Using Network Science to Explore the Organization of Patterns Within Large Music Corpora

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Abstract

Network science can be used to explore and quantify organizational structures within large collections of similar units. Here I demonstrate how to convert music corpora into networks where those units represent short interval patterns. As an example of network structure analysis, I compared patterns within the Weimar Jazz Database to a version of the Essen Folk Song corpus. The analysis showed that both corpora contain mostly patterns that are sparsely organized around much fewer highly connected hubs. This organizational principle has been termed “scale-free” and has been used to describe other types of naturally occurring networks. I further explore potential network growth models previously used in language research to describe how children build vocabulary. Finally, I suggest other types of music related networks based on different connectional principles. Applying network analysis to music corpus research may reveal organizational principles not previously explored.

Keywords: network science, patterns, jazz, improvisation, scale free.

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Previous research in pattern use in jazz improvisations has focused exclusively on the content and frequencies of those patterns. Here I suggest a method for exploring the relationships between interval patterns and how large collections of patterns may be organized through the creation of networks. I employ graph theoretical measures to explore the structure of a large corpus of jazz improvisations on the micro, mesa, and macro levels. I believe this approach is advantageous both for future jazz and other music cognition research. I discuss how many other related types of networks could be created from music data and outline potential research directions including an exciting developmental view. Many of the current ideas are adapted from language research that has employed network science for over 20 years. I start with a quick overview of specific related jazz research, then broaden the scope to describe some prior applications of network science to language research, and finally I demonstrate how to create and analyze networks of music data. The current paper focuses specifically on the musical applications of the network approach while our previous paper explored underlying cognitive principle through a related behavioral experiment (Merseal et al., 2023). A central question left unanswered by Merseal et al. (2023) was whether the specific network based on jazz improvisations has unique organizational principles. Here this question is explored by comparing a network of jazz improvisations to another large music network based on folk melodies and by comparing both music networks to two random networks created using two different construction principles.

Prior research of pattern use in jazz improvisations has largely focused on pattern content and frequency. In an analysis of 48 improvised solos by jazz great, Charlie Parker, I showed that a very large percentage, 82.4%, of notes in the corpus start 4-interval patterns if two or more iterations of the same interval sequence is considered a pattern (Norgaard, 2014). Interval patterns were identified using a moving window approach in which each interval was investigated as a potential onset of a pattern. This same approach is described in detail below as I used the same basic method to create networks. This high percentage of pattern use aligns with an often cited theoretical framework for improvisation in which continuous music is conceived as a concatenation of smaller patterns (Pressing, 1988). I also investigated individual pattern content and listed the most commonly occurring patterns (see Appendix C in Norgaard, 2014). Other researchers have investigated jazz improvisation through the analysis of much larger corpora that includes many jazz artists (Henry et al., 2020; Pfleiderer et al., 2017). Essential to this effort was the creation of the Weimar Jazz Database (WJD) that consists of 456 solos spanning recordings from 1925 to 2009 and includes iconic artists such as Louis Armstrong, Charlie Parker, and John Coltrane (Pfleiderer & Frieler, 2010). An extension of this research produced the even larger Dig That Lick collection that can be explored through an online interface (Henry et al., 2020). However, none of these efforts has specifically investigated the relationships between patterns and organizational principles that apply to the entire collection.

Network Science Terminology

Network science has recently been used to investigate structural organization of very large datasets (Barabási, 2009). Using this approach, entities are defined as nodes and the relations between the entities are referred to as edges. The number of connections emanating from one node is referred to as the *degree* of this node and nodes with high degrees relative to other nodes in the same network are referred to as hubs. In directed networks where edges include directional information, the number of edges pointing to a node is the node's *in-degree*, the number pointing away from the note, *out-degree*. For consistency, this vocabulary and other terms from network science will be used throughout the remainder of this paper.

In a very influential paper, Barabási (1999), showed that many different kinds of real-world networks show structural similarities labeled “scale-free” because no single value describes a common degree. For example, a network of data from the World Wide Web, consisted of nodes representing webpages and edges representing the links between pages. Here the network is considered directed as a link from one webpage to another does not mean a link exists in the opposite direction. One defining feature of this network is that relatively few nodes (e.g. the webpages of Google and New York Times) are connected to a very high number of other nodes, while many more websites have many fewer edges (like this author’s website). Interestingly, Barabási identified several other completely different large real-world networks that also have a similar scale-free organization. These included a network in which nodes are Hollywood actors and edges represent participation in the same movie and other networks describing airline routes, proteins, and scientific collaborations. Barabási argued that many real-world networks exhibit scale-free structure (see Broido & Clauset, 2019 for a counter argument).

Network Science and Language

Language researchers have adapted the network approach creating several types of networks, of which the most common are semantic and phonological networks (for a review, see Siew et al., 2019). Semantic networks are typically based on experimental data in which participants list terms they consider related (e.g. Steyvers & Tenenbaum, 2005). For example, the word, “tiger” may illicit responses such as, “lion,” “savannah,” “Africa,” and so on. Therefore, the network nodes are words and edges represent links to semantically related words. In phonological networks, nodes are also words, but the edges represent connections to words in which only one phoneme is different. For example, the word, “log” is connected to “hog,” “loss,” and “lawn” (see Fig. 1 in Chan & Vitevitch, 2010). Interestingly, semantic networks also exhibit the scale-free structure in which only few words are related to many other words (Steyvers & Tenenbaum, 2005). For phonological networks, some hub structure is also apparent, but they typically have different organizational structures (Siew et al., 2019).

Several influential papers have investigated the development of these networks as children acquire vocabulary. Hills et al. (2009) investigated a network growth model referred to as preferential attachment. In this model, originally suggested as a mechanism that results in a scale-free network, new nodes are more likely to connect to existing nodes that are already highly connected (Barabási & Albert, 1999). Using the WWW example, my new faculty website is more likely to connect to existing hubs (like the SMPC website which is a hub for music cognition researchers) than to other researchers’ individual websites. However, comparing the preferential attachment model to an alternative growth model in which the structure of words in children’s *environment* (as opposed to the structure of their own vocabulary) better aligned with longitudinal data of children’s semantic networks (Hills et al., 2009). This growth model termed preferential acquisition may be of interest to music researchers studying development. However, newer research shows this model does not explain vocabulary development analyzed as phonological networks where sound similarities account for node connections (Siew & Vitevitch, 2020).

Network Construction

Another field employing cognitive network science to investigate the structure of real-word networks is animal research. Of particular interest to the current discussion is an approach employed by Sasahara (2012) and others in which sequentially occurring birdsong syllables are used to create a network. Here the syllables are nodes and edges represent possible continuations to other syllables (Potvin et al., 2019; Sasahara et al., 2012; Weiss et al., 2014). In other words, if birdsong syllables are labeled with numbers and an actual sequence of syllables is 5, 14, 7, 13, then the node 5 would be connected with an edge to node 14, then 7 and so on. In this case the network is considered directed as the connection between 5 to 14 does not imply that the opposite order, 14 to 5, is observed (see Fig. 1 in Sasahara et al., 2012).

In a recently published paper, we adapted this approach to the WJD creating a large network of 5-pitch patterns (Merseal et al., 2023). The goal of this proof-of-concept paper was to demonstrate that network science can be used to analyze music as a complex system. After creating the network, we extracted 5-pitch pattern stimuli pairs and used these in a behavioral experiment in which listeners judged melodic similarity. Importantly, we found the melodic distance within the network related to the similarity judgments. As the melodic distance within the network increased, participants judged the patterns to be less related. This work was based on *pitch* patterns as that allowed for stimuli to be directly adapted from the network. The goal of the current paper is to use the same network construction principle to create a network of *interval* patterns and describe potential musical applications by comparing two large music corpora using network measures. Interval patterns may be a more meaningful representation of pitch relationships (Dowling, 1978). Furthermore, jazz musicians describe patterns in relational terms independent of pitch chroma and specifically practice playing the same interval pattern in different keys (Berliner, 1994; Norgaard, 2011).

Small Music Network Example

To demonstrate how a music network could be created and analyzed, I describe in detail a very small network created from an improvised solo in the WJD by Kid Ory recorded in 1925.

The very short solo can be viewed as a string of intervals (see Fig. 1). Using a moving window approach in which each interval can be the starting point for a pattern, the initial sequence of 2 3 2 3 2 2 -1… can be viewed as a consecutive string of 4-interval patterns as follows: 2 3 2 3; 3 2 3 2; 2 3 2 2…. Adapting the network construction principles from birdsong research, each of these patterns become nodes and observed continuations edges (see Fig. 2).

INSERT FIGURE 1 & 2

The decision to use the 4-interval pattern length is somewhat arbitrary, but previous research, including an example in the Pressing (1988) paper, has used similar length as the smallest meaningful pattern unit (Norgaard, 2014; Weisberg et al., 2004). In the current example, the first eight patterns only have one possible continuation reflecting the initial unique string of notes in the solo. The first pattern that has two different possible continuations is the interval pattern -4 -3 -6 1 (here, the notes E, C, A, D#, E) which occurs in both measure 3 and 7. In the network, this is graphically represented by the two arrows pointing out from the node labeled -4 -3 -6 1 which therefore has an out-degree of 2. Because the node also has two in-degrees, the total number of degrees for this node is 4. Similarly, the node 1 -1 1 0 has two different possible continuations. Looking at the degree distributions, nearly all the nodes in this network have one in-degree and one out-degree except for the two nodes just discussed (-4 -3 -6 1 and 1 -1 1 0) and the nodes at the beginning and end of the solo which only have one degree. Using this measure to describe the entire network, an average degree for all nodes can be calculated. The average of the total number of in and out degrees is 1.027. For the undirected version of the network (i.e. by disregarding the directions of the arrows most nodes are connected to two other nodes), the average degree is 2.05 (see Table 1).

INSERT TABLE 1

To describe the relationship between two nodes in the network, it is useful to simply quantify the fewest number of edges linking the nodes within the network. This measurement is called the *shortest path* between two nodes. For example, the shortest (and only) path between 2 3 2 3 (the first pattern in the solo) and -4 -3 -6 1 is eight. To describe the size of the full network, it is useful to identify the longest possible path through the network. This measure is referred to as the *network diameter* which is 24 for the directed version and 22 for the undirected version of the Kid Ory network. Another useful measurement is the average shortest path between all nodes in the network which is referred to as the *average path length*. Again, the directed (9.17) and undirected (8.38) values are calculated depending on whether path direction is considered. It is often useful to look at the connections between a node's neighbors as a measure of the density of the network. This is measured by the *clustering coefficient* which is the number of connections between a nodes neighbor divided by all possible connections. In the current example, no node has connected neighbors resulting in the clustering coefficient of zero for each node. Notice that all the measurements just described can be calculated for each individual node or pair of nodes describing the micro level of the network but also averaged out for all nodes or paths giving an indication of the structural properties of the entire network - the macro level.

One important measurement of the intermediate, or mesa level, describes the number of "communities" of nodes. In figure 2, five communities within the Kid Ory network are illustrated with different node colors. Importantly, these communities were identified using a standard network *modularity* algorithm (Blondel et al., 2008) implemented using the freely available software package Gephi. This algorithm is completely agnostic to any musical features or relationships, and simply divided the network into communities based on each node's position within the network. In our previous research, this feature was used to divide the WJD pitch pattern network into communities that also appeared to align with musical characteristics (see Fig. 3 in Merseal et al., 2023).

Looking exclusively at the macro level of the network, in addition to the measures listed above, the entire structure of the network can be characterized by the *distribution of degrees*. In the undirected Ory network, all nodes have two degrees except the two nodes with 4 degrees and two with 1, resulting in a highly skewed distribution (Fig. 4a). Finally, a macro-level measurement describes if hubs are connected to other hubs and is referred to as *assortativity*. This is quantified by correlating the number of degrees for each node with the number of degrees of the node's neighbors resulting in a simple r value. In other words, if most hubs are also connected to other hubs, lesser connected nodes are connected to other lesser connected nodes and so on; the assortativity r value would be high.

To make sense of these macro-level measures, it is standard practice to compare networks to randomly created networks of the same size and density. Often networks are compared to a Erdős-Rényi random graph (ER) where random edges are assigned to a specified number of nodes with a given probability. In the current work, ER random graphs were created with the fast_gnp_random_graph algorithm (Batagelj & Brandes, 2005) using NetworkX, a network analysis library for Python (Hagberg et al., 2008). In the current illustration, only one random network was created but in actual investigations, researchers commonly create 1000+ random networks and compare the investigated model to a distribution of values of the measurements of interest from the collection of random networks. Figure 3 shows a random network that has the same number of nodes and edges as the Kid Ory network. Comparing the measurements in Table 1, it is apparent that the main component (ignoring the six nodes that are disconnected from the main part of the graph) of the random network is more highly connected as the diameter and average path length are smaller. Also, due to the random design principle of the Erdős-Rényi random graph, the distribution of degrees roughly approximates the bell curve (see Fig 4b).

INSERT FIGURES 3 & 4

Large Music Network Analysis

Using the same procedure to create a network of all the solos in the WJD results in a very large graph with 41416 nodes and 95516 edges. In this illustration, the WJD network measures are compared to two random networks created with different algorithms and a network created using the same procedure as above but using a version of the Essen folk song database (Table 2). The Essen database has been described in detail in other research (Dahlig, 2020) and is often used to investigate features of western tonal music (e.g. Conklin & Anagnostopoulou, 2006). Below, I describe macro level measurements of the WJD compared to the Essen network and briefly describe what some of these results may mean for musical features of the networks.

INSERT TABLE 2

Exploring the size of the WJD and Essen networks, Essen has more intervals (Essen: 347,246; WJD: 200,353), but fewer nodes (Essen: 15,305; WJD: 41,416) which reflects the uniqueness of the WJD. The WJD contains jazz solos from many eras representing multiple jazz styles including New Orleans, Swing, Bebop, Cool, and Free Jazz. On the other hand, the Essen collection used here includes exclusively European songs. A similar conclusion can be drawn by comparing the network diameters and average path lengths of the two networks, showing that the WJD is “larger” despite being based on fewer total number of intervals.

Both networks are highly distributed and contain degree distributions similar to scale-free networks (see Fig. 5). In other words, not only do both networks have hubs that are relatively rare compared to the size of the network but the high assortativity correlations would indicate that these hubs are connected to each other.

INSERT FIGURES 5 & 6

To further illustrate the unique structure of the WJD, the network was compared to two different types of random networks. As above, the ER (directed) network was created by simply entering the same number of nodes as the WJD and a probability that resulted in the creation of approximately the same number of edges. The WJD and ER networks have similar average degree and path lengths but the diameter of the ER is about half of the WJD (directed) (see Table 2). Interestingly, the distribution of degrees is very different when comparing both music corpora (Fig. 5) with ER (Fig. 6, blue bars). While the music networks consist mostly of nodes connected with two edges (WJD: 22,973 out of 41,416 – 55%; Essen: 6235 out of 15,305 – 41%), the most common degree for nodes in the ER network is 4 degrees (7692 out of 41,416 – 19%). Looking at the two distributions, the music networks have very high initial values that decrease exponentially. The ER distribution is closer to a bell-shaped curve. Therefore, a second random undirected network was constructed using the Barabási–Albert Random model (BA) (Barabási, 2016). Here, nodes are added one at a time and connected to existing nodes using the preferential attachment principle. Each new node is connected to existing nodes with a probability that is proportional to the degree of the existing node. In other words, new nodes are more likely to be connected to existing hubs. Interestingly, a comparison between the degree distribution of the music and BA networks shows very similar, exponentially decreasing shapes (Fig. 5 and Fig. 6, orange bars). Comparing the undirected values of the WJD to the BA model revealed very similar average path length (WJD: 7.2; BA: 5.6) and clustering coefficient (WJD & BA: 0.001), though the average degree (WJD: 2.3; BA: 4) and diameter (WJD: 40; BA: 10) are different. Despite the commonalities between the BA and the music networks, looking at degree assortativity, only the music networks showed any evidence that hubs were linked to other hubs (correlations undirected WJD: 0.39; Essen: 0.34; BA: -0.041). Following standard procedures for directed networks, correlations are listed between all four possible connection types (in-in, in-out, out-in, out-out). Comparing the directed versions of the WJD and the Essen networks again shows evidence that hubs are linked to other hubs as compared to the directed ER random network.

Discussion

Here I proposed a procedure to create musical networks from continuous interval data. This procedure was used as an example of the application of network science to analyze large music collections. My hope is that the current demonstration may inspire new research in which the structure of large corpora of music is analyzed using network science. I described the organizational structure of two very large music networks where nodes represent 4-interval patterns and edges represent possible continuations. I used this example to demonstrate how several common network science measurements can be used to describe musically relevant features of the networks. Below, I summarize the main organizational similarities and differences between four networks, the two music networks and two randomly created networks of similar size. The observations raise questions about the underlying reasons for these differences, which could be the focus of further research. My main goal here is not to answer these questions but to argue that network science as applied to music corpus research can be used to investigate problems that other methodologies can not address.

Results show that both the WJD and Essen networks have degree distributions that are similar to scale-free networks. Here, very few hubs are very highly connected compared to the large majority of nodes that have few neighbors. Indeed, many of the nodes in the music networks (WJD: 55%, Essen: 41%) simply have one in and one out degree, meaning they are simply part of a string of concatenated 4-interval patterns that only occur once in the corpus. A list of the hubs reveals they often consist of scalar interval structures (Supplementary Fig. 1). This makes sense musically as hubs may serve as the "glue" connecting more rare patterns. Indeed, scale fragments have been identified as occurring often in previous corpus research of jazz improvisations (Norgaard, 2014). Interestingly, the scale-free structure has been identified in many real-world networks (Barabási, 2009). Recently, the ubiquity of scale-free networks has been questioned based on an analysis of the tails of the degree distributions (Broido & Clauset, 2019). However, this distinction may be less relevant to descriptions of the macro structure of large music networks. For the current discussion, I believe the main point is the existence of very few, highly connected, hubs. Furthermore, the similar assortativity measures in the two current music networks show that hubs are often connected to other hubs. Future research could further investigate whether western tonal musical rules cause this organization. Indeed, the mode of creation (i.e., improvised solos versus folk melodies) did not appear to influence the overall organization of interval patterns, though this issue should also be investigated further.

Another intriguing line of research explores how these networks came to be. The BA network model was specifically created to mirror a model grown through preferential attachment. However, it is interesting to note that the BA network does not show evidence that hubs are linked to other hubs as the assortativity correlations are low. Interesting comparisons between growing networks of children's semantic networks have shown that preferential attachment is not the best model to describe network growths. Hills et al (2009) showed the growth model termed preferential acquisition best fit experimental data. In this model, it is the connectivity of words in the children's environment that determine in which order words are added to their vocabulary. This is an intriguing idea that could also describe how jazz musicians develop their musical vocabulary of patterns. Indeed, ethnographic accounts of learning based on interviews with well-known jazz musicians place a large emphasis on the listening environment and how patterns may be integrated through an automatic unsupervised learning process (Berliner, 1994). However, developing jazz musicians also consciously add vocabulary through deliberate practice and many pedagogical materials are centered around idiomatic jazz patterns (e.g. Baker, 1988). To avoid confounds related to instrumental technique, these patterns are often transposed to all twelve keys and practiced. Indeed, in interviews concerning a just performed improvisation, musicians often identified the origin of particular patterns as being from mentors or recordings (Norgaard, 2011). To properly investigate whether the vocabulary of individual jazz musicians expand through preferential attachment or preferential acquisition, researchers could analyze solos from developing improvisers at different stages and create related networks.

Looking at broader implications for the music cognition community, it is important to outline other ways in which music networks could be created. Just as semantic language networks are created based on experimental similarity judgments, the large literature on melodic similarity could be used to create networks in which nodes of patterns are connected with edges reflecting these judgments. Edges could also be based on commonly calculated melodic or rhythmic similarity measures (e.g. Eerola & Toiviainen, 2004, p. 41). In addition, the current paradigm only included connections between exact interval patterns. Future research could consider relationships between patterns with similar contour (but not exact interval structure) reflected in a network. That would presumably result in many more edges and a much denser network. Rhythm patterns could also be used to create networks and it is even possible to create “complex” networks in which different types of edges represent different relationships between nodes. One could imagine a network of interval/rhythm patterns where two types of edges represent rhythmic and melodic similarity. Finally, it is possible to create networks in which nodes represent chords, articulations, harmony, or any other musical feature that can be quantified.

In summary, here I demonstrated how to create a network of interval patterns based on possible continuations within a large music corpus. After a short introduction to basic network science terminology, I described selected organizational principles based on node degree distributions. Specifically, the WJD and Essen network organization was compared to two randomly created networks of similar sizes. This analysis showed that a likely growth model for both music networks may be simulated with an algorithm in which new patterns are more likely to be connected to existing hubs. Based on this example, I argued that network analysis could be more broadly adapted to corpus music research.

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Network graphs available as Gephi files here: https://osf.io/st7rj/

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Table 1. Measurements for the undirected and directed versions of the Ory network as well as the similar random directed network.

	Kid Ory network		Erdős-Rényi Random (ER)
Type	Directed (weighted)	Undirected	Directed (unweighted)
Solos/songs	1	1	1
Intervals	42	42	42
Total 4 n-grams	39	39	39
Nodes	37	37	37
Edges	38	38	38
Average Degree	1.027	2.05	1.027
Diameter	24	22	6 (main component)
Avg. Path Length	9.17	8.38	2.65 (main component)
Avg. Clustering Coef	0	0	0
Modularity	0.67		0.54

Table 2. Measurements for the undirected and directed versions of the WJD and Essen networks as well as two random networks.

	WJD		Erdős-Rényi Random (ER)	Barabási–Albert Random (BA, pref att: m=2)	Essen	
Type	Directed (weighted)	Undirected	Directed (unweighted)	Undirected (unweighted)	Directed (weighted)	Undirected
Solos/songs	456				7284	
Intervals	200,353				347,246	
Total 4 n-grams	198,985				325,394	
Nodes	41,416		41,416	41,416	15,305	
Edges	95,516		95,560	82,828	43,047	
Average Degree	2.306		2.307	4	2.813	
Diameter	74	40	33	10	35 G/nx*	22 G/nx
Avg. Path Length	9.24	7.17	12.23	5.62 G/nx	7.47 (nx 7.34)	6.077 G/nx
Avg. Clustering Coef	0.000596 nx	0.001 (798 triangles) 0.00119 nx	0 (0 triangles Gephi, 0 nx) [Undirected: 11 triangles]	0.00127 G/nx (169 triangles)	0.000339 nx	0.001 (213 triangles) 0.000679 nx
Modularity	0.595		0.447	0.516	0.541	
Degree assortativity (r)		0.388		-0.041		0.338
In-in	0.377		0.005		0.319	
In-out	0.417		0.004		0.345	
Out-in	0.328		-0.0002		0.293	
Out-out	0.371		0.002		0.317	

Figure Captions:

Figure 1. Kid Ory solo notation with intervals.

Figure 2. Kid Ory solo represented as a directed network starting with the node in lower left.

Figure 3. A random network with the same number of degrees and edges as the Kid Ory network.

Figure 4. Degree distributions for the Kid Ory network (A, top) and a similar random network (B, bottom).

Figure 5. Degree distributions for the WJD and Essen networks

Figure 6. Degree distributions for the ER (undirected) and BA random networks

Figure 1

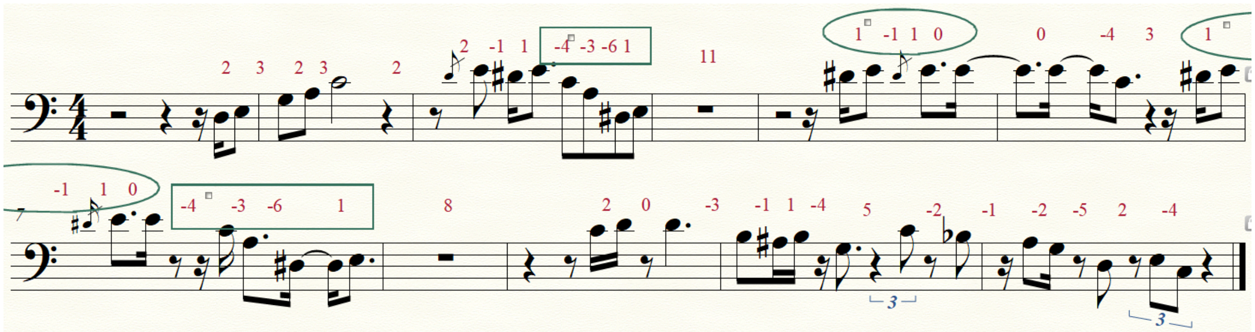


Figure 2

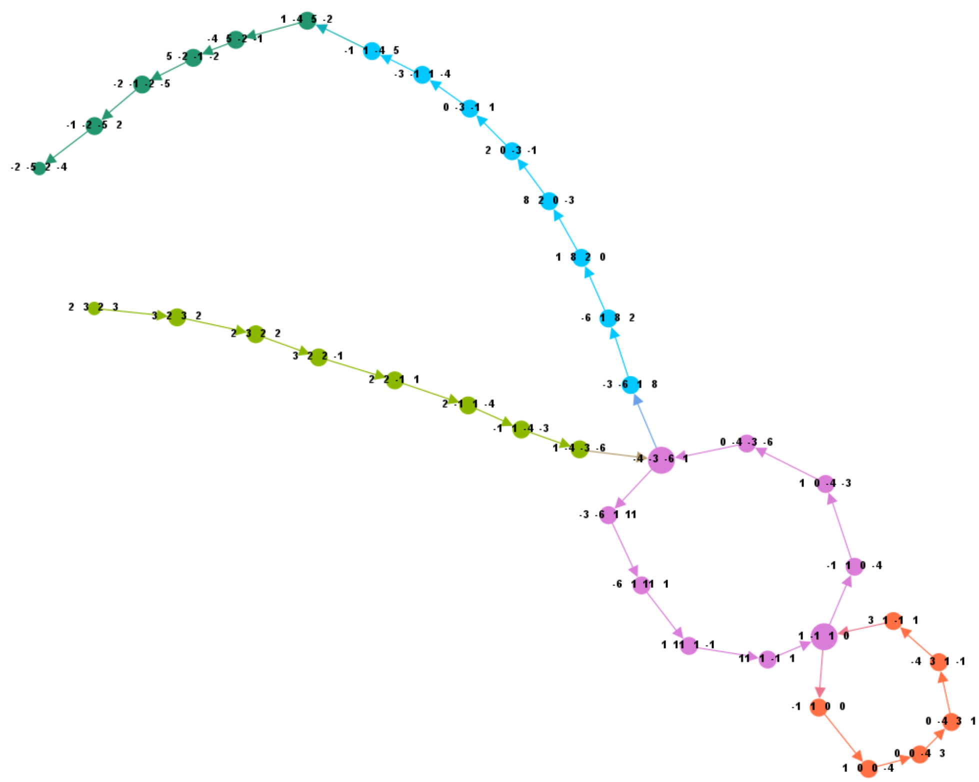


Figure 3

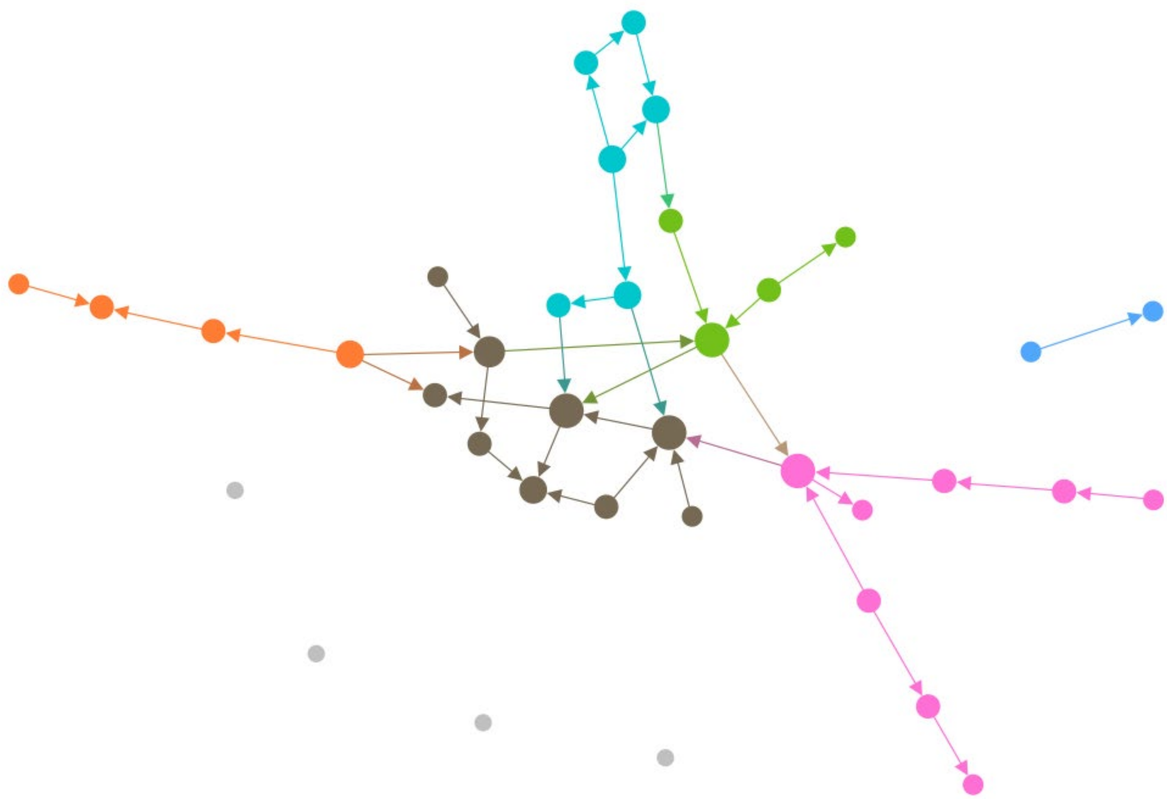


Figure 4

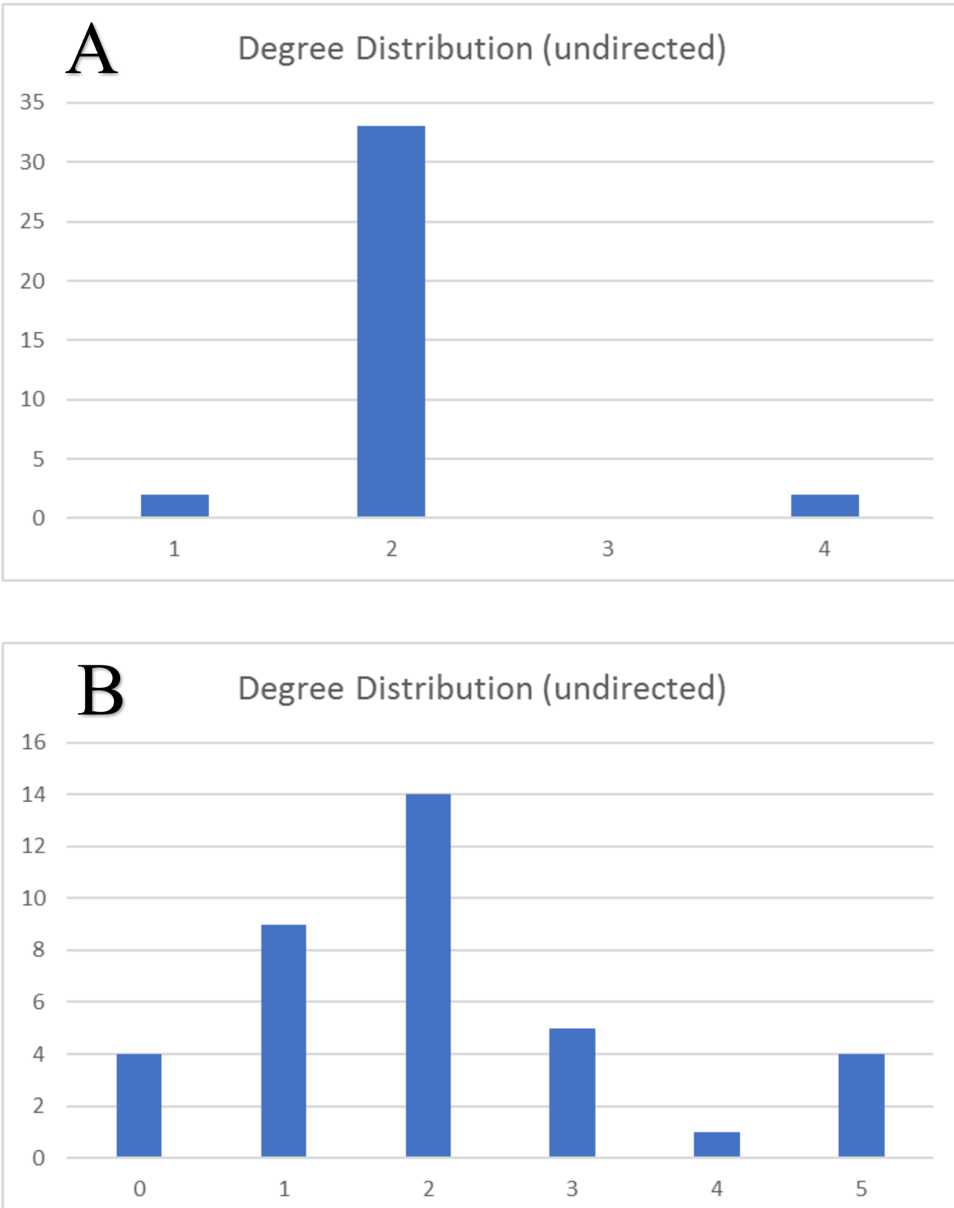

A
Degree Distribution (undirected)
35
30
25
20
15
10
5
0
1
2
3
4
B
Degree Distribution (undirected)
16
14
12
10
8
6
4
2
0
0
1
2
3
4
5

Figure 5

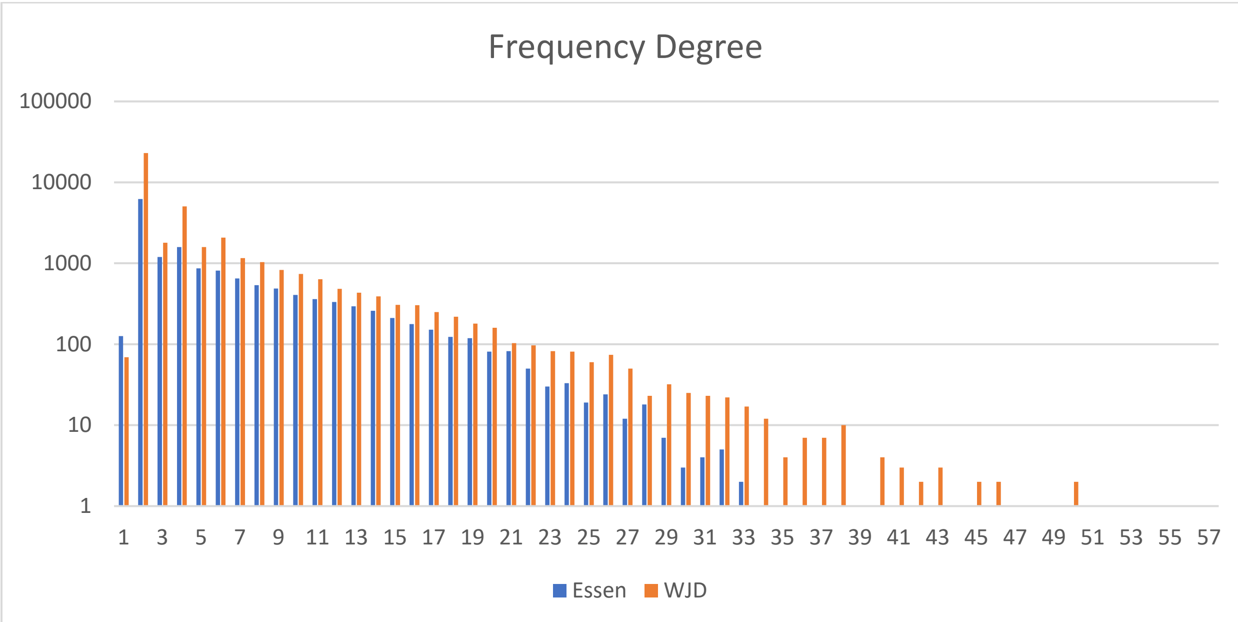
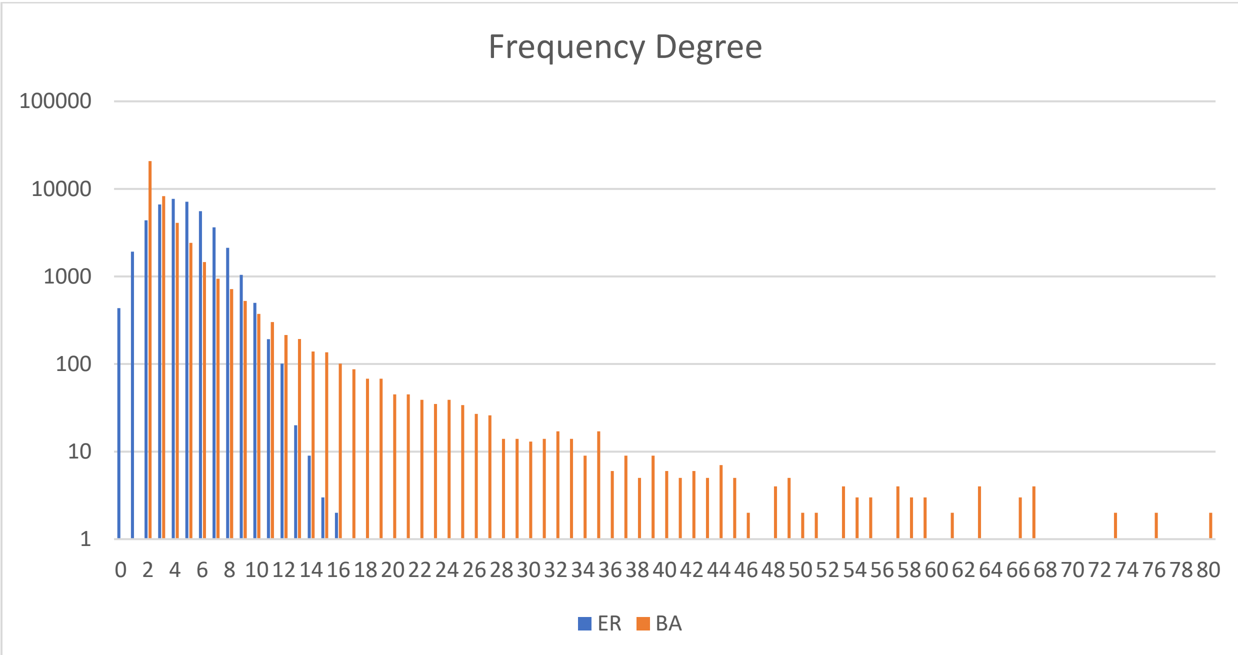

Frequency Degree
100000
10000
1000
100
10
1
1
3
5
7
9
11
13
15
17
19
21
23
25
27
29
31
33
35
37
39
41
43
45
47
49
51
53
55
57
Essen
WJD

Figure 6


Frequency Degree
100000
10000
1000
100
10
1
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38 40 42 44 46 48 50 52 54 56 58 60 62 64 66 68 70 72 74 76 78 80
ER
BA

Supplementary Table 1: The 30 most common 4-interval patterns in WJD.

	Frequency	# of solos in which the pattern appears	4-interval pattern
1	1473	262	-1 -1 -1 -1
2	1025	141	0 0 0 0
3	859	271	2 1 2 2
4	760	249	-2 -2 -1 -2
5	757	251	-2 -1 -2 -2
6	677	234	2 2 1 2
7	654	234	-1 -2 -2 -1
8	652	252	-1 -1 -1 -2
9	628	226	-2 -1 -2 -1
10	599	168	1 1 1 1
11	578	170	1 -1 1 -1
12	551	195	1 2 1 2
13	550	171	2 -2 2 -2
14	512	139	-1 1 -1 1
15	484	145	-2 2 -2 2
16	469	206	-2 -1 -1 -1
17	448	183	-1 -2 -1 -2
18	445	203	-1 -1 -2 -1
19	445	184	2 1 2 1
20	435	195	1 2 2 1
21	399	196	1 -1 -1 -1
22	377	185	-2 -2 -1 -1
23	373	174	1 2 2 2
24	364	186	-1 -2 -2 -2
25	357	147	-3 1 2 1
26	347	176	-2 -2 -2 -1
27	343	192	2 1 -1 -2
28	336	158	2 2 2 1
29	332	183	1 -1 -2 -2
30	313	149	-1 -2 -1 -1