An Extension Of The Mankiw, Romer, and Weil's "Contribution To The Empirics Of Economic Growth" with Recent Data

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Abstract

This paper explores the practical application of the Solow growth model in a number of countries and tests the consistency of the textbook Solow model with the recent data available for those countries. It replicates and expands the 1992 econometric study of the textbook Solow growth model by Mankiw, Romer, and Weil to recent years. Using cross- sectional data, the paper analyzes the impact of saving and population growth, technology and capital depreciation rates on the economic growth of a country. It shows that, in line with the textbook Solow model, higher saving positively affects economic growth, while the population growth rate affects it negatively. However, the paper demonstrates higher values of the OLS estimators of these factors than predicted by the Solow model, which suggests that augmentation of the model is necessary for it to be more successful in explaining the variation in international standards of living.

Introduction

The neoclassical growth model, introduced by Robert Solow in 1956, was a major contribution to economics. Today, it can be called the organizing structure of macroeconomics, as it explains the essential features of the economic growth. In this work, I test the consistency of the Solow growth model with the cross-country data for variation in the standard of living. I replicate the 1992 work of Mankiw, Romer, and Weil by exploring the textbook Solow model for the same set of countries and then expanding the data for those countries to more recent years to test the external validity of the model. This paper uses the cross-sectional dataset provided by Mankiw, Romer, and Weil for replication and more recent cross-sectional data for the expansion of the model. The results agree with the general predictions of the Solow model in how the saving and population growth variables influence the steady-state level of income of a country, showing that the country tends to be richer if its rate of saving increases and poorer if its population grows at a higher rate. However, both the

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replications and the extension of the model to more recent years suggest that the effects of saving and population growth rates on income are too large and do not conform to the Solow model's predictions completely, which calls for an augmentation of the textbook model in further research.

Analytical Model and Research Hypotheses

I provide here a short review of the textbook Solow growth model before determining the analytical model used for the testing in Stata. The production function in the Solow growth model is assumed to be a Cobb-Douglas function with diminishing returns to capital. It takes the form of:

$$Y(t) = K(t)^{\alpha}(A(t)L(t))^{1-\alpha} \qquad 0 < \alpha < 1, \tag{1}$$

with the standard notation, where Y is the output, K capital, L labor, and A is the level of technology. Solow took the rates of L and A as exogenous, with L growing at the rate of n and A growing at the rate of g:

$$L(t) = L(0)e^{nt} \tag{2}$$

$$A(t) = A(0)e^{gt}. \tag{3}$$

Then, the number of effective units of labor, defined by $A(t)L(t)$, grows at the rate of *n+g.* The Solow model also takes the rate of saving as exogenous, therefore assuming that a constant fraction of output, s, is invested. If k is the stock of capital per effective unit of labor, $k = K/AL$, and y is the level of output per effective unit of labor, $y = Y/AL$, then the change in capital is determined by:

$$\dot{k}(t) = sy(t) - (n+g+\delta)k(t) = sk(t)^{\alpha} - (n+g+\delta)k(t), \tag{4}$$

where δ is the rate of capital depreciation – the rate at which capital wears out, breaks or becomes obsolete in the production process due to technological advancements. As Mankiw, Romer and Weil conclude, equation (4) suggests that k converges to a steady-state value k*defined by $sk^{*\alpha} = (n+g+\delta)k^{*}$, or

$$k^{*} = \left[\frac{s}{n+g+\delta}\right]^{\frac{1}{1-\alpha}} \tag{5}$$

The natural conclusion from this is that the saving rate positively affects the steady rate capital-labor ratio, while the rate of the population growth affects it negatively.

In the paper "A Contribution to the Empirics of Economic Growth," Mankiw, Romer and Weil then concentrate on the central predictions of the Solow model that concern the impact of saving and population growth on real income. They substitute equation (5) into the production function and take logs, thereby getting the equation that posits that steady-state income per capita is

$$\ln\left[\frac{Y(t)}{L(t)}\right] = \ln A(0) + gt + \frac{\alpha}{1-\alpha}\ln(s) - \frac{\alpha}{1-\alpha}\ln(n+g+\delta).^{2} \qquad (6)$$

They also point out at the fact that the model, assuming that factors are paid their marginal products, predicts not only the signs but also the magnitudes of the coefficients on saving and population growth. In particular, the model predicts that the elasticity of income per capita with respect to the saving rate is approximately 0.5 and that the elasticity with respect to $(n+g+\delta)$ is approximately –0.5, assuming that the share of profits in output is approximately 1/3.

One of the main purposes of Mankiw, Romer, and Weil's work was to answer the question if it is true that real income is higher in countries with higher saving rates and lower in countries with higher values of $n+g+\delta$. To do that, they assumed constant g and δ across countries, an assumption followed in this paper as well, since the advancement of knowledge, represented by g is not country-specific and there is indeed no reason, as Mankiw, Romer, and Weil say, to expect depreciation rates to vary greatly across countries and also no data to estimate it. However, because A(0) reflects resource endowment, institutions, climate, etc., it can differ across countries. Therefore, they make a specification to the model, assuming that

$$\ln A(0) = \alpha + \epsilon,$$

where α is a constant and ϵ is the error term, specific to the country. Taking time 0 for the purpose of simplicity, Mankiw, Romer and Weil get that log income per capita is

$$\ln\left(\frac{Y}{L}\right) = \alpha + \frac{\alpha}{1-\alpha}\ln(s) - \frac{\alpha}{1-\alpha}\ln(n+g+\delta) + \epsilon. \qquad (7)$$

Another assumption is that s and n are independent of ϵ, which allowed the authors to estimate the equation with ordinary least squares method (OLS) and obtain consistent OLS estimators. The assumption was made partly because, regardless of whether saving and population growth are exogenous or endogenous (like in many standard models of economic growth), if preferences are considered to be isoelastic, s and n are unaffected by ϵ. This means that, under isoelastic utility, permanent differences in the level of technology do not affect saving rates or population growth rates.[3] For the purposes of this work, this paper agrees with this assumption since it is made in many economic growth models and because the Solow model allows for prediction of the magnitudes of the coefficients on saving and population growth.

Equation (7) can thus be expressed by the standard multiple regression model:

[2]Mankiw, Romer, and Weil, (1992. p. 410).
[3]Mankiw, Romer, and Weil, (1992. p. 411).

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + u_i \, , \tag{8}$$

where β_0 is equal to α, β_1 is equal to $\frac{\alpha}{1-\alpha}$, and β_2 is equal to $\left(-\frac{\alpha}{1-\alpha}\right)$. The error term represents ϵ.

To sum up then, our research hypotheses are:

RH1: β_1 is positive, implying that higher saving rate leads to higher real income in countries;

RH2: β_2 is negative, implying that higher values of (n+g+δ) lead to lower real income in countries;

In addition, the paper also assesses the prediction of the Solow growth model that the magnitudes of the coefficients are approximately 0.5 and –0.5, respectively.

Data and Variables

For the purpose of replication, this paper uses the same dataset that Mankiw, Romer, and Weil used in doing their econometric study. This dataset was provided by them in the Appendix to their work.[4] Mankiw, Romer, and Weil measured *n* as the average rate of growth of the working age population, i.e. population of ages 15 to 64. As a measure of *s*, they used the average share of real investment (including government investment) in real GDP, and Y/L as real GDP in 1985 divided by the working-age population in that year. The span of their study covered the years of 1960 to 1985 for almost all of the world countries, except the centrally planned economies at the time.

They also considered three samples of countries: non-oil, intermediate, and OECD countries. The first, non-oil, sample consisted of 98 countries and represented the most comprehensible one. Oil-rentier economies were excluded, according to the authors, because recorded GDP in such countries was largely influenced by resource extraction and did not reflect the value added. Therefore, they did not believe standard growth models could account for measured GDP in those countries. The second sample consisted of 75 countries and the criteria for constructing it were twofold: first, Mankiw, Romer, and Weil excluded small countries (i.e. countries whose populations were less than 1 million in 1960); and second, they excluded the countries whose real income figures were based on extremely little primary data. Thus, they addressed two problems that could arise with the internal validity of the model: the bias of the estimators introduced by outliers (such as very little countries, whose real income can be determined by idiosyncratic factors) and the measurement error caused by little primary data available. As a third sample, they took the 22 OECD countries with

[4]Mankiw, Romer, and Weil, (1992. p. 434-436).

populations over 1 million and admitted in advance that, though the data for these countries appeared to be all of high quality and the variation in omitted country-specific countries was likely to be small, this sample had disadvantages of its small size (less than 30 observations) and lack of statistical leverage.

The replications carried out in the next section used the same samples that the original paper employed. These samples appear to address many of the possible problems with internal validity, such as omitted variable bias (e.g. oil rents influencing a country growth and being correlated with investment and population growth rates), measurement error, outliers effects, etc.; as well as take into consideration the disadvantages of the small sample and are thus prepared for less reliable results from that sample. For the extension of the study with recent data, this paper does not make any alterations to the sample for the purpose of keeping all the countries in the sample that Mankiw, Romer, and Weil used the same in the new data. However, one should be careful doing that, as many of the countries might have acquired different characteristics from what they had in 1960-1985. For example, a country's population could have exceeded 1 million people. It appears, nevertheless, that such population growth, if occurred at all, did not change those countries status as outliers. This gives the grounds for using those samples in the extension of the model in the next section.

Turning back to the replications, it is important to acknowledge that $g+\delta$ are assumed to be 0.05 to match the available data that suggest that g is about 0.02 and δ is about 0.03.[5] The application of the OLS method by Mankiw, Romer, and Weil to obtain the OLS estimators of the coefficients is replicated in the next section.

Replications

The results from regressions in Stata are shown in Table 1. The results of the replications are almost identical to what Mankiw, Romer, and Weil obtained, except for the coefficients on the constant terms, which, although having the same sign, differ in magnitude from theirs. However, this difference can be explained by the different units in which investment share and the population growth, technology growth, and depreciation rates are measured: as a fraction or in percentage terms. This difference piles up on the coefficient of the constant terms, not affecting any other results. Note that coefficients on the investment estimator and population growth estimator are very significant in all the samples except for the small OECD one.

[5]For reference, see Mankiw, Romer, and Weil, (1992. p. 413-414).

Table 1: Estimation of the Textbook Solow Model, 1960-1985

Dependent variable: log GDP per working-age person in 1985			
Sample	Non-oil	Intermediate	OECD
Number of observations	98	75	22
Constant	8.04***	8.57***	9.14***
	(1.26)	(1.23)	(1.60)
ln*(I/GDP)*	1.42***	1.32***	0.50
	(0.13)	(0.16)	(0.34)
ln $(n+g+\delta)$	-1.99***	-2.02***	-0.74
	(0.55)	(0.48)	(1.10)
R^2	0.60	0.60	0.11

Source: Author's calculations based on the data from Mankiw, et. al. (1992).
Note: *** p<0.01, ** p<0.05, * p<0.1. Standard errors are in parentheses. The investment and population growth rates are averages for the period 1960-1985. $(g+\delta)$ is assumed to be 0.05

Table 1 provides strong contradiction to the null hypothesis that neither savings nor population growth affect income per capita at least in two of the three samples. In the third sample, although the coefficients on the estimators of investment and population growth rate are not significant, they still show contradiction to the null hypothesis. Therefore, the alternative hypothesis that saving and population growth rates affect real income per capita is supported by the results.

Another important result is that R^2 is high in two of the three samples (around 0.60), which means that differences in saving and population growth account for a large portion of cross-country variation in income per capita. Consequently, it also means that the fraction of cross-country variation in income per capita explained by the error term (unobserved variables et al.) is quite small, which speaks in favor of the model.

However, the estimated coefficients on saving and working-age population growth are much larger than the model predicts (that is, larger than 0.5 and –0.5 respectively, which is consistent with the share of profits of about one third in the Cobb-Douglas production function above). These predictions of the model are based on the data on factor shares.[6] Therefore, even though the results confirm the general predictions of the Solow model, it appears that the model needs some augmentation in order for it to account for most of the cross-country variation in real income.

[6] Mankiw, Romer, and Weil, (1992. p. 412).

Testing the Model with Recent Data

This section aims at testing the external validity of the econometric model described in the previous sections by applying the model to test its convergence with the data available for the recent years, specifically for the period from 1990 to 2010. The same econometric techniques are applied in order to be consistent with the estimations. The same empirical specifications hold. As mentioned in the above section, the three samples are preserved the same.

The data for the recent years comes from the World Development Indicators of the World DataBank.[7] As in the previous model, n is measured as the average annual growth rate of the working age population for the period of 1990-2010, while s is the average share of real investment (gross fixed capital formation) in real GDP. The ratio Y/L is measured as real GDP in 2010 divided by the working age population in that year. $(g + \delta)$ is assumed to be 0.05.

Table 2: Estimation of the Textbook Solow Model, 1990-2010

Dependent variable: log GDP per working-age person in 2010			
Sample	Non-oil	Intermediate	OECD
Number of observations	93	71	22
Constant	17.07***	17.71***	17.03***
	(1.86)	(2.68)	(3.42)
ln*(I/GDP)*	2.15***	1.55**	-1.06
	(0.41)	(0.76)	(0.75)
ln$(n+g+\delta)$	-7.80***	-7.11***	-1.92
	(0.57)	(0.64)	(1.52)
R^2	0.68	0.56	0.22

Source: Author's calculations based on the data from the World Bank (2016).
Note: *** p<0.01, ** p<0.05, * p<0.1. Standard errors are in parentheses. The investment and population growth rates are averages for the period 1990-2010. $(g+\delta)$ is assumed to be 0.05

Table 2 reports the results. The results obtained are somewhat contradictory. On the one hand, in the first two samples the signs on the coefficients on investment and population growth are in line with what the model predicts them to be and highly significant (at p<0.01 level), representing a strong contradiction to the null hypothesis. The third sample though has the wrong (i.e. unexpected) sign on the coefficient on investment, and both

[7] The World Bank (2016).

coefficients on saving and population growth are insignificant – a problem we already encountered in the replication of Mankiw, Romer, and Weil's dataset. The OECD sample has some disadvantages, as mentioned above, therefore it is not surprising that the results on it lack significance, although the sign on investment coefficient is still troublesome.

One support for the model is that R^2 is quite high in two of the three samples, which means that much of the variation in income can be explained by variation in saving and population growth rate across countries in 1990-2010 as well as in 1960-1985. However, the magnitude of the coefficients still represent a problem for the predictions of the Solow model, as they are very large, more so in the case of the coefficients on population growth rate.

The results of the econometric investigation for the period 1990-2010 on the whole support the idea that the Solow growth model has potential in explaining the variation in real national income across countries. In two of the three samples, it gave us the estimators for the coefficients that had the same sign as the Solow model predicted them to have. However, in this dataset for the period 1990-2010, the problem that the estimators for the coefficients are too large still remains and even intensifies, suggesting that the augmentation of the model to account for other variables, human capital for example as the Mankiw, Romer, and Weil do later in their work, is indeed necessary. The textbook Solow model alone cannot account for all the differences in real income per capita across countries.

Conclusions

The results from the recent data tend to confirm the results of Mankiw, Romer, and Weil on most of the issues, although some variation in the values of the estimators of the coefficients is observed, particularly in the case of the small-size OECD sample. The output of the regressions support the idea that higher saving rate leads to higher real income per capita, while higher population growth rate leads to lower real income per capita. The recent data results suggest that the magnitude of the population growth rate coefficient is higher than the coefficient on saving. This means that population growth affects real income at a larger rate than saving, that is the elasticity of real income is higher in absolute terms with respect to population growth rate than to saving rate.

The major problem with the model arises then with the magnitude of the estimators for the coefficients on the variables. In particular, the textbook Solow model predicts the coefficients on saving and population growth to be 0.5 and -0.5 respectively, while the Stata regression output suggests much larger numbers, which are highly significant. This leads us to suspect the presence of omitted variable bias and calls for inclusion of other variables that could augment the textbook Solow model to account for all of the variation in the international standard of living. One of the primary candidates for inclusion remains a proxy for human capital, as it has long been thought to influence the growth of the economies across the world.

Another possibility to improve the fit of the Solow model can be the use of a bigger sample, which will not have as much missing data as in the samples available now. Inclusion of the countries that switched from state-controlled economies to free market economies since 1985 might be one option for enlarging the sample. With regards to the the third sample, its size of 22 OECD countries is too small to produce unbiased OLS estimators, but has been chosen still because of the high quality of the data provided by those countries (justification chosen by Mankiw, Romer, and Weil initially).

Some other problems can affect the internal validity of the model used. The OLS estimator can be biased also due to functional form misspecification: when the true population regression function is nonlinear but the estimated regression is linear. Here, a linear regression of the logs of the dependent and independent variables was assumed due to the extensive literature that followed this functional form on the study of the Solow model. The problem of the missing data in our model reduces the sample size of the regression (for example, 93 countries instead of 98 in the N sample in Table 2), but does not introduce bias because the data are missing completely at random or based on the value of the regressor – some countries are worse than others at providing the world community with their data.

It has to be admitted that simultaneous causality can constitute a problem for this model, because higher real income can cause higher saving rate and higher or lower population growth rate. This reverse causality can make the OLS estimators biased and inconsistent due to picking up both effects. This problem would be best addressed by finding a good instrumental variable and running an IV regression. Mankiw, Romer, and Weil argue, though, that it is formidable to find instrumental variables that are correlated with the saving and population growth rates, but uncorrelated with the country-specific shift in the production function – the error term in the model.[8]

In doing the regressions for the period of 1990-2010 and in replicating the results of 1960-1985 years, I used robust standard errors – an improvement with regard to the methods of Mankiw, Romer, and Weil in their 1992 paper. Using robust standard error eliminates another threat to internal validity of the model: the heteroskedasticity of the regression error, which makes the use of standard errors not reliable for hypothesis tests and confidence intervals.

Despite the limitations and problems with the model, it is important to keep in mind that, nonetheless, the textbook Solow model does quite a good job

[8] Mankiw, Romer, and Weil, (1992. p. 411).

at predicting the effect of saving and population growth on real income per capita: more than half of the cross-country variation in income per capita can be explained by these two variables alone. Extensive literature has demonstrated the applicability of the Solow model, and the consistency of its predictions with the evidence. In this paper, it was shown that the predictions of the model are generally consistent with the most recent data for a large set of countries. These results therefore, do not allow one to dismiss the Solow model as a valid theory of growth, since it answers the questions it poses correctly in general. Rather, our results call for the improvement of the model and further research on the issue.

References

Mankiw, N. Gregory, David Romer, and David N. Weil. “A Contribution to the Empirics of Economic Growth.” *The Quaterly Journal of Economics* vol.107, No.2 (May 1992): 407–437.

Appendix 1

Table 3: Dataset Summary for 1990-2010 Period

Variable	Obs	Mean	Standard Deviation	Min	Max
N	120	0.817	0.389	0	1
I	120	0.625	0.486	0	1
O	120	0.183	0.389	0	1
GDP in 1990	120	9210.554	13226.530	0.84	53976
GDP in 2010	119	11606.920	17564.100	-2.29	76625
GDP per adult growth (1990-2010)	116	1.550	2.110	-3.67	16.81
Working age population growth	113	3.260	4.505	-0.28	26.83
Investment share (% of GDP)	105	20.416	4.963	9.37	46.97

Source: The World Bank (2016).

Appendix 2

Figure 1 shows two scatter graphs: one representing the relationship between the log of investment share in real GDP and log of GDP in 2010, and the other representing the relationship between ln*(n+g+δ)*and log of GDP in 2010. As expected, the elasticity of Y/L with respect to saving is positive and with respect to *(n+g+δ)* – negative.

Figure 1.