The competitive dynamics of invalid voting in French legislative elections, 2002-2022

Jocelyn Evans (University of Leeds)

Abstract

Despite a recent increase in studies, spoiled and blank ballots remain an under-theorised aspect of voting behaviour. The French case provides an opportunity, conceptually and empirically, to analyse a number of competitive dynamics which should influence rates of vote invalidation. The stable institutional context of two-round legislative elections between 2002 and 2022 provides electoral district-level data to examine how outcomes from the first round encourage vote invalidation at the subsequent run-off. Starting from a multi-level time-series cross-section specification, this paper finds that the more that candidate choices and performance deprive voters of second-round choice, the higher the rates of invalid voting. In particular, electoral thresholds ousting otherwise competitive candidates increase invalid voting. Despite the radical shift in party system array across this 20-year period, however, it finds that the candidate alignments which best characterize those voter pools deprived of choice remain relatively stable over time.

KEYWORDS

Blank and spoiled ballots - France - legislative elections - electoral law - party system

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Introduction

Invalid votes – spoiled or blank ballots – when intentionally cast constitute a rejection of either the political supply in an election, or of the institutional framework within which the election takes place. Despite recent, more formalized work, studies of invalid voting remain piecemeal. The majority of work favours the use of ecological data, at the national (for comparative) and subnational (for country-studies) level. Microstudies using surveys are less prevalent, principally because of the substantial under-reporting of invalid votes in such instruments, as well as because of the often small number of vote invalidations. Explanatory studies of invalid voting are potentially hampered by the confounding variable of erroneous voting – ballots spoiled through incorrect voting choices as defined by an electoral law, or by ballots unintentionally marked with nonetheless illegitimate content.

One country where vote invalidation has risen dramatically in recent years is France. There has been substantial change in the rates of invalid voting at the second, run-off round in the last two national elections, in 2017 and 2022 (see Figure 1). This has been linked to growing dissatisfaction with political supply and executive performance, alongside other protest phenomena such as populist voting, *les gilets*

jaunes and anti-government protests. To date, no-one has taken advantage of the stable institutional framework France offers electorally to understand if there are consistent drivers of invalid voting over time, and the extent to which those account for the increase in invalid voting. The last across-time study of France was published some half-century ago (Rosenthal and Sen 1973).

Second Tourids (2002-2022)

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Figure Again Again

Figure 1 Invalid voting in French legislative elections, first and second rounds (2002-2022)

Source: French Ministry of the Interior election data

France is an auspicious case in this regard. First, it offers two instances of a simple, stable electoral system, namely the presidential and legislative elections, with both featuring two rounds of uninominal majoritarian or plurality voting. This minimizes the risks of erroneous voting, with a simple closed selection system with no voter mark-up. Second, these elections have been stable in their order and scheduling

from 2002 until 2022. Third, the two-round run-off provides an ideal situation to look at how competition may motivate voter behaviour.

Unlike studies which rely upon the election result to characterize prior competition ex post, the outcomes of the first round will logically have a potential influence on voter actions in the second.

The paper, first, reviews the invalid voting literature, to identify the key hypotheses in invalid voting. Second, it characterizes the French electoral and party systems, hypothesising that invalid voting, as a political choice, will be partially a function of the competitive dynamics at play within the electoral cycle, both in terms of a static component – the electoral law – and a dynamic component – the competitive interaction between parties in the election. Third, it details the data set-up across the twenty-year period, and the justification of its starting-point of a two-level time-series cross-section negative binomial regression focusing on the second round of the legislative elections, before moving to election-specific models and other robustness tests. It finds evidence that there is consistent variation in rates of invalid voting related to the outcome of the first round, the subsequent implications for the run-off ballot choice, and the party system dynamics.

Causes and correlates of invalid voting

Studies of invalid voting remain limited, compared to other fields of voting, despite a recent uptick in research in countries where the

phenomenon has increased in prevalence (Kouba and Lysek 2019, 745). Much of the older research has focused on error-based invalidation, ie. where voters intend to vote for a party or candidate, but invalidate their vote through ambiguity of choice or incorrect process, often attributed to complexity of ballot design, and indirectly thereby to cognitive mobilization, literacy, age and socio-economic status (Sinclair and Alvarez 2004; Power and Garand 2007; Herrnson et al 2012; Pachón et al 2017). Whilst relevant in the study of multi-member open list systems, involving multiple marks and ordinal ranking on complex ballot papers, the French case requires a single, printed ballot paper to be placed in an envelope, thereby reducing the risk of unintentional ballot invalidation. A regional analysis of invalid votes in France by Bon and Cheylan (1988) rejected the characterization of invalid votes as simple errors. Similarly, Zulfikarpasic estimated the number of such errors to be very small (2001, 248). In the analysis below, we include some basic controls for possible socioeconomic drivers of unintentional invalidation - which may account for variance in invalid voting - but given the simple ballot design, and in the absence of a means of differentiating between intentional and unintentional invalidation, assume that the latter will be minimal.

Institutional explanations of invalid voting dominate the literature, with compulsory voting in particular identified as a key driver (McAllister and Makkai 1993; Power and Garand 2007; Dejaeghere and Vanhoutte 2016). Compulsory voting is not relevant to the French case, but informal pressures of social conformity have been identified as incentivizing

turnout in more sparsely populated rural areas, where invalid voting rather than abstention has traditionally been more prevalent as a means of expressing discontent (Ranger 1970; Percheron et al 1987; Bussi 1998; Zulfikarpasic 2001, 261). There is evidence that the urban-rural divide is still relevant in contemporary French politics (Brookes and Cappellina 2023) but data limitations mean we can only test indirectly for urban-rural difference, using population density, in the three most recent elections (Appendix 5).

Political party system and competitive measures have been hypothesized to influence invalid voting. However, the evidence has been mixed. In terms of party system array, the number of parties or candidates competing has been suggested as a driver of political satisfaction, through a lack of choice among smaller numbers of candidates / greater choice through larger numbers (Högström et al 2022); or a driver of confusion through complexity of choice among a large number of candidates (Cunow et al 2021). Cohen (2018) finds that both effects obtain – large numbers of candidates associating with higher rates of invalid voting, but across-election increases in choice through higher numbers of candidates associating with lower rates of invalid voting.

Competitivity generally reduces to the margin of victory of the winning party or candidate over second place (Fornos et al 2004; Uggla 2008). In uncompetitive elections with landslide outcomes, many voters are assumed to be turned off by having no tactical choice, and therefore spoil

their votes in protest. Conversely, in close races, where individual votes will count more, fewer voters choose to waste their vote. Any causal direction requires a number of strong assumptions, and we still cannot discount reverse causality whereby a close-run race may to some extent be so because few people chose to spoil their vote for other reasons.

The most formalized analysis of multiple elections in France was carried out by Rosenthal and Sen (1973) on four legislative elections between 1958 and 1968, using a rational choice framework to look at run-off abstention and invalidation using an early aggregate operationalization of Downsian spatial proximity theory (Riker and Ordeshook 1968; Downs 1957). Heinsohn (2018) provides a convincing cross-sectional analysis of invalid votes in the second round of the 2017 presidential election, in particular positing that the proportion of such votes is related to the proportion of losing voters in an area (in this case, commune) at the first round, ie those voters whose first-round candidate did not reach the runoff. For the purposes of this paper, we will refer to such voters using the Pierce-Adams 'thwarted voters' label (1998, 167). As a cross-sectional analysis, however, this necessarily begs the question as to how stable such a 'thwarted' dynamic is across election cycles, and their specific competitive contexts and, as we examine below, how such voters should be identified within the French party and electoral systems. Pons and Tricaud test the effect of two- and three-candidate run-offs on invalidation in the context of understanding expressive voting and the competitive effects of third candidate presence, finding that the presence of a third candidate reduces invalid voting by 3.7% (2018, 1632). Silva and Crisp (2022) look at ballot spoilage as an outcome of choice-limiting ballot structure in local elections. Finally, Kim tests a choice model and loss-aversion for presidential run-offs in 2017, where invalid votes constitute a choice for voters for whom both run-off candidates constitute a loss (2022).

The French electoral space and party system

In terms of competition, the French case presents a number of institutional and competitive dynamics matching those detailed above. The French party system literature is rich in analyses of the changing shape of competition across governing and challenger parties. In keeping with many democratic systems (Duch et al 2010; Hagevi 2015) all French elections have been predicated upon blocs, rather than individual parties. From the so-called 'bipolar quadrille' – a moderate and radical party partnership within Left and Right blocs – of the 1970s and early 1980s (Duverger 1985) through the tripartition of the 1990s and early 2000s (Grunberg and Schweisguth 2003b; Gougou and Labouret 2013)¹ to the apparently bipartisan dominance of 2007 and 2012 (Grunberg and Haegel 2007), the moderate pluralism dynamic of centripetal competition and limited effective multiparty competition has dominated.

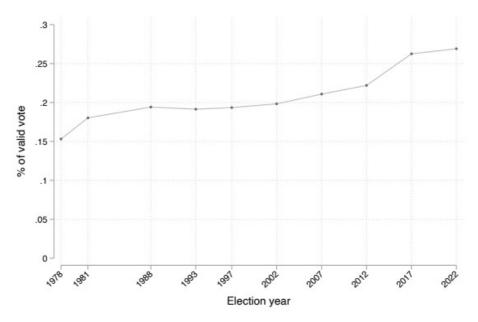
¹ We distinguish here between *tripartition* as an incontestable competitive array of parties at the system level, and *tripartition* as a classification of demographic and value profiles underpinning that, which has been debated (Grunberg and Schweisguth 1997, 2003a; Andersen and Evans, 2005; Gougou and Persico, 2017).

The electoral earthquake of 2017, placing a centrist government between substantial Radical Left (*La France Insoumise* – LFI) and Right (*Rassemblement,* formerly *Front National* – RN) challengers, and minority vestiges of the former Socialist (*Parti* Socialiste – PS) and Gaullist (*Union pour un Mounvement Populaire* – UMP / *Les Républicains* – LR) governing parties, shifted this dynamic to a multi-polar, centrifugal system of polarized pluralism, with a Centre-occupying government (*La République en Marche!* / *Renaissance*) allying with other Centre and Centre-Right parties (*Modem*) to retain a majority in 2022 (Gougou and Persico 2017; Raynaud 2022).

The coincidence between system changes and increased rates of invalid voting at first sight seem linked, but analyses of the latter to date have largely remained descriptive, identifying voter discontent with governing elites, and increased polarization across political parties, as a reason for voters' increased usage of spoiled and blank votes. In terms of testing the competitive dynamics of invalid voting, the presidential election stage of the cycle presents a number of obstacles, not least the single, national district which characterizes competition. Any breakdown of this district into subnational units will define competition through first-round outcomes delineated by arbitrary administrative boundaries. We may identify variation from the national result at a local level, but the competition is nonetheless national.

The legislative race is far more amenable. The spatial area is clearly defined through a set of stable electoral districts (circonscriptions). Competition is locally specific, with varying numbers of candidates, although clearly national parties will determine presence of candidates, but this is competitively ex ante. Most importantly, in all but a handful of cases, progression to the run-off will be determined by local competition alone, according to prescribed thresholds. Specifically, if any candidate wins 50% +1 of the vote, and at least 25% of the registered electorate, they win at the first round. If no candidate satisfies these criteria, the top two candidates, plus any candidate receiving 12.5% or more of the registered electorate, progress to the run-off. Changes made in 1966 and 1976 to the threshold for participation in the second round of the election, moving from 5% of valid vote to 10%, and subsequently 12.5%, of the registered electorate, were designed to limit the number of multicandidate run-offs that would 'distort' straight duels between parties of the governing Right and opposition Left (Cole 2000). Given the increase in abstention across legislative elections, this threshold has ensured that, for third party candidates to progress to the second round, a functional threshold of over 20% of the valid vote has been increasingly the norm across a majority of electoral districts (see Figure 2).

Figure 2 Effective run-off threshold, by year, in French legislative elections, first round (% of valid vote) since imposition of 12.5% threshold



Note: 1986 omitted, due to change to PR in electoral system. Source: author calculations from French Ministry of the Interior data 'Thwartedness' in French legislative elections

The baseline expectation of cross-round voting is that the probability of an invalid vote increases subsequent to a vote for a non-winning candidate in the previous round. Those voters who are thwarted by their candidates' failure to progress to the second round may abstain, vote for one of the remaining candidates, or cast an invalid ballot. As a naïve starting point, we expect that the larger the number of thwarted voters in a district, the larger the number of invalid votes at the second round. Because we rely upon ecological data, we do not control for abstention rate or change across rounds because of its strong positive association with both invalid voting rate and with many of our explanatory variables. Some previous models (e.g. Heinsohn 2018) have controlled for this. Consequently, Appendix B2 presents the same models but controlling for the change in abstention rate. Substantively, the findings remain effectively identical to the models presented here.

We expect that there are different classes of thwarted voter, dependent upon political alignment and upon electoral cycle. The two-round plurality electoral system has been characterized as one where one votes expressively in round one ('with one's heart'), and strategically in round two ('with one's head') (Beaudonnet et al 2014: 25; see also Baujard and Lebon 2022, on the tension between expressive and strategic voting). Yet, in a system characterized as bloc-based under the Fifth Republic, and increasingly two-bloc based until 2017, the absence of the favoured candidate does not necessarily imply that no candidate of any proximity is available if the acceptability of candidates is based upon bloc belonging. Rosenberg and Sen (1973) tested a similar hypothesis in the 1950s and 1960s, with a spatial proximity model using first-round vote distribution of first-round votes and survey-based estimations of party positions. Given the crowded competitive space of recent French legislative elections, with idiosyncrasies of smaller party groupings and candidate associations, as well as the stronger emphasis on political blocs as the structuring factor to the party system, rather than a unimodal centripetal logic, we choose here to adopt a nominal categorization of blocs to identify thwarted voters.

French political space as measured by official Ministry of the Interior (MI) data provides a number of progressively broader ways of identifying thwarted voters. How we adjudicate between codings is discussed later in the methodology section.

i) Candidate

The most restrictive definition of thwarted voter allocates any voter whose candidate does not progress to the run-off round to this category. Such a definition maximises the pool of potential invalid votes, assuming that all voters whose candidate progresses to the subsequent round will support them at the next. Previous models of invalid voting in France have used this simple categorization in their operationalization (Rosenberg and Sen, 1973; Heinsohn, 2018).

ii) Nuance

Candidate *nuance* in French elections is an administratively driven allocation of candidates to a pre-determined set of political labels indicating their main ideological. Candidates pick from a list of political organisations when they register their candidacy, and officials then allocate these labels to a reduced set – between 16 and 22 in the 2002-2022 period – based upon financial and organizational criteria. This may be a political party, where such an organization is standing in multiple districts, or has been a stable political actor over time, or in cases of small or less formalized networks may fall into one of the *divers* ('other') categories. The differences between candidate and nuance codings are relatively small, generally reflecting multiple *divers* codings, often in

specific contexts such as dissident candidates, or regional candidates in Brittany and Corsica.²

iii) Bloc

juin-2022/).

Despite using candidate performance at the district level, we are trying to infer thwarted voter response to candidate supply at the second round. Over time, we can be relatively certain that Left- and Right-wing parties will broadly be mutually exclusive (except in the case of the republican front - see below). Similarly, the cordon sanitaire - the rejection of cooperation with the Radical Right by the Moderate Right - would indicate a strong directive that Moderate and Radical Right blocs be separated. However, a delineation between Radical and Moderate Left is less clear (Grunberg and Schweisguth 2003b). Also, the status of the Centre changes considerably over time. Until 2007, candidates of the UDF and Modem clearly aligned Moderate Right, in terms of coalition participation. However, the 2012 elections saw a rupture after their leader's declaration in favour of the Socialist rather than conservative 2 The Ministry of the Interior categorization has been criticized for not reflecting party groupings and coalitions sufficiently accurately (e.g. http://www.france-politique.fr , http://geoelections.free.fr/). Given the particularly sharp criticism of the MI codings in 2022, Appendix 4 details models including alternative codings for 2022 from the LEGIS-2022 project - https://www.data.gouv.fr/fr/datasets/legis-2022-codage-desnuances-politiques-des-candidats-aux-elections-legislatives-des-12-et-19candidate. By 2017, and the implantation of Macron and LREM's centrism, no such bloc with LR can be assumed. At the demand level, in 2022 opinion polling indicates cross-party shifts were more fluid than in previous elections – for example, a quarter of Nupes voters supporting an RN candidate in RN/Ensemble! run-offs (Ivaldi 2023: 162). However, a larger proportion reported not choosing one of the candidates, suggesting potentially higher levels of invalid voting. In short, we would expect a multi-bloc classification as a better predictor of invalid votes than the two-bloc version. To avoid simply testing arbitrary codings, similarly to Gougou and Persico (2017) we employ those which correspond to the party system literature's posited structure, namely:

- Two-bloc the Left and Right blocs (*bipartisme*)
- Three-bloc two-bloc coding, but with the RN and other Radical
 Right candidates (*tripartition*) separate from a Moderate Right bloc
- Four-bloc three-bloc coding, but with the Centre parties
 separated from the Moderate Right (post-2017 tripolarisation)
- Five-bloc four bloc-coding but with the Radical Left parties separated from the Moderate Left (destabilization, posited in 2002 as well as post-2017)

Other sources of thwartedness

The 12.5% threshold

Research on France and elsewhere has emphasized how increased choice is beneficial in increasing turnout (Pons and Tricaud 2018; Bol and

Ivandic 2022) and in satisfaction with democracy (Högström et al 2022; but see also Dassonneville and McAllister 2020).

Analytically here, we are more interested in how the constraining of choice is democratically negative, in potentially increasing vote invalidation. As shown in Figure 1, rising abstention rates in legislative elections under the Fifth Republic have raised the effective vote threshold required to reach the second round (see also Evans and Ivaldi 2013, 164). Subsequent to the steep increase in invalidation in 2017, commentators specifically highlighted the absence of competition due to the 12.5% threshold (Grunberg and Missika 2021). Yet, in mainstream media, despite the historical decline of *triangulaires* (see Table 1), these have still been presented as a realistic possibility in many districts, rather than a vanishingly uncommon inflection in 1.8% of districts over five elections (and almost two-thirds of these in 2012 alone).

Table 1 Number of run-offs contested by 3 or more candidates (1958-2022)

1958 1962	353 159	106 4-way, 12 5-way and 2 6-way run-offs	5% of votes cast
1967	74	14 4-way run-offs 2 4-way run-offs	10% of
1973	97	1 4-way run-off	registered electate
1978 1981	1 1		12.5% of registered
1988 1993	9 15		electorate
1997	79		
2002 2007	10 1		
2012 2017	33 1		
2022	7		

Source: French Ministry of the Interior data

As Rosenthal and Sen identify, the number of candidates at the run-off should have a strong effect on the level of vote invalidation (1973, 42). Contrary to the context of legislative elections in the 1950s and 1960s, or contemporary local elections, the level of turnout and vote distribution by district ensures that four-way run-offs are non-existent in the new century's legislative elections, and *triangulaires* are extremely rare – 52 districts between 2002 and 2022, with 33 of these in 2012. Contrast this with 353 districts with more than two candidates (two with six candidates) in the run-off in 1958, or 97 in 73. Equally, the trend is not monotonic – in 1978 and 1981, the coordinated blocs of the *quadrille bipolaire* and the change in electoral threshold to 12.5% resulted in only a single three-way run-off for each, rising back to 79 in 1997.

Overall, there should be a number of third-placed candidates who, in terms of their share of the valid vote, appear competitive, and might be expected to progress to the second round, but do not do so, because they fail to reach the 12.5% of registered electors required. We would expect higher rates of vote invalidation among the thwarted voters of higher-profile, strongly performing third candidates than, for want of a better term, 'also-rans'.

Lack of or reduced competitiveness

In each election, a small number of districts see a second round with only one candidate standing, due to one of the second-round candidates withdrawing unilaterally or through a party agreement. As a purely ceremonial second round, we would expect higher rates of spoiled ballots driven by voters deprived of their qualifying candidate, and wishing to vote against the remaining candidate. It might be argued that candidates withdrawing to avoid splitting votes, or to reinforce the symbolic victory of their bloc, would conversely result in positive votes – for the remaining candidate – or simply abstention. This is undoubtedly true, but we would also expect a proportion of the departing candidates' voters to be dissatisfied with this outcome, and express this dissatisfaction accordingly.

A similar dynamic would be expected where, in a possible three-way runoff, one of the qualifying candidates withdraws. This may be to avoid splitting the vote within a single bloc, meaning that the supporters of such a candidate would not then find themselves allocated to any bloc-definition of thwarted voted. Alternatively this may be to avoid splitting the vote in the so-called *front républicain*, with a qualifying candidate from a competing bloc withdrawing to ensure the defeat of an RN or other Extreme Right candidate. In this case, the thwarted bloc will increase. Whilst both cases imply an increase in spoiled ballots due to the proactive withdrawal of a candidate, we test in Appendix 3 whether the *front républicain* results in fewer additional invalid votes than a bloc opposition would anticipate, given the cooperative strategy again the radical opposition. We find only very partial support for this.

Finally, greater rates of invalidation should be expected where the first-placed candidate enjoys a larger margin over the second-placed. Here, we should expect a small increase in invalid voting through a sense of the run-off being a foregone conclusion but voters still wishing to cast a vote, as opposed to close races where a vote will be more likely to make a difference (Fornos et al 2004; Uggla 2008).

Data and operationalisation

Ministry of the Interior data

The main dataset used in this paper is constructed from French Ministry of the Interior election data records, downloaded from data-gouv.fr (full data and transformation syntax available on REPLICATION). Full candidate, nuance and bloc codings have been constructed from the raw data. Bloc codings from listed nuances are available in Appendix 1. Individual election round datasets were merged by district. Our expectation is that vote invalidation should rise in the second round from the first – only 23 district-years varied from that pattern, with a single district (Var, 6th district, 2002) seeing the identical number of invalid votes. Nine of these exceptions were three-way run-offs, where we would expect lower rates of invalid voting; these cases also had a higher mean first-loser differential (8% of registered electorate) than cases where invalid voting was higher (5% of registered electorate).

Departmental-level controls

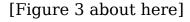
With the exception of a study of the 2017 legislative election,³ French census data are not published at the district level, or indeed in election years. We therefore use departmental census data as Level 2, within which electoral districts are nested. The use of the higher-level unit is supported by previous political geography work on France which has linked departmental-level conditions to invalid voting (Bussi 1998). At the departmental level, we include three predictors derived from the invalid voting literature (Zulfikarpasic 2001; Damore et al 2012; Kouba and Lysek 2019, 755) - age, coded as the proportion of retired inhabitants; education, coded as the proportion of inhabitants aged 25 or over with a university-level qualification; and unemployment, coded as the proportion of the active population without employment in the second quarter of the election year. Because the census years do not align with election years, we impute the three measures using linear interpolation / extrapolation from the INSEE data. Whilst the departmental level is spatially large, previous research has noted how this context still matters in national elections due similar socioeconomic conditions and political culture (Auberger and Dubois 2003; Arzheimer and Evans 2010; Evans and Ivaldi 2021). We include these as controls rather than explanatory variables, to check for any contextual demographic confounders.

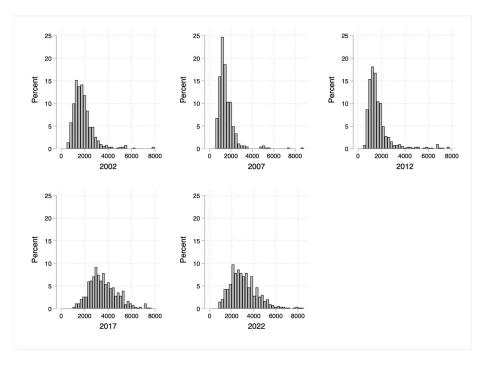
Dependent variable and functional form

³ 'Portraits des circonscriptions législatives Indicateurs économiques et sociodémographiques', https://www.insee.fr/fr/statistiques/6436478

We should recall that invalid voting is a scarce event, accounting for on average 2.0 and 4.3 per cent, by round, of the district's registered electorate. Bounded by 0 and the district size, and with a right-skewed distribution in each year (Figure 3) a count data functional form is more appropriate than a linear regression. The distribution is over-dispersed, breaking the simple Poisson assumption of variance being equal to the mean. Consequently, we use a negative binomial function which is robust to over-dispersion. Given population differences in district size by year, we include an exposure variable for the number of registered electors in the second round.

Figure 3 Distribution of invalid vote count, by year and election district, in French legislative elections, second round (2002-2022)





Because we wish to include socio-economic controls at the departmental level, we specify a mixed two-level model with fixed effects at Level 2. Observations are district-years, with a fixed time variable included to look across the cross-sectional panel for dynamic changes, in particular in the political competitive variables. We subsequently break this down into cross-sectional models, where observations are districts.

Most voting models will use the percentage of valid vote (*exprimés*) as their measure, given this is used to calculate winners of seats / executive positions, and sums to 100% across the candidate pool. Evidently, the total number of voters, valid or otherwise, (*votants*) could be more appropriate to look at invalid voting. Moreover, in looking at behaviour in the second round, based upon first-round predictors, both voters and abstainers from the first round are eligible to vote in the second. Consequently the total number of electors (*inscrits*) suggests itself as the relevant electoral denominator.

Hypotheses

H1 The higher the proportion of thwarted voters in the first round, the higher the rate of invalid voting in the second round.

For our first-round loser blocs, we take the number of voters as a proportion of *inscrits* as the available pool of potential invalid votes. For all bloc codings, we retain a miscellaneous 'Other' category, picking up regionalists, rural parties, and other non-aligned parties, where we have

no theoretical reason to add this usually small group to any of the Left, Centre or Right blocs.

Two restrictions should be noted here. First, we do not test for candidate effects (Sineau and Tiberj 2007; Brouard and Kerrouche 2013). We also do not test *consignes* – where losing candidates advise their supporters whom, if anyone, to support at the second round. Conceptually, we do not believe that such *consignes* will have significant sway over voters intending to spoil their ballot. Either voters intending to cast an invalid ballot will do so irrespective of candidate instruction; or the candidate instruction can be expected to align with precisely the political blocs for which we control. Cases of within-bloc candidates advising an invalid vote would be rare.

H2 The smaller the proportion of registered electors by which the third candidate misses the 12.5% threshold, the higher the rate of invalid voting in the second round.

The third-candidate threshold differential is taken as a proportion of *inscrits*, reflecting the rate of vote increase, including abstainers turning out, that would have allowed the third candidate to stand in the run-off.

H3 The larger the margin of victory of the leading candidate in the first round, the higher the rate of invalid voting in the second round.

We employ a different logic for the margin of victory variable. Here, whilst we avoid as much as possible any strong inference of individual behaviour or motivations from ecological data, we inevitably expect the reported vote for the first placed candidate ahead of the second (ie. the two first-round winners) to weigh on the value of a second-round vote. Consequently, we use the difference between the first- and second-placed candidate as a proportion of the valid vote as our measure.

H4 The rate of invalid voting at the second round will be lower when a third candidate qualifies.

H5 Unopposed candidates in the second round will increase the rate of invalid voting.

H6 The rate of Invalid voting at the second round will be higher when a qualifying candidate withdraws.

These three hypotheses are addressed using two explanatory variables in the model. Following Rosenthal and Sen (1973, 42) we use the reciprocal of the number of candidates which then decreases the marginal change in the dependent variable for the presence of two over one, and three over two candidates (H4 and H5). It also saves a degree of freedom by combining the *triangulaires* and single candidacies. We also include a dummy control for candidate withdrawal, to account for an expected

'bonus' invalidation among voters thwarted because of the pro-active departure of their candidate (H6).⁴

H7 The higher the reduction in the number of candidates between the first and second rounds, the higher the rate of invalid voting at the second round.

We include the number of candidates standing in the first round. Given we also include the reciprocal of the number of second-round candidates, this acts as a change measure. We use the first-round number of invalid votes, logged, to act as a baseline for change in rate at the second round, picking up variance from other, latent level 1 factors which drive invalid voting independent of competitive and institutional dynamic determinants.

Finally, a number of districts had boundary changes in 2010, removing 33 districts and creating 22 new ones across 45 departments, and redistricting others in a further 25 departments. Given we use district-years as observations, and the redistricting did not cross departmental boundaries, this does not affect our two-level specification. However, we cannot discount the possibility that redistricting, being unpopular with

4 This is important not for single-candidate ceremonial rounds, but for two-candidate

run-offs where a qualifying third candidate withdraws.

⁵ Using the change in the number of candidates between rounds resulted in virtually

identical results. Similarly, including the raw number of candidates for both rounds produces identical results.

some politicians and their supporters,⁶ may affect the rate of invalid votes in the elections subsequent to the changes. Consequently, for those departments whose districts were redistricted, we include a shock term for 2012, assuming that by 2017, any effect will have decayed.

Findings

Table 2 presents the pooled election models from 2002 to 2022 by round, with different thwarted codings by candidate, nuance and two-, three-, four- and five-bloc groupings. For the most part, individual parameter estimates confirm our hypotheses. These vary by thwarted specification, which we discuss below, but are generally stable in direction and significance. From a baseline rate of spoiled ballots at the first round, stable across all specifications, the competitive array of districts influences the rate at the second round. The smaller the margin by which the third-placed candidate misses qualification, the greater the rate of vote invalidation (H2). The larger the margin of victory of the first-placed candidate, the greater the rate of vote invalidation (H3). The positive coefficient for the inverse number of second-round candidates shows the fewer the candidates, the higher the rate of invalidation (H4/H5).

The number of candidates in the first round does also have an effect on invalidation, independent of their combined vote-share, which the

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⁶ The Socialist Party in particular opposed the boundary changes https://www.lemonde.fr/politique/article/2009/10/12/les-deputes-socialistes-engagent-la-bataille-contre-le-redecoupage-electoral 1252703 823448.html,

competitive bloc controls for – larger numbers of candidates in the first round associate with slightly higher rates of invalidation for bloc specifications, but are null for candidate and nuance models (H7). There is also some variation in the candidate withdrawal variable (H6). The bloc specifications act in line with our expectations, although noticeably the broader bloc specifications have stronger effects.

[Table 2 about here]

Table 2 TSCS models of invalid voting, 2nd round French legislative elections (2002-2022)

	Five-bloc	Four-bloc	Three-bloc	Two-bloc	Nuance	Candidate
Invalid R1 (log)	0.32^{***}	0.31***	0.28^{***}	0.36***	0.36^{***}	0.35***
	(0.017)	(0.016)	(0.016)	(0.017)	(0.018)	(0.018)
Thwarted bloc	3.91***	4.09^{***}	3.92***	2.71***	4.08***	4.07^{***}
	(0.13)	(0.11)	(0.095)	(0.094)	(0.16)	(0.16)
Threshold gap	-2.67***	-2.52***	-3.11***	-5.85***	-1.41***	-1.36***
	(0.26)	(0.24)	(0.22)	(0.24)	(0.31)	(0.31)
Winner m.o.v.	0.45^{***}	0.53***	0.55^{***}	0.45^{***}	0.57^{***}	0.58^{***}
	(0.062)	(0.058)	(0.055)	(0.062)	(0.063)	(0.063)
# candidates (R1)	0.011^{***}	0.015^{***}	0.016^{***}	0.019^{***}	-0.000088	-0.0010
	(0.0023)	(0.0020)	(0.0019)	(0.0022)	(0.0025)	(0.0025)
1/# candidates	1.79^{***}	2.07^{***}	2.06^{***}	2.13^{***}	1.80^{***}	1.81^{***}
(R2)	(0.10)	(0.090)	(0.086)	(0.10)	(0.11)	(0.11)
Withdrawn	0.12^{***}	0.18^{***}	0.25^{***}	0.32^{***}	-0.096^{*}	-0.090^{*}
candidate	(0.037)	(0.034)	(0.032)	(0.036)	(0.041)	(0.040)
% higher ed.	2.19^{***}	1.42^{***}	1.83***	2.39^{***}	2.84^{***}	2.84^{***}
(dept)	(0.12)	(0.11)	(0.10)	(0.12)	(0.13)	(0.13)
% unemp. (dept)	3.43^{***}	3.85^{***}	3.91^{***}	4.15^{***}	6.68^{***}	6.88^{***}
	(0.50)	(0.44)	(0.43)	(0.50)	(0.50)	(0.50)
% retired (dept)	5.43^{***}	4.39^{***}	5.71***	5.99^{***}	5.78^{***}	5.79^{***}
	(0.41)	(0.35)	(0.36)	(0.43)	(0.47)	(0.47)
Boundary change	-0.25***	-0.18***	-0.29^{***}	-0.29***	-0.34^{***}	-0.35***
	(0.017)	(0.016)	(0.015)	(0.017)	(0.017)	(0.017)
Constant	-8.85***	-8.62***	-8.52***	-8.86***	-9.66***	-9.65***
	(0.16)	(0.14)	(0.14)	(0.16)	(0.16)	(0.16)
log(alpha)	-2.87***	-3.00***	-3.11***	-2.88***	-2.82***	-2.83***
5 · 1	(0.029)	(0.029)	(0.029)	(0.029)	(0.029)	(0.029)
σ^2 department	0.019^{***}	0.011***	0.015^{***}	0.023***	0.029^{***}	0.029^{***}
-	(0.0035)	(0.0022)	(0.0026)	(0.0040)	(0.0050)	(0.0049)
Observations	2523	2523	2523	2523	2523	2523
BIC	38840.6	38486.4	38224.1	38823.2	38995.0	38979.2
Standard errors in parentheses						
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$						

²⁸

The nuance and candidate codings see a reversal of the effect, although with limited significance. There are no collinearity issues in either models, suggesting that the additional thwarted voter share from candidate withdrawal, whatever the bloc belonging, overestimates the increase in invalid vote share. Finally, the shock term for district change in 2010 is negative and significant. In the 2017 election, districts which were modified were associated with lower rates of invalid voting.

Turning to the bloc specifications, all versions are positive and significant. However we choose to identify thwarted voters, higher numbers of them will increase rates of invalidation. However, looking at the BIC for model fit, across all elections the three-bloc fit, distinguishing Left/ Moderate Right + Centre / Radical Right, gives the best estimation of the rate of invalidation across all five elections.

Does this specification hold stable for all election-years? Rather than scrutinize 30 sets of parameter estimates (available nonetheless in Appendix B1), Table 3 presents the BICs as an heuristic across the five election years. Note that BICs can only be compared within year, given the changing number of observations across years. We opt to run separate models for each year, rather than include a time-interaction, as this latter would provide a within-class comparison of the different competitive arrays, and their fit over time, rather than across-class comparison.

Table 3 Alternative thwarted bloc specifications (parsimony indicator fit, BIC)

	2002	2007	2012	2017	2022
Nuance	7515	6550	7451	8361	8288
Candidate	7501	6550	7435	8359	8290
Five-bloc	7527	6548	7409	8333	8386
Four-bloc	7472	6438	7338	8346	8392
Three-bloc	7300	6371	7336	8383	8263
Two-bloc	7312	6382	7276	8416	8409

^{*} Bold = best fit

Note: full models available in appendices.

The three-bloc coding has the best fit in three of the five elections – 2002, 2007 and 2022. For 2012, the two-bloc coding is better; 2017 the five-bloc coding. Given the long-term isolation of the RN from Left-Right competition through the *cordon sanitaire* particularly intensified in 2002 after Jean-Marie Le Pen's progression to the second round of the presidential race, the three-bloc fit aligns with the competitive space. The weakening in 2012 lines up with the so-called *droitisation* (right-shift) of the UMP after 2007, the calls for a rejection of the *cordon sanitaire* from Marine le Pen (Ivaldi 2012) and Nicolas Sarkozy's own appeal to the radical wing in the 2012 presidential election (Evans and Ivaldi 2013: 158-9).

A number of things are worth highlighting. First, recall from Table 2, that any non-random specification of losers improves the model fit – there is a

positive association between the proportion of first-round losers and second-round rates of invalid voting.⁷ For elections taking place in the period of *bipartisme* – 2007 and 2012 – the grouping of the UDF / Modem with its conservative Right coalition partners is expected. For 2022, however, the three-bloc coding – substantially better than the four-bloc coding – belies any notion of a centre-based LREM / Renaissance disproportionately shunned by conservative Right voters.

There is clear evidence that the competitive dynamics of individual elections are reflected in the levels of invalid voting. In 2017, the most fragmented coding of the five elections is present. By 2022, however, the three-category coding is noticeably a better fit once more. The Radical Left / Moderate Left combination reflects the fielding of the Nupes coalition across LFI, PS and Greens; the refusal of LREM to issue a blanket *consigne* for this same coalition in the numerous run-offs against RN; and the implicit *consigne* of the Republicans to support the presidential majority.⁸

2017 presents the greatest heterogeneity in competitive coding, with the full five-category coding separating Centre and moderates from extremes into thwarted groupings. The novel presence of a dominant Centre,

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⁷ There is one exception to this – the relatively poorly fitting two-bloc coding in 2022 finds a negative relationship between invalidation and thwarted voter levels.

⁸ https://www.lefigaro.fr/elections/legislatives/resultats-legislatives-2022-quelles-sont-les-consignes-de-vote-des-partis-pour-le-second-tour-20220612

through LREM, competing with declining Left and Right governing blocs aligns with this coding, alongside the disappointing first-round performances of both the populist radical left and right in the first round of the legislatives (Evans and Ivaldi, 2018).

Table 4 reports the best-fit model for each year. There is variation in magnitude of predictors, but direction and significance are stable. Year-specific nulls are generally explicable. The reciprocal number of candidates is not significant in 2017, given only two districts had run-offs with anything other than two candidates, and the one withdrawal is not a third candidate. As originally expected, the change in candidate numbers from the first round does not have a substantial impact, reaching significance only in 2012. Conversely to 2017, this may be because of the larger number of three-way run-offs providing explicable variance. Finally, the margin of victory for the first candidate seems not to have played a role in invalidation in 2022, perhaps evidence of growing competition.

[Table 4 about here]

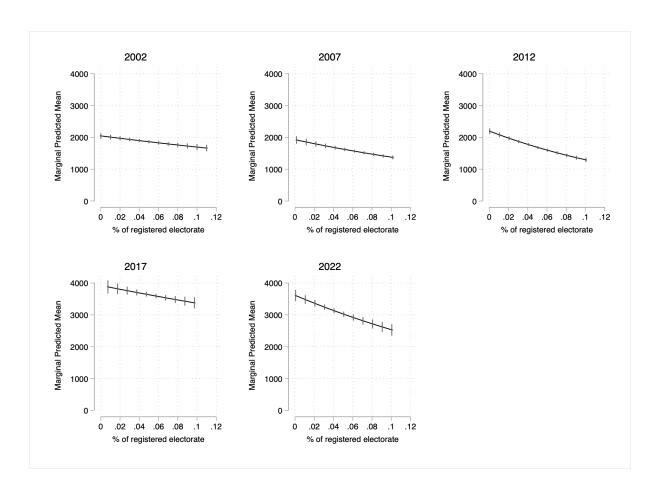
Table 4 Individual year [bloc #] models (best fit) of invalid voting, 2nd round French legislative elections (2002-2022)

Invalid R1 (log)	2002 [3]	2007 [3]	2012 [2]	2017 [5]	2022 [3]
	0.17***	0.21***	0.36***	0.22***	0.34***
	(0.027)	(0.030)	(0.031)	(0.024)	(0.030)
Thwarted bloc	4.94***	5.55***	4.34***	4.21***	3.54***
	(0.27)	(0.32)	(0.23)	(0.26)	(0.26)
Threshold gap	-1.89***	-3.36***	-5.28***	-1.54**	-3.55***
	(0.43)	(0.44)	(0.40)	(0.56)	(0.56)
Winner m.o.v.	0.29**	0.39**	0.63***	0.52***	-0.16
	(0.11)	(0.12)	(0.11)	(0.096)	(0.11)
# candidates (R1)	0.0042	0.0074	0.013**	-0.0034	0.0082
	(0.0038)	(0.0057)	(0.0041)	(0.0029)	(0.0046)
1/# candidates (R2)	1.36***	1.53***	1.88***	0.14	1.83***
Withdrawn candidate	(0.21) 0.21***	(0.28) 0.33***	(0.12) 0.31***	(0.32)	(0.24) 0.18
% higher ed.	(0.039)	(0.066)	(0.056)	0.29	(0.19)
(dept)	0.16	0.33	0.41		0.088
% unemp. (dept)	(0.26)	(0.28)	(0.22)	(0.15)	(0.19)
	0.78	2.03*	2.62***	-1.67**	2.13*
	(0.81)	(1.01)	(0.73)	(0.58)	(0.89)
% retired (dept)	3.66***	4.38***	2.26***	1.78***	1.42**
	(0.54)	(0.64)	(0.55)	(0.41)	(0.51)
Boundary change	(0.51)	(0.01)	-0.052 (0.029)	(0.11)	(0.01)
Constant	-6.57***	-7.12***	-7.89***	-5.69***	-7.28***
	(0.24)	(0.30)	(0.25)	(0.25)	(0.29)
log(alpha)	-3.50***	-3.41***	-3.46***	-3.80***	-3.61***
	(0.071)	(0.077)	(0.071)	(0.068)	(0.068)
σ^2 department	0.0082***	0.011***	0.0057**	0.0032**	0.0076***
	(0.0023)	(0.0032)	(0.0020)	(0.0012)	(0.0021)
Observations	501	445	507	536	534
BIC	7300.4	6370.7	7276.1	8333.1	8263.3

Standard errors in parentheses p < 0.05, ** p < 0.01, *** p < 0.001

Beyond the larger pool of thwarted voters, the mystery of the substantial increase of second-round invalidation in 2017 is illuminated, but not entirely resolved, by the model. The proportion of invalid vote progressing is lower than 2012 and 2022. The threshold gap reduction is shallower than in other years, suggesting that thwarted voters may spoil at consistently higher rates in 2017 than in previous years. Figure 4, plotting fitted rates across increasing threshold differentials suggests this is the case.

Figure 4 Fitted estimates (with 95% CIs) of vote invalidation by year and threshold differential

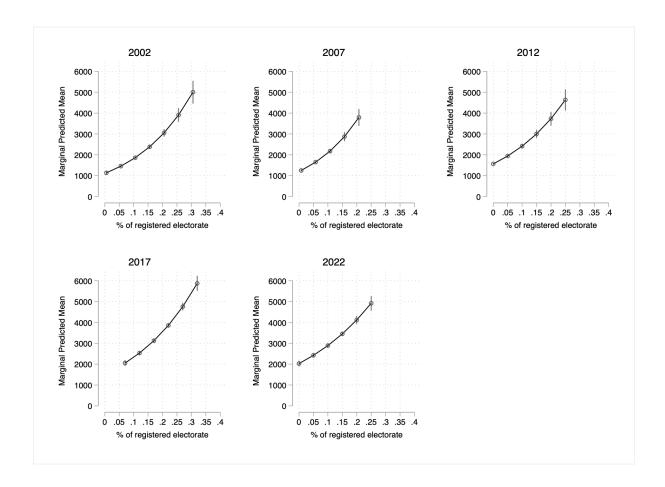


The separation of multiple blocs of thwarted voters (Figure 5) leads to higher marginal and absolute rates of vote invalidation, with less additional variation by threshold differential. The higher rate of invalidation in 2017 is not due to increased retention of first-round invalidation either, as evidenced by a coefficient of the logged first round number lower than both adjacent election-years. Indeed, the most suggestive finding is the constant, which adjusts the intercept downwards to a much lesser degree than for the other elections.

The high rates of invalidation in 2017, then, do appear to be a function of the fragmentation of political supply, and in that respect, Figure 4 points out the similarity with 2002 – an election cycle characterized as *le vote de tous les refus* (a vote of refusal – Perrineau and Ysmal 2003) where anti-system blocs and a rejection of the mainstream, particularly of the Left, characterized the outcome. Overall, breaking down the TSCS models into separate years is worth the loss of efficiency, highlighting some important differences. Nonetheless, the principal explanatory variables are stable across time.

[Figure 5 about here]

Figure 5 Fitted estimates (with 95% CIs) of vote invalidation by year and thwarted voter bloc



Simulations

[Table 5 about here]

Table 5 Simulations of invalid vote share under different competitive conditions

RUN-OFF FORMAT	1 ML v MR	2 MR v XR	3 MR v XR	4 ML v C v XR	5 ML v C
Registered Invalid R1 (log) # Candidates (R1)	77000 6.68 (796 votes) 6				
XL ML C MR XR O	3000 16500 1120 15000 6000 1500	3000 6000 1120 16500 15000 1500	500 9500 5120 14500 13000 500	2000 12800 12100 3200 11500 1520	1630 10425 9855 2606 9366 1238
Valid votes		43120			
Threshold gap Winner m.o.v. # Candidates (R2)	.047 .035 2	.047 .035 2	.002 .035 2	.083 .016 3	.003 .016 2
Spoiled ballots [blocs]	Valid vote % (thwarted, n) Spoiled				
2002 [3]	.097 (7500) 1592 .097 (7500)	.136 (10500) 1930 .136 (10500)	.136 (10500) 2101 .136 (10500)	.020 (1520) 807 .020 (1520)	.138 (10604) 2107 .138 (10604)
2007 [3]	1923 .019	2387 .136 (10500)	2777 .136	854 .020	2778 .016
2012 [2]	(1500) 1504 .151	2499 .151 (11620)	(10500) 3170 .203	(1520) 902 .087	(1238) 1850 .193
2017 [5]	(11620) 2781 0.097 (7500)	2781 0.136 (10500)	(15620) 3710 0.136 (10500)	(6720) 1944 .020 (1520)	(14840) 3517 .138 (10604)

Table 5 presents five scenarios, to assess how identical vote shares result in differing levels of vote invalidation under the different competitive structures across years. For the first four scenarios, the district is assumed to have 77,000 registered electors – around the average size of a French electoral district – and the legislative average of 56% turnout in the second round. The log of first-round invalidation is fixed at 6.68 (793 invalid votes). Level 2 demographics are fixed at their means. For simplicity, we assume a single party stands in each bloc. Unfortunately, we cannot use scenarios which all have real-world equivalents, for example because there is no district where a *triangulaire* was held in all elections. For the two-candidate run-offs, we have ensured that these all correspond in party qualification (but not in ranking) to actual examples. For

Scenario 1 represents a classic Fifth Republic run-off between the Moderate Right and the Moderate Left in a relatively close race, with a third placed Radical Right candidate some distance from qualification.

Unsurprisingly, 2017 and 2022 produce substantially higher rates of

⁹ Since 2002, only the first district of the Aube department has had a three-way run-off in two consecutive elections (2012 and 2017).

¹⁰ For 2002 and 2007, the FN never finished in first place, but did qualify for the second round in 35 districts in 2002, and 2 in 2007.

invalidation, but note that in the case of 2022, the three-bloc definition implies no greater pool of thwarted voters than in 2002 or 2007.

Scenario 2 retains identical vote shares, but reallocates these to form a Moderate Right / Radical Right run-off, with a Moderate Left candidate now in third place and missing qualification by the same margin. For all years except 2017, this increases the loser grouping, with Left voters thwarted. In 2017, given the separation of all parties into their own bloc, who the thwarted voters are doesn't matter - only two blocs are represented, so the predicted rate of invalidation remains the same as Scenario 1. Scenario 3 retains the Moderate Right / Radical Right run-off, by the same lead in terms of share of valid vote, but now with a much stronger Moderate Left candidate who misses the run-off by just 125 votes (0.2% of the registered electorate). Here we see increases across all years, but particularly in 2017, with almost 1000 additional blank and spoiled votes on Scenario 2. The increases are more modest in other years, but still underline the premium that a close-run third place gives, irrespective of losing bloc affiliation.

Scenario 4 provides a stark contrast, with a qualifying third candidate. In this case, three blocs are represented (two for 2012, given the binary coding) thereby limiting the number of thwarted voters, and shifting the effect of the missed candidate to a fourth Moderate Right also-ran, some 8% of the registered electorate (c. 15% of the valid vote) away from

qualification. Invalid vote rates here are at least halved for later years, cut by two-thirds or more in earlier years.

Finally, Scenario 5 looks at the impact of lower turnout, reducing the valid vote by c. 18.5%, but retaining the same proportionate vote shares and margin of victory. As a result of the lower turnout, the Radical Right candidate no longer qualifies, increasing the thwarted pool for all years except 2012, and incurring the close third-place premium. Low turnout has no impact on two winning candidates, even should they themselves fail to pass the 12.5% bar, but for a third party, the lower number of voters for a relatively higher vote share increases the distance from the threshold. This underlines the possible paradox of the vote threshold – an ostensibly excellent third-place score can be further from the qualification threshold than a mediocre performance in a high-turnout district.

One simple but crucial implication of the model is that the 12.5% threshold not only reduces competitive choice at the second round – something noted by Bol and Ivandic (2022) and Pons and Tricaud (2018) in its effect on turnout – but also accentuates the rates of invalid voting. Its imposition in 1976 was brazenly political, to strengthen the position of the governing party in straight run-offs against divided opposition by eliminating centrists, in a period of aligned, even deferential voting. While the high bar has in the past reduced the number of districts where the RN has been able to participate in the run-off, this impact has self-

evidently reduced given the RN's improving performance in 2017 and 2022. (Appendix 2 tests for a possible RN differential in thwarted invalidation.)

Empirically, what effect would the reintroduction of the 10% threshold have? Calculating the number of additional qualifiers is a simple process based upon vote shares, as shown across the five elections in Table 6.

[Table 6 about here]

Table 6 Number of candidates per district winning more than 10% of the registered electorate (automatic qualification in case of run-off for candidates above the dotted line)

No. of candidate	2002	2007	2012	2017	2022
$\frac{s}{0}$	0	0	0	8	4
1	22	54	25	253	108
2	386	456	355	256	311
3	139	45	153	22	114
4	8	0	6	0	2

Evidently, the number of three-way run-offs would increase substantially in all elections, with four-way run-offs also returning in a handful of districts. The impact on spoiled votes would have two routes, at least in

terms of what we can reasonably estimate. First, the share of first-round losers would diminish, thereby reducing the pool of thwarted voters. Whilst the effect will vary across districts, the outcome globally would mechanically lead to fewer invalid votes, under the terms of the model. Second, the 'near miss' category of candidate would not disappear, but rather shift either in terms of bringing a still-losing third-placed candidate closer to the threshold (which we assume would potentially increase the rate of invalidation) or allow the accession of a third-placed candidate, but place a fourth-placed candidate in a similar 'near(er) miss' position. Here, if we assume that the same mechanism would operate for fourth-placed candidates, either the rate of invalid voting could drop substantially, as a third-placed candidate makes the run-off, and a trailing fourth candidate is much further from the threshold; or invalid voting could rise, with a third-placed candidate now much closer to the reduced threshold.

To simulate the change in invalid voting in Table 7, we use year-specific parameter-estimates, as above, and alter observed values for first-round losers, number of candidates and threshold differential variable in scenarios 3 and 5. We also produce a separate scenario, by altering the Centre and Moderate Left vote shares to 7710 and 6920 in turn, 11 to

11

¹¹ A hypothetical run-off between three Centre / Right candidates placing first, second and third place in an observed election would be unusual, but is observed in the data – for example, in the seventh district of Alpes-Maritimes in 2022. The 2022 model substantially overestimates invalid voting in this district, despite two candidates from a single bloc standing in the run-off – an example of where idiosyncrasies of local

understand the effect of a close fourth place by differing competitive arrays. We continue to assume no withdrawal by any eligible candidate between rounds.

Table 7 Simulations of invalid vote share with 10% run-off threshold

Threshold gap	3a .034	5a .067	3b [C]	3c [ML]
# Candidates (R2)		3		
Spoiled ballots [bloc #]	Change (ch	f table 4)	Nui	nber
2002 [3]	-1271	-1292	1402	869
2007 [3]	-1839	-1897	2093	1017
2012 [2]	-2057	-886	1902	1248
2017 [5]	-1662	-1654	1841	1841
2022 [3]	-2048	-2181	2233	1584

Self-evidently, the lowering of the threshold allows the third-placed candidate into the run-off, thereby reducing rates of invalid voting substantially. In terms of our two stable vote scenarios (3a and 5a), there is little noticeable difference between the change except in 2012, but this is principally due to the already low level of invalidation in the original scenario 5 (1850). Models 3b and 3c confirm that, even if we make the very conservative assumption that fourth-placed candidates close to the 10% threshold would produce a similar 'bonus' vote invalidation to current third-placed losers, this is offset by the reduced thwarted voter pool and/or presence of a third candidate. The inclusion of an additional

competition cannot be modelled.

bloc through the Moderate Left qualification, rather than an additional intra-bloc candidate through the Centre qualification, substantially reduces invalidation in all years, except 2017, when the Centre and Moderate Right blocs are distinct. In short, the influence of reducing the threshold is conditional upon the competitive array of the parties in the election.

Discussion

Invalid voting in the run-off of French legislative elections is linked to the competitive and institutional constraints imposed on voters by first-round results. While greater shares of losing voters are associated with higher rates of invalid voting, there is a stronger association between voters denied a choice through the absence of their political bloc (rather than candidate) from the run-off. Those blocs correspond closely to the bloc arrays commonly used to summarise the French party system in its electoral guise. Moreover, where competition is curtailed by institutional constraints (the 12.5% threshold), candidate behaviour (second-round withdrawal) or a dominant first-round winner, rates of invalid voting increase, independent of the political profile of the blocs in competition. Most strikingly, otherwise competitive third-placed candidates who miss the second round because of low turnout produce higher levels of invalidation.

The ecological data used in this analysis mean that we cannot safely infer individual choice mechanisms. To approach the former, individual level data examining the motivations of voters would be required. However, even if the previous levels of under-reporting of invalid voting in large surveys no longer seem to obtain – recent French Election Studies have unweighted samples reporting representative levels of invalid voting – the need to link the competitive dynamics by district represents a data issue. Insufficient cases exist at the district level to allow any statistical power in such tests. Only a survey experiment approach could provide sufficient cases to test the effects of hypothetical competition on vote invalidation as a choice, but with serious concerns over external validity in using such a method. To that extent, across-time ecological approaches probably represent the limit to empirical analysis of such effects.

This article has avoided the normative considerations of institutional constraints and candidate choice which are associated with rates of invalid voting. Clearly, the 12.5% threshold has such implications for increased invalidation, along with the increases in abstention noted by other research (Bol and Ivandic, 2017; Pons and Tricaud, 2018). More recent commentary has begun to suggest the reduction or removal of the 12.5% threshold, principally to eliminate the delegitimised pool of second-round candidates who only reach the run-off because of their automatic promotion rather than sufficient vote share (Grunberg and Missika, 2021). Our analysis suggests that, as well as relegitimising third

candidates in districts with lower turnout rates, at least in terms of the institutional rules, this would also serve to reduce considerably the invalidation rate. To the extent that invalidation is an unequivocal rejection of the democratic race in hand, such a reduction would surely be a positive, even if the resulting competition may not work in an amending executive's favour. As Grunberg and Missika note (2021), the stabilizing benefits of a bipolar system pitching a clear executive majority against a unified opposition are not dynamics that can or should be delivered *au forceps*.

Similarly, this analysis has not focused in any depth on the relationship between invalid voting and other 'negative' vote choices – abstention, and historically Radical Right party voting in particular (see also Uggla, 2008: 1156-7; Aron and Superti 2022). Why some voters choose to invalidate their vote, and others choose not to vote, in potentially identical competitive circumstances is clearly of substantial interest, and has motivated some theoretical and qualitative analyses to date (Zulfikarpasic, 2001; Moualek, 2017). Again, however, the limits of individual-level data mitigate against its investigation. At the correlational level, however, the better overall fit of models separating Radical Right as a bloc from the Moderate Right signposts the artificial exclusion of the RN from run-offs where media have identified a number of expected three-way run-offs well in excess of the number actually achieved, partly due to the 12.5% threshold. The elections years where the *tripartition* coding has worked less well have been those where the

Conservative Right has moved closer to the Radical Right. Whatever the views on the issues of Radical Right presence and representation, exclusion of candidates through the threshold does not seem to have worked to discourage or offset their presence, and may have indirectly contributed to a rise in vote invalidation.

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APPENDICES

Appendix 1 Bloc and nuance coding of legislative elections

Appendix 2 Robustness test 1 – "invalid voting is driven predominantly by thwarted FN/RN voters."

Appendix 3 Robustness test 2 – "invalid voting is limited by the *front républicain* in districts where the RN is present in the second round."

Appendix 4 Robustness test 3 – "nuance codings in the 2022 election were sub-optimal – the reversion to the 3-bloc coding is an artefact of the Nupes 'catch-all' nuance that does not differentiate between radical and moderate Left voters."

Appendix 5 Population density and invalid voting

Appendix B1 Full model specifications by year across six bloc codings

Appendix B2 Full model specifications by year controlling for abstention

Appendix 1	Bloc and nuance coding of legislative elections				
	XL	ML	CEN	MR	XR
200	COM+EXG+LO+LCR	SOC+DVG+VEC+ECO+PRG	UDF	UMP+MPF+RPF+DVD	FN+MNR+EXD
2		+PREP		+DL	
200	COM+EXG	DVG+RDG+ECO+SOC+VEC	UDFD	UMP+DVD+PSLE+MPF	FN+EXD
7				+MAJ	
201	EXG+FG	SOC+RDG+DVG+ECO+VEC	CEN	UMP+MPF+	FN+EXD
2				NouvC+PRV+DVD+ALLI	
201	COM+EXG	SOC+RDG+DVG+ECO	MDM+REM	LR+UDI+DVD	FN+DLF+EXD
7	+FI				
202	DXG	NUP+RDG+DVG+ECO	ENS+DVC	LR+UDI+DVD	DSV+DXD+REC
2					+RN
XL COM LO LCR FG FI EXG/DXG ML SOC VEC ECO PRG/RDG PREP DVG NUPES	Front de Gauch La France Insc Other Extreme Parti Socialiste (Europe Ecology Other ecology Parti Radical d Pôle Républica Other Left	niste Révolutionnaire he pumise Left gie) Les Verts e Gauche in e Union pop.,	UDF(D) Française CEN MODEM(min)/MDM REM ENS DVC MR NCE PR UMP Populaire LR MPF MAJ	Union pour la Dém Le Centre pour la France Mouvement Démocrate La République en Marche Ensemble! Other centre Le Nouveau Centre Parti Radical Union pour un Mouvement Les Républicains Mouvement pour la Franc Majorité présidentielle	nt
écologique et soc. C			DL XR	Démocratie Libérale	
			FN/RN	Front / Rassemblement N	ational

MNR REC DLF/DSV Souverainiste DXD Mouvement National Républicain Reconquête! Debout la France/ Droite

blicain REG CPNT proite DIV/AUT

Regionalist party Chasse Pêche Nature Traditions Other

Other Extreme Right

Other

Appendix 2 Robustness test 1 - "invalid voting is driven predominantly by thwarted FN/RN voters."

The competitive array of legislative elections has undoubtedly been shaped by the 'nuisance potential' of the FN/RN (henceforth RN), and the three-bloc structure dominant in three of the five election years is clear evidence of that. Within our models, it is possible that explained variance linked to the missing gap is instead an RN 'premium', ie heterogeneity in a majority of districts with a third-placed RN candidate, whose supporters convert into invalid voters at a higher rate than other thwarted voters. 32 per cent of district-years had RN candidates placed non-qualifying third, so this party clearly constitutes the modal choice for thwarted voters. If this alternative explanation is correct, we would expect those districts where the RN placed third, but did not qualify, to exhibit higher rates of invalidation than other districts, and a steeper slope to the missing gap.

Figure A2 Differences (with 95% CIs) between non-qualifying FN/RN candidates in invalid voting rates

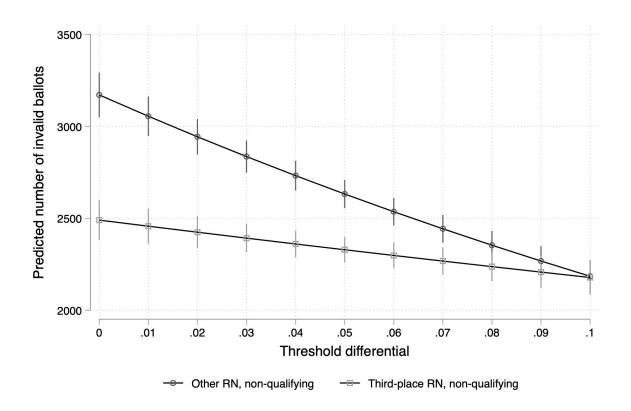


Figure A2 shows that, for the global model (Model A2 below), this is not the case. For the missing gap, the marginal effect of the difference is in fact greater in districts where a party other than the RN placed third. There is still a significant effect for RN districts, but smaller, and consistently lower.

Model A2

Invalid R1 (log)	0.30^{***}
	(0.015)
Three-bloc	4.16^{***}
	(0.094)
Threshold gap	-3.72***
<u> </u>	(0.26)
Third-place RN, non-	-0.24***
qualifying	(0.021)
Third-place RN, non-	2.39^{***}
qualifying # Threshold	(0.38)
gap	
Winner m.o.v.	0.46^{***}
	(0.053)
# candidates (R1)	0.015^{***}
, ,	(0.0018)
1/# candidates (R2)	2.01***
• •	

	(0.083)
Withdrawn candidate	0.22^{***}
	(0.031)
% higher ed. (dept)	1.74^{***}
	(0.10)
% unemp. (dept)	3.69^{***}
	(0.42)
% retired (dept)	5.23^{***}
	(0.35)
Boundary change	-0.25***
	(0.014)
Constant	-8.40^{***}
	(0.13)
log(alpha)	-3.20***
	(0.029)
σ^2 department	0.014^{***}
-	(0.0025)
Observations	2523
BIC	38028.8
Standard errors in parenthe	
* p < 0.05, ** p < 0.01, *** p <	< 0.001

Appendix 3 Robustness test 2 - "invalid voting is limited by the *front républicain* in districts where the RN is present in the second round."

Traditionally, mainstream parties have mobilized against significant RN and other Radical Right opposition at run-offs by issuing *consignes* to their supporters to vote for the party opposing the RN, whatever their political affiliation, or even standing down to present the vote being split. Under such circumstances, is the rate of invalidation lower in districts where the Radical Right reaches the run-off? This proposition is not testable in 2007, when the RN only reached one run-off. We therefore test this for the remaining four years separately, using the best-fitting model for each. We also separate between all districts where the Radical Right fielded a candidate, and those where this was a duel rather than three-way run-off (although there are very few cases of RR three-way run-offs).

Table A3 FN parameter estimates, by year and all / two-way runoffs

	20	02	20	12	20	17	20	22
	All	Duel	All	Duel	All	Duel	All	Duel
RR run- off	.24***	.32***	.13***	.30***	09***	09***	.05*	.05*
n	37	28	60	32	122	121	208	204

* *p* < 0.05, ** *p* < 0.01, *** *p* < 0.001

Other model parameters reported in full table below.

Table A3 presents the dummy variable parameter estimates controlling for Radical Right presence / absence in the run-off, along with descriptives, across the four election years (full models A3a and A3b below). There were no substantive changes in other parameter estimates

after the inclusion of the Radical Right dummy presence. In the three years other than 2017, the effect is the opposite of what one would expect under the *front républicain* – invalid voting rates rise, other things being equal, in the presence of a Radical Right candidate. Moreover, the effect is slightly stronger in duel run-offs. Only in 2017 is there a decline in invalidation. The only year with sufficient cases to run the model in three-way run-offs alone is 2012, where a significant negative effect is found (-.27, p < .01). Therefore, there is weak evidence that in three-way run-offs, the presence of a Radical Right challenger may lower invalidation, but there is no consistent evidence of this for duels.

Model A3aAll second rounds by year, controlling for RN candidate presence

	2002 [3]	2012 [2]	2017 [5]	2022 [3]
Invalid R1 (log)	0.17*** (0.025)	0.34*** (0.031)	0.24*** (0.024)	0.33*** (0.030)
Thwarted bloc	4.51*** (0.27)	3.96*** (0.25)	4.22*** (0.25)	3.47*** (0.26)
RN candidate running	0.24***	0.13***	-0.087***	0.045*
J	(0.036)	(0.035)	(0.019)	(0.020)
Threshold gap	-2.08*** (0.41)	-5.59*** (0.41)	-1.33* (0.54)	-3.49*** (0.56)
Winner m.o.v.	0.15 (0.11)	0.58*** (0.11)	0.50*** (0.095)	-0.16 (0.11)
# candidates (R1)	0.0061	0.013**	-0.0033	0.0094^*
1/# candidates (R2)	(0.0036) 1.76***	(0.0040) 2.06***	(0.0029) 0.13	(0.0046) 1.88***
` ,	(0.21)	(0.12)	(0.31)	(0.24)
Withdrawn candidate	0.20***	0.31***		0.17
	(0.037)	(0.055)		(0.18)
% higher ed. (dept)	0.067	0.39	0.28	0.15
	(0.27)	(0.23)	(0.15)	(0.19)
% unemp. (dept)	-0.069 (0.83)	1.82* (0.80)	-0.77 (0.61)	1.56 (0.94)
% retired (dept)	3.59*** (0.55)	2.46*** (0.57)	1.69*** (0.40)	1.54** (0.52)
Boundary change	(,	-0.058 (0.030)	(===,	()
Constant	-6.69***	-7.75** [*]	-5.88***	-7.27***

	(0.24)	(0.26)	(0.25)	(0.29)				
/								
log(alpha)	-3.62***	-3.51***	-3.84***	-3.62***				
	(0.072)	(0.071)	(0.068)	(0.068)				
σ^2 department	0.0093^{***}	0.0072^{**}	0.0033^{**}	0.0082***				
-	(0.0024)	(0.0022)	(0.0012)	(0.0022)				
Observations	501	507	536	534				
BIC	7262.4	7269.3	8319.3	8264.5				
Standard errors in parentheses								
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$								

Model A3b All second-round duels, controlling for RN candidate presence (final model including only threeway run-offs)

2002 [3]	2012 [2]	2017 [5]	2022 [3]	2012 - three- way only
0.19*** (0.025)	0.34***	0.24***	0.34***	0.47*** (0.073)
4.07***	3.88***	4.21***	3.45***	-1.86 (3.90)
0.32***	0.30***	-0.086***	0.051*	-0.27****
(0.039) -1.91*** (0.42)	(0.037) -3.88*** (0.42)	(0.019) -1.32* (0.55)	(0.020) -2.93*** (0.57)	(0.079) -3.90^{***} (0.74)
0.092 (0.11)	0.34** (0.10)	0.50*** (0.095)	-0.22* (0.11)	-0.25 (0.44)
0.0057 (0.0036)	0.011** (0.0038)	-0.0033 (0.0029)	0.0083 (0.0046)	-0.00033 (0.0062)
1.53***	1.88***	0.098	1.56***	
(0.20) 0.18***	(0.12) 0.22***	(0.32)	(0.25) 0.081	
(0.037)	(0.051)		(0.19)	
0.10	0.39	0.28	0.18	1.08
(0.26)	(0.21)	(0.15)	(0.20)	(0.68)
0.46	2.76^{***}	-0.77	1.64	0.39
(0.83)	(0.76)	(0.62)	(0.96)	(2.04)
3.54*** (0.54)	$2.66^{***} (0.54)$	$1.70^{***} \ (0.40)$	$1.66^{**} \ (0.54)$	5.58* (2.19)
	-0.032 (0.028)			-0.35*** (0.10)
-6.67*** (0.23)	-7.80*** (0.24)	-5.87*** (0.25)	-7.19*** (0.29)	-7.89*** (0.64)
-3.67*** (0.073)	-3.71*** (0.074)	-3.84*** (0.068)	-3.65*** (0.069)	-5.76*** (0.51)
0.0090^{***}	0.0065^{***}	0.0034^{**}	0.0089^{***}	0.015* (0.0064)
				33
				434.5
rentheses	0.30.1	2230.0	0100 . 2	101.0
	0.19*** (0.025) 4.07*** (0.27) 0.32*** (0.039) -1.91*** (0.42) 0.092 (0.11) 0.0057 (0.0036) 1.53*** (0.20) 0.18*** (0.20) 0.18*** (0.26) 0.46 (0.83) 3.54*** (0.54) -6.67*** (0.23) -3.67*** (0.073) 0.0090*** (0.0024) 491 7103.4	0.19*** 0.34*** (0.025) (0.029) 4.07*** 3.88*** (0.27) (0.24) 0.32*** 0.30*** (0.039) (0.037) -1.91*** -3.88*** (0.42) (0.42) 0.092 0.34** (0.11) (0.10) 0.0057 0.011** (0.0036) (0.0038) 1.53*** 1.88*** (0.20) (0.12) 0.18*** 0.22*** (0.037) (0.051) 0.10 0.39 (0.26) (0.21) 0.46 2.76*** (0.83) (0.76) 3.54*** 2.66*** (0.54) (0.54) -0.032 (0.028) -6.67*** -7.80*** (0.23) (0.24) -3.67*** -3.71*** (0.073) (0.074) 0.0090*** 0.0065*** (0.0024) (0.0019) 491 474 7103.4 6753.4	0.19*** 0.34*** 0.24*** (0.025) (0.029) (0.024) 4.07*** 3.88*** 4.21*** (0.27) (0.24) (0.25) 0.32*** 0.30*** -0.086*** (0.039) (0.037) (0.019) -1.91*** -3.88*** -1.32* (0.42) (0.42) (0.55) 0.092 0.34** 0.50*** (0.11) (0.10) (0.095) 0.0057 0.011** -0.0033 (0.0036) (0.0038) (0.0029) 1.53*** 1.88*** 0.098 (0.20) (0.12) (0.32) 0.18*** 0.22*** (0.037) (0.051) (0.098) (0.26) (0.21) (0.15) 0.46 2.76**** -0.77 (0.83) (0.76) (0.62) 3.54*** 2.66**** 1.70*** (0.54) (0.54) (0.40) -0.032 (0.028) -6.67*** -7.80*** -5.87*** (0.23) (0.24) (0.25)	0.19*** 0.34*** 0.24*** 0.34*** (0.025) (0.029) (0.024) (0.030) 4.07*** 3.88*** 4.21*** 3.45*** (0.27) (0.24) (0.25) (0.26) 0.32*** 0.30*** -0.086*** 0.051* (0.039) (0.037) (0.019) (0.020) -1.91*** -3.88*** -1.32* -2.93*** (0.42) (0.42) (0.55) (0.57) 0.092 0.34** 0.50*** -0.22* (0.11) (0.10) (0.095) (0.11) 0.0057 0.011** -0.0033 0.0083 (0.0036) (0.0038) (0.0029) (0.0046) 1.53*** 1.88*** 0.098 1.56*** (0.20) (0.12) (0.32) (0.25) 0.18** 0.22*** 0.081 (0.037) (0.051) (0.19) 0.10 0.39 0.28 0.18 (0.26) (0.21) (0.15) (0.20)

Standard errors in parentheses p < 0.05, ** p < 0.01, *** p < 0.001

Appendix 4 Robustness test 3 - "nuance codings in the 2022 election were sub-optimal - the reversion to the 3-bloc coding is an artefact of the Nupes 'catch-all' nuance that does not differentiate between radical and moderate Left voters."

During candidate registration in May, the Ministry of the Interior did not allow Nupes as one of the closed list of nuances, thereby denying the existence of the Left electoral coalition. In early June 2022, after an appeal by Jean-Luc Mélenchon, the Conseil d'Etat instructed the Ministry to list all component candidates of the Nupes coalition between Socialists, Communists, LFI and Greens under a single nuance. For voters, Nupes ballot papers all carried the 'V' symbol, and the name of the coalition, with different colours indicating the party of choice. While this balanced the presentation of results opposite the presidential majority, *Ensemble!*, also constituted by multiple parties, it potentially aggregates across voters who would be happy to vote Radical Left but not Moderate Left, or vice versa, and were sufficiently informed to recognize the party to which their Nupes candidate belonged. The differentiation visible in 2017 would not be possible in 2022, beyond the Far Left support of the Nouveau Parti Anticapitaliste, Lutte Ouvrière, and other very small formations.

The Cevipof LEGIS-2022 project provides the 2022 results, breaking down the Nupes coalition by its constituent parties, as well as reclassifying other parties, such as the *Parti Pirate*, which were allocated across multiple different nuances by district (divers gauche, divers centre, régionaliste, divers ...), dissident candidacies, and DOM-TOM

idiosyncrasies. Table A4 lists the LEGIS-2022 codings, and how we allocate these to the analytical blocs. Given the mutually exclusive nature of Nupes candidacies – a Moderate Left Nupes candidate will not stand against a Radical Left Nupes candidate in the first round, therefore the 'thwarted' category is moot – we do not expect that the results will differ substantially from the Ministry codings, other than thwarted voters from minor candidates having a differential coding.

Using the separate coding scheme provided by LEGIS-2022, the candidate coding provides almost identical results to the Ministry data (Table A4). The worst-fitting two-bloc model thwarted coefficient notably turns negative (see model A4 below) - but this is in line with the two-bloc specification for the Ministry data. Most notably, the candidate coding following this scheme provides the best fitting model for this version of the 2022 data, followed by the three-bloc coding - the opposite ranking to the Ministry data. This swap is undoubtedly due to differential coding of candidates, as well as our own conversion of these codings into the blocs. But confirming the close validity of a three-bloc coding, mapping the party blocs across Nupes, Moderate Right - including the *Ensemble!* Centre - and the Radical Right grouping of RN, Zemmour's Reconquête! and the sovereignist Right, alongside candidate-specific voter pools, suggests an increased atomization of electoral support and more widespread distribution of voter disenchantment. Invalid voting may be moving from the preserve of thwarted tranches of voters, disenfranchised by a proximate party candidate, towards a second-round protest against

the dominant blocs. That said, we would note that the three-bloc coding using the original Ministry data constitutes the best fit.

Table A4 Bloc coding of 2022 parties using Legis2022 project codings

\mathbf{XL}

Lutte Ouvrière Nouveau Parti Anticapitaliste **NUPES-LFI NUPES-PC** Divers dissident de NUPES Divers extrême gauche (XL)

ML

NUPES-PS NUPES-Ecologiste NUPES-Génération.s Divers écologistes Parti Animaliste Parti Radical de Gauche Socialiste dissident de NUPES Divers dissident de NUPES (ML) Divers gauche

\mathbf{C}

Ensemble-Renaissance Ensemble-Modem **Ensemble-Horizons Ensemble-Autres** Dissident d'Ensemble Divers centre

Les Républicains Divers centriste soutenu par Les Républicains Divers droite

$\mathbf{X}\mathbf{R}$

Rassemblement national Union pour la France - Debout la France Union pour la France - Génération Frexit Union pour la France - les Patriotes Reconquête! Autres souverainistes Divers extrême droite

Other

Parti Pirate Régionalistes Tous unis pour le vivant **UDMF**

Model A4 Bloc specifications for invalid voting using Legis2022 party codings

	Five-bloc	Four-bloc	Three-bloc	Two-bloc	Candidate
Invalid R1 (log)	0.39*** (0.032)	0.39*** (0.033)	0.35*** (0.031)	$0.39^{***} (0.034)$	0.39*** (0.030)
Thwarted bloc	1.99*** (0.25)	1.20*** (0.26)	2.83*** (0.26)	-0.58*** (0.16)	3.30*** (0.28)
Threshold gap	-4.97*** (0.63)	-5.90*** (0.66)	-4.50*** (0.57)	-7.88*** (0.57)	-3.37*** (0.62)
Winner m.o.v.	-0.023 (0.13)	-0.088 (0.13)	-0.19 (0.12)	-0.14 (0.13)	0.16 (0.12)
# candidates (R1)	0.011* (0.0052)	0.017*** (0.0052)	0.011* (0.0048)	0.022*** (0.0053)	0.0026 (0.0049)
1/# candidates (R2)	2.07***	2.20***	1.97***	2.41***	1.76***
Withdrawn	(0.26) 0.34	(0.27) 0.37	(0.25) 0.25	(0.28) 0.57**	(0.25) 0.23
candidate					
% higher ed. (dept)	(0.20) -0.052	(0.21) -0.033	(0.19) 0.024	(0.21) -0.13	(0.19) 0.0078
% unemp. (dept)	(0.18) 3.07***	(0.19) 2.84**	$(0.19) \\ 2.42^{**}$	$(0.20) \ 2.06^*$	(0.17) 3.24***
% retired (dept)	(0.90) 0.93	$(0.97) \\ 1.28^*$	$(0.91) \\ 1.23^*$	$(0.96) \\ 1.41^{**}$	$(0.85) \\ 0.64$
Constant	(0.51) -7.59***	(0.54) -7.56***	(0.52) -7.27***	(0.55) -7.34***	(0.48) -7.74^{***}
	(0.31)	(0.32)	(0.30)	(0.32)	(0.29)
log(alpha)	-3.37*** (0.069)	-3.31*** (0.068)	-3.51*** (0.068)	-3.29*** (0.069)	-3.50*** (0.069)
σ^2 department	0.0060** (0.0022)	0.0076** (0.0025)	0.0075**** (0.0022)	0.0076** (0.0025)	0.0055 ^{**} (0.0019)
Observations	534 8367.6	534 8406.0	534 8310.1	534 8414.3	534 8298.3
Standard errors in na		0100.0	001011	0 1 1 1 1 0	0200.5

Standard errors in parentheses p < 0.05, p < 0.01, p < 0.001

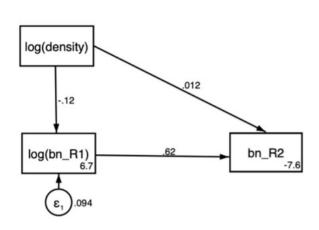
Appendix 5 Population density and invalid voting

Other things being equal, is there a higher rate of invalid voting in more rural districts, due to social conformity pressures disincentivizing abstention? Demographic and other socioeconomic data are not collected methodically by district in France, with the exception of a demographic 'snapshot' INSEE produced for districts just before the 2022 elections (https://www.insee.fr/fr/statistiques/6436315?sommaire=6436478). To derive an index for urban-rural profile of each district, we estimate population density using area of the spatial unit, derived from ESRI shapefiles of the districts, and the size of the registered electorate. Shapefiles for the district boundaries since 2012 are widely available (https://www.insee.fr/fr/statistiques/6441661?sommaire=6436478), allowing the calculation of density (registered electors / hectare). We take the log of this measure given very heavy right skew, due to particularly high densities in the Ile-de-France, Marseilles and Lyons. We were unable to locate an accurate set of shapefiles for the 2002 and 2007 elections.

If rurality does influence invalidation, we would expect this to occur in the first round as well as the second. However, the rurality effect should act indirectly via first-round invalidation. There seems no reason to expect an additional rurality effect at round 2. A simple path model, fitting first- and second-round invalidation with an exogenous density variable, confirms this for the three years we can estimate (Figure A5 and Model A5). The expected negative relationship between population

density and first-round invalidation holds (p < .001), but fails to reach significance in its direct effect on second-round invalidation (p = .101). For the three years we can estimate, then, the absence of this control is not problematic.

Figure A5 Path diagram of direct and indirect effects of district populations density on first- and second-round vote invalidation



Note: GSEM estimation using negative binomial with mean log link for 2012, 2017 and 2022

Model A5 Path model of district population density on first- and second-round vote invalidation

Population density (log)	0.012
Invalid R1 (log)	(0.0073) 0.62***
Constant	(0.035) -7.57***
Invalid R1 (log)	(0.23)
Population density (log)	-0.12***
Constant	(0.0043) 6.74^{***}
	(0.0084)
log(alpha)	-1.78*** (0.035)

```
\sigma^2 Invalid R1(log) 0.094*** (0.0033)
Observations 1577
BIC 27200.1
Standard errors in parentheses p < 0.05, ** p < 0.01, *** p < 0.001
```

Appendix B1 Full model specifications by year across six bloc codings

2002	5-bloc	4-bloc	3-bloc	2-bloc	Nuance	Candidate
Invalid R1 (log)	0.21^{***}	0.21^{***}	0.17^{***}	0.22^{***}	0.23^{***}	0.22^{***}
. 37	(0.033)	(0.031)	(0.027)	(0.027)	(0.032)	(0.032)
Thwarted bloc	1.85***	2.56***	4.94***	3.63***	2.31***	2.40***
	(0.26)	(0.24)	(0.27)	(0.20)	(0.29)	(0.27)
Threshold gap	-3.27***	-2.71***	-1.89***	-3.98***	-1.97**	-1.79**
g	(0.57)	(0.52)	(0.43)	(0.42)	(0.63)	(0.62)
Winner m.o.v.	0.39**	0.39**	0.29**	0.15	0.43**	0.45^{***}
***************************************	(0.14)	(0.13)	(0.11)	(0.11)	(0.14)	(0.13)
# candidates (R1)	0.00021	-0.0010	0.0042	0.0039	-0.011*	-0.013**
" canalactes (111)	(0.0048)	(0.0045)	(0.0038)	(0.0038)	(0.0050)	(0.0050)
1/# candidates	2.20***	2.17***	1.36***	1.71***	2.09***	2.04***
(R2)	2.20	2.17	1.50	1./1	2.03	2.04
(112)	(0.26)	(0.24)	(0.21)	(0.22)	(0.26)	(0.26)
Withdrawn	0.12^*	0.13**	0.21	0.20***	-0.0066	-0.012
candidate	0.12	0.15	0.21	0.20	-0.0000	-0.012
Canadate	(0.052)	(0.048)	(0.039)	(0.039)	(0.057)	(0.056)
% higher ed.	0.023	0.072	0.16	-0.18	0.096	0.12
(dept)	0.023	0.072	0.10	-0.10	0.030	0.12
(dept)	(0.32)	(0.30)	(0.26)	(0.29)	(0.33)	(0.33)
% unemp. (dept)	2.63**	2.74^{**}	0.78	2.02^*	4.25***	4.56***
% unemp. (dept)	(0.97)	(0.89)	(0.81)	(0.87)	(0.99)	(0.98)
% retired (dept)	3.13***	3.30***	3.66***	2.30***	2.69***	2.74^{***}
% retired (dept)	(0.65)	(0.61)			(0.68)	(0.67)
Constant	-7.01****	-7.04***	(0.54) -6.57***	(0.59) -6.46^{***}	-7.25***	-7.20***
Constant	(0.30)	(0.28)			(0.30)	(0.29)
	(0.30)	(0.20)	(0.24)	(0.25)	(0.30)	(0.29)
log(alpha)	-3.03***	-3.13***	-3.50***	-3.51***	-3.07***	-3.10***
log(diplia)	(0.072)	(0.072)	(0.071)	(0.072)	(0.071)	(0.071)
σ² department	0.011**	0.0091**	0.0082***	0.011***	0.013***	0.013***
o dopar minorit	(0.0037)	(0.0031)	(0.0023)	(0.0029)	(0.0038)	(0.0037)
Observations	501	501	501	501	501	501
BIC	7526.8	7471.7	7300.4	7311.6	7515.4	7502.1
Ctandard arrare in no		/ 1 / 1 . /	7500.1	/511.0	7010.4	/ 502.1

Standard errors in parentheses p < 0.05, ** p < 0.01, *** p < 0.001

2007	5-bloc	4-bloc	3-bloc	2-bloc	Nuance	Candidate 0.26***
Invalid R1 (log)	0.28***	0.27***	0.21***	0.24***	0.27***	
Thwarted bloc	(0.037) 3.42*** (0.37)	(0.033) 4.71*** (0.31)	(0.030) 5.55*** (0.32)	(0.031) 5.64*** (0.34)	(0.037) 4.55*** (0.50)	(0.037) 4.54^{***} (0.49)
Threshold gap	-2.85***	-2.63***	-3.36***	-3.22***	0.23	0.24
	(0.59)	(0.49)	(0.44)	(0.45)	(0.81)	(0.81)
Winner m.o.v.	0.14 (0.15)	0.094 (0.13)	0.39** (0.12)	0.43*** (0.12)	0.18 (0.15)	0.15 (0.15)
# candidates (R1)	0.0047	0.0087	0.0074	0.0084	-0.014	-0.014
	(0.0069)	(0.0061)	(0.0057)	(0.0058)	(0.0076)	(0.0075)
1/# candidates (R2)	2.24***	1.95***	1.53***	1.17***	1.70***	1.71***
Withdrawn	(0.32)	(0.28)	(0.28)	(0.29)	(0.35)	(0.35)
candidate	0.22**	0.27***	0.33***	0.32***	-0.14	-0.14
% higher ed.	(0.082)	(0.071)	(0.066)	(0.066)	(0.098)	(0.098)
(dept)	-0.24	-0.18	0.33	0.13	0.16	0.17
% unemp. (dept)	(0.33)	(0.30)	(0.28)	(0.29)	(0.34)	(0.34)
	1.78	3.44**	2.03*	3.87***	1.96	1.97
	(1.21)	(1.11)	(1.01)	(1.07)	(1.24)	(1.24)
% retired (dept)	2.87***	3.57***	4.38***	3.64***	2.98***	3.02***
	(0.77)	(0.70)	(0.64)	(0.67)	(0.78)	(0.78)
Constant	-7.72***	-7.78***	-7.12***	-6.98***	-7.81***	-7.80***
	(0.36)	(0.32)	(0.30)	(0.31)	(0.37)	(0.37)
log(alpha)	-3.00***	-3.26***	-3.41***	-3.40***	-3.00***	-3.00***
	(0.076)	(0.077)	(0.077)	(0.077)	(0.076)	(0.076)
σ^2 department	0.017***	0.014***	0.011***	0.014***	0.018***	0.018***
	(0.0047)	(0.0039)	(0.0032)	(0.0036)	(0.0048)	(0.0048)
Observations BIC Standard errors in pa $p < 0.05$, ** $p < 0.01$,	445 6548.0 rentheses	445 6437.6	445 6370.7	445 6382.0	445 6550.1	445 6549.7

2012	5-bloc	4-bloc	3-bloc	2-bloc	Nuance	Candidate
Invalid R1 (log)	0.29*** (0.035)	0.33*** (0.033)	0.33*** (0.033)	0.36^{***} (0.031)	0.31*** (0.037)	0.31*** (0.037)
Thwarted bloc	4.31***	4.88***	5.01***	4.34***	3.64***	3.81***
Threshold gap	(0.31) -2.29***	(0.29) -1.60**	(0.30) -1.41**	(0.23) -5.28***	(0.31) -1.61**	(0.30) -1.26^*
3.1	(0.52)	(0.49)	(0.49)	(0.40)	(0.59)	(0.59)
Winner m.o.v.	0.39^{**}	0.44^{***}	0.46^{***}	0.63^{***}	0.57^{***}	0.60^{***}
	(0.13)	(0.12)	(0.12)	(0.11)	(0.13)	(0.13)
# candidates (R1)	0.0046	0.0095^{*}	0.012^{**}	0.013^{**}	-0.011^*	-0.012^*
	(0.0047)	(0.0043)	(0.0043)	(0.0041)	(0.0051)	(0.0050)
1/# candidates (R2)	1.71***	2.15***	2.07^{***}	1.88****	1.99***	1.97***
	(0.14)	(0.11)	(0.11)	(0.12)	(0.14)	(0.14)
Withdrawn candidate	0.10	0.13^{*}	0.10	0.31****	-0.11	-0.12
	(0.065)	(0.060)	(0.060)	(0.056)	(0.072)	(0.071)
% higher ed. (dept)	0.46^*	0.67**	0.73**	0.41	0.50	0.49
(111 F 3)	(0.22)	(0.22)	(0.23)	(0.22)	(0.26)	(0.26)
% unemp. (dept)	1.67*	3.10^{***}	2.70***	2.62***	3.93***	4.32***
	(0.78)	(0.74)	(0.77)	(0.73)	(0.89)	(0.87)
% retired (dept)	2.67^{***}	3.04^{***}	3.24^{***}	2.26^{***}	2.08^{**}	1.92^{**}
	(0.57)	(0.56)	(0.58)	(0.55)	(0.66)	(0.64)
Boundary change	-0.031	-0.035	-0.029	-0.052	-0.045	-0.043
	(0.031)	(0.030)	(0.030)	(0.029)	(0.035)	(0.034)
Constant	-7.87***	-8.53***	-8.49***	-7.89***	-8.22***	-8.29***
	(0.28)	(0.26)	(0.27)	(0.25)	(0.31)	(0.30)
log(alpha)	-3.16***	-3.32***	-3.33***	-3.46***	-3.12***	-3.14****
	(0.069)	(0.070)	(0.070)	(0.071)	(0.070)	(0.069)
σ^2 department	0.0050^*	0.0053^{**}	0.0062^{**}	0.0057^{**}	0.0091^{**}	0.0082^{**}
	(0.0020)	(0.0020)	(0.0021)	(0.0020)	(0.0029)	(0.0026)
Observations	507	507	507	507	507	507
BIC	7409.2	7337.5	7336.3	7276.1	7450.5	7435.5
Standard errors in pa * $p < 0.05$, ** $p < 0.01$	rentneses					
p < 0.05, p < 0.01	, p < 0.001					

5-bloc	4-bloc	3-bloc	2-bloc	Nuance	Candidate 0.20*** (0.025) 4.19***
0.22***	0.24***	0.24***	0.24***	0.20***	
(0.024)	(0.025)	(0.026)	(0.027)	(0.025)	
4.21***	2.58***	1.62***	1.46***	4.13***	
(0.26)	(0.16)	(0.12)	(0.1 <i>2</i>)	(0.27)	(0.28)
-1.54**	-3.71***	-4.86***	-5.94***	-1.09	-1.08
(0.56)	(0.51)	(0.52)	(0.53)	(0.59)	(0.59)
0.52***	0.68***	0.81***	0.74***	0.52***	0.52***
(0.096)	(0.095)	(0.098)	(0.10)	(0.099)	(0.099)
(0.0029)	(0.0028)	(0.0029)	(0.0030)	(0.0032)	-0.013*** (0.0032)
					0.23 (0.32)
0.29	0.36	0.27	0.21	0.27*	0.28*
(0.15)	(0.22)	(0.22)	(0.23)	(0.14)	(0.14)
-1.67**	-1.23	-0.21	-0.94	-1.01	-0.97
(0.58)	(0.80)	(0.82)	(0.86)	(0.55)	(0.54)
1.78***	2.88***	3.30***	3.18***	1.61***	1.58***
(0.41)	(0.56)	(0.57)	(0.60)	(0.38)	(0.37)
-5.69***	-5.88***	-6.06***	-5.76***	-5.57***	-5.55***
(0.25)	(0.28)	(0.29)	(0.30)	(0.25)	(0.25)
-3.80***	-3.91***	-3.83***	-3.78***	-3.71***	-3.71***
(0.068)	(0.068)	(0.068)	(0.068)	(0.068)	(0.068)
0.0032^{**} (0.0012)	(0.0023)	(0.0024)	0.013*** (0.0027)	0.0018 (0.0010)	0.0016 (0.00099)
536 8333.1 rentheses *** p < 0.001	536 8346.4	536 8383.4	536 8415.9	536 8360.9	536 8359.0
	0.22*** (0.024) 4.21*** (0.26) -1.54** (0.56) 0.52*** (0.096) -0.0034 (0.0029) 0.14 (0.32) 0.29 (0.15) -1.67** (0.58) 1.78*** (0.41) -5.69*** (0.25) -3.80*** (0.068) 0.0032** (0.0012) 536 8333.1 rentheses	0.22*** (0.024) (0.025) 4.21*** 2.58*** (0.26) (0.16) -1.54** -3.71*** (0.56) (0.51) 0.52*** 0.68*** (0.096) (0.095) -0.0034 0.0088** (0.0029) (0.0028) 0.14 0.53 (0.32) (0.32) 0.29 0.36 (0.15) (0.22) -1.67** -1.23 (0.58) (0.80) 1.78*** 2.88*** (0.41) (0.56) -5.69*** -5.88*** (0.25) (0.28) -3.80*** (0.28) -3.80*** -3.91*** (0.068) 0.0032* (0.0012) (0.0023) 536 8333.1 8346.4 rentheses	0.22*** 0.24*** 0.24*** (0.024) (0.025) (0.026) 4.21**** 2.58*** 1.62**** (0.26) (0.16) (0.12) -1.54** -3.71*** -4.86*** (0.56) (0.51) (0.52) 0.52**** 0.68*** 0.81*** (0.096) (0.095) (0.098) -0.0034 0.0088** 0.0094** (0.0029) (0.0028) (0.0029) 0.14 0.53 1.11*** (0.32) (0.32) (0.32) (0.29) 0.36 0.27 (0.15) (0.22) (0.22) -1.67** -1.23 -0.21 (0.58) (0.80) (0.82) 1.78*** 2.88*** 3.30*** (0.41) (0.56) (0.57) -5.69*** -5.88*** -6.06*** (0.25) (0.28) (0.29) -3.80*** -3.91*** -3.83*** (0.068) (0.068) (0.068) 0.0032** 0.011**** 0.011*** (0.0012)<	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	0.22*** 0.24*** 0.24*** 0.24*** 0.20*** (0.024) (0.025) (0.026) (0.027) (0.025) 4.21*** 2.58*** 1.62**** 1.46*** 4.13*** (0.26) (0.16) (0.12) (0.12) (0.27) -1.54** -3.71*** -4.86*** -5.94*** -1.09 (0.56) (0.51) (0.52) (0.53) (0.59) 0.52*** 0.68*** 0.81*** 0.74*** 0.52*** (0.096) (0.095) (0.098) (0.10) (0.099) -0.034 0.008** 0.0094** 0.0076** -0.012*** (0.0029) (0.0028) (0.0029) (0.0030) (0.0032) 0.14 0.53 1.11*** 1.10** 0.25 (0.32) (0.32) (0.32) (0.34) (0.32) (0.29) 0.36 0.27 0.21 0.27* (0.15) (0.22) (0.22) (0.23) (0.14) -1.67** -1.23

p < 0.01, p < 0.00

2022	5-bloc	4-bloc	3-bloc	2-bloc	Nuance	Candidate
Invalid R1 (log)	0.39^{***}	0.39^{***}	0.34^{***}	0.38^{***}	0.39^{***}	0.40^{***}
	(0.033)	(0.033)	(0.030)	(0.034)	(0.030)	(0.030)
Thwarted bloc	2.00^{***}	1.80^{***}	3.54^{***}	-0.73***	3.57^{***}	3.61^{***}
	(0.31)	(0.30)	(0.26)	(0.17)	(0.28)	(0.29)
Threshold gap	-4.72***	-4.93***	-3.55***	-8.03***	-2.91***	-2.84***
	(0.69)	(0.70)	(0.56)	(0.57)	(0.62)	(0.63)
Winner m.o.v.	-0.037	-0.046	-0.16	-0.14	0.20	0.20
	(0.13)	(0.13)	(0.11)	(0.13)	(0.12)	(0.12)
# candidates (R1)	0.016^{**}	0.016^{**}	0.0082	0.022***	0.0013	0.0012
	(0.0051)	(0.0051)	(0.0046)	(0.0052)	(0.0049)	(0.0049)
1/# candidates	2.12***	2.14^{***}	1.83^{***}	2.44***	1.73***	1.73^{***}
(R2)	(0.05)	(0.07)	(0.04)	(0.00)	(0.05)	(0.05)
TA71.3 3	(0.27)	(0.27)	(0.24)	(0.28)	(0.25)	(0.25)
Withdrawn	0.30	0.31	0.18	0.60^{**}	0.19	0.19
candidate	(0.21)	(0.21)	(0.10)	(0.21)	(0.10)	(0.10)
0/ 1: 1 1	(0.21)	(0.21)	(0.19)	(0.21)	(0.19)	(0.19)
% higher ed.	0.015	0.015	0.088	-0.16	0.063	0.060
(dept)	(0.20)	(0.20)	(0.19)	(0.19)	(0.17)	(0.17)
% unomp (dont)	3.34***	3.20**	2.13*	2.18*	3.27***	(0.17) 3.38***
% unemp. (dept)	(0.97)	(0.97)	(0.89)	(0.95)	(0.86)	3.36 (0.85)
% retired (dept)	1.44^{**}	1.43**	1.42**	1.40^{**}	0.69	0.63
% retired (dept)	(0.54)	(0.55)	(0.51)	(0.54)	(0.49)	(0.49)
Constant	-7.80***	-7.77***	-7.28***	-7.31***	-7.84***	-7.87***
Constant	(0.32)	(0.33)	(0.29)	(0.32)	(0.29)	(0.29)
	(0.52)	(0.55)	(0.29)	(0.52)	(0.29)	(0.23)
log(alpha)	-3.36***	-3.34***	-3.61***	-3.30***	-3.53***	-3.52***
iog(diplia)	(0.068)	(0.068)	(0.068)	(0.069)	(0.069)	(0.069)
σ^2 department	0.0079**	0.0079**	0.0076***	0.0073**	0.0060**	0.0059**
5 11 P 11 11 11 11 11 11 11 11 11 11 11 1	(0.0024)	(0.0025)	(0.0021)	(0.0025)	(0.0020)	(0.0019)
Observations	534	534	534	534	534	534
BIC	8385.5	8392.0	8263.3	8408.7	8288.0	8289.8
Standard errors in pa						
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$					
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Appendix B2 Full model specifications by year controlling for abstention

Use of abstention as a control variable in models of French invalid voting varies (models of abstention, conversely, tend not to control for invalidation). Heinsohn (2018) controls for turnout change between presidential rounds. Rosenthal and Sen (1973) and Pons and Tricaud (2018) run separate specifications for abstention and invalid voting. Both outcomes are clear choice alternatives to competitive dynamics, and may be driven by similar motivators, conditioned by exogenous factors – selective benefits, cost of voting, psychological factors. None of these can be controlled for satisfactorily in an ecological model. Endogeneity and spuriousness are also issues – abstention does not cause invalid voting, and their association is compositional. Higher choice rates of abstention suggest lower rates of invalidation; yet, empirically, they tend to be positively associated, given similar underlying dynamics.

There are methodological issues too. A change measure is warranted, given that, like invalid voting, a baseline of non-participation across any election is needed to look at the shift in turnout between rounds.

Negative values prevent the use of a logged value for a right-skewed measure, so we are forced to use a percentage change.

Consequently, we have omitted abstention from the main models. Here, we report the effects of including this measure. Overall, the models remain stable. The explanatory variables upon which we focus change

only marginally, with the threshold differential, number of candidates and withdrawal effects remaining significant in the expected direction. The baseline of invalid voting in round 1 also remains stable. The main change is in the margin of victory variable, which washes out or switches direction. Shared variance across abstention and margin of victory can be rationalized – races which seem foregone conclusions deincentivize participation, at least in plurality systems (Blais 2006, 120). However, it is difficult to map this in path terms through abstention as a mediator, or explain the direct effect.

Finally, in terms of the overall model fits, the 2017 best-fit model shifts from five-blocs to four blocs, though again the relative change is slight. This suggests a link between abstention and thwarted left bloc voters, but we refrain from speculating further given the impossibility of unpicking this dynamic with the current dataset.

2002 Invalid R1 (log)	5-bloc 0.24***	4-bloc 0.23***	3-bloc 0.18***	2-bloc 0.23****	Nuance 0.25***	Candidate 0.24***
Change in abstention (%)	(0.031) 4.20***	(0.030) 3.38***	$(0.027) \\ 1.22^*$	(0.027) 0.73	$(0.031) \ 4.22^{***}$	(0.031) 3.96***
abstention (70)	(0.53)	(0.53)	(0.49)	(0.52)	(0.51)	(0.51)
Thwarted bloc	1.50***	2.05***	4.64^{***}	3.46***	1.99***	2.00***
	(0.24)	(0.24)	(0.29)	(0.24)	(0.27)	(0.26)
Threshold gap	-2.60***	-2.36***	-1.84***	-3.87***	-1.42^{*}	-1.40^{*}
	(0.54)	(0.51)	(0.43)	(0.43)	(0.59)	(0.58)
Winner m.o.v.	-0.13	-0.022	0.15	0.071	-0.090	-0.044
	(0.14)	(0.14)	(0.12)	(0.12)	(0.14)	(0.14)
# candidates (R1)	-0.0034	-0.0037	0.0032	0.0033	-0.013**	-0.014^{**}
	(0.0045)	(0.0043)	(0.0038)	(0.0038)	(0.0046)	(0.0047)
1/# candidates (R2)	1.08***	1.29***	1.08***	1.55***	0.95**	1.00***
	(0.30)	(0.28)	(0.24)	(0.25)	(0.29)	(0.29)
Withdrawn candidate	0.12*	0.13**	0.20***	0.20***	0.0037	0.0064
	(0.048)	(0.045)	(0.039)	(0.039)	(0.053)	(0.052)
% higher ed. (dept)	-0.23	-0.13	0.074	-0.21	-0.17	-0.14
(1)	(0.33)	(0.32)	(0.28)	(0.29)	(0.34)	(0.33)
% unemp. (dept)	3.51***	3.44***	1.08	2.21*	4.91^{***}	5.10***
r (r (r)	(1.01)	(0.97)	(0.85)	(0.89)	(1.00)	(0.99)
% retired (dept)	4.24***	4.22^{***}	3.98***	2.54^{***}	3.85***	3.82***
(1-1)	(0.69)	(0.67)	(0.58)	(0.62)	(0.70)	(0.69)
Constant	-6.81***	-6.88***	-6.52***	-6.45***	-7.01***	-6.99***
	(0.29)	(0.28)	(0.24)	(0.25)	(0.29)	(0.29)
/	` ,	, ,	, ,	, ,	, ,	, ,
log(alpha)	-3.20***	-3.26***	-3.53***	-3.52***	-3.24***	-3.24***
3、1 /	(0.073)	(0.073)	(0.072)	(0.072)	(0.072)	(0.072)
σ^2 department	0.014^{***}	0.013***	0.0094***	0.012***	0.015***	0.014^{***}
•	(0.0041)	(0.0038)	(0.0026)	(0.0030)	(0.0040)	(0.0038)
Observations	501	501	501	501	501	501
BIC	7470.4	7438.0	7300.4	7315.8	7454.6	7450.0
Standard errors in pa						
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$					
_	•					

2007	5-bloc	4-bloc	3-bloc	2-bloc	Nuance	Candidate
Invalid R1 (log)	$0.29^{***} (0.034)$	0.28*** (0.032)	0.22*** (0.031)	$0.25^{***} \ (0.031)$	0.28*** (0.034)	$0.27^{***} (0.034)$
Change in abstention (%)	5.79***	3.09***	1.81**	2.00**	5.86***	5.93***
Thwarted bloc	(0.65) 2.10***	(0.69) 3.67***	(0.67) 4.88***	$(0.68) \\ 4.86^{***}$	(0.62) 3.00***	(0.62) 3.08***
Threshold gap	(0.36) -1.86*** (0.54)	(0.38) -2.24*** (0.48)	(0.40) -3.02^{***} (0.45)	(0.42) -2.86*** (0.45)	(0.47) 0.37 (0.73)	(0.47) 0.52 (0.73)
Winner m.o.v.	-0.52*** (0.15)	-0.25 (0.15)	0.15 (0.15)	0.15 (0.15)	-0.50*** (0.15)	-0.53*** (0.15)
# candidates (R1)	0.0062 (0.0064)	0.0088 (0.0060)	0.0077 (0.0057)	0.0086 (0.0057)	-0.0061 (0.0070)	-0.0066 (0.0069)
1/# candidates (R2)	0.42	1.06**	1.05**	0.68*	0.022	-0.026
Withdrawn candidate	(0.36) 0.12	$(0.34) \\ 0.20^{**}$	(0.33) 0.29***	(0.33) 0.27***	(0.36) -0.13	(0.36) -0.14
candidate	(0.073)	(0.069)	(0.067)	(0.067)	(0.087)	(0.087)
% higher ed. (dept)	-0.48	-0.32	0.19	-0.0090	-0.23	-0.22
% unemp. (dept)	(0.38) 4.77*** (1.40)	(0.33) 4.65*** (1.23)	(0.30) 2.90** (1.12)	(0.32) 4.59*** (1.16)	(0.38) 4.82*** (1.37)	(0.38) 4.85*** (1.38)
% retired (dept)	4.56*** (0.88)	4.34*** (0.77)	4.75*** (0.69)	4.13*** (0.73)	4.65*** (0.86)	4.70*** (0.86)
Constant	-7.14*** (0.36)	-7.45*** (0.33)	-7.00*** (0.31)	-6.88**** (0.32)	-7.19**** (0.36)	-7.17*** (0.36)
/ log(alpha)	-3.28*** (0.079)	-3.36*** (0.079)	-3.46*** (0.079)	-3.46*** (0.079)	-3.29*** (0.079)	-3.30*** (0.079)
σ^2 department	0.027*** (0.0065)	0.079) 0.019*** (0.0050)	0.079) 0.014*** (0.0039)	0.017*** (0.0044)	0.026*** (0.0063)	0.026*** (0.0063)
Observations BIC	445 6475.1	445 6423.3	445 6369.3	445 6379.2	445 6469.9	445 6466.1

Standard errors in parentheses p < 0.05, ** p < 0.01, *** p < 0.001

2012	5-bloc	4-bloc	3-bloc	2-bloc	Nuance	Candidate
Invalid R1 (log)	0.30^{***}	0.33^{***}	0.33^{***}	0.36^{***}	0.32^{***}	0.32^{***}
. 5,	(0.034)	(0.032)	(0.032)	(0.030)	(0.036)	(0.035)
Change in abstention	3.04***	2.21***	2.13***	-1.04	3.51***	3.36***
(%)						
	(0.53)	(0.52)	(0.53)	(0.59)	(0.53)	(0.53)
Thwarted bloc	3.75***	4.43***	4.56***	4.64^{***}	3.02***	3.22***
	(0.32)	(0.31)	(0.32)	(0.29)	(0.31)	(0.30)
Threshold gap	-2.38***	-1.81***	-1.66***	-5.35***	-1.87**	-1.52**
9P	(0.51)	(0.49)	(0.49)	(0.40)	(0.58)	(0.58)
Winner m.o.v.	0.14	0.26^{*}	0.28*	0.73***	0.26	0.30*
	(0.13)	(0.12)	(0.12)	(0.12)	(0.13)	(0.13)
# candidates (R1)	0.0057	0.0098*	0.012**	0.013**	-0.0073	-0.0084
,	(0.0045)	(0.0042)	(0.0042)	(0.0041)	(0.0049)	(0.0048)
1/# candidates	1.12***	1.66***	1.60***	2.07***	1.28***	1.29***
(R2)						
,	(0.17)	(0.16)	(0.16)	(0.16)	(0.17)	(0.17)
Withdrawn	0.12	0.14^{*}	0.11^{*}	0.31***	-0.042	-0.060
candidate						
	(0.062)	(0.058)	(0.058)	(0.056)	(0.070)	(0.069)
% higher ed.	0.32	0.56^{*}	0.62^{*}	$\stackrel{\cdot}{0}.44^{*}$	0.34	0.34
(dept)						
• •	(0.24)	(0.24)	(0.24)	(0.21)	(0.27)	(0.26)
% unemp. (dept)	1.65*	2.90**	2.55**	2.60***	3.62^{***}	3.99***
1 • 1	(0.81)	(0.79)	(0.82)	(0.71)	(0.90)	(0.86)
% retired (dept)	2.98***	3.32***	3.52***	2.09^{***}	2.55***	2.37***
_	(0.59)	(0.59)	(0.61)	(0.54)	(0.66)	(0.64)
Boundary change	-0.043	-0.044	-0.037	-0.048	-0.058	-0.056
, ,	(0.032)	(0.031)	(0.032)	(0.028)	(0.035)	(0.034)
Constant	-7.59***	-8.25***	-8.20***	-7.97***	-7.88***	-7.95***
	(0.28)	(0.27)	(0.28)	(0.25)	(0.30)	(0.30)
log(alpha)	-3.25***	-3.38***	-3.40***	-3.45***	-3.22***	-3.24***
_	(0.070)	(0.071)	(0.071)	(0.071)	(0.071)	(0.070)
σ^2 department	0.0066^{**}	0.0073^{**}	0.0082^{***}	0.0049^*	0.010^{***}	0.0087^{**}

Observations BIC Standard errors in pare $p < 0.05$, ** $p < 0.01$, *		(0.0024) 507 7325.9	(0.0025) 507 7326.4	(0.0019) 507 7279.2	(0.0030) 507 7414.1	(0.0027) 507 7401.4
2017 Invalid R1 (log) Change in abstention	5-bloc 0.26*** (0.022) 5.69***	4-bloc 0.27*** (0.021) 5.75***	3-bloc 0.27*** (0.022) 5.87***	2-bloc 0.27*** (0.022) 6.39***	Nuance 0.24*** (0.022) 5.91***	Candidate 0.24*** (0.023) 5.90***
(%) Thwarted bloc	(0.43) 2.87***	(0.41) 1.82***	(0.44) 1.01***	(0.41) 1.00****	(0.44) 2.69***	(0.44) 2.71***
Threshold gap	(0.24) -1.38** (0.48) -0.12	(0.15) -2.76*** (0.43) -0.029	(0.11) -3.62*** (0.45) 0.040	(0.10) -4.15*** (0.45) -0.064	(0.26) -1.23* (0.50) -0.14	(0.26) -1.25* (0.50) -0.14
Winner m.o.v. # candidates (R1)	-0.12 (0.094) -0.0048 (0.0025)	(0.029 (0.093) 0.0036 (0.0024)	(0.10) 0.0037 (0.0025)	(0.097) 0.0022 (0.0025)	-0.14 (0.097) -0.010*** (0.0027)	(0.097) -0.011*** (0.0027)
1/# candidates (R2)	-1.48*** (0.31)	-1.24*** (0.30)	-0.85** (0.32)	-1.05**** (0.31)	-1.49*** (0.31)	-1.49*** (0.31)
% higher ed. (dept)	(0.20)	0.14	0.072	0.019 (0.24)	(0.19)	0.064
% unemp. (dept)	-0.46 (0.72)	-0.18 (0.81)	0.50 (0.85)	0.14 (0.86)	0.068 (0.70)	0.084 (0.70)
% retired (dept) Constant	2.35*** (0.50) -5.25***	3.12*** (0.56) -5.37***	3.40*** (0.59) -5.48***	3.34*** (0.60) -5.24***	2.29*** (0.49) -5.13***	2.28*** (0.49) -5.11***
/ log(alpha)	(0.25) -4.23***	(0.26) -4.30***	(0.27) -4.20***	(0.27) -4.23***	(0.25)	(0.25) -4.17***
σ^2 department	(0.070) 0.0090*** (0.0020)	(0.069) 0.013*** (0.0024)	(0.069) 0.014*** (0.0027)	(0.069) 0.015*** (0.0027)	(0.070) 0.0082*** (0.0019)	(0.070) 0.0081*** (0.0019)
Observations	536	536	536	536	536	536

BIC Standard errors in part $p < 0.05$, ** $p < 0.01$,		8183.0	8235.3	8225.0	8211.4	8211.4
2022	5-bloc	4-bloc	3-bloc	2-bloc	Nuance	Candidate
Invalid R1 (log)	0.40^{***}	0.41^{***}	0.34^{***}	0.41^{***}	0.40^{***}	0.40^{***}
	(0.033)	(0.033)	(0.030)	(0.033)	(0.030)	(0.030)
Change in	2.61^{**}	2.73^{**}	-0.012	5.61***	1.48	1.67^{*}
abstention (%)	(0.05)	(0, 0,0)	(0,00)	(0, 02)	(0.70)	(0.70)
Thwarted bloc	(0.85) 1.63***	$(0.86) \\ 1.40^{***}$	$(0.80) \ 3.54^{***}$	(0.82) -1.07***	(0.76) 3.39***	(0.76) 3.41***
Thwarted bloc	(0.33)	(0.32)	(0.29)	(0.17)	(0.30)	(0.30)
Threshold gap	-4.37***	-4.60***	-3.55***	-6.35****	-2.65***	-2.53***
Tiffeshold gdp	(0.70)	(0.70)	(0.57)	(0.60)	(0.63)	(0.64)
Winner m.o.v.	-0.23	-0.25	-0.16	-0.50***	0.085	0.068
	(0.14)	(0.14)	(0.12)	(0.14)	(0.13)	(0.13)
# candidates (R1)	0.017^{**}	0.017^{***}	0.0082	0.023^{***}	0.0023	0.0022
	(0.0051)	(0.0051)	(0.0046)	(0.0050)	(0.0049)	(0.0049)
1/# candidates	1.79^{***}	1.79^{***}	1.83****	1.70^{***}	1.55^{***}	1.53^{***}
(R2)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
XA7:+11	(0.29)	(0.29)	(0.26)	(0.29)	(0.26)	(0.26)
Withdrawn candidate	0.24	0.24	0.18	0.44^*	0.15	0.14
Garraraavo	(0.21)	(0.21)	(0.19)	(0.21)	(0.19)	(0.19)
% higher ed.	0.0015	-0.0021	0.088	-0.21	0.058	0.055
(dept)						
	(0.20)	(0.20)	(0.19)	(0.20)	(0.18)	(0.18)
% unemp. (dept)	3.74***	3.62***	2.13*	3.67***	3.59***	3.74^{***}
0/ 1/1	(0.98)	(0.99)	(0.91)	(0.98)	(0.88)	(0.88)
% retired (dept)	1.38*	1.37*	1.42**	1.29*	0.70	0.63
Constant	(0.55) -7.72***	(0.55) -7.68***	(0.51) -7.28***	(0.55) -7.33***	(0.49) -7.82***	(0.49) -7.85^{***}
Constant	(0.32)	(0.33)	(0.29)	(0.31)	-7.82 (0.29)	(0.29)
1	(0.52)	(0.55)	(0.23)	(0.51)	(0.23)	(0.23)
log(alpha)	-3.38***	-3.37***	-3.61***	-3.41***	-3.54***	-3.54***

	(0.068)	(0.068)	(0.068)	(0.069)	(0.069)	(0.069)
σ^2 department	0.0082^{***}	0.0082^{***}	0.0076^{***}	0.0085^{***}	0.0064^{**}	0.0062^{**}
_	(0.0025)	(0.0025)	(0.0021)	(0.0026)	(0.0020)	(0.0020)
Observations	534	534	534	534	534	534
BIC	8382.4	8388.4	8269.6	8369.1	8290.4	8291.2
Standard errors in p	arentheses					
* $p < 0.05$, ** $p < 0.0$	1, *** $p < 0.001$					