Renormalization and Singular Percolation Theory

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1 Dimensional Analysis is Incredibly Useful

The Buckingham Pi Theorem [?] asserts that any equation that completely describes the relation between a collection of physical quantities takes the form,

$$\Pi = f(\Pi_0, \Pi_1, ..., \Pi_n) = 0$$

where the Π_is are all the independent dimensionless products that may be formed from the given quantities. As an example of it's utility, consider the diffusion equation in one dimension,

$$\partial_t u(x,t) = \frac{1}{2}\kappa \partial_{xx} u(x,t)$$

with an initial condition $u(x,0) = \frac{A_0}{\sqrt{2\pi l^2}} e^{-x^2/2l^2}$. By forming the dimensionless quantities,

$$\Pi = \frac{u\sqrt{\kappa t}}{A_0} \quad \Pi_1 = \frac{x}{\sqrt{\kappa t}} \quad \Pi_2 = \frac{l}{\sqrt{\kappa t}}$$

we may immediately infer a solution of the form,

$$u = \frac{A_0}{\sqrt{\kappa t}} f\left(\frac{x}{\sqrt{\kappa t}}, \frac{l}{\sqrt{\kappa t}}\right)$$

which forms a starting point for futher inquiry.

1.1 Similarity Solutions

Once the dimensional form of the equation is established, it is often productive to examine similarity solutions. In these functions, the arguments occur such that that length and time scales are interdependent. (As an example $u(x,t) = t^\alpha f(xt^\beta)$.) By reducing the number of arguments of the scaling function f, similarity solutions often allow us to convert PDEs to ODEs greatly facilitating their solution. In addition, the long term behavior of a system is often given by similarity solutions which may be evidence of natural stabilization [?]. We call this regime, where times/distances are large enough that boudary/initial values no longer influence the system but the system is still far from equilibrium **intermediate asymptotics**