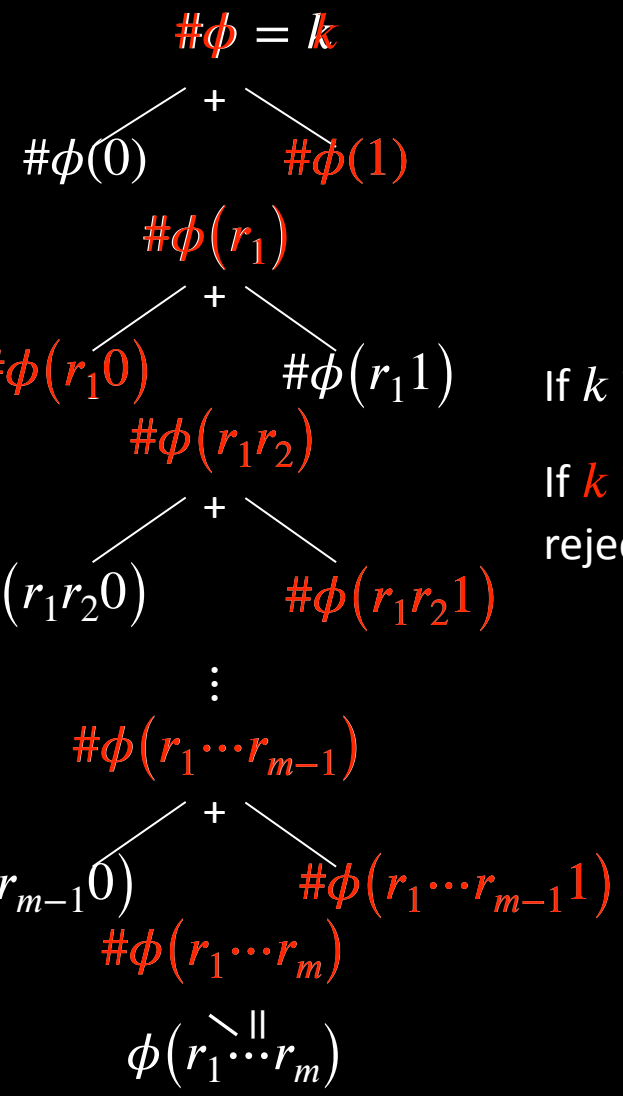

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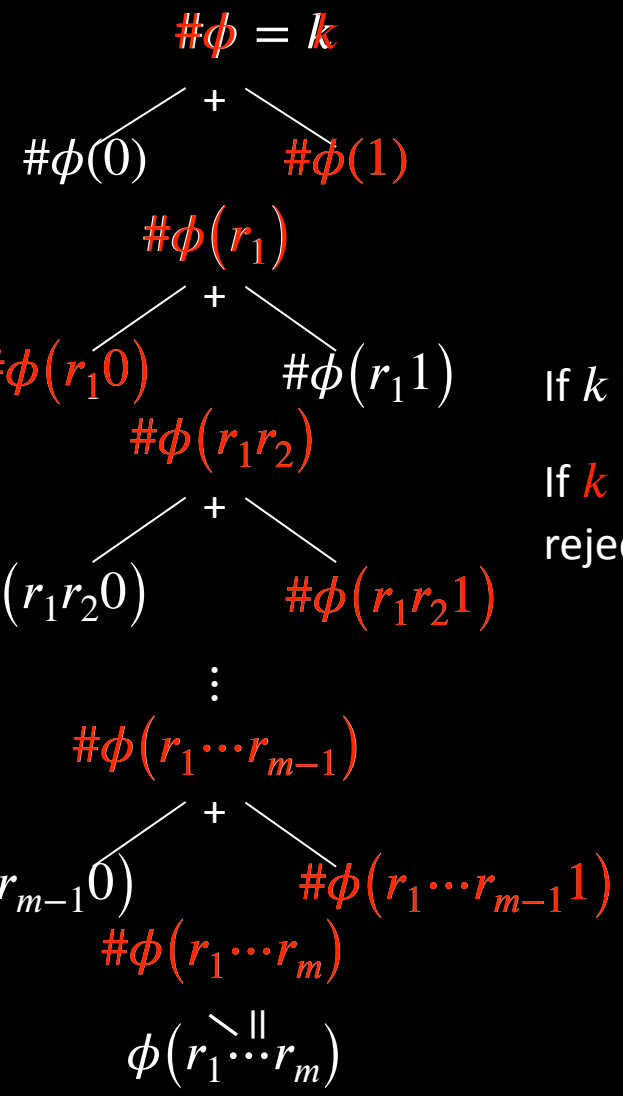

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Best wishes for the holidays and the New Year!