The Special Theory of Employment, Exchange Rate, and Money

With the Focus on Inflation and Technological Progress

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Abstract

We introduce the quantity theory of money into the Harrod–Balassa–Samuelson effect model. Our policy rule specifies the impossibility of perfect exchange rate stability with monetary policy, as Friedman (1953) suggests. We discover the importance of inflation to technological progress, while the rent-seeking behaviors in firms foster their productivity slowdowns and disinflations. Their forward-looking behaviors, like animal spirit (Keynes, 1936/1997), control outputs under the marginal productivity hypothesis with the Cobb-Douglus production function. Baumol's (1959) sales revenue maximization hypothesis explains full employment and deflations but breaks the marginal productivity hypothesis. We briefly argue the downward stickiness of nominal wages (incomes).

Keywords: downward stickiness of nominal wage (income), exchange rate, forward-looking behavior, heterogeneity, quantity theory of money, technological progress, unemployment

JEL classification: F31, F16, E31, E52, E24, J63, J24, J64

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In this paper, I show that a policy rule of the exchange rate (relative price) can be derived for a home country's monetary policy given a foreign country's monetary policy by introducing the modified Harrod–Balassa–Samuelson effect model (Samuelson, 1964; Obstfeld & Rogoff, 1997, for its modern treatment). This model includes the existence of unskilled and skilled workers and is combined with the quantity theory of money. We get this model under the traditional *profit* maximization hypothesis. At the same time, we obtain the *price differential* between the home and foreign countries with the (inverse, i.e., inverse effects of the standard) Harrod–Balassa–Samuelson effect under the *sales revenue* maximization hypothesis, as we will note below.

We briefly argue the necessary *revival* of *sales revenue* maximization hypotheses (Baumol, 1959) under the *constant* and *increasing* returns to scale production functions, using the *Krush-Kuhn-Tucker* (KKT) *condition* (e.g., Dixit, 1990) with the *resource constraints* (i.e., labor inputs constraints). This revival justifies *full employment* and supports short-run *deflations* only under this full employment. We should respect the insights of Friedman (1969/2006) (e.g., Ljungqvist & Sargent, 2000, for the other detailed explanations of his insights).

Our result tells us that (short-run) deflations should only be justified by full employment. So, unlike the traditional *competitive equilibrium* concept (i.e., the profit maximization hypothesis), we successfully separate the full employment from the marginal productivity hypothesis (i.e., the profit maximization hypothesis; e.g., Binger & Hoffman, 1998, for their traditional connections).

In short, this sales revenue maximization hypothesis breaks the marginal productivity hypothesis. It introduces the *average productivity* hypothesis to keep full employment, whose economic interpretations are ensured and micro-founded by our new (un)employment theory (e.g., Becker, 1975/1976). Firms' objectives, such as the sales revenue maximization hypothesis or the profit maximization hypothesis, can distinguish the state of the macroeconomy as well as the nominal wage distributions for workers. However, instead of achieving full employment, the average productivity hypothesis might admit to the unequal incomes of workers in our macroeconomy under the sales revenue maximization hypothesis.

On the other hand, this aggregate (or representative agent) wage determination theory and our new (un)employment theory suggest that the employment level (i.e., unemployment [rate]) should be *indeterminate* under the *constant* returns to scale production function. It should be determined by the firm's optimistic or pessimistic *forward-looking* behaviors, like animal spirit (Keynes, 1936/1997). In line with this inference, Cuba-Borda and Singh (2024) argue the essentiality of expectations in the macroeconomy, for example.

This is our micro-foundation for the aggregate labor demand theory, which supposes indeterminate labor input levels with constant returns to scale production function leading to *diminishing equilibria* under the profit maximization hypothesis. These diminishing equilibria satisfy the optimal conditions that the aggregate (or representative) firm's profit maximization problem and our new (un)employment theory based on Rogerson et al. (2005) suggest.

In short, we build our new (un)employment theory to provide a micro-foundation for the aggregate (representative agent) wage determination theory (i.e., the marginal productivity hypothesis under the profit maximization hypothesis). Because the aggregate (or representative agent) wage determination theory just treats unemployment as a stochastic error. However, we would like to construct a structural model for this unemployment in this paper.

Furthermore, the (nominal) wage is *not* connected with output gaps (business cycles) from the beginning due to the usual aggregate (or representative agent) wage determination theory (i.e., the marginal productivity hypothesis).

Thus, in usual recessions (under the profit maximization hypothesis), the firms mainly focus on *layoffs* (leading to a higher unemployment [rate], i.e., *diminishing equilibria*) due to their forward-looking expectations for sales and outputs and do not cut the (nominal) wages so much as in *economic crises*. If any, they only cut a minor part of the nominal wages, such as overtime pay (e.g., Yashiro, 1997). This is one of the origins of the *downward stickiness* of nominal wage, as our new (un)employment theory suggests. Therefore, the nominal wage (rate) fluctuations should not attributed initially to output gaps, as long as the (nominal) wage contracts are not enforced to connect with output gaps (business cycles) by *law*, for example, as Taylor (1979, 1980) suggested. Alternatively, the equation (#5) or(#6) suggests the loose relationship between output gaps and nominal wage rates through the Phillips curve.

Also, we show that this aggregate wage determination theory will admit *rent-seeking* behaviors in the firms (e.g., Tirole, 1988) that also satisfy the optimal conditions derived from the above firm's profit maximization problem (hypothesis). We should escape from such a diminishing equilibrium with the (pessimistic) forward-looking expectations (behaviors) of the public (e.g., Cuba-Borda & Singh, 2024, for the importance of the expectation in the macroeconomy).

Then, as Masuda (2024b) suggests the profound role of individuals in society and history, we need the correct knowledge and forecasting about the macroeconomy led by *technological progress* and optimistic forward-looking expectation of the public, that is, the *human capital* accumulation for the former (Barro & Sala-i-Martin, 1995; Lucas, 2002), created and mentored by the distinguished *authorities* (e.g., Takahashi, 1993). Such a macroeconomy led by distinguished authorities should generate a high-pressure economy (e.g., Samuelson & Solow, 1960).

Moreover, our theory of employment has two theories: Our new (un)employment theory (Rogerson et al., 2005) and the labor force participation theory (e.g., Flinn & Heckman, 1982; Heckman, 1979). This paper connects these two theories to explain employment determination in the macroeconomy. This connection makes us accurately consider the labor input (demand) growth in the macroeconomy.

Overall, we insist that forward-looking behaviors in society are critical for improving this diminishing equilibrium (i.e., unemployment [rate]; e.g., Cuba-Borda & Singh, 2024, for the critical role of forward-looking expectation in the macroeconomy) to generate a high-pressure economy (e.g., Samuelson & Solow, 1960). Thus, this paper provides the logical basis for the so-called neoclassical synthesis (Samuelson, 1948; Samuelson & Solow, 1960), but our policy idea is not its simple repetition.

Then, we mathematically analyze the effects of monetary policy based on money motives such as *transaction*, *speculative*, and *precautionary* (*asset*; Friedman, 1969/2006; e.g., Dornbusch et al., 2008, for details) in the quantity theory of money on nominal and real interest rates together with helicopter money, a supply of money by printing monies (e.g., Bernanke, 2002; Friedman, 1969/2006). We will mathematically prove Friedman's (1970) statement: "Inflation is always and everywhere a monetary phenomenon in the sense that it is and can be produced only by a more rapid increase in the quantity of money than in output."

Finally, we briefly argue the effectiveness of the approximated Sen's (1976) poverty measure in estimating the heterogeneous real income shocks in the *inflationary environments* and the economic crises instead of the output gaps in terms of *relative poverty*. The above heterogeneous real income shocks in the inflationary environments should be the so-called *consequential trickle-down* (real income) growth or its equivalent, consequential *regressive* real income growth (Son, 2004; see Appendix A for the detailed definition of this consequential trickle-down [real income] growth).

The quantity theory of money explicitly considers the whole country's general price level and can represent the *exchange rates* as the objective of the home and foreign monetary policies. Empirical studies sometimes use this formulation to examine the Purchasing Power Parity hypothesis (hereafter, PPP; e.g., Isard, 1995/2001). Here, PPP is the idea that the exchange rate should be the relative price of general price levels between two countries. Moreover, PPP is the nominal exchange rate theory because it includes the home and foreign price levels and quantities of money, the nominal variables. Except for this point, this theory is not different from the real exchange rate theory.

Samuelson (1964) and Obstfeld and Rogoff (1997) discussed the Harrod–Balassa–Samuelson effect. This Harrod–Balassa–Samuelson effect model is a famous exchange rate model that explains the relative price (exchange rate) between a home country and a foreign country based on the differences in total factor productivities (technological progress) of the tradable and nontradable goods industries of both countries.

We get this model under the traditional *profit* maximization hypothesis. This Harrod–Balassa–Samuelson effect model also represents the exchange rates established on PPP, although this model treats only the real side (e.g., Obstfeld & Rogoff, 1997). As noted, under the sales revenue maximization hypothesis, we obtain the price differential between the home and foreign countries characterized as a kind of inverse Harrod–Balassa–Samuelson effect model. Here, "inverse" means the inverse effect of each technological progress compared with the normal Harrod–Balassa–Samuelson effect model. We can also obtain the hybrid price differential equation between the country under the sales revenue maximization hypothesis and the other under the profit maximization hypothesis and vice versa.

This paper interchangeably uses these three words: Technological progress, productivity, and total factor productivity. As Barro and Sala-i-Martin (1995) characterize, practically, these notions are primarily similar. At least, we should not take care of the differences, even considering the existence of Hicks neutral or labor-augmenting technology, once we take the natural logarithm (and assume the linear production function) in this paper.

More precisely, human capital accumulation should be interpreted as labor-augmenting technology, as Barro and Sala-i-Martin (1995) and Lucas (2002) suggest. Readers should be careful that this paper does not examine this separation between labor-augmenting technological progress and human capital accumulation so much.

Back to the story, however, few studies have examined the relationship between the Harrod–Balassa–Samuelson effect and the quantity theory of money. Assuming that the quantity theory of money holds for home and foreign countries together with the Harrod–Balassa–Samuelson effect model, my model will explicitly consider monetary policy rule when the Harrod–Balassa–Samuelson effect and the quantity theory of money theoretically coexist. In this paper, I analyze this resulting monetary policy rule of the exchange rate in detail.

Furthermore, introducing the Harrod–Balassa–Samuelson effect model into the quantity theory of money enables us to consider monetary expansion, like helicopter money (e.g., Bernanke, 2002; Friedman, 1969/2006). This expansion is more than Friedman's (1960) famous money growth rule suggests; it should induce domestic inflation and the resulting technological progress of the home tradable goods industry. In this model, even with the quantity theory of money, unlike itself, the Harrod–Balassa–Samuelson effect derives a positive connection between the technological progress of the tradable goods industry and the domestic inflation rate.

Thus, our study theorizes the positive, favorable connection between monetary policy, that is, the inflation rate and technological progress. Our study should resemble the previous approaches to support the optimality of the positive inflation rate by discussing the inflation tax theory such as Summers (1991) and the zero lower bounds of nominal interest rate such as Orphanides and Wieland (1998) with the new Keynesian model.

However, regarding the inflation tax theory, some critics like Erosa and Ventura (2002), who insist that the inflation tax should be regressive, also exist. Still, the usual inflation tax theory argues that the loss from the inflation tax is negligible using Harberger's triangle, as Summers (1991) stated (see Lucas, 2000, for a similar result about the cost of inflation). We will get in touch with this critic through the heterogeneous effects of inflation on real incomes in the inflationary environments analyzed in this paper. Also, this critic holds even considering some traditional costs of inflation proposed by Romer (1996), like the certainty equivalence (see Masuda, 2024a, for the other inflation's harm that higher inflations induce the resource misallocations).

Appendix A briefly provides the approximated Sen's (1976) poverty measure to estimate the above heterogeneous (inflation) effects both in inflationary environments (the consequential *trickle-down* [real income] growth; Son, 2004), as supposed by Erosa and Ventura (2002), and in economic crises. Relative poverty is a heterogeneous phenomenon, and economic crises are also known to cause heterogeneous income reductions (recall the unemployment surge in the Great Depression; e.g., Galbraith, 1997/2008; Krugman, 2002). Hereafter, in this paper, we refer to poverty as relative poverty (e.g., Sen, 1997/2000).

Here, consumption is usually equal to permanent income (Friedman, 1957), so we can measure poverty by consumption as well as income (e.g., Deaton & Paxson, 1997). However, in this paper, Appendix D shows that the current consumption depends only on the current income. Taking the household's, that is, consumer's usual dissatisfaction with inflations into account, the consumer's welfare, that is, the social welfare solely depends on their consumption. Because, from our theory, inflations should *temporarily* impair their real incomes and resulting consumption even if the monetary authority takes continual monetary expansions.

Therefore, this poverty measure should be as beneficial in estimating the negative effects of inflationary environments and economic crises as output gaps. Moreover, as we will explain later, this paper suggests that we should care about precautionary (asset) motives only in economic crises, as Friedman and Schwartz (1963/2008) supposed. Thus, this poverty measure will support estimating the need to conduct monetary expansions when the precautionary (asset) motive that introduces the monetary contraction matters.

Also, I briefly argue the role of money motives like transaction, speculative, and precautionary (asset, and excluding speculative in the literature of optimum quantity of money of the monetarist tradition; Friedman, 1969/2006) motives in the quantity theory of money using Keynesian and monetarist perspectives (e.g., Dornbusch et al., 2008). Following and based on these famous three motives for holding money rooted in Keynesian tradition (e.g., Dornbusch et al., 2008), I analyze the effects of monetary policy. This analysis should include the effects of changing the quantity of money on the velocity of money and nominal and real interest rates with mathematical expressions. In this paper, the motives above are intended by real factors such as income growth, interest rate movement, and (fluctuating) conditional heteroscedastic variances of and the increased sensitiveness to output and interest rate (and price) (Dornbusch et al., 2008, for the former two; see Appendix B for the latter). They are shown as a bundle that consists entirely of Friedman's (1960) money growth rule.

Furthermore, regarding interest rates, for the convenience of the above discussions, we can assume *any positive* level for the *natural rates* of real interest (e.g., Holston et al., 2016), that is, the interest rates that keep inflation constant in the macroeconomy. This is because this paper mainly treats the money demand, which does not usually include the natural rate of interest in its formulation, and the investment-saving (IS) relation where the natural rate of interest is essential is behind this money demand. Thus, we explicitly touch this natural rate of interest only when we should inevitably mention it. Note that we cannot accept a nonpositive (natural) rate of real interest to ensure a stable equilibrium (see Masuda, 2024a).

On the other hand, helicopter money itself is a nominal phenomenon (e.g., Bernanke, 2002; Friedman, 1969/2006). More precisely, we define *helicopter money* as the quantity not demanded by the three above motives but supplied like *mana* by printing monies to scatter from heaven. In this paper, as noted, we insist that even this helicopter money, based on not real demands like the above three money motives but just printing money to scatter, can cause inflations to induce the technological progress of the tradable goods industry and be critical.

Note that three motives for holding money are mostly verbally explained in macroeconomics textbooks, such as by Dornbusch et al. (2008). Still, few studies have specified the function of the quantity of money in the quantity theory of money as mathematical expressions. Again, this function refers to the transaction, speculative, precautionary, or asset motive corresponding to the third precautionary motive (Friedman, 1969/2006).

The exceptions are the Baumol–Tobin type transactions model (Baumol, 1952; Tobin, 1956) for the above transaction motive (e.g., Dornbusch et al., 2008, for the verbal explanations; Niehans, 1978, for its modern mathematical treatment) and the money demand equation with the precautionary (asset) motive. For example, the latter captures the randomness of the cash withdrawal by Alvarez and Lippi (2009) (e.g., Blanchard & Fischer, 1989; Boonekamp, 1978, for the other applications of the precautionary [asset] motive).

Lucas (1988) and Niehans (1978) argued that the micro- and logical foundations are given for the money demand equation based on the above transaction motive discussed in this paper. Alvarez and Lippi (2009) also stand on this transaction motive-based money demand equation. Therefore, to analyze the above three motives, we should choose the traditional Baumol–Tobin type transactions cost model (Baumol, 1952; Tobin, 1956), as Niehans (1978) explained.

This traditional *transactions cost* model admits the semi-log type money demand equation that is a more general functional form than the log-log type money demand equation (e.g., Miyao, 2002), which Lucas (1988) gave the micro-foundation. Furthermore, this paper clarifies the relationship between the Taylor reaction function proposed by Alvarez et al. (2001) and the above traditional money demand equation (see Taylor, 1993, for the original Taylor reaction function). We conclude that they have mostly the same functional forms but have different signs of their parameter, depending on the different market views.

Note that words like the reaction functions and the rules are tentatively similarly used in macroeconomics. This paper will separate these words rigorously and use the *reaction function* when we derive such a function based on consistent *mathematics* like a Taylor reaction function (e.g., Orphanides, 2003). At the same time, we apply the *rule* when we obtain such a function from the consistently *logical* foundations and not necessarily from the robust mathematical optimizations, like Friedman's (1960) money growth rule (this paper treats the quantity theory of money as the *identity*).

Furthermore, from our calculation and inference, the corresponding significant part of money demand growth to the output growth represents the transaction motive. Thus, naturally, this Baumol–Tobin type transactions cost model (Baumol, 1952; Tobin, 1956) covers the money growth rule. In short, the other two speculative and precautionary (asset) motives should be considered similar to the transaction motive in this money growth rule because the summed-up effect usually represents the resulting positive real money demand.

Then, note that accurately speaking, Friedman (1969/2006) excludes the speculative motive for the optimum quantity of money and suggests that the precautionary (asset) motive in regular times should be explored more than what it is. We will explain what Fridman (1969/2006) meant with our mathematical calculations in Appendix B. Appendix B introduces the precautionary (asset) motive as the conditional heteroscedastic variance of outputs and nominal interest rates (and general price levels) with the reduced real income (but increased general price level) sensitivity in a mathematical expression with the multivariate second-order Taylor expansion differently from Alvarez and Lippi (2009).

In addition, in the text, we briefly argue that this precautionary (asset) motive should induce *self-fulfilling* monetary disturbances due to the difficulty of forecasting the conditional heteroscedastic variance explained in Appendix B. Furthermore, this reduced sensitivity to the real income requires less money to output movements than the transaction motive. Summed-up effects are *asymmetric*; the monetary authorities should not respond less in the booms, while they should respond more in the recessions (i.e., economic crises).

In short, monetary authorities should not adjust to this motive except for economic crises, as our intuitions suggest (similarly, e.g., Friedman & Schwartz, 1963/2008; Orphanides, 2003). Forecasting the conditional heteroscedastic variance is usually tricky for policymakers, excluding special situations like *economic crises* (Friedman & Schwartz, 1963/2008).

Thus, as we will show, this motive analysis should complement my analysis of the power of monetary policy on the corresponding technological progress of the home tradable goods industry to the caused domestic inflations, based on the established theoretical discoveries such as Samuelson (1964).

In short, as we noted and will show in the later sections, helicopter money is enough to control our new exchange rate stabilization (monetary policy) rule. We should hide the roles of three motives behind the helicopter money discussions for this inflation's effects on real factors. Friedman's (1960) money growth rule entirely explains and substitutes these roles, and this paper mainly treats a more significant quantity of money than this money growth rule suggests.

Additionally, we refer to the revival of the sales revenue maximization hypothesis suggested by Baumol (1959). First, because our production function is assumed to have mainly the constant returns to scale property, we cannot decide the *absolute* (*concrete*) levels of labor inputs only with the standard profit maximization hypothesis. Second, we should consider *resource constraints* (e.g., labor input constraints) for the profit maximization problem. This is because our problem in this paper treats the *aggregate* (*macroeconomic*) problem with limited resources.

Furthermore, this profit maximization problem cannot avoid the existence of *markup-like* terms to exhaust the existing resources. Recall the *shadow price* discussion for the KKT multiplier (see Dixit, 1990); we have a similar discussion about the *Cournot competition* example for the *finite correction* of the price set by the *finite* number of firms. In this Cournot competition example, we assume that the firms can control their sales outputs and have price control powers. However, markup disappears when the number of firms reaches *infinite*, which means that resource constraints are not implicitly binding then (Jehle & Reny, 2001). However, in this case with markup-like terms, mathematically, we cannot get the optimal solution with KKT conditions.

Alternatively, this profit maximization problem should admit the *idle* resources to ensure the marginal productivity (i.e., marginal cost) hypothesis under *competitive markets* once we explicitly consider these (aggregate) resource constraints (with the increasing and constant returns to scale production functions). In this case, we need the firm's forward-looking expectation for its sales and outputs (and the new additional (un)employment theory) to determine the *absolute* levels of labor demands (inputs) like ours in this paper. Furthermore, the zero profit condition (under both the sales revenue maximization hypothesis and the profit maximization hypothesis) can also bring the *average cost* pricing (i.e., average productivity) hypothesis for the industry with *increasing* returns to scale production function (i.e., the natural monopoly).

This paper indicates the existence of a *diminishing equilibrium* in the labor market. This diminishing equilibrium in the labor market is based on characteristics like not the absolute but the derived (i.e., indeterminate in this paper) *relative* labor demand determined under the *constant* returns to the scale production function. Furthermore, firms cannot employ more than currently available resources. In short, unlike the sales revenue maximization hypothesis, the profit maximization hypothesis produces this unemployment, a diminishing equilibrium even with the firm's *forward-looking* behaviors based on their future sales expectation, like *animal spirit* (the *optimistic* forward-looking behavior; Keynes, 1936/1997; see also Cuba-Borda & Singh, 2024).

Furthermore, this paper also indicates the relationship between unemployment and nominal wages (incomes), that is, the *downward stickiness* of the *(segmented average*) nominal incomes in the macroeconomy. It also clarifies the trick of the *average concept*, like the *conditional average* of *the employed*, which is always used around the world when labor statistics measure the nominal wage in the macroeconomy. Our new (un)employment theory generates this inference, which we will explain mathematically in Appendix C.

For our new (un)employment theory, we discuss it from the job search theoretic standpoint with the matching theory to get a micro-foundation for our aggregate (or representative agent) wage determination theory (i.e., the marginal productivity hypothesis under the profit maximization hypothesis). This theory is a variant of the search theoretic model introduced by Rogerson et al. (2005). Note that our (un)employment theory assumes that the heterogeneous workers accept or reject the job offers from the homogeneous firms in this paper.

On the other hand, the sales revenue maximization hypothesis can lead to *full employment*, which is the usual, traditional result in competitive markets. This case specifies that the current labor force (i.e., labor supply) should decide the absolute labor input (labor demand) levels. For that reason, this hypothesis supports the insight of Friedman (1969/2006), although it is limited to the short-run case: The optimality of *deflations*. Furthermore, this hypothesis *rejects* the marginal productivity hypothesis and *accepts* the *average productivity* hypothesis (i.e., zero profit condition), which is justified by our new (un)employment theory to introduce dynamic aspects like *turnovers*.

However, instead of achieving full employment, the average productivity hypothesis might admit to the unequal asset accumulation of workers in our macroeconomy under the sales revenue maximization hypothesis. Overall, intuitively, if the firm's sales growth is at least equal to or less than the economic growth of the macroeconomy (or each industry as in this paper), then such a firm's sales growth should be sustainable.

Note that because we use the Karush-Kuhn-Tucker condition, we can easily prove the existence of the unique equilibrium in this sales revenue maximization hypothesis suggested by Baumol (1959). For example, Masuda (2022) proved the unique existence of the equilibrium in this kind of formulation with the Karush-Kuhn-Tucker approach (e.g., Sundaram, 1996). However, we do not try such proof in this paper.

This paper has a *solution set* based on the sales revenue and profit maximization hypotheses (von Neumann & Morgenstern, 1953/2009, for a solution set concept). Still, we suggest the deficit of the profit maximization hypothesis behaviors with our *intuitive* aggregate resource constraints in terms of the macroeconomy. At the same time, in the microeconomics context, without the necessity to consider the aggregate resource constraints, which are different from our above intuition, the profit maximization hypothesis can also be justified to establish an equilibrium. Furthermore, we obtain a *unique* equilibrium under each profit and sales revenue maximization hypothesis behavior.

As a result, our model in this paper has multiple equilibria for individual firms corresponding to the firm's purposes (i.e., choices of constraints and objectives). This is because the sales revenue and profit maximization objectives should be adapted quite differently, corresponding to the firm's ages from its foundation (e.g., Keynes, 1926), and dominate the ideas (i.e., the course of actions) of the firm's leaders and managers forming their company cultures and so on. Therefore, as a result, such a setting should not allow the *mixed strategy* Nash equilibrium (Gibbons, 1992/1995; Nash, 1950a, 1951; Osborne & Rubinstein, 1994). Thus, for establishing an equilibrium, the vested interests and status quo induced by various speculations and the firm age are critical (see the essential role of the vested interest and status quo in the later section).

Suppose that we describe the choices of these objective functions and constraints using stage games, like the choices of these objective functions and constraints as one of the rational behaviors forming solutions. In that case, we can construct these constrained maximization problems by formulating the general objective functions with different constraints. Thus, we can have a solution set as we explained above, similar to von Neumann and Morgenstern (1953/2009).

Still, Baumol (1962) and Tirole (1988) suggest a similar sales *growth* maximization hypothesis that maximizes the *discounted sum* of the current and future sales revenues with the firm's *growth rate*. However, this paper *only* specifies the sales *revenue* maximization hypothesis because this sales revenue maximization hypothesis is *enough* for our purpose.

After the discussion introducing the sales revenue maximization hypothesis, we mention that the failure of the organization (e.g., Arrow, 1974), the scarcity of managerial resources, and so on, can induce the *rent-seeking behaviors* of leaders and managers in firms (e.g., Drazen, 2000; Tirole, 1988, for the rent-seeking behaviors) *only* under the profit maximization hypothesis. Such phenomena in firms sometimes make them take harmful and unsustainable behaviors, that is, *productivity slowdown*, to realize their own rent-seeking objectives. Such a behavior should raise a firm's risk of being forced to exit from the market. This result is consistent with the original profit (but *not* sales revenue) maximization behaviors.

Furthermore, it generates *deflationary* (at least *disinflationary*) pressures for *market wages* in the labor markets with this productivity slowdown. This continued productivity slowdown leads to the tendency for disinflation (deflation) even for the general price level related to this market wage reduction. Suppose the firms have pessimistic forward-looking expectations for their sales and outputs through the Phillips curve and are threatened by inflation pressures to avoid these pressures. In that case, they might choose productivity slowdowns to lower the velocity of money instead of enduring these inflation pressures. This is why we emphasize the role of rent-seeking behaviors in the firms in this paper.

In this line of disinflationary pressure under the profit maximization hypothesis, Ricardian equivalence (no difference of the fiscal finance between taxes and debts; e.g., Romer, 1996) can also contribute to the (temporal) inflation without technological progress or with disinflationary pressures like the intended recessions by the (continuing) demand deficit (i.e., the negative or decreasing output gaps) with the future tax burden threat after that. The intended recessions usually result in related productivity slowdowns in empirical applications. This is because the wage rates, in reality, might be connected with output gaps by their enforced wage rate contracts by law. Alternatively, our equation (#5) or (#6) should suggest the loose relationships between output gaps and nominal wage rates instead of the above low enforcement.

Again, from our theory, these diminishing equilibria or more unemployment can be induced by the firm's optimistic or pessimistic future sales expectations, like animal spirit (Keynes, 1936, 1997). The decline in nominal wage induced by disinflation or deflation will naturally generate pessimistic forward-looking expectations for their sales and outputs. In our theory, the animal spirit under the profit maximization hypothesis cannot coincide with the full employment under the sales revenue maximization hypothesis.

Moreover, this diminishing equilibrium should contribute to the disinflation or deflation tendencies or decide the degree of the corresponding chronic unemployment (i.e., natural unemployment rate) like the above Ricardian equivalence case through the negative output gaps. In this case, disinflation or deflation harms the macroeconomy compared with those under full employment in the short run, as we will explain in this paper. Here, note that we mention productivity slowdown with different meanings from the modern ones, like supply shocks due to technological spillovers and the scarcity of resources (e.g., Goldin et al., 2024; Olson, 1988).

In reality, separating workers by their skill levels is essential in the firms for their profit maximization and sales revenue maximization objectives, as the basic human capital theory suggests (Becker, 1975/1976). In this paper, we concretely use the Harrod–Balassa–Samuelson effect model with two tradable goods by skilled and unskilled workers and the nontradable goods, based on Obstfeld and Rogoff (1997). Obstfeld and Rogoff (1997) only partly show this skilled and unskilled worker model, but I show its detailed derivation and slightly different extensions in this paper. Hereafter, we call a bundle of nontradable goods that are qualitatively indistinguishable in terms of wage or price simply as the *nontradable goods*. Furthermore, *unskilled* means lower schooling than skilled workers. Moreover, this *unskilled* does not mean that no education exists in this paper.

In addition, we briefly analyze the effect of biased technological progress in this Harrod–Balassa–Samuelson effect model on the industrial structure of the domestic economy (and the foreign economy), contrasting to that under the quantity theory of money. Unlike the usual balanced growth result, our theory suggests that biased technological progress and its resulting economic growth make the industrial structure in favor of each industry.

On the other hand, such technological progress and its resulting economic growth work asymmetrically for the tradable and nontradable industries (e.g., Baumol, 1967). However, unlike us, Baumol (1967) argues that economic growth should vanish to the limit under his unbalanced growth model. However, as Appendix D shows, the *general equilibrium* setting prevents ever-continuing one-sided, biased technological progress in our theory. It may obscure our conclusions about the change in the industrial structure in favor of biased technological progress, depending on its complicated parameter setting and structure.

Note that our main conclusion in this paper is something related to the famous *Stolper–Samuelson theorem* that emphasizes that the cost of some goods will rise once its price increases such that the price of the production factor intensively used for that goods will rise (e.g., Stolper & Samuelson, 1941). We depend on this kind of inference that the factor price for some goods will rise once its price rises when we ignore the demand structure in our Harrod–Balassa–Samuelson effect model.

Again, this explains the movements along the supply curve, and introducing the demand structure should be another attempt to extend this inference to the more general case. However, once we consider the demand structure, as detailed in the AD-AS analysis in Masuda (2024a) and shown in Appendix D (recall the existence of the tâtonnement process), this inference should not hold as it is.

Still, to clarify our arguments, we ignore the existence of *tariffs* in this paper for the simplicity of our discussions, different from Stolper and Samuelson (1941). Other essential practical observations (and resultant theoretical problems), like the pass-throughs and the J-curve effects of the exchange rate changes as such, are also ignored to clarify our theoretical discoveries. They are sometimes captured by the *lag* (time-series persistency) structure problems in the *macroeconomic* perspectives.

Notably, in the price index literature, the *hedonic regression* (e.g., Berndt & Rappaport, 2001) separates the quality changes for goods and services in the price index from the actual price changes. As we will discuss in the text again, we insist that inflations should cause technological progress with some delay, even in the short run with the sticky price (and nominal wage). Thus, this hedonic regression could empirically test our attempt to connect inflations with technological progress in this paper.

Furthermore, we use the game theoretic approach to understand our inflation theory in the case of walking inflation and the utility of long-run inflation expectation.

We have some empirical studies on the stability of long-run inflation expectations. First, Carvalho et al. (2023) explain that the long-run inflation expectations are anchored, as we assume in this paper, when the central bank succeeds in making the public believe its inflation target and stability (i.e., the successful monetary policy is needed).

Second, Beechey et al. (2011) empirically suggest that the long-run inflation expectation is well-anchored in the U.S. and the Euro area, mostly in developed countries. These findings are consistent with Kiley (2008), who theoretically argues for the stability (i.e., anchoring) of long-run inflation expectations and the essential role of monetary policy in achieving that stability. To illustrate the critical role of this long-run inflation expectation, we pick up two kinds of stagflations that represent those in the 1970s and the 2020s, such as those created by the Ukrainian war.

As for stability, another long-running controversy exists between Keynesians and monetarists about whether the velocity of money is stable. This paper briefly picks up Friedman's (1969/2006) discussion that the stability of the velocity of money depends on the definition of the quantity of money and mainly depends on his empirical findings for the stability. On the other hand, we care about the counter opinion against Friedman (1969/2006) suggested by De Long (2000) that its stability in the U.S. might be doubtful (see Masuda, 2024a, for this theoretical meaning of the discussion by

Friedman, 1969/2006).

Recall that the intended productivity slowdowns can reduce the velocity of money, keeping other things equal, even under the expansionary monetary policy from the quantity theory of money. The quantity theory of money under the expansionary monetary policy must suppress the velocity of money given the sticky price and the decrease in output caused by this intended productivity slowdown. Also, with the fragile banking system (the *shrinking* money multiplier) and so on, the large quantity of *ineffective* and *dormant* money in past Japan that did not lead to inflation should be explained as the *constancy* (or possibly *reduction*) of the *circulating* money in the macroeconomy due to this reduction of the velocity of money.

Regarding the empirical aspects of the quantity theory of money itself, examples are Lucas (1980), Lucas (1988), and Sargent and Surico (2011). Lucas (1988) discussed the unit income elasticity, stability, and other characteristics of the money demand equation in the United States (e.g., de Bondt, 2009, for the Euro area). On the other hand, Sargent and Surico (2011) discussed that with more recent data, Lucas' (1980) result becomes destabilized, assuming the new Keynesian structural model behind the quantity theory of money. The U.S. evidence proposed by Lucas (1988) suggests a small estimate for the interest rate elasticity of money in the money demand equation (i.e., the log-log type; Miyao, 2002, and see the later section).

Together with this *evidence*, this paper provides another view supporting Friedman (1969/2006), which is that active monetary and fiscal policies are sometimes taken in developed countries. As a result, they mostly succeeded in not misjudging or did not fail to estimate their domestic macroeconomic performances correctly. This would help stabilize the velocity of money in recent developed countries; as evidenced by some exceptional events like the Lehman shock, no economic crises in developed countries during these two or three decades did not exist. Note that we exclude the Greek sovereign debt crisis as an exception because the countries influenced by this crisis are difficult to recognize as *developed countries*.

Moreover, I propose the problems related to empirical data to argue the essential practical aspects of our monetary policy operations, such as the stochastic terms and uncertainties of economic growth, business cycles, and measurements. Orphanides (2001) explained these practical aspects, such as data uncertainty and information lags in policymaking. With the presence of autocorrelations in outputs and measurement errors in productivities of tradable and nontradable goods industries (e.g., Solow, 1956, for the Total Factor Productivity discussion), the velocities of money and the labor force participation, including the unemployment rate, often observed in economic data, this consideration yields different insights from investigating only the theoretical, deterministic model.

For example, we discuss the benefits and potentiality of the floating exchange rate and the fixed exchange rate in terms of the business cycles and their synchronizations between the two countries in question. Furthermore, we argue the famous *impossibility* of exchange rate stabilization (e.g., Friedman, 1953) based on factors such as the wrong ideas on the side of policymakers (Romer & Romer, 2013), the degrees of business cycle synchronization, measurement errors in actual policymaking, persistent old policy disciplines, and the timely importance of policy objectives.

Regarding our (un)employment theory and the labor force participation theory, we mainly argue about them in view of measurement errors, as well as with a brief sketch of our new (un)employment theory, which explains unemployment as our diminishing equilibria from the micro, heterogeneous perspective (see also Masuda, 2024c). This heterogeneous aspect of our new (un)employment theory structurally supports our revival of the average productivity hypothesis for full employment under the sale revenue maximization hypothesis.

Still, we have Heckman (1979), Flinn and Heckman (1982) for the concrete labor force participation theories like Probit and Heckit (Heckman, 1979), Rogerson et al.

(2005) for search and matching theory, and Stoker (1984, 1993) for the aggregation problem of Probit and Heckit with (semi-)macro data for the measurement error issue. In this paper, we use the results of these previous studies.

This paper supposes that firms in the macroeconomy should employ the available labor force in our labor market with *slacks*, as the profit maximization hypothesis derives this result in the text. Thus, this paper tries to connect this labor force participation theory with our new (un)employment theory. Furthermore, for our analytical purpose, we will briefly sketch the general equilibrium effects in labor force participation theories for home and foreign countries through the exchange rate change itself in this paper.

The remaining previous studies we have not introduced yet are as follows. As for the exchange rate theory from the mathematical perspective, Obstfeld and Rogoff (1997) did comprehensive work in this field. Also, for the broad introduction of the exchange rate theory, mainly in terms of its history, relevant actual policymaking, and empirical studies, we pick up Isard (1995/2001) again. In addition to the standard theoretical view of the exchange rates theory, such as PPP, which we have stated, he concisely explains the existing empirical studies about the exchange rate determinants and the theoretical importance and problems of the exchange rate regimes, stabilities, and policy coordination for exchange rate stabilization.

Harvey (1981/1985) briefly argued the relationship between time series models and the spectrum density function. Hamilton (1994) also briefly and concisely explained the spectral analysis. Christiano and Fitzgerald (1999) show the band-pass filter example. Lucas (1980) argued that the quantity theory of money holds even when using spectrum analysis. Furthermore, Friedman (1953) and Heckman (2000) wrote fundamental papers discussing the classical stance on the exchange rate and the essentiality of heterogeneity.

There are few studies about sectoral inflations and the role of monetary policy. Kreamer (2022) analyzes sectoral inflation using the new Keynesian model. However, his interest is in variances that affect the social welfare represented by the second-order terms and is different from our study in this paper. Because our study emphasizes the first-order importance of the inflation effect on technological progress.

The remainder of this paper is organized as follows. In The Monetary Policy Rule of the Exchange Rate by Integrating the Quantity Theory of Money and the Harrod–Balassa–Samuelson Effect Model With Two Tradable Goods Produced by Skilled Workers and Unskilled Workers and Nontradable Goods section, I introduce the quantity theory of money briefly and the sales revenue and profit maximization hypotheses for our Harrod–Balassa–Samuelson effect model in details. Furthermore, we discuss the necessity of the sales revenue maximization hypothesis suggested by Baumol (1959) for full employment instead of the usual profit maximization hypothesis. With this hypothesis, we briefly argue the revival of the average productivity hypothesis. The profit maximization hypothesis cannot accomplish full employment. The usual profit maximization hypothesis and zero profit condition suggest these three establishments: The distribution law with the equivalent factor prices (i.e., real wage) to the marginal productivities under the competitive market, the average cost pricing under the increasing returns to scale production function, and indeterminacy of resource levels under the constant returns to scale production function. We can obtain the Harrod–Balassa–Samuelson effect model under the profit maximization hypothesis by analyzing these establishments. On the other hand, we get the price differential equation between the home and foreign countries with the (inverse) Harrod–Balassa–Samuelson effect under the sales revenue maximization hypothesis case. This paper develops the above profit maximization hypothesis case only. Additionally, I briefly argue the function of money in the quantity theory of money using Keynesian and monetarist perspectives, such as three motives: Transaction, speculative, and precautionary (asset). Also, we explain why three money motives can represent the money growth rule suggested by

Friedman (1960) and briefly argue the difference in monetary policy conducts between regular times and economic crises. Moreover, we briefly discuss the self-fulfilling nature of monetary authority's misperception of the macroeconomy. Developing a model with the Harrod–Balassa–Samuelson effect, I introduce the quantity theory of money into this Harrod–Balassa–Samuelson effect model with skilled and unskilled workers in the tradable goods industry that is partly and a little differently explained in Obstfeld and Rogoff (1997). This introduction should combine such models with the fact that both theories explain the general price levels (PPP). Then, we get our new policy rule for exchange rate stabilization. We also explain this exchange rate rule and its relation to domestic inflations in detail. Furthermore, we explain the heterogeneous income increases (and temporal reductions) under this policy rule, as Erosa and Ventura (2002) show. Moreover, we argue how the rent-seeking behaviors of the firms' leaders and managers only under the usual profit maximization hypothesis will bring disinflationary or deflationary pressure for the general price level, notwithstanding the monetary authority's (massive) monetary injection for generating inflation. Furthermore, we briefly argue the negative effects of supply shocks through the import price inflation with the exchange rate depreciation that is a case of stagflations (Blanchard, 2000; Friedman, 1978) and get in touch with one of the origins of the downward stickiness of the segmented average nominal wages. Note that this paper explains the existence of two types of stagflation later. In The Monetary Policy Rule From the Model Analysis in the Previous Section: Conclusions in the Deterministic World section, I briefly summarize the role of the home country's monetary policy based on our deterministic model for the exchange rate in The Monetary Policy Rule of the Exchange Rate by Integrating the Quantity Theory of Money and the Harrod–Balassa–Samuelson Effect Model With Two Tradable Goods Produced by Skilled Workers and Unskilled Workers and Nontradable Goods section. Furthermore, this section details two kinds of stagflations with the anticipated inflations and the real (permanent and transitory) income fluctuations related to the long-run inflation expectations. Also, we explain that the above disinflationary or deflationary pressure will result in a large amount of ineffective and dormant money in the macroeconomy. This result also leads to no effect of monetary expansion on inflations and technological progress, similarly under the Ricardian equivalence. In The Difficulties of the Policy Rule on Exchange Rate Derived in the Previous Section in Policy Practice Dominated by the Risks section, I discuss the practical policy responses and exchange rate fluctuations when there are autocorrelations and serial correlations in outputs, as well as measurement errors in the productivities of tradable and nontradable goods industries, the velocities of money, and labor force participation with unemployment. These factors are the exchange rate determinants obtained in this paper. Finally, I briefly argue the implications for the stability and controllability of the exchange rate with the precautionary (asset) motive example and show the difference between our theory and the neoclassical synthesis (Samuelson, 1948; Samuelson & Solow, 1960). Appendix A briefly explains the approximated Sen's (1976) poverty measure to estimate the heterogeneous negative impacts of real income shocks in inflationary environments and economic crises. A simple analysis of the precautionary (asset) motive in the money demand equation is presented in Appendix B. This appendix derives the precautionary (asset) money demand with the multivariate second-order Taylor expansion. Appendix C suggests the mathematical skeleton of our new (un)employment theory to argue the indeterminate labor demand under the constant returns to scale production function structurally, based on Rogerson et al. (2005). This theory should be applied when we structurally understand with the micro-based, heterogeneous agents perspective that the leader's optimistic or pessimistic forward-looking behaviors in firms decide the labor demand under the profit maximization hypothesis in the macroeconomy (e.g., Cuba-Borda & Singh, 2024). Appendix D shows that our conclusions about the effect of the biased technological progress on the industrial structure with the Harrod–Balassa–

Samuelson effect model should become possibly mathematically ambiguous once we adopt not the *partial* but the *general* equilibrium setting considering the *demand side* in which Obstfeld and Rogoff (1997) provided.

The Monetary Policy Rule of the Exchange Rate by Integrating the Quantity Theory of Money and the Harrod–Balassa–Samuelson Effect Model With Two Tradable Goods Produced by Skilled Workers and Unskilled Workers and Nontradable Goods

After constructing the equation for determining the exchange rate based on the quantity theory of money, in this section, I introduce another exchange rate determination model with the Harrod–Balassa–Samuelson effect as explained mathematically by Samuelson (1964) and Obstfeld and Rogoff (1997). Then, I combine these two models to derive a relational equation for the money stocks of the home country and a foreign country. In the later section, I will show that this leads to a new policy rule for exchange rate determination.

Note that we assume that the money stock is controllable by monetary authorities for some reasons. We refer to these reasons in discussing the definition of money, the money multiplier, the circulating quantity of money for these money stocks, and the relation to the Taylor reaction function later in this paper (Masuda, 2024a).

The Equation for Determining the Exchange Rate Based on the Quantity Theory of Money

As shown below, considering the exchange rate under the quantity theory of money in the home currency, the quantity of money *does* affect the exchange rate. According to Isard (1995/2001), this kind of empirical application of the quantity theory of money to investigate whether the actual exchange rates should be established on PPP or not existed numerously in the past. Also, much literature exists on the quantity theory of money itself (e.g., Friedman,1956, 1969/2006; Lucas, 1980, 1988; Niehans, 1978). Still, I refer to any macroeconomics textbooks that introduce the fundamental equation of the quantity theory of money as follows.

The quantity theory of money means that $M_t V_t = P_t Y_t$, where M_t is the nominal quantity of money at the period t, V_t is the velocity of money at the period t, P_t is the general price level at the period t, and Y_t is the output (income) at the period t, yielding $\frac{dM_t}{M_t} + \frac{dV_t}{V_t} = \frac{dP_t}{P_t} + \frac{dY_t}{Y_t}$ where M_t, V_t, P_t, and $Y_t \in \mathbb{R}_+$ for any t. The above model is discrete with respect to time, but continuous in those cross-sectional model variables. The other models in this paper have the same property.

From this quantity theory of money, one can conclude that (other things being equal) the quantity of money, M_t, becomes Γ times more significant, then the general price level, P_t, must also increase by Γ times. Note that from the additive relationship $Y_t = Y_{TU,t} + Y_{TS,t} + Y_{N,t}$, where $Y_{TU,t}$ is the output of the tradable goods industry produced by unskilled workers, $Y_{TS,t}$ is the output of the tradable goods industry produced by skilled workers, and $Y_{N,t}$ is the output of the nontradable goods industry, and $Y_{TU,t}$, $Y_{TS,t}$, and $Y_{N,t} \in \mathbb{R}_+$ for any t, one can obtain the equation:

$$\frac{dY_t}{Y_t} = \frac{Y_{TU,t}}{Y_t}\frac{dY_{TU,t}}{Y_{TU,t}} + \frac{Y_{TS,t}}{Y_t}\frac{dY_{TS,t}}{Y_{TS,t}} + \frac{Y_{N,t}}{Y_t}\frac{dY_{N,t}}{Y_{N,t}} =$$

$$\frac{Y_{TU,t}}{Y_t}\left(\frac{dA_{T,t}}{A_{T,t}} + \frac{dF_{TU,t}}{F_{TU,t}}\right) + \frac{Y_{TS,t}}{Y_t}\left(\frac{dA_{T,t}}{A_{T,t}} + \frac{dF_{TS,t}}{F_{TS,t}}\right) + \frac{Y_{N,t}}{Y_t}\left(\frac{dA_{N,t}}{A_{N,t}} + \frac{dF_{N,t}}{F_{N,t}}\right),$$

(this equation will be used later) because $Y_{TU,t} = A_{T,t}F_{TU,t}$, $Y_{TS,t} = A_{T,t}F_{TS,t}$, and $Y_{N,t} = A_{N,t}F_{N,t}$, where $A_{T,t}$ is the common total factor productivity (technological progress) of the tradable goods industry for skilled and unskilled workers at the period t, $F_{TU,t}$ is the production relationship of tradable goods industry for unskilled workers at the period t, $F_{TS,t}$ is the production relationship of tradable goods industry for skilled workers at the period t, $A_{N,t}$ is the total factor productivity (technological progress) of nontradable goods industry at the period t, and $F_{N,t}$ is the production relationship of nontradable goods industry at the period t.

This paper assumes only the labor input as the production factor for the above production relationship (we will see this fact in detail in the later section). The production relationship of the tradable goods industry for skilled workers at the period

t, $F_{TS,t}$ can have the increasing returns to scale property with respect to this labor input. However, this assumption is not critical here, and we do not emphasize it, unlike in the later sections. Furthermore, $A_{T,t}$ and $A_{N,t} \in \mathbb{R}_+$ for any t.

Symmetrically, the foreign variables can be expressed by putting an asterisk (*) on their right shoulders and we also assume P_t^*, M_t^*, V_t^*, Y_t^*, $Y_{TU,t}^*$, $Y_{TS,t}^*$, $Y_{N,t}^*$, $A_{T,t}^*$, and $A_{N,t}^* \in \mathbb{R}_+$ for any t. Then, if the quantity theory of money holds symmetrically in both countries, we obtain:

$$\frac{dP_t}{P_t} - \frac{dP_t^*}{P_t^*} = \left(\frac{dM_t}{M_t} + \frac{dV_t}{V_t} - \frac{dY_t}{Y_t}\right) - \left(\frac{dM_t^*}{M_t^*} + \frac{dV_t^*}{V_t^*} - \frac{dY_t^*}{Y_t^*}\right)$$

$$= \left\{\frac{dM_t}{M_t} + \frac{dV_t}{V_t} - \left[\frac{Y_{TU,t}}{Y_t}\left(\frac{dA_{T,t}}{A_{T,t}} + \frac{dF_{TU,t}}{F_{TU,t}}\right) + \frac{Y_{TS,t}}{Y_t}\left(\frac{dA_{T,t}}{A_{T,t}} + \frac{dF_{TS,t}}{F_{TS,t}}\right) + \frac{Y_{N,t}}{Y_t}\left(\frac{dA_{N,t}}{A_{N,t}} + \frac{dF_{N,t}}{F_{N,t}}\right)\right]\right\} -$$

$$\left\{\frac{dM_t^*}{M_t^*} + \frac{dV_t^*}{V_t^*} - \left[\frac{Y_{TU,t}^*}{Y_t^*}\left(\frac{dA_{T,t}^*}{A_{T,t}^*} + \frac{dF_{TU,t}^*}{F_{TU,t}^*}\right) + \frac{Y_{TS,t}^*}{Y_t^*}\left(\frac{dA_{T,t}^*}{A_{T,t}^*} + \frac{dF_{TS,t}^*}{F_{TS,t}^*}\right) + \frac{Y_{N,t}^*}{Y_t^*}\left(\frac{dA_{N,t}^*}{A_{N,t}^*} + \frac{dF_{N,t}^*}{F_{N,t}^*}\right)\right]\right\}.$$

Based on this equation, that is, with the quantities of money supplied by the home country and foreign countries, changes in velocities of the home and foreign countries, changes in technological progress and inputs in the home country's tradable goods industry (-) and nontradable goods industry (-), and changes in technological progress and inputs in the foreign country's tradable goods industry (+) and nontradable goods industry (+), the exchange rate can be determined. Note that the + and - symbols in parentheses indicate the directions of the effect of the technological progress and inputs on the growth rate of the exchange rate (+ means the depreciation and the – means the appreciation of the exchange rate in the home currency).

Conversely, when the exchange rate changes due to these factors, the country in question can respond by changing its quantity of money (i.e., monetary policy), M_t, to counteract these effects to stabilize the exchange rate due to the money growth rule (i.e., k% rule) and so on. We will use the word *money growth* rule in Friedman's (1960) original meaning from here that the money growth should respond to *k%* that is equivalent to the *entire* economic *growth rate* (or, equivalently, the difference of the output in the macroeconomy from the previous period itself) because we specify not the *monetary base* but the *money stock* for M_t here.

On the other hand, the dynamics of relative prices based on home and foreign general prices (i.e., exchange rates) are as follows (this result will be used frequently in the following sections):

$$\frac{dP_t}{P_t} - \frac{dP_t^*}{P_t^*} = \frac{P_{TU,t}Y_{TU,t}}{P_tY_t}\left(\frac{dv_{TU,t}}{v_{TU,t}} + \frac{dP_{TU,t}}{P_{TU,t}}\right) + \frac{P_{TS,t}Y_{TS,t}}{P_tY_t}\left(\frac{dv_{TS,t}}{v_{TS,t}} + \frac{dP_{TS,t}}{P_{TS,t}}\right) + \frac{P_{N,t}Y_{N,t}}{P_tY_t}\left(\frac{dv_{N,t}}{v_{N,t}} + \frac{dP_{N,t}}{P_{N,t}}\right) - \left[\frac{P_{TU,t}^*Y_{TU,t}^*}{P_t^*Y_t^*}\left(\frac{dv_{TU,t}^*}{v_{TU,t}^*} + \frac{dP_{TU,t}^*}{P_{TU,t}^*}\right) + \frac{P_{TS,t}^*Y_{TS,t}^*}{P_t^*Y_t^*}\left(\frac{dv_{TS,t}^*}{v_{TS,t}^*} + \frac{dP_{TS,t}^*}{P_{TS,t}^*}\right) + \frac{P_{N,t}^*Y_{N,t}^*}{P_t^*Y_t^*}\left(\frac{dv_{N,t}^*}{v_{N,t}^*} + \frac{dP_{N,t}^*}{P_{N,t}^*}\right)\right]$$

$$= \left(\frac{P_{TU,t}Y_{TU,t}}{P_tY_t} - \frac{Y_{TU,t}}{Y_t}\right)\left(\frac{dA_{T,t}}{A_{T,t}} + \frac{dF_{TU,t}}{F_{TU,t}}\right) + \frac{P_{TU,t}Y_{TU,t}}{P_tY_t}\frac{dP_{TU,t}}{P_{TU,t}} + \left(\frac{P_{TS,t}Y_{TS,t}}{P_tY_t} - \frac{Y_{TS,t}}{Y_t}\right)\left(\frac{dA_{T,t}}{A_{T,t}} + \frac{dF_{TS,t}}{F_{TS,t}}\right) + \frac{P_{TS,t}Y_{TS,t}}{P_tY_t}\frac{dP_{TS,t}}{P_{TS,t}} + \left(\frac{P_{N,t}Y_{N,t}}{P_tY_t} - \frac{Y_{N,t}}{Y_t}\right)\left(\frac{dA_{N,t}}{A_{N,t}} + \frac{dF_{N,t}}{F_{N,t}}\right) + \frac{P_{N,t}Y_{N,t}}{P_tY_t}\frac{dP_{N,t}}{P_{N,t}} - \left[\left(\frac{P_{TU,t}^*Y_{TU,t}^*}{P_t^*Y_t^*} - \frac{Y_{TU,t}^*}{Y_t^*}\right)\left(\frac{dA_{T,t}^*}{A_{T,t}^*} + \frac{dF_{TU,t}^*}{F_{TU,t}^*}\right) + \frac{P_{TU,t}^*Y_{TU,t}^*}{P_t^*Y_t^*}\frac{dP_{TU,t}^*}{P_{TU,t}^*} + \left(\frac{P_{TS,t}^*Y_{TS,t}^*}{P_t^*Y_t^*} - \frac{Y_{TS,t}^*}{Y_t^*}\right)\left(\frac{dA_{T,t}^*}{A_{T,t}^*} + \frac{dF_{TS,t}^*}{F_{TS,t}^*}\right) + \frac{P_{TS,t}^*Y_{TS,t}^*}{P_t^*Y_t^*}\frac{dP_{TS,t}^*}{P_{TS,t}^*} + \left(\frac{P_{N,t}^*Y_{N,t}^*}{P_t^*Y_t^*} - \frac{Y_{N,t}^*}{Y_t^*}\right)\left(\frac{dA_{N,t}^*}{A_{N,t}^*} + \frac{dF_{N,t}^*}{F_{N,t}^*}\right) + \frac{P_{N,t}^*Y_{N,t}^*}{P_t^*Y_t^*}\frac{dP_{N,t}^*}{P_{N,t}^*}\right].$$

In the above equation, the general price level is defined as $P_tY_t = P_{TU,t}Y_{TU,t} + P_{TS,t}Y_{TS,t} + P_{N,t}Y_{N,t}$ where $P_{TU,t}$, $P_{TS,t}$, and $P_{N,t} \in \mathbb{R}_+$ for any t (#1). The above formula is based on the definition of the general price index, which is similar to that of the Divisia price index concept in a continuous model. However, our model is discrete and does not use discrete corrections for such an index (see any introductory statistics textbooks for the definition and discrete corrections of the Divisia price index).

Note that we obtain:

$$\frac{dP_t}{P_t} = \frac{P_{TU,t}Y_{TU,t}}{P_tY_t}\left(\frac{dv_{TU,t}}{v_{TU,t}} + \frac{dP_{TU,t}}{P_{TU,t}}\right) + \frac{P_{TS,t}Y_{TS,t}}{P_tY_t}\left(\frac{dv_{TS,t}}{v_{TS,t}} + \frac{dP_{TS,t}}{P_{TS,t}}\right) + \frac{P_{N,t}Y_{N,t}}{P_tY_t}\left(\frac{dv_{N,t}}{v_{N,t}} + \frac{dP_{N,t}}{P_{N,t}}\right),$$

where the real weights of the outputs produced by tradables goods industry with unskilled workers and skilled workers, and by nontradable goods industry are $v_{TU,t} = \frac{Y_{TU,t}}{Y_t}$, $v_{TS,t} = \frac{Y_{TS,t}}{Y_t}$, and $v_{N,t} = \frac{Y_{N,t}}{Y_t}$.

Thus, after $v_{TU,t}$ is differentiated by time, t, the equation:

$$\frac{dv_{TU,t}}{v_{TU,t}} = \frac{dY_{TU,t}}{Y_{TU,t}} - \frac{dY_t}{Y_t}$$

$$= \frac{dA_{T,t}}{A_{T,t}} + \frac{dF_{TU,t}}{F_{TU,t}} - \left[\frac{Y_{TU,t}}{Y_t}\left(\frac{dA_{T,t}}{A_{T,t}} + \frac{dF_{TU,t}}{F_{TU,t}}\right) + \frac{Y_{TS,t}}{Y_t}\left(\frac{dA_{T,t}}{A_{T,t}} + \frac{dF_{TS,t}}{F_{TS,t}}\right) + \frac{Y_{N,t}}{Y_t}\left(\frac{dA_{N,t}}{A_{N,t}} + \frac{dF_{N,t}}{F_{N,t}}\right)\right],$$

is obtained and can be used. Similarly, for $v_{TS,t}$ and $v_{N,t}$, the following result are applied:

$$\frac{dv_{TS,t}}{v_{TS,t}} = \frac{dA_{T,t}}{A_{T,t}} + \frac{dF_{TS,t}}{F_{TS,t}} - \left[\frac{Y_{TU,t}}{Y_t}\left(\frac{dA_{T,t}}{A_{T,t}} + \frac{dF_{TU,t}}{F_{TU,t}}\right) + \frac{Y_{TS,t}}{Y_t}\left(\frac{dA_{T,t}}{A_{T,t}} + \frac{dF_{TS,t}}{F_{TS,t}}\right) + \frac{Y_{N,t}}{Y_t}\left(\frac{dA_{N,t}}{A_{N,t}} + \frac{dF_{N,t}}{F_{N,t}}\right)\right],$$

$$\frac{dv_{N,t}}{v_{N,t}} = \frac{dA_{N,t}}{A_{N,t}} + \frac{dF_{N,t}}{F_{N,t}} - \left[\frac{Y_{TU,t}}{Y_t}\left(\frac{dA_{T,t}}{A_{T,t}} + \frac{dF_{TU,t}}{F_{TU,t}}\right) + \frac{Y_{TS,t}}{Y_t}\left(\frac{dA_{T,t}}{A_{T,t}} + \frac{dF_{TS,t}}{F_{TS,t}}\right) + \frac{Y_{N,t}}{Y_t}\left(\frac{dA_{N,t}}{A_{N,t}} + \frac{dF_{N,t}}{F_{N,t}}\right)\right].$$

For foreign countries, defining the functions, $F_{TU,t}^*$, $F_{TS,t}^*$, and $F_{N,t}^*$, as the corresponding foreign ones to $F_{TU,t}$, $F_{TS,t}$, and $F_{N,t}$, they are:

$$\frac{dv_{TU,t}^*}{v_{TU,t}^*} = \frac{dA_{T,t}^*}{A_{T,t}^*} + \frac{dF_{TU,t}^*}{F_{TU,t}^*} - \left[\frac{Y_{TU,t}^*}{Y_t^*}\left(\frac{dA_{T,t}^*}{A_{T,t}^*} + \frac{dF_{TU,t}^*}{F_{TU,t}^*}\right) + \frac{Y_{TS,t}^*}{Y_t^*}\left(\frac{dA_{T,t}^*}{A_{T,t}^*} + \frac{dF_{TS,t}^*}{F_{TS,t}^*}\right) + \frac{Y_{N,t}^*}{Y_t^*}\left(\frac{dA_{N,t}^*}{A_{N,t}^*} + \frac{dF_{N,t}^*}{F_{N,t}^*}\right)\right],$$

$$\frac{dv_{TS,t}^*}{v_{TS,t}^*} = \frac{dA_{T,t}^*}{A_{T,t}^*} + \frac{dF_{TS,t}^*}{F_{TS,t}^*} - \left[\frac{Y_{TU,t}^*}{Y_t^*}\left(\frac{dA_{T,t}^*}{A_{T,t}^*} + \frac{dF_{TU,t}^*}{F_{TU,t}^*}\right) + \frac{Y_{TS,t}^*}{Y_t^*}\left(\frac{dA_{T,t}^*}{A_{T,t}^*} + \frac{dF_{TS,t}^*}{F_{TS,t}^*}\right) + \frac{Y_{N,t}^*}{Y_t^*}\left(\frac{dA_{N,t}^*}{A_{N,t}^*} + \frac{dF_{N,t}^*}{F_{N,t}^*}\right)\right],$$

$$\frac{dv_{N,t}^*}{v_{N,t}^*} = \frac{dA_{N,t}^*}{A_{N,t}^*} + \frac{dF_{N,t}^*}{F_{N,t}^*} - \left[\frac{Y_{TU,t}^*}{Y_t^*}\left(\frac{dA_{T,t}^*}{A_{T,t}^*} + \frac{dF_{TU,t}^*}{F_{TU,t}^*}\right) + \frac{Y_{TS,t}^*}{Y_t^*}\left(\frac{dA_{T,t}^*}{A_{T,t}^*} + \frac{dF_{TS,t}^*}{F_{TS,t}^*}\right) + \frac{Y_{N,t}^*}{Y_t^*}\left(\frac{dA_{N,t}^*}{A_{N,t}^*} + \frac{dF_{N,t}^*}{F_{N,t}^*}\right)\right],$$

respectively.

In addition, the above equation for the change in general prices is also valid even if general prices are redefined using the normalization $P_{TU,t} = 1$ for any t (see The Effect of the Normalization of the Price of Tradable Goods by Unskilled Workers section). This redefined result will be intensively used later.

Some readers may wonder if this is the theory of not the *real* exchange rate but the *nominal* exchange rate (we owe this point to Professor Akihiko Kaneko's suggestion). However, the *real* exchange rate is only determined by the *real* terms on the right-hand side (i.e., terms except for the quantity of money) in the above nominal exchange rate theory (e.g., Isard, 1995/2001). Any qualitative discussions in this paper do *not* change with this separation of exchange rates, as we will discuss below. This is our foundation for combining the Harrod–Balassa–Samuelson effect model with the quantity theory of money.

The existing empirical applications of the quantity theory of money treat the nominal exchange rates in line with PPP (e.g., Isard, 1995/2001). Obstfeld and Rogoff (1997) do not separate real and nominal exchange rate theory, which is also in line with PPP when discussing a model with the Harrod–Balassa–Samuelson effect. The real and nominal difference in their model vanishes because their Harrod–Balassa–Samuelson effect model treats the real side only and money as a *veil*.

Actually, we can normalize the nominal exchange rate to equate it with the real exchange rate in the initial equilibrium. Thus, as we will see, we can consistently discuss these nominal and real exchange rate theories in this paper.

The Impact of Unbalanced Development of Industries

Economic development is not limited to the case of balanced growth, in which both tradable and nontradable goods industries develop equally (e.g., Baumol, 1967). For example, only the tradable goods industry might develop ahead of the others, or the nontradable goods industry might grow more significantly than the tradable goods industry, assuming that the amounts of labor inputs do not change. Many cases of unbalanced development exist, corresponding to that degree.

Suppose that the technological progress in the nontradable goods industry is slower than that in the tradable goods industry, $\frac{dA_{T,t}}{A_{T,t}} > \frac{dA_{N,t}}{A_{N,t}} (= 0$ for simplicity), although the amounts of labor input do not change. Then, from the output identity, $Y_t = Y_{T,t} + Y_{N,t}$ where $Y_{T,t} = Y_{TU,t} + Y_{TS,t}$, the change in the share of the nontradable goods industry over time can be expressed as $\frac{dv_{N,t}}{v_{N,t}} = -\frac{Y_{T,t}}{Y_t}\frac{dA_{T,t}}{A_{T,t}}$ because $v_{N,t} = \frac{Y_{N,t}}{Y_t} \in (0,1)$ and $\frac{dv_{N,t}}{v_{N,t}} = \left(1 - \frac{Y_{N,t}}{Y_t}\right)\left(\frac{dA_{N,t}}{A_{N,t}} + \frac{dF_{N,t}}{F_{N,t}}\right) - \left(\frac{Y_{TU,t}}{Y_t} + \frac{Y_{TS,t}}{Y_t}\right)\frac{dA_{T,t}}{A_{T,t}} - \left(\frac{Y_{TU,t}}{Y_t}\frac{dF_{TU,t}}{F_{TU,t}} + \frac{Y_{TS,t}}{Y_t}\frac{dF_{TS,t}}{F_{TS,t}}\right)$, if the other variables stay unchanged. In other words, the industrial structure of this country over time becomes biased toward the increasing share of the tradable goods industry, and that degree is gradually enlarged.

Inversely, if $\frac{dA_{T,t}}{A_{T,t}} = 0$(for simplicity)$< \frac{dA_{N,t}}{A_{N,t}}$, that is, the nontradable goods industry develops faster than the tradable goods industry, and the other variables stay unchanged, then $\frac{dv_{N,t}}{v_{N,t}} = \frac{Y_{T,t}}{Y_t}\frac{dA_{N,t}}{A_{N,t}}$ where $\frac{Y_{T,t}}{Y_t} = 1 - \frac{Y_{N,t}}{Y_t} \in (0,1)$. As a result, the industrial structure of this country becomes biased toward the increasing share of nontradable goods industry over time, but that degree is gradually lessened. Note that in both cases, the *path dependency* of industrial structure evolutions exists (see also Appendix D; e.g., Ljungqvist & Sargent, 2000, for the path dependency discussion).

Together with the previous (nominal) exchange rate formula under the quantity theory of money, this implies that in a world where only the quantity theory of money holds, regardless of whichever technological progress, respective positive changes in such technological progress surely have a positive effect on the industrial structure evolution in favor of its side.

However, each effect of this technological progress of the tradable goods industry or nontradable goods industry on the exchange rate is negative (i.e., causing appreciations) by generating deflations (recall the simple formulae: $\frac{dP_t}{P_t} - \frac{dP_t^*}{P_t^*} = \left[\frac{dM_t}{M_t} + \frac{dV_t}{V_t} - \frac{dY_t}{Y_t}\right] - \left[\frac{dM_t^*}{M_t^*} + \frac{dV_t^*}{V_t^*} - \frac{dY_t^*}{Y_t^*}\right]$ and $\frac{dY_t}{Y_t} = \frac{Y_{TU,t}}{Y_t}\left[\frac{dA_{T,t}}{A_{T,t}} + \frac{dF_{TU,t}}{F_{TU,t}}\right] + \frac{Y_{TS,t}}{Y_t}\left[\frac{dA_{T,t}}{A_{T,t}} + \frac{dF_{TS,t}}{F_{TS,t}}\right] + \frac{Y_{N,t}}{Y_t}\left[\frac{dA_{N,t}}{A_{N,t}} + \frac{dF_{N,t}}{F_{N,t}}\right]$). Because the aggregated technological progress itself is determined by the weighted average of those technological progresses of the tradable goods industry and the nontradable goods industry. It is different from the Harrod–Balassa–Samuelson effect model that explains that the technological progress of home tradable goods industry only inversely raises domestic inflation and bring the exchange rate depreciation among the factors in home country.

The only thing that matters is that the growth of the whole output by any productivity change induces deflation and the exchange rate appreciation under the pure quantity theory of money. However, the home industrial structure surely changes in favor of the industry with more rapid technological progress asymmetrically, even if unbalanced technological progress exists. Furthermore, in the above, the possible property of $F_{TS,t}$, that is, the increasing returns to scale, does not play a serious role again. We focus on technological progress only and exclude labor input changes here.

Note that the uniform (nominal) wage (technically without that of skilled workers, as we assume later) in the domestic labor market remains established once the price of nontradable goods is considered (see the following sections). This is because perfect mobility in the domestic labor market for the tradable goods industry for unskilled workers and the nontradable goods industry is assumed, as will be seen in the discussion of the Harrod–Balassa–Samuelson effect model.

The Relationship With the Money Demand Equation: Our Logical Foundation

As for the quantity theory of money, the money demand equation is sometimes used in place of the equation $M_tV_t = P_tY_t$ as Friedman (1956) suggested. In the usual money demand equation, such as that of Miyao (2002), this equation is specified as $\ln(M_t) - \ln(P_t) \cong \beta_1 - \beta_2 i_t + \ln(Y_t)$, where $\ln$ means the natural logarithm, β_1 is the parameter representing a part of the inverse velocity of money, β_2 is the positive parameter, i_t is the nominal interest rate at the period t.

Recall the result of Lucas (1988) for unit elasticity of income in the money demand equation (see also Niehans, 1978). Hereafter, r_t represents the real interest rate at the period t, and i_t and $r_t \in \mathbb{R}_+$ for any t. In the above, I use the convenient first-order approximated relation around some positive fixed value, i_0, $\ln(1 + i_t) \cong \ln(1 + i_0) + \frac{1}{1+i_0} i_t$ (see also Appendix B; Masuda, 2024a, for our specific reason not to choose the usual approximation around $i_0 = 0$ at this time). We suppose that β_1 and β_2 in the above include these additional coefficients generated by this first-order approximation.

This formula, similar to those of Lucas (1980), Lucas (1988), and Sargent and Surico (2011), is rewritten as $M_t = P_t \exp(\beta_1) i_t^{-\beta_2} Y_t$, and thus, $V_t^{-1} = \exp(\beta_1) i_t^{-\beta_2}$. In the above money demand equation, we use i_t instead of $\ln(i_t)$. Because the Baumol–Tobin type money demand equation (Baumol, 1952; Tobin, 1956) admits the (*gross*) variable, $1 + i_t$, instead of the simple (*net*) variable, i_t, to extend this model incorporating the *zero* or *negative* interest rates. That model uses the *net loss* case using the gross variable, $1 + i_t$, and the *nonlinear* transactions cost (the nonlinear marginal money holding period; e.g., Niehans, 1978). This is our logical(micro)-foundation.

The above formulation can include the zero (negative) interest rate policy and is more general than the log-log type function (see Miyao, 2002). Furthermore, Niehans (1978) can provide the logical foundation for this money demand equation. However, we do not concretely treat the zero (negative) interest rate policy where we assume the positive natural rate of real interest in this section (we leave it for Appendix B and Masuda, 2024a).

The Relationship of Our Money Demand Equation to the Taylor Reaction Function

Note that, solving for the nominal interest rate in the above, the above money demand equation could be transformed into the following:

$$i_t = b_1 + b_2\ln(Y_t) + b_2\ln(P_t) - b_2\ln(M_t)$$

$$\cong [b_1 + b_2\ln(Y_0) + b_2\tau_0 + b_2\ln(\bar{P}_0) + b_2\bar{\pi}_0 - b_2\bar{\rho}_0 - b_2\ln(\bar{M}_0)] + b_2(\tau_t - \tau_0) +$$

$$b_2(\pi_t - \bar{\pi}_0) - b_2(\rho_t - \bar{\rho}_0),$$

where $b_1 = \frac{\beta_1}{\beta_2}$, and $b_2 = \frac{1}{\beta_2}$ are the parameters, Y_0, $\bar{M}_0$, and $\bar{P}_0 \in \mathbb{R}_+$ are some arbitrally real output at the period 0, the quantity of money specified by the corresponding money growth rule to Y_0 at the period 0, and the general price level controlled by $\bar{M}_0$ at the period 0, $\tau_t \in \mathbb{R}$ for any t and $\tau_0 \in \mathbb{R}$ are the real income growth rate at the period t from the initial period 0 and the real income growth rate in the initial equilibrium, $\pi_t \in \mathbb{R}$ for any t and $\bar{\pi}_0 \in \mathbb{R}$ are the inflation rate at the period t from the initial period 0 and the inflation rate set by the quantity of money growth rate that is controlled by the money growth rule, $\bar{\rho}_0$, in the initial equilibrium, and $\rho_t \in \mathbb{R}$ for any t and $\bar{\rho}_0 \in \mathbb{R}$ are the money growth rate at the period t from the initial period 0 and the money growth rate specified by the corresponding money growth rule to τ_0 in the initial equilibrium.

Our above rule means that the nominal interest rate should result in rising, accompanied by positive real income growth from the initial nominal interest rate decrease in the (sticky price) short run, when the monetary authority increases the quantity of money more than Friedman's (1960) money growth rule. Empirical analyses determine this story (e.g., McCallum & Nelson, 1999; see also the following discussions with three money motives). Given the IS and Phillips curves, Orphanides (2003) derived his Taylor reaction function with *linear-quadratic control* theory using the new Keynesian structural model. His result should support the above *story*.

Remember this transformed equation for discussing the difference from the model of Alvarez et al. (2001) and its short-run and long-run meanings, as explained in the later paragraphs. Notice that the first term, $b_1 + b_2\ln(Y_0) + b_2\tau_0 + b_2\ln(\bar{P}_0) + b_2\bar{\pi}_0 - b_2\bar{\rho}_0 - b_2\ln(\bar{M}_0)$, in the above formulation *is* the natural rate of *nominal* interest in *this* Taylor rule (Holston et al., 2016) and should take any level including the *negative finite* rate (recall the deflation; Masuda, 2024a) as we assumed.

In our previous money demand equation, without considering the initial equilibrium, we obtain $b_1 + b_2\ln(Y_t) + b_2\ln(\bar{P}_t) - b_2\ln(\bar{M}_t)$ where $\bar{P}_t$ is the general price level under the quantity of money specified by the money growth rule, $\bar{M}_t$. This quantity of money specified by the money growth rule neutralizes inflation as its steady state value such as zero or two percent given other variables as the natural rate of nominal interest. This natural rate of nominal interest rate could have any real value, including the negative finite value.

Thus, the natural rate of *real* interest in our money demand equation must coincide with the logarithm of the nominal output growth adjusted by the money growth multiplied by b_2 plus the inverse velocity of money not affected by nominal interest rate minus the steady inflation rate, such as *zero* or *two percent* or so.

This paper suggests that real income growth should be positive without economic crises when the (real) quantity of money should be required to be a more significantly positive amount (that means rapid positive money growth) than the money growth rule usually requires due to avoiding the monetary contractions caused by the precautionary (asset) motive (and usually the transaction motive), as we will mention later in Appendix B like that.

Regarding negative real income growth, we can also admit technical recessions whose negative real income growth continues for at least two consecutive quarters. They should not usually be cared for by mobilizing economic policies as long as they are not considered the severe beginnings of prolonged recessions, which result in a negative annual growth rate and are (nearly) equal to economic crises.

From the above, it is natural that we assume the positive (real) growth rate of the quantity of money (or equivalently, under the stable money multiplier assumption, the monetary base) as our money growth rule without any exceptional times.

Recall that usual recessions are represented by negative output gaps. In contrast, economic crises are represented by the surge of income reductions, that is, the technological *regress* like wars, natural disasters, false and sloppy accounting, and so on, accompanied by or represented by the huge persistent negative output gaps that ultimately make potential outputs lower (remember the filter problem in the later section and the natural unemployment rate), from our wage determination theory explained in this paper (see the later sections). Such a phenomenon as the persistent negative output gaps is often observed empirically.

Now, suppose that we add the real output term, such as the output gap term, to Alvarez et al. (2001) model. Then, as we saw, we safely obtained a policy rule similar to the usual Taylor reaction function (e.g., McCallum & Nelson, 1999) that explicitly considers the particular relation of the open market operation. Alternatively, to obtain that result in the usual Taylor reaction function, we can suppose that the money growth rule should control inflations, assuming that the inflation rate is a function of the quantity of money like Friedman (1970) such that $\pi_t = \pi(M_t - \bar{M}_t)$ where $\bar{M}_t \in \mathbb{R}_+$ for any t is the money growth that satisfies, say, zero or two percent inflation when $M_t = \bar{M}_t$ given by the money growth rule, $\bar{M}_t$. With the assumption of the stable money multiplier below, that *is* equivalent to our traditional money demand equation in this paper.

As we will explain, our money growth rule specified by Friedman (1969/2006) should not require skillful monetary policy even due to the difference term in the sticky price short run, $b_2(\tau_t - \tau_0) + b_2(\pi_t - \bar{\pi}_0) - b_2(\rho_t - \bar{\rho}_0)$. That is, at least π_t and $\bar{\pi}_0$ are defined as $\pi_t = \pi(M_t - \bar{M}_t)$ and $\bar{\pi}_0 = \pi(\bar{M}_0 - \bar{M}_0)$ where $\bar{M}_t \in \mathbb{R}_+$ for any t which is the specific money growth neutralizing the inflation as zero percent or at least constant such as two percent given by the money growth rule, and are very small (like two percent), or zero, as we will assume so later. However, situations change significantly if we consider the business cycle fluctuations, like Keynesians. This is because additionally, they obtain $\tau_t = \tau(r_t)$ and use the monetary policy, M_t, for their fine-tunings, as we will mention later. Remember that we will detaily analyze the complex relationships among i_t, r_t, and $M_t - \bar{M}_t$ later, assuming that τ_t is fixed.

Thus, taking our conclusion in advance, a solution for conducting this skillful monetary policy should be a Taylor reaction function, recalling its derivation by Alvarez et al. (2001) and Orphanides (2003), for example. This result is always symmetrical for foreign countries. Note that ρ_t and $\bar{\rho}_0$ are the money growth rates based on the monetary base, that is, the quantity of money in the macroeconomy given the stable money multiplier (i.e., money stock), as we will also explain later.

The difference between the genuine Taylor reaction function and our above Taylor-type rule based on the (traditional) money demand equation is that the former focuses on both the inflation and the output gaps and pursues zeros for them as much as possible. However, the latter pursues steady inflations (usually and traditionally, zero inflations) by the money growth rule only because monetarism thinks that suppressing the short-run (real business) fluctuations should instead propagate their volatilities. This fact means that it does not care about the short-run output fluctuations, as we will explain later. Such different policy ideas exist behind these policy reactions.

Furthermore, we should reflect on the modern fact that the monetary authority should use not the interest rate but the quantity of money as the policy tool (i.e., the monetary base [the high-powered money]) in the open market operation. For this, we should see the following discussion about the relation between the quantity of money and the monetary base (e.g., Friedman, 1984). Our specification assuming the quantity of money as the control variable for the monetary authority in this paper is justified by the existence of (*monetizing* and *purchasing*) *bonds* in the open market operation. Then, the discounted sum of government debt will increase by just the right amount of the increase in the quantity of money in this paper.

Still, regarding the discussion of the above bonds, we owe Alvarez et al. (2001) for their bond equation. Bernenake and Blider (1988) and Blanchard and Fischer (1989) are the different applications using the existence of open market operations from us. More precisely, the former treats the bonds as the imperfect substitute of the loans and represents the *credit view*, which this paper does not choose (this paper is based on the *money view*). The latter explains the essentiality of the federal funds rate from the policy implication.

Technically, suppose that foreign countries (investors) buy this bond. In that case, we should consider the summed-up effect of the amount of bond absorbed within that country and the reduced amount of that currency for the foreign countries to buy the bonds of that country (see Céspedes & Chang, 2024, for the critical role of this foreign reserves for the central banks). That is, we just consider the amount of bond that affects the domestic quantity of money.

Thus, to grasp the nominal quantity of money correctly, we should consider the foreign government's actions for rebalancing its portfolios. Because this kind of foreign government's behavior should affect the money multiplier of the home country and become one of the sources of fluctuating the nominal quantity of money in the macroeconomy, or money stock. This is one of the reasons why fiscal policy can affect exchange rates through the monetary policy tool. However, we do not investigate this fiscal aspect further in this paper.

Alvarez et al. (2001) specify that the *positive* velocity shock by open market operation will initially *raise* the nominal interest rate. However, suppose that the velocity shock in our case should be represented by the fluctuations of b_1, that is, the natural logarithm of the *inverse* velocity of money in the quantity theory of money specification as before. This is because we ignore the error terms in our formulation for simplicity for the time being.

In that case, as we will see, we assume that the *positive* velocity shock in the open market operation will *decrease* the nominal interest rate first. Because *inflation* in the macroeconomy does *not* respond to this *positive* velocity shock, that is, this *positive* change in the *circulated* quantity of money in the open market operation due to the sticky price. This is the *natural* logic in the *market*.

Our short-run and long-run *money demand* equation discussion regarding the effects of the changing quantity of money on the nominal and real interest rates will explain this difference between the short-run and long-run cases later in detail. Recall that the quantity theory of money like that Friedman (1969/2006) supposed should require the money growth rule represented by the following three money motives, and this money growth rule is mostly connected with the real income movements (see the explanations in the later section).

Still, the average nominal interest rate in the macroeconomy differs from the federal funds rate, as Masuda (2024a) stated. Moreover, treasury bills (bonds) are connected more to the Secured Overnight Financing Rates than the federal funds rate but move parallel to the federal funds rate (Bernanke & Blinder, 1992). In short, the monetary base and the quantity of money are different. In the real economy, the *bridge* between the federal funds rate and the average nominal interest rate in the macroeconomy, that is, between the monetary base and the quantity of money circulating in the macroeconomy, is essential (Bernanke & Blinder, 1992). Note that the above average interest rate differs from the federal funds rate by the risk premium in the macroeconomy, that is, the term structure, and so on (e.g., Masuda, 2024a).

More precisely, we should extend our argument to the real economy. Therefore, for this *bridge* between those monetary bases and the quantity of money circulating in the macroeconomy, we need the stable *money multiplier* (e.g., Miron et al., 1994, or see any introductory macroeconomics textbooks for this money multiplier) and must always assume it. Our money demand equation in this paper treats not the federal funds *rate* in the open market operation directly but the *quantity* of *money* that actually focuses on the macroeconomy more than the federal funds rate itself does. In short, we mention the view of not the *financial markets* but the *macroeconomy*.

Furthermore, this money multiplier is included in the velocity of money as usual (e.g., Friedman, 1969/2006; Masuda, 2024a). The Alvarez et al. (2001) model does not need this money multiplier relation. Because their simple model excludes the firm's optimization problem and the banking system. We suppose this latter simplified assumption of Alvarez et al. (2001) hereafter, due to a different reason from them, that is, the observable facts about the stability of the money multiplier by Friedman (1969/2006). However, some (empirical) critics may exist (e.g., De Long, 2000).

In other words, our money demand equation in this paper treats not the federal funds rate but the weighted average nominal interest rate in the macroeconomy specified as the average of the returns of all the existing financial assets (and debts) in the macroeconomy, including mortgages. As for the (household's) portfolio approach (i.e., speculative motive), we should also treat the mortgage rates as the adjustments of such returns of real assets, making them positive or (sometimes) even negative. Note that our idea is similar to Flavin and Yamashita's (2006) approach.

For the above kind of Taylor reaction function (rule), the policy interest rate should be the function of inflation as before: The quantity of money (see the long-run inflation expectation discussion later for this functional relation). Thus, as Friedman (1970) stated, "Inflation is always and everywhere a monetary phenomenon in the sense that it is and can be produced only by a more rapid increase in the quantity of money than in output," even in the new Keynesian literature. Because the Taylor reaction function usually includes the quantity of money combined with the open market operation and the stable money multiplier. However, the new Keynesian model literature does not always emphasize this fact (e.g., an exception is Sargent & Surico, 2011). Still, for including temporarily unstable, time-varying financial intermediaries, we have Gertler and Karadi (2011) in the new Keynesian literature.

The Effects of Quantity of Money on Nominal and Real Interest Rates in the Short Run and Long Run Given the Money Growth Rule in Mathematical Expressions, and the Relations of Three Money Motives to This Money Growth Rule

In this section, we use dM_t/M_t, M_t, P_t, and M_t/P_t instead of $d\widetilde{M}_t/\widetilde{M}_t$, $\widetilde{M}_t$, $\tilde{P}_t$, and $\widetilde{M}_t/\tilde{P}_t$. However, the latter notation with tilde only means that they are normalized by the price of tradable goods produced by unskilled workers (the numeraire) and remember that these different notations do not change our conclusions in this section at all.

As we will see, this money demand equation is affected by three motives for holding money: The *transaction* motive, the *speculative* motive, and the *precautionary* (*asset*) motive, following Keynesian tradition (e.g., Dornbusch et al., 2008). Furthermore, the monetarist interpretation specifies the third precautionary motive as the asset motive and excludes the speculative motive when specifying the optimum quantity of money (e.g., Friedman, 1969/2006), accurately speaking.

In this paper, to understand the power of monetary policy, we should separate the cases of changes in the quantity of money as monetary policy based on demand-side factors, such as these three motives, from supply-side factors, such as helicopter money (e.g., Bernanke, 2002; Friedman, 1969/2006). Then, we derive their implications for the effects of monetary policy on the velocity of money and nominal and real interest rates.

Thus, in the following, we further argue the effects of increases in nominal and real quantities of money on the nominal and real interest rates separately under the sticky price assumption, with their motives, and compare the effects of those motives and so on under the flexible price long run with them.

As we will discuss later, the sticky price should be realized through some mechanism based on the staggered (nominal) wage (income) contract (e.g., Masuda, 2024b; Taylor, 1979, 1980; Walsh, 1998) and mostly constant or nondecreasing average nominal wage level (see Appendix C). In the short run, this constant or nondecreasing *average* nominal wage level is magically realized from the delicate but solid balance of the employed and the unemployed in our theory. Even in that case with our new (un)employment theory, as we will explain, our following inference will not change at all.

Additionally, as in the following paragraphs, some causal relationships behind variables in this quantity theory of money (money demand) equation can be specified, as in Sargent and Surico (2011). This is the main point of the long controversy (e.g., De Long, 2000) in the tradition of Friedman's (1956) (monetarist's) quantity theory of money. Again, this paper assumes the stable money multiplier (e.g., Friedman, 1969/2006, for the sound banking system; recall the rent-seeking behaviors).

First, I explain the Keynesian tradition regarding the effects of the quantity of money on nominal and real interest rates. In the standard new Keynesian model, that is, the structural equation model, like Woodford (2003), the monetary authority should control the real interest rate, r_t, and smooth the variation of output, Y_t, formulated as $Y_t = Y(r_t)$, where Y_t is assumed to be the decreasing function of real interest rate, r_t. $Y_t = Y(r_t)$ is the so-called conceptual IS relationship, and many concrete formulations exist (e.g., Masuda, 2024a; Woodford, 2003). We adopt the general and abstract one in this section.

For simplicity, this paper can specify any positive real interest rate level as the natural rate of real interest (e.g., Holston et al., 2016) defined by the money growth rule. We will later explain the relationship among the transaction motive, other motives, and this money growth rule. Note that this natural rate of real interest has both Keynesian and monetarist definitions, which we will discuss later.

Regarding the *positive* natural rate of real interest case in our Taylor rule, we should subtract some positive interest rate (i.e., the natural rate of real interest), $\bar{r} \cong b_1 + b_2\ln(Y_0) + b_2\tau_0 + b_2\ln(\bar{P}_0) + (b_2 - 1)\bar{\pi}_0 - b_2\bar{\rho}_0 - b_2\ln(\bar{M}_0)$, from the real interest rate in the above mathematical IS equation also specified by our Taylor rule. As a result, this subtraction expresses that the corresponding equivalent real interest rate to the natural rate of real interest has no positive or negative effect on the macroeconomy. However, this consideration complicates our following discussions without any essential differences. In this paper, we do not explicitly assume (i.e., ignore) the existence of a natural rate of (nominal and real) interest in our formulation. Because the nominal or real interest rate does not play such a serious role in our inflation theory, which is suggested in this paper.

Note that according to the quantity theory of money, given the *price* is *sticky*, $\frac{\partial i(M_t, P_t)}{\partial M_t} < 0$ when $dM_t - d\bar{M}_t > 0$ and $\bar{M}_t \in \mathbb{R}_+$ for any t where $d\bar{M}_t$ represents the k% or the corresponding money growth to neutralize inflations to the real income growth and so on. That money growth rate specifies the natural rate of real interest in our previous Taylor rule type money demand equation, $b_1 + b_2\ln(Y_0) + b_2\tau_0 + b_2\ln(\bar{P}_0) + (b_2 - 1)\bar{\pi}_0 - b_2\bar{\rho}_0 - b_2\ln(\bar{M}_0)$.

As mentioned above, that natural rate can be assumed to be any positive percent. Because, assuming the stcky price, the (real) quantity of money should increase due to at least the k% rule (the money growth rule), which forces the monetary authority to adjust the (real) growth of the quantity of money with the development of the real output to $d\bar{M}_t$ (e.g., Friedman, 1960). Again, $d\bar{M}_t$ is redefined as the necessary quantity of money to put dP_t staying unchanged given some real constant economic (income) growth, due to at least the k% rule (the money growth rule, though it must remain constant that is more than zero).

This means that the natural rate of nominal interest should specified by the growth of the nominal quantity of money provided by the money growth rule, $d\bar{M}_t/M_t$, again. This rate should usually be positive except in the case of deflation. However, in that deflation case, the natural rate of real interest must always be positive (Masuda, 2024a).

On the other hand, inflation rates do not respond much to changes in the quantity of money in the short run, as we assume in the above. More precisely, this is also because the money market immediately responds to the policy change, but the inflation rate does not. However, in the long run, this term, $\partial i(M_t, P_t)/\partial M_t$, might be positive. Because the inflation rates fully respond to changes in the quantity of money such as $dM_t - d\bar{M}_t > 0$. This paper mainly ignores this long-run case, and the short-run case is assumed.

To be more precise, assume the Fisher equation for the nominal interest rate, $i_t \cong r_t + \left(\frac{P_t}{P_{t-1}} - 1\right)$, and the price (inflation) stickiness. Furthermore, even if an increase of dM_t does not result from at least the k% rule (the money growth rule) but is due to the supply side factor like the helicopter money such that if $dM_t - d\bar{M}_t > 0$, we obtain the same result as $\frac{dr(M_t/P_t)}{dM_t} < 0$ and $\frac{\partial i(M_t,P_t)}{\partial M_t} < 0$ from the above sticky price assumption even in the short run.

In the above, our main point is that the *sticky price* assumption works to get the above conclusions under helicopter money (e.g., Bernanke, 2002; Friedman, 1969/2006). Otherwise, that is, under the *flexible price* assumption fixed by rigorous monetarists, even in the short run, we get different conclusions under the helicopter money such as $\frac{dr(M_t/P_t)}{dM_t} = 0$ and $\frac{\partial i(M_t,P_t)}{\partial M_t} \geq 0$, if $dM_t - d\bar{M}_t \geq 0$. Otherwise, naturally $\frac{dr(M_t/P_t)}{dM_t} > 0$. Because dP_t changes decreasingly to the shortage like deflation, $dM_t - d\bar{M}_t < 0$ such that we assume $d\left(\frac{M_t}{P_t}\right) < 0$, and the first-order derivative of M_t/P_t with respect to P_t is negative and its second-order derivative is positive. Here, we suppress the pathological possibility that $\frac{\bar{M}_t}{\bar{P}_t} < \frac{M_t}{P_t}$ when $^{\exists}M_t < \bar{M}_t$ where $\bar{P}_t$ is the general price level under $M_t = \bar{M}_t$ even if we assume the nonlinear relationship between dM_t/M_t and dP_t/P_t. Otherwise, this phenomenon disappears. Recall the quantity theory of money and the later discussion on the long-run inflation expectation. Furthermore, $\partial i(M_t, P_t)/\partial M_t$ is uncertain.

Thus, note that there are two kinds of dM_t increases: $d\bar{M}_t$, the increase of the nominal quantity of money due to the k% rule (the money growth rule) and the pure expansion of the nominal quantity of money, $dM_t - d\bar{M}_t > 0$, such as helicopter money. The former is originally real, but the latter is a nominal phenomenon and becomes real only under the sticky price assumption. Thus, helicopter money is enough to lead to expansionary monetary policy, particularly in the short run (with sticky prices).

Furthermore, consider the more realistic case. In the transition to the long run with the pent-up real demand (i.e., a part of the business cycle), when the full price change does finally work (i.e., prices imperfectly respond to the increasing quantity of money) and the nominal interest rate affects the velocity of money, we obtain

$\frac{dr(M_t/P_t)}{dM_t} < 0$ ($\frac{dr[M_t/P_t]}{d[M_t/P_t]} < 0$) and $\partial i(M_t, P_t)/\partial M_t$ is uncertain with the expansionary monetary policy anticipating this pent-up demand at the beginning of the period. It is when dM_t means the more increase of M_t/P_t for the monetary authority. Because due to the misestimation of the k% rule (the money growth rule), they estimate the pent-up real demand below with some positive erroneous (preemptive) expectations.

Recall that the money growth rule represents three money motives, as we will explain later. Here, we assume the positive expectational error of the monetary authority. As indicated below, *forecasting* the money demand due to the *precautionary* (*asset*) motive to stabilize the inflation rates is *difficult*. Because the monetary authority should predict its *conditional heteroscedastic* variances to estimate the true money demand, together with the real income movements (and the general price level movements if the monetary authority is not Keynesian), notwithstanding its reduced sensitivity to outputs (see Appendix B).

Thus, in the transition, the sign of $dr(M_t/P_t)/dM_t$ is not always equal to that of $\partial i(M_t, P_t)/\partial M_t$ because according to the quantity theory of money and the Fisher equation, dM_t increases (or possibly decreases) i_t. Because an increase in M_t by $dM_t - d\overline{M}_t > 0$ induces a the resulting increase in P_t and i_t (Fisher equation effect: inflation effect), whereas this increase in M_t means by a simultaneous increase in

$d(M_t/P_t)$. In such a case, $dM_t - dP_t > 0$ (the sticky price assumption) and $dM_t - d\bar{M}_t > 0$ decreases i_t due to (the misestimation of) the real money demand more than the above k% rule (the money growth rule) induced by the attempt to satisfy the (pent-up) real investment (demand) schedule with the markets or banks. Whichever is more significant, the former or latter about the direction of i_t depends on the relationship between the quantity of money and the resulting inflation rate, and should be determined empirically.

Note that the latter phenomenon (the decrease in i_t) should occur, even if we consider the fact that the inverse velocity of money decreases with the nominal interest rate increase when the corresponding output increase is unbalanced from the quantity theory of money. Because the decrease in the real interest rate stimulates the output.

This example is the result mitigated by the real and nominal effects when the above pent-up real money demand and misestimated helicopter money due to the erroneous preemptive motive (expectation) in the monetary authority coexist. In this example, the output fluctuations and the precautionary (asset) money demand in the same period, which is difficult for the monetary authority to estimate in advance, as shown in Appendix B, should occur. Furthermore, the above pent-up real demand case assumes that real money demand is caused by real demand (investment).

However, increasing output growth stimulated by the decreased real interest rate with the exceeding (supplied) quantity of money should satisfy more than this real money demand by the money growth rule as if the monetary authority's expectation is *self-fulfilled* (see Orphanides, 2003, for its possibility as one example). However, in fact, the transaction (speculative and also precautionary [asset]) motive(s) induced by the fluctuating output growth must generate the necessary real money demand to keep the general price level constant.

Under the *sticky* price assumption (but the *nonzero* inflations in the above realistic example), an increase only in the *nominal* (supplied) quantity of money, can be enough for the monetary authority's erroneous expectations and misestimates more than this money growth rule related to these three money motives to cause more unnecessary real fluctuations (and resulting inflations) through the Phillips curve or the quantity theory of money. This holds except for the long run under the fully flexible price assumption. However, as Friedman (1970) noted, this long-run case with the too exceeding nominal quantity of money empirically gives unnecessary higher and more volatile inflations. Ropele et al. (2024) indicate that these higher inflations will also cause resource misallocations that are naturally harmful to the country in question.

As we will explain later, our theory suggests that even in this case, such higher inflation should cause higher technological progress in the flexible price long run. On the other hand, from the natural unemployment rate hypothesis (Friedman, 1978), such higher inflation should cause just wage inflations through technological progress of the tradable goods industry induced by the general price level increases (i.e., price inflations through the quantity theory of money) and should not have any effects on the output gaps (business cycles), as our theory also indicates (see also the walking inflation discussion in the later section).

Here, as we will repeatedly use this relation, we assume the separability between the productive efficiency (economic growth) represented for the potential output and the capacity utilization rates (business cycles) represented for the output gaps under the constant and the increasing returns to scale production functions (e.g., Burnside et al., 1988; Hall, 1973; Havik et al., 2014). The constant and the increasing returns to scale production functions can successfully separate potential outputs and output gaps (i.e., capacity utilization rates or business cycles).

In this case, the only difference between the constant and the increasing returns to scale production functions is that the former has a constant returns effect of output gap fluctuations on outputs. However, the latter has an increasing return effect of output gap fluctuations on outputs. For example, suppose the degree of increasing returns to scale is two. In that case, its impact of output gap fluctuations on output is two times as significant as that under the constant returns to scale production function.

Regarding the relationship between the real interest rate and the output, in the short run, if the output responds to the real interest rate negatively, to smooth the increase (or decrease) of output, Y_t, the real interest rate, r_t, should be increased (or decreased) to set the output, Y_t, as constant, and $\frac{dV^{-1}}{di_t} = \frac{d\exp(\beta_1)i_t^{-\beta_2}}{di_t} < 0$, $i_t \cong r_t + \left(\frac{P_t}{P_{t-1}} - 1\right)$, $\frac{\partial i(M_t,P_t)}{\partial M_t} < 0$, and $\frac{dr(M_t/P_t)}{dM_t} < 0$ due to the staggered P_t dynamics, as we noted before. As a result, the quantity of money, M_t, should be decreased (or increased) to smooth the increase in $Y(r_t)$ from the money demand equation, $M_t = P_t\exp(\beta_1)i_t^{-\beta_2}Y_t$, even considering the degree of $\frac{dV^{-1}}{di_t} = \frac{d\exp(\beta_1)i_t^{-\beta_2}}{di_t} (< 0)$.

That is, different from the partial equilibrium setting, tightening the quantity of money not from the k% rule (the money growth rule) accompanies the rise of the velocity of money due to the increase in the nominal interest rate, as we saw in the previous realistic transition case and the above. The resulting M_tV_t due to the tightened monetary policy is slightly more significant than that before this adjusting because the velocity of money increases, adjusting to the corresponding (nominal) interest rate rise.

On the other hand, tightening the quantity of money *due to* the *k% rule* does lead to a direct reduction in the circulating quantity of money (i.e., the quantity of money itself in this case) by just the right amount. Because r_t and $\frac{P_t}{P_{t-1}} - 1$ stay unchanged due to the zero effect that P_t and Y_t are unchanged by this k% rule (the money growth rule) even in the long run. Thus, this k% rule is equivalent to specifying the natural rate of nominal interest in our Taylor rule if the monetary authority controls the money growth rate to become this k% rule even in the short run.

Furthermore, it is essential to remember the intricate relation between wage and price inflations justified by Masuda (2024b), Taylor (1979, 1980), and Walsh (1998). In short, the Taylor's (1979, 1980) staggered (nominal) wage contract model holds (e.g., Masuda, 2024b). Suppose that such a short-run tradeoff between inflation and unemployment (output gap) exists only under the profit maximization hypothesis, as in the Phillips curve (based on the staggered wage above).

Then, even the increase (or decrease) of the quantity of money, M_t (i.e., $dM_t - d\bar{M}_t \geq 0$ or $dM_t - d\bar{M}_t < 0$), not from the money growth rule should lower (or raise) the real interest rate given the *sticky* inflation rate and increase (or decrease) the output, Y_t. This effect breaks the natural unemployment rate hypothesis in the short run, which recalls the *stagflation* situations with the importance of expectations in the long run (e.g., Masuda, 2024b; recall the accurate term, $\pi(M_t - \bar{M}_t) - \bar{\pi}_0$, in the Phillips curve; we define two types of stagflations later. This stagflation is similar to that in the 1970s).

Initially, the k% rule (the money growth rule) specifies $dr_t > 0$ when $dM_t - d\bar{M}_t < 0$ and $dr_t = 0$ when $dM_t - d\bar{M}_t \geq 0$ in the long run when the price is (completely and instantly) *flexible*.

Again, in this structural interpretation of the quantity theory of money, the up or down of output, Y_t, like business cycles, should be accompanied by the increase or decrease of the quantity of money, M_t (i.e., the gain or reduction of dM_t/M_t). This relation should be discretionarily counteracted in the broader sense. That behavior is included in the policy reaction function, such as the Taylor reaction function, considering the changes in the velocity of money, V_t (or its inverse V_t^{-1}), IS curve, and the short-run Phillips curve. This is the case that Sargent and Surico (2011) presented in the Keynesian tradition. Moreover, such consideration for background relations is not included in the monetarist-like rigorous rule, such as the k% rule (the money growth rule).

Thus, in the monetarist tradition of the *pure* quantity theory of money, one should only react to the growth of output, Y_t, with the increase of the quantity of money, M_t (i.e., the increase of dM_t/M_t), given $\frac{dV_t}{V_t} = 0$. To be more precise, the quantity of money, M_t (i.e., the increase of dM_t/M_t), should be increased, from the k% rule (the money growth rule) which is equal to $d\bar{M}_t$ (i.e., $d\bar{M}_t/M_t$). Recall the result of Lucas

(1988) for unit elasticity of income in the money demand equation. In this case, one should not consider the specific (complicated) causal relationship behind the money demand equation to be a wise way.

This is the tradition of Friedman's (1960) money growth rule (originally, it means the k% rule). We should distinguish the true output growth, which is neither affected by business cycles nor temporal output growth variations. For example, Friedman (1960) suggested that the money stock should not respond to seasonal variations. In the modern interpretation, Friedman's (1960) statement ensures that not the *output gap* fluctuations but the *potential output* growths are critical (e.g., Havik et al., 2014). This means that we should use this potential output growth concept for the transaction (and possibly precautionary [asset]) motive(s).

More precisely, regarding interest rate controlling policy, (new) Keynesians suggest that the monetary authority should respond to the additional *output gap* fluctuations to the *potential output* fluctuations when they decide (real) interest rate levels (i.e., monetary policy). Such an idea specifies the *short-run* new Keynesian policy idea for interest rates.

On the other hand, monetarists only consider the potential output fluctuations when examining the (real) interest rate levels. This is the corresponding long-run idea to interest rate controlling policy. This difference also depends on whether the *precautionary* (*asset*) *motive* further caused by the business cycles (i.e., output gap fluctuations) and so on in addition to the usual transaction motive is essential or not, as the natural result of respect to even the temporal output fluctuations (i.e., even the smaller output gap fluctuations; *economic crises* should cause *more significant* output gaps due to its effect on the permanent and transitory parts of real incomes in empirical analysis), as we will describe later (e.g., the conditional heteroscedastic variance; see Appendix B).

Regarding the speculative motive, I have considered the nominal interest rate and the money demand caused by it, which will fluctuate with the nominal interest rate movements. As we will note, this motive empirically affects the demand for money only a little. Moreover, I present the *precautionary* (*asset*) motive. However, they work as if they are relatively minor enough to be ignored at regular times (see Appendix B) so that I safely make the transaction motive represent the other two motives (i.e., represent the k% rule, the money growth rule in this paper). Therefore, I do not explicitly consider them except for Appendix B in this paper. Initially, Friedman (1969/2006) argued only the transaction and *asset* motives (the precautionary motive mainly corresponded to this asset motive) when he specified the optimum quantity of money. Furthermore, Friedman and Schwartz (1963/2008) emphasized the importance of the precautionary motive in economic crises. On the other hand, all three motives are used in the Keynesian tradition.

For example, the new Keynesian loss (hereafter, social welfare) function tells us that this is worse for the public when inflation and real output fluctuations increase, which respects the variances (the second-order term) of inflation and output gap (e.g., Woodford, 2003). However, helicopter money has first-order importance and matters even during transitions (e.g., Fischer, 1979).

Which is the Correct Policy Tool, Nominal Interest Rate or Quantity of Money?

The official discount rate is usually outdated, particularly in regular times, and the open market operation is the primary tool for the monetary authorities in developed countries. The interest rate controlling policy cannot help using the open market operation; it depends only on the quantity of money.

Therefore, suppose that the money demand equation is solved for the nominal interest rate (i.e., the Taylor rule; see the previous discussions and Masuda, 2024a). Taylor (1993) found that the Taylor reaction function is the best response of the monetary authority to the macroeconomy. In that case, we know that the quantity of money should be used by the monetary authority to target the nominal interest rate more clearly, which provides the public with information about the macroeconomy.

More precisely, Cukierman (1992) argued that the monetary authority had the interest rate smoothing motive (i.e., financial stability motive). Because the banks usually have many bank loans fixed with long-term interest rates, as well as debts like deposits fixed with short-term interest rates. Furthermore, this fact will generate bank health concerns when the short-term interest rates in the financial market, like the federal funds rate, fluctuate more than expected.

Thus, according to Cukierman (1992), these interest rate fluctuations affect social welfare in their objective function as one of the penalty terms. The monetary authority respects suppressing and smoothing the nominal interest rate movements. Therefore, it is probable that the monetary authority should take more care of financial stabilization than inflation stabilization, depending on the situation. Cukierman (1992) argued that this motive for financial stability also led to inflation bias in the monetary authority.

Thus, in the *long run*, not M_t but i_t should be used as a direct policy instrument for monetary policy to send a clear message to the market or the macroeconomy according to the above inference for the more straightforward conduct of monetary policy to clarify the monetary authority's objectives.

Even in the short run, i_t should be chosen because the price or the inflation rate does not change (so much due to the sticky price assumption), but the money multiplier fluctuates in the short run (De Long, 2000; Friedman, 1969/2006). This fluctuation will inevitably cause further misestimation and erroneous expectations of the monetary authority. Also, households or consumers will similarly at least misestimate or go beyond their understanding to grasp the meaning of such an incrementally fluctuating quantity of money. Brückner and Shabert (2006) explain the relationship between money and interest rates in some different literature.

Thus, disseminating the correct knowledge about which policy tool is significant, the interest rate or the quantity of money (in the short or the long run) is critical. Furthermore, forward-guiding its future path through the newspapers, etc., for the monetary authority is quite essential for the market participants and the public to understand their economic future. It is convenient for the policymakers to operate their policy by clarifying their policy objectives again (e.g., Blinder et al., 2024).

Furthermore, the advances in Fintech should increase this kind of essentiality for the public. For example, technological progress like Fintech should affect the money demand such that more saved cash should be stored in savings accounts to invest in risk assets like stocks and securities to some degree than otherwise. In line with this example, Alvarez and Lippi (2009) derive the precautionary (asset) motive for the random cash withdrawal of households. Such technological progress should also make the public aware of the essentiality of interest rates. Note that the nominal interest rate is not a control variable for households or consumers but an exogenous, given variable like from heaven (in reality, from the monetary authority).

Friedman's (1969/2006) Velocity of Money Discussion and Other Topics

Furthermore, Friedman (1969/2006) proposes the seemingly contradictory suggestion that (a) the velocity of money is affected by the fluctuating nominal interest rate (even in the short run), but (b) the velocity of money is stable. However, even though we can consider the effect of nominal interest rates on the velocity of money, it is usually small (e.g., de Bondt, 2009, for the Euro area; Lucas, 1988, for the U.S. case). In developed countries, the monetary authorities usually conduct careful policymaking and mostly succeed in smoothing inflation and output fluctuations (i.e., its parameter is small; e.g., Cukierman, 1992, for the interest rate smoothing motive), as we will also mention in Appendix B.

Thus, the above points (a) and (b) are not contradictory. They *are* essentially *empirical* problems. Friedman (1969/2006) empirically studied these characteristics of the velocity of money in detail. Moreover, Friedman (1969/2006) insists that these seemingly contradictory arguments entirely depend on the definition of money.

Similarly, theoretically, we can indeed explain this empirical problem as the definition of money (see Masuda, 2024a). In short, the money (credit) multiplier is rooted in the definition of money and affects measuring the velocity of money.

Moreover, my focus is connected with that of Lucas (1988), who suggested the unit income elasticity and the stability of the money demand equation in the United States. Lucas's (1980) case is similar to this monetarist tradition. However, he orthodoxically focused on the quantity theory of money relationship between the nominal interest rate, inflation rate, and money supply. On the other hand, this paper mainly focuses on another quantity theory of money relationship in that growing income requires more money from the k% rule (i.e., the money growth rule).

This correctly increased quantity of money does not cause inflation under both the flexible (and the sticky price) assumptions. The k% rule specifies the natural rate of real interest if the money growth is specified by this k% rule. This is because the k% rule (nearly) represents the corresponding potential output growth in the monetarist tradition (remember the speculative and the precautionary [assety] motives). Only economic crises can make the monetary authority deviate from this k%, although this fact is also included in the monetarist tradition (e.g., Friedman & Schwartz, 1963/2008). Recall the Phillips curve discussion (the difference between output gaps and potential output growth) and the long-run inflation expectation in the later section.

Finally, these policy suggestions also lead to somewhat contradictory monetary policy operations between Keynesians and monetarists and the case of The Difficulties of the Policy Rule on Exchange Rate Derived in the Previous Section in Policy Practice Dominated by the Risks section. These considerations do not affect the conclusions in The Monetary Policy Rule of the Exchange Rate by Integrating the Quantity Theory of Money and the Harrod–Balassa–Samuelson Effect Model With Two Tradable Goods Produced by Skilled Workers and Unskilled Workers and Nontradable Goods section and The Monetary Policy Rule From the Model Analysis in the Previous Section: Conclusions in the Deterministic World section but provide the theoretical foundations for the discussions of The Difficulties of the Policy Rule on Exchange Rate Derived in the Previous Section in Policy Practice Dominated by the Risks section, specifically for removing the randomness of outputs. Again, note that in the above, we can safely assume any positive level for the natural rate of interest.

The Harrod–Balassa–Samuelson Effect Model With Skilled and Unskilled Workers

Our Setting and FOCs for the Firm's Sales Revenue and Profit Maximization Problems With the Karush-Kuhn-Tucker Approach

In this section, we formulate the firm's sales revenue and profit maximization problems with the Karush-Kuhn-Tucker approach. More precisely, for the firm's profit maximization problem, we conveniently assume a competitive market that requires *level demands*. However, as Masuda (2024a) suggests, the quantity theory of money proposed by Friedman (1969/2006) breaks this kind of *Say's law*.

Say's law means that the level demand exists and is never affected by prices (inflations; see Masuda, 2024a), and the above break means the *rejection* of this level demand. Instead, in Appendix D, we construct the concrete general equilibrium model with household or consumer demand and derive the downward-sloping demand curve between the real consumption (income) and prices.

Thus, we know that the level demand curve is *nonsense* in our theory, but this section assumes this level demand for convenience. This is because the other market structures like oligopoly and monopoly should only add the *constant* markup term that never affects our theory in this paper, as long as we assume the *stable* (*constant* through time) demand.

Now, the labor force in the market is assumed to be $L_{1,t} \geq L_{TU,t} + L_{N,t}$, $L_{2,t} \geq L_{TS,t}$, $L_{TU,t}$, $L_{TS,t}$, and $L_{N,t} \in \mathbb{R}_+$ for any t such that $L_t = L_{1,t} + L_{2,t}$ and L_t, $L_{1,t}$, and $L_{2,t} \in \mathbb{R}_+$ for any t where $L_{1,t}$: The number of unskilled workers available for the tradable goods industry and workers available for nontradable goods industry at the period t, that is, the labor force of unskilled workers for the tradable goods industry and workers for the nontradable goods industry at the period t, $L_{2,t}$: The number of skilled workers available for the tradable goods industry at the period t, that is, the labor force of skilled workers for the tradable goods industry at the period t and its supply in the market is assumed to be restricted by educational capacity and so forth such that this leads to the *other assumption* of the *segmented* tradable goods industry (and nontradable goods industry) markets for unskilled and skilled workers in this paper, $L_{TU,t}$: The number of firm's labor demand for unskilled workers in the tradable goods industry at the period t, $L_{TS,t}$: The number of firm's labor demand for skilled workers in the tradable goods industry at the period t, $L_{N,t}$: The number of firm's labor demand for workers in nontradable goods industry at the period t.

Following the tradition of Samuelson (1964) and other trade theorists as in the previous section, suppose that we consider the case in which both the tradable goods industry for skilled and unskilled workers and the nontradable goods industry have only one input good: Labor. In that case, we can assume the linear production relation with the unique production factor such as labor input, except for the nonlinear production function of the tradable goods industry for the skilled workers due to the increasing returns to scale property of its production function, as noted before.

Then, our *sales revenue* maximization hypothesis under the *firm's profit* constraint and the *resource* constraints gives the following firm's optimization problem (see this kind of formulation in Baumol, 1959):

$$\max_{L_{TU,t}, L_{TS,t}, L_{N,t}} \sum_{l=TU,TS,N} P_{l,t} Y_{l,t},$$

$$\text{subject to } \sum_{l=TU,TS,N} P_{l,t} Y_{l,t} \geq W_t L_{TU,t} + (1+g) W_t L_{TS,t} + W_t L_{N,t} \ \ (\lambda),$$

$$W_t \in \mathbb{R}_+ \text{ for any } t, \text{ and } {}^\forall g \in \mathbb{R}_+,$$

$$Y_{TU,t} = A_{T,t} F(L_{TU,t}),\ Y_{TS,t} = A_{T,t} F(L_{TS,t}),\ Y_{N,t} = A_{N,t} F(L_{N,t}),$$

$$L_{1,t} \geq L_{TU,t} + L_{N,t} \ \ (\mu_1), \text{ and } L_{2,t} \geq L_{TS,t} \ \ (\mu_2).$$

The *Greek* alphabet with parentheses on the right-hand sides of the above constraints shows the KKT multipliers. The first line of the above constraints indicates the firm's profit constraint, the third line indicates the production functions, and the final line shows the resource constraints explained above.

The variables used in the above formulation is as follows (we explain some variables repeatedly for the reader's convenience); $P_{TU,t}$ is the price of tradable goods produced by unskilled workers at the period t, $Y_{TU,t}$ is the output of the tradable goods industry produced by unskilled workers at the period t, $A_{T,t}$ is the common total factor productivity (technological progress) of the tradable goods industries for skilled and unskilled workers at the period t, $F(L_{TU,t}) = L_{TU,t}$ is the linear production relationship of the tradable goods industry for unskilled workers at the period t, W_t is the common nominal wage for unskilled workers in the tradable goods industry and workers in the nontradable goods industry at the period t, $P_{TS,t}$ is the price of tradable goods produced by skilled workers at the period t, $Y_{TS,t}$ is the output of the tradable goods industry produced by skilled workers at the period t, $F(L_{TS,t}) = L_{TS,t}^{1+\gamma}$, $\gamma > 0$ is the nonlinear production relationship of the tradable goods industry for skilled workers at the period t, $(1+g)W_t$ is the nominal wage for skilled workers in the tradable goods industry at the period t with the constant premium g, $P_{N,t}$ is the price of nontradable goods at the period t, $Y_{N,t}$ is the output of nontradable goods industry at the period t, $A_{N,t}$ is the total factor productivity (technological progress) of nontradable goods industry at the period t, and $F(L_{N,t}) = L_{N,t}$ is the linear production relationship of nontradable goods industry at the period t.

We should use the following relations for the above optimization problem, $P_{TS,t} = P_{TS,t}^{*}\frac{P_t}{P_t^{*}}$, and $P_{TU,t} = P_{TU,t}^{*}\frac{P_t}{P_t^{*}}$, or $P_{TS,t}^{*} = P_{TS,t}\frac{P_t^{*}}{P_t}$, and $P_{TU,t}^{*} = P_{TU,t}\frac{P_t^{*}}{P_t}$, where P_t^{*} is the general price level of foreign country at the period t, and P_t is the general price level of home country at the period t, regarding the prices of tradable goods industry by unskilled and skilled workers. In short, as we will explain in detail in Appendix D, the firms in the home country should *export* their tradable goods such that they must face *foreign* prices for their sales and outputs. Here, we suppose the law of one price for $P_{TU,t}$ and $P^*_{TU,t}$, and $P_{TS,t}$ and $P^*_{TS,t}$.

Thus, in the above optimal problem formulation, the firms in the home tradable goods industry should consider not the home price but the foreign price like $P^*_{TS,t} = P_{TS,t}\frac{P^*_t}{P_t}$ for their sales, for example, at a glance. That is, accurately speaking, we should multiply the additional term, P^*_t/P_t, to $P_{TS,t}$ in the above optimal problem for example. However, this paper omits this additional term, following the tradition. We can assume that the home tradable goods industry firms should evaluate their sales and outputs only in their home currency. So, the home currency exchange rate should adjust their sales and outputs. This assumption safely exclude the above additional exchange rate term, P^*_t/P_t, by multiplying with the term, P_t/P^*_t.

Of course, as long as we express $P_{TU,t}$ explicitly in this section different from the later sections, we should apply the law of one price for this price, $P_{TU,t}$, as we noted. This further consideration also does not change our conclusions. Recall that we transform the other price variables with normalizing by $P_{TU,t}$ in the later sections because we also suppose that $P_{TU,t}$ is the numeraire for the home country (the same thing goes for the foreign country and $P^*_{TU,t}$). However, our results in this section are unchanged even in those cases.

Solving this optimization problem with the *Karush-Kuhn-Tucker approach* (e.g., Dixit, 1990), we obtain the following FOCs:

$$P_{TU,t}A_{T,t}(1-\lambda)+\lambda W_t-\mu_1=0,$$

$$P_{N,t}A_{N,t}(1-\lambda)+\lambda W_t-\mu_1=0,$$

$$P_{TS,t}A_{T,t}(1+\gamma)L_{TS,t}^{\gamma}(1-\lambda)+\lambda(1+g)W_t-\mu_2=0,$$

$$P_{TU,t}A_{T,t}L_{TU,t}+P_{TS,t}A_{T,t}L_{TS,t}^{1+\gamma}+P_{N,t}A_{N,t}L_{N,t}=W_tL_{TU,t}+(1+g)W_tL_{TS,t}+W_tL_{N,t}\ \ (\lambda>0),$$

$$L_{1,t}=L_{TU,t}+L_{N,t}\ \ (\mu_1>0),$$

$$L_{2,t}=L_{TS,t}\ \ (\mu_2>0).$$

Because we can exclude the case of $\lambda=0$ from our economic intuition that the firm's profit should be exhausted (i.e., held the zero profit condition [and symmetry, if necessary, as we will later describe]). As a result, we can ensure the positivity of μ_1 and μ_2 (i.e., excluding the case that μ_1 and μ_2 are both or is individually null; we will discuss these null cases in the later) by combining the above FOCs under the sales revenue maximization hypothesis. Therefore, in equilibrium under the sales revenue maximization hypothesis, the solution in the above formulation with the Karush-Kuhn-Tucker approach must satisfy the above FOCs and the distribution laws in the first below.

Once the distribution laws are satisfied with $\lambda>0$ due to the zero profit condition principle (i.e., the profit should be entirely exhausted by or distributed to production factors) and segmented market assumption for unskilled and skilled workers in the tradable goods produced by skilled workers market, the skilled worker's wage, $(1+g)W_t$, should be equal to the average labor productivity, $P_{TS,t}A_{T,t}L_{TS,t}^{1+\gamma}/L_{TS,t}$, under our sales revenue maximization hypothesis.

This result is different from the optimal condition of the usual profit maximization problem under the increasing returns to scale production function, which explains that $(1+\gamma)P_{TS,t}\frac{A_{T,t}L_{TS,t}^{1+\gamma}}{L_{TS,t}} = (1+g)W_t$. It is rather similar to average cost pricing (i.e., average productivity) hypothesis in the natural monopoly case as such in the standard microeconomics textbooks. After four pages, we will show our distribution laws in a mathematical expression.

Because we verify that $\lambda = 1 > 0$ when $P_{TU,t}A_{T,t} \neq P_{N,t}A_{N,t}$ and $\mu_1 = W_t > 0$, we obtain the result that if $\mu_2 = (1+g)W_t > 0$, then $L_{TS,t} = L_{2,t}$, and if $\mu_2 = 0$, then $L_{TS,t} < L_{2,t}$. Also, because γ is assumed to be the time-invariant variable, our convenient assumption in this paper that $dL_{2,t}$ and $dL_{TS,t}$ are zero or some minor constant is practically justified (this convenient assumption is for here, for the time being. See Appendix C for the another possibility). It is so even when $\mu_2 > 0$ and the skilled worker's labor force, $L_{2,t}$, is perfectly employed. Because we assume that the number of skilled worker's labor force reaches its ceiling ($L_{2,t}$). This ceiling typically changes to the total population growth and conveniently we can assume $dL_{2,t} = 0$ or some minor constant again (see Appendix C for the other possibility under the profit maximization hypothesis).

From the FOCs that $L_{1,t} = L_{TU,t} + L_{N,t}$ and $L_{2,t} = L_{TS,t}$, we obtain not the *indeterminate* labor input levels but the *absolute* labor input levels for $L_{TU,t}$, $L_{TS,t}$, and $L_{N,t}$ such that we can also get the *absolute* sales levels. It appears that each absolute level is indeterminate for $L_{TU,t}$ and $L_{N,t}$. At the same time, $L_{TU,t}$ and $L_{N,t}$ have the *adding-up* restriction to equal to $L_{1,t}$. As a result, from this adding-up restriction, we can specify the absolute levels of sales and output levels for the tradable goods industry with the unskilled workers and the nontradable goods industry.

More precisely, the concrete levels of $L_{TU,t}$ and $L_{N,t}$ should be uniquely determined by the concrete levels of the nontradable goods price, the productivities of the tradable and nontradable goods industries (see the following distribution law equation). Taking our conclusion in advance, those concrete levels are empirically given and also theoretically given if the initial values are provided (see the later price differential equation discussion).

Regarding $\mu_2 > 0$, this inequality holds under the sales revenue maximization hypothesis (we will explain the case under the profit maximization hypothesis in the later section). More precisely, we can technically assume that $\mu_2 = 0$ but this assumption (diminishing equilibrium in the skilled worker market) will be denied under the sales revenue maximization hypothesis. Note that if μ_2 is zero, this means that this system admits the *diminishing equilibrium* of labor demand and labor supply and more unemployment in the labor market for the skilled workers in the tradable goods industry under the sales revenue maximization hypothesis.

However, as in the later sections, this is the counter-intuitive result that we assume the full employment equilibrium in labor markets, which is possible even with the existence of the aggregate resource constraints under the sales revenue maximization hypothesis. We think that our benevolent leaders and managers (i.e., typically the young entrepreneurs) in the firms under the sales revenue maximization hypothesis assumed in this paper do not allow slack (the smaller labor demand than the currently available labor force [supply]), that is, (involuntary) unemployment in the labor market under the sales revenue maximization hypothesis, like the *traditional* competitive market concept.

At least, this paper can treat the possibility of diminishing equilibrium due to insufficient (labor) demand compared with the currently available labor force (supply) in the labor market. However, we deal with such a diminishing equilibrium case intensively only under the profit maximization hypothesis. As we will see later, the usual (traditional) Harrod–Balassa–Samuelson effect model holds only in this diminishing equilibrium under the profit maximization hypothesis.

Note that we admit the possibility for $L_{TU,t}$ and $L_{N,t}$ to ensure their positive (or, its more minor case, negative) growths case with assuming the positive (or, negative) growth rate for $L_{1,t}$. Because otherwise (i.e., $dL_{1,t} = 0$), suppose that $dL_{TU,t}$ and $dL_{N,t}$ are not zero. In that case, we must have the growth tradeoff between $L_{TU,t}$ and $L_{N,t}$ due to the result of exhausting the existing resources (i.e., labor force) like under the traditional competitive markets.

Still, this paper supposes that our labor force participation theory in the later sections specifies the available amounts of labor inputs (i.e., the labor forces or supplies), $L_{1,t}$ and $L_{2,t}$. Furthermore, they are perfectly employed by firms in the macroeconomy, under the above sales revenue maximization hypothesis like the traditional competitive market results. Thus, the *labor force* (*labor supply*) growth *partly* specifies economic growth in our theory. Here, *partly* means that technological progress *exists* as the *engine* of economic growth in our theory (e.g., Solow, 1956).

The Distribution Law Equations Under the Sales Revenue Maximization Hypothesis

and the Profit Maximization Hypothesis

Using the segmented market assumption and the zero profit condition, the following distribution law from the *sales revenue* maximization hypothesis is obtained:

$$P_{TU,t}Y_{TU,t} + P_{N,t}Y_{N,t} = P_{TU,t}A_{T,t}F(L_{TU,t}) + P_{N,t}A_{N,t}F(L_{N,t}) = W_t(L_{TU,t} + L_{N,t}),$$

$$P_{TS,t}Y_{TS,t} = P_{TS,t}A_{T,t}F(L_{TS,t}) = (1+g)W_tL_{TS,t}.$$

On the other hand, we obtain the following distribution laws under the *profit* maximization hypothesis:

$$P_{TU,t}Y_{TU,t} = P_{TU,t}A_{T,t}F(L_{TU,t}) = W_tL_{TU,t},$$

$$P_{TS,t}Y_{TS,t} = P_{TS,t}A_{T,t}F(L_{TS,t}) = (1+g)W_tL_{TS,t},$$

$$P_{N,t}Y_{N,t} = P_{N,t}A_{N,t}F(L_{N,t}) = W_tL_{N,t}.$$

In the above distribution laws under the profit maximization hypothesis, we suppose that the profit from each industry is *entirely separated* by its labor input. Furthermore, each *independent* industry accomplishes *zero profit* under the profit maximization hypothesis. Here, *zero profit* does not mean that the current account is always balanced. However, it means that the earned profit should be exhausted for distribution to all the production factors, as usual in microeconomics. Note that under the profit maximization hypothesis, the zero profit condition and the marginal productivity hypothesis coincide if the production function has constant returns to scale property. This is a well-known result (e.g., Binger & Hoffman, 1998).

On the other hand, these distribution laws under the sales revenue maximization hypothesis cannot accept this zero profit condition for each labor market for the tradable goods industry by unskilled workers and the nontradable goods industry. The total profit of the tradable goods industry for unskilled workers and the nontradable goods industry can only satisfy this zero profit condition (we will discuss this point further in a later section).

Note that the above second equation of the distribution laws under the profit maximization hypothesis is the same as that under the sales revenue maximization hypothesis. It is directly derived from the *zero profit* condition due to its increasing returns property, even if we do not consider the above *resource constraints*. They recall the average productivity hypothesis under the industry with the increasing returns to scale production function like the natural monopoly (see any introductory microeconomics textbooks; e.g., Binger & Hoffman, 1998).

However, the solution under the profit maximization hypothesis should not satisfy the aggregate resource constraint. So, skilled worker unemployment always exists in the labor market for the tradable goods industry (see Appendix C for its steady state case).

The distribution laws under the sales revenue maximization hypothesis hold when the *KKT multipliers* μ_1 and μ_2 should work to become *positive* to exhaust the existing resources (i.e., labor force; see the below discussion for the indeterminate levels of labor inputs under the profit maximization problem). Even in this sales revenue maximization hypothesis, as long as we assume that resource constraints are not satisfied, we can use the newly derived and usual FOCs under the profit maximization problem as if we assume that the profit maximization problem (principle) should be given instead of the sales revenue maximization problem (hypothesis).

However, this is not the sales revenue maximization hypothesis (i.e., the profit maximization hypothesis) case, which could not satisfy resource constraints like those under the profit maximization hypothesis. Moreover, the above distribution law for skilled workers is not justified without applying the average productivity hypothesis to increasing returns to scale production function case.

Again, the FOCs under the profit maximization problem with *exhausting* the (aggregate) resource constraints mean that the prices need *markup-like* terms (i.e., μ_1 and μ_2 are positive). However, we cannot obtain the optimal solution mathematically in this case, where the existing resources are perfectly engaged in production. For the above case, recall the *shadow price* discussion for the KKT multiplier (e.g., Dixit, 1990) and the similar discussion about the *Cournot competition* for the *finite correction* (i.e., markup) of the price set by the *finite* number of firms.

In this Cournot competition example, we assume that the firms can control their sales outputs and have price control powers. However, this markup term disappears when the number of firms reaches *infinite* (e.g., Jehle & Reny, 2001), similar to our following case with being able to ignore the aggregate resource constraints. That is, the particular case in which infinite resources *are* available must be possible in this Cournot competition example. Actually, this analogy should hold, although the above Cournot competition example assumes the traditional profit maximization hypothesis *without* resource constraints (see the additional benevolent condition in the later discussion).

On the contrary, as we noted, under the profit maximization problem or the sales revenue maximization problem with the profit maximization hypothesis, once KKT multipliers μ_1 and μ_2 should become *null*, even in this case, we can have some plausible equilibrium like that with the marginal productivity hypothesis (the traditional competitive equilibrium). However, these resource constraints hold *with inequality* such that we *must admit* the existence of *idle* resources even in this *equilibrium*.

In short, in terms of the aggregate resource satisfaction problem, the above *mathematics* (i.e., the KKT condition) indicates that we should use the *sales revenue* maximization hypothesis suggested by Baumol (1959) instead of the simple profit maximization hypothesis for accomplishing full employment. It must be so when we explicitly consider the (aggregate) *resource constraints* in the macroeconomy and would like to keep the competitive market properties in the *usual sense* that resources are exhausted for our results. Note that we can consult any other mathematical economics textbooks that provide optimization methods like the KKT conditions in mathematical detail (e.g., Sundaram, 1996; Dixit, 1990).

Of course, we can admit unlimited labor inputs (resources in general) when firms need not care about the limits of available resources and those resource constraints are not critical. Then, we can have the usual traditional optimal conditions for the marginal productivity hypothesis (i.e., the marginal cost hypothesis) with full employment *without* any excuses, even under the constant returns to scale production function. However, even in this case, the absolute employed production factor levels should be indeterminate.

With or without resource constraints, under the profit maximization hypothesis, idle resources should be generally assumed to maintain the marginal productivity hypothesis. However, we do not exclude the fortunate probability that the determined absolute employment level is equal to the existing fully available resource levels by chance as long as the resource constraints are *not* a problem (see also the additional benevolent condition in the later discussion). Then, under the *finite* resources, that probability *must* be zero (impossible).

Resource constraints are *inevitable* when considering the *aggregate* problem in the *macroeconomy*, such as ours in this paper. *Macroeconomics* should treat the *limit* as well as the *scarcity* of available resources. In the later section, we will consider and ensure that the above discussion due to the mathematics (i.e., *sales revenue* maximization hypothesis with the *Karush-Kuhn-Tucker approach*) should be *correct*, with (verbally and mathematically) arguing the indeterminacy of labor input levels under the constant returns to scale property of the production functions under the profit maximization hypothesis. Because we use the Karush-Kuhn-Tucker condition, we can easily prove the existence of the unique equilibrium in this sales revenue maximization hypothesis suggested by Baumol (1959) (e.g., Masuda, 2022, for the proof of this kind of model with the Karush-Kuhn-Tucker condition).

We do not try such proof in this paper. Note that in the above maximization problem, we assume the nonlinearity for the objective function due to the increasing returns to scale of the production function of the tradable goods industry for skilled workers. In some sections of this paper, we safely assume the complete linear objective function (firm's profit) with the constant returns to scale production function for skilled workers in the tradable goods industry.

In such a case, we can get the boundary optimum from the particular property of the linear objective function (without the nonzero second-order derivative) with linear programming (e.g., Dorfman et al., 1958/1987). Thus, recall that this kind of linear programming is connected to activity analysis of production and allocation in business administration.

Anyway, the sum of each nominal output equals the sum of each wage fund, which the zero profit condition suggests (see the previous distribution law equations). Any introductory microeconomics textbooks explain the average productivity hypothesis for the increasing returns to scale production function case (e.g., Binger & Hoffman, 1998).

The Further Revival of the Average Productivity Hypothesis and Its Relation to Our New (Un)employment Theory

As for the relationship between the profit maximization hypothesis and the sales revenue maximization hypothesis, we should go further with respect to this point. Previously, we have shown that the *condition*, $P_{TU,t}A_{T,t} \neq P_{N,t}A_{N,t}$, must be satisfied for the establishment of the sales revenue maximization hypothesis. Furthermore, to investigate the establishment of this condition thoroughly, we obtain the following critical inference.

Seeing the previous distribution law equation under the sale revenue maximization hypothesis, the *zero profit* condition forces workers in the tradable goods industry for unskilled workers and the nontradable goods industry to accept the *average* wage hypothesis. That is, such workers should enjoy a real wage equal to their *average* productivity. Though they enjoy *different* real wages, as shown in the above condition, on average, they should enjoy the real wage that the marginal productivity is equivalent to. Seeing the individual worker's real wages, the marginal productivity hypothesis *does not* work under the sales revenue maximization hypothesis.

Then, does this mean that some workers constantly enjoy the real wage surplus while others always enjoy the real wage deficit, compared with their productivity? The answer is *no*. The reason is presented in the following.

Recall that the workers suffering from this real wage deficit decided on labor participation because the *expected* real wage rate is more significant than their real *reservation* wages (see Appendix C and Our Labor Force Participation Theory section). However, their real wages for current jobs are insufficient to satisfy their productivity; some workers might want to change their jobs. We should be aware that the turnovers, the dynamic job search explained in Appendix C, must exist behind this *average* (real) *wage* hypothesis. Our new (un)employment theory represents such a worker's choice.

Moreover, the *aggregate preference* optimization problem in Appendix D *fixes* the demand structure for the consumption of such industries, that is, the outputs. Thus, the tradable or nontradable goods industry that provides the *lower* real wage should *not* be forced to exit the country in question. The demand structure of the whole country suggests such *insurance* for these industries.

Then, first, we can assume that their real wages are averaged *intertemporally* using their turnovers, as they dislike the real wage deficit and like the real wage surplus, such that they always search for the more payable jobs in the labor market, like the following *human capital* theory suggests (Becker, 1975/1976).

Note that our theory based on the average productivity hypothesis is *not* the first example of violating the marginal productivity hypothesis (recall Keynes' [1936/1997] denial of the first postulate of the classical theory of employment). As noted, the human capital theory proposed by Becker (1975/1976) suggests that workers cannot receive the corresponding real wages to their productivity, particularly in their youth. His human capital theory similarly assumes *intertemporal rationality* as our one example (we have other examples like *monopoly* and *oligopoly* for this violation due to the existence of the *markup* terms, although we ignore them here).

Intertemporal rationality means that workers should consider the discounted sum of real incomes from the current job. In short, this means that the discounted sum of real income from all the job hoppings in their lifetime should be at least equal to their discounted sum of productivity in their lifetime, different from the Japanese seniority wage (income) system (e.g., Doepke & Gaetani, 2024, for the ineffectiveness of human capital theory to Japamese seniority wage system).

Furthermore, our workers maximize their incomes to satisfy their real reservation incomes, and the market structure of required productivity for each industry (i.e., the output share of each industry in the macroeconomy) is stochastically changing (see the following paragraph). As a result, incorporating all the expected turnovers and changing labor market structures in their lifetime at each decision point into consideration is impossible for us as well as our workers.

Thus, on a temporal average or aggregate basis, our average productivity hypothesis should hold like the market equilibrium condition in Appendix D, and the individual expected intertemporal real income balance is out of our theory from the beginning, different from the human capital theory proposed by Becker (1975/1976). Notwithstanding, this inference must be justified by the special property of our new (un)employment theory that the worker's real reservation incomes are ensured to be satisfied at any time when they get any job, even in such a case.

This means that the *individual rationality* for labor market participation for those workers is at least satisfied. However, the individual remaining discounted sum of real wage surplus and real wage deficit from their jobs in their lifetime is not an aggregate (or a representative agent) problem. This regressive asset accumulation based on the unbalanced real wage relations to their productivity and the resulting regressive real wage growth should be measured with (approximated) Sen's (1976) poverty measure for the future redistribution policies, as proposed in Appendix A. That is our standpoint in this paper.

Thus, our model in this section seemingly defines the static equilibrium. However, interpreting its solution should require us to consider the dynamic structure behind this model implicitly, specifically under the sales revenue maximization hypothesis.

Of course, we can assume the following for the individual reason to accept such a real wage deficit: labor productivity is not visible to the workers (and firm leaders and managers). In this case, the real wage differential in the tradable goods industry for unskilled workers and the nontradable goods industry might be fixed for these workers because they are *unaware* of severe *deviations* of these real wages from their productivity. This alternate hypothesis accepts the fixed wage deficit, unlike the previous one. However, this hypothesis should further assume the additional formula of wage (income) determination for our new (un)employment theory, like the learning process or the expectation equations for finding their true productivity. Thus, this paper does not choose this alternative hypothesis.

As we have already argued in this section, the real wage differential is easily *changeable* for unskilled workers in the tradable goods industry and workers in the nontradable goods industry, as the above condition and our hypothesis allow. So, we can assume that sometimes the real wage of one industry is more significant than that of the other and vice versa stochastically on other occasions.

The Utility of Solution Set Proposed by von Neumann and Morgenstern (1953/2009)

Based on the results of previous sections, we have a *solution set* based on the sales revenue maximization and profit maximization hypotheses. Still, remember that we should determine the absolute output level for firms in terms of social welfare (see the discussion in the later section for rent-seeking behaviors). From that point, we should exclude the possibility of the *unique* equilibrium under the profit maximization hypothesis only with our *intuitive* aggregate resource constraints in our game.

Instead, from our above intuition, the profit maximization hypothesis can also be justified to establish *equilibria* without satisfying the aggregate resource constraints (or unique to its capital-labor ratio in the model with capital case). As a result, our model in this paper has multiple equilibria that correspond to the firm's objectives. This is because the objectives *differ*, including the rent-seeking objective of leaders and managers in the firms explained later.

However, remember that both hypotheses can be specified as the *constrained* maximization problem with the general functional form for the objective functions and the *changing* number of the *binding* constraints. Thus, if we describe these objective functions and constraints as the choices in some stage games, we can have a *solution set* as we explained above, *similar* to the solution set concept of von Neumann and Morgenstern (1953/2009). The single solution is ruled out from this solution set by the economic intuition for leaders' and managers' objectives like the sales revenue maximization hypothesis, the profit maximization hypothesis, the rent-seeking behaviors, and the benevolent (right) motives responding to the founding purposes of the firms and so on (e.g., Keynes, 1926; Tirole, 1988), as von Neumann and Morgenstern (1953/2009) suggested.

Naturally, that is due to our above setting, which should not allow the *mixed strategy* Nash equilibrium (Gibbons, 1992/1995; Nash, 1950a, 1951; Osborne &

Rubinstein, 1994). The sales revenue and profit maximization objectives should be considered quite different ideas adapted to each firm's age from its foundation and dominate the course of actions of the firm's leaders and managers forming their company cultures and so on (e.g., Keynes, 1926).

Again, in this case, we can form a kind of game that includes the first stage when players (firms) can choose their maximizing objectives for their utility functions, and the second stage when they choose the constraints. Thus, such *rational behaviors* could result in forming a *solution set*, Ω_t; this solution set, Ω_t, is defined as that the solution set, Ω_t, consists of all the elements that are not dominated by elements in Ω_t (von Neumann & Morgenstern, 1953/2009, for further discussions).

The Effect of the Normalization of the Price of Tradable Goods by Unskilled Workers

The general price level is the weighted average of the prices of tradable goods produced by skilled and unskilled workers and the price of nontradable goods. However, the price of tradable goods produced by unskilled workers is standardized to unity using its numeraire assumption in this paper. We also assume that the existence of international trade markets makes the law of one price hold for the prices of tradable goods produced by skilled and unskilled workers for both home and foreign countries, adjusting with the exchange rate.

Therefore, $\frac{P_t}{P_{TU,t}}Y_t = Y_{TU,t} + \frac{P_{TS,t}}{P_{TU,t}}Y_{TS,t} + \frac{P_{N,t}}{P_{TU,t}}Y_{N,t}$ and this equation is redefined as $\tilde{P}_t Y_t = Y_{TU,t} + \tilde{P}_{TS,t}Y_{TS,t} + \tilde{P}_{N,t}Y_{N,t}$ where $\tilde{P}_t = \frac{P_t}{P_{TU,t}}$, $\tilde{P}_{TS,t} = \frac{P_{TS,t}}{P_{TU,t}}$, and $\tilde{P}_{N,t} = \frac{P_{N,t}}{P_{TU,t}}$. As a result, we obtain:

$$\frac{d\tilde{P}_t}{\tilde{P}_t} = \frac{Y_{TU,t}}{\tilde{P}_t Y_t}\frac{dv_{TU,t}}{v_{TU,t}} + \frac{\tilde{P}_{TS,t}Y_{TS,t}}{\tilde{P}_t Y_t}\left(\frac{d\tilde{P}_{TS,t}}{\tilde{P}_{TS,t}} + \frac{dv_{TS,t}}{v_{TS,t}}\right) + \frac{\tilde{P}_{N,t}Y_{N,t}}{\tilde{P}_t Y_t}\left(\frac{d\tilde{P}_{N,t}}{\tilde{P}_{N,t}} + \frac{dv_{N,t}}{v_{N,t}}\right),$$

after taking the (natural) logarithm of both sides and differentiating by the time, t. Note that this is essentially the same as:

$$\frac{dP_t}{P_t} = \frac{P_{TU,t}Y_{TU,t}}{P_t Y_t}\left(\frac{dv_{TU,t}}{v_{TU,t}} + \frac{dP_{TU,t}}{P_{TU,t}}\right) + \frac{P_{TS,t}Y_{TS,t}}{P_t Y_t}\left(\frac{dv_{TS,t}}{v_{TS,t}} + \frac{dP_{TS,t}}{P_{TS,t}}\right) + \frac{P_{N,t}Y_{N,t}}{P_t Y_t}\left(\frac{dv_{N,t}}{v_{N,t}} + \frac{dP_{N,t}}{P_{N,t}}\right).$$

Also, the above unity assumption means that $dP_{TU,t}/P_{TU,t}$ is null.

Suppose that the (natural) logarithm of both sides of the distribution law equation derived from the profit maximization problem of the firm is taken and differentiated with respect to the time, t. Again, suppose that the price of tradable goods by unskilled workers is standardized to be *unity* for normalizing the other prices as the numeraire at any period t. Then, we obtain $\tilde{P}_{TS,t} = \frac{P_{TS,t}}{P_{TU,t}}$, $\tilde{P}_{N,t} = \frac{P_{N,t}}{P_{TU,t}}$, and $\widetilde{W}_t = \frac{W_t}{P_{TU,t}}$. Furthermore, the following equation is obtained:

$$\frac{d\widetilde{W}_t}{\widetilde{W}_t} = \frac{dA_{T,t}}{A_{T,t}} - \left(\frac{dL_{TU,t}}{L_{TU,t}} - \frac{dF_{TU,t}}{F_{TU,t}}\right),$$

$$\frac{d\tilde{P}_{TS,t}}{\tilde{P}_{TS,t}} + \frac{dA_{T,t}}{A_{T,t}} = \frac{d\widetilde{W}_t}{\widetilde{W}_t} + \frac{dL_{TS,t}}{L_{TS,t}} - \frac{dF_{TS,t}}{F_{TS,t}},$$

$$\frac{d\tilde{P}_{N,t}}{\tilde{P}_{N,t}} + \frac{dA_{N,t}}{A_{N,t}} = \frac{d\widetilde{W}_t}{\widetilde{W}_t} + \frac{dL_{N,t}}{L_{N,t}} - \frac{dF_{N,t}}{F_{N,t}},$$

Then, we obtain the relations (#2):

$$\frac{d\tilde{P}_{TS,t}}{\tilde{P}_{TS,t}} = \frac{dL_{TS,t}}{L_{TS,t}} - \frac{dF_{TS,t}}{F_{TS,t}} - \left(\frac{dL_{TU,t}}{L_{TU,t}} - \frac{dF_{TU,t}}{F_{TU,t}}\right),$$

$$\frac{d\tilde{P}_{N,t}}{\tilde{P}_{N,t}} = \frac{dA_{T,t}}{A_{T,t}} - \frac{dA_{N,t}}{A_{N,t}} - \left(\frac{dL_{TU,t}}{L_{TU,t}} - \frac{dF_{TU,t}}{F_{TU,t}}\right) + \frac{dL_{N,t}}{L_{N,t}} - \frac{dF_{N,t}}{F_{N,t}}.$$

In addition, the above result for $d\tilde{P}_t/\tilde{P}_t$ is essentially the same as the above dP_t/P_t equation as before, and $\frac{M_tV_t}{P_{TU,t}} = \widetilde{M}_tV_t = \tilde{P}_tY_t = \frac{P_tY_t}{P_{TU,t}}$ where $\widetilde{M}_t = \frac{M_t}{P_{TU,t}}$ according to the quantity theory of money, $M_tV_t = P_tY_t = P_{TU,t}Y_{TU,t} + P_{TS,t}Y_{TS,t} + P_{N,t}Y_{N,t}$. Furthermore, this quantity theory of money equation has the same dynamics in case of $d\tilde{P}_t/\tilde{P}_t$ as that of dP_t/P_t by only replacing prices with prices in tilde even when this equation is differentiated with respect to the time, t. Our assumption that $\frac{dP_{TU,t}}{P_{TU,t}} = 0$ is easily obtained by this mathematical operation.

Also, the formulae in the previous sections should be kept the same for our policy rule based on the quantity theory of money by this mathematical operation. Moreover, because the uniform price $P_{TU,t} = 1$ is always assumed also in the policy rule specified by our combined quantity theory of money with the Harrod–Balassa–Samuelson effect model in the next section of this paper, the fact that this assumption holds for both home and foreign countries, is not explicitly mentioned hereafter.

The Derivation of the Harrod–Balassa–Samuelson Effect

Using the equation (#1), (#2) and the relations, $\frac{dF_{TU,t}}{F_{TU,t}} = \frac{dL_{TU,t}}{L_{TU,t}}$, $\frac{dF_{TS,t}}{F_{TS,t}} = (1 + \gamma)\frac{dL_{TS,t}}{L_{TS,t}}$, and $\frac{dF_{N,t}}{F_{N,t}} = \frac{dL_{N,t}}{L_{N,t}}$, and so on as we explained, the additional equation:

$$\frac{d\tilde{P}_t}{\tilde{P}_t} = \frac{Y_{N,t}}{Y_t}\left(\frac{dA_{T,t}}{A_{T,t}} - \frac{dA_{N,t}}{A_{N,t}}\right) - \left(\frac{Y_{TU,t}}{Y_t} - \frac{Y_{TU,t}}{\tilde{P}_t Y_t}\right)\frac{dL_{TU,t}}{L_{TU,t}} - \left[\frac{Y_{TS,t}}{Y_t}(1+\gamma) - \frac{\tilde{P}_{TS,t}Y_{TS,t}}{\tilde{P}_t Y_t}\right]\frac{dL_{TS,t}}{L_{TS,t}} - \left(\frac{Y_{N,t}}{Y_t} - \frac{\tilde{P}_{N,t}Y_{N,t}}{\tilde{P}_t Y_t}\right)\frac{dL_{N,t}}{L_{N,t}},$$

is obtained. Then, we will first show our model with the traditional competitive equilibrium case under the profit maximization hypothesis. This is because this traditional case is standard and familiar to most economists. The following will show our new case under the full employment under the sales revenue maximization hypothesis after that traditional case.

Note that the relations, $\frac{dF_{TU,t}}{F_{TU,t}} = \frac{dL_{TU,t}}{L_{TU,t}}$ and $\frac{dF_{N,t}}{F_{N,t}} = \frac{dL_{N,t}}{L_{N,t}}$, are also derived from the following equation:

$$\frac{dF_{l,t}}{F_{l,t}} = \frac{d\ln(F_{l,t})}{dt} = \frac{d\ln(F_{l,t})}{dF_{l,t}}\frac{dF_{l,t}}{dL_{l,t}}\frac{dL_{l,t}}{dt} = \frac{L_{l,t}}{F_{l,t}}\frac{dF_{l,t}}{dL_{l,t}}\frac{dL_{l,t}}{L_{l,t}},$$

where ln means the natural logarithm for $l = TU, TS, N$.

To get the differentiation, $\frac{dF_{TS,t}}{F_{TS,t}} = (1+\gamma)\frac{dL_{TS,t}}{L_{TS,t}}$, we use the fact that $F(L_{TS,t}) = L_{TS,t}^{1+\gamma}$, together with the above relational equation.

Here, if $F_{l,t}$ is assumed to be a linear function of $L_{l,t}$ for $l = TU, TS, N$, then the right-hand side becomes $dL_{l,t}/L_{l,t}$ for $l = TU, TS, N$. This result applies to both the tradable goods industry and nontradable goods industry, provided the natural conditions where the only input is labor. This result is also independent of the distinction between home and foreign countries.

The Traditional Competitive Equilibrium Case (The Profit Maximization Hypothesis). Because the same is true for foreign countries, the foreign variables can be expressed by putting an asterisk (*) on their right shoulders, and we obtain:

$$\frac{d\tilde{P}_t}{\tilde{P}_t} - \frac{d\tilde{P}_t^*}{\tilde{P}_t^*} = \frac{Y_{N,t}}{Y_t}\left(\frac{dA_{T,t}}{A_{T,t}} - \frac{dA_{N,t}}{A_{N,t}}\right) - \left(\frac{Y_{TU,t}}{Y_t} - \frac{Y_{TU,t}}{\tilde{P}_t Y_t}\right)\frac{dL_{TU,t}}{L_{TU,t}} - \left[\frac{Y_{TS,t}}{Y_t}(1+\gamma) - \frac{\tilde{P}_{TS,t}Y_{TS,t}}{\tilde{P}_t Y_t}\right]\frac{dL_{TS,t}}{L_{TS,t}} - \left(\frac{Y_{N,t}}{Y_t} - \frac{\tilde{P}_{N,t}Y_{N,t}}{\tilde{P}_t Y_t}\right)\frac{dL_{N,t}}{L_{N,t}} - \left\{\frac{Y_{N,t}^*}{Y_t^*}\left(\frac{dA_{T,t}^*}{A_{T,t}^*} - \frac{dA_{N,t}^*}{A_{N,t}^*}\right) - \left[\frac{Y_{TS,t}^*}{Y_t^*}(1+\gamma^*) - \frac{\tilde{P}_{TS,t}^*Y_{TS,t}^*}{\tilde{P}_t^* Y_t^*}\right]\frac{dL_{TS,t}^*}{L_{TS,t}^*} - \left(\frac{Y_{TU,t}^*}{Y_t^*} - \frac{Y_{TU,t}^*}{\tilde{P}_t^* Y_t^*}\right)\frac{dL_{TU,t}^*}{L_{TU,t}^*} - \left(\frac{Y_{N,t}^*}{Y_t^*} - \frac{\tilde{P}_{N,t}^*Y_{N,t}^*}{\tilde{P}_t^* Y_t^*}\right)\frac{dL_{N,t}^*}{L_{N,t}^*}\right\}.$$

In the above, $1+\gamma^*$ where $\gamma^* > 0$ means the degree of increasing returns to scale of the foreign production function. This result indicates that the exchange rate is affected by the so-called Harrod–Balassa–Samuelson effect, which refers to the difference in relative technological progress between the home and foreign tradable and nontradable goods industries. The model in this paper follows a similar derivation to that of Obstfeld and Rogoff (1997) and has the same implications, although the details of the model differ.

For example, the higher home technological progress of the tradable goods industry depreciates the exchange rate, while the higher home technological progress of the nontradable goods industry appreciates it. The lower foreign ones have the same effects. Moreover, the lower home and higher foreign ones have the inverse effects of the above higher home ones. Regarding the effects of technological progress of different industries, in the later section, we change our focus to the industrial structure and domestic balance of technological progress in this model.

From our results in the Impact of Unbalanced Development of Industries section, saying our result in advance, the effect of each technological progress on the industrial structure in favor of each industry should be determined by the increasing share of the real output of the tradable goods industry to the total (in the long run). This is so in our version of the Harrod–Balassa–Samuelson effect model, which is similar to the quantity theory of money case.

In other words, the technological progress of the nontradable goods industry will weaken the effect of that technological progress on it by decreasing the share of the nontradable goods industry. Moreover, the technological progress of the tradable goods industry should strengthen this effect in favor of the tradable goods industry by increasing the share of the tradable goods industry. Both effects depend on their path dependency characteristics (e.g., Ljungqvist & Sargent, 2000, for this path dependency discussion).

The Full Employment Equilibrium Case (The Sales Revenue Maximization Hypothesis). The second case is for the sales revenue maximization hypothesis case, that is, the full employment case. This is different from the above traditional case. First, we differentiate the distribution law equation under the sales revenue maximization hypothesis with respect to the time, t and use the relations, $\frac{dF_{TU,t}}{F_{TU,t}} = \frac{dL_{TU,t}}{L_{TU,t}}$, $\frac{dF_{TS,t}}{F_{TS,t}} = (1+\gamma)\frac{dL_{TS,t}}{L_{TS,t}}$, and $\frac{dF_{N,t}}{F_{N,t}} = \frac{dL_{N,t}}{L_{N,t}}$, we obtain:

$$\frac{d\tilde{P}_{TS,t}}{\tilde{P}_{TS,t}} + \frac{dA_{T,t}}{A_{T,t}} + \gamma\frac{dL_{TS,t}}{L_{TS,t}} = \frac{d\widetilde{W}_t}{\widetilde{W}_t},$$

$$\frac{A_{T,t}}{Y_{TU,t}+\tilde{P}_{N,t}Y_{N,t}}\frac{dA_{T,t}}{A_{T,t}} + \frac{\tilde{P}_{N,t}}{Y_{TU,t}+\tilde{P}_{N,t}Y_{N,t}}\frac{d\tilde{P}_{N,t}}{\tilde{P}_{N,t}} + \frac{A_{N,t}}{Y_{TU,t}+\tilde{P}_{N,t}Y_{N,t}}\frac{dA_{N,t}}{A_{N,t}}$$

$$= \frac{d\widetilde{W}_t}{\widetilde{W}_t} + \left(\frac{L_{TU,t}}{L_{1,t}} - \frac{L_{N,t}}{Y_{TU,t}+\tilde{P}_{N,t}Y_{N,t}}\right)\frac{dL_{TU,t}}{L_{TU,t}} + \left(\frac{L_{N,t}}{L_{1,t}} - \frac{L_{N,t}}{Y_{TU,t}+\tilde{P}_{N,t}Y_{N,t}}\right)\frac{dL_{N,t}}{L_{N,t}}.$$

The first line is the same as the above traditional competitive equilibrium case. The second line is original.

From the above first-line equation, we infer that the rise in the technological progress of the tradable goods industry will raise the nominal wage, other things being equal. Then, from the second-line equation, these rises should balance this equation with the rise in the price of the nontradable goods industry and/or the rise in the technological progress of the nontradable goods industry. This is the favorable inflation-technological progress correlation induced by *autonomous* technological progress.

Then, other things being equal, consider the rise in the price of the nontradable goods industry in the second-line equation: The *inflation* case. This will induce the nominal wage rise, and the first-line equation increases the technological progress of the tradable goods industry due to this nominal wage rise, with other things being equal.

This further technological progress will connect to the above autonomous technological progress in the first line equation case. This is the *long-run* case.

On the other hand, instead in the case above, we can assume the *constant* nominal wage in the *short-run* case. However, this case should bring about technological *regress* and result in the *inflation* of the tradable goods produced by skilled workers. This case will bring a *real* wage reduction and is not suitable for the country under the sales revenue maximization hypothesis inducing the *high-pressure* economy (e.g., Samuelson & Solow, 1960). This case represents the traditional one that specifies inflation as *harmful*, like in the classical state.

Moreover, consider the *decline* in the price of the nontradable goods industry in the second-line equation: The *deflation* case. Then, from the above second-line equation, we infer that the other things being equal, this decline in the nontradable goods price induces technological progress in the tradable goods industry, the nontradable goods industry, or both, assuming the *constant* nominal wage as in the *short run*. Then, suppose that the technological progress in the tradable goods industry occurs in the second-line equation. In that case, this leads to the deflation of the prices of the tradable goods produced by skilled workers and the resulting rise in the real wages, or the above successive favorable inference with the autonomous technological progress of the tradable goods industry in the first-line equation, if we admit the change in the nominal wage for the latter.

Furthermore, as the third case, we can think that the nominal wage is assumed not to be constant but to decline with the decreasing price of the nontradable goods industry in the second-line equation as in the *long run*. Then, the first-line equation tells us that the decline in the nominal wage induces the technological regress of the tradable goods industry and lures the fearful *deflationary spiral* from the above successive *inverse* inference for the autonomous technological progress of the tradable goods industry in the first-line equation, other things being equal (that is, $\tilde{P}_{TS,t}$ is also constant).

Thus, the assumption of the *constant* nominal wage is the key. This allows us to think that the time perspective is crucial, like the short run or the long run, when the nominal wage is constant or not. In short, considering the path dependency of the above differential equation version distribution law equation (e.g., Ljungqvist & Sargent, 2000, for this path dependency discussion), *only* under the *full employment*, *deflations* in the *short run* will do good for the macroeconomy. However, its positive effect will not remain in the *long run*. In this point, the insight of Friedman (1969/2006), that is, the optimality of deflations, should be respected (e.g., Ljungqvist & Sargent, 2000).

Then, even by combining the previous general price equation with the above equation, we cannot obtain the general price equation without the other price variables. Therefore, such an equation is not solved for the general price level, and we cannot obtain the traditional Harrod–Balassa–Samuelson effect model only in real terms (i.e., without the price variable) in this case.

Instead, we substitute $d\widetilde{W}_t/\widetilde{W}_t$ by combining these distribution law equations and solve for $d\tilde{P}_{TS,t}/\tilde{P}_{TS,t}$. We obtain:

$$\frac{d\tilde{P}_{TS,t}}{\tilde{P}_{TS,t}} = \frac{\tilde{P}_{N,t}}{Y_{TU,t}+\tilde{P}_{N,t}Y_{N,t}}\frac{d\tilde{P}_{N,t}}{\tilde{P}_{N,t}} + \left(\frac{A_{T,t}}{Y_{TU,t}+\tilde{P}_{N,t}Y_{N,t}} - 1\right)\frac{dA_{T,t}}{A_{T,t}} + \frac{A_{N,t}}{Y_{TU,t}+\tilde{P}_{N,t}Y_{N,t}}\frac{dA_{N,t}}{A_{N,t}} + \left(\frac{L_{TU,t}}{Y_{TU,t}+\tilde{P}_{N,t}Y_{N,t}} - \frac{L_{TU,t}}{L_{1,t}}\right)\frac{dL_{TU,t}}{L_{TU,t}} + \left(\frac{L_{N,t}}{Y_{TU,t}+\tilde{P}_{N,t}Y_{N,t}} - \frac{L_{N,t}}{L_{1,t}}\right)\frac{dL_{N,t}}{L_{N,t}} - \gamma\frac{dL_{TS,t}}{L_{TS,t}}.$$

Representing this combined distribution law equation referring to the definition of the general price level, we obtain the following:

$$\frac{d\tilde{P}_{N,t}}{\tilde{P}_{N,t}} = \frac{\tilde{P}_tY_t}{\tilde{P}_{TS,t}Y_{TS,t}}\frac{Y_{TU,t}+\tilde{P}_{N,t}Y_{N,t}}{\tilde{P}_{N,t}}\left(\frac{d\tilde{P}_{T,t}}{\tilde{P}_{T,t}} - \left[\frac{Y_{TU,t}}{\tilde{P}_tY_t}\frac{Y_{N,t}}{Y_t} + \frac{\tilde{P}_{TS,t}Y_{TS,t}}{\tilde{P}_tY_t}\left(\frac{A_{T,t}}{Y_{TU,t}+\tilde{P}_{N,t}Y_{N,t}} - 1 + \frac{Y_{N,t}}{Y_t}\right)\right]\frac{dA_{T,t}}{A_{T,t}} - \left[\frac{\tilde{P}_{TS,t}Y_{TS,t}}{\tilde{P}_tY_t}\frac{A_{N,t}}{Y_{TU,t}+\tilde{P}_{N,t}Y_{N,t}} - \left(\frac{Y_{TU,t}}{\tilde{P}_tY_t} + \frac{\tilde{P}_{TS,t}Y_{TS,t}}{\tilde{P}_tY_t}\right)\frac{Y_{N,t}}{Y_t}\right]\frac{dA_{N,t}}{A_{N,t}} - \left[\frac{Y_{TU,t}}{\tilde{P}_tY_t}\left(1 - \frac{Y_{TU,t}}{Y_t}\right) + \frac{\tilde{P}_{TS,t}Y_{TS,t}}{\tilde{P}_tY_t}\frac{L_{TU,t}}{L_{1,t}} - \frac{\tilde{P}_{TS,t}Y_{TS,t}}{\tilde{P}_tY_t}\frac{Y_{TU,t}}{Y_t} + \frac{\tilde{P}_{TS,t}Y_{TS,t}}{\tilde{P}_tY_t}\frac{L_{TU,t}}{Y_{TU,t}+\tilde{P}_{N,t}Y_{N,t}}\right]\frac{dL_{TU,t}}{L_{TU,t}} - \left[\frac{\tilde{P}_{TS,t}Y_{TS,t}}{\tilde{P}_tY_t}\left(\frac{L_{N,t}}{Y_{TU,t}+\tilde{P}_{N,t}Y_{N,t}} - \frac{Y_{N,t}}{Y_t}\right) + \frac{\tilde{P}_{TS,t}Y_{TS,t}}{\tilde{P}_tY_t}\frac{L_{N,t}}{L_{1,t}} - \frac{Y_{TU,t}}{\tilde{P}_tY_t}\frac{Y_{N,t}}{Y_t}\right]\frac{dL_{N,t}}{L_{N,t}} + \left\{\frac{Y_{TU,t}}{\tilde{P}_tY_t}\frac{Y_{TS,t}}{Y_t} + \frac{\tilde{P}_{TS,t}Y_{TS,t}}{\tilde{P}_tY_t}\left[\frac{Y_{TS,t}}{Y_t}(1+\gamma) - 1\right]\right\}\frac{dL_{TS,t}}{L_{TS,t}}\right),$$

where $\frac{d\tilde{P}_{T,t}}{\tilde{P}_{T,t}} = \frac{Y_{TU,t}}{\tilde{P}_tY_t}\frac{dv_{TU,t}}{v_{TU,t}} + \frac{\tilde{P}_{TS,t}Y_{TS,t}}{\tilde{P}_tY_t}\left(\frac{d\tilde{P}_{TS,t}}{\tilde{P}_{TS,t}} + \frac{dv_{TS,t}}{v_{TS,t}}\right).$

We find that the price of the nontradable goods industry is a function of the tradable goods industry price, $\tilde{P}_{T,t}$, which obeys the law of *one price* in the general price level, and other real variables. Combining with the foreign one, the price differentials between the home and foreign countries, represented by the home and foreign prices of the nontradable goods industry, must be determined by the prices holding for the law of one price.

Suppose that $\frac{Y_{TU,t}}{\tilde{P}_t Y_t}\frac{Y_{N,t}}{Y_t}+\frac{\tilde{P}_{TS,t}Y_{TS,t}}{\tilde{P}_t Y_t}\left(\frac{A_{T,t}}{Y_{TU,t}+\tilde{P}_{N,t}Y_{N,t}}-1+\frac{Y_{N,t}}{Y_t}\right)>0$ (recall that it is quite probable). In that case, technological progress in the tradable goods industry decreases the price of the nontradable goods industry, that is, the price differential equation between the home and foreign countries while technological progress in the nontradable goods industry increases this price. In the above, we assume that the term $\frac{\tilde{P}_{TS,t}Y_{TS,t}}{\tilde{P}_t Y_t}\frac{A_{N,t}}{Y_{TU,t}+\tilde{P}_{N,t}Y_{N,t}}-\left(\frac{Y_{TU,t}}{\tilde{P}_t Y_t}+\frac{\tilde{P}_{TS,t}Y_{TS,t}}{\tilde{P}_t Y_t}\right)\frac{Y_{N,t}}{Y_t}$ should be negative and it is also quite probable. Thus, we can construct the other new monetary policy rule to keep these price differentials between the home and foreign countries constant.

However, this paper does not focus on constructing this price differential equation as in the above. This is because such an attempt will bring just the complicated result that is inversely but not essentially different from those in this previous traditional competitive equilibrium case. Above all, we should have ample empirical evidence that makes us believe that the previous traditional competitive equilibrium case should be dominant in reality. Therefore, hereafter, we do not extend this version and mostly focus only on the traditional competitive equilibrium case. In short, we focus on the case with not full employment but unemployment, that is, slack in the macroeconomy, which depends on the forward-looking behaviors of the leaders and managers in firms like that, as we will explain later.

The Mixed Case With the Full Employment Equilibrium and the Traditional Competitive Equilibrium Case. Furthermore, note that the above price differential equation must only hold under the full employment equilibrium case. Thus, we should care that the above case does not treat the mixed case between the countries with full employment equilibrium and traditional competitive equilibrium. For that, we should modify the traditional competitive equilibrium case. To see the price differential equation with the mixed case, this modification should make the price equation for the tradable goods produced by skilled workers (the equation [#7] in the following section) in the tradable goods industry to exclude and other remaining parts for the numeraire price that should be ignorable to add.

Now, we will briefly explain the mixed case. For simplicity, we only focus on the effect of technological progress on the price differential equation. For example, in this mixed case, the price differential between the home country in the full employment equilibrium and the foreign country in the traditional competitive equilibrium should be enlarged when the technological progress of the home and foreign nontradable goods industries rises faster than the other. On the other hand, it should be shrunk when those of the home and foreign tradable goods industries rise faster than the other, and so on.

Furthermore, suppose the inverse case that the home country is under the traditional competitive equilibrium and the foreign country is under the full employment equilibrium. In that case, the above results should be applied only inversely. However, these are easy; we will leave their details to the readers.

Note that even in this theory, the exchange rate can be assumed to be *nominal* because we treat the difference in the growth rates of two countries' general price levels (or two countries' prices of the nontradable goods industries) as nominal variables. This is the same application of PPP, similar to previous studies' applications of the quantity theory of money as the exchange rate model, as Isard (1995/2001) noted.

Thus, we combine the real world with money as a *veil*, like the (traditional or inverse price differential version) Harrod–Balassa–Samuelson effect model, with the nominal world, like the quantity theory of money through the common (nominal) variables, the general price levels. This is one of the main concepts in this paper. This assumption is (even empirically) justified by normalizing the nominal variables unity in the initial equilibrium where the real and nominal worlds are combined without the loss of generality.

The difference between the result in the previous section, The Equation for Determining the Exchange Rate Based on the Quantity Theory of Money section, and that in this section is that the effect of changes in technological progress in the tradable (or nontradable goods) goods industries of the home country and foreign country on the exchange rate is *opposite*.

The Impact of Unbalanced Development of Industries

Our Harrod–Balassa–Samuelson effect model under the profit maximization hypothesis indicates an entirely different effect of technological progress and asymmetry from the previous one. Suppose that the technological progress in the tradable goods industry is faster than in the nontradable goods industry, although the amounts of labor inputs do not change. In this case, the industrial structure of the home country becomes biased in favor of the tradable goods industry over time with the more rapid increase in its share, similar to the former quantity theory of money case.

However, the equation (#2) and the Harrod–Balassa–Samuelson effect model as its application suggests that any technological progress of the nontradable goods industry should accompany the *simultaneous* technological progress of the tradable goods industry without the short-run case, other things being equal. Because, in that case, the nominal wage (i.e., the technological progress of the tradable goods industry) *cannot* change abruptly due to its discrete (frequent) revisions of their nominal income (wage) contract as the *basis* of the non-vertical Phillips curve (e.g., Masuda, 2024b; Taylor 1979, 1980; Walsh, 1998). This is irrelevant to our above conclusions for the biased technological progress, similar to what we have seen in the previous section about the quantity theory of money.

Furthermore, the above results regarding the industrial structure are not independent of each country's demand structure (see Masuda, 2024a, and Appendix D for the effect of demands in our discussions in this paper; Obstfeld & Rogoff, 1997). Thus, in some general equilibrium settings, we should be afraid that our conclusion for the industry evolution generated by each technological progress in this section will become ambiguous. Actually, the direction of industrial structural changes caused by biased technological progress in this general equilibrium case will be uncertain.

The implication of this section for these changes in technological progress is that if the Harrod–Balassa–Samuelson effect is present, changes in the technological progress of different industries significantly affect the exchange rate differently. For example, when technological progress in the *home nontradable* goods industry is more substantial than in the home tradable goods industry, the exchange rate (relative price) *appreciates*. On the other hand, when technological progress in the *home tradable* goods industry is more substantial than that in the home nontradable goods industry, the exchange rate (relative price) *depreciates*.

The above discussion concerns the relative technological progress between domestic industries. However, according to the quantity theory of money, the *general* technological progress of the entire sector that brings income growth always results in the exchange rate *appreciation*. This result is not affected by the relative difference in technological progress. Also, here, general technological progress means the positive direction of the whole technological progress, even caused by each domestic industry's *nonnegative* technological progress, which is weighted by its share.

Thus, suppose that the *aggregated* general technological progress brings the resultant exchange rate appreciation. In that case, its unbalanced industrial structure evolution tendency in favor of the particular one-sided technological progress should be recognized theoretically in advance, in the case of the Harrod–Balassa–Samuelson effect model. This favorable situation should not happen under the quantity theory of money case. However, the demand structure prevents its ever-continuing one-sided technological progress (recall the existence of the real consumption of tradable and nontradable goods).

Furthermore, again, even in the Harrod–Balassa–Samuelson effect model case, the directions of growth rates of shares of $Y_{TU,t}$, $Y_{TS,t}$, and $Y_{N,t}$ to the total output, Y_t, become uncertain in general equilibrium setting with the demand structure different from the above (see Appendix D). Unlike Baumol (1967), we do not assume some specific function for technological progress in our setting, and we also get general results even in this section.

Japan has suffered from a lower price level than usual or deflation. Together with the empirical evidence of its lower economic growth, this is because the productivity of the home tradable goods industry has been lower than that of the home nontradable goods industry, following this theory. Because the Harrod–Balassa–Samuelson effect model suggests that a country with higher productivity in the tradable goods industry than in the nontradable goods industry has a higher general price level or inflation than otherwise (e.g., Obstfeld & Rogoff, 1997).

Alternatively, note that if appreciations and deflations occur, the general price level of the home, that is, the technological progress of the home tradable goods industry, might be lower than that of the foreign country. Moreover, if other things are equal, the higher home technological progress of the nontradable goods industry than that of the foreign one should also cause deflation and appreciation. However, even in this latter case, the following pessimistic forward-looking behaviors should be applied (e.g., Cuba-Borda & Singh, 2024). Such a phenomenon has not been observed in Japan.

As we will see, rent-seeking behaviors in firms like Japan should generate negative output gaps and lower technological progress. This rent-seeking behavior case must occur only under the profit maximization hypothesis. Then, unemployment should be generated, and the negative real output gaps should also cause deflations or disinflations from the Phillips curve relation.

As discussed in the later section, such disinflation or deflation tendency should be continuously realized under the (self-fulfilling) pessimistic forward-looking expectations of the public (firms) for their sales and outputs (e.g., Cuba-Borda & Singh, 2024). Thus, the appreciation occurs even from the negative output gaps.

Moreover, the law of one price should be dominated even in the home country. Thus, the rent-seeking behaviors in the nontradable goods industry should cause the technological *regress* of both the tradable goods and the nontradable goods industries (and resulting price deflation of the nontradable goods industry in the long run) if the price of the nontradable goods industry is constant (or sticky) in the short run. This keeps the exchange rate *neutral* (or *appreciate*; see the equation [#5] or [#6]).

Unlike the above pessimistic case, deflations under the sales revenue maximization hypothesis are not always risky because the sales revenue maximization hypothesis ensures the active course of actions of leaders and managers in the firms that must not forgive them for receding in the whole economy, like what the pessimistic forward-looking expectations suggest (e.g., Cuba-Borda & Singh, 2024). However, the average productivity hypothesis might admit to the unequal incomes of workers in our macroeconomy under the sales revenue maximization hypothesis.

The Derivation of the Policy Rule for Monetary Policy: Integration of the Quantity Theory of Money and the Harrod–Balassa–Samuelson Effect

In this section, I discuss what monetary policy should be when each theory of exchange rates (relative prices), which was established separately in The Equation for Determining the Exchange Rate Based on the Quantity Theory of Money section and The Harrod–Balassa–Samuelson Effect Model With Skilled and Unskilled Workers section, is in equilibrium in both theories, as we repeatedly mentioned in the previous sections. In this section, we assume that the exchange rates establish both approaches. In other words, our threshold in this section is the equilibrium where the nominal and real exchange rates are equivalent.

From the results of The Equation for Determining the Exchange Rate Based on the Quantity Theory of Money section and The Harrod–Balassa–Samuelson Effect Model With Skilled and Unskilled Workers section, one can conclude that:

$$\frac{d\tilde{P}_t}{\tilde{P}_t} - \frac{d\tilde{P}_t^*}{\tilde{P}_t^*} = \frac{Y_{N,t}}{Y_t}\left(\frac{dA_{T,t}}{A_{T,t}} - \frac{dA_{N,t}}{A_{N,t}}\right) - \left(\frac{Y_{TU,t}}{Y_t} - \frac{Y_{TU,t}}{\tilde{P}_t Y_t}\right)\frac{dL_{TU,t}}{L_{TU,t}} - \left[\frac{Y_{TS,t}}{Y_t}(1+\gamma) - \frac{\tilde{P}_{TS,t}Y_{TS,t}}{\tilde{P}_t Y_t}\right]\frac{dL_{TS,t}}{L_{TS,t}} - \left(\frac{Y_{N,t}}{Y_t} - \frac{\tilde{P}_{N,t}Y_{N,t}}{\tilde{P}_t Y_t}\right)\frac{dL_{N,t}}{L_{N,t}} - \left\{\frac{Y_{N,t}^*}{Y_t^*}\left(\frac{dA_{T,t}^*}{A_{T,t}^*} - \frac{dA_{N,t}^*}{A_{N,t}^*}\right) - \left(\frac{Y_{TU,t}^*}{Y_t^*} - \frac{Y_{TU,t}^*}{\tilde{P}_t^* Y_t^*}\right)\frac{dL_{TU,t}^*}{L_{TU,t}^*} - \left[\frac{Y_{TS,t}^*}{Y_t^*}(1+\gamma^*) - \frac{\tilde{P}_{TS,t}^* Y_{TS,t}^*}{\tilde{P}_t^* Y_t^*}\right]\frac{dL_{TS,t}^*}{L_{TS,t}^*} - \left(\frac{Y_{N,t}^*}{Y_t^*} - \frac{\tilde{P}_{N,t}^* Y_{N,t}^*}{\tilde{P}_t^* Y_t^*}\right)\frac{dL_{N,t}^*}{L_{N,t}^*}\right\}$$

$$= \left(\frac{d\tilde{M}_t}{\tilde{M}_t} + \frac{dV_t}{V_t} - \left\{\frac{Y_{TU,t}}{Y_t}\left(\frac{dA_{T,t}}{A_{T,t}} + \frac{dL_{TU,t}}{L_{TU,t}}\right) + \frac{Y_{TS,t}}{Y_t}\left[\frac{dA_{T,t}}{A_{T,t}} + (1+\gamma)\frac{dL_{TS,t}}{L_{TS,t}}\right] + \frac{Y_{N,t}}{Y_t}\left(\frac{dA_{N,t}}{A_{N,t}} + \frac{dL_{N,t}}{L_{N,t}}\right)\right\}\right) - \left(\frac{d\tilde{M}_t^*}{\tilde{M}_t^*} + \frac{dV_t^*}{V_t^*} - \left\{\frac{Y_{TU,t}^*}{Y_t^*}\left(\frac{dA_{T,t}^*}{A_{T,t}^*} + \frac{dL_{TU,t}^*}{L_{TU,t}^*}\right) + \frac{Y_{TS,t}^*}{Y_t^*}\left[\frac{dA_{T,t}^*}{A_{T,t}^*} + (1+\gamma^*)\frac{dL_{TS,t}^*}{L_{TS,t}^*}\right] + \frac{Y_{N,t}^*}{Y_t^*}\left(\frac{dA_{N,t}^*}{A_{N,t}^*} + \frac{dL_{N,t}^*}{L_{N,t}^*}\right)\right\}\right) \quad (\#3).$$

Note that the assumption that $P_{TU,t}$ and $P_{TU,t}^*$ are normalized to unity as the numeraire and $dP_{TU,t}/P_{TU,t}$ and $dP_{TU,t}^*/P_{TU,t}^*$ are null is maintained as before. Given $R_{1,t}$ as the right-hand side of the Harrod–Balassa–Samuelson effect model and $R_{2,t}$ as the remaining (real) factors without $\frac{d\tilde{M}_t}{\tilde{M}_t} - \frac{d\tilde{M}_t^*}{\tilde{M}_t^*}$ on the right-hand side of the quantity theory of money, the above equation (#3) can also be redefined as $\frac{d\tilde{P}_t}{\tilde{P}_t} - \frac{d\tilde{P}_t^*}{\tilde{P}_t^*} = R_{1,t} = \frac{d\tilde{M}_t}{\tilde{M}_t} - \frac{d\tilde{M}_t^*}{\tilde{M}_t^*} + R_{2,t}$. This simplified relationship is used later.

For the above, with the usual assumptions of monetarists, $\frac{dV_t}{V_t} = \frac{dV_t^*}{V_t^*} = 0$, because not the *business cycles* (*output gap*) phenomena as in The Difficulties of the Policy Rule on Exchange Rate Derived in the Previous Section in Policy Practice Dominated by the Risks section but the *economic growth* (*potential output* growth) cases are examined for the time being in this section (recall the previous natural rate of interest discussion; Havik et al., 2014).

Please dare to suppose the (new) Keynesian case here. For example, let us consider the velocity of money explicitly based on the money demand equation as in the later section. Then, we must have the additional positive small fluctuations like $\beta_2(dr_t + d\pi_t)$ for the home country (and the same for the foreign country) in our following rule. Because we have $di_t \cong dr_t + d\pi_t$. However, again, from the thought experiment later, we know that we should ignore this minor fluctuation, which will complicate and bring nothing productive to our discussions regarding simplicity. Such a consideration only obscures our conclusions in this paper.

Thus, hereafter, for the simplicity of our discussions, we mostly ignore this term, $\beta_2(dr_t + d\pi_t)$, and the corresponding foreign term even when we argue the (new) Keynesian responses to suppress the short-run fluctuations later, *except* when discussing the velocity of money *is* critical. This does not mean that monetarism ignores the effect of nominal interest rate on the velocity of money, as in Say's situation (see Masuda, 2024a). All that we can say is that, as Friedman (1969/2008) suggests, empirically, this fluctuation is (practically) minor.

Solving for $\frac{d\widetilde{M}_t}{\widetilde{M}_t} - \frac{d\widetilde{M}_t^*}{\widetilde{M}_t^*}$, the following equation (#4) is obtained:

$$\frac{d\widetilde{M}_t}{\widetilde{M}_t} - \frac{d\widetilde{M}_t^*}{\widetilde{M}_t^*} = \left(\frac{dA_{T,t}}{A_{T,t}} - \frac{dA_{T,t}^*}{A_{T,t}^*}\right) + \left[\frac{1}{\tilde{P}_t}\frac{Y_{TU,t}}{Y_t}\frac{dL_{TU,t}}{L_{TU,t}} - \frac{1}{\tilde{P}_t^*}\frac{Y_{TU,t}^*}{Y_t^*}\frac{dL_{TU,t}^*}{L_{TU,t}^*}\right] + \left(\frac{\tilde{P}_{TS,t}}{\tilde{P}_t}\frac{Y_{TS,t}}{Y_t}\frac{dL_{TS,t}}{L_{TS,t}} - \frac{\tilde{P}_{TS,t}^*}{\tilde{P}_t^*}\frac{Y_{TS,t}^*}{Y_t^*}\frac{dL_{TS,t}^*}{L_{TS,t}^*}\right) + \left(\frac{\tilde{P}_{N,t}}{\tilde{P}_t}\frac{Y_{N,t}}{Y_t}\frac{dL_{N,t}}{L_{N,t}} - \frac{\tilde{P}_{N,t}^*}{\tilde{P}_t^*}\frac{Y_{N,t}^*}{Y_t^*}\frac{dL_{N,t}^*}{L_{N,t}^*}\right).$$

This equation represents a kind of *rule* for the monetary policy. I describe the implications of this policy rule in this section and The Monetary Policy Rule From the Model Analysis in the Previous Section: Conclusions in the Deterministic World section. Note that the home monetary policy, $d\widetilde{M}_t/\widetilde{M}_t$, given the foreign monetary policy, $d\widetilde{M}_t^*/\widetilde{M}_t^*$, should obey this rule to keep the exchange rate constant. Because the quantity theory of money specifies this monetary policy for the stability of the current growth rate of the exchange rate.

To be more precise, from the previous simplified relational equation, the home monetary policy given the foreign monetary policy, $\frac{d\widetilde{M}_t}{\widetilde{M}_t} - \frac{d\widetilde{M}_t^*}{\widetilde{M}_t^*}$, is determined as $\frac{d\widetilde{M}_t}{\widetilde{M}_t} - \frac{d\widetilde{M}_t^*}{\widetilde{M}_t^*} = R_{1,t} - R_{2,t}$. Here, we have $\frac{d\tilde{P}_t}{\tilde{P}_t} - \frac{d\tilde{P}_t^*}{\tilde{P}_t^*} = R_{1,t} = \frac{d\widetilde{M}_t}{\widetilde{M}_t} - \frac{d\widetilde{M}_t^*}{\widetilde{M}_t^*} + R_{2,t}$, to maintain constant exchange rate (see [#3] for the related terms).

Therefore, if the real factors, $R_{1,t} - R_{2,t}$, change (that is, the [real] exchange rate changes), monetary policies should ideally respond to this change of the real factors to keep the (nominal) exchange rate constant. I use this implication intensively in The Difficulties of the Policy Rule on Exchange Rate Derived in the Previous Section in Policy Practice Dominated by the Risks section.

In the later section, remember that we focus on not three money motives but the helicopter money (supply of money; e.g., Bernanke, 2002; Friedman, 1969/2006). This means that the three motives for money do *not* play a serious role in our later discussions. They are only related to the money growth rule that does not generate inflations or deflations from the quantity theory of money equation and should be safely assumed always to be satisfied hereafter.

The Implications of Our Exchange Rate Stabilization Rule for the Domestic General Price Level (Domestic Inflation)

In this section, we first explain how inflation causes technological progress. Then, we discuss that this technological progress might be disturbed by the rent-seeking behaviors of leaders and managers in the firms (e.g., Tirole, 1988). After that, we mention the relation of Ricardian equivalence to inflation. We argue that our new (un)employment theory relates to our necessary separation and discussions on potential outputs and output gaps. Moreover, we briefly discuss the effect of negative supply shocks caused by exchange rate depreciation and the relations of the above discussions to three money motives. Finally, we argue that the general equilibrium effects of exchange rate fluctuations on labor force participation and the nominal variables matter in our policy rule.

Inflations Cause Technological Progress

In our policy rule, the active changes in $\frac{d\widetilde{M}_t}{\widetilde{M}_t} - \frac{d\widetilde{M}_t^*}{\widetilde{M}_t^*}$ not from the k% rule (the money growth rule) but from the helicopter money should cause changes in $\frac{d\tilde{P}_t}{\tilde{P}_t} - \frac{d\tilde{P}_t^*}{\tilde{P}_t^*}$ due to the changes in $d\tilde{P}_{N,t}/\tilde{P}_{N,t}$ and resultantly in $dA_{T,t}/A_{T,t}$. Because $\frac{dA_{T,t}}{A_{T,t}} = \frac{d\tilde{W}_t}{\tilde{W}_t} + \left(\frac{dL_{TU,t}}{L_{TU,t}} - \frac{dF_{TU,t}}{F_{TU,t}}\right)$, $\frac{d\tilde{P}_{TS,t}}{\tilde{P}_{TS,t}} + \frac{dA_{T,t}}{A_{T,t}} = \frac{d\tilde{W}_t}{\tilde{W}_t} + \frac{dL_{TS,t}}{L_{TS,t}} - \frac{dF_{TS,t}}{F_{TS,t}}$, and $\frac{d\tilde{P}_{N,t}}{\tilde{P}_{N,t}} + \frac{dA_{N,t}}{A_{N,t}} = \frac{d\tilde{W}_t}{\tilde{W}_t} + \frac{dL_{N,t}}{L_{N,t}} - \frac{dF_{N,t}}{F_{N,t}}$. These relations (#5) are more obvious when we exclude the labor input growth by assuming the constant returns to scale in the unskilled and skilled workers in the tradable goods industry and workers in the nontradable goods industry production functions.

In that case, we obtain the relationships (#6) $\frac{dA_{T,t}}{A_{T,t}} = \frac{d\tilde{W}_t}{\tilde{W}_t}$, and $\frac{d\tilde{P}_{N,t}}{\tilde{P}_{N,t}} + \frac{dA_{N,t}}{A_{N,t}} = \frac{d\tilde{W}_t}{\tilde{W}_t}$ (and, $\frac{d\tilde{P}_{TS,t}}{\tilde{P}_{TS,t}} = 0$). Note that here, we can use the optimal conditions (#6) from the profit maximization problem, but we cannot derive the same optimal conditions (#5 & #6) from our sales revenue maximization problem in the previous section.

Thus, for example, increases in $dL_{TU,t}/L_{TU,t}$, $dL_{TS,t}/L_{TS,t}$, and $dL_{N,t}/L_{N,t}$ (and $dA_{T,t}/A_{T,t}$) will lead to increases in $\frac{d\tilde{M}_t}{\tilde{M}_t} - \frac{d\tilde{M}_t^*}{\tilde{M}_t^*}$. This generates a kind of *money growth* rule in the sense that the exchange rate (i.e., the general price level) is kept constant (i.e., no inflations). From the previous section, this fact that the monetary expansion influences the general price level through the increasing price level of nontradable goods industry and leads to the higher technological progress of the tradable goods industry through nominal wage increases is given, if we exclude the firm's effect of its discrete (frequent) revisions of nominal income contracts (see the rent-seeking behavior in the following section). Then, the monetary authority should enact monetary expansion when they try to cause inflation (higher general price level) and induce technological progress of the home tradable goods industry (the Phillips curve suggests the resulting positive output gaps, too).

Note that if we assume the *increasing* returns to scale for the production relation, $F_{TS,t} = L_{TS,t}^{1+\gamma}$ where $\gamma > 0$, while we assume the *constant* returns to scale (linearity) to the other production relation, $F_{TU,t}$ and $F_{N,t}$, we just obtain the reduced relation such as $\frac{d\tilde{P}_{TS,t}}{\tilde{P}_{TS,t}} = -\gamma \frac{dL_{TS,t}}{L_{TS,t}}$ (#7). This relation simply shows that the labor input of skilled workers in the tradable goods industry should grow to reduce the price of their tradable goods that need skills. Skilled workers' labor input growth in the tradable goods industry should bring *deflation pressures*.

However, this paper mainly specifies the effect of inflation on technological progress and vice versa. Furthermore, note that the effect of the skilled labor input growth on inflation should be more minor than the above inflationary effect of technological progress, according to the related discussions in this paper. That is that $dL_{TS,t}/L_{TS,t}$ affects $d\tilde{P}_{TS,t}/\tilde{P}_{TS,t}$ but we usually assume that the share of $dL_{TS,t}/L_{TS,t}$ or $d\tilde{P}_{TS,t}/\tilde{P}_{TS,t}$ is small enough in this theory section and other sections from the empirical results in general (see labor and price statistics in the developed countries).

In this case, we additionally impose the assumption that the supply of skilled workers, $L_{2,t}(= L_{TS,t})$, is restricted by its constrained (educational) capacity to produce in the macroeconomy. Thus, its growth is assumed to be the corresponding constant to the total population growth or a minor constant (or null) that does not affect our conclusions for simplicity (see the alternative discussion for this point under the profit maximization hypothesis in Appendix C).

Discussions of the previous k% rule (the money growth rule demanded by three money motives) targeted the *entire* demand growth of money specified by the real output growth in the macroeconomy (e.g., Friedman, 1960, 1969/2006). Thus, suppose that we follow the exchange rate stabilization rule derived in this section. In that case, suppose that we would like to compensate for the negative real output growth caused by negative labor input growths and technological regress with the quantity of money to generate positive economic growth. Then, we should consider supplying a more significant quantity of money than that required by the k% rule (the money growth rule demanded by three motives).

Again, these negative growths show the necessary negative money demand caused by the *entire* reduced real demand levels themselves, as indicated by the money growth rule. Here, labor input growth in the above means labor force growth under the sales revenue maximization hypothesis or labor demand growth under the profit maximization hypothesis.

Note that we should take a temporal real income reduction when we induce technological progress with the (continual) price inflation of the nontradable goods industry by (continual) monetary injection (see the equation [#6]). The nominal wage (i.e., the technological progress of the tradable goods industry) cannot catch up with this (continual) price inflation of the nontradable goods industry in reality with staggered wage adjustments (i.e., the discrete revisions of nominal income contracts) such as the basis of the not-vertical Phillips curve (e.g., Taylor, 1979, 1980).

However, finally, we should have a higher aggregate real income than before, even in a similar case with an aggregate real income increase induced by the autonomous technological progress of the tradable goods industry (see also the previous sections and Appendix D for the additional discussions of the industrial structures. Also see Appendix C for the decline of the nominal incomes in the booms).

Note that the above conclusion is solely based on the short-run possible difference between the stickiness of the general price level and nominal wage that must vanish in the long run when the completely flexible general price and nominal wage prevail.

Recall that $P_{TU,t}$ and $P^*_{TU,t}$ and $P_{TS,t}$ and $P^*_{TS,t}$ follow the law of one price in this paper. Also, we assume the symmetry between the two countries as in Appendix D. Thus, without the U.S., where we are not bothered by the currency standard, this result never depends on the definition of real income to measure with which price should be used, the import (foreign) price for the international comparison with PPP or the export (domestic) price for the time series comparison in home currency. Here, the key is only the stickiness of the general price levels in these two countries.

Again, in this and the following sections, we assume the staggered wage adjustments and continual monetary injection. Furthermore, we do not suppose the (temporal) downward shift of the general price level by firms with their actual behaviors for their productivity slowdowns in the below and like that through the nominal (or real) wage reduction in the (autonomous) technological regress case (recall the FOCs under the profit maximization hypothesis in the above) to soothe the created inflations against this conclusion without the next section.

As we will see, our theory supports Summers' (1991) intuitive and Orphanides and Wieland's (1998) numerical discussion of the optimality of the positive inflation rate, which differs from such inflation tax theory (e.g., Lucas, 2000) and existing zero lower bounds of the nominal interest rate approach. Note that, as explained in the opening section, the *hedonic regression* (e.g., Berndt & Rappaport, 2001) should be used to empirically test our theoretical attempt to connect inflations with the technological progress of the tradable goods industry in this section.

The Disinflation or Deflation by the Productivity Slowdown Caused by the Rent-Seeking Behaviors in the Firms

In this section, we assume that leaders and managers in the firms could employ not the (aggregate) labor inputs, $L_{l,t}$, only but the (aggregate) labor inputs *with* human capital, $A_{l,t}L_{l,t}$, for $l = TU, TS, N$ and $A_{TU,t} = A_{TS,t} = A_{T,t}$ in this paper. With this critical assumption, we proceed with our discussion in this section.

Until now, we have ignored the (firm's) downshift of the price of nontradable goods with the real productivity slowdown like labor misallocations for keeping the status quo and vested interests (e.g., a decrease in $dA_{N,t}/A_{N,t}$) and so on as a kind of rent-seeking behaviors (the failure of the organization or the scarcity of the management resources; e.g., Arrow, 1974/1999; Tirole, 1988).

Suppose the firms have pessimistic forward-looking expectations for their sales and outputs through the Phillips curve and are threatened by inflation pressures. To avoid these pressures, they might choose productivity slowdowns to lower the velocity of money instead of enduring inflation pressures. Recall the *shrinkflation* that reduces the quality and quantity of their goods, keeping their prices constant.

Moreover, the decline in nominal wage induced by productivity slowdowns will naturally generate pessimistic forward-looking expectations for their sales and outputs (here, we take care of the relationship between productive efficiency and capacity utilization). This is why we emphasize the role of rent-seeking behaviors in the firms in this paper. Ropele et al. (2024) argue that higher inflations will induce resource misallocations. However, our theory in this section suggests that resource (labor) misallocations are not attributed only to higher inflation.

Then, assume the simple relationship like $P_t A_t = W_t$ for our convenience. Recall the law of one price and the (normally assumed) establishment of the factor price equalization theorem, at least in the long run (holding only for the expected values; see any introductory textbooks for trade theory regarding this theorem).

Now, this simple relationship is derived from the firm's profit maximization *problem* (i.e., aggregate or representative agent problem) where A_t is assumed to be the weighted average of $A_{T,t}$ and $A_{N,t}$ as in the later paragraphs. Here, we assume that the labor input for the skilled workers is constant because the production function for the skilled workers is supposed to have increasing returns to scale property and a limited supply for simplicity.

Also, remember the equation (#1) and our theoretical results under the *profit* maximization hypothesis in the previous section. As we saw, this increasing returns to scale calls for deflationary pressure for the general price level through the decreasing price of tradable goods produced by skilled workers. However, this is caused by the increasing returns to scale, not by the above rent-seeking behaviors by the firms' leaders and managers. Therefore, we ignore its effect by assuming its minor constant labor input growth.

Thus, if A_t is constant, the above relationship between prices and wages should be reduced to $P_t = W_t$. However, if A_t decreases as we suppose the rent-seeking behaviors in the above, at least, in the inflationary environments, W_t should rise less than P_t. In short, no inflation ($dP_t = 0$) due to the sticky price and the disinflation (or deflation) of the market wage, W_t, should occur. Instead, the market wage deflation should occur.

Even in the case of a monopoly, such a firm specifies its price to equate the marginal revenue and the marginal cost. Then, saying our conclusion in this section in advance, a *decrease* in A_t would lead to the general price level *slowing down* or *decreasing*, that is, disinflation or *deflation* through the nominal wage deflation with the equation (#5) or (#6) (which is also through the Phillips curve by case; e.g., Masuda, 2024b), assuming *stable demand* in the market and constant returns to scale production functions for other two industries.

More precisely, from the equation (#5) or (#6), if the technological regress of the nontradable goods industry occurs, the nominal wage should decrease, and/or the price of nontradable goods should rise. The former case should cause the technological regress of the tradable goods industry, and the exchange rate (i.e., the general price level) should remain constant.

However, once the price of the nontradable goods industry even partly increases, the nominal wage and the resulting technological level of the tradable goods industry decrease less than that degree of the technological regress of the nontradable goods industry above. Then, the exchange rate should generate appreciation (i.e., the general price level decreases), other things being equal.

Furthermore, we can realize the same situation with the above simple wage-price relationship as with the not-vertical Phillips curve for our purpose in this paper. Note that to incorporate the not-vertical Phillips curve like that in Masuda (2024b), we might assume that there is no perfect competition but monopoly or monopolistic competition as usual. However, it is as easy as the above.

Our above model is based on the representative labor force model, which treats the firm's organization as a single player in this section. In the following, we exclude the possibility of coordination failure (e.g., Cooper, 1999), where cooperation is needed once the organization consists of *heterogeneous* workers. Here, we assume the heterogeneity of the payoffs due to the different characteristics of players, like the battle of sexes game.

Then, suppose that the firm's productivity is the weighted average of each heterogeneous worker's productivity level for simplicity, as in the above, for the time being. Then, the intended selections from the lower (at least, not higher) productivity workers for the higher status (higher positions in the firms) that have the more significant weights (distributions) in the productivity of the firms could naturally lower the productivity of the firms. This selection will occur, for example, to keep vested interests and status quo by recruiting workers with lower (not higher) productivity (talents) but with the same graduate schools of the same universities as those of the firms' leaders and managers. This means that the firms employ workers only with their schooling, not confirming their talents.

Recall that the signaling model for schooling does not need to be backed by the higher productivity of the workers with the higher (schooling) signals (e.g., Gibbons, 1992/1995). Leaders and managers who benefit from these rent-seeking behaviors sometimes have a lower discount factor than usual (see Drazen, 2000). This is an example of the (intended) productivity slowdown by rent-seeking behaviors. Furthermore, such rent-seeking behaviors should be revealed when they recruit other workers in firms as well as when they recruit their own successors.

Note that this paper assumes that the leaders and managers in the firms could employ not the (aggregate) labor inputs, $L_{l,t}$, only but the (aggregate) labor inputs *with* human capital, $A_{l,t}L_{l,t}$, for $l = TU, TS, N$ and $A_{TU,t} = A_{TS,t} = A_{T,t}$, again. Thus, as we will note, for example, leaders and managers in the firms do *not* have enough ability to grasp the structural changes in the labor market such that this misperception will bring the (aggregate) labor inputs with *low* technology levels (i.e., lower human capital accumulation), $A'_{l,t}L_{l,t}$ with $A_{l,t} > A'_{l,t}$ for any l. This is the result of our rent-seeking behaviors represented by vested interests and status quo.

First of all, as we noted, such rent-seeking behaviors cause *deflationary* (at least, *disinflationary*) pressures on *market wages* (and *general price* levels) with their induced (intended) productivity slowdown. Recall that we assume $P_tA_t = W_t$, and the equation (#1). Moreover, as we stated, such productivity slowdowns might cause even technological *regress* and *disinflation* (*deflation*) tendencies in the macroeconomy.

In the above productivity slowdown case, to support such lower (not higher) productivity managers or leaders, other workers with higher productivity that improves the firm's productivity but without any significant positions (distributions) in those firms must continue to remain in such firms and work harder than those with labor productivity that affects the firm's productivity but with higher positions in such inefficient firms. Such behaviors should occur to compensate for losses from low productivity to keep the weighted average productivity of such firms at least constant. Otherwise, such firms should be forced to exit from the markets.

Similarly, suppose that such leaders fail to recruit more new talented workers or the majority of these newborn workers are less qualified or educated, as well as their educated but not-talented successors, etc. Here, education represents human capital accumulation as labor-augmenting technological progress (Barro & Sala-i-Martin, 1995; Lucas, 2002).

However, the talents of workers determine their human capital accumulation speeds, their depreciation speeds, or their accumulation degrees if they are given the same schooling years. The above argument says that higher productivity should not always back higher schooling if *monetary* terms measure effort efficiency or disutility. This leads to the above implication of the signaling game suggested by Gibbons (1992/1995).

Here, we assume that productivity is so correlated not to *schooling* itself but to *talents* with schooling, and efforts to get schooling correlate to schooling costs. This fact would support this implication if the schooling should require higher tuitions and longer prep-school provisions. The firms employ workers only with their schooling, not confirming their talents.

Alternatively, suppose that as the worst case that we noted, they should be willing to recruit unqualified or uneducated workers without considering the current structural changes in the labor markets to keep or satisfy their vested interests and status quo or from their inabilities to grasp such actual critical changes in the labor market.

Then, such rent-seeking behaviors (and resultant vested interests and status quo (e.g., Tirole, 1988) would not be sustainable. Takahashi (1993) argues that authority is essential in organizations, including firms. As Arrow (1974/1999) describes, authority without responsibility induces unnecessary mistakes.

The above equations (#5) and (#6) represent the optimal (conditions required for) behaviors derived from not the sales revenue maximization hypothesis but the usual profit maximization hypothesis. However, the sales revenue maximization hypothesis implies that full labor force (supplies) employment decides such a firm's demands. It can exclude the possibility of the above rent-seeking behaviors by deciding the arbitrally *absolute* level of the labor inputs and specifying not the relative, *indeterminate,* but the arbitrally *absolute* output and sales revenue levels again.

This means that the profit maximization hypothesis should promise full employment with chance only when resource constraints are not a problem or when benevolent leaders and managers nearly realize such a situation (see below). However, it should ensure disinflation or deflation through the nominal wage deflations caused by technological regress due to the above rent-seeking behaviors of leaders and managers in the firms.

In short, recall that under the Cobb–Douglas production function specifying constant returns to scale, in general, the optimal absolute output *level* is *uncertain* because only the capital-labor *ratio* is determined (e.g., Jehle & Reny, 2001). Even in our firm's profit maximization setting, the labor input is indeterminate under this constant returns to scale production function. Again, the above firm's behaviors regarding the productivity slowdown and idle capacities in the macroeconomy (that is, unemployment) are not contradicted by the above optimal conditions (#5) and (#6) obtained only under the profit maximization hypothesis.

This means that the profit maximization hypothesis can always admit the rent-seeking behaviors to employ the low productivity workers and insufficient employment, which affect the firm's productivity and keep the diminishing equilibrium of the labor market (that is, generate unemployment). These behaviors naturally lead to price disinflations or deflations, as we saw. However, we do not explicitly assume such rent-seeking behaviors afterward in this paper.

Imagine the leader's and manager's other (not the firm's) rent (profit) maximization objective, in addition to the optimal behaviors specified under the usual firm's profit maximization problem, as the stage game explained in the opening section. Then, to avoid rent-seeking behaviors, a different formulation from this troublesome profit maximization problem, like the sales revenue maximization hypothesis (Baumol, 1959) for the firm's objective in this paper, is required.

This hypothesis means that a firm maximizes not the profit but the sales revenue. Otherwise, to keep the marginal productivity hypothesis for factor prices (i.e., real wages) under the competitive market while avoiding the rent-seeking behaviors, excluding the resource constraints (i.e., admitting the unlimited resources) is insufficient, as we indicated before. We need the additional assumption of benevolent leaders and managers considering the firm's quite optimistic future (forward-looking) sales expectations like that under the sales revenue maximization hypothesis (i.e., full employment). Furthermore, to explain the result of this rent-seeking behavior of not benevolent leaders and managers who do not try to achieve full employment, we need a new (un)employment theory (Appendix C), too.

In short, this benevolent case can avoid the above leader's and manager's rent-seeking behaviors. As we noted, such rent-seeking behaviors should (continuously) try to employ the lower (not higher) productivity workers for their own sake, together with satisfying our new (un)employment theory accounting for their insufficient employment that accomplishes the diminishing equilibrium. This is due to the indeterminacy of labor input and output levels based on the profit maximization hypothesis. However, we emphasize that for achieving full employment to avoid rent-seeking behaviors, the above benevolent leader's and manager's assumption is *unnecessary* once we consider the resource constraints in the macroeconomy and apply the sales revenue maximization hypothesis.

Still, this paper clarifies that our labor force participation theory in the later sections specifies the available amounts of the labor force, $L_{1,t}$ and $L_{2,t}$, perfectly to be employed by firms in the macroeconomy, as our sales revenue maximization hypothesis derives. Also, in the later section, we will discuss the connections among these labor inputs (demands), our new (un)employment theory based on Rogerson et al. (2005) and Appendix C, and the labor force participation theory suggested by Flinn and

Heckman (1982) and Heckman (1979).

For this, suppose the above sales revenue maximization hypothesis to be nearly established under the profit maximization hypothesis with optimistic forward-looking behaviors. In that case, to avoid the rent-seeking behaviors by inflation pressures and the sticky price, we should further implicitly assume not the temporal but the continual inflation pressures (i.e., continual monetary injections) by the monetary authority to ensure their objectives.

This monetary expansion will support inflations and give firms optimistic, forward-looking expectations for their sales (the Phillips curve suggests the resulting positive output gaps). In this monetary expansion case, our new (un)employment theory results in reduced unemployment through self-fulfilling inflation (see the previous section for our self-fulfilling monetary disturbance example and recall the Phillips curve relation) if we exclude the firm's effects of its discrete (frequent) revisions of nominal income contracts to induce technological progress of the tradable goods industry. This realizes the *high-pressure economy* that Samuelson and Solow (1960) explain.

However, suppose that such *persistent* rent-seeking behaviors connected to productivity slowdowns remain and significant monetary injections continue. In that case, the large amount of seemingly *ineffective* and *dormant money* will only exist in the macroeconomy (see the later section for the long-run inflation expectation and the velocity of money).

This will be realized under disinflations or deflations, which support the pessimistic forward-looking expectations for firms' sales and result in a more diminishing equilibrium under the profit maximization hypothesis (e.g., Cuba-Borda & Singh, 2024). Thus, the sales revenue maximization hypothesis or the optimistic forward-looking expectation is critical.

Last, the above discussion mainly illustrates the rent-seeking behaviors in the nontradable goods industry. However, these rent-seeking behaviors should also occur in the tradable goods industry.

Inflations, Technological Progress, and Ricardian Equivalence

Furthermore, according to our conclusions in this paper, the same discounted sum increase in the government bond as the quantity of money specified by the money growth rule would not cause inflation at all (Friedman, 1960). Only the amount of monetized government bonds exceeding this money growth rule can induce inflation and the resulting technological progress (Friedman, 1970).

On the other hand, when the Ricardian equivalence (no difference of the fiscal finance between taxes and debts; e.g., Romer, 1996) holds, such a kind of (temporal) increase in government bond (the quantity of money) financing the additional fiscal spending should cause the corresponding real demand decrease, anticipating their future tax burden. The newly issued bonds often only soften the future tax burden with the decreased real demands by absorbing the money from the household's budget originally for consumption.

More precisely, the domestic savings of households buying such bonds will increase by the Ricardian equivalence motive and cause the diminished real demand to continue. That depends on the sizes of this (temporal) fiscal spending and the gradually revealing number of households with the above Ricardian equivalence motive. This counter-action to offset the existing positive effect of the (temporal) fiscal spending by anticipating the future tax burden by households would be decided by the effective contents of fiscal spending, the household's time horizon, and the household's correct knowledge of the macroeconomy and so on, as we saw in the previous section. For example, newspapers can disseminate this concrete (effective) content of fiscal spending as it is to the public.

Under the profit maximization hypothesis, our above discussion suggests that this kind of inflation may only generate the temporal one when Ricardian equivalence partially holds in reality at the first period. Furthermore, the real demands continue to diminish little by little due to the gradually revealed Ricardian household behaviors after that (temporal) fiscal spending together with its decreasing multiplier effect.

Suppose that (a) the real demand changes (fiscal shocks) have persistently decaying autocorrelations due to the decreasing multiplier effects, (b) the number of the gradually revealed Ricardian is at least continually constant (or increasing), and (c) the employed amount of labor inputs due to the above real demand decrease stays constant or decreasing. Then, it calls for and fosters post disinflations (or deflations) with a kind of *recessions* due to the decreasing real demands after the temporal fiscal expansion. In this case, the technological progress induced by inflations would not be realized again.

Because the output increase by such a fiscal expansion is related not to technological progress, but to output gaps. Output gaps are not generally related to technological progress (see the equation [#6]), although they might induce technological progress through the inflations caused by the Phillips curve. Of course, some exceptions exist, such as the nominal income contracts enforced by law (see Appendix C). However, in this paper, this case in Appendix C is not related to our case, where output gaps will become negative soon, and we exclude this low enforcement case.

We have treated the deterministic world with *full employment* or diminishing equilibrium in the labor market until the previous section. In this section, our discussion uses the latter fact that the *short-run slackness* in the macroeconomy is caused only by the profit maximizing behavior of firms. Once we admit this kind of slackness as a short-run phenomenon, we will observe the *negative* output gaps in the short run and the related disinflations (or deflations) through the Phillips curve without affecting the technological progress instead of the productivity slowdowns.

In short, assuming the not-vertical short-run Phillips curve as usual, these negative output gaps will *not* cause productivity slowdowns but naturally *generate* disinflations (or deflations) due to the demand shortage.

Furthermore, such slackness should *not* be maintained in the *long run*. *Persistent slackness* should be considered the *permanent stagnation* of economic growth, which leads to the *overwhelming unemployment* of the *lower* labor *productivity* workers due to the *lower* educational attainments in the society caused by the *lower* economic (income) growth (expectations), etc. (see Appendix C). Such a phenomenon as persistent slackness is often observed empirically.

Recall that firms with low technologies should be eliminated from the market or must explore their small place to survive by struggling with other firms holding similar low technologies (recall the Stackelberg duopoly game as one of the oligopoly [disease]; e.g., Gibbons, 1992/1995; Mas-Collel et al., 1995).

Ultimately, the *productivity slowdowns* should be realized once negative output gaps become permanent as we usually observe empirically and from the (moving) average concept for estimating the potentital output (growth). Recall that the potential output growth will deteriorate and be diminished by the newly generated productivity slowdowns with leading these negative output gaps to the normal output gap states (Havik et al., 2014).

Actually, in the subsequent sections, we will introduce the *unemployment rate* (and not in the labor force rate) as a *stochastic* factor with *zero* mean after taking a natural logarithm with the normalization. These stochastic factors might explain a kind of not only the *recessions* but also the productivity slowdowns in the above due to the decreasing real demands. The general price or (segmented) average nominal income stickiness, as well as disinflations or deflations under the profit maximization hypothesis in the short run when facing negative pressures like recessions, should usually induce more unemployment than usual, as our following new (un)employment theory (and the Phillips curve) predict(s) (see the next section and Appendix C).

Our New (Un)employment Theory

Unemployment never happens under the firm's sales revenue maximization behavior. However, the profit maximization hypothesis should admit the diminishing equilibrium and its resulting unemployment. Then, the new (un)employment theory (Appendix C) should specify the firm's strategic behaviors in recessions to choose either the wage (income) cuts representing the flexible nominal wages (incomes), the layoffs reflecting the sticky nominal wages (incomes), or some other alternatives only under the firm's profit maximization hypothesis (e.g., Masuda, 2024c).

For example, the Japanese employment system in the 1980s and 1990s was famous for long-term employment (i.e., *rare* layoffs in economic downturns and low labor mobility). The resulting excessive labor hoardings inside the firms showed the (relatively) *flexible* nominal wages (incomes) with significant working hours adjustments related to overtime pay in recessions (e.g., Yashiro, 1997).

Thus, layoffs for the low productivity workers in the firms should mean the *constant* (or possibly even rising) average of the nominal wages (incomes) of the *remaining workers* instead of giving up the wage (income) cuts of all the currently employed workers that should *push down* that average of the nominal wages (incomes). As a result, they become one of the causes generating the *downward rigidity* of the (average) nominal wage (incomes), in addition to the theoretical downward stickiness of the segmented average nominal incomes shown in Appendix C and other psychological obstacles from the wage (income) cuts. In short, the average of the nominal wages (incomes) in the macroeconomy only represents the *conditional average* of *the employed*.

The significant wage (income) cuts usually mean the near future bankruptcy of such firms in typical recessions. Because this phenomenon shows their critical technological regress due to the false and sloppy accounting or forecasting or so, from our wage determination equation in the previous section. Moreover, recall that workers can compare their nominal wages (incomes) with the corresponding nominal market wages (incomes) to their abilities, experiences, etc., like their *outside options*. We have Appendix C for this outside option issue, which belongs to the search theory and owes to Rogerson et al. (2005). However, our point is that this outside option is a decreasing (or at least nonincreasing) function of the worker's ages, at least in Japan.

Recall the relationship between the reservation wage (income) of workers and the low labor mobility in Japan promoted by the Japanese seniority wage (income) system where such a seniority wage (income) must be realized like the human capital theory suggests (e.g., Becker, 1975/1976). Furthermore, this seniority wage (income) is actually realized *only within* their similarly long belonging firms in Japan (e.g., Doepke & Gaetani, 2024). At the same time, recall that the number of job offers for aged workers should usually be small in Japan.

Furthermore, the firm's optimistic or pessimistic future sales expectations, that is, the leader's and manager's optimistic or pessimistic *forward-looking behaviors* for their firm's future sales, such as *animal spirit* (for the former optimistic case; Keynes, 1936/1997; e.g., Cuba-Borda & Singh, 2024), could specify more or less the *diminishing* equilibrium labor demand. We remember that the labor demand derived from the constant returns to scale production function cannot define its absolute level. Furthermore, it can admit any labor demand level that is less than its current labor force (supply). Because firms cannot employ more than currently available resources. Moreover, this paper should further extend such a new (un)employment theory that explains the coexistence of unemployment, sticky wages, and outside options, as shown in Appendix C.

In addition, we can incorporate the fluctuations in working hours in our new (un)employment theory. This inclusion of working hours will work like the wage (income) premium for the skilled workers in the mathematical model in Appendix C and be done in our mathematical model in other sections. The positive premiums in the booms and the (nearly) zero premiums in the recessions alternately occur with some attached positive probabilities like the Markov switching model (e.g., Hamilton, 1989).

Also, this inclusion will specify the fluctuations in working hours as an additional stochastic factor to the unemployment rate and not in the labor force rate, as we will explain later. Working hours are consistently recognized as a kind of utilization rate in macroeconomics because the total labor inputs are essential (e.g., Burnside et al., 1988). Due to their nature as the initial endowment for households, working hours fluctuations should not have some one-sided, upward-sloping growth rates and should remain stationary from the beginning.

Such a modification of our new (un)employment theory should enable us to explain the above working hours adjustments related to overtime pay in recessions under the Japanese employment system. In Appendix C, our new (un)employment theory and our related stochastic model will explain this.

Still, working hours should usually have the eight hours in a day restriction as its lower bound, which forces us to reconsider the Tobit-like (left-)censored formulation instead of our innocent no-restricted formulation (see Masuda, 2024d, for this kind of careful consideration). Because, for the concrete, not normalized example that Masuda (2024d) supposes, we might obtain the positive or negative huge finite number of working hours shocks like “googol” even with the minor positive probabilities.

However, together with the existence of paid and sick leaves, furlough, as well as overtime work, we can assume that the original lognormal distribution for the working hours has the mean as the eight hours (regular working hours a day; see the later section for the unemployment as the stochastic errors). Then, with the (statistical) normalization after taking the logarithm, this modification need not bother us with this (Tobit-like) difficulty in this section and later sections; we should only obtain some more complicated background stochastic factors with their innocent no-restricted formulations as a result.

Anyway, Masuda (2024c) empirically observes that such a firm's behavior, like severe employment policies (i.e., the vacancy rate and/or involuntary unemployment rate policies) in recessions in deciding the diminishing equilibrium labor demand in the market, would affect the worker's life satisfaction, that is, the worker's welfare. This relationship between the worker's life satisfaction or welfare should be backed by the relationship between *stochastic choices* and *cardinal utilities* (e.g., Debreu, 1958).

Moreover, considering our conclusion in advance, the natural (unemployment rate) concept should only be considered in the usual profit maximization hypothesis. However, the *full employment* concept proposed by Friedman (1953) should be realized under the *sales revenue* maximization hypothesis. In the former case, as we pick up in this paper, the temporal slackness due to the firm's profit maximizing behaviors will converge to the original diminishing equilibrium: Chronic slackness or natural unemployment rate in the long run.

Naturally, this chronic slackness or natural unemployment rate, as well as labor force participation, like under the sales revenue maximization hypothesis, should also include the friction ones that the tradition emphasizes (see any introductory macroeconomics textbooks). However, labor force participation for the latter should be ignored in this paper.

Again, the temporal fluctuations like that of the above working hours are sustainable due to their stationarity and mean-reverting properties. Recall the existence of the time trend and its characteristic error term property in our later formulation. The productivity slowdown pressures (i.e., the technological regress pressures) will *only remain* to lead to the resultant corresponding disinflations (or deflations) to the (*diminishing*) macroeconomy (diminishing real demands related to output gaps). We model the above unemployment and downward stickiness of *segmented average* nominal incomes characteristic to the usual recessions (and the Japanese employment system) in Appendix C and so on.

Negative Supply Shocks and Inflations

Note that this paper connects inflations with technological progress, as we have discussed so far. Still, inflation should inevitably be caused by the exchange rate's depreciation and its resulting higher import prices as *negative supply* shocks. Recall that our two-country model in Appendix D specifies that foreign exports must be equivalent to home imports and vice versa. From the beginning of this discussion, we excluded import price inflations based on the demand and supply in the market or, equivalently, its scarcity as natural resources.

Such higher import prices embarrass domestic households (and industries) through the importing necessities (see Appendix D for the specification like $\tilde{P}_{TS,t} = \tilde{P}^*_{TS,t}\frac{\tilde{P}_t}{\tilde{P}^*_t}$; here, we drop the symmetry assumption; e.g., Masuda, 2021, for the latter case). This higher inflation affects households' (equivalently, consumers', as seen in Appendices A and D) welfare, that is, social welfare.

Thus, the monetary authority should balance the merit and deficit of increasing real incomes through technological progress induced by inflations and temporal reductions in their real incomes through the higher price of imported necessities and so on. However, our above inference suggests that in the end, the increases in households' real incomes due to technological progress induced by inflation should overcome the reductions in their real incomes through higher import prices and so on if the cause of higher import prices is only the exchange rate depreciation.

As we stated, if its balance is lost not in favor of households like in the short run, this is one of the typical *stagflations* in our case (Blanchard, 2000; see The Role of The Long-run Inflation Expectation and Its Relation to the Velocity of Money section later for the *second kind* of stagflation case). Then, if necessary, they are enough to force policymakers to suppress their monetary expansions and often revert to a moderate, optimal monetary policy.

Suppose that this moderate, optimal policy case is accomplished. Then, the technological progress induced by inflations should be suppressed. Such a policy results in pursuing zero or pretty moderate inflations considering the measurement error issues (the upward bias of the Consumer Price Index; Boskin et al., 1996) and responding to a short-sighted consumer's welfare (Laibson, 1997), as we will state later.

Engemann (2019) suggested the reasons for the positive inflation target, like the upward bias of the Consumer Price Index and the increase in the room for monetary policy to cut nominal interest rates. Actually, from this practical perspective, some economists argue that a positive inflation rate, like two percent or more, should be *desirable* in the Taylor reaction function literature. For example, Boskin et al. (1996) discussed the upward bias of the Consumer Price Index. Blanchard et al. (2010) argued the increase in the room for nominal interest rate cuts with a higher inflation target than *two percent* as the theoretical and empirical suggestions. Orphanides (2003) illustratively discussed the *two percent* inflation target as the theoretical one.

Our main point in this paper is that, aside from the above practical issues, we prove that positive (moderate) inflation is *theoretically* desirable, like what Summers (1991) (and Orphanides, 2003) suggested.

It is off the subject, but in the previous section, we introduced the existence of the households with the Ricardian motive. Regarding such tax policies for saving poverty by distributing the incomes in inflationary environments to the entire country, all that we can say easily in this paper is as follows. Similar to the Inflation tax literature that we refer to and discuss in this paper, we suggest that the *redistribution policy*, like taxing the rich and distributing this tax collection to people experiencing poverty, should usually be *ineffective*. Because the intended moderate inflation by the monetary authority should result in higher real incomes for all in the country in the *long run* than otherwise.

In this sense, the previously mentioned consequential *trickle-down growth* will deserve better for the country than the redistribution policy targeted for the heterogeneous inflation effect in the above sense (remember that we do *not deny* all the redistribution policies in this paper). *This kind* of redistribution policy should be unnecessary in general. However, we should also care about the *short-run* high political pressure caused by the existence of *short-sighted* households or consumers in reality, as noted above.

Also, the above discussion discards further theoretical problems, like the distortion of the rich's incentives by taxation for redistribution and the different consumption propensity problems (e.g., Ampudia et al., 2024) between the rich and people experiencing poverty. For example, the lump sum tax does not affect the work incentives of households (a hold up problem; e.g., Salanié, 1994/1997). However, it decreases the rich's welfare by reducing their real consumption instead of raising real consumption, resulting in a welfare increase for people experiencing poverty (see Appendix D). Such a policy is *not* Pareto improving (e.g., Friedman, 1991). In this sense, this kind of redistribution policy represents a kind of *false egalitarianism* or *reverse discrimination*.

As another possibility, we might provide a policy for *intertemporally* smoothing these heterogeneous income fluctuations caused by inflations with fiscal funds by borrowing debts. This debt policy might accomplish the *Pareto improving* redistribution which smoothes the heterogeneous income fluctuations caused by inflations in terms of *intertemporal substitution*. However, we exclude the role of debt to evaluate it here. Because various complex practical problems like the benefit principle and debt ceiling to be solved exist.

In addition, we must tackle this incentive and its resulting macroeconomic problems to treat the effects of the above redistribution policy earnestly with a rigorous theoretical approach. However, we give them up in this paper because constructing such models is far beyond our scope. Thus, the above discussion is a convenient and tentative one in this sense.

The Relations to Three Money Motives

Furthermore, the above arguments have no relation to the discussions separated by money motives in the previous section. Instead, our main point is that our above discussions focus on the helicopter money (e.g., Bernanke, 2002; Friedman, 1969/2006). In reality, the staggered price dynamics (the Phillips curve) exists and the dynamics of $\frac{d\widetilde{M}_t}{\widetilde{M}_t} - \frac{d\widetilde{M}_t^*}{\widetilde{M}_t^*}$ will not directly accompany with the simultaneous movements of $d\tilde{P}_{N,t}/\tilde{P}_{N,t}$. Therefore, $d\tilde{P}_{N,t}/\tilde{P}_{N,t}$, and $d\widetilde{W}_t/\widetilde{W}_t$ will not increase so much as $\frac{d\widetilde{M}_t}{\widetilde{M}_t} - \frac{d\widetilde{M}_t^*}{\widetilde{M}_t^*}$ due to the existence of Phillips curve which is not vertical at least in the short run and its staggered (nominal) wage (income) contracts (see the equation [#6]; Taylor, 1979, 1980; e.g., Masuda, 2024b, for the discussions of the Phillips curve).

However, with the helicopter money, both in the short run and long run, our above arguments, like the technological progress induced by inflations, the diminishing equilibrium, and disinflations by the rent-seeking behaviors or the firm's forward-looking behaviors from our new (un)employment theory (Appendix C) and the Phillips curve, and the import prices inflations about the monetary expansion discussions, completely holds with both the not-vertical and vertical Phillips curves (see the equation [#6]; Masuda, 2024b).

The sticky price assumption does contribute to realizing the real effects of monetary expansions like helicopter money for inflation. Still, the relation between inflation and the quantity of money should be ensured even under the flexible price assumption in the long run.

Now, the three motives are represented by the transaction motive, such as the Baumol–Tobin type transactions cost model (Baumol, 1952; Tobin, 1956).

For example, the speculative motive should be defined as the movement of the (nominal) interest rate (Dornbusch et al., 2008; Friedman, 1969/2006) and be included in the above transactions model due to the (estimated) parameter of its control variable, the nominal interest rate. Other things being equal, it is inevitable that this speculative motive should usually allow the *negative* effect of the nominal interest rate rises on the velocity of money due to its portfolio rebalance effect through indirect finance, that is, the banking system or through direct finance.

The resulting negative parameter of the nominal interest rate in our money demand equation supports our following inference and conclusion. For example, when we obtain more than or less than 0.5 for the estimated parameter of the nominal interest rate in the traditional Baumol–Tobin type transactions cost model (Baumol, 1952; Tobin, 1956), as Niehans (1978) explained, it means the existence of this speculative motive.

Recall the bank deposits with their nominal interest rates as the *broad money* (i.e., safe assets) and the other rises in the return rates of the existing *risky assets*. Higher borrowing rates reduce real investment, resulting in shrinking money multipliers procyclical to business cycles (e.g., Friedman, 1969/2006). However, consider the empirical portfolio rebalance effects of households, for example (e.g., Flow of Funds Accounts in developed countries; de Bondt, 2009, for the Euro area; Lucas, 1988, for the U.S. case). Then, the effect of the nominal interest rate on the velocity of money and the quantity of money should be minor enough even for this speculative motive, compared with the usual transaction motive. Thus, the nominal interest rate parameter in our money demand equation should remain negative, as suggested by the above inference and the transaction motive.

On the other hand, we will explain the precautionary (asset) motive in detail in

Appendix B. Briefly to summarize that result, the precautionary (asset) motive could work for the money demand in a different *negative* direction from the transaction motive. This is because not only the *direction* but also the *volatility* of the real income matters for this precautionary motive, and economic crises usually respect this (surge of) real income volatility more than the nominal interest rate (and the general price level) volatility(ies) as well as its negative real income movements, as we will explain in Appendix B.

Furthermore, this is why the monetary authority should *not* accommodate the *monetary contraction* caused by this precautionary motive *and* the transaction motive induced by the (temporal) negative real income growth in economic crises. In this case, if the velocity of money *shrinks* due to the fragile banking system and/or panics like the unemployment fear in the public, the monetary policy should seem *helpless* in the short run from the quantity theory of money because of the shrinking velocity of money.

In short, expansionary monetary policy *must* be only the *necessary condition* in such a case in the short run (but does not become a *sufficient* condition), like pushing on a piece of string. However, even in this short-run case, expansionary monetary policy will also help the recovery of the circulated quantity of money through the quantity theory of money. Because our money demand equation proposes using a more significant quantity of money than usual in that situation due to the *increase* in the *inverse* velocity of money (i.e., the shrunk velocity of money). It will ultimately lead the velocity of money to its original higher state than now.

Anyway, the sign and magnitude of the precautionary motive should be empirically examined and determined. Actually, this motive fluctuates more with very small movements, even in the short run under regular times, due to its conditional heteroscedastic variance property. Furthermore, we could hardly grasp it empirically (see Appendix B).

At least in the economic crises when monetarism should force us to consider this motive, it is inevitable that the precautionary (asset) motive would work negatively for the money demands, similar to the transaction motive. However, we should also judge that we should *not regularly* consider this precautionary (asset) motive (Appendix B; Friedman & Schwartz, 1963/2008). Therefore, our previous discussions are still the same, even explicitly considering these additional speculative and precautionary (asset) motives to the transaction motive.

General Equilibrium Effect of Exchange Rate Fluctuations on the Labor Inputs

We should consider the possibility that exchange rate fluctuation would generate an *additional* increase or decrease in labor inputs from *abroad* and *home*. The exchange rate fluctuations should have a *feedback* effect on the home country in the sense of *general equilibrium* (see equation [#4]). As we will state our labor participation theory in the later section, for that labor participation theory, we additionally incorporate the effect of exchange rate fluctuations on the other different net real home wages.

That is the real home market wage adjusted by the exchange rate in the *foreign* currency, which is the original home real wage adjusted by the exchange rate in the foreign currency normalized by $P^*_{TU,t}$, $\frac{\widetilde{W}_t}{\tilde{P}_t}\left(\frac{\tilde{P}^*_t}{\tilde{P}_t}\right)$, minus the foreign worker's positive adjustment *costs*, $\tilde{c}^*_{LM,t}$ like linguistic and moving in the *foreign* currency also normalized by $P^*_{TU,t}$, as $\frac{\widetilde{W}_t}{\tilde{P}_t}\left(\frac{\tilde{P}^*_t}{\tilde{P}_t}\right) - \tilde{c}^*_{LM,t}$.

We should focus on the individual worker's decision-making. Let us consider the individual foreign worker ξ, where the integer variable $\xi \in [1, NP^*_t]$ and NP^*_t is the number of the foreign adult population. That is, the numbering ξ represents the ascending numbering of the existing foreign people who can work. The foreign person who do not decide working in either the home or the foreign country should compare his or her real wage in the foreign country, $\widetilde{W}^*_{\xi,t}/\tilde{P}^*_t$, the above comparable real wage earnable in the home country, $\frac{\widetilde{W}_{\xi,t}}{\tilde{P}_t}\left(\frac{\tilde{P}^*_t}{\tilde{P}_t}\right) - \tilde{c}^*_{LM,\xi,t}$ and his or her positive real reservation wage, $\widetilde{RW}^*_{\xi,t}/\tilde{P}^*_t$. If $\frac{\widetilde{W}_{\xi,t}}{\tilde{P}_t}\left(\frac{\tilde{P}^*_t}{\tilde{P}_t}\right) - \tilde{c}^*_{LM,\xi,t} \geq \frac{\widetilde{W}^*_{\xi,t}}{\tilde{P}^*_t} \geq \frac{\widetilde{RW}^*_{\xi,t}}{\tilde{P}^*_t}$ or $\frac{\widetilde{W}_{\xi,t}}{\tilde{P}_t}\left(\frac{\tilde{P}^*_t}{\tilde{P}_t}\right) - \tilde{c}^*_{LM,\xi,t} \geq \frac{\widetilde{RW}^*_{\xi,t}}{\tilde{P}^*_t} \geq \frac{\widetilde{W}^*_{\xi,t}}{\tilde{P}^*_t}$, this foreign worker ξ would decide to work in the home country.

If $\frac{\tilde{W}_{\xi,t}^{*}}{\tilde{P}_{t}^{*}} \geq \frac{\tilde{W}_{\xi,t}}{\tilde{P}_{t}}\left(\frac{\tilde{P}_{t}^{*}}{\tilde{P}_{t}}\right) - \tilde{c}_{LM,\xi,t}^{*} \geq \frac{R\tilde{W}_{\xi,t}^{*}}{\tilde{P}_{t}^{*}}$ or $\frac{\tilde{W}_{\xi,t}^{*}}{\tilde{P}_{t}^{*}} \geq \frac{R\tilde{W}_{\xi,t}^{*}}{\tilde{P}_{t}^{*}} \geq \frac{\tilde{W}_{\xi,t}}{\tilde{P}_{t}}\left(\frac{\tilde{P}_{t}^{*}}{\tilde{P}_{t}}\right) - \tilde{c}_{LM,\xi,t}^{*}$, this foreign worker would remain in the foreign country to work there. Otherwise, this foreign worker would not choose to work anywhere. In the later section, we treat this relation in terms of the *aggregate view* again.

Note that we should determine the (foreign) worker's choice in the case where three terms, $\frac{\tilde{W}_{\xi,t}}{\tilde{P}_{t}}\left(\frac{\tilde{P}_{t}^{*}}{\tilde{P}_{t}}\right) - \tilde{c}_{LM,\xi,t}^{*}$, $\frac{\tilde{W}_{\xi,t}^{*}}{\tilde{P}_{t}^{*}}$, and $\frac{R\tilde{W}_{\xi,t}^{*}}{\tilde{P}_{t}^{*}}$ are all equal. In such a case, we should assume that (foreign) workers want (choose) to work in their own country (see the later section for our labor force participation theory). However, we do not clarify this point in this paper because such a case is rare and do not affect our conclusions at all in this paper.

In addition, to connect the above theory to our new (un)employment theory as in the later sections, accurately speaking, we should take the expectation of the real wages in our formulation for the labor force participation theory. We will explain this necessity in detail in Appendix C.

The above relation also suggests the possible increase or decrease of home labor inputs due to the foreign worker's and the domestic worker's additional labor participation in the home and foreign countries. Depending on the net real foreign wage adjusted by the exchange rate in the domestic currency, the opposite case of home workers wanting to work abroad exists, too.

The above is the theoretical possibility. In reality, once the original real wages in the home country are not so higher than those net real wages in the foreign country and the exchange rate in domestic currency depreciates more, the unskilled workers will first escape from the home country to get more payable jobs in the foreign countries (here, we discard the symmetry assumption in Appendix D). In short, the skilled workers can remain in their home country by domestic firms manipulating the adjusted premium term, $1 + g$, to the changes in the exchange rates.

In this case, with the new adjusted wage term in the distribution law for the tradable goods industry for the skilled workers under the profit maximization hypothesis, $(1 + g_t)\widetilde{W}_t$, the term, $d\widetilde{W}_t/\widetilde{W}_t$, in our original distribution law equation should be substituted by $\frac{dg_t}{1+g_t} + \frac{d\widetilde{W}_t}{\widetilde{W}_t}$. In this case, dg_t means its changes responding to the exchange rate fluctuations accompanied usually by inversely fluctuating labor demand growth rates to keep the growth of total wage ($[1 + g_t]\widetilde{W}_t$) growths zero, accurately speaking.

However, a once-and-for-all change could replace these time-varying wage premium changes responding to these exchange rate fluctuations. Such a once-and-for-all change always ensures a higher real wage with less labor demand for skilled workers in the home country than in foreign countries, not to escape for foreign countries, even when the exchange rate in the home currency has a depreciation tendency. This case must admit the unemployment in the tradable goods for skilled workers' labor market, even though their shares are minor to be ignored. We can ignore this change for the sake of simplicity in our arguments.

Note that under the profit maximization hypothesis, the unemployment in the tradable goods industry for skilled workers' labor market always remains, as we know from the distribution law equation in The Distribution Laws Under the Firm's Sales Revenue and Profit Maximization Problems With the Karush-Kuhn-Tucker Approach (see also Appendix C). Some readers might doubt that the profit and sales revenue maximization hypotheses assume the same average productivity hypothesis for the tradable goods industry for skilled workers and that the existence of unemployment in this labor market is wrong. However, it is because the profit maximization hypothesis does not mathematically ensure that the labor demand is equal to the labor supply, while the sales revenue maximization hypothesis does.

Also, remember that the (real) wage rise caused by the technological progress of the tradable goods industry should increase labor force participation through the nominal (real) wage rise, clearly from the above inference (and that in the later section for our labor force participation theory). This technological progress of the tradable goods industry induces the exchange rate depreciation in the Harrod–Balassa–Samuelson effect model. Furthermore, this technological progress causes the above general equilibrium effect that will lure many workers in the home country to escape for the more payable jobs in foreign countries and reduce those in the foreign country who want to work in the home country, as we will see later. However, the home real wage should ultimately rise through inflation. Recall the case of the varying $\tilde{c}^{*}_{LM,\xi,t}$ to the exchange rate fluctuation. Thus, exchange rate depreciation and technological progress of the tradable goods industry as its original cause will complicate labor force participation in the home country.

Anyway, we do not explicitly explore this *international* labor mobility effect on the exchange rate fluctuations further and explore this possibility in this section.

Do Nominal Variables Appear in Our Model?

Finally, recall that the relative price variables appear in this new policy rule (#4), combining the quantity theory of money with the Harrod–Balassa–Samuelson effect model. In the later section, I treat these variables combined with the real variables, like shares, as nominal variables in practice. Thus, in practice, one should treat the nominal variables in our monetary policy rule, but in theory, one should treat them separately as relative prices and shares by real variables.

The Monetary Policy Rule From the Model Analysis in the Previous Section: Conclusions in the Deterministic World

I draw the following two conclusions about the policy rule for monetary policy from the model in The Monetary Policy Rule of the Exchange Rate by Integrating the Quantity Theory of Money and the Harrod–Balassa–Samuelson Effect Model With Two Tradable Goods Produced by Skilled Workers and Unskilled Workers and Nontradable Goods section.

Note that exchange rate stabilization has the merit suggested by Isard (1995/2001): It induces the reductions of uncertainties and fluctuations in the macroeconomy. This stability should always increase social welfare (household's welfare) in the standard new Keynesian models. However, we do not try to prove this idea with concrete models in this paper. Thus, on the side of the home country, to stabilize the exchange rate:

(a) when the technological progress in foreign tradable goods industries is more rapid than that in the home country's tradable goods industries at present, the growth in the home country's quantity of money must be by the amount multiplied by its nominal output share less than that at the previous period, other things being equal.

(b) suppose that the growth of (output of) the tradable goods industry for unskilled workers or the nontradable goods industry is more significant due to an increase in the home country's labor inputs at present than those of the foreign country. Then, the growth in the home country's quantity of money must exceed that of the previous period by the amount multiplied by its nominal output share, other things being equal. Conversely, if that home labor input decreases, the home country's quantity of money should also be reduced from the money growth rule (the k% rule in the previous section, i.e., entire labor input growth matters, as we noted) by the amount multiplied by its nominal output share. The growth of the home tradable industry for skilled workers is assumed to be a minor constant that does not affect our conclusions as it is.

The paradigm governed by the Harrod–Balassa–Samuelson effect model influences the above results even in this policy rule (the combined quantity theory of money).

Also, we assume that our labor force participation theory proposed by Flinn and Heckman (1982) and Heckman (1979) in the later section specifies the available amount of labor inputs for firms to achieve full employment or diminishing equilibrium in the macroeconomy. Recall that the full labor supply (force) always decides the labor input (demand) by establishing the sales revenue maximization hypothesis in the above discussions. This means that the labor inputs should be decided not by the crosspoints of labor demand and supply but by labor supply only.

Furthermore, considering the labor input movements, as we noted, we should consider the other possibility that exchange rate fluctuation would generate the increase or decrease of labor inputs from home as well as abroad once we consider *international* labor mobility. However, we ignore this general equilibrium (feedback) effect for simplicity in this section because this general equilibrium (feedback) effect will be critical only in developing countries. In other countries, it will not (e.g., Stolper & Samuelson, 1941). We do not assume any condition for the real wages of the home and foreign countries.

The above results conclude that the mechanisms of the impact of technological progress on exchange rates (relative prices) differ asymmetrically between home and foreign countries, even if we assume a symmetric equilibrium between home and foreign countries. In that sense, as indicated by the Harrod–Balassa–Samuelson effect, we insist that the *impact* of technological progress on exchange rates (relative prices) differs depending on which technological progress in the tradable goods industries between a home country and a foreign country is faster. Thus, the above results report that the technological progress, more precisely, the technological progress of tradable goods industries in both countries, asymmetrically matters. Furthermore, the labor input of each sector in each country is critical.

Note that the Harrod–Balassa–Samuelson effect predicts that a kind of *heterogeneity* will significantly impact monetary policy (see, e.g., Heckman, 2000, for the importance of this heterogeneity). In addition, the new policy rule seemingly has nominal variables in practice (but never in theory).

As a result, using insight from active domestic monetary policy in our policy rule, we conclude that domestic monetary expansion leads to exchange rate depreciation through mechanisms like the rising price of the nontradable goods industry, assuming the stable velocity of money (no staggered pricing here). The Phillips curve suggests the resulting positive output gaps. Furthermore, nominal wage increases and the resulting technological progress of the home tradable goods industry increases, supposing the equation (#5) or (#6) and if we exclude the firm's effect of its discrete (frequent) revisions of nominal wage (income) contracts.

From our theory, (primary) trends or long-run inflations in the world are initially assumed in our theory, as Beechey et al. (2011) and Esfahani et al. (2024) empirically suggest. Furthermore, these world inflations induce technological progress in the global tradable goods industry. Thus, the home country suffers from the exchange rate appreciation if the domestic technological progress for the tradable goods industry is less than that of the foreign (world) one and other things being equal. Based on empirical facts like the stable long-run inflation expectations in most developed countries suggested by Beechey et al. (2011), this assumption is crucial in our theory. Note that the inflation tendency worldwide should also be attributed to other causes, such as accounting problems suggested by Braun and Di Tella (2004) (see Masuda, 2024a).

In the short run, at least with the *staggered price* dynamics, suppose that the price of nontradable goods does not rise corresponding to the helicopter money such that the real income (i.e., domestic technological progress) does not change correspondingly. Then, as a result, the velocity of money could be decreased to balance our policy rule, different from our previous assumption based on empirical findings for the (long-run) stability of the velocity of money by Friedman (1969/2006). That is, in the short run, we can assume that the velocity of money would fluctuate to ebb. Notwithstanding, seemingly contradictory, this result remains consistent with our previous assumption of a constant velocity of money in the long run.

Note that, as before, we suppose the continual monetary injections in the above discussion to exclude the counter-intuitive possibility that the firms try to offset the inflation pressures by rent-seeking behaviors, even following the optimal conditions. That is *not* based on the sales revenue maximization hypothesis *but* based on the profit maximization hypothesis behavior that lacks the determinant of output level and so on in nature, which is different from the sales revenue maximization hypothesis.

Furthermore, this monetary expansion should be only a *pure* monetary injection, like the helicopter money explained in the previous sections (e.g., Bernanke, 2002; Friedman, 1969/2006). Because inflation induces technological progress and the positive output gaps through the Phillips curve, these pressures might induce the leaders and managers in the firms to deviate from rent-seeking behaviors, with assuming the stable velocity of money and if we exclude the firm's discrete (frequent) revisions of nominal income contracts. This should be held everywhere around the world, including Japan.

Now, we should turn to the labor input case and its relationship with the quantity of money. This paper specifies that firms should increase their corresponding labor inputs to the growth of the labor force under full employment due to the sales revenue maximization hypothesis or the growth of the labor demands under diminishing equilibrium due to the profit maximization hypothesis. From the money growth rule (the k% rule in the former section), the quantity of money should be reduced by just the amount multiplied by its nominal output share when the labor demand decreases, following our theory. As noted, the labor input growth for skilled workers is assumed to be a minor constant and not to affect our conclusions as it is.

Once the monetary authority thinks that this labor input decrease should be compensated by the technological progress of the home tradable goods industry, the increase in the quantity of money, that is, the helicopter money, should be adopted at least to the degree of the decreasing amount of the labor input. However, at the same time, the (continual) impact of, its degree of, and the time perspective for such a monetary expansion should be critical (see Masuda, 2024a). As noted in the following section, our theory does not recommend walking inflations.

The Role of the Long-Run Inflation Expectation and Its Relation to the Velocity of Money

Suppose that the helicopter money is too much (recall Friedman's statement in the opening section; Friedman, 1970). In that case, the possibility of walking inflation exists. In short, the peril of high inflation and the resulting stagflation exists. *Stagflation* is only a problem of the *Phillips curve*, which specifies the tradeoff between inflation and output gaps. The Phillips curve does not usually say anything about the relationship between inflation and technological progress (see Masuda, 2024b, for its exception).

Our theory suggests that the popular stagflations in the 1970s should have been the worst complex, with the higher anticipated inflations (the surge of long-run inflation expectations due to the supply shortage) and technological regress derived from this abruptly generated scarcity (supply shortage) of natural resources (oil shock; see Olson, 1988). Recall Friedman's (1978) natural unemployment rate hypothesis (see also Masuda, 2024b). Therefore, *expectation* is critical.

Then, we propose our theory to explain two different stagflation cases and the different behaviors of the U.S. and Japan in responding to the vast amounts of (helicopter) money that should induce walking inflation. Walsh (1998) argued the game structure between the monetary authority and the public in the literature of inflation bias due to Barro and Gordon (1983a, 1983b). According to him, the (given anchored) *long-run* inflation *expectation* in the Phillips curve (e.g., Kiley, 2008, for the concise discussion of this anchored expected inflation) should not fully accommodate this large amount of (helicopter) money in the short run. Because not the long-run inflation expectation but the *current* inflation rate itself is determined by the changes in the supplied quantity of money in this game. As before, we assume the controllable monetary base and the stable money multipliers (Miron et al., 1994; Walsh, 1998, for details).

Regarding the long-run inflation expectations, Beechey et al. (2011) argue that the long-run inflation expectations in most developed countries are empirically stable. Carvalho et al. (2023) empirically suggest that the long-run inflation expectation is well-anchored when the central bank successfully conducts monetary policy to make the public believe its inflation target and stability.

Cukierman (2000) and Ruge-Murcia (2003) reject the Barro and Gordon (1983a, 1983b) type *symmetric* objective function for the monetary authority *empirically* but *support* some kind of *asymmetric* objective function with respect to ups and downs of inflations and unemployment rates. However, we develop our theory in this section based on the theoretically standard discussion with this traditional *symmetric* objective function for simplicity. Because this asymmetric objective function includes the symmetric objective function in the particular case.

Furthermore, Dennis (2000) suggested that the above objective function is attributed to the chairman of the U.S. Federal Reserve, for example. In addition to the necessity of forward-looking behaviors (Lucas, 1972), this is one of our reasons to emphasize the profound role of individuals in society and history, as Masuda (2024b) suggests.

Thus, corresponding to this increased quantity of money, even if the velocity of money does not change, or at least corresponds to the degree of reduction in the velocity of money, still, in the short run, possibly higher inflations would occur. Furthermore, more real incomes (real permanent and transitory incomes; e.g., Friedman, 1957) would also be realized from the Harrod–Balassa–Samuelson effect through technological progress and the Phillips curve based on optimistic forward-looking sales expectations in firms caused by this induced inflation (recall the Phillips curve; e.g., Masuda, 2024b).

This induced inflation should introduce a lower unemployment rate than the natural one, which might also fluctuate if we consider the effects and resulting increased real incomes and labor force participation. However, in this finite but far reach of this process, the public should notice the monetary authority's incentive for the *inflation bias* to generate not the induced technological progress in the *long run* but the positive output gaps (the cyclical booms) in the *short run* (Friedman, 1978). Note that we have other motives like the financial stability motive and others for this inflation bias (e.g., Cukierman, 1992).

These *anticipated* inflations that prevent generating positive output gaps by raising the long-run inflation expectations and the *unlucky simultaneous* negative supply shocks induced by *wars* in the resource-rich countries result in *stagflations* (e.g., Olson, 1988). In the *first kind* of stagflations, price expectation is crucial for the Phillips curve, which is the key to understanding the mechanism of these stagflations and its welfare loss from the new Keynesian social welfare function (e.g., Woodford, 2003). Recall our inflation and technological progress discussion, the Phillips curve, and the natural unemployment rate hypothesis with our previous discussions and Friedman (1978) (see also Masuda, 2024b).

More precisely, the first kind of stagflation that Friedman (1978) explained had technological regress caused by negative supply shocks with the abrupt supply shortage in the resource market. This shortage simultaneously induces the nominal wage decline caused by this technological regress (of the tradable goods industry). These technological regress introduces the lower potential output growth. Moreover, this nominal wage decline, together with the technological regress of the nontradable goods industry, leads to higher inflations for the price of the nontradable goods industry (see the equation [#5] or [#6]). Also, this shortage raises the price of imported necessities and leads to higher inflation.

This further causes higher long-run inflation expectations due to the potential output decrease, supposing that the current expected output declines together with the resulting decline of potential output in the macroeconomy, whose current output has become relatively high at that time. Then, the nominal wage decline means the demand shortage that affects the firm's forward-looking expectation for their sales and outputs, and more unemployment should be generated. This leads to decreasing or even negative output gaps, in addition to the famous inference of Friedman (1978). Thus, the higher long-run inflation expectation and negative output gaps (and the lower potential output growth) coexist in this first kind of stagflation.

However, the second kind of stagflation also exists. This second case does not accompany the vast supply shortage of resources and only implies inflation with exchange rate depreciation (see the previous section for The Negative Supply Shocks and Inflations section). Still, in the second case, the above real transitory income represents the part of real income related to the output gap (Friedman, 1957): The utilization rates of production factors, like the unemployment rate, which are specified not under the sales revenue hypothesis but only under the profit maximization hypothesis.

In short, higher inflation not caused by negative supply shock should encourage the technological progress of the tradable goods industry and lead to more growth in real incomes. Through the Phillips curve, this inflation also decreases the unemployment rate and increases the part of real income related to the output gaps (recall the overtime pay discussion). Furthermore, from our theory, in total, real incomes should continue to decrease in the short run when inflation does not soothe, although real incomes should grow more in the end (in the long run). This is the standard result in the current U.S.

If the output gaps prior to inflations start to decrease with the disinflationary policy, this case seems like the second kind of stagflation from our theory due to the decreased (not always negative) output gaps, for example. However, economists can separate this phenomenon from the first kind of stagflation in the 1970s by looking at the steadily growing real incomes (GDPs) and the existence of stable long-run inflation expectations.

In addition, the current stubborn U.S. inflation in 2024, irrelevant to the tight monetary policy, should be the lag problem of the slow price expectation adjustments in the Phillips curve and the rising natural rate of real interest due to the increasing velocity of money with steady real income growth. In fact, these are unifyingly interpreted as the high velocity of money problem in the quantity theory of money that is made to realize the high state of the circulated quantity of money that may lead to the equity boom for enlarging the possibility of further technical progress in society (i.e., exploring the *unicorn* stocks) with slow price expectation adjustments. Of course, the (temporal) *meme* stocks example also *exists* in the financial market, as Keynes (1936/1997) suggested as the *beauty contest* example.

Note that higher inflations should make some inflation costs more serious. More precisely, the public should undoubtedly dislike (higher) inflations due to their preference for certainty equivalence. Because (higher) inflations put the public's retirement plans in more unstable and unpredictable conditions. For example, Niehans (1978) and Romer (1996) provided detailed explanations of these inflation costs. Engle (1982) and Friedman (1978) provided empirical evidence of higher inflations with higher variances. Also, as Ropele et al. (2024) and Braun and Di Tella (2004) suggest, higher inflations, as well as the rent-seeking behaviors of the leaders and managers in the firms, induce the misallocation of resources (see also Masuda, 2024a). This will bring disastrous future negative effects to the macroeconomy.

As another view of inflation costs, Erosa and Ventura (2002) propose that the inflation tax should be a regressive tax for the macroeconomy. In this point, when we review the dynamism of inflation-induced technological progress, we should understand that inflation affects real wages heterogeneously (i.e., makes them grow regressively).

For this point, we can consult the discussions for the general equilibrium effects of exchange rate changes on the labor inputs and the once-and-for-all change in the nominal wage of the skilled workers in the tradable goods industry. However, such a phenomenon is explained as the *temporal* (i.e., those in transition [not in steady state] or short-run) phenomenon from our theory.

As we noted, persistent inflations always have a solid pressure to suppress the real incomes in this process, and households should evaluate the current monetary policy by their real purchasing powers (see Appendix D). Thus, this kind of regressive real income growth may lure the critics of the monetary authority's policies in reality. Still, Appendix A will briefly provide the approximated Sen's (1976) poverty measure to estimate these heterogeneous (inflation) effects both in inflationary environments (i.e., consequential trickle-down, regressive real income growth; Son, 2004) and economic crises.

The helicopter money and its induced considerable reduction of the velocity of money should coexist to keep the macroeconomy stable or stagnate, at least in the short run, as in Japan. Also, recall the possibility of rent-seeking or shrinking bank behavior (Masuda, 2024a, for the latter). Note that we partly explained the minor increased velocity of money case accompanied by tightening the quantity of money as the inverted case in The Relationship With the Money Demand Equation section.

In short, as a result, more or less, only a large amount of *ineffective* and *dormant money* will be in the central bank account and the macroeconomy. Recall the stable money multiplier assumption and remember that the velocity of money is stable in the long run as a counterexample (Friedman, 1969/2006).

On the above, here, we explore further the other interpretation for the latter; for example, the money multiplier reduction represented by the reduction of the velocity of money is due to shrinking credits (money) provided by the existing (unstable) banking system. Furthermore, this fragility of the banking system will also bring a rise in risk premia in society, which will somehow increase the velocity (while the unemployment fear decreases the velocity of money). Under the profit maximization hypothesis, deflations or recessions should be the threshold for the macroeconomy to become pessimistic for future sales, and diminishing equilibria strengthened by the tighter monetary policy will make the money multiplier shrink more (recall the procyclicality of the velocity of money; Friedman. 1969/2006).

The velocity of money will decrease totally in the above case with the nominal interest rate effect on the velocity of money (e.g., Miron et al., 1994, for the effect of the banking system on the money multiplier). However, we do not mention it any more in this paper. Because we do not mainly focus on the velocity shocks brought by the banking system and others except for the nominal interest rate (again, see Masuda, 2024a, for the theoretical relations about the velocity of money, banking system, and money multiplier). Note that in the above and this paper, we exclude the role of the error term in the money demand equation for simplicity (see the previous section and Appendix B).

Overall, it is natural to us that our theory supports Summers' (1991) discussion about the optimality of the positive inflation rate using the inflation tax theory, similar to the existing approach from the existence of zero lower bounds of nominal interest rate in the new Keynesian model such as Orphanides and Wieland (1998). However, Summers' (1991) discussion (and Lucas, 2000) is inevitably connected to that of Erosa and Ventura (2002).

Again, we partly explain this phenomenon proposed by Erosa and Ventura (2002) with the real wage fluctuations in transition heterogeneously affected by induced inflations by helicopter money. For this point, again, we can also consult the previous discussions for the general equilibrium effects of exchange rate changes on the labor inputs and the once-and-for-all change in the nominal wage of the skilled workers in the tradable goods industry. Similarly, we can use the approximated Sen's (1976) poverty measure to estimate these heterogeneous (inflation) effects both in the inflationary environments in question (i.e., consequential trickle-down, regressive real income growth; Son, 2004) and economic crises (see Appendix A).

The Empirical Basis for the Constant Returns to Scale Production Function

Finally, regarding the labor input, if we do not specify linear production relations for tradable and nontradable goods industries, then estimating the effect of labor input changes on the exchange rate is very difficult for policymakers. However, once we can assume the linear production relations for all, such an estimation becomes much easier, and policymaking, such as changing the quantity of money by the amount multiplied by its nominal output share, might become a less onerous task than before.

For example, some empirical papers investigate the concrete functional forms for production and sometimes specify the constant returns to scale. However, some controversial discussions remain about the empirical result of the constant returns to scale for the production function and have not entirely settled down. Because such empirical investigations suggest some instability of the result of the constant returns to scale for the production function and different empirical findings (see Burnside et al., 1988; Greene, 2003; Greene, 1997, for the detailed frontier production function discussions).

The Difficulties of the Policy Rule on Exchange Rate Derived in the Previous Section in Policy Practice Dominated by the Risks

The Assumption of Stochastic Process of Outputs, $\ln(Y_t)$ and $\ln(Y_t^*)$, due to Empirical Analyses and Its Implications

Until the previous section, I have dealt with the deterministic world. However, in this section, I will gradually incorporate stochastic factors such as business cycles and measurement errors into the determinants of the previous exchange rate model and discuss how the implications of this model change in terms of the objective of minimizing exchange rate fluctuations (its merit is explained in Isard, 1995/2001).

First, in this section, I consider the case in which the errors in the natural logarithms of output levels, $\ln(Y_t)$ and $\ln(Y_t^*)$, vary. This case means that the output levels, $\ln(Y_t)$ and $\ln(Y_t^*)$, include a (actual) portion that grows at a stochastically variable rate due to technological progress (probably leapfrogging; e.g., Barro & Sala-i-Martin, 1995) and a random factor based on the production factor utilization changes and measurement errors.

The main reason to treat stochastic factors in this section is the practical one that policymakers always face the stochastic variations of technological progress, the velocity of money, and the capacity utilization rate of production factors for such as $L_{l,t}$

for $l = TU, TS, N$, etc., like not in the labor force rate and the unemployment rate, and measurement errors (we can incorporate the additional working hours fluctuations to the above, too). We will explain $ur_{l,t}$ for $l = TU, TS, N$ representing the above stochastic factors for these variables in the later section.

The latter unemployment rate is stochastic under the profit maximization hypothesis, in addition to only not in the labor force rate in the sales revenue maximization hypothesis. However, the profit maximization hypothesis needs a new (un)employment theory (Appendix C) structurally to determine absolute labor input levels, like ours in this paper. This new (un)employment theory suggests that the unemployment rate depends on the firm's forward-looking behaviors for their sales (outputs; e.g., Cuba-Borda & Singh, 2024). Because the labor input levels are indeterminate under the profit maximization hypothesis with the constant returns to scale production function.

In other words, setting $L^{+}_{l,t}$ for $l = TU, TS, N$ as the positive deterministic part and the mean of the stochastic part of labor input, $L_{l,t}$ for $l = TU, TS, N$, one can further decompose each of the variables in The Equation for Determining the Exchange Rate Based on the Quantity Theory of Money section as follows:

$$L_{l,t} = ur_{l,t} L^{+}_{l,t} \text{ for } l = TU, TS, N,$$

where the mean of the stochastic variable, $\ln(ur_{l,t})$ for $l = TU, TS, N$, is assumed to be zero. Because this stochastic variable is assumed to follow the lognormal distribution normalized to become the standard normal distribution after taking a natural logarithm for simplicity. This assumption does not affect the results in The Harrod–Balassa–Samuelson Effect Model With Skilled and Unskilled Workers section.

Assume the autoregressive moving average (ARMA) process for entire output levels, $\ln(Y_t)$ and $\ln(Y_t^*)$, in this section. This is equivalent to assuming that the above stochastic process of the utilization rate, $ur_{l,t}$ for $l = TU, TS, N$, etc., and/or the stochastic processes of the technological progress, $dA_{T,t}/A_{T,t}$ and $dA_{N,t}/A_{N,t}$, and so on are included in this ARMA process.

Moreover, $L_{l,t}^{+}$ for $l = TU, TS, N$ is considered to be a constant weight in this paper for simplicity. Assuming this time-varying $L_{l,t}^{+}$ for $l = TU, TS, N$, for example, only makes us get the complicated formula with more possible measurement errors. Therefore, we should ignore this time-varying property safely for our convenience in this paper.

Note that this stochastic variable, $ur_{l,t}$ for $l = TU, TS, N$, the utilization rate with the measurement error (we will explain it later) in the above should represent both the not in the labor force rate and the unemployment rate case by case. The not in the labor force rate can also mean the utilization rate of the adult population due to the stochastic nature of our labor force participation theory.

To be more precise, following Hamilton (1994), the covariance stationary ARMA process with the absolutely summable autocovariance for output, $\ln(Y_t)$, is assumed, like:

$$\ln(Y_t) = a_1 + a_2 t + \sum_{j=0}^{p} \delta_{j+1} \ln(Y_{t-j-1}) + \sum_{k=0}^{q} \varphi_{k+1} u_{t-k-1},$$

where a_1 is constant from the normalization of u_t, a_2, δ_{j+1} whose integer variable, $j \in [0, p]$, and φ_{k+1} whose integer variable, $k \in [0, q]$, are parameters, t is the time trend, and $u_t \sim i.i.d. N(0, \sigma^2)$. Regarding that equation in the above for the home country case and the below equation for the foreign country case, we assume that parameters are constant and do not have any uncertainties when we consider the following *filter* problem.

According to Harvey (1981/1985), the filter is defined as:

$$X(\theta) = \frac{1 + \sum_{j=0}^{q} \varphi_{j+1} \exp(-i\theta j)}{1 - \sum_{j=0}^{p} \delta_{j+1} \exp(-i\theta j)},$$

where i is the imaginary number.

Then, the population spectrum of output level, $\ln(Y_t)$, is expressed as $f_y(\theta) = \frac{\sigma^2}{2\pi} \frac{\left|1 + \sum_{j=0}^{q} \varphi_{j+1} \exp(-i\theta j)\right|^2}{\left|1 - \sum_{j=0}^{p} \delta_{j+1} \exp(-i\theta j)\right|^2}$, supposing the population spectrum. However, the sample periodogram can be used. This difference exists in the estimation problem (e.g., Hamilton, 1994). Note that the complex-conjugate is used to calculate the absolute values of complex variables powered by two (see any introductory textbooks of complex analysis) and have $\cos(\theta) = 0.5[\exp(i\theta) + \exp(-i\theta)]$ from the Euler's formula (e.g., Harvey, 1981/1985).

Then, following Hamilton (1994), one can define and obtain:

$$f_x(\theta) = h[\exp(-i\theta)]h[\exp(i\theta)]f_y(\theta),$$

where $x_t = 100[\ln(Y_t) - \ln(Y_t)]$, $h[\exp(-i\theta)]h[\exp(i\theta)] = 2 - 2\cos(\theta)$.

Hamilton (1994) described that the above time series, $\ln(Y_t)$, could be represented by the stochastic process with sums of the error terms multiplied by sine and cosine of any fixed frequency, θ, in the domain $[0, \pi]$ (the *spectral representation* theorem). However, I do not show the proof. The above procedure leads to *band-pass* filters (e.g., Christiano & Fitzgerald, 1999). In short, it makes business cycles represented by these ARIMA models show *cycles* for sure.

Thus, the models in this section should express *business cycles* based on this fact. These band-pass filters are sometimes used to estimate business cycle components, that is, output gaps and resulting low-frequency components (i.e., potential outputs). For another example, Lucas (1980) uses this spectrum analysis to analyze the quantity theory of money with empirical data.

Similarly, for $\ln(Y_t^*)$, one can define:

$$\ln(Y_t^*) = a_1^* + a_2^* t + \sum_{n=0}^{b} \delta_{n+1}^* \ln(Y_{t-n-1}^*) + \sum_{o=0}^{v} \varphi_{o+1}^* u_{t-o-1}^*,$$

where a_1^* is constant from the normalization of u_t^*, a_2^*, δ_{n+1}^* whose integer variable, $n \in [0, b]$, and φ_{o+1}^* whose integer variable, $o \in [0, v]$, are parameters, t is the time trend, and $u_t^* \sim i.i.d.N(0, {\sigma^*}^2)$.

Not only u_t and u_t^* but also (temporal) economic growth, $a_2 t$ and $a_2^* t$, and so on could cause random fluctuations from our discussion above. Because the (time-varying) technological progress and such could cause (time-varying) changes of a_2 and a_2^* for the home and foreign growth of outputs in every period t.

They are originally treated as the stochastic parameters (i.e., parameter uncertainty) and interpreted the remaining residual excluded that are already explained by nonstationary parts (e.g., $a_2 t$) as utilization rates (innovations) and measurement errors, as we will note in the following. Note that in the above, we define innovations and measurement errors as united *unique* variables, u_t and u_t^*, for simplicity.

In short, in this paper, it is safe and convenient to mix a measurement error and not in the labor force rate (this rate is not always equal to the *population growth* itself) or an additional unemployment rate to it as *one* error term in our exchange rate rule as such. This is done by assuming the positive constant as its expected value before normalizing these error terms only without a measurement error whose mean is initially null for our analytical purpose. This assumption should not complicate our discussions.

These are some causes of the real business cycles (Kydland & Prescott, 1982; e.g., Romer, 1996, for its concise explanations) and our monetary fluctuations due to the monetary authority's misperceptions of them and from the desirability of positive inflations and its policy response by the monetary authority (i.e., inflation bias; see the previous discussion). Thus, considering the labor force participation theory and so on and the measurement error issues explained in the next section, the misperception of $a_2 t$ and $a_2^* t$ in the above sense, for example, will also induce the misconduct of monetary policy in reality if the perspectives of this section are applied. In the following sections, I discuss this kind of misconduct by decomposing and misestimating the effects of changing a_2 and a_2^* substantially.

As shown in the following discussions, the order of the autoregressive moving average process for $\ln(Y_t)$ and $\ln(Y_t^*)$ is not essential in theory. However, controversy regarding this order might empirically matter, more precisely, for the estimation problem and the concrete measurement of policy effects (e.g., Sims, 1971).

Also, as above, we assume the stationary time series for the outputs of home and foreign countries. Note that the trend stationarity and the difference stationarity are indistinguishable in theory. This is empirically essential but theoretically not, at least in our theory, which assumes stationarity (e.g., Sims, 1971).

The Practical Case of Randomness of and Measurement Errors in the Growth Rates of Outputs and Productivities of Tradable and Nontradable Goods Industries, Velocities of Money, and Labor Force Participation in Two Countries

The policy rule for monetary policy derived in The Monetary Policy Rule of the Exchange Rate by Integrating the Quantity Theory of Money and the Harrod–Balassa–Samuelson Effect Model With Two Tradable Goods Produced by Skilled Workers and Unskilled Workers and Nontradable Goods section can be rearranged simply as follows:

$$\frac{d\widetilde{M}_t}{\widetilde{M}_t}-\frac{d\widetilde{M}_t^*}{\widetilde{M}_t^*}=\left(\frac{dA_{T,t}}{A_{T,t}}-\frac{dA_{T,t}^*}{A_{T,t}^*}\right)+\frac{dV_t^*}{V_t^*}-\frac{dV_t}{V_t}+\left(\frac{Y_{TU,t}}{\tilde{P}_tY_t}\frac{dL_{TU,t}}{L_{TU,t}}-\frac{Y_{TU,t}^*}{\tilde{P}_t^*Y_t^*}\frac{dL_{TU,t}^*}{L_{TU,t}^*}\right)+\left(\frac{\tilde{P}_{TS,t}Y_{TS,t}}{\tilde{P}_tY_t}\frac{dL_{TS,t}}{L_{TS,t}}-\frac{\tilde{P}_{TS,t}^*Y_{TS,t}^*}{\tilde{P}_t^*Y_t^*}\frac{dL_{TS,t}^*}{L_{TS,t}^*}\right)+\left(\frac{\tilde{P}_{N,t}Y_{N,t}}{\tilde{P}_tY_t}\frac{dL_{N,t}}{L_{N,t}}-\frac{\tilde{P}_{N,t}^*Y_{N,t}^*}{\tilde{P}_t^*Y_t^*}\frac{dL_{N,t}^*}{L_{N,t}^*}\right).$$

In this section, we consider the velocity of money explicitly, implicitly implying the additional positive small fluctuations like $\beta_2(dr_t+d\pi_t)$ based on the money demand equation for the home country (and the same for the foreign country), suggested in the previous section. However, we never discuss this additional term explicitly and in detail. Because such a consideration should only complicate our discussions again.

Recall that the monetary policy in this paper is defined as controlling the growth rate of the quantity of money in the first place. However, even if the above policy rule is correct, it is unrealistic to assume that policymakers always have perfect and accurate knowledge of (the growth rate of) the technological progress of tradable goods and nontradable goods industries, given the practical difficulties of measuring them from industry-specific data.

Additionally, (the growth rates of) the velocities of money at home and abroad are challenging to measure accurately from empirical data. Furthermore, they dynamically fluctuate to the output changes by the interest rate controlling policy, equivalent to controlling quantities of money in the short run to smooth the output variations if the monetary authority is Keynesian (see the previous discussion about the policy tools for the monetary authority). In the above formula, the following is typically used:

$$\frac{dV_t}{V_t} = \mathrm{E}(c_t),\ \frac{dA_{T,t}}{A_{T,t}} = \mathrm{E}(e_{T,t}),\ \frac{dA_{N,t}}{A_{N,t}} = \mathrm{E}(e_{N,t}),\ \frac{dV_t^*}{V_t^*} = \mathrm{E}(c_t^*),\ \frac{dA_{T,t}^*}{A_{T,t}^*} = \mathrm{E}(e_{T,t}^*),\ \frac{dA_{N,t}^*}{A_{N,t}^*} =$$

$$\mathrm{E}(e_{N,t}^*),\ \frac{dL_{TU,t}}{L_{TU,t}} = \mathrm{E}(\omega_{TU,t}),\ \frac{dL_{TS,t}}{L_{TS,t}} = \mathrm{E}(\omega_{TS,t}),\ \frac{dL_{N,t}}{L_{N,t}} = \mathrm{E}(\omega_{N,t}),\ \frac{dL_{TU,t}^*}{L_{TU,t}^*} = \mathrm{E}(\omega_{TU,t}^*),$$

$$\frac{dL_{TS,t}^*}{L_{TS,t}^*} = \mathrm{E}(\omega_{TS,t}^*), \text{ and } \frac{dL_{N,t}^*}{L_{N,t}^*} = \mathrm{E}(\omega_{N,t}^*),$$

where E is the expectation operator, and c_t, $e_{T,t}$, $e_{N,t}$, c_t^*, $e_{T,t}^*$, $e_{N,t}^*$, $\omega_{TU,t}$, $\omega_{TS,t}$, $\omega_{N,t}$, $\omega_{TU,t}^*$, $\omega_{TS,t}^*$, and $\omega_{N,t}^*$ are the stochastic variables whose means are constant.

Note that c_t and c_t^* are initially the functions of the home and foreign nominal interest rates from the previous discussion of the money demand equation like the term

$\beta_2(dr_t + d\pi_t)$ and its foreign country's version term. However, this section specifies these terms as stochastic errors because these fluctuations are empirically minor. For example, if we separate these effects by case, we have three cases.

(A) if $dM_t - d\bar{M}_t > 0$, $dr_t < 0$ and $d\pi_t \geq 0$ in the short run, while $dr_t \leq 0$ and $d\pi_t > 0$ in the long run. The summed-up effect for $dr_t + d\pi_t$ depends on the magnitudes of the injected quantity of money and the price movements (recall the IS and Phillips curves), and is empirically determined.

(B) if $dM_t - d\bar{M}_t = 0$, $dr_t = 0$ and $d\pi_t = 0$ both in the short run and in the long run.

(C) if $dM_t - d\bar{M}_t < 0$, $dr_t > 0$ and $d\pi_t \leq 0$ in the short run, while $dr_t > 0$ and $d\pi_t < 0$ in the long run. The summed-up effect for $dr_t + d\pi_t$ depends on the magnitudes of the injected quantity of money and the price movements as above, and is empirically determined.

These separations by three cases are quite complicated, and their fluctuations depend on the concrete states of the financial markets and the macroeconomy. All are empirically small. Thus, these facts theoretically allow us to represent these resulting fluctuations as random errors.

In this section, unlike the previous sections, we incorporate stochastic factors for every variable in our exchange rate rule. After taking the expectations of both sides and transforming the above theoretical policy rule into the equation with the empirically obtainable variables, the following equation (#8) is obtained:

$$\mathrm{E}\left(\frac{d\tilde{M}_t}{\tilde{M}_t}\right) - \mathrm{E}\left(\frac{d\tilde{M}_t^*}{\tilde{M}_t^*}\right) = \mathrm{E}(c_t^*) - \mathrm{E}(c_t) + \frac{Y_{N,t}}{Y_t}\mathrm{E}(e_{T,t} - e_{N,t}) - \frac{Y_{N,t}^*}{Y_t^*}\mathrm{E}(e_{T,t}^* - e_{N,t}^*) + \frac{dY_t}{Y_t} - \frac{dY_t^*}{Y_t^*} + \left[\left(\frac{Y_{TU,t}}{\tilde{P}_t Y_t} - \frac{Y_{TU,t}}{Y_t}\right)\mathrm{E}(\omega_{TU,t}) - \left(\frac{Y_{TU,t}^*}{\tilde{P}_t^* Y_t^*} - \frac{Y_{TU,t}^*}{Y_t^*}\right)\mathrm{E}(\omega_{TU,t}^*)\right] + \left[\left(\frac{\tilde{P}_{TS,t}Y_{TS,t}}{\tilde{P}_t Y_t} - \frac{Y_{TS,t}}{Y_t}\right)\mathrm{E}(\omega_{TS,t}) - \left(\frac{\tilde{P}_{TS,t}^* Y_{TS,t}^*}{\tilde{P}_t^* Y_t^*} - \frac{Y_{TS,t}^*}{Y_t^*}\right)\mathrm{E}(\omega_{TS,t}^*)\right] + \left[\left(\frac{\tilde{P}_{N,t}Y_{N,t}}{\tilde{P}_t Y_t} - \frac{Y_{N,t}}{Y_t}\right)\mathrm{E}(\omega_{N,t}) - \left(\frac{\tilde{P}_{N,t}^* Y_{N,t}^*}{\tilde{P}_t^* Y_t^*} - \frac{Y_{N,t}^*}{Y_t^*}\right)\mathrm{E}(\omega_{N,t}^*)\right],$$

where $\frac{dY_t}{Y_t} \cong \ln(Y_t) - \ln(Y_{t-1}) = a_1 + a_2 t + \sum_{j=0}^{p} \delta_{j+1} \ln(Y_{t-j-1}) + \sum_{k=0}^{q} \varphi_{k+1} u_{t-k-1} - [a_1 + a_2(t-1) + \sum_{j=0}^{p} \delta_{j+1} \ln(Y_{t-j-2}) + \sum_{k=0}^{q} \varphi_{k+1} u_{t-k-2}]$,

$\frac{dY_t^*}{Y_t^*} \cong \ln(Y_t^*) - \ln(Y_{t-1}^*) = a_1^* + a_2^* t + \sum_{n=0}^{b} \delta_{n+1}^* \ln(Y_{t-n-1}^*) + \sum_{o=0}^{v} \varphi_{o+1}^* u_{t-o-1}^* - [a_1^* + a_2^*(t-1) + \sum_{n=0}^{b} \delta_{n+1}^* \ln(Y_{t-n-2}^*) + \sum_{o=0}^{v} \varphi_{o+1}^* u_{t-o-2}^*]$.

In this formulation, the technological progress of the nontradable goods industry reappears, different from the deterministic model in The Monetary Policy Rule of the Exchange Rate by Integrating the Quantity Theory of Money and the Harrod–Balassa–Samuelson Effect Model With Two Tradable Goods Produced by Skilled Workers and Unskilled Workers and Nontradable Goods section. Thus, limiting interest to the mostly empirically observable variables makes this policy rule similar to the original Harrod–Balassa–Samuelson model introduced by Samuelson (1964) and Obstfeld and Rogoff (1997).

Furthermore, in the above, considering the need for simplicity, we assume the linearity of the production relationships. Suppose that we assume the *increasing* returns to scale for the production relation, $F_{TS,t} = L_{TS,t}^{1+\gamma}$, where $\gamma > 0$ while we assume the *constant* returns to scale (linearity) to the other production relation, $F_{TU,t}$ and $F_{N,t}$. Then, we just obtain the reduced relation such as $\frac{d\tilde{P}_{TS,t}}{\tilde{P}_{TS,t}} = -\gamma \frac{dL_{TS,t}}{L_{TS,t}}$ (#7).

However, it is natural that this result does not matter in the above policy rule due to the minor constancy of γ and $dL_{TS,t}/L_{TS,t}$, at least not-so-increasing-property due to the constrained capacity in the macroeconomy for the latter, in the theoretical meanings (see the discussion in Appendix C for the other possibility of the time-varing γ). For example, even if we assume the increasing returns to scale to the production relation for skilled workers in the tradable goods industry only as before, we have the alternative term:

$$\left\{\left[\frac{\tilde{P}_{TS,t}Y_{TS,t}}{\tilde{P}_t Y_t} - (1+\gamma)\frac{Y_{TS,t}}{Y_t}\right]\mathrm{E}(\omega_{TS,t}) - \left[\frac{\tilde{P}^*_{TS,t}Y^*_{TS,t}}{\tilde{P}^*_t Y^*_t} - (1+\gamma^*)\frac{Y^*_{TS,t}}{Y^*_t}\right]\mathrm{E}(\omega^*_{TS,t})\right\},$$

for the $\mathrm{E}(\omega_{TS,t})$ and $\mathrm{E}(\omega^*_{TS,t})$ terms in our expected version exchange rate stabilization rule (#8). This fact does not change the following conclusions. Such a simplification excluding an increasing return to scale does not alter our conclusions even because the same conclusion in the previous section holds.

The Difficulty of Predicting the Labor Input Growth

First, we focus on the labor input growths $\omega_{TU,t}$, $\omega_{TS,t}$, $\omega_{N,t}$, $\omega^*_{TU,t}$, $\omega^*_{TS,t}$, and $\omega^*_{N,t}$. These variables are predicted by the theory of labor force participation (e.g., Flinn & Heckman, 1982) and our new (un)employment theory. Remember that we explain this labor force participation theory in the previous section. Here and hereafter, we mainly argue its aggregate, empirical, and stochastic natures.

For this, we initially need the microdata for empirical analysis, which the statistical agency often keeps secret from other governmental agencies. Because the labor force participation theory usually insists on the importance of the nominal *reservation wage*, RW_t. However, that is hard to be estimated. After all, this reservation wage is sometimes stochastic (e.g., Maddala, 1983; Masuda, 2024a, for this kind of theoretical discussion), which decides one's labor force participation once the real market wage exceeds that real one.

Hereafter, we interchangeably describe this reservation wage as the aggregate, averaged like the other nominal wages and treated like so in case of obtaining the marginal effect in the (segmented) aggregate equations shown in the following and implications for our policy rule. This paragraph and the previous section propose the theory for individuals, and the aggregate one will be mathematically explained in detail later.

Due to the above empirical complications, this theory still needs to be completed. Predicting these variables theoretically in advance is difficult even in developed countries with data-rich environments (e.g., Bernanke & Boivin, 2003), together with the above secrecy situation. In this paper, we refer to this theory less in actual policymaking discussions.

In reality, not using the rigorous approach of microdata with microeconometrics but mainly using the cohort trends of the categorized labor force participation rate in the labor force survey is still essential in practice (e.g., Shackleton, 2018). For example, Montes (2018), who provides a sound basis for Shackleton's (2018) potential output estimation method with the labor force participation, argues that Shackleton (2018) uses 60,000 samples from the Current Population Survey in the U.S. for its monthly publication and segments them with the total 516 pairs by age-sex-education-race.

As Stoker (1984) suggests, to estimate the micro-relational parameters from the aggregate data, we should require sufficient observations on the *consistent* statistical moments estimates necessary for estimating these labor force participation equations. Because these labor force participation equations are usually represented as the *Probit* (or Heckit, as its extension; e.g., Heckman, 1979).

Thus, we have the *aggregation problem* for estimating labor force participation

(Stoker, 1993). Notwithstanding, in this estimation, Stoker (1984) argues that segmentation of the aggregate data (the segmented aggregate data) would not cause a serious estimation problem in recovering the micro-relational parameters. Thus, the current labor force participation theory results in one branch of ordinal estimates. By representing it as the bundle of measurement errors issue, we can safely recognize the stochastic nature of this labor force participation theory and the stochasticity of an additional unemployment rate to this not in the labor force rate.

Our Labor Force Participation Theory

As explained similarly in the previous section about the foreign worker ξ where the integer variable $\xi \in [1, NP_t^*]$ case (i.e., the case with foreign people who can work in the home country), our labor force participation theory has two steps: Individual and aggregate. First, take the *individual* domestic worker case as our example for the more simplified function without considering the entries in the foreign labor market, which is slightly different from the previous section about foreign workers. This case is simpler and easier to understand than the previous section.

Our theory in this section uses only industry segmentation, including three industries in this paper, and does not use more segmentations, as suggested by Shackleton (2018), without a loss of generality. In addition, as noted above, to connect the labor force participation theory in this section to our new (un)employment theory in Appendix C and so on, we should take the expectation of the real wages in our formulation of this section. However, we ignore such a mathematical operation here for our notational convenience. That randomness will be the other source of labor force participation fluctuations that we focus on for the (erroneous) monetary policy conducts in this paper. We will explain the necessity of this randomness assumption in detail in Appendix C.

Then, the *individual* labor force participation function for the unskilled worker ι in the tradable goods industry, $\omega_{TU,\iota,t}$, takes *one* when $\frac{\widetilde{W}_{\iota,t}}{\tilde{P}_t} \geq \frac{\widetilde{RW}_{TU,\iota,t}}{\tilde{P}_t}$, and *zero* otherwise. This labor force participation theory holds symmetrically for workers in other industries in both home and foreign countries.

Thus, we should consider the *aggregation* such that:

$$\omega_{TU}\left(\frac{\widetilde{W}_t}{\tilde{P}_t} \colon \frac{\widetilde{RW}_{TU,t}}{\tilde{P}_t}\right) = \omega_{TU}(t) = \frac{\sum_{\iota=1}^{NP_{TU,t}} \omega_{TU}\left(\frac{\widetilde{W}_t}{\tilde{P}_t} \colon \frac{\widetilde{RW}_{TU,\iota,t}}{\tilde{P}_t}\right)}{NP_{TU,t}} \text{ with } \frac{\widetilde{RW}_{TU,t}}{\tilde{P}_t} = \frac{\sum_{\iota=1}^{NP_{TU,t}} \frac{\widetilde{RW}_{TU,\iota,t}}{\tilde{P}_t}}{NP_{TU,t}},$$

$$\omega_{TS}\left[\frac{(1+g)\widetilde{W}_t}{\tilde{P}_t} \colon \frac{\widetilde{RW}_{TS,t}}{\tilde{P}_t}\right] = \omega_{TS}(t) = \frac{\sum_{\varsigma=1}^{NP_{TS,t}} \omega_{TS}\left[\frac{(1+g)\widetilde{W}_t}{\tilde{P}_t} \colon \frac{\widetilde{RW}_{TS,\varsigma,t}}{\tilde{P}_t}\right]}{NP_{TS,t}}$$

$$\text{with } \frac{\widetilde{RW}_{TS,t}}{\tilde{P}_t} = \frac{\sum_{\varsigma=1}^{NP_{TS,t}} \frac{\widetilde{RW}_{TS,\varsigma,t}}{\tilde{P}_t}}{NP_{TS,t}},$$

$$\omega_{N}\left(\frac{\widetilde{W}_t}{\tilde{P}_t} \colon \frac{\widetilde{RW}_{N,t}}{\tilde{P}_t}\right) = \omega_{N}(t) = \frac{\sum_{\varrho=1}^{NP_{N,t}} \omega_{N}\left(\frac{\widetilde{W}_t}{\tilde{P}_t} \colon \frac{\widetilde{RW}_{N,\varrho,t}}{\tilde{P}_t}\right)}{NP_{N,t}} \text{ with } \frac{\widetilde{RW}_{N,t}}{\tilde{P}_t} = \frac{\sum_{\varrho=1}^{NP_{N,t}} \frac{\widetilde{RW}_{N,\varrho,t}}{\tilde{P}_t}}{NP_{N,t}},$$

for each worker ι, ς, and ϱ in the industry $l = TU, TS, N$ case, where for $l =$

TU, TS, N, $\widetilde{RW}_{l,t} = \frac{RW_{l,t}}{P_{TU,t}}$, $RW_{l,t} \in \mathbb{R}_+$ for any t.

$NP_{TU,t} \in \mathbb{R}_+$ for any t is the number of the adult population in the home country as the potential unskilled workers for the tradable goods industry, $NP_{TS,t} \in \mathbb{R}_+$ for any t is the number of the adult population in the home country as the potential skilled workers for the tradable goods industry, and $NP_{N,t} \in \mathbb{R}_+$ for any t is the number of the potential adult population in the home country as the potential workers for the nontradable goods industry such that $NP_{1,t} = NP_{TU,t} + NP_{N,t}$ where $NP_{1,t}$ is the total number of the adult population in the home country as the potential unskilled workers for the tradable goods industry and potential workers for the nontradable goods industry.

One's labor force participation decision might be heterogeneously related to monetary policy. However, excluding the time-varying g_t case, the reservation wages, $\widetilde{RW}_{TU,t}/\tilde{P}_t$, $\widetilde{RW}_{TS,t}/\tilde{P}_t$, and $\widetilde{RW}_{N,t}/\tilde{P}_t$, in the aggregate functions are the arguments of the (nonlinear) aggregate labor force participation functions and are not in linear in these aggregate functions such that they should not be equivalent to those in the individual functions, $\widetilde{RW}_{TU,\iota,t}/\tilde{P}_t$, $\widetilde{RW}_{TS,\varsigma,t}/\tilde{P}_t$, and $\widetilde{RW}_{N,\varrho,t}/\tilde{P}_t$, in general. However, for our convenience, in the above, we assume this linearity without the loss of generality and represent them with the averages of the individual reservation wages, respectively. Note that this is a kind of aggregation for obtaining the marginal effects of the Probit (see Stoker, 1984, for the segmentation of the aggregate data above; Greene, 2003). This procedure does not impair the attempt for the recovery of micro-relational parameters such that it is well known that we can avoid the aggregation problem with this step.

Furthermore, $\widetilde{RW}_{TU,t}/\tilde{P}_t$ is not always the same as $\widetilde{RW}_{N,t}/\tilde{P}_t$. Because our labor force (supply) participation theory admits the further heterogeneity between unskilled workers in the tradable goods industry and workers in the nontradable goods industry in terms of the reservation wage. It does not contradict the homogeneity assumption regarding their labor demands (see Appendix C).

At last, the corresponding aggregate functions to the above individual labor force participation theory are represented as the weighted shares such that we obtain:

$$\omega_{TU,t} = \omega_{TU}\left(\frac{\widetilde{W}_t}{\tilde{P}_t} : \frac{\widetilde{RW}_{TU,t}}{\tilde{P}_t}\right)\frac{dNP_{TU,t}}{NP_{TU,t}},$$

$$\omega_{TS,t} = \omega_{TS}\left[\frac{(1+g)\widetilde{W}_t}{\tilde{P}_t} : \frac{\widetilde{RW}_{TS,t}}{\tilde{P}_t}\right]\frac{dNP_{TS,t}}{NP_{TS,t}},$$

$$\omega_{N,t} = \omega_{N}\left(\frac{\widetilde{W}_t}{\tilde{P}_t} : \frac{\widetilde{RW}_{N,t}}{\tilde{P}_t}\right)\frac{dNP_{N,t}}{NP_{N,t}},$$

where $0 \leq \omega_{l,t} \leq 1$ for $l = TU, TS, N$, and the total adult population in the home country is defined as $NP_t = \sum_{l=TU,TS,N} NP_{l,t}$.

Because we assume that our labor force participation theory in this section specifies the shares (i.e., average labor force participation rates) of the available labor force in the populations for firms to represent full employment, labor supply, in the macroeconomy. Thus, because the labor supply is determined by the above labor force participation theory, we obtain $L_{1,t} = \omega_{TU}(t)NP_{TU,t} + \omega_N(t)NP_{N,t}$, $L_{2,t} = \omega_{TS}(t)NP_{TS,t}$, and $L_t = \omega_{TU}(t)NP_{TU,t} + \omega_N(t)NP_{N,t} + \omega_{TS}(t)NP_{TS,t}$. These are equal to those under the sales revenue maximization hypothesis in this paper such that $L_{TU,t} = \omega_{TU}(t)NP_{TU,t}$, $L_{TS,t} = \omega_{TS}(t)NP_{TS,t}$, and $L_{N,t} = \omega_N(t)NP_{N,t}$ (here, we ignore the friction unemployment).

Thus, for $l = TU, TS, N$, we obtain the not in the labor force rate defined as $uet_{l,t} = 1 - \omega_l(t)$ and the overall not in the labor force rate, uet_t, is calculated as $1 - \frac{\sum_{l=TU,TS,N}[1-\omega_l(t)]NP_{l,t}}{NP_t}$ where $0 \leq 1 - \frac{\sum_{l=TU,TS,N}[1-\omega_l(t)]NP_{l,t}}{NP_t} \leq 1$. Furthermore, $uu_{l,t}$ includes the unemployment rate (in our above sense, i.e., in the labor force), $urt_{l,t}$, in the industry l for $l = TU, TS, N$ in addition to the not in the labor force rate, $uet_{l,t}$, in the industry l for $l = TU, TS, N$. Then, the overall unemployment rate in the usual *labor statistics*, uu_t, is defined as $uu_t = \frac{\sum_{l=TU,TS,N}\omega_l(t)urt_{l,t}NP_{l,t}}{NP_t}$ where $0 \leq \frac{\sum_{l=TU,TS,N}\omega_l(t)urt_{l,t}NP_{l,t}}{NP_t} \leq 1$. Again, we can reinterpret $uu_{l,t} = \omega_l(t)urt_{l,t} = (1 - uet_{l,t})urt_{l,t}$ for $l = TU, TS, N$ as the unemployment rate in the macroeconomy for $l = TU, TS, N$, as in the usual labor statistics.

Moreover, once we consider international labor mobility, the above relational functions for the foreign workers are modified as in the previous section. Based on the previous explanations for the individual labor force participation theory of the foreign workers, after aggregating such functions similarly in the above standard labor force participation theory, we should add the following equations to the above formulae:

$$\omega^*_{H,TU,t} = \omega^*_{H,TU}\left[\frac{\widetilde{W}_t}{\tilde{P}_t}\left(\frac{\tilde{P}^*_t}{\tilde{P}_t}\right) - \tilde{c}^*_{LM,TU,t}, \frac{\widetilde{W}^*_t}{\tilde{P}^*_t} : \frac{\widetilde{RW}^*_{TU,t}}{\tilde{P}^*_t}\right]\frac{dNP^*_{TU,t}}{NP^*_{TU,t}}$$

$$\text{with } \omega^*_{H,TU}(t) = \omega^*_{H,TU}\left[\frac{\widetilde{W}_t}{\tilde{P}_t}\left(\frac{\tilde{P}^*_t}{\tilde{P}_t}\right) - \tilde{c}^*_{LM,TU,t}, \frac{\widetilde{W}^*_t}{\tilde{P}^*_t} : \frac{\widetilde{RW}^*_{TU,t}}{\tilde{P}^*_t}\right],$$

$$\omega^*_{H,TS,t} = \omega^*_{H,TS}\left[\frac{(1+g)\widetilde{W}_t}{\tilde{P}_t}\left(\frac{\tilde{P}^*_t}{\tilde{P}_t}\right) - \tilde{c}^*_{LM,TS,t}, \frac{(1+g^*)\widetilde{W}^*_t}{\tilde{P}^*_t} : \frac{\widetilde{RW}^*_{TS,t}}{\tilde{P}^*_t}\right]\frac{dNP^*_{TS,t}}{NP^*_{TS,t}}$$

$$\text{with } \omega^*_{H,TS}(t) = \omega^*_{H,TS}\left[\frac{(1+g)\widetilde{W}_t}{\tilde{P}_t}\left(\frac{\tilde{P}^*_t}{\tilde{P}_t}\right) - \tilde{c}^*_{LM,TS,t}, \frac{(1+g^*)\widetilde{W}^*_t}{\tilde{P}^*_t} : \frac{\widetilde{RW}^*_{TS,t}}{\tilde{P}^*_t}\right],$$

$$\omega^*_{H,N,t} = \omega^*_{H,N}\left[\frac{\widetilde{W}_t}{\tilde{P}_t}\left(\frac{\tilde{P}^*_t}{\tilde{P}_t}\right) - \tilde{c}^*_{LM,N,t}, \frac{\widetilde{W}^*_t}{\tilde{P}^*_t} : \frac{\widetilde{RW}^*_{N,t}}{\tilde{P}^*_t}\right]\frac{dNP^*_{N,t}}{NP^*_{N,t}}$$

$$\text{with } \omega^*_{H,N}(t) = \omega^*_{H,N}\left[\frac{\widetilde{W}_t}{\tilde{P}_t}\left(\frac{\tilde{P}^*_t}{\tilde{P}_t}\right) - \tilde{c}^*_{LM,N,t}, \frac{\widetilde{W}^*_t}{\tilde{P}^*_t} : \frac{\widetilde{RW}^*_{N,t}}{\tilde{P}^*_t}\right],$$

where $\omega^*_{H,TU,t}$, $\omega^*_{H,TS,t}$, $\omega^*_{H,N,t}$, $\tilde{c}^*_{LM,TU,t}$, $\tilde{c}^*_{LM,TS,t}(> \tilde{c}^*_{LM,TU,t})$, and $\tilde{c}^*_{LM,N,t}(\neq \tilde{c}^*_{LM,TS,t} > \tilde{c}^*_{LM,TU,t})$ show the concrete level of each labor input (force) growth with each additional home labor input (force) share from abroad multiplied by each foreign population growth and the foreign worker's linguistic and moving costs for three industries in the home country normalized by $P^*_{TU,t}$. The foreign worker's normalized reservation wage is $\widetilde{RW}^*_{l,t} = \frac{RW^*_{l,t}}{P^*_{TU,t}}$, $RW^*_{l,t} \in \mathbb{R}_+$ for $l = TU, TS, N$ and for any t in the above. These reservation wages in the above formulation are averaged ones, similar to the former only domestic case (we do not show them mathematically to avoid the trivial repetitions).

In the above formulation, we use the first right bottom subscript HO for the foreign case (and FO for the home case afterward) to recognize our representations easily for the convenience of the readers. Furthermore, we assume that $NP^*_{l,t} \in \mathbb{R}_+$ for $l = TU, TS, N$ and any t, and $\tilde{c}^*_{LM,TU,t}$, $\tilde{c}^*_{LM,TS,t}$, and $\tilde{c}^*_{LM,TU,t}$ are positive for any t. $\widetilde{W}^*_t/\tilde{P}^*_t$ is the real market wage in the foreign country and $1 + g^*$ ($^\forall g^* \in \mathbb{R}_+$) is the wage premium in the foreign labor market for the skilled workers.

$NP^*_{l,t}$ for $l = TU, TS, N$ is the number of the foreign adult population in question respectively such that the total foreign adult population is defined as $NP^*_t = \sum_{l=TU,TS,N} NP^*_{l,t}$. Therefore, the *net* real home wages adjusted by the exchange rate in the foreign currency are defined as these real home wages adjusted by the exchange rate in the foreign currency *minus* the foreign worker's linguistic and moving costs in the foreign currency.

Note that the functions of the additional home labor input (force) shares from abroad as well as the original home labor inputs should be taken the share to the total or detrended for actual estimations. Situations are the same for functions of the below additional foreign labor (force) input shares from home and the original foreign ones.

Also, the better the net real home wage adjusted by the exchange rate in the foreign currency is, the more significant the share of those who work in the home country from abroad becomes. Furthermore, the variable linguistic and moving costs by foreign workers should exist such that comparing the net real home wage adjusted by the exchange rate in the foreign currency with the real foreign wage and the foreign reservation wage for workers from abroad is critical in the above.

Equivalently, we can define:

$$\omega_{FO,TU,t} = \omega_{FO,TU}\left[\frac{\widetilde{W}_t^*}{\tilde{P}_t^*}\left(\frac{\tilde{P}_t}{\tilde{P}_t^*}\right) - \tilde{c}_{LM,TU,t}, \frac{\widetilde{W}_t}{\tilde{P}_t} : \frac{\widetilde{RW}_{TU,t}}{\tilde{P}_t}\right]\frac{dNP_{TU,t}}{NP_{TU,t}}$$

$$\text{with } \omega_{FO,TU}(t) = \omega_{FO,TU}\left[\frac{\widetilde{W}_t^*}{\tilde{P}_t^*}\left(\frac{\tilde{P}_t}{\tilde{P}_t^*}\right) - \tilde{c}_{LM,TU,t}, \frac{\widetilde{W}_t}{\tilde{P}_t} : \frac{\widetilde{RW}_{TU,t}}{\tilde{P}_t}\right],$$

$$\omega_{FO,TS,t} = \omega_{FO,TS}\left[\frac{(1+g)\widetilde{W}_t^*}{\tilde{P}_t^*}\left(\frac{\tilde{P}_t}{\tilde{P}_t^*}\right) - \tilde{c}_{LM,TS,t}, \frac{(1+g)\widetilde{W}_t}{\tilde{P}_t} : \frac{\widetilde{RW}_{TS,t}}{\tilde{P}_t}\right]\frac{dNP_{TS,t}}{NP_{TS,t}}$$

$$\text{with } \omega_{FO,TS}(t) = \omega_{FO,TS}\left[\frac{(1+g)\widetilde{W}_t^*}{\tilde{P}_t^*}\left(\frac{\tilde{P}_t}{\tilde{P}_t^*}\right) - \tilde{c}_{LM,TS,t}, \frac{(1+g)\widetilde{W}_t}{\tilde{P}_t} : \frac{\widetilde{RW}_{TS,t}}{\tilde{P}_t}\right],$$

$$\omega_{FO,N,t} = \omega_{FO,N}\left[\frac{\widetilde{W}_t^*}{\tilde{P}_t^*}\left(\frac{\tilde{P}_t}{\tilde{P}_t^*}\right) - \tilde{c}_{LM,N,t}, \frac{\widetilde{W}_t}{\tilde{P}_t} : \frac{\widetilde{RW}_{N,t}}{\tilde{P}_t}\right]\frac{dNP_{N,t}}{NP_{N,t}}$$

$$\text{with } \omega_{FO,N}(t) = \omega_{FO,N}\left[\frac{\widetilde{W}_t^*}{\tilde{P}_t^*}\left(\frac{\tilde{P}_t}{\tilde{P}_t^*}\right) - \tilde{c}_{LM,N,t}, \frac{\widetilde{W}_t}{\tilde{P}_t} : \frac{\widetilde{RW}_{N,t}}{\tilde{P}_t}\right],$$

for each labor input (force) growth with the additional foreign labor inputs (force) share from home for each three industries in the foreign country multiplied by each home population growth, where $\tilde{c}_{LM,TU,t} > 0$, $\tilde{c}_{LM,TS,t}(> \tilde{c}_{LM,TU,t})$, and $\tilde{c}_{LM,N,t}(\neq \tilde{c}_{LM,TS,t} > \tilde{c}_{LM,TU,t})$ is the home worker's linguistic and moving costs in the domestic currency for three industries in the foreign country. $\widetilde{W}_t/\tilde{P}_t$ and $1+g$ are defined similarly as we explained before. As a result, we obtain:

$$\frac{dL_{TU,t}}{L_{TU,t}} = [\omega_{TU}(t) - \omega_{FO,TU}(t)]\frac{dNP_{TU,t}}{NP_{TU,t}} + \omega^*_{HO,TU}(t)\frac{dNP^*_{TU,t}}{NP^*_{TU,t}},$$

$$\frac{dL_{TS,t}}{L_{TS,t}} = [\omega_{TS}(t) - \omega_{FO,TS}(t)]\frac{dNP_{TS,t}}{NP_{TS,t}} + \omega^*_{HO,TS}(t)\frac{dNP^*_{TS,t}}{NP^*_{TS,t}},$$

$$\frac{dL_{N,t}}{L_{N,t}} = [\omega_{N}(t) - \omega_{FO,N}(t)]\frac{dNP_{N,t}}{NP_{N,t}} + \omega^*_{HO,N}(t)\frac{dNP^*_{N,t}}{NP^*_{N,t}}.$$

This formulation incorporates the total effects of exchange rate fluctuations on the labor force participation because this incorporation constructs the general equilibrium model. In this case, we also obtain:

$$L_{TU,t} = [\omega_{TU}(t) - \omega_{FO,TU}(t)]NP_{TU,t} + \omega^*_{HO,TU}(t)NP^*_{TU,t},$$

$$L_{TS,t} = [\omega_{TS}(t) - \omega_{FO,TS}(t)]NP_{TS,t} + \omega^*_{HO,TS}(t)NP^*_{TS,t},$$

$$L_{N,t} = [\omega_{N}(t) - \omega_{FO,N}(t)]NP_{N,t} + \omega^*_{HO,N}(t)NP^*_{N,t},$$

and other foreign relations should be taken in the similar ways above.

The unemployment rate under the profit maximization hypothesis, different from the above, that is, the labor input ratio to the total population, in this case is defined similarly as before such as the formula considering only the number of the employed (including the foreign ones) to the home total population in that country as many current labor statistics do. Alternatively, it is defined as the formula modifying and adding the highest domestic and foreign possible employment for working abroad term. In this case, we omit mathematically representing this unemployment rate because its derivation is straightforward.

Thus, remember that the (nominal, i.e., resulting real) wage rise caused by the technological progress of the tradable goods industry should increase domestic labor force participation. However, this technological progress in the tradable goods industry induces the exchange rate depreciation in the Harrod–Balassa–Samuelson effect model. Furthermore, this technological progress causes the above general equilibrium effect that will lure many workers in the home country to escape for the more payable jobs in foreign countries and reduce those in the foreign country who want to work in the home country. As a result, exchange rate depreciation and technological progress of the tradable goods industry as its cause will have complicated effects on the labor force participation in the home country (and the foreign country), as we showed above.

Furthermore, consider the technological progress induced by inflation. In that case, this case also ultimately supports the above autonomous technological progress case. However, in the transition of real wage reductions to their ultimate rises, these real wage reductions should decrease labor force participation in the home country. Furthermore, the exchange rate depreciation causes the escape of the home workers to foreign countries and the refrain of the foreign workers from working in the home country.

Note that the unemployment modification also needs additional new (un)employment theory (Appendix C) structurally to determine its absolute levels like ours in this paper, as we stated. However, even if we assume this new (un)employment theory, the unemployment rate itself should be contained in error terms for our convenience in analyzing our new policy rule, as we suggested before. Because the unemployment rate should usually be interpreted as random shocks. This operation and interpretation do not theoretically alter any results in this paper.

Notwithstanding, suppose that these random shocks have a nonzero mean, different from the previous assumption. In that case, the labor demand should determine the labor input level with the firm's severe employment policies (firm's vacancy and unemployment rate policy; see Appendix C) like that, although Masuda (2024c) mainly focuses on (the threat of) the involuntary one.

Thus, the labor input growth rate should have random factors, even in the above case. This nonzero mean property should only be realized for transitions to the other natural unemployment rates. Furthermore, this nonzero mean property of the unemployment rate is finally statistically adjusted by its normalization. Therefore, such a modification should keep our conclusion in this section the same.

Again, in this case, we should look at the stochastic errors of labor force participation as the one, including the unemployment fluctuations, which are normalized to have zero means. The expected constant term generated by the normalizations of errors, $\mathrm{E}(\omega_{l,t})$ for $l = TU, TS, N$, should be modified to include the additional natural unemployment rate as the positive constant.

The domestic labor input growth, both in total and in each industry, does not always have any positive or negative relations to the foreign ones in the developed countries (even if the exchange rate changes more or less). In this paper, it is sufficient to note that labor input (force) growths in separated sectors are difficult to predict accurately and promptly according to the existing theories and data environments, even without considering this general equilibrium effect. Our above theory can theoretically avoid the aggregation problem suggested by Stoker (1984, 1993), as we noted in the previous discussion about its empirical estimation problem.

Suppose that we exclude the counterintuitive possibility of firm leaders' and managers' rent-seeking behaviors only under the profit maximization hypothesis. Then, regarding labor force participation, our inflation theory insists that monetary policy, such as even helicopter money, should work to increase the *nominal* (and, resultantly, *real*) *wage* even in the short run when the nominal wage staggers to fluctuate, which raises labor force participation. Therefore, considering the above complex effects on labor force participation (and unemployment rates), predicting labor force (input) growth is quite tricky for policymakers in terms of theory as well as empirics.

The Heterogeneity and the Difficulty of Measurements of Technological Progress, etc.

Next, in the above policy rule, the variables $e_{T,t}$, $e_{N,t}$, $e^*_{T,t}$, and $e^*_{N,t}$ are not weighted by the industry outputs in question: $Y_{TU,t}$, $Y_{TS,t}$, $Y_{N,t}$, $Y^*_{TU,t}$, $Y^*_{TS,t}$, and $Y^*_{N,t}$ with the entire output levels, Y_t and Y^*_t. Therefore, other things being equal and given the foreign variables, policymakers are forced to react quite differently depending on whether (a) the same randomness (i.e., the aggregate innovations) is occurring in the output growths (that is, technological progresses) of the tradable and nontradable goods industries of the home country, or (b) the different randomness (i.e., the industry-specific innovations) is occurring in the technological progresses of the tradable and/or nontradable goods industries of the home country.

Hereafter, note that the word *shock* instead of *innovation* in the time series literature is usually called the stochastic error in this paper, which means the randomness with the concrete nonzero (not normalized) averages in this section, following the macroeconomics tradition.

If (a) is true, given the foreign variables, one can predict the growth rate of the entire output with aggregate shocks that have nonzero means for $\frac{dY_t}{Y_t} - \frac{dY_t^*}{Y_t^*}$ and the technological progress for the tradable and nontradable industries to some extent (that is, in the expected terms) by looking at empirical data. However, suppose that (b) is correct, given the foreign variables.

Then, different industry-specific shocks with nonzero means occur in the technological progress of the tradable goods industry and the nontradable goods industry of the home country. In this case, policymakers will need to capture the industry-specific shocks in the technological progress of both the tradable goods industry and the nontradable goods industry of the home country separately in addition to the aggregate shocks. However, again, this task is naturally quite tricky for policymakers.

The above discussion implicitly includes the difficulty of predicting labor input growth as a part of output fluctuations, as in the previous part of our arguments. Again, (the growth rates of) the velocities of money at home and abroad are difficult to measure directly from empirical data and dynamically fluctuate to the output changes by the interest rate controlling policy to smooth the output variations, according to the Keynesian idea.

However, at least in the long run, these velocities of money are relatively stable according to monetarists. This stability issue also owes to the definition of money, as we noted (e.g., Friedman, 1969/2006; Masuda, 2024a). However, the continuing controversy between monetarists and Keynesians about the stability of the velocity of money exists (e.g., De Long, 2000). Thus, it is unrealistic to easily assume that policymakers always correctly specify the technological progress (for each industry), the labor inputs of the tradable goods industry for skilled and unskilled workers and nontradable goods industries, and the velocities of money from empirical data. This unreality also provides further evidence of the practical difficulty of exchange rate stabilization by controlling the quantity of money.

From now on, we admit measurement errors only in real outputs (by sectors) and ignore other factors like the velocity of money and others hereafter. Orphanides (2001) suggested that *overall* measurement errors are essential in actual policymaking. Notwithstanding, this consideration simplifies our theory without a loss of generality.

Justifications of Floating and Fixed Exchange Rate Regimes

Suppose that the magnitudes of the growth rates and the business cycles in two countries are equal (a_1 and a_1^*, a_2t and a_2^*t, δ_{j+1} for $j = 0,1,\ldots,p$ and δ_{n+1}^* for $n = 0,1,\ldots,b$, φ_{k+1}, and for $k = 0,1,\ldots,q$ and φ_{o+1}^* for $o = 0,1,\ldots,v$ are equal for all factors). Here, the business cycles are defined as stochastic fluctuations from the mean values of technological progress, the velocity of money, the unemployment rate, and the labor force participation without measurement errors, particularly under our profit maximization hypothesis. If we choose the sales revenue maximization hypothesis, we could just exclude the above *unemployment* fluctuations.

Then, the higher the correlation coefficient (i.e., the degree of the synchronization) between the two countries' business cycles (u_t and u_t^*) becomes, the smaller the excess variation of the home monetary policy becomes, given the countercyclical, cooperated foreign monetary policy with the home monetary policy. Furthermore, the smaller the fluctuation of the (growth rate of) exchange rate becomes.

This makes policy cooperation easier and promotes international coordination for exchange rate stabilization. In short, further strengthening of this policy coordination leads to a *fixed* exchange rate between these two countries and the *optimal currency area* (e.g., Mundell, 2000, for its original idea; Obstfeld & Rogoff, 1995, for its modern treatment). However, note that to develop this theory further, we should require the theory of fiscal policy coordination; that is another story.

Then, suppose that the correlation coefficient falls between null and negative unity. In that case, the domestic and foreign monetary policies should be conducted more and more independently or even inversely and cannot cooperate. This is the following floating exchange rate case.

To be more precise, suppose the countercyclical foreign monetary policy cooperated with the home monetary policy. Then, the more home and foreign business cycles move in opposite directions (i.e., the smaller the correlation coefficient between u_t and u_t^*), the more significant becomes the excess variation of the home monetary policy, the more significant the fluctuation of the (growth rate of) exchange rate becomes. This reduces the incentive for international policy coordination because the home and foreign monetary policies should become more independent. In short, a smaller correlation coefficient supports the *floating* exchange rate in this theory.

When the magnitudes of two business cycles are not equal, and if these two aggregate shocks (cycles) are perfectly synchronized, the random volatility of the more minor home business cycle will not exceed the random volatility of the more significant foreign business cycle. However, suppose that the foreign country's monetary policy is more imprecise than otherwise due to the monetary authority's misperception of their business cycles like that. Then, the excess variation of the home monetary policy and the fluctuations of (growth rate of) exchange rate will be more significant than that to stabilize the exchange rate correctly, even if the home monetary policy is synchronized with the foreign one as possible.

Then, suppose that the monetary policies of the two countries are perfectly reactive and optimal for exchange rate stabilization. In that case, no exchange rate fluctuations will exist, even if the two countries' business cycles are different in size and ideally move inversely. Later, I discuss the theoretical possibility that the policymakers will not spontaneously implement the optimal policy for the exchange rate stabilization and that the home (or the foreign or both) monetary policy naturally fails to stabilize the exchange rate.

In addition, once we assume another heterogenous idiosyncratic shock for the firms in each industry of home and foreign countries (although we ignore this possibility entirely in other parts of this paper) and they are somehow correlated, the above discussions will be further complicated. Thus, the exchange rate fluctuations usually bring more uncertainty than usual for policymakers to respond to if we pursue the reality, which is different from the above theoretical clear-cut considerations.

Regarding less or negative correlation, we infer that the floating exchange rate is suitable for such cases as financially independent or heterogeneous countries. However, this is one example of reality that only focuses on exchange rate stabilization. Because the developed countries have other reasons to adopt the floating exchange rate, such as getting *free hands* for their *domestic* economic problems, as Friedman (1953) suggested.

This is why some countries, like the developed countries, adopt the floating exchange rate regime.

In Appendix D, we argue symmetric trade between two countries as the example of developed countries. This symmetry will weaken the above correlation to become mostly no correlation. Because the trade balance should be net out from the symmetry (recall the GDP identity). However, the actual trade between the two countries should become deeper with voluminous exports and imports than the trade balance data shows. This holds for the little correlation case in the above examples again. This theoretical result also supports our above inference for an independent policy case.

Overall, from the macro perspective, Isard (1995/2001) argues that exchange rate stabilization helps reduce this uncertainty for (randomness affecting) the macroeconomy. Policy coordination would be beneficial to suppress this uncertainty rooted in the business cycles if such coordination does not force the policymakers to keep the impossible exchange rate levels from their fundamentals or its *misalignments*.

This misalignment case will occur even in our exchange rate stabilization rule when the monetary authority misjudges (or misperceives) that the real factors or large persistent real shocks like economic crises happen as such. More precisely, misalignment should occur when the nominal exchange rate does not fit the real state of the macroeconomy. Maintaining such inadequate, misjudged levels by monetary policy is disastrous from our policy rule. As we stated before, such inconsistent behavior becomes one of our essential reasons for avoiding walking inflations or deflations. Note that these misalignments are caused by wrong policy ideas on the side of policymakers, stochastic operational errors, and so on, as explained in the following section.

This paper supposes that reducing uncertainties and fluctuations in the macroeconomy is the merit of keeping our exchange rate stabilization objective, as Isard (1995/2001) suggests. However, this paper does not construct the general equilibrium model confirming such a benefit concretely (e.g., Clarida et al., 2002). In some cases, social welfare will increase with this reduction of uncertainties and fluctuations by policy coordination. However, it might not be improved in the Paretian sense, so future research will be needed. Furthermore, such an attempt is beyond the scope of this paper.

The Main Causes of Monetary Fluctuations Like Wrong Policy Ideas, Data Uncertainty, Information Lags, and Human Errors and Their Solutions

Another point should be noted: The stochastic operational errors in monetary policy. Suppose that the theoretically optimal monetary policy is implemented in both countries. However, practically, that might be out of synchronization with the real business cycle due to delays in policymakers' decision-making and/or information lags and measurement errors in data (Orphanides, 2001). Then, the (growth rate of) exchange rate will fluctuate due to such monetary factors and those real business cycles.

Furthermore, other operational errors exist in monetary policy, such as inefficient or unsophisticated institutional structures. In that case, no matter how the policymakers of the two countries direct theoretically optimal and perfect monetary policies for exchange rate stabilization, some practical errors like the obstinate persistence to the convention, misunderstandings, and unsophistication to the new policy ideas in institutions will exist in such monetary policy conducts themselves.

Of course, as Romer and Romer (2013) suggested, additional examples of wrong policy ideas on the part of policymakers should exist. However, we do not get in touch with their (critical) effects in this paper. If we mention just a little, our prediction for Romer and Romer's (2013) case is the resulting misalignment like that from our theory.

More precisely, as mentioned before, suppose that the business cycles and economic growth, like growth rates of real business fluctuations (that stable part is equivalent to the economic growth or the potential output growth; Havik et al., 2014), extend to the entire economy (aggregate shocks with nonzero means) and the tradable and/or nontradable goods industries (industry-specific shocks with nonzero means).

Then, it becomes desirable for monetary policy that policymakers check these business cycles (and economic growth) by examining detailed data. However, such data examinations are not immune to human and nonhuman errors, no matter how hard the policymakers try to deal with these data, not to mention the data uncertainty and information lags proposed by Orphanides (2001). This leads to misconduct and delays in policymakers' decision-making. In such a case, for example, human capital accumulation in policymakers will help reduce the above operational errors by avoiding mismeasurement and misconduct.

Therefore, even if policymakers decide the theoretically optimal monetary policy concerning (growth rates of) real business fluctuations (and economic growth), the actual monetary policy is only partially perfectly operated due to these practical errors, as discussed before (Orphanides, 2001). The (growth rate of) exchange rate will fluctuate due to these operational factors generated by central banks themselves, that is, monetary policies themselves. This will affect the stabilization of (the growth rate of) the exchange rate. Recall the example of self-fulfilling monetary disturbances in the previous section (e.g., Orphanides, 2003, for its possibility as one example).

Thus, again, the human capital accumulation in policymakers and the public should decide the effective sizes of policy effects. Human capital accumulation in the public will also support understanding the policy effects and intentions and lead to their forward-looking behaviors (e.g., Lucas, 1972) and a better future of the macroeconomy—human capital matters (e.g., Becker, 1975/1976).

Furthermore, even if the social pressure to make the public *behave homogeneously* due to the existence of social capital exists, as Masuda (2022) suggests, we should have the *authority* to avoid the above operational errors. Masuda (2024b) suggested the profound role of individuals in society and history. Takahashi (1993) argued the role of authority and the efficiency of hierarchy. This also holds for *firms* that we discussed in the rent-seeking behaviors. Even in any organization, including the firms, it is well known that quite *a few* persons dominate and lead such organizations, as Lippmann (1922/1987) suggested.

Again, monetary policy should be developed subject to mismeasurement of (the growth rates of) the total factor productivities of the tradable and nontradable goods industries, implicitly including (the growth rates of) the labor inputs and the velocities of money fluctuations in the home and foreign countries. More precisely, the difficulty of measuring technological progress, the velocity of money, and the labor input fluctuation with empirical data available for the policymakers exist.

This difficulty is also due to the innate stochastic properties of available empirical data with measurement errors, information lags, and data examinations with human and nonhuman errors, in addition to original stochastic operational errors of home monetary policy and foreign monetary policy, such as inefficient and unsophisticated institutional structures such as the wrong ideas on the side of policymakers, the obstinate persistence to the convention, misunderstandings, and unsophistication to the new policy ideas in organizations, etc. These causes foster the mismeasurements and instabilities of monetary policy, exchange rate, and macroeconomy.

Our Findings of the Policy Rule in Theory and Practice So Far

Keynesians suppose that the monetary policy should control the randomness of output, which refers to its level, as new Keynesians do (see The Equation for Determining the Exchange Rate Based on the Quantity Theory of Money section for details). In this section, policymakers can also be assumed to use the growth rate of the quantity of money, $d\widetilde{M}_t/\widetilde{M}_t$, to control the *expected* output growth, $\ln(Y_t) - \ln(Y_{t-1})$, represented by the term, $\mathrm{E}\left\{\alpha_1 - \alpha_2\left[r\left(\frac{\widetilde{M}_t}{\widetilde{P}_t}\right) - r\left(\frac{\widetilde{M}_{t-1}}{\widetilde{P}_{t-1}}\right)\right]\right\}$, assuming the IS curve behind the expected output growth where E is the expectation operator, α_1 and α_2 are parameters different from a_2. The real interest rate is the function of $\widetilde{M}_t/\widetilde{P}_t$, which can be a policy tool to stabilize the exchange rate directly.

Policymakers can do so given the foreign monetary policy, $d\widetilde{M}_t^*/\widetilde{M}_t^*$, its effect on $d\widetilde{P}_t^*/\widetilde{P}_t^*$, and $\ln(Y_{t-1})$, instead of the term, $a_1 + a_2 t$, in the $\ln(Y_t)$ equation of the previous section where $\frac{dr(\widetilde{M}_t/\tilde{P}_t)}{d\widetilde{M}_t} < 0$ in the *short run* with the sticky price when $d\widetilde{M}_t - d\widetilde{\overline{\overline{M}}}_t > 0$ (see also the previous section).

Now, recall that we assume the uncertainties of parameters, a_1 and a_2, and, of course, α_1 and α_2, except for the case treating the filter problem in this paper. Furthermore, in the above case, $\widetilde{\overline{\overline{M}}}_t$ means the variable, $\overline{M}_t$, given by the money growth rule adjusted by the price of tradable goods produced by unskilled workers (the numeraire), and we assume $\alpha_2 > 0$.

To be more precise, based on this new Keynesian model, as in The Equation for Determining the Exchange Rate Based on the Quantity Theory of Money section, like Sargent and Surico (2011), policymakers should consider the monetary effects of the growth rates of the quantity of money (monetary stocks) on the entire output. This should be so according to the effects of the real interest rates and inflation rates using the IS curve and the staggered new Keynesian Phillips curve (e.g., Masuda, 2024b), with the modified quantity theory of money (i.e., the combined Harrod–Balassa–Samuelson effect) model given in The Harrod–Balassa–Samuelson Effect Model With Skilled and Unskilled Workers section.

Overall, based on the above, such a policy rule is challenging to apply in practice if not only economic (output) growth rate but also the business cycle fluctuations (growth rate) is considered for output growth rates, dY_t/Y_t and dY_t^*/Y_t^*. This empirical practice makes the situation more difficult for policymakers. However, suppose that the Phillips curve is vertical, as in the long run. In that case, this other relationship, like the Phillips curve, does not need to be considered, as Friedman (1978) suggested in his natural unemployment rate hypothesis (recall the previous stagflation discussion).

Again, in summary, this theory of exchange rate determination based on the quantity theory of money and the Harrod–Balassa–Samuelson effect model, that is, the policy rule to stabilize exchange rates presented in this paper, suggests as follows. It is theoretically possible to derive a monetary policy rule to stabilize the (growth rates of) exchange rates from the existing theories, which emphasize heterogeneity and represent PPP. Still, it is tough to implement this rule perfectly in practice.

Furthermore, another causal factor will harm the exchange rate stabilization, which even policymakers in the home and foreign countries cannot theoretically control, at least as long as they follow their multiple standard policy objectives, as I will describe in the next section.

Some Causes of Deviations From the Exchange Rate Stabilization: The Multiple Objectives Tradeoff and Exchange Rates Problems of Policymakers

In the previous section, I described the causes of misconduct in monetary policy to stabilize the exchange rate. These causes will prevent policymakers from implementing optimal monetary policies for exchange rate stabilization. However, these are entirely stochastic and somewhat unintentional for the policymakers in the sense that the central banks themselves are not always aware of them and do not indicate any incentive to stop the optimal policy for exchange rate stabilization spontaneously.

In the following discussion, based mainly on Drazen (2000), I show the tradeoff problem of policymakers that the optimal monetary policy should change to the phases by phases of their domestic economies. In some situations, a multiple-objectives tradeoff problem might exist for the policymakers to deviate from such an optimal monetary policy for exchange rate stabilization. This theoretical tradeoff problem for policymakers also strengthens the conclusion that conducting the optimal monetary policy for exchange rate stabilization is fairly difficult, as similarly presented in the previous section.

Note that in advance, as his conclusion, Drazen (2000) suggested that one of the causes of this tradeoff problem is that there are fewer numbers of the existing policy instruments than that of the economic issues that the policymakers must face. For example, many developed countries initially adopt the *floating* exchange rate to get free hands for their corresponding domestic economic goals (e.g., Friedman, 1953).

According to Drazen (2000), this is because the policy instruments have already been exhausted and will not be newly invented. However, homogeneous countries will behave in the same ways such that they will face the same economic problems, my answer is as follows: This tradeoff problem is initially *impossible* for policymakers to solve due to the *heterogeneity* of the existing macroeconomies, such as developed and developing countries, resource-rich and industrial countries, *large* and *small* countries such as domestic demand-driven (in addition, with some tendency to substitute imports by their domestic industries) and export-driven countries, and so on. Furthermore, as we will note, the tradeoff between the short-run and the long-run (i.e., different time perspectives) gains in social welfare does exist. Finally, this divergence also means that the policymaker's *target* exchange rates should not be single but *multiple*.

Note that we ignore a kind of heterogeneity generated by *idiosyncratic shocks* for the firms in the industries of home and foreign countries that we have discussed in the previous section, excluding the reservation wage case in our labor participation theory. Such heterogeneity just complicates our discussion hereafter, and we safely ignore it for the firms in the industries of home and foreign countries for the simplicity of our following discussion.

In developed countries, policymakers have the incentive to implement inflationary monetary policies. That is, $d\widetilde{M}_t/\widetilde{M}_t$ becomes more than its steady-state value from the money demand equation (money growth rule; recall the discussion about the relation to its Taylor reaction function in this paper). Such a policy can increase employment, improve real factors, and deviate from the standard inflation-fighter stance. For the above discussion, we need to specify the concrete IS curve and the objective function of the monetary authority. We should see the previous section for that (Barro & Gordon, 1983b; Kydland & Prescott, 1977; Masuda, 2024b).

Furthermore, the capital flow problem is generally severe in countries other than developed countries. Therefore, similar to policymakers in developed countries, policymakers in such countries would have the incentive to implement an inflationary monetary policy to make the current account surplus (see Drazen, 2000, for the importance of this capital flow problem).

The point in the above cases is that policymakers have multiple objectives in different economic phases, such as inflation stabilization, money growth objective, output stabilization, employment improvement, financial stability motive, capital flow improvement, and capital flow stabilization (Cukierman, 1992; Drazen, 2000; Woodford, 2003). However, our exchange rate model proposes another exchange rate stabilization objective for policymakers. Furthermore, these objectives are contradictory in nature.

Here is another example of the short-run and the long-run (i.e., different time perspectives) gains in social welfare. This paper connects inflations with technological progress, as we discussed with our exchange rate stabilization policy rule. In short, inflation should inevitably be caused by the exchange rate's depreciation, resulting in higher import prices as *negative supply* shocks. From the beginning of this discussion, we exclude the import price inflations based on the market's demand and supply or its scarcity of natural resources. Notwithstanding, this might be a typical second kind of stagflation in our case as follows (Blanchard, 2000).

Such higher import prices suffer the domestic households (and industries) through the importing necessities (see Appendix D; e.g., Masuda, 2021). Our inference in this paper suggests that real income increases by technological progress induced by inflations will ultimately overcome the real income reductions caused by this kind of import price inflation.

However, suppose that the public claims disinflations due to daily inflation-induced problems like real income reductions. Recall that a household's utility function depends on not their real income but their real consumption, as stated in the opening section and formulated in Appendix D. Then, the public's claims might be enough to force policymakers to suppress their monetary expansions and revert to a moderate, optimal monetary policy in the short run, risking the possibility to lose the further induced technological progress of the tradable goods industry (e.g., Laibson, 1997, for the hyperbolic discounting for the consumers). Thus, based on the different time perspectives, the tradeoff between the short-run and the long-run (i.e., different time perspectives) gains in social welfare exists.

Furthermore, in addition, the policymaker's *target* exchange rates are usually not single but *multiple*, and the priorities about which exchange rate should be primarily stabilized may annoy policymakers.

Originally, the exchange rate stabilization objective that follows this rule can soothe the unnecessary uncertainties and fluctuations of the macroeconomy, as Isard (1995/2001) suggests. However, it differs from the above existing policymakers' objectives (with the different time perspectives). The tradeoffs among these objectives (with the different time perspectives) can give the policymakers an opportunistic-like stance with the incentive to weigh some objectives that are the most critical at that time, sacrificing others and weighing others at a different time. Furthermore, this stance depends on which benefit the policymakers respect more, the long-run or short-run goal, the economic majority or the economic minority, considering the changing (domestic) economic phases.

This problem will not be solved by simultaneously optimizing all those objectives with adequate *constant weights*. This *adequacy* itself is indeterminate in theory and empirically depends on leaders (i.e., policymakers) at that time, who might have some changeable finite employment terms and sometimes face different or similar economic phases by chance from or to their interests and/or expertise (Masuda, 2024b, for the profound role of individuals in history).

Furthermore, even once the policymakers can concentrate on the exchange rate stabilization objective, the next problem about which exchange rate they should stabilize may emerge due to the above heterogeneous countries issues.

This multiple objective problem (with the different time perspectives) has appeared in many relatively restrictive but similar discussions, such as the historical and empirical approaches by Obstfeld et al. (2004) and Eichengreen and Esteves (2019) regarding the famous trilemma of international economics. This trilemma specifies the incompatibility of the fixed exchange rate, perfect capital mobility, and independent monetary policy.

As we repeatedly discussed, Isard (1995/2001) argues that exchange rate stabilization is essential for reducing the macroeconomy's uncertainty. However, in short, the possibility of policymakers' incentive to deviate from the respective optimal policies for their above multiple policy objectives suggests that these tradeoffs of benefits and costs among many policy objectives should naturally disturb the establishment of only the exchange rate stabilization objective. The multiple exchange rate stabilization problem fosters this difficulty, too.

As a result, many (developed) countries do not always think that this exchange rate stabilization is the most critical policy objective (even from different time perspectives). At least, it depends on the economic situation at that time, as it is indicated in reality that developed countries usually take a floating exchange rate regime. In such a case, the above simultaneous optimizations with adequate constant weights mean the meaningless remedy in reality because such a procedure should be right only *ex ante*.

Thus, if these tradeoffs exist in multiple objectives, some countries' policymakers might deviate from the optimal monetary policy to stabilize the exchange rate. Other countries' policymakers must forecast this possible deviation of foreign policymakers to stabilize their exchange rates. Furthermore, as we noted, the policymaker's target exchange rates are usually not single but multiple.

Additionally, these forecasts of policymakers would be innately erroneous because policymakers may be experts only in their domestic economy and continuously use the inherently imperfect (international) macroeconometric models to be improved (e.g., Frankel & Rockett, 1988). This forecasting problem should also foster the difficulties of stabilizing the exchange rates (and their timings) and which exchange rates should be stabilized for those policymakers.

The Summary of This Section

Remember that the stationary processes for the outputs, $\ln(Y_t)$ and $\ln(Y_t^*)$, are assumed in the first place. The errors, u_t and u_t^*, will always occur randomly with various magnitudes by positive and negative signs and sometimes positively or negatively correlated in every period t and in time series (autocorrelation problem). These errors prevent policymakers from implementing optimal policies.

The difficulty of predicting sectoral technological progress, the velocity of money, and labor input growths in separated sectors like tradable and nontradable industries should foster this prevention by its innate various uncertainties. As Masuda (2024d) suggests, the total factor productivity should contain natural disasters, wars, and false and sloppy accountings, which should be interpreted as a *technological regress*. These causes should also increase uncertainties and be attributed to the existence of the Knightian uncertainty (in that case, mathematically interpreted as calculable risks; e.g., Mas-Colell et al., 1995).

Thus, in the classical monetarist perspective, dampening such randomness with the (mostly impossible) artistic monetary policy is unnecessary if such randomness is manageable for the macroeconomy to endure, often enough to be politically tolerable. Moreover, such an artificial flexible monetary policy could cause even more critical (unnecessary) business cycles. However, political patience should not usually be equivalent to the best the standard theory suggests (e.g., see the social welfare discussion referring to Laibson, 1997, in the previous section).

The tradeoff, that is, the incentive problem of policymakers for the multiple policy objectives, theoretically strengthens the conclusion in this section. This incentive problem of policymakers naturally indicates the possibility of deviating from the original exchange rate stabilization objective and the difficulty of stabilizing the exchange rate. It explains the reasonable cause for one of the fundamental errors of monetary policy that makes it impossible for policymakers to achieve this objective consistently and flawlessly.

Some economists would like to suppose that the monetary authority correctly recognizes these problematic situations surrounding policymaking with empirical data that should be accurately separated from (measurement) errors in real time. However, even if so, conducting policymaking in such a situation is pretty tricky for the monetary authority in actual environments (Orphanides, 2001). Therefore, again, we must admit that an attempt to conduct such an ideal monetary policy is often called an *art* (e.g., Blinder, 1999).

As noted before, monetarism denies this *artistic* monetary policy conduct. Because monetarism respects the possibility of the *failure* of this kind of policy rather than the success not from the individual *heroism risks* but from the stylized fact about the limiting situations for the policymakers that only permit them with the limited, lagged information and their resulting inability to control the current subtle and complicated situations. Furthermore, this monetarism insists that such an attempt should usually *lose* the *luck* for the correct responses.

For example, regarding the precautionary (asset) motive, as we stated in the text and explained in Appendix B, the *conditional heteroscedastic* variance brought by this precautionary (asset) motive might complicate the *correct* money demand forecasts by monetary authorities. Then, this kind of *monetary disturbance* will generate (unnecessary) output fluctuations, too (recall its self-fulfilling characteristics indicated in the text; e.g., Orphanides, 2003, for its possibility as one example).

However, our discussion in Appendix B should *not deny* the role of *forward-looking expectations*. As Masuda (2024a) argues, forward-looking expectations remain the key (e.g., Cuba-Borda & Singh, 2024; Lucas, 1972), irrelevant to our mathematical result in Appendix B. In this paper, we insist that forecasting the temporal and fluctuating money demands, like the conditional heteroscedastic variances, is quite tricky. Furthermore, such forecasting might sometimes be unnecessary in terms of macroeconomic stability, taking the policy misjudgments and sizes of these variances into account if we think about the policy conduct as if we are monetarists. On the other hand, as mentioned, forward-looking, preemptive behavior is the key to economic crises, where the conditional heteroscedastic variance as well as the transaction motive plays a serious role, as Friedman and Schwartz (1963/2008) suggested (see also Lucas, 1972).

In short, forecasting not the fluctuating temporal state of the macroeconomy but its *steady states* in advance and adjusting our behaviors with the *backward inductions* from these future steady states, like the forward-looking behaviors of policymakers, leaders in firms, and many other leading figures in the public, remains *essential* (Lucas, 1972). *Authority* is the key to the hierarchy in organizations and societies (e.g., Takahashi, 1993).

For such persons (i.e., authorities), it is well known that in addition to forward-looking behaviors, we sometimes need the optimal combination of self-control (e.g., Gul & Pesendorfer, 2001) and tolerance for risk-taking such as when they plan the project finance (Merton, 1969; Samuelson, 1969; see Masuda, 2024b, for the profound role of individuals in history). The above tolerance means the relative risk aversion under the constant relative risk aversion utility function. This risk aversion is equivalent to the inverse of the elasticity of intertemporal substitution, which means that the higher this elasticity of intertemporal substitution becomes, the less that person respects the future results, which results in not avoiding the risks (Deaton, 1992).

The previous rent-seeking discussion will successfully connect to the above discussions because the risk here means doing *nothing new* and its (*absurd*) *continuity*. Suppose that the leaders and managers in firms earnestly consider their firm's future facing the turbulent states of the macroeconomy (recall the lower discount factor for the account of the rent-seeking behaviors), that is, have a lower elasticity of intertemporal substitution. In that case, they should naturally avoid such (downward) risks.

Back to the story, thus, we ignore this precautionary (asset) motive in regular situations. Furthermore, our above intuition, similar to Friedman and Schwartz (1963/2008), should be correct even in some special situations, including *economic* *crises*, regarding the correct conduct of monetary policy. As Friedman and Schwartz (1963/2008) described, those economic crises cut real incomes through more significant layoffs than usual and usually let the general price levels and nominal interest rates remain nearly constant (or abruptly rise). As shown with mathematical expressions in Appendix B, the (real) money demand caused by this precautionary (asset) motive (and the transaction motive) will induce *monetary contractions*.

Monetary authorities must avoid such contractions with more significant supplying quantities of money than usual to provide sufficient liquidity to the confused market and macroeconomy by generating inflationary tendencies to prevent from the post fearful *deflationary spirals* (Friedman & Schwartz, 1963/2008; see the previous discussion about the *dormant money* in Japan). Appendix A briefly provides the approximated Sen's (1976) poverty measure to estimate these heterogeneous (real) income shocks in economic crises.

Therefore, according to the above discussions about random and intended misconducts of the monetary policy by the monetary authority, monetary policy can only partially manage the stability of (the growth rate of) the exchange rate. As we noted in the previous section, we may be theoretically able to stabilize the exchange rate as fixed in case of the synchronized business cycles between two countries, like optimal currency areas. However, this generally does not hold empirically without some remarkable exceptions like the European Union. Alternatively, even if so, many other possible difficult obstacles might exist that need to be solved, such as the need for fiscal policy coordination and so on.

Isard (1995/2001) also suggests that exchange rate stabilization aims to suppress the (harmful) uncertainties and fluctuations that the unstable exchange rate brings to the macroeconomy. However, as has been suggested for decades, it is better and possible for monetary policy not to target the stability of (the growth rate of) the exchange rate as a policy objective. Many developed countries initially adopt the *floating* exchange rate to get free hands for their many corresponding domestic economic goals, as conventional wisdom indicates (e.g., Friedman, 1953).

These suggest theoretical inquiries for estimating the alternative returns to replace the loss from fluctuating exchange rates. In short, as we noted, once we adopt the floating exchange rate to get free hands for treating domestic economic problems, this loss from (harmful) fluctuations should inevitably occur. As we discussed before, policymakers should ponder these returns and losses from the exchange rate stabilization. So, even under the floating exchange rate regime, some kind of international cooperation, such as (loose, i.e., holding only for the expected values) policy coordination, should be desirable. This is so due to the merit of the exchange rate stabilization objective, as Isard (1995/2001) noted, as long as it does not impose an impossible exchange rate level or a misalignment on the countries in question.

Our conclusion for the above practical impossibility of the solid exchange rate stabilization does not indicate that monetary expansion would be potentially useless in preventing a lower domestic general price level. We should remember that domestic monetary expansion cannot stabilize the exchange rate perfectly but generally results in the induced technological progress of the home tradable goods industry (and more real income) through a higher domestic general price level (i.e., a higher price level growth: inflation).

Note that this paper connects inflations with technological progress, as discussed with our policy rule applications and the hedonic regression as our illustrative example for empirical analyses. Still, inflation might inevitably be caused by the exchange rate's depreciation, resulting in higher import prices as *negative supply* shocks. Recall that in our model, the *foreign exports* mean the *home imports* and vice versa. Such higher import prices suffer the domestic households (and industries) through the importing necessities. Here, we only assume that domestic inflation and exchange rate depreciations are the sources of higher import prices (e.g., Masuda, 2021). Thus, the monetary authority should balance the merits and deficits of increasing real incomes through inflation in the long run and the short-run temporal reductions in their real incomes.

Then, if necessary, they are enough to force policymakers to suppress their monetary expansions and often revert to a moderate, optimal monetary policy in the short run when the public claims disinflations due to the intolerable short-run real income reductions (see Laibson, 1997, for the short-sighted consumer case represented by the hyperbolic discounting). This alternative optimal monetary policy is the traditional one that does not generate inflations (i.e., the induced technological progress of the tradable goods industry by inflations) as well as deflations (e.g., Marshall, 1923/2003).

Based on the discussion supported by the policymakers (e.g., Engemann, 2019), some economists argue that a positive (moderate) inflation rate is theoretically desirable, as we do (e.g., Orphanides, 2003, for his discussion of a *two percent* inflation target as the theoretical cause; Summers, 1991). In contrast, as Engemann (2019) argues, Boskin et al. (1996) and Blanchard et al. (2010) say that this should be so considering the upward bias of the Consumer Price Index and the increase in the room for monetary policy to cut interest rates as a practical matter. The former should support positive inflation from the practical issues. However, the latter should theoretically support our expansionary monetary policy, comparing the benefits and deficits of inflation.

Thus, the expansionary monetary policy is desirable for the development of the macroeconomy. At the same time, its (welfare, or equivalently, political) cost might prevent the monetary authority from reckless and irresponsible conduct of expansionary monetary policy. Moreover, policymakers should always try to persuade the public about the timely legitimacy of their current policies at every opportunity, based on scientific policy simulations, for example.

Moreover, as to deflations, we should note that deflations are only justified under full employment in the short run, that is, the firm's (confident) optimistic forward-looking behavior bringing favorable results to the macroeconomy under the sales revenue maximization hypothesis (Baumol, 1959; e.g., Samuelson & Solow, 1960, for the *high-pressure* economy). Furthermore, our discussion provides an entirely different motive from the *inflation bias* in line with its *traditional* understanding, such as the employment improvement motive, financial stability motive, and others (Barro & Gordon, 1983a, 1983b; Kydland & Prescott, 1977; e.g., Cukierman, 1992, for the latter). Our discussion provides another foundation for such monetary authority's positive attitude toward inflation under the profit maximization hypothesis: Inducing technological progress and higher (nominal and resulting real) wages.

Finally, our policy idea expressed in this paper is similar to the so-called *neoclassical synthesis* (Samuelson, 1948). However, some policy ideas presented in this paper differ from the *traditional* understandings of this neoclassical synthesis, such as fine-tuning policies.

More precisely, we do *not* need the traditional neoclassical synthesis *assumption* that we need Keynesian aggregate demand management policy under the equilibrium *with unemployment*. At the same time, under *full employment*, the classical state with flexible prices prevails (Samuelson & Solow, 1960). This paper specifies that we need Keynesian aggregate management policy under the world *with* the profit maximization hypothesis.

In contrast, the sales revenue maximization hypothesis establishes full employment, making even deflations better for the macroeconomy in the short run, like the classical state. However, this hypothesis should not support Say's law like the classical state. Instead, as Masuda (2024a) suggests, Keynesian aggregate management policy *must* be needed even under the sales revenue maximization hypothesis.

As Masuda (2024a) argues, the Keynesian aggregate management policy should be justified by breaking Say's law with Friedman's (1969/2006) quantity theory of money and liquidity trap by the negative interest rate policy. This negative interest rate policy provides the negative finite floor whose absolute value is less than that of the average risk premium in the macroeconomy to make the average nominal interest rate in the macroeconomy positive.

Our main point in this paper is that, as we explained in detail before, the firm's objectives should decide the state of the macroeconomy. Furthermore, again, even among Keynesian demand management policies, this paper suggests that monetary policy led by *our* monetarism and supported by enhanced *human capital* should be *better* and *desirable*.

Thus, readers can easily find that this paper never repeats the existing idea of neoclassical synthesis, only mechanically. Instead, readers will find that our policy idea in this paper is mainly rooted in *monetarism*. In other words, this paper supports the *revival* of monetarism, incorporating the *essence* of Keynesian ideas.

Appendix A.

The Approximated Sen's (1976) Poverty Measure

In the text, we argue that poverty measures should be used to estimate the heterogeneous real income shocks caused by inflationary environments and economic crises. In this appendix, we theoretically analyze the effects of heterogeneous real income shocks with relative poverty measures such as Sen's (1976) poverty measure and so on. Again, this is because inflationary environments and economic crises bring (severe temporal relative) poverty, that is, severe heterogeneous real income shocks or heterogeneous consumption shocks (e.g., Deaton & Paxson, 1997), as in the opening section.

This appendix mainly analyzes Sen's (1976) poverty measure. This is because it can include the meaning of the changes in the income distribution of people experiencing poverty, although other famous and standard measures, like the headcount ratio and the average income gap, do not. Considering the distribution of people experiencing poverty, we can examine the state of people experiencing poverty in detail.

However, in Sen's (1976) poverty measure, the Gini coefficient of people experiencing poverty is used and is not additive. We cannot investigate the contribution analysis with this measure as is well known (e.g., Sen, 1997/2000). Furthermore, in Sen's (1976) poverty measure, we do not use the Gini coefficient of *the total* as Son (2004) did, but instead, we use the Gini coefficient of people *experiencing poverty* again. Hence, the contributions are more obscure than those discussed by Son (2004), who investigates how to identify economic growth as the pro-poor type, like trickle-down growth, using the Lorenz curve (i.e., Gini coefficient).

In this appendix, we briefly explain our findings, remembering these facts. Our main point in this appendix is that we can use Sen's (1976) poverty measure for the contribution analysis by linearizing it, which differs from the existing literature. This improvement should solve our difficulty in analyzing consequential trickle-down growth (e.g., Ravallion & Chen, 2003, for their axiom 1 and 2), as Son (2004) has mentioned above, focusing on the Gini coefficient of people *experiencing poverty* and the *contributions* of average income gaps and headcount ratios differently from Son (2004).

Some examples for analyzing poverty with this approximated poverty measure to consider the future income redistribution policies exist based on the subjects proposed by this paper. First, suppose the situation that the economic growth induced by inflation from our theory reduces poverty by Sen's (1976) poverty measure with this contribution analysis after some laps from the initial equilibrium. In that case, *trickle-down* growth is defined as the economic growth (or regress) that reduces (or increases) the headcount ratio, the average income gap, and the Gini coefficient among this (approximated) Sen's (1976) poverty measure (Ravallion & Chen, 2003). In short, the economic growth that *ultimately* reduces Sen's (1976) poverty measure with *any* of its arguments is called *consequential* trickle-down growth in this paper.

Then, we discussed that under the sales revenue maximization hypothesis, the real wages are not equivalent to their productivity and are distributed heterogeneously in the labor market for unskilled workers in the tradable goods industry and workers in the nontradable goods industry. The (approximated) Sen's (1976) poverty measure will help analyze the effects of this heterogeneous distribution on the incomes of households or consumers. This is the second example of the application of our (approximated) Sen's (1976) poverty measure in this paper.

We assume that the government can correctly estimate the (real) incomes, consumptions, assets, and household structure (attributes) of all the households in society with data like the Population Census or the Current Population Survey. Here, we assume our model in Appendix D, which equates current income to current consumption.

However, some readers might say that the Population Census is insufficient for our timely policy objective because it is published every five years. In that case, we should use individual surveys, including the real income, the real consumption, and the household attributes data, for timely specific policy purposes, such as measuring poverty. Note that the Current Population Survey is published monthly in the U.S. and includes income and household structure data.

For this analytical purpose, we should use the *equivalence scales* for household real incomes (and consumptions) adjusted by the number of household members (e.g., Burniaux et al., 1998). This procedure enables us to treat not poverty based on households that have various structures and are not easily compared with each other but poverty based on the individuals, that is, consumers. As a result, people experiencing poverty are judged only by the (individual) income standard (the poverty line; e.g., Shrider & Creamer, 2023, for the U.S. case). These make it possible to analyze relative poverty using detailed microdata for households with poverty indices, including Sen's (1976) poverty measure.

Introduction of Sen's (1976) Poverty Measure and Its Approximation

Following Sen (1976), his poverty measure at the period t, S_t, is calculated as $S_t = H_t[IG_t + (1 - IG_t)G_t]$, where H_t is the ratio of the people under the poverty line, Z_t, to the total population at the period t. This is called the headcount ratio. G_t means the Gini coefficient of income of people experiencing poverty at the period t. Moreover, IG_t is the average income gap, the people's average ratio of income falling short of the poverty line at the period t, Z_t (Sen, 1976).

Following Sen (1997/2000), here, let MI_t be the average income of people experiencing poverty. Furthermore, let NP_t, $NQ(Z_t)$, and $y_{\epsilon,t}$ be the total population at the period t, the number of the population of people experiencing poverty under the poverty line at the period t, Z_t, and the individual ϵ's real income at the period t. Then, H_t is calculated as $H_t = \frac{NQ(Z_t)}{NP_t}$. IG_t is calculated as $IG_t = \frac{\sum_{\epsilon \in NQ(Z_t)}(Z_t - y_{\epsilon,t})}{NQ(Z_t)Z_t}$ (e.g., Sen, 1976). The Gini coefficient is calculated as $G_t = \frac{1}{2NQ(Z_t)^2 MI_t} \sum_{\epsilon=1}^{NQ(Z_t)} \sum_{\varepsilon=1}^{NQ(Z_t)} |y_{\epsilon,t} - y_{\varepsilon,t}|$ (see Sen, 1997/2000).

Another derivation of the Gini coefficient (Sen, 1997/2000) exists:

$$G_t = 1 + \frac{1}{NQ(Z_t)} - \frac{2\{NQ(Z_t)y_{1,t} + [NQ(Z_t)-1]y_{2,t} + \cdots + y_{NQ(Z_t),t}\}}{NQ(Z_t)^2 MI_t},$$

where $y_{NQ(Z_t),t} \geq y_{NQ(Z_t)-1,t} \geq \cdots \geq y_{2,t} \geq y_{1,t}$.

We apply this alternative derivation in the next section. Poverty becomes worse when the Gini coefficient is significant. It should be in $[0,1]$, and zero means perfect equality and one means perfect inequality. The distribution is more skewed when the Gini coefficient becomes more considerable. At the same time, the headcount ratio, H_t, and the income gap, IG_t, increase, the poverty increases. Therefore, when this Sen's (1976) poverty measure becomes more significant, the poverty changes worse.

As Anand (1977) pointed out, Sen's (1976) poverty measure summarizes all of "the distribution of income," "the number actually in the poverty," and "the amounts by which the incomes of the poor fall short of the poverty line," different from the headcount ratio, H_t, and the income gap, IG_t, respectively. To be more precise, the Gini coefficient summarizes "the distribution of income" and has the normative aspect showing the distance from perfect equality. Furthermore, Sen's (1976) poverty measure incorporates this desirable normative property of the Gini coefficient.

In addition, the headcount ratio and the income gap cannot grasp the transfer from the poorer to the richer within people experiencing poverty. In this case, poverty should be interpreted as enlarged in general, but these measures say nothing about this impoverished transfer. However, our distribution-sensitive Sen's (1976) poverty measure can evaluate this transfer (Sen, 1997/2000). Needless to say, estimating the skewness of the distribution among people experiencing poverty is essential when we measure poverty. This is the merit of using Sen's (1976) poverty measure.

Of course, the headcount ratio, H_t, and the income gap, IG_t, are helpful to measure the poverty conveniently, and they can be easily used for the contribution analysis. Note that the Gini coefficient used in Sen's (1976) poverty measure is based on the distribution of only people experiencing poverty. Therefore, this is different from Son's (2004) case, in which he uses the Gini coefficient for the *total population*.

The Differential Analysis of Gini Coefficient

In the previous paragraph, we introduce Gini coefficient with：

$$G_t = 1 + \frac{1}{NQ(Z_t)} - \frac{2\{NQ(Z_t)y_{1,t} + [NQ(Z_t)-1]y_{2,t} + \cdots + y_{NQ(Z_t),t}\}}{NQ(Z_t)^2 MI_t},$$

where $y_{NQ(Z_t),t} \geq y_{NQ(Z_t)-1,t} \geq \cdots \geq y_{2,t} \geq y_{1,t}$, $NQ(Z_t)$ is the number of people experiencing poverty under the poverty line at the period t, Z_t (Sen, 1997/2000).

This formula is transformed into $G_t = 1 + \frac{1}{NQ(Z_t)} - \frac{2\sum_{\epsilon=1}^{NQ(Z_t)}[NQ(Z_t)-\epsilon+1]y_{\epsilon,t}}{NQ(Z_t)^2 MI_t}$. Using the definition $MI_t = \frac{\sum_{\epsilon=1}^{NQ(Z_t)} y_{\epsilon,t}}{NQ(Z_t)}$ and differentiating this G_t with respect to $y_{\epsilon,t}$ yields:

$$\frac{\partial G_t}{\partial y_{\epsilon,t}} = -\frac{2}{NQ(Z_t)^3 MI_t^2}\left\{[NQ(Z_t)-\epsilon+1]\sum_{\epsilon=1}^{NQ(Z_t)} y_{\epsilon,t} - \sum_{\epsilon=1}^{NQ(Z_t)}[NQ(Z_t)-\epsilon+1]y_{\epsilon,t}\right\}.$$

The above $[NQ(Z_t)-\epsilon+1]\sum_{\epsilon=1}^{NQ(Z_t)} y_{\epsilon,t} - \sum_{\epsilon=1}^{NQ(Z_t)}[NQ(Z_t)-\epsilon+1]y_{\epsilon,t}$ term is always positive under the standard condition that $NQ(Z_t)-\epsilon+1$ is large, that is, the number of people experiencing poverty, $NQ(Z_t)$, is large. Then, the above derivative is always negative. Therefore, when the income, $y_{\epsilon,t}$, with the smaller number in the descending order increases, G_t will decrease. In short, when the poorer in people experiencing poverty are assisted, the Gini coefficient reduces. In other words, the poverty (inequality) reduces.

Linearizing Sen's (1976) Poverty Measure Around the Initial Values

As we see, because Sen's (1976) poverty measure uses the Gini coefficient, we can get a normative interpretation of (relative) poverty measured by this index.

However, this measure is nonlinear and complex to understand intuitively. Moreover, it is too complicated to get a simple insight from the factor contribution to the measured poverty.

So, we take a natural logarithm of this measure and linearize around the initial values, IG_0 and G_0, with the first-order Taylor approximation, to get the contribution of three constituent indices at two points in time series (Chiang, 1984, for two variable Taylor approximation).

Then, we have:

$$\ln(S_t) \cong \ln(H_t) + \ln[IG_0 + (1 - IG_0)G_0] + \frac{1-G_0}{IG_0+(1-IG_0)G_0}(IG_t - IG_0) + \frac{1-IG_0}{IG_0+(1-IG_0)G_0}(G_t - G_0).$$

Because we have:

$$\ln(S_0) = \ln(H_0) + \ln[IG_0 + (1 - IG_0)G_0],$$

the change of this measure becomes:

$$d\ln(S_t) \cong d\ln(H_t) + \frac{1-G_0}{IG_0+(1-IG_0)G_0}dIG_t + \frac{1-IG_0}{IG_0+(1-IG_0)G_0}dG_t.$$

The contributions of the right-hand side variables to the log-difference of Sen's (1976) poverty measure, $d\ln(S_t)$, are expressed as the above equation. Furthermore, this is because we have IG_0 and G_0 $\in [0,1]$ from the definition, and the terms, $\frac{1-G_0}{IG_0+(1-IG_0)G_0}$ and $\frac{1-IG_0}{IG_0+(1-IG_0)G_0}$, are positive constants.

Note that the logarithmic function is infinitely differentiable, and the domains for the above Sen's (1976) poverty measure are assumed to be nonnegative real numbers. On the other hand, the domains of the approximated Sen's (1976) poverty measure include even the negative domains due to their difference and logarithm properties.

Therefore, we show that Sen's (1976) poverty measure is the linear combination of the headcount ratio, the average income gap, and the Gini coefficient among people experiencing poverty. Again, we can get the contribution of these three factors at two points to the log-difference of Sen's (1976) poverty measure in time series. Still, Haughton and Khandker (2009) introduce the other approximation. However, the details are unknown.

Sub-Group Decomposition

As Sen (1997/2000) noted, *sub-group consistency* requires a strong argument that people are entirely divided by some attributes and show superiority in such attributes. This is a very strong argument, and we cannot ignore it.

Note that the Gini coefficient does not require such a strong argument. So, we use other measures when we need to decompose poverty from this standpoint (sub-group decomposition). Then, the above decomposition of Sen's (1976) original poverty measure should be used to analyze the contribution of poverty dynamics in the entire macroeconomy.

We can decompose poverty for consumer-type sub-groups with these measures. Suppose that each consumer-type group has $NP_{\chi,t}$ numbers in the sub-group χ at the period t and $NQ_{\chi}(Z_t)$ numbers of people experiencing poverty in this sub-group χ at the period t.

Then, if $NP_{\chi,t}$ grows up to $NP^{+}_{\chi,t}$, $NQ_{\chi}(Z_t)$ should grow up to $NQ^{+}_{\chi}(Z_t) = NQ_{\chi}(Z_t)\frac{NP^{+}_{\chi,t}}{NP_{\chi,t}}$. Also, from the definition, the headcount ratio and the average income gap change with this proportional change, $NQ^{+}_{\chi}(Z_t)$. This is the *share effect*. If this *ex post* $NQ_{\chi}(Z_t)$ is not equal to $NQ^{+}_{\chi}(Z_t) = NQ_{\chi}(Z_t)\frac{NP^{+}_{\chi,t}}{NP_{\chi,t}}$, this difference means the effect of *within-generation effect*.

We should initially calculate poverty numerically to see these share and within-generation effects. Thus, we can decompose the share effect and within-generation effect with the sub-group consistency of the additive poverty measures. Fortunately, the headcount ratio and average income gap have this desirable property.

The Theoretical Results With the Approximated Sen's (1976) Poverty Measure

We assume that there are two types of consumers in society: People experiencing poverty and the rich (the nonpoor). We focus on the effects of fluctuating these two types of consumers on relative poverty, theoretically as seen in Sen's (1976) poverty measure in the inflationary environments and economic crises in this appendix. Our theoretical findings in this appendix are summarized as follows.

Headcount Ratio

In the headcount ratio, we know how many percent shares of consumers are under the poverty line. Our result indicates whether, among all the consumers, the number of consumers who make ends meet under the poverty line is increasing or not. If this ratio gets more significant, poverty will get worse.

Average Income Gap

This measure shows how much income shortage the actual people experiencing poverty under the poverty line live. If this gap gets more significant, poverty will get worse. Even if the incomes of consumers that are all under the poverty line should increase more heterogeneously with supporting the relatively richer among people experiencing poverty (that is, if the inequality among people experiencing poverty is enlarged), this average income gap will improve (Haughton & Khandker, 2009). Furthermore, the poverty measured by this index will be reduced when the government provides uniform benefits that do not improve the income distribution or inequality of the society to all the consumers, including poorer consumers. However, this measure is suitable for grasping the income shortage of the people experiencing poverty from a macroeconomic perspective.

Gini Coefficient of the Poor

The Gini coefficient measures inequality. If the Gini coefficient increases due to the impoverished transfers, the distribution of people experiencing poverty becomes skewed, and inequality among people experiencing poverty expands. This means the enlarged poverty, even if the headcount ratio and the average income gap do not respond to such transfers.

Sen's (1976) Poverty Measure

Sen's (1976) poverty measure connects the inequality represented by the Gini coefficient among people experiencing poverty and the poverty represented by the headcount ratio and average income gap (see the previous sections). This is the distribution-sensitive measure, different from the previous first two measures. If this measure gets more significant, poverty will get worse. Among these four indices, this poverty measure is most suitable for analyzing the (consequential) trickle-down growth defined in this paper.

Appendix B.

The Analysis of Precautionary (Asset) Motive in the Money Demand Equation

For this appendix, I analyze the effects of the precautionary (asset) motive on the Baumol–Tobin type transactions cost equation (Baumol, 1952; Tobin, 1956) mathematically (Dornbusch et al., 2008, for detailed verbal explanations of the precautionary [asset] motive and other motives). Alvarez and Lippi (2009) is the previous study for another different derivation of the precautionary (asset) motive from us (e.g., Blanchard & Fischer, 1989; Boonekamp, 1978, as the other applications).

The money demand equation in the text is the same type and modifies the Baumol–Tobin type transactions cost equation (Baumol, 1952; Tobin, 1956) mathematically to include the zero (negative) interest rate policy. In our interpretation, our money demand equation is mostly equivalent to the Taylor reaction function (e.g., Alvarez et al., 2001) or is a kind of *Taylor rule*. As a result, our discussion in this section holds for our Taylor rule.

However, in the text and this appendix, we do not explicitly consider zero or negative nominal and real interest rates. Considering the international low interest rate environments not so long ago, we should formulate our money demand equation to incorporate the zero interest rate policy or negative interest rate policy into the analysis of this section. In that case, we should change the domain of the *nominal* interest rate to incorporate the zero and negative finite rates. However, in this case, we must assume a positive natural rate of interest to ensure a stable equilibrium in the macroeconomy, as we suggested in the opening section (Masuda, 2024a). Also, we cannot change the domain of the *real* interest rate to incorporate the nonpositive region (Masuda, 2024a). Such a change does not alter our following inferences and conclusions. Thus, we do not explicitly respecify such domains here.

Our Setting and Model for the Precautionary Motive Based on Baumol–Tobin Type Transactions Cost Equation (Baumol, 1952; Tobin, 1956)

Now, I assume the stochasticity for independent variables in the money demand equation that we propose in the text. I use the second-order Taylor expansion with the nominal interest rate and real income around some fixed values, i_0 and $Y_0 \in \mathbb{R}_+$, in the money demand equation in the text. Suppose that I consider neither null nor negative values for them. Because I should take the natural logarithm. Then, this fixed value of interest rate should be a fairly small positive number (near the lower bound) as some value, i_0, as in Masuda (2024a). We can consult Masuda (2024a) for the interest rate problem of taking the natural logarithm for the negative values in economics. Also, suppose that for real income, I take the (real) potential GDP as some fixed value, Y_0, which should be the potential output estimated by the band-pass filters (see the filter problem in the text).

Then, we obtain, after taking the expectation of both sides:

$$\mathrm{E}[\ln(M_t/P_t)] \cong C - \frac{(1+2i_0)\beta_2}{1+i_0}\mathrm{E}(i_t) + \frac{\beta_2}{2(1+i_0)}\mathrm{Var}(i_t) + \frac{2}{Y_0}\mathrm{E}(Y_t) - \frac{1}{2Y_0^2}\mathrm{Var}(Y_t),$$

where β_1 and β_2 are parameters of the money demand equation as in the text, and $C = \beta_1 - \beta_2(1+i_0)\ln(1+i_0) + \frac{\beta_2}{2(1+i_0)} i_0^2 + \beta_2 i_0 + \ln(Y_0) - \frac{3}{2}$ (as usual in the above, we can use $\widetilde{M}_t$ and $\tilde{P}_t$ instead of M_t and P_t and obtain the same result). Here, we assume that the $[\mathrm{E}(Y_t)]^2/(2Y_0^2)$ and $[\mathrm{E}(i_t)]^2/[2(1+i_0)^2]$ terms is so small that we can ignore it.

Note that, in the above, the term, $\mathrm{E}[\ln(M_t/P_t)]$, is nonlinear and stochastic due to the existence of the money multiplier. Still, we know that the logarithmic function is infinitely differentiable in its domains and the strictly monotonically increasing function when M_t and P_t are positive and $\mathrm{E}[\ln(M_t/P_t)]$ is the one to one onto mapping of $\mathrm{E}(M_t/P_t)$ (see any elementary textbooks of mathematics for the properties of logarithmic function).

Also, we take a natural logarithm for the variable, M_t/P_t, in the left-hand side of the above equation. Furthermore, this solves the general difficulty like Siegel's paradox (Siegel, 1972) that $\mathrm{E}(1/P_t)$ is not equal to $1/\mathrm{E}(P_t)$, only assuming $\mathrm{Cov}(M_t, P_t) = 0$ where M_t is exogenous as monetarists assume and P_t is (exogenously) given in the short run in this paper (we will explain the endogenous price case later).

Then, we can safely interpret the above equation as usual, as if we use the standard variables without a natural logarithm. Thus, as in the text, we obtain one similar to the quantity theory of money (i.e., the money demand equation) formula above.

Last, we investigate the effect of the nominal interest rate on the velocity of money in the above formulation. Unlike the previous sections on the quantity theory of money in the text, this section explicitly considers this effect.

$\mathrm{Var}(i_t)$ and $\mathrm{Var}(Y_t)$ are the risks of nominal interest rate and real income, respectively. In other words, in countries where interest rate risk is high so that its volatility is high, real money demand in the money demand equation increases, and the velocity of money decreases (see the discussion about $V_t^{-1} = \exp[\beta_1] i_t^{-\beta_2}$ in the text). This would generate more significant growth of the quantity of money in the macroeconomy than usual, although it depends on empirical parameters for its degree.

The real income volatility enters negatively, although its concrete magnitude also depends on the actual data. This represents that the real demand for money decreases as temporary real income volatility increases. That is, as the business cycle (i.e., real income) fluctuations become smaller, the inverse of the velocity of money increases.

However, the real demand for money should always become smaller than that without considering the business cycle variance, other things being equal. This means that due to the existence of this real income volatility, a tight monetary policy should be required to hold other things equal by comparing it with the state without this volatility when considering the decreased inverse velocity of money even due to this increased real income volatility.

Note that we obtain the *less* significant parameter for the real income under the precautionary (asset) motive. Alternatively, this precautionary (asset) motive formulation uses the *output gap* term instead of the real income *level* term. This means that the precautionary (asset) motive also fluctuates the real money demand *less* than that only under the transaction motive. Thus, if the monetary authority should be sensitive to this precautionary (asset) motive, the monetary authority needs to care less about the income movement than usual.

However, our conclusion in the text should remain unchanged as long as the precautionary (asset) motive has the real income volatility term, which remains difficult for the monetary authority. Note that this precautionary (asset) motive due to the real income movement and volatility contributes to the money demand *asymmetrically* in the booms and recessions, including economic crises from the above formulation.

Inclusion of the General Price Level Risk Into the Above Model

Note that the above *precautionary* (*asset*) *motive* is the factors related only to the *real* money demand, $\mathrm{E}[\ln(M_t/P_t)]$. If we consider the *nominal* money demand,

$\mathrm{E}[\ln(M_t)]$, there should exist another explanatory variable, *price*, $\mathrm{E}[\ln(P_t)]$, and *general price level risk* as the additional part of the precautionary (asset) motive to the above as Boonekamp (1978) suggested. This price volatility should be similarly derived from the relation with the second-order Taylor expansion as:

$$\mathrm{E}[\ln(P_t)] \cong \ln(P_0) + \frac{1}{P_0}\mathrm{E}(P_t - P_0) - \frac{1}{2P_0^2}\mathrm{E}(P_t - P_0)^2,$$

where P_0 is some fixed positive value in $\mathbb{R}_+$ that is in the steady state like the initial value, mostly nomalized to unity.

Then, we have the new constant term:

$$C_{nominal} = \beta_1 - \beta_2(1 + i_0)\ln(1 + i_0) + \frac{\beta_2}{2(1+i_0)}i_0^2 + \beta_2 i_0 + \ln(Y_0) + \ln(P_0) - 3,$$

in the nominal money demand equation with the new explanatory variables in the right-hand side multiplied by its parameters, $\frac{2}{P_0}\mathrm{E}(P_t) - \frac{1}{2P_0^2}\mathrm{Var}(P_t)$. As usual, we can use $\widetilde{M}_t$ and $\tilde{P}_t$ instead of M_t and P_t and have the same result. Here, again, we assume that the $[\mathrm{E}(P_t)]^2/(2P_0^2)$ term is so small that we can ignore it.

This risk should work *similarly* to the real income risk, and its increase (decrease) brings the decrease (increase) in nominal money demand. However, similar to the real income risk, the nominal money demand is less than that without it.

Note that we generally obtain the *more* significant parameter for the general price level under the precautionary (asset) motive, which differs from the real income case.

Furthermore, suppose the monetary authority should be sensitive to this precautionary (asset) motive. In that case, the monetary authority needs to care more about the general price level movement as well as its volatility than usual. However, this paper assumes the sticky price in the short run. So, this price movement and its risk should be minor or underestimated.

Recall that in the text, we should consider the structural relationships *behind* the variables in the money demand equation, as Sargent and Surico (2011), if we should become Keynesians. For example, assuming the first-order approximation around some steady state value P_0, the general price level should be correlated to the nominal interest rate because we obtain its non-zero correlation as:

$$\mathrm{Cov}[\ln(P_t), \ln(1+i_t)] \cong \mathrm{Cov}\left\{\frac{P_t}{P_0}, \frac{1}{r_0+\left(\frac{P_0}{P_{t-1}}-1\right)}\left[r_t+\left(\frac{P_t}{P_{t-1}}-1\right)\right]\right\}$$

$$= \frac{1}{\left[r_0+\left(\frac{P_0}{P_{t-1}}-1\right)\right]P_0 P_{t-1}}\mathrm{Var}(P_t) \neq 0,$$

where the nominal interest rate is defined as $i_t \cong r_t + \left(\frac{P_t}{P_{t-1}} - 1\right)$ as in the text.

Moreover, in this case, we should further consider the structural model that specifies the correlation in question, $\mathrm{Cov}[\ln(P_t), \ln(1+i_t)]$, behind in this money demand equation with the precautionary (asset) motive, like Keynesian. Thus, for the detailed relationship between quantity of money, inflation (i.e., the general price level), and nominal interest rate, we can consult the structural discussions in the previous section of the text. On the other hand, the usual Keynesian assumption for the sticky price (i.e., constant price in the extreme) in the short run, as in this paper, denies or at least should underestimate the existence of this general price level movement and risk again.

More precisely, recall that we can have another three additional relations such as $\mathrm{Cov}[\ln(1+i_t), \ln(Y_t)]$, $\mathrm{Cov}[\ln(P_t), \ln(Y_t)]$, and $\mathrm{Cov}[\ln(M_t), \ln(P_t)]$ in our money demand equation to the above correlation. Remembering the IS relation, the usual Phillips curve with the inflation rate, $\pi_t = P_t - P_{t-1}$, and the quantity of money determining this inflation rate as in the text, we can consider the more complicated causal relationships as the (Keynesian) structural model for the money demand, as suggested in the above.

However, the monetarist tradition should recommend ignoring these structural relationships behind the money demand equation, as we stated in the text. Again, as we will mention later, Keynesians should consider these additional four complicated relations to (at most) three time-varying variances when deciding the quantity of money if Keynesians should be loyal to their ideas. In other words, as we noted, monetarists ignore this precautionary (asset) motive in regular times, but Keynesians do not.

Different Views for the Precautionary Motive Between Keyneasin and Monetariss

Anyway, the precautionary (asset) motive due to the increased real income volatility (and general price level risk) will induce a smaller real (nominal) quantity of money than usual. In contrast, the increased nominal interest rate risk will bring a more significant real (nominal) quantity of money than usual. The overall effect of the precautionary (asset) motive is the sum of the above nominal interest rate and real income volatilities (and general price level risk plus constant term change).

Empirically, to measure their impacts accurately, for example, we should use the standard partial regression coefficients. However, this is another story. Due to the precautionary (asset) motive, this paper assumes that this money demand is *negative* or asymmetry between booms and recessions. Because the sum of the increased volatilities should decrease the money demand, irrelevant to the booms and recessions, while the real income (and general price level) movement(s) should fluctuate to the booms and recessions. This motive is always so because not only the *direction* but also the *volatility* of the real income matters for this motive, which is different from the transaction motive. We can estimate and confirm the demand for money required for this conclusion when different situations like inflationary environments and economic crises happen. However, even if not, our main conclusion does not change only by inversely changing the relations that the above conclusion should have.

In developed countries, the monetary authority cares greatly about smoothing both interest rates and real incomes (e.g., Cukierman, 1992). So, the increase or decrease in the real amount of money required due to such changes in the velocity of money should become slight (e.g., small parameters). Recall that the velocity of money is affected by the nominal interest rate in our formulation of the money demand equation. In fact, in the Keynesian tradition assuming the *sticky price*, we should successfully smooth the nominal interest rates and real incomes using the *nominal* quantity of money, including helicopter money (e.g., Bernanke, 2002; Friedman, 1969/2006), if we can control such variables skillfully.

However, in contrast to the above discussion, many Keynesians traditionally believe that the velocity of money is unstable (e.g., De Long, 2000, for the skeptics about and the difficulty of its measurement). On the other hand, in the monetarism tradition, the velocity of money should be constant, and this fluctuation is ignorable, assuming the flexible price (in the long run).

As discussed above, the velocity of money fluctuations should be minor, at least in developed countries. Because the real income and nominal interest rate volatilities and related parameters, such as the nominal interest rate elasticity, should be empirically minor in developed countries during regular times (e.g., de Bondt, 2009, for the Euro area; Lucas, 1988, for the U.S. case), as monetarists assume.

In the theoretical view, this evidence is enough to suggest that we should mainly respect the fluctuating *real* quantity of money due to the transaction motive for analyzing the property of such motives, as we discussed in the text (recall the negative real income shocks in economic crises). Here, the property of such motives represents the total effect of three money motives: The money growth rule (e.g., Friedman, 1969/2006). This is why I do not explicitly argue the precautionary (asset) motive and treat it as similar to the transaction motive in the text.

Recall that in regular times, we should ignore the precautionary (asset) motive, while in economic crises, the precautionary (asset) motive (mainly due to real income volatility) works much negatively on money demand, similar to the transaction motive.

Furthermore, the above equation is still the *static* equation, while we treat the *growth* of the quantity of money and so on in the text. Thus, for the precautionary (asset) motive to work dynamically, the terms, $\mathrm{Var}(i_t)$ and $\mathrm{Var}(Y_t)$ (and $\mathrm{Var}[P_t]$), should be changing through the time and show the *conditional heteroskedasticity* (Engle, 1982), as previously we briefly mentioned in the text. Naturally, we assume these conditional heteroscedastic variances when referring to the precautionary (asset) motive in this paper.

Our Result Adapted to the Taylor Rule

Finally, for the Taylor rule case, we can apply the transformation in the text to the above money demand equation. We can obtain the precautionary motive variables, $\frac{1}{2(1+2i_0)}\mathrm{Var}(i_t) - \frac{1+i_0}{2\beta_2(1+2i_0)Y_0^2}\mathrm{Var}(Y_t) - \frac{1+i_0}{2\beta_2(1+2i_0)P_0^2}\mathrm{Var}(P_t)$. This term should be included as the constant term in our Taylor rule in the text. The monetary authority should smooth these constant term fluctuations by making minor adjustments (fine-tunings) to the nominal interest rate with the quantity of money as indicated in the text, if they are Keynesians.

Appendix C.

The Micro-Foundation for Our New (Un)employment Theory in the Text With the Simple Stochastic Search-Theoretic Model Based on Rogerson et al. (2005)

This appendix is based on Moen (1997) and Rogerson et al. (2005). It briefly explains the mathematical skeleton of our new (un)employment theory, which is verbally explained in the text. This appendix mainly focuses on the *downward stickiness* (*not* rigidity) of the (individual and average) nominal income and explains this fact with our new (un)employment theory.

More precisely, we clarify that originally, only technological progress affects the nominal wage rates from our aggregate (or representative agent) wage rate determination theory, and output gaps (i.e., business cycles) affect only the employment level, that is, unemployment (rate). Furthermore, like in Japan, even if the nominal wage rates are connected with output gaps enforced by *laws* (*contracts*) or partly with the price of nontradable goods industry indicated in the equation (#5) or (#6), that should be attributed to the *overtime pay* fluctuations (or through the Phillips curve for the latter), and the firms would *not* consider the significant *wage cut* in recessions. They are *only* interested in business cycles (output gaps), that is, unemployment (rate).

Moreover, technological *regress* causing the nominal wage rate decline (i.e., wage cut) would happen due to false and sloppy accounting and forecasting, which typically exist in *economic crises*. This is why we often observe that firms with significant wage cuts should usually go bankrupt in the macroeconomy. Also, we briefly explain the observable fact worldwide that real wage rates decrease during booms and rise during recessions with our new (un)employment theory. Note that our (un)employment theory assumes that the heterogeneous workers accept or reject the job offers from the homogeneous firms in this paper.

Still, for our convenience, this appendix mostly assumes the constant returns to scale production functions for three industries. In the later section of this appendix, we include the increasing returns to scale production function case for skilled workers in the tradable goods industry and briefly explain its applications and implications for our new (un)employment theory.

Also, we should remember that this new (un)employment theory only specifies the *market tightness* (we will explain it later) *instead of* the absolute levels of vacancy and unemployment rates. Thus, this theory is *consistent* with our aggregate (or representative agent) wage rate (income) determination theory to determine the distribution laws equations in the text.

Our Setting and Model Based on Rogerson et al. (2005)

Suppose that $c_{l,t} \in \mathbb{R}_+$ for any t is the queuing cost of industry l for $l = TU, TS, N$ for workers to get jobs, $q_{l,t} \in \mathbb{R}_+ \cup \{0\}$ for any t is the intensity of vacancy equal to the ratio of unemployment rate, $urt_{l,t}$, to the vacancy rate, $vr_{l,t}$, in the industry l for $l = TU, TS, N$, that is, $vr_{l,t}/urt_{l,t}$, $p_e(q_{l,t})$ is the new offer arrival rate of industry l for $l = TU, TS, N$ for the employer with vacancies and is the increasing function of $urt_{l,t}$, $A_{l,t} \in \mathbb{R}_+$ for any t is the productivity of industry l for $l = TU, TS, N$ (we assume $A_{TU,t} = A_{TS,t} = A_{T,t}$ as in the text) and is the firm's control variable, $H_{l,t}$ is the hiring cost for firms in the industry l for $l = TU, TS, N$, $w_{l,t} \in \mathbb{R}_+$ for any t is the nominal income for the workers in the industry l for $l = TU, TS, N$, assumed to be the function of $A_{T,t}$ as in the text (recall the Harrod–Balassa–Samuelson effect model; e.g., Rogerson et al., 2005, for the Nash bargaining solution, and see the following discussion), and the firm's control variable (or strategy in the game theoretic sense; see the later paragraphs), $L_{l,t}$ is the labor input (demand) in industry l for $l = TU, TS, N$, $V_{l,t}$ is the firm's profit in the industry l for $l = TU, TS, N$ from the hiring at the period t, and $\lambda_q \in \mathbb{R}_+$ is the mean of the Poisson distribution as the quitting rate and assumed to be constant, for the time being.

(Firms)

Then, $V_{l,t}$ and $H_{l,t}$ are represented as:

$$V_{l,t} = -c_{l,t} + p_e(q_{l,t})[H(A_{l,t}L_{l,t} - w_{l,t}) - V_{l,t}] \text{ for } l = TU, TS, N,$$

$$H(A_{l,t}L_{l,t} - w_{l,t}) = A_{l,t}L_{l,t} - w_{l,t} + \lambda_q[V_{l,t} - H(A_{l,t}L_{l,t} - w_{l,t})] \text{ for } l = TU, TS, N.$$

After some arithmetic, we obtain:

$$c_{l,t} = \frac{p_e(q_{l,t})(A_{l,t}L_{l,t} - w_{l,t}) - [1 + \lambda_q + p_e(q_{l,t})]V_{l,t}}{1 + \lambda_q} \text{ for } l = TU, TS, N,$$

$$\therefore c_{l,t} = \frac{p_e(q_{l,t})(A_{l,t}L_{l,t} - w_{l,t})}{1 + \lambda_q} \text{ due to free entry } (V_{l,t} = 0) \text{ for } l = TU, TS, N.$$

(Workers)

The workers can take the additional outside options for those in the industry l for $l = TU, TS, N$ at the period t (we explain this option further in detail later), $b_{l,t}$, to the expected income, $p_w(q_{l,t})[WF(w_{l,t}) - UB_{l,t}]$. Denote $UB_{l,t}$ as the benefit for the unemployed in the industry l for $l = TU, TS, N$ at the period t and WF as the expected incomes for the workers and a function of the nominal income, $w_{l,t}$, in the industry l for $l = TU, TS, N$. In the text, this function works to generate the wage premium, g, including overtime pay like that. Recall the loose (long-run, like holding only for the expected values) establishment of the factor price equalization theorem, assumed in the text. $p_w(q_{l,t})$ is the new offer arrival rate for the unemployed and is the decreasing function of $urt_{l,t}$ for $l = TU, TS, N$.

Then, we obtain:

$$UB_{l,t} = p_w(q_{l,t})[WF(w_{l,t}) - UB_{l,t}] + b_{l,t} \text{ for } l = TU, TS, N,$$

$$WF(w_{l,t}) = w_{l,t} + \lambda_q[UB_{l,t} - WF(w_{l,t})] \text{ for } l = TU, TS, N,$$

where $b_{l,t} \in \mathbb{R}$ for $l = TU, TS, N$ and for any t is the unemployment insurance for workers at the period t, including the nonmonetary benefits like the home production and leisure, and so on, as Rogerson et al. (2005) suggested. For this interpretation for $b_{l,t}$ for $l = TU, TS, N$, we can assume that that for workers varies with their ages and belonging industries as in the text.

Note that we can interpret this unemployment insurance as his or her outside option or a part of his or her reservation wage that is mainly constructed of the summation of the discounted maximum value of the benefit from the unemployed or the expected nominal income (e.g., Rogerson et al., 2005, for the latter interpretation). This is not always equal to (usually less than) their own (heterogeneous) technological progress (level). Following Rogerson et al. (2005), in this paper, we define the reservation income of the workers in the industry l for $l = TU, TS, N$, $RI_{l,t}$, as the one satisfying the equation, $WF(RI_{l,t}) = UB_{l,t}$, using the nominal income function, WF.

In this appendix, we assume the industry-wide reservation incomes different from the individual ones (though it is wage *rates*) in our labor force participation theory in the text. This assumption makes the unemployment rates fluctuate not in the small continuous values but in the zero-one (rough) discrete values for each industry as such. However, for our analytical purpose in this appendix, this assumption is safe for the convenience of our discussions. The above idea for simplification of the segmentation is similar to the one for the minor segmentation by three industries in our labor force participation theory in the text.

This $b_{l,t}$ changes if workers change his or her belonging industry from the tradable goods industry for unskilled workers to the nontradable goods industry and from the nontradable goods industry to the tradable goods industry for unskilled workers. However, we should ban the worker's stream from the tradable goods industry for skilled workers to others and vice versa, as we assume in the text as the segmented market assumption. Moreover, note that the expected income for the workers,

$WF(w_{l,t})$, should be more significant than this $b_{l,t}$ under the human capital theory, more precisely, under the seniority wage system in Japan which is provided for the long-term employed only (e.g., Doepke & Gaetani, 2024, for the empirical evidence of its deficit with the U.S. and German data).

Thus, after some arithmetic, we obtain (#C1):

$$UB_{l,t} = \frac{p_w(q_{l,t})w_{l,t} + (1+\lambda_q)b_{l,t}}{1+\lambda_q+p_w(q_{l,t})} \text{ such that}$$

$$w_{l,t} = -\frac{1+\lambda_q}{p_w(q_{l,t})}b_{l,t} + \frac{1+\lambda_q+p_w(q_{l,t})}{p_w(q_{l,t})}UB_{l,t} \text{ for } l = TU, TS, N.$$

The above equation shows the relationship between the nominal income and the market tightness, $p_e(q_{l,t})$. Given the other variables as fixed, the nominal income should move with λ_q, if we consider that $urt_{l,t}$ is the function of λ_q (see Moen, 1997).

As long as $q_{l,t}$ for $l = TU, TS, N$ is fixed, if $w_{l,t}$ for $l = TU, TS, N$ and also the outside option for the unemployed in the industry l for $l = TU, TS, N$, $b_{l,t}$, decreases, $UB_{l,t}$ for $l = TU, TS, N$ decreases. Remember that for $l = TU, TS, N$, $UB_{l,t} \geq b_{l,t}$ because $w_{l,t} \geq b_{l,t}$ from our previous assumption. Moreover, suppose that λ_q rises. In that case, other things being equal, $w_{l,t}$ for $l = TU, TS, N$ should be at least constant or rise, as we showed (recall that $p_w[q_{l,t}]$ is the increasing function of λ_q).

Once we examine this equation in this way, we can understand that our conditional average inference for the nominal wage rate under the recessions is consistent with this theory, as explained below. Saying further, in addition to the trick by labor statistics we explained repeatedly in the text, our new (un)employment theory suggests that not only the average wage rate but also individual nominal income also has downward stickiness.

Firm's Profit Maximization Problem and Competitive Search Equilibrium in Our Model

Using the relation $p_e(q_{l,t}) = q_{l,t}p_w(q_{l,t})$ for $l = TU, TS, N$, the profit maximization problem of the firms in the industry l, given $p_e(q_{l,t})(A_{l,t}L_{l,t} - w_{l,t})$ as its profit, under $UB_{l,t}$ constraint for $l = TU, TS, N$ becomes (#C2):

$$\max_{q_{l,t}} \frac{p_e(q_{l,t})(A_{l,t}L_{l,t} - UB_{l,t})}{1+\lambda_q} - q_{l,t}(UB_{l,t} - b_{l,t}) \text{ for } l = TU, TS, N.$$

FOC:

$$\frac{p_e'(q_{l,t})(A_{l,t}L_{l,t} - UB_{l,t})}{1+\lambda_q} = UB_{l,t} - b_{l,t} \text{ for } l = TU, TS, N.$$

Thus, as Rogerson et al. (2005) suggested, all the firms in the industry l for $l = TU, TS, N$ choose the same $q_{l,t}$. Then, using the above relationships, we obtain:

$$\frac{p_e'(q_{l,t}) + 1 + \lambda_q}{p_e(q_{l,t}) - q_{l,t}p_e'(q_{l,t})} = \frac{A_{l,t}L_{l,t} - b_{l,t}}{c_{l,t}} \text{ for } l = TU, TS, N.$$

The above relational equation should be satisfied in an equilibrium of our new (un)employment theory. Based on Rogerson et al. (2005), the interpretation of this equation is that an increase of $b_{l,t}$ makes the market tightness decrease, that is, the unemployment rate rise, and drives workers to accept the better $w_{l,t}$, while such an attempt increases his or her risk for being unemployed. Thus, our model implies the tradeoff between the higher nominal incomes and the lower probabilities of being unemployed, as Rogerson et al. (2005) argued.

Furthermore, *competitive search* equilibrium suggested by Moen (1997) can allow the *zero profit* condition under the marginal productivity hypothesis (e.g., Rogerson et al., 2005). Our above model setting and derivations should satisfy the equilibrium proposed by Moen (1997) and proposition 1 in Moen (1997). Of course, its (Pareto) optimality is ensured by assuming the Hosios condition (Hosios, 1990) for our model safely, where $q_{l,t}$ for $l = TU, TS, N$ should be equal to the universal one in the (decentralized) market that the social planner will set. Thus, our model safely establishes competitive search equilibrium (e.g., Alvarez and Veracierto, 1999, for the other but similar competitive equilibrium notion satisfying the Pareto optimality with the different settings).

However, as we will note again, for $l = TU, TS, N$, this competitive search equilibrium is unique in the ratio of the vacancy rate to the unemployment rate, $q_{l,t}$, and also allows the indeterminacy of the absolute levels of the unemployment rate,

$urt_{l,t}$, and the vacancy rate, $vr_{l,t}$, as in the text.

Finally, following Rogerson et al. (2005), we assume that the workers *maximize* the (expected individual) *income* under our new (un)employment theory in this appendix. On the other hand, in Appendix D, we construct the general equilibrium model and use the *aggregate preference* of the households or consumers without labor disutility, given their (nominal) incomes determined by their labor inputs and nominal wage rates (see Masuda, 2021, 2022, for the definitions of the aggregate, representative agent, and heterogeneous agents [individual]).

We only assume that (heterogeneous) workers have one unit of (heterogeneous) labor (input and technology level) and buy three types of optimal consumption goods in this paper. Note that our (un)employment theory assumes that the heterogeneous workers accept or reject the job offers from the homogeneous firms in this paper. In short, this paper proposes the following *three-stage game* for households or consumers and firms or the social planner.

(A) the *first step* is for workers to decide on labor force participation (labor supply) by comparing the (potentially acceptable *expected*) market wage rates with their reservation wage rates. Suppose these (potentially acceptable *expected*) market wage rates are more significant than their reservation wage rates. In that case, they decide to work (i.e., to supply their labor) and wait for the concrete wage rate offers.

(B) then, as the *second step*, based on the (expected segmented average) *income maximization* and *given* the *labor demand* by firms, they *accept* the concrete (segmented average) nominal income offers with their labor input levels determined by firms. Suppose that they *reject* the (segmented average) nominal income offer at this stage. In that case, they remain unemployed (i.e., they want to work [they are in labor force participation] but get no jobs) and wait for the other (segmented average) nominal income offers with being unemployed.

(C) in the third step, based on the *aggregate preference*, households or consumers (i.e., the above workers) decide their aggregate consumption levels, ignoring their (segmented average) nominal wage rates given the (segmented average) nominal incomes determined in the second step.

The utility maximization based on the aggregate preference in the third step differs from the (expected segmented average) income maximization in the other steps but is *not contradictory*. Because this aggregate preference represents the *social welfare* function rather than the *representative agent* preference (see Appendix D), while the above (expected segmented average) income maximization is based on the *individual* (risk-neutral) preference that is (possibly heterogeneous and) not aggregated.

In other words, individual workers decide their incomes in the second step after deciding on their labor force participation in the first step. Moreover, in the third step, the *social planner* decides their aggregate real demands (consumptions) in the social welfare problem specifying their aggregate demands, ignoring or irrelevant to the (segmented average) nominal wage rates, absolute levels of the labor inputs, and the resulting (segmented average) nominal incomes specified in the second stage (see the combined FOCs of the Harrod–Balassa–Samuelson effect model and recall the indeterminacy discussion for labor inputs in Appendix D).

Remember that the second step mostly specifies a competitive search equilibrium that coincides with the social planner's solutions (we will explain this point further in a later section). Thus, this story does not always mean that the same person *consistently* fixes their decisions to accept the (expected individual or segmented [aggregate]) incomes and determine their (aggregate) real consumption.

Recall that the optimal aggregate real consumption specified by the aggregate preference does not consider the concrete structure of this optimal aggregate real consumption. This concrete structure will vary based on individual preferences, and they might be heterogeneous, similar to their reservation incomes (or reservation wage rates) (remember the heterogeneous reservation incomes or unemployment benefits in this appendix). Thus, the former aggregate theory represents the aggregate of the latter heterogeneous agent theory (see Appendix D). This is our justification for using different utility maximization problems in the different stage games for the households or consumers and the firms or the social planner in this paper.

Note that the above inference holds for the rent-seeking behavior in the text because the rent-seeking behavior satisfies the optimal conditions derived from the firm's profit maximization problem under the profit maximization problem specified in the Harrod–Balassa–Samuelson effect model.

Then, we should arrange the relationship between the two-stage game for the firms in the text and the above three-stage game for households or consumers and firms in this appendix. In short, the above three-stage game works, given the two-stage game in the text. Because the firm's labor demand should be given for the households or consumers optimization problem, as the tradition of the general equilibrium suggests (the indeterminacy of labor inputs enables this structure). Thus, we should formulate and interpret our (total) *game* as the one that the households or consumers optimization problem should be solved given the firm's optimization problem solution.

Finally, here, in this appendix, we indicate the word (segmented average) nominal income offer and (segmented average) nominal income contract as not W_t but $W_t L_t$.

Because W_t means the (segmented average) nominal *wage rate* and W_tL_t means the (segmented average) *nominal income* in this paper. To emphasize their separation, we use the word the (segmented average nominal) *wage rate* instead of the (segmented average nominal) *wage* only in this appendix.

The Laws of Motion for the Unemployment Rate and the Vacancy Rate

Note that following Rogerson et al. (2005), from the above relationships, we can logically obtain the following flow equation (law of motion) of the unemployment rate in our theory:

$$durt_{l,t} = \lambda_q(1 - urt_{l,t}) - p_w(q_{l,t})[1 - G(RI_{l,t})]urt_{l,t} \text{ for } l = TU, TS, N,$$

$$durt_t = \sum_{l=TU,TS,N} vu_{l,t} durt_{l,t},$$

where $urt_{l,t} \in [0,1]$ for any t is the unemployment rate in the industry l for $l = TU, TS, N$ at the period t, $urt_t \in [0,1]$ for any t is the total unemployment rate at the period t, that is, the weighted sum of unemployment rate in the industry l with the weights, $G(RI_{l,t})$ is the probability of workers in the industry l rejecting the offered nominal income based on their reservation nominal income at the period t, $vu_{l,t} \in [0,1]$ for any t and $l = TU, TS, N$, calculated as each ratio of each number of the unemployed in each industry to the total number of the unemployed, at the period t, and $p_w(q_{l,t}) \in [0,1]$ is the worker's contact rate to firms in the industry l for $l =$

TU, TS, N at the period t again.

This unemployment rate is the function of the nominal income (and the productivity) and leaders' and managers' forward-looking behaviors in the firms which are firm's control variables because of the definition that $p_w(q_{l,t}) = \frac{mg(urt_{l,t}, vr_{l,t})}{urt_{l,t}}$ where $mg(urt_{l,t}, vr_{l,t})$ is the later defined matching function in the industry l for $l = TU, TS, N$ at the period t and the previous transformed FOC (#C2), as we noted. Recall that the labor input is assumed to accompany human capital accumulation (i.e., labor-augmenting technology) as the schooling suggests, as noted in The Disinflation or Deflation by the Productivity Slowdown Caused by the Rent-Seeking Behaviors in the Firms section.

Note that the quitting rate, λ_q, should be affected by the firm's severe employment policy (i.e., a kind of firm's labor-market policy to reduce their vacancy rate and offered nominal incomes, to increase the unemployment, and so on) as well as the worker's attributes in this paper. In this paper, a firm's layoffs should affect the quitting. Furthermore, no difference between voluntary and involuntary terminations like $\lambda_q = \lambda_{q,vo} + \lambda_{q,in}$ where $\lambda_{q,vo}$ is the voluntary quitting rate and $\lambda_{q,in}$ is the involuntary one is explicitly assumed in our theory of this section. Here, we use the reproductive property of exponential families (see any textbooks about mathematical statistics).

However, this difference should be empirically estimated, although Masuda (2024c) mainly focuses on (the threat of) the involuntary one (the voluntary one has a neoclassical basis; e.g., Ljungqvist & Sargent, 2000). Still, the above involuntary quitting rate could also be defined as the function whose control variables can include the following *idle capacity* variable (i.e., the firm's control variable; recall the previous transformed FOC. We could use this FOC to solve for λ_q; e.g., Ljungqvist & Sargent, 2000, for the voluntary quitting).

Again, the *average* quitting rate, λ_q, is assumed to follow the Poisson distribution (recall the law of *small numbers*). This provides one of the foundations for Taylor's (1979, 1980) argument that nominal income (wage rate) contracts should be overlapped and staggered, as assumed above.

Regarding the flow of this vacancy rate, according to Blanchard and Diamond (1990), we also logically obtain the following law of motion in our theory from the above:

$$dvr_{l,t} = durt_{l,t} + p_{f,l,t} I_{l,t} - p_{n,l,t} vr_{l,t} \text{ for } l = TU, TS, N,$$

$$dvr_t = \sum_{l=TU,TS,N} vv_{l,t} dvr_{l,t},$$

where $vr_{l,t} \in [0,1]$ for any t is the vacancy rate in the industry l for $l =$

TU, TS, N at the period t, $vr_t \in [0,1]$ for any t is the total vacancy rate to add up those of all the existing industries with the adequate weights, $vv_{l,t} \in [0,1]$ for any t and $l = TU, TS, N$, calculated as each ratio of each number of the vacancy in each industry to the total number of the vacancy, $p_{f,l,t} \in [0,1]$ is the probability of the flow from unproductive to the productive in the industry l for $l = TU, TS, N$ at the period t, $I_{l,t} \in \mathbb{R}$ is the number of the unfilled jobs with no vacancy posted (*idle capacity*) in the industry l for $l = TU, TS, N$ at the period t and the firm's control variable, and $p_{n,l,t} \in [0,1]$ is the probability of the flow from productive to the unproductive in the industry l for $l = TU, TS, N$ at the period t.

Following Barro and Sala-i-Martin (1995), these equations could be transformed into the following *summation* equations (#C3):

$$urt_t = \sum_{l=TU,TS,N}\left[\sum_{\varpi=1}^{t} vu_{l,\varpi}\left\{\frac{1}{1+\lambda_q+[1-G(RI_{l,\varpi})]p_w(q_{l,\varpi})}\right\}^{t-\varpi+1}\lambda_q + \prod_{\varpi=1}^{t}\frac{vu_{l,\varpi}}{1+\lambda_q+[1-G(RI_{l,\varpi})]p_w(q_{l,\varpi})}urt_{l,0}\right],$$

$$vr_t =$$

$$\left[\sum_{l=TU,TS,N}\left\{\sum_{\varpi=1}^{t} vv_{l,\varpi}\left(\frac{1}{1+p_{n,l,\varpi}}\right)^{t-\varpi+1}\left[\frac{vu_{l,\varpi-1}}{1+\lambda_q+[1-G(RI_{l,\varpi-1})]p_w(q_{l,\varpi-1})}\right]^{t-\varpi} vu_{l,\varpi}\left\{\frac{1}{1+\lambda_q+[1-G(RI_{l,\varpi})]p_w(q_{l,\varpi})} - 1\right\}\lambda_q + \prod_{\varpi=1}^{t-1}\frac{vu_{l,\varpi}}{1+\lambda_q+[1-G(RI_{l,\varpi})]p_w(q_{l,\varpi})}\left\{\frac{vu_{l,\varpi+1}}{1+\lambda_q+[1-G(RI_{l,\varpi+1})]p_w(q_{l,\varpi+1})} - 1\right\}urt_{l,0} + \sum_{\varpi=1}^{t} vv_{l,\varpi}\left(\frac{1}{1+p_{n,l,\varpi}}\right)^{t-\varpi+1} p_{f,l,\varpi}I_{l,\varpi} + \prod_{\varpi=1}^{t}\frac{vv_{l,\varpi}}{1+p_{n,l,\varpi}}vr_{l,0}\right\}\right],$$

where $urt_{l,0}$ and $vr_{l,0} \in [0,1]$ for each industry l for $l = TU, TS, N$ are the initial values.

Unemployment (rate) should occur accompanied by the (nominal) income movements to satisfy the equation (#C1). More precisely, from the above unemployment rate equation, we know that the (nominal) income movements and unemployment rate (i.e., $q_{l,t}$ for $l = TU, TS, N$) should be the firm's control variables, as in the above, although we assume the constant quitting rate as exogenous. Thus, we successfully formulate the relationship between nominal income and the unemployment rate in our theory.

The Downward Stickiness and Rigidity of Individual (Segmented Average) Income and Average Nominal Wage Rates

The relationship, $w_{l,t} \geq b_{l,t}$ or $w_{l,t} = b_{l,t}$ for $l = TU, TS, N$, depends on the situations of the macroeconomy like booms and recessions. Still, $w_{l,t} < b_{l,t}$ never happens. Because that case indicates that only the unemployed prevail in the labor market and no one can get a job, we exclude such an extreme case from this paper.

Here, we assume that λ_q is not the constant different from other parts of this appendix and treat it like the control variable. In recessions where $w_{l,t} = b_{l,t}$ for $l = TU, TS, N$, from the equation (#C1), $w_{l,t}$ is the increasing function of λ_q. This means that the (nominal) income, $w_{l,t}$ for $l = TU, TS, N$, is increasing when λ_q rises. Still,

λ_q specifies the unemployment rate as in the equation (#C3). Moreover, the Phillips curve specifies the relationship between the general price level and the unemployment rate. Thus, summarizing our ideas, we obtain the following two cases for $l = TU, TS, N$.

(A) in recessions, we have $d\lambda_q > 0$, $dw_{l,t} > 0$ (from the equation [#C1]) and $durt_{l,t} > 0$ (from the equation [#C3], i.e., output gaps become negative) and $dP_t < 0$ (from the usual Phillips curve), for $l = TU, TS, N$. The real wage will rise.

(B) in booms, we have $d\lambda_q < 0$, $dw_{l,t} < 0$ (from the equation [#C1]) and $durt_{l,t} < 0$ (from the equation [#C3], i.e, output gaps become positive) and $dP_t > 0$ (from the usual Phillips curve), for $l = TU, TS, N$. The real wage will decline.

Note that in the latter Phillips curve inference for the above two cases, we use the fact that the unemployment rate, $urt_{l,t}$ for $l = TU, TS, N$, is the increasing function of the quitting rate, λ_q (see the equation [#C3]). If we consider this relationship suggested by the equation (#C3), the direction of the real wages in our theory is rising in booms and declining in the recessions. This is consistent with the universally observed empirical phenomenon all around the world that the real wage decreases in booms and increases in recessions,

Recall that we assume the positive premium in booms and (nearly) zero premium in recessions for nominal wage rates as working hours fluctuate in the Markov switching model for overtime pay. This means that, if any, the nominal wage rate (income) cut should be relatively small in usual recessions. Because this wage rate (income) cut should only remain the overtime pay cut (and productivity slowdowns do not occur early) in usual recessions when we assume that the nominal wage rates (incomes) are related to output gaps. The above theoretical inference coincides with the empirical fact that unemployment rather than significant wage (income) reductions should often be observed in usual recessions. Furthermore, that is what the Japanese employment system does.

Moreover, firms do not usually cut (significant) nominal wage rates (incomes) because nominal wage rates (incomes) are related to not only output gaps but also technological progress. The output gap fluctuations influence such nominal wage rates (incomes) procyclically, and the effect of the quitting rate rise with the negative output gaps in recessions on the average nominal income is *positive*. In usual recessions, its beginnings do not usually show technological *regress*, and *negative* output gaps are a problem. The concrete effect on the nominal wage rate should be empirically examined, although the rough sketch of data justifies its constancy (at least, downward stickiness) in recessions.

Thus, the above result is only related to the quitting rate, which controls the unemployment rate specified as one of the sources for *output gap* fluctuations only. The quitting rate is assumed to be mainly affected by employees' *involuntary causes* due to the firm's severe employment policy, like vacancy rate cuts and the unemployment rate rises (e.g., Masuda, 2024d). The aggregate (nominal) wage rate determination theory should be specified by the firm's problem for our distribution laws in the text. In short, again, in our aggregate (nominal) wage rate determination theory, nominal wage rate (income) rises should be determined not by factors like the quitting rate but by factors specifying the *potential output* growth such as technological progress, market structure, etc., as shown in the text and this appendix. This modeling will theoretically explain the firm's behaviors in recessions, like nominal wage rate (income) cuts or layoffs, as our new (un)employment theory.

In short, in usual recessions, firms do not consider technological progress but are interested in deciding on labor inputs. According to our theory, this is why not only average wage rate but also individual (segmented average) nominal incomes show downward stickiness (recall the conditional average of the nominal wage rates for the employed as the trick discussion).

From our nominal wage rate determination equation and (indeterminate) derived labor demands in usual recessions in the text, controlling output gaps (unemployment) is a problem for smoothing business cycles. Here, remember the productive efficiency and capacity utilization discussion, which assumes the production function for separating the potential outputs and output gaps in the text.

Of course, output gaps can also affect nominal *wage rates*. However, as Taylor (1979, 1980) noted, this relation should not only be the result of economic theory like the equation (#5) or (#6) suggests, but also the assumption itself (like necessary terms in [nominal] wage rate contracts enforced by *law* and so on) until now.

To clarify this point further, remember that $W_t = P_t A_t$ and $W_t L_t = P_t A_t L_t$ in our aggregate (or equivalently, the representative agent) theory in production functions specified for the distribution law equation formulations in the text. This theory suggests that output gaps are only included in L_t, and W_t, the nominal wage *rate*, accurately speaking, is not related to output gaps in our original formulation for the derived nominal wage rate determination equation in the aggregate (or the representative agent) theory. This is why we refer to the role of *law* in the above. Alternatively, our equation (#5) or(#6) shows the loose relationship between the output gaps and nominal wage rates through the Phillips curve.

More precisely, our new (un)employment theory suggests that the quitting rate and the nominal *income* (not wage rate) should be related, like the combination of our aggregate wage rate determination theory and the indeterminate labor input (demand) condition in the text imply.

Then, note that because economic crises induce technological regress (e.g., the false and sloppy accounting and so on), the nominal wage rate dominated by technological progress should fall significantly, different from usual recessions (see the Nash bargaining solution application in the following section). Thus, in our theory of this paper, the real income reduction in economic crises should be caused by the combined critical effects of the nominal wage rate and employment reductions (i.e., wage rate deflation and more unemployment). Recall the firm's pessimistic forward-looking expectation for its sales and outputs induced by technological regress: Nominal wage rate decline.

Moreover, remember our discussion of the rent-seeking behaviors in the text. In this behavior, we assume that technological progress and labor inputs are the control variables for the firms. Because firms provide workers with nominal wage rate (income) contracts, and their revisions are not continuous but random and frequent. This frequency is controlled by the quitting rate in this appendix. Because we assume that the renewed nominal wage rate (income) contracts mean the new nominal wage rate (income) contract between firms and workers, which results from voluntary quittings and renegotiations.

Therefore, we should care that our new (un)employment theory suggests that the *downward stickiness* of nominal income due to the *inverse* relationship between the quitting rate and the nominal income to our intuition *generally* prevails. However, our new (un)employment theory does *not deny* that, as in the economic crises, the nominal wage (rate and the accompanying unemployment benefit) decrease due to the technological *regress* might happen in reality.

This appendix mainly focuses on labor input, that is, unemployment changes. Influencing technological progress, or equivalently, nominal wage rate fluctuations should be (partly) caused by (expansionary) monetary policy (if we exclude the firm's discrete [frequent] revisions of nominal income contracts). This is another story in this appendix, and we repeatedly mention it in the text as our conclusion.

On the other hand, under the sales revenue maximization hypothesis, full labor supply must always determine the labor input. Furthermore, even if any, the matching function should only determine the new hire equal to just the right amount of labor supply (input) growth at this period. In this case, the labor supply should determine the ceilings and the absolute levels of the labor demands in the macroeconomy. Only friction unemployment will exist under the sales revenue maximization hypothesis, that is, full employment, although we exclude such friction unemployment in the text for the simplicity of our discussions. Thus, this (un)employment theory also supports our aggregate (or representative agent) theory under the sales revenue maximization hypothesis.

Last, some economists may think that this section assumes the general equilibrium (un)employment theory, and we should explicitly and mathematically construct the concrete and complete general equilibrium model for our new (un)employment theory. However, such an attempt will complicate our discussions and bring only some minor contributions to them, such as the quitting rate being also the function of the vacancy rates determined by firms that are the function of the quitting rate, and so on. This is why we extend our discussions based on the relatively Taylor-made partial equilibrium model in this appendix.

Matching Technology, Nash Bargaining Solution, and Our Diminishing Equilibrium in the Text

Together with the previous summation equations, based on Rogerson et al. (1995), we can define the following *matching technology* function:

$$mg = mg(urt_t, vr_t) = \sum_{l=TU,TS,N} vmg_{l,t} mg(urt_{l,t}, vr_{l,t}),$$

where urt_t and vr_t are defined as in the above, $vmg_{l,t} \in [0,1]$ for $l = TU, TS, N$ and any t is the weight calculated as each ratio of the respective new hire to the total new hire at the period t, and $mg(0, vr_t) = mg(urt_t, 0) = 0$ (suppose the $urt_t = 0$ or $vr_t = 0$ cases).

The relation of the above *segmented* matching functions in the industry l for $l = TU, TS, N$, $mg(urt_{l,t}, vr_{l,t})$, to the *aggregate* matching function, $mg(urt_t, vr_t)$, here is characterized by the property of saving the *additivity* such as *linearity* regarding their arguments and respective functions. The segmented matching functions are assumed to be constant returns to scale functions, so the aggregate matching function, $mg(urt_t, vr_t)$, must have the above formulation to keep this constant returns to scale property. Because this formulation should maintain the property of constant returns to scale functions to hold the original concept of Blanchard and Diamond (1990) as well.

The matching functions determine the new hires for the respective industries, that is, changes in the equilibrium labor inputs, $dL_{l,t}$ for $l = TU, TS, N$. This is our micro-foundation of the matching technology function in the matching theory. Again, note that this function can have constant reruns to scale property but is determined empirically, as Rogerson et al. (2005) suggested (see also Blanchard & Diamond, 1990).

Then, based on this matching function, we can specify the concrete nominal income level as the result of Nash bargaining solution (Nash, 1950b), as Rogerson et al. (2005) discussed. Based on Rogerson et al. (2005), this is $w_{l,t} = RI_{l,t} + \vartheta(A_{l,t}L_{l,t} - RV_{l,t} - RI_{l,t})$ for $l = TU, TS, N$ where $\vartheta \in (0,1)$ is the worker's bargaining power and $RV_{l,t}$ is the reservation profit levels for workers and firms for $l = TU, TS, N.$

However, this paper omits further calculations of the explicit formula and equilibrium of this Nash bargaining solution. Because its detailed derivation is just a repetition of that of Rogerson et al. (2005). Here, our point is that the technological progress (output) specified above determines the nominal income even in the standard matching theory, as in the text.

Note that the above Nash bargaining solution ensures that the nominal income should be equivalent to the reservation income when the reservation profit is zero. The output level is equal to the reservation income. Our distribution law equations show that the (nominal or equivalently) wage rate levels that the sales revenue maximization hypothesis ensures are determined based on average productivity. Some workers might have lower productivity than average due to the heterogeneous distribution of human capital accumulation.

However, as long as the reservation income is equal to or smaller than their output level, as seen before, the nominal income is equal to or higher than the reservation income. Then, such workers should not resign or cause turnovers any more. Thus, our inference in the text that even the lower productivity workers than its average can get a job is not economically contradictory, as our aggregate (or representative agent) wage rate determination theory suggests. Recall that our reservation income is calculated to satisfy $WF(RI_{l,t}) = UB_{l,t}$ (Rogerson et al., 2005, for details). Of course, we assume zero for the reservation profit in this case.

In short, the nominal income level that workers enjoy should depend on the firm's offers for this nominal income level as well as the worker's offers themselves through their bargaining. In this sense, nominal income level determination is the firm's task, and the trade union will partly decide the resulting nominal income level by strengthening or weakening this bargaining process on the side of workers.

The role of the trade union in this process should depend on its objectives, such as protecting the rights of the existing workers or raising the minimum wage rate within its belonging industry (International Labour Organization, 2023, for the latter).

Furthermore, it is natural in theory that a rise in the minimum wage rate will not induce corresponding increasing unemployment, as Card and Krueger (1994) suggested empirically. The rise in the minimum wage rate will increase unemployment only when this minimum wage rate is *over* the *equilibrium* wage rate. Furthermore, if the labor is

Giffen goods and this minimum wage rate is over the equilibrium wage rate, the minimum wage rate also *reduces* unemployment, although this case is *pathological*). In that case, setting the minimum wage rates becomes only a distribution problem between consumer surplus and producer surplus.

However, our new (un)employment theory also indicates that equilibrium employment should be determined by the firm's forward-looking expectations similarly under the profit maximization hypothesis (e.g., Samuelson & Solow, 1960; Cuba-Borda & Singh, 2024). Furthermore, this theory and hypothesis should permit the existence of idle resources.

As a result, we can find our new (un)employment theory, which explains the trick of the segmented average wage rate of the employed or individual nominal income, that is, segmented average nominal income stickiness (downward stickiness; e.g., recall the overtime pay), the nominal income movement, and the generated unemployment. As in the text, the generated unemployment depends on the nominal (real) income (e.g., the nominal income offer rejection of the worker) and could depend on idle capacity (vacancy) in this appendix through the intensity of vacancy (i.e., market tightness) variable, which are the firm's strategic variables. These strategic variables entirely depend on the firm's optimistic or pessimistic forward-looking behaviors, including animal spirit (Keynes, 1936/1997; e.g., Cuba-Borda & Singh, 2024), even though we assume the homogeneous behavior of firms within each industry.

The Relationship Between Marginal Productivity Hypothesis in the Text and Competitive Search Equilibrium in This Appendix

Suppose that we solve the profit maximization problem with constant returns to scale production function. In that case, the FOCs *cannot* determine the absolute level of the derived demand for labor input and its output level, as we discussed. Furthermore, this labor demand (undetermined by marginal productivity hypothesis and) decided by leaders and managers (forward-looking behaviors) in the firms should be represented as the function of the unemployment rate under the profit maximization hypothesis:

$L_{l,t} = (1 - urt_{l,t})L_{g,t}$ for $l = TS, TU, N$ and the corresponding $g = 2,3,4$,

where $L_t = \sum_{l=TU,TS,N} L_{l,t}$, $L_{1,t} = L_{3,t} + L_{4,t}$, $L_{3,t}$ is the labor force of unskilled workers in the tradable goods industry, and $L_{4,t}$ is the labor force of workers in the nontradable goods industry.

This relation means that the unemployment rate is the control variable of firms due to their forward-looking behaviors. Furthermore, this unemployment rate is the function of the quitting rate from the previous specifications in this appendix. Thus, this fact makes us assume that the quitting rate is the control variable for firms, which is different from most of this appendix, where we mainly assume that this rate is exogenous. Moreover, this alternative assumption does not change our conclusions in this paper without a loss of generality. Again, in most of this appendix, we only suppose that the quitting rate is *given* for the simplicity of our discussions, and we only see what results we can obtain under such a convenient assumption.

The matching function determines the *new hires* (Blanchard & Diamond, 1990). Still, we should remember the aggregate relation, $A_t = \sum_{l=TU,TS,N} v_{l,t} A_{l,t}$, where $A_{l,t}$ is the industry l's productivity for $l = TU, TS, N$ at the period t, and $v_{l,t}$ is the share of industry l's output for $l = TU, TS, N$ at the period t. In the text, we define the common, same technology level for tradable goods industry for skilled and unskilled workers.

Thus, our marginal productivity hypothesis insisting on the existence of unemployment is just combined with the labor force participation theory as representing the *slack*. Furthermore, we assume that this additional unemployment is a random error due to the forward-looking behaviors of the leaders and managers in the firms. However, our new (un)employment theory based on search and matching theory treats literally the unemployment rate, not the labor force participation, with constructing the mathematically structural model for these slacks, the random errors as the unemployment rate. Our labor force participation theory based on Flinn and Heckman (1982) is assumed before establishing this new (un)employment theory.

Furthermore, Moen (1997) suggests that our search model has a *competitive search* equilibrium related to the Pareto optimality by satisfying the Hosios condition (Hosios, 1990; Rogerson et al., 2005). In the text, we argue the competitive market, at least the marginal productivity hypothesis, for our labor markets. This marginal productivity hypothesis also ensures the Pareto optimality from the first welfare theorem (e.g., Mas-Colell et al., 1995). Recall that we can specify the optimal consumption in our equilibrium from the result of Appendix D (e.g., Masuda , 2022). These facts mean that we assume the common decentralized markets for our labor markets under the profit maximization hypothesis in the text and this appendix, also identified by this common Pareto optimality.

In short, our new (un)employment theory determines the unemployment rate from micro, heterogeneous perspectives. In contrast, the marginal productivity hypothesis determines this unemployment from macro, aggregate perspectives with the representative agent. Thus, our new (un)employment theory suggests the miniature for the proposed slacks in the aggregate or representative agent terms by the marginal productivity hypothesis in the text. The latter macro theory is the aggregated or averaged one of the former micro theory (see our labor force participation theory and Appendix D).

Furthermore, most of this appendix assumes the constant returns to scale property of the production function, even for skilled workers in the tradable goods industry. However, we can make the following inferences once we return to the increasing returns to scale assumption.

First, the zero profit condition does not ensure the Pareto optimality for the tradable goods industry for skilled workers market due to its increasing returns to scale production function assumption. In contrast, the competitive search equilibrium for this market seemingly establishes the Pareto optimality in the unemployment rate determination mechanism suggested by our new (un)employment theory. However, this is due to the alternate constant returns to scale (segmented) matching function assumption, which differs from the increasing returns to scale assumption above.

Recall that the matching function is the function of the intensity of vacancy (i.e., market tightness). Therefore, it is natural that the increasing returns to scale assumption in the aggregate or representative agent theory of the text should also keep this property in its corresponding segmented matching function. Then, if we assume the increasing returns to scale for the segmented matching function, the Hosios condition breaks, and the Pareto optimality is lost from such an equilibrium (e.g., Rogerson et al., 2005).

Thus, unlike the easy assumption of the constant returns to scale in the opening of this appendix, the existence of increasing returns to scale in the tradable goods industry for skilled workers loses the Pareto optimality in its equilibrium. Because of this negative property, assuming the increasing returns to scale for the segmented matching function is considered nonsense (e.g., Rogerson et al., 2005).

Next, for the tradable goods industry for skilled workers, employment, that is, the labor demand, should be constant due to its increasing returns to scale property and the zero profit condition. Here and in the text, we assume that employment, in this case, should always be constant in a steady state.

The assumption of the time-varying degree of the increasing returns to scale γ makes the growth of $dL_{TS,t}$ possible to coincide with the growth of $dL_{2,t}$, even with the existence of the fluctuating (or increasing) unemployment (*not* the unemployment *rate*) of the skilled workers in the tradable goods industry market responding to the growth of $dL_{2,t}$. For this supposition, we need to assume that the degree of increasing returns to scale, γ, should be decreasing with respect to time, t to generate more employment (see Burnside et al., 1988, for the existence of the slight positive increasing returns to scale in the macroeconomy). Still, we also exclude the case that the labor demand for skilled workers is higher than the labor supply for those.

Saying further, increasing returns to scale production function does not allow for the indeterminacy of labor input level due to the rent-seeking behaviors in the firms. The equilibrium under the zero profit condition motivates and produces the sales revenue maximization hypothesis due to the different reasons for establishing the average productivity hypothesis from analyzing the rent-seeking behaviors or pessimistic forward-looking expectations (e.g., Cuba-Borda & Singh, 2024). However, it is not equal to a competitive search equilibrium nor accomplished as a competitive search equilibrium. Therefore, such an equilibrium cannot maintain Pareto optimality or be called a competitive search equilibrium.

Justifications for the Existence of Workers With Heterogeneous Technology Levels Like the Rent-Seeking Behaviors Suggested in the Text

In the text, we argue that the aggregate or representative agent model initially assumes heterogeneous workers but derives macroeconomic (i.e., average) behaviors, such as the rent-seeking behaviors of leaders and managers in the firms, irrelevant to this heterogeneity. As a result, firms should employ workers with heterogeneous technologies in the labor markets, but they are averaged to see their effects in the macroeconomy.

Thus, the text assumes the labor markets for workers with heterogeneous technology. Furthermore, the firms should employ workers with (at least) the same technological level as the real wage rate. This technological level for respective workers should be related to (but more than) their reservation wage rates, that is, their reservation incomes, and so on (see the previous discussion for the determination of the reservation incomes). Recall that we assume the heterogeneous reservation wage rates in the text. Our new (un)employment theory in this appendix also represents the bargaining process for accepting the nominal income offer: Labor force employment.

In short, we do not assume the homogeneous labor market but the heterogeneous labor market as the mathematical formulation of our new (un)employment theory. Again, that should be shown as their various reservation incomes with various technology levels, as noted. The rent-seeking behaviors in the firms suggested in the text should be formulated that the leaders and managers in the firms try to employ the workers with low technology levels from their additional (not public but private) benefit or inability to grasp the labor market structural changes. Due to its property to satisfy the optimal conditions under the profit maximization hypothesis, our new (un)employment theory in this appendix also justifies our discussion about rent-seeking behaviors in the text.

Furthermore, in the discussion of rent-seeking behaviors in the text, we argue the relationship between human capital accumulation (i.e., labor-augmenting technology; Barro & Sala-i-Martin, 1995; Lucas, 2002) and rent-seeking behaviors, that is, employing workers with lower technology, illustrated with signaling games (e.g., Gibbons, 1992/1995). This means that the firms employ workers only with their schooling, not confirming their talents. Human capital accumulation is the key.

Thus, in recessions, the macroeconomy, which presses (potential) workers to push into the labor market earlier than their sufficient preparation accomplished, like schooling that ensures the higher accumulation of human capital for them, should have economic slowdowns in the long run. Because employing workers without ample human capital accumulation will do good for the macroeconomy by satisfying the employment shortage in the short run. However, in the long run, such workers cannot accept the higher technology for more efficient production. As a result, this worker's embedded inability impairs the chances of steady or higher economic growth for such a macroeconomy.

Therefore, in developed countries like the U.S., recessions should produce a more significant number of new students who will enter professional schools like that, even borrowing debts reflecting the current severe labor market conditions and be employed with higher schooling (that is, provided the more human capital accumulation) and enjoy the resulting higher salaries in the future like booms or advantageous positions even in the recessions. This is the *favorable* cycle for economic growth in society (e.g., Lucas, 2002). Burdett and Smith (2002) explain the necessary mechanism, like coordinating actions between the firms and the workers, to establish this favorable cycle.

For example, Doepke and Gaetani (2024) suggest that long-term employment protection reduces workers' incentive to obtain better schooling and increases their firm-specific human capital (i.e., relational) investments with the U.S. and German data. This example should illustrate the *failure* of the Japanese seniority wage system, where firms limit their long-term benefits like seniority wages and employment insurance to their *own* employees only.

Appendix D.

The General Equilibrium Analysis of the Changing Industrial Structure in Our Model With the Harrod–Balassa–Samuelson Effect

In this appendix, I mathematically analyze the effects of biased technological progress on the industrial structure in our Harrod–Balassa–Samuelson effect model when considering the demand structure specified by consumers' (households') aggregate preference. This appendix is entirely indebted to Obstfeld and Rogoff (1997).

Suppose that $C^*_{TU,t}$, $C^*_{TS,t}$, and $C_{N,t}$ are *home consumptions* of *foreign* tradable goods produced by unskilled workers, *foreign* tradable goods produced by skilled workers, and *home* nontradable goods at the period t. Note that we assume the *symmetric* two countries case that specifies the *foreign* tradable goods as the *home* consumption for the tradable goods and vice versa.

We often observe such bilateral relationships in the contemporary world, although some readers may doubt how trade can occur in such a *symmetric* case. However, data supports this idea. For example, even trade theorists recognize that more *homogeneous* countries like the U.S. and the Euro area have more significant trade volumes than what the comparative advantage theory suggests, like that more *heterogeneous* countries such as the U.S. and India should trade more with each other (see any introductory textbooks about trade or international economy).

Here, we pick up the U.S. and India as the *classic example* of trade theory. No harm is intended, but we must admit that global situations change significantly daily, and today's common sense might be *tomorrow's nonsense*. At least, this observable fact in the trade data remains one of the *mysteries* in the current trade theory.

Aggregate Household Maximization Problem

Anyway, then, we assume the following *home* and *foreign* aggregate household (or consumer) maximization problems with constant elasticity of substitution utility function, U, subject to the budget constraint:

(Home country)

$$\max_{C^*_{TU,t}, C^*_{TS,t}, C_{N,t}} U\left(C^*_{TU,t}, C^*_{TS,t}, C_{N,t}\right) = \left[\psi_{TU}^{\frac{1}{\upsilon}}\left(C^*_{TU,t}\right)^{\frac{\upsilon-1}{\upsilon}} + \psi_{TS}^{\frac{1}{\upsilon}}\left(C^*_{TS,t}\right)^{\frac{\upsilon-1}{\upsilon}} + (1-\psi_{TU}-\psi_{TS})^{\frac{1}{\upsilon}} C_{N,t}^{\frac{\upsilon-1}{\upsilon}}\right]^{\frac{\upsilon}{\upsilon-1}},$$

$$\text{subject to } \tilde{P}_t Y_t = C^*_{TU,t} + \tilde{P}_{TS,t} C^*_{TS,t} + \tilde{P}_{N,t} C_{N,t},$$

where parameters ψ_{TU}, ψ_{TS}, $1-\psi_{TU}-\psi_{TS}$ are in $(0,1)$ and $\upsilon > 0$, and $C^*_{TU,t}$, $C^*_{TS,t}$, and $C_{N,t} \in \mathbb{R}_+$ for any t.

(Foreign country)

$$\max_{C_{TU,t}, C_{TS,t}, C^*_{N,t}} U\left(C_{TU,t}, C_{TS,t}, C^*_{N,t}\right) = \left[\psi_{TU}^{\frac{1}{\upsilon}} C_{TU,t}^{\frac{\upsilon-1}{\upsilon}} + \psi_{TS}^{\frac{1}{\upsilon}} C_{TS,t}^{\frac{\upsilon-1}{\upsilon}} + (1-\psi_{TU}-\psi_{TS})^{\frac{1}{\upsilon}}\left(C^*_{N,t}\right)^{\frac{\upsilon-1}{\upsilon}}\right]^{\frac{\upsilon}{\upsilon-1}},$$

$$\text{subject to } \tilde{P}^*_t Y^*_t = C_{TU,t} + \tilde{P}^*_{TS,t} C_{TS,t} + \tilde{P}^*_{N,t} C^*_{N,t},$$

where parameters ψ_{TU}, ψ_{TS}, $1-\psi_{TU}-\psi_{TS}$ are in $(0,1)$ and $\upsilon > 0$, and $C_{TU,t}$, $C_{TS,t}$, and $C^*_{N,t} \in \mathbb{R}_+$ for any t.

Note that we formulate the aggregate utility function in the above because we have the aggregate laws of motion for labor inputs and others. Moreover, we do not show the representative agent version utility functions in this paper for the simplicity of our discussions. However, we can easily specify the representative agent laws of motion by adjusting the aggregate laws of motion by population growth rate.

In such a case, we have a different interpretation for the above model, like the existence of the *almighty average* person who works for all the industries, from the *heterogeneous agent* interpretation, such as the example for the inflation tax in the text. We only assume that (heterogeneous) workers have one unit of (heterogeneous) labor (input and technology level) and buy three types of optimal consumption goods in this paper. Thus, we take the *aggregate* interpretation for our utility function even in this appendix.

We should notice that the home budget constraint might include foreign tradable goods prices. However, we can justify using the home tradable goods prices instead of the foreign tradable goods prices once we consider the law of one price assumption and the fact that the home consumptions should be evaluated in its home currency as in the text. For example, from the law of one price assumption, we have $\tilde{P}_{TS,t} = \tilde{P}^{*}_{TS,t}\frac{\tilde{P}_t}{\tilde{P}^{*}_t}$ and $\tilde{P}^{*}_{TS,t} = \tilde{P}_{TS,t}\frac{\tilde{P}^{*}_{t}{}_t}{\tilde{P}_t}$.

Furthermore, the deep parameters in the above utility functions are assumed to be the same between the home and foreign countries because of the mere simplification of our following calculations with the symmetry assumption. This convenient assumption is enough to show our conclusion that the specific parameter selection might change our conclusions in the text.

Following Obstfeld and Rogoff (1997), we exclude the labor input term, that is, the disutility term, from the labor input in the above utility function, together with our reason that such formulation only gives the same but unnecessarily more complicated result than that in this appendix.

The FOCs of the Previous Maximization Problem, Its Dynamics, and Market Equilibrium Condition

Solving this optimization problem and transforming the FOC in terms of the home country with our *symmetry* assumption that also specifies $C_{TU,t} = C^*_{TU,t}$, $C_{TS,t} = C^*_{TS,t}$, and $C_{N,t} = C^*_{N,t}$ in equilibrium, we obtain the following related equations to the home country (#D1):

$$C_{TU,t} = \frac{\psi_{TU}\tilde{P}_t}{\psi_{TU}+\psi_{TS}\tilde{P}_{TS,t}^{1-\upsilon}+(1-\psi_{TU}-\psi_{TS})\tilde{P}_{N,t}^{1-\upsilon}} Y_t,$$

$$C_{TS,t} = \frac{\tilde{P}_{TS,t}^{-\upsilon}\psi_{TS}\tilde{P}_t}{\psi_{TU}+\psi_{TS}\tilde{P}_{TS,t}^{1-\upsilon}+(1-\psi_{TU}-\psi_{TS})\tilde{P}_{N,t}^{1-\upsilon}} Y_t,$$

$$C_{N,t} = \frac{\tilde{P}_{N,t}^{-\upsilon}(1-\psi_{TU}-\psi_{TS})\tilde{P}_t}{\psi_{TU}+\psi_{TS}\tilde{P}_{TS,t}^{1-\upsilon}+(1-\psi_{TU}-\psi_{TS})\tilde{P}_{N,t}^{1-\upsilon}} Y_t.$$

In the above, we omit the foreign case because we can obtain the similar symmetric results to the home case and their derivations are trivial. From the symmetry, these formulae clearly show that the consumption $C_{TU,t}$, $C_{TS,t}$, and $C_{N,t}$ are the function of Y_t, that is, $A_{T,t}$, $A_{N,t}$, $L_{TU,t}$, $L_{TS,t}$, and $L_{N,t}$. Thus, if $A_{T,t}$, $A_{N,t}$, and/or

$L_{TU,t}$, $L_{TS,t}$, and $L_{N,t}$ increase, $C_{TU,t}$, $C_{TS,t}$, $C_{N,t}$, and Y_t increase.

Note that we obtain the usual *downward-sloping* demand curve between the *general price* levels and the real *aggregate consumption* (real *aggregate income*) relationship, assuming parameters and variables that *ensure* this downward-sloping demand curve in the above specifications. This assumption should seem technical, but some messy calculations prove that it is not artificial and pathological.

Log differentiating (#D1) with respect to the time, t, we obtain the following dynamic equations for $C_{TU,t}^*$, $C_{TS,t}^*$, and $C_{N,t}$ (#D2):

$$\frac{dC_{TU,t}}{C_{TU,t}} = \frac{d\tilde{P}_t}{\tilde{P}_t} + \frac{dY_t}{Y_t} - \frac{\psi_{TU}\psi_{TS}(1-\upsilon)\tilde{P}_{TS,t}^{1-\upsilon}}{[\psi_{TU}+\psi_{TS}\tilde{P}_{TS,t}^{1-\upsilon}+(1-\psi_{TU}-\psi_{TS})\tilde{P}_{N,t}^{1-\upsilon}]^2}\frac{d\tilde{P}_{TS,t}}{\tilde{P}_{TS,t}} - \frac{\psi_{TU}(1-\psi_{TU}-\psi_{TS})(1-\upsilon)\tilde{P}_{N,t}^{1-\upsilon}}{[\psi_{TU}+\psi_{TS}\tilde{P}_{TS,t}^{1-\upsilon}+(1-\psi_{TU}-\psi_{TS})\tilde{P}_{N,t}^{1-\upsilon}]^2}\frac{d\tilde{P}_{N,t}}{\tilde{P}_{N,t}},$$

$$\frac{dC_{TS,t}}{C_{TS,t}} = \frac{d\tilde{P}_t}{\tilde{P}_t} + \frac{dY_t}{Y_t} - \left[\upsilon + \frac{(1-\upsilon)\tilde{P}_{TS,t}^{1-\upsilon}}{\psi_{TU}+\psi_{TS}\tilde{P}_{TS,t}^{1-\upsilon}+(1-\psi_{TU}-\psi_{TS})\tilde{P}_{N,t}^{1-\upsilon}}\right]\frac{d\tilde{P}_{TS,t}}{\tilde{P}_{TS,t}} - \frac{\tilde{P}_{TS,t}^{-\upsilon}\psi_{TS}(1-\psi_{TU}-\psi_{TS})(1-\upsilon)\tilde{P}_{N,t}^{1-\upsilon}}{[\psi_{TU}+\psi_{TS}\tilde{P}_{TS,t}^{1-\upsilon}+(1-\psi_{TU}-\psi_{TS})\tilde{P}_{N,t}^{1-\upsilon}]^2}\frac{d\tilde{P}_{N,t}}{\tilde{P}_{N,t}},$$

$$\frac{dC_{N,t}}{C_{N,t}} = \frac{d\tilde{P}_t}{\tilde{P}_t} + \frac{dY_t}{Y_t} - \frac{\tilde{P}_{N,t}^{-\upsilon}(1-\psi_{TU}-\psi_{TS})\tilde{P}_t\psi_{TS}(1-\upsilon)\tilde{P}_{TS,t}^{1-\upsilon}}{[\psi_{TU}+\psi_{TS}\tilde{P}_{TS,t}^{1-\upsilon}+(1-\psi_{TU}-\psi_{TS})\tilde{P}_{N,t}^{1-\upsilon}]^2}\frac{d\tilde{P}_{TS,t}}{\tilde{P}_{TS,t}} - \left[\upsilon + \frac{(1-\upsilon)\tilde{P}_{N,t}^{1-\upsilon}}{\psi_{TU}+\psi_{TS}\tilde{P}_{TS,t}^{1-\upsilon}+(1-\psi_{TU}-\psi_{TS})\tilde{P}_{N,t}^{1-\upsilon}}\right]\frac{d\tilde{P}_{N,t}}{\tilde{P}_{N,t}}.$$

These are the home country's related results, but as usual, we can obtain similar results for the foreign country symmetrically. From now on, to simplify and concentrate on our arguments in this appendix, we only focus on the case of the home country. After some messy arithmetics, the dynamics of the nominal output, $\tilde{P}_t Y_t$, in the text is as follows (#D3):

$$\frac{d\tilde{P}_t}{\tilde{P}_t} + \frac{dY_t}{Y_t} = \frac{dA_{T,t}}{A_{T,t}} + \frac{dF_{TU,t}}{F_{TU,t}} + \frac{Y_{TU,t}}{\tilde{P}_t Y_t}\frac{dL_{TU,t}}{L_{TU,t}} + \frac{\tilde{P}_{TS,t}Y_{TS,t}}{\tilde{P}_t Y_t}\frac{dL_{TS,t}}{L_{TS,t}} + \frac{\tilde{P}_{N,t}Y_{N,t}}{\tilde{P}_t Y_t}\frac{dL_{N,t}}{L_{N,t}}.$$

Also, from the derived equations (#2) in the text:

$$\frac{d\tilde{P}_{TS,t}}{\tilde{P}_{TS,t}} = \frac{dL_{TS,t}}{L_{TS,t}} - \frac{dF_{TS,t}}{F_{TS,t}} - \left(\frac{dL_{TU,t}}{L_{TU,t}} - \frac{dF_{TU,t}}{F_{TU,t}}\right),$$

$$\frac{d\tilde{P}_{N,t}}{\tilde{P}_{N,t}} = \frac{dA_{T,t}}{A_{T,t}} - \frac{dA_{N,t}}{A_{N,t}} - \left(\frac{dL_{TU,t}}{L_{TU,t}} - \frac{dF_{TU,t}}{F_{TU,t}}\right) + \frac{dL_{N,t}}{L_{N,t}} - \frac{dF_{N,t}}{F_{N,t}},$$

have been obtained.

Because the demand and supply are equated in tradable goods markets ($C_{TU,t} = Y_{TU,t}$ and $C_{TS,t} = Y_{TS,t}$) and nontradable goods market ($C_{N,t} = Y_{N,t}$), respectively, we obtain the following dynamic relationships between $C_{TU,t}$ and $Y_{TU,t}$, between $C_{TS,t}$ and $Y_{TS,t}$, and between $C_{N,t}$ and $Y_{N,t}$, (#D4) from this market equilibrium conditions:

$$\frac{dC_{TU,t}}{C_{TU,t}} = \frac{dY_{TU,t}}{Y_{TU,t}} = \frac{dA_{T,t}}{A_{T,t}} + \frac{dF_{TU,t}}{F_{TU,t}},$$

$$\frac{dC_{TS,t}}{C_{TS,t}} = \frac{dY_{TS,t}}{Y_{TS,t}} = \frac{dA_{T,t}}{A_{T,t}} + \frac{dF_{TS,t}}{F_{TS,t}},$$

$$\frac{dC_{N,t}}{C_{N,t}} = \frac{dY_{N,t}}{Y_{N,t}} = \frac{dA_{N,t}}{A_{N,t}} + \frac{dF_{N,t}}{F_{N,t}}.$$

Our model with the Harrod–Balassa–Samuelson effect holds from these conditions even considering the demand (structure). However, as we will see in (#D5) and (#D6), we have the same results with Masuda (2024a). Masuda (2024a) uses the AD-AS framework (graphically) and also insists on the *essentiality* of *demand* even for this model with the Harrod–Balassa–Samuelson effect (recall the tâtonnement process). Thus, as stated in the opening section, our result of the above model, including the above demand structure, is quite different from that of the model proposed by Stolper and Samuelson (1941), as detailed discussed also in the opening section and Masuda

(2024a).

The Dynamics of the Labor Input Growths for $L_{TU,t}$, $L_{TS,t}$, and $L_{N,t}$

Combining (#D2), (#D3), (#2), and (#D4), we obtain the following relationship for the labor input growths for $L_{TU,t}$, $L_{TS,t}$, and $L_{N,t}$ of our economy (#D5):

$$\frac{dL_{TU,t}}{L_{TU,t}}=\frac{-\frac{\psi_{TU}(1-\psi_{TU}-\psi_{TS})(1-\upsilon)\tilde{P}_{N,t}^{1-\upsilon}}{\left[\psi_{TU}+\psi_{TS}\tilde{P}_{TS,t}^{1-\upsilon}+(1-\psi_{TU}-\psi_{TS})\tilde{P}_{N,t}^{1-\upsilon}\right]^{2}}}{1-\frac{Y_{TU,t}}{\tilde{P}_tY_t}}\left(\frac{dA_{T,t}}{A_{T,t}}-\frac{dA_{N,t}}{A_{N,t}}\right)+\frac{1}{1-\frac{Y_{TU,t}}{\tilde{P}_tY_t}}\left\{\frac{\tilde{P}_{TS,t}Y_{TS,t}}{\tilde{P}_tY_t}+\frac{\gamma\psi_{TU}\psi_{TS}(1-\upsilon)\tilde{P}_{TS,t}^{1-\upsilon}}{\left[\psi_{TU}+\psi_{TS}\tilde{P}_{TS,t}^{1-\upsilon}+(1-\psi_{TU}-\psi_{TS})\tilde{P}_{N,t}^{1-\upsilon}\right]^{2}}\right\}\frac{dL_{TS,t}}{L_{TS,t}}+\frac{\frac{\tilde{P}_{N,t}Y_{N,t}}{\tilde{P}_tY_t}}{1-\frac{Y_{TU,t}}{\tilde{P}_tY_t}}\frac{dL_{N,t}}{L_{N,t}},$$

$$\frac{dL_{TS,t}}{L_{TS,t}}=\frac{-\frac{\tilde{P}_{TS,t}^{-\upsilon}\psi_{TS}(1-\psi_{TU}-\psi_{TS})(1-\upsilon)\tilde{P}_{N,t}^{1-\upsilon}}{\left[\psi_{TU}+\psi_{TS}\tilde{P}_{TS,t}^{1-\upsilon}+(1-\psi_{TU}-\psi_{TS})\tilde{P}_{N,t}^{1-\upsilon}\right]^{2}}}{\left[\upsilon+\frac{(1-\upsilon)\tilde{P}_{TS,t}^{1-\upsilon}}{\psi_{TU}+\psi_{TS}\tilde{P}_{TS,t}^{1-\upsilon}+(1-\psi_{TU}-\psi_{TS})\tilde{P}_{N,t}^{1-\upsilon}}\right]\gamma+1+\gamma-\frac{\tilde{P}_{TS,t}Y_{TS,t}}{\tilde{P}_tY_t}}\left(\frac{dA_{T,t}}{A_{T,t}}-\frac{dA_{N,t}}{A_{N,t}}\right)+\frac{\frac{Y_{TU,t}}{\tilde{P}_tY_t}}{\left[\upsilon+\frac{(1-\upsilon)\tilde{P}_{TS,t}^{1-\upsilon}}{\psi_{TU}+\psi_{TS}\tilde{P}_{TS,t}^{1-\upsilon}+(1-\psi_{TU}-\psi_{TS})\tilde{P}_{N,t}^{1-\upsilon}}\right]\gamma+1+\gamma-\frac{\tilde{P}_{TS,t}Y_{TS,t}}{\tilde{P}_tY_t}}\frac{dL_{TU,t}}{L_{TU,t}}+\frac{\frac{\tilde{P}_{N,t}Y_{N,t}}{\tilde{P}_tY_t}}{\left[\upsilon+\frac{(1-\upsilon)\tilde{P}_{TS,t}^{1-\upsilon}}{\psi_{TU}+\psi_{TS}\tilde{P}_{TS,t}^{1-\upsilon}+(1-\psi_{TU}-\psi_{TS})\tilde{P}_{N,t}^{1-\upsilon}}\right]\gamma+1+\gamma-\frac{\tilde{P}_{TS,t}Y_{TS,t}}{\tilde{P}_tY_t}}\frac{dL_{N,t}}{L_{N,t}},$$

$$\frac{dL_{N,t}}{L_{N,t}}=\frac{1-\left[\upsilon+\frac{(1-\upsilon)\tilde{P}_{N,t}^{1-\upsilon}}{\psi_{TU}+\psi_{TS}\tilde{P}_{TS,t}^{1-\upsilon}+(1-\psi_{TU}-\psi_{TS})\tilde{P}_{N,t}^{1-\upsilon}}\right]}{1-\frac{\tilde{P}_{N,t}Y_{N,t}}{\tilde{P}_tY_t}}\left(\frac{dA_{T,t}}{A_{T,t}}-\frac{dA_{N,t}}{A_{N,t}}\right)+\frac{\frac{Y_{TU,t}}{\tilde{P}_tY_t}}{1-\frac{\tilde{P}_{N,t}Y_{N,t}}{\tilde{P}_tY_t}}\frac{dL_{TU,t}}{L_{TU,t}}-\frac{1}{1-\frac{\tilde{P}_{N,t}Y_{N,t}}{\tilde{P}_tY_t}}\left\{\frac{\tilde{P}_{N,t}^{-\upsilon}(1-\psi_{TU}-\psi_{TS})\tilde{P}_t\psi_{TS}(1-\upsilon)\tilde{P}_{TS,t}^{1-\upsilon}}{\left[\psi_{TU}+\psi_{TS}\tilde{P}_{TS,t}^{1-\upsilon}+(1-\psi_{TU}-\psi_{TS})\tilde{P}_{N,t}^{1-\upsilon}\right]^{2}}(-\gamma)+\frac{\tilde{P}_{TS,t}Y_{TS,t}}{\tilde{P}_tY_t}\right\}\frac{dL_{TS,t}}{L_{TS,t}}.$$

Note that the production relations, $F_{TU,t}$, $F_{TS,t}$, and $F_{N,t}$, are assumed to be constant returns to scale or increasing returns to scale, respectively, corresponding to the situations that we assume. If all these functions are supposed to be constant returns to scale, we have $\frac{dL_{TU,t}}{L_{TU,t}} = \frac{dF_{TU,t}}{F_{TU,t}}$, $\frac{dL_{TS,t}}{L_{TS,t}} = \frac{dF_{TS,t}}{F_{TS,t}}$, and $\frac{dF_{N,t}}{F_{N,t}} = \frac{dL_{N,t}}{L_{N,t}}$ as we have explained in the text. However, in the above, we use increasing returns to scale characteristics only for $F_{TS,t}$ as also in the text.

The Dynamics of Labor Input Growth for $L_{N,t}$ in Terms of Relative Technological Progress and Its Implications

Further representing dynamic equations about $L_{TU,t}$ and $L_{TS,t}$ with $A_{T,t}$, $A_{N,t}$, and $L_{N,t}$ and combining these three dynamic equations, we obtain the following much complicated dynamic equation for $L_{N,t}$ (#D6):

$$\frac{dL_{N,t}}{L_{N,t}} = \left(\frac{1-\left[\upsilon+\frac{(1-\upsilon)\tilde{P}_{N,t}^{1-\upsilon}}{\psi_{TU}+\psi_{TS}\tilde{P}_{TS,t}^{1-\upsilon}+(1-\psi_{TU}-\psi_{TS})\tilde{P}_{N,t}^{1-\upsilon}}\right]}{1-\frac{\tilde{P}_{N,t}Y_{N,t}}{\tilde{P}_tY_t}} + \frac{\frac{Y_{TU,t}}{\tilde{P}_tY_t}}{1-\frac{\tilde{P}_{N,t}Y_{N,t}}{\tilde{P}_tY_t}} A1_{LTU,t} + \right.$$

$$\frac{1}{1-\frac{\tilde{P}_{N,t}Y_{N,t}}{\tilde{P}_tY_t}} \left\{ \frac{\tilde{P}_{N,t}^{-\upsilon}(1-\psi_{TU}-\psi_{TS})\tilde{P}_t\psi_{TS}(1-\upsilon)\tilde{P}_{TS,t}^{1-\upsilon}}{[\psi_{TU}+\psi_{TS}\tilde{P}_{TS,t}^{1-\upsilon}+(1-\psi_{TU}-\psi_{TS})\tilde{P}_{N,t}^{1-\upsilon}]^2}(-\gamma) + \right.$$

$$\left. \frac{\tilde{P}_{TS,t}Y_{TS,t}}{\tilde{P}_tY_t} \right\} \left\{ \frac{-\frac{\tilde{P}_{TS,t}^{-\upsilon}\psi_{TS}(1-\psi_{TU}-\psi_{TS})(1-\upsilon)\tilde{P}_{N,t}^{1-\upsilon}}{\left[\psi_{TU}+\psi_{TS}\tilde{P}_{TS,t}^{1-\upsilon}+(1-\psi_{TU}-\psi_{TS})\tilde{P}_{N,t}^{1-\upsilon}\right]^2}}{\left[\upsilon+\frac{(1-\upsilon)\tilde{P}_{TS,t}^{1-\upsilon}}{\psi_{TU}+\psi_{TS}\tilde{P}_{TS,t}^{1-\upsilon}+(1-\psi_{TU}-\psi_{TS})\tilde{P}_{N,t}^{1-\upsilon}}\right]\gamma+1+\gamma-\frac{\tilde{P}_{TS,t}Y_{TS,t}}{\tilde{P}_tY_t}} + A1_{LTU,t} \right\} \Bigg) \Bigg/$$

$$\left(1 - \left\{ \left(\frac{\frac{Y_{TU,t}}{\tilde{P}_tY_t}}{1-\frac{\tilde{P}_{N,t}Y_{N,t}}{\tilde{P}_tY_t}} + \frac{1}{1-\frac{\tilde{P}_{N,t}Y_{N,t}}{\tilde{P}_tY_t}} \left\{ \frac{\tilde{P}_{N,t}^{-\upsilon}(1-\psi_{TU}-\psi_{TS})\tilde{P}_t\psi_{TS}(1-\upsilon)\tilde{P}_{TS,t}^{1-\upsilon}}{[\psi_{TU}+\psi_{TS}\tilde{P}_{TS,t}^{1-\upsilon}+(1-\psi_{TU}-\psi_{TS})\tilde{P}_{N,t}^{1-\upsilon}]^2}(-\gamma) + \frac{\tilde{P}_{TS,t}Y_{TS,t}}{\tilde{P}_tY_t} \right\} \right) A2_{LTU,t} + \right. \right.$$

$$\left. \left. \frac{\frac{\tilde{P}_{N,t}Y_{N,t}}{\tilde{P}_tY_t}}{\left[\upsilon+\frac{(1-\upsilon)\tilde{P}_{TS,t}^{1-\upsilon}}{\psi_{TU}+\psi_{TS}\tilde{P}_{TS,t}^{1-\upsilon}+(1-\psi_{TU}-\psi_{TS})\tilde{P}_{N,t}^{1-\upsilon}}\right]\gamma+1+\gamma-\frac{\tilde{P}_{TS,t}Y_{TS,t}}{\tilde{P}_tY_t}} \right\} \right) \left(\frac{dA_{T,t}}{A_{T,t}} - \frac{dA_{N,t}}{A_{N,t}} \right),$$

$$\text{where } A1_{LTU,t} = \left(\frac{-\frac{\psi_{TU}(1-\psi_{TU}-\psi_{TS})(1-\upsilon)\tilde{P}_{N,t}^{1-\upsilon}}{\left[\psi_{TU}+\psi_{TS}\tilde{P}_{TS,t}^{1-\upsilon}+(1-\psi_{TU}-\psi_{TS})\tilde{P}_{N,t}^{1-\upsilon}\right]^2}}{1-\frac{Y_{TU,t}}{\tilde{P}_tY_t}} + \frac{1}{1-\frac{Y_{TU,t}}{\tilde{P}_tY_t}} \left\{ \frac{\tilde{P}_{TS,t}Y_{TS,t}}{\tilde{P}_tY_t} + \right. \right.$$

$$\left.\frac{\gamma\psi_{TU}\psi_{TS}(1-\upsilon)\tilde{P}_{TS,t}^{1-\upsilon}}{\left[\psi_{TU}+\psi_{TS}\tilde{P}_{TS,t}^{1-\upsilon}+(1-\psi_{TU}-\psi_{TS})\tilde{P}_{N,t}^{1-\upsilon}\right]^{2}}\right\}\frac{-\frac{\tilde{P}_{TS,t}^{-\upsilon}\psi_{TS}(1-\psi_{TU}-\psi_{TS})(1-\upsilon)\tilde{P}_{N,t}^{1-\upsilon}}{\left[\psi_{TU}+\psi_{TS}\tilde{P}_{TS,t}^{1-\upsilon}+(1-\psi_{TU}-\psi_{TS})\tilde{P}_{N,t}^{1-\upsilon}\right]^{2}}}{\left[\upsilon+\frac{(1-\upsilon)\tilde{P}_{TS,t}^{1-\upsilon}}{\psi_{TU}+\psi_{TS}\tilde{P}_{TS,t}^{1-\upsilon}+(1-\psi_{TU}-\psi_{TS})\tilde{P}_{N,t}^{1-\upsilon}}\right]\gamma+1+\gamma-\frac{\tilde{P}_{TS,t}Y_{TS,t}}{\tilde{P}_{t}Y_{t}}}\Bigg)$$

$$\Bigg/\left\{1-\frac{1}{1-\frac{Y_{TU,t}}{\tilde{P}_{t}Y_{t}}}\left[\frac{\tilde{P}_{TS,t}Y_{TS,t}}{\tilde{P}_{t}Y_{t}}+\right.\right.$$

$$\left.\left.\frac{\gamma\psi_{TU}\psi_{TS}(1-\upsilon)\tilde{P}_{TS,t}^{1-\upsilon}}{\left[\psi_{TU}+\psi_{TS}\tilde{P}_{TS,t}^{1-\upsilon}+(1-\psi_{TU}-\psi_{TS})\tilde{P}_{N,t}^{1-\upsilon}\right]^{2}}\right]\frac{\frac{Y_{TU,t}}{\tilde{P}_{t}Y_{t}}}{\left[\upsilon+\frac{(1-\upsilon)\tilde{P}_{TS,t}^{1-\upsilon}}{\psi_{TU}+\psi_{TS}\tilde{P}_{TS,t}^{1-\upsilon}+(1-\psi_{TU}-\psi_{TS})\tilde{P}_{N,t}^{1-\upsilon}}\right]\gamma+1+\gamma-\frac{\tilde{P}_{TS,t}Y_{TS,t}}{\tilde{P}_{t}Y_{t}}}\right\},$$

$$A2_{LTU,t}=\left(\frac{\frac{\tilde{P}_{N,t}Y_{N,t}}{\tilde{P}_{t}Y_{t}}}{1-\frac{Y_{TU,t}}{\tilde{P}_{t}Y_{t}}}+\frac{1}{1-\frac{Y_{TU,t}}{\tilde{P}_{t}Y_{t}}}\left\{\frac{\tilde{P}_{TS,t}Y_{TS,t}}{\tilde{P}_{t}Y_{t}}+\right.\right.$$

$$\left.\left.\frac{\gamma\psi_{TU}\psi_{TS}(1-\upsilon)\tilde{P}_{TS,t}^{1-\upsilon}}{\left[\psi_{TU}+\psi_{TS}\tilde{P}_{TS,t}^{1-\upsilon}+(1-\psi_{TU}-\psi_{TS})\tilde{P}_{N,t}^{1-\upsilon}\right]^{2}}\right\}\frac{\frac{\tilde{P}_{N,t}Y_{N,t}}{\tilde{P}_{t}Y_{t}}}{\left[\upsilon+\frac{(1-\upsilon)\tilde{P}_{TS,t}^{1-\upsilon}}{\psi_{TU}+\psi_{TS}\tilde{P}_{TS,t}^{1-\upsilon}+(1-\psi_{TU}-\psi_{TS})\tilde{P}_{N,t}^{1-\upsilon}}\right]\gamma+1+\gamma-\frac{\tilde{P}_{TS,t}Y_{TS,t}}{\tilde{P}_{t}Y_{t}}}\right)$$

$$\Bigg/\left(1-\frac{1}{1-\frac{Y_{TU,t}}{\tilde{P}_{t}Y_{t}}}\left\{\frac{\tilde{P}_{TS,t}Y_{TS,t}}{\tilde{P}_{t}Y_{t}}+\right.\right.$$

$$\left.\left.\frac{\gamma\psi_{TU}\psi_{TS}(1-\upsilon)\tilde{P}_{TS,t}^{1-\upsilon}}{\left[\psi_{TU}+\psi_{TS}\tilde{P}_{TS,t}^{1-\upsilon}+(1-\psi_{TU}-\psi_{TS})\tilde{P}_{N,t}^{1-\upsilon}\right]^{2}}\right\}\frac{\frac{Y_{TU,t}}{\tilde{P}_{t}Y_{t}}}{\left[\upsilon+\frac{(1-\upsilon)\tilde{P}_{TS,t}^{1-\upsilon}}{\psi_{TU}+\psi_{TS}\tilde{P}_{TS,t}^{1-\upsilon}+(1-\psi_{TU}-\psi_{TS})\tilde{P}_{N,t}^{1-\upsilon}}\right]\gamma+1+\gamma-\frac{\tilde{P}_{TS,t}Y_{TS,t}}{\tilde{P}_{t}Y_{t}}}\right).$$

Our main point is that in this equation, the dynamics of the labor input in the nontradable goods industry depends on its historical path and complicated parameter setting, as usual in most general equilibrium models (the path dependency; e.g., Ljungqvist & Sargent, 2000, for this path dependency discussion; recall each production function).

Thus, we get the same conclusion as that of Obstfeld and Rogoff (1997): Biased technological progress does not always surely bring biased labor input allocation in its favor in the general equilibrium setting. This depends on its historical path and the complicated empirical parameter setting. As the above equation shows, this conclusion safely holds under the increasing returns to scale production functions, which differs from Obstfeld and Rogoff (1997).

Still, some reders may wonder if the above dynamics of $L_{N,t}$ indicates the determinacy of the absolute level of $L_{N,t}$. However, this $L_{N,t}$ is determined by the relative magnitude of $\frac{dA_{T,t}}{A_{T,t}} - \frac{dA_{N,t}}{A_{N,t}}$. This means the same interpretation in the text that specifies the indeterminate level of labor inputs, including $L_{N,t}$.

Finally, using the above relations for the labor inputs growth equations, our general equilibrium model also specifies our policy rule in the text only with relative technological progress term as the above equation of $L_{N,t}$ shows. However, this reformulation does not change our conclusions in the text at all (see the following discussion); the monetary expansion and the relative technological progress are completely positively combined.

The Dynamics of the Industrial Structure for $L_{N,t}$ and Our Implications in This Appendix

Finally, by combining the equations for p42–43 in the text, $v_{N,t} = \frac{Y_{N,t}}{Y_t}$ and

$$\frac{dv_{N,t}}{v_{N,t}} = \left(\frac{Y_{TU,t}}{Y_t} + \frac{Y_{TS,t}}{Y_t}\right)\left(\frac{dA_{N,t}}{A_{N,t}} - \frac{dA_{T,t}}{A_{T,t}}\right) - \frac{Y_{TU,t}}{Y_t}\frac{dL_{TU,t}}{L_{TU,t}} - \frac{Y_{TS,t}}{Y_t}(1+\gamma)\frac{dL_{TS,t}}{L_{TS,t}} + \left(1 - \frac{Y_{N,t}}{Y_t}\right)\frac{dL_{N,t}}{L_{N,t}},$$

(#D5), and (#D6), the industrial structure in view of the nontradable goods industry, that is, the share of output of the nontradable goods industry to the total is defined as (#D7):

$$
\begin{aligned}
\frac{dv_{N,t}}{v_{N,t}} = &-\Bigg\{ \frac{-\frac{\tilde{P}_{TS,t}^{-\upsilon}\psi_{TS}(1-\psi_{TU}-\psi_{TS})(1-\upsilon)\tilde{P}_{N,t}^{1-\upsilon}}{\left[\psi_{TU}+\psi_{TS}\tilde{P}_{TS,t}^{1-\upsilon}+(1-\psi_{TU}-\psi_{TS})\tilde{P}_{N,t}^{1-\upsilon}\right]^2}}{\left[\upsilon+\frac{(1-\upsilon)\tilde{P}_{TS,t}^{1-\upsilon}}{\psi_{TU}+\psi_{TS}\tilde{P}_{TS,t}^{1-\upsilon}+(1-\psi_{TU}-\psi_{TS})\tilde{P}_{N,t}^{1-\upsilon}}\right]\gamma+1+\gamma-\frac{\tilde{P}_{TS,t}Y_{TS,t}}{\tilde{P}_tY_t}}\frac{Y_{TS,t}}{Y_t}+\left(\frac{Y_{TS,t}}{Y_t}+\frac{Y_{TU,t}}{Y_t}\right)A1_{LTU,t}- \\
&\left(1-\frac{Y_{N,t}}{Y_t}\right)+\Bigg\{2A2_{LTU,t}+\frac{\frac{\tilde{P}_{N,t}Y_{N,t}}{\tilde{P}_tY_t}}{\left[\upsilon+\frac{(1-\upsilon)\tilde{P}_{TS,t}^{1-\upsilon}}{\psi_{TU}+\psi_{TS}\tilde{P}_{TS,t}^{1-\upsilon}+(1-\psi_{TU}-\psi_{TS})\tilde{P}_{N,t}^{1-\upsilon}}\right]\gamma+1+\gamma-\frac{\tilde{P}_{TS,t}Y_{TS,t}}{\tilde{P}_tY_t}}- \\
&\left(1-\frac{Y_{N,t}}{Y_t}\right)\Bigg\}\Bigg(\frac{1-\left[\upsilon+\frac{(1-\upsilon)\tilde{P}_{N,t}^{1-\upsilon}}{\psi_{TU}+\psi_{TS}\tilde{P}_{TS,t}^{1-\upsilon}+(1-\psi_{TU}-\psi_{TS})\tilde{P}_{N,t}^{1-\upsilon}}\right]}{1-\frac{\tilde{P}_{N,t}Y_{N,t}}{\tilde{P}_tY_t}}+\frac{\frac{Y_{TU,t}}{\tilde{P}_tY_t}}{1-\frac{\tilde{P}_{N,t}Y_{N,t}}{\tilde{P}_tY_t}}A1_{LTU,t}- \\
&\frac{\gamma}{1-\frac{\tilde{P}_{N,t}Y_{N,t}}{\tilde{P}_tY_t}}\Bigg[\frac{\tilde{P}_{N,t}^{-\upsilon}(1-\psi_{TU}-\psi_{TS})\tilde{P}_t\psi_{TS}(1-\upsilon)\tilde{P}_{TS,t}^{1-\upsilon}}{\left[\psi_{TU}+\psi_{TS}\tilde{P}_{TS,t}^{1-\upsilon}+(1-\psi_{TU}-\psi_{TS})\tilde{P}_{N,t}^{1-\upsilon}\right]^2}(-\gamma)+ \\
&\frac{\tilde{P}_{TS,t}Y_{TS,t}}{\tilde{P}_tY_t}\Bigg]\Bigg\{\frac{-\frac{\tilde{P}_{TS,t}^{-\upsilon}\psi_{TS}(1-\psi_{TU}-\psi_{TS})(1-\upsilon)\tilde{P}_{N,t}^{1-\upsilon}}{\left[\psi_{TU}+\psi_{TS}\tilde{P}_{TS,t}^{1-\upsilon}+(1-\psi_{TU}-\psi_{TS})\tilde{P}_{N,t}^{1-\upsilon}\right]^2}}{\left[\upsilon+\frac{(1-\upsilon)\tilde{P}_{TS,t}^{1-\upsilon}}{\psi_{TU}+\psi_{TS}\tilde{P}_{TS,t}^{1-\upsilon}+(1-\psi_{TU}-\psi_{TS})\tilde{P}_{N,t}^{1-\upsilon}}\right]\gamma+1+\gamma-\frac{\tilde{P}_{TS,t}Y_{TS,t}}{\tilde{P}_tY_t}}+A1_{LTU,t}\Bigg\}\Bigg)\Bigg/\Bigg(1- \\
&\Bigg\{\Bigg(\frac{\frac{Y_{TU,t}}{\tilde{P}_tY_t}}{1-\frac{\tilde{P}_{N,t}Y_{N,t}}{\tilde{P}_tY_t}}+\frac{1}{1-\frac{\tilde{P}_{N,t}Y_{N,t}}{\tilde{P}_tY_t}}\Bigg\{\frac{\tilde{P}_{N,t}^{-\upsilon}(1-\psi_{TU}-\psi_{TS})\tilde{P}_t\psi_{TS}(1-\upsilon)\tilde{P}_{TS,t}^{1-\upsilon}}{\left[\psi_{TU}+\psi_{TS}\tilde{P}_{TS,t}^{1-\upsilon}+(1-\psi_{TU}-\psi_{TS})\tilde{P}_{N,t}^{1-\upsilon}\right]^2}(-\gamma)+\frac{\tilde{P}_{TS,t}Y_{TS,t}}{\tilde{P}_tY_t}\Bigg\}\Bigg)A2_{LTU,t}+ \\
&\frac{\frac{\tilde{P}_{N,t}Y_{N,t}}{\tilde{P}_tY_t}}{\left[\upsilon+\frac{(1-\upsilon)\tilde{P}_{TS,t}^{1-\upsilon}}{\psi_{TU}+\psi_{TS}\tilde{P}_{TS,t}^{1-\upsilon}+(1-\psi_{TU}-\psi_{TS})\tilde{P}_{N,t}^{1-\upsilon}}\right]\gamma+1+\gamma-\frac{\tilde{P}_{TS,t}Y_{TS,t}}{\tilde{P}_tY_t}}\Bigg\}\Bigg)\Bigg\}\left(\frac{dA_{T,t}}{A_{T,t}}-\frac{dA_{N,t}}{A_{N,t}}\right).
\end{aligned}
$$

We derive the same conclusion as the above labor input allocation path for our industrial structure discussion: Its direction is not always sure under the general equilibrium setting and depends on its historical path (e.g., Ljungqvist & Sargent, 2000) and specific empirical parameter setting.

Once the production functions are used with constant and increasing returns to scale, focusing on the wage equations and some additional messy arithmetic should produce the exact figure, as shown in Figure 4.6 in Obstfeld and Rogoff (1997). Furthermore, it should show that technological progress is the main engine projecting the income expansion path, as in Figure 4.6 in Obstfeld and Rogoff (1997) (see also [#D1] equations). However, our assumption of symmetry excludes the *trade imbalance* in our case, which differs from that of Obstfeld and Rogoff (1997). We can easily exclude the pathological case that the increase in $A_{N,t}$ is smaller than the decrease in $L_{N,t}$, even if the (#D7) equation holds.

From the above, under the technological progress like the increase in $A_{N,t}$, we know that the direction of each industry's output share might not always be sure, but the (absolute) level itself should increase. Again, note that the income expansion path itself could work under the increasing returns to scale production assumption, as we showed, which is different from Obstfeld and Rogoff (1997), who assumed only the constant returns to scale production functions.

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