Principle of Automatic Control II
（自动控制原理II）

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学时：40学时

考试：闭卷

References

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Chapter 1. Introduction

Chapter 2. Mathematical Models of Linear Systems

Chapter 3. Analyses in Time Domain for Linear Systems

Chapter 4. Root Locus

Chapter 5. Analyses in Frequency Domain for Linear Systems

Chapter 6. Controller Design for Linear Systems

Chapter 7 Analysis and Design of Linear Discrete-Time System （Sampled-data System）

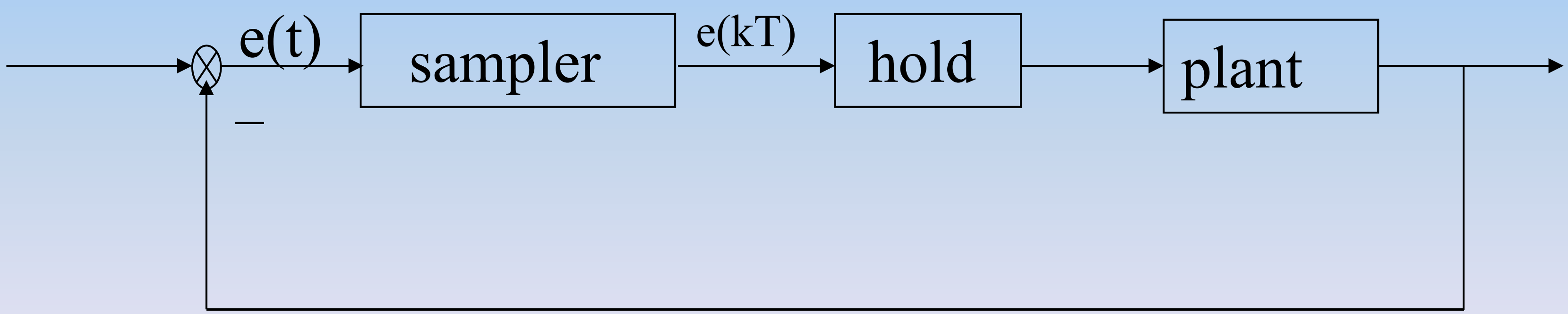
7.1 Introduction

Discrete-Time Systems:

Types:
- Sampling systems: Discrete Time, Continuous Value
- Digital systems: Discrete Time, Quantized Value

Digital System: There is one or more impulse series or digital signals in the system.

Sampled-Data System: a system that is continuous except for one or more sampling operations.



Sampled-data control system

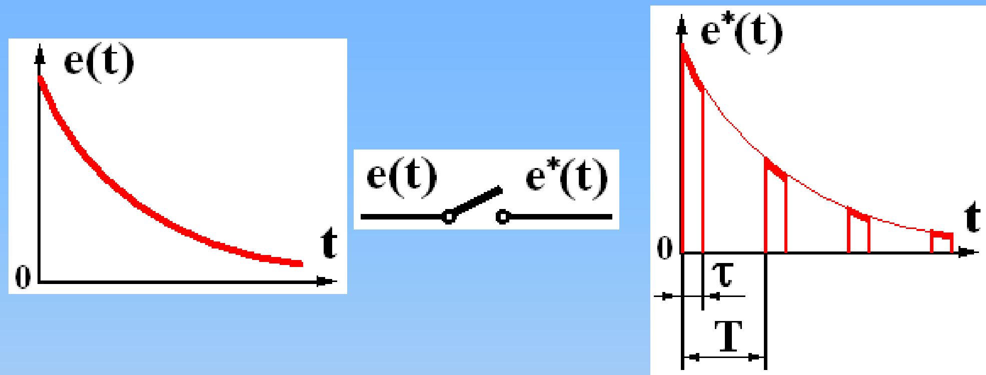
e(kT) is obtained by sampling a continuous signal e(t).

A/D : analog to digital converter

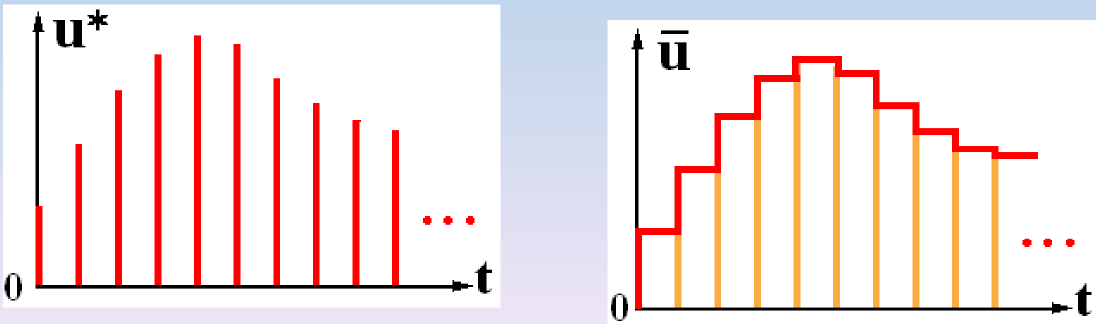
D/A : digital to analog converter

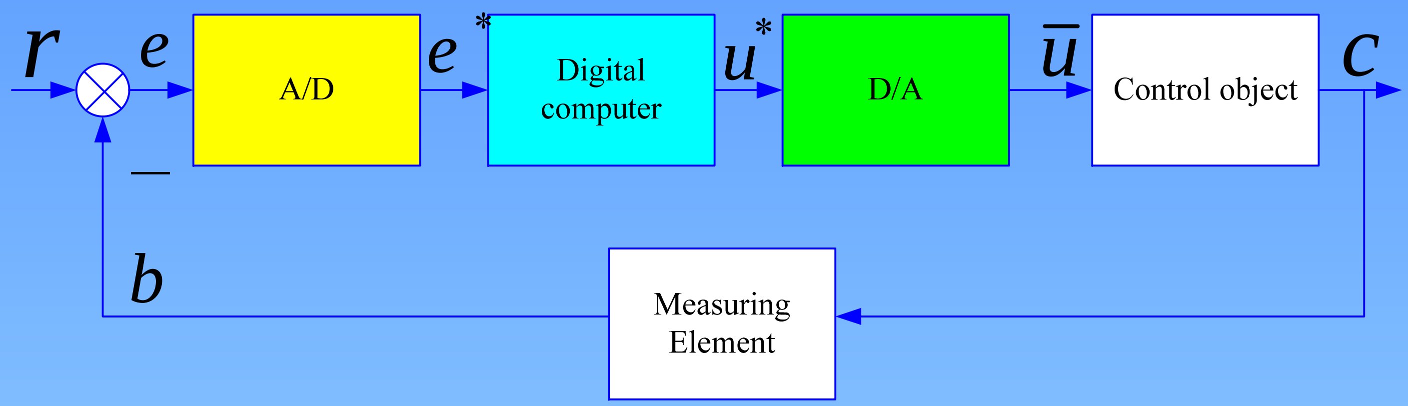
A/D process

- Sampling — Time sampled
- Quantization — Value quantized

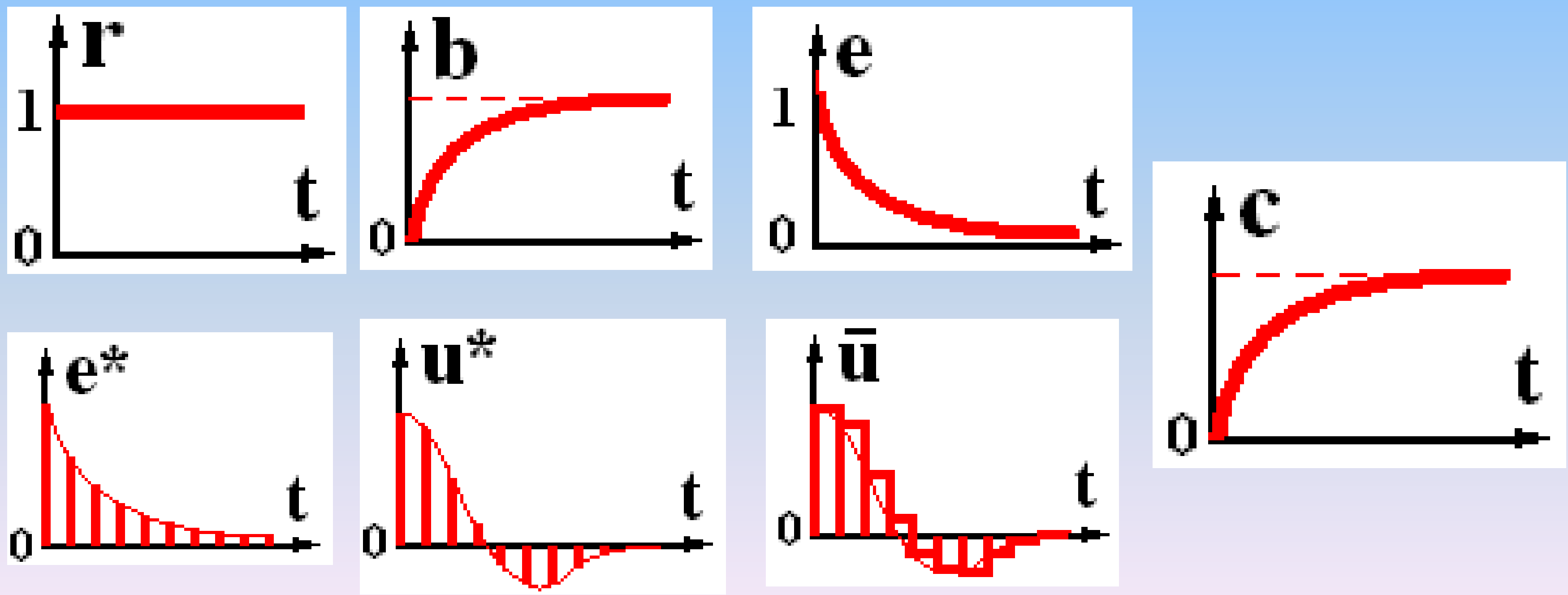


D/A process





Computer Controlled Systems



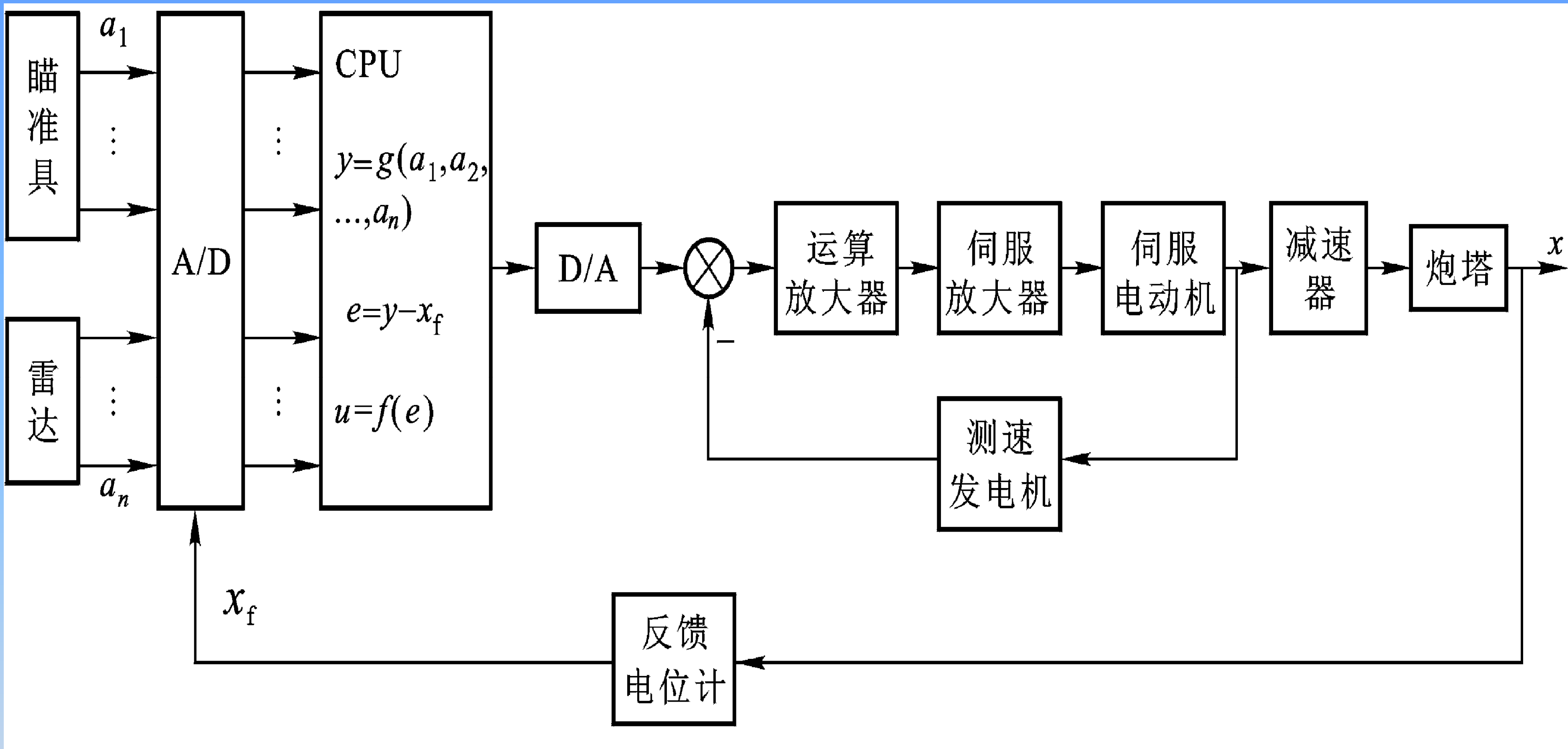


Fig 7-1 机载火力控制系统原理

History of Discrete-time system (p. 196-197)

DDC-Direct Digital Control （直接数字控制系统)

SCC- Surveillance Computer Control System(计算机监督控制系统)

TDE- Total and Distributed Control(集散控制系统)：muti-agent robots

Advantages and Disadvantages

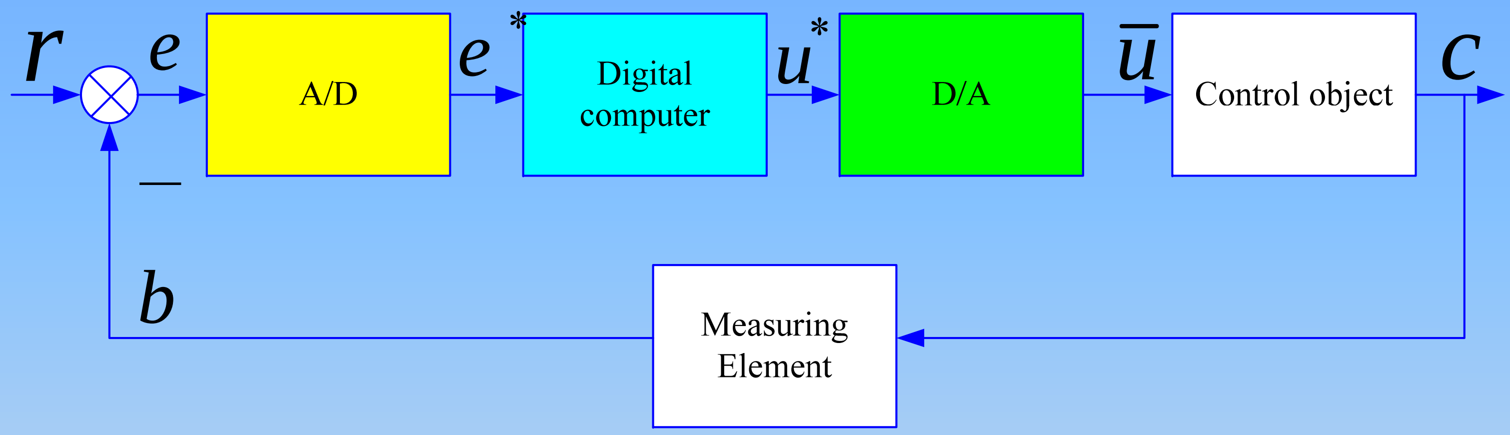
Computer Control System

- (1) Calculations are performed in the software. Easy for modification.
  (2) Complex control laws easily realized;
  (3) Reduced sensitivity to noise;
  (4) One computer for multi-tasks, high utilization ;
  (5) Network for process automation, macro-management and remote control.

- (1) Information between samples is lost. Compared with continuous system in the similar condition, the performance is reduced;
  (2) Needs A/D and D/A conversion devices.

7.2 The Sampling Process and Sampling Theorem

7.2.1 The Sampling Process

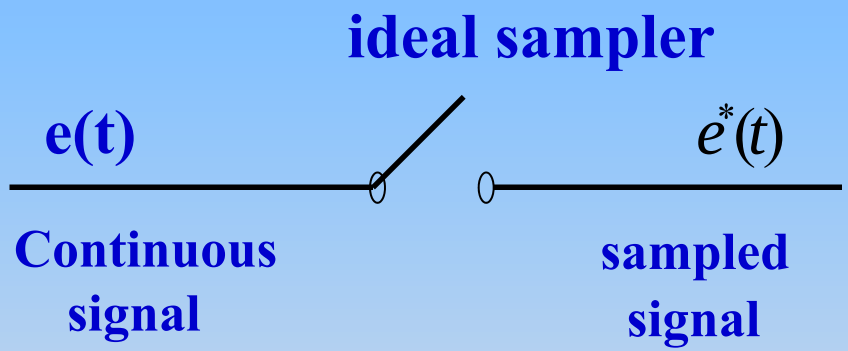


Computer Control System

Question: In the above computer control system, which signals are discrete, which signals are continuous?

- **Sampling Process: Continuous signal → Discrete Signal**
- **Holding Process: Discrete Signal → Continuous Signal.**
- **The two are inverse process to each other.**

Sampler: A switch which closes every T seconds for one instant of time.



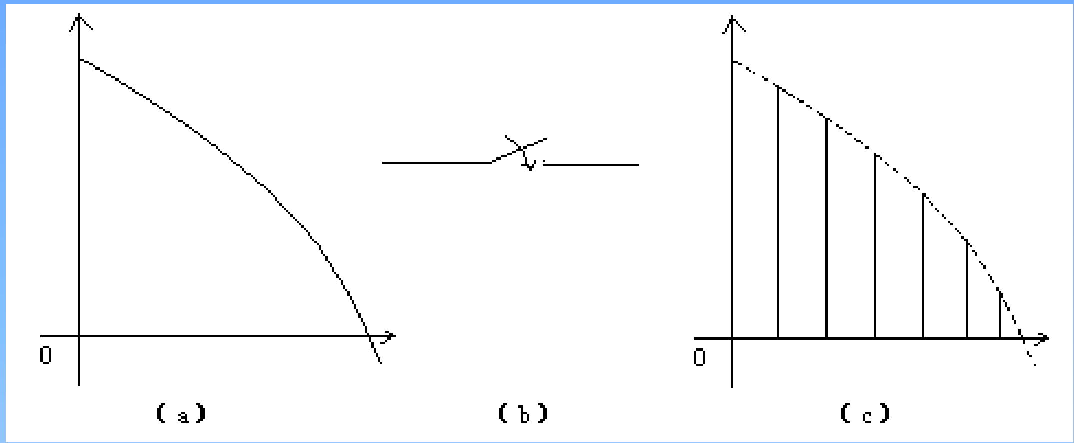
Where T is called the sampling period.

Ideal sampling process:

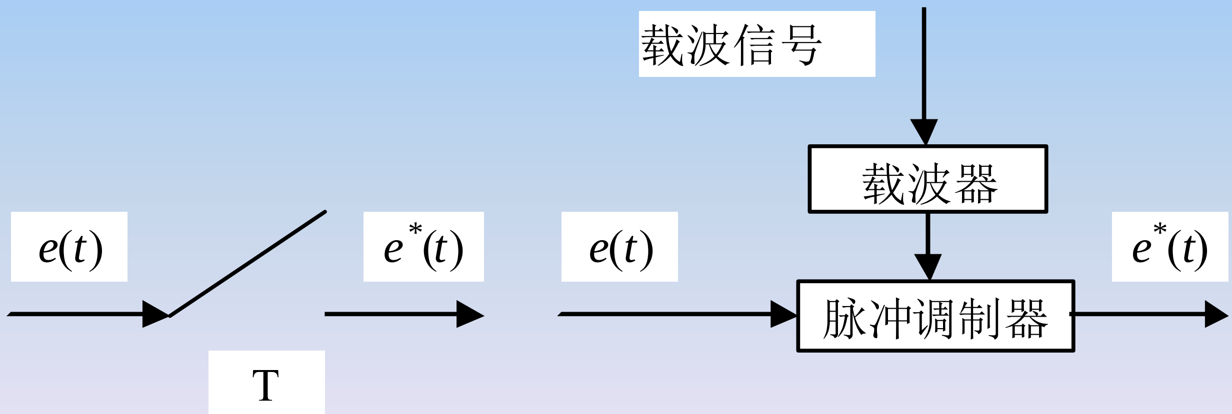
(1) t <<T. The sampling process is completed instantaneously

(2) Word Length is enough, thus e*(Kt)=e(Kt)

Types of Samplers: ideal, periodical, random,…



Sampling Process



7.2.2 Mathematical Model for sampling Signals

1、 Some ideal assumptions

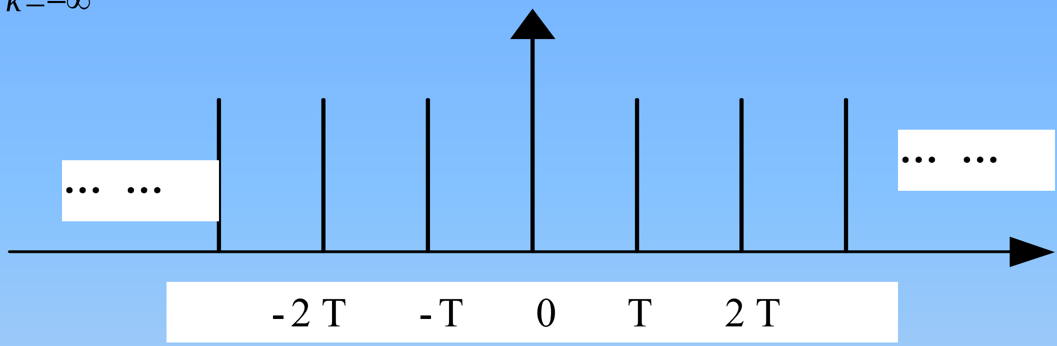
- The sampling process is completed instantaneously；
- The signals in and out the sampler have no difference;
- $\tau \ll T$, that is $\tau \to 0$;
- The output is constant when sampler shuts down;
- Sample Period T is a constant。

2、 Unit Impulsive Signal $\delta(t)$

$$\delta(t) = \begin{cases} 1 & t = 0 \\ 0 & t \neq 0 \end{cases}$$

3、Unit Impulse Sequence (Unit Impulse Train)

$$\delta_T(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT) = \delta(t) + \delta(t - T) + \cdots + \delta(t - kT) + \cdots$$



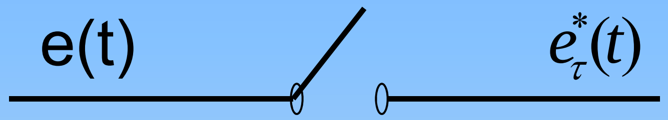
Unit Impulse sequence

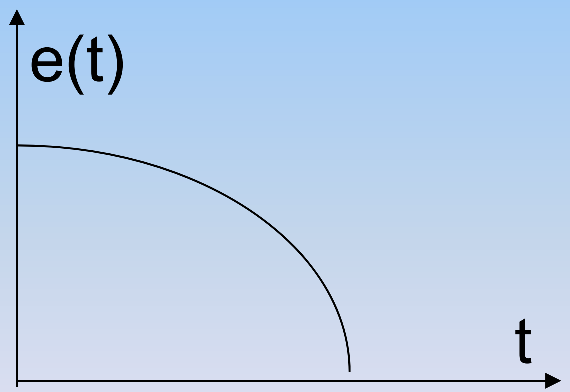
4、Sampling Signal

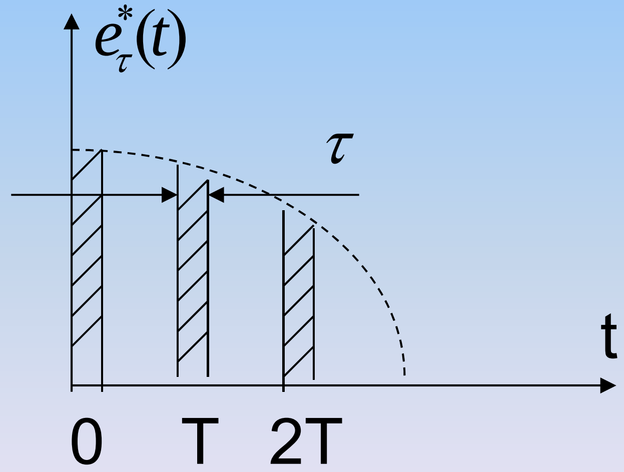
$$e^*(t) = \sum_{k=-\infty}^{\infty} e(t)\delta(t - kT)$$

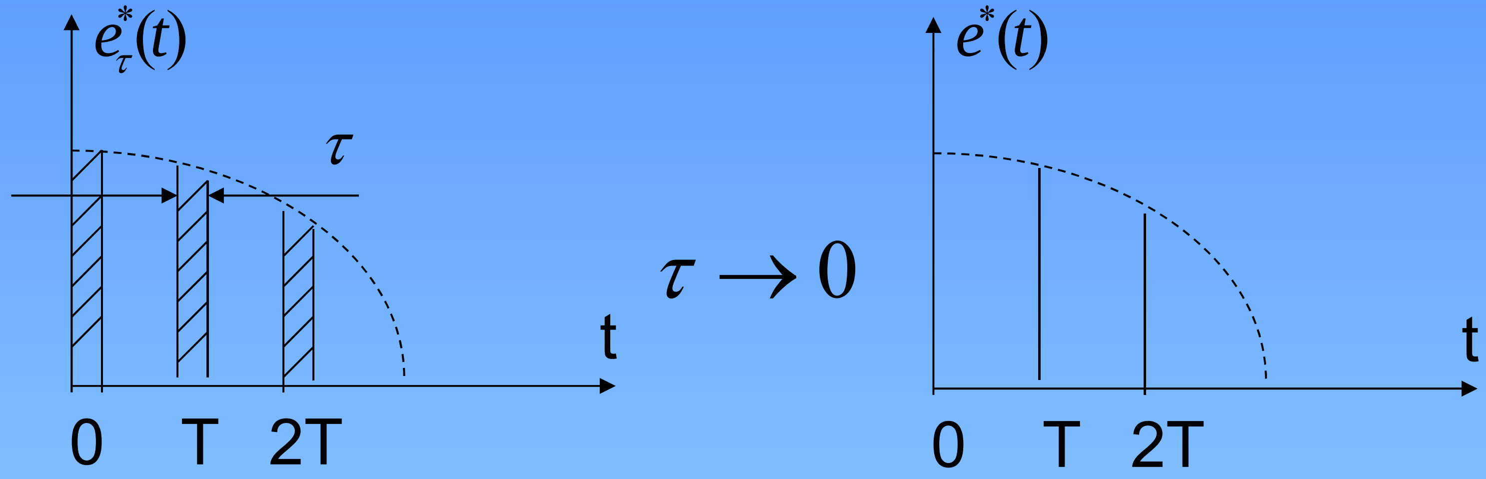
$$e^*(t) = \sum_{k=0}^{\infty} e(kT)\delta(t-kT)$$

real sampler









So the sampling operation can be expressed as

$$e^*(t) = \sum_{k=0}^{+\infty} e(kT)\cdot\delta(t-kT)$$

or

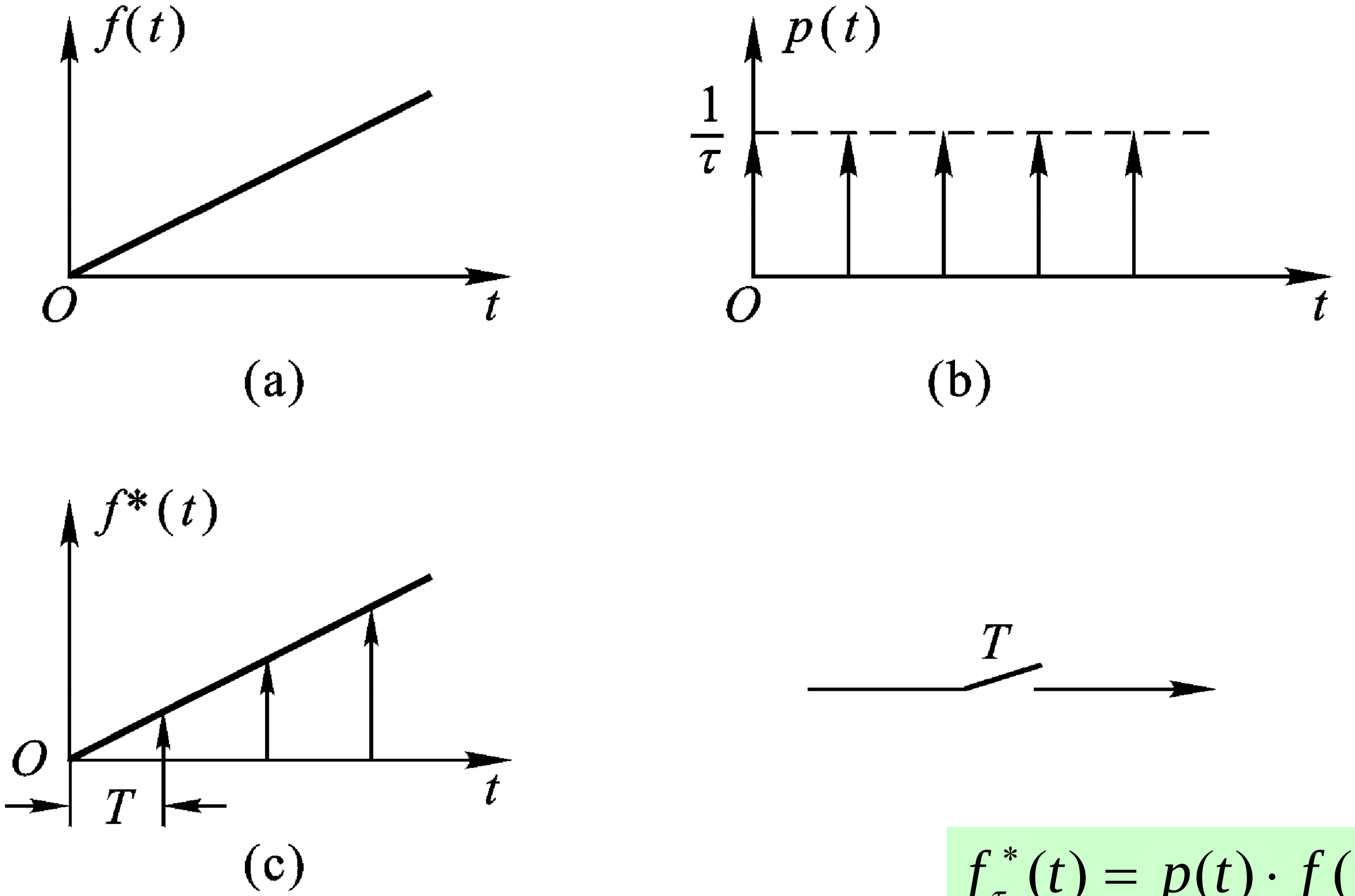
$$e^*(t) = e(t)\cdot\sum_{k=0}^{\infty}\delta(t-kT)$$

or

$$e^*(t) = e(t)\cdot\delta_T(t)$$

where

$$\delta_T(t) = \sum_{k=0}^{\infty}\delta(t-kT)$$



$$f_\tau^*(t) = p(t) \cdot f(t)$$

Fig 7 - 3 Sampling Process

Laplace Transformation

(6) **位移定理**：

a.实域中的位移定理，若原函数在时间上延迟 τ，则其象函数应乘以 $e^{-\tau\cdot s}$

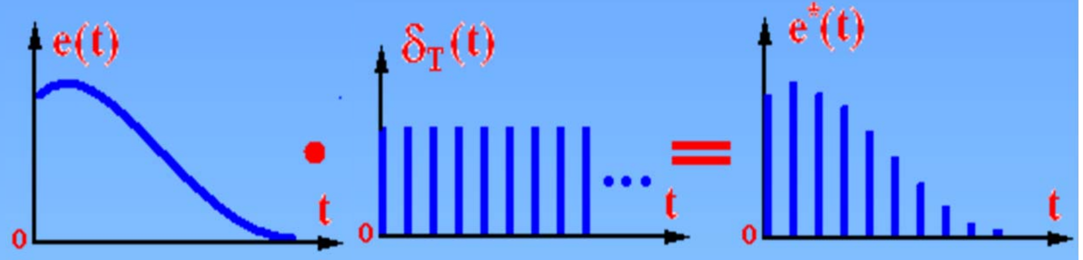
$$L[f(t-\tau)] = e^{-\tau\cdot s}F(s)$$

b.复域中的位移定理，象函数的自变量延迟a，原函数应乘以 e^{at}，即

$$L[e^{at}f(t)] = F(s-a)$$

Ideal sampling sequence

$$\delta_T(t)=\sum_{k=0}^{\infty}\delta(t-kT)$$



$$e^*(t)=e(t)\cdot\delta_T(t)$$

$$=e(t)\cdot\sum_{k=0}^{\infty}\delta(t-kT) \qquad =\sum_{k=0}^{\infty}e(kT)\cdot\delta(t-kT)$$

(2) *L* :

$$E^*(s)=L\left[e^*(t)\right]$$

$$=L\left[\sum_{k=0}^{\infty}e(kT)\cdot\delta(t-kT)\right] \qquad =\sum_{k=0}^{\infty}e(kT)\cdot e^{-kTs}$$

$$E^*(s)=\sum_{k=0}^{\infty} e(kT)\cdot e^{-kTs}$$

Example 7-1 $e(t)=1(t)$ Obtain $E^*(s)$

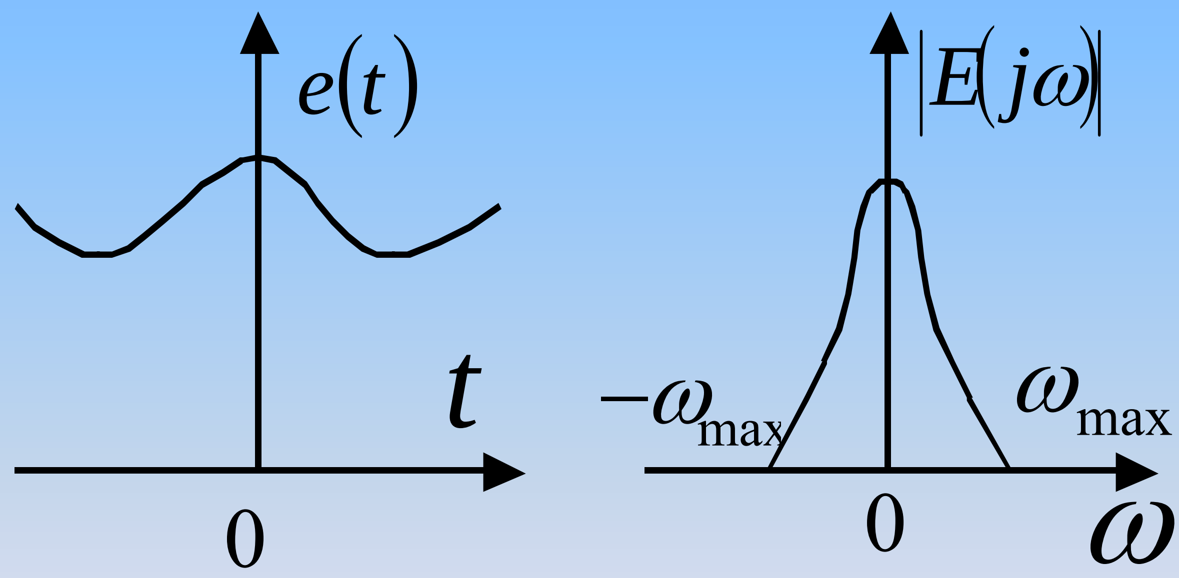
Solution
$$E^*(s)=\sum_{k=0}^{\infty} 1\cdot e^{-kTs}$$
$$=1+e^{-Ts}+e^{-2Ts}+\cdots=\frac{1}{1-e^{-Ts}}=\frac{e^{Ts}}{e^{Ts}-1}$$

Example 7-2 $e(t)=e^{-at}$ Obtain $E^*(s)$

Solution
$$E^*(s)=\sum_{k=0}^{\infty} e^{-akT}\cdot e^{-kTs}=\sum_{n=0}^{\infty} e^{-(s+a)kT}$$
$$=\frac{1}{1-e^{-(s+a)T}}=\frac{e^{Ts}}{e^{Ts}-e^{-aT}}$$

7.2.3 Frequency Spectrum Analysis of Sampled Signal

Consider a continuous signal and its amplitude spectrum are:



The Fourier-series expansion of $\delta_T(t)$ is

$$\delta_T(t) = \frac{1}{T}\sum_{k=-\infty}^{\infty} e^{jk\omega_s t}$$

So the sampled signal is

$$e^*(t) = \frac{1}{T}\cdot\sum_{k=-\infty}^{\infty} e(t)\cdot e^{jk\omega_s t}$$

which Laplace transform is $E^*(s) = \frac{1}{T}\sum_{k=-\infty}^{\infty} E(s + jk\omega_s)$

$$E^*(j\omega) = \frac{1}{T}\sum_{k=-\infty}^{+\infty} E[j(\omega + k\omega_s)]$$

where the operator s is replaced by $j\omega$

$$E^*(s) = \sum_{n=0}^{\infty} e(nT) \cdot \mathrm{e}^{-nTs}$$

① shows the value relation of E*(s) and e(t) on the sampling point;

② can be written into the closed form;

③ can be used to obtain the time response and the Z transform of e*(t)

$$E^*(s) = \frac{1}{T} \sum_{n=-\infty}^{\infty} E(s + jn\omega_s)$$

① shows relationship of E*(s) and E(s);

② can not be written as close form;

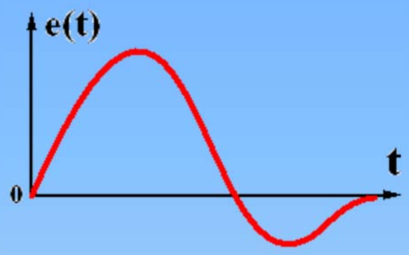
③ can be used for the frequency spectrum analysis of e*(t).

The frequency spectrum analysis of continuous signal e(t) and discrete signal e*(t)

Frequency spectrum — Frequency expansion of the signal

$e(t)$

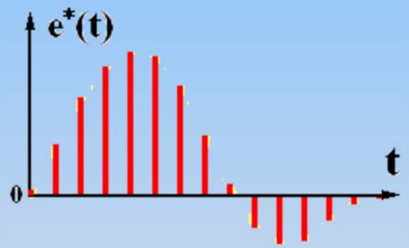
continuous signal



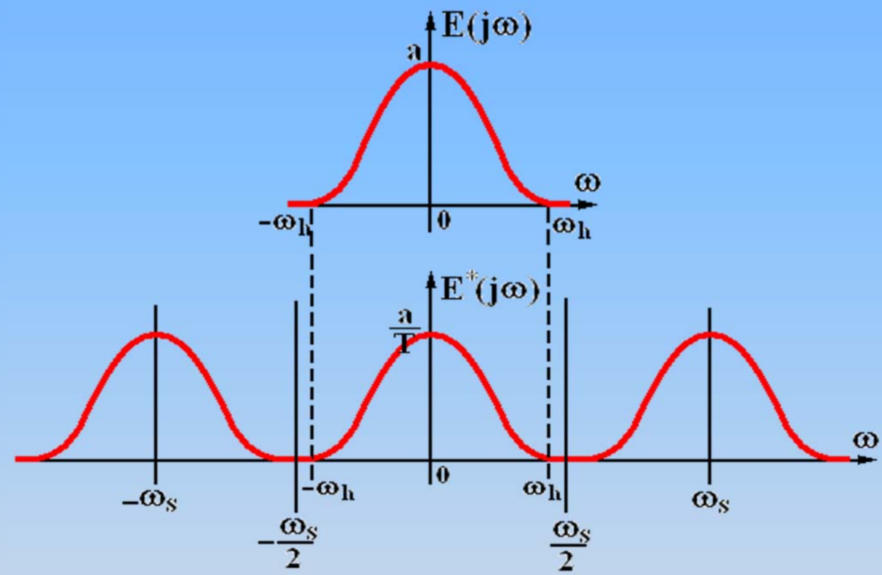
$\xrightarrow{\mathcal{F}}$

$e^*(t)$

discrete signal



$\xrightarrow{\mathcal{F}}$



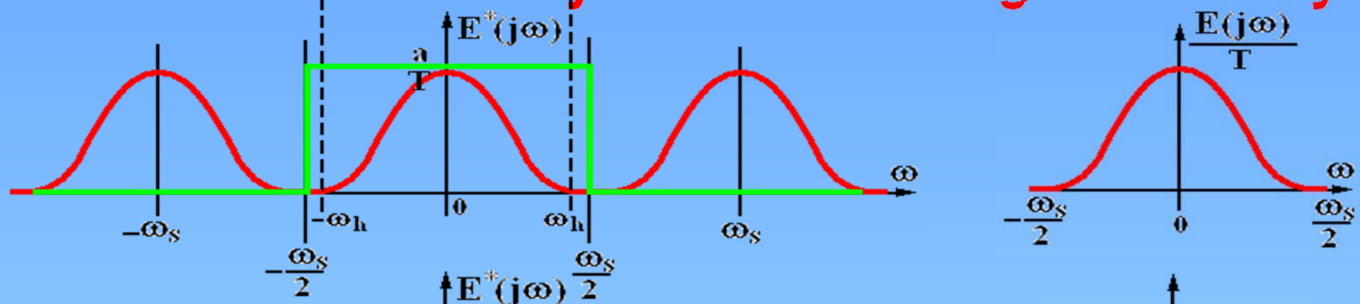
$$E^*(s) = \sum_{n=0}^{\infty} e(nT)\cdot e^{-nTs}$$

$$E^*(s) = \frac{1}{T}\sum_{n=-\infty}^{\infty} E(s + jn\omega_s)$$

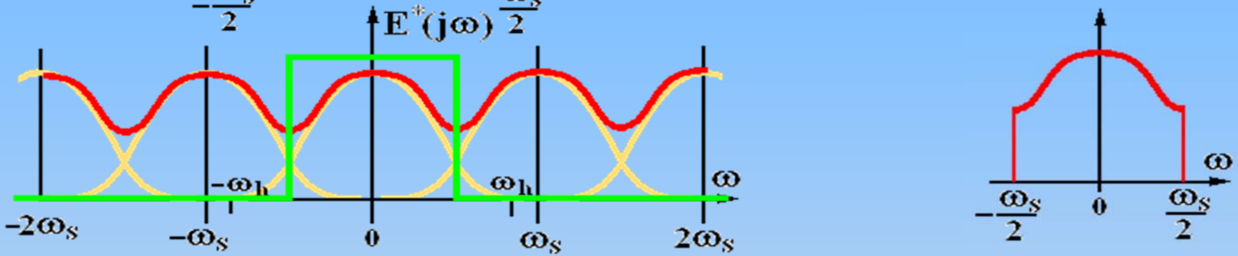
Shannon Sampling Theorem—

The Necessary Condition for signal recovery

$$\omega_s = \frac{2\pi}{T} > 2\omega_h$$

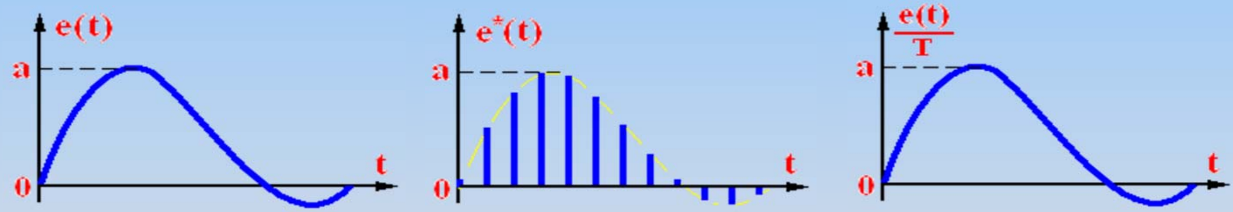


$$\omega_s = \frac{2\pi}{T} < 2\omega_h$$



$$\boxed{\omega_s = \frac{2\pi}{T} > 2\omega_h}$$

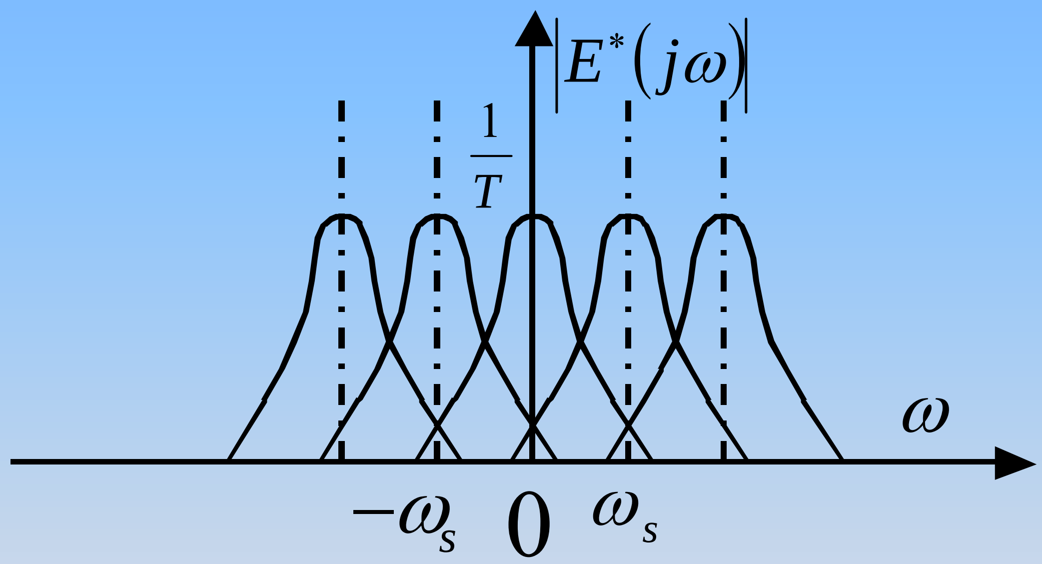
$$\boxed{T < \frac{\pi}{\omega_h}}$$



$$\omega_s > 2\omega_h$$

Sampling switch **Ideal filter**

there are no overlap of each component, so the input signal can be recovered approximately. The is called sampling theorem or Shannon's Theorem



In the figure the input signal can't be recovered.