8 : Matrices par blocs

$$q+r=p \qquad l+m=n$$

$$\begin{array}{c|c}
 & \overbrace{\hphantom{a_{11} \cdots a_{1q}}}^{q} \quad \overbrace{\hphantom{a_{1,q+1} \cdots a_{1,q+r}}}^{r} \\
\begin{array}{c} l \\[4em] m \end{array} &
\left(\begin{array}{ccc|ccc}
a_{11} & \cdots & a_{1q} & a_{1,q+1} & \cdots & a_{1,q+r} \\
a_{21} & & \vdots & \vdots & & \vdots \\
\vdots & & & & & \\
a_{l1} & & a_{l,q} & a_{l,q+1} & & a_{l,q+r} \\
\hline
a_{l+1,1} & \cdots & a_{l+1,q} & a_{l+1,q+1} & \cdots & a_{l+1,q+r} \\
\vdots & & \vdots & \vdots & & \vdots \\
a_{l+m,1} & \cdots & a_{l+m,q} & a_{l+m,q+1} & \cdots & a_{l+m,q+r}
\end{array}\right) = A \quad n \\
 & \underbrace{\hphantom{a_{11} \cdots a_{1q} \quad a_{1,q+1} \cdots a_{1,q+r}}}_{p}
\end{array}$$

A_{11} A_{12}

A_{21} A_{22}

$$A_{11} = (a_{ij})_{\substack{1 \leq i \leq l \\ 1 \leq j \leq q}}$$

$$A_{12} = \left(a_{ij}\right)_{\substack{1 \le i \le l \\ q+1 \le j \le q+r}} = \left(a_{i,q+j}\right)_{\substack{1 \le i \le l \\ 1 \le j \le r}}$$

$$\in \mathcal{M}_{l,r}(\mathbb{R})$$

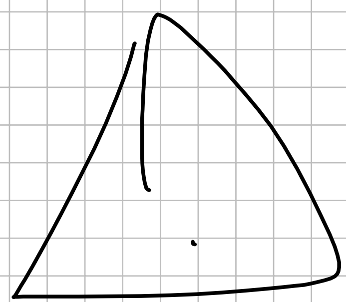
$$A_{21} = \left(a_{l+i,j}\right)_{\substack{1 \le i \le m \\ 1 \le j \le q}}$$

$$A_{22} = \left(a_{l+i,q+j}\right)_{\substack{1 \le i \le m \\ 1 \le j \le r}}$$

$$A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}$$

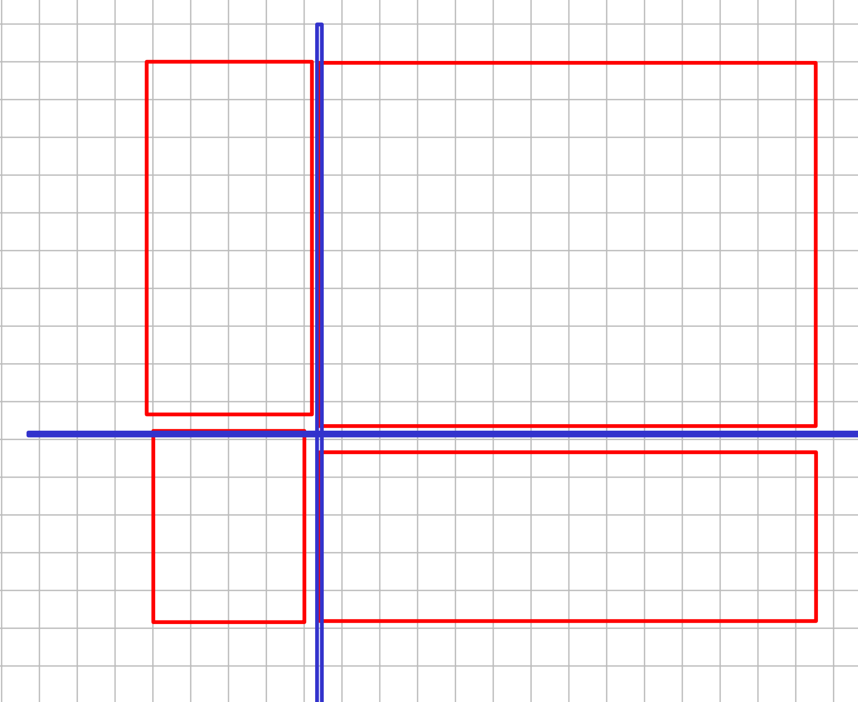
$$A = \begin{array}{cc}
\begin{array}{c} \ell_1 \\ \ell_2 \\ \ell_3 \\ \\ \end{array} & \left(\begin{array}{c|c|c|c}
A_{11} & A_{12} & A_{13} & \\ \hline
 & & & A_{24} \\ \hline
 & A_{32} & & \\ \hline
 & & A_{43} & \\ \hline
 & A_{52} & &
\end{array}\right) \\
 & \begin{array}{cccc} q_1 & q_2 & \phantom{A_{13}} & \phantom{A_{24}} \end{array}
\end{array}$$

$$A_{32} = \left(a_{\ell_1+\ell_2+i,\, q_1+j} \right)_{\substack{1 \le i \le \ell_3 \\ 1 \le j \le q_2}} .$$



$$A = \begin{pmatrix} \times & \times & \times & \times \\ \times & \times & \times & \times \\ \times & \times & \times & \times \\ \times & \times & \times & \times \end{pmatrix}$$

<u>INTERDIT</u>



Somme :

Ex :

$$\overbrace{\begin{pmatrix} 1 & 2 & \vline & 4 \\ 0 & -1 & \vline & 2 \\ \hline 3 & 1 & \vline & 5 \end{pmatrix}}^{A} + \overbrace{\begin{pmatrix} -1 & 0 & \vline & 5 \\ 2 & 1 & \vline & 6 \\ \hline 4 & 2 & \vline & 1 \end{pmatrix}}^{B} = \begin{pmatrix} 0 & 2 & \vline & 9 \\ 2 & 0 & \vline & 8 \\ \hline 7 & 3 & \vline & 6 \end{pmatrix}$$

$$\begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} + \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix} = \begin{pmatrix} A_{11}+B_{11} & A_{12}+B_{12} \\ A_{21}+B_{21} & A_{22}+B_{22} \end{pmatrix}$$

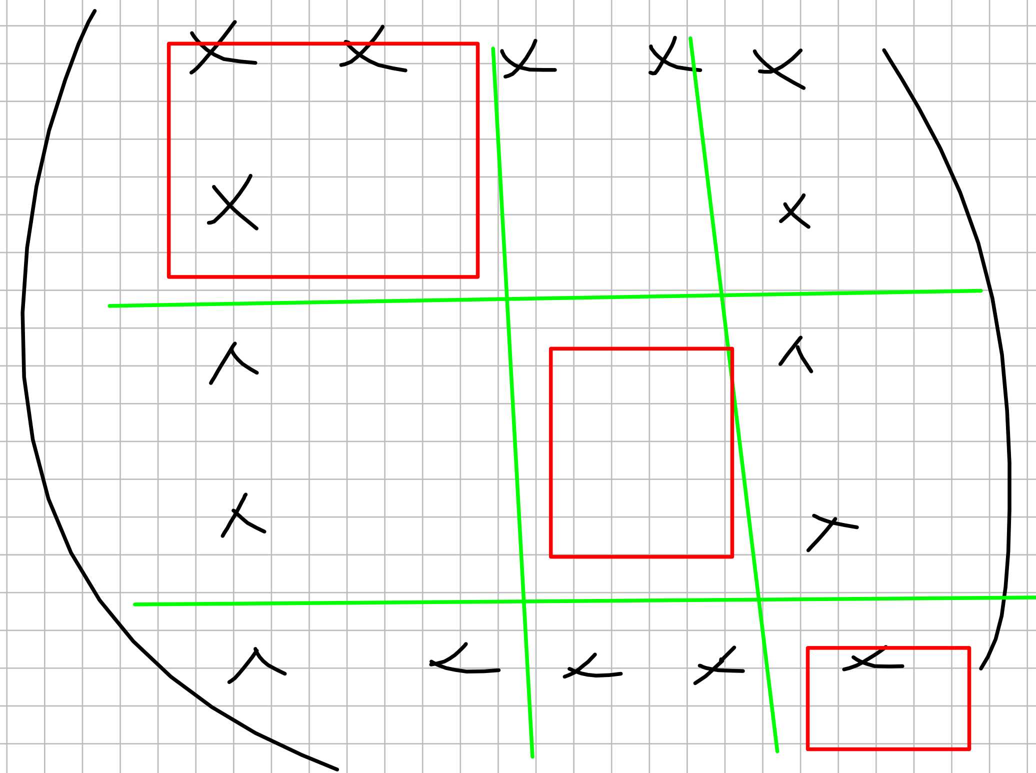
Ex :

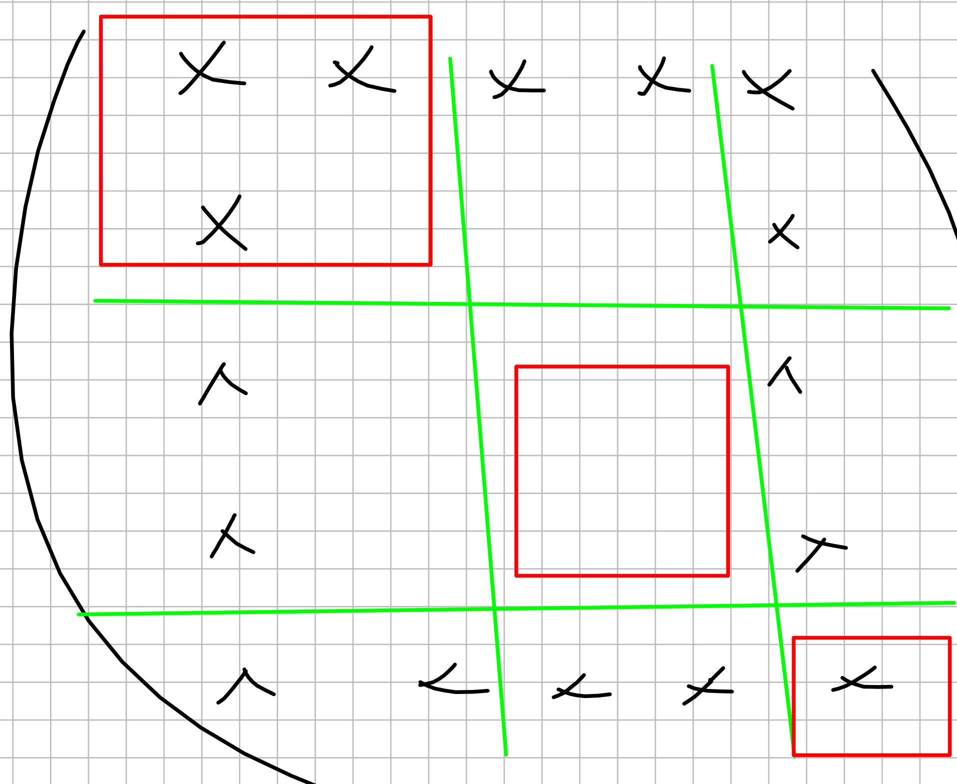
$$\begin{pmatrix} I_{9900} & \vline & 0 \\ \hline 0 & \vline & -I_{100} \end{pmatrix} \qquad \begin{pmatrix} \overbrace{\begin{matrix} 1 & & \\ & \ddots & \\ & & 1 \end{matrix}}^{9900} & \vline & \overbrace{\phantom{\begin{matrix} 0 \end{matrix}}}^{100} \\ \hline & \vline & \begin{matrix} -1 & & \\ & \ddots & \\ & & -1 \end{matrix} \end{pmatrix}$$

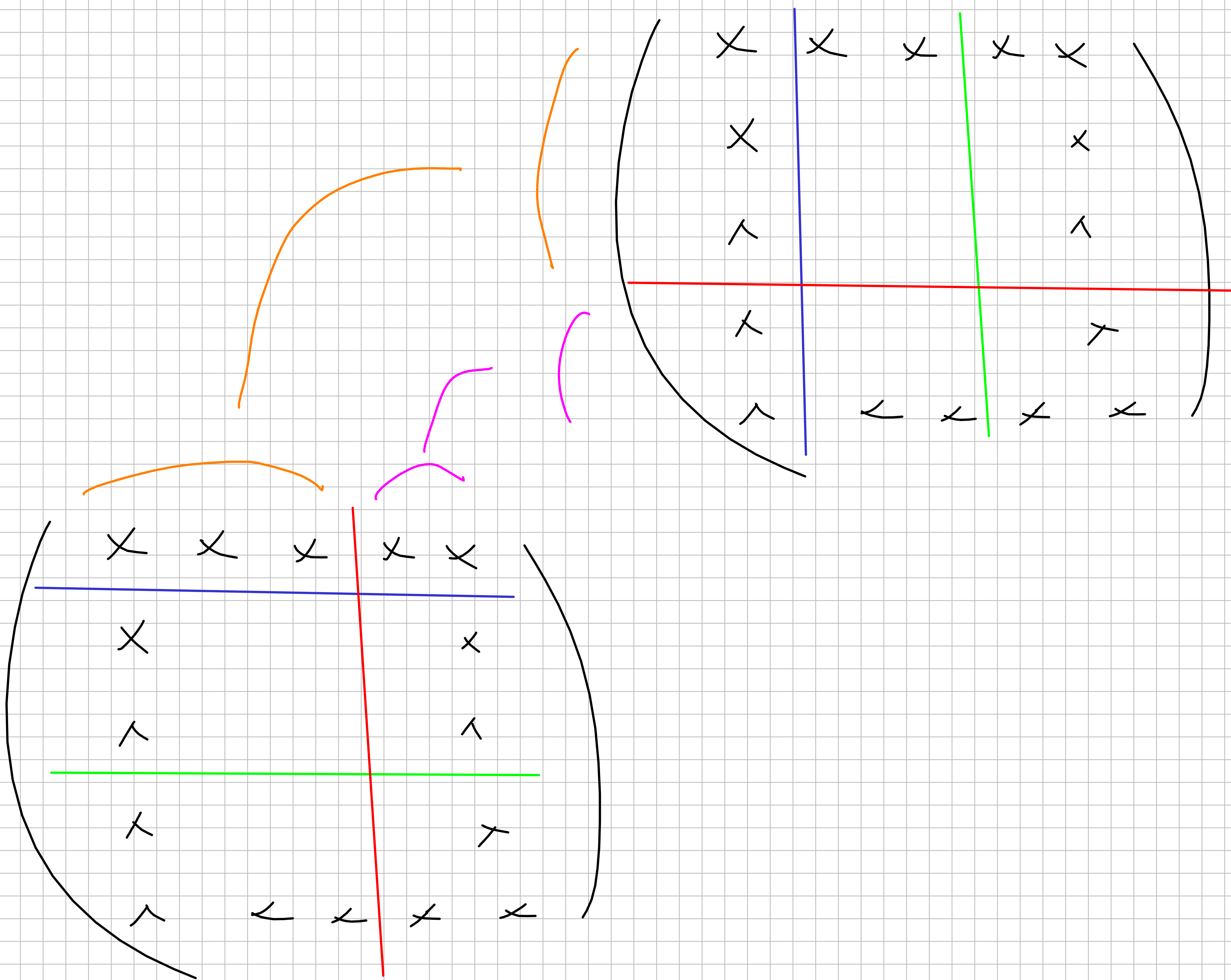
$$A \times B = \begin{pmatrix} 19 & 10 & 21 \\ 6 & 3 & -4 \\ 19 & 22 & 27 \end{pmatrix}$$

$$\left(\begin{array}{c|c} A_{11} & A_{12} \\ \hline A_{21} & A_{22} \end{array}\right) \left(\begin{array}{c|c} B_{11} & B_{12} \\ \hline B_{21} & B_{22} \end{array}\right) = \left(\begin{array}{c|c} A_{11}B_{11}+A_{12}B_{21} & A_{11}B_{12}+A_{12}B_{22} \\ \hline A_{21}B_{11}+A_{22}B_{21} & A_{21}B_{12}+A_{22}B_{22} \end{array}\right)$$

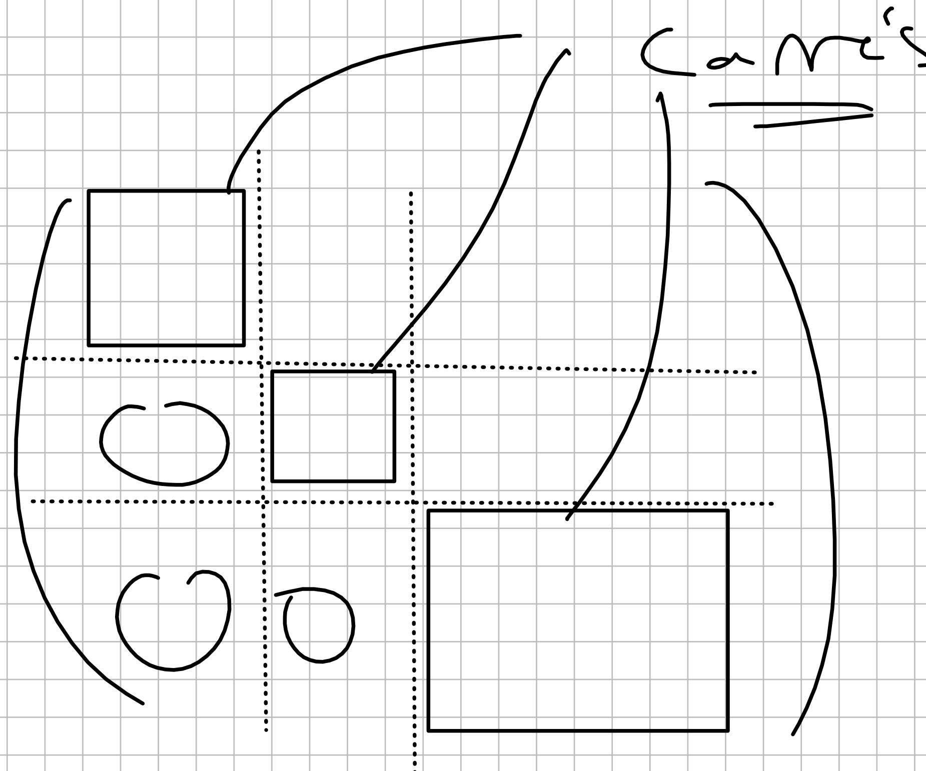
⚠ $\underbrace{\left(\begin{array}{ccc|c} \times & \times & \times & \times \\ \hline \times & \times & \times & \times \\ \times & \times & \times & \times \end{array}\right)}_{A} \times \underbrace{\left(\begin{array}{cc} \times & \times \\ \times & \times \\ \hline \times & \times \\ \times & \times \end{array}\right)}_{B}$: AB existe, le produit par blocs n'est pas possible.







Voc :



cette matrice est triang sup par blocs.

Ex :

$$\left(\begin{array}{ccc|cc} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 1 & 1 \\ 1 & 2 & -1 & 1 & 1 \\ \hline 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 2 & 2 \end{array}\right)$$

est triang par blocs !

Matrices diag. par blocs : idem.

→ le produit de 2 matrices triang sup. par blocs est triang sup par blocs.

(idem avec diag).

→ 1 matrice triang par blocs est inv. ssi tous les blocs de la diag. st inv.

→ Si 1 matrice triang sup. par blocs est inv., son inverse est triang. sup. par blocs.

$$A = \begin{array}{c} \begin{array}{cc} \quad q \quad & \quad r \quad \end{array} \\ \begin{pmatrix} A_{11} & \boxed{A_{12}} \\ A_{21} & A_{22} \end{pmatrix} \end{array} \begin{array}{l} \}\, n \quad \}\, \mathcal{C}_1 \\ \}\, \ell \end{array}$$

$u(\mathcal{B}_2)$

$$A = \mathrm{Mat}_{\mathcal{B},\mathcal{C}}(u)$$

$$\mathcal{B} = \underbrace{\mathcal{B}_1}_{q \text{ vect}} \uplus \underbrace{\mathcal{B}_2}_{r \text{ vect}} \quad ; \quad \mathcal{C} = \underbrace{\mathcal{C}_1}_{n \text{ vect}} \uplus \underbrace{\mathcal{C}_2}_{\ell \text{ vect}}$$

alors : $A_{ij} = \mathrm{Mat}_{\mathcal{B}_j,\mathcal{C}_i}(u_{ij})$

??

quel est le lien entre u et u_{ij} ??

$$U_{ij} = u\Big|_{Vect(\mathcal{B}_j)}^{Vect(\mathcal{C}_i)}$$

$$= \pi_i \circ u\Big|_{Vect(\mathcal{B}_j)}$$

où π_i est la projec° sur $Vect(\mathcal{C}_i)$

// à $Vect(\mathcal{C} \setminus \mathcal{C}_i)$

<u>Ex:</u> $A = \left(\begin{array}{cc|c} 1 & 1 & 3 \\ \hline 2 & -1 & 1 \end{array}\right) = Mat_{\mathcal{B},\mathcal{C}}(u)$

$$u\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \boxed{x+y}+3z \\ 2x-y+z \end{pmatrix}$$

$$u_{11} : \begin{pmatrix} x \\ y \end{pmatrix} \mapsto (x+y) : u\Big|_{\mathrm{Vect}(e_1, e_2)}^{\mathrm{Vect}(f_1)}$$

$$u_{12} : (z) \mapsto 3z$$

$$u_{21} : \begin{pmatrix} x \\ y \end{pmatrix} \mapsto (2x - y)$$

$$u_{22} : (z) \mapsto z$$