Percolation in Random Resistor Networks

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1 Introduction

Percolation deals with the properties of clusters created by some probabilistic means. Generally, past a threshold an infinite cluster will form that spans the lattice and we consider this a phase transition. While the process that generates this cluster is simply stated, its properties are quite rich, resisting analytic treatment and exhbiting complex phenomena such as multifractal scaling. The marriage of conceptual simplicity and complex behavior makes percolation a popular introduction to phase transitions, and the generality of the model makes it applicable to a wide variety of physical problems from superconductivity to forest fires.

This coverage aims to give an introduction to percolation theory with particular attention to the problem as a conduction transition in a random resistor network. We will follow our coverage of the Ising model, first introducing percolation and giving brief attention to it's variants and correspondence to physical transitions. Then we will review exact solutions on simple lattices, namely in one dimension and the Bethe lattice. Next we will review mean field theory techniques away from the transition and then scaling relations derivable from assumptions about the form of the infinite cluster. Finally, we will review applications of renormalization, and give a summary of extensions to more complex examples.

1.1 The Basic Percolation Problem

Consider a finite cubic lattice where neighboring sites are not yet connected. At every potential edge, place a bond with a probability, or **concentration** p. As we increase this concentration from 0, at some point instantiations of the lattice will contain clusters that connect from the top to the bottom and we say the lattice **percolates**. As the size of the lattice increases, the point at which a percolating cluster forms becomes more sharply defined and in the infinite limit we can define a critical concentration p_c beyond which an infinite cluster exists.

We may consider a similar problem where bonds are all occupied but *sites* are initially vacant. Each site is occupied with a probability p and clusters are collections of occupied sites connected by bonds.

1.2 Critical Exponents

1.3 Physical Correspondences

2 Exact Solutions

2.1 One-Dimension

2.2 Bethe Lattice

3 Mean-Field Techniques

3.1 Mean Field Theory

3.2 Effective Medium Theory

4 Scaling Relations

5 Renormalization

6 Extensions