# Address Challenges Markowitz (1952) Faces: A New Measure of Asset Risk

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Current version: Sep 2025

## Abstract

Markowitz (1952) asset risk (MAR) has long been challenged. First, asset volatility captures the noise of asset risk over maturity that improperly reflects asset risk (which has to be measured over a unit of time length to be cumulative since asset holder's risk approaches zero as the holding time length approaches zero). Second, we argue that asset risk approaches zero as payment distance approaches zero, and that asset risk drives volatility, but not vice versa, implying that asset risk cannot be diversified away. Third, support to MAR appears to arise from a confusion between asset value and wealth utility—the law of diminishing marginal utility supports that volatility reduces the latter. The above causes explain why CAPM and Fama-French models have long been struggling to price asset volatilities. To address the challenges, we propose that the gross volatility (variance), which includes the excess risk ALPHA, of realized (expected) asset value approaches zero as distance approaches zero. We delineate expected asset value (which asset risk impacts without a distribution) and risk volatility (which does not affect the former while following a blended normal distribution we proposed). Our asset risk for a specific asset excludes the macrorisk in Nie (2024a) that is tied to assets with a face value denominated by the currency. We show that equity price is the present value of a spanning bond, a payment spanning over the predictable lifetime of firm performance, and that options are equity bonds. Our examples show how to compute a firm's debt and equity risk. Our asset risk, captured as risk premium, thus solves issues that have long been challenging agency theories, thereby redefining firm misvaluation theory.

Keyword: Asset risk; firm risk; asset pricing; agency theory; firm misvaluation theory.
JEL Classification: G34, J33, M12.

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## Conflict-of-interest disclosure statement

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I have nothing to disclose

## 1. Introduction

*Holding a stock over a minute or a year,*

*your risk remains the same.*

This is what schools around the world have been effectively teaching over 70 years, as Jenson and Meckling (1976) and Markowitz (1952) asset risk (henceforth, MAR), in effect, claim that asset holder's risk remains the same regardless of the holding time length. [1] A comparable case is the value of a condominium building that is irrelevant to the value of the units you buy: the former remains the same, regardless of the number of the units you buy. In contrast to the risk of one-time activities, such as lottery drawings, the risk of activities continuing over a period of time, such as asset holding, is cumulative like a salary. For instance, the risk of two-day fighting on the Ukrainian battlefield is about twice that of one-day fighting. As illustrated in Figure 1, asset holder's longitudinal risk (LR) approaches zero as the holding time length $T$ approaches zero. Our LR usually ignores the asset value thereby assuming one unit of the currency.

**[Please insert Figure 1 about here]**

One could argue that a relevant asset risk does not have to be a risk over a unit of time length (which is a year in this study). For example, a Tesla's driving risk can be meaningfully tagged as Tier 1, Tier 2, or Tier 3, which does not mean that your risk remains the same regardless how many hours you drive. This argument, however, ignores the fact that asset pricing models use returns over a unit of time length to price volatility that reflects the noise of asse risk over maturity (which we will show in Section 2). To be relevant, we have to measure the risk and the return both

[1] You have the same volatility, which is not cumulative, regardless you hold the asset over a minute or a year.

over a unit of time length or both over maturity. [2] MAR thus prevents scholars from measuring asset holder's risk over a period, such as managerial risk that captures managerial skin in the game (which plays a key role in agency and contracting theories). A manager's risk over the contracting period motivates, thereby reflecting, her effort over the holding period. This is why a firm usually requires managers to hold firm assets over certain periods. [3]

Yet another challenge to MAR is that volatility of a distribution does not decrease asset value although the former is positively associated with asset risk. In this study, asset volatility excludes the excess risk ALPHA (i.e., $A$) that leads to the excess return alpha (i.e., $\alpha$) when we suggest that the volatility follows a blended normal distribution we proposed (see Section 2.5). A more serious issue is that assets more uncorrelated to the current portfolio are likely to incur higher cognition cost—the current portfolio holder is likely to be less informed of the assets' risk. This explains why asset pricing models have long been struggling to relate volatility to asset returns (e.g., Fama and French, 2015, 1993; Lintner, 1965; Sharpe, 1964). Longman Dictionary defines risk as "the possibility that something bad, unpleasant, or dangerous may happen". For investors, indeed, volatility may not be bad or dangerous if it is not associated with a permanent loss of asset value. We believe that volatility is positively associated with asset risk. However, we argue that asset risk causes volatility, but not vice versa. For example, the number of birds hanging out on a city is highly correlated to the amount of bird droppings on the ground. However, the former drives the latter, but not vice versa. Similarly, hold maturity constant, asset risk is related to volatility proportionately. The latter, however, does not affect the former, which cannot be diversified away.

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[2] A decrease in a firm's risk results in a relatively smaller decrease in equity volatility, because the former raises firm maturity, which we will show in Section 2.

[3] In this study, firm assets refer to a firm's bonds, equity, and options.

The argument that a diversification strategy reduces asset risk, therefore, is as unjustifiable as the argument that cleaning workers' effort to decrease the quantity of bird droppings on the ground reduces the number of birds hanging out on the city.

Finally, support to MAR appears to come from a confusion between asset value and wealth utility, because the law of diminishing marginal utility (LDMU) suggests that wealth volatility reduces the latter, rather than the former. To illustrate, we suppose that an investor has wealth $w_0$ that includes a risk-free asset and a risky asset whose price follows a normal distribution. Given two opposite impacts that change the price of risky asset $P$ by $x$ and $-x$, respectively, $\forall\, x \in (0, P)$, we examine the two impacts' net effect on the wealth utility that reflects the effect of volatility of a normal distribution on the expected asset value. As illustrated in Figure 2, given the investor's wealth utility function $U$, LDMU indicates that the first order differential is positive (i.e., $U' > 0$) and that the second order differential is negative (i.e., $U'' < 0$). Obviously, the net change in wealth utility from the two opposite impacts on the risky asset, which is negative, captures the net impact:

$$\Delta U = \int_{w_0}^{w_0+x} U'(w)\, dw - \int_{w_0-x}^{w_0} U'(w)\, dw < 0. \tag{1}$$

Therefore, LDMU supports that the utility loss from a decrease in the risky asset's value by $x$ (i.e., the height of brown pole) is always greater than the utility gain from an increase in asset value by $x$ (i.e., the height of green pole). In short, asset volatility reduces wealth utility rather than asset value. Further, we observe that the loss is greater for poor people—the loss is greater when we reduce the risk-free asset's value such that the wealth decreases from $w_0$ to $w_1$. In fact, our conclusion remains intact if we reduce the values of the risky and risk-free assets by the same fraction such that the overall wealth decreases from $w_0$ to $w_1$. To sum up, the more volatile the wealth, the greater the utility loss; the lower the wealth, the greater the impact.

**[Please insert Figure 2 about here]**

In case of a lognormal distribution, the net change in wealth utility from the two opposite impacts of a lognormal distribution can be written as:

$$\Delta U = \int_{e^{w_0}}^{e^{w_0+x}} U'(w)\, dw - \int_{e^{w_0-x}}^{e^{w_0}} U'(w)\, dw. \tag{2}$$

Diversification in effect is a strategy that reduces the magnitudes of temporary losses at a cost of reduced sizes of temporary gains. Even in terms of wealth utility, the benefit of a diversification strategy is likely to be insignificant economically, because volatility is more harmful to the wealth utility of poor people (who can hardly hold risky assets greatly when diversification is a simple choice of their own) than to that of rich people. In addition, we argue that that assets more uncorrelated to the current portfolio incur higher cognition cost. Therefore, Black and Scholes (1973) effectively treat options as risk-free, because asset volatility effectively raises option value as we will show in Section 2.

To address the above challenges, we expunge maturity from volatility to measure asset risk over a unit of time length. As in Nie (2024), we convert the central bank's rate into a continuously compounding rate for estimation purpose. This is because the rates in our models are continuously compounding whereas those from the central bank and the market are annual rates. As in Nie (2024a), the risk-free rate is equal to the central bank's rate plus the constant risk which captures the depreciation cost of interbank lending. All rates in this study related to equity and corporate debt, however, are all continuously compounding.

We assume that the noise of asset risk follows a blended normal distribution we proposed. Since volatility of a distribution is not a risk to asset value, our asset risk impacts asset value without a distribution, reflecting the magnitude and/or possibility of a decrease in the expected asset value. In particular, we separate the expected asset value (which the risk affects without a distribution) from the volatility (which does not impact the former while following a blended lognormal distribution). Our analyses highly depend on the strong logic of an exclusive disjunction: one must be right if the other is wrong. All assets with a face value carry the inflation risk or the investment opportunity cost that mirrors the country's T-bill risk in Nie (2024a). To be comparable across assets, a corporate bond's risk (i.e., micro-risk or default risk) is defined as a one-year payment's risk. We show that equity price is actually the present value of a spanning bond—a payment spanning over the predictable lifetime of firm performance (i.e., firm maturity). A spanning bond's risk thus is also defined as a one-year payment's risk that approaches zero as maturity approaches zero. We show that options are equity bonds paid with equity, in contrast to T-bills and corporate bonds that are cash bonds paid with cash. Our asset risk is a one-year payment's risk, whereas our longitudinal risk is the risk accumulated over asset maturity or the holding time length. [4] We show that the gross volatility (variance), which includes the excess risk ALPHA, of realized (expected) asset value approaches zero as the measuring distance approaches zero. We reject the notion that uncertainty is a risk since unexpected increase in asset value is not a risk.

Asset pricing models, such as CAPM and Fama-French ones, have long been struggling to explain their asset risk. This study proposes a new measure of asset risk to address these challenges. Applying our risk measure to a principal-agent relationship, we illustrate that the agent's risk over the contracting period represents the agent's skin in the game that motivates, thereby capturing,

[4] Our reported asset risk thus is defined as a simple risk premium in a year, whereas a longitudinal risk is a risk continuously compounding over maturity or the holding time length. We often refer the two simply as a risk.

the agent's effort. The agent's effort thus is observable to both parties, opposite to the literature claims (e.g., Holmstrom, 1979; Harris and Raviv, 1976; Mirrlees, 1974, 1976).

Our methodology has to be developed into software to be widely usable, which is almost impossible decades ago when computers and internet are largely inaccessible. We contribute to several strands of literatures. First, we contribute to the asset pricing literature with a new measure of asset risk (e.g., Markowitz, 1952; Lintner, 1965; Jensen and Meckling, 1979; Black, 1972; Black and Scholes, 1973; Sharpe, 1964; Fama and French, 1997, 2015; Fama, 1968; Samuelson, 1965; Samuelson and Merton, 1969). Our asset risk, captured as risk premium, negatively impacts expected asset value without a distribution. Removing maturity from asset volatility, we produce a risk that investors truly care about. We argue that uncertainty is not a risk, suggesting that the literature improperly measures option risk. We propose that asset risk causes volatility, but not vice versa, arguing that asset risk cannot be diversified away.

Second, our asset risk allows us to compute managerial risk of inside debt holdings, option grants, and stockholdings, thereby solving issues that have long been challenging agency and contracting theories. (e.g., Harris and Raviv, 1976; Stiglitz, 1975; Spence and Zeckhauser, 1971; Murphy, 1985; Ross, 1973; Jensen and Meckling, 1976, 1979; Holmstrom, 1979; Lambert and Larcker, 1986; Baker and Wurgler, 2002; Loughran and Ritter, 1995). MAR prevents scholars from calculating the longitudinal risk. Our asset risk, in contrast, allows us to obtain a manager's risk that captures her effort over the period.

Third, our asset risk is defined as a one-year payment's risk, which approaches zero as maturity approaches zero. We propose that the gross volatility (variance) of realized (expected) asset value approaches zero as measuring distance approaches zero. We show that equity value reflects a spanning bond's present value, which is a payment spanning over firm maturity, transforming equity price volatility in a year into the spanning bond's baseline risk.

Fourth, our equity price movement delineates the spanning bond's excess risk ALPHA (i.e., $A$), which causes the excess return alpha (i.e., $\alpha$), and volatility that follows a blended normal distribution (which includes the put and the call distribution). We suggest that an option is an equity bond paid with equity, and that the difference between the option's price and present value captures the overcharge or the transaction cost.

Finally, our asset risk is directly related to asset value, thereby redefining firm misvaluation theories (e.g., Dong, Hirshleifer, and Teoh, 2012; Loughran and Ritter, 1995). We assume that managers have information advantage about the risk of their firm and future changes in the risk.

The remainder of this paper is organized as follows. Section 2 deducts a new measure of asset risk that is captured as risk premium, measuring the gross volatility (variance) of realized (expected) asset value. Section 3 simply shows how our measure of asset risk solves problems that have long been challenging agency and firm misvaluation theories. Section 4 concludes.

## 2. A new measure of asset risk

### *2.1 Asset risk definition*

To capture asset risk properly, we start from people's response to the increased risk for a product or task—higher premium. This also explains why scholars and practitioners take bond spread as compensation for the increased risk. Therefore, risk premium, which is cumulative over the holding time length, is our ideal proxy for asset risk. A premium-proxied asset risk thus reflects the possibility/magnitude of a permanent loss in the expected value which does not follow a distribution. We argue that asset risk causes the volatility (variance) of realized (expected) asset value, but not vice versa, suggesting that asset risk cannot be diversified away. We support that the volatility (i.e., the noise) of asset risk is positively associated with asset risk, following a blended normal distribution (which we will discuss in Section 2.5). We thus delineate the realized asset value (which asset risk affects without a distribution) and volatility (which does not impact the former while following a distribution). [5] We expect that asset return (volatility) reflects the market's (ir)rational perception of asset risk over maturity: the more lasting (temporary) of a price movement, the more likely to be treated as a return (volatility). Asset volatility thus reflects the predictable uncertainty whereas asset risk, which is positively associated with the former, captures the unpredictable possibility and/or magnitude of a lasting loss in asset value. Consistent with the debt risk in Nie (2024a), our asset risk is defined as:

**Proposition 1** *Asset risk is a payment's risk that approaches zero as maturity approaches zero.*

*2.2 The risk and the risk-free rate*

As in Nie (2024a), the continuously compounding return of a constant rate *r* is given by: [6]

$$\hat{Y}_T(r) = \int_0^T \left(e^{r(t)} - 1\right)dt = \int_0^T (e^r - 1)dt = T(e^r - 1). \tag{3}$$

In contrast, conventional concept gives the return of a constant rate *r* over *T* as:

$$\hat{Y}_T(r) = \int_0^T re^{rt}dt = e^{rT} - 1. \tag{4}$$

[5] Previously, we propose a quasi-normal distribution that actually lies between a normal and a lognormal distribution.

[6] For instance, the annual rate of 4.58% gives the continuously compounding rate as ln(1+0.0458) = 4.478%.

Nie (2024a) splits US and Canada T-bill yield into the inflation risk (i.e., macrorisk) $r'_f(t)$ and the risk-free rate $r_f(t)$. The paper argues that the risk-free rate largely reflects the expected inflation rate, and that the inflation risk mirrors the unexpected inflation effect or the investment opportunity cost for assets with a face value at maturity. The risk-free rate reflects the central bank's short-term rate plus a risk constant over maturity that captures the depreciation cost of the interbank lending rate. Thus, the nondepreciating asset value over $T$ is equal to the asset value with the risk-free return over $T$. An independence model, however, supports that the risk-free return of a nondepreciating value, does not produce a risk return, and vice versa, as implied by the dependence models. In Nie(2024a), the market expects the current monetary policy to move the current rate $r_{f0}$ towards the neutral rate $r_{fn}$ over the period $T_m$. To be comparable across assets, the inflation risk $r'_f$ is measured as a 1-year payment's risk. Then, the inflation risk at time $t$ can be written as:

$$r'_f(t) = r'_f(T-t),\ \ t \in (0, T). \tag{5}$$

On top of the macrorisk, a bond's microrisk (i.e., default risk or firm risk) can be expressed as:

$$r_d(t) = r_d(T-t),\ \ t \in (0, T). \tag{6}$$

The central bank's rate (i.e., the risk-free rate) at time $t$ can be written as:

$$rf(t) = \begin{cases} r_{f0} + \frac{(r_{fn}-r_{f0})t}{T_m} \\ r_{fn},\ \ (t > T_m) \end{cases}. \tag{7}$$

In case $T \leq T_m$, the risk-free return over $T$ ignoring the constant risk is given by:

$$\hat{Y}_T(r_f) = \int_0^T (e^{r_f(t)} - 1)\, dt$$

$$= \frac{e^{r_{f0}}\, T_m}{r_{fn}-r_{f0}} \left( e^{\frac{(r_{fn} - r_{f0})T}{T_m}} - 1 \right) - T. \tag{8}$$

The expected risk-free rate is equal to the central bank's effective (expected) rate $r_f(t)$ plus the constant risk $r_s$ that our formulas often ignore. In case $T > T_m$, the risk-free return beyond $T_m$ is:

$$\hat{Y}_{T_{m0}}(r_f) = \int_{T_m}^{T} (e^{r_f(t)} - 1)\, dt$$

$$= (T - T_m)(e^{r_{fn}} - 1). \tag{9}$$

According to formula (8), the return within $T_m$ is $\hat{Y}_{T_m}(r_f)$. As Nie (2024a), we allow the risk-free return $\hat{Y}_T(r_f)$ generated within $T_m$ to generate return beyond $T_m$:

$$\hat{Y}_T(r_f) = \hat{Y}_{T_m}(r_f) + \hat{Y}_{T_{m0}}(r_f) + \hat{Y}_{T_m}(r_f) \times \hat{Y}_{T_{m0}}(r_f)$$

$$= \left(\hat{Y}_{T_m}(r_f) + 1\right)\left(\hat{Y}_{T_{m0}}(r_f) + 1\right) - 1$$

$$= \left(\frac{e^{r_{f0}\, T_m}(e^{r_{fn}-r_{f0}}-1)}{r_{fn} - r_{f0}} - T_m + 1\right)\left((T - T_m)(e^{r_{fn}} - 1) + 1\right) - 1. \tag{10}$$

As in Nie (2024a), $\hat{Y}_T(r_f)$ is the predicted asset return in this study, where the subscript $T$ is maturity or the time length over which the return is measured, and $r_f$, the variable in the bracket, is the highest level of return-factor including the applicable lower levels. A return can have as many as 3 levels (from the lowest to the highest): 1. The risk-free rate $r_f$ (plus the constant risk $r_s$); 2. The inflation risk for assets with a face value at maturity: $r'_f$; 3. The corporate debt risk: $r_d$, or the spanning bond's risk: $r_e$, or the risk for holding a spanning bond over time: $r_{e0}$. We use $R_T(r'_f)$ to reflect the longitudinal risk over maturity $T$, where the variable in the bracket is the risk computed. The 4-factor independence model in Nie (2024a) gives the longitudinal risk of $r'_f$ over $T$ as:

$$R_T(r'_f) = \int_0^T \left(e^{\,r'_f(T\,-\,t)} - 1\right) dt$$

$$= \frac{e^{r'_f T} - 1}{r'_f} - T. \tag{11}$$

Given a T-bill's risk $r'_f$ and maturity *T*, the return can be expressed as:

$$\hat{Y}_T(r'_f) = \hat{Y}_T(r_f) + R_T(r'_f). \tag{12}$$

Therefore, a corporate bond's return over $T$ can be written as:

$$\hat{Y}_T(r_d) = \hat{Y}_T(r'_f) + R_T(r_d). \tag{13}$$

The 4-factor independence model in Nie (2024a) reveals strong power in explaining T-bill returns, on average predicting 99.1% of US T-bill returns (including those reported in Table 1), and 99.3% excluding the partially continuously compounding (PCC) regime, that is, $\hat{Y}_{T_m}(r_f) \times \hat{Y}_{T_{m0}}(r_f)$ in formula (10). The paper concludes that conventional continuously compounding rates and annual rates entangle the risk with the risk-free rate. This study uses the 4-factor independence model to compute the T-bill metrics including PCC. The inflation risk $r'_f$ is tied to assets with a face value denominated by the depreciating currency. [7] This unexpected inflation effect is not a risk for assets without a face value because unexpected inflation only increases their market value. Given the asset price and the T-bill metrics, we can solve the microrisk with repeated trials. Figure 3 illustrates how a bond's return is generated over maturity. The expected value at time *t*, that is, $D_t$ for debt or $E_t$ for equity, is equal to the current price plus the return, which reflects the asset holding cost, over *t*. The marginal asset holding cost of the bond declines over time because a

[7] We expect that a 10-year inflation risk approximately reflects the long-term inflation effect. For instance, our limited T-bill metrics data from the 4-factor independence model in Nie (2024a) indicate that the mean Canada (US) 10-year inflation risk is about 1.9% (3.7%) from Jan 2024 to July 2024 when the mean inflation rate reported by the government over the period is 1.9% (3.6%), which is available at https://tradingeconomics.com.

payment's risk approaches zero as maturity approaches zero regardless if the risk-free rate is constant over time. Figure 4 illustrates how a market noise of firm risk moves a corporate bond's price, or alternatively, how a bond's volatility reflects the noise of asse risk over maturity. A noise raises debt risk by $\Delta r_d$, thereby reducing the price by $\Delta D$ that mirrors the increased longitudinal risk over $T$: $R_T(r_d + \Delta r_d) - R_T(r_d)$. Thus formula (11) allows us to obtain $\Delta r_d$ given $\Delta D$. We will propose in Section 2.8 that the gross volatility (variance) of realized (expected) asset value approaches zero as distance approaches zero.

**[Please insert Figures 3 and 4 about here]**

*2.3 An example of firm debt risk computation*

We extract the price data of an Apple corporate bond from investing.com over the period from 28 Nov 2023 to 27 Nov 2024, [8] which matures on 6 May 2044 paying a semi-annual interest of 2.25% on 11 March and 11 September each year. As an example, we compute the bond's daily risk. To this end, we first compute the distance of the first interest payment as:

$$T_0 = \frac{(11\,Marc\quad 2024 - 28\,November\,2023)}{365} = 0.2849. \tag{14}$$

The distance of principal payment can be obtained as:

$$T = \frac{(6\,May\,2044 - 28\,November\,2023)}{365} = 20.4521. \tag{15}$$

Thus, the number of interest payments over the period is given by:

[8] We observe that bond data have 7 more observations on Sundays. We thus drop 7 days data such that the stock data and the bond data both have 253 observations over the sample period. We treat the risk on 28 Nov 2023 as the baseline risk to examine the daily changes in the baseline risk over the calendar year.

$$N = 1 + INT\left(\frac{T - T_0}{0.5}\right) = 41. \tag{16}$$

where function INT rounds a real number down to the nearest integer.

To examine the daily changes in the risk over a year (i.e., 252 trading days), we use formula (13) obtain the present value of each payment. We then use formula (8) to obtain $\hat{Y}_T(r_f)$ for the payment in case $T < T_m$. For instance, we compute the present value of the principal $D$ as:

$$P = \frac{D}{1 + \hat{Y}_{T - \frac{i}{252}}(r_{d,i})}. \tag{17}$$

where $T - \frac{i}{252}$ is the distance of the principal on day $i$, and $r_{d,i}$ is debt risk on day $i$.

The number of the remaining interest payments on day $i$ can be expressed as:

$$M = 1 + INT\left(\frac{T - T_0}{0.5}\right) + INT\left(\frac{T_0 - \frac{i}{252}}{0.5}\right). \tag{18}$$

where $1 + INT\left(\frac{T - T_0}{0.5}\right)$ is the number of interest payments to receive prior to the 1st trading day, and $INT\left(\frac{T_0 - \frac{i}{252}}{0.5}\right)$ is minus the number of interest payments that have been received up to day $i$.

The distance of the future $k$th interest payment can be written as:

$$T_0 + 0.5INT\left(\frac{i}{126}\right) - \frac{i}{252} + 0.5(k - 1), \quad k \in (1, M). \tag{19}$$

where $T_0 + 0.5INT\left(\frac{i}{126}\right)$ is the span of the next interest payment, then $T_0 + 0.5INT\left(\frac{i}{126}\right) - \frac{i}{252}$ is the distance of the next interest payment on day $i$.[9]

[9] We observe that a minimal inaccuracy in the distance may lead to incorrect count of the interest payments on a day. After carefully checking, we do not find unusual risk around the interest payment day (i.e., 11 Mar and 11 Sep 2024).

Then, the bond price on day $i$ reflects the present value of a bunch of bonds (i.e., payments):

$$P_i = \frac{D}{1+\hat{Y}_{T-\frac{i}{252}}(r_{d,i})} + \sum_{k=1}^{1+INT\left(\frac{T-T_0}{0.5}\right)+INT\left(\frac{T_0-\frac{i}{252}}{0.5}\right)} \left( \frac{D_I}{1+\hat{Y}_{T_0+0.5INT\left(\frac{i}{126}\right)-\frac{i}{252}+0.5(k-1)}(r_{d,i})} \right). \quad (20)$$

where $D$ is the face value of the principal, and $D_I$ is the face value of an interest payment.

We convert all rates from annual rates to continuously compounding rates for estimation purpose. We compute the US T-bill metrics manually. The risk on 28 Nov 2023 (i.e., the day prior to the examining year) is the baseline risk $r_d$. Initially we apply T-bill metrics on 3 days to compute the bond's daily risk over the calendar year. Observing that the estimated risk numbers appear to be very high or very low over some specific days, we rework on the T-bill metrics either by trying to reduce the MAV (i.e., the mean absolute value of T-bill prediction errors) or by generating more recent T-bill metrics. For instance, we extract T-bill data on 29 May 2024 to obtain more recent T-bill measures, finding that the bond's gross risk noise, that is, $\Delta r_{D,i} = r_{d,i} - r_d$, on day $i$ for each of the 252 trading days, for June 2024 appears to be abnormally low. This is the main reason that the T-bill dates reported in Table 1 are not equally distributed over the sample year. We finally obtain T-bill metrics on 13 days including those on 3 May 2024 reported in Nie (2024a). Table 1 shows that our delineation of the risk and the risk-free rate for US T-bills on these dates on average predicts 99.1% of the bill's yield. Definitely, we can estimate asset risk more robustly with weekly or daily T-bill metrics. Table 2 reports our final results of the gross risk noise of the Apple corporate bond over the year, even if the high (low) numbers turn suspiciously higher (lower) with more recent, and/or better estimated, T-bill metrics. All the reported rates are continuously compounding unless otherwise indicated. We find that the baseline risk is 14.30 bps, that the mean gross risk noise of 1.14 bps, and that the standard deviation is 6.21 bps.

We can easily obtain the bond's risk on a given day with repeated trials, since the price monotonically decreases with higher risk. A challenge of the analysis is to manually compute the US T-bill metrics. We strongly believe that the 4-factor independence model can predict over 99.5% T-bill returns (i.e., MAV < 0.5%) with software developed. Another challenge is to apply formula (20) free of mistakes. We obtain qualitatively similar results without PCC. We would provide more detailed data and formulas upon request.

**[Please insert Tables 1 and 2 about here]**

*2.4 A spanning bond*

First, we show that equity price reflects the present value of a payment spanning over equity maturity (i.e., a spanning bond) that mirrors the predictable lifetime of the firm's performance. An example to explain firm maturity, beyond which a payment has no present value. Suppose that a firm's debt risk $r_d$ plus the inflation risk $r'_f$ is equal to 1%, and that the firm's long-term nondepreciating profitability is 10%. Then our debt risk theory supports that: 1) the risk is almost zero for a payment in a day; 2) the risk reaches 1%, 5%, and 10% by the end of the 1st, 5th, and 10th year, respectively; 3) the nondepreciating yield of a 10-year bond is 5% while the marginal debt cost reaches 10% at the end of the 10th year. Thus, firm maturity in terms of the firm's debt is 10 years where the nondepreciating performance (i.e., payment) accounting for the risk decreases to zero—a point beyond which a payment has no present value because such a payment is unsustainable and expected to default. We define a firm's bond reaching this point as firm maturity bond (FMB). Firm maturity captures the predictable lifetime of firm performance, reflecting the maximum risk acceptable to the market. In short, equity price represents the present value of the firm's spanning bond. To help understand, we can treat equity value as the present value of a series of "daily nondepreciating payments" spanning over equity maturity $T_e$, where a daily payment reflects the nondepreciating value that we expect the firm to produce on a daily basis. Therefore, equity value reflects the present value that the market expects the firm to continuously pay over $T_e$, whereas the price of a corporate bond is the present value of the one-time payment at maturity $T$. Therefore, volatility improperly reflects asset risk because volatility is positively associated with maturity.

The present value of the continuous payment is highly correlated to, but different from, the firm's earnings and profits. For example, Apple's performance is below the market expectation if the nondepreciating price remains the same in a month since the latter price has less present value after accounting for the asset holding risk. Today is 24 Oct 2024, the risk $r_e$ mirrors the risk of a nondepreciating payment on 24 Oct 2025. Thus, the risk at $t$ can be written as:

$$r_e(t) = r_e t, \quad t \in (0, T_e). \tag{21}$$

As illustrated in Figure 5, the continuous payment's risk moves from current zero to maximum at $T_e$, while that of the one-time payment moves from current maximum to zero at $T$.

For a spanning bond's return, the unexpected inflation effect acts like a risk-free return rather than a risk in raising the market value denominated by the depreciating currency. Therefore, the return of a spanning bond is similar to that of a corporate bond ignoring the inflation risk:

$$\begin{aligned}\hat{Y}_{T_e}(r_e) &= \hat{Y}_{T_e}(r_f) + \int_0^{T_e} (e^{r_e t} - 1)\, dt \\ &= \hat{Y}_{T_e}(r_f) + \frac{e^{r_e T_e - 1}}{r_e} - T_e \\ &= \hat{Y}_{T_e}(r_f) + R_{T_e}(r_e). \end{aligned} \tag{22}$$

where $\hat{Y}_{T_e}(r_f)$ is given by formula (10).

To analyze a spanning bond's value and risk, we cannot confuse the expected growth in equity value over time with the value that we expect the spanning bond to generate over maturity. The former is like the railway on which the train of spanning bond moves with the wheels $E_0$ and $r_{e0}$ on the track. The return independence assumption allows us to ignore the effect of the payment's risk-free return $r_f(t)$ that reflects the inflation effect on the asset's market value. Ignoring the risk-free return, the spanning bond's marginal present value (which we expect the firm to generate over maturity $T_e$) is nondepreciating. Remember, the marginal present value generated by the firm declines to zero at $T_e$ after accounting for the payment risk $r_e t$, which increases from current zero to maximum of $r_e T_e$ at maturity $T_e$ (see, Figure 6). Then, the spanning bond's present value is:

$$E_0 = E_0 \int_0^{T_e} (e^{r_e T_e} - e^{r_e t}) dt. \tag{23}$$

$$\Rightarrow \quad 1 = T_e e^{r_e T_e} + \frac{1 - e^{r_e T_e}}{r_e}. \tag{24}$$

Formula (24) allows us to easily obtain $T_e$ with repeated trials given $r_e$, as $T_e$ monotonically increases with lower $r_e$. As demonstrated in Figure 6, equity price $E_0$ reflects the firm's lifetime performance in excess of the asset holding cost, and that the marginal present value produced by the firm declines to zero at $T_e$ when the spanning bond's marginal holding cost overtakes the marginal performance. Thus, $E_0$ mirrors the present value of the firm's lifetime performance.

**[Please insert Figures 5 and 6 about here]**

The risk for holding the spanning bond over time can be computed as a value-weighted risk:

$$r_{e0} = \int_0^{T_e} (e^{r_e T_e} - e^{r_e t}) r_e t dt$$

$$= \int_0^{T_e} e^{r_e T_e} r_e t dt - \int_0^{T_e} e^{r_e t} r_e t dt$$

$$= 0.5 r_e {T_e}^2 e^{r_e T_e} - \frac{\int_0^{r_e T_e} e^{r_e t} r_e t d\ _e t}{r_e}$$

$$= 0.5 r_e {T_e}^2 e^{r_e T_e} + \frac{e^{r_e T_e} - r_e T_e e^{r_e T_e} - 1}{r_e}$$

$$= 0.5 r_e {T_e}^2 e^{r_e T_e} - 1. \tag{25}$$

Then the expected equity value at time *T* is given by:

$$E_T = E_0 + E_0 \int_0^T (e^{r_f(t) + r_s} + e^{r_{e0}} - 2) dt. \tag{26}$$

Without developing software to delineate the risk and the risk-free rate of T-bills on a daily basis, we cannot examine the risk noise that accounts for the noise of inflation risk and the risk-free rate (which affects the depreciating value of various assets). Therefore, we assume that the macrorisk's noise influences asset value through only through the firm's microrisk, and that management actions, such as restructures of firm debt or capital, affect asset risk as a permanent change only in the long-term. In the short-term, therefore, our price error on day *i* is caused by the gross risk noise $\Delta r_{E,i}$ (which includes the excess risk ALPHA *A* and the risk noise $\Delta r_{e,i}$), rather than the change in the expected asset value such as a change in the long-term growth rate or a loss of value from the firm's accident or legal issue (which affect asset price through another channel). We expect that the equity holding risk $r_{e0}$ remains unchanged when the gross noise $\Delta r_{E,i}$ changes equity maturity $T_e$ by $\Delta T_{e,i}$ and equity price $E_i$ by $\Delta E_i$, as in Figure 7. Then we have:

$$\left(r_e + \Delta r_{E,i}\right)\left(T_e + \Delta T_{e,i}\right) = r_e T_e. \tag{27}$$

$$\Rightarrow \quad \Delta T_{e,i} = \frac{-\Delta r_{E,i} T_e}{r_e + \Delta r_{E,i}}. \tag{28}$$

Thus, the change in equity price, or the price error, on day $i$, can be expressed as:

$$\Delta E_i/E_i = \int_0^{T_e+\Delta T_{e,i}} (e^{r_e T_e} - e^{(r_e+\Delta r_{E,i})t})dt - \int_0^{T_e} (e^{r_e T_e} - e^{r_e t})dt$$

$$= \Delta T_{e,i} e^{r_e T_e} + \frac{1-e^{(r_e+\Delta r_{E,i})(T_e+\Delta T_{e,i})}}{r_e+\Delta r_{E,i}} - \frac{1-e^{r_e T_e}}{r_e}. \qquad (29)$$

We solely rely on equity price movement over a year to obtain a spanning bond's baseline risk $r_e$ (the risk on the day prior to the examining year), since limited access to a firm's financial data may not allow us to estimate $r_e$ as rigorously as we compute a corporate bond's risk. To this end, we use formula (29) to transform the gross volatility of equity price over in the year into the premium-based risk $r_e$, expecting that equity price movement reveals the gross volatility that is positively related to $r_e$. Observing that the ratio well captures a 10-year payment's risk of a spanning bond, we simply obtain the baseline risk as the ratio of the standard deviation of equity price to the mean price in a year. In particular, we compute the risk as:

$$r_e = \frac{V_e}{10\ {}_e}. \qquad (30)$$

where $V_e$ is the standard deviation of price in the year, and $P_e$ is the mean price in the year.

Although the gross volatility yields the baseline risk, a diversification strategy cannot reduce a portfolio's risk, because a less correlated diversifying firm is likely to raise the cognition cost(risk).

Intuitively, the risk of a payment without a face value is riskier than that with a face value, because equity holders are actually responsible for the firm's debt payment. Figure 7 suggests that the value curve of a spanning bond revolves counter clockwise when the risk increases. As illustrated in Figure 8, the more (less) informed market can raise (reduce) the longitudinal risk $R_{T_e}(r_e)$ over time whereas the decrease (increase) in the expected asset value at maturity is not according to formula (27). To be more specific, the market can reduce (raise) the risk and/or the growth rate, which reduces (raises) the expected bond value at $T_e$ thereby rotating the curve (counter-)clockwise. Curve $E_{0,r_f}$ represents the present value discounting the risk-free return, whereas curve $E_0$ additionally discounts the longitudinal risk $R_{T_e}(r_e)$, that is, $r_e$ over $T_e$. Curve $E_{0,1}$ is the equity value when the market treats the negative event wholly as an increase in the spanning bond's risk by $\Delta n$. The decrease in equity price is given by:

$$\Delta N = \frac{E_{T_e}}{1+\hat{Y}_{T_e}(r_e)} - \frac{E_{T_e}}{1+\hat{Y}_{T_{e,1}}(r_e+\Delta n)}. \tag{31}$$

where $\hat{Y}_{T_e}(r_e)$ and $\hat{Y}_{T_{e,1}}(r_e + \Delta n)$ are spanning bond's returns given by formula (22).

As indicated by curve $E_{0,2}$, the market treats the event wholly as a decrease in the firm's current wealth by $\Delta N$ thereby reducing the expected value at $T_e$ by $\Delta N(1 + \hat{Y}_{T_e}(r_f))$, where $\hat{Y}_{T_e}(r_f)$ reflects the inflation effect over $T_e$. Thus, the curve can turn between the two lines given $\Delta r_e \in (0, \Delta n)$ even if the price remains unchanged. The transformation of a risk (noise) in the real market can be more complicated than the above example. Thus, we use (30), rather than $r_e$ plus the gross risk noise (i.e., $\Delta r_E$) at the end of the year, to obtain the baseline risk of the next calendar year.

**[Please insert Figures 7 and 8 about here]**

Given $r_e$ of a spanning bond, and equity price change $\Delta E_i$ on day $i$, we can easily obtain the gross risk noise $\Delta r_{E,i}$ with repeated trials, as we can move the trial value of $\Delta r_{E,i}$ indefinitely close to the true value of $\Delta r_{E,i}$ that increases monotonically with lower $\Delta E_i$.

*2.5 A blended normal distribution*

An example to explain a blended normal distribution. Suppose the ACT scores of US high school students perfectly follow a normal distribution with a median (mean) of 21. Then high schools change the scoring policy by awarding each point above 21 with two more points. We propose that the new scores follow a blended normal distribution—the standard deviation of scores above the median is different from that of scores below the median. This study assumes that the noise of asset risk follows a blended normal distribution—the asset price thus follows a blended lognormal distribution. The blended ratio (BR) with $2n$ or $2n+1$ observations can be written as:

$$\mathrm{BR}=\frac{\sum_{i=1}^{n}(a_i-s_m)}{\sum_{i=1}^{n}(s_m-b_i)}. \tag{32}$$

where $s_m$ is the median, $a_i$ and $b_i$ are the $i$th scores above and below the median, respectively.

We define the standard deviation of the lower (upper) normal distribution as the put (call) standard deviation of a blended normal distribution, a key determinant of a put (call) option's exercise probability. We artificially generate observations that mirror the put (call) scores to obtain the put (call) standard deviation of the put (call) normal distribution.

As asset risk is likely to change over a period, we use the excess risk ALPHA (i.e., $A$) to capture the systematic risk, such as the price error prior to, or the net change in the risk over, the examining period. ALPHA causes the excess return alpha (i.e., $\alpha$) in the opposite direction, as suggested by formulas (13) and (25). For a spanning bond, we compute alpha as the change in the holding risk driven by ALPHA as suggested by formula (25):

$$\alpha = 0.5(r_e + A)(T_e + \Delta T_e)^2 e^{(r_e+A)(T_e+\Delta T_e)} - 0.5 r_e {T_e}^2 e^{r_e T_e}. \tag{33}$$

We argue that asset volatility following a distribution is not a risk although it is positively associated with asset risk, supporting that the excess risk ALPHA represents the true asset risk that does not follow a distribution and thus cannot be diversified away. Our asset risk theory delineates ALPHA (which is unpredictable and undiversifiable) and the risk noise (which is predictable and diversifiable while following a blended normal distribution). We thus expect that the risk noise, which accounts for ALPHA, has zero median. In case of even observations, such as 252 trading days which is usually considered to be a calendar year, our ALPHA value requires that BR is equal to the minus ratio of the smallest positive score to the smallest negative score.

Asset volatility ***at*** a time reflects the noise of asset risk over maturity. Proposition 2 describes that the impact of gross volatility (variance), which includes the excess risk ALPHA, on the realized (expected) asset value reflects the gross noise of asset risk ***over*** the distance:

**Proposition 2** *The gross volatility (variance) of realized (expected) asset value approaches zero as distance approaches zero.*

As a spanning bond's maturity rolls over, we need a reference point, which is usually the price prior to the examining period, to measure asset volatility ***over*** the distance. The gross volatility (variance) of asset value, regardless of potential change in equity maturity, is likely to be zero in a second, minimal in a minute, and significant in a week. [10] However, we cannot use the excess risk $A$, which neither follows a distribution nor is necessarily proportionately related to the risk, to obtain $r_e$. For a spanning bond or a corporate bond, the gross risk noise on day $i$ is given by:

$$\varDelta r_{E,i} = \frac{iA}{252} + \varDelta r_{e,i},\ \ i \in (1, 252). \tag{34}$$

As indicated by formula (34), our theory supports that $\varDelta r_{e,i}$ approaches zero as the unit of time length approaches zero (such as one second). As the excess risk $A$ determines zero median of the

[10] The trading volume, which likely to be zero in a second and significant in a week, may better capture the distance.

risk noise, we can apply repeated trials to solve $A$ and $\Delta r_{e,i}$ over the calendar year, thereby obtaining the put (call) distribution of the risk noise. A more advanced model of blended normal distribution that reflects the gross volatility of formula (29) rather than the simply ACT score model definitely better captures the distribution of risk noise, which we leave for future research. For a perfect blended normal distribution, BR should be equal to the ratio of the call standard deviation to the put standard deviation. To measure the efficiency, we compute the BR error as:

$$\text{BR Err} = \frac{PUT\_DISTRIBUTION}{CALL\_DISTRIBUTION \times BR} - 1. \tag{35}$$

We expect the BR error, which is sensitively high with a small number of observations, to be zero with a large number of observations that perfectly follow a blended normal distribution.

*2.6 Equity risk computation examples*

As examples, we compute the gross risk noise of Apple, Tesla, and Nvidia spanning bonds from 29 Nov 2023 to 27 Nov 2024—252 trading days is usually a calendar year. We extract market data from the Center for Research in Security Prices (CRSP) and adjust the prices for dividend and splits and/or capital gain distribution during the year. We use formula (8) rather than (10) to obtain $\hat{Y}_T(r_f)$ of a firm's expected equity price since $T < T_m$. As given by formula (26), we use the most recent risk-free rate in Table 1 to compute the expected equity price on the day. We convert all annual rates in Table 1 to continuously compounding rates for our estimation purpose.

To obtain the gross risk noise over a year, we first use formula (30) to obtain the baseline risk (i.e., $r_e$) prior to the examining year. Then, formula (25) gives the spanning bond's holding risk, or equity holding risk, that is, $r_{e0}$. Next, we use the equity price errors produced by formula (26) to obtain the gross risk noise on each trading day with repeated trials, as suggested by formula

(24). Table 3 reports the results. All rates are continuously compounding unless otherwise indicated. We find that Apple spanning bond has a baseline risk of 109.9 bps with maturity 12.87 years, and that the gross risk noise over the year has a mean of −0.38 bps with standard deviation of 10.12 bps. For Tesla spanning bond, we obtain a baseline risk of 197.2 bps with maturity 9.46 years. The gross risk noise has a mean of 49.15 bps with standard deviation 60.01 bps. The results indicate that Nvidia spanning bond reveals a baseline risk of 296.0 bps with maturity 7.62 years. The gross noise has a mean of −128.2 bps with standard deviation 56.48 bps.

**[Please insert Table 3 about here]**

*2.7 Delineation of the excess risk and the risk noise*

In this section, we employ repeated trials to delineate the excess risk and the risk noise for the gross risk noise reported in Tables 2 and 3. In particular, we separate the daily gross risk noise into the excess risk ALPHA (i.e., $A$), which leads to the excess return alpha (i.e., $\alpha$), and the risk noise which follows a blended normal distribution. We artificially generate the positive (negative) scores for the put (call) distribution to obtain the put (call) standard deviation. Finally, we plot the results in Figure 9. Panels A and B illustrate the figures for the Apple corporate bond. Panels C to H display the figures for Apple, Tesla, and Nvidia spanning bonds.

Figure 9 shows that the Apple corporate bond has a baseline risk of 14.30 bps with an excess risk −4.67 bps which produces the excess return of 4.67 bps in the year. The standard deviation of the put (call) distribution is 7.73 (4.92) bps. For Apple spanning bond, the baseline risk is 109.9 bps with the baseline risk (i.e., $r_{e0}$) 4.77%. We find that the excess risk of 8.02 leads to the excess return of −16.41 bps. In contrast, we observe that the risk of Tesla and Nvidia spanning bonds prior to the examining year is 197.2 bps and 296.0 bps, respectively. The excess risk of −116.3 bps and −278.6 bps is associated with the excess return 219.9 bps and 572.2 bps, respectively. Tesla spanning bond has the highest BR error 6.95%. A more advanced computation methodology and/or a blended normal distribution model more advanced than the ACT score will improve the estimation efficiency definitely, which we leave for future work.

**[Please insert Figure 9 about here]**

*2.8 Option pricing*

We can treat corporate bonds and T-bills as cash bonds because they are paid with cash at maturity. Then options are actually equity bonds paid. Intuitively, the risk of an equity payment is zero in a second, minimal in an hour, and significant in a week—the risk approaches zero as maturity approaches zero. One may confuse the risk of holding the underlying share with that of holding the option. The former is expected to be $r_{e0}$ over time, whereas the latter decreases to zero at maturity $T$ when the option holder obtains either zero equity value or certain equity value that she can sell immediately. In case she holds the equity obtained from the option exercise, she makes another investment. Therefore, the spanning bond's risk $r_e$, which is a one-year payment's risk, reflects the option's risk. However, the former increases from current zero to maximum at $T_e$ while the latter decreases from current maximum to zero at $T$. Therefore, the risk of an option with 100% exercise possibility, where the option holder holds the full distribution, can be expressed as:

$$r_e(t) = r_e(T-t),\ t \in (0, T). \tag{36}$$

Next, we show how to obtain an option's present value, where the asset holder owns only part of the distribution that is determined by the exercise price $C_0$. Given the current equity price $E_0$, the spanning bond's risk $r_e$, maturity $T$, and the put (call) standard deviation, we expect that the option buyer believes that she has expertise in predicting the firm's excess risk. Then, we can compute the bottom line, or benchmark excess risk $A_c$ of the investor, such that the asset's present value is not below the option price. To this send, we use the risk-free rate and the equity holding risk to obtain the expected equity price $E_T$. Then, we scale the gross exercise risk $\Delta r_{E,c}$, which is equal to $TA_c + \Delta r_{e,c}$, with the call (put) standard deviation to obtain the standard gross exercise risk $\Delta r_{E,c0}$, as suggested by formula (26). As illustrated in Figure 10, the standard option value is:

$$\phi = \int_0^{+\infty} pdf(v + \Delta r_{E,c0})dv = \int_{\Delta r_{E,c0}}^{+\infty} pdf(v)dv. \tag{37}$$

where $\phi \in (0, 1)$, and *pdf* is the probability density function of a standard normal distribution.

An equity bond and a spanning bond have the same return formula (22). Given the current stock price $E_0$, the option's value at $T$ is equal to the value of the equity bond that pays at maturity:

$$O_T = \phi E_0\big(1 + \hat{Y}_T\,(r_e\,)\big). \tag{38}$$

As in Figure 10, the option actually reduces the share value from 1 to $\phi$, thereby condensing the equity bond's risk from $r_e$ to $\frac{r_e}{\phi}$. Then, the equity bond's present value can be computed as:

$$O_0 = \frac{\phi E_0(1 + \hat{Y}_T\,(r_e\,))}{1 + \hat{Y}_T\left(\frac{r_e}{\phi}\right)}. \tag{39}$$

Formula (39) allows us to obtain $A_c$ and $\Delta r_{e,c}$ with repeated trials. Alternatively, we can assume that the firm's excess risk $A$ remains unchanged over option maturity $T$, thereby computing the overcharge or transaction cost of the option, which represents the difference between the option price and the present value. In contrast to Black and Scholes (1973), our option value is positively related to $T$ that raises the gross risk volatility thereby raising the option value.

**[Please insert Figure 10 about here]**

## 3. Agency and firm misvaluation theories

### *3.1 Risk sharing in a principal-agent relationship*

In this section, we use our asset risk theory to calibrate the agent's skin in the game, showing how our asset risk addresses challenges that have long been facing the literature. The previous agency and contracting theories actually predict that the agent's effort remains the same even if the payoff is risk-free or near risk-free (e.g., Jensen and Meckling, 1976; Holmstrom, 1979), because MAR in effect prevents scholars from properly measuring a manager's risk that is tied to her firm with untradable asset holdings. [11] [12] This tied risk represents the agent's skin in the game, that motivates, thereby proxying, the agent's effort over the period, which the literature has to ignore entirely. Studies thus raise the above issue by treating the payoff or the asset value as the agent's risk when exploring optimal risk sharing rules in a principal-agent relationship. [13] They actually miss two of the three orthogonal dimensions of the longitudinal risk illustrated in Figure 1 by treating the agent's effort as unobservable to the principal (e.g., Mirrlees, 1974, 1976; Holmstrom, 1979). Our risk measure allows us to calibrate the agent's risk that reflects the agent's

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[11] We treat the macrorisk or T-bill risk as a form of risk-free rate when examining managerial risk in debt-like holdings.

[12] We argue that Dittmann and Maug (p. 309, 2007), while introducing a risk premium on the firm's stock, improperly claim that the CEO cannot obtain the market risk premium when not allowed to trade in the market. Instead, the trading restriction forces the CEO to take the risk premium that represents the CEO's risk.

[13] See, for example, Holmstrom (1977), Harris and Raviv (1976), Mirrlees (1974, 1976), Stiglitz (1975), Williamson (1975), Wilson (1968), Luenberger (1969), Gjesdal (1976), Spence and Zeckhauser (1971), Jensen and Meckling (1976, 1979), Lambert and Larcker (1986), Pratt (1964), Pratt and Zeckhauser (1987), Smith and Zimmerman (1976), Noreen and Wolfson (1981), Aboody (1996), Carpenter (2000), Kahl, Liu, and Longstaff (2003), Hall and Murphy (2002), Murphy (1985), and Ross (1973).

effort over the contracting period. Our managerial risk ignores the macrorisk of debt-like assets that is out of managerial control.

We compute the longitudinal risks of various assets in the spirit of formulas (13) and (26). The agent's risk motivates, thereby capturing, the agent's effort, implying that the agent's risk (i.e., effort) approach zero when the holding time length approaches zero. The effort-outcome dynamics suggest that the agent's effort and payoff are observable to both parties, where the payoff reflects the outcome or the expected value with a distribution given the agent's effort. Thus, we can aggregate and compare the holding risks of various assets over different holding time lengths. For instance, a firm's total equity holding risk in a year can be written as:

$$\mathfrak{R}_{e0} = Capitalization \times R_{e0}\,(1). \tag{40}$$

where *Capitalization* is the firm's market capitalization, and $R_{e0}\,(1)$ is equal to $e^{r_{e0}} - 1$.

A major challenge to agency theory is the privacy obstacles, which means that we can hardly obtain a usable utility function of a manager. We need to measure a manager's utility in exact numbers such as a payment of $1,000.

*3.2 Corporate financing and market misperception of firm risk*

The risk determines the value, but not vice versa: one goes up, the other goes down. This is what MAR fundamentally obscures and asset pricing models have long been struggling to prove. For instance, market timing hypothesis suggests that managers conduct a seasoned equity offering (SEO) when their firm is overvalued, or a share repurchase when their firm is undervalued (e.g., Baker and Wurgler, 2002; Loughran and Ritter, 1995; Dong, Hirshleifer, and Teoh, 2012; Khan, Kogan, and Serafeim, 2012). In contrast, we argue that market misperception of firm risk, rather than firm value, drives firm financing decisions. The literature also ignores the fact that the market may correct debt risk as well when it corrects equity risk, thereby missing the cost of the equivalent debt repurchase (issuance) associated with the SEO (share repurchase). Managers not only affect the risk of the firm that they manage, they are also better informed of the risk and its future changes. A firm thus always profits from an equity issuance (repurchase) when firm risk observed by the market is lower (higher) than the true risk. Since a firm's equity risk is always much higher than its debt risk, the value gained from a market correction of firm risk is always greater than the value lost from the equivalent debt repurchase (issuance). For instance, the Apple corporate bond has a mean risk of 15.44 bps in Table 2, [14] in contrast to a proximate mean risk of 4.77% for equity holding in the examining year in Figure 9. [15] Therefore, the cost caused by the change in the former is negligible compared to that caused by the change the spanning bond's risk.

## 5. Conclusion

We argue that asset risk has to be cumulative because the longitudinal risk approaches zero as the holding time length approaches zero. We show that asset volatility reflects the noise of asset risk over maturity, which improperly reflects the risk of market participants although we do not care about the holding time length that are up to themselves. A manager's risk, however, captures her skin in the game that motivates, thereby reflecting, her effort in the principal-agent relationship. MAR thus prevents scholars from measuring managerial risk. We argue that asset risk causes

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[14] As reported in Table 2, the microrisk prior to the examining year is 14.30 bps while the mean risk noise in the year is 1.14. Thus, the mean risk over the year is equal to 14.30 + 1.14 = 15.44 bps.

[15] The mean gross risk noise of Apple spanning bond over the examining year is −0.38 bps (see, Table 3) whereas the excess risk ALPHA (i.e., $A$) is 8.02 bps in Figure 9. Thus, formula (33) suggests the mean risk for holding the spanning bond over the year is slightly higher than the reported equity holding risk (i.e., $r_{e0}$) of 4.77%.

volatility, but not vice versa, supporting that asset risk cannot be diversified away as diversification can incur additonal cognition cost. Finally, support to MAR appears to emerge from a confusion between asset value and wealth utility: LDMU suggests that asset volatility reduces the latter. In fact, diversification decreases the magnitudes of temporary losses at a cost of reduced sizes of temporary gains. Speculative investors, especially option investors, actually depend on volatility to profit. Even in terms of wealth utility, we expect that the benefit of a diversification solution is economically insignificant, because poor people, whom LDMU affects significantly, can hardly afford to hold risky assets largely when diversification is a simple choice of their own. These causes explain why CAPM and Fama-French models have largely failed to price asset volatility that reflects the noise of asset risk over maturity. To address these challenges, we separate expected asset value (i.e., which asset risk impacts without a distribution) from volatility (which does not affect the former while following a blended lognormal distribution we proposed).

Our asset risk for a specific asset ignores the macrorisk of Nie (2024a) that is tied to assets with a face value denominated by the depreciating currency. Our nondepreciating asset value accounts for the expected risk-free rate, which is captured as the expected inter-bank short-term rate plus the constant risk (i.e., the depreciation cost of the short-term rate). Our equity risk is a payment's risk that approaches zero as maturity approaches zero, reflecting a one-year payment's risk to be comparable across assets. We propose that equity value represents the present value of a payment spanning over the firm's predictable lifetime. We transform the gross volatility of equity price into the underlying spanning bond's baseline risk, expecting that the former is positively associated with the premium-based latter. As an example, we demonstrate how to compute the daily risk over a year of an Apple corporate bond and three firms' spanning bonds. We show that the gross volatility of realized asset value reflects the variance of expected asset value at that distance, that options are equity bonds paid with equity, and that the difference between the option price and the present value captures the option's overcharge or the transaction cost.

Our premium-based asset risk addresses challenges that have long been challenging MAR and agency theories. In a principal-agent relationship, we show that optimal risk sharing point represents optimal payoff sharing point between the two parties, and that the agent's effort and payoff thus are observable to both parties, in contrast to the literature postulation that the agent's effort is unobservable to the principal (e.g., Holmstrom, 1977, 1979; Mirrlees, 1974, 1976). Our risk theory also redefines firm misvaluation theories. We show that a firm always profits from equity issuance (repurchase) when its risk perceived by the market is lower (higher) than its true risk that managers are better informed of. As a major player impacting firm risk, firm management also have information advantage about future changes in firm risk.

A main disadvantage of our theory is that we need to develop complicated software to estimate asset risk. A promising future work is to examine asset risk noise using software to delineate the risk and the risk-free rate of T-bills, thereby separating the noise of inflation risk and the risk-free rate from the noise that drives the (non)depreciating value of various assets. It is also promising to develop wealth utility functions that address the major privacy challenges.

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**Figure 1. The three orthogonal dimensions of a longitudinal risk**

Figure 1 uses firm debt as an example to illustrate the three orthogonal dimensions of a longitudinal risk that approaches zero as any one of the three dimensions approaches zero. As in Nie (2024a), the risk-free rate $r_f(t)$ is the expected short-term rate of the central bank capturing the inflation effect over time. The debt risk $r_d$ reflects the asset's risk in addition to the inflation risk. Given asset value $V$, the asset holder's risk over $T_h$ (i.e., longitudinal risk) can be expressed as:

$$LR = V \times R_{T_h}(r_d). \tag{42}$$

where $R_{T_h}(r_d)$ is given by formula (11).

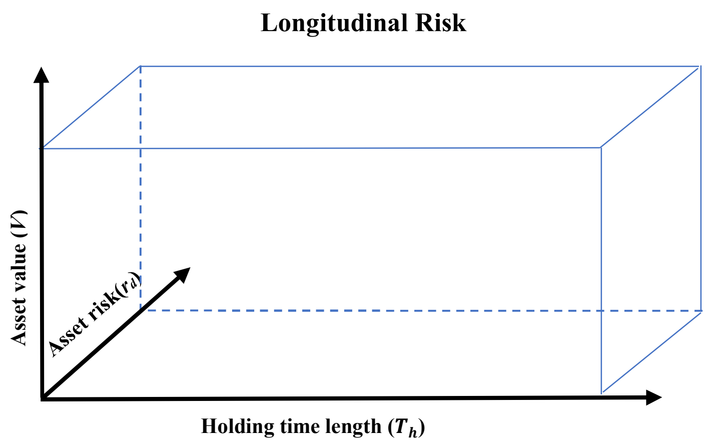


**Figure 2. Asset volatility and wealth utility: the law of diminishing marginal utility**
This figure shows that asset volatility decreases asset holder's wealth utility rather than asset value, as suggested by the law of diminishing marginal utility (LDMU). A confusion between asset value and wealth utility thus lends support to MAR, which argues that a diversification strategy reduces asset risk by decreasing volatility. Given asset holder's utility function $U(w)$, LDMU gives $U' > 0$ and $U'' < 0$. Given asset holder's wealth $w_0$, the figure illustrates the loss of wealth utility from asset volatility following a normal distribution, as captured by a change in $w_0$ by $x$ and $-x$ (two equally possible changes). In particular, the utility loss from a decrease in the risky asset value by $x$ (i.e., the height of brown pillar) is always greater than the utility gain from an increase in the asset value by $x$ (i.e., the height of green pillar). The net loss in wealth utility $\Delta U$ thus can be expressed as:

$$\Delta U = \int_{w_0}^{w_0+x} U'(w)\, dw - \int_{w_0-x}^{w_0} U'(w)\, dw < 0. \tag{1}$$

Therefore, the more volatile the wealth (i.e., the greater the random variable $x$), the greater the impact of LDMU (i.e., the greater the utility loss). Also, the impact of LDMU on wealth utility is greater when we reduce the asset holder's wealth from $w_0$ to $w_1$. In another word, the lower the wealth, the greater the impact of LDMU.

**Asset Volatility and Wealth Utility**

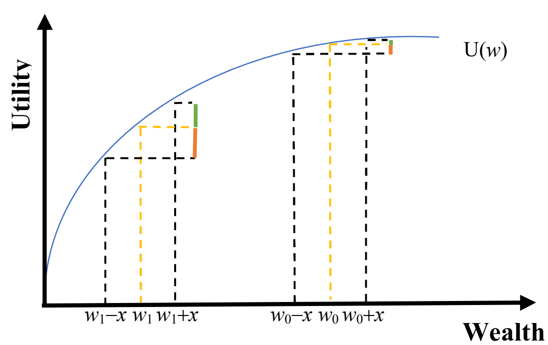


**Figure 3. The return of a corporate bond**

This figure illustrates the return of a corporate bond, where the risk of a payment approaches zero as maturity approaches zero, implying that the gross volatility(variance) of realized (expected) asset value approaches zero when distance approaches zero, as the gross volatility (variance) of realized (expected) asset value is positively related to asset risk over the distance. The bond's expected price at time *t* reflects the present value plus the return over *t*:

$$D_t = D_0\left(1 + \hat{Y}_t(r_d)\right),\ t \in (0, T). \tag{43}$$

where $\hat{Y}_t(r_d)$ given by formula (13), and $r_d$ can be expressed as:

$$r_d(t) = r_d(T - t). \tag{6}$$

**The Return of a Corporate Bond**

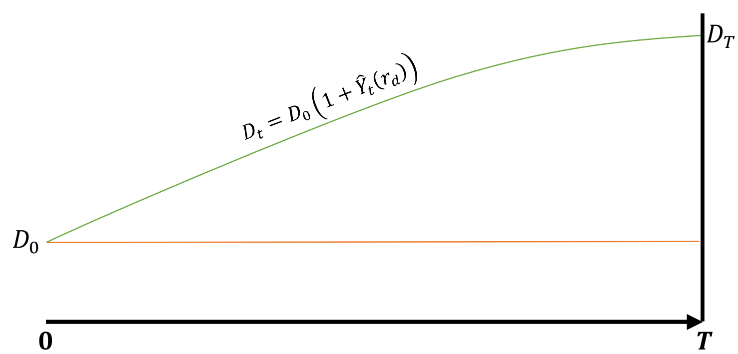


**Figure 4. A corporate bond's price movement with a market noise of firm risk**
This figure illustrates the dynamics of a corporate bond's price movement when a market noise of firm risk temporarily impacts the bond's micro-risk $r_d$. A bond's return over maturity $\hat{Y}_T(r_d)$ is given by formula (13). To simplify the illustration, we can omit the macrorisk $r'_f$ and treat asset value as nondepreciating to account for the risk-free rate $r_f(t)$ over $T$. We assume that the asset value follows a blended lognormal distribution, that is, $D_0 \sim \mathcal{B}(\frac{D_T}{1+\hat{Y}_T(r_d+\sigma_t Z)}, \sigma_t)$, where $D_T$ is the bond's face value at $T$, $Z$ is a random variable following a blended normal distribution, and $\sigma_t$ is the debt risk volatility. Given a risk noise that raises $r_d$ by $\Delta r_d$, the ratio of the price change to the original price is:

$$\frac{D_0 - \Delta D}{D_0} = \frac{1+\hat{Y}_T(r_d)}{1+\hat{Y}_T(r_d+\Delta r_d)}. \tag{41}$$

**A Corporate Bond's Price Movement with A Market Noise**

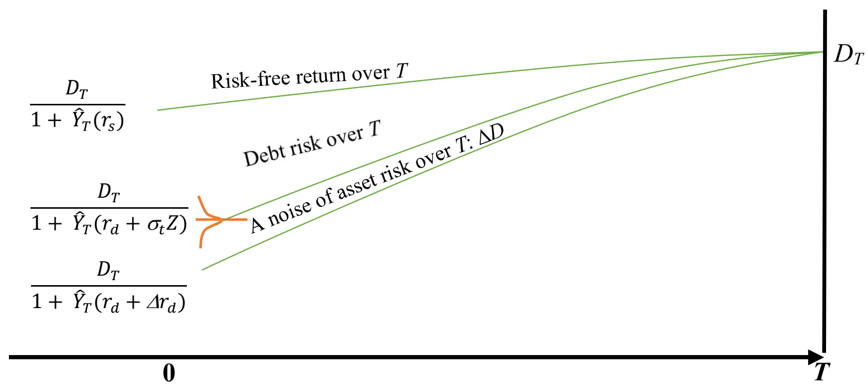


**Figure 5. The maximum marginal cost of asset holding: a firm maturity bond and a spanning bond**
This figure illustrates that an asset's firm maturity is determined by the maximum marginal cost of asset holding, which reflects the firm's expected performance in the long-term. Equity price reflects the present value of the firm's lifetime performance, where the risk $r_e(t)$ moves from current zero to maximum at maturity $T_e$. In contrast, the risk of a firm maturity bond (FMB), which includes the macrorisk $r'_f(t)$ and the microrisk $r_d(t)$, moves from current maximum to zero at $T$. Any payment beyond firm maturity has no present value since it is expected to default. The risk returns account for the risk-free return thereby nondepreciating. FMB's risk and the spanning bond's risk at $t$ is:

$$r'_f(t) = r'_f(T-t),\ t \in (0, T). \tag{5}$$

$$r_d(t) = r_d(T-t),\ t \in (0, T). \tag{6}$$

$$r_e(t) = r_e t,\ t \in (0, T_e). \tag{21}$$

**A Firm Maturity Bond and A Spanning Bond**

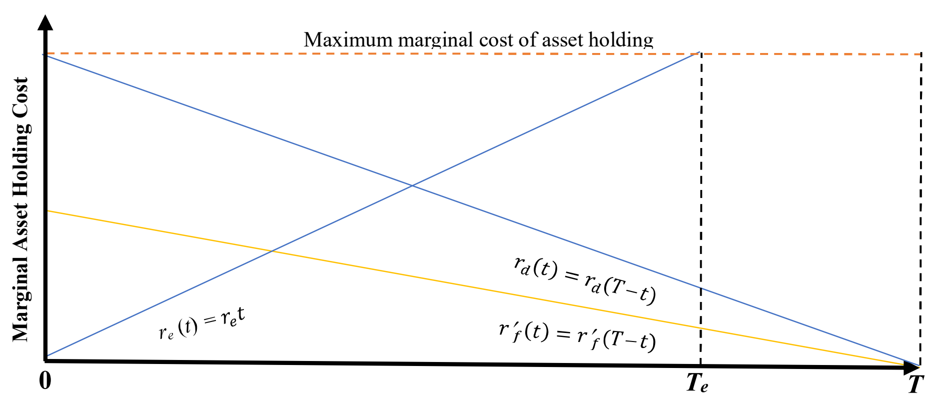


**Figure 6. A spanning bond's marginal present value and nondepreciating return over maturity**
Figure 6 shows that equity price represents the firm's lifetime nondepreciating performance (which accounts for the risk-free return) in excess of the asset holding cost (i.e., the risk return) over maturity. The market expects the firm to generate the nondepreciating value with a constant rate of $r_e T_e$ in the long-term, until the marginal asset holding cost reaches the firm's marginal performance at $T_e$. Then the total present value produced by the firm can be computed as:

$$E_0 = E_0 \int_0^{T_e} (e^{r_e T_e} - e^{r_e t}) dt. \tag{23}$$

$$\Rightarrow \quad 1 = T_e e^{r_e T_e} + \frac{1 - e^{r_e T_e}}{r_e}. \tag{24}$$

**A Spanning Bond's Present Value and Return**

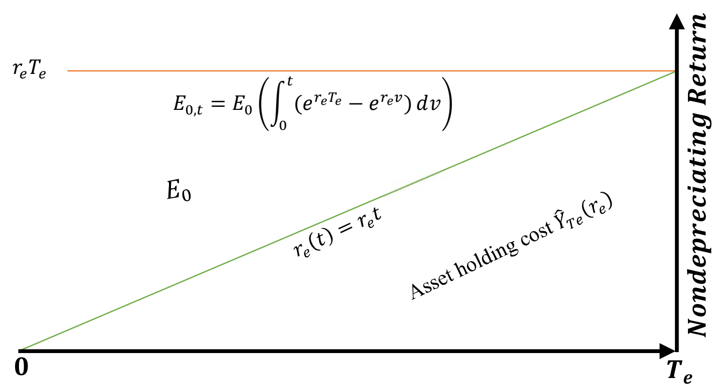


**Figure 7. Equity price movement with a market noise**

This figure shows that the firm's stock value $E_0$ decreases by $\mathit{\Delta}E_i$ when the gross noise of firm risk on Day $i$ raises the spanning bond's risk by $\mathit{\Delta}r_{E,i}$, thereby reducing maturity by $\mathit{\Delta}T_{e,i}$, where $Z$ is a random variable following a blended normal distribution. The nondepreciating value generated over maturity accounts for the risk-free return over time. As indicated by formula (29), $\mathit{\Delta}E_{i,1}$ is the change in the bond's present value over $T_e$ that is caused by the increase in the risk. In contrast, $\mathit{\Delta}E_{i,2}$ is the change in the present value caused by the decrease in maturity.

**A Change in Equity Price with a Market Noise**

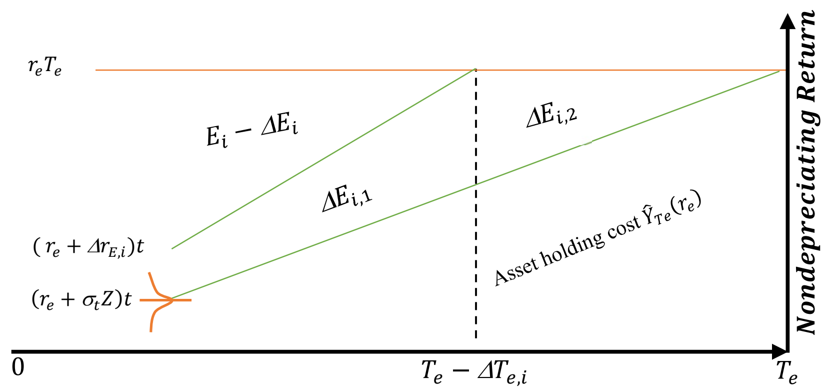


**Figure 8. A transformation between longitudinal risk and asset value: a negative event**

This figure uses a negative event to show how a (counter-)clockwise turn of a spanning bond's value curve reflects a transformation of the spanning bond's risk $r_e$ over $T_e$, that is, the longitudinal risk $R_{T_e}(r_e)$, into the reduced current wealth or the reduced expected growth rate. Curve $E_{0,r_f}$ is the present value discounting the risk-free return, whereas curve $E_0$ additionally discounts the longitudinal risk of $r_e$ over $T_e$. Curve $E_{0,1}$ shows that the uninformed market interprets the event wholly as an increase in the risk by $\Delta n$, thereby reducing the bond's present value by $\Delta N$ and the expected value at $T_e$ by $\Delta N(1+\hat{Y}_{T_e}(r_f))$. Curve $E_{0,2}$ shows that the informed market treats the event wholly as a decrease in the bond's current wealth that also reduces the present value by $\Delta N$. Thus, the market can turn the curve (counter-)clockwise when it becomes more (less) informed of the risk over time, where the risk noise $\Delta r_e \in (0, \Delta n)$. Formula (22) gives $\hat{Y}_{T_e}(r_e)$ and $\hat{Y}_{T_{e,1}}(r_e+\Delta n)$. Given the risk noise $\Delta n$, the decrease in the present value $\Delta N$ is:

$$\Delta N = \frac{E_{T_e}}{1+\hat{Y}_{T_e}(r_e)} - \frac{E_{T_e}}{1+\hat{Y}_{T_{e,1}}(r_e+\Delta n)}. \tag{31}$$

**The Impact of a Piece of Negative News**

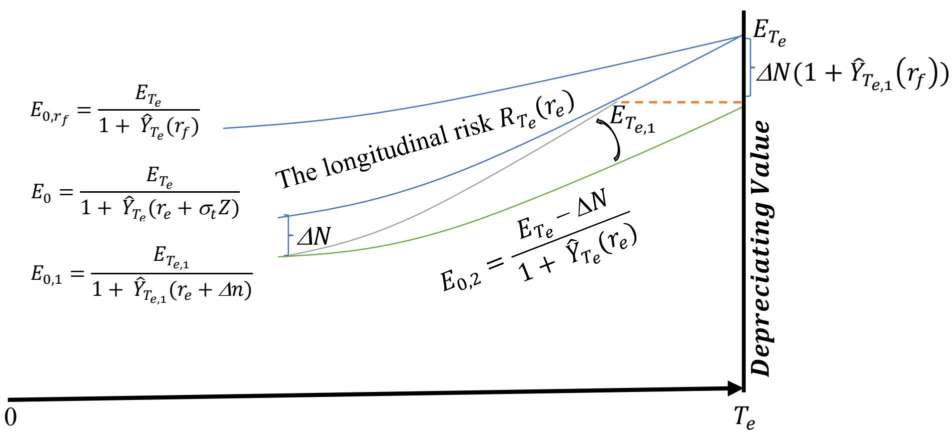
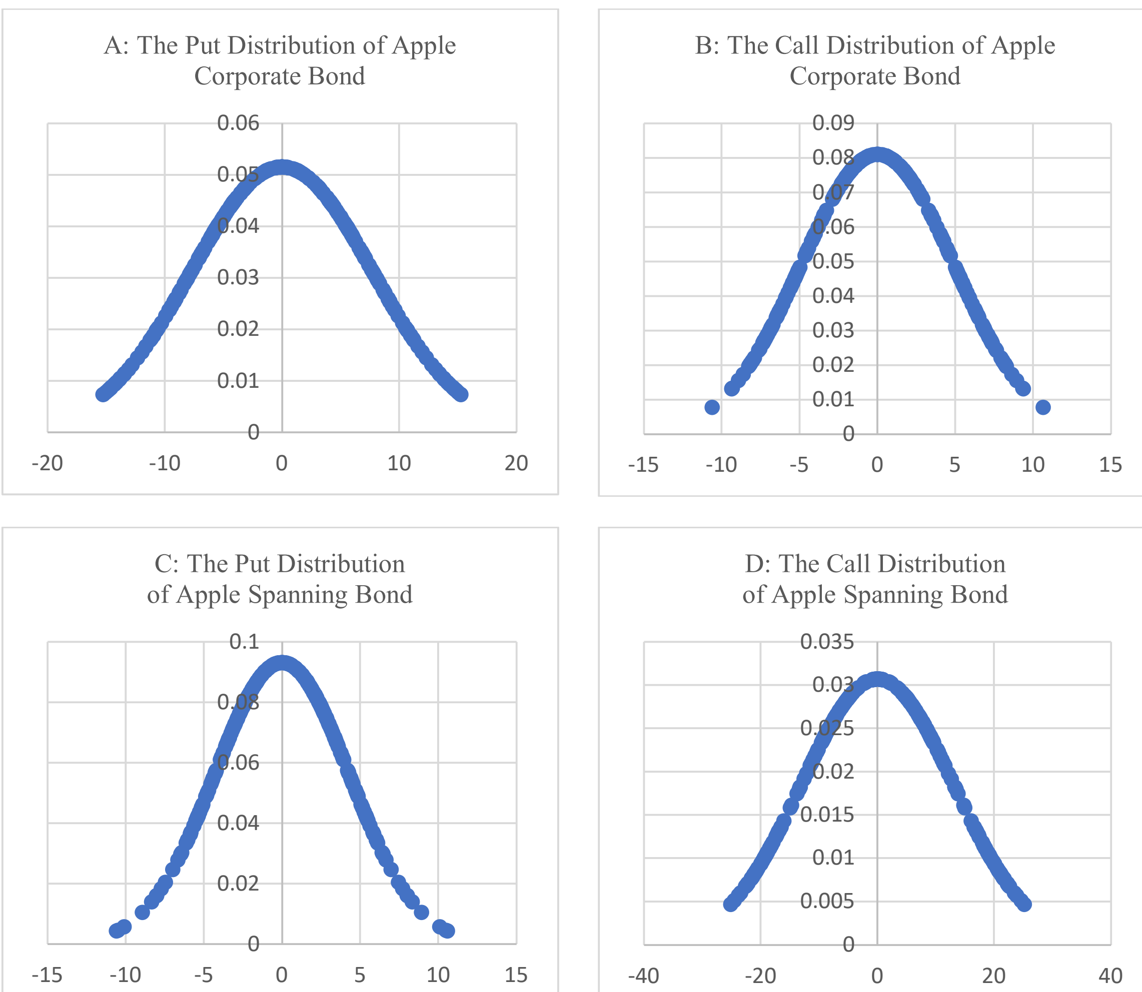


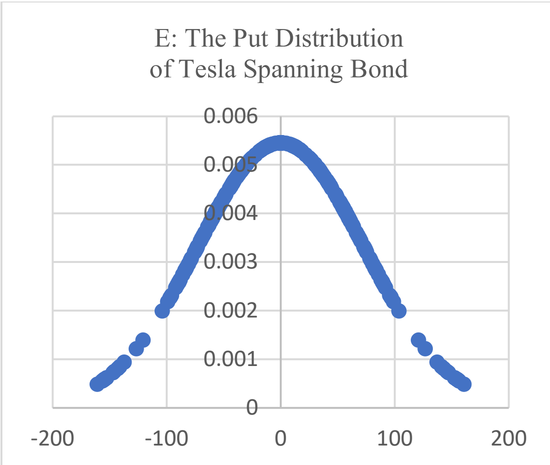
**Figure 9. The put (call) distribution of a corporate bond and three spanning bonds**
This figure uses formula (34) to delineate the excess risk ALPHA (i.e., $A$) and the risk noise (which follows a blended normal distribution) of the daily gross risk noise reported in Tables 2 and 3. BR is the blended ratio of a blended normal distribution given by formula (32). The assets include Apple corporate bond maturing on 6 May 2044 and the spanning bonds of Apple, Tesla, and Nvidia firms. We use zero median of the blended normal distribution to obtain the excess risk ALPHA (i.e., $A$). Then, we artificially generate the positive (negative) scores of the put (call) distribution of each spanning bond to compute the put (call) standard deviation. All numbers are in basis points and continuously compounding unless otherwise suggested. A corporate bond's the excess return alpha (i.e., $\alpha$) is opposite the excess risk ALPHA (i.e., $A$). As suggested by formula (25), a spanning bond's excess return $\alpha$ is computed as:

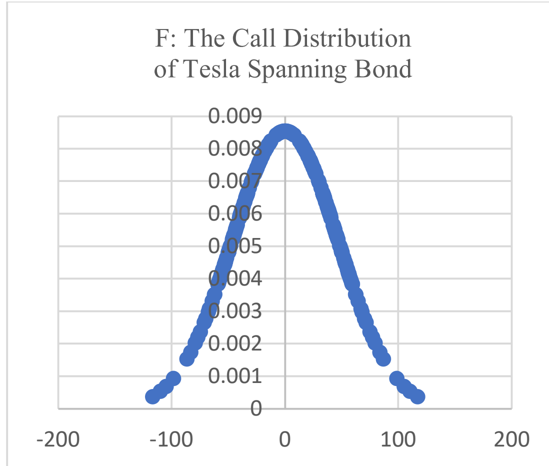
$$\alpha = 0.5(r_e + A)(T_e + \Delta T_e)^2 e^{(r_e+A)(T_e+\Delta T_e)} - 0.5 r_e {T_e}^2 e^{r_e T_e}. \tag{33}$$

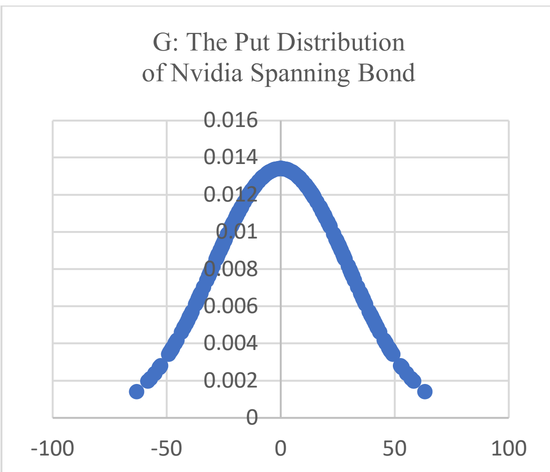
where $A$ is the excess risk, $T_e$ is equity maturity, and $\Delta T_e$ is the change in $T_e$ given by formula (24).

| | $r_e$ / $r_d$ | $r_{e0}$ | $A$ | $\alpha$ | BR | Put STD | Call STD | BR Er |
|---|---|---|---|---|---|---|---|---|
| Apple corporate Bond | 14.30 | – | −4.67 | 4.67 | 0.63 | 7.73 | 4.92 | −1.03% |
| Apple Spanning Bond | 109.9 | 4.77% | 8.02 | −16.41 | 3.15 | 4.06 | 12.9 | −0.87% |
| Tesla Spanning Bond | 197.2 | 6.31% | −116.3 | 219.9 | 0.69 | 72.4 | 46.2 | 6.95% |
| Nivida Spanning Bond | 296.0 | 7.66% | −278.6 | 572.2 | 1.98 | 28.7 | 54.6 | 3.90% |




E: The Put Distribution
of Tesla Spanning Bond
0.006
0.005
0.004
0.003
0.002
0.001
0
-200
-100
0
100
200


F: The Call Distribution
of Tesla Spanning Bond
0.009
0.008
0.007
0.006
0.005
0.004
0.003
0.002
0.001
0
-200
-100
0
100
200


G: The Put Distribution
of Nvidia Spanning Bond
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
-100
-50
0
50
100

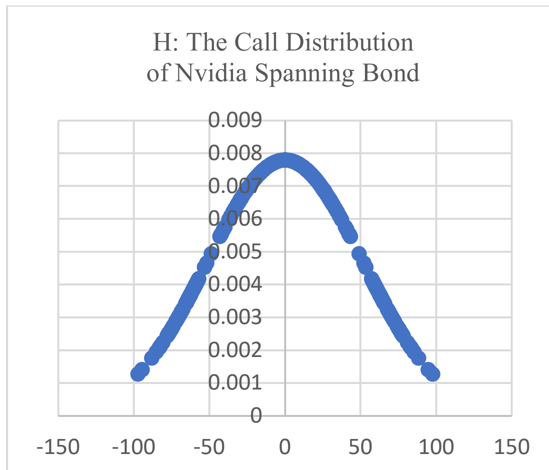

H: The Call Distribution
of Nvidia Spanning Bond
0.009
0.008
0.007
0.006
0.005
0.004
0.003
0.002
0.001
0
-150
-100
-50
0
50
100
150

**Figure 10. The standard option value and condensed asset risk**

This figure uses an put option as example to show that an option's exercise value reflects the fraction of the asset price movement, which decreases equity value from 1 to $\phi$ thereby condensing the risk from $r_e$ to $\frac{r_e}{\phi}$. Or alternatively, an option creates a new share that is far riskier than the stock. The standard option value can be written as:

$$\phi = \int_0^{+\infty} pdf(v + \Delta r_{E,c0})dv = \int_{\Delta r_{E,c0}}^{+\infty} pdf(v)dv. \tag{37}$$

where *pdf*($v$) is the probability density function of the put distribution's risk noise, and $\Delta r_{E,c0}$ is the standard gross exercise risk equal to the ratio of $TA_c + \Delta r_{e,c}$ to the put standard deviation of the spanning bond.

Given the exercise price $C_0$, and the current stock price $E_0$, the equity bond's expected return over $T$ is equal to $\hat{Y}_T(r_e)$. Then, the expected option value at $T$ is given by:

$$O_T = \phi E_0 \left(1 + \hat{Y}_T(r_e)\right). \tag{38}$$

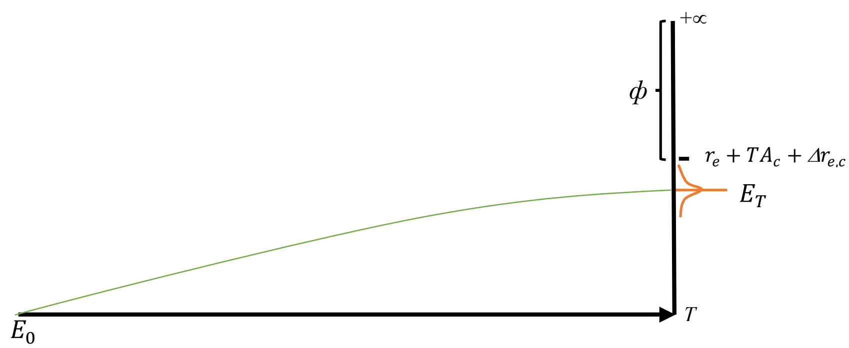


**Table 1. The risk and the risk-free rate used to compute asset risk**

This table reports the T-bill metrics obtained with the 4-factor independence model in Nie (2024a) over 13 days, which reflects the sample period from 28 November 2023 to 27 November 2024. Panel A describes the T-bill data. Panel B reports the results from separation of the risk and the risk-free rate of the T-bills on these days. Each analysis of T-bill metrics includes 9 T-bills. The inflation risk $r'_f(t)$, or alternatively the macrorisk, is measured as a one-year payment's risk, which approaches zero as maturity approaches zero. The risk-free rate is equal to the central bank's expected effective rate plus the depreciation cost (i.e., the constant risk). All rates are annual rates unless otherwise stated. We use these metrics as an example to compute the daily gross risk noise for a corporate bond in Table 2 and three spanning bonds in Table 3. Assuming that the market expects the current monetary policy to take $T_m$ years to move the current effective rate $r_{f0}$ to the neutral level $r_{fn}$, the 4-factor independence model in Nie (2024a) is specified as:

$$\hat{Y}_T(r'_f) = \int_0^T (e^{r_{f0} + r_s + \frac{(r_{fn} - r_{f0})t}{T_m}} - 1)\, dt + \int_0^T (e^{r'_f(T-t)} - 1)\, dt$$

$$= e^{r_s}\, \hat{Y}_T(r_f) + \frac{e^{r'_f T} - 1}{r'_f} - T. \qquad (44)$$

where $\hat{Y}_T(r_f)$ is the risk-free return of formula (10), $T$ is maturity, and $r_s$ is the constant risk reflecting the depreciating cost of the interbank short-term lending rate $r_f(t)$.

Panel A

| Maturity (years) | 1/12 | 1/4 | 1/2 | 1 | 2 | 3 | 5 | 10 | 30 | |
|---|---|---|---|---|---|---|---|---|---|---|
| Date | Market Yield % | | | | | | | | | US Fed effective rate |
| 28-Nov-2023 | 5.53 | 5.47 | 5.42 | 5.21 | 4.73 | 4.49 | 4.29 | 4.34 | 4.52 | 5.33% |
| 11-Dec-2023 | 5.55 | 5.47 | 5.40 | 5.14 | 4.71 | 4.42 | 4.25 | 4.23 | 4.32 | 5.33% |
| 18-Dec-2023 | 5.52 | 5.46 | 5.36 | 4.95 | 4.43 | 4.15 | 3.94 | 3.95 | 4.05 | 5.33% |
| 9-Feb-2024 | 5.49 | 5.44 | 5.26 | 4.86 | 4.48 | 4.25 | 4.14 | 4.17 | 4.37 | 5.33% |
| 1-Apr-2024 | 5.49 | 5.44 | 5.36 | 5.06 | 4.72 | 4.51 | 4.34 | 4.33 | 4.47 | 5.33% |
| 2-Apr-2024 | 5.49 | 5.42 | 5.34 | 5.05 | 4.70 | 4.51 | 4.35 | 4.36 | 4.51 | 5.33% |
| 2-May-2024 | 5.47 | 5.46 | 5.43 | 5.21 | 4.96 | 4.79 | 4.64 | 4.63 | 4.74 | 5.33% |
| 3-May-2024 | 5.36 | 5.39 | 5.36 | 5.12 | 4.83 | 4.66 | 4.51 | 4.52 | 4.67 | 5.33% |
| 29-May-2024 | 5.50 | 5.46 | 5.43 | 5.20 | 4.96 | 4.79 | 4.63 | 4.61 | 4.74 | 5.33% |
| 2-Jul-2024 | 5.48 | 5.47 | 5.36 | 5.07 | 4.74 | 4.54 | 4.36 | 4.43 | 4.60 | 5.33% |
| 2-Aug-2024 | 5.54 | 5.29 | 4.88 | 4.33 | 3.88 | 3.70 | 3.62 | 3.80 | 4.11 | 5.33% |
| 15-Oct-2024 | 4.93 | 4.73 | 4.42 | 4.18 | 3.95 | 3.86 | 3.86 | 4.03 | 4.32 | 4.83% |
| 18-Oct-2024 | 4.92 | 4.73 | 4.45 | 4.19 | 3.95 | 3.86 | 3.86 | 4.08 | 4.38 | 4.83% |

Panel B

| Maturity (years) | 1/12 | 1/4 | 1/2 | 1 | 2 | 3 | 5 | 10 | 30 | T-bill metrics | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Date | Prediction error (%) | | | | | | | | | MAV | $T_m$ (yrs) | $r_{fn}$ | $r'_f$ | $r_s$ |
| 28-Nov-2023 | 3.20 | 0.00 | -1.17 | -1.39 | 0.00 | -0.16 | 2.33 | 2.88 | -1.93 | 1.13% | 2.35 | 3.15% | 34 bps | 10 bps |
| 11-Dec-2023 | -0.05 | 0.07 | -0.62 | 0.37 | 1.34 | 0.00 | -1.31 | 0.00 | 0.00 | 0.42% | 3.03 | 2.66% | 34 bps | 10 bps |
| 18-Dec-2023 | 2.67 | 1.51 | 0.00 | 1.20 | -1.36 | -1.53 | 0.00 | 0.89 | 0.00 | 1.02% | 1.92 | 2.64% | 29 bps | 23 bps |
| 9-Feb-2024 | 1.16 | -0.08 | 0.05 | 1.45 | -2.37 | -1.39 | 0.00 | 2.66 | 0.00 | 1.02% | 1.70 | 2.95% | 34 bps | 12 bps |
| 1-Apr-2024 | 0.41 | -0.04 | -0.60 | 1.12 | 0.07 | -2.19 | -1.53 | 1.07 | 0.00 | 0.78% | 2.67 | 2.95% | 37 bps | 38 bps |
| 2-Apr-2024 | 0.09 | 0.00 | -0.57 | 0.92 | 0.01 | -2.49 | -1.73 | 0.87 | 0.00 | 0.74% | 2.61 | 2.86% | 38 bps | 5 bps |
| 2-May-2024 | 1.14 | 0.33 | -0.58 | 0.68 | 0.00 | -1.69 | 0.00 | 2.92 | 0.00 | 0.82% | 2.92 | 3.21% | 40 bps | 8 bps |
| 3-May-2024 | 1.55 | -0.07 | -1.06 | 0.44 | 0.00 | -1.45 | 0.00 | 2.71 | 0.00 | 0.75% | 1.61 | 3.99% | 40 bps | -7 bps |
| 29-May-2024 | 0.00 | -0.18 | -0.97 | 0.70 | 0.27 | -1.21 | 0.00 | 3.06 | -0.07 | 0.72% | 3.16 | 3.18% | 40 bps | 5 bps |
| 2-Jul-2024 | 0.06 | -1.01 | -0.85 | 1.00 | 0.42 | -1.18 | 0.00 | 1.88 | -0.64 | 0.78% | 2.63 | 3.03% | 38 bps | 4 bps |
| 2-Aug-2024 | -1.90 | -1.23 | 0.78 | 0.00 | -3.49 | -2.23 | 0.02 | 0.96 | 0.00 | 1.18% | 1.11 | 2.63% | 32 bps | 6 bps |
| 15-Oct-2024 | -1.12 | 0.29 | 2.97 | 0.00 | -2.86 | -1.75 | 0.00 | 2.00 | 0.00 | 1.22% | 1.12 | 3.00% | 35 bps | -1 bps |
| 18-Oct-2024 | -0.77 | 0.45 | 2.46 | 0.00 | -2.18 | -0.83 | 0.66 | 2.19 | 0.00 | 1.06% | 1.11 | 3.02% | 36 bps | 0 bps |
| | | | | | | | | | Mean | 0.90% | | | | |

**Table 2. The daily gross risk noises of Apple corporate bond maturing on 6 May 2044**

Table 2 reports the daily gross risk noises (which includes the excess risk ALPHA and the risk noise) of Apple corporate bond (which matures on 6 May 2044) over the 252 trading days from 29 Nov 2023 to 27 Nov 2024 (which is approximately a calendar year). Row 1 column 1 the first day is 29 Nov 2023. Row 28 column 9 the final day is 27 Nov 2024. The bond pays two 2.25% semi-annual interests. See formula (20) for the computation methodology. The bold underlined numbers indicate T-bill metrics update dates as reported in Table 1. All rates are in basis points and continuously compounding unless otherwise indicated.

| No. 252 | $r_d$ | Min | Mean | Median | Max | STD |
|---|---|---|---|---|---|---|
| | 14.30 | -12.75 | 1.14 | 1.77 | 12.59 | 6.47 |

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 1 | **<u>1.98</u>** | 0.00 | 1.23 | -3.05 | -3.09 | -0.39 | -0.31 | -0.77 | **<u>-4.13</u>** |
| 2 | -8.43 | -8.58 | -6.68 | -6.81 | -7.39 | -5.64 | -4.71 | -3.83 | -7.99 |
| 3 | -6.40 | -5.24 | -5.21 | -3.59 | -2.51 | **<u>-0.64</u>** | 0.86 | 0.51 | -2.80 |
| 4 | -3.01 | -3.66 | -5.41 | -3.86 | -1.00 | -0.67 | **<u>8.65</u>** | 7.16 | 5.20 |
| 5 | 8.04 | 6.98 | 7.18 | 7.21 | 5.76 | 4.22 | 2.82 | 1.85 | 8.47 |
| 6 | 6.45 | 7.67 | 8.96 | 9.09 | **<u>7.25</u>** | 11.58 | 8.25 | 8.27 | 9.28 |
| 7 | 9.65 | 8.25 | 10.38 | 8.42 | 7.95 | 8.18 | 9.30 | 8.72 | 7.49 |
| 8 | 8.48 | 8.06 | 5.36 | 5.06 | 5.34 | 4.10 | 1.06 | 1.85 | 3.00 |
| 9 | 3.23 | 2.81 | 2.46 | 1.69 | 0.10 | 2.00 | 2.17 | 1.46 | 0.13 |
| 10 | 0.19 | 2.25 | **<u>-0.37</u>** | 0.14 | 1.66 | 1.04 | -0.60 | 3.45 | 5.14 |
| 11 | 3.23 | 2.81 | 2.46 | 1.69 | 0.10 | 2.00 | 2.17 | 1.46 | 0.13 |
| 12 | 8.08 | 6.43 | 8.62 | 7.78 | 6.58 | 3.08 | 2.48 | -7.23 | -7.43 |
| 13 | -6.05 | -5.75 | -7.00 | -9.53 | -8.19 | -8.56 | -7.47 | **<u>-8.53</u>** | **<u>-7.75</u>** |
| 14 | -6.55 | -7.00 | -6.97 | -5.92 | -3.05 | -5.13 | -6.77 | -10.47 | -11.14 |
| 15 | -12.36 | -11.92 | -8.86 | -8.14 | -9.52 | -10.97 | -12.38 | **<u>-12.73</u>** | -11.28 |
| 16 | -12.75 | -12.02 | -12.47 | -11.31 | -11.53 | -12.02 | -8.34 | -10.45 | -8.06 |
| 17 | -8.47 | -6.93 | -8.24 | **<u>1.22</u>** | -0.34 | -1.35 | -1.37 | -1.52 | -2.84 |
| 18 | -3.74 | -2.94 | -4.23 | -3.87 | -2.63 | -1.69 | -1.15 | -0.26 | 1.26 |
| 19 | 0.35 | -2.11 | -2.89 | -2.95 | -4.36 | -3.97 | -9.19 | **<u>8.37</u>** | 10.54 |
| 20 | 11.11 | 12.14 | 12.59 | 10.39 | 10.21 | 8.35 | 7.54 | 7.98 | 6.15 |
| 21 | 4.84 | 5.92 | 6.05 | 3.94 | 4.23 | 4.82 | 4.86 | 5.78 | 6.40 |
| 22 | 6.80 | 7.55 | 5.08 | 4.17 | 3.10 | 4.65 | 2.53 | -1.76 | -1.86 |
| 23 | -1.80 | -4.18 | -3.32 | -3.05 | -1.68 | -2.35 | -2.49 | -1.92 | 0.00 |
| 24 | -0.38 | -0.91 | 0.36 | -1.05 | -0.19 | 1.62 | 2.74 | 3.57 | 5.09 |
| 25 | 5.18 | 5.84 | 5.07 | 3.88 | **<u>-1.56</u>** | 0.08 | 0.11 | **<u>4.16</u>** | 6.19 |
| 26 | 7.79 | 4.75 | 5.93 | 7.00 | 6.36 | 6.62 | 5.46 | 10.39 | 7.75 |
| 27 | 5.86 | 9.82 | 6.12 | 5.31 | 5.59 | 8.89 | 9.61 | 9.74 | 11.29 |
| 28 | 10.12 | 9.82 | 9.89 | 9.89 | 10.51 | 6.24 | 6.82 | 6.99 | 5.63 |

**Table 3. The gross risk noise of Apple, Tesla and Nvidia spanning bonds**

This table presents the gross risk noise (which includes the excess risk ALPHA or $A$) of Apple, Tesla, and Nivida's spanning bonds from 29 Nov 2023 to 27 Nov 2024 in Panels A, B, and C, respectively. The bold underlined numbers are T-bill metrics update dates as reported in Table 1. Row 1 column 1 is 29 Nov 2023, and row 28 column 9 is 27 Nov 2024. A spanning bond's baseline risk $r_e$ is given by formula (30). Formula (24) gives the relationship between the risk $r_e$ and maturity $T_e$. All numbers are in basis points and continuously compounding unless otherwise indicated.

Panel A: Apple spanning bond

| $r_e$ | $T_e$ | Min | Mean | Median | Max | STD |
|---|---|---|---|---|---|---|
| 109.9 | 12.87(yrs) | -15.32 | -0.38 | -1.94 | 21.77 | 10.12 |

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 1 | **<u>0.64</u>** | 0.35 | -0.35 | 0.74 | -1.50 | -0.83 | -1.89 | -2.64 | **<u>-1.19</u>** |
| 2 | -2.00 | -3.73 | -3.77 | -3.43 | -2.48 | -3.01 | -1.81 | -1.68 | -1.03 |
| 3 | -0.68 | -0.69 | -0.89 | -0.25 | 3.87 | **<u>4.78</u>** | 6.30 | 6.81 | 4.10 |
| 4 | 4.41 | 3.81 | 4.22 | 4.06 | 5.53 | 6.18 | **<u>2.56</u>** | 0.89 | -0.41 |
| 5 | -1.09 | -0.66 | -0.43 | 0.61 | 1.05 | 3.28 | 5.56 | 4.08 | 4.75 |
| 6 | 3.67 | 2.75 | 2.73 | 3.42 | **<u>3.15</u>** | 4.22 | 5.42 | 6.02 | 6.25 |
| 7 | 7.28 | 7.81 | 7.37 | 6.13 | 7.33 | 8.26 | 7.36 | 8.18 | 8.67 |
| 8 | 9.43 | 12.59 | 16.22 | 17.01 | 17.15 | 15.91 | 14.49 | 14.20 | 15.77 |
| 9 | 14.46 | 14.78 | 14.04 | 12.43 | 10.71 | 15.89 | 15.28 | 16.37 | 17.27 |
| 10 | 14.68 | 16.06 | **<u>17.18</u>** | 18.12 | 17.56 | 18.24 | 17.72 | 18.62 | 17.75 |
| 11 | 19.24 | 13.93 | 12.92 | 15.71 | 18.22 | 19.32 | 20.11 | 21.77 | 21.15 |
| 12 | 20.37 | 18.79 | 18.18 | 18.68 | 15.62 | 18.00 | 18.83 | **<u>16.10</u>** | **<u>9.05</u>** |
| 13 | 10.19 | 9.78 | 9.60 | 8.47 | 9.50 | 7.31 | 6.64 | 5.28 | 5.26 |
| 14 | 5.28 | 4.62 | 3.89 | 4.80 | 7.30 | 5.44 | 5.48 | **<u>5.34</u>** | 4.78 |
| 15 | 4.26 | 3.25 | 3.11 | 2.28 | 3.12 | 1.79 | 4.01 | -3.66 | -6.56 |
| 16 | -7.08 | -6.20 | -8.16 | -6.99 | -4.69 | -3.54 | -3.83 | -4.26 | -6.29 |
| 17 | -6.66 | -4.92 | -7.84 | **<u>-9.43</u>** | -9.97 | -12.05 | -12.64 | -12.97 | -14.72 |
| 18 | -12.42 | -13.64 | -15.19 | -15.32 | -12.83 | -10.76 | -10.78 | -10.59 | -11.02 |
| 19 | -8.05 | -7.53 | -7.71 | -7.80 | -8.02 | -9.48 | -7.74 | **<u>-8.40</u>** | -3.23 |
| 20 | -2.14 | -3.43 | -5.13 | -6.50 | -7.08 | -8.89 | -9.05 | -10.36 | -10.90 |
| 21 | -10.79 | -11.03 | -10.94 | -10.08 | -11.06 | -11.17 | -11.50 | -10.79 | -12.18 |
| 22 | -11.80 | -9.03 | -8.12 | -8.78 | -8.03 | -8.03 | -7.62 | -8.75 | -8.76 |
| 23 | -8.60 | -5.68 | -5.86 | -7.66 | -11.26 | -10.94 | -10.15 | -10.50 | -10.03 |
| 24 | -10.49 | -10.58 | -12.75 | -9.81 | -10.02 | -9.49 | -9.95 | -7.62 | -9.42 |
| 25 | -11.03 | -10.78 | -10.09 | -11.67 | **<u>-12.70</u>** | -11.80 | -11.93 | **<u>-13.08</u>** | -13.65 |
| 26 | -13.36 | -11.20 | -11.08 | -11.41 | -12.21 | -12.29 | -10.74 | -8.87 | -7.48 |
| 27 | -7.03 | -7.66 | -9.53 | -9.39 | -9.13 | -7.87 | -7.83 | -8.20 | -9.54 |
| 28 | -8.19 | -9.49 | -9.57 | -9.85 | -9.60 | -10.16 | -11.40 | -12.28 | -12.19 |

Panel B: Tesla spanning bond

| $r_e$ | $T_e$ | Min | Mean | Median | Max | STD |
|---|---|---|---|---|---|---|
| 197.2 | 9.46(yrs) | -43.11 | 49.15 | 50.22 | 161.2 | 60.01 |

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 1 | **2.18** | 5.64 | 6.80 | 9.71 | 7.09 | 6.63 | 3.97 | 3.08 | **6.60** |
| 2 | 9.04 | 7.17 | -2.31 | -4.10 | -2.92 | -6.72 | 1.15 | -4.50 | -2.92 |
| 3 | -5.90 | -9.35 | -3.13 | 0.63 | 0.77 | **9.14** | 9.68 | 10.17 | 7.71 |
| 4 | 12.59 | 13.61 | 19.94 | 28.31 | 27.36 | 32.01 | **36.09** | 35.86 | 39.75 |
| 5 | 39.47 | 41.07 | 74.07 | 73.28 | 62.52 | 61.75 | 67.82 | 65.73 | 67.18 |
| 6 | 77.31 | 71.44 | 68.01 | 65.37 | **60.04** | 67.60 | 73.64 | 67.03 | 51.67 |
| 7 | 52.41 | 60.50 | 59.28 | 55.97 | 63.26 | 53.67 | 53.37 | 50.62 | 50.93 |
| 8 | 50.11 | 69.29 | 80.33 | 87.05 | 83.83 | 89.27 | 85.47 | 85.97 | 99.57 |
| 9 | 112.46 | 110.58 | 92.59 | 96.92 | 89.78 | 94.63 | 98.16 | 95.22 | 87.05 |
| 10 | 83.76 | 90.35 | **91.41** | 106.43 | 103.41 | 98.75 | 110.04 | 95.82 | 89.49 |
| 11 | 98.16 | 93.49 | 99.66 | 117.39 | 126.29 | 129.88 | 142.08 | 148.88 | 161.22 |
| 12 | 154.86 | 117.12 | 102.38 | 105.88 | 65.77 | 81.35 | 86.57 | **86.66** | **84.93** |
| 13 | 79.61 | 90.55 | 95.77 | 100.59 | 106.91 | 100.99 | 91.62 | 97.66 | 96.36 |
| 14 | 92.15 | 96.43 | 78.23 | 88.28 | 98.87 | 89.92 | 94.10 | **95.15** | 91.02 |
| 15 | 92.30 | 95.37 | 98.05 | 97.79 | 93.05 | 93.93 | 100.24 | 105.82 | 94.62 |
| 16 | 86.47 | 93.70 | 79.19 | 83.17 | 88.38 | 86.25 | 87.05 | 79.93 | 67.32 |
| 17 | 66.03 | 65.53 | 50.65 | 27.81 | 14.09 | 9.87 | 8.80 | 1.51 | 0.90 |
| 18 | 19.26 | 13.07 | 9.50 | 6.43 | 13.12 | 12.60 | 21.49 | 10.88 | 15.30 |
| 19 | 45.31 | 40.72 | 41.31 | 28.77 | 38.50 | 29.00 | 44.97 | **55.79** | 67.09 |
| 20 | 64.89 | 77.14 | 67.49 | 66.07 | 69.53 | 56.37 | 64.60 | 49.11 | 46.97 |
| 21 | 39.83 | 41.67 | 39.46 | 53.73 | 42.83 | 50.94 | 55.80 | 60.16 | 59.61 |
| 22 | 50.33 | 54.56 | 44.56 | 33.36 | 54.74 | 48.39 | 37.74 | 35.83 | 34.22 |
| 23 | 33.84 | 37.52 | 36.49 | 37.28 | 21.31 | 26.60 | 16.18 | 12.68 | 10.53 |
| 24 | 12.91 | 7.96 | 7.13 | 10.08 | 17.66 | 25.22 | 16.94 | 25.26 | 22.01 |
| 25 | 25.24 | 27.46 | 49.20 | 47.77 | **47.42** | 45.57 | 46.15 | **46.46** | 48.62 |
| 26 | 49.71 | 54.81 | 9.59 | 2.98 | 8.16 | 10.61 | 12.28 | 18.83 | 19.67 |
| 27 | 25.24 | 17.72 | -9.82 | -15.04 | -28.75 | -42.54 | -32.35 | -33.16 | -23.04 |
| 28 | -28.15 | -37.08 | -40.38 | -38.49 | -37.31 | -43.11 | -36.68 | -36.45 | -33.80 |

Panel C: Nvidia spanning bond

| $r_e$ | $T_e$ | Min | Mean | Median | Max | STD |
|---|---|---|---|---|---|---|
| 296.0 | 7.62(yrs) | -187.6 | -128.2 | -147.7 | 16.20 | 56.48 |

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 1 | **-1.80** | 6.80 | 7.20 | 15.50 | 8.70 | 16.20 | 9.00 | 3.30 | **8.60** |
| 2 | 3.20 | -0.20 | -1.20 | -4.60 | -11.30 | -8.20 | 0.70 | -4.50 | -3.10 |
| 3 | -5.90 | -6.90 | -6.80 | 1.40 | 5.40 | **3.00** | -3.30 | -21.10 | -25.60 |
| 4 | -31.20 | -33.70 | -32.80 | -34.90 | -40.50 | -38.80 | **-43.40** | -53.20 | -53.90 |
| 5 | -54.60 | -60.30 | -61.50 | -58.60 | -64.30 | -65.40 | -60.20 | -65.90 | -76.70 |
| 6 | -86.70 | -83.10 | -88.50 | -87.50 | **-94.30** | -94.70 | -93.90 | -98.80 | -95.50 |
| 7 | -95.60 | -86.00 | -79.70 | -109.80 | -110.70 | -111.10 | -110.20 | -107.40 | -111.50 |
| 8 | -117.80 | -124.10 | -125.30 | -130.30 | -137.40 | -128.00 | -124.60 | -135.70 | -133.75 |
| 9 | -128.33 | -128.10 | -129.20 | -130.70 | -132.30 | -134.25 | -139.10 | -140.33 | -136.15 |
| 10 | -131.70 | -132.00 | **-131.73** | -130.15 | -128.83 | -123.10 | -127.10 | -125.10 | -121.50 |
| 11 | -125.00 | -131.50 | -127.03 | -122.50 | -125.23 | -118.35 | -119.67 | -99.80 | -107.93 |
| 12 | -114.30 | -108.03 | -114.50 | -125.20 | -125.03 | -122.30 | -115.63 | **-120.70** | **-126.33** |
| 13 | -132.60 | -129.70 | -129.30 | -126.10 | -128.27 | -129.00 | -130.70 | -136.20 | -135.85 |
| 14 | -132.50 | -137.25 | -136.50 | -150.15 | -153.50 | -159.40 | -163.13 | **-163.70** | -158.73 |
| 15 | -157.43 | -163.77 | -165.43 | -171.70 | -170.27 | -169.93 | -171.03 | -169.90 | -174.07 |
| 16 | -178.00 | -180.30 | -179.40 | -183.23 | -186.93 | -179.07 | -175.20 | -166.40 | -174.67 |
| 17 | -174.78 | -171.77 | -170.83 | **-175.56** | -175.93 | -173.87 | -175.93 | -178.87 | -181.95 |
| 18 | -175.20 | -176.70 | -175.73 | -173.73 | -165.07 | -168.43 | -164.83 | -170.70 | -170.00 |
| 19 | -160.33 | -158.03 | -158.67 | -156.70 | -146.13 | -163.17 | -153.75 | **-150.73** | -146.50 |
| 20 | -138.73 | -147.70 | -147.17 | -153.30 | -161.97 | -163.83 | -168.60 | -168.53 | -170.23 |
| 21 | -170.20 | -175.43 | -172.90 | -173.97 | -169.23 | -174.40 | -173.77 | -171.03 | -162.33 |
| 22 | -164.33 | -150.20 | -147.83 | -148.80 | -142.70 | -147.70 | -149.80 | -160.87 | -163.23 |
| 23 | -163.03 | -160.30 | -158.73 | -156.30 | -161.73 | -159.50 | -159.70 | -164.50 | -167.30 |
| 24 | -167.77 | -164.77 | -164.57 | -159.73 | -162.03 | -166.23 | -168.03 | -170.93 | -175.90 |
| 25 | -175.33 | -177.43 | -177.30 | -174.67 | **-174.60** | -177.95 | -179.20 | **-179.93** | -184.33 |
| 26 | -184.23 | -181.15 | -181.70 | -182.47 | -181.73 | -182.10 | -180.67 | -174.77 | -177.03 |
| 27 | -177.60 | -180.65 | -185.07 | -187.60 | -186.63 | -184.87 | -186.99 | -185.73 | -185.83 |
| 28 | -182.17 | -180.40 | -185.70 | -184.70 | -185.23 | -181.65 | -176.53 | -177.33 | -175.90 |