Feuille d'exercice n° 05 : Calculs sur les complexes – Fiche d'entraînement – Correction

Exercice 1

1)
$$\mathscr{C}(1,1) \cap \mathscr{C}\left(-1,\frac{3}{2}\right) = \left\{\frac{5}{16} - i\frac{15}{16}, \frac{5}{16} + i\frac{15}{16}\right\}$$

2)
$$\mathscr{C}(i,2) \cap \mathscr{C}(1,1) = \left\{ \frac{5 - \sqrt{7}}{4} - i \frac{1 + \sqrt{7}}{4}, \frac{5 + \sqrt{7}}{4} + i \frac{-1 + \sqrt{7}}{4} \right\}$$

3)
$$\mathscr{C}(i,1) \cap \mathscr{C}(1+4i,3) = \{1+i\}$$

4)
$$\mathscr{C}(3i,1) \cap \mathscr{C}(5,2) = \emptyset$$

Exercice 2

1)
$$\pm (1-3i)$$

2)
$$\pm (5+i)$$

3)
$$\pm \left(\sqrt{\frac{\sqrt{58}+3}{2}} - i\sqrt{\frac{\sqrt{58}-3}{2}}\right)$$

4)
$$\pm \left(\sqrt{\frac{\sqrt{29}+5}{2}} + i\sqrt{\frac{\sqrt{29}-5}{2}}\right)$$

Exercice 3

1)
$$\{3-i; -2+2i\}$$

2)
$$\{4; 1-2i\}$$

3)
$$\{i; 7-i\}$$

4)
$$\{5i; 2+i\}$$

Exercice 4

1)
$$(-2)^n \sin^n\left(\frac{x}{2}\right) \sin\left(\frac{nx}{2} + \frac{n\pi}{2}\right)$$

2)
$$\frac{\sin(nx)\sin((n+1)x)}{2\sin(x)}$$
 si $x \neq 0 \ [\pi], 0$ sinon.

Exercice 5

1)
$$\frac{1}{4}\sin(2x) - \frac{1}{8}\sin(4x)$$

2)
$$\frac{19}{16} + \frac{47}{32}\cos(2x) + \frac{5}{16}\cos(4x) + \frac{1}{32}\cos(6x)$$

3)
$$\frac{1}{8} - \frac{1}{8}\cos(4x)$$

4)
$$\frac{3}{4}\sin(x) + \frac{3}{32}\sin(2x) + \frac{3}{4}\sin(3x) - \frac{1}{32}\sin(6x)$$

Exercice 6

1)
$$8\cos^4(x) - 8\cos^2(x) + 1$$

2)
$$63\cos^7(x) - 112\cos^5(x) + 56\cos^3(x) - 7\cos(x)$$

3)
$$8\cos^5(x) - 10\cos^3(x) + 3\cos(x)$$

4)
$$-16\cos^6(x) + 24\cos^4(x) - 8\cos^2(x)$$