بسم الله الرحمن الرحیم

تمرین‌های رابطه بازگشتی - درس ریاضیات گسسته نیمسال دوم ۹۲-۹۳ – دانشگاه شریف

تمرین های الزامی:

8 Solve the recurrence

$$Q_0 = \alpha; \qquad Q_1 = \beta;$$
$$Q_n = (1+Q_{n-1})/Q_{n-2}, \qquad \text{for } n > 1.$$

Assume that $Q_n \neq 0$ for all $n \geqslant 0$. *Hint:* $Q_4 = (1+\alpha)/\beta$.

16 Use the repertoire method to solve the general four-parameter recurrence

$$g(1) = \alpha;$$
$$g(2n+j) = 3g(n) + \gamma n + \beta_j, \qquad \text{for } j = 0, 1 \quad \text{and} \quad n \geqslant 1.$$

Hint: Try the function $g(n) = n$.

۹.۱.۹ فرض کنید a_n تعداد راههای پوشاندن جدولی $n \times ۲$ با موزاییکهای ۲ × ۱ و ۲ × ۲ باشد. رابطه‌ای بازگشتی برای a_n بیابید.

۱۶.۱.۹ فرض کنید t_n تعداد کلمات n حرفی با حروف a و b باشد که شامل دو حرف a متوالی هستند. رابطه‌ای بازگشتی برای t_n بیابید.

تمرین امتیازی:

20 Use the repertoire method to solve the general five-parameter recurrence

$$h(1) = \alpha;$$
$$h(2n+j) = 4h(n) + \gamma_j n + \beta_j, \qquad \text{for } j = 0, 1 \quad \text{and} \quad n \geqslant 1.$$

Hint: Try the functions $h(n) = n$ and $h(n) = n^2$.

برای تمرین بیشتر و آمادگی برای کوییز:

2 Find the shortest sequence of moves that transfers a tower of n disks from the left peg A to the right peg B, if direct moves between A and B are disallowed. (Each move must be to or from the middle peg. As usual, a larger disk must never appear above a smaller one.)

6 Some of the regions defined by n lines in the plane are infinite, while others are bounded. What's the maximum possible number of bounded regions?

10 Let Q_n be the minimum number of moves needed to transfer a tower of n disks from A to B if all moves must be *clockwise*—that is, from A to B, or from B to the other peg, or from the other peg to A. Also let R_n be the minimum number of moves needed to go from B back to A under this restriction. Prove that

$$Q_n = \begin{cases} 0, & \text{if } n = 0; \\ 2R_{n-1}+1, & \text{if } n > 0; \end{cases} \qquad R_n = \begin{cases} 0, & \text{if } n = 0; \\ Q_n+Q_{n-1}+1, & \text{if } n > 0. \end{cases}$$

(You need not solve these recurrences; we'll see how to do that in Chapter 7.)

11 A Double Tower of Hanoi contains $2n$ disks of n different sizes, two of each size. As usual, we're required to move only one disk at a time, without putting a larger one over a smaller one.

a How many moves does it take to transfer a double tower from one peg to another, if disks of equal size are indistinguishable from each other?

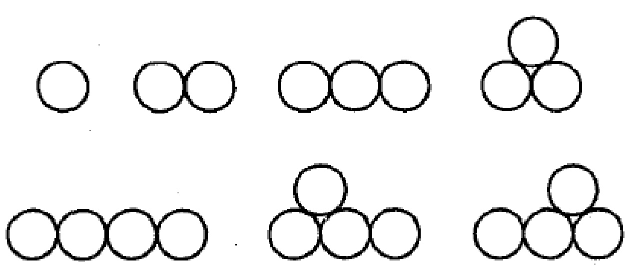
12 Let's generalize exercise 11a even further, by assuming that there are n different sizes of disks and exactly m_k disks of size k. Determine $A(m_1, \ldots, m_n)$, the minimum number of moves needed to transfer a tower when equal-size disks are considered to be indistinguishable.

15 Josephus had a friend who was saved by getting into the next-to-last position. What is $I(n)$, the number of the penultimate survivor when every second person is executed?

21 Suppose there are $2n$ people in a circle; the first n are "good guys" and the last n are "bad guys." Show that there is always an integer m (depending on n) such that, if we go around the circle executing every mth person, all the bad guys are first to go. (For example, when $n = 3$ we can take $m = 5$; when $n = 4$ we can take $m = 30$.)

۱۰.۱.۹ فرض کنید a_n تعداد راههای بالا رفتن از n پله باشد، به‌طوری‌که در هر گام مجاز به بالا رفتن از یک یا دو پله هستیم. رابطه‌ای بازگشتی برای a_n بیابید.

***۱۲.۱.۹** n سکهٔ یکسان در اختیار داریم. این سکه‌ها را در یک ردیف یا دو ردیف می‌چینیم که در ردیف دوم هر سکه درست با دو سکه زیرش در تماس باشد. فرض کنید a_n تعداد راههای چیدنِ این n سکه به‌صورت موردنظر باشد. مثلاً $a_1 = 1$، $a_2 = 1$، $a_3 = 2$ و $a_4 = 3$.



رابطه‌ای بازگشتی برای a_n بیابید.