빅데이터 혁신공유대학

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한동대학교 전산전자공학부

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Data Structures in Python
Chapter 5 - 1

- Binary Search
- Recursive Binary Search
- Bubble sort
- Selection sort
- **Insertion sort**

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Agenda & Readings

- Agenda
  - Insertion sort algorithm
  - Time complexity
  - Summary - sorting basics
  - Empirical Analysis
- Reference:
  - Problem Solving with Algorithms and Data Structures
    - Chapter 5 Search, Sorting and Hashing

Insertion Sort Algorithm

- Given is a list L of n value {L[0], ⋯ , L[n−1]}
  - Divide list into sorted (left - initially only one element) and sorted part (right):
    `Sorted: {L[0]}                    Unsorted: {L[1], … , L[n-1]}`
  - In each pass, **take left most element from unsorted part** and place it into correct position of sorted part.
  - Reduce size of unsorted part by one and increase size of sorted part by one.
    After i–th pass:
    `Sorted: {L[0],…,L[i]}     Unsorted: {L[i+1], … , L[n-1-i]}`
  - Repeat until unsorted part is an empty list - then all elements are sorted.

←—— Unsorted ——→

29	10	14	13	18	List to sort
10	29	14	13	18	PASS 1 (1 Comp, 1 Shift)
10	14	29	13	18	PASS 2 (2 Comp, 1 Shift)
10	13	14	29	18	PASS 3 (3 Comp, 2 Shift)
10	13	14	18	29	PASS 4 (2 Comp, 1 Shift)

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10	13	14	18	29	PASS 4 (2 Comp, 1 Shift)

29	10	14	13	18	
10	29	14	13	18	pick 10 & inserted
10	14	29	13	18	pick 14 & inserted
10	13	14	29	18	pick 13 & inserted
10	13	14	18	29	pick 18 & inserted

Insertion Sort – Exercise

	54	26	93	17	77	31	44	55	20	List to sort
pick 26 & inserted	26	54	93							PASS 1 (Comp, Shift)
pick 93 & inserted										PASS 2 (Comp, Shift)
pick 17 & inserted										PASS 3 (Comp, Shift)
pick 77 & inserted										PASS 4 (Comp, Shift)
pick 31 & inserted										PASS 5 (Comp, Shift)
pick 44 & inserted										PASS 6 (Comp, Shift)
pick 55 & inserted										PASS 7 (Comp, Shift)
pick 20 & inserted										PASS 8 (Comp, Shift)

Total PASS 8 (26 Comp, 20 Shift)

Insertion Sort – Exercise

	35	34	26	90	37	28	10	27	36	List to sort
pick 34 & inserted	34	35								PASS 1 (1 Comp, 1 Shift)
pick 26 & inserted										PASS 2 (Comp, Shift)
pick 90 & inserted										PASS 3 (Comp, Shift)
pick 37 & inserted										PASS 4 (Comp, Shift)
pick 28 & inserted										PASS 5 (Comp, Shift)
pick 10 & inserted										PASS 6 (Comp, Shift)
pick 27 & inserted										PASS 7 (Comp, Shift)
pick 36 & inserted										PASS 8 (Comp, Shift)

Insertion Sort - making room for the element to be inserted

6	28	34	35	37	90	**10**	27	36

- For example, to insert 10 into the sorted part of the list we need to store 10 into a temporary variable and move all the elements which are bigger than 10 up one position, then insert 10 into the empty slot.

temp = 10

6	28	34	35	37	90	__	27	36

Shift 5 list values

6	__	28	34	35	37	90	27	36

6	**10**	28	34	35	37	90	27	36

Insertion Sort Code

```
def insertion_sort(a):
    for i in range(1, len(a)):
        ivalue = a[i]
        while i > 0 and a[i - 1] > ivalue:
            a[i] = a[i - 1]              # this is "shift", not swap
            i = i - 1
        a[i] = ivalue
        #print(i, "-", a) # enable to see each pass

if __name__ == '__main__':
    a = [54, 26, 93, 17, 77, 31, 44, 55, 20]
    print("before: ", a)
    insertion_sort(a)
    print(" after: ", a)
```

```
before:  [54, 26, 93, 17, 77, 31, 44, 55, 20]
 after:  [17, 20, 26, 31, 44, 54, 55, 77, 93]
```

```
def swap(a, i, j):
    temp = a[i]
    a[i] = a[j]
    a[j] = temp
```

```
def swap(a, i, j):
    a[i], a[j] = a[j], a[i]
```

Insertion Sort - Big O

- For a list with n elements:
  - The number of comparisons in the WORST CASE?
  - pass 1 pass 2 pass 3 … last pass
    1 2 3 ⋯ n-3 n-2 n-1

    $1 + 2 + \cdots + (n-3) + (n-2) + (n-1) = \frac{1}{2}(n^2 - n)$

- Big O of the insertion sort is $O(n^2)$
  - The number of data increases 10 times, then it takes a 100 times longer.

- Note 1: Best case **O(n) … when does this occur?**
- Note 2: The number of shifts is equal or one smaller than the number of comparisons, so same order of magnitude.

Insertion Sort - Big O

- What if the data is **already sorted**?
  - Move elements?
  - Comparisons?

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10	14	29	13	18	PASS 2 (2 Comp, 1 Shift)
10	13	14	29	18	PASS 3 (3 Comp, 2 Shift)
10	13	14	18	29	PASS 4 (2 Comp, 1 Shift)

	5	10	14	32	35	List to sort
pick 10 & inserted	5	10	14	32	35	PASS 1 (Comp, Shift)
pick 14 & inserted	5	10	14	32	35	PASS 2 (Comp, Shift)
pick 32 & inserted	5	10	14	32	35	PASS 3 (Comp, Shift)
pick 35 & inserted	5	10	14	32	35	PASS 4 (Comp, Shift)

Insertion Sort - Big O

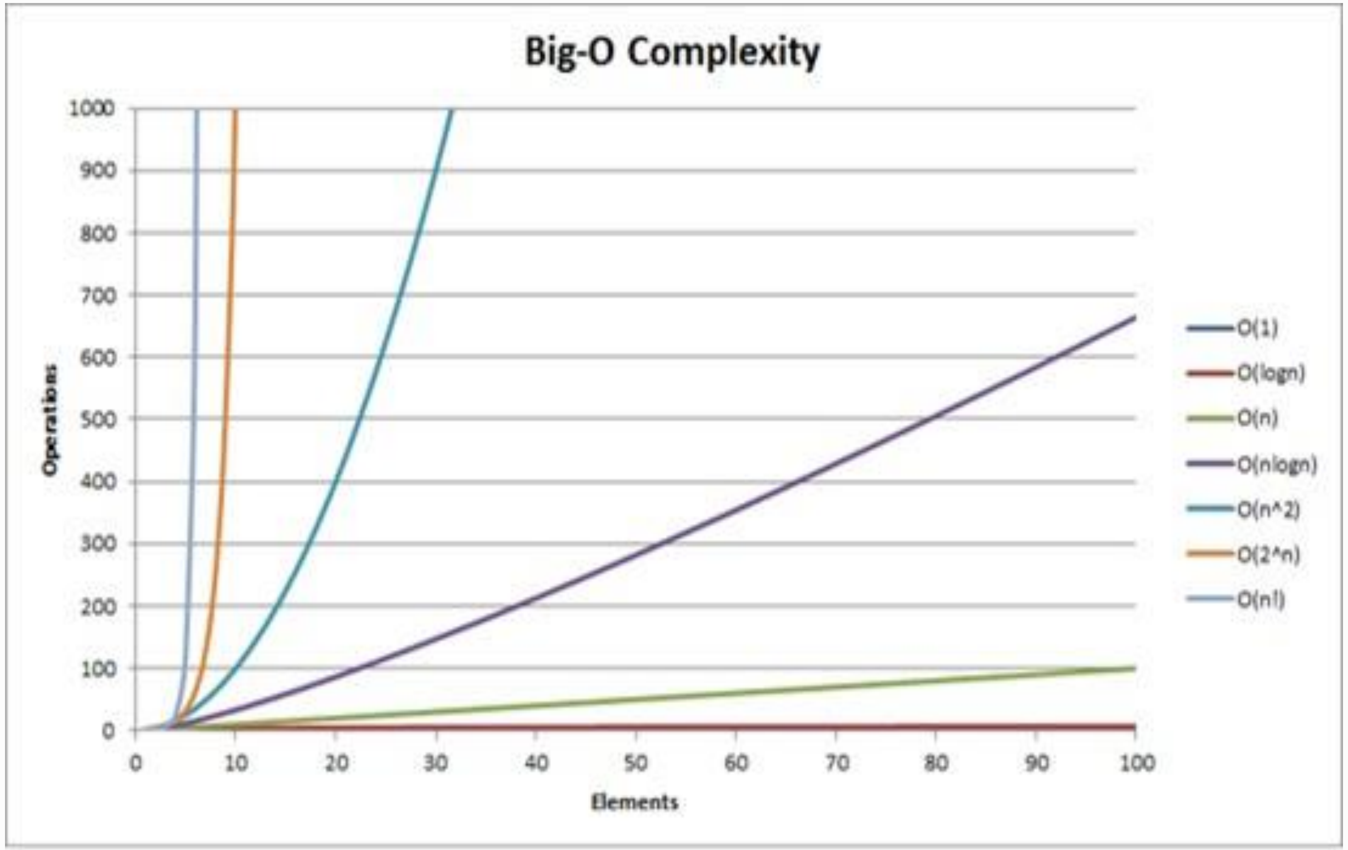
- What if the data is in **reverse order**?
  - Move elements?
  - Comparisons?

29	10	14	13	18	List to sort
10	29	14	13	18	PASS 1 (1 Comp, 1 Shift)
10	14	29	13	18	PASS 2 (2 Comp, 1 Shift)
10	13	14	29	18	PASS 3 (3 Comp, 2 Shift)
10	13	14	18	29	PASS 4 (2 Comp, 1 Shift)

	35	32	14	10	5	List to sort
pick 10 & inserted	32	35	14	10	5	PASS 1 (Comp, Shift)
pick 14 & inserted	14	32	35	32	35	PASS 2 (Comp, Shift)
pick 32 & inserted	10	14	32	35	35	PASS 3 (Comp, Shift)
pick 35 & inserted	5	10	14	32	35	PASS 4 (Comp, Shift)

Running Time Matters

- The usefulness of an algorithm in practice depends on the data size n and the complexity (Big O) of the algorithm (time and memory).
- In general algorithms with linear, logarithmic or low polynomial running time are acceptable.
  - $O\ (\log n)$
  - $O\ (n)$
  - $O\ (n^k)$ where k is a small constant, (in many cases $k\ < 2$ is ok)
- Algorithms with exponential or high polynomial running time are often of limited use.
  - $O\ (n^k)$ where k is a large constant, say $>\ 3$
  - $O\ (2^n), O\ (n^n)$



Summary

- Insertion sort is a good middle-of-the-road choice for sorting lists of a few thousand items or less.
- **Insertion sort is known faster than selection sort** on average.
- For small lists, the insertion sort is appropriate due to its simplicity.
  For **almost sorted** lists, the insertion sort is **a good choice.**
- For large lists, all $O(n^2)$ algorithms, including the insertion sort, are prohibitively inefficient.

Summary - Simple Sorting Algorithms

- All sorting algorithms (bubble, selection, insertion sorts) discussed so far had an $O(n^2)$ average and the worst-case complexity
  → In practice for large lists it is **too slow.**

- The **Timsort** algorithm (written in C - not using the Python interpreter) used by Python combines elements from **Merge sort** and **Insertion sort.**
  - Worst case and average case complexity $O(n \log n)$
  - Very fast for almost sorted lists
- All comparison-based sorting algorithms require at least $O(n \log n)$ time in the worst and average case.

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