

# Collaboration Transcript

## S<sub>2</sub>P Search Paper

AI-Assisted Academic Writing Session  
Lance Fortnow

### Models

Google Antigravity using Gemini 3 Pro. The italics are the author's inputs.

### Initial Planning

*Let's plan a paper showing that finding a S<sub>2</sub>P witness is equivalent to TFNP<sup>NP</sup>. The paper's author is Lance Fortnow (me) and should be written in my style.*

I've drafted a plan for the paper, outlining the argument that finding an S<sub>2</sub>P witness is equivalent to TFNP<sup>NP</sup>. The plan is saved in c:/Users/lance/Dropbox/Papers/S2P Search/plan.md.

### Plan Overview

The core argument rests on two directions:

1. **S<sub>2</sub>P-Search ∈ TFNP<sup>NP</sup>:** This is the straightforward direction. Verifying a witness (strategy) requires checking a universal quantifier ( $\forall z P(x, y, z)$ ), which is a coNP query. Since S<sub>2</sub>P guarantees a witness always exists, the problem is total.
2. **TFNP<sup>NP</sup> ⊆ S<sub>2</sub>P-Search:** This is the main technical contribution. We need to show that any total search problem with an NP-oracle verifier can be reduced to the game-theoretic setting of S<sub>2</sub>P. This likely involves encoding the P<sup>NP</sup> verification circuit into the symmetric alternation structure.

### Next Steps

- Review the Plan: Does the structure in plan.md look right to you?
- Start the Draft: I can initialize a paper.tex file with this structure and your standard preamble.
- Flesh out the Proofs: We can start sketching the reduction for the hard direction (TFNP<sup>NP</sup> ⊆ S<sub>2</sub>P-Search) in more detail.

### Adding Citations and Corollaries

*Don't forget to cite Cai's paper that S<sub>2</sub>P is in ZPP<sup>NP</sup>. Add as a corollary that reducing search to decision for S<sub>2</sub>P would put  $\Sigma_2^P \cap \Pi_2^P$  in ZPP<sup>NP</sup>.*

I've updated the plan to include the corollary and the citation for Jin-Yi Cai's paper.

### Updates

**Section 4:** Added the corollary: "If search reduces to decision for  $S_2^P$ , then  $\Sigma_2^P \cap \Pi_2^P \subseteq ZPP^{NP}$ ."

**References:** Added "Cai, J.-Y. (2001).  $S_2^P \subseteq ZPP^{NP}$ ."

The plan is now updated in c:/Users/lance/Dropbox/Papers/S2P Search/plan.md. Ready to proceed with drafting the content whenever you are.

*Mention the corollary in the abstract, and note the stark difference between search and decision.*

I've updated the abstract in plan.md to include the corollary and emphasize the stark difference between the search and decision complexities of  $S_2^P$ , specifically referencing Cai's result. The plan is now fully updated with your requirements. Would you like to proceed with creating the LaTeX file for the paper?

## Creating the LaTeX Document

*Proceed.*

I've created the paper.tex file in c:/Users/lance/Dropbox/Papers/S2P Search/ with the content we discussed. It includes the abstract, introduction, preliminaries, the equivalence proof sketch, the corollary regarding search-to-decision, and the references. You can now compile this LaTeX file to generate the PDF. Would you like me to try and compile it for you (if a LaTeX environment is available), or is there anything else you'd like to add to the paper first?

## Technical Corrections

*It's open whether  $\Sigma_2^P \cap \Pi_2^P \subseteq S_2^P$ . The point is that  $\Sigma_2^P \cap \Pi_2^P$  can be reduced to  $TFNP^{NP}$ .*

I've corrected the proof in paper.tex. It now correctly states that the search problem for  $\Sigma_2^P \cap \Pi_2^P$  is in  $TFNP^{NP}$ , which (via our main result) connects it to  $S_2^P$ -Search and subsequently  $ZPP^{NP}$ . Is there anything else you'd like to refine in the paper?

## Refining the Writing Style

*Avoid statements like "it is known" and put in details instead.*

I've expanded the proof in paper.tex to explicitly detail why the search problem for  $\Sigma_2^P \cap \Pi_2^P$  falls into  $TFNP^{NP}$ , replacing the "it is known" statement. The proof now walks through the definitions of membership in  $\Sigma_2^P$  and  $\Pi_2^P$ , the existence of witnesses for either case, and the verification complexity using an NP oracle. Is there anything else you'd like to modify or add?

*Similarly give the details of predicate Q.*

I've updated the paper with the detailed construction of the predicate Q and the logic for the reduction. This should make the proof much more rigorous.

The paper now contains:

1. Equivalence of  $S_2^P$ -Search and  $TFNP^{NP}$ .

2. Detailed reduction construction.
3. Corollary connecting search-to-decision for  $S_2^P$  with  $\Sigma_2^P \cap \Pi_2^P \subseteq ZPP^{NP}$ .
4. Corrected proof for the corollary.

Is there anything else you'd like to refine?

## Finalizing the Title

*Change the title to "Search versus Decision for  $S_2^P$ " and adjust the abstract and introduction appropriately.*

I have read paper.tex and understood the necessary changes. Now I am creating the implementation plan. I have created an implementation plan to update the paper's title, abstract, and introduction. Please review it. I have verified the changes in paper.tex. The title, abstract, and introduction have been correctly updated. Now I am creating the walkthrough artifact. I have updated the paper's title, abstract, and introduction as requested. You can review the changes in paper.tex.

## Compiling to PDF

*Can you convert the tex file to pdf?*

I have successfully compiled paper.tex to paper.pdf. You can find the output at paper.pdf.

## Updating Citations

*Check Google Scholar that we have the latest versions of the papers in the citation.*

I have checked the citations on Google Scholar. I found updated journal versions for Canetti (IPL 1996) and Cai (JCSS 2007). Russell-Sundaram (1998) and Megiddo-Papadimitriou (1991) are already correct. I found updated journal versions for Canetti and Cai. I have created a plan to update the citations. Please review it. I have verified the updated citations in paper.tex. They are correct. I have updated the citations for Canetti and Cai to their journal versions and recompiled the paper. You can check paper.pdf for the latest version.

## Adding Acknowledgment

*Add an acknowledgment section: "While the results are fully due to the author, this paper was generated using the large language model Gemini 3 Pro with prompting from the author. The author takes full responsibility for its contents."*

I have added the acknowledgement section and recompiled the paper. You can check paper.pdf for the final version.