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A realized volatility measure reflecting prospect theory investors' sentiments is empirically seen to improve volatility forecast accuracy

1. Introduction

Not every investor behaves according to expected utility theory. The most famous approach accounting for behavior contradicting the expected utility theory is known as prospect theory developed by Kahneman and Tversky (1979) and generalized as cumulative prospect theory (CPT) in Tversky and Kahneman (1992). According to CPT, a given lottery is analyzed by a representation and a valuation step. In the first step the outcome probabilities from the gamble are weighted. The second step evaluates the corresponding gains and losses. A typical result is that individuals overweight the probabilities of extremely rare events, which helps to explain why individuals seek for insurance and participate in lotteries. Although the results of Tversky and Kahneman are robust in several laboratory experiments, indicating that people tend to behave according to prospect theory, the existence of prospect theory investors on financial markets is hard to verify, as probabilities of possible future outcomes are unknown on financial markets and therefore the probability weighting step is unobservable (see Barberis *et al*. 2016). To overcome this problem, Barberis *et al*. (2016) assume investors to mentally represent an asset by its past return distribution, which might be an intuitive proxy for future return outcome probabilities. By imposing this assumption, Barberis *et al*. (2016) confirm that a part of the investors evaluates stocks according to CPT. The CPT investors are assumed to hold a portfolio different to the mean-variance optimal tangency portfolio and stocks which are attractive for CPT investors are shown to be systematically overpriced.

In this paper I adopt this idea and investigate the influence of CPT investors on stock market volatility. If investors have distinct perceptions of asset risk their reactions to stock market movements may differ. For the classic expected utility investor negative and positive returns have the same effect on the variance, as long as both are equally far away from the mean. In contrast, a CPT investor will be more strongly scared by large negative returns than a professional investor. Thus, the CPT investor is more likely to rebalance her portfolio which generates additional asset volatility. In order to exploit this behavior for volatility modeling, I construct a new behavioral variance measure based on CPT that aims at reflecting the volatility perception of those investors. The measure is constructed similar to a monthly realized variance (see Merton 1980 and French *et al*. 1987). I show that this measure has significant explanatory power for asset volatility and improves forecasts. As model framework I adopt the heterogeneous autoregressive (HAR) model of Corsi (2009) and an extension proposed by Patton and Sheppard (2015), which are the workhorse models for realized variance modeling.

The remainder is organized as follows: Section 2 gives a brief introduction into (cumulative) prospect theory and introduces a new behavioral volatility measure. Section 3 presents the model and different extensions. Section 4 presents

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estimation results and a forecast comparison. Section 5 concludes.

2. Behavioral realized volatility

In this section, I introduce the behavioral realized variance (BRV) measure that is designed to capture the asset volatility evaluation of prospect theory investors. It is constructed similar to the prospect theory stock value of Barberis *et al*. (2016). In the first step, the investor observes objective probabilities p_i for an outcome i, with $\sum p_i = 1$, and uses those to form her subjective probabilities π_i via a probability weighting function, defined below. In contrast to gambles in the laboratory, the true p_i is unknown on financial markets, such that the true return distribution has to be approximated by the investors. In the second step, possible outcomes are valuated via a valuation function.

In the present paper, I consider how CPT investors evaluate asset risk rather than asset returns. Since the CPT investor is assumed to be non-professional it would be implausible to construct the CPT variance measure from high frequency data as high-frequency traders are usually experts that are less likely to behave according to CPT. Instead, the assumed CPT investor observes past daily returns (r_t), which are used to form her mental representation of the volatility, as fluctuations of the return series provide a simple impression whether an asset is volatile and hence risky.

I follow the assumption of Barberis *et al*. (2016), that CPT investors construct the future return distribution from the distribution of past returns, where they assign to each past return the same probability. Formally, the investors observe past Lreturns, where $m$returns are smaller zero and $n$returns are equal or greater zero and $L = m + n$, and set the unknown objective probabilities p_i equal to $1/L$. This assumption seems fairly restrictive, but as discussed in Barberis *et al*. (2016), the past Lprice changes are easily accessible and for non-professional CPT investors without deeper knowledge of financial econometrics, the empirical distribution of past returns is one of the most intuitive predictors for the future return distribution. The p_i's are weighted, such that the probability of extreme negative and extreme positive returns is overweighted. The original weighting formula proposed by Tversky and Kahneman (1992) obtains as

$$\pi_i = \begin{cases} \begin{array}{l} w^+(p_i + \ldots + p_n) \\ -w^+(p_{i+1} + \ldots + p_n) \end{array} & \text{for} \quad 0 \le i \le n, \\ \begin{array}{l} w^-(p_{-m} + \ldots + p_i) \\ -w^-(p_{-m} + \ldots + p_{i-1}) \end{array} & \text{for} \quad -m \le i < 0, \end{cases} \tag{1}$$

with

$$w^+(P) = \frac{P^{\gamma^+}}{(P^{\gamma^+} + (1-P)^{\gamma^+})^{1/\gamma^+}},$$
$$w^-(P) = \frac{P^{\gamma^-}}{(P^{\gamma^-} + (1-P)^{\gamma^-})^{1/\gamma^-}}, \tag{2}$$

where p_{-m} denotes the objective probability of the most negative outcome, p_{-m+1} of the second most negative outcome and so on, p_n corresponds to the probability of the most positive outcome and Pdenotes the cumulated outcome probabilities. Here, investors are assumed to use the empirical distribution in order to obtain the outcome probabilities and they set $p_i = 1/L,\ \forall i = -m, \ldots, n$, such that equation (1) simplifies to

$$\pi_i = \begin{cases} w^+\left(\frac{n-i+1}{L}\right) - w^+\left(\frac{n-i}{L}\right) & \text{for} \quad 0 \le i \le n, \\ w^-\left(\frac{i+m+1}{L}\right) - w^-\left(\frac{i+m}{L}\right) & \text{for} \quad -m \le i < 0, \end{cases} \tag{3}$$

where $w^+(\cdot)$ and $w^-(\cdot)$ are defined as in equation (2). Figure 1 shows three possible shapes of the difference of probability weighting functions for varying parameter values and $p_i = 0.01$ (corresponding to $L = 100$ days), for $i = -m, \ldots, n$. For $\gamma \in (0,1)$, the weighting functions generate overweighting of the tails of the distribution and an underweighting of the center. The effect is the stronger the smaller the value of γ. and $w^+(\cdot)$ and $w^-(\cdot)$ are defined as in equation (2).

For the evaluation step, I apply an adjusted version of the valuation function of Tversky and Kahneman (1992). In the classical CPT, the player evaluates a gamble where she can win or lose an amount x, such that the valuation function takes positive and negative values, accounting for the fact that the investor is more sensitive to losses than to gains of the same size. The original value function $v^*(x)$ proposed by Tversky and Kahneman (1992) obtains as

$$v^*(x) = \begin{cases} x^{\beta^*} & \text{for} \quad x \ge 0 \\ -\lambda\,(-x)^{\beta^*} & \text{for} \quad x < 0, \end{cases} \tag{4}$$

and $\beta^* \in (0,1)$. In the context of prospect theory investors, $x$does not denote the gain or loss of a lottery but the return of an asset. In order to apply the valuation function for determining asset volatility, the function needs to be adjusted since asset risk is a positive quantity by definition. To account for the asymmetric reaction of investors' risk perception to positive and negative returns (see, e.g. Black 1976, Christie 1982, Nelson 1991 or Yu 2005), the proposed valuation function $v(x)$ is adjusted such that $v(-a) > v(a)$, where $a > 0$and $v(x) > 0\ \forall x \in \mathbb{R}$:

$$v(x) = \begin{cases} \left(x^2\right)^{\beta} & \text{for} \quad x \ge 0 \\ \lambda\left(x^2\right)^{\beta} & \text{for} \quad x < 0, \end{cases} \tag{5}$$

where x is now squared in order to proxy asset volatility. The slope of the original valuation function is retained by setting $\beta = \beta^*/2$. Figure 2 shows the shape of the original and the adjusted valuation functions for $\beta = 0.3$, $\beta^* = 0.6$ and $\lambda = 2.25$. Both functions generate asymmetric responses to positive and negative inputs, which reflects that investors are more scared of losses than they are pleased by a gain of the same magnitude. For returns greater or equal zero, it holds that $v(x) = v^*(x)$, while returns smaller zero imply $v(x) = |v^*(x)|$. Thus, the new value function generates a higher value for negative return observations than for positive ones with the same absolute value which is in line with the aforementioned literature on leverage effects in asset volatility.

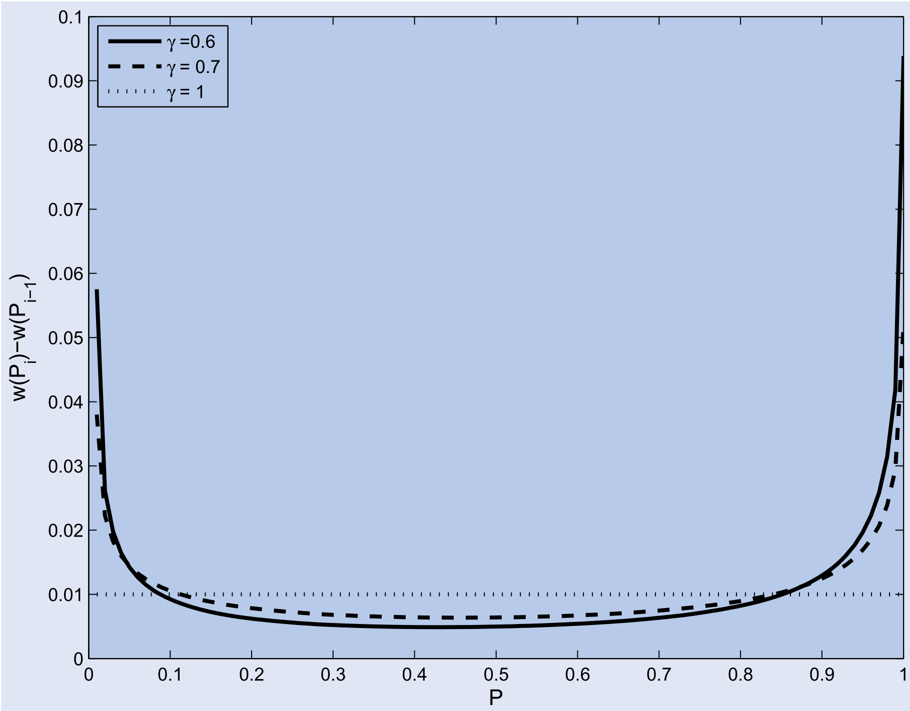

Figure 1. CPT weights.

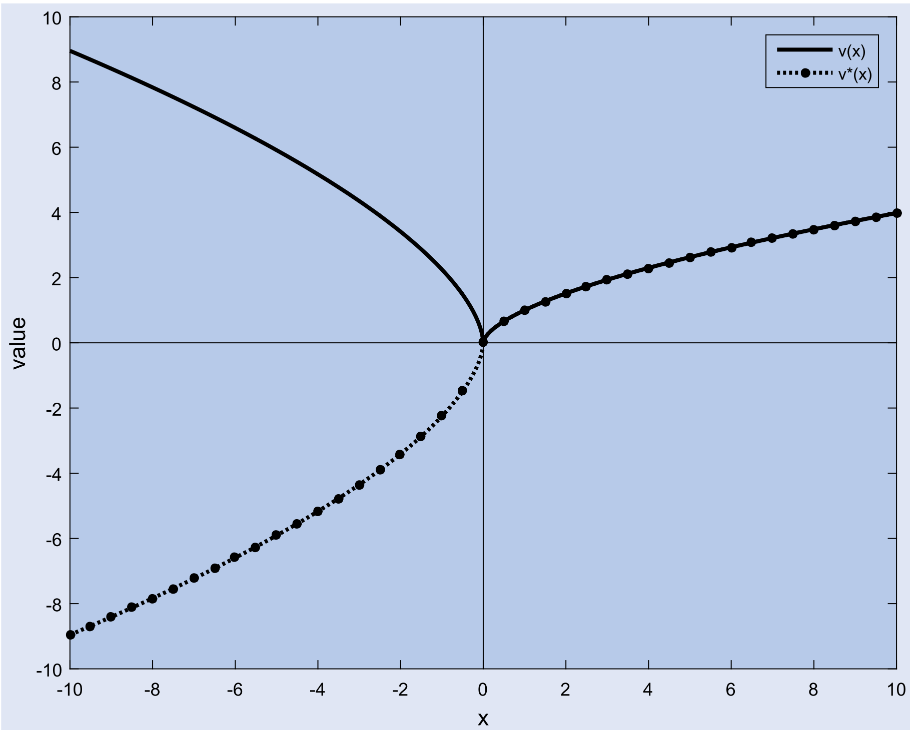

Figure 2. Original value function $v^*(x)$ with $\beta^* = 0.6$ and $\lambda = 2.25$ (dotted line with circle markers) and for adjusted value function $v(x)$ with $\beta = 0.3$ and $\lambda = 2.25$ (solid line).

Having completed the brief review of CPT, I now turn to the description of the BRV measure. The BRV measure of day t is constructed from daily returns over the period $t - L$ to $t - 1$. In this period, the investor observes L returns, denoted as $r_{t-L:t-1}$. Those returns are ordered by size, which results in m returns smaller zero and n returns greater or equal zero, with $L = n + m$ and

$$
\begin{aligned}
r_{-m,t-L:t-1} &\le r_{-m+1,t-L:t-1} \le \cdots \\
&\cdots \le r_{-1,t-L:t-1} < r_{0,t-L:t-1} \le r_{1,t-L:t-1} \le \cdots \\
&\cdots \le r_{n-1,t-L:t-1} \le r_{n,t-L:t-1},
\end{aligned}
$$

where $r_{0,t-L:t-1} = 0$. The notation $r_{i,t-L:t-1}$ denotes the i'th highest return, $i = -m, \ldots, n$, during the period $t-L$ to $t-1$. Finally, the new BRV measure in period t is the weighted arithmetical mean of the valuation function from equation (5) evaluated at the squared returns observed during the last L periods, where the weights are given by the subjective probabilities π_i generated by the probability weighting function defined in equation (3):

$$\mathrm{BRV}_{t-1:t-L} = \sum_{i=-m}^{-1} \pi_i v(r_{i,t-L:t-1}) + \sum_{i=1}^{n} \pi_i v(r_{i,t-L:t-1}). \quad (6)$$

The number of considered lags L may depend on the application. As stated by Barberis *et al*. (2016), CPT investors do not evaluate their portfolios on high frequency, such that for daily variance modeling $L \geq 22$ (corresponds to one month) seems sensible. I follow Barberis *et al*. (2016), and set γ^+, γ^- and λ equal to the values obtained in Tversky and Kahneman (1992), whereas the value of β is set equal to the half of the corresponding value from the original study:

$$\beta = 0.44, \quad \gamma^+ = 0.61, \quad \gamma^- = 0.69, \quad \lambda = 2.25. \quad (7)$$

These values have been found to be robust in different studies, see, e.g. Gonzalez and Wu (1999) or Abdellaoui (2000). This parametrization implies overweighting of the tails of the return distribution and leads to a higher increase of volatility in succession of negative returns than in succession of positive returns.†

3. Model

A famous approach for modeling asset volatility is the realized variance suggested by Andersen and Bollerslev (1998). Let price of an asset P_t follow the simple stochastic process

$$\mathrm{d}\log(P_t) = \mu_t \mathrm{d}t + \sigma_t \mathrm{d}W_t, \quad (8)$$

where μ_t denotes the drift, σ_t the instantaneous volatility and W_t a standard Brownian motion. The true Integrated Variance at day t obtains as

$$\mathrm{IV}_t = \int_{t-1}^{t} \sigma_s^2 \mathrm{d}s. \quad (9)$$

Now let $r_{t,i} = \log(P_{t-1+i\Delta}) - \log(P_{t-1+(i-1)\Delta})$ denote the ith intraday return over a period of length Δ and assume that $M = 1/\Delta$ intraday returns are available. Without jumps in the price process, the integrated volatility can be consistently estimated by the realized variance measure

$$\mathrm{RV}_t = \sum_{i=1}^{M} r_{t,i}^2. \quad (10)$$

However, the IV is measured with error and Barndorff-Nielsen and Shephard (2002) showed that

$$\mathrm{RV}_t = \mathrm{IV}_t + \eta_t, \quad \eta_t \sim N(0, 2\Delta \mathrm{IQ}_t), \quad (11)$$

where $\mathrm{IQ}_t = \int_{t-1}^{t} \sigma_s^4 \mathrm{d}s$ is the Integrated Quarticity.

In order to test the predictive power of the BRV for asset volatility I include it in the HAR model by Corsi (2009). The HAR was designed to account for the strong persistence of realized measures in a simple way and it is motivated by the behavior of investors with different investment horizon H. Following Corsi (2009), the latent partial integrated volatilities (IV_t) on the daily (*day*), weekly (*week*, 5 days) and monthly (*mon*, 22 days) horizon can be modeled as follows:

$$\begin{aligned}
\mathrm{IV}_t^{\mathrm{mon}} &= c^{\mathrm{mon}} + \phi^{\mathrm{mon}} \mathrm{RV}_{t-1:t-22} + w_t^{\mathrm{mon}} \\
\mathrm{IV}_t^{\mathrm{week}} &= c^{\mathrm{week}} + \phi^{\mathrm{week}} \mathrm{RV}_{t-1:t-5} + E(\mathrm{IV}_t^{\mathrm{mon}} \mid \mathcal{F}_{t-1}) + w_t^{\mathrm{week}} \\
\mathrm{IV}_t^{\mathrm{day}} &= c^{\mathrm{day}} + \phi^{\mathrm{day}} \mathrm{RV}_{t-1} + E(\mathrm{IV}_t^{\mathrm{week}} \mid \mathcal{F}_{t-1}) + w_t^{\mathrm{day}},
\end{aligned}$$

where $w_t^{\mathrm{mon}}, w_t^{\mathrm{week}}, w_t^{\mathrm{day}}$ denote iid zero mean noise variables, $\mathcal{F}_t = \{\mathrm{RV}_t, \ldots, \mathrm{RV}_1\}$ and

$$\mathrm{RV}_{t-1:t-H} = \frac{1}{H} \sum_{i=1}^{H} \mathrm{RV}_{t-i}.$$

This dependence structure is motivated by the results of Müller *et al*. (1997), who find that long run volatility affects the short run component but not vice versa. Simple recursive substitution and using $IV_t^{\mathrm{day}} = \mathrm{RV}_t^{\mathrm{day}} + \zeta_t^{\mathrm{day}}$, where ζ_t^{day} captures measurement errors, leads to the basic HAR model. It obtains as

$$\mathrm{RV}_t = \alpha_0 + \alpha_1 \mathrm{RV}_{t-1} + \alpha_2 \mathrm{RV}_{t-1:t-5} + \alpha_3 \mathrm{RV}_{t-1:t-22} + u_t, \quad (12)$$

where $u_t \sim (0, \sigma_u^2)$. However, the derivation of the HAR requires that investors have the same expectations when evaluating $E(IV_t^{\mathrm{mon}} \mid \mathcal{F}_{t-1})$ and $E(IV_t^{\mathrm{week}} \mid \mathcal{F}_{t-1})$. As shown by Barberis *et al*. (2016), a fraction of investors evaluate stock returns according to CPT, such that they do not hold the mean-variance-efficient tangency portfolio. Here, guided by this result, I further assume that the fraction of CPT investors does not only affect asset returns but also asset return variance and that not all investors have the same rational expectations about future variance outcomes, but rather that $E(IV_t^{\mathrm{mon}} \mid \mathcal{F}_{t-1})$ is determined by rational and CPT investors, such that

$$E(\mathrm{IV}_t^{\mathrm{mon}} \mid \mathcal{F}_{t-1}) = c^{\mathrm{mon}} + \phi^{\mathrm{mon}} \mathrm{RV}_{t-1:t-22} + \psi \mathrm{BRV}_{t-1:t-22}. \quad (13)$$

This assumption translates into the derivation of the HAR model and leads to the following extended HAR:

$$\begin{aligned}
\mathrm{RV}_t = \alpha_0 &+ \alpha_1 \mathrm{RV}_{t-1} + \alpha_2 \mathrm{RV}_{t-1:t-5} + \alpha_3 \mathrm{RV}_{t-1:t-22} \\
&+ \alpha_4 \mathrm{BRV}_{t-1:t-22} + u_t, \quad (14)
\end{aligned}$$

with $u_t \sim (0, \sigma_u^2)$. The behavioral variance is only included for the long-term volatility (*mon*, 22 days), since the CPT investors are assumed to be non-professional investors who don't rebalance their portfolios at higher frequency.

† As pointed out by an anonymous referee, given the probabilities π it is possible to estimate β and λ using QMLE. The estimation results reveal that the choice of β is sensible (mean value not significantly different from 0.44 at the 5%-level), while λ is estimated to be higher for several assets. As higher values of λ imply a steeper shape of the left-hand side of the valuation function, these results do not contradict the intuition of the behavioral volatility measure and 2.25 can be understood as lower bound for possible choices of λ. Insample and forecasting results for different values of λ are available upon request.

Consequently, the BRV measure is not included for the weekly or daily frequency. In order to control for asymmetric reactions of variance on positive and negative returns the HAR includes a leverage term $\mathrm{RV}_{t-1}I_{\{r_{t-1}<0\}}$, where $I_{\{r_{t-1}<0\}} = 1$ for $r_{t-1} < 0$ and zero else. As squared daily returns are known to be informative for realized variances (see, e.g. Engle and Gallo 2006 or Hansen and Lunde 2012), the considered HAR further contains the simple arithmetical mean of the past $L = 22$ daily returns $\sum_{\ell=1}^{22} r_{t-\ell}^2/22$. Summarizing, I consider the following extended HAR model:

$$\begin{aligned}\mathrm{RV}_t &= \alpha_0 + \alpha_1 \mathrm{RV}_{t-1} + \alpha_2 \mathrm{RV}_{t-1:t-5} + \alpha_3 \mathrm{RV}_{t-1:t-22} \\ &\quad + \alpha_4 \mathrm{BRV}_{t-1:t-22} + \alpha_5 \mathrm{RV}_{t-1} I_{\{r_{t-1}<0\}} \\ &\quad + \alpha_6 \sum_{\ell=1}^{22} \frac{r_{t-\ell}^2}{22} + u_t, \end{aligned} \tag{15}$$

where $u_t \sim (0, \sigma_u^2)$. The complete model in equation (15) will be denoted as HARB, whereas the same model without the behavioral variance will simply be denoted as HAR.

Patton and Sheppard (2015) have proposed an adjustment of the HAR that allows for asymmetric effects as it disentangles the effect of variance generated by negative and positive returns:

$$\begin{aligned}\mathrm{RV}_t &= \alpha_0 + \alpha_1^- \mathrm{RV}_{t-1}^- + \alpha_1^+ \mathrm{RV}_{t-1}^+ + \alpha_2 \mathrm{RV}_{t-1:t-5} \\ &\quad + \alpha_3 \mathrm{RV}_{t-1:t-22} + \alpha_4 \mathrm{BRV}_{t-1:t-22} + \alpha_5 \mathrm{RV}_{t-1} I_{\{r_{t-1}<0\}} \\ &\quad + \alpha_6 \sum_{\ell=1}^{22} \frac{r_{t-\ell}^2}{22} + u_t, \end{aligned} \tag{16}$$

where

$$\mathrm{RV}_t^- = \sum_{j=1}^{M} r_{t,j}^2 I_{\{r_{t,j}<0\}} \quad \text{and} \quad \mathrm{RV}_t^+ = \sum_{j=1}^{M} r_{t,j}^2 I_{\{r_{t,j}\geq 0\}},$$

and $u_t \sim (0, \sigma_u^2)$. Note that the asymmetric HAR has also been extended by $\mathrm{RV}_{t-1}I_{\{r_{t-1}<0\}}$ and $\sum_{\ell=1}^{22} r_{t-\ell}^2/22$ in order to control for leverage effects and allow to distinguish the effect of squared returns and the behavioral variance. The model in equation (16) is denoted as PSHARB, the version without the behavioral variance measure as *PSHAR*.

For robustness checks of the results, I also consider the effect of behavioral investors on asset variance in a squareroot version of (15) and (16), where RV_t is replaced by $\sqrt{\mathrm{RV}_t}$, $\sum_{\ell=1}^{22} r_{t-\ell}^2/22$ by $\sqrt{\sum_{\ell=1}^{22} r_{t-\ell}^2/22}$ and BRV_t by $\sqrt{\mathrm{BRV}_t}$. Furthermore, I analyze the models in a logarithms, such that RV_t is replaced by $\log(\mathrm{RV}_t)$, $\sum_{\ell=1}^{22} r_{t-\ell}^2/22$ by $\log(\sum_{\ell=1}^{22} r_{t-\ell}^2/L)$ and BRV_t by $\log(\mathrm{BRV}_t)$. This results in six different model specifications, where $\sqrt{\mathrm{(PS)HAR(B)}}$ denotes the (PS)HAR(B) in roots and $\log(\mathrm{(PS)HAR}(B))$ the (PS)HAR(B) in logs.

4. Empirical application

4.1. Data

The data contains daily returns and realized variances of 60 assets. It is obtained from QuantQuote. All considered stocks were at some point of the sample period part of the S&P 500. Following Barberis *et al.* (2016), the analysis focuses on stocks with small market capitalization since those are expected to be stronger affected by individual, typically non-professional CPT investors, than medium or high cap stocks. The sample starts at 2 January 2001 and ends on 30 June 2015, which results in 3645 observations per asset. I use open-to-close returns and realized variances computed from 5-minute returns as recommended by Liu *et al.* (2015). The intraday data is cleaned following the instructions in Barndorff-Nielsen *et al.* (2009). Table 1 provides the means of the realized variance and the behavioral variance and their correlation for each series, and figure 3 shows the realized variance of the Schlumberger stock. The mean of the behavioral variance measure is smaller in most cases, as the value function $v(x)$ weights down extreme returns. There exist medium positive correlation between realized variance and the behavioral variance.

4.2. Estimation results

Since the HAR has been extensively studied in the literature I omit a deep analysis of the parameters α_0 to α_3 and rather focus on the estimates of α_4. Table 2 provides detailed estimation results for Apple (AAPL) and Gap (GPS), who are exemplarily chosen as stocks with high and low market capitalization. The significance levels are obtained using robust standard errors. For AAPL, the BRV measure is only significant in the HARB at the 10% level not significant (10% level) for the remaining specifications. In contrast, for GPS α_4 is found to be significant at the 1% level for all considered specifications. Interestingly, α_1 is not significant at the typical levels in the HARB which may be explained by peaks in the series of realized variance that are weaker pronounced for the model in roots or logs. The leverage parameter α_5 is positive and highly significant indicating that negative returns in $t-1$increase the volatility in t. However, the effect becomes less important when including the behavioral volatility measure which does not only include the different reaction to positive and negative returns but also a valuation of the return size. The negative sign of α_6 may be explained by the interaction with the behavioral measure which already generates additional persistence. Including the mean over past squared returns without the behavioral measure results in mostly positive signs of α_6. However, in most cases α_6 is not significant at the 5% level. Figure 4 presents a boxplot of the estimates of α_4 for all six models, while table 3 presents the mean and median over the 60 assets and how many times out of 60 α_4 is found to be significant at the 10%, 5% and 1% level.† For the models in levels (HARB and PSHARB) the median estimate of α_4 is 0.69, for the models in squareroots ($\sqrt{\mathrm{HARB}}$ and $\sqrt{\mathrm{PSHARB}}$) around 0.24 and around 0.18 in the log(HARB) and 0.17 in the log(PSHARB), respectively.

† Note that some of the regressors are highly correlated (e.g. $\mathrm{Corr}(\mathrm{RV}_{t-1:t-22}, \sum_{\ell=1}^{2} 2r_{t-\ell}^2) \approx 0.93$), which could lead to invalid inference due (multi)collinearity. Nevertheless, as the considered samples are large and the regressors show distinct variation, the standard errors are reasonably small (see Maddala and Lahiri 2009, for a discussion of multicollinearity and possible solutions.)

Table 1. Descriptive statistics.

Symbol	Comp. name	$\overline{\mathrm{RV}}$	$\overline{\mathrm{BRV}}$	Corr(RV, BRV)
AA	Alcoa Inc.	5.2532	2.3902	0.5552
AAPL	Apple Inc.	4.7212	2.4057	0.5071
ABT	Abbott Laboratories	2.0391	1.6906	0.4803
AES	AES Corp.	11.0239	2.9245	0.6569
AXP	American Express Co	4.2459	2.1661	0.5262
BA	Boeing Co.	2.8204	1.9057	0.5839
BK	BNY Mellon	5.1632	2.2679	0.3553
BMY	Bristol-Myers Squibb	2.6625	1.8648	0.5028
C	Citigroup Inc.	7.3324	2.6356	0.5356
CB	Chubb Ltd.	2.3624	1.8263	0.5612
CI	Cigna Corp.	4.4407	2.2231	0.4951
COST	Costco Co.	2.5599	1.8089	0.6411
CSCO	Cisco Systems	3.9801	2.2861	0.6281
CVX	Chevron Corp.	2.1284	1.7184	0.4925
DGX	Quest Diagnostics	2.9705	1.9684	0.5381
DIS	The Walt Disney Co.	2.9944	1.9954	0.5720
DO	Diamond Offshore Drilling	5.7234	2.7360	0.5391
DOW	Dow Chemical	3.9216	2.3104	0.5249
DUK	Duke Energy	2.5137	1.7668	0.5392
ED	Consolidated Edison	1.4967	1.3661	0.5002
FDX	FedEx Corporation	2.6578	1.8871	0.5556
FISV	Fiserv Inc.	3.1589	1.8792	0.6757
GE	General Electric	3.0614	2.0381	0.5890
GPS	Gap Inc.	4.9350	2.4422	0.4879
HAS	Hasbro Inc.	3.4351	2.1334	0.5170
HD	Home Depot	3.1527	2.1455	0.6013
INTC	Intel Corp.	3.7863	2.2683	0.6741
IP	International Paper Co.	4.2670	2.2923	0.6482
IR	Ingersoll-Rand PLC	3.6994	2.3902	0.5187
JWN	Nordstrom	5.3889	2.8272	0.6417
K	Kellogg Co.	1.4012	1.6995	0.5473
KLAC	KLA-Tencor Corp.	5.3729	2.7754	0.6718
LEN	Lennar Corp.	9.0475	3.4777	0.6748
LLY	Eli Lilly and Co.	1.9714	2.1961	0.4708
MAS	Masco Corp.	5.2701	2.8378	0.6011
MMM	3M Company	1.8089	1.9619	0.5011
NTAP	NetApp	9.0250	3.4372	0.6753
ORCL	Oracle Corp.	4.3322	2.7603	0.7123
PAYX	Paychex Inc.	3.3269	2.4006	0.7017
PEP	PepsiCo Inc.	1.4657	1.7073	0.4471
PKI	PerkinElmer	4.6847	3.0266	0.5728
RL	Polo Ralph Lauren Corp.	4.2063	2.7309	0.5255
ROK	Rockwell Automation Inc.	4.1470	2.7283	0.4923
SCHW	Charles Schwab Corp.	6.3929	2.9323	0.5189
SLB	Schlumberger Ltd.	4.5148	2.7055	0.5799
SO	Southern Co.	1.6280	1.7545	0.4182
SPLS	Staples Inc.	4.4704	2.5023	0.6178
STJ	St. Jude Medical, Inc.	2.5492	2.4545	0.3405
SYK	Stryker Corp.	2.1346	2.2334	0.4061
TMO	Thermo Fisher Scientific	2.6153	2.2575	0.5054
TRV	The Travelers Companies	3.0076	2.2477	0.5020
TWX	Time Warner Inc.	3.9482	2.4590	0.6350
TXN	Texas Instruments	5.2730	2.5589	0.6955
UNH	United Health Group Inc.	3.3691	2.2821	0.5647
VTR	Ventas Inc.	4.1257	2.2641	0.6909
WBA	Walgreens Boots Alliance	2.6370	2.0004	0.4323
WFC	Wells Fargo	4.4317	2.1068	0.6576
XOM	Exxon Mobil Corp.	2.0439	1.6381	0.4235
XRX	Xerox Corp.	6.0518	2.8042	0.4595
YUM	Yum! Brands Inc.	2.7563	2.2680	0.5168

At the 5% level, the BRV measure is significant for 46 (HARB) to 53 assets (log(HARB)). This shows that the behavioral variance measure helps to model the dynamics of asset volatility.

Most of the high parameter values are found for stocks with small market capitalization. To test this formally, I run a regression from the estimated α_4 from each series on the weights of the corresponding asset in the S&P 500. Since

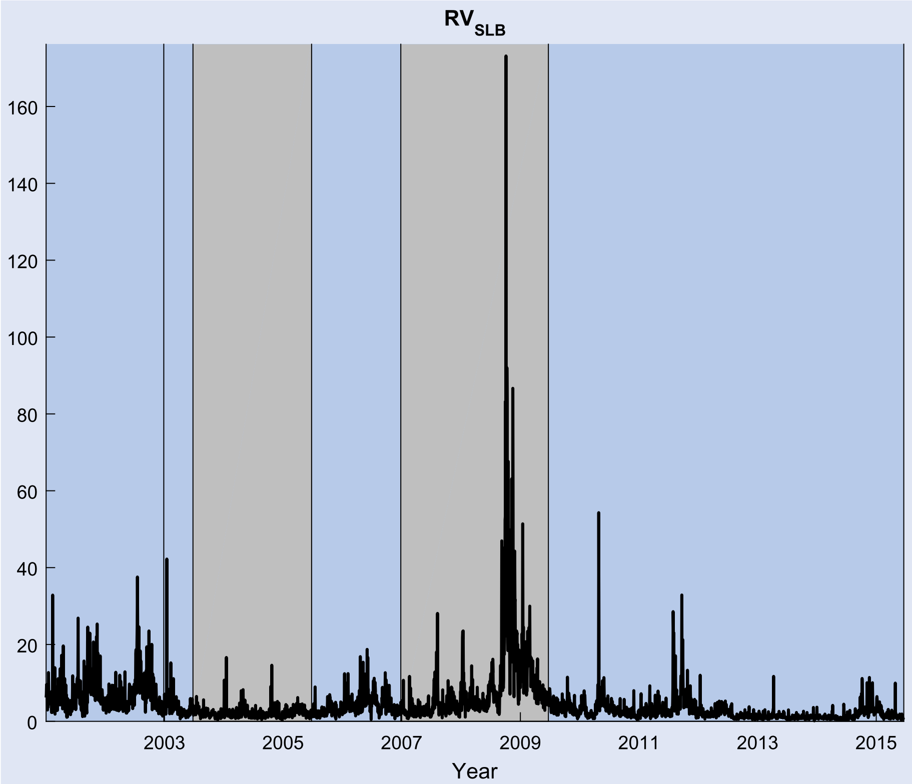

Figure 3. The light-shaded area shows the full sample forecasting period, the dark-shaded area shows the considered subperiods.

Table 2. Parameter estimates.

	AAPL			GPS		
	HARB	$\sqrt{\text{HARB}}$	log(HARB)	HARB	$\sqrt{\text{HARB}}$	log(HARB)
α_0	−0.1565	0.0372	0.0253	−1.2697**	−0.2010**	−0.0108
α_1	0.0743	0.2778***	0.3461***	0.0943	0.2299***	0.2603***
α_2	0.5875***	0.3700***	0.2718***	0.2888***	0.3246***	0.2999***
α_3	0.2902*	0.3101***	0.2812***	0.3488***	0.3786***	0.3690***
α_4	0.3115*	0.0511	−0.0053	1.0224***	0.2926***	0.1138***
α_5	0.1679***	0.0749***	0.0799***	0.2529**	0.0494***	0.0406***
α_6	−0.1832	−0.0589	0.0305	−0.1287*	−0.0939**	−0.0338
	PSHARB	$\sqrt{\text{PSHARB}}$	log(PSHARB)	PSHARB	$\sqrt{\text{PSHARB}}$	log(PSHARB)
α_0	−0.1787	0.0341	0.2744***	−1.3256**	−0.2042**	0.1931***
α_1^-	−0.2364	0.0974	0.1796***	−0.0677	0.1137**	0.1245***
α_1^+	0.2846	0.2806***	0.1556***	0.2282	0.2261***	0.1538***
α_2	0.5846***	0.3696***	0.2806***	0.2840***	0.3105***	0.2838***
α_3	0.2944*	0.3084***	0.2753***	0.3368**	0.3731***	0.3619***
α_4	0.3167	0.0527	−0.0106	1.0590***	0.2943***	0.1149***
α_5	0.2633***	0.0995***	0.0789***	0.3070*	0.0652**	0.0444***
α_6	−0.1790	−0.0560	0.0391	−0.1338**	−0.0895**	−0.0296

Notes: Table 2 shows the estimation results for all models for the stock of Apple (AAPL) and GAP (GPS). $***$ indicates significance at the 1% level, $**$ at the 5% level and $*$ at the 10% level. The results are obtained using robust standard errors.

some of the assets were not part of the S&P 500 during the complete sample period I use the weights from December 2014.† However, choosing a different year and omitting assets that are not part of the S&P 500 during the chosen year does not change the results qualitatively. Table 5 presents the estimation results. For the coefficients of the root and log specification the results are as expected: the higher the market capitalization of a stock the lower is the impact of behavioral investors on asset volatility, which is in line with the results of Barberis *et al*. (2016), who found CPT investors to have stronger influence on the prices of stocks with low market capitalization.

4.2.1. Robustness analysis. The estimation results are obtained for $L = 22$ days, which assumes that CPT investors

† The weight data is obtained from Siblis Research.

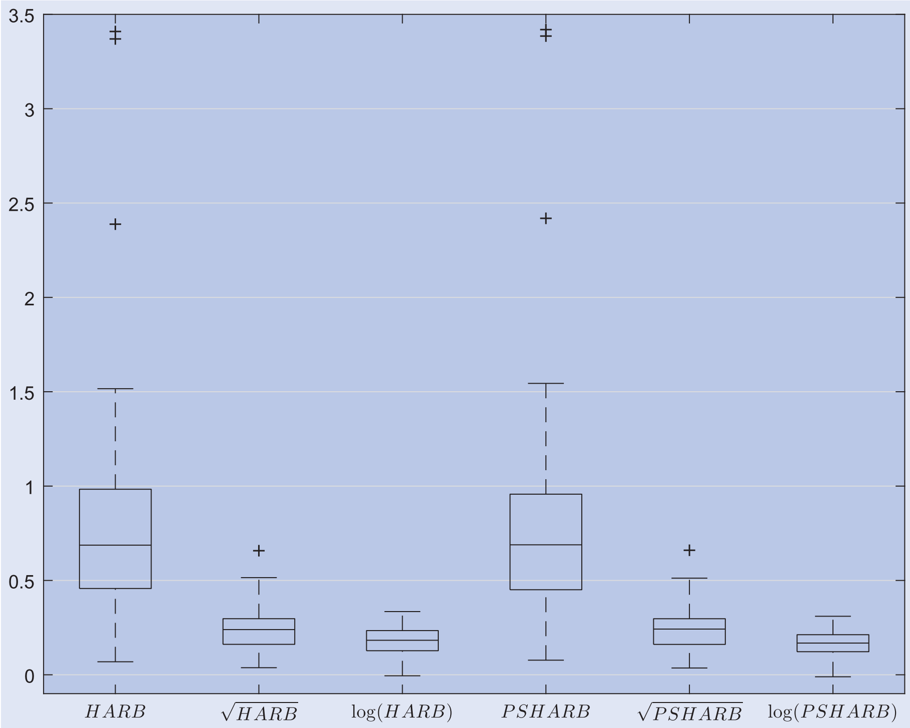

Figure 4. Boxplots for the parameter estimates of α_4.

Table 3. α_4 Estimation results.

	HARB	$\sqrt{\text{HARB}}$	log(HARB)	PSHARB	$\sqrt{\text{PSHARB}}$	log(PSHARB)
Mean($\hat{\alpha}_4$)	0.8175	0.2437	0.1780	0.8116	0.2440	0.1649
Median($\hat{\alpha}_4$)	0.6869	0.2395	0.1828	0.6888	0.2422	0.1680
10%	48	54	54	48	54	53
5%	46	49	53	46	50	50
1%	38	34	45	40	35	42

Notes: Table 3 presents the mean, the median and significance of α_4. 10%, 5% and 1% shows how often out of 60 times α_4 is found to be significant at the respective level. The results are obtained using robust standard errors.

rebalance their portfolio on monthly frequency. To check the robustness of my results, I also consider a higher frequency (two weeks, $L = 10$) and a lower frequency (three months, $L = 66$). For $L = 66$, one has to replace $\text{BRV}_{t-1:t-22}$ and $\sum_{\ell=1}^{22} r_{t-\ell}^2/22$ by $\text{BRV}_{t-1:t-66}$ and $\sum_{\ell=1}^{66} r_{t-\ell}^2/66$, respectively. For $L = 10$, the derivation of the empirical model changes: As the CPT investors now rebalance their portfolio on higher than monthly frequency, equation (13) does not hold anymore. Instead, $L = 10$ implies

$$E(\text{IV}_t^{\text{mon}} \mid \mathcal{F}_{t-1}) = c^{\text{mon}} + \phi^{\text{mon}}\text{RV}_{t-1:t-22}$$

but

$$E(\text{IV}_t^{\text{week}} \mid \mathcal{F}_{t-1}) = c^{\text{week}} + \phi^{\text{week}}\text{RV}_{t-1:t-5} + E(\text{IV}_t^{\text{mon}} \mid \mathcal{F}_{t-1}) + \psi\,\text{BRV}_{t-1:t-5},$$

such that the CPT investors have influence on the weekly volatility component.

The estimation results are given in table 4. For $L = 10$, the results are similar to the results for $L = 22$, while there are fewer significant coefficients for $L = 66$. This indicates that the influence of CPT investors on volatility is higher for the bi-weekly and monthly frequency and that a large fraction of their trading happens on the monthly and lower frequency. I conclude that the results are robust against lowering the considered period length L but the importance of the behavioral measure decreases for increasing values of L.

4.3. Forecasting performance

In this section I investigate the predictive information contained in the behavioral measure. The forecast horizon spans from 2 January 2003 to 30 June 2015 resulting in 3145 out of sample forecasts. Additionally, I consider two subperiods: a crisis period spanning from 3 January 2007 to 30 June 2009 (628 observations) and a tranquil period that starts at 1 July 2003 and ends at 30 June 2005 (505 observations). The different forecasting periods are shown in figure 3, where the dark-shaded areas display the subperiods. The forecasting performance is evaluated using the mean squared error

Table 4. Robustness analysis α_4 estimation results.

	HARB	$\sqrt{\text{HARB}}$	log(HARB)	PSHARB	$\sqrt{\text{PSHARB}}$	log(PSHARB)
				$L = 10$		
Mean($\hat{\alpha}_4$)	1.2272	0.2765	0.1570	1.3161	0.3289	0.1826
Median($\hat{\alpha}_4$)	1.0702	0.2637	0.1522	1.2298	0.3271	0.1803
10%	47	49	58	48	54	59
5%	44	45	56	46	50	57
1%	37	38	50	39	45	52
				$L = 66$		
Mean($\hat{\alpha}_4$)	0.2603	0.0923	0.0813	0.3017	0.1116	0.0840
Median($\hat{\alpha}_4$)	0.2106	0.0900	0.0960	0.2622	0.1086	0.0933
10%	43	33	32	47	39	32
5%	37	24	25	41	32	26
1%	22	11	12	33	20	13

Notes: Table 4 presents the mean, the median and significance of α_4. 10%, 5% and 1% shows how often out of 60 times α_4 is found to be significant at the respective level. The results are obtained using robust standard errors.

Table 5. Estimation results market capitalization.

	HARB	$\sqrt{\text{HARB}}$	log(HARB)	PSHARB	$\sqrt{\text{PSHARB}}$	log(PSHARB)
b_0	0.8708***	0.2726***	0.1984***	0.8668***	0.2726***	0.1862***
b_1	− 0.1268**	− 0.0687***	− 0.0485***	− 0.1315**	− 0.0681***	− 0.0508***

Notes: Table 5 shows the estimation results for a regression of the parameter estimates of α_4 on the weight of the assets in the S&P 500: $\hat{\alpha}_j = b_0 + b_1 cap_j + \epsilon_j$. $***$ indicates significance at the 1% level, $**$ at the 5% level and $*$ at the 10% level. The results are obtained using robust standard errors.

(MSE) criterion: $MSE = \sum_{t=1}^{T}(Y_t - \hat{Y}_t)^2/T$, where Y denote the value of interest and $\hat{Y}$ the corresponding forecasts. For better comparability, I use all specifications to predict realized variances and not its squareroot or its logarithm. In order to have an analytical expression for the conditional expectation $E(\mathrm{RV}_{t+1} \mid \mathcal{F}_t)$ using the model in roots or logs, I assume for the squareroot and log version of equations (15) and (16) Gaussian error terms $u_t \sim N(0, \sigma_u^2)$. Using results for the non-central chi-squared and the log-normal distribution, the resulting forecasts of the model in roots and logs are given by:

$$E(\mathrm{RV}_{t+1} \mid \mathcal{F}_t) = E(\sqrt{\mathrm{RV}_{t+1}} \mid \mathcal{F}_t)^2 + \hat{\sigma}_u^2 \tag{17}$$

and

$$E(\mathrm{RV}_{t+1} \mid \mathcal{F}_t) = \exp\{E(\log \mathrm{RV}_{t+1} \mid \mathcal{F}_t) + \hat{\sigma}_u^2/2\}. \tag{18}$$

The considered models are the HAR(B) and the PSHAR(B) in levels, squareroots and logs. Since $\sum_{\ell=1}^{22} r_{t-\ell}^2/22$ was mostly found to be insignificant for the insample analysis it is omitted in the forecasting experiment. The models are re-estimated daily for an expanding window. Table 6 presents the mean and the median of the 60 MSEs. The MSEs are expressed relative to the HAR model, such that values smaller unity indicate superior forecasting performance. Additionally, the lines *#Min* and #MCS show how many times out of 60 the particular model generates forecasts with the smallest MSE and how often the model falls into the model confidence set (MCS) with $\alpha = 0.1$ (Hansen *et al.* 2011).

Overall, the models including the behavioral variance show the smallest mean MSEs but the effects are rather minor for the full sample and crisis periods. The same holds for the number of appearances in the model confidence set: the corresponding behavioral model specification has always a higher number but the differences ranging from 3 (e.g. PSHAR and PSHARB in the full period or for these models in the crisis period) to 12 (HAR and HAR for the tranquil period). The number of minimal MSE values gives the clearest result: the highest number corresponds for all considered periods to a model that includes the behavioral variance measure. A reason for the weak improvements may be found in the time horizon of the BRV measure: it is constructed from the past 22 daily returns and adapts therefore only slowly to changes in the volatility. During the tranquil period the volatility is rather smooth and consequently the inclusion of the BRV measure improves forecasting performance stronger than for the full sample or the crisis period. Surprisingly, except for the specification in logs, the more flexible PSHAR does not outperform the HAR model and generates in several cases worse forecasts. This does not translate that clearly into the behavioral versions of the models: For the squareroot specifications in the tranquil period the smallest mean and median MSE are obtained by the PSHARB. Summarizing, the inclusion of BRV performs never worse and provides in several cases slightly superior forecasts, which indicates that parts of the volatility dynamics are explained by the behavioral variance measure that builds on the risk sentiment of CPT investors.

Table 6. Forecast results.

		HAR	PSHAR	HARB	PSHARB
		Full sample			
Level	Mean	1.0000	1.0047	**0.9981**	1.0052
	Median	1.0000	1.0029	**0.9987**	1.0021
	#Min	17	8	**26**	9
	#MCS	56	54	**60**	57
Roots	Mean	1.0000	1.0015	0.9992	**0.9980**
	Median	1.0000	1.0007	0.9988	**0.9928**
	#Min	16	11	**19**	14
	#MCS	48	50	58	**59**
Logs	Mean	1.0000	1.0004	**0.9929**	0.9972
	Median	1.0000	1.0006	**0.9935**	0.9974
	#Min	11	8	**23**	18
	#MCS	55	56	59	**60**
		Crisis period			
Level	Mean	1.0000	1.0056	**0.9979**	1.0030
	Median	1.0000	1.0032	**0.9970**	0.9998
	#Min	14	8	**25**	13
	#MCS	57	56	**60**	59
Roots	Mean	1.0000	1.0025	**0.9969**	0.9992
	Median	1.0000	1.0026	**0.9984**	1.0001
	#Min	15	6	**25**	14
	#MCS	57	56	**60**	**60**
Logs	Mean	1.0000	1.0008	**0.9961**	0.9968
	Median	1.0000	1.0011	**0.9967**	0.9978
	#Min	10	8	**26**	16
	#MCS	55	54	**60**	**60**
Level	Mean	1.0000	1.0021	**0.9718**	0.9763
	Median	1.0000	1.0000	**0.9881**	0.9906
	#Min	11	10	**20**	19
	#MCS	45	43	**57**	50
Roots	Mean	1.0000	0.9987	0.9836	**0.9822**
	Median	1.0000	0.9990	0.9932	**0.9897**
	#Min	6	16	18	**20**
	#MCS	41	40	53	**54**
Logs	Mean	1.0000	0.9980	0.9992	**0.9965**
	Median	1.0000	0.9986	0.9997	**0.9956**
	#Min	11	18	11	**20**
	#MCS	50	51	**54**	**54**

Notes: Table 6 shows the forecast results for the full forecast sample (1 January 2003 to 30 June 2015), for the crisis period (3 January 2007 to 30 June 2009) and for the tranquil period (1 July 2003 to 30 June 2005). The losses are computed relative to the loss of the HAR with leverage. The lines Mean and Median show the mean and median over the 60 assets. #Min denotes how many times out of 60 the loss of the corresponding model was the smallest; #MCS shows how many times the model falls into the model confidence set with $\alpha = 0.1$, blocklength 15 and 1000 bootstrap replications. The best value is displayed in bold.

5. Conclusion

This paper considers the effect of investors who evaluate asset volatility according to cumulative prospect theory on daily volatility proxied by realized volatility. A simple specification of a behavioral volatility measure in the spirit of Barberis *et al*. (2016) is found to be a significant explanatory variable for realized variance, controlling for different investment horizons, long-run effects, asymmetry and leverage. This result is robust against transformations of the variance. The positive effect of CPT investor volatility on daily volatility is shown to be stronger for assets with low market capitalization. Extending the workhorse volatility models by the behavioral measure leads to forecast improvements in several cases during different periods. As the basic version of the behavioral variance already helps to predict asset volatility, future research may consider more advanced methods for constructing this measure, e.g. include a weighting how recent a return observation is. This may lead to further improvements of the forecasting performance.

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