Mobile Data Offloading with Uniform Pricing and Overlaps

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Abstract—Mobile data offloading is an emerging technology to alleviate cellular network congestion and improve user service quality. In this paper, we investigate the *economics* of mobile data offloading through access points (APs) deployed by small cell service providers (SSPs), implementing uniform volume prices for all the mobile users (MUs) in each SSP's coverage including the overlapping area. In particular, we consider a data offloading game with a single mobile network operator (MNO) and two SSPs with overlapping coverage areas, where each SSP announces a uniform price for serving the cellular traffic within its coverage, and the MNO determines the traffic volumes to offload. We show that there is no pure Nash equilibrium (PNE) under such price competition, and determine the corresponding mixed strategy Nash equilibrium (MNE) using price randomization. As a practical solution, we propose a simple one shot auction mechanism that is easy to implement and has PNEs which is payoff equivalent with the MNE under price competition. We believe that this simple mechanism due to its simplicity of determining the equilibrium prices could be used in the negotiation between the SSPs and the MNO to determine the average service prices. Finally, we study the strategic topological infrastructure placement problem using a 1-dimension (1D, linear) user traffic flow model and a 2-dimension (2D) user traffic flow model when SSPs compete assuming uniform price competition as above. We show that the first mover in the placement problem will deploy its APs to cover more than half of the total flow volume and has an advantage to obtain a higher equilibrium payoff.

Index Terms—Mobile data offloading, uniform pricing, mixed strategy nash equilibrium, auction, strategic topological infrastructure placement

1 Introduction

We are witnessing an unprecedented worldwide growth of mobile data traffic that is expected to continue at an annual rate of 45 percent over the next few years, surpassing 30 exabytes per month by 2020 [1]. Traditional network capacity expansion methods such as network technology upgrades and additional spectrum acquisition are costly, time-consuming, and outpaced by the continuing traffic increase. Mobile data offloading is a promising approach to utilize certain complementary transmission technologies to deliver mobile traffic originally targeted to cellular networks. A large number of studies have investigated the potential benefits of mobile data offloading and various innovative schemes have been proposed to better manage data flows including WiFi [2], [3], [4], [5], femtocells [6], [7], [8], and opportunistic offloading [9], [10]. It is shown that in a typical urban environment, WiFi can offload about 65 percent cellular traffic and save 55 percent battery energy for mobile users (MUs) [3]. This performance gain can be further enlarged with the use of delaying transmission [11], [12]. The authors in [13], [14] investigated the network operators' profit gains from offering dual services through both macrocells and femtocells. All of these studies have shown that mobile data offloading is a cost-effective and energy-prudent approach to resolve network congestion and improve user service quality.

However, the ubiquitous deployment of WiFi or femtocell access points (APs) by the mobile network operators (MNOs) themselves is costly and often impractical due to the limitations of additional site spaces and backhaul cost. An alternative option for the MNOs is to employ existing WiFi and femtocell APs already deployed by third-party small cell service providers (SSPs) (as O2 did with BT [15]), instead of deploying their own offloading networks. This outsourcing method is attractive due to the high population of WiFi or femtocell users [16] as well as the technology innovations (e.g., Hotspot 2.0 protocol). Nevertheless, without proper economic incentives, the SSPs are expected to be reluctant to admit the cellular traffic since offloading cellular traffic will consume their resource and increase serving cost. Thus, studying the economic interactions of the involved entities is an important step towards the realization of this promising technology.

In this paper, we develop a framework to define the fair prices for contracts between large SSPs and the MNO for the data offloading service. In particular, we consider the common case where

- The SSPs compete to offload data from the same MNO because of the overlapping geographical coverage.
- A single uniform price independent of specific location is to be negotiated between the SSPs and the MNO.

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As we show, due to these overlapping areas where the customers of the macro operator can be served by both SSPs, there might be no pure Nash equilibrium (PNE) when the SSPs compete using prices that are uniform over his whole service area. This result suggests that simple uniform pricing mechanism is unstable and impractical. To remedy that in a competitive market, we propose a simple one-shot auction mechanism between the SSPs and the MNO with the following properties:

- It's easy to perform based on observable system parameters (properties of the coverage area of the SSPs and average customer traffic densities).
- It has a pure NE that is payoff-equivalent for both SSPs and the MNO with the mixed strategy Nash equilibrium (MNE) in the case of free market price competition.

Hence this mechanism can be used as a basis for a practical and sustainable price determination between various stakeholders, and can be used off-line in price negotiation exploiting average historical measurements of traffic offloading density at various locations served by the SSPs.

As a natural extension of the price determination mechanism, we study the strategic topological infrastructure placement problem of SSPs competing for offloading. Given the cellular flow information[1] of a specific network (assume that there is no AP deployed in the area at the beginning), both SSPs compete to deploy new APs in order to optimize their offloading payoffs in the subsequent data offloading pricing game. Particularly, in this work we study the AP placement problem using a simple 1-dimension (1D, linear) user traffic flow model, where the cellular traffic is uniformly distributed on some given interval, and further extend it to a 2-dimension (2D) flow model. We show that the first mover in the placement problem does not want to "over covers" an area and has an advantage to gain a higher payoff.

The main contributions of this work are summarized as follows:

- We study a general market model where two SSPs compete to offload cellular flow from a single MNO, and characterize the uniform pricing strategies of the SSPs in the data offloading game.
- We show that there is no PNE in the data offloading game with uniform prices, and compute the MNE using price randomization.
- Since MNE can not be implemented in practice, we further propose a payoff-equivalent one shot auction mechanism that is simple to implement and has PNEs. The SSP who fails the auction is allowed to quit the competition and set the monopoly offloading price to maximize its payoff, and the SSP who wins the auction will use a price same as its bid for users in both the overlapping and non-overlapping areas of coverage.
- Based on the above auction mechanism and the corresponding market equilibrium, we further investigate the strategic topological placement problem considering a simple linear cellular flow model and a 2D flow model.

1. In this work, we assume that the cellular flow information is common knowledge for the SSPs, where they can get from the MNO or from long term history interactions.

The rest of this paper is organized as follows. Section 2 briefly reviews the existing work. In Section 3, we describe the data offloading problem and analyze the uniform pricing scheme for the SSPs. In Section 4, we investigate the Nash equilibria of the data offloading game with uniform prices, including the PNE and the MNE. In Section 5, we introduce a one shot auction mechanism and prove the existence of the PNEs of the auction offloading game. In Section 6, we study the strategic topological placement problem of a 1D cellular flow model and a 2D flow model using results derived from previous sections. Numerical results and analysis of the offloading game and the auction game are presented in Section 7. Section 8 concludes the paper.

2 Related Work

There are many works considering the interactions between SSPs and MNOs from an economic point of view [17], [18], [19], [20], [21], [22], [23], [24]. In [17] and [18], the authors considered the incentive framework for user-initiated data offloading, where MUs decide when and where to offload their traffic, and hence the MUs offer necessary incentives in order to access to the APs. In this paper, we consider the network-initiated data offloading through APs deployed by third-party SSPs, and we assume that the MUs are either (i) willing to offload their traffic exactly as the networks intended, or (ii) unaware of the offloading process at all (i.e., data offloading is totally transparent to MUs) [20]. The authors in [21] proposed an iterative double auction mechanism to manage the marketplace where MNOs competed to lease multiple (possibly overlapping) APs for data offloading. The work presented in [22] formulated the offloading problem as a combinatorial reverse auction for realistic scenarios in which only part of the data traffic can be offloaded. In [23], a distributed market pricing framework was proposed for mobile data flows to price the offloading service. The payment for a specific flow is shared proportionally among all APs according to the amount of data offloaded to each AP.

Most of the previous works focus on the scenarios involving (i) SSPs with non-overlapping coverage areas, thus they can both serve as a monopoly, i.e., no competition between SSPs. (ii) SSPs with overlapping APs and non-uniform prices, i.e., that charge different prices for users in the overlapping and non-overlapping areas determined by rather complex auction mechanisms. Another shortcoming is that the pricing schemes proposed in the existed literature only focused on the volume of cellular data that is assigned to each APs, without considering the geographical locations of the MUs that generated these data. We believe that in the practical setting of future 5G deployment, due to the high population and density of small cell APs, it's important to develop simple pricing schemes whose complexity does not grow with the number of APs operated under a single SSP. To the best of our knowledge, this is the first work that considers a mobile data offloading framework with overlapping SSPs and a uniform pricing scheme.

Additionally, the deployment of small cells has been identified as one of the future-proof solutions to cope with the increasing demand for higher data rates and ubiquitous access in mobile networks [25], [26], [27]. There have been some studies on WiFi deployment problems [28], [29], [30],

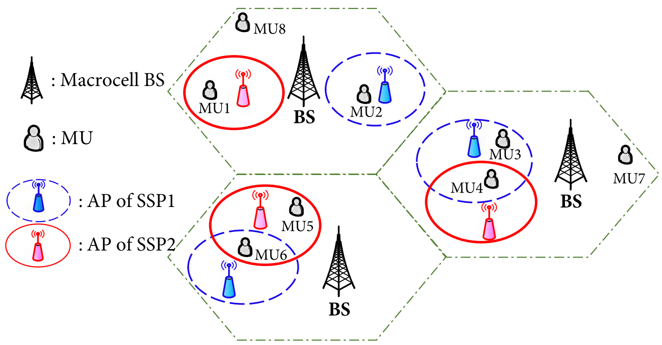


Fig. 1. An instance of the AP-offloading scenario with three macro BSs serving eight mobile users (MUs) and two SSPs. Each SSP has deployed three APs and can offload the traffic generated by those MUs within its coverage.

[31]. Wang et al. [29] proposed WiFi deployment algorithms based on realistic mobility characteristics. Even though their algorithms significantly improve the continuous coverage for mobile users while reducing the required number of APs, they regarded WiFi as a separated network and did not consider the objective of mobile data offloading. Dimatteo et al. [2] quantifies the number of APs required for WiFi offloading with different quality of service for data delivery. Further in [30], the authors proposed a mathematical approach to find the minimum required number of WiFi APs for efficient offloading. However, none of them considered the economic interactions between the deployment of WiFi network and cellular network, and the possible influences on the offloading results.

Specifically, in this work we try to answer the question that how should the SSPs deploy their APs strategically in order to maximize their payoffs assuming they compete using uniform pricing schemes in the offloading game, and what kind of deployment equilibrium can we observe? The results in this work can provide some insights for the SSPs about how and where to deploy the new APs and the possible equilibrium deployment scenarios.

3 Problem Description

In this section, we first describe the system model and the corresponding flow model of the AP-based cellular data offloading problem. Then the uniform pricing strategies of the SSPs are investigated, and we show that SSPs have no incentive to price below some threshold value.

3.1 Network Model

We consider a cellular network that includes one MNO with multiple macro cellular base stations (BSs) and two SSPs with multiple deployed APs. The APs deployed by different SSPs may be overlapping with each other. The MNO serves a set of MUs. The traffic of a MU can be offloaded to an AP only if it is in the coverage of that AP. Each MU can be served by no more than one AP simultaneously. Fig. 1 illustrates such a two-tier network.

As shown in Fig. 1, the MNO can offload the traffic generated by MU1 and MU5 merely to the APs deployed by SSP 2, and the traffic generated by MU2 and MU3 only to the APs of SSP 1. However, the traffic generated by MU4 and MU6 can be offloaded to either SSP 1 or SSP 2 since they are located in the overlapping coverage area of both

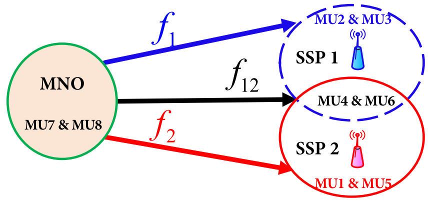


Fig. 2. Mobile data flow model corresponding to the network model in Fig. 1, where f_i , $i \in \{1, 2\}$ is the aggregate traffic generated by users covered only by SSP i, and f_{12} is the aggregate traffic generated by users in the overlapping area.

SSPs. In addition, MU7 and MU8 are not within the coverage of either SSP, so they can only be served by the MNO. Assume that time is slotted and we study the game for one time period. The location and traffic volumes of MUs registered to the MNO may change over time but are considered fixed within each time slot.[2] According to the MUs' offloading possibilities, we can combine the traffic volumes of the MUs into cellular data flows that can be offloaded only to SSP 1, SSP 2, and both SSPs. Therefore, we can map the network model to an offloading flow model as shown in Fig. 2, where $f_i, i \in \{1, 2\}$ is the "*monopoly flow*" of SSP i, i.e., the aggregate traffic volume generated by the users located only in the coverage of SSP i, and f_{12} is the "*overlapping flow*", i.e., the volume of aggregate traffic generated by users located in the overlapping area of both SSPs.

3.2 Pricing Problem of the SSPs

We consider the uniform pricing scheme, where SSP i sets a uniform offloading price $p_i \geq 0$ for all the MUs in its coverage, which will cause price competition between the two SSPs in the overlapping areas, and the MNO will choose the available SSP with a lower price to maximize its payoff. In the case when the two SSPs have the same offloading price, each SSP will serve half of the flow in the overlapping area. Let c_M denote the marginal cost of the MNO, which represents the decrease of MNO's serving cost induced by offloading to some SSP one unit of its customer traffic. For SSPs with non-overlapping APs, each SSP acts as a monopoly supplier in its own coverage area and thus it will set a *monopoly offloading price* c_M to maximize its payoff.[3] In the following, we focus on the scenario where the coverage area of the APs deployed by the two SSPs are overlapping with each other, which is more reasonable due to the ultra-dense deployment of APs in future networks. For simplicity, we assume that the SSPs have the same offloading cost c for each unit of traffic served and the offloading capacity of the APs is unlimited.

3.2.1 Payoff of MNO

Given the offloading price p_i announced by SSP $i, i \in \{1, 2\}$, the MNO's payoff is given by

$$u_{\text{MNO}}(p_1, p_2) = f_1(c_M - p_1) + f_2(c_M - p_2) \\ + f_{12}(c_M - \min\{p_1, p_2\}), \tag{1}$$

2. The time period can be one hour, one day, or one month, and the corresponding flow can then represent the average volume of flow in this time period.

3. Even when $p_i = c_M$, and MNO makes zero profit, it still prefers to offload the cellular traffic since the saving capacity can be used to serve other users and relieve the network congestion in the BSs.

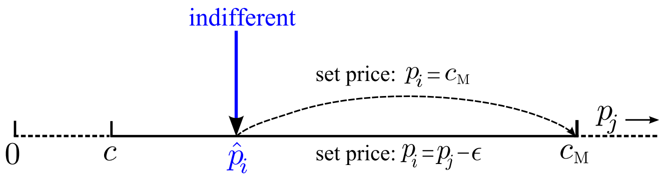


Fig. 3. Uniform pricing strategy of SSP i when p_j varies on $[c, c_M]$: If $p_j \in (\hat{p}_i, c_M]$, SSP i will compete and set a price $p_i = p_j - \epsilon$; when $p_j \in [c, \hat{p}_i]$. SSP i will not compete by reducing price any further and set monopoly price c_M.

where $p_i \in [c, c_M]$. We assume that the MNO will always prefer to offload as long as $p_i \leq c_M$.

3.2.2 Payoff of SSPs

Given the announced price of SSP j as $p_j, j \in \{1, 2\}$, the payoff function of SSP i with price p_i is given by

$$u_i(p_i, p_j) = \begin{cases} f_i(p_i - c) & \text{if} \quad p_i > p_j, \\ (f_i + f_{12})(p_i - c) & \text{if} \quad p_i < p_j, \\ (f_i + 0.5 f_{12})(p_i - c) & \text{if} \quad p_i = p_j, \end{cases} \tag{2}$$

where $p_i \in [c, c_M]$, for $i \in \{1, 2\}$ and $j \neq i$.

3.2.3 Uniform Pricing Scheme

In the data offloading game, the flow $f_i, i \in \{1, 2\}$ can only be offloaded by SSP i, therefore SSP i can always set the monopoly price c_M to offload at least f_i and obtain a *monopoly payoff* u_i^m given by

$$u_i^m = f_i(c_M - c). \tag{3}$$

To compete for serving the users in the overlapping area, i.e., flow f_{12}, each SSP will lower their offloading prices down from c_M. There is a trade-off between the volume of offloading flow and the offloading price. When the price p_i drops to some value $\hat{p}_i$ such that SSP i is indifferent between offloading merely f_i with price c_M and offloading $f_i + f_{12}$ with price $\hat{p}_i$, which yields

$$(f_i + f_{12})(\hat{p}_i - c) = u_i^m,$$

hence $\hat{p}_i$ can be written as

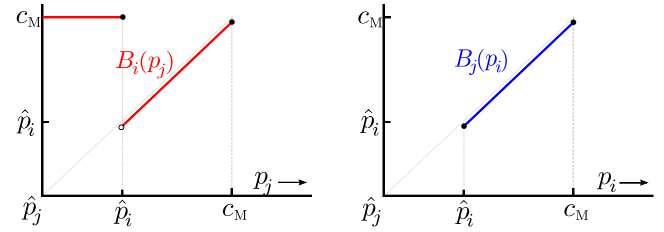
$$\hat{p}_i = \frac{1}{f_i + f_{12}}(c_M f_i + c f_{12}). \tag{4}$$

If SSP i continues to lower its price, the loss caused by the lower price will exceed the profit gain by offloading the extra f_{12}. Thus it is better off by simply setting the monopoly price c_M to offload merely its monopoly amount f_i.

Therefore, $\hat{p}_i$ is a *threshold price* for SSP i in its choice of pricing strategies, see Fig. 3. Straightforwardly, we have the following proposition to state the pricing rules of the SSPs.

Proposition 1. *For each SSP i, such that $i \in \{1, 2\}$, any price p_i smaller than $\hat{p}_i$ in (4) is a strictly dominated strategy for SSP i.*

Proof. If $p_i \in [c, \hat{p}_i)$, for any given p_j the maximal payoff SSP i can obtain will be $(f_i + f_{12})(p_i - c)$, when SSP i wins the overlapping flow. Since $u_i^m > (f_i + f_{12})(p_i - c)$, $\forall p_i \in [c, \hat{p}_i)$, thus SSP i can always be better off by merely offloading f_i at monopoly price c_M. □



(a) SSP i's best response function. (b) SSP j's best response function.

Fig. 4. SSPs' best response functions in the mobile data offloading game, assuming $f_i \geq f_j$ and the small circle indicates a point is excluded.

3.2.4 Characteristics of the Threshold Price

Subsequently, we analyze the characteristics of the threshold price $\hat{p}$ defined in (4) as follows

(a) If overlapping flow $f_{12} \to 0$ and $\frac{f_i}{f_{12}} \to \infty$, i.e., there is no overlapping between the two SSPs, each SSP will serve as a monopoly in its coverage area and set the monopoly offloading price c_M.

(b) If overlapping flow $f_{12} \to \infty$ and $\frac{f_i}{f_{12}} \to 0$, i.e., APs deployed by different SSPs are almost co-located, the offloading game will degenerate to the Bertrand competition [32], where the game has a single PNE, in which each SSP charges the price c.

(c) For any fixed overlapping flow f_{12}, the SSP with a larger monopoly flow f_i will prefer to set a higher offloading price. The SSP with a smaller f_i is willing to set a smaller offloading price and is more in the price competition.

(d) For any fixed monopoly flow f_i, such that $i = \{1, 2\}$, a larger overlapping flow (or overlapping area) f_{12} will aggravate the competition and drive down the offloading prices.

4 Analysis of Nash Equilibria

In this section, we analyze the Nash equilibria in the data offloading game described in Section 3. We show that there is no PNE and prove the existence of the MNE with price randomization. Without loss of generality, we assume that $f_i \geq f_j (i \neq j)$, then $\hat{p}_i \geq \hat{p}_j$.

4.1 Analysis of PNE

Since a strictly dominated action is not used in any Nash equilibria, we can thus eliminate from consideration all the strictly dominated strategies and only consider the strategy profile $p_i \in [\hat{p}_i, c_M]$ for SSP i, $i \in \{1, 2\}$. Next we analyze the PNE in this game using the best response functions.

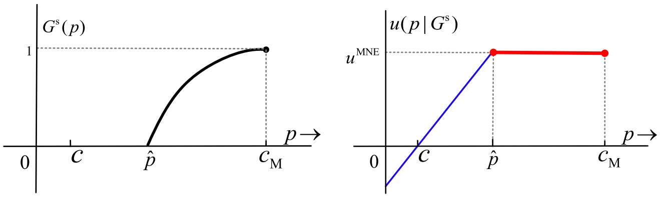
Given SSP j's price strategy as $p_j \in [\hat{p}_j, c_M]$, the best response function of SSP i is given by

$$B_i(p_j) = \begin{cases} c_M & \text{if} \quad p_j \in [\hat{p}_j, \hat{p}_i], \\ p_j - \epsilon & \text{if} \quad p_j \in (\hat{p}_i, c_M]. \end{cases} \tag{5}$$

Also, given SSP i's price strategy as $p_i \in [\hat{p}_i, c_M]$, SSP j's best response function is

$$B_j(p_i) = p_i - \epsilon \,, \qquad \text{for } p_i \in [\hat{p}_i, c_M] \,. \tag{6}$$

Fig. 4 shows the SSPs' best response functions, from which we can observe that the best response functions of



(a) MNE strategy G^{s}. (b) SSPs' expected MNE payoff.

Fig. 5. MNE in the symmetric data offloading game, where the mixed strategy G^{s} shown in (a) assigns positive probability only on the interval $[\hat{p}, c_{\mathrm{M}}]$, and any price on the interval $[\hat{p}, c_{\mathrm{M}}]$ yields the SSP the same expected payoff as shown in (b).

the two SSPs have no intersections, hence we state that there is no PNE in this game by the following proposition.

Proposition 2. *The data offloading game with uniform pricing schemes has no pure strategy Nash equilibrium.*

Proof. We can further prove that there is no pure Nash equilibrium by the following arguments:

- No pair of strategies (p, p) with $p \in [\hat{p}_i, c_{\mathrm{M}}]$ is a Nash equilibrium because SSP j can always be better off by slightly decreasing its price, and when $p > \hat{p}_i$, either SSP is better off by slightly decreasing its price.
- No pair of strategies (p_i, p_j) with $p_i \neq p_j$, $p_i \in [\hat{p}_i, c_{\mathrm{M}}]$ and $p_j \in [\hat{p}_j, \hat{p}_i]$ is a Nash equilibrium because SSP j can always be better off by increasing its price p_j as long as it is still smaller than p_i.
- No pair of strategies (p_i, p_j) with $p_i \neq p_j$, $p_i \in [\hat{p}_i, c_{\mathrm{M}}]$ and $p_j \in (\hat{p}_i, \hat{c}_{\mathrm{M}}]$ is a Nash equilibrium because the SSP whose price is higher can increase its payoff by decreasing its price to some value that is slightly less than the other SSP's price. □

4.2 Analysis of MNE

In this section, we compute the MNE in the data offloading game. For a game in which each player has a continuum of actions in some interval, we can identify each player's mixed strategy with a cumulative probability distribution (CDF) on the action interval [33]. In our data offloading game, SSP i, $i \in \{1, 2\}$ can choose any price in the interval $[c, c_{\mathrm{M}}]$, and the mixed strategy of SSP i is a CDF denoted by $G_i(p)$ over the interval $[c, c_{\mathrm{M}}]$, where $G_i(p_i)$ represents the probability that SSP i sets a price less than or equal to p_i.

4.2.1 Symmetric Data Offloading Game

First, we investigate the case when $f_i = f_j = f$, and $\hat{p}_i = \hat{p}_j = \hat{p}$. For any price pair (p_i, p_j), such that $p_i, p_j \in [c, c_{\mathrm{M}}]$, SSP i's payoff $u_i(p_i, p_j)$ satisfies $u_i(p_i, p_j) = u_j(p_j, p_i)$, thus the offloading game is a symmetric game. We establish the existence of the MNE in such a symmetric data offloading game by the next proposition.

Proposition 3. *The symmetric data offloading game with $f_1 = f_2 = f$ has a mixed strategy Nash equilibrium pair $(G^{\mathrm{s}}, G^{\mathrm{s}})$, and G^{s} is given by*

$$G^{\mathrm{s}}(p) = \begin{cases} \frac{f+f_{12}}{f_{12}} \frac{p-\hat{p}}{p-c} & \text{if} \quad p \in [\hat{p}, c_{\mathrm{M}}]\,, \\ 0 & \text{if} \quad p \in [c, \hat{p}]\,, \end{cases} \tag{7}$$

where $\hat{p} = \frac{1}{f+f_{12}}(c_{\mathrm{M}} f + c f_{12})$.

Proof. To investigate the possibility of such an equilibrium, consider a symmetric mixed strategy pair (G, G), each SSP's expected payoff with $p \in [c, c_{\mathrm{M}}]$ is given by

$$u(p) = G(p)(p-c)f + (1 - G(p))(p-c)(f + f_{12}), \tag{8}$$

we look for an equilibrium such that

$$\begin{aligned} &G(\hat{p}) = 0\,, \\ &G(c_{\mathrm{M}}) = 1\,, \\ &u(p|p \in [\hat{p}, c_{\mathrm{M}}]) \geq u(p|p \in [c, \hat{p}])\,, \end{aligned} \tag{9}$$

and any price in the interval $[\hat{p}, c_{\mathrm{M}}]$ yields the SSP the same expected payoff. Therefore, the first order condition (FOC) of Eq. (8) is given by

$$\frac{du(p)}{dp} = -f_{12}[G(p) + G'(p)(p-c)] + f_1 + f_{12} = 0\,.$$

Solve the FOC yields

$$G^{\mathrm{s}}(p) = \frac{f + f_{12}}{f_{12}} \frac{p - \hat{p}}{p - c}, \quad \text{for } p \in [\hat{p}, c_{\mathrm{M}}]\,,$$

and SSP's expected payoff at equilibrium is give by

$$u(p) = \begin{cases} (f + f_{12})(p - c) & \text{if} \quad p \in [c, \hat{p})\,, \\ (f + f_{12})(\hat{p} - c) & \text{if} \quad p \in [\hat{p}, c_{\mathrm{M}}]\,. \end{cases} \tag{10}$$

Thus, $(G^{\mathrm{s}}, G^{\mathrm{s}})$ as defined in (7) is a mixed strategy Nash equilibrium. □

Fig. 5 shows the MNE strategy as defined in (7) and the SSPs' expected payoff at the corresponding equilibrium. It is easy to observe that the MNE strategy G^{s} assigns zero probability to any strictly dominated strategies in $[c, \hat{p})$, and is convex on the interval when $p \geq \hat{p}$. Any price $p \in [\hat{p}, c_{\mathrm{M}}]$ gives the SSP the same expected payoff as $u^{\mathrm{MNE}} = (f + f_{12})(\hat{p} - c)$, which is greater than the expected payoff when SSPs' set any price less than $\hat{p}$.

4.2.2 Asymmetric Data Offloading Game

For the case where $f_i \neq f_j$, we assume that $f_i > f_j$ and $\hat{p}_i > \hat{p}_j$. We can establish the MNE of the asymmetric data offloading game by the next proposition.

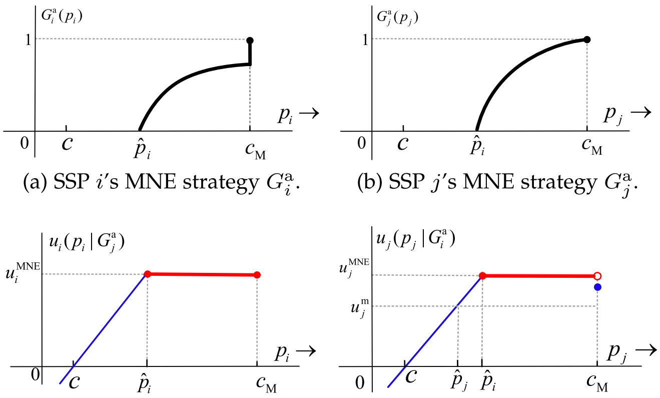
Proposition 4. *The asymmetric data offloading game with $f_i > f_j$ has a mixed strategy Nash equilibrium pair $(G_i^{\mathrm{a}}, G_j^{\mathrm{a}})$, where G_i^{a} and G_j^{a} are given by*

$$G_i^{\mathrm{a}}(p) = \begin{cases} 1 & \text{if} \quad p = c_{\mathrm{M}}\,, \\ \frac{f_j+f_{12}}{f_{12}} \frac{p-\hat{p}_i}{p-c} & \text{if} \quad p \in [\hat{p}_i, c_{\mathrm{M}})\,, \\ 0 & \text{if} \quad p \in [c, \hat{p}_i), \end{cases} \tag{11}$$

$$G_j^{\mathrm{a}}(p) = \begin{cases} \frac{f_i+f_{12}}{f_{12}} \frac{p-\hat{p}_i}{p-c} & \text{if } p \in [\hat{p}_i, c_{\mathrm{M}}] \\ 0, & \text{if? } p \in [c, \hat{p}_i)\,. \end{cases} \tag{12}$$

Proof. See Appendix A. □

Fig. 6 illustrates the asymmetric MNE as defined in (11) and (12), and the SSPs' corresponding expected payoffs at MNE. We can observe from Fig. 6b that SSP j also has no incentive to price on the interval $[\hat{p}_j, \hat{p}_i)$, since for SSP j the strategy $p_j = \hat{p}_i - \epsilon$ always strictly dominates any price in



(a) SSP i's MNE strategy G_i^a. (b) SSP j's MNE strategy G_j^a.

(c) Expected payoff of SSP i. (d) Expected payoff of SSP j.

Fig. 6. MNE in the asymmetric mobile data offloading game. Both SSPs assign zero probability for any price below $\hat{p}_i$; SSP i's expected payoff at MNE is the same to its monopoly payoff, and SSP j's expected payoff is greater than its monopoly payoff.

$[\hat{p}_j, \hat{p}_i)$ when $p_i \in [\hat{p}_i, c_M]$. At equilibrium, SSP i's expected payoff u_i^{MNE} is equal to its monopoly payoff u_i^m as defined in (3), and SSP j's expected payoff is greater than its monopoly payoff. In particular, we have the following remarks.

Remark 1. In the data offloading game with $f_i \geq f_j$, at the mixed strategy equilibrium, the expected payoffs of the SSPs are given by

$$u_i^{MNE} = (\hat{p}_i - c)(f_i + f_{12}), \tag{13}$$

$$u_j^{MNE} = (\hat{p}_i - c)(f_j + f_{12}), \tag{14}$$

and $u_i^{MNE} \geq u_j^{MNE}$, i.e., the player with a larger monopoly flow has a higher equilibrium payoff in the open offloading market with uniform pricing.

Remark 2. In the asymmetric data offloading game with $f_i > f_j, i \neq j$, for any price $p \in (\hat{p}_i, c_M)$, the probability that SSP j sets an offloading price no more than p is greater than SSP i.

This conclusion can be observed directly from Fig. 6 and the insights behind it is that the SSP with a smaller monopoly is more aggressive and willing to set a lower offloading price. Note that the notion G^a of the asymmetric MNE in Proposition 4 is a little different from the symmetric case. First, SSP i's equilibrium strategy G_i^a always assigns a positive probability to c_M which equals to $\frac{f_i - f_j}{f_i + f_{12}}$, and zero probability for any price below $\hat{p}_i$. Second, SSP j's equilibrium strategy G_j^a always assigns zero probability to c_M since the expected payoff at c_M is smaller than u_j^{MNE} (refer to Appendix A).

5 One Shot Auction Mechanism

In Section 4, we have proved that there is no PNE in the data offloading game, and the MNE cannot be implemented in practice, due to the instability of the uniform pricing scheme. This result motivates the introduction of the following payoff-equivalent one shot auction mechanism that is simple to implement and has pure NEs which yield the SSPs the same equilibrium payoffs to the expected payoffs obtained at the above MNE. Due to its payoff equivalence and simplicity, we expect this to be a "fair" choice by all market participants (MNO and SSPs).

5.1 One Shot Auction Mechanism

At the beginning of the time slot, both SSPs simultaneously submit their bidding prices to the MNO, denoted as $b_i, i \in \{1, 2\}$, where $b_i \in [\hat{p}_i, c_M]$. The SSP with a smaller bid will win the auction and set the offloading price equal to his bid. And the SSP who fails the auction will quit the competition and set the monopoly offloading price c_M. In the following, we assume that perfect flow information is available for the SSPs, where each SSP can observe the flow knowledge of his competitor.[4] Based on this auction mechanism, given the bidding strategy of SSP j as b_j, the payoff of SSP i with a bid b_i is given by

$$u_i(b_i, b_j) = \begin{cases} f_i(c_M - c) & \text{if} \quad b_i > b_j, \\ (f_i + f_{12})(b_i - c) & \text{if} \quad b_i < b_j, \\ (f_i + 0.5 f_{12})(b_i - c) & \text{if} \quad b_i = b_j, \end{cases} \tag{15}$$

where $b_i \in [\hat{p}_i, c_M], \forall i \in \{1, 2\}$. This auction mechanism can guarantee that each SSP always has a payoff no less than his monopoly payoff u_i^m as defined in (3).

5.2 Existence of Nash Equilibria

Similar to the analysis in Section 4.1, assume that $f_i > f_j$ and $\hat{p}_i > \hat{p}_j$, given SSP i's bid b_i, such that $b_i \in [\hat{p}_i, c_M]$, the best response function of SSP j is given by

$$B_j(b_i) = b_i - \epsilon \qquad \text{if } b_i \in [\hat{p}_i, c_M], \tag{16}$$

and the best response function of SSP i given b_j is given by

$$B_i(b_j) = \begin{cases} b_j - \epsilon & \text{if} \quad b_j \in (\hat{p}_i, c_M], \\ (\hat{p}_i, c_M] & \text{if} \quad b_j = \hat{p}_i, \\ [\hat{p}_i, c_M] & \text{if} \quad b_j \in [\hat{p}_j, \hat{p}_i). \end{cases} \tag{17}$$

Note that, when $b_j = b_i = \hat{p}_i$, SSP i's payoff is given as

$$u_i(b_j = \hat{p}_i, b_i = \hat{p}_i) = (f_i + 0.5 f_{12})(\hat{p}_i - c),$$

which is smaller than u_i^m. Therefore, SSP i can be better off by bidding greater than $\hat{p}_i$, and is indifferent between any value on $(\hat{p}_i, c_M]$, because it will loss the auction anyway.

We plot the SSPs' best response functions relative to different axes in Fig. 7, we can see that the one shot auction game has two pure NEs, which can be concluded by the following proposition.

Proposition 5. *In the one shot auction offloading game, there exist two pure Nash equilibrium* $(b_i^*, b_j^*) = (\hat{p}_i + \epsilon, \hat{p}_i)$ *and* $(b_i^*, b_j^*) = (\hat{p}_i, \hat{p}_i - \epsilon)$.

We can make the following observations from Fig. 7:

(a) $(b_i^*, b_j^*) = (\hat{p}_i + \epsilon, \hat{p}_i)$ is the first equilibrium that SSPs will achieve when they lower their bidding prices down from c_M. The SSP with a larger monopoly flow f_i will fail the auction and quit the competition to set the monopoly offloading price c_M. Therefore, at

4. This can be the result of long term interactions with each other or the SSPs can obtain the flow information from the MNO.

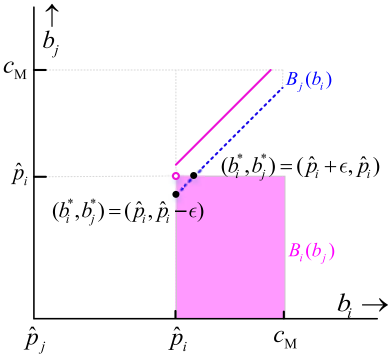


Fig. 7. SSPs' best response functions in the one shot auction game with $f_i > f_j$. The best response function of SSP i is the solid line and the rectangle area; that of SSP j is the dash line. There exist two NEs in this auction game: $(b_i^*, b_j^*) = (\hat{p}_i + \epsilon, \hat{p}_i)$ and $(b_i^*, b_j^*) = (\hat{p}_i, \hat{p}_i - \epsilon)$.

equilibrium, the offloading prices in the market are give by

$$(p_i^*, p_j^*) = (c_{\mathrm{M}}, \hat{p}_i)\,.$$

(b) $(b_i^*, b_j^*) = (\hat{p}_i, \hat{p}_i - \epsilon)$ is the second equilibrium and the SSP with a larger monopoly flow still fails the auction, thus the equilibrium offloading prices are

$$(p_i^*, p_j^*) = (c_{\mathrm{M}}, \hat{p}_i - \epsilon)\,.$$

At both equilibria, the SSP with a larger monopoly flow (here SSP i) will set the monopoly price c_{M} and obtain a payoff as

$$u_i^{\mathrm{A}} = (c_{\mathrm{M}} - c) f_i\,, \tag{18}$$

which is equal to the monopoly payoff u_i^{m}. However, SSP j's payoff will be slightly different at the two PNEs, and there exists a dominant equilibrium for SSP j, which can be demonstrated by the next proposition.

Proposition 6. *For the two equilibria in the one shot auction game with $f_i > f_j$, the equilibrium $(b_i^*, b_j^*) = (\hat{p}_i + \epsilon, \hat{p}_i)$ is dominated by the equilibrium $(b_i^*, b_j^*) = (\hat{p}_i, \hat{p}_i - \epsilon)$ for SSP j.*

Proof. As shown in Fig. 8, we plot the payoff of SSP j as it varies with b_i when it plays the two equilibrium strategies.

It's easy to see that, when $b_i = \hat{p}_i$, the payoff of SSP j when it plays $b_j^* = \hat{p}_i$ and $b_j^* = \hat{p}_i - \epsilon$ can be denoted as

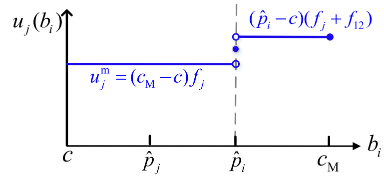
$$u_j(b_i = \hat{p}_i; b_j^* = \hat{p}_i) = (\hat{p}_i - c)(f_j + 0.5 f_{12})\,,$$
$$u_j(b_i = \hat{p}_i; b_j^* = \hat{p}_i - \epsilon) = (\hat{p}_i - \epsilon - c)(f_j + f_{12})\,,$$

since ϵ is arbitrary small, hence $u_j(b_i = \hat{p}_i; b_j^* = \hat{p}_i - \epsilon) > u_j(b_i = \hat{p}_i; b_j^* = \hat{p}_i)$. □

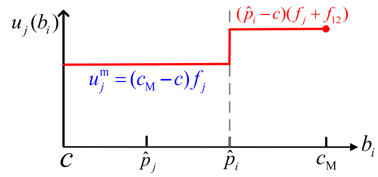
Remark 3. The expected payoffs SSPs can obtain at MNE are same to the equilibrium payoffs by one shot auction denoted by u_i^{A}, i.e.,

$$u_i^{\mathrm{MNE}} = u_i^{\mathrm{A}}, \quad \text{for } i \in \{1, 2\}. \tag{19}$$

Proof. In the offloading game using one shot auction mechanism, assuming $f_i \geq f_j$, the equilibrium offloading prices in the market are $(p_i^*, p_j^*) = (c_{\mathrm{M}}, \hat{p}_i)$, thus SSPs' equilibrium payoffs are given by



(a) When $b_j^* = \hat{p}_i$.



(b) When $b_j^* = \hat{p}_i - \epsilon$.

Fig. 8. SSP j's payoff varies with b_i while SSP j plays his equilibrium strategies b_j^*: When $b_i = \hat{p}_i$, SSP j can be better off by playing $b_j^* = \hat{p}_i - \epsilon$ than $b_j^* = \hat{p}_i$.

$$u_i^{\mathrm{A}} = (c_{\mathrm{M}} - c) f_i\,, u_j^{\mathrm{A}} = (\hat{p}_i - c)(f_j + f_{12})\,.$$

Compare with Remark 1, it is easy to see that $u_i^{\mathrm{MNE}} = u_i^{\mathrm{A}}$, for $i \in \{1, 2\}$. □

This result suggests that such a mechanism is of real interest to be accepted in practice, since both the MNO and SSPs obtain the same expected equilibrium payoff as in the case of classical price competition, and can serve as a simple and effective economic tool in the offloading game.

6 Strategic Topological Placement Problem

The above simple auction mechanism allows us to investigate the complex question related to the strategic topological placement of competing SSPs. An interesting question is how should the SSPs deploy their APs strategically assuming they compete in the future over uniform prices?

In practice, infrastructure deployment is not simultaneously, hence it makes sense to assume a first mover in the placement problem. The logical timeline of this problem is given in Fig. 9, where in Phase I and Phase II, the two SSPs deploy their APs,[5] and in Phase III, given the topological locations of the APs and the cellular flow information, the two SSPs compete in prices to do offloading. We summarize the results we need in Phased III from the previous sections, as the equilibrium payoffs of the two competing SSPs with monopoly flow of f_1, f_2, and overlapping flow f_{12} are given by

$$\begin{aligned} u_1 &= (\hat{p}_1 - c)(f_1 + f_{12}) = f_1 (c_{\mathrm{M}} - c)\,,\\ u_2 &= (\hat{p}_1 - c)(f_2 + f_{12}) = f_1 (c_{\mathrm{M}} - c)\,\frac{f_2 + f_{12}}{f_1 + f_{12}}\,, \end{aligned} \tag{20}$$

where $f_1 \geq f_2$. In the following, we will investigate this strategic AP placement problem using a linear flow model and a general 2D flow model, respectively. For simplicity, we do not consider the AP deployment cost in this work. We show the first mover in the AP placement game can obtain a higher equilibrium payoff by deploying his APs to cover more than half of the total cellular flow.

5. We assume that there is no AP deployed at the start of Phase I.

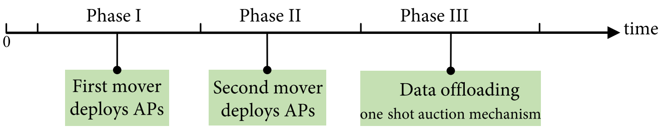


Fig. 9. Logical timeline for the strategical topological placement problem.

6.1 1D Flow Model

To study the placement behaviors of the SSPs, we first consider a specific 1-dimension (linear) flow model, as shown in Fig. 10, where the mobile cellular traffic (from single MNO) is uniformly distributed only on some interval $[0, L]$, with total flow volume normalizes to 1. Assume that $L = 1$, both SSPs $\{1, 2\}$ intend to deploy APs on this interval $[0, 1]$ to offload cellular traffic from the MNO. APs deployed by both SSPs are of similar characteristics, with a maximum coverage length $R = 2r$, where $0.5 < R < 1$. As such, each SSP deploys only one AP. The "placement" of the AP is defined by its center (middle point) x, i.e., if the AP is placed at some point $x \in [0, 1]$, it covers the interval $[x - r, x + r]$, and the total cellular traffic within its coverage is given as

$$f_{\mathrm{AP}} = \min(x + r, 1) - \max(0, x - r)\,, \tag{21}$$

since there is no cellular flow outside the interval $[0, 1]$. Subsequently, we study the Nash equilibrium placement strategies for the SSPs by finding the best response functions.

6.1.1 *Best Response Functions of SSPs*

Assume that $x_i \in [0, 1]$ is the center of the AP deployed by SSP i, such that $i = 1, 2$. Given SSP 1's AP center $x_1 \in [0, 1]$, then SSP 2's best response function $B_2(x_1)$ can be analyzed from the following different cases:

i. If $x_1 \in [0, 1 - 3r]$, i.e., $r \leq x_1 + r \leq 1 - 2r$, there is at least $2r$ uncovered line of flow and the best SSP 2 can do is to simply deploy its AP without overlapping with SSP 1. Therefore, SSP 2's best response function $B_2(x_1)$ is given by

$$B_2(x_1) \in [x_1 + 2r, 1 - r] \quad \text{if } x_1 \in [0, 1 - 3r]\,. \tag{22}$$

ii. If $x_1 \in (1 - 3r, 0.5 - r]$, i.e $x_1 + r \in (1 - 2r, 0.5]$, SSP 1 covers no more than half of the flow line. As long as SSP 2 deploys its AP to cover the remaining half flow line, the monopoly flow of SSP 2 will always be $f_2 \geq 0.5$, which yields SSP 2 to be the player with a larger amount of monopoly flow. Therefore, SSP 2 will be indifferent from not overlapping or overlapping part with SSP 1. As a result, the best response function $B_2(x_1)$ is given by

$$B_2(x_1) \in [1 - r, x_1 + 2r] \quad \text{if } x_1 \in (1 - 3r, 0.5 - r]\,. \tag{23}$$

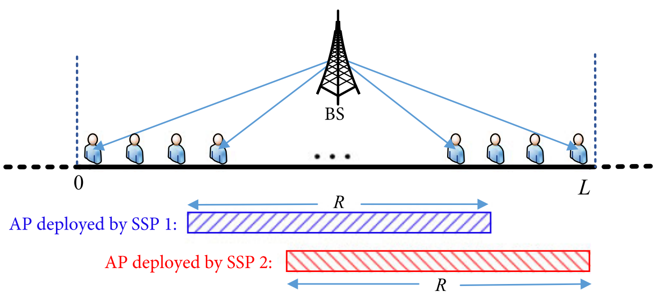


Fig. 10. The linear flow mode in the placement problem, where cellular flow is uniformly distributed on the interval $[0, L]$, $L = 1$. APs deployed by the SSPs have a maximum coverage length of R, and $0.5 < R < 1$.

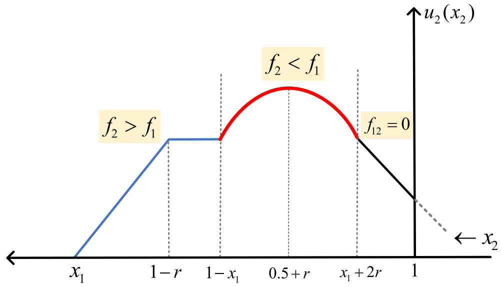


Fig. 11. SSP 2's payoff varies with the location of his AP in case iii: Overlapping befits SSP 2 first, then the payoff loss caused by excessive overlap may exceed the gain and drive down his payoff.

iii. If $x_1 + r > 0.5, x_1 - r \leq 0$, SSP 1 covers more than half of the flow line, but less than the size of the AP. When $x_2 - r \geq x_1 + r$, i.e., $x_2 \geq x_1 + 2r$, there is no overlapping with SSP 1 and SSP 2's payoff is given by

$$u_2^{\mathrm{non}}(x_2) = [1 - (x_2 - r)](c_{\mathrm{M}} - c) \quad \text{if } x_2 \in [x_1 + 2r, 1]\,,$$

and the maximal payoff can be obtained at $x_2 = x_1 + 2r$, which yields

$$u_2^{\mathrm{non,max}} = [1 - (x_1 + r)](c_{\mathrm{M}} - c)\,.$$

When $x_2 \leq x_1 + 2r$, the coverage area of the two SSPs starts to overlap. As shown in Fig. 11, when x_2 decreases (moving to the left side), SSP2's payoff $u_2(x_2)$ will increase at first, i.e., overlapping benefits SSP 2, since he will win the overlapping flow in the auction. However, excessive overlapping (when $x_2 < 0.5 + r$) will aggravate the price competition between the two SSPs, and drive down equilibrium offloading prices and payoffs. The optimal payoff can be achieved at $x_2 = 0.5 + r$ (see Appendix B.1 for the proof). Therefore, SSP 2's best response function can be denoted as

$$B_2(x_1) = 0.5 + r \quad \text{if } x_1 \in (0.5 - r, r]\,. \tag{24}$$

iv. When $x_1 - r > 0$ and $x_1 \leq 0.5$, if $x_2 \geq x_1 + 2r$, there is no overlapping between the two SSPs. Similarly, the maximal payoff of SSP 2 can be obtained at $x_2 - r = x_1 + r$ and yields

$$u_2^{\max} = [1 - (x_1 + r)](c_{\mathrm{M}} - c) \quad \text{if} \quad x_2 \in [x_1 + 2r, 1].$$

When $x_2 \leq x_1 + 2r$, as x_2 decreases, its coverage starts overlapping with SSP 1's. And the maximal payoff can be obtained at $x_2 = 0.5(1 + x_1 + r)$ (see Appendix B.2 for proof). And SSP 2's best response function is given by

$$B_2(x_1) = \frac{1}{2}(1 + (x_1 + r)) \quad \text{if} \quad x_1 \in [r, 0.5]\,. \tag{25}$$

When $x_1 > 0.5$, the analysis of the best response functions will be similar to the cases i to case iv. In summary, given the

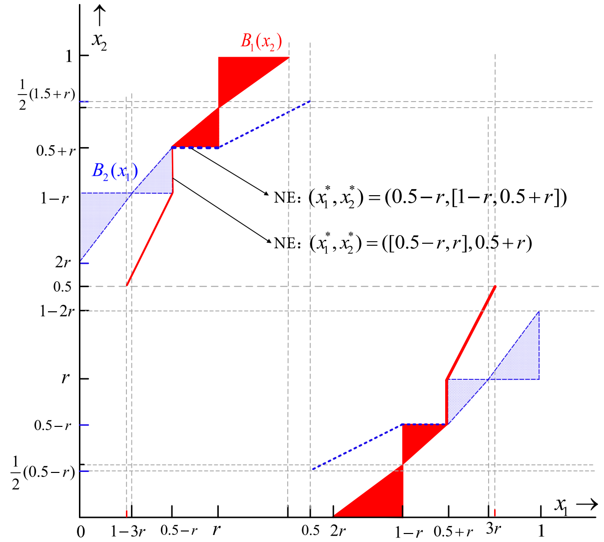


Fig. 12. SSPs' best response functions in the strategic placement game. The best response function of SSP 1 is solid and that of SSP 2 is dash. When $x_1 \leq 0.5$, there exist two equilibrium placement scenarios.

strategy of SSP 1 $x_1 \in [0,1]$, the corresponding best response function of SSP 2 $B_2(x_1)$ can be summarized as follows:

$$B_2(x_1)\begin{cases} \in [x_1+2r, 1-r] & \text{if} \quad x_1 \in [0, 1-3r], \\ \in [1-r, x_1+2r] & \text{if} \quad x_1 \in (1-3r, 0.5-r], \\ = 0.5+r & \text{if} \quad x_1 \in (0.5-r, r], \\ = \frac{1}{2}[1+(x_1+r)] & \text{if} \quad x_1 \in (r, 0.5], \\ = \frac{1}{2}(x_1-r) & \text{if} \quad x_1 \in (0.5, 1-r], \\ = 0.5-r & \text{if} \quad x_1 \in (1-r, 0.5+r], \\ \in [x_1-2r, r] & \text{if} \quad x_1 \in (0.5+r, 3r], \\ \in [r, x_1-2r] & \text{if} \quad x_1 \in (3r, 1]\,. \end{cases} \tag{26}$$

Note that in case i and case ii, when the first mover (SSP 1) covers no more than half of the flow line, not overlapping is always a best response strategy for SSP 2. And in case iii and case iv, when the SSP 1 takes more than half of the flow line, SSP 2 will prefer to overlap with SSP1 properly to avoid excessive overlapping.

6.1.2 Analysis of NE

We also can obtain the best response function of SSP 1 given the center of SSP 2's AP x_2 with a symmetric formula of (26), and plot the above best response functions in Fig. 12 to establish the Nash equilibria of the strategic AP placement game. It is easy to observe that there exist two equilibrium placement scenarios when $x_1 \leq 0.5$ as : $(x_1^*, x_2^*) = (0.5-r, [1-r, 0.5+r])$ and $(x_1^*, x_2^*) = ((0.5-r, r], 0.5+r)$. Here, we only analyze the equilibria when $x_1 \leq 0.5$, since the equilibria placement scenarios when $x_1 > 0.5$ are symmetric to the case of $x_1 \leq 0.5$.

Fig. 13 shows the equilibrium placement scheme corresponding to $(x_1^*, x_2^*) = (0.5-r, [1-r, 0.5+r])$. It can be observed that, if SSP 1 places his AP at $x_1^* = 0.5-r$, i.e., he only covers half of the flow line, then at equilibrium SSP 2 will be indifferent from not overlapping or overlapping as long as he covers the whole other half. In this case, SSP 2 will always be the one with a higher amount of monopoly flow, therefore, as long as he covers the remaining half line, overlapping has no effects on his expected payoff, which is given by

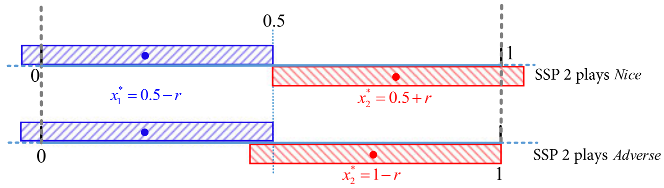


Fig. 13. Equilibrium AP placement schemes in the line flow model, when SSP 1 plays *Nice* by setting $x_1^* = 0.5-r$.

$$u_2^{1D} = 0.5(c_M - c)\,, \tag{27}$$

which equals u_2^{1D}. However, as the overlapping areas increases, the payoff of SSP 1 will decrease. Particularly, if the SSP 2 plays *Nice* by choosing $x_2^* = 0.5+r$, i.e., not overlapping with SSP 1, then the equilibrium payoff of SSP 1 is give by

$$u_1^{1D}(x_1^* = 0.5-r, x_2^* = 0.5+r) = 0.5(c_M - c)\,.$$

And if SSP 2 plays *Adverse* by overlapping as much as he can and maintains the same payoff, i.e., $x_2^* = 1-r$, then SSP 1's equilibrium payoff is given by

$$u_1^{1D}(x_1^* = 0.5-r, x_2^* = 1-r) = \frac{1}{4r}0.5(c_M - c),$$

and $4r > 1$, hence $u_1^{1D}(x_1^* = 0.5-r, x_2^* = 1-r) < u_1^{1D}(x_1^* = 0.5-r, x_2^* = 0.5+r)$, i.e., SSP 1 is worse off at the equilibrium. To reduce the possibility of payoff loss, as the first mover, SSP 1 can choose $x_1^* \in (0.5-r, r]$, specially if he plays *Adverse* by setting $x_1^* = r$ as shown in Fig. 14, in which equilibrium, SSP 2 will set $x_2^* = 0.5+r$, thus SSP 1 is guaranteed to have a payoff equal to $u^* = 0.5(c_M - c)$. We can illustrate the strategic placement game with SSPs' possible actions as {*Adverse*, *Nice*} using Fig. 15, where the values in each box are the SSPs' payoffs to the strategy file to which the box corresponds, with the first mover's payoff listed first.

It is easy to observe that if the first movers plays *Nice* by setting $x_1^* = 0.5-r$, then the second mover can play *Adverse* ($x_2^* = 1-r$) to make the first player worse off without reducing his own profit; if the first movers plays *Adverse* by setting $x_1^* = r$, then the second mover can play *Nice* obtaining a payoff $\frac{1}{2R}u^*$, or play *Adverse* with payoff $2(1-R)u^*$, where $2(1-R) < 1/2R$, thus his only best response of the second mover is to play *Nice*. It's easy to observe that there exist two pure NEs in this game: (*Adverse*, *Nice*) and (*Nice*, *Adverse*), but only one sub-game perfect NE (SPNE): (*Adverse*, *Nice*) using backward induction. Therefore, we can state that the first mover in this game has the advantage to obtain a higher payoff by the following proposition.

Proposition 7. *The first mover in the strategic placement game of the linear user traffic model can deploy his AP to cover more than half of the flow line and obtain a higher equilibrium payoff.*

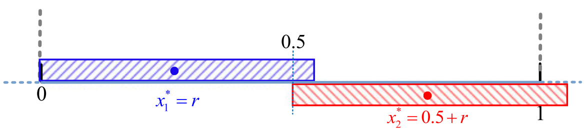


Fig. 14. Equilibrium AP placement schemes in the line flow model, when SSP 1 plays *Adverse* by setting $x_1^* = r$.

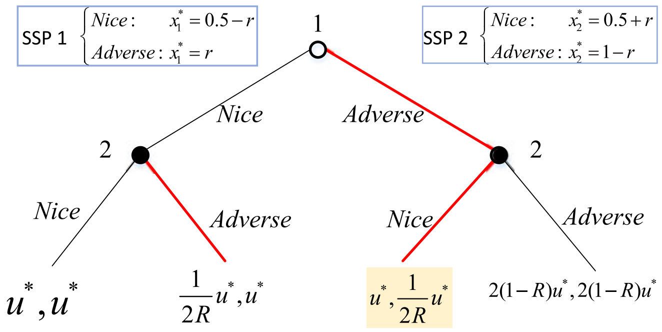


Fig. 15. The strategic placement game with SSPs' possible actions as {*Adverse*, *Nice*}: If the first movers plays *Nice* by setting $x_1^* = 0.5 - r$, then the second mover can play *Nice* or *Adverse* and obtain the same payoff as u^*; if the first mover plays *Adverse* by setting $x_1^* = r$, then the best response of the second mover is to play *Nice*.

6.2 2D Flow Model

In this section, we study the strategic topological placement game using a 2-dimensional (2D) flow model. Consider a 2D area with normalized cellular traffic 1, in this case, we do not investigate how the individual AP is designed or deployed, instead we're only interested in the optimal volume of cellular flow that the SSPs choose to cover.[6]

As shown in Fig. 16, assume that the first SSP enters the market and deploys his APs to cover f_1 cellular traffic, where $f_1 \in [0, 1]$:

a. If the second SSP chooses to not overlap with the first one by covering all the remaining area and offloading at monopoly price c_{M}, the payoff of the second SSP is given by

$$u_2^{\mathrm{non,2D}} = (1 - f_1)(c_{\mathrm{M}} - c). \tag{28}$$

b. If the second SSP chooses to overlap with the first player such that the overlapping amount of flow to be $f_{12} \in (0, f_1]$. Then we have the renewed monopoly flow $f_1' = f_1 - f_{12}$ and $f_2' = 1 - f_1$. As the overlapping flow f_{12} increases, the monopoly flow of the first mover decreases, and the monopoly flow of the second mover remains the same. When $f_{12} > 2f_1 - 1$, we have $f_2' > f_1'$, and the corresponding payoff of the second mover is the same as the non-overlapping case $u_2^{\mathrm{non,2D}}$. Therefore, the payoff of the second SSP is summarized by

$$u_2(f_{12}) = \begin{cases} \frac{f_2' + f_{12}}{f_1' + f_{12}} f_1'(c_{\mathrm{M}} - c) & \text{if} \quad f_{12} \le 2f_1 - 1, \\ f_2'(c_{\mathrm{M}} - c) & \text{if} \quad f_{12} > 2f_1 - 1, \end{cases}$$

yields the maximal payoff of the second SSP as

$$u_2^{\mathrm{2D}} = \frac{c_{\mathrm{M}} - c}{4f_1}, \text{ when } f_{12}^* = f_1 - 0.5. \tag{29}$$

Given (29), the optimal coverage flow for the first SSP is $f_1^* > 0.5$, and the renewed monopoly flow of the first mover is always $f_1' = 0.5$. The equilibrium payoffs for the two SSPs are given by

$$\begin{aligned} u_1^{\mathrm{2D}} &= 0.5(c_{\mathrm{M}} - c), \quad \text{where } f_1^* > 0.5, \\ u_2^{\mathrm{2D}} &= \frac{c_{\mathrm{M}} - c}{4f_1^*}. \end{aligned} \tag{30}$$

Thus, the best response function of SSP 2 in terms of f_{12}^* given $f_1 \in [0, 1]$ can be give as

$$f_{12}^*(f_1) = \begin{cases} [0, f_1] & \text{if} \quad f_1 \in [0, 0.5], \\ f_1 - 0.5 & \text{if} \quad f_1 \in (0.5, 1]. \end{cases} \tag{31}$$

It's easy to observe that:

- If the first mover plays *Nice* by only covering half of the total flow, i.e., $f_1^* = 0.5$, then he will be worse off if the second mover plays *Adverse* by greedily overlapping with $f_{12} = f_1$.
- As long as the first mover covers more than half of the flow volume, excessive covering will not increase his payoff but will harm the profit of the second SSP.

Note that if we consider about the deployment cost of the APs, then the first mover will not be too greedy, since excessive deployment will only increase his deploy cost.

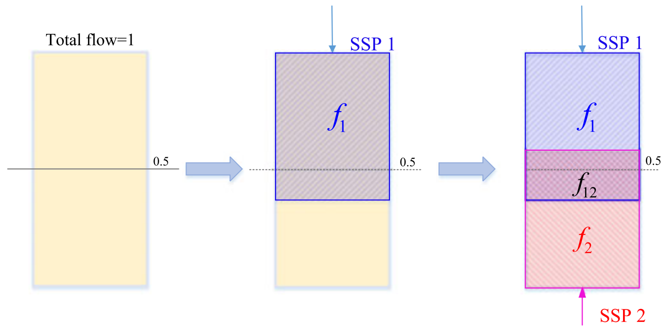


Fig. 16. Deployment game in the 2D flow model: SSP 1 enters the market first and choose to deploy his APs in order to cover $f_1 \in [0, 1]$ amount of cellular flow; then, SSP 2 enters the market and deploy his APs, where f_{12} is the overlapping amount of flow.

7 Numerical Results

This section presents some of numerical results to verify our theoretical analysis of the data offloading and the auction game.

7.1 Existence of MNE

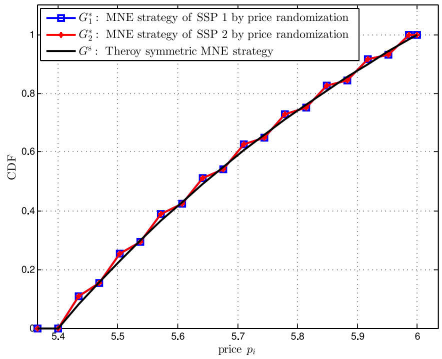
7.1.1 Symmetric MNE

We discretize the price on interval $[c, c_{\mathrm{M}}]$ to find the mixed strategy Nash equilibrium and compare with G^{s} defined in (7). In our simulation, we fix $f_1 = f_2 = 70$ and investigate the equilibrium strategy with a small overlapping flow $f_{12} = 30$ in Fig. 17a, and a large overlapping flow $f_2 = 130$ in Fig. 17b. From these figures, we observe that the threshold price decreases as f_{12} increases, and the MNE obtained by price randomization match well with the MNE G^{s} as defined in (7).

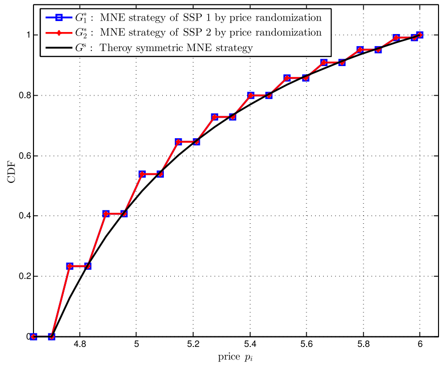
7.1.2 Asymmetric MNE

We discretize the price to find the mixed strategy Nash equilibrium in the asymmetric offloading game when $f_1 > f_2$, and compare with the equilibrium strategy defined in (11)

6. Here, we do not care about the cellular flow distribution in the 2D area, we only know that the total volume of traffic equals 1. And, we assume zero deployment cost.

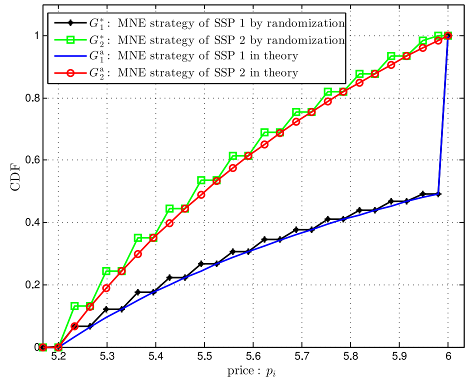


(a) Here, $f_{12} = 30$.

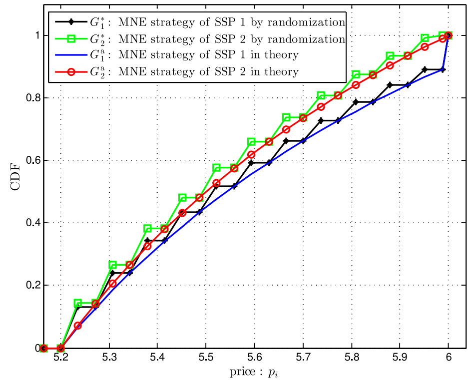


(b) Here, $f_{12} = 130$.

Fig. 17. Symmetric MNE strategy by price discretization comparing with theory MNE strategy G^{s} as defined in Eq. (7).



(a) Here, $f_2 = 10$.



(b) Here, $f_2 = 50$.

Fig. 18. Asymmetric MNE strategy by price discretization comparing with theory MNE strategy G^{a} as defined in Eqs. (11) and (12).

and (12). In our simulation, we fix $f_1 = 60, f_{12} = 40$, and study the MNE varying with the volume of $f_2 \in \{10, 50\}$, as shown in Fig. 18. It is easy to observe that, for any value $p \in [\hat{p}_1, c_{\mathrm{M}}]$, the cumulative probability that SSP 2 prices below this value is greater than SSP 1. Because with a smaller monopoly flow, SSP 2 is more aggressive in the competition, and willing to set a smaller price to win the overlapping flow.

7.2 Existence of PNE in the One Shot Auction Game

This section illustrates the pure equilibria in the one shot auction offloading game as shown in Fig. 19, where $f_1 = 20, f_2 = 15, f_{12} = 5, c = 4,\ c_{\mathrm{M}} = 6$, and $\epsilon = 0.01$. The threshold prices are $\hat{p}_1 = 5.60$ and $\hat{p}_2 = 5.50$. There exist three PNEs in this auction game: $(b_1^*, b_2^*) = (5.60, 5.59)$, $(b_1^*, b_2^*) = (5.61, 5.60)$ and $(b_1^*, b_2^*) = (5.62, 5.61)$. We can conclude the NEs in Table 1, where b_i^*, p_i^*, and u_i denote the equilibrium bid, the equilibrium offloading price, and the corresponding payoff for the SSPs, respectively. The first two equilibria in the simulation are consistent with the theory NEs as claimed in Proposition 5. The third equilibrium $(b_1^* = \hat{p}_1 + 2\epsilon,\ b_2^* = \hat{p}_1 + \epsilon)$ results from the value of the parameter ϵ.

In the previous analysis, we assume that ϵ is some infinite small positive number. However, in realistic charging and pricing systems, ϵ cannot be infinitely small. In our numerical results, we consider ϵ as the smallest price interval based on which the SSPs can adjust theirs bids. Therefore, in the simulation, due to different value of ϵ, it is possible that there are more than two equilibria.

7.3 Linear Deployment Game Simulation

In this section, we provide the simulation results of the linear deployment game. We consider the equilibrium placement scheme where $x_1^* = 0.5 - r, x_2^* \in [1 - r, 0.5 + r]$. When the first player chooses to cover only half of the total flow, the second mover can play *Nice* by not overlapping or *Adverse* by overlapping as much as he can, which can be observed from Fig. 20, as the overlap increases, the equilibrium payoff of the second mover remains the same, while the equilibrium payoff of the first player decreases. Therefore, as the first player enters the offloading market, to reduce the possibility of profit loss, the first mover should cover more than half of the flow to obtain a higher payoff.

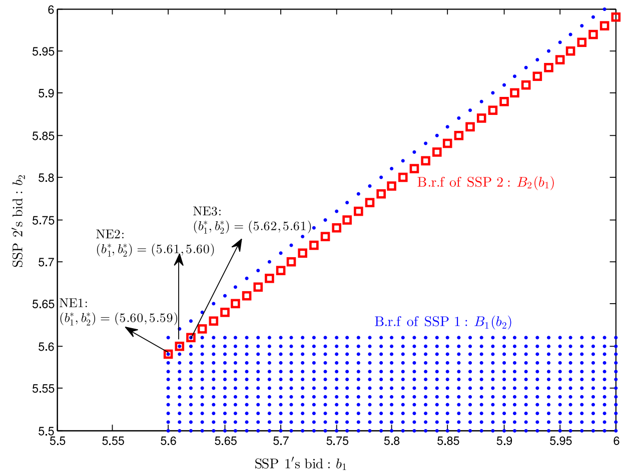


Fig. 19. Pure equilibria in the one shot auction offloading game, when $f_1 = 20, f_2 = 15, f_{12} = 5, c = 4, c_{\mathrm{M}} = 6$, and $\epsilon = 0.01$. This game has three NEs: $(b_1^*, b_2^*) = (5.60, 5.59)$, $(b_1^*, b_2^*) = (5.61, 5.60)$, and $(b_1^*, b_2^*) = (5.62, 5.61)$.

TABLE 1
Three Pure Equilibria in the One-Shot Auction Offloading Game, and the Corresponding Equilibrium Bids b_i^*, Offloading Prices p_i^*, and Payoffs u_i of the SSPs

SSPs	NE1			NE2			NE3		
	$(\hat{p}_1, \hat{p}_1 - \epsilon)$			$(\hat{p}_1 + \epsilon, \hat{p}_1)$			$(\hat{p}_1 + 2\epsilon, \hat{p}_1 + \epsilon)$		
	b_i^*	p_i^*	u_i	b_i^*	p_i^*	u_i	b_i^*	p_i^*	u_i
SSP1	5.60	6	40	5.61	6	40	5.62	6	40
SSP2	5.59	5.59	31.8	5.60	5.60	32	5.61	5.61	32.2

7.4 2D Deployment Game Simulation

In this section, we consider a 2D flow model as discussed in Section 6.2 and show that the first mover in the strategic deployment game has the advantage to obtain a higher payoff. As shown in Fig. 21, given the first mover's strategy $f_1 \in [0,1]$, the second player can obtain the maximal payoff u_2 by playing his best response strategies. When the first mover plays *Nice* by covering only half of the area, i.e., $f_1 = 0.5$, if the second SSP plays *Nice* by non-overlapping, they both obtain the same payoff; If the second SSP plays *Adverse* by overlapping as much as possible, i.e., $f_{12}^* = f_1$, then u_1 drops below u_2. Therefore, the first SSP enters this market will place his APs to cover more than half of the total traffic to avoid the possible payoff loss.

8 Conclusion

This paper studies the economics of mobile data offloading through SSPs. We consider a cellular network with single MNO and two SSPs with overlapping coverage areas implementing uniform pricing schemes, where each SSP charges a uniform price for serving all the users within its coverage. We show that (a) a larger overlapping area will intensify the price competition between the SSPs and drive down the offloading prices in the market. (b) Simple uniform pricing scheme might be unstable and impractical to perform. (c) The proposed one shot auction mechanism is simple to implement and payoff-equivalent with the MNE in the standard free competition market. (d) The first mover in the strategic topological placement will deploy his APs to cover more than half of the cellular flow and obtain a higher equilibrium payoff.

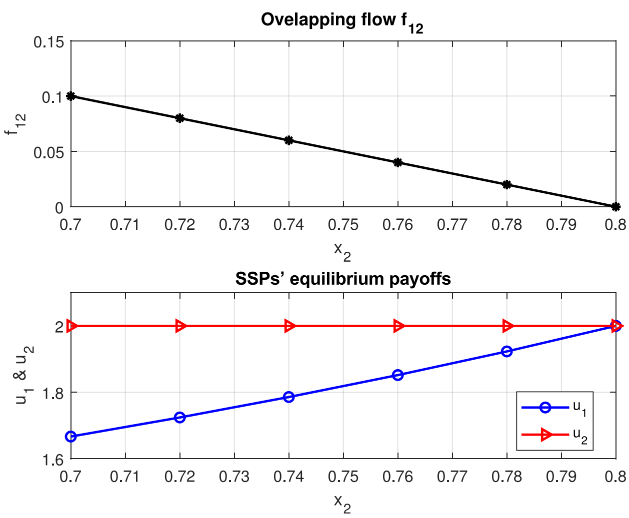


Fig. 20. The corresponding overlapping flow f_{12} and SSPs' equilibrium payoffs u_1, u_2 vary as $x_2^* \in [1-r, 0.5+r]$ and $x_1^* = 0.5 - r$, where $r = 0.3, L = 1$.

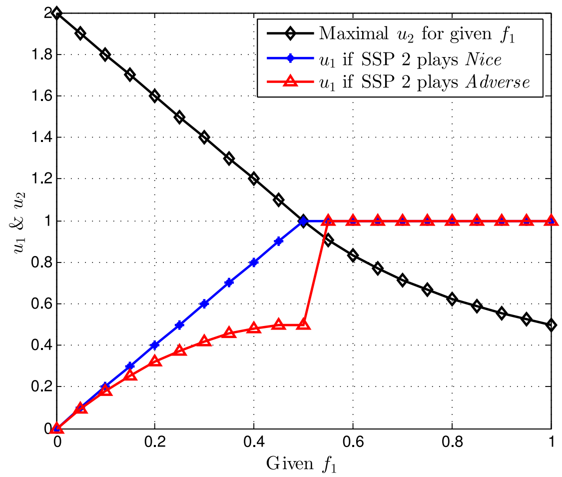


Fig. 21. The SSPs' deployment game in 2D flow model: SSP 1 chooses coverage $f_1 \in [0,1]$, and the second mover SSP 2 plays his best response strategies to maximize his payoff, which he can choose between *Nice* and *Adverse* for the same payoff.

There are a few limitations in our work. Our results rely on the assumption that the network traffic f_1, f_2, f_{12} information is available (as common knowledge) for all the players and the offloading capacity of the APs is unlimited, which may not be practical in the realistic scenarios. Therefore, investigating the problem when there is only partial flow information available for the SSPs and constrained offloading capacity of APs will be interesting, which we leave as a future work. Another future work is to consider the scenario where the serving cost of SSPs are different, and that the deployment cost of new APs is not negligible.

Appendix A
Proof of Proposition 4

Given SSP i's mixed strategy defined as (11), the payoff of SSP j with a price p_j is given by

$$u_j(p_j|G_i^{\mathrm{a}}) = \begin{cases} (f_j + f_{12})(p_j - c) & \text{if} \quad p_j \in [c, \hat{p}_i), \\ (f_j + f_{12})(\hat{p}_i - c) & \text{if} \quad p_j \in [\hat{p}_i, c_{\mathrm{M}}), \\ [f_j + 0.5 f_{12}(1 + f_j/f_i)](\hat{p}_i - c) & \text{if} \quad p_j = c_{\mathrm{M}}. \end{cases}$$

When $p_j \in [\hat{p}_i, c_{\mathrm{M}})$. As shown in Fig. 6, the expected payoff of SSP j is

$$u_j^{\mathrm{MNE}} = (f_j + f_{12})(\hat{p}_i - c) > u_j^{\mathrm{m}} = (f_j + f_{12})(\hat{p}_j - c),$$

and $u_j^{\mathrm{MNE}} > u_j(c_{\mathrm{M}}|G_i^{\mathrm{a}})$. Similarly to the symmetric game, given SSP j's mixed strategy as G_j^{a}, for any $p_i \in [c, c_{\mathrm{M}}]$, the expected payoff of SSP i is given by

$$u_i(p_i) = \begin{cases} (f_i + f_{12})(p_i - c) & \text{if} \quad p_i \in [c, \hat{p}_i), \\ (f_i + f_{12})(\hat{p}_i - c) & \text{if} \quad p_i \in [\hat{p}_i, c_{\mathrm{M}}]. \end{cases} \tag{32}$$

Thus, the mixed strategy pair $(G_i^{\mathrm{a}}, G_j^{\mathrm{a}})$ is a MNE.

APPENDIX B
PROOFS PF NE IN LINE FLOW MODEL

B.1 SSP 2's Best Response Function in Case III

When $x_2 \le x_1 + 2r$, the coverage area of the two SSPs starts to overlap.

(a) When $1 - x_1 < x_2 \le x_1 + 2r$, the coverage area of the two SSPs overlap in the interval $[x_2 - r, x_1 + r]$. The volume of the overlapping flow and the monopoly flow become $f_{12} = (x_1 + r) - (x_2 - r)$, $f_1 = x_2 - r$ and $f_2 = 1 - (x_1 + r)$. If $x_2 > 1 - x_1$, i.e., $f_1 > f_2$, SSP 2 is the player with a smaller amount of monopoly in the offloading game and using the results in (20), the payoff of SSP 2 is given by

$$u_2^{\mathrm{a}}(x_2) = (x_2 - r)\frac{1 - (x_2 - r)}{x_1 + r}(c_{\mathrm{M}} - c) > u_2^{\mathrm{non,max}},$$

and the maximization point can be obtained at $x_2 = 0.5 + r$, which yields

$$u_2^{\mathrm{a,max}} = \frac{1}{4}(x_1 + r)(c_{\mathrm{M}} - c).$$

(b) When $1 - r < x_2 \le 1 - x_1$, SSP 2 will become the one with a larger volume of monopoly flow, i.e., $f_1 \le f_2$, according to (20), the payoff of SSP 2 can be written as

$$u_2^{\mathrm{b}}(x_2) = [1 - (x_1 + r)](c_{\mathrm{M}} - c) = u_2^{\mathrm{non,max}}.$$

Note that, when $x_2 \ge 1 - r$, the overlapping flow $f_{12} = x_1 - x_2 + 2r$ increases as x_2 decreases, the monopoly flow $f_2 = 1 - (x_1 + r)$ remains the same.

(c) When $x_1 \le x_2 < 1 - r$, SSP 2 will still be the player with a larger monopoly flow $f_2' = x_2 - x_1$. However, f_2 starts to decrease as x_2 decreases and the payoff decreases as well and is given by

$$u_2^{\mathrm{c}}(x_2) = (x_2 - x_1)(c_{\mathrm{M}} - c) \quad \text{if } x_2 \in [x_1, 1 - r).$$

Note that, when $x_2 = x_1$, the coverage of the two SSPs are collocated, which will drive the offloading price close to c.

(d) When $0 \le x_2 < x_1$, the monopoly flow of SSP 2 becomes zero and its payoff can be written as

$$u_2^{\mathrm{d}}(x_2) = (x_1 - x_2)\frac{x_2 + r}{x_1 + r}(c_{\mathrm{M}} - c) \quad \text{if } x_2 \in [0, x_1),$$

which yields the maximal point at $x_2 = 0$, and the corresponding maximal payoff is given by

$$u_2^{\mathrm{d,max}} = \frac{x_1 r}{x_1 + r}(c_{\mathrm{M}} - c).$$

Therefore, SSP 2's best response function can be denoted as

$$B_2(x_1) = 0.5 + r \quad \text{if } x_1 \in (r, 0.5]. \tag{33}$$

B.2 SSP 2's Best Response Function in Case IV

(a) When $1 - r \le x_2 \le x_1 + 2r$, there is overlapping in $[x_1 + r, x_2 - r]$, and the monopoly flows are $f_1 = (x_2 - r) - (x_1 - r) = x_2 - x_1$ and $f_2 = 1 - (x_1 + r)$. Since $f_1 \ge f_2$, SSP 2 is the player with a smaller amount of monopoly flow with payoff function given by

$$u_2^{\mathrm{a}}(x_2) = (x_2 - x_1)\frac{1 - (x_2 - r)}{2r}(c_{\mathrm{M}} - c) > u_2^{\mathrm{non,max}},$$

and the maximization can be achieved at $x_2 = \frac{1}{2}(1 + x_1 + r)$ with

$$u_2^{\mathrm{a,max}} = \frac{1}{8r}(1 + r - x_1)^2(c_{\mathrm{M}} - c).$$

(b) When $x_1 \le x_2 \le 1 - r$, the monopoly flows are $f_1 = (x_2 - r) - (x_1 - r) = x_2 - x_1$ and $f_2 = (x_2 + r) - (x_1 + r) = x_2 - x_1$. Moreover, we have $f_{12} + f_1 = 2r$ and $f_{12} + f_2 = 2r$, which yields a symmetric offloading game with payoff given by

$$u_2^{\mathrm{b}}(x_2) = (x_2 - x_1)(c_{\mathrm{M}} - c) \quad \text{if } x_2 \in [x_1, 1 - r),$$

which decreases as x_2 decreases and turns to zero when $x_2 = x_1$ and the two APs are collocated.

(c) When $r \le x_2 < x_1$, we have a symmetric game with $f_1 = f_2 = x_1 - x_2$ and the payoff of the SSP 2 is given by

$$u_2^{\mathrm{c}}(x_2) = (x_1 - x_2)(c_{\mathrm{M}} - c) \quad \text{if } x_2 \in [r, x_1),$$

which yields an increasing function as x_2 reduces.

(d) When $x_2 - r < 0$, i.e., $x_2 \in [0, r)$, the monopoly flows are $f_1 = x_1 - x_2$ and $f_2 = x_1 - r$. In addition, we have $f_2 + f_{12} = x_2 + r$ and $f_1 + f_{12} = 2r$, such that SSP 1 is the player with a larger amount of monopoly flow, and the payoff of SSP 2 is given by

$$u_2^{\mathrm{d}}(x_2) = \frac{x_1 - x_2}{2r}(x_2 + r)(c_{\mathrm{M}} - c) \quad \text{if } x_2 \in [0, r),$$

and the maximization can be achieved at $x_2 = \frac{x_1 - r}{2}$, which yields

$$u_2^{\mathrm{d,max}} = \frac{(x_1 + r)^2}{8r}(c_{\mathrm{M}} - c).$$

Therefore, SSP 2's best response function is given by

$$B_2(x_1) = \frac{1}{2}(1 + (x_1 + r)) \quad \text{if } x_1 \in [r, 0.5]. \tag{34}$$

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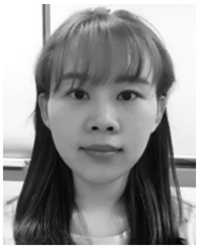

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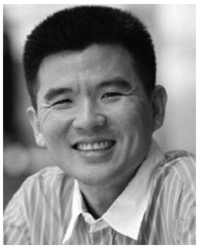

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