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Bachelor's Thesis

FINITE AUTOMATA IPAD EDITOR

Marek Fořt

Czech Technical University, Faculty of Information Technology Department of Theoretical Informatics Supervisor: Ing. Jiří Trávníček, Ph.D. April 20, 2021

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Abstract

Write abstract here!

Keywords iPad application, finite automata, interactive editor, AlgorithmsLibrary Toolkit, Composable Architecture, Swift

Chapter 1

Introduction

1.1 Motivation, Focus of Thesis

Theory of finite automata is an important part of computer science curriculum at FIT CTU in Prague and other universities around the world. And although there is a lot of resources one can learn from, there is a lack of those that utilize modern tools. One of such modern tools is iPad (and touch devices in general). This thesis will fill in this gap as the result will be a finite automata editor application for iPad.

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Furthermore, I will expand on the recent work done at FIT CTU in Prague concerning development of algorithms library and, more importantly for this thesis, finite automata algorithms including simulating input. This library is named Algorithms Library Toolkit (ALT) [1] and it has been open sourced.

The main motivation of this thesis is to improve how students learn finite automata and more specifically, enhance the current course BI-AAG that is taught at FIT CTU in Prague. It is also an opportunity to try out algorithms library in practice and create a concrete example of how it can be leveraged.

1.2 Thesis Goals

The main goal of this thesis is to implement a prototype of an automata editor for iPad. This application should enable users to create and edit finite automata with emphasis on touch-based input. I will also study the web interface of ALT [2] [3], ALT itself focusing on design and drawing of finite automata, the possibilities of strokes detection on touch devices, and the approaches of shape detection, especially those used in automata drawing on iOS platform.

After the initial study of current approaches and theory, I will implement a prototype of a finite automata editor iPad app that will be capable of recognizing finite automaton elements from strokes and simulating input.

I will then conduct usability testing to assess the usability and shortcomings of the prototype.

1.3 Thesis Structure

Let me now introduce you to the structure of the rest of the thesis:

■ In **Chapter 2** I will go over the theoretical concepts to properly explain terms and concepts on which it will be built upon later.

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■ Chapter 3 is concerned with the analysis of already existing solutions of creating automata editor, the existing ALT web interface and ALT itself.

- **Chapter 4** is about the design of the editor itself.
- In **Chapter 5** I will write about the implementation.
- Chapter 6 will go into the specifics of user testing and its outcomes.
- Conclusion is the last chapter of this thesis where I will assess the success of fulfilling aforementioned goals and lay out possible future development.

Chapter 2

Theory

Firstly, I will need to define terms and formal definitions concerning mainly finite automata theory, as that is the main subject of this thesis, and then machine learning as some of its concepts were important during the implementation.

2.1 Formal Languages and Grammars

The following definitions are taken from Automata and Grammars by Eliška Šestáková [4], Introduction to Automata Theory, Languages, and Computation [5], and materials from BIE-AAG course [6].

2.1.1 Formal Languages

▶ **Definition 2.1.** Alphabet (conventionally denoted by Σ) is a finite set whose elements are called symbols.

Alphabets therefore can be:

- $\Sigma = \{0, 1\}$
- $\Sigma = \{a, b, c, d, e\}$
- $\Sigma = \{\text{one, two}\}\$
- ▶ Definition 2.2. String (word) over an alphabet is a finite sequence of symbols from that alphabet.
- \bullet ϵ empty string (string with zero occurrences of symbols)
- Σ^* set of all strings over Σ
- Σ^+ set of all nonempty strings over Σ

For a binary alphabet $\Sigma = \{0, 1\} \epsilon$, 1001, 100, 1, 001 are all strings over the alphabet Σ .

▶ **Definition 2.3.** Formal language L over an alphabet Σ is any subset of all the strings over Σ - i.e., $L \subseteq \Sigma$

For a binary alphabet $\Sigma = \{0, 1\}$ a formal language over Σ is then subsets of *all* binary strings. We can denote the language either by:

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• enumeration notation where all possible strings in the language are listed, e.g.: $L_1 = \{\epsilon\}$, $L_2 = \{1\}$, $L_3 = \{0,00,000,01\}$.

■ set-builder notation where the languages are described in the following way: $\{ w \mid \text{something about } w \}$. Examples are: $L_4 = \{ w \mid w \in 0, 1^* \land |w| \mod 2 = 0 \}$, $L_5 = \{ 0^n 1^n : n \in \mathbb{N}_0 \}$.

2.1.2 Grammar

Grammars are used to describe languages. Below you can find how they are defined:

- ▶ **Definition 2.4.** Grammar is a quadruple of $G = (N, \Sigma, P, S)$ where:
- \blacksquare N is a finite non-empty set of nonterminal symbols.
- Σ is a finite set of terminal symbols $(\Sigma \cap N = \emptyset)$. Note that $N \cap \Sigma = \emptyset$.
- P is a finite set of production rules, assuming the following form:

$$\alpha A\beta \to \gamma \ (\alpha, \beta, \gamma \in (N \cup \Sigma)^*)$$

The following is an example of a grammar that describes the language $L = \{01^n0 : n \in \mathbb{N}_0\}$: Grammar $G = (\{A, S\}, \{0, 1\}, P, S)$ where P:

- $-S \rightarrow 0A$
- $\blacksquare A \rightarrow 1A$
- $\blacksquare A \rightarrow 0$

2.1.3 Chomsky Classification of Grammars

Grammars are divided into four classes where they differ in their production rules.

- ▶ **Definition 2.5.** Let $G = (N, \Sigma, P, S)$. We say that G is:
- **1.** Unrestricted grammar (type 0), if every rule is in the form of: $\alpha A \beta \to \gamma \ (\alpha, \beta, \gamma \in (N \cup \Sigma)^*, A \in N)$
- **2.** Context-sensitive (type 1), if every rule is in the form of: $\gamma A\delta \to \gamma \alpha\delta \ (\gamma, \delta \in (N \cup \Sigma)^*, \ a \in (N \cup \Sigma)^+)$

or in the form of $S \to \epsilon$ if S is not present on the right hand side of any rule of a given grammar.

- **3.** Context-free grammar (type 2) if every rule is in the form of: $A \to \alpha \ (A \in \mathbb{N}, \alpha \in (\mathbb{N} \cup \Sigma)^*)$
- **4.** Regular grammar (type 3), if every rule is in the form of: $A \to a$ or $A \to aB$ ($a \in \Sigma, A, B \in N$)

or in the form of $S \to \epsilon$ if S is not present on the right hand side of any rule of a given grammar.

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2.1.4 Classification of Languages

Classification of languages, also known as the Chomsky hierarchy, has the following definition:

- ▶ **Definition 2.6.** We say that language is:
- 1. formal if it is a formal language but is neither regular, context-free, context-sensitive, nor recursive enumerable. These languages are not recognized by Turing machine.
- 2. recursively enumerable if and only if \exists unrestricted grammar which generates it
 - recognized by Turing machine
- 3. context-sensitive if and only if \exists context-sensitive grammar which generates it
 - recognized by linear bounded Turing machine
- **4.** context-free if and only if \exists context-free grammar which generates it
 - recognized by a nondeterministic pushdown automaton
- **5.** regular if and only if \exists regular grammar that generates it
 - recognized by finite automaton

In this thesis we will be mainly interested in regular languages / grammars since those are recognized by finite automata. Finite automata will be defined in the following section.

2.2 Finite Automata

The final editor app will be for finite automata, therefore they are very important for this thesis. Informally, a finite automaton is a model for simple computation. States, that serve as memory, and transitions together form a *control unit*. Along with a control unit, the finite automaton has a *read-only input tape*, which is divided into individual cells, and the *head* that scans the input tape as the automaton continuously reads it, cell by cell. Automaton starts in its initial state and with head pointing at the first cell. As the input is read, the head moves until it has read all of the input tape. If there is a missing transition for an input, the automaton does not accept the input. Otherwise, it accepts the input if it is in an end state at the end of the input.

Let's define a finite automaton formally:

- ▶ **Definition 2.7.** Finite automaton is a quintuple $M = (Q, \Sigma, \delta, q_0, F)$ where:
- Q is a finite non-empty set of states
- Σ is a finite input alphabet
- ullet δ is the transition function (the exact definition is determined by which type of finite automaton it is see below)
- $= q_0 \in Q$ is the initial state
- $lue{F} \subseteq Q$ is the set of initial states

Finite automaton can also be either deterministic finite automaton or nondeterministic finite automaton. This dictates the exact definition of δ - transition function. For deterministic finite automaton (DFA) the definition of δ is:

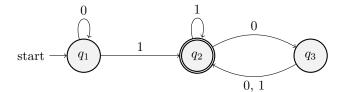
 δ is a mapping from $Q \times \Sigma$ to Q

 δ for nondeterministic finite automaton (NFA) is defined as:

 δ is a mapping from $Q \times \Sigma$ into the set of all subsets Q (denoted by 2^Q)

Expanding upon the difference between the definition of the transition function:

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■ Figure 2.1 FA graph representation

δ_{NFA}	0	1
$\rightarrow S$	S	S, A
A	B	
$\leftarrow B$		

Figure 2.2 FA table representation

- DFA can only transition from one state to another, e.g. from q_0 to q_1 ($q_0, q_1 \in Q$)
- NFA can transition to a set of states, e.g. from q_0 to q_1 $(q_0, q_1, q_2 \in Q)$

If we change the definition of NFA's δ to a mapping from $Q \times (\Sigma \cup \{\epsilon\})$ we allow, what are called, ϵ -transitions that allow us to move to a different state while not reading any input from the tape. This finite automaton is then called *nondeterministic finite automaton with* ϵ - transitions.

2.2.1 Representation of Finite Automata

Finite automata's transition functions δ are generally represented in the form of:

Formal notation (NFA) $\delta(S, 0) = \{S, A\}$ (transition from the state S and symbol 0 to the states S and A) (DFA) $\delta(A, 0) = B$ (transition from the state A and symbol 0 to a single state, not a set of states, B)

■ Weighted directed graph (state diagram)

Automata can be represented graphically as directed weighted graphs. Each state is represented as a vertice in the graph and final states are recognized by being a double circle, instead of a single one. Initial state is the one with an incoming edge. The transitions are

■ Table

in 2.1

Table representation has in the first column all states where initial state is marked with \rightarrow while final states is marked with \leftarrow . In the first row, excluding the first column, there are symbols of the alphabet, Σ . In the rest of the rows are states (or a set of states) that will be transitioned to on a given input (defined in the first row). You can see an example of it in 2.2.

then directed edges between states. You can see FA represented as weighted directed graph

In this thesis we will mostly be working with the representation in form of weighted directed graph as that is what will the user edit in the app. This also concludes theory about finite automata and formal languages.

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2.3 Machine Learning

Machine learning does not have an exact definition but e.g. in a book Foundations of Machine Learning it's loosely defined as "computational methods using experience to improve performance or to make accurate predictions" [7]. Experience means something we know from the past that we can leverage for making predictions in the future. Usually, this experience comes in the form of data. The book Foundations of Machine Learning [7] and materials from BIE-VZD from FIT CTU in Prague [8] will be used further in this section to define terms and concepts necessary for this thesis.

2.3.1 Classification

Machine learning, in order to cluster problems that can be solved in a similar way, have defined a few learning scenarios, most notably supervised and unsupervised learning. Learning scenario is a basic description of what type of data we have, how we receive the data and the test data that we use to evaluate the learning algorithm.

- supervised learning: Our goal is to explain variable Y given independent variables $X_0, X_1, ..., X_{p-1}$. We do this by finding a "function" for which most of its examples the following holds: $Y \approx f(X_0, X_1, ..., X_{p-1})$
- unsupervised learning: Our goal is to find structures of "similar" data. We do not predict any class and there is no clear way to assess the quality of an unsupervised learning algorithm since it is not clearly defined what the end result should be.

In this thesis we will be only interested in the supervised learning. We can also divide common problems that machine learning is trying to solve by learning tasks - that includes classification, regression, ranking, clustering, etc. Let's look more closely at classification which will be later used in the implementation.

■ Classification is a problem of assigning a category to each item.

It is also a problem solved via supervised learning. To expand on the definition of supervised learning from above, classification is a special case where Y has only a few (countable amount) of values. The simplest example of classification is binary classification. E.g. we want to predict whether a patient has flu and our data - gender of a patient, person can leave the bed - can be represented in a binary format (yes/no).

Chapter 3

Analysis

In the analysis I will study the following:

- existing applications that enable users edit finite automata
- ALT itself, focusing on design and drawing of finite automata

possibilities of detection of strokes on touch devices.

3.1 Existing Applications

This section will be concerned with the study of existing applications - be it applications for mobile or web.

3.1.1 ALT Web Interface

ALT web interface has been built as a part of bachelor's thesis made by Michael Vrána [3] leveraging work already done in ALT itself. ALT web interface uses Pipe-and-Filter [9] architecture to easily combine input and ouputs of the individual algorithms that ALT offers which can be seen in figure 3.1. Apart from ALT algorithms it also includes finite automata editor done by Petr Svoboda [2].

This finite automata editor is called Statemaker and you can see a screenshot of how it looks in figure 3.2. To summarize its capabilities - users can:

- add states, as well as initial and final states
- add transitions between states
- edit transition string
- mark state as initial or final
- remove states and transitions
- import and export automaton in supported formats
- automatic positioning of transitions and states

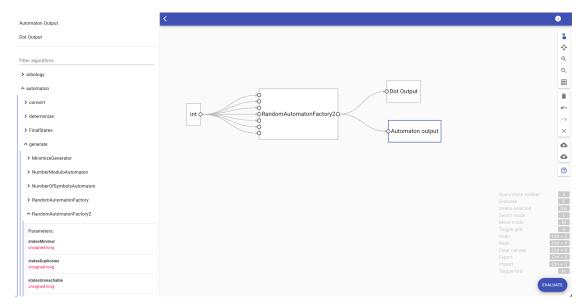


Figure 3.1 ALT web interface screenshot

All of the above features work reliably and are done in intuitive manner - user can quickly understand how to work with all the components. The most notable missing feature is easy simulation of input - this can be done via ALT web interface but if someone is looking for only editing FAs and simulating whether input string is accepted, they have to transition between two interfaces. The benefit is that they can then tap into all the other functionality that ALT offers. The author of Statemaker has chosen React and Typescript as underlying technologies [2].

3.1.2 Other Existing Applications

As the main goal of this thesis is to write a finite automata editor for iPad, in this part I will study existing applications mainly for touch devices.

One of such applications is TuringSim [10]. Although, it is not for FAs but for a Turing machine, it also consists of an editor where user can add and edit states and transitions, thus making it similar to a FA editor. You can see its interface in figure 3.3. This editors lets users add and edit automaton's states and transitions. Users can also simulate input on the Turing machine's read-and-write tape. Editing of the automaton is done only via tap gestures which is similar to Statemaker with the difference that there are no distinct buttons for those actions. Therefore, it does not fully utilize the potential of touch devices as the UX is very similar to what would one experience on the web. Unfortunately, the app on iPad is broken at the moment as it is missing bottom toolbar for simulating input.

There is also app called Finite Automata [11]. In this app user can not edit automata in their weighted graph representation but instead has to use a command line that takes individual command which are described in the app. This app does not utilize touch device features at all.

There are also apps available as desktop applications. One is a Finite Automaton Editor by Jaime Rangel-Mondragon that is available as interactive Wolfram notebook [12]. This app allows you to edit the automaton via a transition table and does not allow to simulate any input. There is also Automata Editor by Max Shawabkeh [13]. In this desktop applications users can create and edit their automaton either via a table representation or regular expression. There are also features such as NFA determinization, evaluating automata on strings, and minimizing DFA. Thus it has a powerful feature set but one has to be already familiar with FA theory. It should also be noted that the feature set is a subset of what ALT web interface offers.

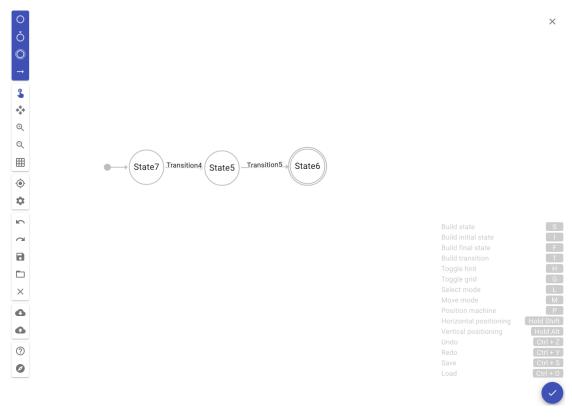
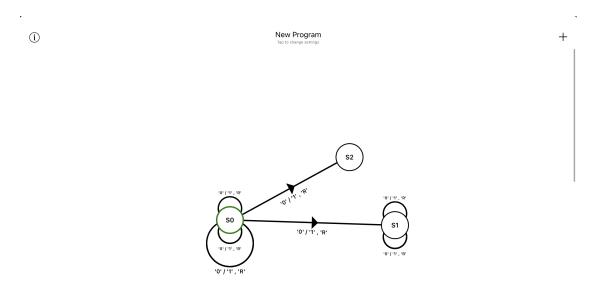


Figure 3.2 Statemaker screenshot



■ Figure 3.3 TuringSim interface screenshot

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3.2 **ALT**

Now I will go over ALT and its features that can be leveraged for simulating FA input. The code of ALT is available on GitLab [14] where there are multiple repositories in group Algorithms Library Toolkit (webui-client, infrastructe, etc.) - that includes repository Algorithms Library Toolkit Core [15], a library written in C++ [16]. There we can find algorithms that can later be used for FA editor. The code is divided into multiple modules that are then built and linked together using CMake [17]. The most important module for this thesis are alib2data and alib2algo where alib2data contains FA models and alib2algo algorithms for simulating input.

3.2.1 FA Model

Multiple FA types are supported by ALT - that includes deterministic and nondeterministic finite automaton as well as nondeterministic finite automaton with $\epsilon-transitions$. There is also extended NFA that has regular expressions as their transitions. These models serve as a definition of an automaton - its states, transitions, etc. To create e.g. NFA with $\epsilon-transitions$ one can use its constructor where it is possible to specify its states and input alphabet. For adding transitions there is a method called addTransition. Both can be seen in figure 3.1.

Code snippet 3.1 EpsilonNFA example methods

```
// Creates a new instance of the Automaton
// with a concrete initial state.
explicit EpsilonNFA (
    ext::set < StateType > states,
    ext::set < SymbolType > inputAlphabet,
    StateType initialState,
    ext::set < StateType > finalStates
);
// Add a transition to the automaton.
bool addTransition (
    StateType from,
    common::symbol_or_epsilon < SymbolType > input,
    StateType to
);
```

3.2.2 FA Algorithms

ALT offers multitude of algorithms that can be run on finite automata - such as minimization, determinization, and simulating input. Simulating input can be found in Accept.h and Run.h. Where the former is able to determine whether an input is accepted and the latter does the same but a part of its output is also e.g. in which states did the simulation end in.

3.3 Strokes Recognition

The final prototype will include recognizing automaton elements from drawing. In this section I will go over available methods of how to achieve it.

3.3.1 Google ML Kit

Google offers a framework called ML Kit that includes what they call "Digital Ink Recognition". This lets you construct a stroke from points drawn on the screen and create Ink object from

Strokes Recognition 13

those strokes. It also includes base models for recognizing text and even some basic shapes like arrow and rectangle. If you want to create your own model with TensorFlow Lite [18], you are forced to use "Image Labeling". Since the editor should support creating cycles, it is necessary to create a custom model because that shape is not supported by any of the base models for Digital Ink Recognition.

3.3.2 Core ML

Apple's CoreML framework supports variety of use cases - analysis of images, processing text, converting audio to text, and identifying sounds in audio [19]. It does not, however, support anything like Digital Ink Recognition. For the editor it is suitable to use analysis of images because it is possible to create an image from the screen and pass that to the model. Apple also provides some models already in CoreML format [20]. Not any of them are applicable for the FA editor's use case. Therefore, a custom model for CoreML would be necessary as well.

Considering that CoreML is bundled in the system and Google ML Kit needs to be installed separately, increasing app's size and incurring maintenance burden, I opted for CoreML. This decision was also made based on the fact that both frameworks do support TensorFlow, although, for CoreML it needs to be first converted to its format.

3.3.3 Creation of CoreML Model

There are multiple ways how to create CoreML model, though, they generally fall into two categories:

- ML model created by ML libraries that are not from Apple such as TensorFlow or Keras [21] and then converted with coremltools [22].
- ML model created by framework or application that outputs CoreML directly.

For creating CoreML models directly there is either Create ML [23] or turicreate [24]. Create ML, at the time of writing, supports only image classification, whereas turicreate has built-in support for drawing classification. Although, both image classification and drawing classification operate on images, the important distinction is that drawing classification 28x28 grayscale bitmap as input. The drawing classification is also tailored for inputs created by Apple Pencil [25], thus I have chosen to use it instead of libraries such as TensorFlow. It should be noted, though, that turicreate leverages TensorFlow as a lower-level framework and it should serve to streamline development of CoreML models.

Chapter 4

Automata Editor Design

In this chapter I will go over some decisions made, such as which technology I have decided to use, and over the design of the editor - how the app will look and how users will interact with the editor.

4.1 Touch Device

The main reason why this prototype is meant for a touch device is to simulate as much as possible the experience of drawing FAs on a paper. There were three main options that were possible:

- create touch-friendly web interface
- implement app for Android
- target iPad devices

Creating a touch-friendly web interface would have the benefit of being universal and not tied to a specific platform. But native apps offer better precision and developers can tap into OS APIs that are tailored for touch. The choice between Android and iPad was less clear but iPad has the benefit of Apple Pencil [26] that offers high precision that will make the user experience better.

4.2 Used Technologies

I will now cover what technologies are used in the app and why I have chosen them. A lot of decisions have been influenced by focusing on iPad, and even more specifically, usage of the app on iPad along with Apple Pencil.

4.2.1 Language

Choosing a language in which one will write the application is an important first step. For iOS applications I could have used:

- Objective-C
- Swift
- cross-platform framework

Objective-C was designed by Brad J. Cox at the start of 1980s and was then licensed by NeXT Software in 1988. Then in 1996 NeXT Software was acquired by Apple - along with Objective-C. Apple have then chosen Objective-C as a main language for OS X and in 2007 for the new operating system iOS [27]. On the Apple developer website it is described as "superset of the C programming language and provides object-oriented capabilities and a dynamic runtime" [28]. Objective-C has been thus the main programming language for years. Nowadays, Objective-C is not anymore that popular and ranks at the 23rd place as per TIOBE index [29]. Apple has rather shifted their focus to Swift and some of the new frameworks, like SwiftUI [30], are only available in Swift. Swift is thus a much better option to choose if one is starting a new iOS app.

Using a cross-platform framework - such as React Native [31] or Flutter [32] - was also a possibility. But to leverage Apple Pencil fully it was necessary to use PencilKit [33] and for this one would have to write native code. Therefore, I have decided to use Swift as the main language.

4.2.2 UI Framework

At the time of writing it is possible to use two UI frameworks offered by Apple to write UI code. Those are UIKit [34] or, already mentioned, SwiftUI [30]. SwiftUI is a newer framework than UIKit, release in 2019 [35]. In Thinking in SwiftUI book it is described as a "radical departure from UIKit, AppKit, and other object-oriented UI frameworks" [36]. SwiftUI offers a more declarative approach, quite similar to React [37] used in the web development. Declarative UIs have the benefits of less code since it enables the framework to do more on behalf of the developers. This comes at a cost of lesser control. Getting back to SwiftUI, specifically, one of its major drawbacks is that not all components, that are written in UIKit, are available in SwiftUI. But there is very strong support for SwiftUI-UIKit interoperability [38] and thus it is always possible to use UIKit when necessary. The fact that SwiftUI offers faster development due to its declarative nature and also SwiftUI previews [39] has made it a better candidate than UIKit, especially for a prototype. Therefore, I decided to use SwiftUI as the main UI framework.

4.2.3 Architecture

There is no recommended architecture by Apple for apps written on top of SwiftUI frameworks. It is also entirely possible to create an app without adhering to any architecture. This code, though, is more difficult to maintain for a longer period.

One possible architecture is MVVM [40]. Diagram of this architecture can be seen in figure 4.1. It enables developers to have a clear boundary between UI code and business logic and is a good option for either UIKit or SwiftUI applications. One of its drawbacks is that it can sometimes lead to imperative code where developers call a function and act based on its inputs. The alternative are (among others) architectures inspired by Redux. Diagram of Redux is depicted in figure 4.2. The main difference between Redux and MVVM is that MVVM is event-driven whereas Redux is data-driven. Data-driven approach is much more closer to declarative programming since the state of the application describes how it should look. Thus I have decided that Redux-like architecture will be a better option.

One of concrete implementations of Redux architecture is the Composable Architecture by Point-Free [41]. This architecture is based on Redux but it has some modifications such as handling of side effects. It also makes testing more exhaustive by asserting that no action that you do not expect is run as well as that the state is not changed in any other way than you describe in your test. Considering all of the points given, I have chosen to use the Composable Architecture.

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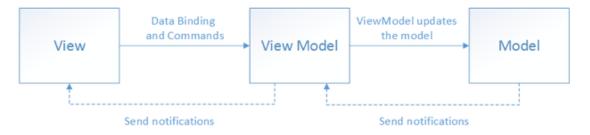


Figure 4.1 MVVM architecture diagram [40]

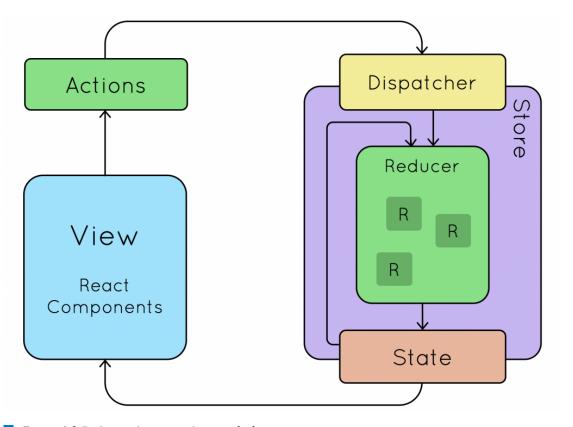


Figure 4.2 Redux architecture diagram [42]

4.3 ALT Integration

I have already talked about ALT in chapter 3. I have not discussed there, however, how ALT will be integrated in the application. That is now possible as I have stated that the app will be written in Swift. As already mentioned, ALT is a library written in C++. There exists a Swift-C++ interoperability manifesto [43]. This manifesto goes over what it would take to make C++ and Swift interoperable but not even all functionalities of C++ have their discussions of how they could be ported to Swift. However, there is a well-supported interoperability between Swift and Objective-C [44]. For Objective-C and C++ interoperability there is a language iteration of Objective-C called Objective-C++. It is even e.g. possible to "include pointers to Objective-C objects as data members of C++ classes" [45]. I first tried to integrate ALT directly and compile it right via Xcode but due to the fact that ALT is built via CMake [17] and does not have a simple setup, I then resorted to pre-building and then bundle the already built frameworks in the application. I will go over the details in chapter 5.

4.4 User Interface

As a final section of this chapter I will go over the design of UI. The design has been heavily influenced by the fact that one of the main goals was to imitate the experience of drawing FAs on a piece of paper. The app should let users to:

- create FA states, transitions, and cycles
- delete and rearrange all of the above
- name states
- specify symbols for transitions
- simulate input and see whether the input was or was not accepted by the automaton

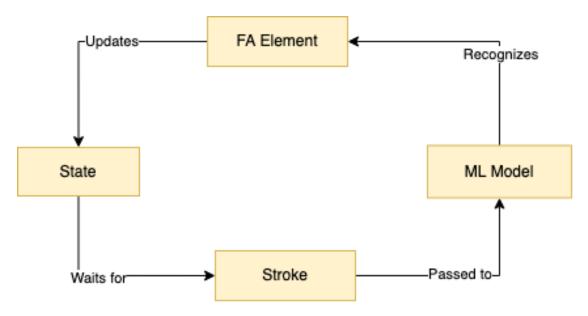
4.4.1 **Canvas**

Canvas is the most important part of the editor since it is the space where user can draw FA elements. After each individual stroke, a function will be run that will evaluate the stroke to determine which FA state the user has drawn. The flow of recognizing the FA elements is graphically represented in figure 4.3. The app first waits for the user to make a stroke, after a stroke is made its representation is sent to the ML model which recognizes the type of FA element, a state is updated with the new element and it is drawn on the canvas. Then the app again waits for another stroke.

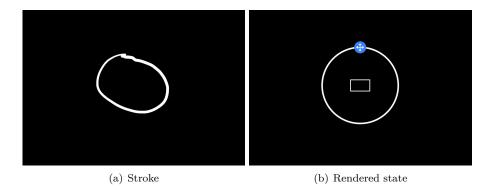
4.4.2 **State**

The app needs to be able to work with three FA elements - state, transitions and a cycle (a special case of transition that starts and ends in the same state). These elements should be represented the same way as they are in the weighted directed graph representation. That means a state will be rendered as a circle. But it is necessary to also enable user to edit the name of the state. Thus, a text field in the center of the state will be shown. You can see example of a stroke that should be rendered as a state and how it looks like after being recognized in figure 4.4. Notice also a button at the top of the circle - this button is for dragging the state. To indicate that the state is final, user should be able to draw another circle where the stroke contains the center of the state that should be final. In figure 4.5 you should see examples of strokes that should be then rendered as a final state.

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■ Figure 4.3 Flow of recognizing FA elements from strokes



■ Figure 4.4 Example of a state stroke (a) and how it is rendered (b)

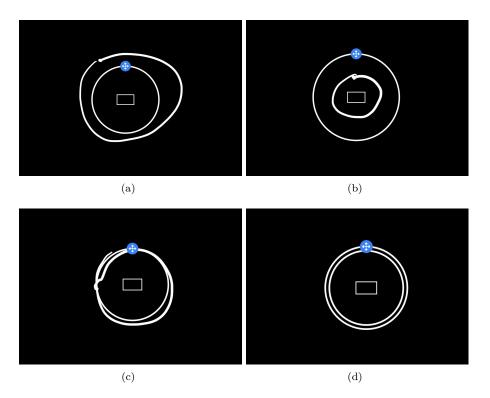


Figure 4.5 (a), (b), (c) are example strokes that should be rendered as a final state (d)

4.4.3 Transition

A transition is represented as a directed edge between vertices which is a shape of an arrow. The stroke can either be as an arrow or, more conveniently, just a straight line which is faster and easier to draw, especially when the shape must be drawn with a single stroke. Example strokes and a rendered transition are in figure 4.6. Note that the transition can be drawn in no matter which direction. The transition has also a text field positioned above its middle point. Apart from this text field where users can write symbols that the transition should occur on there is also a button with a plus icon. This button allows users to add multiple symbols to a single transition. Leveraging a delimiter, such as comma, was also considered but that could inhibit discoverability. Users can easily remove the transition symbols by tapping a cross symbol beside the symbol. To enable drawing FA with $\epsilon-transitions$ there is also a button with ϵ . When it is tapped, it is added as another symbol for that particulat transitions. A transition with multiple symbols and with $\epsilon-transition$ are in figure 4.7. Similar to a state there is a drag button to drag the middle point of a transition. This is especially useful when having multiple states on the same horizontal line with a transition going from the leftmost to the rightmost state.

4.4.4 Cycle

Cycle must be handled differently than a normal transition due to its shape - that means that it must be recognized as a different class in ML model. The business logic tied to a cycle will not be the same, too. Alike a regular transition, it can be drawn in whichever orientation. The ML model should also support multiple variants of a cycle shape to accommodate most of the cycle strokes drawn by users. It should, however, have the same text field, button for ϵ - transition, as well as multiple transition symbols as a regular transition. The possible strokes and how the

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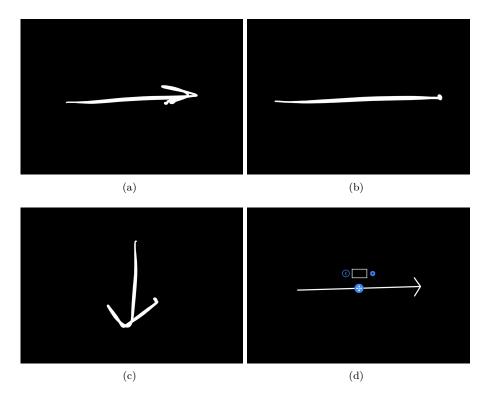


Figure 4.6 (a), (b), (c) are example strokes that should be rendered as a transition (d)

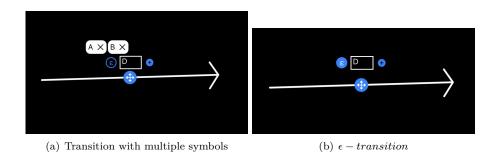


Figure 4.7 Transitions with symbols

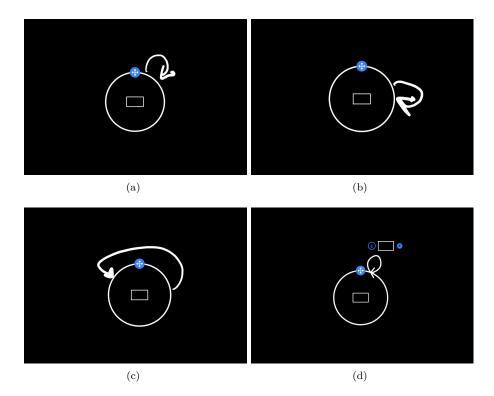


Figure 4.8 (a), (b), (c) are example strokes that should be rendered as a a cycle (d)

cycle should then look like are in figure 4.8.

4.4.5 Connecting Transitions and States

So far, I have only described how transitions and states will be recognized in isolation. In order to create a FA, though, it is necessary to create valid connections between transitions and states. There are multiple cases that the app should support. Similarly to Statemaker [2] it should allow users to create a transition between two existing states. But because the app strives to immitate the experience of using a simple paper, it should be less restrictive and enable creating a transition:

- without any state
- without a start state
- without an end state
- with both start and end states

For transitions that do not have states on both sides it must be possible to connect a new state to their end. E.g. if user draws a state close to a transition's end and it has no state there, it should connect it and make the transition an incoming edge of that state. Note that if user draws a transition without a start state, it is a valid transition as it will be interpreted as the marking of an initial state.

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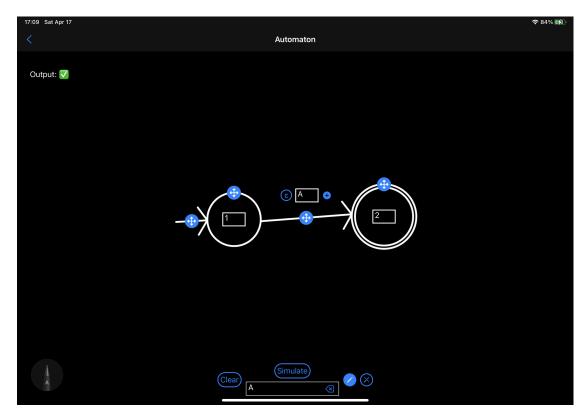


Figure 4.9 Accepted input

4.4.6 Simulating Input

The only missing functionality that I have not yet touched upon is simulating input. Considering that a FA is drawn, user should be able to write their desired input into a text field and see whether the input string has or has not been accepted. That can be done via tapping a button with a title "Simulate". If FA accepts the input, the output is denoted with a checkmark emoji (4.9) and if not, it is denoted with cross emoji (4.10). For convenience there is a button to erase the last character.

4.4.7 Erasing

As you might have noticed in 4.9 and 4.10 there are two buttons right of the input text field. The left is for drawing mode and the right one lets users to erase specific elements. If a user made a stroke over an element in the eraser mode, it would be deleted. On the left side there is a "Clear" button that clears the whole canvas whenever it is tapped.

4.4.8 Text Fields

One thing I have purposefully omitted is how the user will interact with the app's text fields. There are two options:

- using keyboard
- via Scribble feature

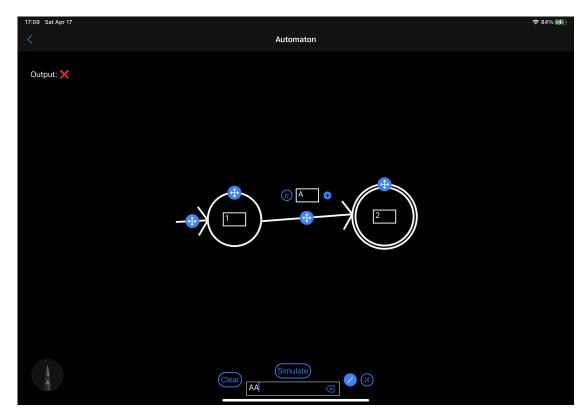


Figure 4.10 Rejected input

Using keyboard is the standard way of input for text fields. But this breaks the flow of using the Apple Pencil and does not fulfill the goal of immitating real life experience. Fortunately, from iOS 14 it is possible to use Apple Pencil for writing in text fields [46]. This means that it was not necessary to use a custom ML model in order to achieve the same effect. It does have a drawback and that is the Scribble feature is only available from iOS 14.0 and only if the device's language is in English or Traditional and Simplified Chinese. For other scenarios, users can use a keyboard as a fallback.

4.4.9 Document Based Apps

Users should also be able to save their automata and even share them with other people. For this I have leveraged the benefits of the Composable Architecture and saved the whole state of editor to JSON. Then this JSON could be saved as a document in what Apple calls document based apps [47]. In document based apps it is also possible to save changes done by the user as they happen, ensuring no progress is ever lost.

Chapter 5

Implementation

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I have laid out a basic overview of how the app is designed to look and work. In this chapter I shall go into details of how the most important parts of the app have been implemented.

. . . .

5.1 ML Model

ML model for recognizing FA elements is a core part of the app. I will go over how it has been implemented and integrated.

5.1.1 Creating ML Model

For creating a ML model I have chosen to use turicreate [24] as discussed in chapter 3. The user guide [25] describes how to create a ML model for drawing classification using QuickDraw dataset [48]. This dataset has millions of drawings of various objects, shapes. The shapes do not include, however, arrow and what would resemble a cycle. Therefore, I had to create a custom dataset. To make the ML model as accurate as possible I used the same input method for creating the dataset as will be used for drawing the elements in the prototype. For this I created an MNIST Maker app [49]. This app lets you to draw shapes with either a finger or Apple Pencil, scales it down to the desired size (I have used 28x28), and converts it to grayscale to make the image as small as possible and it is also what is used when creating the ML Model.

Once I have created a dataset, I could train the ML model. I have used the guide from turicreate and made some modifications like working with grayscale images exclusively (originally, the guide works with RGB images) and specifying which classes to work with. To make iterating on the model easy and quick I have used a Jupyter notebook [50]. The final Jupyter notebook with the data used is available on Github. Once an ML model is trained, one can use turicreate's visualization to see how the ML model worked on the test dataset. In figure 5.1 you can see the visualization. From the results it can be seen that the model did not make a mistake in recognizing an arrow but it had mistaken a cycle for a circle and vice versa. This is due to the fact that circle and cycle do have a similar shape.

The final product, once all the commands in the Jupyter notebook are run, is a file *AutomataClassifier.mlmodel*. The extension hints at the fact that it is a ML model that can be used by CoreML framework.

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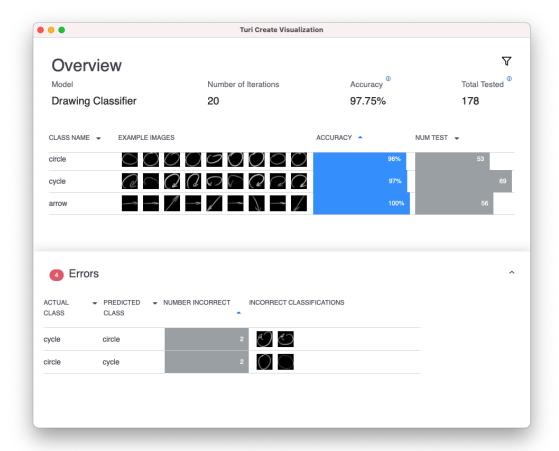


Figure 5.1 Test dataset ML model prediction results in turicreate

5.1.2 Integration of ML Model

To integrate the ML model one has to simply drag and drop the file into Xcode [51], Apple's IDE, to create a reference in the project file. This will automatically bundle the model with the app and synthesize Swift code that can be later leveraged to interact with the model. The code for recognizing a stroke is located in AutomataClassifierService and the main logic is in a code snippet 5.1.

■ Code snippet 5.1 Automata classifier

```
// Convert stroke to cgImage
// ...
let input = try AutomataClassifierInput(
    drawingWith: cgImage
)
let classifier = try AutomataClassifier(
    configuration: MLModelConfiguration()
)
let prediction = try classifier.prediction(input: input)
guard
```

5.2 Drawing FA Elements

Before FA elements can be recognized, they also must be drawn by the user, converted to a model that can be processed by the AutomataClassifierService and then re-drawn with a more exact shape once it is known which element the user has made.

5.2.1 Canvas

To enable drawing with Apple Pencil Apple offers PencilKit [33]. One of the components this framework defines is a PKCanvasView - in the documentation described as a "view that captures Apple Pencil input and displays the rendered results in an iOS app" [52]. To use it in SwiftUI View I had to wrap it into a view I called CanvasView conforming to UIViewRepresentable since PKCanvasView is only available in UIKit. Instance of CanvasView was then added to EditorView that is the main view of the app.

5.2.2 Strokes

When drawing in PKCanvasView the individual strokes are represented with PKStroke. It contains all of the available information about the given stroke - such as its bounds, type of ink (PKInt), and path (PKStrokePath). In order not to use PencilKit directly and also to make it easier to work with, I wrapped PKStroke into a custom model Stroke that contains control points (array of CGPoint) of PKStroke. That is because PKStrokePath contains infinite amount of points. To get only a subset of them one can use a method interpolatedPoints(by: CGFloat). The most recent PKStroke is passed to AutomataClassifierService and from which an image is created as can be seen in 5.2 where modelImage() method simply converts the image from PKDrawing to grayscale and a desired size.

Code snippet 5.2 Automata classifier

```
let image = PKDrawing(strokes: [stroke.pkStroke()])
.image(
    from: stroke.pkStroke().renderBounds,
    scale: 1.0
)
.modelImage()
```

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5.2.3 Drawing State

If the ML classifier makes a prediction that the stroke being analyzed is a state, it is necessary to draw it more precisely than a user. State is represented as a circle, so to draw it the app needs to know its center and radius. To obtain a center I make an average of all the points available. Once the center is computed, radius is then calculated as an average of distances of all the points from the center. A new stroke can then be created as shown in 5.3.

It is also necessary to connect a state to an existing transition. There is a method for this called closestTransitionWithoutEndState that, as the name suggests, finds the closest transition that does not have an end state. If such a transition exists, a center is moved in the direction of the tip of the transition, so they an intersection point.

A final state is marked with a double circle. Therefore, if a stroke is a state, before making a new one I check whether a center of a different state is contained in a frame of the new state. If it is so, then the state is marked as final and a new circle around the previous one is made and the state is denoted in the internal state as a final one.

■ Code snippet 5.3 Circle stroke

5.2.4 Transition

To draw a transition I take its first and last point (tip point). At the tip point two additional lines must be made, so the final shape looks like an arrow. For this I needed vectors, so it works for whichever orientation. Vectors are not offered by any of the bundled frameworks, so I heavily inspired my implementation from a repository called VectorMath [53]. Once vectors were available, I was able to compute a perpendicular vector to the line from the start point to the tip point that has one common point with that line and that is a point on the transition's line in a pre-defined distance from the tip point. The bottom and top point are then on that vector with a pre-defined distance from the line as well. To add additional points top and bottom vectors are created by connecting top and bottom points with the tip point as in 5.4.

Similarly to states, it is necessary to handle the case when a new transition should be connected to an existing state. Firstly, the closest states to the start point and to the tip point are computed. Then the closer one is chosen as a start state or end state, respectively - if the distance is smaller than a given threshold.

Code snippet 5.4 Computation of top and bottom points, vectors

```
let vector = Vector(flexPoint ?? startPoint, tipPoint)
let anchorPoint = vector.point(
```

```
distance: -arrowSpan / 3,
   other: tipPoint
)
let perpendicularVector = vector.rotated(by: .pi / 2)
let topPoint = perpendicularVector.point(
   distance: -arrowSpan / 2,
   other: anchorPoint
)
let bottomPoint = perpendicularVector.point(
   distance: arrowSpan / 2,
   other: anchorPoint
)
let topVector = Vector(tipPoint, topPoint)
let bottomVector = Vector(tipPoint, bottomPoint)
```

5.2.5 Cycle

The last class that the ML model recognized and that has not been discussed how to draw is a cycle. A cycle is different from a state and a transition since it can not exist without being connected to a state. If there is no state to connect it to, the stroke is just erased. Otherwise, once the closest state is, the cycle's shape can be created. Its shape is more complicated than the previous - but it is possible to use PencilKit's behavior that smoothes out a curve based on the points given. Thus, to recreate it I use the point at the intersection with the state and the state's center in 5.5.

Code snippet 5.5 Cycle stroke

```
extension Array where Element == CGPoint {
    static func cycle(
        _ point: CGPoint,
        center: CGPoint
    ) -> Self {
        let vector = Vector(point, center)
        let topPoint = vector.point(distance: -70, other: point)
        let startToTopVector = Vector(point, topPoint)
        let finalPoint = startToTopVector
            .rotated(by: .pi * 0.4)
            .point(distance: 5, other: point)
        return [
            point,
            startToTopVector
                .rotated(by: -.pi / 3)
                .point(distance: 10, other: point),
            startToTopVector
                .rotated(by: -.pi / 4)
                .point(distance: 40, other: point),
            topPoint,
            startToTopVector
                .rotated(by: .pi / 4)
                .point(distance: 40, other: point),
            startToTopVector
                .rotated(by: .pi / 3)
                .point(distance: 10, other: point),
```

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5.3 ALT Integration

ALT integration has been discussed both in chapter 3 and chapter 4. In chapter 4

User Testing

Chapter 7

Conclusion

7.1 Goals Assessment

Assess my goals.

7.2 Thesis Contribution

The contribution of this thesis is testing Algorithms Library Toolkit in practice and can now be pointed to for users who want to see its capabilities. The editor can now also be recommended in the course of BI-AAG at FIT CTU (and other universities) - for students and teachers alike.

7.3 Future Work

Algorithms Library Toolkit is an extensive library and there are still capabilites that are not implemented in the editor.

The choice of developing a native iOS application has resulted in good UX, but in the future it would be beneficial to broaden the possible audience and either develop a similar Android app or create a more ubiquituous web interface.

Chapter 8

List of Abbreviations

- DFA Deterministic Finite Automaton
 - FA Finite Automaton
- NFA Nondeterministic Finite Automaton
- ALT Algorithms Library Toolkit
 - UI User Interface
- ML Machine Learning

Příloha A **Nějaká příloha**

Sem přijde to, co nepatří do hlavní části.

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Obsah přiloženého média

ı	readme.txt	stručný popis obsahu média
		adresář se spustitelnou formou implementace
ı	src	1
Ī	impl	zdrojové kódy implementace
I	thesis	zdrojové kódy implementace zdrojová forma práce ve formátu L ^A T _E X
		text práce
		text práce ve formátu PDF