Extracting initial parameters from an existing Holt-Winter forecasting model

Fortune Walla

November 29th 2015

Objective

The model is known but the initial parameters need to be found.

Introduction

You are hired by a local company to help them improve their forecasting capabilities. You are tasked with coming up with quarterly forecasts for an item that appears to have level, seasonality, and trend. The good news is that the company has an existing Holt-Winter forecasting model. The bad news is that no one knows what the parameters (Alpha, Beta, or Gamma) are.

You do have some information. For example, you know that historically, the demand in each quarter follows this distribution:

- Q1 (January through March) = 50% of average quarterly demand
- Q2 (April through June) = 75% of average quarterly demand
- Q3 (July through September) = 150% of average quarterly demand
- Q4 (October through December) = 125% of average quarterly demand.

You just ran the forecast at the end of September (end of 2014Q3) and you have the following estimates:

For level: $\hat{a}_{2014Q3}$ = 1052 units

For trend: $\hat{b}_{2014Q3}$ = 46.2 units per quarter

Q1. What is the forecast for demand for 2014Q4?

We know that

$$\hat{x}_{t,t+\tau} = (\hat{a}_t + \tau\hat{b}_t)\hat{F}_{t+\tau-P}$$

$$\hat{a}_t = \alpha\left(\frac{x_t}{\hat{F}_{t-P}}\right) + (1-\alpha)(\hat{a}_{t-1} + \hat{b}_{t-1})$$

$$\hat{b}_t = \beta(\hat{a}_t - \hat{a}_{t-1}) + (1-\beta)\hat{b}_{t-1}$$

$$\hat{F}_t = \gamma\left(\frac{x_t}{\hat{a}_t}\right) + (1-\gamma)\hat{F}_{t-P}$$

	x	a^	b^	F^	x^	F^t-P
2014Q3		1052	46.2		1372.75	1.25 (2013Q4)
2014Q4						

Q2. Suppose the actual demand in 2014Q4 is 1100 units. What is the smallest & largest possible value for your estimate for level, $a^{\wedge}_{2014Q4}$?

The fourth quarter level estimate = $a^{\wedge}_{2014Q4} = x_{2014Q4} / F_{2013Q4}$

Without seasonality, level estimate = $a^{\wedge}_{2014Q4} = (a^{\wedge}_{2014Q3} + b^{\wedge}_{2014Q3})$

	Alpha		x	a^	b^	F^	x^	F^t-P
		2014Q3		1052	46.2		1372.75	1.25 (2013Q4)
Smallest	1	2014Q4	1100	880				
Largest	0	2014Q4	1100	1098.2				

Q3. The model was run at the end of 2014Q4. It provided you with the most recent estimates of each pattern. A) The estimate for level, $a^{\wedge}_{2014Q4}$ was 1065.5. What is value of alpha? B) Estimate of trend, $b^{\wedge}_{2014Q4}$ = 42.9, what is value of beta? C) Estimate of seasonality is $F^{\wedge}_{2014Q4}$ = 1.239

Estimate	**Given**	**Equation**	**Parameter**	**Solved**
$a^{\wedge}_{2014Q4}$	1065.5	$1065.5 = \alpha\left(\frac{1100}{1.25}\right) + (1-\alpha)(1052 + 46.2)$	Alpha	0.15
$b^{\wedge}_{2014Q4}$	42.9	$42.9 = \beta(1065.5 - 1052) + (1-\beta)46.2$	Beta	0.1
$F^{\wedge}_{2014Q4}$	1.239	$1.239 = \gamma\left(\frac{1100}{1065.5}\right) + (1-\gamma)1.25$	Gamma	0.05

Q4. What is your forecast for demand for the 1st quarter of 2015? That is $a^{\wedge}_{2014Q4,2015Q1}$?

$x^{\wedge}_{2014Q4,2015Q1}$ =($a^{\wedge}_{2014Q1}$ + $b^{\wedge}_{2014Q1}$) $F^{\wedge}_{2014Q4}$

We have the unnormalized seasonality factor, $F^{\wedge}_{2014Q4}$ = 1.239

Since the sum of the most recent season estimates (0.500, 0.750, 1.500, and 1.239 for Q1, Q2, Q3, and Q4) adds up to 3.98912, , we need to normalize $F^{\wedge}_{2014Q4}$ before we use it in our calculations.

We use the formula

$$F^{\wedge}_{iadj} = F^{\wedge}_{iold}\frac{P}{\sum F^{\wedge}_{i}}$$

So we have $F^{\wedge}_{2014Q1}$ = 0.500*(4.000/3.989) = 0.50136

$x^{\wedge}_{2014Q4,2015Q1}$ =($a^{\wedge}_{2014Q1}$ + $b^{\wedge}_{2014Q1}$) $F^{\wedge}_{2014Q1}$

$x^{\wedge}_{2014Q4,2015Q1}$ = (1065.5 + 42.9)(0.501) = 555.3084 = 555.31

. If you did not normalize the seasonality factor you would have gotten = (1065.5 + 42.9)(0.500) = 554.20. Normalizing the seasonality factors prevents the estimates from drifting. In this case, it is a small drift - but over time it would grow.