Feuille d'exercice n° 20 : Analyse asymptotique – quelques corrigés

Exercice 14 ()

2)
$$\frac{1}{6}$$

5)
$$\frac{2}{3}$$

6)
$$\frac{a^3}{b^3}$$

8)
$$-\frac{1}{2\sqrt{2}}$$

10)
$$\frac{1}{\pi}$$

12)
$$\frac{\sqrt{2}}{8x^3}$$

13)
$$\frac{x^2}{2}$$

14)
$$-\frac{3}{2}\left(x-\frac{\pi}{4}\right)$$

Exercise 15 ()
$$\lim_{x \to 1} \frac{x^x - 1}{\ln x} = 1$$

$$\lim_{x \to 0} \left(\frac{x^2}{\ln(\cos x)} + \frac{2}{x^2} \sin^2 x \right) = 0$$

$$\lim_{x \to \frac{\pi}{4}} \frac{\ln(\sin^2 x)}{(\frac{\pi}{2} - x)^2} = -\frac{16 \ln 2}{\pi^2}$$

$$\lim_{x \to \frac{\pi}{2}} \frac{\ln(\sin^2 x)}{(\frac{\pi}{2} - x)^2} = -1$$

$$\lim_{x \to +\infty} \frac{\ln x}{(\frac{\pi}{2} - x)^2} = -1$$

$$\lim_{x \to +\infty} \sin \frac{1}{x} \tan \left(\frac{2\pi x}{4x + 3} \right) = \frac{8}{3\pi}$$

$$\lim_{x \to 0^+} \ln x \tan(\ln(1 + x)) = 0$$

Exercice 17 (🖎)

 $\lim_{x \to \infty} (\ln x)^{\tan \frac{\pi x}{2e}} = e^{-\frac{2}{\pi}}$

1)
$$\ln(\cos x) = -\frac{1}{2}x^2 - \frac{1}{12}x^4 - \frac{1}{45}x^6 + o(x^6)$$

2)
$$\tan(x) = x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + o(x^5)$$

3)
$$\sin(\tan(x)) = x + \frac{1}{6}x^3 - \frac{1}{40}x^5 + o(x^5)$$

4)
$$(\ln(1+x))^2 = x^2 - x^3 + \frac{11}{12}x^4 + o(x^4)$$

5)
$$\exp(\sin(x)) = 1 + x + \frac{1}{2}x^2 + o(x^3)$$

6)
$$\sin^6(x) = x^6 - x^8 + o(x^9)$$

Exercice 18 ()

1)
$$\sqrt{x+1} = \sqrt{x} + \frac{1}{2\sqrt{x}} - \frac{1}{8x^{3/2}} + o\left(\frac{1}{x^{3/2}}\right)$$
;

2)
$$x \ln(x+1) - (x+1) \ln x = 1 - \ln(x) - \frac{1}{2x} + \frac{1}{3x^2} + o\left(\frac{1}{x^2}\right)$$
;

3)
$$\left(\frac{x+1}{x}\right)^x = e^{-\frac{1}{2}} \frac{e^1}{x} + \frac{11}{24} \frac{e}{x^2} + o\left(\frac{1}{x^2}\right)$$
;

4) Arctan
$$x = \frac{\pi}{2} - \frac{1}{x} + \frac{1}{3x^3} + o\left(\frac{1}{x^4}\right)$$
.

Exercice 19 ()

1)
$$\frac{\arctan x - x}{\sin x - x} = 2 - \frac{11}{10}x^2 + o(x^2)$$
;

2)
$$\ln \sin x = \ln(\sqrt{2}/2) + (x - \pi/4) - (x - \pi/4)^2 + \frac{2}{3}(x - \pi/4)^3 + o((x - \pi/4)^3)$$
;

3)
$$(1+x)^{\frac{1}{x}} = e\left(1 - \frac{1}{2}x + \frac{11}{24}x^2 - \frac{7}{16}x^3\right) + o(x^3)$$
;

4)
$$x(\sqrt{x^2 + \sqrt{x^4 + 1}} - x\sqrt{2}) = \frac{\sqrt{2}}{8x^2} + o\left(\frac{1}{x^2}\right).$$

Exercice 22 (🎾)

1)
$$\sqrt[3]{x^3 + x^2} - \sqrt[3]{x^3 - x^2} = x\left(\sqrt{1 + \frac{1}{x}} - \sqrt{1 - \frac{1}{x}}\right) = 1 + \frac{1}{8x^3} + o(1/x^3).$$

2)
$$\ln(\sqrt{1+x}) = \frac{1}{2}\ln x + \frac{1}{2x} + o(1/x)$$
.

Exercice 23 ()

1) a)
$$\frac{\cos x}{\sqrt{1+x}} = 1 - \frac{x}{2} - \frac{x^2}{8} - \frac{x^3}{16} + \frac{49x^4}{384} + o(x^4)$$

b)
$$\frac{\sqrt{1+x}}{\cos x} = 1 + \frac{x}{2} + \frac{3x^2}{8} + \frac{5x^3}{16} + \frac{41x^4}{384} + o(x^4)$$

c)
$$\frac{\ln(1+x)}{\cos x} = x - \frac{x^2}{2} + \frac{5x^3}{6} - \frac{x^4}{2} + o(x^4)$$

d)
$$\frac{1+\cos x}{2+\sin x} = 1 - \frac{x}{2} + \frac{x^3}{12} - \frac{x^4}{16} + o(x^4)$$

e)
$$\frac{\sin(x/2)}{e^{2x}} = \frac{x}{2} - x^2 + \frac{47x^3}{48} - \frac{5x^4}{8} + o(x^4)$$

f)
$$\frac{\ln(1+x)}{2-\cos x} = x - \frac{x^2}{2} - \frac{x^3}{6} + o(x^4)$$

2) a)
$$\frac{\sin(2x - \pi/4)}{\cos x} = 1 + 3(x - \frac{\pi}{4}) + \frac{3}{2}(x - \frac{\pi}{4})^2 + \frac{3}{2}(x - \frac{\pi}{4})^3 + \frac{19}{8}(x - \frac{\pi}{4})^4 + o((x - \frac{\pi}{4})^4)$$

b)
$$\frac{\cos(x-1)}{\ln(1+x)} = \frac{1}{\ln 2} - \frac{1}{2(\ln 2)^2}(x-1) + \frac{1}{\ln 2} \left(-\frac{1}{2} + \frac{1}{8\ln 2} + \frac{1}{4(\ln 2)^2} \right) (x-1)^2 + o((x-1)^2)$$

c)
$$\frac{e^{x-1}}{\ln x} = \frac{1}{x-1} + \frac{3}{2} + \frac{11}{12}(x-1) + \frac{3}{8}(x-1)^2 + \frac{71}{720}(x-1)^3 + \frac{41}{1440}(x-1)^4 + o((x-1)^4)$$

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Exercice 24

- 2) $-\frac{1}{2}$ 3) $-\frac{e}{2}$ 4) $\frac{1}{12}$ 5) $-\frac{1}{6}$ 6) $-\frac{1}{2}$