

빅 데이터 혁신 공유 대학

파이썬으로 배우는 데이터 구조

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Data Structures in Python

Chapter 3 - 1

- Stack Concept and ADT
- Stack Example - Matching
- **Stack Example - Postfix**

Agenda

- Postfix Calculation
- Conversion from Infix to Postfix

Infix, postfix and prefix expressions

Stacks can be used to implement algorithms involving Infix, postfix and prefix expressions.

- Infix:
 - An infix expression is a single letter, or an operator, preceded by one infix string and followed by another infix string.
 - A , $A + B$, $(A + B) + (C - D)$
- Prefix:
 - A prefix expression is a single letter, or an operator, followed by two prefix strings. Every prefix string longer than a single variable contains an operator, first operand and second operand.
 - A , $+ A B$, $++ A B - C D$
- Postfix:
 - A postfix expression (also called Reverse Polish Notation) is a single letter or an operator, preceded by two postfix strings. Every postfix string longer than a single variable contains first and second operands followed by an operator.
 - A , $A B +$, $A B + C D - +$

Infix, postfix and prefix expressions

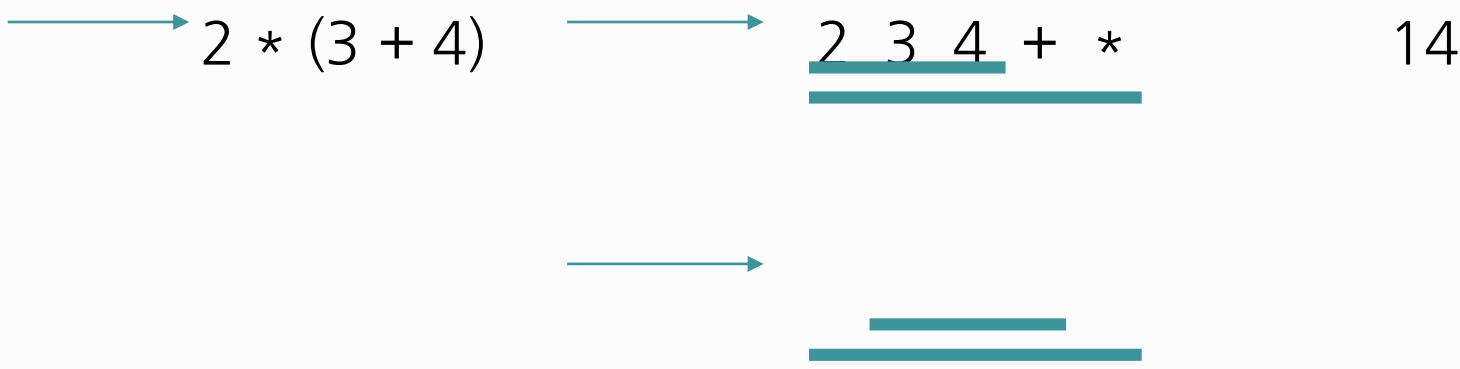
- Prefix and postfix notations are methods of writing mathematical expressions without parenthesis.
 - Why:** Time to evaluate a postfix and prefix expression is $O(n)$, where n is the number of elements in the array.

Infix	Prefix	Postfix
A + B	+ A B	A B +
A + B - C	- + A B C	A B + C -
(A + B) * C - D	- * + A B C D	A B + C * D -

Postfix Calculator

- Computation of arithmetic expressions can be efficiently carried out in Postfix notation with the help of stack.

infix	infix	postfix	Result
$2 * 3 + 4$	$(2 * 3) + 4$	$2 \ 3 \ * \ 4 \ +$	10



Postfix Calculator

- Requires you to enter postfix expressions.
 - Example: 2 3 4 + *

Algorithm:

- When an **operand** is entered,
 - the calculator pushes it onto a stack
- When an **operator** is entered,
 - the calculator applies it to the top **two operands** of the stack
 - Pops the top two operands from the stack
 - Pushes the result of the operation on the stack

Postfix Calculator - Algorithm

■ Example 1: Evaluating the expression: 2 3 4 + *

<u>Key entered</u>	<u>Calculator action</u>		<u>Stack(bottom to top)</u>
2	push 2		2
3	push 3		2 3
4	push 4		2 3 4
+	operand2 = pop stack	(4)	2 3
	operand1 = pop stack	(3)	2
	result = operand1 + operand2	(7)	2
	push result		2 7
*	operand2 = pop stack	(7)	2
	operand1 = pop stack	(2)	
	result = operand1 * operand2	(14)	
	push result		14

Postfix Calculator

- Example 2: Evaluating the expression: 2 3 * 4 +

Key entered Calculator action Stack(bottom to top)

2	push 2	2
3	push 3	2 3

*	operand2 = pop stack (3)	2
	operand1 = pop stack (2)	
	result = operand1 * operand2 (6)	
	push result	6
4	push 4	6 4

+	operand2 = pop stack (4)	6
	operand1 = pop stack (4)	
	result = operand1 + operand2 (10)	
	push result	10

Postfix Calculator

- Example 3: Evaluating the expression: 12 3 - 3 /

Key entered Calculator action Stack(bottom to top)

12	push 12	12
3	push 3	12 3

-	operand2 = pop stack (3)	12
	operand1 = pop stack (12)	
	result = operand1 + operand2 (9)	
	push result	9
3	push 3	9 3

/	operand2 = pop stack (3)	6
	operand1 = pop stack (9)	
	result = operand1 / operand2 (3)	
	push result	3

The order of operand1 and operand2 is very important.

Postfix Calculator - Exercise 2

- Evaluate the expression: 10 4 2 - 5 * + 3 -

Key entered Calculator action

Stack(bottom to top)

Postfix Calculator

- Coding

```
def evaluate_postfixList(postfixList):  
    stack = Stack()  
    operators = "+-/*"  
    for op in postfixList:  
        if op in operators:    #operator  
            if stack.size() > 1:  
                num2 = stack.pop()  
                num1 = stack.pop()  
                result = compute(int(num1), int(num2), op)  
                stack.push(result)  
            else:  
                return "Failed while parsing postfix expression"  
        else: #operand  
            stack.push(op)  
    return stack.pop()
```

Write your own compute() function
to make this code work properly.

```
#Sample Run:  
evaluate_postfixList(['3', '4', '7', '*', '+'])  
31
```

Conversion from Infix to Postfix

- Examples:
 - $2 * 3 + 4 \rightarrow 2\ 3\ * \ 4\ +$
 - $2 + 3 * 4 \rightarrow 2\ 3\ 4\ * \ +$
 - $1 * 3 + 2 * 4 \rightarrow 1\ 3\ * \ 2\ 4\ * \ +$
- Algorithm Concept:
 - Operands always stay in the same order with respect to one another.
 - An operator will move only “to the right” with respect to the operands.
 - All parentheses are removed.

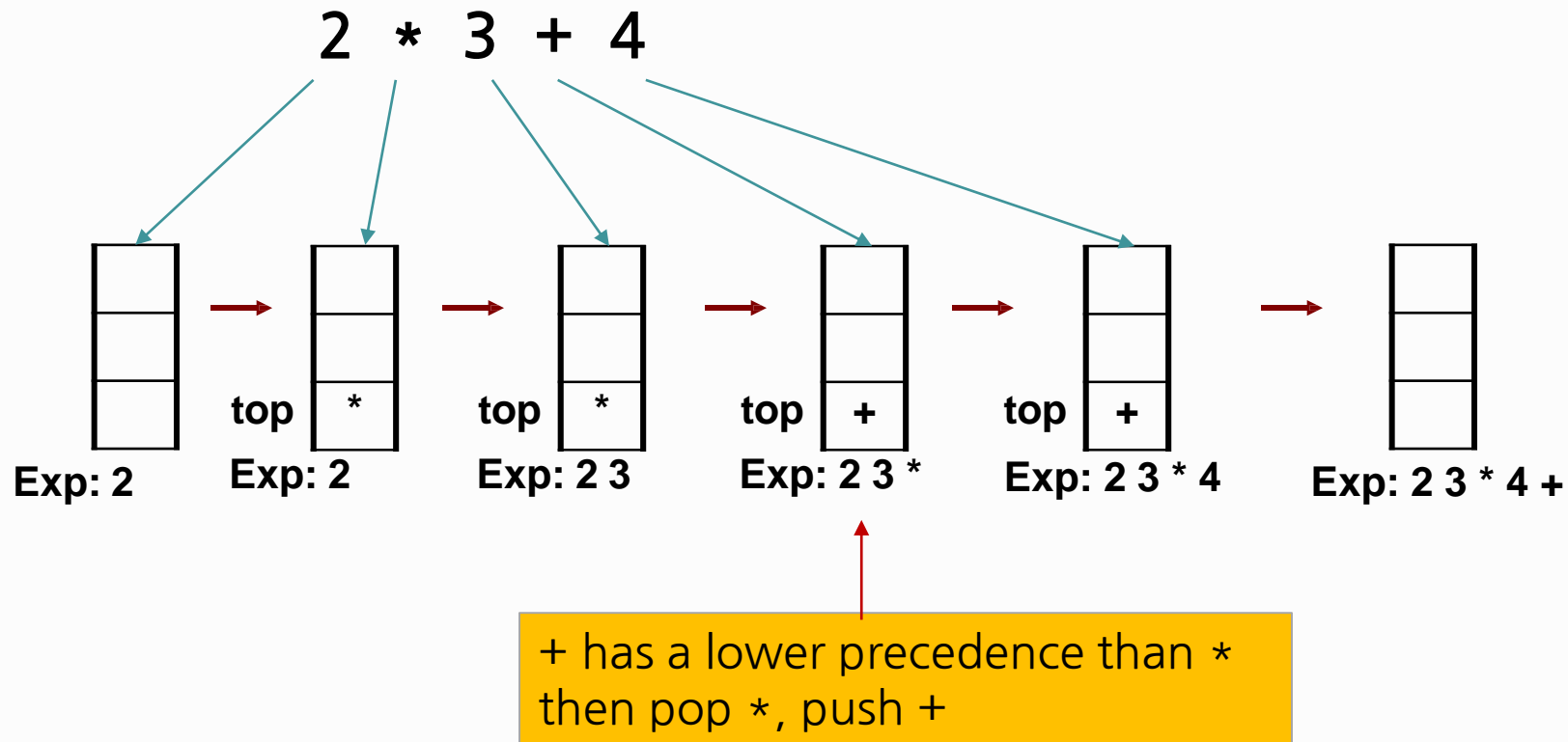
Conversion from Infix to Postfix

Algorithm:

- operand - output it to postfixExp.
- “(“ - push onto the stack.
- “)” - pop the operators off the stack and append them to the end of postfixExp until encounter the match “(“.
- operator
 - For **higher** precedence operator, push it onto the stack.
 - For **lower or equal** precedence operator, pop them until "(" and push it onto the stack.
- End of the string
 - append the remaining contents of the stack to postfixExp.

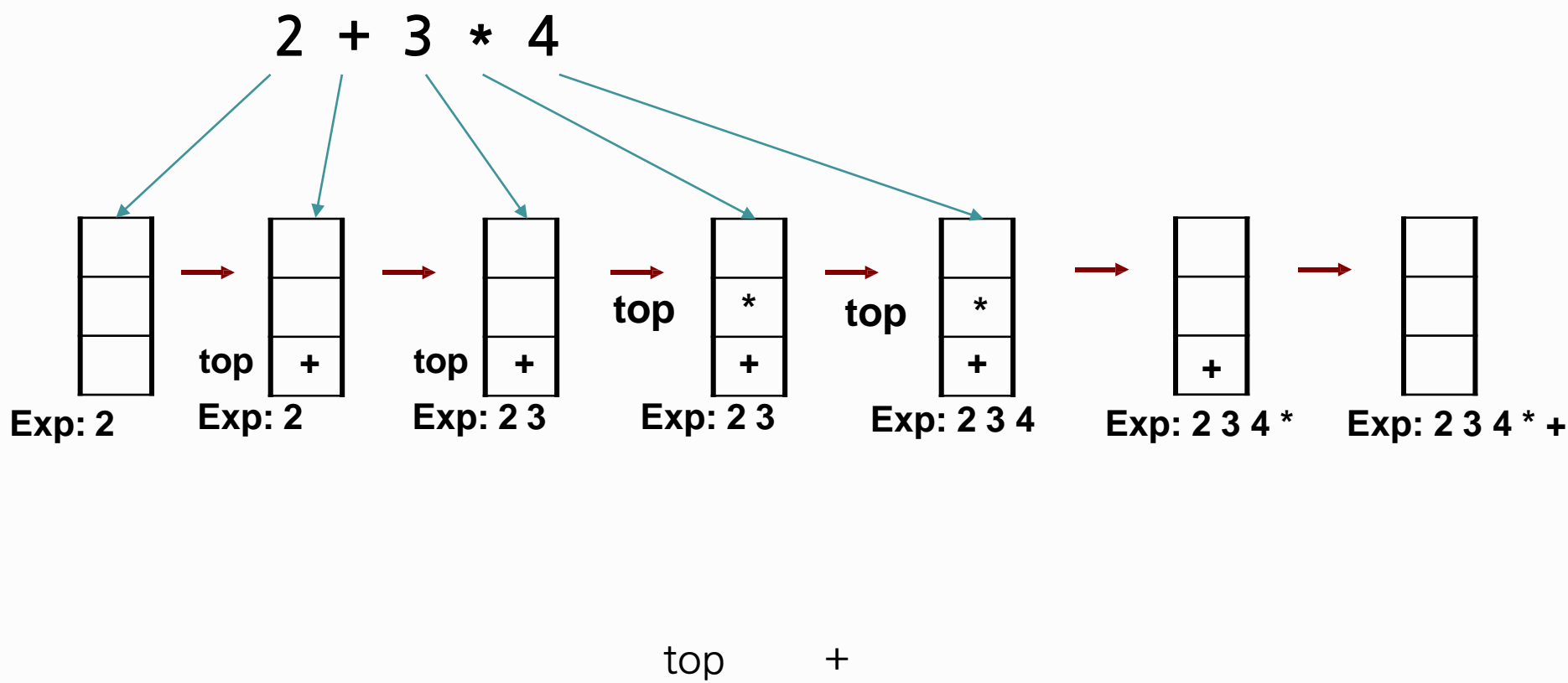
Conversion from Infix to Postfix

- Example 1



Conversion from Infix to Postfix

- Example 2:



Conversion from Infix to Postfix

- Example 3: $a - (b + c * d) / e$

<u>token</u>	<u>stack</u>	<u>postfix</u>	
a		a	
-	-	a	
(-(a	
b	-(ab	
+	-(+	ab	
c	-(+	abc	
*	-(+*	abcd	
)	-(+	abcd*	get operators
	-(abcd*+	from stack to
	-	abcd*+	postfix until "("
/	-/	abcd*+	
e	-/	abcd*+e	get operators
			from stack to
			postfix until empty

bottom top

Conversion from Infix to Postfix - Exercise 3

- Debug the following program.

```
def get_postfix(infixList):
    precedence = {"*":3, "/":3, "+":2, "-":2, "(":1 }
    operators = "+-/*"
    op_stack = Stack()
    postfixList = []
    for op in infixList:
        if op in operators:
            while (not op_stack.is_empty()) and (precedence[op_stack.peek()] >= precedence[op]):
                postfixList.append(op_stack.pop())
            op_stack.push(op)
        elif op == "(":
            op_stack.push(op)
        elif op == ")":
            op = op_stack.pop()
            while not op == "(":
                postfixList.append(op)
            op = op_stack.pop()
        else: #operand
            postfixList.append(op)
    while not op_stack.is_empty():
        postfixList.append(op_stack.pop())
    return " ".join(postfixList), postfix
```

#Sample Run:

```
a, b = get_postfixList(['3', '+', '4', '*', '7'])
print(a)
print(b)
```

```
3 4 7 * +
['3', '4', '7', '*', '+']
```

Conversion from Infix to Postfix - Exercise 4

- Converting the infix expression to postfix: $(B - C) * (D - E)$
token stack postfix

Conversion from Infix to Postfix - Exercise 4 solution

- Converting the infix expression to postfix: $(B - C) * (D - E)$

<u>token</u>	<u>stack</u>	<u>postfix</u>
((
B		B
-	(-	B
C	(-	BC
)	(BC-
)	BC-
*	*	BC-
(*(BC-
D	*(BC-D
-	*(-	BC-D
E	*(-	BC-DE
)	*(BC-DE-
	*	BC-DE-
		BC-DE-*

```
a, b = get_postfix(['(', 'B', '-', 'C', ')', '*', '(', 'D', '-', 'E', ')'])
print(a)
print(b)
```

3 4 7 * +
['3', '4', '7', '*', '+']

Summary

- Stacks are used in applications that manage data items in LIFO manner, such as:
 - Checking for Balanced Braces
 - Matching bracket symbols in expressions
 - Evaluating postfix expressions
 - Conversion from Infix to Postfix

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