

بسم الله الرحمن الرحيم

نظريه علوم کامپیوتر

نظريه علوم کامپیوتر - بهار ۱۴۰۰-۱۴۰۱ - جلسه هشتم: خودتولیدکنندگی

Theory of computation - 002 - S08 - self-reproducibility

Self-reproduction Paradox

Self-reproduction Paradox

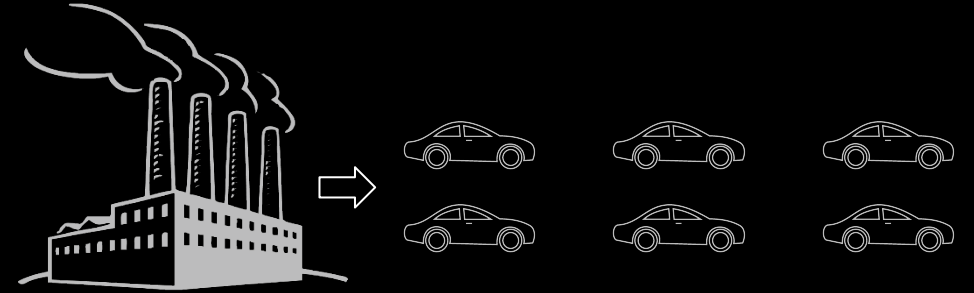
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Suppose a Factory makes Cars

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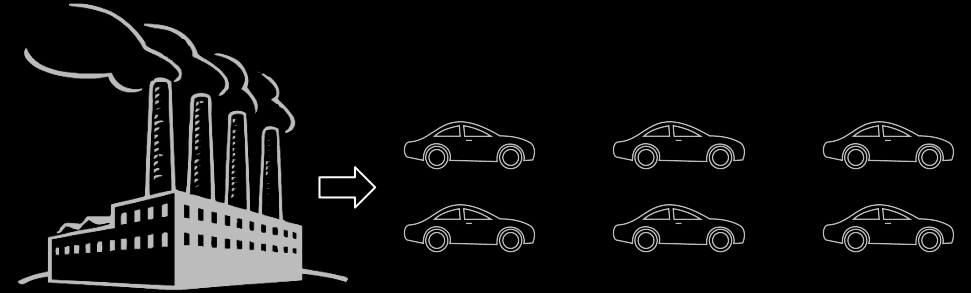


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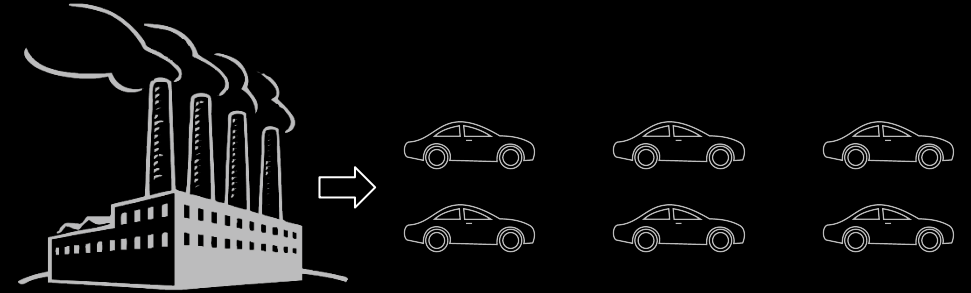
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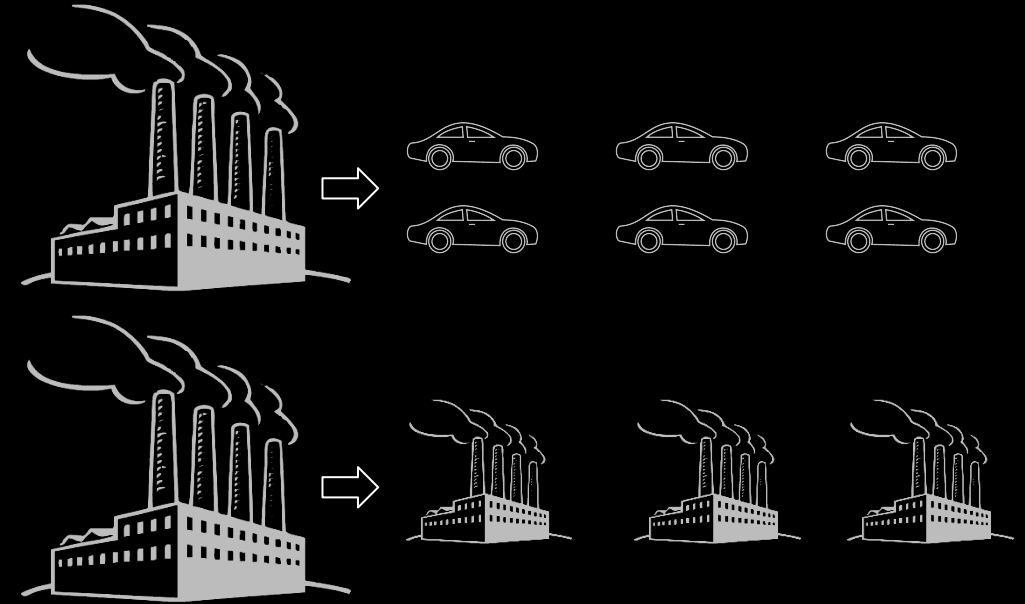
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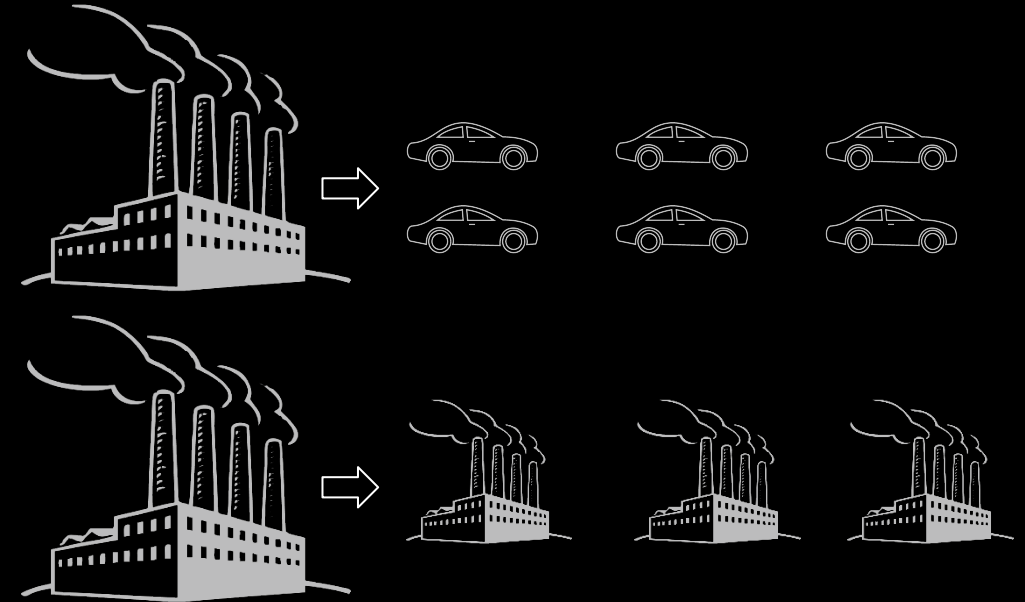
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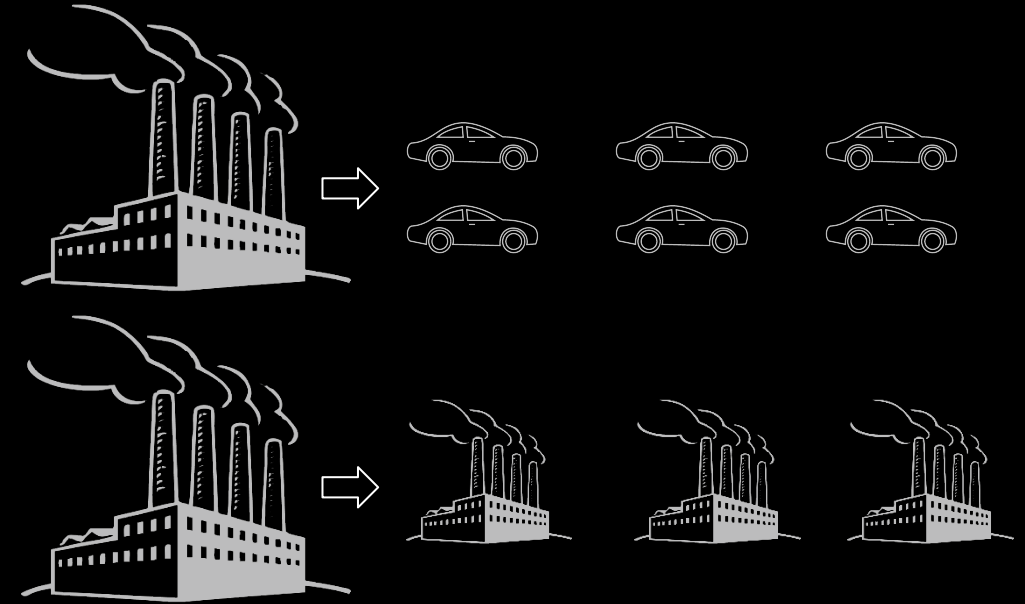
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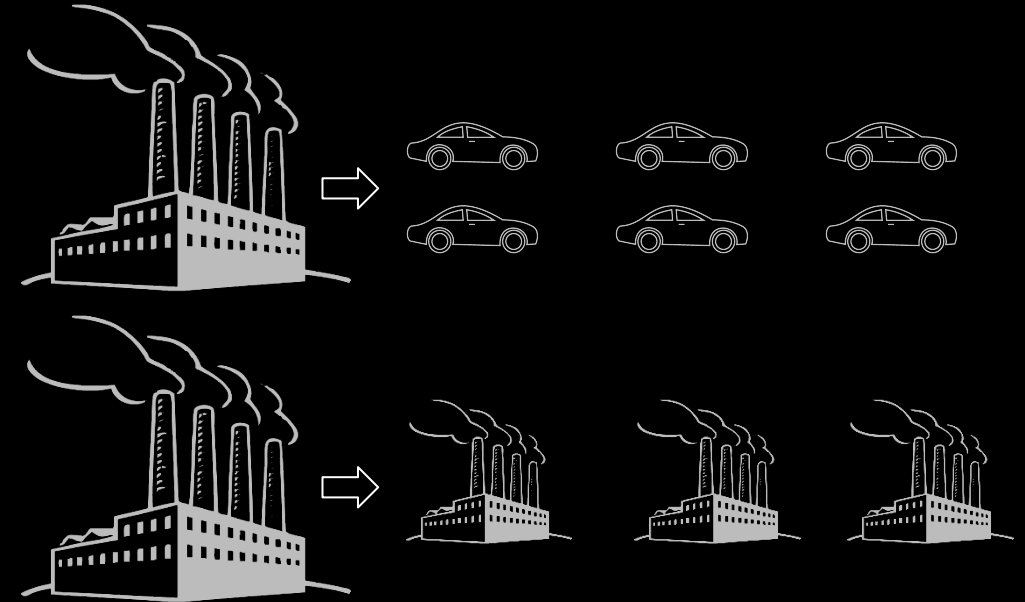
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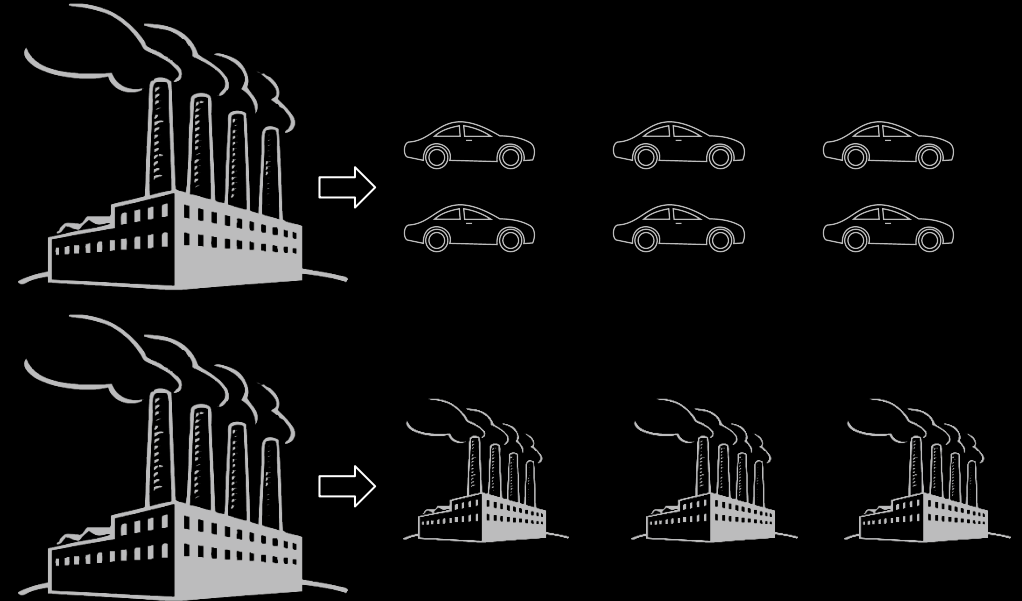
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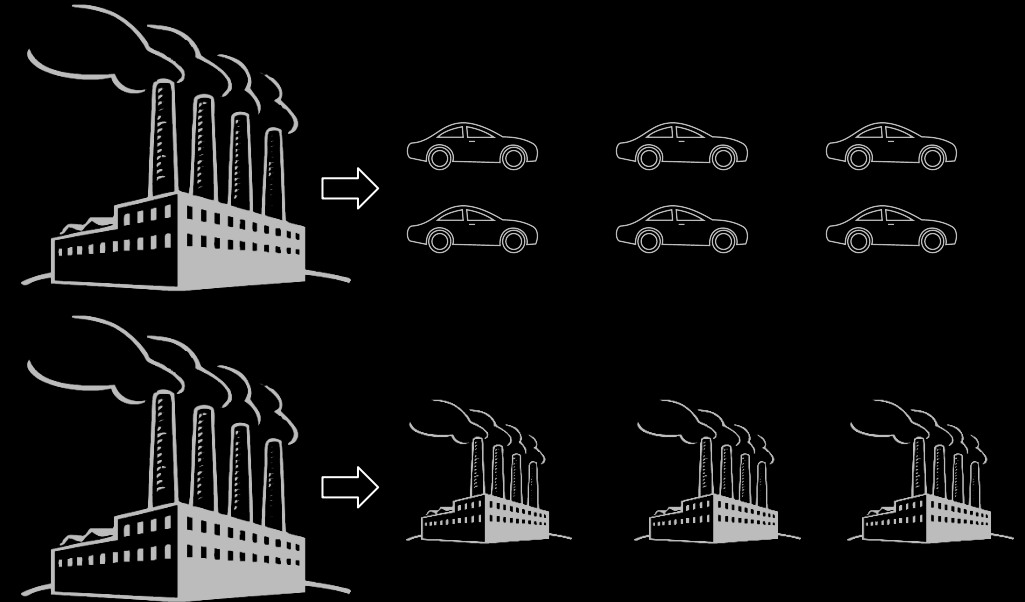
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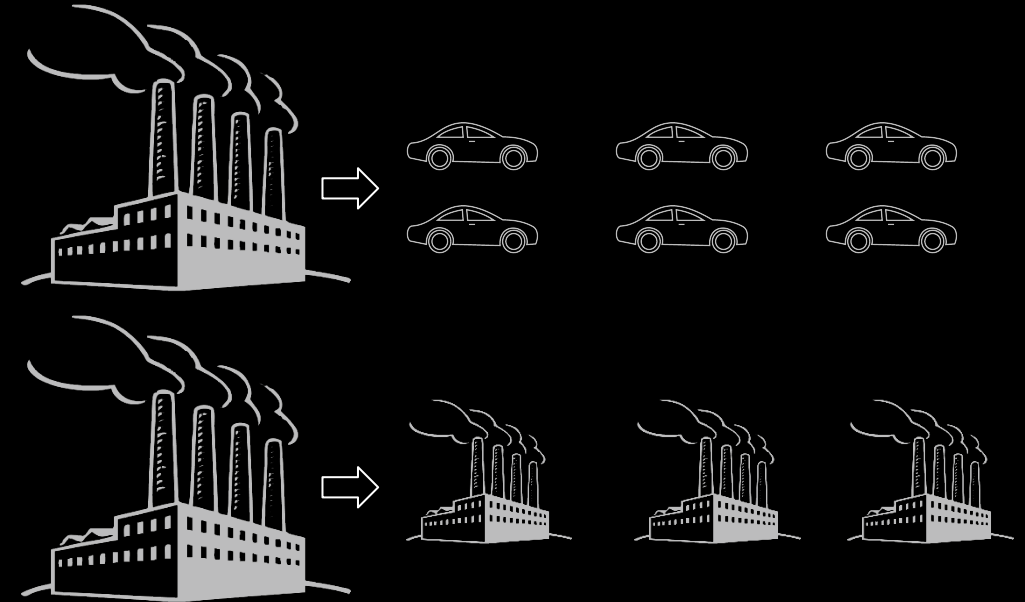
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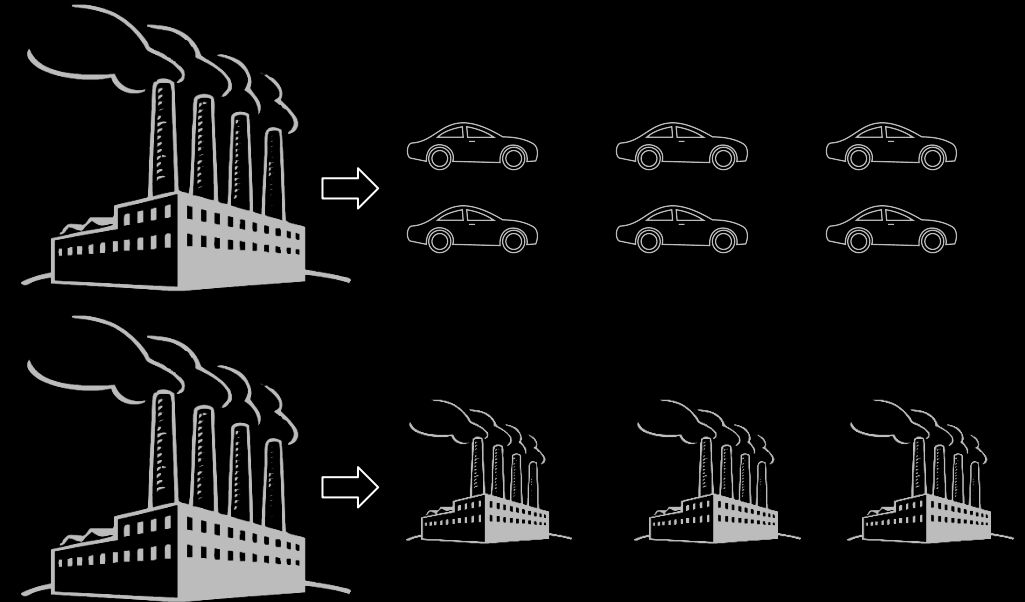
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A Self-Reproducing TM

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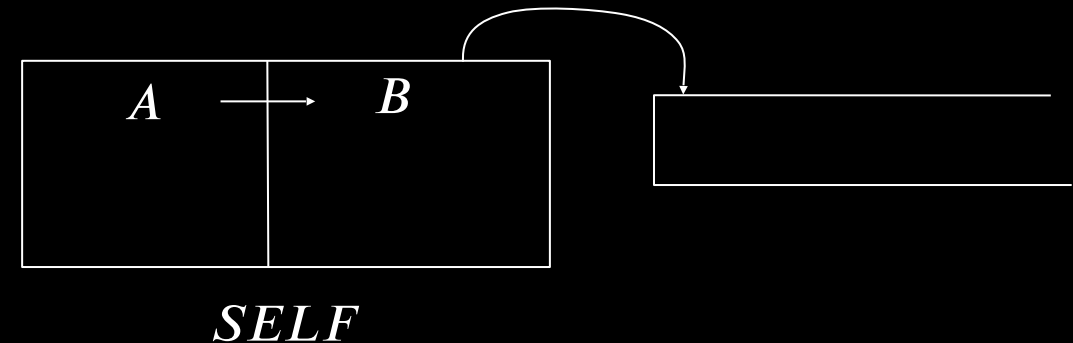
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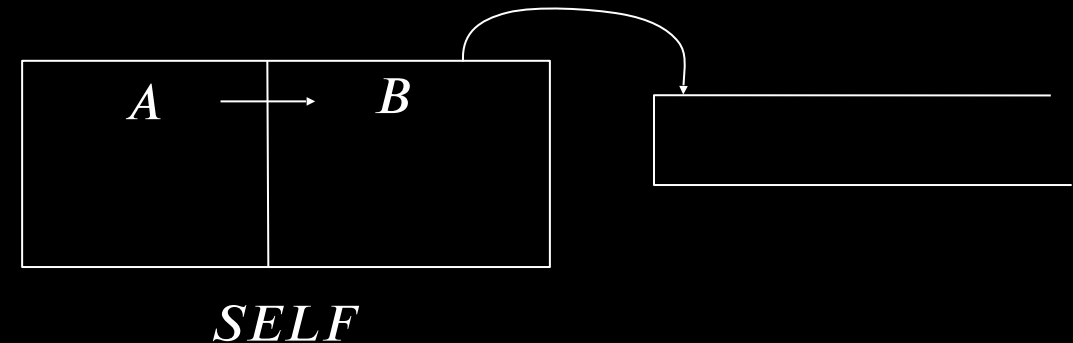
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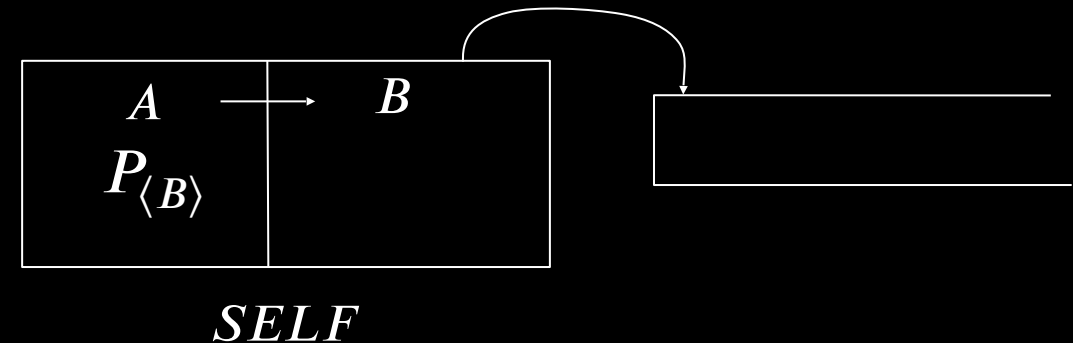
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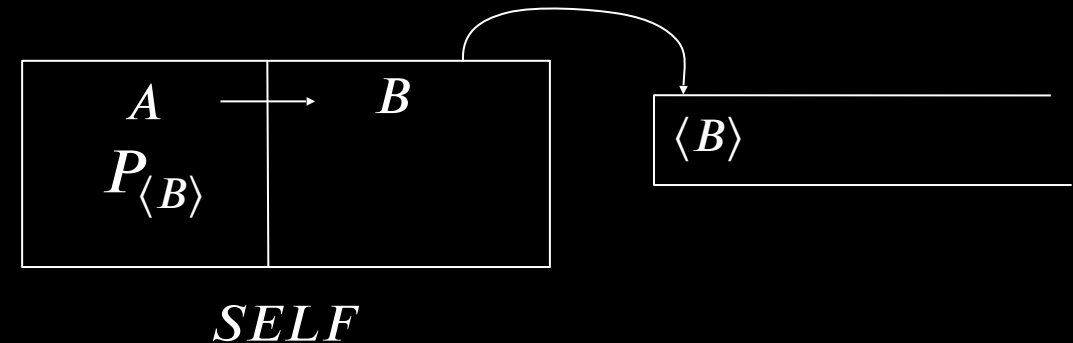
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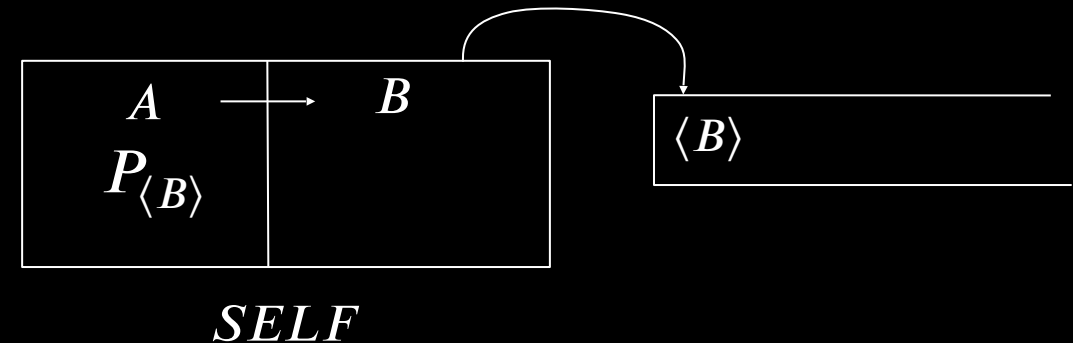
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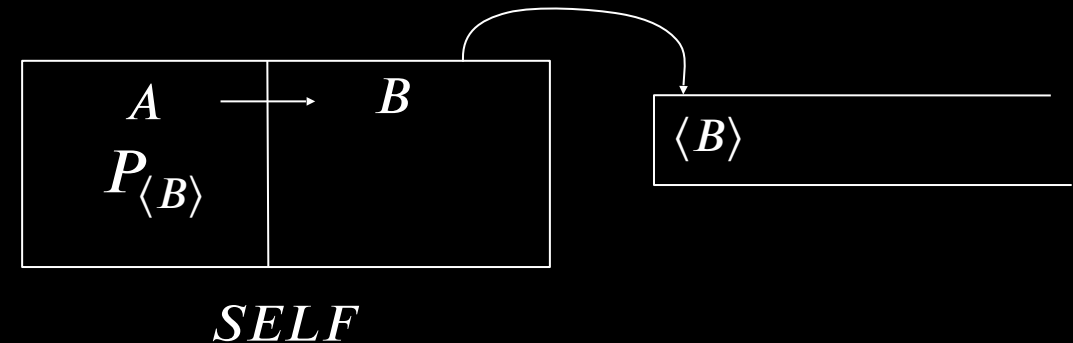
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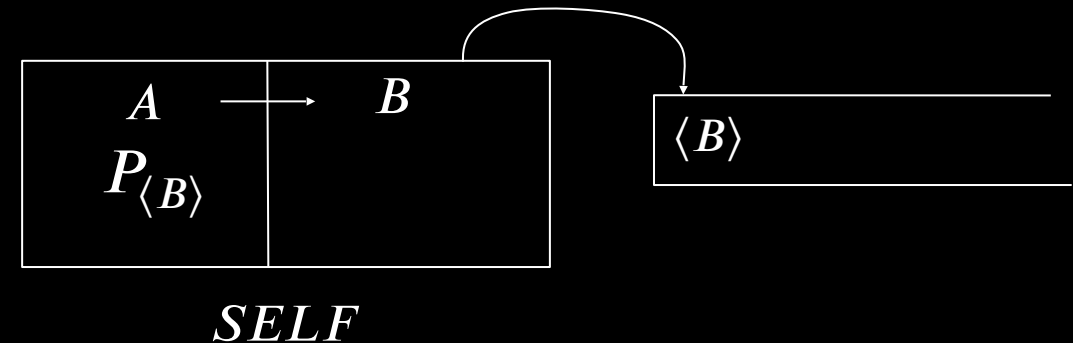
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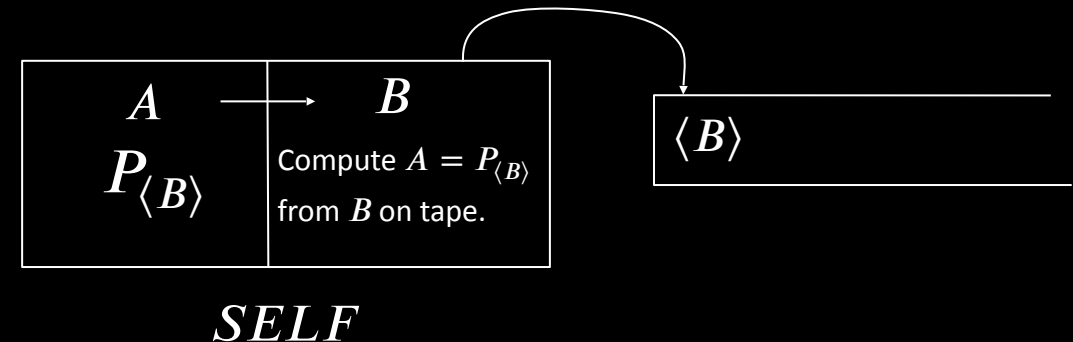
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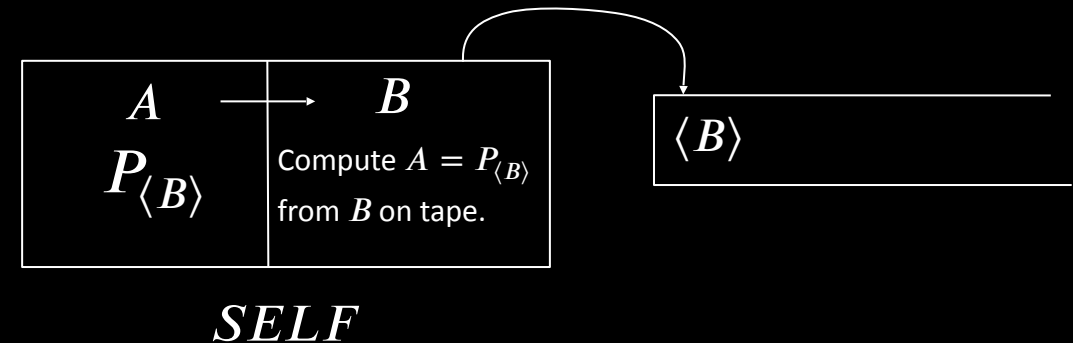
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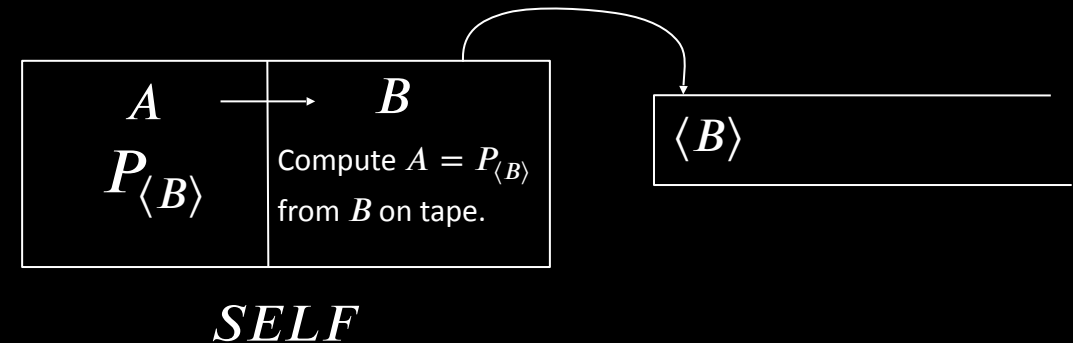
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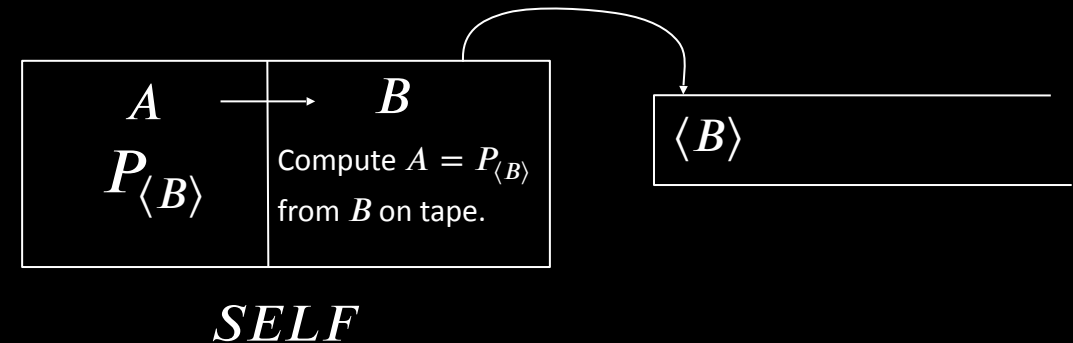
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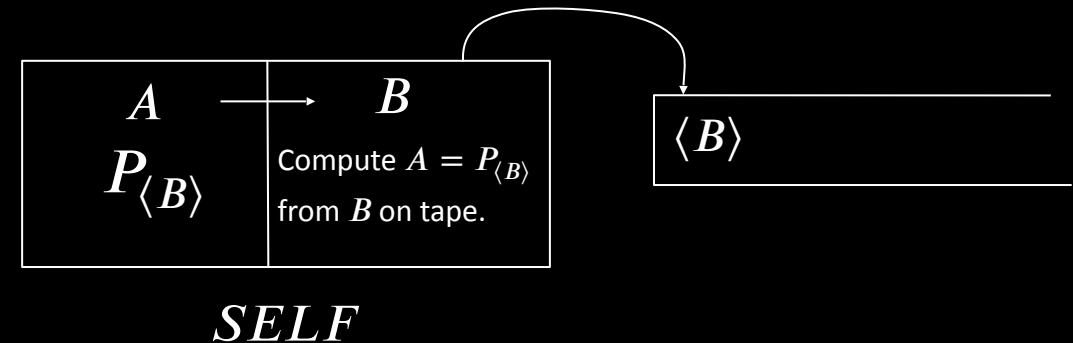
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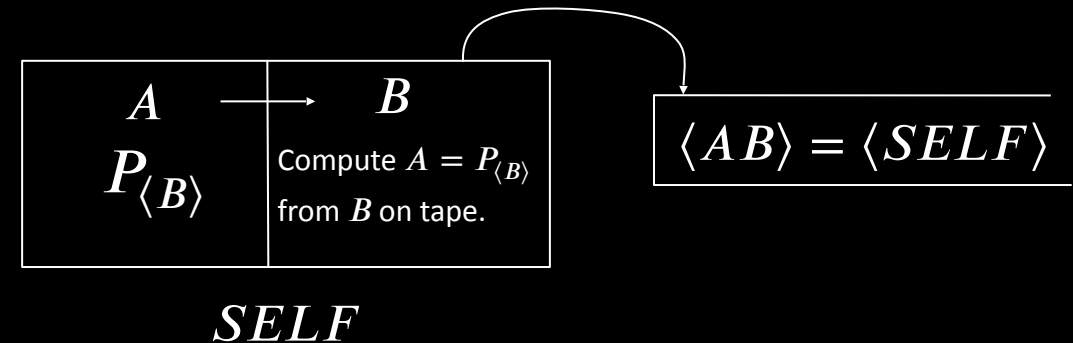
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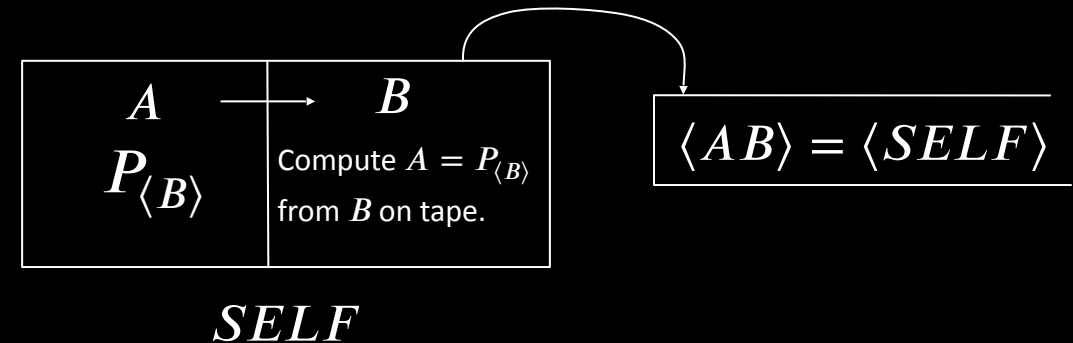
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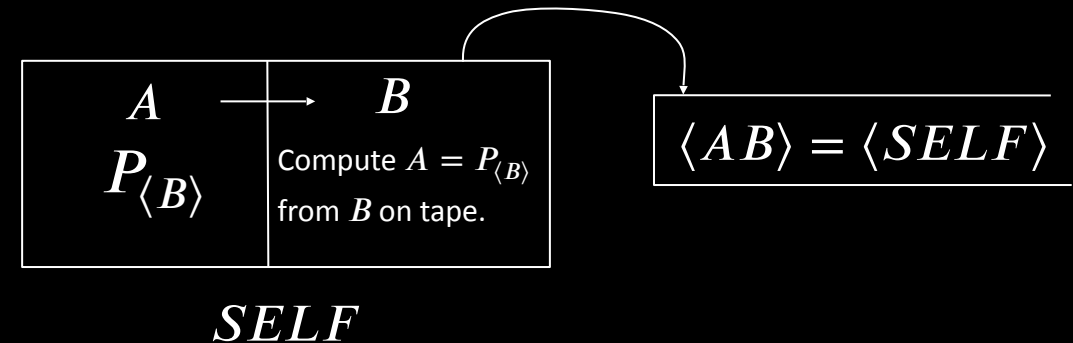
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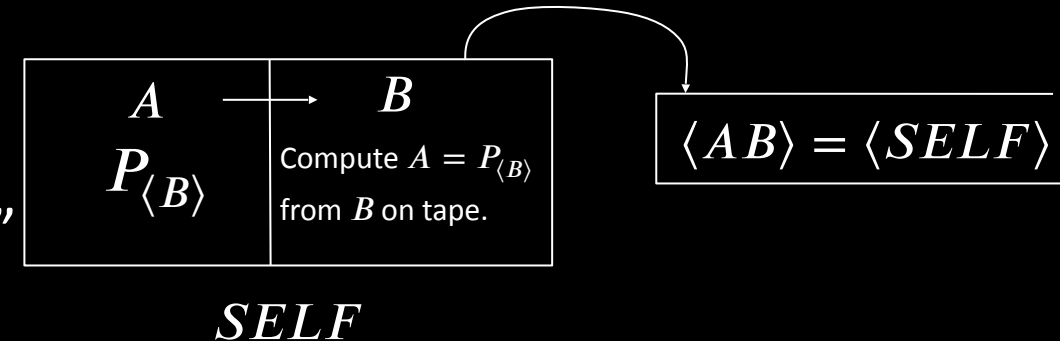
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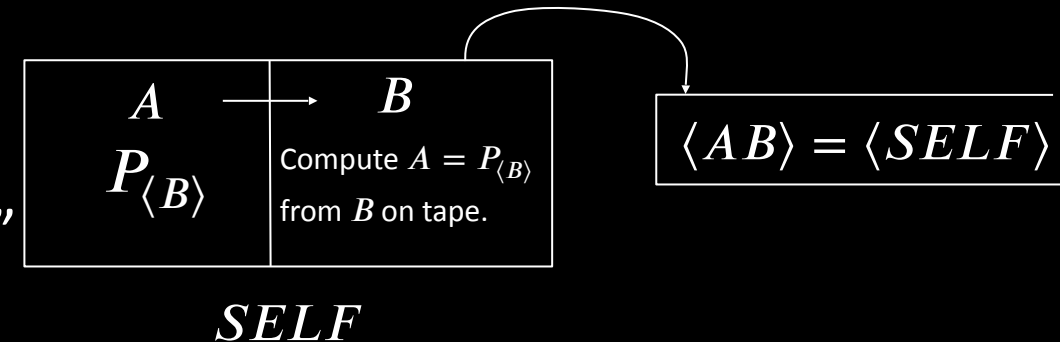
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Check-in 11.1

Implementations of the Recursion Theorem have two parts, a Template and an Action. In the TM and English implementations, which is the Action part?

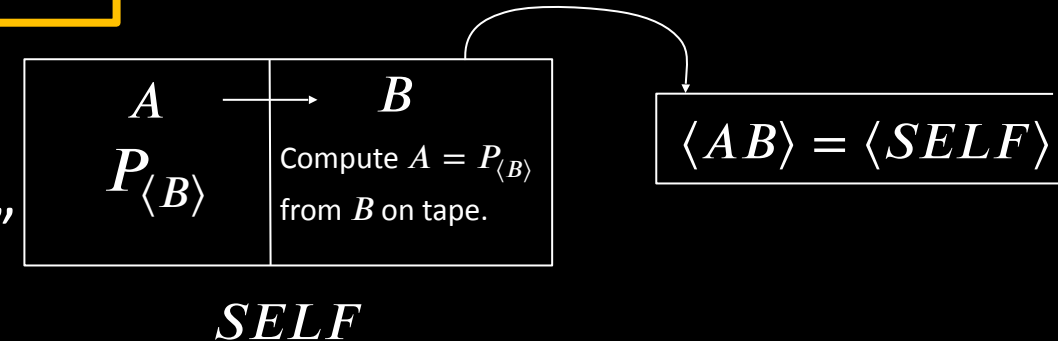
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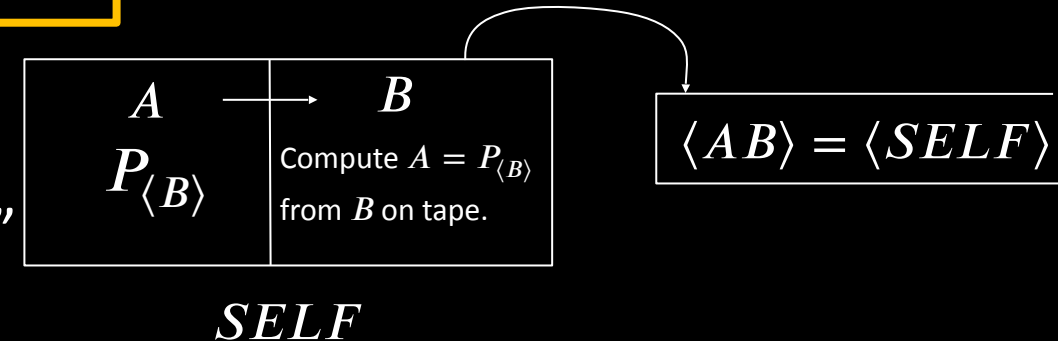
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Note on Pset Problem 6: Don't need to worry about quoting.



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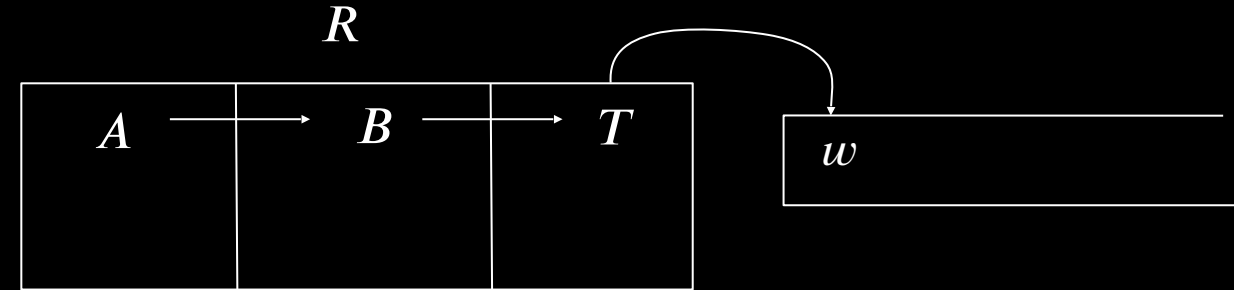
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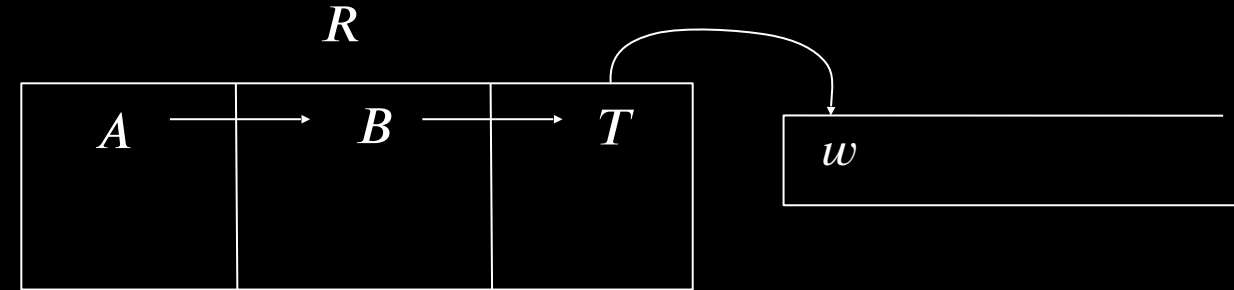
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A compiler which implements “compute your own description” for a TM.

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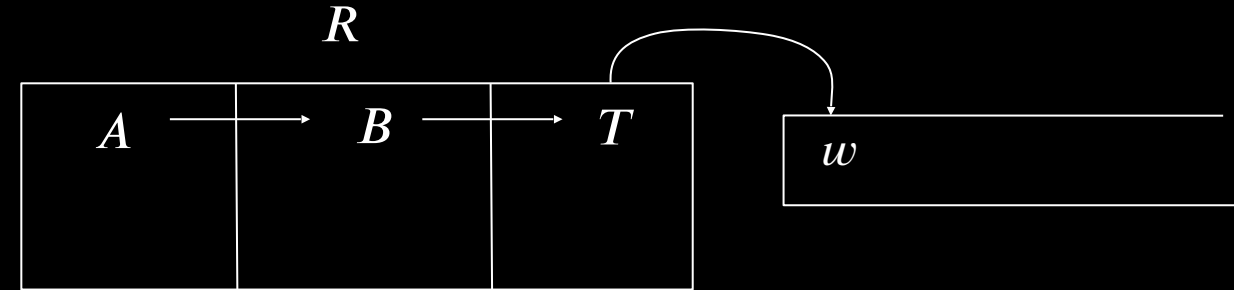
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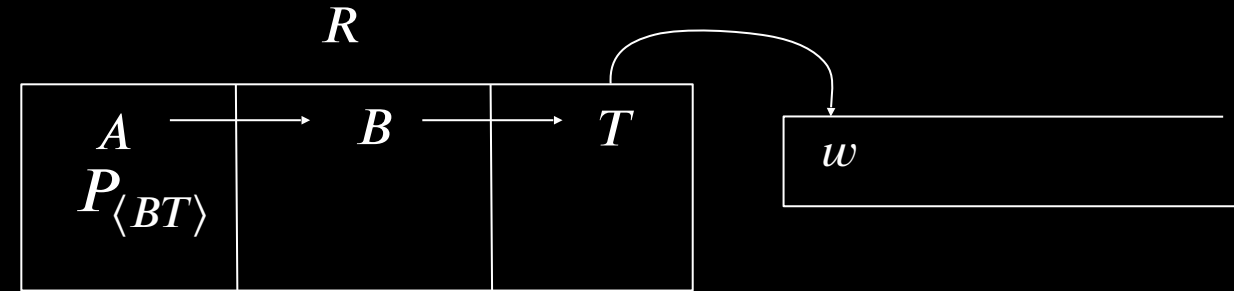
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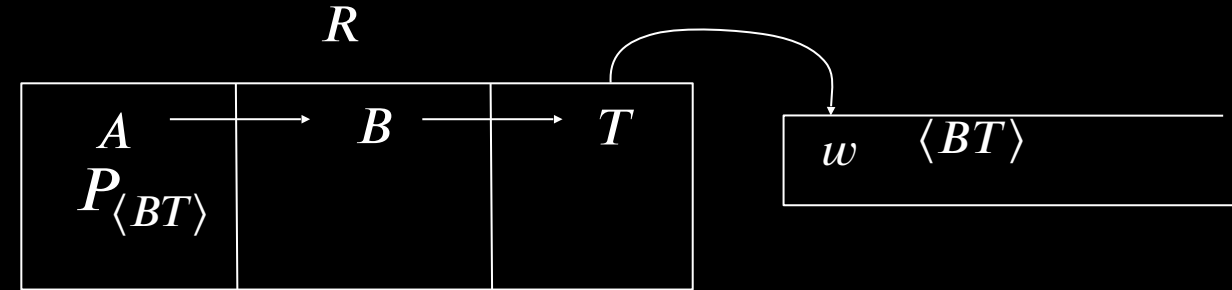
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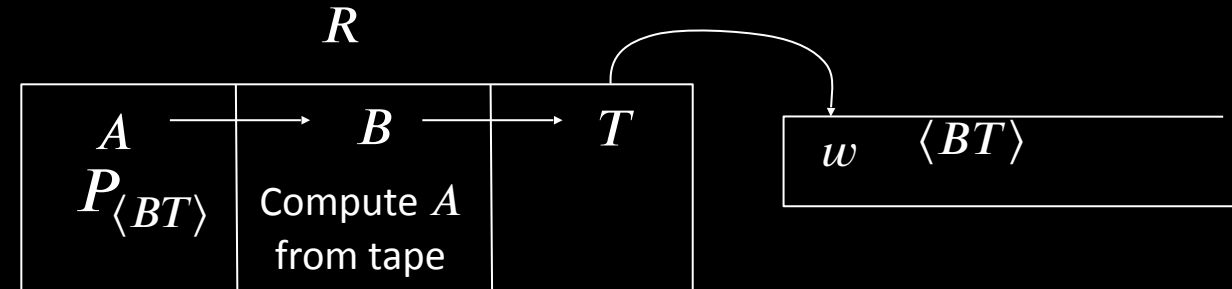
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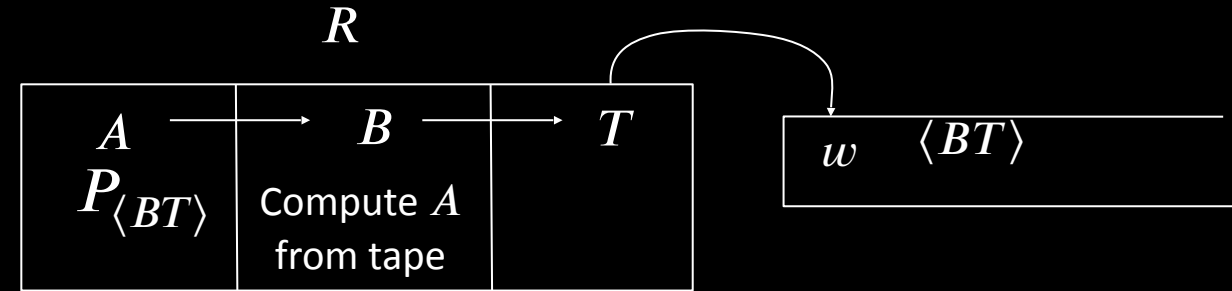
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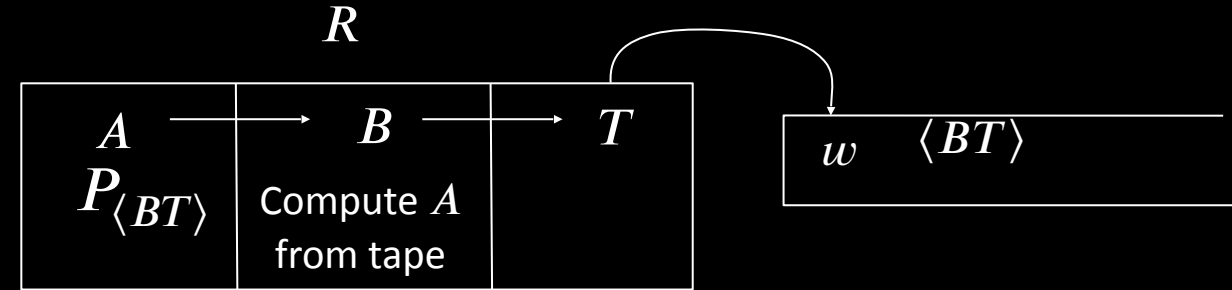
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- $B =$
1. Compute $q(\text{tape contents after } w)$ to get A .
 2. Combine with BT to get $ABT = R$.



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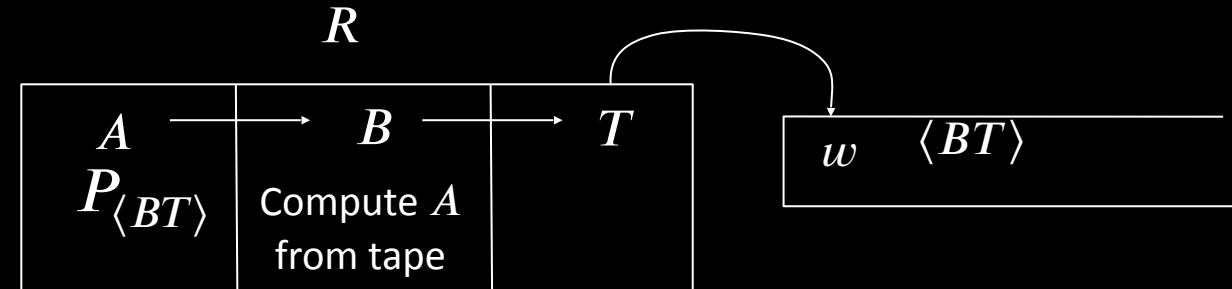
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1. Compute $q(\text{tape contents after } w)$ to get A .
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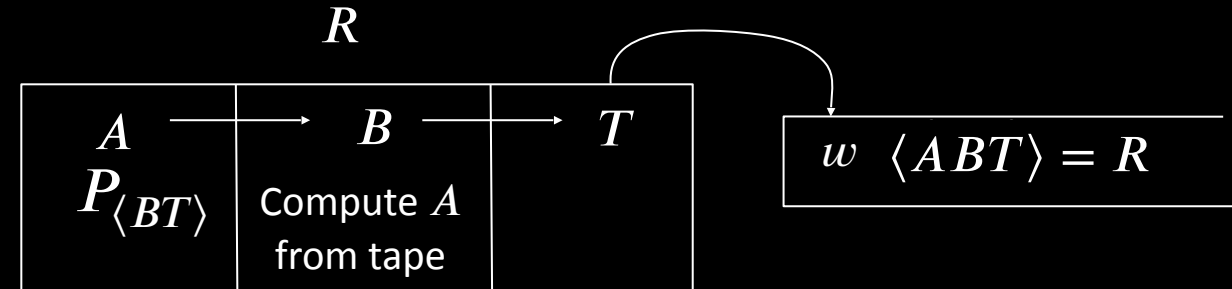
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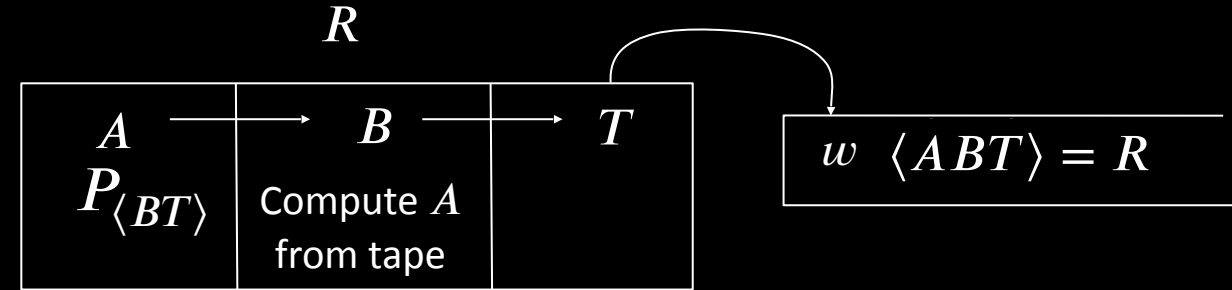
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Moral: You can use “compute your own description”
in describing TMs.

The Recursion Theorem

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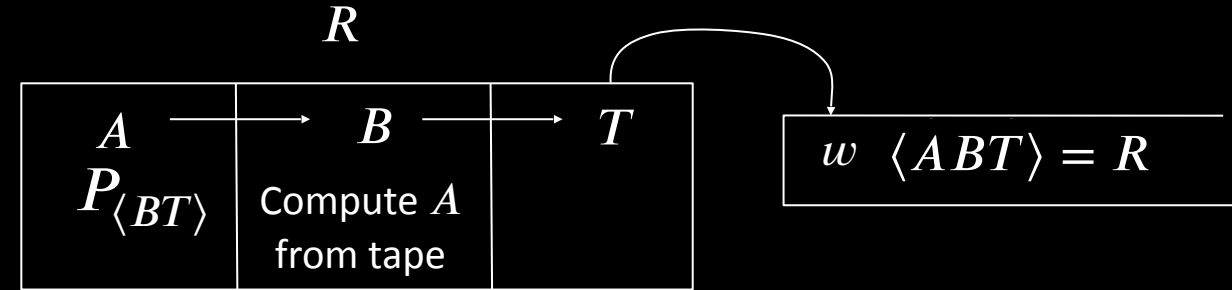
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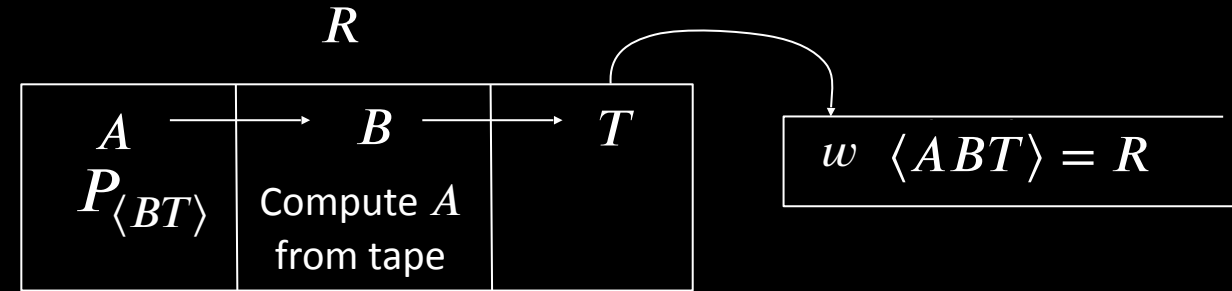
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Check-in 11.2

Can we use the Recursion Theorem to design a TM T where $L(T) = \{\langle T \rangle\}$?

- (a) Yes.
- (b) No.

Moral: You can use “compute your own description” in describing TMs.

Check-in 11.2

Ex 1: *ATM* is undecidable - new proof

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Ex 1: ATM is undecidable - new proof

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Theorem: ATM is not decidable

Proof by contradiction: Assume some TM H decides ATM .

Consider the following TM R :

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1. Get own description $\langle R \rangle$.
2. Use H on input $\langle R, w \rangle$ to determine whether R accepts w .

Ex 1: ATM is undecidable - new proof

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Proof by contradiction: Assume some TM H decides ATM .

Consider the following TM R :

$R =$ “On input w

1. Get own description $\langle R \rangle$.
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3. Do the opposite of what H says.”

Ex 1: ATM is undecidable - new proof

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