

Ex 27: $g(x) = a + bx + cx^2 + o(x^2)$

$$\begin{aligned} (\sin x)^3 &= \left(x - \frac{x^3}{6} + \frac{x^5}{120} + o(x^5) \right)^3 = x^3 \left(1 - \frac{x^2}{6} + \frac{x^4}{120} + o(x^4) \right)^3 \\ &= x^3 \left(1 - 3 \times \frac{x^2}{6} + 3 \times \frac{x^4}{120} + 3 \times \frac{x^4}{36} + o(x^4) \right) \\ &= x^3 - \frac{x^5}{2} + \frac{13}{120} x^7 + o(x^7) \end{aligned}$$

$$\begin{aligned} g(x) &= \frac{x - \frac{x^3}{3} + \frac{x^5}{5} + o(x^5)}{x^3 - \frac{x^5}{2} + \frac{13}{120} x^7 + o(x^7)} - \frac{1}{x^2} = \frac{1 - \frac{x^2}{3} + \frac{x^4}{5} + o(x^4)}{x^2 - \frac{x^4}{2} + \frac{13}{120} x^6 + o(x^6)} - \frac{1}{x^2} \\ &= \frac{1}{x^2} \times \frac{1 - \frac{x^2}{3} + \frac{x^4}{5} + o(x^4)}{1 - \frac{x^2}{2} + \frac{13}{120} x^4 + o(x^4)} - \frac{1}{x^2} \\ &= \frac{1}{6} + \frac{7}{40} x^2 + o(x^2) \end{aligned}$$