The Risk and Risk-free Rate of T-bills

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Abstract

This study argues that a payment's risk approaches zero as maturity approaches zero, and that the central bank's short-term rate best captures the risk-free rate of various assets. Expecting that the T-bill risk largely reflects a country's inflation risk or macrorisk, we measure the risk as a one-year payment's risk to be comparable across various assets. We that the risk-free rate reflects the expected inflation effect, and that the inflation risk reflects the unexpected inflation effect or the investment opportunity cost. To simplify the formulas, we also examine if the two returns can be computed independently. We employ two factors to model the risk-free rate, which the market expects the current monetary policy to bring towards the neutral level over a certain period. The 4-factor models include a risk constant over maturity to capture the depreciation cost of inter-bank short-term lending. We thus use different models to split T-bill returns into the risk and the risk-free rate with repeated trials to minimize the prediction errors, thereby solving the model factors. The 4-factor independence model outperforms all the other models on average explaining 99.3% of the US T-bill returns. The T-bill metrics is the gateway to the risks of various corporate assets. We have to develop the 4-factor independence model into software to analyze the noise of the risk and the risk-free rate that probably follows a blended normal distribution of Nie (2023).

Keyword: Macrorisk; bond risk; monetary policy; risk-free rate; neutral interest rate.

JEL Classification: G34, J33, M12.

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I have nothing to disclose

1. Introduction

We argue that a payment's risk approaches zero as maturity approaches zero, [1] and that the central bank's short-term rate best captures the risk-free rate. Intuitively, the more distant a payment, the less informed the risk, thus the riskier the payment. This appears to be the most plausible explanation of the literature evidence that a treasury bond's yield is positively associated with maturity (see, e.g., Mishkin, p. 124, 2016). This also challenges conventional "risk-free" rate, as captured by T-bill yield that actually decreases over time. Therefore, conventional "risk-free" rate cannot be risk-free, because the competing argument is obviously doubtful—a bill's risk increases, or remains the same, whereas the "risk-free" rate declines, over maturity. An increase in the central bank's short-term rate, for instance, reduces a T-bill's value. Since the risk approaches zero at maturity, the return immediately prior to maturity is the most risk-free. The difference between the rate at the start and that at maturity, therefore, represents the difference between the risks of the two points of T-bill maturity. The short-term rate of central bank, where maturity is as short as one day, therefore, best reflects the risk-free rate of various assets. The risk-free rate can be seemingly higher than T-bill yield when the market expects the interest rate to decrease soon. Obviously, a T-bill holding can lose thousands of times more than inter-bank short-term lending in case of an interest rate increase, which casts serious doubt on conventional risk-free rate.

We treat a country's T-bill risk as the macrorisk that largely reflects the inflation risk in the country. To find the microrisk of a corporate asset, we have to compute the risk-free rate and the

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[1] This is the risk over a unit of time length, which is in contrast to asset holding risk that approaches zero as the holding time length approaches zero even when the risk over a unit of time length remains unchanged over time (Nie, 2023). The returns of our analyses are continuously compounded whereas the market and the central banks use annual rates.

macrorisk of T-bills in the country. We calibrate the T-bill risk as a one-year payment's risk to be comparable across assets, modeling the linear changes in risk and risk-free rate. A corporate payment with a face value carries the T-bill risk that may cause noise in the payment's micro-risk (i.e., default risk or firm risk). [2] This study delineates the risk and risk-free rate of US and Canada T-bills, accounting for the future changes as expected by the market. For instance, US and Canada treasury bonds with long maturity in May 2024 yield significantly lower than the risk-free rate because the market expects the central banks to start rate decreases in the second half of the year.

We also examine if we can assume that the risk and the risk-free returns are independent of each other, which significantly simplify, thereby instrumenting, the formulas. To solve the model factors, we use repeated trials to minimize the mean absolute value (MAV) of prediction errors. In addition, our continuously compounding return of a constant rate $r(t)$ is given by: [3]

$$\hat{Y}_T(r) = \int_0^T \left(e^{r(t)} - 1\right)dt = T(e^r - 1). \tag{1}$$

where T is maturity, and r is the return rate.

Our rates are different from conventional continuously compounding return of a constant rate:

$$\hat{Y}_T(r) = \int_0^T re^{rt}dt = e^{rT}. \tag{2}$$

As a result, annual rate in this study is also different from conventional annual rate. A constant rate estimated in this study should be slightly higher than a conventional annual rate, as our continuous compounding rate appear to be a simple rate that never compounding. Readers may suggest that we transform our rates into conventional ones. Unfortunately, our evidence indicates

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[2] In this study, a bond's face value is the principal plus the interest (i.e., the return or the asset holding cost over maturity).

[3] For instance, given annual rate 4.58%, the continuously compounded rate is equal to ln(1+0.0458) = 4.478%.

that conventional rates probably mix the risk with the risk-free rate since a partially continuously compounding (henceforth, PCC) strategy reduces the models' power in predicting T-bill returns.

As the fundamentals of our asset risk theory, this study argues that a payment's risk approaches zero as maturity approaches zero, and that the central bank's rate best reflects the risk-free rate, thereby addressing a conventional misconception about the risk-free rate. The results strongly support our modeling of the risk and the risk-free rate. Our first round of trials yields an MAV above 15%, in contrast to a mean MAV 0.7% from the 4-factor independence model (see, Table 4), which suggests that the model on average explains 99.3% of the T-bill returns. Thus, the model represents the gateway to asset risk analyses. We strongly believe that mean MAV can drop below 0.3% with advanced software developed.

The remainder of this study is organized as follows. Section 2 describes the 4-factor and 3-factor models. Section 3 describes the computation methodology. Section 4 presents the results from examining the yield of recent US and Canada T-bills. Section 5 concludes.

2. The Methodology and Models

2.1 The independence versus dependence returns

We can compute a bond's the risk $r'_f(t)$ and the risk-free rate $r_f(t)$ returns over maturity T independently as:

$$\hat{Y}_T(r'_f) = \int_0^T (e^{r_f(t)} + e^{r'_f(t)} - 2)\, dt. \tag{3}$$

Obviously, an independence model implies that the risk-free return does not generate a risk return, and vice versa, over maturity. We can also compute the return dependently as:

$$\hat{Y}_T(r'_f) = \int_0^T (e^{r_f(t) + r'_f(t)} - 1)\, dt. \quad (4)$$

We define interaction return as the difference between the returns obtained from (3) and (4): [4]

$$\text{Interaction Return} = \int_0^T (e^{r_f(t) + r'_f(t)} - e^{r_f(t)} - e^{r'_f(t)} + 1)\, dt. \quad (5)$$

Due to the return interaction, it is more difficult to delineate the two returns over a period of time than at a point of time. The interaction return is negligible for small returns especially over a short period such as a year. [5] As it is more difficult to obtain asset risk from dependence returns, we horserace the independence and dependence models by examining their power in predicting US and Canada T-bill returns.

2.2 The linearly declining risk of T-bills

We argue that a payment's risk approaches zero as maturity approaches zero. To be comparable across various assets, we measure the T-bill risk as a one-year payment's risk, that is, $r'_f(1) = r'_f$. Given maturity *T*, a T-bill's risk at time *t* is given by:

$$r'_f(t) = r'_f(T - t), \quad t \in (0, T). \quad (6)$$

As illustrated in Figure 1, a T-bill's return declines over maturity *T* because the bond's risk approaches zero as maturity approaches zero. The risk $r'_f(t)$ and conventional "risk-free" rate, that

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[4] A simple example to obtain the interaction return. Suppose we have three 10-year bonds. The first one is the risk bond with a yield 2%, the second one is the risk-free bond with a yield 3%, and the third bond combine the both characters with a yield 5%. Then the interaction return is given by: $(1 + 5\%)^{10} - (1 + 2\%)^{10} - (1 + 3\%)^{10} - 1 = 6.6\%$.

[5] A plausible separation of the two returns should mirror the proportions of $\int_0^T (e^{r_f} - 1)\, dt$ and $\int_0^T (e^{r'_f(t)} - 1)\, dt$, or the proportions of $r_{f(t)}$ and $r'_{f(t)}$ over *T*. To separate the two parts, we can distribute the interaction return by their proportions of returns over maturity (i.e., $\frac{\int_0^T (e^{r'_f(t)} - 1)\, dt}{\int_0^T (e^{r_f(t)} - 1)\, dt + \int_0^T (e^{r'_f(t)} - 1)\, dt}$ and $\frac{\int_0^T (e^{r_f(t)} - 1)\, dt}{\int_0^T (e^{r_f(t)} - 1)\, dt + \int_0^T (e^{r'_f(t)} - 1)\, dt}$), or the mean rates over *T* (i.e., $\frac{\overline{r'_f}}{\overline{r_f} + \overline{r'_f}}$ and $\frac{\overline{r_f}}{\overline{r_f} + \overline{r'_f}}$). To simplify the analysis, we can treat $\int_0^T e^{r_f + r'_f}\, dt - \int_0^T e^{r'_f}\, dt$ as the risk return and $\int_0^T (e^{r_f} - 1)\, dt$ as the risk-free return, thereby sorting the interaction return wholly into the risk.

is, $r_f(t) + r'_f(t)$, early (late) in maturity are both higher (lower) than their means over T. Also, we expect that a corporate bond's micro-risk (i.e., firm risk or default risk) approaches zero as maturity approaches zero (See, Figure 2). [6]

[Please insert Figures 1 and 2 about here]

2.3 The linearly moving risk-free rate

For US and Canada T-bills in early 2024, we observe that a T-bill's yield is negatively (positively) related to maturity when maturity is shorter (longer) than 5 years. The yield pattern suggests that the market expects the monetary policy to bring the current interest rate r_{f0} to the neutral level r_{fn}. Also, we expect that the market cannot predict the direction of the next monetary policy. Therefore, the linearly moving risk-free rate can be specified as:

$$rf(t) = \begin{cases} r_{f0} + \frac{(r_{fn} - r_{f0})t}{T_m} \\ r_{fn},\ (t > T_m) \end{cases}. \tag{7}$$

where r_{f0} is the current effective interest rate, and T_m is the number of years for the current monetary policy to move from r_{f0} to r_{fn}.

In case $T \leq T_m$, the risk-free return over T can be expressed as:

$$\hat{Y}_T(r_f) = \int_0^T (e^{r_f(t)} - 1)\, dt$$

$$= \int_0^T e^{r_{f0} + \frac{(r_{fn} - r_{f0})t}{T_m}}\, dt - T$$

[6] We assume that a country's treasury bills are issued under the same terms regardless of their issue dates, suggesting that a country's treasury bonds have the same risk at a given time.

$$= \frac{T_m}{r_{fn} - r_{f0}} \int_{e^{r_{f0}}}^{e^{r_{f0} + \frac{(r_{fn} - r_{f0})T}{T_m}}} de^{r_{f0} + \frac{(r_{fn} - r_{f0})t}{T_m}} - T$$

$$= \frac{e^{r_{f0}} T_m}{r_{fn} - r_{f0}} \left(e^{\frac{(r_{fn} - r_{f0})T}{T_m}} - 1 \right) - T. \quad (8)$$

In case $T > T_m$, we set *T* in formula (8) to T_m to obtain $\hat{Y}_{T_m}(r_f)$. Then the return beyond T_m is:

$$\hat{Y}_{T_{m0}}(r_f) = \int_{T_m}^{T} (e^{r_f(t)} - 1)\, dt$$

$$= (T - T_m)(e^{r_{fn}} - 1). \quad (9)$$

We expect that the risk-free rate reflects the expected inflation effect over *T*, where the constant risk captures the depreciation cost of the interbank short-term lending rate. Therefore, asset value including the risk-free return mirrors the nondepreciating asset value over *T*.

2.4 The partially continuously compounding (PCC)

As in in formulas (8) and (9), the continuously compounding rates and the annual rates are more like simple rates in this study. To examine the models' robustness, we allow the risk-free return generated in the first period to generate a return in the second period (i.e., partially continuously compounding). Then, the risk-free independence return (which includes the constant risk) over *T* can be computed as:

$$\hat{Y}_T(r_f) = \hat{Y}_{T_m}(r_f) + \hat{Y}_{T_{m0}}(r_f) + \hat{Y}_{T_m}(r_f) \times \hat{Y}_{T_{m0}}(r_f)$$

$$= \frac{(T - T_m)T_m(e^{r_{fn}} - 1)(1 - T_m + e^{r_{fn}} - e^{r_{f0}})}{r_{fn} - r_{f0}} - 1. \quad (10)$$

Or alternatively, we can treat r_{fn} as a conventional continuously compounding rate:

$$\hat{Y}_{T_{m0}}(r_f) = e^{r_{fn}(T-T_m)} - 1. \tag{11}$$

To examine if the inclusion of PCC improves the model's power in explaining T-bill returns, we repeat all the tests by excluding $\hat{Y}_{T_m}(r_f) \times \hat{Y}_{T_{m0}}(r_f)$ from the risk-free returns:

$$\hat{Y}_T(r_f) = \hat{Y}_{T_m}(r_f) + \hat{Y}_{T_{m0}}(r_f). \tag{12}$$

The risk-free return $\hat{Y}_T(r_f)$ of 4-factor models additionally includes the constant risk r_s, which captures the depreciation cost of the interbank lending rate as a correction to the risk-free rate.

2.5 The models

The 4-factor independence model

The model includes a depreciation cost r_s constant over maturity to reflect the additional depreciation rate of inter-bank short-term lending. We use r_s as a correction to, or an error of, the current (expected) risk-free rate $r_f(t)$. Then the T-bill's return can be written as:

$$\hat{Y}_T(r'_f) = \int_0^T (e^{r_{f0} + r_s + \frac{(r_{fn} - r_{f0})t}{T_m}} - 1)\, dt + \int_0^T (e^{r'_f(T-t)} - 1)\, dt$$

$$= e^{r_s}\, \hat{Y}_T(r_f) + \frac{e^{r'_f T} - 1}{r'_f} - T. \tag{13}$$

The 4-factor dependence model

Similar to formulas (8) and (9), we obtain the returns within T_m as:

$$\hat{Y}_T(r'_f) = \int_0^T (e^{r_{f0} + r_s + \frac{(r_{fn} - r_{f0})t}{T_m} + r'_f(T-t)} - 1)\, dt$$

$$= \frac{e^{r_{f0} + r_s}}{\frac{r_{fn} - r_{f0}}{T_m} - r'_f} \int_{r'_f T}^{\frac{(r_{fn} - r_{f0})T}{T_m}} e^{\frac{(r_{fn} - r_{f0})t}{T_m} + r'_f(T-t)}\, d\left(\frac{(r_{fn} - r_{f0})t}{T_m} + r'_f(T-t)\right) - T$$

$$= \frac{e^{r_{f0} + r_s}}{\frac{r_{fn} - r_{f0}}{T_m} - r'_f} \left(e^{\frac{(r_{fn} - r_{f0})T}{T_m}} - e^{r'_f T} \right) - T. \tag{14}$$

In case $T > T_m$, we compute the return beyond T_m as:

$$\hat{Y}_{T_{m0}}(r'_f) = \int_{T_m}^{T} (e^{r_{fn} + r_s + r'_f(T-t)} - 1)\, dt$$

$$= T_m - T + \frac{-e^{r_{fn} + r_s}}{r'_f} \int_{r'_f(T - T_m)}^{0} de^{r'_f(T-t)}$$

$$= \frac{e^{r_{fn} + r_s}}{r'_f} (e^{r'_f(T - T_m)} - 1) + T_m - T. \tag{15}$$

We use formula (14) to obtain $\hat{Y}_{T_m}(r'_f)$. Then the dependence return over *T* is given by:

$$\hat{Y}_T(r'_f) = \left(1 + \hat{Y}_{T_m}(r'_f)\right)\left(1 + \hat{Y}_{T_{m0}}(r'_f)\right) - 1$$

$$= \hat{Y}_{T_m}(r'_f) + \hat{Y}_{T_{m0}}(r'_f) + \hat{Y}_{T_m}(r'_f)\, \hat{Y}_{T_{m0}}(r'_f). \tag{16}$$

The 3-factor independence model

The return over *T* drops the constant risk r_s:

$$\hat{Y}_T(r'_f) = \hat{Y}_T(r_f) + \int_0^T (e^{r'_f(T-t)} - 1)\, dt$$

$$= \hat{Y}_T(r_f) - \frac{1}{r'_f} \int_{r'_f T}^{0} e^{r'_f(T-t)}\, d\left(r'_f(T-t)\right) - T$$

$$= \hat{Y}_T(r_f) + \frac{e^{r'_f T} - 1}{r'_f} - T. \tag{17}$$

The 3-factor dependence model

Similar to the return of the 4-factor dependence model, the T-bill's return within and beyond T_m ignoring r_s can be computed respectively as:

$$\hat{Y}_T(r'_f) = \frac{e^{r_{f0}}}{\frac{r_{fn} - r_{f0}}{T_m} - r'_f}\left(e^{\frac{(r_{fn} - r_{f0})T}{T_m}} - e^{r'_f T}\right) - T. \tag{18}$$

$$\hat{Y}_{T_{m0}}(r'_f) = \frac{e^{r_{fn}}}{r'_f}\left(e^{r'_f(T - T_m)} - 1\right) + T_m - T. \tag{19}$$

We convert the rates drawn from the central bank and the market into our continuously compounding rates for estimation purpose and reverse the conversion to report the rates. Figure 3 sketches the risk and risk-free returns of US T-bills. Given the market yield y, we first obtain the predicted yield $\hat{y}$. Then, we compute the prediction error as:

$$\text{Prediction Error} = \frac{\hat{y} - y}{y} = \frac{10^{\frac{\log\left(\hat{Y}_T(r'_f) + 1\right)}{T}} - 1}{y} - 1. \tag{20}$$

Similar to independence models, we finally remove $\hat{Y}_{T_m}(r'_f)$ $\hat{Y}_{T_{m0}}(r'_f)$ from dependence models to examine the effect of PCC strategy on the models' performance. For future work, we have a 3-factor model without the risk-free rate break point:

$$\hat{Y}_T(r'_f) = \int_0^T \left(e^{r_{fn} + \frac{(r_{f0} - r_{fn})}{1 + kt} + r'_f(T - t)} - 1\right) dt. \tag{21}$$

where k is the factor controlling the slope of the risk-free rate curve $r_{fn} + \frac{(r_{f0} - r_{fn})}{1 + kt}$.

The risk-free rate reflects the expected inflation effect, whereas the inflation risk captures the unexpected inflation effect or the investment opportunity cost that approaches zero as distance approaches zero. Therefore, the nondepreciating value over a period is defined as the asset value accounting for the risk-free return that includes the interbank lending rate plus the depreciation cost of the interbank short-term lending rate. The risk-free return of a nondepreciating value, however, does not generate a risk return, and vice versa, as implied by the dependence models.

3. The Computation Methodology

This study assumes that the solution of model factors is the one amongst all solution candidates that produces the lowest MAV. To this end, we use repeated trials to minimize the mean errors to solve the model factors T_m, r'_f, r_{fn}, and, if applicable, r_s. To be a solution candidate, we require that any change in any one of the factors cannot reduce the MAV by 10^{-8} (which we can set to 10^{-9} or 10^{-10} with software developed. An experimenter may miss some potential candidates, thereby missing the candidate that has the lowest MAV. We observe that dependence models are more likely to have multiple candidates, thereby requiring advanced trial strategies. Generally, a candidate's MAV monotonically increases if any one of the factors moves away from the point. However, this may not the case for T_m as MAV can decrease over some periods of *T*. Usually, we can locate a new candidate when we observe that the second order differential of the model factor (e.g., $\frac{\partial^2 MAV}{\partial {T_m}^2}$) changes the sign. This finding provides a clue for more complicated candidate searches, which is out of the scope of this study.

A manually computing experimenter is more likely to miss some candidates when shortcuts are critical to the process. To analyze the impact of such failure, our experiments are focused on the 4-factor independence model with US T-bill data on 2 Aug 2024 when the model generally reveals high MAVs. To analyze the impact of such failure, Table 1 reports the candidates that our

4-factor independence model produces. [7] We observe that the T-bill risk r'_f, the most important factor of our estimation, mostly remains unchanged although T_m and/or MAV show different values. However, the T-bill risk decreases by about 20% for the highest MAV, which provides an important clue for prospect software developers. [8] Therefore, we can safely use a solution that even fails to obtain the lowest MAV. A rule of thumb is that the mean MAV is below 2.0% for 9 T-bills for instance. For US T-bills with the 4-factor independence model using formula (12) rather than (10), as reported in Table 4, the mean MAV from our manual computation is 0.78%, which can drop below 0.3% with advanced software developed. According to our experience, a reasonable T_m is between 0.5 to 2.5 years. To locate a solution efficiently, for example for US and Canada T-bills, we set the start point as: T_m = 0.5 years, r'_f= 25 bp, r_{fn} = 2.5%, and r_s = 4 bp. Or alternatively, we can use the previous solution as the start point of the current experiment.

4. The Results

Tables 2 and 3 report the results from the independence and dependence models, respectively. For Tables 2 and 3, Panels A and B present the results from US and Canada T-bills, respectively. The (non)bold numbers are results from the (3-)4-factor models. The results reveal strong power of the 4-factor independence model in predicting T-bill returns, on average explaining 99.1% of US T-bill returns on the two days initially examined in this paper. More

[7] We choose the Canada data, which appear to be noisier than the US data. The former produces a mean MAV 1.2% whereas the latter 0.9%.

[8] For independence returns, one candidate has T_m 1.87 yrs and r'_f 99.9 bps whereas another has T_m 0.90 yrs and r'_f 81.6 bps. We report the worst candidate to reveal potential issues. A test of several points (i.e., positions) between the possible range of 0.5 and 2.0, such as 0.5, 0.8, 1.1, 1.4, 1.7, and 2.0, is likely to solve such a problem. In particular, focus first on changing the other model factors to minimize MAV when examining these points. For more advanced analysis, one can similarly test each of the other model factors over the possible range of the factor.

importantly, the model similarly explains 99.1% of the returns on including on the 13 dates of US T-bills reported in Nie (2023). The Canada data we extracted from ycharts.com yield a mean MAV of 1.4%, which appear to be noisier than the US data we obtained from the US Fed website. In particular, we observe that the results from the 4-factor (in)dependence model have a mean MAV of (1.1%)1.2%, supporting that on average the 4-factor independence model have stronger power in predicting T-bill yields than the 4-factor dependence model. We also find that the 3-factor (in)dependence model yields a mean MAV of (2.0%)1.3%. The 4-factor (in)dependence model produces a T-bill risk of (37)35 bp for US data on 3 May 2024 and (40)32 bp for US data on 11 Feb 2025. For Canada T-bill data, the 4-factor (in)dependence model yields a risk of (20)18 bp on 3 May 2024 and (20)20 bp on 11 Feb 2025. The inflation risk reflects the market concern regarding the future face value, whereas the inflation rate released by the government captures the expected inflation effect on the asset's face value. The results from the 4-factor model produces a interbank lending depreciation cost between −0.1 and −8.8 bps for US bills and a depreciation rate between −3.7 and −23.3 bp for Canada bills. The 4-factor (in)dependence model predicts that the US Fed will move the effective rate of 5.33% on 3 May 2024 to the neutral rate of (3.40%)3.92% in (2.7)1.9 years, and move the rate of 4.33% on 11 Feb 2025 to the neutral of (3.63%)4.09% in (1.9)1.1 years. The 4-factor (in)dependence model also predicts that Bank of Canada will bring the effective rate of 5.02% on 3 May 2024 to the neutral rate of (2.90%)3.23% in (2.4)1.9 years, and move the effective rate of 3.02% on 11 Feb 2025 to the neutral rate of (2.57%)2.64% in (2.0)0.86 years. The T-bill risk, the interbank lending depreciation rate, and the neutral rate predicted by the 4-factor independence and dependence models are qualitatively similar in this study. The results in Nie (2023) probably support that the independence model performs better than the dependence model, as 13 dates represents a more comprehensive analysis. As expected, the results suggest that the 3-factor models have weaker power in explaining T-bill returns. We obtain qualitatively similar results when defining the prediction error as the deviate to the market return (i.e., $\frac{\hat{Y}-Y}{Y}$), where Y is the market return.

Table 4 tabulates the more important evidence of this paper: the 4-factor independence model explains 99.3% of US bill returns on 14 dates after excluding the PCC, which reduces the magnitude of overall prediction errors by about 20%. This important finding indicates that conventional rates probably entangle the risk with the risk-free rates. For four reasons, we do not report the results from the 4-factor dependence model. First, we do not expect that the dependence model can possibly outperform the independence model. Second, as the dependence model is more difficult to handle, the manual computation process can easily involve hidden mistakes. Third, the 4-factor dependence model generally yields higher MAV with the PCC. Fourth, the results may inconclusive as the model examines T-bills on only 4 days.

[Please insert Tables 2 to 4 about here]

The return independence can significantly simplify the formulas in our future asset risk analyses. The results support that the 4-factor independence model explains 99.3% of US bill returns after removing the PCC. Including the PCC, the model only predicts 98.6% of Canada T-bill returns and 99.1% of US T-bill returns initially reported in this paper and 13 days' data in Nie (2023). To sum up, the evidence strongly supports our argument that a country's T-bill risk approaches zero as maturity approaches zero. As illustrated in Figure 3, we find that a T-bill's risk approaches zero as maturity approaches zero, and that the risk-free rate linearly moves toward the neutral level r_{fn} in T_m years. A T-bill's risk reflects the return in excess of the central bank's rate after accounting for the baseline cost of inter-bank lending, although a T-bill's yield can be lower than the current effective rate when the market expects the latter to significantly decline in the near future.

5. Conclusion

This study proposes that the central bank's short-term interest rate best captures the risk-free rate of various assets, and that a payment's risk approaches zero as maturity approaches zero. We measure the risk as a one-year payment's risk to be comparable across assets. We use two other factors to model that the market expects the current monetary policy to linearly move the central bank's rate towards the neutral level over a certain period. The 4-factor model additionally includes a depreciation cost constant over maturity to capture the cost of inter-bank lending. The 4-factor independence model examines how the return independence assumption impacts the model's power in predicting T-bill returns. Thus, the 3- and 4-factor models separate US and Canada T-bill yield into the risk and the risk-free rate. We use US and Canada T-bill yield on 3 May 2024 and 11 Feb 2025 to solve the model factors. In particular, we minimize the prediction errors with repeated trials. The 4-factor independence model excluding the PCC explain 99.3% of US T-bill yield on average. We also observe that the depreciation cost constant over bond maturity significantly reduces the prediction errors for both dependence and independence models. The models can predict the differently number of years for the central bank to move the current effective rate to the neutral level, they generally predict inflation risk and neutral rate similarly.

The results strongly support our argument that the central bank's short-term rate best reflects the risk-free rate of the country's various assets. The T-bill risk reflects the inflation risk or the macrorisk for assets with a face value denominated by the depreciating currency, acting as inflation premium for assets without a face value. We argue that a payment's risk approaches zero as maturity approaches zero. The delineation of the risk and the risk-free rate in this study represents the gateway to the risk of various assets. Since T-bill risk captures a country's inflation risk, a corporate bond's risk in excess of the T-bill risk is a firm's default risk, which also approaches zero as maturity approaches zero.

We expect that the risk-free rate captures the expected inflation effect while the risk reflects the unexpected inflation effect or the investment opportunity cost, supporting that asset value accounting for the risk-free return, which includes interbank short-term rate and the constant risk or the depreciation cost of the interbank lending rate, represents nondepreciating asset value over time. According to our knowledge, our models and the computation methodology have never been used in any mathematical or financial literatures. Without developing the 4-factor independence model into software, we cannot examine the noise of the risk and the risk-free rate, which follows a blended normal distribution as proposed in Nie (2023).

REFERENCES

Mishkin, Frederic S., 2016, The economics of money, banking, and financial markets (11th edition), The Pearson Series in Economics, (Columbia University, US).

Nie, George Y., 2023, Address challenges Markowitz (1952) faces: A new measure of asset risk, Available at SSRN https://ssrn.com/abstract=4506410.

Figure 1. The risk-free rate and the T-bill risk

This figure shows that a T-bill's returns include the risk-free rate $r_f(t)$, which is captured the central bank's short-term rate, and the T-bill risk (i.e., the macrorisk or inflation risk) which is indicated by the green line $r_f(t) + r'_f(t)$. The T-bill returns decline over maturity T because the T-bill risk $r'_f(t)$ approaches zero as maturity approaches zero. The risk r'_f is measured as a one-year payment's risk to be comparable across assets:

$$r'_f(t) = r'_f(T - t),\ t \in (0,\ T). \tag{6}$$

Given the current bond price D_0 and face value D_T, the market return $\hat{Y}_T(r'_f)$ is equal to $\frac{D_T}{D_0} - 1$, the risk-free return can be computed as:

$$\hat{Y}_T(r_f) = \int_0^T (e^{r_f(t)} - 1)\, dt. \tag{8}$$

Then the predicted independence return is given by:

$$\hat{Y}_T(r'_f) = \int_0^T (e^{r_f(t)} + e^{r'_f(t)} - 2)\, dt. \tag{4}$$

The bond's price can be computed as:

$$D_0 = \frac{D_T}{1 + \hat{Y}_T(r'_f)}. \tag{23}$$

The Risk-free Rate and T-bill Return

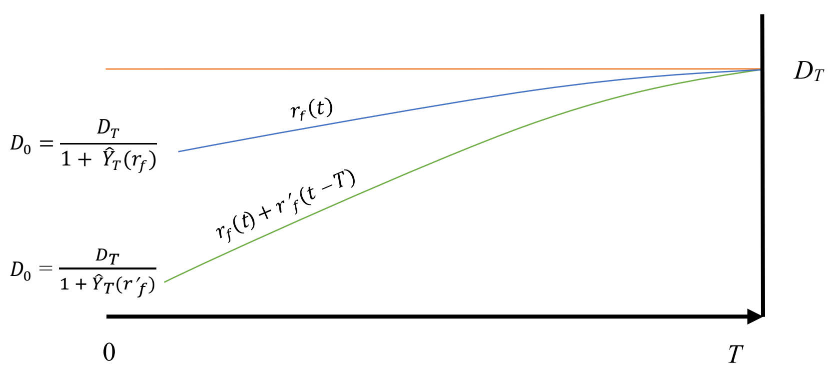

Figure 2. A corporate bond's risks declining over maturity
This figure illustrates the linearly declining risks of a corporate bond, where $r_f(t)$ is the risk-free rate captured by the short-term rate of the central bank. The T-bill risk $r'_f(t)$ is the macrorisk (i.e., inflation risk) and the default risk r_d is the corporate bond's micro-risk. Asset value is nondepreciating after accounting for the risk-free return. The two risks are measured as a one-year payment's risk:

$$r'_f(t) = r'_f(T - t), \quad t \in (0, T). \tag{6}$$

$$r_d(t) = r_d(T - t), \quad t \in (0, T). \tag{22}$$

The Linearly Declining Risks of a Corporate Bond

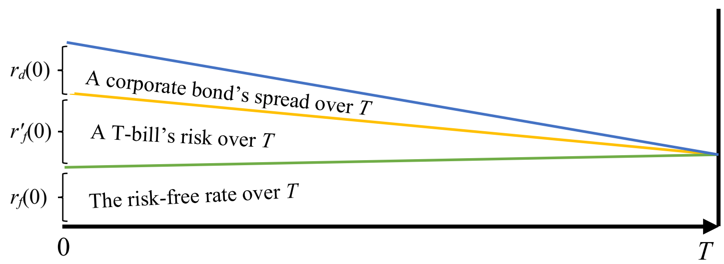

Figure 3. The linearly declining risk of US treasury bills

This figure shows that a US treasury bond's risk $r'_f(t)$ approaches zero as maturity approaches zero, and that the market expects the risk-free rate $r_f(t)$, that is, the Fed's short-term rate, as indicated by the green line, to take T_m years to linearly decline to the neutral level r_{fn}. To obtain the risk, we use US T-bill yields on May 3, 2024 (see, Table 2 Panel A) to solve the model's three factors of formulas (18) to (19). In particular, $r'_{f,k}(0)$ is a bond's risk at the start of k-year maturity, where r'_f is the country's T-bill risk. We convert r_{f0} into continuously compounding rate for the model and reversely convert the rates to report in Table 2, which are still different from conventional concept.

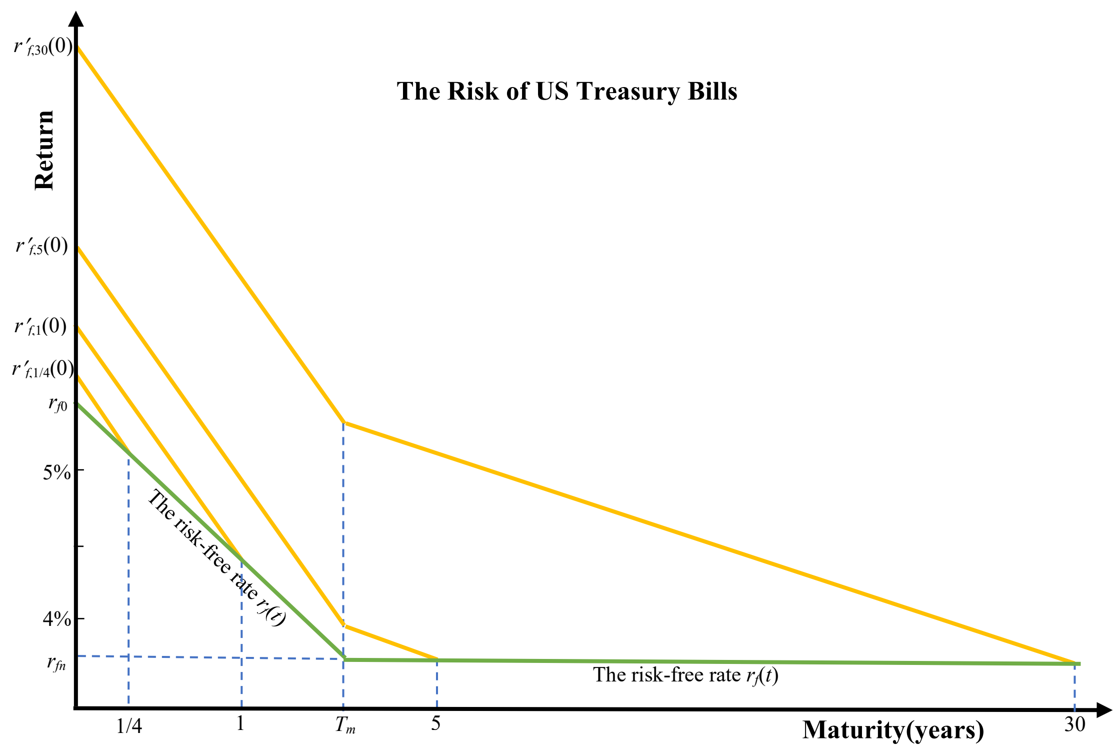

Table 1. Sample solution candidates of independence return

This table presents solution candidates obtained from the 4-factor independence models, which assume that the risk and the risk-free returns are independent of each other. The solution is in bold numbers that has the lowest MAV. The model assumes that the market expects the current monetary policy to take T_m years to move the current effective rate r_{f0} to the neutral level r_{fn}, where r'_f is the T-bill risk. The T-bill risk reflects a one-year payment's risk to be comparable across assets. The candidates are obtained from the 4-factor models for Canada T-bills on 3 May 2024, where MAV the mean absolute value of prediction errors, and r_s is the interbank lending depreciation cost constant over T-bill maturity T. Formulas (13), (17) and (18) give the T-bill return. The central bank's short-term rate can be expressed as:

$$rf(t) = \begin{cases} r_{f0} + \frac{(r_{fn} - r_{f0})t}{T_m} \\ r_{fn}, (t > T_m) \end{cases} . \tag{7}$$

where r_{f0} is the current effective rate of the central bank's rate, r_{fn} is the neutral rate, and T_m is the number of years that the central bank is expected to bring the current rate to the neutral rate.

The market return over T is equal to $(1 + y)^T - 1$, where y is the market yield. Then, the error of a bond's predicted return is defined as the ratio of deviation to the market yield:

$$\text{Prediction Error} = \frac{\hat{y} - y}{y} = \frac{10^{\frac{log(\hat{Y}+1)}{T}} - 1}{y} - 1. \tag{20}$$

Maturity (years)	1/12	1/4	1/2	1	2	3	5	10	30
	The 4-factor dependence model: US T-bills on 2 Aug 2024, r_{f0} = 5.33%								
Market Yield %	5.54	5.29	4.88	4.33	3.88	3.7	3.62	3.80	4.11
	-1.89	**-1.22**	**0.79**	**0.00**	**-1.66**	**0.00**	**1.95**	**1.25**	**-2.50**
	-1.94	-1.26	0.75	-0.03	-1.61	0.02	1.83	0.80	-3.34
Prediction error%	-1.69	0.08	3.91	7.22	1.63	0.29	0.53	0.57	0.29
	-3.28	-1.80	1.54	3.81	-1.12	-1.21	0.00	0.51	0.00
	-4.00	-3.00	-0.47	0.00	-1.63	0.00	1.86	0.98	-2.97

	1.25%		**1.04yrs**		**28.8 bp**		**2.83%**		**6.0 bp**
	1.29%		1.04yrs		28.0 bp		2.85%		6.0 bp
MAV =	1.80%	T_m =	1.60yrs	r'_f =	33.1 bp	r_{fn} =	2.48%	r_s =	4.1 bp
	1.47%		1.41yrs		32.5 bp		2.62%		3.0 bp
	1.66%		1.10yrs		28.4 bps		2.95%		-5.5 bp

Table 2. The risk and risk-free returns of US and Canada T-bills: the independence models

This table reports the risk and risk-free returns, assuming that the risk and risk-free returns are independent of each other. Panels A and B reports returns for US and Canada T-bills, respectively. Formulas (13), (16) and (17) give the returns of the 3- and 4-factor models. The models assume that the market expects the current monetary policy to take T_m years to move the current effective rate r_{f0} to the neutral level r_{fn}, where r'_f is the inflation risk or T-bill risk, and MAV is the mean absolute value of prediction errors. The results from the 4-factor models are in bold numbers, which includes a interbank lending depreciation cost r_s constant over the bond's maturity.

Panel A

Maturity (years)	1/12	1/4	1/2	1	2	3	5	10	30
US T-bills on 3 May 2024, r_{f0} = 5.33%									
Market Yield %	5.36	5.39	5.36	5.12	4.83	4.66	4.51	4.52	4.67
Prediction error%	**0.12**	**-1.23**	**-1.84**	**0.41**	**1.75**	**0.00**	**-1.02**	**0.95**	**0.00**
	1.59	-0.05	-1.07	0.36	0.00	-2.08	-1.01	1.84	0.00
MAV =	**0.81%**	T_m =	**2.5 yrs**	r'_f =	**18.8 bp**	r_{fn} =	**3.86%**	r_s =	**-7.8bp**
	0.89%		1.98 yrs		37.1 bp		3.84%		N/A
US T-bills on 11 Feb 2025, r_{f0} = 4.33%									
Market Yield %	4.38	4.35	4.31	4.24	4.28	4.30	4.34	4.51	4.71
Prediction error%	**-0.58**	**-0.31**	**0.00**	**0.20**	**-1.83**	**0.00**	**1.26**	**3.19**	**-1.06**
	13.3	0.00	-2.75	-2.31	-1.97	0.00	1.16	2.61	-2.29
MAV =	**0.94%**	T_m =	**1.7 yrs**	r'_f =	**39 bp**	r_{fn} =	**3.65%**	r_s =	**-5.7 bp**
	2.93%		0.13yrs		37 bp		3.85%		N/A

Data source, US Federal Reserve, accessed on May 1, 2025: https://fred.stlouisfed.org/series/DGS10.

Panel B

Maturity (years)	1/12	1/4	1/2	1	2	3	5	10	30
Canada T-bills on 3 May 2024, r_{f0} = 5.02%									
Market Yield	4.95	4.90	4.82	4.36	4.16	4.04	3.67	3.65	3.58
Prediction error%	**0.52**	**-0.02**	**-0.70**	**4.79**	**0.00**	**-3.83**	**1.00**	**0.01**	**0.00**
	3.10	2.34	1.34	6.26	0.00	-4.13	0.53	-0.35	0.21
MAV =	**1.21%**	T_m =	**2.0 yrs**	r'_f =	**19 bp**	r_{fn} =	**3.28%**	r_s =	**-3.4bp**
	2.03%		1.9 yrs		19 bp		3.20%		N/A
Canada T-bills on 11 Feb 2025, r_{f0} = 3.02%									
Market Yield	3.00	2.87	2.82	2.77	2.72	2.71	2.80	3.12	3.32
Prediction error%	**-6.17**	**-2.22**	**-0.95**	**-0.08**	**0.01**	**1.38**	**1.19**	**-2.66**	**0.00**
	0.00	-4.82	-5.03	-3.41	0.00	2.29	2.59	-1.51	0.00
MAV =	**1.63%**	T_m =	**2.0yrs**	r'_f =	**20 bp**	r_{fn} =	**2.57%**	r_s =	**-23.3bp**
	2.18%		0.02yrs		20 bp		2.54%		N/A

Data source, accessed on May 1, https://ycharts.com/.

Table 3. The risk and risk-free rate of US and Canada T-bills: the dependence models

This table presents the risk and risk-free rate of US (Panel A) and Canada (Panel B) T-bill yields on 3 May 2024 and 11 Feb 2025. The models assume that the market expects the current monetary policy to take T_m years to move the current effective rate r_{f0} to the neutral level r_{fn}. The model factors' values are computed with repeated trials to minimize the mean absolute value (MAV) of prediction errors. The results in bold numbers are those from the 4-factor model, which includes the interbank lending depreciation cost. A bond's dependence returns can be expressed as:

$$\hat{Y}_T(r'_f) = \int_0^T \left(e^{r_f(t)+r'_f(t)} - 1\right) dt$$

$$= \hat{Y}_{Tm}(r'_f) + \hat{Y}_{Tm0}(r'_f) + \hat{Y}_{Tm}(r'_f) \times \hat{Y}_{Tm0}(r'_f). \quad (16)$$

where $\hat{Y}_{Tm}(r'_f)$ and $\hat{Y}_{Tm0}(r'_f)$ are given by (18) and (19) for the 3-factor model and (14) and (15) for the 4-factor model.

Panel A

Maturity (years)	1/12	1/4	1/2	1	2	3	5	10	30
				US T-bills on 3 May 2024, r_{f0} = 5.33%					
Market Yield %	5.36	5.39	5.36	5.12	4.83	4.66	4.51	4.52	4.67
Prediction error%	**1.55**	**-0.07**	**-1.06**	**0.44**	**0.00**	**-1.45**	**0.00**	**2.71**	**0.00**
	1.60	-0.02	-1.02	0.47	0.00	-1.45	0.00	2.71	0.00
MAV =	**0.81%**	T_m =	**1.9 yrs**	r'_f =	**35 bp**	r_{fn} =	**3.92%**	r_s =	**-0.1bp**
	0.81%		1.9 yrs		35 bp		3.92%		N/A
				US T-bills on 11 Feb 2025, r_{f0} = 4.33%					
Market Yield %	4.38	4.35	4.31	4.24	4.28	4.30	4.34	4.51	4.71
Prediction error%	**-0.59**	**-0.03**	**0.72**	**1.77**	**-0.71**	**0.00**	**0.00**	**0.40**	**-4.85**
	0.95	1.50	2.21	3.18	0.00	-0.85	-0.97	1.56	0.00
MAV =	**1.01%**	T_m =	**1.1 yrs**	r'_f =	**32 bp**	r_{fn} =	**4.09%**	r_s =	**-6.3bp**
	1.25%		1.6 yrs		38 bp		3.86%		N/A

Panel B

Maturity (years)	1/12	1/4	1/2	1	2	3	5	10	30
				Canada T-bills on 3 May 2024, r_{f0} = 5.02%					
Market Yield	4.95	4.90	4.82	4.36	4.16	4.04	3.67	3.65	3.58
Prediction error%	**2.19**	**1.43**	**0.44**	**5.31**	**-0.94**	**-4.82**	**0.00**	**-0.72**	**-0.05**
	2.87	1.67	0.00	3.38	-4.07	-6.12	0.06	0.00	0.00
MAV =	**1.77%**	T_m =	**1.9 yrs**	r'_f =	**18 bp**	r_{fn} =	**3.23%**	r_s =	**-4.1 bp**
	2.02%		1.5 yrs		18 bp		3.26%		N/A
				Canada T-bills on 11 Feb 2025, r_{f0} = 3.02%					
Market Yield	3.00	2.87	2.82	2.77	2.72	2.71	2.80	3.12	3.32
Prediction error%	**-2.26**	**1.19**	**1.51**	**0.00**	**-0.59**	**0.53**	**0.28**	**-3.31**	**0.00**
	0.82	2.52	0.00	-2.94	-1.72	0.00	0.19	-3.20	0.00
MAV =	**1.08%**	T_m =	**0.86yrs**	r'_f =	**20 bp**	r_{fn} =	**2.64%**	r_s =	**-11.3bp**
	1.27%		0.49yrs		20 bp		2.49%		N/A

Table 4. The performance of the 4-factor independence model excluding the PCC

This table reports the results from separation of the risk and the risk-free rate of US treasury bills excluding the PCC, which stands for the partially continuously compounding return $\hat{Y}_{T_m}(r_f) \times \hat{Y}_{T_{mo}}(r_f)$ in formula (10). Panel A describes the T-bill data on the 14 days. Panel B reports the T-bill metrics computed from the T-bill data on these days. The results indicate that the exclusion of this return in formula (10) reduces the mean MAV from 0.9% to 0.7%. This suggests that the model on average explains 99.3% of US T-bill returns without the PCC, in contrast to 99.1% on 14 days as reported in Nie (2023) and Table 2 of this paper.

Panel A

Maturity (years)	1/12	1/4	1/2	1	2	3	5	10	30	
Date	Market Yield %									US Fed effective rate
28-Nov-2023	5.53	5.47	5.42	5.21	4.73	4.49	4.29	4.34	4.52	5.33%
11-Dec-2023	5.55	5.47	5.40	5.14	4.71	4.42	4.25	4.23	4.32	5.33%
18-Dec-2023	5.52	5.46	5.36	4.95	4.43	4.15	3.94	3.95	4.05	5.33%
9-Feb-2024	5.49	5.44	5.26	4.86	4.48	4.25	4.14	4.17	4.37	5.33%
1-Apr-2024	5.49	5.44	5.36	5.06	4.72	4.51	4.34	4.33	4.47	5.33%
2-Apr-2024	5.49	5.42	5.34	5.05	4.70	4.51	4.35	4.36	4.51	5.33%
2-May-2024	5.47	5.46	5.43	5.21	4.96	4.79	4.64	4.63	4.74	5.33%
3-May-2024	5.36	5.39	5.36	5.12	4.83	4.66	4.51	4.52	4.67	5.33%
29-May-2024	5.50	5.46	5.43	5.20	4.96	4.79	4.63	4.61	4.74	5.33%
2-Jul-2024	5.48	5.47	5.36	5.07	4.74	4.54	4.36	4.43	4.60	5.33%
2-Aug-2024	5.54	5.29	4.88	4.33	3.88	3.70	3.62	3.80	4.11	5.33%
15-Oct-2024	4.93	4.73	4.42	4.18	3.95	3.86	3.86	4.03	4.32	4.83%
18-Oct-2024	4.92	4.73	4.45	4.19	3.95	3.86	3.86	4.08	4.38	4.83%
11-Feb-2025	4.38	4.35	4.31	4.25	4.29	4.28	4.37	4.54	4.75	4.33%

Panel B

Maturity (years)	1/12	1/4	1/2	1	2	3	5	10	30		T-bill metrics				
Date	Prediction error (%)										MAV	T_m (yrs)	r_{fn}	r'_f (bps)	r_s (bps)
28-Nov-2023	0.09	-0.08	-1.02	-0.78	1.61	0.00	-0.01	0.91	0.00		0.50%	2.65	2.99%	39	9
11-Dec-2023	0.00	0.10	-0.63	0.31	1.14	0.00	-1.70	-0.48	0.01		0.49%	2.67	2.92%	35	10
18-Dec-2023	0.62	-0.06	-0.87	1.77	2.15	0.00	-0.31	-0.15	0.00		0.66%	2.25	2.82%	30	12
9-Feb-2024	0.43	-0.46	0.19	2.71	0.00	-0.36	0.00	2.21	0.00		0.71%	1.85	3.13%	35	8
1-Apr-2024	0.00	-0.25	-0.51	1.83	2.04	0.00	-1.68	0.00	0.18		0.72%	2.89	2.96%	38	4
2-Apr-2024	-0.04	0.06	-0.22	1.90	2.24	0.00	-0.55	1.12	0.00		0.68%	2.64	3.15%	38	4
2-May-2024	0.66	0.00	-0.70	0.97	1.11	0.00	-1.34	0.68	0.00		0.61%	3.15	3.24%	42	5
3-May-2024	1.17	-0.43	-1.41	0.11	0.00	-1.22	0.00	2.59	0.00		0.77%	2.38	3.52%	40	-1
29-May-2024	0.11	0.00	-0.70	1.16	1.11	0.00	-0.78	1.53	0.00		0.60%	3.07	3.31%	42	5
2-Jul-2024	0.92	-0.16	0.00	1.87	1.28	-0.40	0.00	1.68	-0.30		0.73%	2.37	3.24%	39	8
2-Aug-2024	-0.13	0.27	1.85	0.00	-3.51	-2.25	0.00	0.95	0.00		0.99%	1.02	2.65%	32	16
15-Oct-2024	-1.14	0.27	2.96	0.00	-2.81	-1.73	0.00	2.00	0.00		1.21%	1.04	3.13%	35	-1
18-Oct-2024	-0.92	0.32	2.37	0.00	-2.61	-1.45	0.00	1.71	0.00		1.04%	1.06	3.12%	36	-1
11-Feb-2025	-0.57	-0.02	0.71	1.73	0.00	-0.14	-0.09	2.21	0.00		0.61%	2.40	3.61%	20	-6
										Mean	**0.74%**				