

Collaboration Transcript

S_2^P Search Paper

AI-Assisted Academic Writing Session
Lance Fortnow

Models

Google Antigravity using Gemini 3 Pro. The italics are the author's inputs.

Initial Planning

Let's plan a paper showing that finding a S_2^P witness is equivalent to $TFNP^{NP}$. The paper's author is Lance Fortnow (me) and should be written in my style.

I've drafted a plan for the paper, outlining the argument that finding an S_2^P witness is equivalent to $TFNP^{NP}$. The plan is saved in c:/Users/lance/Dropbox/Papers/S2P Search/plan.md.

Plan Overview

The core argument rests on two directions:

- 1. S_2^P -Search $\in TFNP^{NP}$:** This is the straightforward direction. Verifying a witness (strategy) requires checking a universal quantifier ($\forall z P(x, y, z)$), which is a coNP query. Since S_2^P guarantees a witness always exists, the problem is total.
- 2. $TFNP^{NP} \subseteq S_2^P$ -Search:** This is the main technical contribution. We need to show that any total search problem with an NP-oracle verifier can be reduced to the game-theoretic setting of S_2^P . This likely involves encoding the P^{NP} verification circuit into the symmetric alternation structure.

Next Steps

- Review the Plan: Does the structure in plan.md look right to you?
- Start the Draft: I can initialize a paper.tex file with this structure and your standard preamble.
- Flesh out the Proofs: We can start sketching the reduction for the hard direction ($TFNP^{NP} \subseteq S_2^P$ -Search) in more detail.

Adding Citations and Corollaries

Don't forget to cite Cai's paper that S_2^P is in ZPP^{NP} . Add as a corollary that reducing search to decision for S_2^P would put $\Sigma_2^P \cap \Pi_2^P$ in ZPP^{NP} .

I've updated the plan to include the corollary and the citation for Jin-Yi Cai's paper.

Updates

Section 4: Added the corollary: "If search reduces to decision for S_2^P , then $\Sigma_2^P \cap \Pi_2^P \subseteq ZPP^{NP}$."

References: Added "Cai, J.-Y. (2001). $S_2^P \subseteq ZPP^{NP}$."

The plan is now updated in c:/Users/lance/Dropbox/Papers/S2P Search/plan.md. Ready to proceed with drafting the content whenever you are.

Mention the corollary in the abstract, and note the stark difference between search and decision.

I've updated the abstract in plan.md to include the corollary and emphasize the stark difference between the search and decision complexities of S_2^P , specifically referencing Cai's result. The plan is now fully updated with your requirements. Would you like to proceed with creating the LaTeX file for the paper?

Creating the LaTeX Document

Proceed.

I've created the paper.tex file in c:/Users/lance/Dropbox/Papers/S2P Search/ with the content we discussed. It includes the abstract, introduction, preliminaries, the equivalence proof sketch, the corollary regarding search-to-decision, and the references. You can now compile this LaTeX file to generate the PDF. Would you like me to try and compile it for you (if a LaTeX environment is available), or is there anything else you'd like to add to the paper first?

Technical Corrections

It's open whether $\Sigma_2^P \cap \Pi_2^P \subseteq S_2^P$. The point is that $\Sigma_2^P \cap \Pi_2^P$ can be reduced to $TFNP^{NP}$.

I've corrected the proof in paper.tex. It now correctly states that the search problem for $\Sigma_2^P \cap \Pi_2^P$ is in $TFNP^{NP}$, which (via our main result) connects it to S_2^P -Search and subsequently ZPP^{NP} . Is there anything else you'd like to refine in the paper?

Refining the Writing Style

Avoid statements like "it is known" and put in details instead.

I've expanded the proof in paper.tex to explicitly detail why the search problem for $\Sigma_2^P \cap \Pi_2^P$ falls into $TFNP^{NP}$, replacing the "it is known" statement. The proof now walks through the definitions of membership in Σ_2^P and Π_2^P , the existence of witnesses for either case, and the verification complexity using an NP oracle. Is there anything else you'd like to modify or add?

Similarly give the details of predicate Q.

I've updated the paper with the detailed construction of the predicate Q and the logic for the reduction. This should make the proof much more rigorous.

The paper now contains:

1. Equivalence of S_2^P -Search and $TFNP^{NP}$.

2. Detailed reduction construction.
3. Corollary connecting search-to-decision for S_2^P with $\Sigma_2^P \cap \Pi_2^P \subseteq ZPP^{NP}$.
4. Corrected proof for the corollary.

Is there anything else you'd like to refine?

Finalizing the Title

Change the title to "Search versus Decision for S_2^P " and adjust the abstract and introduction appropriately.

I have read paper.tex and understood the necessary changes. Now I am creating the implementation plan. I have created an implementation plan to update the paper's title, abstract, and introduction. Please review it. I have verified the changes in paper.tex. The title, abstract, and introduction have been correctly updated. Now I am creating the walkthrough artifact. I have updated the paper's title, abstract, and introduction as requested. You can review the changes in paper.tex.

Compiling to PDF

Can you convert the tex file to pdf?

I have successfully compiled paper.tex to paper.pdf. You can find the output at paper.pdf.

Updating Citations

Check Google Scholar that we have the latest versions of the papers in the citation.

I have checked the citations on Google Scholar. I found updated journal versions for Canetti (IPL 1996) and Cai (JCSS 2007). Russell-Sundaram (1998) and Megiddo-Papadimitriou (1991) are already correct. I found updated journal versions for Canetti and Cai. I have created a plan to update the citations. Please review it. I have verified the updated citations in paper.tex. They are correct. I have updated the citations for Canetti and Cai to their journal versions and recompiled the paper. You can check paper.pdf for the latest version.

Adding Acknowledgment

Add an acknowledgment section: "While the results are fully due to the author, this paper was generated using the large language model Gemini 3 Pro with prompting from the author. The author takes full responsibility for its contents."

I have added the acknowledgement section and recompiled the paper. You can check paper.pdf for the final version.